Dual-Tone Multi-Frequency (DTMF) Detection using Goertzel algorithm

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Overview

Motivation of the Project

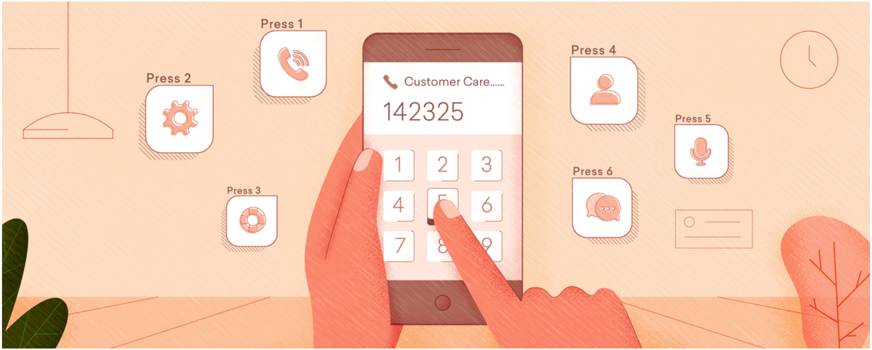

Figure: IVR services have the requirement to detect which key is being pressed

Conventional Telephone Dialer Pads

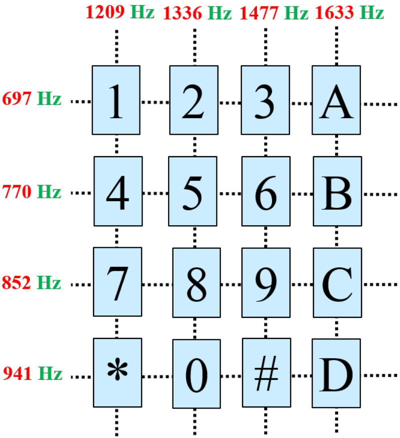

Figure: Telephone Dialer pad with the frequencies associated with it

Visualizing Dialer pad as 2-D Frequency Matrix

- Each key on the dialer pad is characterized by by a a pair of frequencies. One being the row frequency and the other being the column frequency.
- The signal generated when a button is pressed is the sum of two sinusoids corresponding to row and column frequency (hence the name *Dual-Tone*).
- The rows of the frequency matrix take values from a set of 697 Hz, 770 Hz, 852 Hz, 941 Hz and are referred as as low frequency group (F_L).
- Similarly, the columns of the frequency matrix take values from a set of 1209 Hz, 1336 Hz, 1477 Hz, 1633 Hz and are referred as high frequency group (F_H).
- The frequencies were chosen to avoid harmonics: no frequency is a multiple of another, the difference between any two frequencies does not equal any of the frequencies, and the sum of any two frequencies does not equal any of the frequencies.

Single Tone Generation

Analysis Equations

Z-transform function of a sinusoidal sequence sin($n\Omega_o$)

$$\mathrm{H(z)} = \frac{z sin\Omega_o}{z^2 - 2z cos\Omega_o + 1} = \frac{z^{-1} sin\Omega_o}{1 - 2z^{-1} cos\Omega_o + z^{-2}}$$

Inverse Z-transform to the transfer function

$$\mathrm{y(n)} = \sin\Omega_o x(n-1) + 2cos\Omega_o y(n-1) - y(n-2)$$

Impulse Response of the system

$$\mathrm{Z}^{-1}(H(z)) = Z^{-1}\left(\frac{z sin\Omega_o}{z^2 - 2z cos\Omega_o + 1}\right) = sin(n\Omega_o) = \sin\left(\frac{2\pi F_o n}{F_s}\right)$$

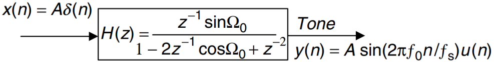

Figure: Single-tone generator

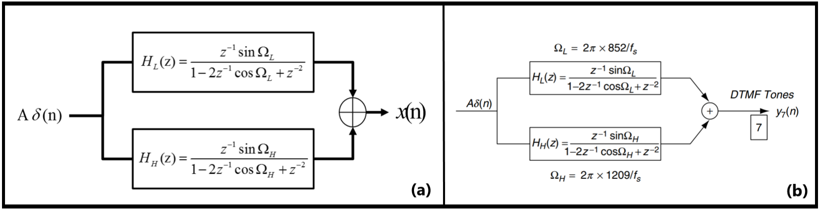

Figure: (a) DTMF generator (general) (b) DTMF generation for key *7*

- When a key on the keypad is pressed, it results in the generation of two sinusoids, one with row frequency Ω_L and the other with column frequency Ω_H.
- The theory of single tone generation can be extended to dual tone sinusoids. The dual-tone generation process can be summarized as in the figures above.

Goertzel Algorithm

- The Goertzel algorithm is a filtering method for computing the DFT coefficients X(k) at the specified frequency bin k with the given N digital data x(0), x(1), ..., x(N-1).

Transfer Function of modified digital Goertzel filter

$$\mathrm{H}_k(z) = \frac{V_k(z)}{X(z)} = \frac{1}{1 - 2cos(\frac{2\pi k}{N})z^{-1} + z^{-2}}; n = 0, 1, 2..., N-1$$

And we define x(N) = 0.

Taking the inverse Z-Transform

$$\mathrm{v}_k(N) = 2cos(\frac{2\pi k}{N})v_k(N-1) - v_k(N-2) + x(n)$$

with the initial conditions $\mathrm{v}_k(-2) = v_k(-1) = 0$

Goertzel Algorithm

X(K) from Goertzel Algorithm

$$|X(k)|^2 = v_k^2(N) + v_k^2(N-1) - 2cos(\frac{2\pi k}{N})v_k(N-1)v_k(n)$$

Why Goertzel Algorithm over FFT ?

- We can apply the algorithm for computing the DFT coefficient X(k) for a specified frequency bin k. Unlike the fast Fourier transform (FFT) algorithm, all the DFT coefficients are computed once it is applied.
- If we want to compute the spectrum at frequency bin k, that is, |X(k)|,from above equations we can see that we need to process recursive equation of $v_k(n)$ for N+1 times and then compute $|X(k)|^2$. The operations avoid complex algebra.

Design principle for DTMF tone detection

- When the digitized DTMF tone x(n) is received, it has two nonzero frequency components from the following eight: 679, 770, 852, 941, 1209, 1336, 1477, 1633 Hz.
- We can apply the modified Goertzel algorithm to compute eight spectral values, which correspond to the eight frequencies. The single-sided amplitude spectrum is computed as $A_k = \frac{2}{N}|X(k)|^2$.
- Since the modified Goertzel algorithm is used, there is no complex algebra involved. Ideally, there are two nonzero spectral components. We will use these two nonzero spectral components to determine which key is pressed.

Design principle for DTMF tone detection

- The frequency bin number (frequency index) can be determined based on the sampling rate F_s, and the data size of N via the following relation: k $= \frac{F}{F_s}$ X N (rounded off to nearest integer).
- Given F_S and N, we can determine the frequency bin at which peak will occur. In our project we consider, $F_s = 8000$ Hz and N $= 205$. The frequency bins for each Ω_L and Ω_H are summarized in next slide.
- The threshold value can be the sum of all eight spectral values divided by a factor of 4. Since there are only two nonzero spectral values, hence the threshold value should ideally be half of the individual nonzero spectral value.
- If the spectral value is larger than the threshold value, it's digital value is 1; otherwise, it's digital value is 0. Based on the 8-bit binary pattern received, we can decode the key pressed.

DTMF Frequency (Hz)	Frequency Bin: k
697	18
770	20
852	22
941	24
1209	31
1336	34
1477	38
1633	42

Table: DTMF frequencies and their frequency bins

DTMF detector using the Goertzel algorithm

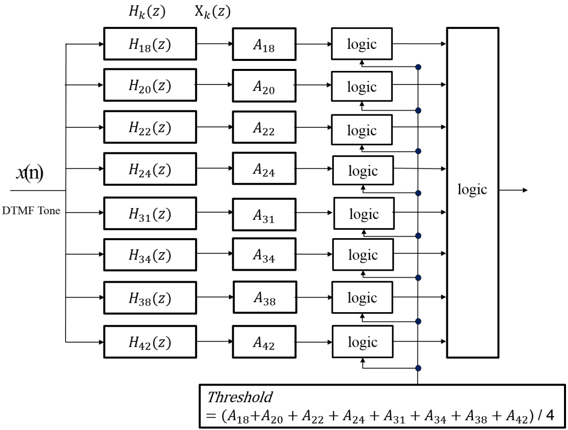

Figure: DTMF detector using the Goertzel algorithm

MATLAB Code Implementation Procedure

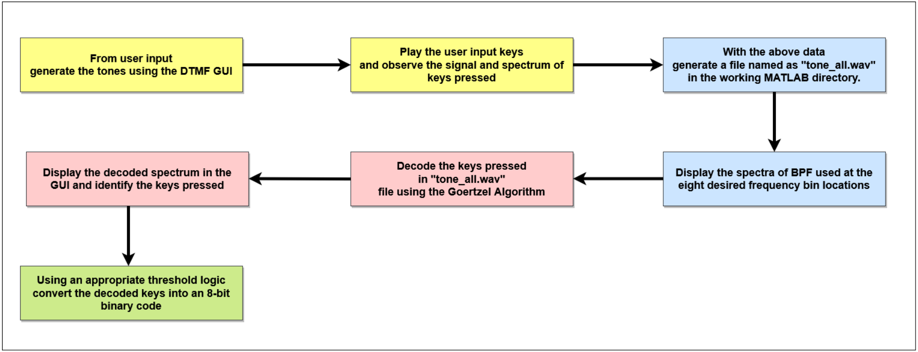

Figure: Steps involved in MATLAB Code design

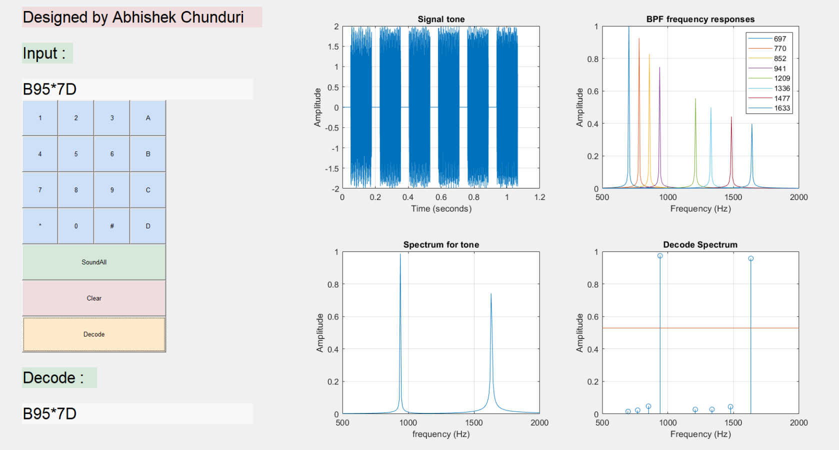

Figure: GUI designed for DTMF Detection and Decoding

References

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- *Texas Instruments*. "Modified Goertzel algorithm for DTMF using the MS320C80", application report spra066. 1996.
- *Guner Arslan and Prof. Brian Evans* .Dual-Tone Multiple Frequency (DTMF) Detector Implementation

Thank You