An Efficient and Flexible Window Function for Memristor Model and its Analog Circuit Application

Chandra Prakash Singh, Raghvendra and Saurabh Kumar Pandey

Abstract— **The memristor is a nanostructure resistive tuning two terminal novel electronic device that has been widely explored in the area of neuromorphic computing systems, memories, digital circuits, analog circuits, and many more new applications. In this article an efficient and flexible window function is presented for linear drift memristor model. The proposed parametric cubic parabolic window function provides a unique feature (controllable window function discontinuity at the boundaries) to linear drift memristor model by which DPHL (Distorted Pinched Hysteresis Loop) problem has been resolved and also improved the number of programming resistance states of the memristor. The five control parameters have been introduced in the presented window function, in order to fix the pre-existing problems (such as boundary effect, boundary lock and inflexibility) and able to provide asymmetric non-linearity at the boundaries of the device, that makes it feasible for tracking the resistive switching dynamic of futuristic oxide based memristive device with different inert electrodes. The proposed model has been validated with solution processed ZnO-based fabricated memristive device. The programmable analog gain amplifier circuit has been ultimately executed to instantiate the utilization of evolved memristor model and investigated the effect of memristance resolution.**

Index Terms— **Window function discontinuity, Control parameter, Boundary effect, Boundary lock, Linear Drift Memristor Model.**

1 INTRODUCTION

In recent decades, memristor becomes emerging nano-scale electronic device in the area of nanotechnology due to their unique feature of resistive tuning, non-volatility, simple structure, high speed, low cost, and COMS-compatibility. Firstly, Chua postulated the concept of memristor by describing the relationship between charge and flux of the device in 1971 and discovered the fourth missing fundamental passive circuit component [1]. First time, in 2008 a team of HP labs fabricated a nano-scale Metal-Insulator-Metal (MIM) structured device using Titanium dioxide (TiO_2) sandwiched between two Platinum (Pt) metal electrodes and discovered the resistive switching phenomena [2]. Since then, industries and academia have attracted much more attention to explore the physics behind the resistive switching memristive device.

Therefore, various mathematical concepts have been proposed to describe the resistive switching phenomena in nano-scale memristive device, like Non-linear model [3], Linear Drift model [2], exponential model [4], Simmons tunneling model [5], Threshold adaptive model (TEAM) [6], Variable threshold adaptive model (VTEAM) [7], Conductive filament based model [8] , and some SPICE macro models [9]-[10]. Although, several models for resistive switching device have been proposed in the literature, but any exact model has not been appeared so far. Accordingly, development of memristor model is still an emerging research domain for the research society.

At the first time, a research group from HP lab's proposed a mathematical concept for describing the resistive switching phenomena in memristive device, which is known as linear drift model. In this model, a window function has been included to introduce non-linearity and physical limiting conditions in the device as well as tracking the actual resistive switching activity of the memristive device.

As of now, apart from mathematical analysis of linear drift memristor model, it have also been investigated in numerous applications, for example, as a memory cell in high density nonvolatile random access memory [11], as an artificial synapses in the neuromorphic systems [12], applicable in spiking neural network [13], in multilayer neural network [14], and as a programmable resistive element in the analog circuit in which its resistance value can tuned by applied electrical pulses [15]-[17]. In linear drift memristor model, window function is an essential part for capturing the resistive switching dynamics of memristive device. In recent time, various kinds of materials and deposition techniques are being used by research communities to fabricate new memristive devices [18]-[20]. Hence, to develop an appropriate and efficient window function is become crucial aspect for the researchers.

In last few years, various window functions have been analyzed in the literature for linear drift memristor model. Most of the problems associated with reported window functions are boundary effect problem, boundary lock

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problem, DPHL problem, and inflexibility of control parameter. In the initial time, different kind of window functions was presented for line model by Strukov *et al.* [2], Joglekar *et al.* [21], Biolek *et al.* [22] and Prodromakis *et al.* [23]. Even though, boundary effect problem and non-linearity effect were taken care by Strukov, Joglekar and Prodromakis but failed to resolve boundary lock problem. Accordingly, Biolek was presented another window function to fix the boundary lock problem by including device current parameter in the window function, but the model was inflexible.

In recent past, for the advancement of linear drift memristor model, numerous window functions have been proposed by research community. For instance, Chen *et al.* [24] proposed a controllable window function to provide better non-linearity, Maldenov *et al.* [25] proposed a window function with activation threshold, Chowdhury *et al.* [26] presented a Prodromakis modified trigonometric window function to improve the flexibility of the memristor model, but these window functions have not considered device current parameter, hence these are failed to resolve boundary lock problem. Further, Zha *et al.* [15,17] and Shi *et al.*[27] proposed some window functions by combining the feature of Biolek and Prodromakis window function, which is able to resolve most of the problems related to window functions, although DPHL problem not have been fixed.

In recent time, some general window function has been proposed by Li *et al.* [13] and Wen *et al.* [14] with terribly restricted and more number of control parameters and it has been also analyzed in spiking neural network and multilayer neural networks. After rigorous study of all the reported window functions, we can conclude that, window function discontinuity is responsible for resolving boundary lock problem and this is also leads to DPHL problem. Till now, adverse effect of window function discontinuity at the boundary not has been discussed in the literature.

After considering the importance of reported window function and the shortcoming with it, we have proposed an efficient and flexible window function for linear drift memristor model with five control parameters, which is able to fix boundary lock, boundary effect, and inflexibility problem. Additionally, our proposed window function have unique feature to control functional discontinuity by which DPHL problem has been resolved and also improves the memristance resolution, as per author knowledge this has not been examined so far in the literature. Finally, evolved window function has been implemented in programmable analog circuit application and analyzed its performance.

This article is organized as follow. In section 2, basic physics of linear drift memristor model is described along with importance of several reported window functions with their excellence and drawbacks. In section 3, evaluation of newly proposed window function is done and shown its control parameters range as well as effects over device characteristics as well as model validation with experimental result of solution processed ZnO-based memristor with Ag and FTO electrode. In section 4, evolved memristor model is implemented in gain amplifier analog circuit and examined its performance with respect to window function control parameter. Finally, in section 5 we concluded our proposed work.

2 PHYSICS OF LINEAR DRIFT MODEL WITH WINDOW FUNCTION

In 2008, a research group from HP Labs manufactured a nano-scale memritive device as well as proposed a mathematical model for describing resistive switching dynamics of the memristive device. This model assumes that insulating layer is separated in two regions, one is undoped region and other is oxygen vacancy doped region (more conductive than undoped region) as well as total resistance of the device is calculated as summation of resistance of these two regions. In Fig.1, schematic of linear drift memristor model is given, where total device length is ‘D’ , width of the oxygen vacancy doped region is ‘w(t)’, R_1 is the resistance of the oxygen vacancy doped region, R_2 is the resistance of the undoped region and R_{eq} is the total instantaneous resistance of the device. Moreover, R_{ON} and R_{OFF} are the limiting resistances of the

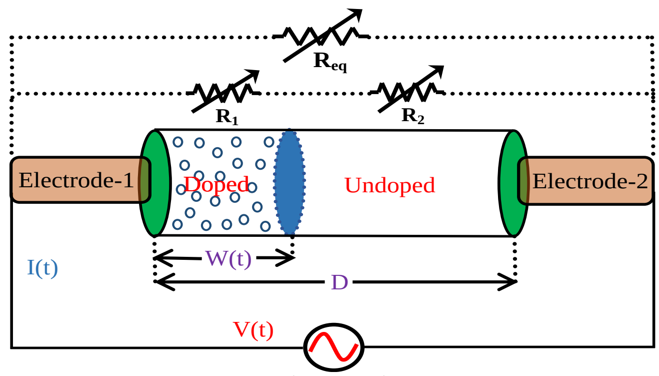

Fig. 1. Schematic of linear drift memristor model.

device for fully doped [w(t) = D] and fully undoped [w(t) = 0] case and ‘x = w(t)/D’ is the normalized width of the device that is called as internal state parameter and it is restricted between ‘0’ and ‘1’ due to physical boundary limitation of the device. The total instantaneous resistance of the nano-scale memristive device can be calculated from expression-1, as mention in [2].

$$R_{eq} = \{R_{ON} * (x) + R_{OFF} * (1 - x)\} \qquad \ldots..(1)$$

In this model, when we are exerting electrical stimuli across the device, then drifting of interface between doped and undoped region occurs. Moreover, the drifting of the interface is formulated as expression-2 by considering basic physics of drift velocity. From expression-2 we can see that, internal state parameter ‘x’ is the linear function of charge flow through the device, that is why this model is known as linear drift model.

$$\frac{dx}{dt} = \mu_v * \frac{R_{ON}}{D^2} * I(t) \qquad \ldots..(2)$$

Where ‘μ_v’ is the average mobility of oxygen vacancy (dopant) in the proposed memristive device. In this linear drift model, if applying high voltage or low frequency then its doping region goes beyond its physical limit (i.e. boundary effect problem), which is practically impossible. Furthermore, even after exerted small voltage across nano-scale devices it experiences high electric fields, which leads to non-linearity in the device characteristics [2]. Accordingly, researchers incorporated a window function in this linear drift model to overcome boundary effect problem and introducing non-linearity in this model. After that, interface dynamic expression is modified as expression-3.

$$\frac{dx}{dt} = \mu_v * \frac{R_{ON}}{D^2} * f(x) * I(t) \qquad \ldots..(3)$$

To understand the advantages of proposed window function, firstly, we have discussed several reported window functions with their properties and drawbacks. First parabolic window function was proposed by Strukov *et al.* [2], that is given in expression-4.

$$f(x) = x - x^2 \qquad \ldots..(4)$$

This window function is capable to resolves the boundary effect problem, because at the boundary of the device (i.e. x=1 or x=0) interface drifting become zero with f(x) = 0. However, it provides only fixed non-linearity to the device. So, Joglekar et al. [21] proposed another window function with one control parameter to tune the non-linearity of the device, which is shown in expression-5.

$$f(x) = 1 - (2x - 1)^{2p} \qquad \ldots..(5)$$

Where, 'p' should be positive integer only. Even then, it suffers from boundary lock issue, means that, if ones interface hits any boundary of the device (x = 0 or 1), it is not capable to tune the resistance of the memristive device from its terminal resistance (R_{ON} or R_{OFF}). Thereafter, Biolek *et al.* [22] proposed a new window function to resolve the boundary lock issue by including device current parameter in window function. This window function is represented by expression-6, which have one control parameter.

$$f(x) = 1 - [\,x - stp(-i)]^{2p} \qquad \ldots..(6)$$

Where, 'p' is positive integer only and $stp(.)$ is a step function which has discontinuity at one point. Additionally, it has restricted flexibility in the window function due to utilizing only one control parameter. By considering above limitations, Prodromakis *et al.* [23] recommended a parabolic window function with two control parameter which has given in expression-7.

$$f(x) = j * [1 - \{(x - 0.5)^2 + 0.75\}]^p \qquad \ldots..(7)$$

Where, 'p' and 'j' belongs to positive real number, which provides better flexibility to the window function. But, there is no device current parameter involved in the window, so it can't handle with boundary lock issue. After taking these constraints into account, an appropriate window function has been provided by Zha *et al.* [15] with two control parameter including device current in it, which has formulated as expression-8.

$$f(x) = j * \left[1 - \left\{0.25 * \left(x - stp(-i)\right)^2 + 0.75\right\}\right]^p \qquad \ldots..(8)$$

Where, 'p' and 'j' are positive real numbers. This is able to resolve boundary lock issue and also provide good flexibility. Although, this window function model have fixed discontinuity at its boundary, which results in DPHL problem for low frequency or high amplitude exerted external stimuli. To improve the flexibility in Zha *et al.*[15] window function, Shi *et al.*[27] proposed another window function model with one additional control parameter(a_s), that is responsible for controlling the maximum value of window [$f(x)_{max}$] only up to one, as mentioned in expression-9.

$$f(x) = j * \left[1 - \left\{a_s{}^2 * \left(x - stp(-i)\right)^2 + (1 - a_s{}^2)\right\}\right]^p \qquad \ldots..(9)$$

Where 'p' and 'j' are positive real numbers and a_s is restricted between '0' and '1'. Further, Zha *et al.* [17] again reported another window function with three control parameter, where some of existing window functions is special case of this window function, which has given in expression-10.

$$f(x) = \left[1 - \left\{a * \left(x - stp(-i)\right)^{2q} + (1 - a)\right\}\right]^p \qquad \ldots..(10)$$

Where, 'p' and 'j' are positive real numbers but 'q' should be positive integer only and 'a' is bounded between '0' and '1'. This window function has resolved most of the problems associated with previous window functions, even than functional discontinuity present here, that creates DPHL problem (after applying bit high voltage or low frequency input, state variable 'x' reaches its limit very quickly which results in distortion in current-voltage response of the device). In addition, this window model has only symmetric non-linearity at the both ends of the device, which is practically always not possible. For addressing the DPHL problem, Anusudha *et al.* [16] defined a cubic parabolic window function in expression-11.

$$f(x) = j * [1 - 2(x^3 - x + 1)^p] \qquad \ldots..(11)$$

Where $j \in R^+$, and 'p' is positive integer only. Although, this window function has failed to resolve boundary effect problem, because at the boundary of the device interface drifting is not zero (i.e. at x=0 or 1; $f(x) \neq 0$) and it has not discussed the boundary lock issue.

3 NEWLY PROPOSED WINDOW FUNCTION

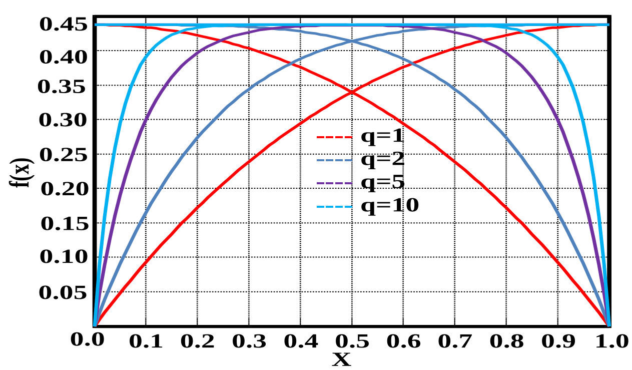

Fig. 2. Effect of 'q' on proposed window (12) with a=0.25, p=2 and j=1.

After rigorous analysis of several reported window functions, in this section, we come up with an efficient and flexible window function model and also discussed about its superiorities over existing window functions. This window function has five control parameters with considering device current, which helps to resolve boundary effect, boundary lock and inflexibility. Proposed window function have a unique property of controlling window function discontinuity, by which it is able to resolves DPHL problem without removing device current parameter from window function and also improves the programming resistance state of memristor. It is fulfilled all the necessary criteria of window function which has mention in [17]. Our proposed window function is

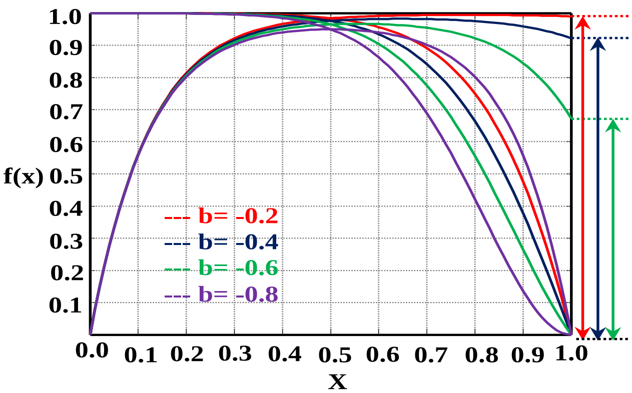

Fig. 3. Effect of 'b' on function discontinuity by proposed window (12) with a=0.8, p=5, q=1 and j=1.

formulated as expression-12.

$$f(x) = j * [1 - \{(b * x^2 + a) * \{x - stp(-i)\}^{2q} + (1 - a - b * x^2)\}]^p \qquad \ldots\ldots(12)$$

Where 'p' and 'j' are any positive real numbers but 'q' should be only positive integer. Other two control parameter is restricted as $a \in (0, 1]$ and $b \in [-a, 0]$ to fulfill the necessity

TABLE -1
SUBSET OF PROPOSED WINDOW FUNCTION

Control Parameter Value	Inherited Window Function
j=1, a=1, q=1, b=0.	$f(x) = 1 - [\,x - stp(-i)]^{2p}$
a=0.25, q=1, b=0.	$f(x) = j * \left[1 - \left\{0.25 * (x - stp(-i))^2 + 0.75\right\}\right]^p$
$a=a_s^2$, q=1, b=0.	$f(x) = j * \left[1 - \left\{a_s^2 * (x - stp(-i))^2 + (1 - a_s^2)\right\}\right]^p$
b=0.	$f(x) = j * \left[1 - \left\{a * (x - stp(-i))^{2q} + (1 - a)\right\}\right]^p$

criteria of window function. Proposed window function is able to overcome DPHL problem by controlling the window function discontinuity at their boundary and also improving programming resistance resolution of memristor, which can enhance the performance of programmable analog circuit in practical implementation.

From Table-1, we can conclude that, our proposed window function is able to produce some of the reported window functions [15,17,22,27] by adjusting its control parameter values. Hence, properties of these reported window functions are inbuilt in our proposed window function model. The impact of control parameter 'a', 'j' and 'p' of memristive model characteristics have been already discussed in literature [15,17].Thereafter, from Fig. 2 we can observe that, the control parameter 'q' only modulates the non-linearity of the device without affecting maximum value of window function with considering others window control parameters are as fixed. When we increase the value of 'q' then the non-linearity of the window is confined to their boundary. Further, we can evident from Fig. 3, the control parameter 'b' is able to control the functional discontinuity at the boundary of the window function which helps to resolve the DPHL problem. In next section, we will see by adjusting window function discontinuity at the boundary, DPHL issue has been resolved.

4 EVALUATION OF PROPOSED WINDOW FUNCTION MODEL

To evaluate the proposed newly window function we have

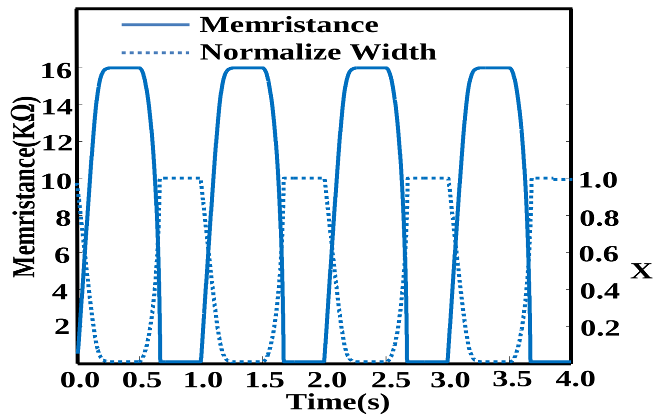

Fig. 4. Response of the normalized width and memristance with high amplitude input voltage

studied boundary effect issue, boundary lock issue, non-linearity and DPHL issue in the device by taken parameter value as R_{ON} = 100Ω, R_{OFF} = 16KΩ, $R_{initial}$ = 0.5KΩ, D = 10nm, $\mu_v = 10^{-14}\ m^2 s^{-1} V^{-1}$, a = 0.8, p = 5, q = 1,b = -0.6, to simulate in SPICE. Where $R_{initial}$ is represents the initial resistance of the memristive device.

4.1 Study of Boundary effect Issue

We can observe that in Fig. 4, after applying sufficient high

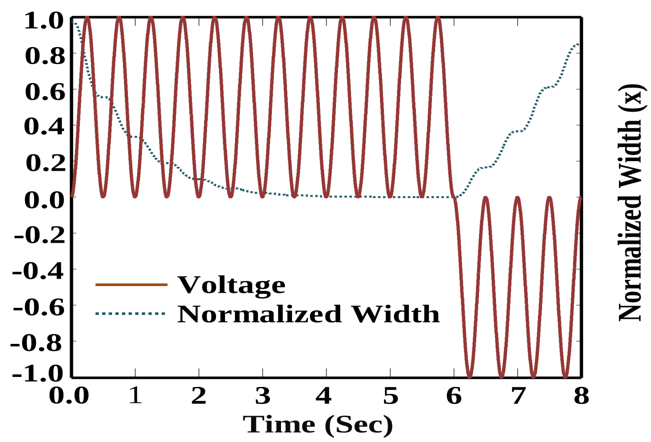

Fig. 5. Response of the normalized width with the input voltage.

amplitude voltage [like as Vm= 10sin(2πf*t)] to the memristive device, it operates between its terminal resistance range (R_{ON} and R_{OFF}) and never goes beyond their own physical limits (means 'x' is restricted between 0 to 1) .Hence, now we can conclude that our proposed window function is free from boundary effect problem.

4.2 Study of Boundary Lock Issue

After applying positive and negative sinusoidal voltage to the memristive device, the response of the normalized width has reported in the Fig. 5. So from Fig. 5, we can observe that, when polarity of the applied voltage changes then its normalized width (state value) modulated from its boundary. Therefore, we can say that our proposed window model is successfully resolved boundary stick issue.

4.3 Study of DPHL Issue

When applied electrical stimuli have low frequency or voltage amplitude is high then interface reaches the boundary of the device very quickly, means 'x' suddenly achieves their limiting value (0 or 1). Hence, memristive device current

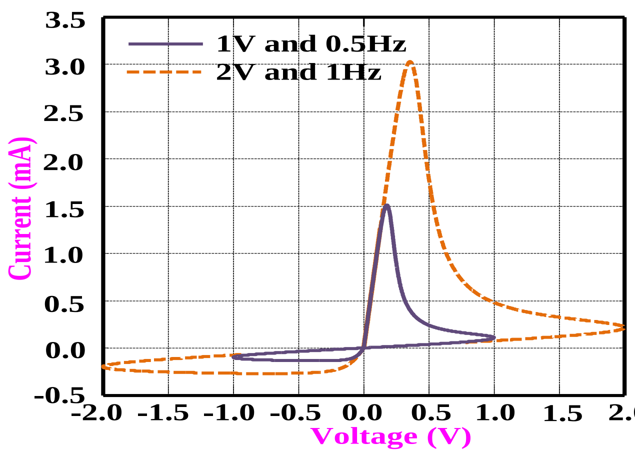

Fig. 6. Distorted I-V response of memristor for low frequency and high amplitude input voltage with b=0.

voltage response gets distorted, that is known as distorted pinched hysteresis loop (DPHL) issue, which has been undesirable for any application. From Fig. 6, we can see that I-

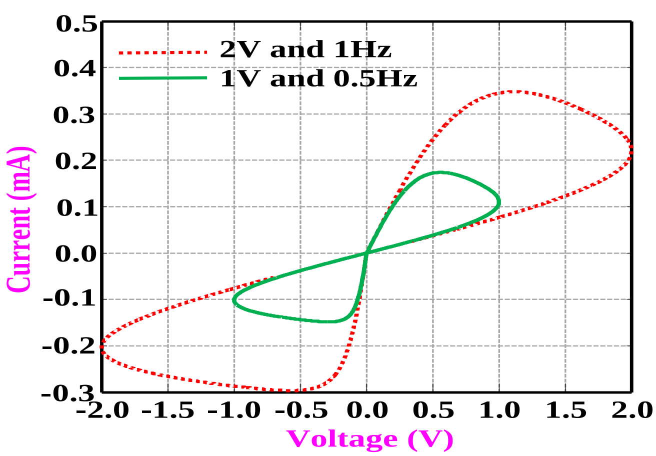

Fig. 7. Desired I-V response of memristor for low frequency and high voltage input voltage with b= -0.6.

V response of the proposed model with b=0, which is distorted in nature, for 1Volt and 0.5Hz applied stimuli and 2Volt and 1Hz applied stimuli. Further, we can evident from Fig.3, our proposed model is able to resolve DPHL issue by adjusting the window function discontinuity at its boundary with different value of control parameter 'b'. In Fig. 7, we demonstrated the I-V response of the proposed model with 'b= -0.6' which is free from DPHL problem, even at low frequency or high amplitude applied voltage.

4.4 Study of Memristance Resolution

The idea of memristance resolution was proposed in [28], where the value of memristance can be tuned by periodic pulses. Although, in the process of memristance tuning by periodic pulses, some certain error exists due to the number of applied pulses should be an integer. Accordingly, designing of memristor model with high precision is very much important. As mention in [17], control parameter 'a' is able to tune the resolution of the memristance. From Fig.8 we can see that, the

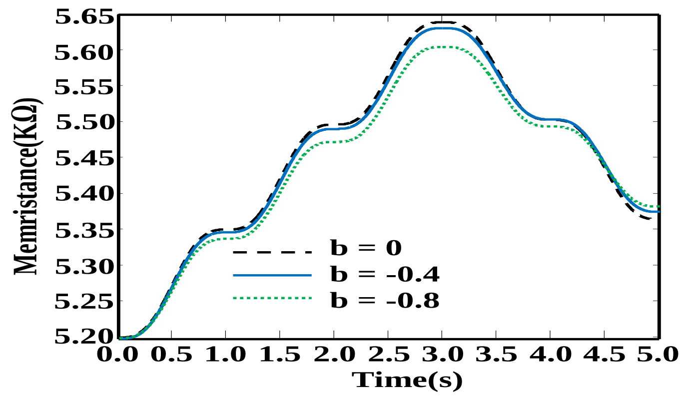

Fig. 8. Effect of 'b' on transient behavior of memristance for the unbalanced input voltage with a=0.8, p=5, q=1.

additional control parameter 'b' is able to improve the memristance resolution with constant value of 'a'. In our proposed model memristance tuning is analyzed by using unbalanced five sinusoidal periodic signal $v(t)= +v_o\sin^2(2\pi f*t)$ for first three pulses and $v(t)= -v_o\sin^2(2\pi f*t)$ for next two pulses with a= 0.8, v_o= 0.05 and f= 1Hz for different value of control parameter 'b', as demonstrated in Fig.8. That is why it is clear that, if resolution is high, then it is able to attain more number of programmable resistance states of memristor.

The existing window functions have boundary lock, boundary effect problem, and inflexibility, but presented window function is free from these problems and also provides very good flexibility with five control parameter. The DPHL (Distorted Pinched Hysteresis Loop) problem was firstly addressed by Anusudha in 2018 [16]. To resolve DPHL problem, Anusudha presented a cubic parabolic window function but this window function is failed to fix the boundary effect problem and also not discussed about boundary lock problem. Hence, our proposed window function is able to fix DPHL problem by controlling window function discontinuity at the boundary and also improve the resolution of the proposed memristor model. In addition, all the reported window functions have only symmetric non-linearity at the both ends of the device, which is practically always not possible but proposed window function has capability to provide asymmetric non-linearity at the boundaries of the device by adjusting its control parameters.

The comparative report of proposed window model with respect to published window function model has been shown in Tabel-2. So from Table-2 we can evident that, all the issues related to the window function has been resolved by our proposed memristor window function model.

TABLE -2
COMPARISON WITH REPORTED WINDOW FUNCTION

Existing Window Function	Strukov [2]	Joglekar [21]	Biolek [22]	Prodromakis [23]	Shi [27]	Zha [15]	Anusudha [16]	Proposed Function
Resolve boundary effect	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes
Resolve boundary stick	No	No	Yes	No	Yes	Yes	Yes	Yes
Scalability	No	No	Limited	Yes	Yes	Yes	Limited	Yes
No. of control Parameter	Zero	One	One	Two	Two	Four	Two	Five
Linkage with linear dopant drift model	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Resolve DPHL	NA	NA	No	NA	No	No	Yes	Yes

4.5 Reproducing the Response of ZnO Based Memristor

In this subsection, we have reproduced the I-V characteristics of solution processed ZnO based memristive device by simulating proposed model. In [29], Patil has discussed all the step of solution processed ZnO memristive device fabrication with 34µm thickness and Ag and FTO as its electrodes. In order to obtained the I-V response of fabricated ZnO memristor, applied voltage has been swept from 0V to +0.8V, +0.8V to 0V, 0V to -0.8V and -0.8V to 0V, and proposed model has been simulated with following parameters: D = 34µm, x= 0.88, R_{ON} = 300Ω and R_{OFF} = 8KΩ along with control parameters a = 0.8, b = -0.2, p = 10 and q = 2. In Fig.9, experimental and simulated I-V response of the ZnO memristive device with red and black curve has been demonstrated. So we can clearly see that, I-V loop for positive current and negative current are not symmetrical means that oxygen vacancy drifting are not symmetrical with respect to device current direction and also our model is able to track the switching mechanism of fabricated device but some short of deviations are present near the peak of I-V response. As we know that, when memristor is used for programmable resistance then resistance value is important at a certain voltage that is known as read voltage, hence, deviations near the peaks are not much significant for programmable resistance application.

Further, in the next section we will investigate the potency of our proposed model in analog gain amplifier circuit.

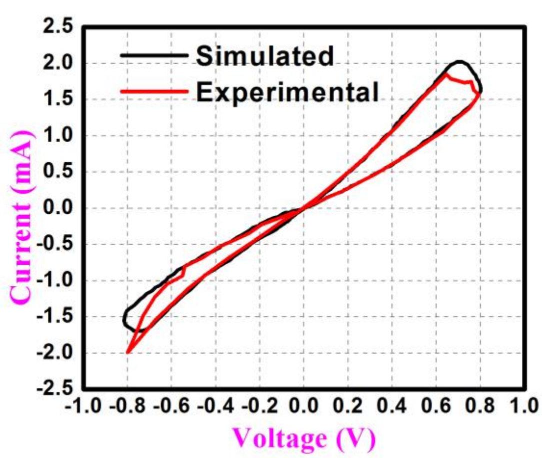

Fig. 9. Simulated and experimental I-V characteristics of ZnO based memristive device.

TABLE -3
EFFECT OF MEMRISTANCE RESOLUTION ON PROGRAMMING STAGE AMLIFIER GAIN

Number of Pulses		Memristance (KΩ)	Gain (dB)
For b=0.0	For b=-0.6		
initial	initial	0.1	52.1
836	954	4.0	41.6
5623	5897	16.0	25.2

5 IMPLIMENTATION IN ANALOG GAIN AMPLIFIER CIRCUIT

According to previous discussion, an efficient and flexile window function has been built for liner drift memristor model to resolve DPHL by adjusting discontinuity at the window function boundary and improve the resolution of memristance. Now, in this section we will illustrate the efficacy of proposed efficient window function linear drift memristor model in analog differential gain amplifier circuit.

In ordinary analog circuits, after making the circuit it is very difficult to tune the value of resistance which restricts their practical implementation. For resolving this issue, instead of resistor we introduce a memristor in the analog circuit which

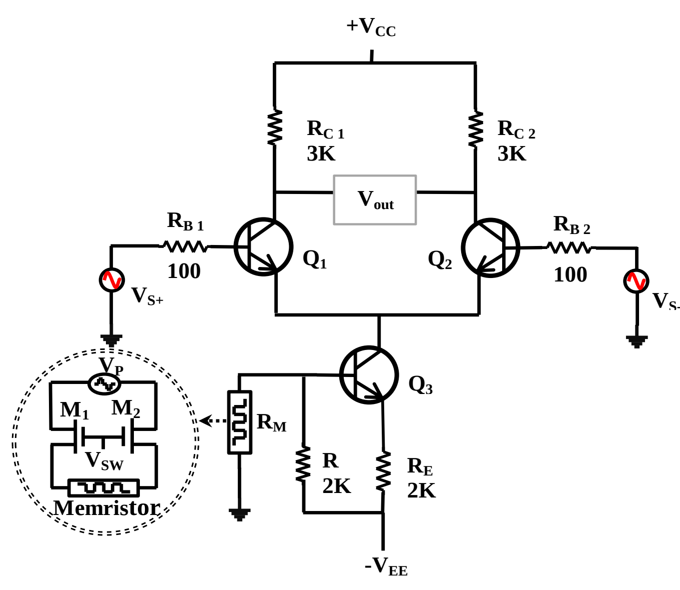

Fig. 10. A programmable analog gain amplifier

is able to tune their own resistance by applying electric pulses over it and makes it programmable [30]. In Fig.10, memristor (programmable resistor) is placed instead of resistor to control the emitter current of transistor 'Q_3' in the analog amplifier circuit, which is responsible for tuning the gain of the analog amplifier and makes it programmable gain amplifier. This gain amplifier works in two modes: amplification and programming mode. In programming mode, resistance of the memristor is tuned to a target value by applying electrical pulse by V_{SW} is set to 1 between its terminal resistance (R_{ON} and R_{OFF}). In amplification mode, after achieving target resistance of memristor, V_{SW} is set to 0, then circuit goes into amplification mode and memristor works as resistor and makes a voltage divider circuit to the base of transistor 'Q_3', that is deciding factor for gain of the amplifier by tuning emitter current of the transistor 'Q_3'. By studying small signal circuit analysis, analog amplifier differential voltage gain is formulated as expression-13.

$$A_{VD} = g_m * R_c = \left({}^{I_c}/_{V_t}\right) * R_c = \left(\frac{I_{e3}}{2 * V_t}\right) * R_c \qquad \ldots..(13)$$

$$I_{e3} \approx \frac{V_{EE}\left(1 - \frac{R_M}{R_M + R}\right) - V_{be}}{R_E} \qquad \ldots..(14)$$

Where R_c=R_{c1}=R_{c2}= 3KΩ, R_E= 2KΩ, R= 2KΩ and 'I_{e3}' is the emitter current in the transistor 'Q_3' which has expressed in the expression-14. In addition, memristance programming electrical pulses is same, as mentioned in section 4.4.The analog voltage gain circuit is simulated in SPICE where the parameter a= 0.5 and R_{intial}= 100Ω with b= 0.0 and b= -0.6. Moreover, differential mode input voltages are V_{s+}= V_a sin(2πf*t) and V_{s-}= -V_a sin(2πf*t), where V_a = 20mV and f = 1KHz. As Table-3 shows, resistance of the memristor and the gain of the circuit closely related to the applied number of electric pulses as well as shows, for attending same memristance value, different number of electrical pulses required with different value of control parameter 'b'. Since, b= -0.6 model required more electrical pulses then with b= 0.0. Hence, it is clear that, b= -0.6 model can adopt more number of discrete memristace states. Therefore, we can conclude that, control parameter 'b' improves the resolution of the memristor, which is desirable for any programmable application.

6 CONCLUSION

This article introduced a newly flexible and efficient window function for linear drift memristor model with unique feature of controllable window function discontinuity at the boundaries. Proposed window function is able to reproduces some existing window functions and also resolve all the issues associated with reported window functions. By introducing a key parameter in this proposed model, functional discontinuity of the window function become controllable, by which DPHL issue has been resolved without removing device current parameter form window function and also improved the resolution of programming resistance of memristor. This model have potential to the track resistive switching behavior of solution processed ZnO-based fabricated memristive device with Ag and FTO electrodes and also able to provide asymmetric non-linearity in oxygen vacancy drifting with respect to current direction of the memristive device. Ultimately, the proposed window function has been equipped in analog gain amplifier circuit and makes it programmable with enhanced number of programming gain stage of differential gain amplifier circuit; it has been validated by SPICE simulation.

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