AMS 231: Nonlinear Control Theory: Winter 2018

Homework #6

Name: ....

Due: March 15, 2018

NOTE: Please show all the steps in your solution. Turn in a hard copy of your HW stapled with this as cover sheet with your name written in the above field. Please submit your HW in class on the due date.

Problem 1

Feedback Linearization **($20+20+20+20+(10+2+3)+5 = 100$ points)**

In this exercise, you will apply the Theorem (pg. 14) and step-by-step recipe (pg. 15–16) for feedback linearization given in Lecture 18 notes. Consider the nonlinear system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} x_3\,(1+x_2) \\ x_1 \\ x_2\,(1+x_1) \end{pmatrix} + \begin{pmatrix} 0 \\ 1+x_2 \\ -x_3 \end{pmatrix} u, \qquad \underline{x} \in \mathbb{R}^3, \quad u \in \mathbb{R}. \tag{1}$$

(a) Prove that the system is locally feedback linearizable around $\underline{x} = \underline{0}$. (Hint: see Step 1 in pg. 15, Lecture 18 notes.)

(b) Show that a solution for $\lambda(\underline{x})$ in the Theorem (pg. 14, Lecture 18 notes) is given by $\lambda(\underline{x}) = x_1$. (Hint: Notice that Step 2 in pg. 15, Lecture 18 notes gives a system of $n-1$ first order PDEs, and one PDE not-equal-to-zero condition, where $\underline{x} \in \mathbb{R}^n$. You will need to consider all of them simultaneously. Also, the solution for $\lambda(\underline{x})$ for this PDE system is non-unique – which is a good thing since our Theorem only requires existence but there can be multiple feedback linearizing controllers corresponding to different admissible $\lambda(\underline{x})$.)

(c) By directly computing relative degree, prove that the state equation (1) above, augmented with the output equation $y = \lambda(\underline{x}) = x_1$, indeed has relative degree 3 (that is, satisfies $r = n$ condition) at the point $\underline{x} = \underline{0}$.

(d) Use your answer in part (b), to compute the feedback linearizing transformation tuple $(\underline{\tau}(\cdot), \alpha(\cdot), \beta(\cdot))$. (Hint: use steps 3 and 4 in pg. 16, Lecture 18 notes.)

(e) Show that the Jacobian of $\underline{\tau}$ is non-singular at $\underline{x} = \underline{0}$. What does it mean in terms of feedback linearization? Submit a plot of the surface of the form $x_3 = \phi(x_1, x_2)$ where the Jacobian of $\underline{\tau}(\cdot)$ is singular.

(f) Clearly write down the feedback linearized control system in new co-ordinates with state $\underline{z}$ and control v, where $\underline{z} = \underline{\tau}(\underline{x})$ and $u = \alpha(\underline{x}) + \beta(\underline{x})v$.