SPICE Modeling of Memristor and Memristive Systems: A Review

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ABSTRACT

Memristor, the fourth fundamental passive circuit element, was first postulated by Prof. Leon Chua in 1971. Physical implementation of novel device Memristor and its mathematical model was first demonstrated by HP Labs research team in 2008.Memristor is getting a considerable attention due to its diversified areas of applications from computing to neuromorphic areas. For its reliable implementation in complex circuits various models are discussed. This paper is a brief review on Memristive systems such as current controlled Memristive system (CCMS) and voltage controlled Memristive system (VCMS). A comparative study of different SPICE modeling of memory resistor (Memristor):Non-linear dopant drift model, linear dopant drift model is discussed in this paper.

Keywords: Memristor, SPICE, CCMS, VCMS, Drift models.

I. INTRODUCTION

Memristor was a novel concept postulated by Prof. Leon Chua in 1971 as the fourth missing circuit element in the list of three fundamental electric circuit elements such as Resistor,Capacitor,Inductor.The special feature of this proposed device was its ability to remember and compute logic simultaneously. It is because of this ability Memristor is intensely studied as a next generation non volatile memory. For its future reliable applications, modeling of the device is an essential part. Memristor being a non-linear device, its modeling can be handled by simulator like SPICE efficiently. SPICE (Simulation Program with Integrated-Circuit Emphasis) is a general – purpose circuit simulator capable of performing nonlinear circuit analysis. The purpose of this paper is to discuss SPICE modeling of Memristive system on the basis of the mathematical definitions and model demonstrated by HP labs [1].

Section II, of this paper will discuss fundamental definitions of Memristor and Memristive system. A schematic of SPICE model and mathematical modeling of SPICE based HP Memristor model is discussed in Section III. SPICE models of CCMS and VCMS are discussed in section IV. A comparative study of different SPICE models is reviewed in section V.

II. MEMRISTOR FUNDAMENTALS

In circuit theory, for nearly 180 years, it has been accepted that there are three fundamental passive circuit elements, the resistor, the capacitor and the inductor discovered in 1827, 1745 and 1831 respectively. These passive two-terminal devices do not store information. They describe the relations between voltage, current, charge and flux[1-2].

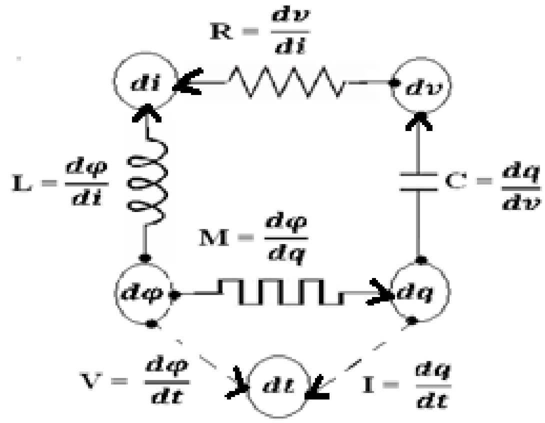

Fig. 1 Fundamental circuit elements

Resistance relates voltage and current (dv=R.di), capacitance relates charge and voltage (dq=C.dv) and inductance relates flux and current (dφ=L.di), respectively. On this background as shown in Fig. 1, on the basis ofsymmetry ,Prof. Chua described Memristor as a non linear resistor relating flux and charge with equation dφ/dq = M(q), where M was termed as Memristance [1][3].

Basic mathematical relations for the fundamental circuit elements shown in Fig 1 are summarized in Table 1.

Element	Derivative Function	Definition	Integral Function	Equations In Terms of voltage
R	dv / di	dv = R di	V = $\int R\,di$	V= IR
L	dφ / di	dφ = L di	φ= $\int V\,dt$	$\int V\,dt$ = LI
C	dq /dv	dq = C dv	q = $\int I\,dt$	$\frac{1}{C}\int I dt$ = V
M	dφ/ dq	$d\varphi$ = Mdq	$\varphi = \int V dt$	$\int V\,dt$ = M$\int I\,dt$

Table 1: Basic Mathematical Equations for Circuit Elements

1. Memristance for Memristive System

A general Memristive system is describe by set of two equations (1) and (3).

v = R (x ,i) i .... (1)

Where; v = voltage, i= current and R(x, i) is instantaneous resistance dependent on the internal state variables of the device.

By definition of Current, i = $\frac{dq}{dt}$

By faradays law of induction, v = $\frac{d\varphi}{dt}$

Equation (1) can be modified to expressrelationship between magnetic flux linkage (φ) and electric charge (q).

R(x ,i) = $\frac{v}{i} = \frac{d\varphi}{dt} \Big/ \frac{dq}{dt} = \frac{d\varphi}{dq}$ .... (2)

The Memristive system is further described by equation(3)which represents the relation betweenthe changing state variable (x) and its dependence on the amount of charge flowing through the device.

$\frac{dx}{dt} = f(x, i)$.. (3)

Where $\frac{dx}{dt}$ is the time derivative of state variable and $f(x, i)$ is a continuous function of the respective variable.The Memristive Systemis different from the Memristor because Memristance of an element depends not only on the state of the system, but it is also affected by the instantaneous values of voltage or current. This contradicts the original definition of Memristor, which states that Memristance depends only on the state of the system.

III. MODELING OF HP MEMRISTOR

A physical model of HP Memristor [9] [10], consists of a thin semiconductor film of TiO_2 and TiO_{2-X} regions. The region with high concentration of dopant(TiO_{2-x})is called RON which behaves like a low resistance and the stoichiometricregion with low concentration of dopant (TiO_2)called as ROFF with high resistanceare sandwiched between two platinum electrodes (Fig. 2).

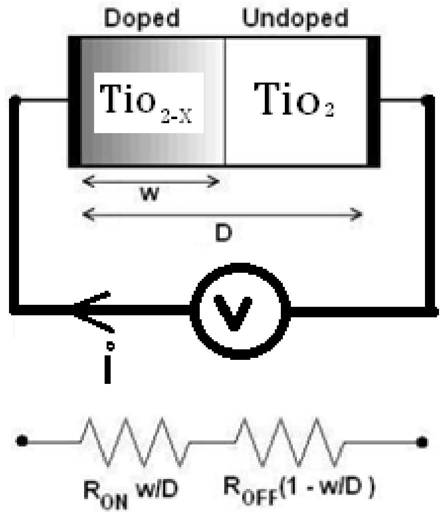

Fig. 2 Physical model of TiO_2 Memristor

After applying voltage, the width of doped region is modulated proportional to the amount of charge passing through the device.The state variable x(t) is found to be dependent on average carrier mobility (µv), on state resistance (R_{ON}), thickness of the device (D) and the amount of charge (q).The total resistance of the device is given by

R(t) = R_{ON} X(t) + R_{OFF} (1- X(t))……………………….(4)

Voltage-current relationship is given by;

v(t) = R(t) * i(t).………………………………………...(5)

v(t) = ($R_{ON}\frac{W(t)}{D}$ + $R_{OFF}(1 - \frac{W(t)}{D}$)) i(t)…...……………..(6)

Where

$\frac{W(t)}{D} = X(t)$= state variable

The function of charge w(t) is expressed as

w(t) = µv $\frac{RON}{D}$q(t) …………………………...………..(7)

Differentiating the equation with respect to‘t’

$\frac{dW(t)}{dt}$ = µv $\frac{RON}{D}$i(t) …………………………..………..(8)

Rate of change of state variable is proportional to current. Let D = Thickness of the sandwiched area and w = doped area (oxygen deficient area) in a HP Memristor, µv = dopant mobility.

Memristance of the system is given by;

M (q) = R_{OFF} (1 - $\frac{\mu v\, R_{ON}}{D^2}$ $q(t)$)………………………….(9)

Where R_{ON}<< R_{OFF}

1. SPICE MODEL OF MEMRISTOR

A structure of SPICE based Memristor model [7] is as shown in Fig.3.

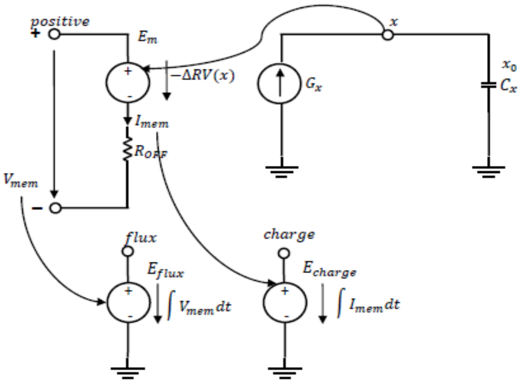

Fig. 3 Structure of SPICE model

Where V_{mem} is the input voltage and I_{mem} is the current flowing through the Memristor. The flux can be calculated by integrating the voltage V_{mem}andcharge can be calculated by integrating current $I_{mem.}$

The relation between the voltage and current is modeled on the basis of modified equation (4) as ,

$R_{mem}(x) = R_{OFF} - x\Delta R,\ \Delta R = R_{OFF} - R_{ON}$………......... (10)

R_{OFF} is the resistance connected in series withthe voltage source $E_m$whose terminal voltage can be controlled using formula $-x\Delta R$.The G_X is the current source whose current is controlled according to the equation $I_{mem}f(v(x))$ where v(x) is the voltage across capacitor C_X and it models normalized width x of the doped layer. The initial state of the normalized width of the doped layer x_0 , which is defined as initial voltage of capacitor, is determined by the initial resistance $R_{INITIAL}$of Memristor derived from (9) [6].

$$X_o = (R_{OFF} - R_{INITIAL}) / \Delta R$$

This model is implemented as a SPICE sub-circuit with significant mention of parameters such as, **$R_{INITIAL}$**,ROFF and RON as resistance states, D as the width of the thin film, the dopant mobility µv.
The SPICE model of Memristor in Fig.3 shows how V_{mem} can be substituted by E_{flux} and I_{mem} can be substituted as E_{charge}. The port and Constitutive equations for VCMS and CCMS is summarized in Table 2.

Element type	Port equation	Constitutive Equation
VCMS	**$v = R_m(q)i$**	**$R_m(q) = \frac{d\varphi}{dq}$**
CCMS	**$i = G_m(\varphi)v$**	**$G_m(\varphi) = \frac{dq}{d\varphi}$**

Table 2:Port and Constitutive equations of Memristive system

IV. SPICE MODELING OF MEMRISTIVE SYSTEM

SPICE model of Memristive system basically correlates port and state equations. Where the port block interacts with state through its terminal voltage and current. the state equations are modeled using integrators and the integrators are implemented by G-type current sources, driving grounded 1F capacitors and also enabled the adjustment of initial conditions .one resistor parallel with capacitor is used to define DC path to ground [12].

The SPICE model for current controlled Memristive system (CCMS) and voltage controlled Memristive system (VCMS) as shown in Fig.4, Fig.5, Fig.6 and Fig.7.

1. Current Controlled Memristive System (CCMS)

Current controlled Memristor, also termed as charge controlled Memristor is mathematically expressed as

$\varphi = f(q)$…………………………………………….. (11)

Differentiating equation (9) with respect to t

$$\frac{d\varphi}{dt} = \frac{df(q)}{dq}\frac{dq}{dt} \quad \text{………………………….……(12)}$$

Substituting $v = \frac{d\varphi}{dt}$, $i = \frac{dq}{dt}$

Equation () may be expressed as

$v(t) = M(q)\ i(t)$………………………………...……(13)

where $M(q) = \frac{df(q)}{dq} = R_m$

M(q) = Memristance (for Memory Resistance)

The Current controlled Memristive System (CCMS), on port side, can be represented by voltage source or current source is as shown in Fig.4 and Fig.5.

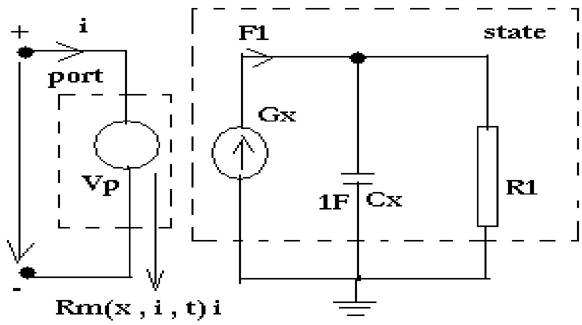

Fig. 4 Current controlled voltage source Memristive system

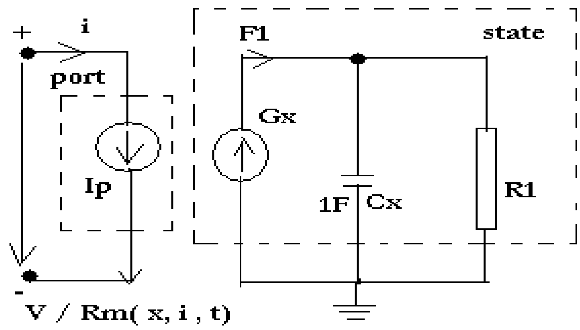

Fig. 5 current controlled current source memristive system

The relation between port voltage and current is determined by the Memristance (R_m) is summarized in Table 3.

SYSTEM TYPE	PORT EQUATION	STATE EQUATION
CCvsMS	**$v = R_m(x, i, t)\ i$**	**$\dot{x} = f(x, i, t)$**
CCcsMS	**$i = v / R_m(x, i, t)$**	**$\dot{x} = f(x, i, t)$**

Table 3: Port and State Equations of CCvsMS and CCcsMS.

2. Voltage controlled Memristive system (VCMS)

A voltage controlled Memristor is also termed as flux controlled Memristor is mathematically expressed as

$q = f(\varphi)$ ……………………………………………..(14)

Differentiating equation 12 with respect to t

$$\frac{dq}{dt} = \frac{df(\varphi)}{d\varphi}\frac{d\varphi}{dt}$$

But, $i = \frac{dq}{dt}$, $v = \frac{d\varphi}{dt}$

$i(t) = G(\varphi)\, v(t)$

Where $G(\varphi) = \frac{df(\varphi)}{d\varphi} = G_m$

$G(\varphi)$ = Memductance (for Memory Conductance), which is reciprocal of Memristance.The voltage controlled Memristive System (VCMS), on port side, can be represented by voltage source or current source.the relation between port voltage and current is determined by the Memductance (G_m) is as shown in Fig. 6 and Fig.7..

The port equations and state equations of Voltage controlled voltage source (VCvsMS) and voltage controlled current source (VCcsMS) is summarized in Table4.

SYSTEM TYPE	PORT EQUATION	STATE EQUATION
VCvsMS	$V_m = i / G_m(x, v, t)$	$\dot{x} = f(x, v, t)$
VCcsMS	$I_m = G_m(x, v, t)$	$\dot{x} = f(x, v, t)$

Table 4: Port and State equations of VVvsMS and VCcsMS.

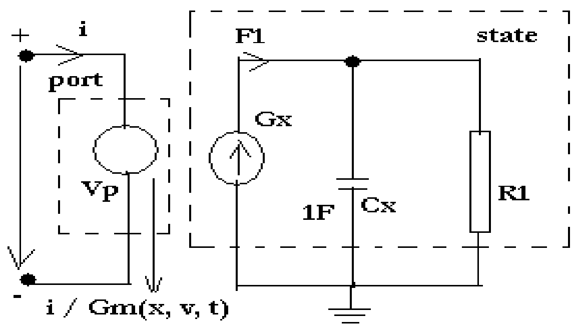

Fig.6 voltage controlled voltage source memristive system

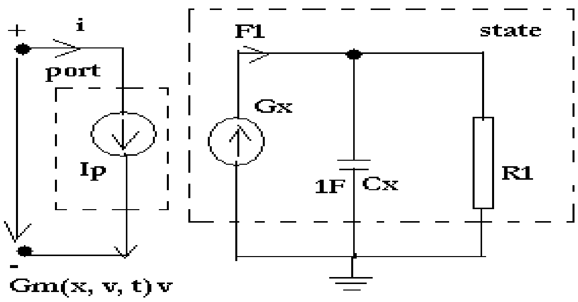

Fig. 7 voltage controlled current source memristive system

V. METHODS OF SPICE MODELING IN MEMRISTOR

To design any circuit with memristor, the behavior of model should be sufficiently accurate, computationally efficient, simple, intuitive and closed form so that it can be tuned to suit different technologies of memristor. In this paper we are discussed two different models i.e. linear and nonlinear ion dopant drift model.

1. Linear Ion Dopant Drift Model

A linear ion dopant drift model assumes a uniform electric field across the device which corresponds to a linear relationship between drift diffusion velocity and net electric field. Where the ions have equal average ion mobilityμv . this model exhibits the property of Memristor [12].

The state equation in (5) can be written as;

$\frac{1}{D}\frac{dw(t)}{dt} = \mu v \ \frac{R_{ON}}{D^2}\ i(t)$………………………………………..(14)

Integrating equation (14) gives

$\frac{w(t)}{D} = \frac{w(t_o)}{D} + \mu v \ \frac{R_{ON}}{D^2}\ q(t)$…………………………………(15)

Where $w(t_0)$ = initial length of w.

The drift speed under a uniform electric field across device is given as

$E_D = \frac{dw(t)}{dt}$ where E_D = drift speed = $\frac{D}{t}$ for assumed to be uniform electric field.

Linear drift model is applicable for boundaries from w $(t_o) = 0$ to w(t_o) = D with certain about of charge (Q_D) required to move the boundary.

$Q_D = \frac{D^2}{\mu v\, R_{ON}}$ Substituting in (15)

$\frac{w(t)}{D} = \frac{w(t_o)}{D} + \frac{q(t)}{Q_D}$……………………………………..(16)

As $x(t) = \frac{w(t)}{D}$ equation(16) can be written as

$x(t) = x(t_0) + \frac{q(t)}{Q_D}$

Thus, linear drift model can be applied only for current controlled Memristive system.

2. Nonlinear Ion Dopant Drift Model

After experimentation of memristor device fabrication it was observed that the linear ion dopant drift model was significantly deviating.for this reason, a model based on experimental results suggested a relationship between current and voltage as in equation(17)[4][13].

$i(t) = w(t)^n\, \beta \sinh(\alpha\, v(t)) + \delta[\exp(\gamma\, v(t)) - 1]$(17)

Where $\alpha, \beta, \gamma, \delta$ are parameters used in experiment and nis the parameter that shows the influence of the state variable on the current[14]. This model assumes an asymmetrical switching behavior and a voltage controlled memristor exhibits a nonlinear dependence on voltage in the state variable differential equation as given in equation (18)[4].

$$\frac{dw(t)}{dt} = a\,.\,f(w).\,v(t)^m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(18)$$

Where a and m are constants , m is an odd integer, and $f(w)$is a window function.In this model, the state variable w is a normalized parameter within the interval (0,1).Introduction of window function is a significant feature of nonlinear ion dopant drift model due to boundary effects. The comparative study of different memristor models summarized in Table 5[13].

Parameters	Linear Dopant Drift Model	Non Linear Dopant Drift Model
Type Of Memristor Used For Control Mechanism	Current Controlled Memristor	Voltage Controlled Memristor
State Variable (Interval)	$0 \leq W \leq D$	$0 \leq W \leq 1$
Region With Width	Doped Region With Physical Width	Doped Region With Normalized Width

Table 5: Comparison of Different Memristor Models

CONCLUSION

In this paper we have discussed basic structure of the SPICE model of memristor and have summarized the port and state equations of memristive system with fundamental equations of memristor. We have discussed CCMS and VCMS memristive system with the aspect of voltage and current source at port side. Linear ion dopant drift model and nonlinear ion dopant drift model are discussed with equation conclude that linear dopant drift model can be applicable for CCMS and nonlinear ion dopant drift model can be applicable foe VCMS.

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