Structure Preserving Power Systems Model

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Dynamics

We consider a power network with $n > 1$ generators, where the ith generator's dynamics is modeled as

$$\dot{\theta}_i = \omega_i, \tag{1a}$$

$$m_i\dot{\omega}_i = P_i - \gamma_i\omega_i - \sum_{j=1}^{n} k_{ij}\sin(\theta_i - \theta_j - \varphi_{ij}). \tag{1b}$$

The state variables are the **rotor angles** $\theta_i \in \mathbb{S}^1 \equiv [0, 2\pi)$, and the **rotor angular velocities** $\omega_i \in \mathbb{R}$ for $i = 1, ..., n$. Recall that the n-torus $\mathbb{T}^n = \underbrace{\mathbb{S}^1 \times \ldots \times \mathbb{S}^1}_{n \text{ times}}$. Thus the state space for (1) is $\mathbb{T}^n \times \mathbb{R}^n$.

Parameters

The parameters $m_i > 0, \gamma_i > 0$ respectively denote the inertia and damping coefficient for the ith generator.

The other three parameters: the **effective power input** P_i, the **phase shift** $\varphi_{ij} \in [0, \frac{\pi}{2})$, and the **coupling** $k_{ij} \geq 0$, depend on the network reduced admittance matrix $Y \in \mathbb{C}^{n\times n}$.

Specifically,

$$P_i = P_i^{\text{mech}} - E_i^2 \Re\left(Y_{ii}\right), \tag{2a}$$

$$\varphi_{ij} = \begin{cases} -\arctan\left(\dfrac{\Re\left(Y_{ij}\right)}{\Im\left(Y_{ij}\right)}\right), & \text{if } i \neq j, \\ 0, & \text{otherwise}, \end{cases} \tag{2b}$$

$$k_{ij} = \begin{cases} E_iE_j|Y_{ij}|, & \text{if } i \neq j, \\ 0, & \text{otherwise}, \end{cases} \tag{2c}$$

where P_i^{mech} is the mechanical power input, and E_i is the internal voltage (magnitude) for generator i.

What we need

The IEEE 14 bus has 5 generators. We want $m_i, \gamma_i, P_i^{\text{mech}}, E_i$ and the 5×5 admittance matrix Y for the IEEE 14 bus test system.

1 Generator Parameters

Important notes:

- **angles θ and rotor speeds ω must be in rad (NOT deg).**

Table 1: Generator Parameters for GENCLS Model ($\omega_0 = 2\pi \times 60$Hz)

Generator	Bus	H	D	$m = 2\omega_0 H$	$\gamma = \omega_0 D$
1	1	2.64	4.00	1.9905×10^3	1.5080×10^3
2	2	2.61	4.00	1.9679×10^3	1.5080×10^3
3	3	3.42	4.00	2.5786×10^3	1.5080×10^3
4	6	2.45	4.00	1.8473×10^3	1.5080×10^3
5	8	3.59	4.00	2.7068×10^3	1.5080×10^3

- **everything is in per unit (p.u)**. For example, $P_{\text{elec}} = 0.8$p.u. with baseMVA $= 100$ means $P_{\text{elec}} = 80$MVA.

Question: where is the voltage magnitude $|V_i|$ of bus i? (1) only has angles.

GENCLS Model:

$$\dot{\theta} = \omega_0 \omega \tag{3a}$$

$$\dot{\omega} = \frac{1}{2H}(P_{\text{mech}} - D\omega - P_{\text{elec}}) \tag{3b}$$

Comparing GENCLS model with (1):

$$m = 2\omega_0 H \tag{4a}$$

$$\gamma = \omega_0 D \tag{4b}$$

Other parameters are in CSV files

case14.m Matpower casefile,

get_parameters_case14.m Matlab script to get all scripts and write into csv files

gen_parameters.csv everything in Table 1

bus-init.csv about initial values of θ_i

line-parameters.csv everything about lines, i, j, k_{ij}, ϕ_{ij}

When simulating line failures, you can remove any one of 20 lines of the 14-bus system (any line in line-parameters.csv).

2 Load Modeling

The most commonly used load modeling is ZIP load:

- Z: constant impedance Z
- I: constant current I
- P: constant power P

So given the voltage at bus i, the real load is calculated as

$$P_i^{\text{load}} = P_i + \text{Real}(V_i I_i^\dagger) + \text{Real}(V_i * (V_i/Z_i)^\dagger) \tag{5}$$

Note we are ignoring reactive load (imaginary parts) in the swing equation.

2.1 Load Models in Kron Reduction

In Florian's slides, they are using constant current and impedance loads (see Figure on page 8/41).

- the constant impedance part is modeled as self-loops in Kron reduction (see Page 33/41), then integrated in the Y_{red} matrix
- the constant current load can be considered in the Kron reduction, more details below
- I believe Kron reduction has trouble dealing with constant power load, so Florian (and us) don't consider it (for load buses).

2.2 Constant Current Load

Assume all load at non-generator buses are modeled as constant current load, with current I_i $i \in \mathcal{L}$.

Original equations with constant current load

$$M_i\ddot{\theta} + D_i\dot{\theta}_i = P_{mech,i} - P_{load,i} - \sum_j |V_i||V_j||Y_{ij}|\sin(\theta_i - \theta_j + \phi_{ij}) \qquad i \in \mathcal{G} \text{ buses with generators} \tag{6a}$$

$$0 = -\text{Real}(V_i I_i^\dagger) - \sum_j |V_i||V_j||Y_{ij}|\sin(\theta_i - \theta_j + \phi_{ij}) \qquad i \in \mathcal{L} \text{ buses without generators} \tag{6b}$$

$$\tag{6c}$$

Note: $P_{mech,i} \in \mathbb{R}$, $P_{load,i} \in \mathbb{R}$ and $I_i \in \mathbb{C}$ are all give parameters; load current is a complex number.

After Kron reduction

$$Y_{red} = Y_{\mathcal{G}\mathcal{G}} - Y_{\mathcal{G}\mathcal{L}}Y_{\mathcal{L}\mathcal{L}}^{-1}Y_{\mathcal{L}\mathcal{G}} \tag{7}$$

$$I_{red} = Y_{\mathcal{G}\mathcal{L}}Y_{\mathcal{L}\mathcal{L}}^{-1}I \tag{8}$$

$$M_i\ddot{\theta} + D_i\dot{\theta}_i = P_i - \sum_{j=1, j\neq i}^{n} |V_i||V_j||Y_{red,ij}|\sin(\theta_i - \theta_j + \phi_{red,ij}) - Y, \tag{9}$$

$$i \in \mathcal{G} \text{ buses with generators} \tag{10}$$

where

$$P_i = P_{mech,i} - P_{load,i} - |V_i|^2\text{Real}(Y_{red,ii}) + |V_i||I_{red,i}|\cos(\theta_i - \alpha_i) \tag{11}$$

These files/parameters are given

gen_parameters.csv everything in Table 1

bus-init.csv about initial values of θ_i

line-parameters.csv everything about lines, i, j, k_{ij}, ϕ_{ij}

Additional:

case14.m Matpower casefile,

get_parameters_case14.m Matlab script to get all scripts and write into csv files