

Finite Horizon Optimal Density Regulation for Nonlinear Systems

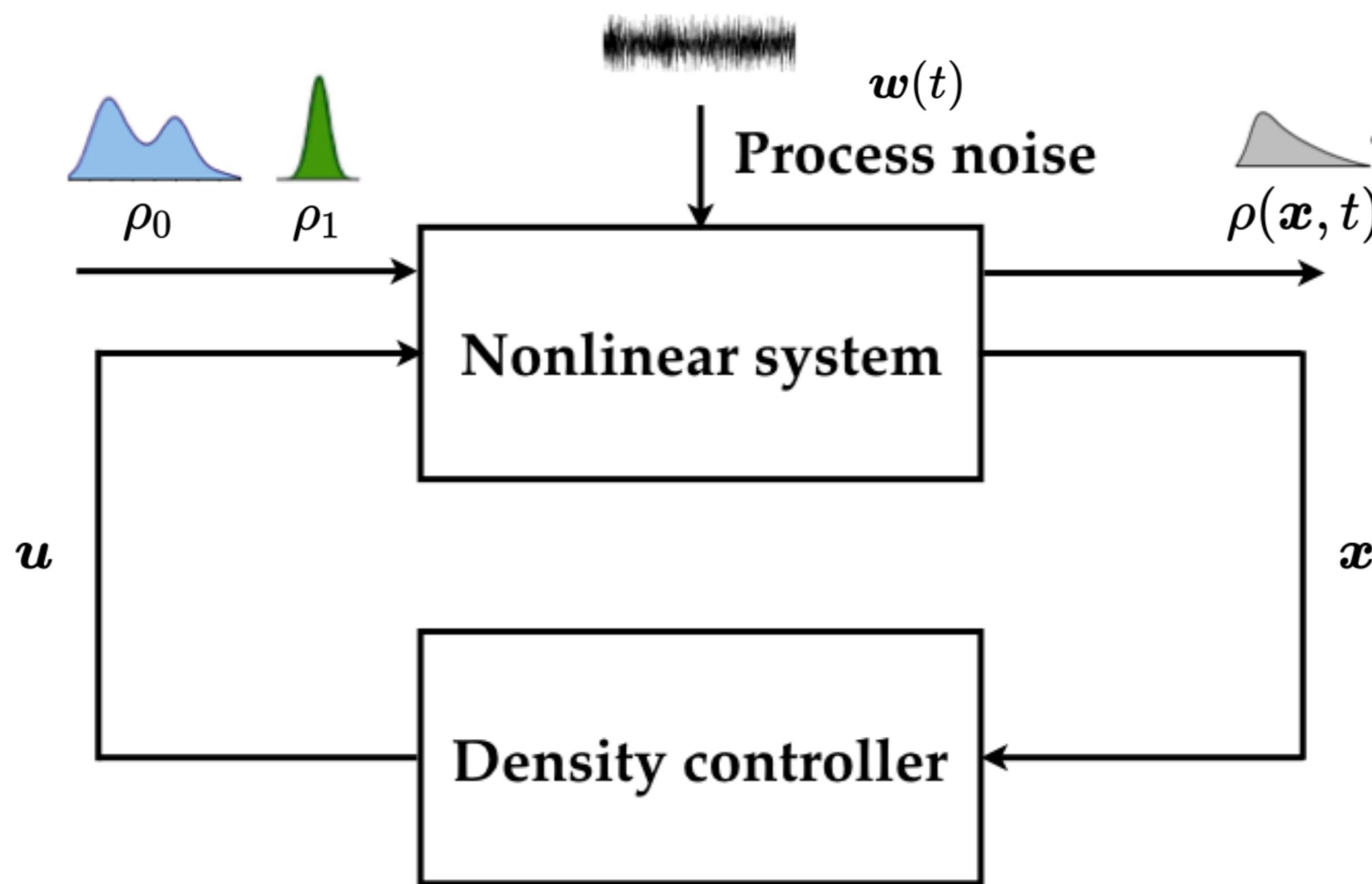
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Santa Cruz, CA 95064

Joint work with students and collaborators



Density Regulation via State Feedback



Motivating Applications

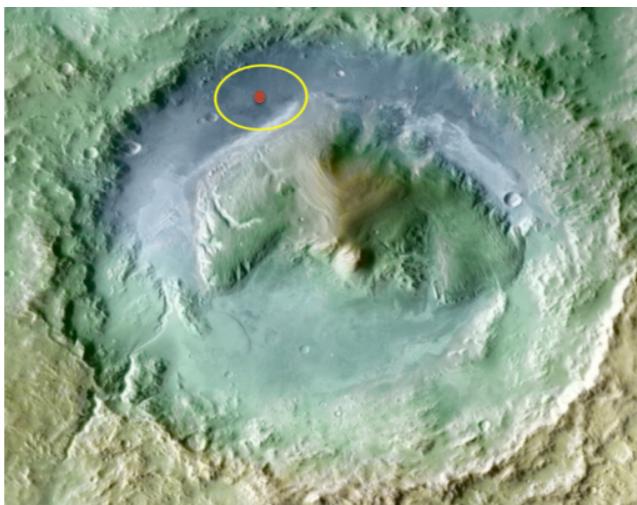
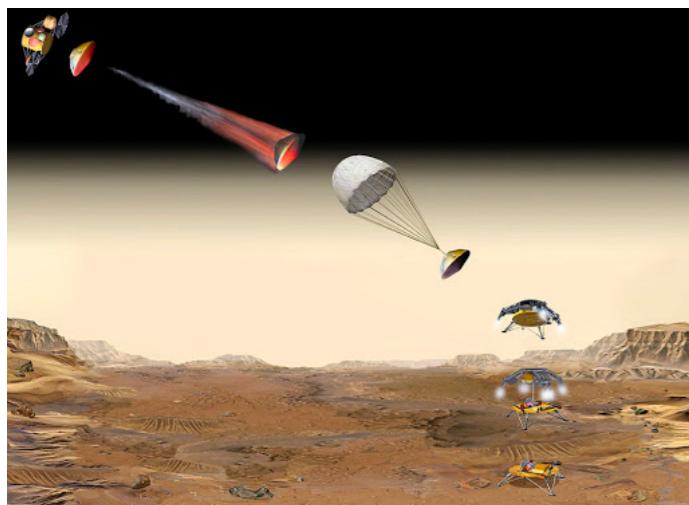
Distribution \sim Probability

Distribution \sim Population

Motivating Applications

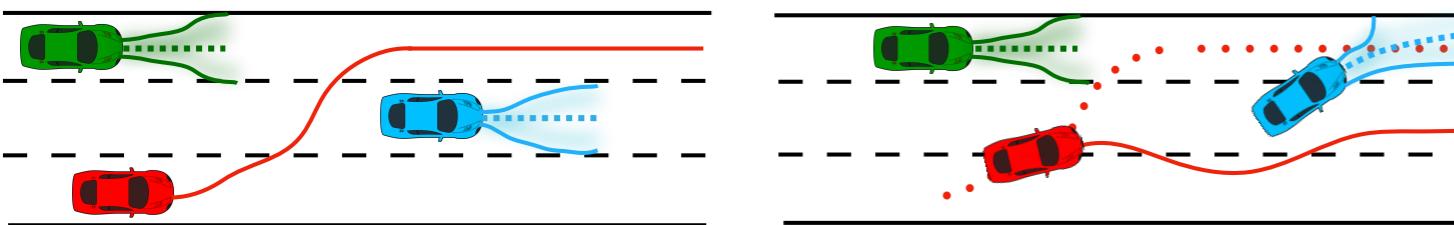
Distribution ~ Probability

Spacecraft landing with desired statistical accuracy



Distribution ~ Population

Risk management for automated driving in multi-lane highways

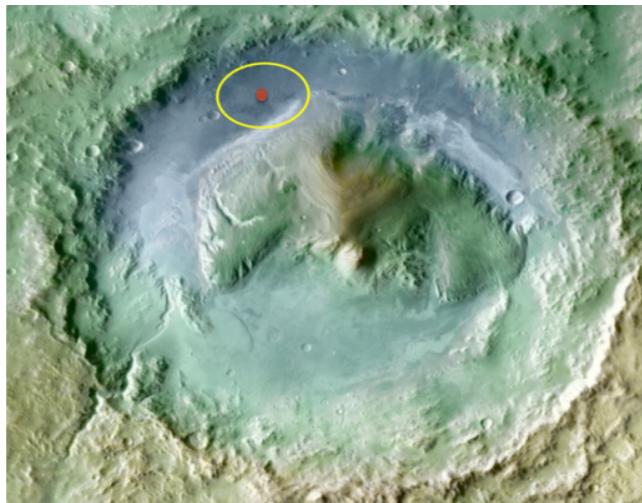
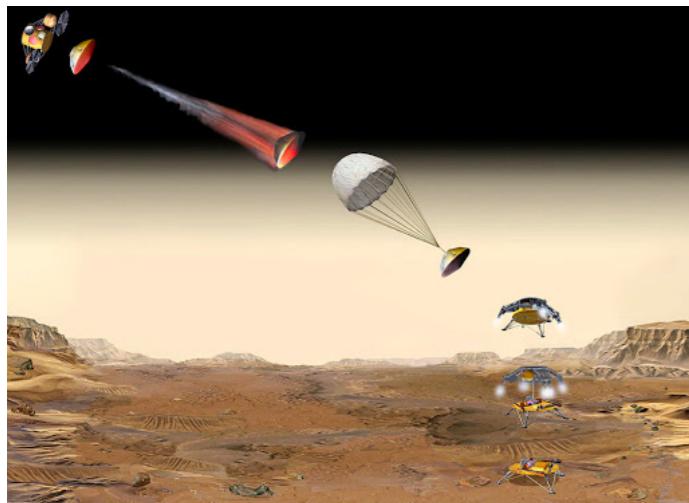


Control of uncertainties

Motivating Applications

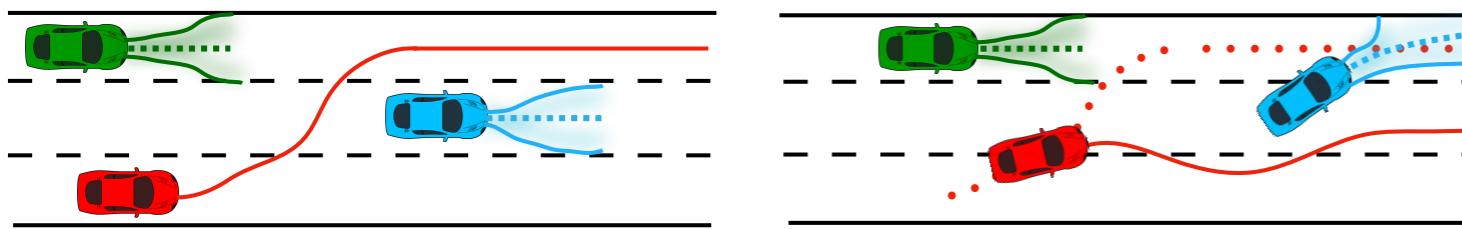
Distribution ~ Probability

Spacecraft landing with desired statistical accuracy



Gale Crater (4.49S, 137.42E)

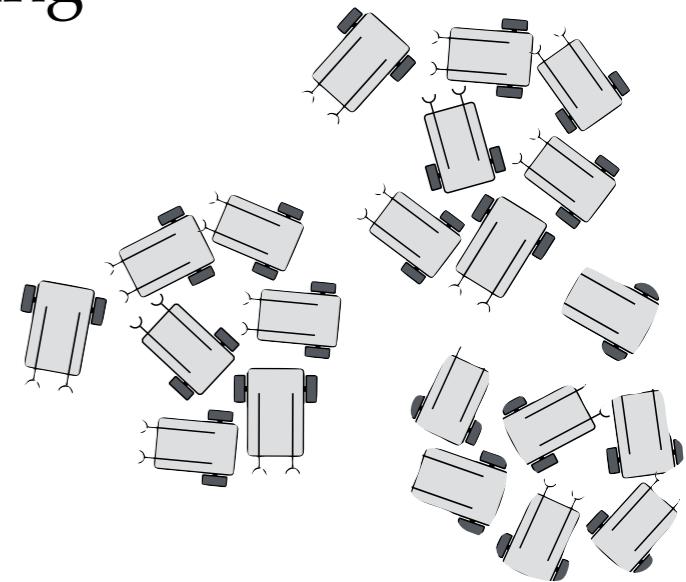
Risk management for automated driving in multi-lane highways



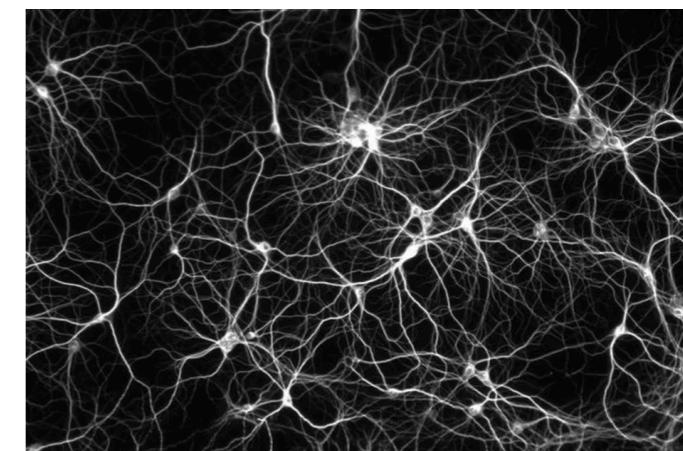
Control of uncertainties

Distribution ~ Population

Dynamic shaping of swarms



Feedback sync. and desync. of neuronal population



Control of ensemble

Growing Interest in Systems-Control

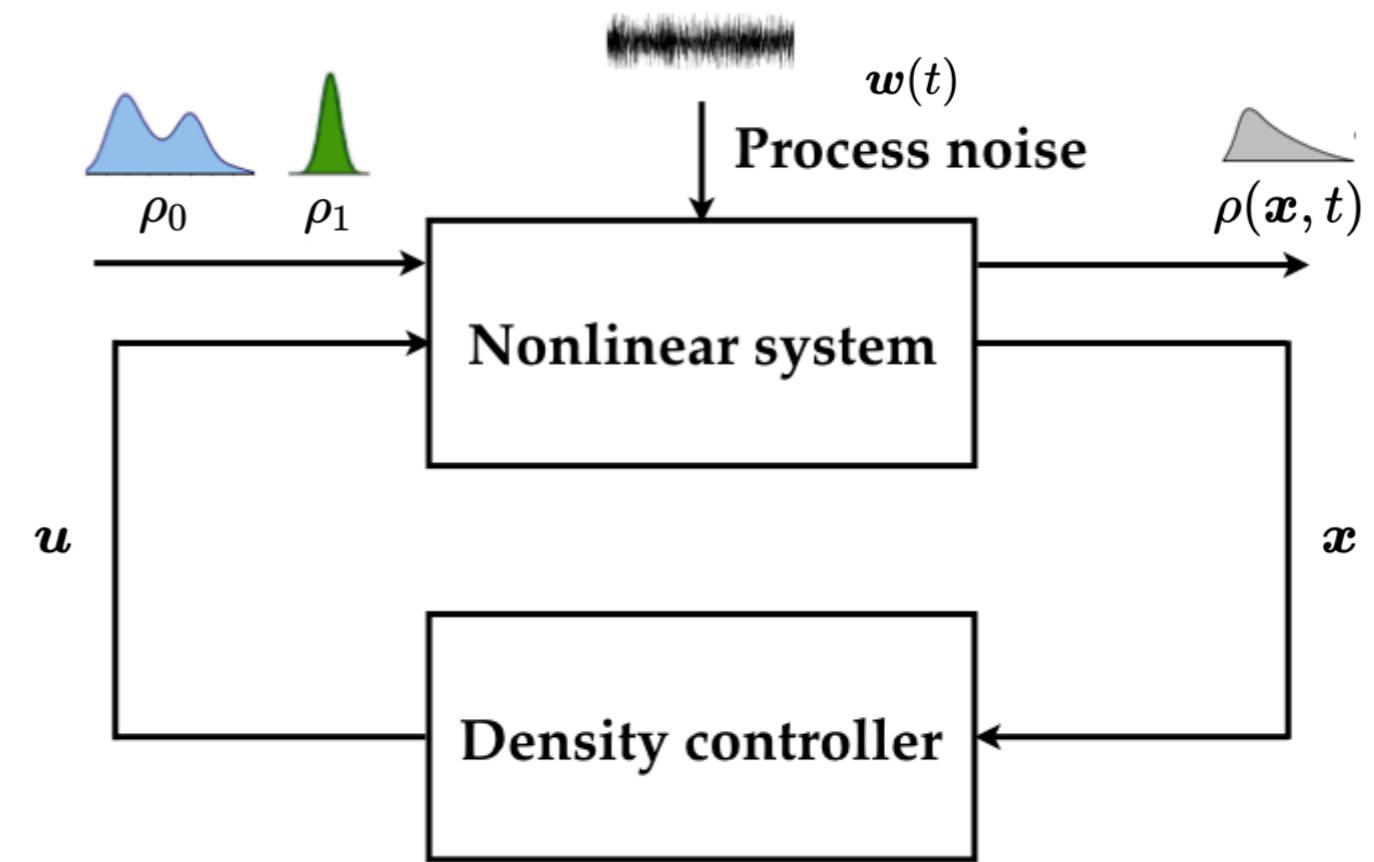
- Early literature in covariance control: Skelton et. al. [late '80s and early '90s]
- Finite horizon linear quadratic Gaussian steering: Chen-Georgiou-Pavon [2014-18]
- Input-constrained covariance steering: Bakolas [2018], Okamoto-Tsiotras [2019]
- Linear quadratic Gaussian steering with terminal cost: Wendel-Halder [2016], Balci-Bakolas [2020], Balci-Halder-Bakolas [2021]
- State constraints: soft probabilistic: Tsiotras et. al. [2018-20], hard deterministic: Caluya-Halder [2021]

Growing Interest in Systems-Control (contd.)

- **Density steering with nonlinear drift:** Chen-Georgiou-Pavon [2015], Caluya-Halder [2020-22], Nodozi-Halder [2022]
- **Application to automated driving:** Haddad-Caluya-Halder-Singh [2021]
- **Application to self-assembly:** Nodozi-O'Leary-Mesbah-Halder [2022]

State Feedback Density Steering

Steer joint state PDF via feedback control over finite time horizon



Common scenario: $G \equiv B$

$$\underset{u \in \mathcal{U}}{\text{minimize}} \quad \mathbb{E} \left[\int_0^1 \left(\frac{1}{2} \|u(t, x_t^u)\|_2^2 + q(t, x_t^u) \right) dt \right]$$

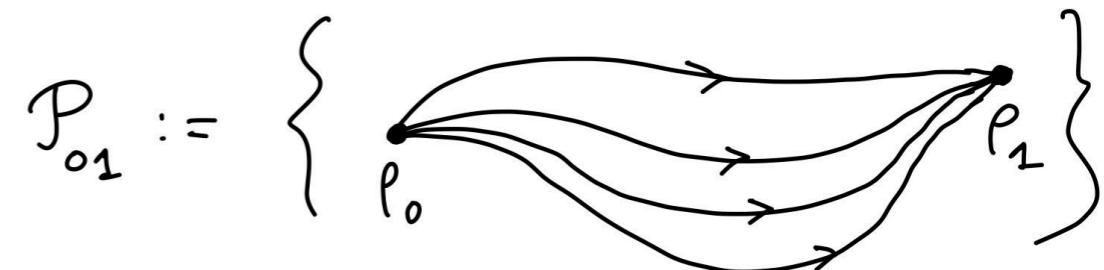
subject to

$$dx_t^u = \{f(t, x_t^u) + B(t, x_t^u)u\}dt + \sqrt{2}G(t, x_t^u)dw_t$$

$$x_0^u := x_t^u(t=0) \sim \rho_0, \quad x_1^u := x_t^u(t=1) \sim \rho_1$$

Optimal Control Problem over PDFs

Diffusion tensor: $D := GG^\top$



Hessian operator w.r.t. state: Hess

$$\inf_{(\rho,u) \in \mathcal{P}_{01} \times \mathcal{U}} \int_{\mathbb{R}^n} \int_0^1 \left(\frac{1}{2} \|u(t, x_t^u)\|_2^2 + q(t, x_t^u) \right) \rho(t, x_t^u) \, dt \, dx_t^u$$

subject to

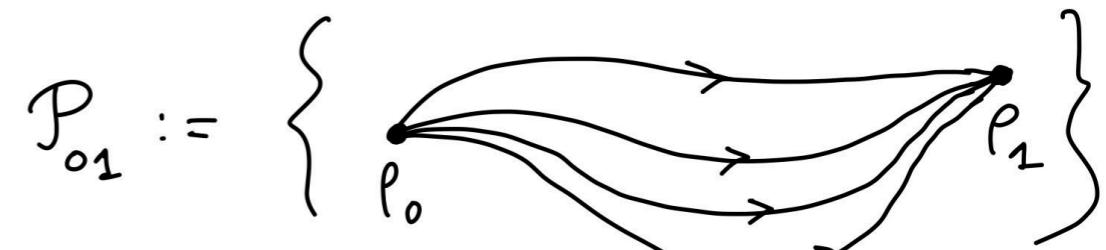
$$\frac{\partial \rho}{\partial t} + \nabla \cdot ((f + Bu) \rho) = \langle \text{Hess}, D\rho \rangle$$

$$\rho(t=0, x_0^u) = \rho_0, \quad \rho(t=1, x_1^u) = \rho_1$$

Controlled Fokker-Planck or Kolmogorov's forward PDE

Zero process noise \rightsquigarrow Optimal mass transport

Dynamic optimal mass transport
with prior dynamics f



$$\inf_{(\rho,u) \in \mathcal{P}_{01} \times \mathcal{U}} \int_{\mathbb{R}^n} \int_0^1 \left(\frac{1}{2} \|u(t, x_t^u)\|_2^2 + q(t, x_t^u) \right) \rho(t, x_t^u) \, dt \, dx_t^u$$

subject to

$$\frac{\partial \rho}{\partial t} + \nabla \cdot ((f + Bu) \rho) = \cancel{\langle \text{Hess}, D\rho \rangle} \quad 0$$

$$\rho(t=0, x_0^u) = \rho_0, \quad \rho(t=1, x_1^u) = \rho_1$$

Controlled Liouville PDE

Optimal Control Problem over PDFs

Existence-uniqueness needs regularity assumptions

Are known to hold for many practical classes of nonlinearities

This talk: will focus on a few important classes

Necessary Conditions of Optimality (Assuming $G \equiv B$)

Coupled nonlinear PDEs + linear boundary conditions

Controlled Fokker-Planck or Kolmogorov's forward PDE

$$\frac{\partial \rho^{\text{opt}}}{\partial t} + \nabla \cdot ((f + D\nabla \psi) \rho^{\text{opt}}) = \langle \text{Hess}, D\rho \rangle$$

Hamilton-Jacobi-Bellman-like PDE

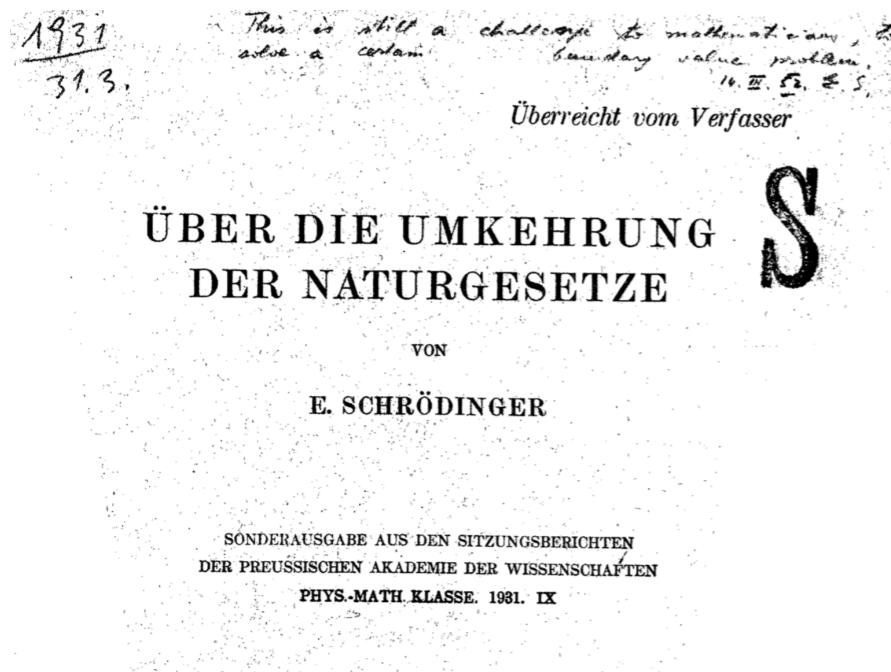
$$\frac{\partial \psi}{\partial t} + \langle \nabla \psi, f \rangle + \langle D, \text{Hess}(\psi) \rangle + \frac{1}{2} \langle \nabla \psi, D \nabla \psi \rangle = q$$

Boundary conditions:

$$\rho^{\text{opt}}(\cdot, t=0) = \rho_0, \quad \rho^{\text{opt}}(\cdot, t=1) = \rho_1$$

Optimal control: $u^{\text{opt}} = B^\top \nabla \psi$

Feedback Synthesis via the Schrödinger System



Sur la théorie relativiste de l'électron
et l'interprétation de la mécanique quantique

PAR

E. SCHRÖDINGER

I. — Introduction

J'ai l'intention d'exposer dans ces conférences diverses idées concernant la mécanique quantique et l'interprétation qu'on en donne généralement à l'heure actuelle ; je parlerai principalement de la théorie quantique relativiste du mouvement de l'électron. Autant que nous pouvons nous en rendre compte aujourd'hui, il semble à peu près sûr que la mécanique quantique de l'électron, sous sa forme idéale, que nous ne possédons pas encore, doit former un jour la base de toute la physique. A cet intérêt tout à fait général, s'ajoute, ici à Paris, un intérêt particulier : vous savez tous que les bases de la théorie moderne de l'électron ont été posées à Paris par votre célèbre compatriote Louis de BROGLIE.



Hopf-Cole a.k.a. Fleming's logarithmic transform:

$$(\rho^{\text{opt}}, \psi) \mapsto (\hat{\varphi}, \varphi) \quad \text{— Schrödinger factors}$$

$$\hat{\varphi}(x, t) = \rho^{\text{opt}}(x, t) \exp(-\psi(x, t))$$

$$\varphi(x, t) = \exp(\psi(x, t)) \quad \text{for all } (x, t) \in \mathbb{R}^n \times [0, 1]$$

Feedback Synthesis via the Schrödinger System

2 coupled nonlinear PDEs → boundary-coupled linear PDEs!!

Uncontrolled forward-backward Kolmogorov PDEs:

$$\frac{\partial \hat{\varphi}}{\partial t} = -\nabla \cdot (\hat{\varphi} f) + \langle \text{Hess}, D\hat{\varphi} \rangle - q\hat{\varphi}, \quad \hat{\varphi}_0 \varphi_0 = \rho_0,$$

$$\frac{\partial \varphi}{\partial t} = -\langle \nabla \varphi, f \rangle - \langle \text{Hess}(\varphi), D \rangle + q\varphi, \quad \hat{\varphi}_1 \varphi_1 = \rho_1,$$

Optimal controlled joint state PDF: $\rho^{\text{opt}}(x, t) = \hat{\varphi}(x, t)\varphi(x, t)$

Optimal control: $u^{\text{opt}}(x, t) = 2B^\top \nabla_x \log \varphi(x, t)$

Feedback Synthesis via the Schrödinger System

2 coupled nonlinear PDEs → boundary-coupled linear PDEs!!

Uncontrolled forward-backward Kolmogorov PDEs:

$$\frac{\partial \hat{\varphi}}{\partial t} = -\nabla \cdot (\hat{\varphi} f) + \langle \text{Hess}, D\hat{\varphi} \rangle - q\hat{\varphi}, \quad \hat{\varphi}_0 \varphi_0 = \rho_0,$$

$$\frac{\partial \varphi}{\partial t} = -\langle \nabla \varphi, f \rangle - \langle \text{Hess}(\varphi), D \rangle + q\varphi, \quad \hat{\varphi}_1 \varphi_1 = \rho_1,$$

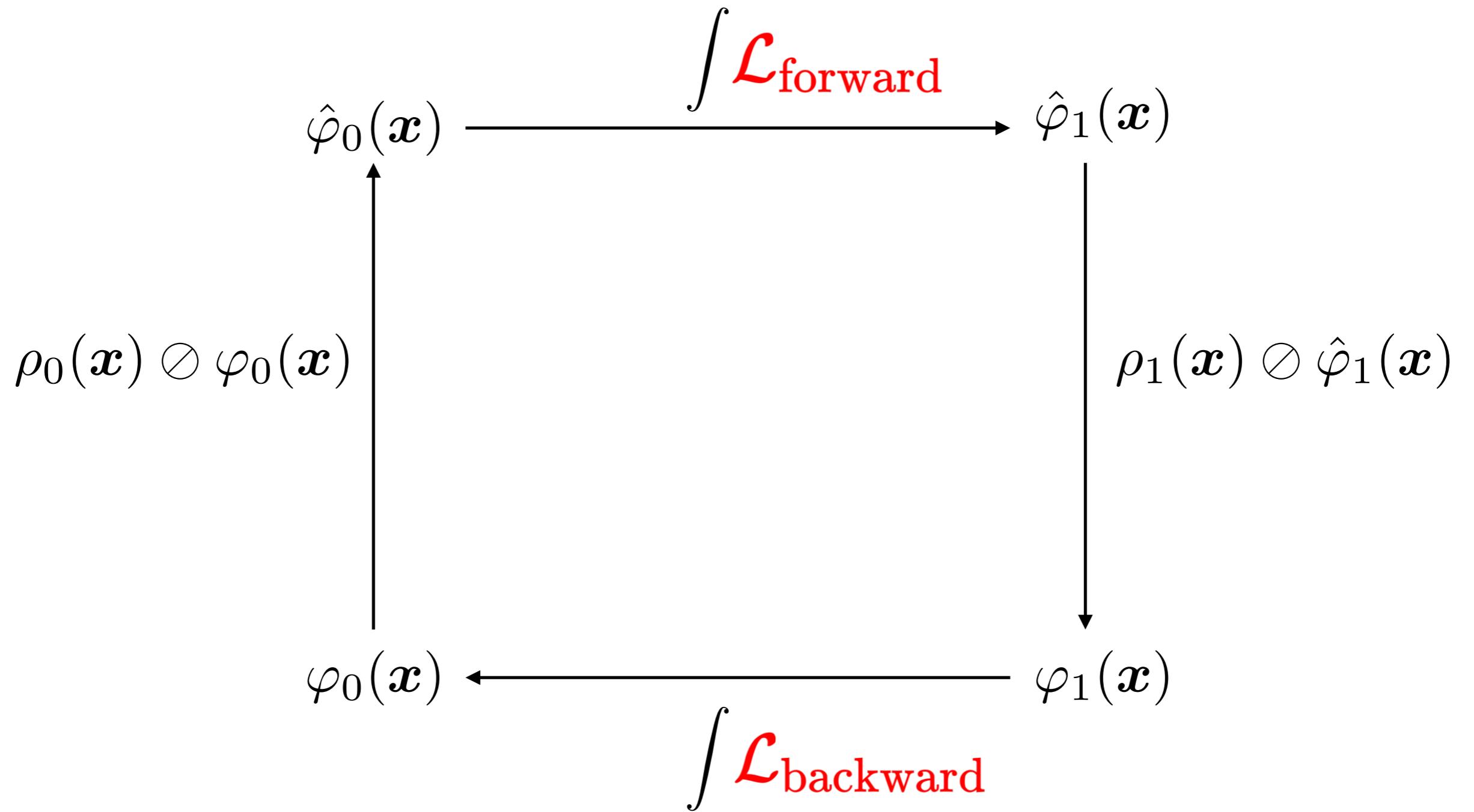
$\mathcal{L}_{\text{forward}} \hat{\varphi}$ $\mathcal{L}_{\text{backward}} \varphi$

Optimal controlled joint state PDF: $\rho^{\text{opt}}(x, t) = \hat{\varphi}(x, t)\varphi(x, t)$

Optimal control:

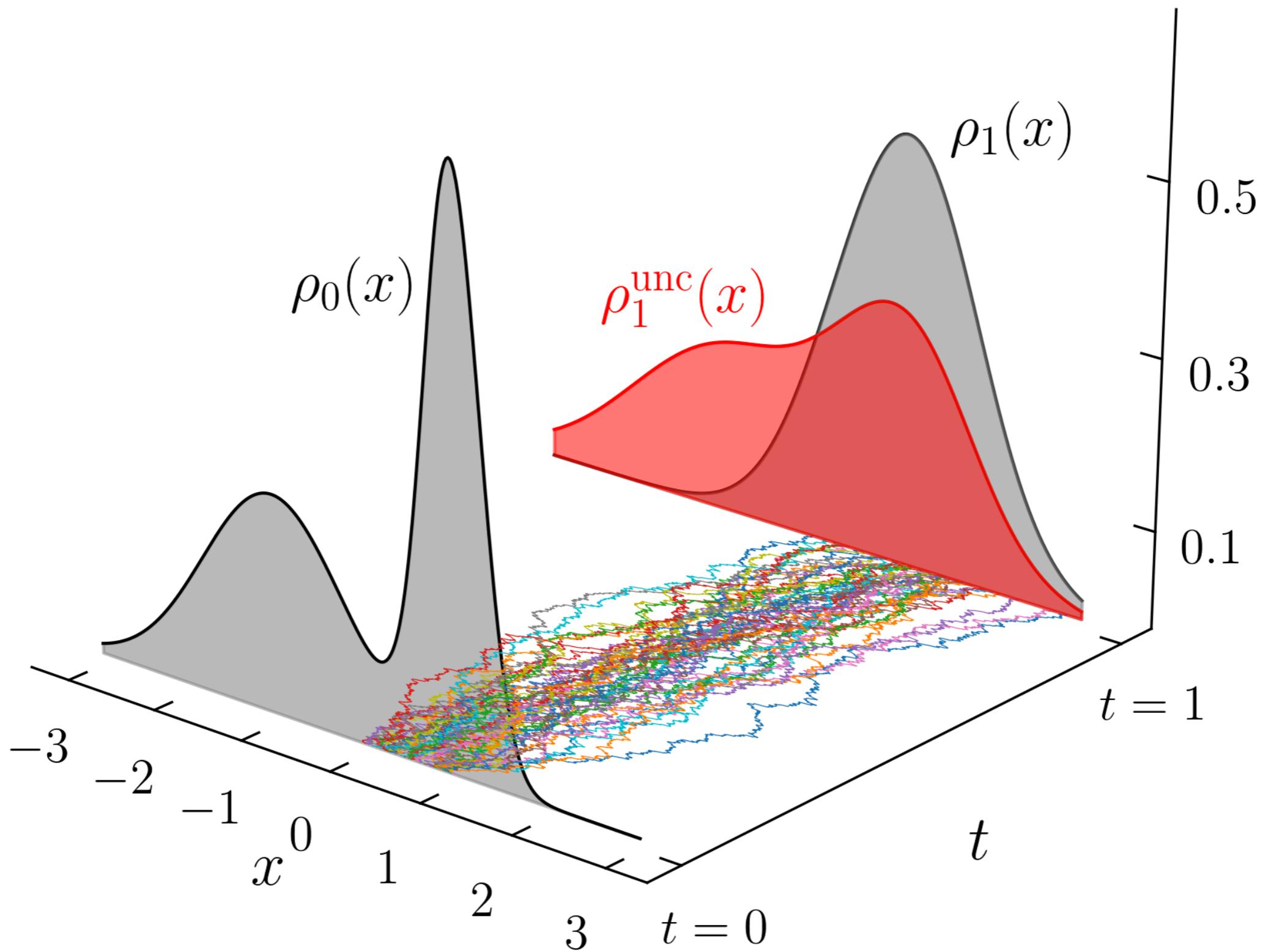
$$u^{\text{opt}}(x, t) = 2B^\top \nabla_x \log \varphi(x, t)$$

Fixed Point Recursion Over Pair $(\varphi_1, \hat{\varphi}_0)$



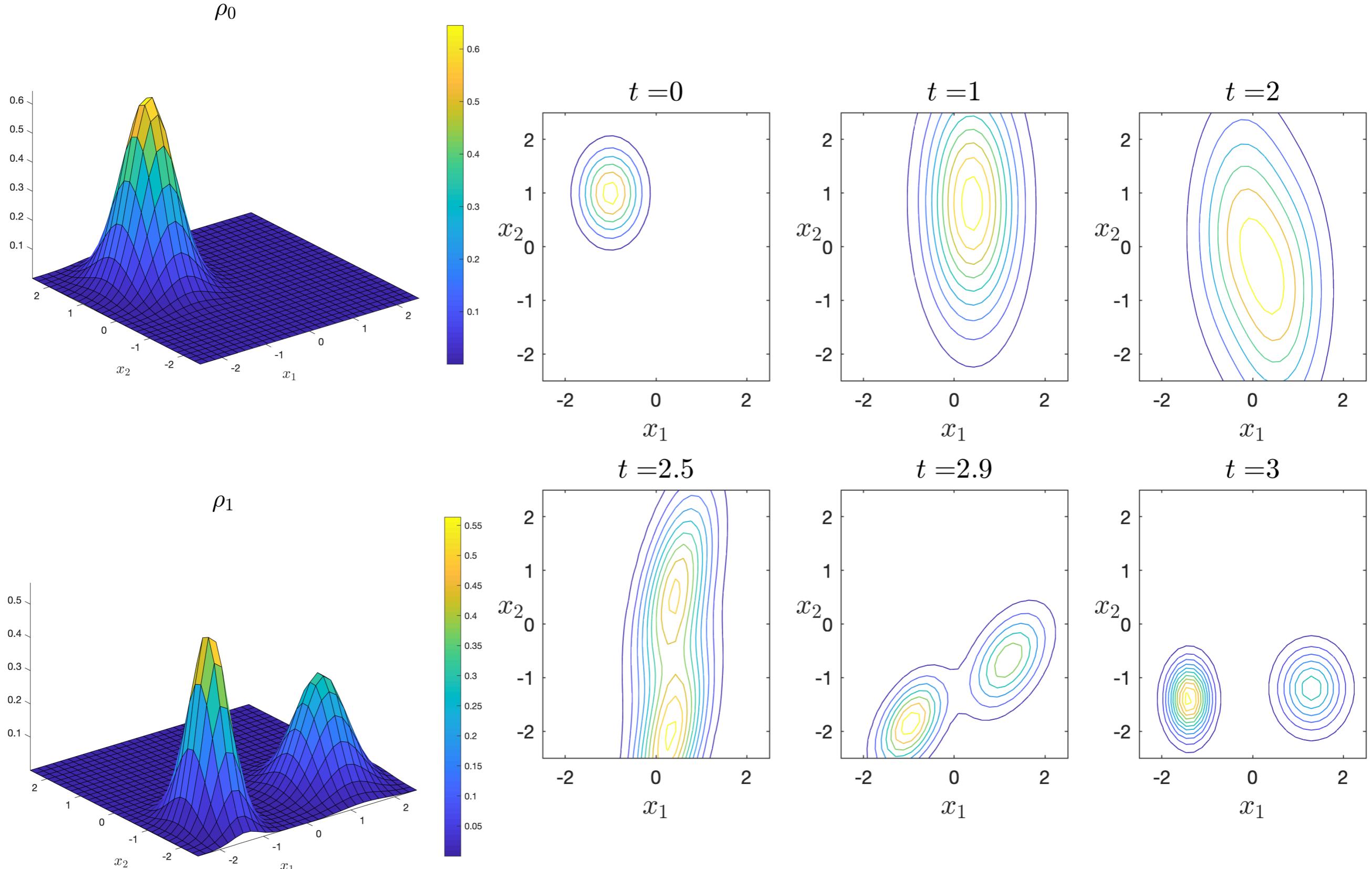
This recursion is contractive in the Hilbert metric!!

Feedback Density Control: $f \equiv 0, B = G \equiv I, q \equiv 0$



Zero prior dynamics

Feedback Density Control: $f \equiv Ax, B = G, q \equiv 0$



Linear prior dynamics

In general ...

Need (uncontrolled) forward AND backward
Kolmogorov solvers

Bad news: need two different solvers

Good news: sometimes one solver suffices!

Even better: it is possible to design generalized gradient flow
solvers based on point clouds!!

Brief Detour: Generalized Gradient Flow

PDE Initial Value Problem:

$$\frac{\partial \rho}{\partial t} = \mathcal{L}\rho, \quad \rho(\cdot, t = 0) = \rho_0$$



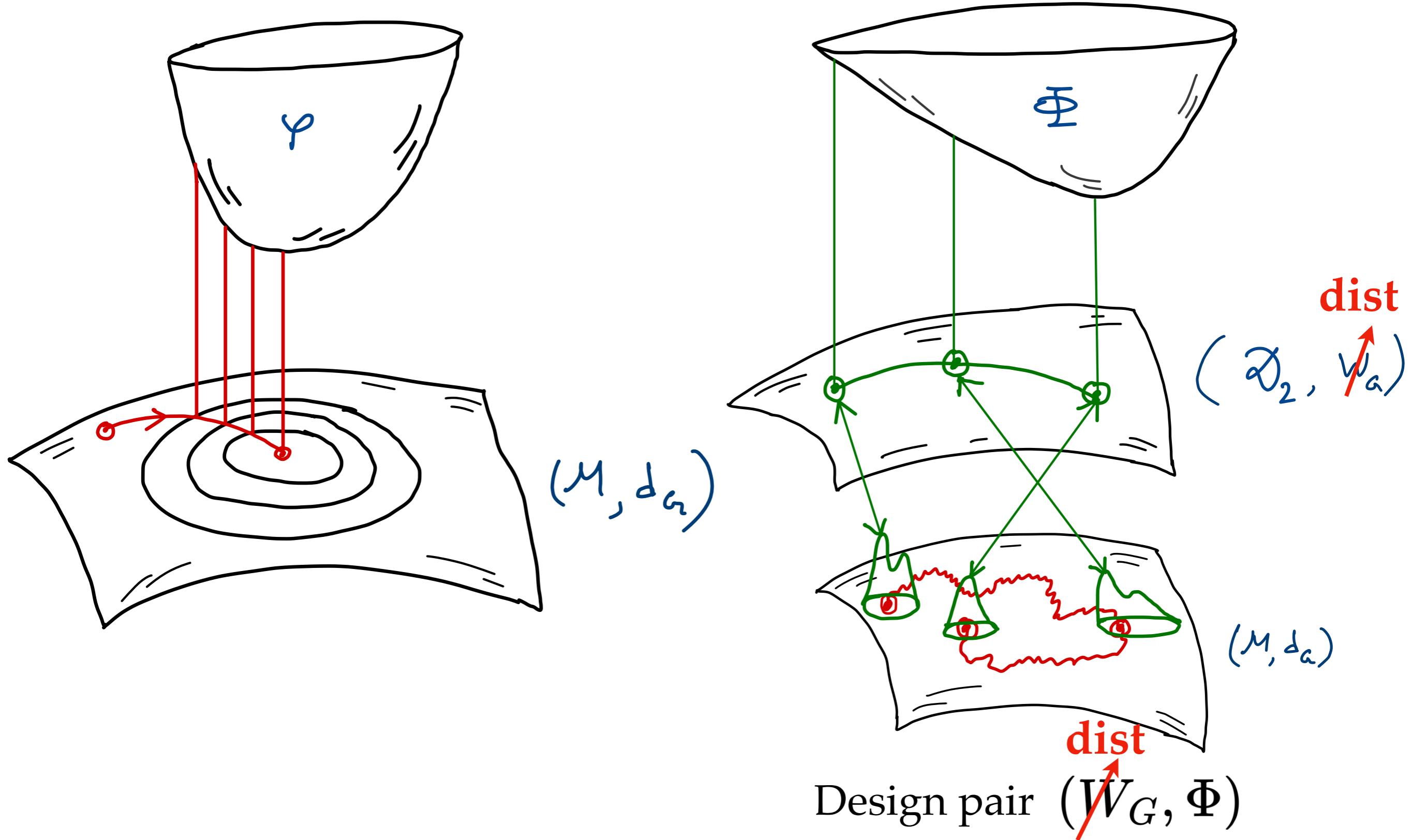
Proximal Recursion:

$$\varrho_k = \text{prox}_{\tau\Phi}^d(\varrho_{k-1}) := \arg \inf_{\varrho} \left\{ \frac{1}{2} \text{dist}^2(\varrho, \varrho_{k-1}) + \tau\Phi(\varrho) \right\}, \quad k \in \mathbb{N}$$

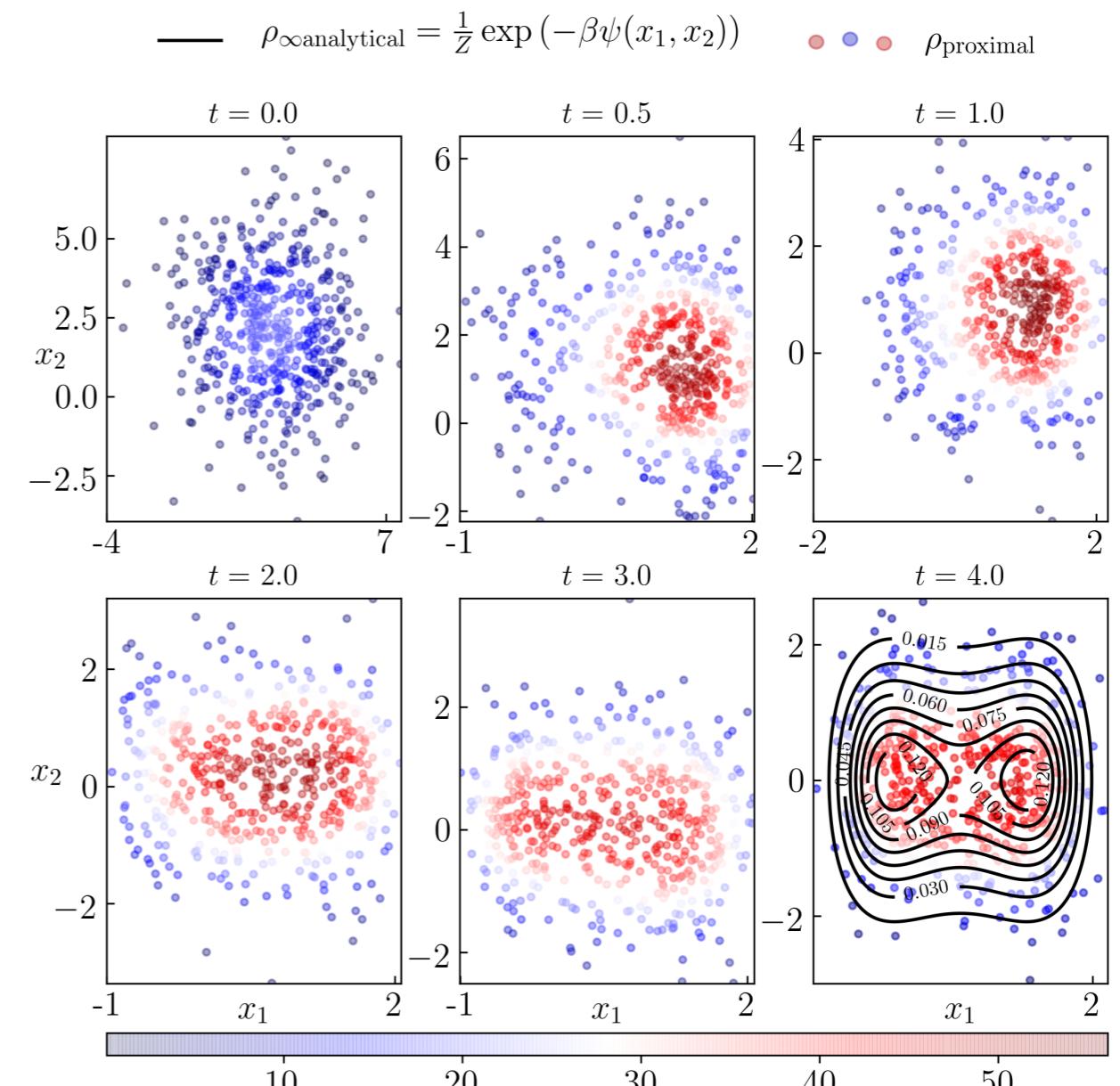
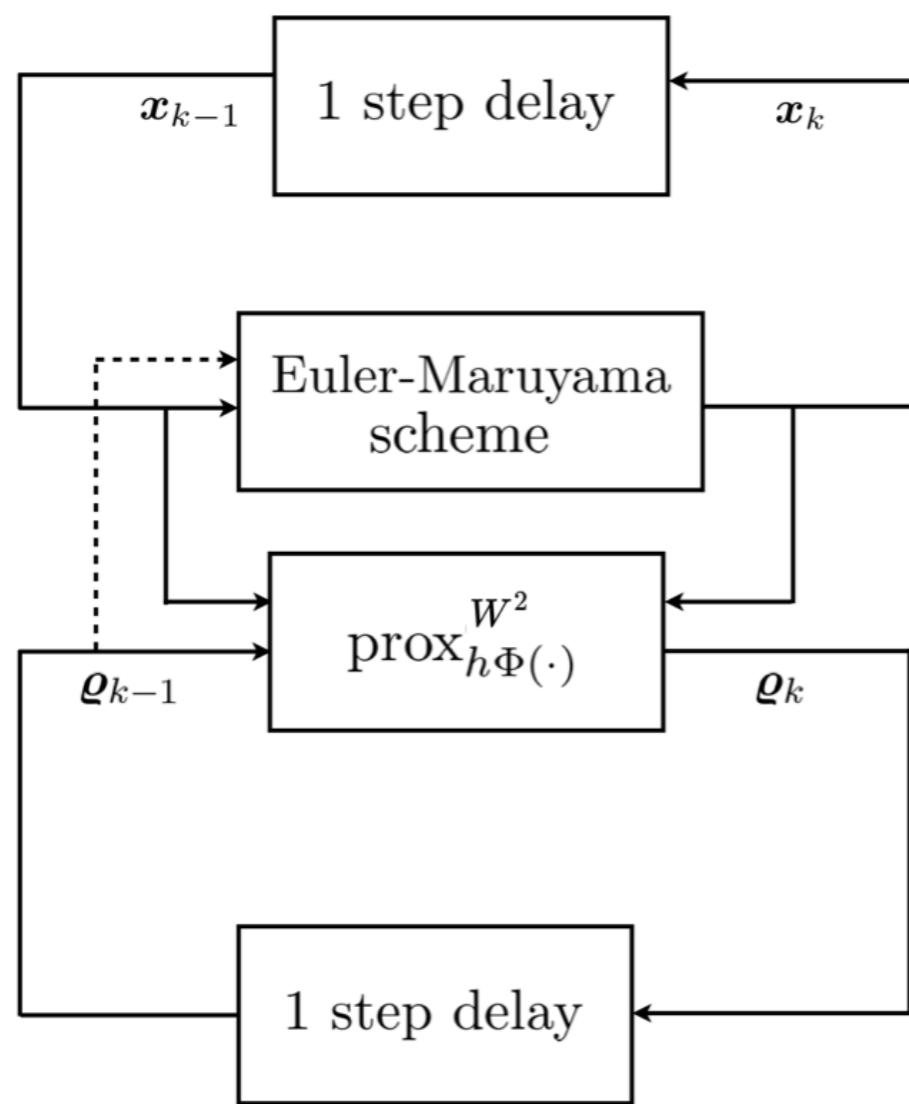
Given the operator \mathcal{L} , design the pair (dist, Φ) such that

$$\varrho_k \xrightarrow{\tau \downarrow 0} \rho(\cdot, t = k\tau)$$

Generalized Gradient Flow



Brief Detour: Generalized Gradient Flow



Details:

- K.F. Caluya, and A.H., Gradient flow algorithms for density propagation in stochastic systems, Vol. 65, No. 10, pp. 3991–4004, *IEEE Trans. Automatic Control*, 2019.
- K.F. Caluya, and A.H., Proximal recursion for solving the Fokker-Planck equation, ACC 2019.
- A.H., K.F. Caluya, B. Travacca, S.J. Moura, Hopfield neural network flow: a geometric viewpoint, Vol. 31, No. 11, pp. 4869–4880, *IEEE Trans. Neural Networks and Learning Systems*, 2020.

Single solver suffices for ...

Example: gradient drift

$$dx = \{-\nabla V(x) + u(x, t)\} dt + \sqrt{2\epsilon} dw$$

Assume: $x \in \mathbb{R}^n, V \in C^2(\mathbb{R}^n)$

Example: mixed conservative-dissipative drift

$$\begin{pmatrix} d\xi \\ d\eta \end{pmatrix} = \begin{pmatrix} \eta \\ -\nabla_\xi V(\xi) - \kappa\eta + u(x, t) \end{pmatrix} dt + \sqrt{2\epsilon\kappa} \begin{pmatrix} 0_{m \times m} \\ I_{m \times m} \end{pmatrix} dw$$

Assume: $\xi, \eta \in \mathbb{R}^m, x := (\xi, \eta)^\top \in \mathbb{R}^n, n = 2m, V \in C^2(\mathbb{R}^m), \inf V > -\infty, \text{Hess}(V) \text{ unif. bounded}$

Feedback Density Control: Gradient Drift, $q \equiv 0$

Theorem

For $t \in [0, 1]$, let $s := 1 - t$.

Define the change-of-variables $\varphi \mapsto q \mapsto p$ as

$$q(x, s) := \varphi(x, s) = \varphi(x, 1 - t),$$

$$p(x, s) := q(x, s) \exp(-V(x)/\epsilon).$$

Then the pair $(\hat{\varphi}, p)$ solves

$$\frac{\partial \hat{\varphi}}{\partial t} = \nabla \cdot (\hat{\varphi} \nabla V) + \epsilon \Delta \hat{\varphi}, \quad \hat{\varphi}(x, 0) = \hat{\varphi}_0(x),$$

$$\frac{\partial p}{\partial s} = \nabla \cdot (p \nabla V) + \epsilon \Delta p, \quad p(x, 0) = \varphi_1(x) \exp(-V(x)/\epsilon).$$

Feedback Density Control: Mixed Conservative-Dissipative Drift, $q \equiv 0$

Theorem

For $t \in [0, 1]$, let $s := 1 - t$. Also, let $\vartheta := -\eta$.

Define the change-of-variables $\varphi \mapsto q \mapsto \tilde{p} \mapsto p$ as

$$q(\xi, \eta, s) := \varphi(\xi, \eta, s) = \varphi(\xi, \eta, 1 - t),$$

$$\tilde{p}(\xi, -\eta, s) := q(\xi, \eta, s) \exp\left(-\frac{1}{\epsilon}\left(\frac{1}{2}\|\eta\|_2^2 + V(\xi)\right)\right),$$

$$p(\xi, \vartheta, s) := \tilde{p}(\xi, -\eta, s).$$

Then the pair $(\hat{\phi}, p)$ solves

$$\frac{\partial \hat{\phi}}{\partial t} = -\langle \eta, \nabla_\xi \hat{\phi} \rangle + \nabla_\eta \cdot (\hat{\phi} (\nabla_\xi V(\xi) + \kappa \eta)) + \epsilon \kappa \Delta_\eta \hat{\phi},$$

$$\frac{\partial p}{\partial s} = -\langle \vartheta, \nabla_\xi p \rangle + \nabla_\vartheta \cdot (p (\nabla_\xi V(\xi) + \kappa \vartheta)) + \epsilon \kappa \Delta_\vartheta p,$$

$$\hat{\phi}(\xi, \eta, 0) = \hat{\phi}_0(\xi, \eta),$$

$$p(\xi, \vartheta, 0) = \varphi_1(\xi, -\vartheta) \exp\left(-\frac{1}{\epsilon}\left(\frac{1}{2}\|\vartheta\|_2^2 + V(\xi)\right)\right).$$

Feedback Density Control via Wasserstein prox.

Design proximal recursions over discrete time pair:

$(t_{k-1}, s_{k-1}) := ((k-1)\tau, (k-1)\sigma)$, $k \in \mathbb{N}$, and τ, σ are step-sizes.

The recursions are of the form:

$$\begin{pmatrix} \hat{\phi}_{t_{k-1}} \\ \varpi_{s_{k-1}} \end{pmatrix} \mapsto \begin{pmatrix} \hat{\phi}_{t_k} \\ \varpi_{s_k} \end{pmatrix} := \begin{pmatrix} \arg \inf_{\hat{\phi} \in \mathcal{P}_2(\mathbb{R}^n)} \frac{1}{2} d^2(\hat{\phi}_{t_{k-1}}, \hat{\phi}) + \tau F(\hat{\phi}) \\ \arg \inf_{\varpi \in \mathcal{P}_2(\mathbb{R}^n)} \frac{1}{2} d^2(\varpi_{s_{k-1}}, \varpi) + \sigma F(\varpi) \end{pmatrix}$$

Consistency guarantees:

$$\hat{\phi}_{t_{k-1}}(x) \rightarrow \hat{\phi}(x, t = (k-1)\tau) \quad \text{in } L^1(\mathbb{R}^n) \quad \text{as } \tau \downarrow 0,$$

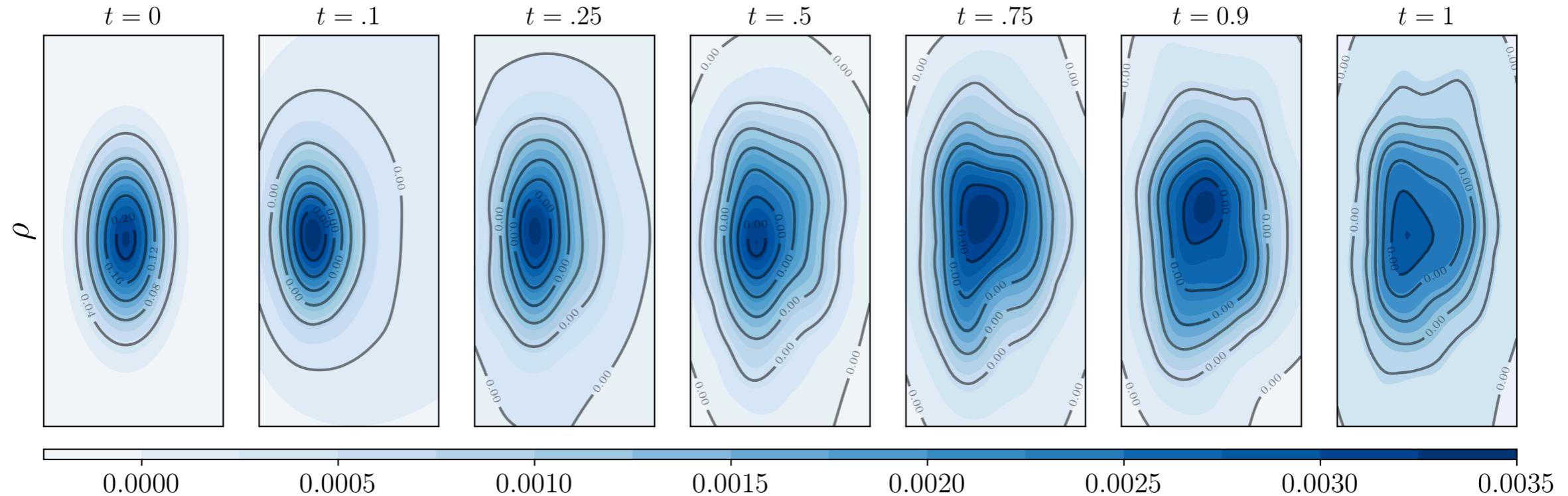
$$\varpi_{s_{k-1}}(x) \rightarrow p(x, s = (k-1)\sigma) \quad \text{in } L^1(\mathbb{R}^n) \quad \text{as } \sigma \downarrow 0.$$

Details:

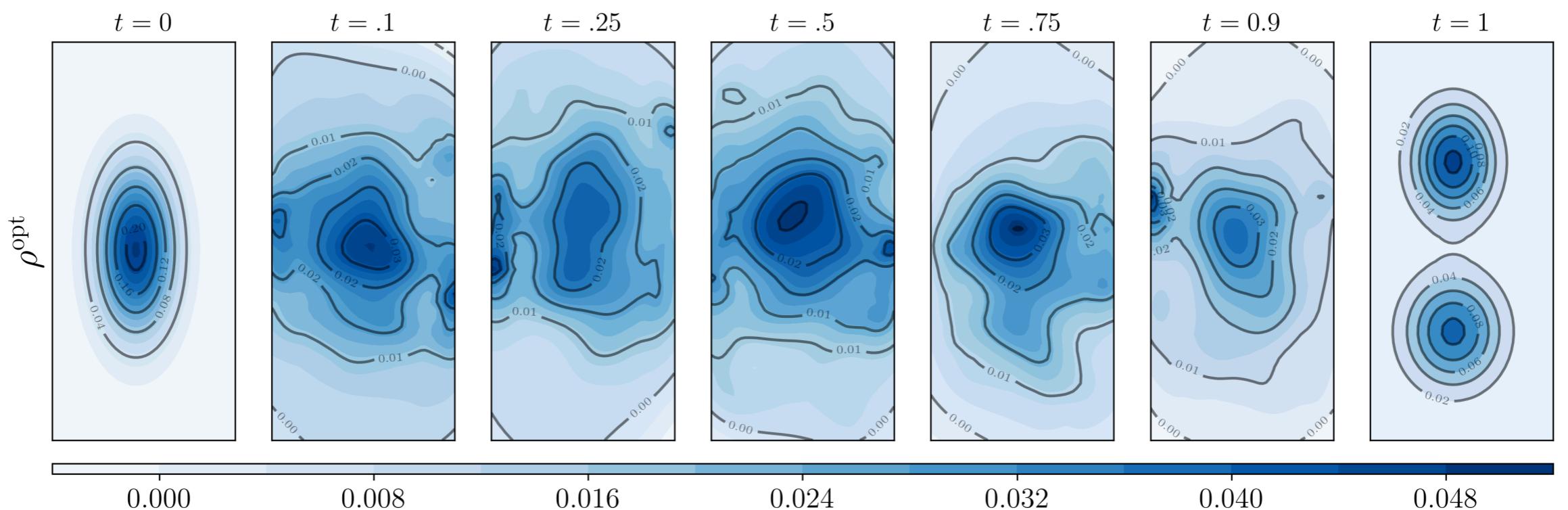
— K.F. Caluya, and A.H., Wasserstein Proximal Algorithms for the Schrödinger Bridge Problem: Density Control with Nonlinear Drift, *IEEE TAC* 2021.

Feedback Density Control: Gradient Drift

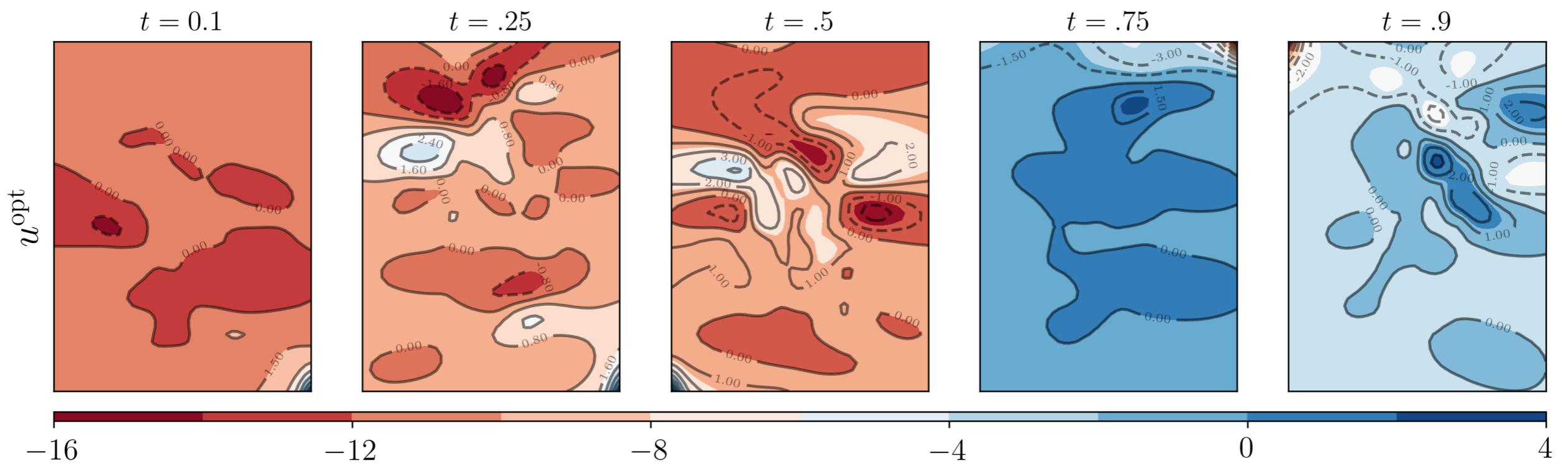
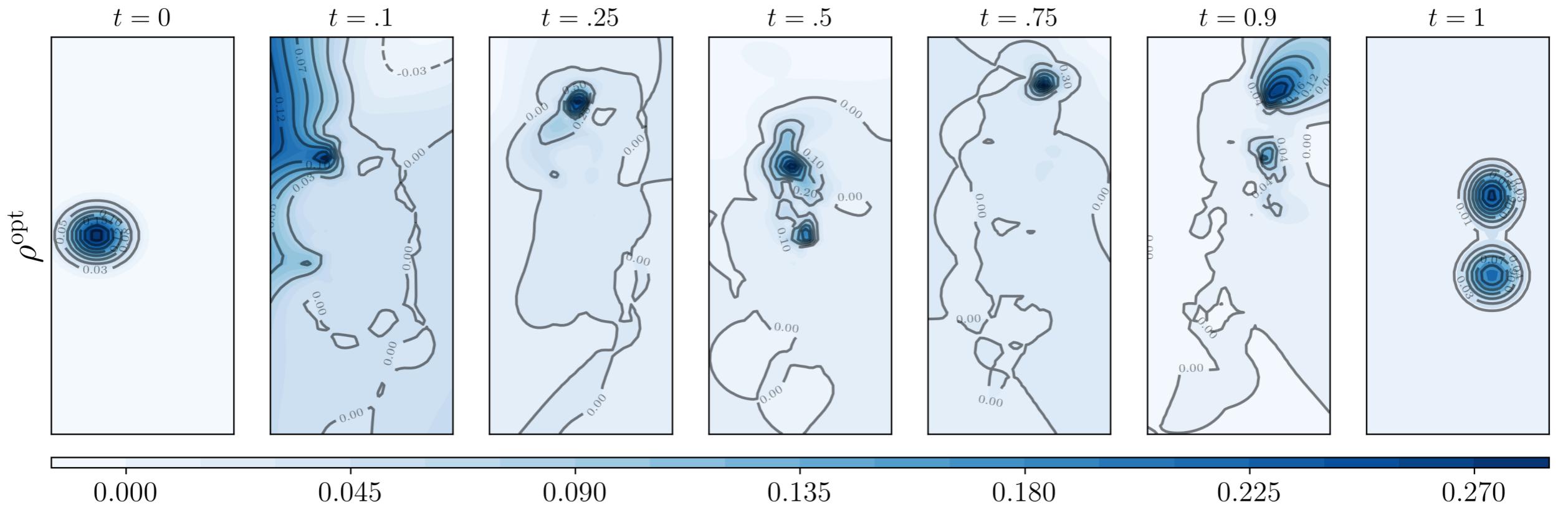
Uncontrolled joint PDF evolution:



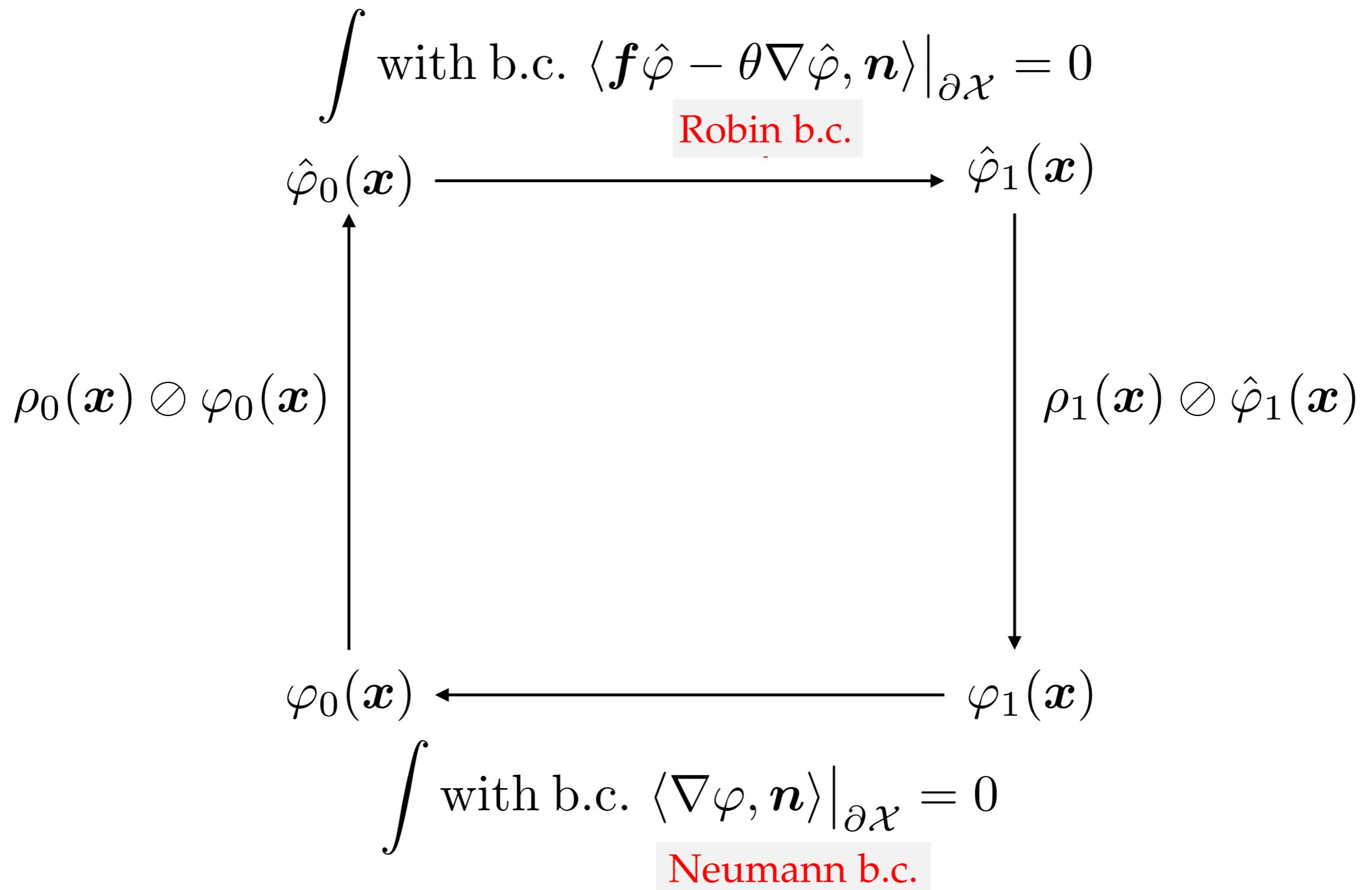
Optimal controlled joint PDF evolution:



Feedback Density Control: Mixed Conservative-Dissipative Drift

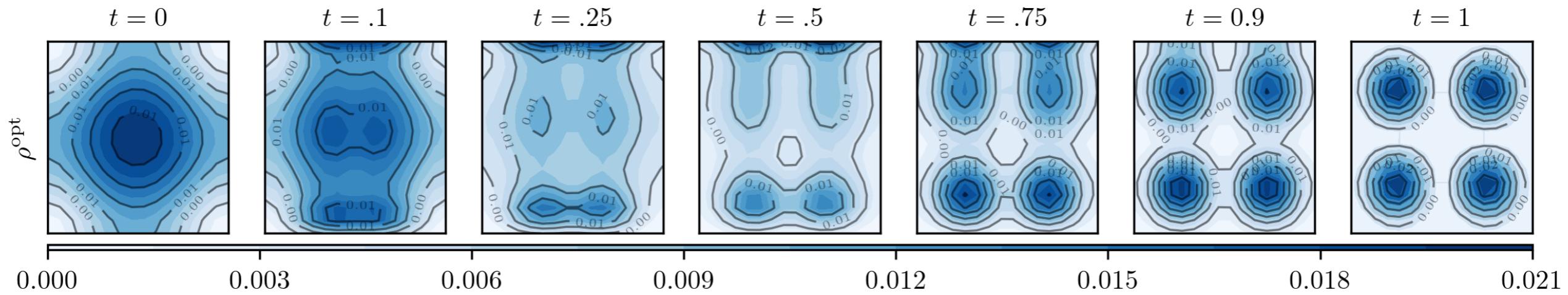


Nonlinear Density Steering with Deterministic Path Constraints

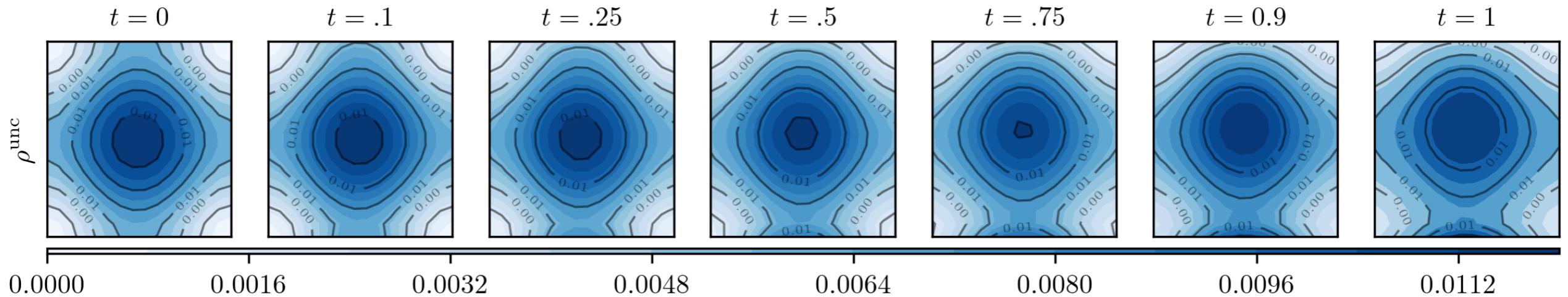


Nonlinear Density Steering with Deterministic Path Constraints

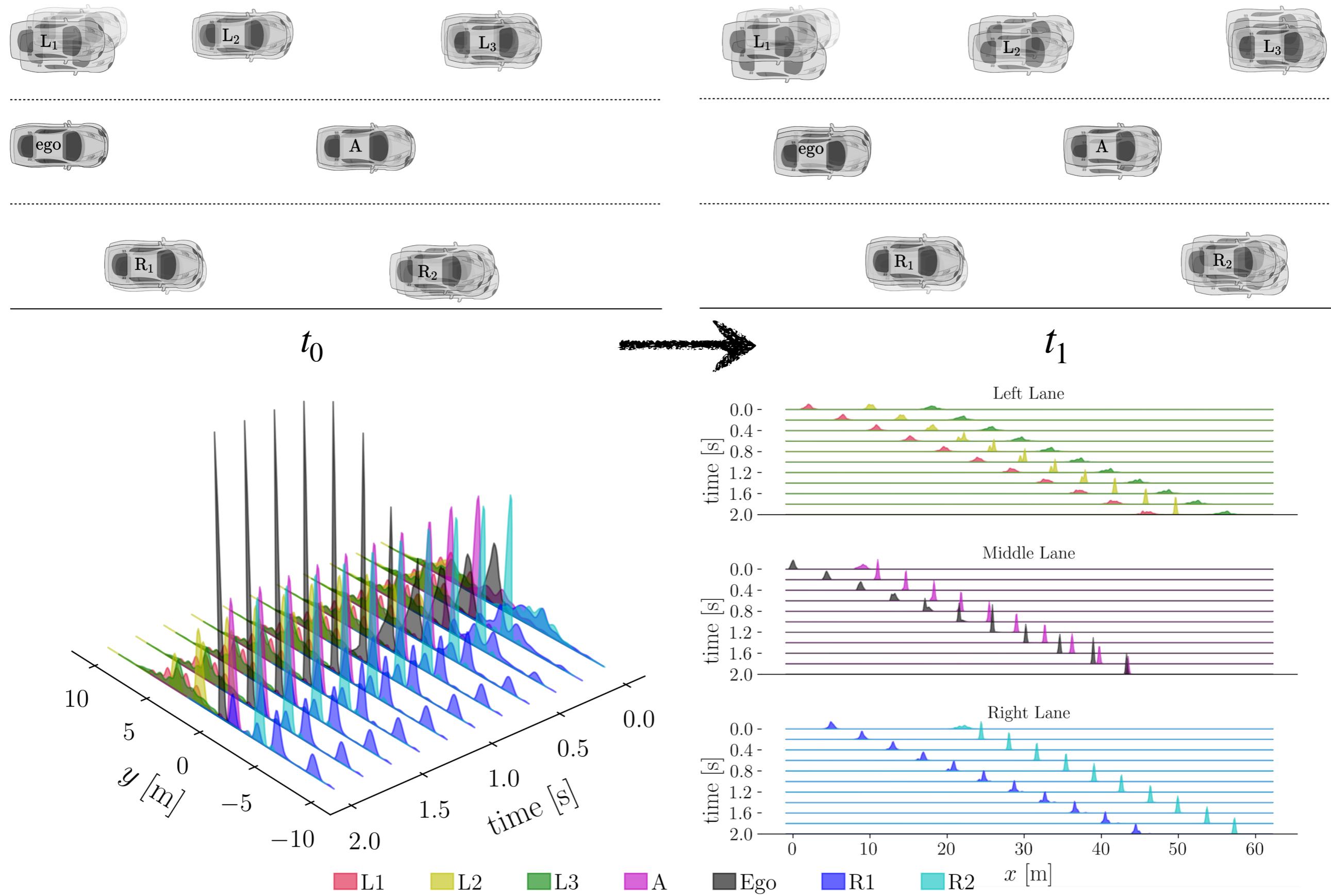
Optimal controlled state PDFs: $V(x_1, x_2) = (x_1^2 + x_2^3)/5$, $\overline{\mathcal{X}} = [-4, 4]^2$



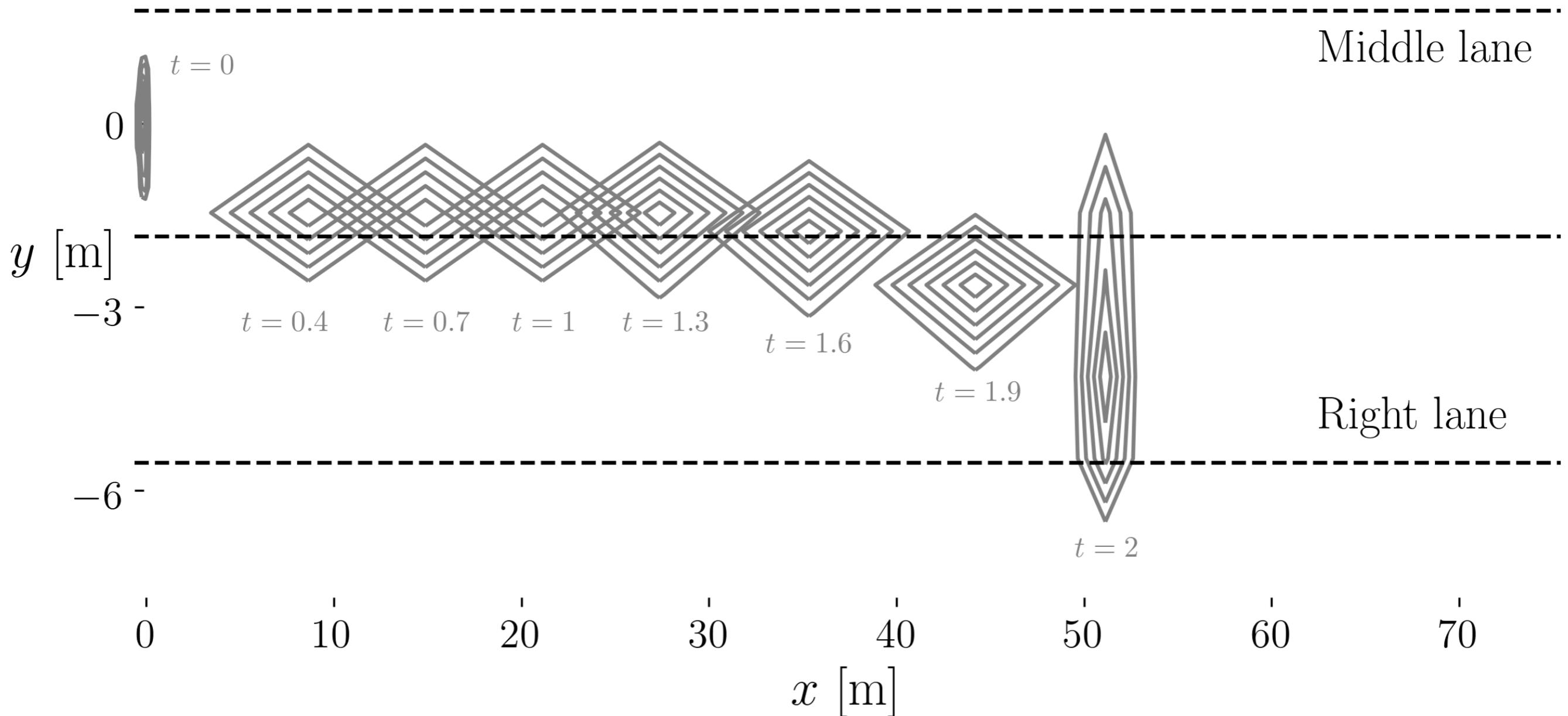
Uncontrolled state PDFs:



Application: Multi-lane Automated Driving



Application: Multi-lane Automated Driving



- S. Haddad, K.F. Caluya, A.H., and B. Singh, Prediction and Optimal Feedback Steering of Probability Density Functions for Safe Automated Driving, *IEEE Control Systems Letters*, 2021.
- S. Haddad, A.H., and B. Singh, Density-based Stochastic Reachability Computation for Occupancy Prediction in Automated Driving, *IEEE Transactions on Control Systems Technology*, 2022.

Application: Multi-lane Automated Driving

Exploit differential flatness: density steering in (Brunovsky) normal coordinates

Markov kernel available but ill-conditioned controllability Gramian

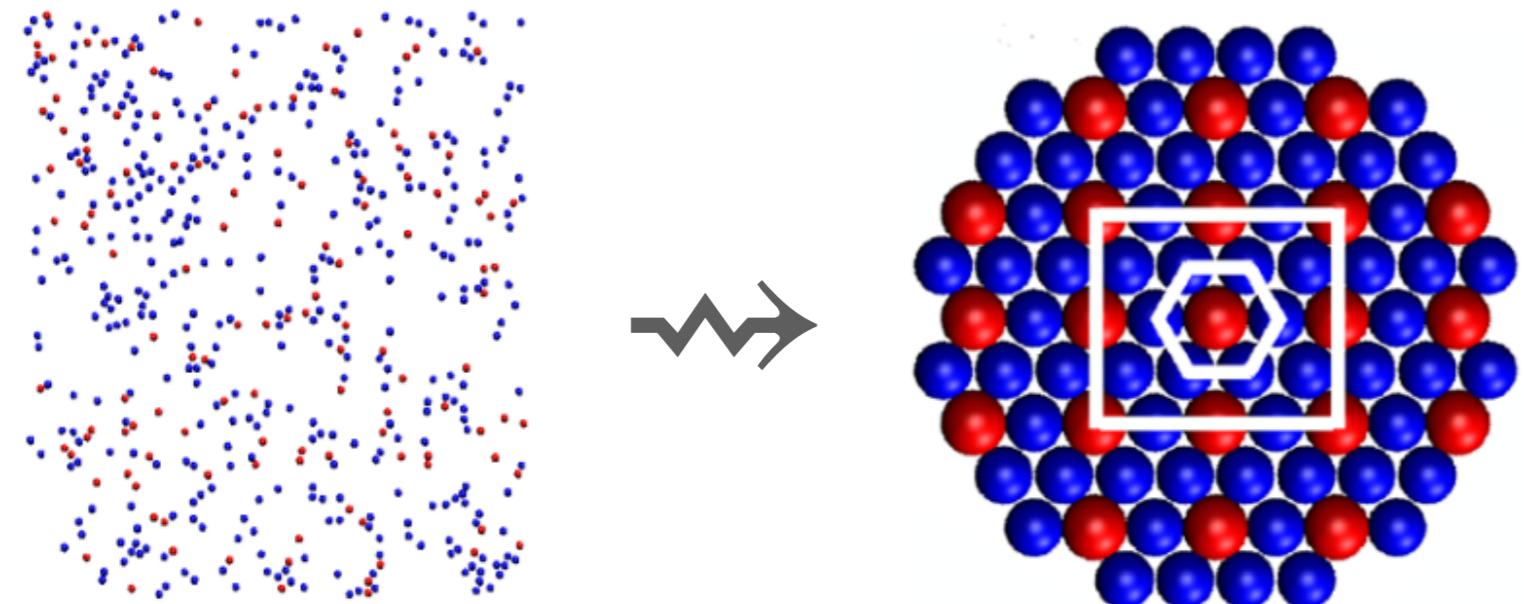
Derived analytical formula for the elements of Gramian inverse

Vector relative degree π	Computational time [s]	
	using Lyapunov ODE	using Theorem
$(2, 2)^\top$	1.9556	0.2995
$(3, 2)^\top$	49.7869	6.9294

Details

- S. Haddad, K.F. Caluya, A.H., and B. Singh, Prediction and Optimal Feedback Steering of Probability Density Functions for Safe Automated Driving, *IEEE Control Systems Letters*, 2020.

Application: Controlled Self-assembly



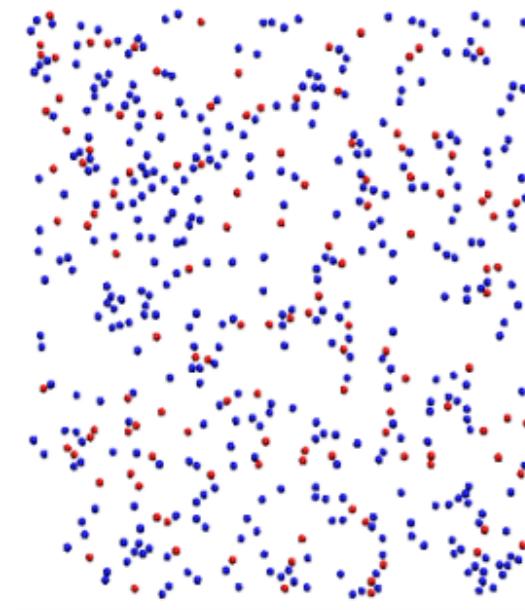
Dispersed particles

Ordered structure

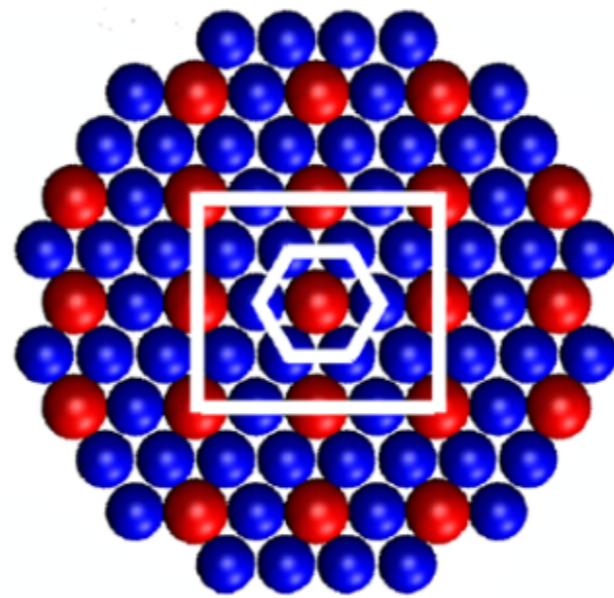
Applications:

Precision (e.g., sub nm scale) manufacturing
of materials with advanced electrical,
magnetic or optical properties

Application: Controlled Self-assembly



Dispersed particles



Ordered structure

Typical state variable: $\langle C_6 \rangle \in (0,6)$

Average number of hexagonally close packed neighboring particles in 2D assembly \rightsquigarrow measure of crystallinity order

Typical control variable: u

Electric field voltage

Technical challenge:

Nonlinear + noisy molecular dynamics



$\langle C_6 \rangle$ is a controlled stochastic process

Applications:

Precision (e.g., sub nm scale) manufacturing of materials with advanced electrical, magnetic or optical properties

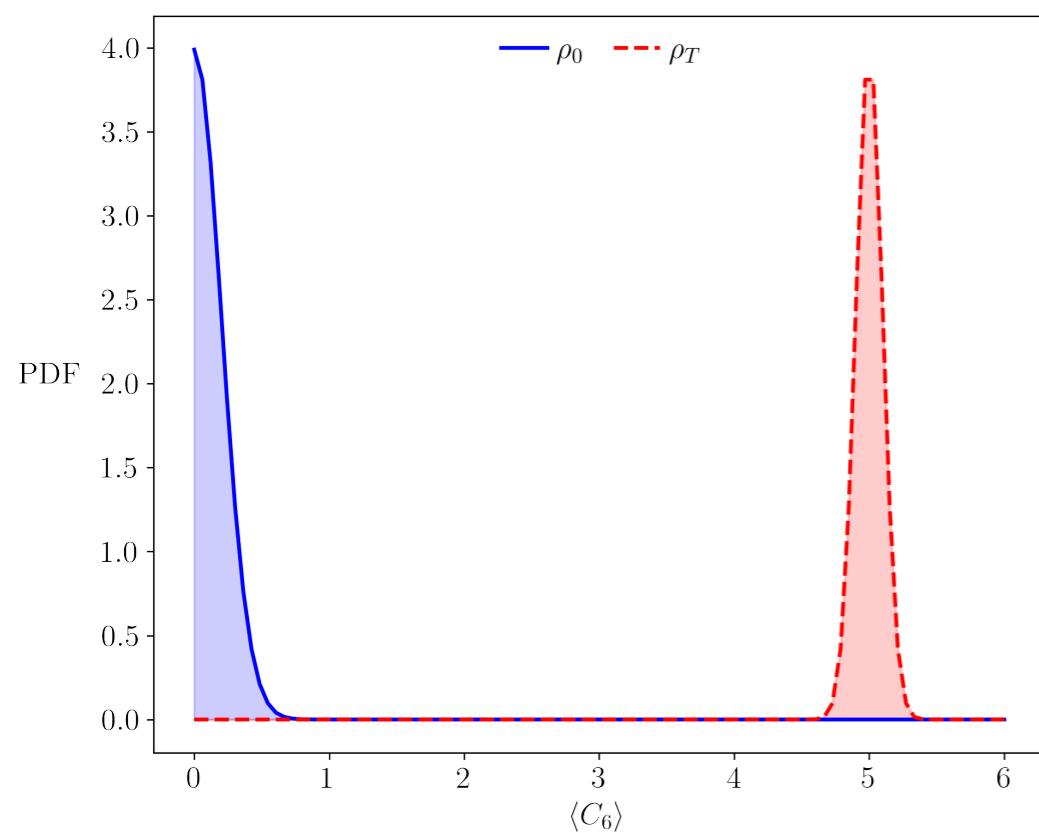
Controlled Self-assembly as PDF Steering

Intuition: $\langle C_6 \rangle \approx 0 \Leftrightarrow$ Crystalline disorder

$\langle C_6 \rangle \approx 5 \Leftrightarrow$ Crystalline order



Steer the PDF of the stochastic state $\langle C_6 \rangle$ from disordered at $t = t_0 \equiv 0$ to ordered at $t = T \equiv 200$ s



Typical prescribed finite horizon for controlled self-assembly

Endpoint PDF constraints: $\langle C_6 \rangle(t = t_0) \sim \rho_0$ (given)

$\langle C_6 \rangle(t = T) \sim \rho_T$ (given)

**Control policy to accomplish
the PDF steering:**

$$u = \pi(\langle C_6 \rangle, t)$$

Underdetermined

Minimum Effort Self-assembly

Proposed formulation:

$$\inf_{u \in \mathcal{U}} \mathbb{E}_{\mu^u} \left[\int_0^T \frac{1}{2} u^2 dt \right], \quad \mu^u \ll dx^u$$

drift landscape	diffusion landscape	free energy landscape
$D_1(x^u, u) := \frac{\partial}{\partial x} D_2(x^u, u)$	$- \frac{\partial}{\partial x} F(x^u, u) \frac{D_2(x^u, u)}{k_B \theta}$	

either from model or learnt from MD simulation data

subject to $dx^u = D_1(x^u, u) dt + \sqrt{2D_2(x^u, u)} dw$,

$\curvearrowleft \langle C_6 \rangle \quad \curvearrowright \text{standard Wiener process}$

$$x^u(t=0) \sim d\mu_0 = \rho_0 dx^u, \quad x^u(t=T) \sim d\mu_T = \rho_T dx^u$$

Nonlinear in state, non-affine in control

Conditions for Optimality

$$\frac{\partial \psi}{\partial t} = \frac{1}{2} (\pi^{\text{opt}})^2 - \frac{\partial \psi}{\partial x} D_1 - \frac{\partial^2 \psi}{\partial x^{u2}} D_2$$

HJB PDE

$$\frac{\partial \rho^u}{\partial t} = - \frac{\partial}{\partial x^u} (D_1 \rho^u) + \frac{\partial^2}{\partial x^{u2}} (D_2 \rho^u)$$

Controlled FPK PDE

$$\pi^{\text{opt}}(x^u, t) = \frac{\partial \psi}{\partial x^u} \frac{\partial D_1}{\partial u} + \frac{\partial^2 \psi}{\partial x^{u2}} \frac{\partial D_2}{\partial u}$$

Optimal policy

$$\rho^u(x^u, t=0) = \rho_0, \quad \rho^u(x^u, t=T) = \rho_T$$

Boundary conditions

value function	optimally controlled PDF	optimal policy
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to be solved for the triple: $\psi(x^u, t), \rho^u(x^u, t), \pi^{\text{opt}}(x^u, t)$

Solve via PINN: Losses for Training

Loss term for HJB PDE

$$\mathcal{L}_\psi = \frac{1}{n} \sum_{i=1}^n \left(\frac{\partial \psi}{\partial t} \Big|_{x_i} - \frac{1}{2} (\pi^{\text{opt}})^2 \Big|_{x_i^u} - + \frac{\partial \psi}{\partial x^u} D_1 \Big|_{x_i^u} - + \frac{\partial^2 \psi}{\partial x^{u2}} D_2 \Big|_{x_i^u} \right)^2$$

Loss term for FPK PDE

$$\mathcal{L}_{\rho^u} = \frac{1}{n} \sum_{i=1}^n \left(\frac{\partial \rho^u}{\partial t} \Big|_{x_i^u} + \frac{\partial}{\partial x^u} (D_1 \rho^u) \Big|_{x_i^u} - \frac{\partial^2}{\partial x^{u2}} (D_2 \rho^u) \Big|_{x_i^u} \right)^2$$

Loss term for policy equation

$$\mathcal{L}_{\pi^{\text{opt}}} = \frac{1}{n} \sum_{i=1}^n \left(\pi^{\text{opt}} \Big|_{x_i^u} - \frac{\partial \psi}{\partial x^u} \frac{\partial D_1}{\partial u} \Big|_{x_i^u} - \frac{\partial^2 \psi}{\partial x^{u2}} \frac{\partial D_2}{\partial u} \Big|_{x_i^u} \right)^2$$

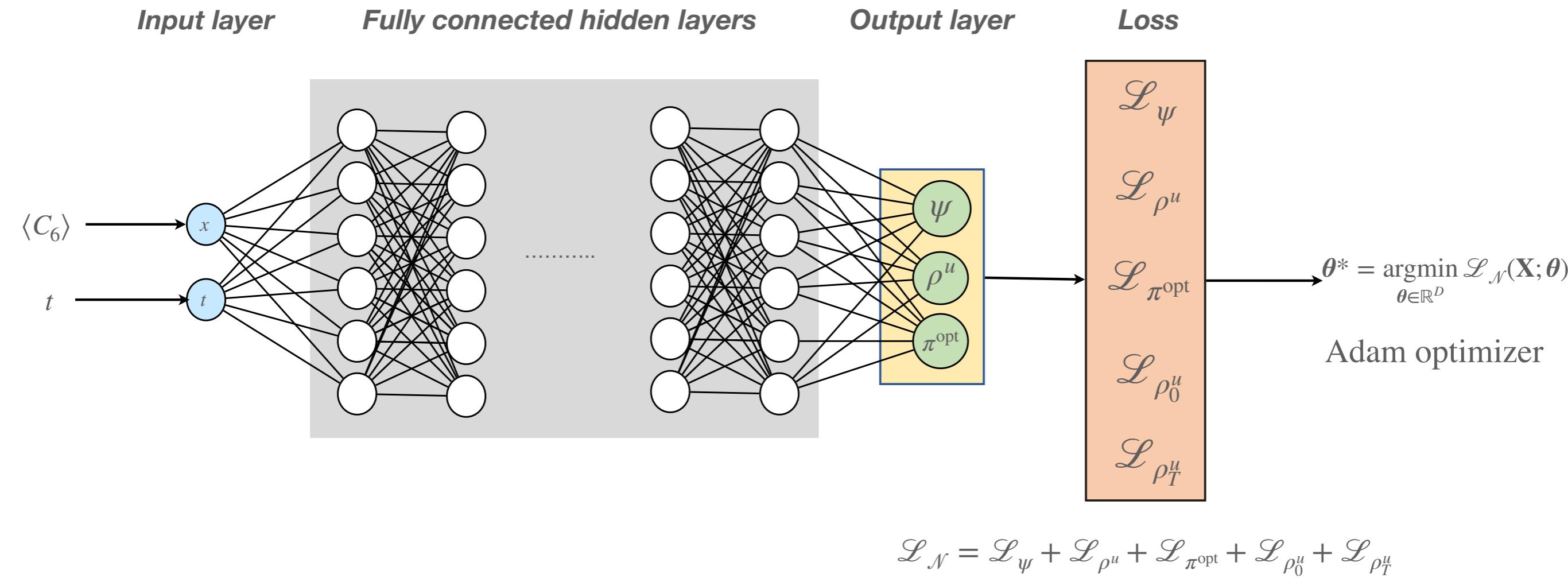
Loss term for initial condition

$$\mathcal{L}_{\rho_0^u} = \frac{1}{n} \sum_{i=1}^n \left(\rho^u \Big|_{t=0} - \rho_0^u(x) \right)^2$$

Loss term for terminal condition

$$\mathcal{L}_{\rho_T^u} = \frac{1}{n} \sum_{i=1}^n \left(\rho^u \Big|_{t=T} - \rho_T^u(x) \right)^2$$

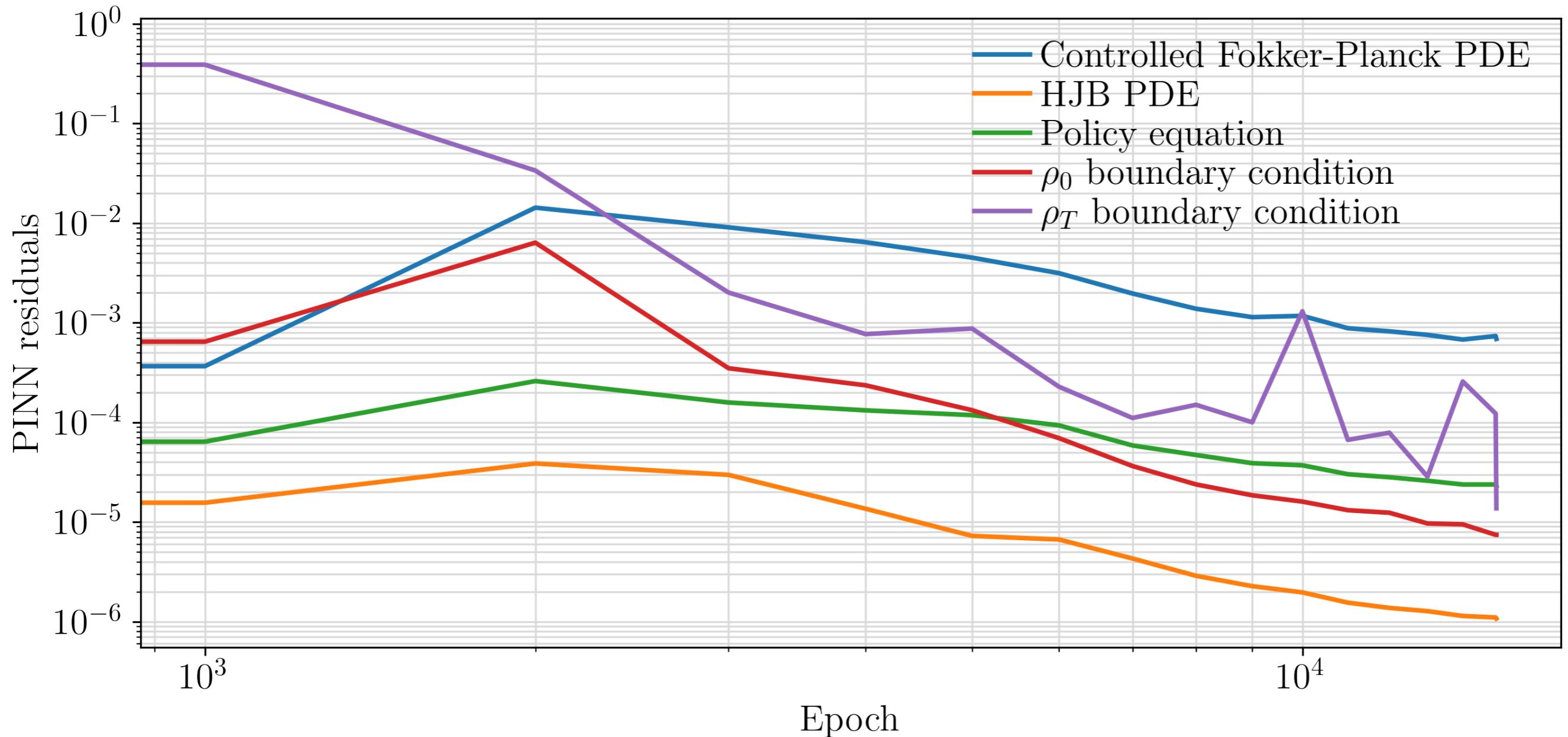
PINN Architecture



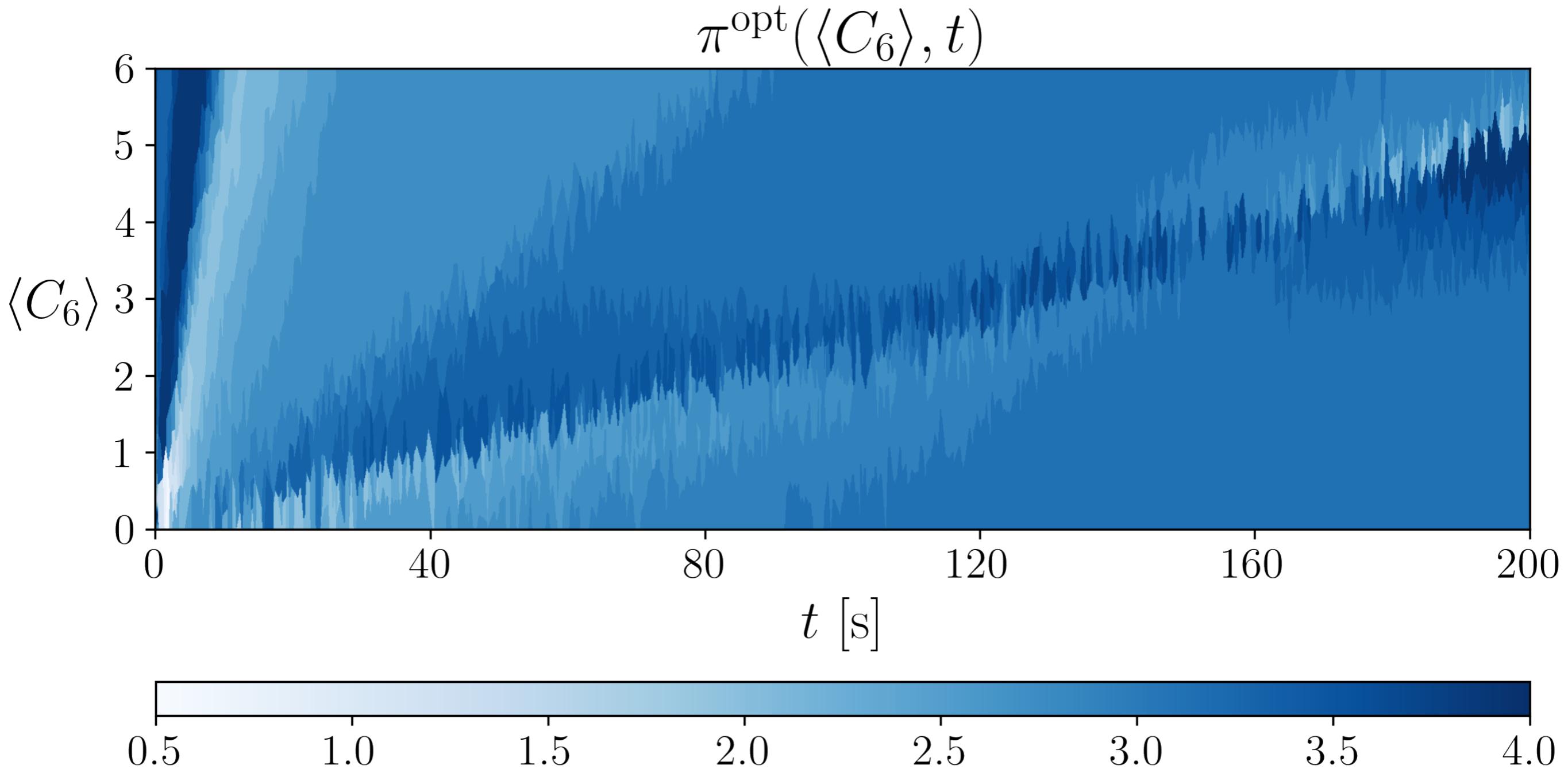
[Lu Lu, et al, 2021] [Niaki, et al, 2021]

Training of the PINN

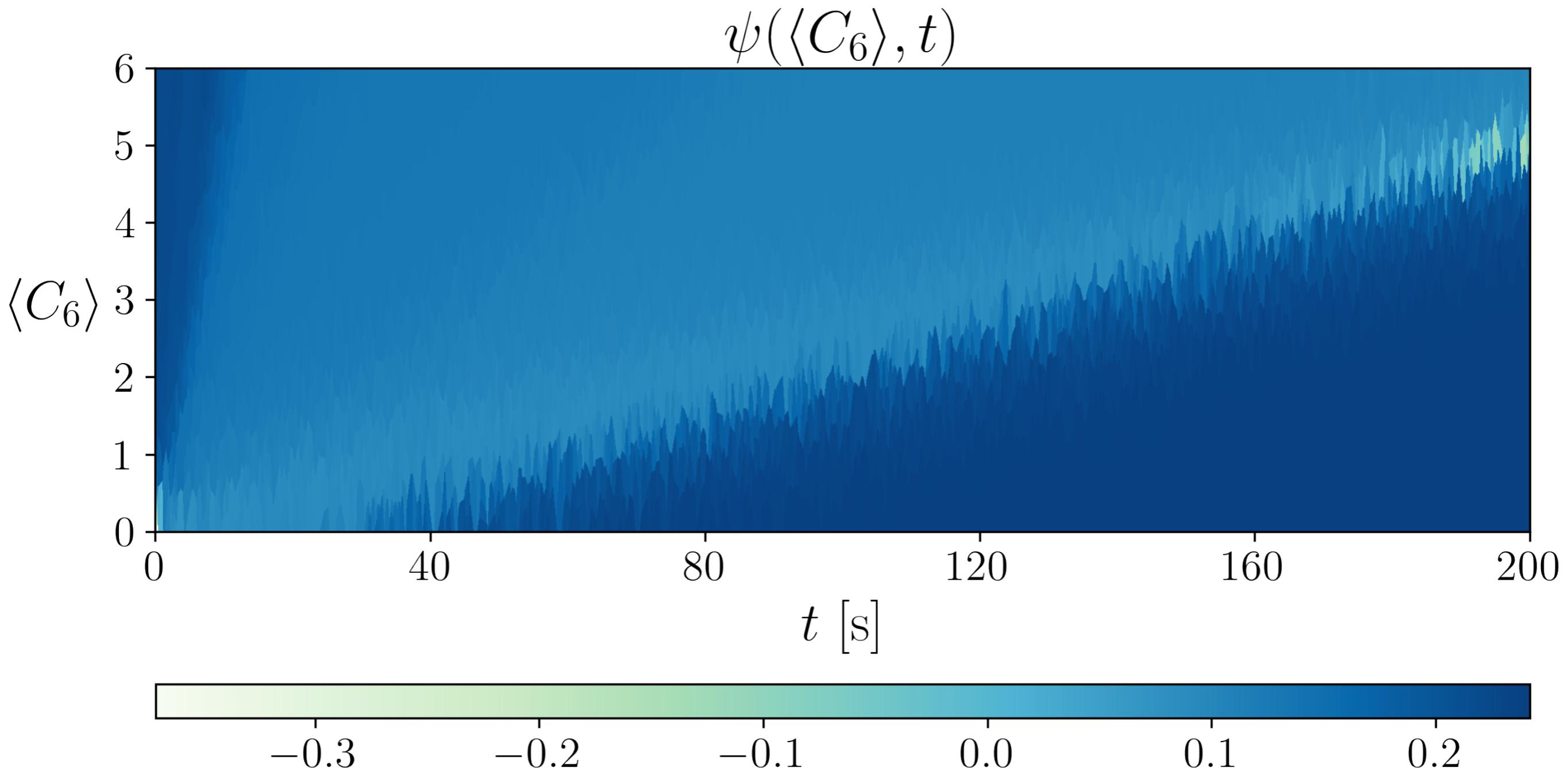
Benchmark controlled self-assembly system: [Y Xue, et al, *IEEE Trans. Control Sys. Technology*, 2014]



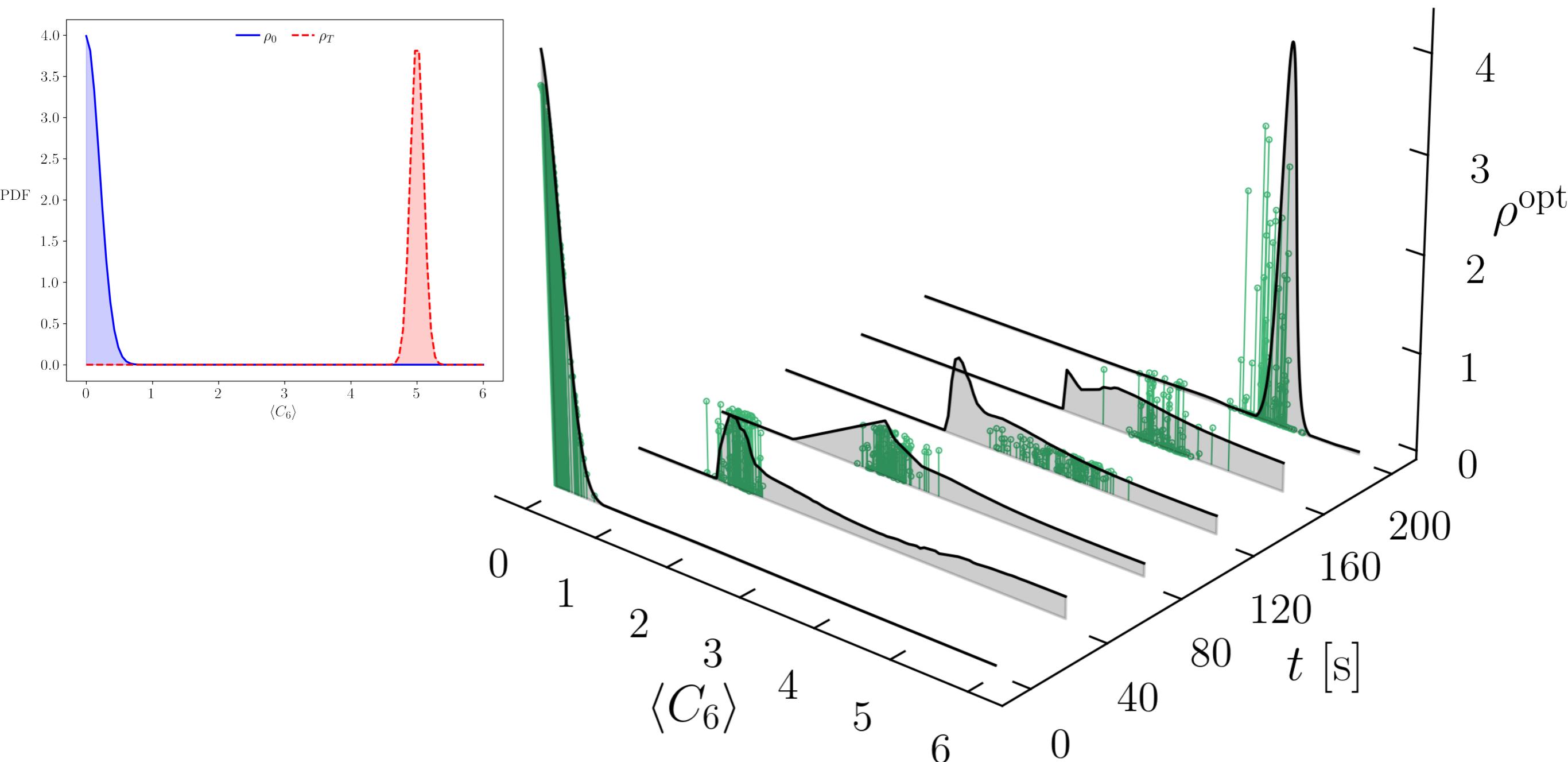
Optimal Policy



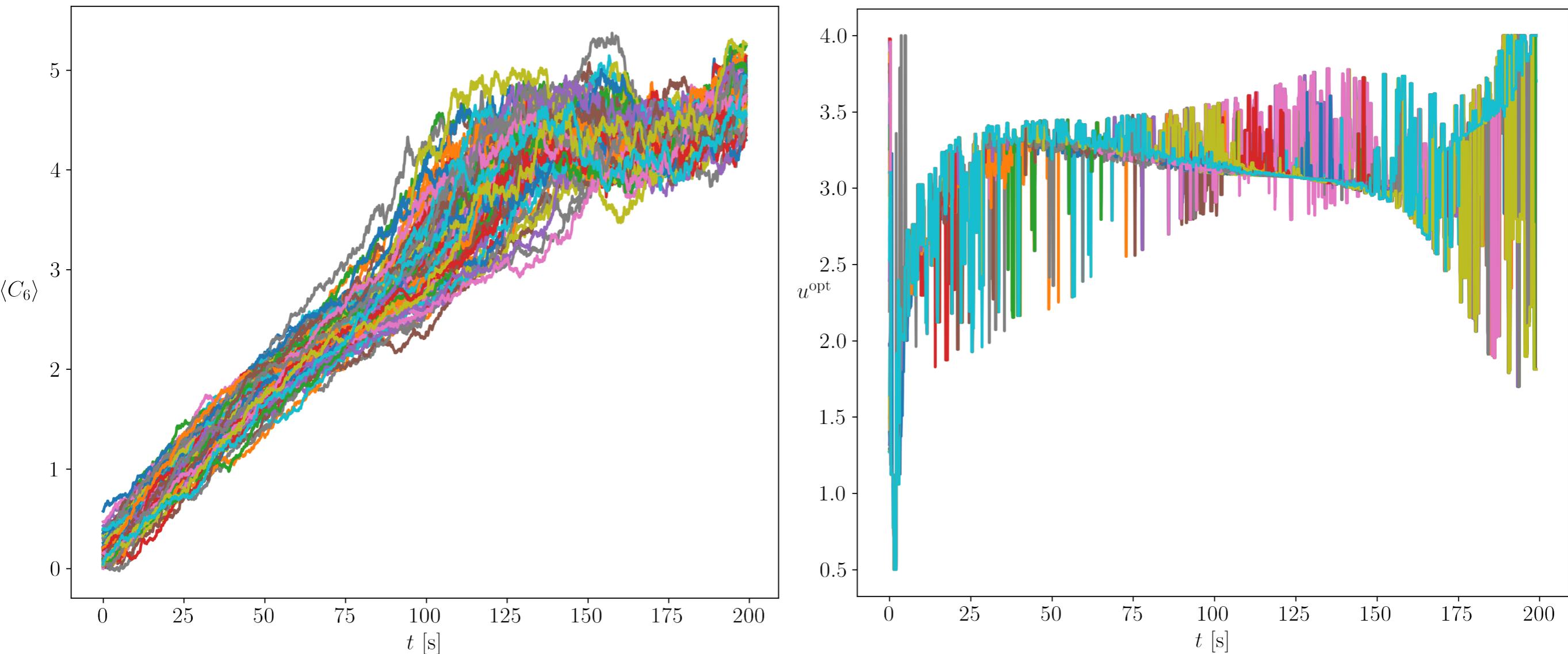
Value Function



Optimally Controlled State PDFs



Optimal State and Optimal Control Sample Paths



— I. Nodozi, J. O'Leary, A. Mesbah, and A.H., A Physics-informed Deep Learning Approach for Minimum Effort Stochastic Control of Colloidal Self-Assembly, *arXiv:2208.09182*.

Outlook

- Distribution control undergoing rapid developments
- Lots of interesting theory, algorithms and applications to be done
- Rapidly growing community in systems-control
- Excellent intersections with related communities: learning, information theory, robotics, systems biology, smart manufacturing

We are hiring for Grad students and Postdocs



- Strong mathematical background / interests
- Interdisciplinary team
- Exciting projects at the intersection of control and ML

Thank You

Support:



CITRIS
PEOPLE AND
ROBOTS

