① $f(x_1, x_2, x_3) = 3x_1 + 2x_2 + x_3$

$\because x_1 \geq 3$

$\because 3x_2 + x_1 \geq 9$

$\because x_3 \geq 12$

$\hookrightarrow$ if $f(\theta \bar{x}_1 + (1-\theta)\bar{x}_2) \leq \theta f(\bar{x}_1) + (1-\theta) f(\bar{x}_2)$

$\bar{x}_1 = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \quad \bar{x}_2 = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix}$

$\hookrightarrow$ LHS $= f(\theta(\bar{x}_1 - \bar{x}_2) + \bar{x}_2)$

$$= f(\theta(X_1 - Y_1) + Y_1,\ \theta(X_2 - Y_2) + Y_2,\ \theta(X_3 - Y_3) + Y_3)$$

$$= 3[\theta(X_1 - Y_1) + Y_1] + 2[\theta(X_2 - Y_2) + Y_2] + [\theta(X_3 - Y_3] + Y_3]$$

$$= \theta[3X_1 - 3Y_1 + 2X_2 - 2Y_2 + X_3 - Y_3] + (3Y_1 + 2Y_2 + Y_3)$$

RHS $= \theta f(\bar{x}_1) + (1-\theta) f(\bar{x}_2)$

$$= \theta f(X_1, X_2, X_3) + (1-\theta) f(Y_1, Y_2, Y_3)$$

$$= \theta[3X_1 + 2X_2 + X_3] + (1-\theta)[3Y_1 + 2Y_1 + Y_3]$$

$$= \theta[3X_1 - 3Y_1 + 2X_2 - 2Y_2 + X_3 - Y_3] + [3Y_1 + 2Y_1 + Y_3]$$

$\therefore$ LHS = RHS

② $c^T x + d \leftarrow$ min.

st. $Gx \leq h$

$Ax = b$

$c = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad \therefore d = 0$

$G = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -3 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $\quad h = \begin{bmatrix} -3 \\ -9 \\ -12 \end{bmatrix}$

$A = 0$; $b = 0$.

EIFML Assign ②

(3) Lagrange function

$$L_p(x_1, x_2, x_3; \lambda) = \underset{\text{funct}^n}{\underbrace{f(x_0)}} + \underset{\text{ineq. constraints}}{\underbrace{\sum_{i=1}^{m} \lambda_i f(x)}} + \underset{\text{equality constraints}}{\underbrace{\sum_{i=1}^{n} v_i f(x)}}$$

$$= (3x_1 + 2x_2 + x_3) + \lambda_1(-x_1 - 3) + \lambda_2(-3x_2 - x_1 + 9) + \lambda_3(-x_3 - 12)$$

(4) Dual function

$$L_D(\lambda, v) = \inf_{x \in D} \left[L(x, \lambda, v) \right]$$

$$= \inf_{x \in D} \left[f(x_0) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{n} v_i f(x) \right]$$

$$L_D(\lambda, v) = \inf_{x \in D} \left[(3x_1 + 2x_2 + x_3) + \lambda_1(-x_1 - 3) + \lambda_2(-3x_2 - x_1 + 9) + \lambda_3(-x_3 - 12) \right]$$

Dual Problem :

$$d^* = \max_{\lambda, v} L_D(\lambda, v)$$

KKT conditions :

$$\lambda \geq 0$$

$$\lambda . f(x) = 0$$

$$f(x) \leq 0$$

(5) Solution

$$L_p = (3x_1 + 2x_2 + x_3) + \lambda_1(-x_1+3) + \lambda_2(-3x_2 - x_1 + 9) + \lambda_3(-x_3+12)$$

$$\{ L_p(x_1, x_2, x_3, \lambda) \}$$

$$\left.\begin{aligned} \frac{\partial L_p}{\partial x_1} &= 3 - \lambda_1 - \lambda_2 = 0 \\ \frac{\partial L_p}{\partial x_2} &= 2 - 3\lambda_2 = 0 \\ \frac{\partial L_p}{\partial x_3} &= 1 - \lambda_3 = 0 \end{aligned}\right\} \quad \begin{aligned} \lambda_1 &= 7/2 \\ \lambda_2 &= 2/3 \\ \lambda_3 &= 1 \end{aligned}$$

$$\left.\begin{aligned} \frac{\partial L_p}{\partial \lambda_1} &= -x_1 + 3 = 0 \\ \frac{\partial L_p}{\partial \lambda_2} &= (-3x_2 - x_1 + 9) = 0 \\ \frac{\partial L_p}{\partial \lambda_3} &= -x_3 + 12 = 0 \end{aligned}\right\} \quad \begin{aligned} x_1 &= 3 \\ x_2 &= 2 \\ x_3 &= 12 \end{aligned}$$

(6) K.K.T conditions

(i) $\lambda_i \geq 0$ $\{ \lambda_1 = 7/2 ; \lambda_2 = 2/3 ; \lambda_3 = 1 \}$

$\therefore \lambda \geq 0$

(ii) $\lambda_i f(x_i) = 0$

$7/2(-x_1+3) = 0$

$2/3(-3x_2 - x_1 + 9) = 0$

$1\,\cancel{12}\,(-x_3+12) = 0$