AMS 231: Nonlinear Control Theory: Winter 2018

Homework #5

Name: ....

Due: March 06, 2018

NOTE: Please show all the steps in your solution. Turn in a hard copy of your HW stapled with this as cover sheet with your name written in the above field. Please submit your HW in class on the due date.

Problem 1

Stabilizing Controllers **(**$10+10+(5+5)+10+(5+5)+10+10 = 70$ **points)**

Consider simple scalar control system

$$\dot{x} = -x^3 + u, \qquad x, u \in \mathbb{R}.$$

We want to design (static) state feedback control $u = u(x)$ such that origin of the closed-loop system is G.A.S. We will design multiple stabilizing controllers for this system, and compare their performance.

(a) Design a **feedback linearizing controller** $u_{\mathrm{FL}}(x)$ by applying "cancel the nonlinearity and get a stable linear closed-loop system" idea.

(b) Prove that a **linear feedback controller** $u_{\mathrm{L}}(x) = -x$ makes the origin of the closed-loop system G.A.S. (Hint: use $V(x) = \frac{1}{2}x^2$ and the Barbashin-Krasovskii theorem.)

(c) Give two reasons why the controller $u_{\mathrm{L}}(x)$ in part (b) is a better controller than $u_{\mathrm{FL}}(x)$ in part (a). (Hint: think rate-of-convergence of the closed-loop system, and magnitude of control signal for large x.)

(d) The answer in part (c) tells us that it is better not to kill "friendly nonlinearity". Consider another design idea: **doing nothing controller**, i.e., $u_0(x) \equiv 0$ for all $x \in \mathbb{R}$. Prove that $u_0(x)$ also makes the origin G.A.S.

(e) Give one advantage and one disadvantage of $u_0(x)$ compared to $u_{\mathrm{L}}(x)$. (Hint: again think in terms of the hint in part (c)).

(f) Design another stabilizing controller $u_{\mathrm{S}}(x)$ using **Sontag's formula**. (Hint: use the Lyapunov function in part (b) as the CLF.)

(g) From your answer in part (f), argue that near $x = 0$, we have $u_{\mathrm{S}}(x) \approx u_{\mathrm{L}}(x)$; and for $|x| \to \infty$, we have $u_{\mathrm{S}}(x) \approx u_0(x)$, and therefore, $u_{\mathrm{S}}(x)$ outperforms all the previous controllers.

Problem 2

Integrator Backstepping (30 **points**)

Consider the following 3 state control system which is a modification of the worked out example in Lecture 16 with an additional integrator at the input side:

$$\begin{aligned}
\dot{x}_1 &= x_1^2 - x_1^3 + x_2, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= u.
\end{aligned}$$

Design an integrator backstepping controller to make the origin G.A.S.