DATA 557: Applied Statistics and Experimental Design
Homework 2

Abhishek Saini

February 2022

Question 1

The data are from a study of the association between exposure to lead and IQ. The study was conducted in an urban area around a lead smelter. A random sample of 124 children who lived in the area was selected. Each study participant had a blood sample drawn in both 1972 and 1973 to assess blood concentrations of lead. The children were grouped based on their blood concentrations as follows:
Group 1: concentration < 40 mg/L in both 1972 and 1973
Group 2: concentration > 40 mg/L in both 1972 and 1973 or > 40 mg/L in 1973 alone (3 participants)
Group 3: concentration > 40 mg/L in 1972 but < 40 mg/L in 1973
Each participant completed an IQ test in 1973. (A subset of the IQ scores from this study were used in HW 1, Question 3.) The variables in the data set are listed below.
ID: Participant identification number SEX: Participant sex (1=M or 2=F) GROUP: As described above (1, 2, or 3) IQ: IQ score

0.1 The first goal is to compare the mean IQ scores for males and females. Use a 2-sample t-test for this comparison. What is the p-value?

The null hypothesis for our test is that the means of IQ scores are same for males and females. We will use a level of significance 0.05. Using 2 sample t-test for unequal variance (Welch t test) we get a p-value of 0.8777.

```
> lead = read.csv("lead_study.csv")
> with(lead,
+      t.test(IQ[SEX == 1], IQ[SEX == 2], var.equal=F))

        Welch Two Sample t-test
```

```
data:  IQ[SEX == 1] and IQ[SEX == 2]
t = 0.1541, df = 146.58, p-value = 0.8777
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -4.258377  4.978637
sample estimates:
mean of x mean of y
 91.23684  90.87671
```

0.2 State the conclusion from your test.

Since the p-value is greater than 0.05, we fail to reject the null hypothesis.

0.3 Are the independence assumptions valid for the t-test in this situation? Give a brief explanation.

If the samples are randomly sampled, we should expect similar proportion of Group 1, 2, 3 in both males and females (assuming the sex of person doesn't impact whether they are in Group 1, 2 or 3). Checking for this we get the following distribution,

```
> lead_M = lead[lead$SEX==1,]
> lead_F = lead[lead$SEX==2,]
> table(lead_M$GROUP)
 1  2  3
46 17 13

> table(lead_F$GROUP)
 1  2  3
41 15 17
```

There is variance between number of people in groups 1,2,3 for males and females but doesn't seem to be alarmingly different. Also, the data might be biased towards people from that particular area but the question states that there are 124 random samples which seems to be the case.

0.4 The second goal is to compare the mean IQ scores in the 3 groups. State in words the null hypothesis for this test.

The null hypothesis is that all three groups have the same.
$H_0 : \mu_1 = \mu_2 = \mu_3$

0.5 State in words the alternative hypothesis for this test.

The alternate hypothesis is that not all the three groups have the same mean.
H_1 : The means are not all equal.

0.6 What method should be used to perform the test?

We should use (Analysis of Variance) ANOVA to perform this test because it is suitable for comparing means between more than 2 groups.

0.7 Perform the test. Report the p-value.

The p-value for the test is 0.0242

```
> summary(aov(lead$IQ ~ factor(lead$GROUP), data=lead))
                    Df Sum Sq Mean Sq F value Pr(>F)
factor(lead$GROUP)   2   1491   745.4   3.816 0.0242 *
Residuals          146  28522   195.4
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

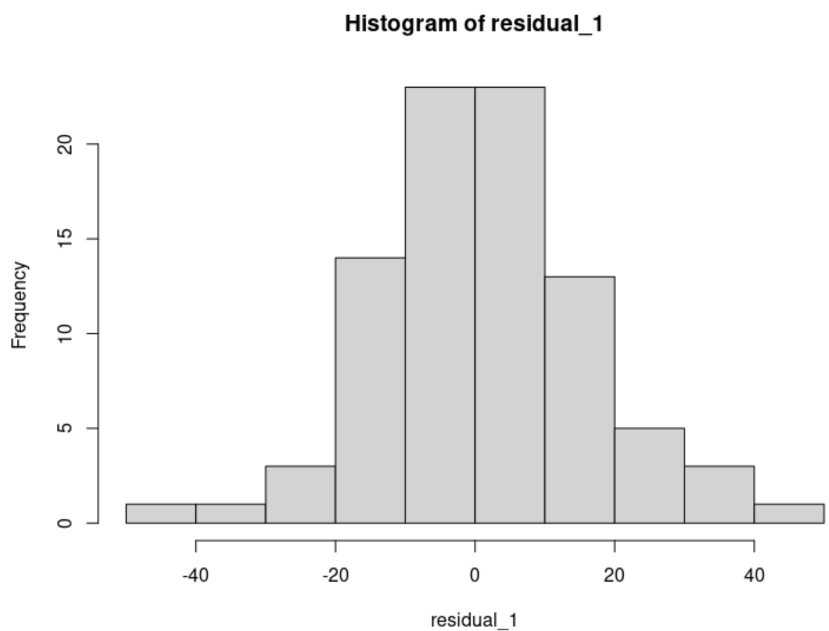
0.8 State your conclusion about the evidence for an association between lead exposure and IQ.

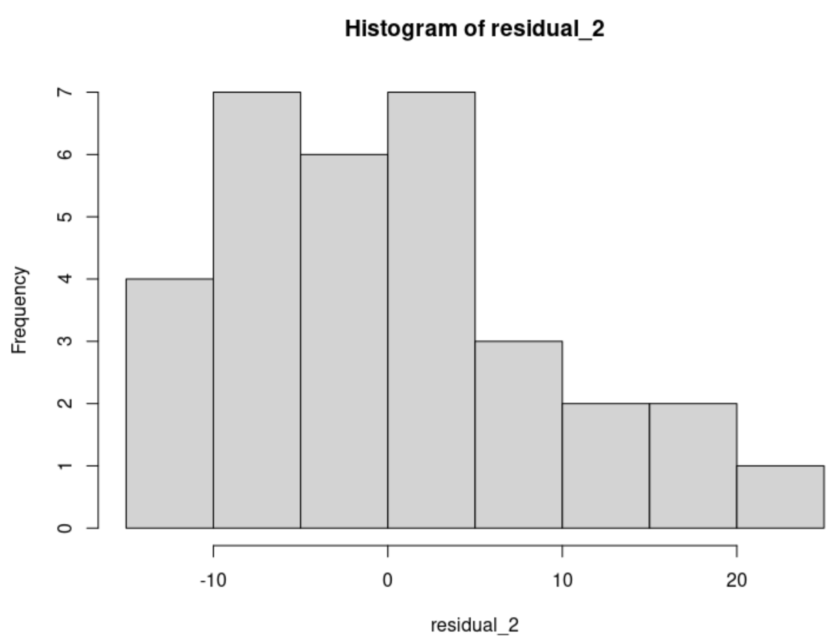
Since the p-value is below our significance level of 0.05, we reject the null hypothesis that means of all groups are the same.

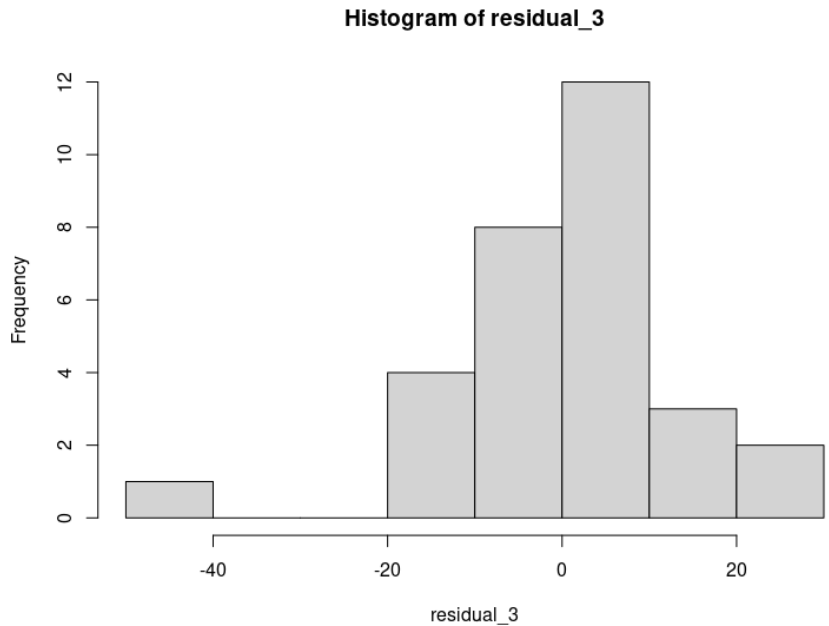
0.9 Are there strong reasons to believe that the assumptions of this test are not met? Briefly justify your answer.

Performing regression diagnostics, we see that the residuals have different variances.
Further, the plots of residuals don’t look normal for all groups.
Independence assumption seems to hold as we saw in question 3.

```
> residual_1 = lead[lead$GROUP==1,]$IQ - with(lead[lead$GROUP==1,],tapply(IQ, GROUP, mean))
> residual_2 = lead[lead$GROUP==2,]$IQ - with(lead[lead$GROUP==2,],tapply(IQ, GROUP, mean))
> residual_3 = lead[lead$GROUP==3,]$IQ - with(lead[lead$GROUP==3,],tapply(IQ, GROUP, mean))
> var(residual_1)
[1] 242.4346
> var(residual_2)
[1] 90.29738
> var(residual_3)
[1] 168.0333
```


Histogram of residual_1
Frequency
0
5
10
15
20
-40
-20
0
20
40
residual_1


Histogram of residual_2
Frequency
0
1
2
3
4
5
6
7
-10
0
10
20
residual_2



0.10 Conduct all pairwise comparison of group means. Report the p-values.

The p-value on comparing group 1 with group 2 is 0.01205
The p-value on comparing group 1 with group 3 is 0.023
The p-value on comparing group 2 with group 3 is 0.8131

```
> IQ_1 = lead[lead$GROUP==1,]$IQ
> IQ_2 = lead[lead$GROUP==2,]$IQ
> IQ_3 = lead[lead$GROUP==3,]$IQ
> t.test(IQ_1, IQ_2)

        Welch Two Sample t-test

data:  IQ_1 and IQ_2
t = 2.5622, df = 90.607, p-value = 0.01205
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  1.363464 10.772312
sample estimates:
mean of x mean of y
 93.72414  87.65625
```

```
> t.test(IQ_1, IQ_3)

        Welch Two Sample t-test

data:  IQ_1 and IQ_3
t = 2.3333, df = 60.024, p-value = 0.023
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
  0.9643448 12.5505977
sample estimates:
mean of x mean of y
 93.72414  86.96667

> t.test(IQ_2, IQ_3)

        Welch Two Sample t-test

data:  IQ_2 and IQ_3
t = 0.23761, df = 52.997, p-value = 0.8131
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
 -5.131548  6.510715
sample estimates:
mean of x mean of y
 87.65625  86.96667
```

0.11 What conclusion about the association between lead and IQ would you draw from the pairwise comparisons of group means? Does it agree with the conclusion in Q8? (Consider the issue of multiple testing in your answer.)

Using Bonferroni's correction our corrected significance level for these three tests is $0.05/3 = 0.0166$. Since the p-value on comparing group 1 and group 2 is less than 0.0166, we reject the null hypothesis that the means of group 1 and group 2 are equal.

This agrees with our conclusion in Q8 where we rejected the null hypothesis that the means of all groups are equal. (Bonferroni correction is used to account for inflated p-values due to multiple tests)

0.12 Now we wish to compare the 3 group means for males and females separately. Display some appropriate descriptive statistics for this analysis.

Here are the means and sd of the 3 groups across males and females.

```
> with(lead[lead$SEX==1,],tapply(IQ, GROUP, mean))
       1        2        3
92.93478 90.17647 86.61538
> with(lead[lead$SEX==2,],tapply(IQ, GROUP, mean))
       1        2        3
94.60976 84.80000 87.23529
> with(lead[lead$SEX==1,],tapply(IQ, GROUP, sd))
       1        2        3
15.42351 11.13124 17.32791
> with(lead[lead$SEX==2,],tapply(IQ, GROUP, sd))
        1         2         3
15.877465  6.471917  8.898942
```

0.13 Perform tests to compare the mean IQ scores in the 3 groups for males and females separately. Report the p-values from the two tests.

Performing ANOVA separately for males and females, we get the following p-values. The p-value for comparing the means of the 3 groups for males is 0.387. The p-value for comparing the means of the 3 groups for females is 0.0235.

```
> lead_M = lead[lead$SEX==1,]
> lead_F = lead[lead$SEX==2,]
> summary(aov(lead_M$IQ ~ factor(lead_M$GROUP), data=lead_M))
                     Df Sum Sq Mean Sq F value Pr(>F)
factor(lead_M$GROUP)  2    429   214.7   0.962  0.387
Residuals            73  16290   223.2
> summary(aov(lead_F$IQ ~ factor(lead_F$GROUP), data=lead_F))
                     Df Sum Sq Mean Sq F value Pr(>F)
factor(lead_F$GROUP)  2   1351   675.3    3.96 0.0235 *
Residuals            70  11937   170.5
```

0.14 What can you conclude about the association between lead and IQ from these tests? Does it agree with the result in Q8 and Q11? (Consider multiple testing.)

Using Bonferroni's correction our corrected significance level for these two tests is $0.05/2 = 0.025$. Since the p-value on comparing group means for females is less than 0.025, we reject the null hypothesis

that the means of group 1 and group 2 are equal.
This agrees with our conclusion in Q8 and Q11 where we rejected the null hypothesis that the means of all groups are equal. (Bonferroni correction is used to account for inflated p-values due to multiple tests)

0.15 Now perform all 3 pairwise comparisons of groups for males and females separately. Report the p-values from these tests?

The p-value on comparing group 1 with group 2 for males is 0.4392
The p-value on comparing group 1 with group 3 for males is 0.2503
The p-value on comparing group 2 with group 3 for males is 0.5259
The p-value on comparing group 1 with group 2 for females is 0.001831
The p-value on comparing group 1 with group 3 for females is 0.02927
The p-value on comparing group 2 with group 3 for females is 0.3796

0.16 What do you conclude about the association between lead and IQ from the results in Q13? Does your conclusion change from previous conclusions made in Q8, Q11 and Q14?

Using Bonferroni's correction our corrected significance level for these six tests is $0.05/6 = 0.00833$. Since the p-value on comparing group means for females is less than 0.00833, we reject the null hypothesis that the means of group 1 and group 2 are equal.
This agrees with our conclusion in Q8, Q11 and Q14 where we rejected the null hypothesis that the means of all groups are equal. (Bonferroni correction is used to account for inflated p-values due to multiple tests)