Proof Mn Sn Gaussian

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$p(w) \to$ prior $= \mathcal{N}(w | m_0, S_0)$ ↑Gaussian

$e^{-x} \times e^{-2y} = e^{-(x+2y)}$

$p(t|w) = \prod_{n=1}^{N} \mathcal{N}(t_n | w^T \phi(x_n), \beta^{-1})$

$N \to$ no of data points

$p(\underline{w}|t) \propto p(\underline{w})\, p(t|w)$

$p(\underline{w}) \to \mathcal{N}(\underline{w} | \underline{m_0}, \underline{S_0})$

$= \mathcal{N}(w | m_N, S_N)$

$e^{-\frac{(w-m)^2}{2\sigma^2}}$

$\boxed{w = \underline{w}}$ small + underline vb; small + bold

$\boxed{m_N = ??}$

$$\Downarrow$$

$$\left[e^{-\frac{1}{2}(\underline{w} - \underline{m_N})^T S_N^{-1} (\underline{w} - \underline{m_N})} = e^{-\frac{1}{2}(\underline{w} - \underline{m_0})^T S_0^{-1} (\underline{w} - \underline{m_0})} \times e^{-\left(\frac{\beta}{2}\right)(\underline{t} - \underline{\phi}^T \underline{w})^T (\underline{t} - \underline{\phi}^T \underline{w})} \right] \quad (1)$$

$\underline{w}^T \phi = \phi^T \underline{w}$

$\phi^T \phi = \phi^T \phi$

$\beta w^T \underbrace{\phi \phi^T} w$

$\mathcal{N}(t_1 | w^T \phi(x_1), \beta^{-1})$

$$e^{-\frac{(t_1 - w^T \phi(x_1))^2}{2\beta^{-1}}}$$

$$e^{-\frac{\beta}{2}(t_1 - w^T \phi(x_1))^2} \times e^{-\frac{\beta}{2}(t_2 - w^T \phi(x_2))^2}$$

$$\boxed{\underline{w}^T \underline{x} = \underline{x}^T \underline{w} = \sum w_i x_i} \longrightarrow \text{logic}$$

$$\underline{w} = \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix}$$

$$\underline{w}^T \underline{x}$$

Compare the co-efficients in (1) on left & right

$$\underline{w}^T \underline{S}_N^{-1} \underline{w} = \underline{w}^T \underline{S}_0^{-1} \underline{w} + w^T \beta(\phi^T \phi) \underline{w}$$

$$\underline{w}^T \underline{S}_N^{-1} \underline{w} = w^T \left(S_0^{-1} + \beta \phi^T \phi \right) \underline{w}$$

$$\boxed{S_N^{-1} = S_0^{-1} + \beta \phi^T \phi}$$

$$m_N = ??$$