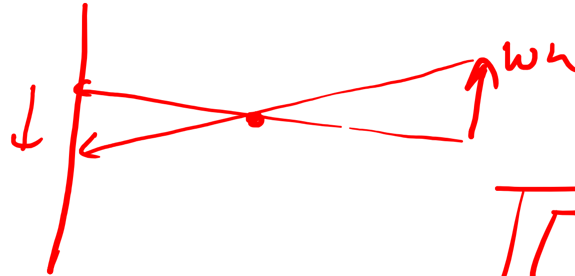
$\underbrace{CM = [Int][Ext]}$,



Single view Geometry
- Camera matrix

$$\boxed{[ch_i] = [CM][wh]}$$

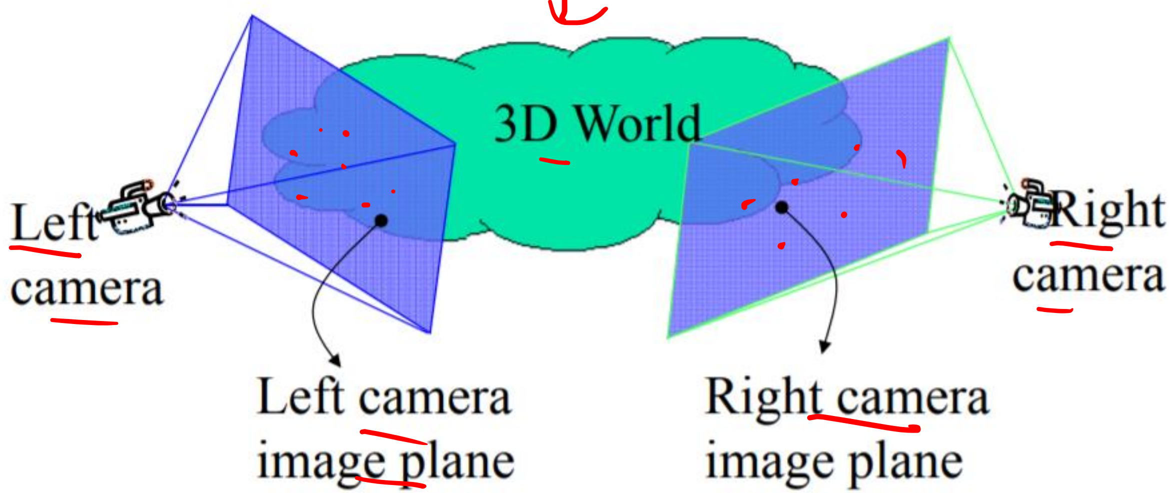
EPIPOLAR GEOMETRY FOR PAIR OF IMAGES

$x = ch_1/ch_4$
$y = ch_2/ch_4$

Multi View Geometry.

- → Fundamental matrix.
- → Essential matrix.

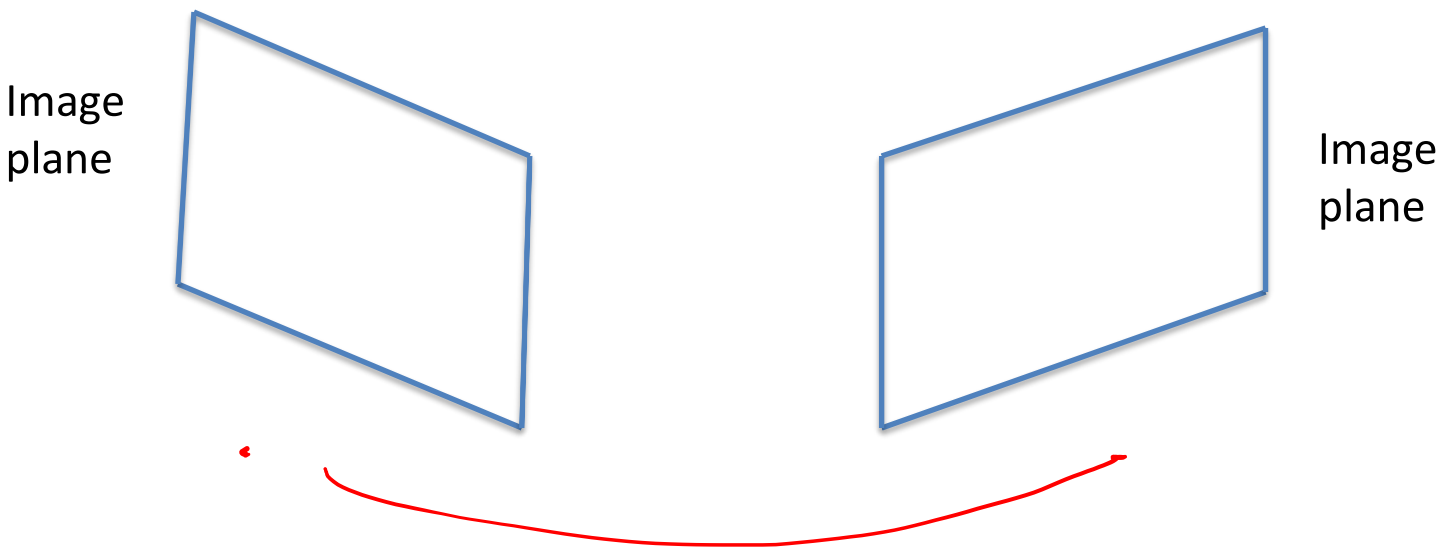
- Defined for two static cameras

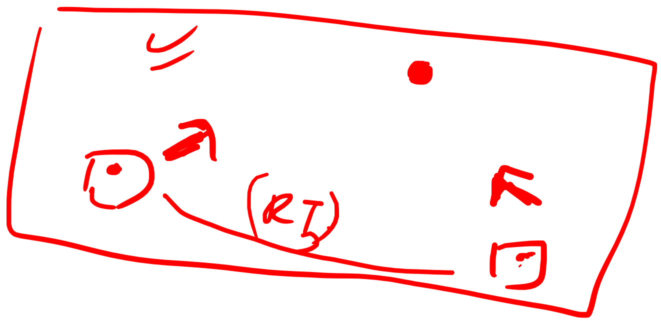


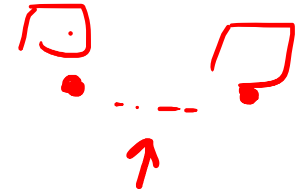
Multiple views.

SET UP

- Static Scene (Camera Moves)
- Two pictures of the same scene, taken at the same time, from two different cameras







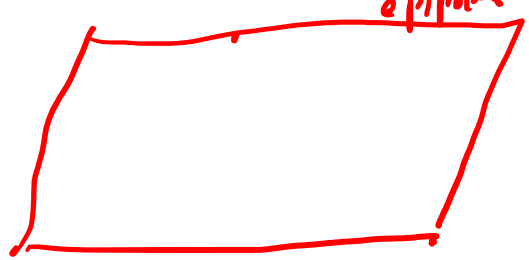
epipolar lines

Left-.

Right-.

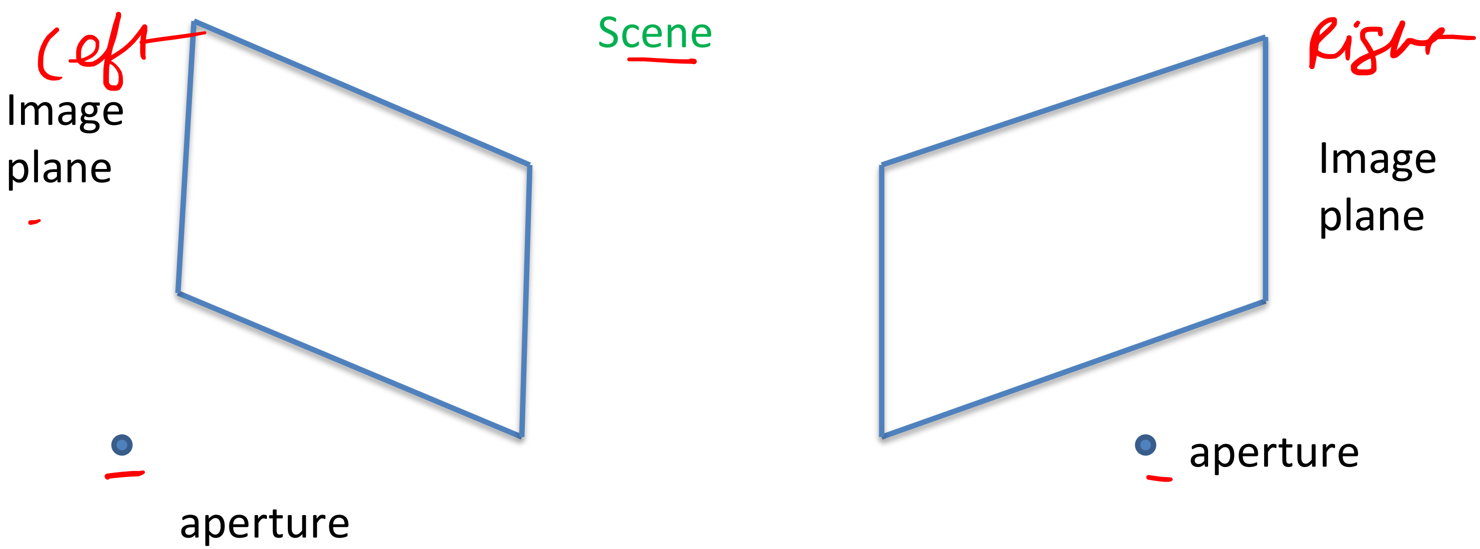
(a) (b)

epipolar



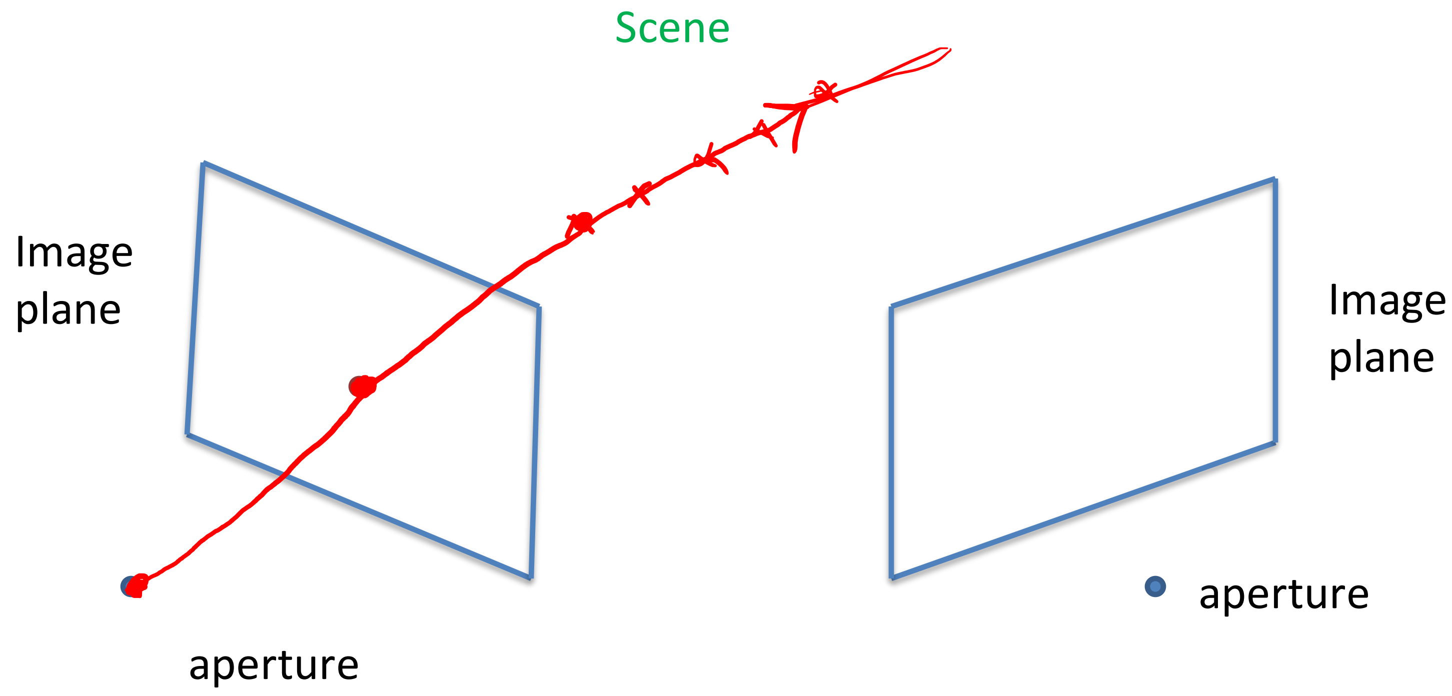
SET UP

- Static Scene (Camera Moves)
- Two pictures of the same scene, taken at the same time, from two different cameras



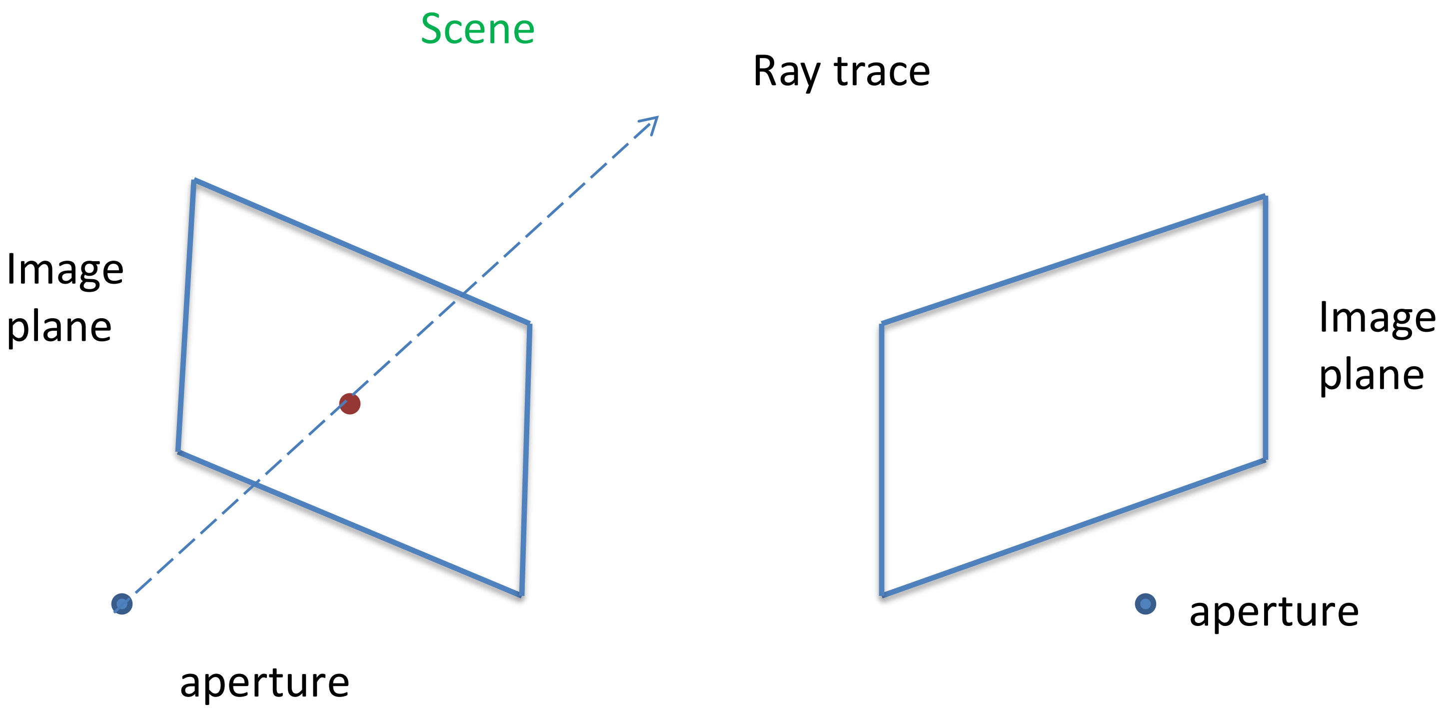
SET UP

- Static Scene (Camera Moves)
- Two pictures of the same scene, taken at the same time, from two different cameras



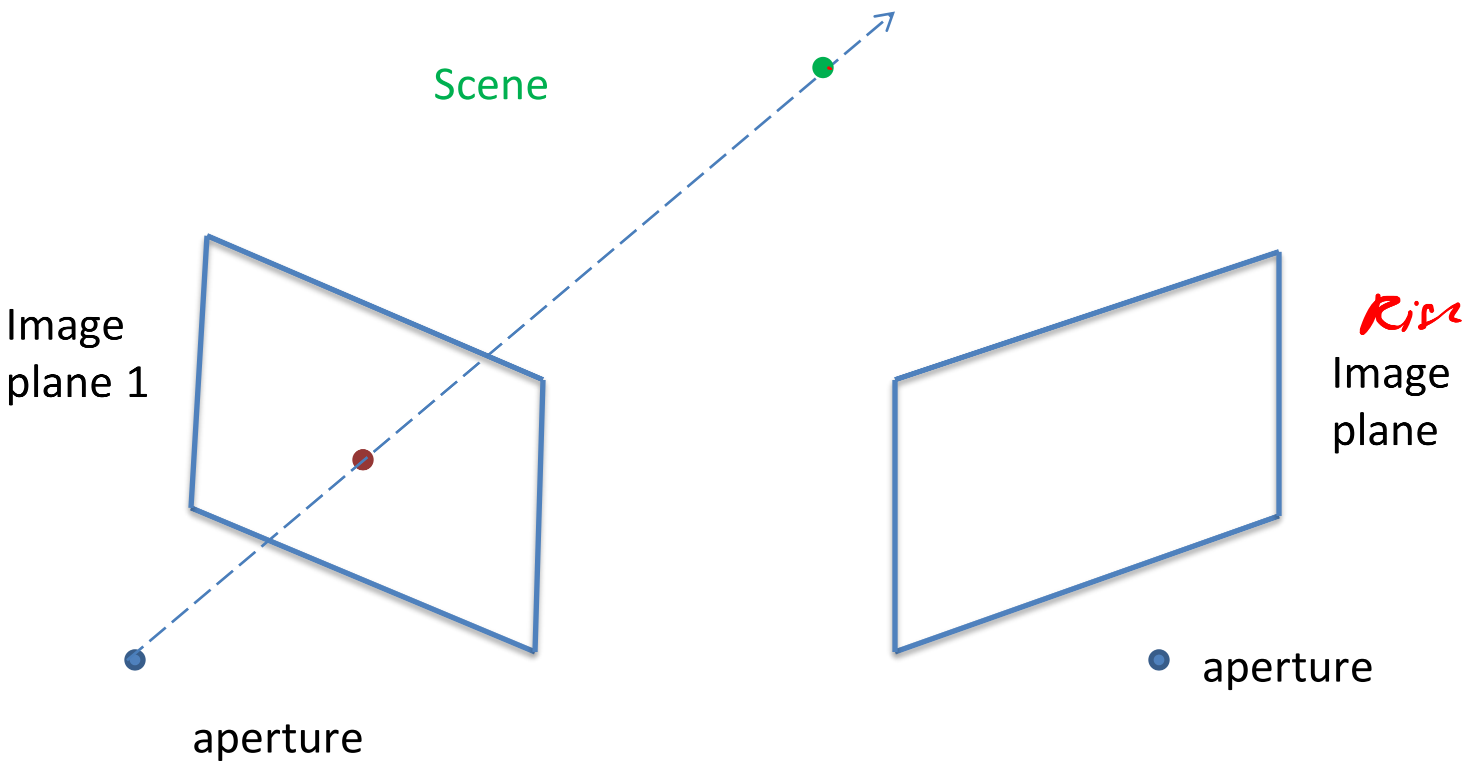
SET UP

- Finding correspondence for image plane 1



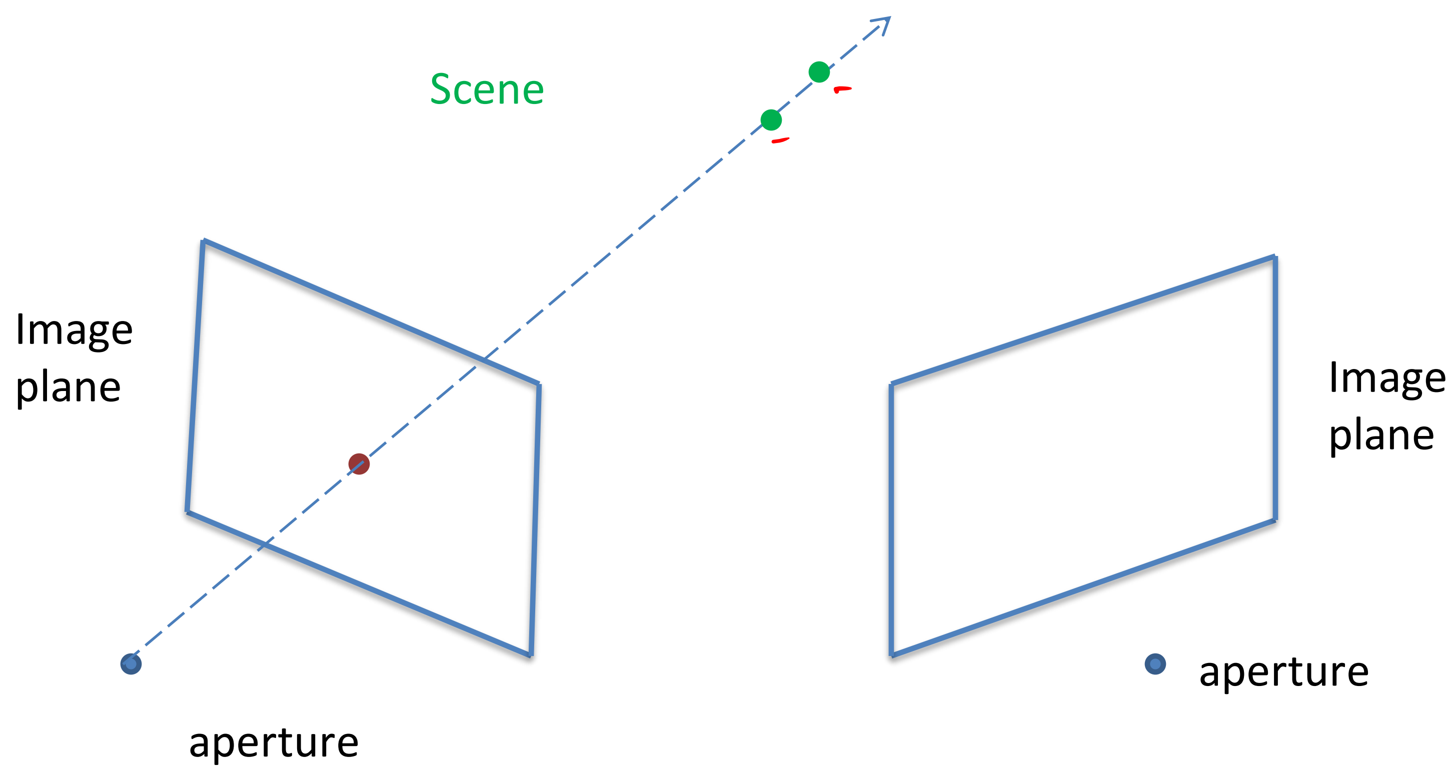
SET UP

- Finding correspondence for image plane 1



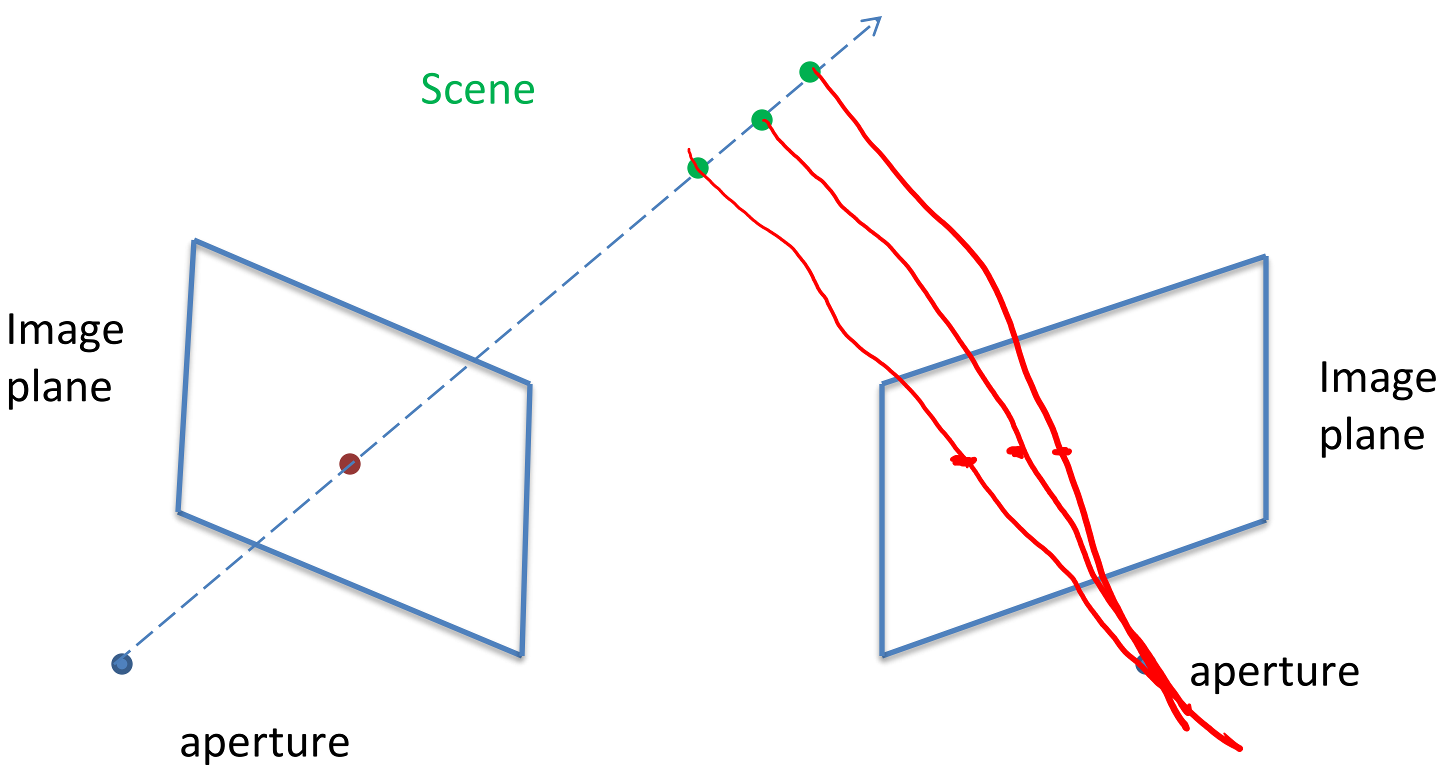
SET UP

- Finding correspondence for image plane 1



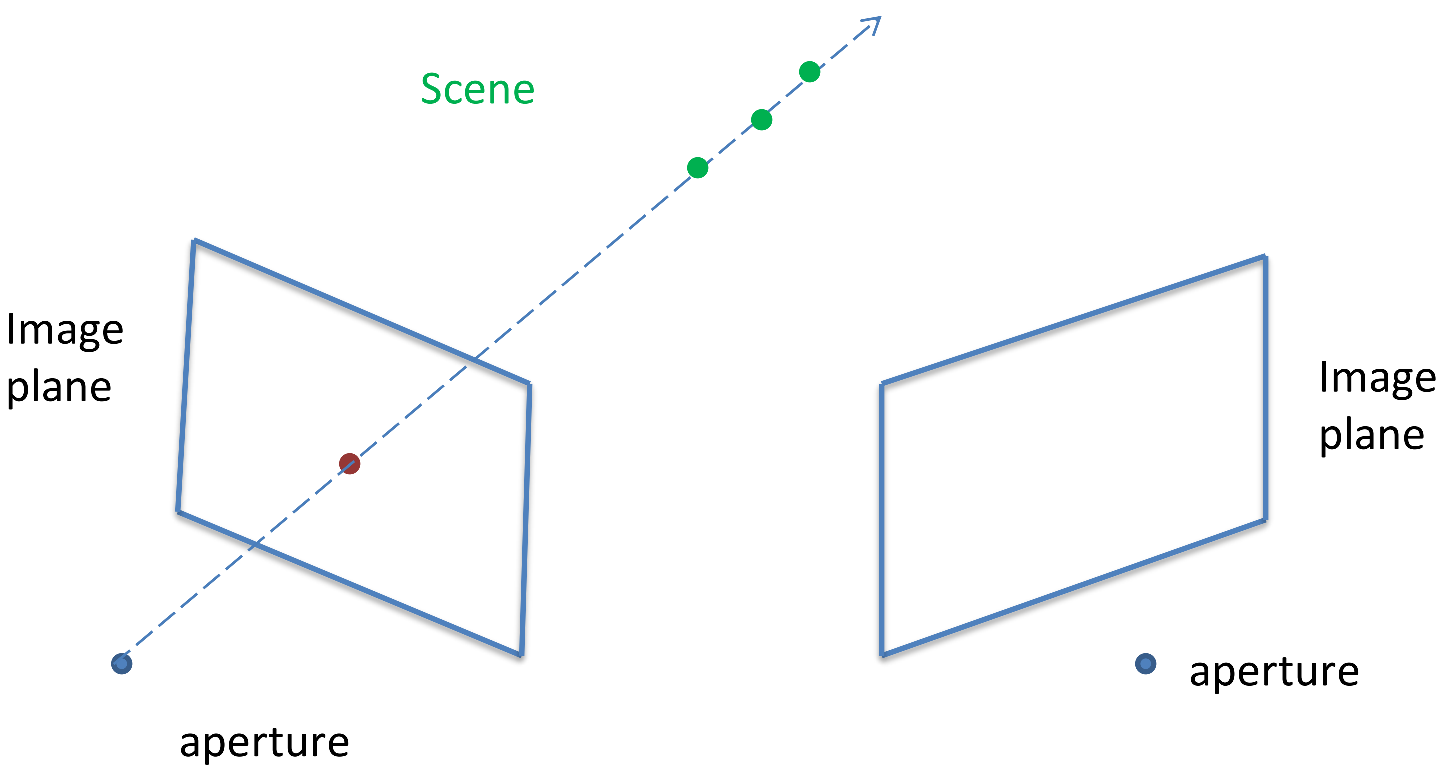
SET UP

- Finding correspondence for image plane 1



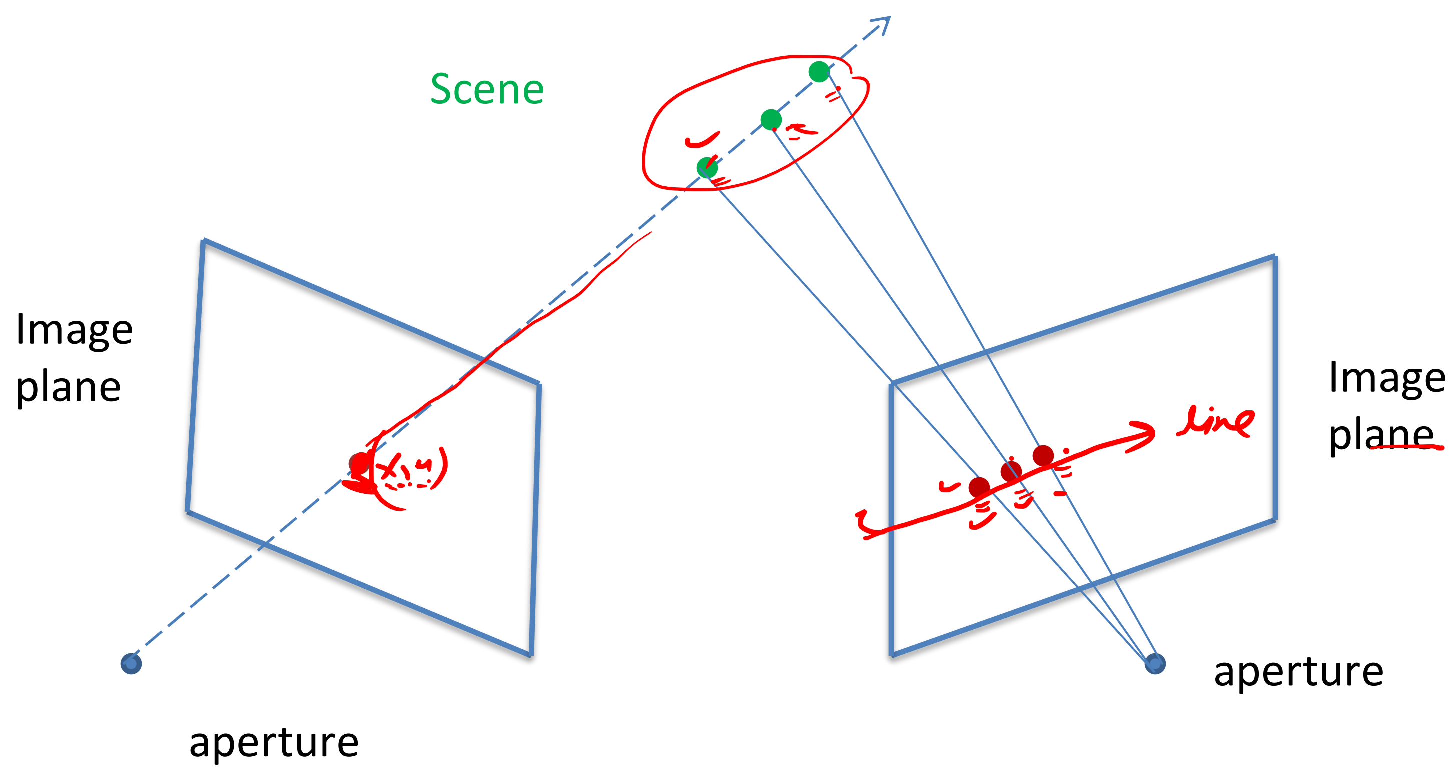
SET UP

- Finding correspondence for image plane 1



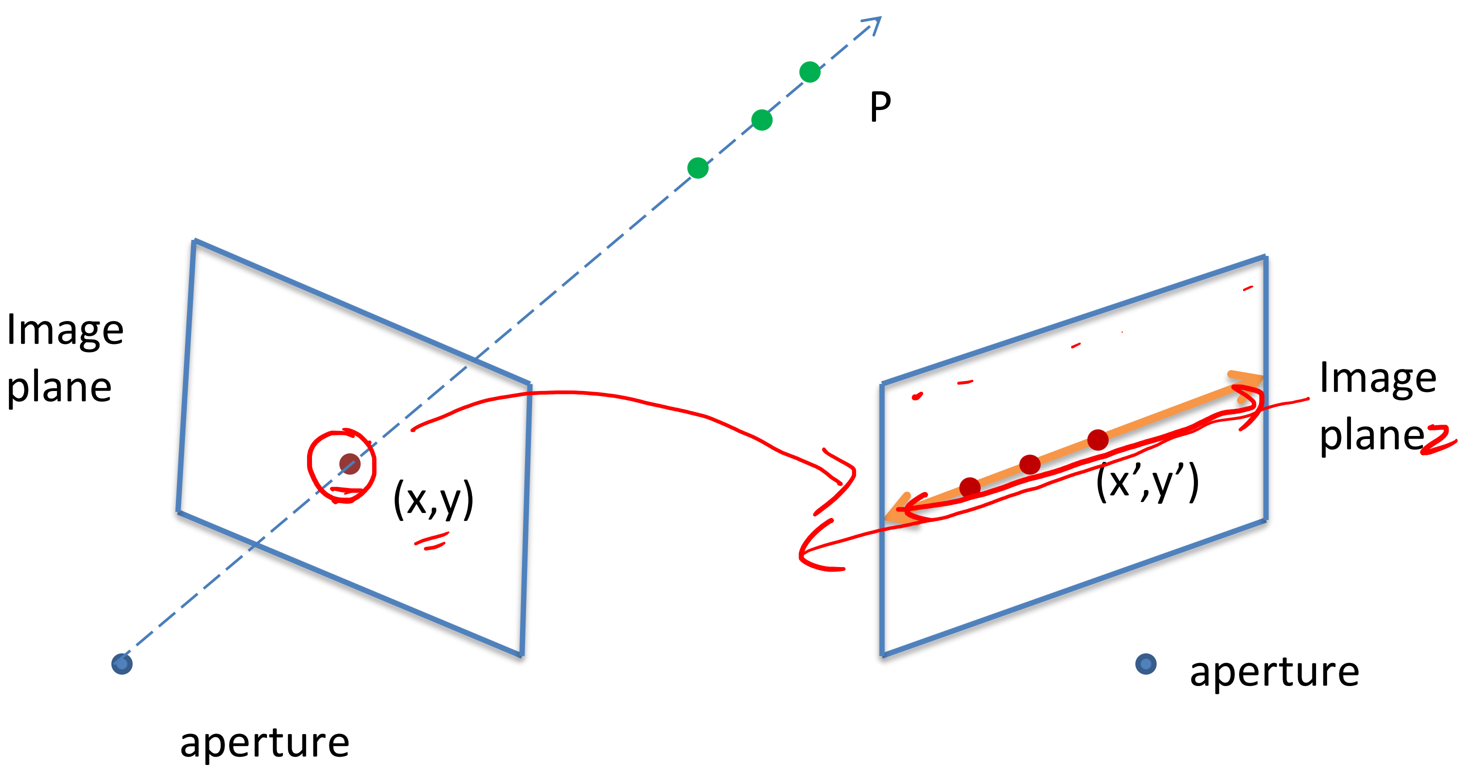
SET UP

- Finding correspondence for same points in by projecting back on image plane 2



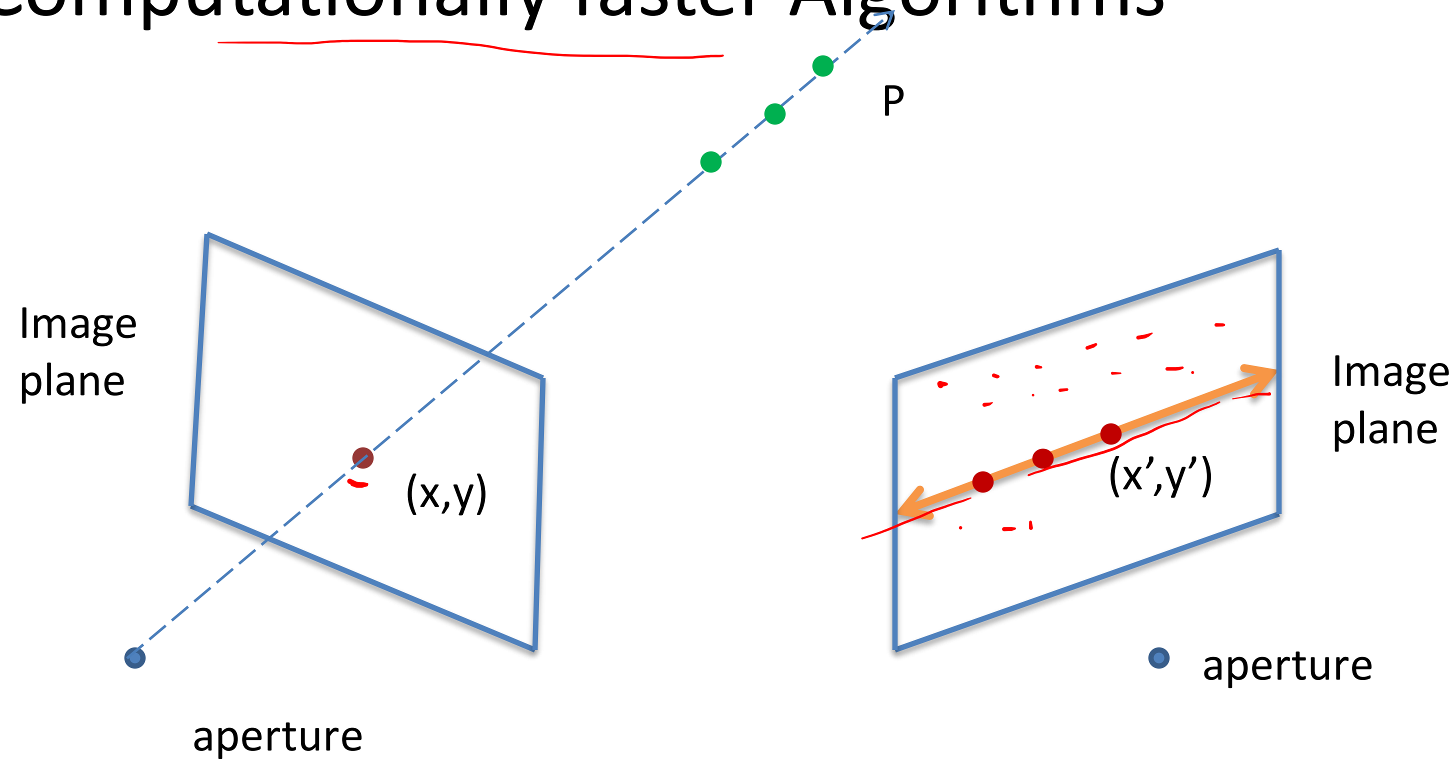
SET UP

- Point in image plane 1 will have a matching correspondence on the line in image plane 2.



SET UP

- Don't need to search the entire plane space for match…only the line……
- Computationally faster Algorithms

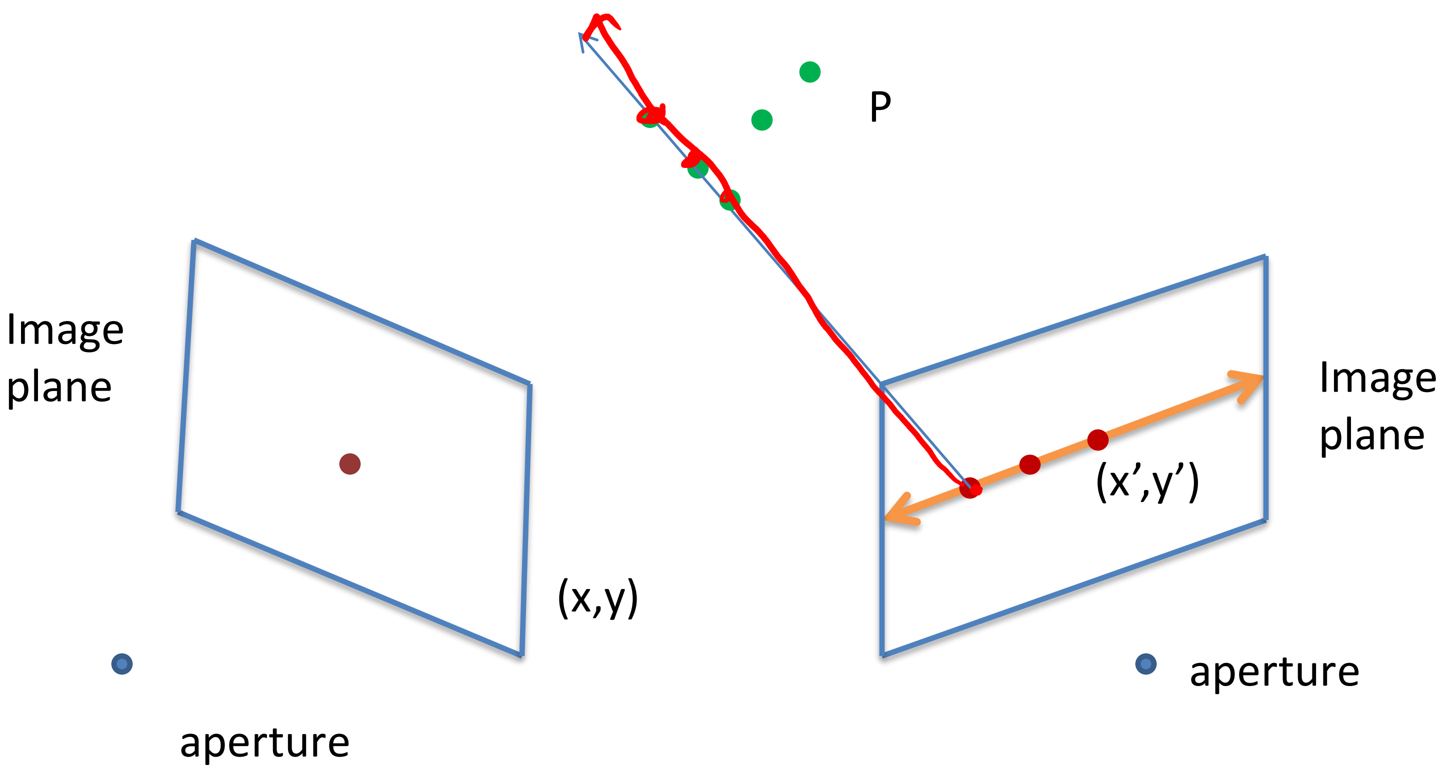


Note

- There exits a parallel and mutual relationship between the two images

EPIPOLAR LINES

- There exits a parallel and mutual relationship between the two images



EPIPOLAR LINES

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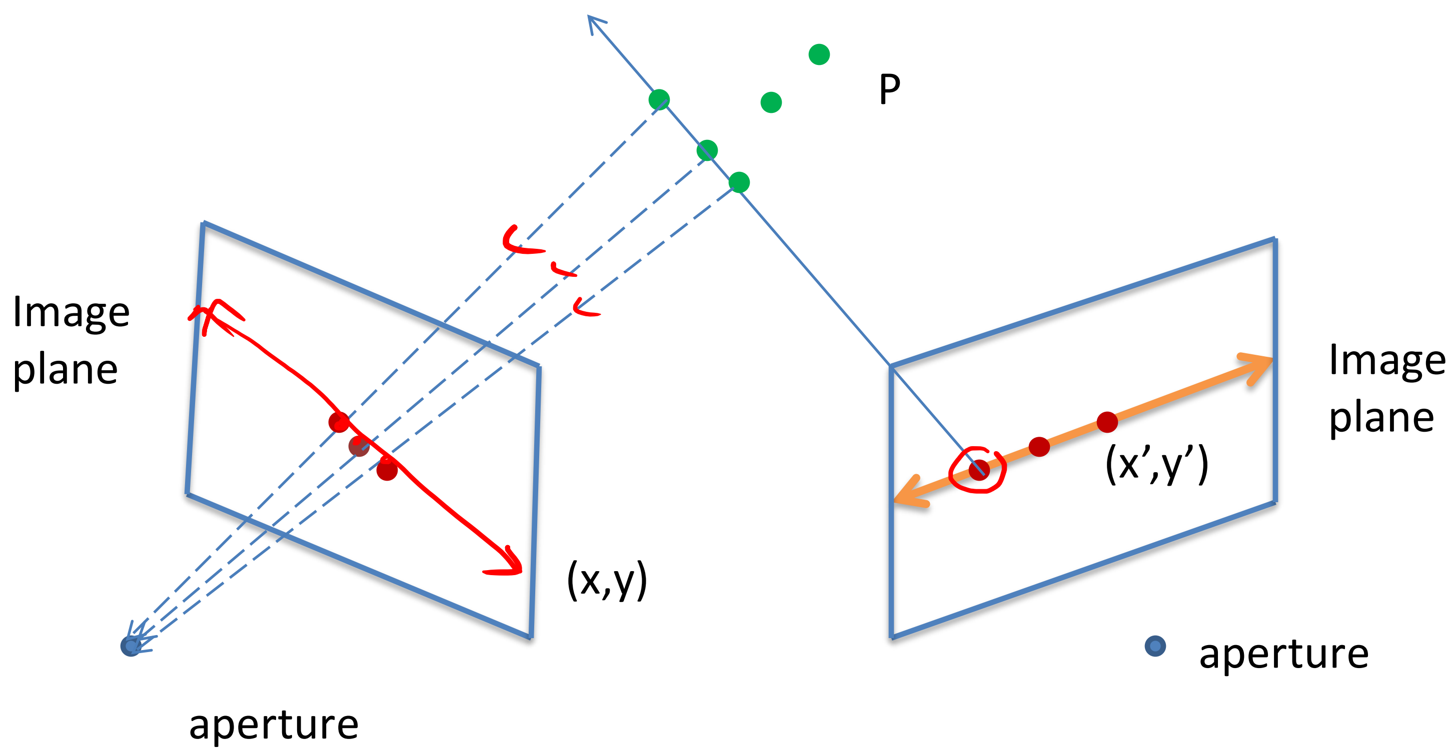

EPIPOLAR LINES

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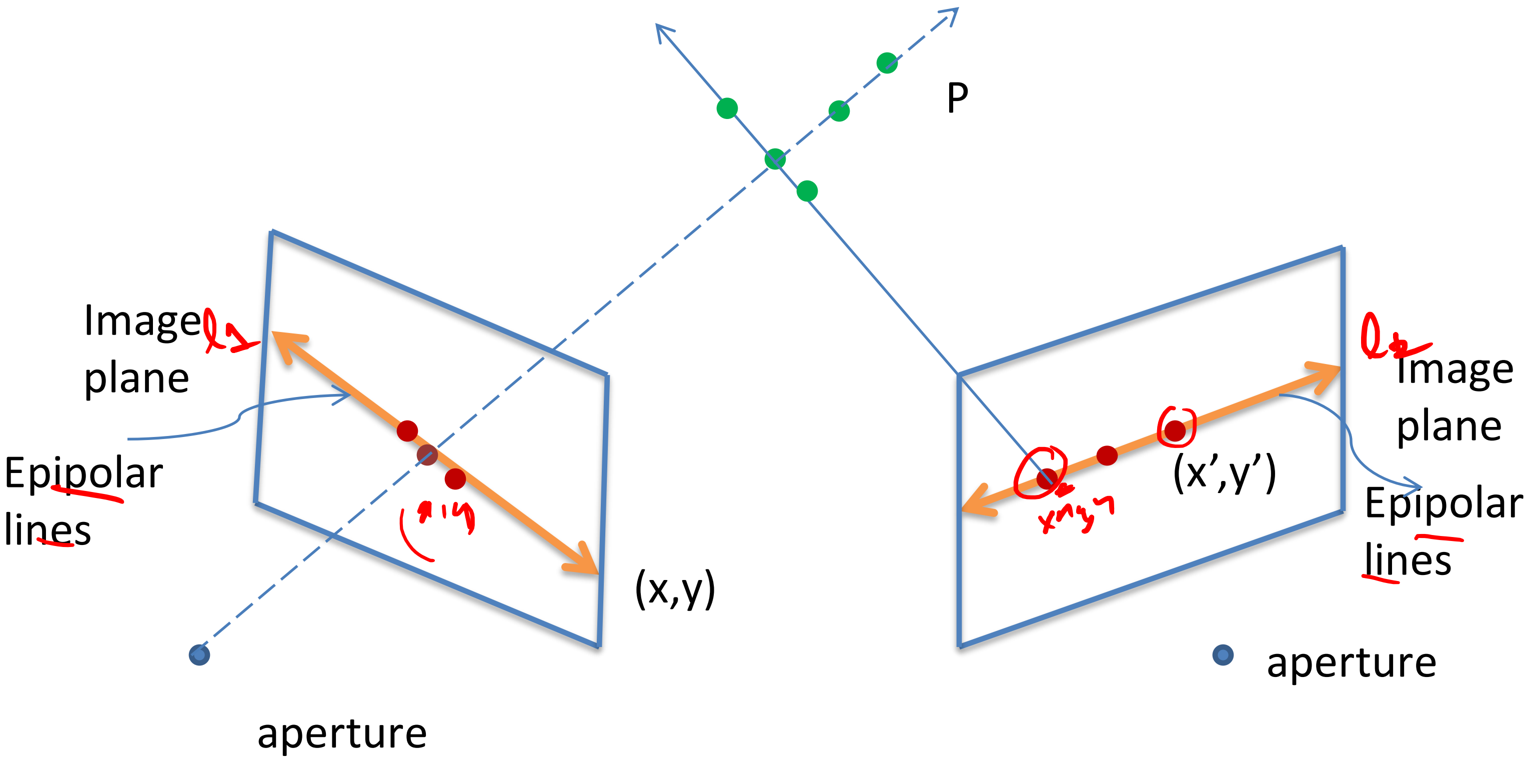
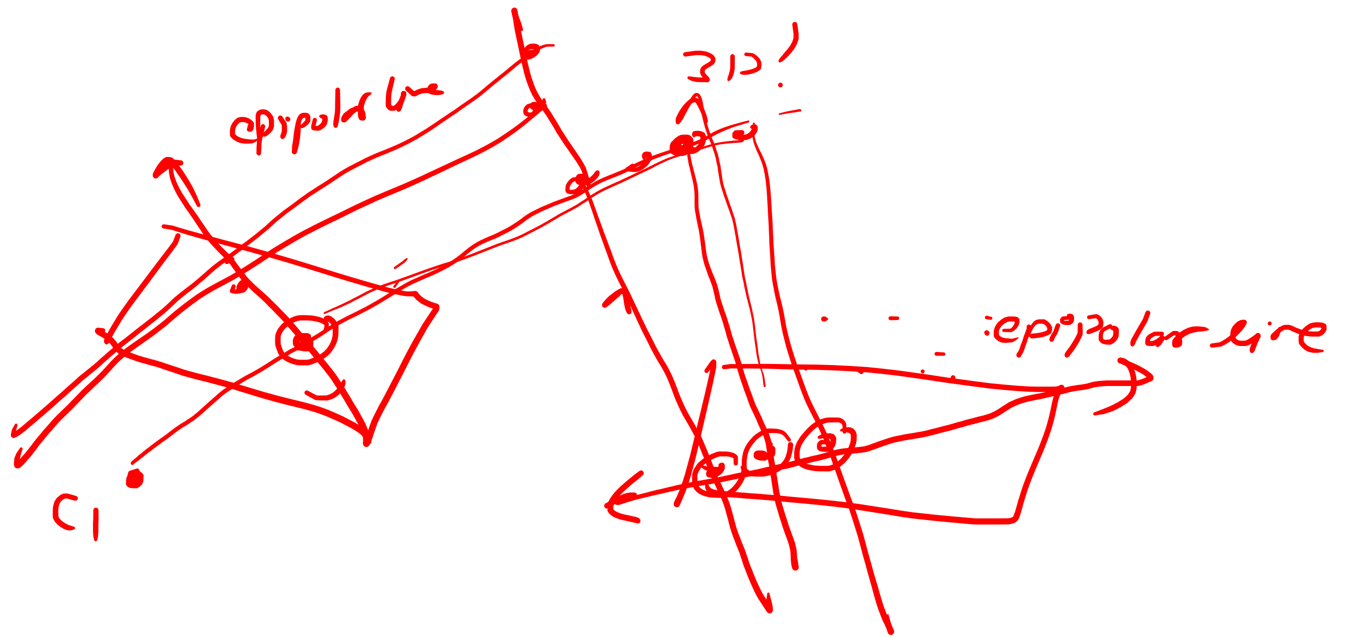
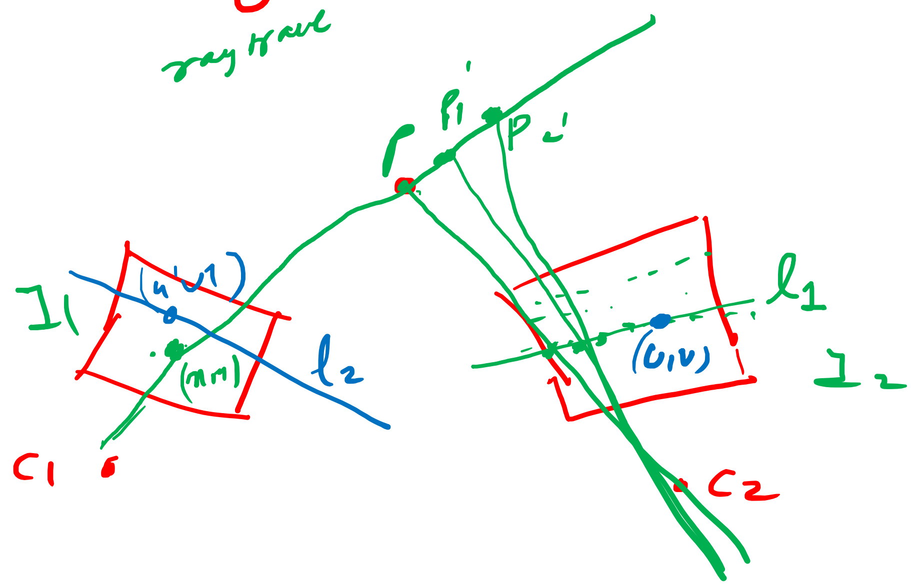



epipolar line
C1
3D?
epipolar line

- ~~Epilar~~ Epipolar lines –
- Epipolar plane –
- epipoles –

Epipolar Geometry (Multiview Geometry)



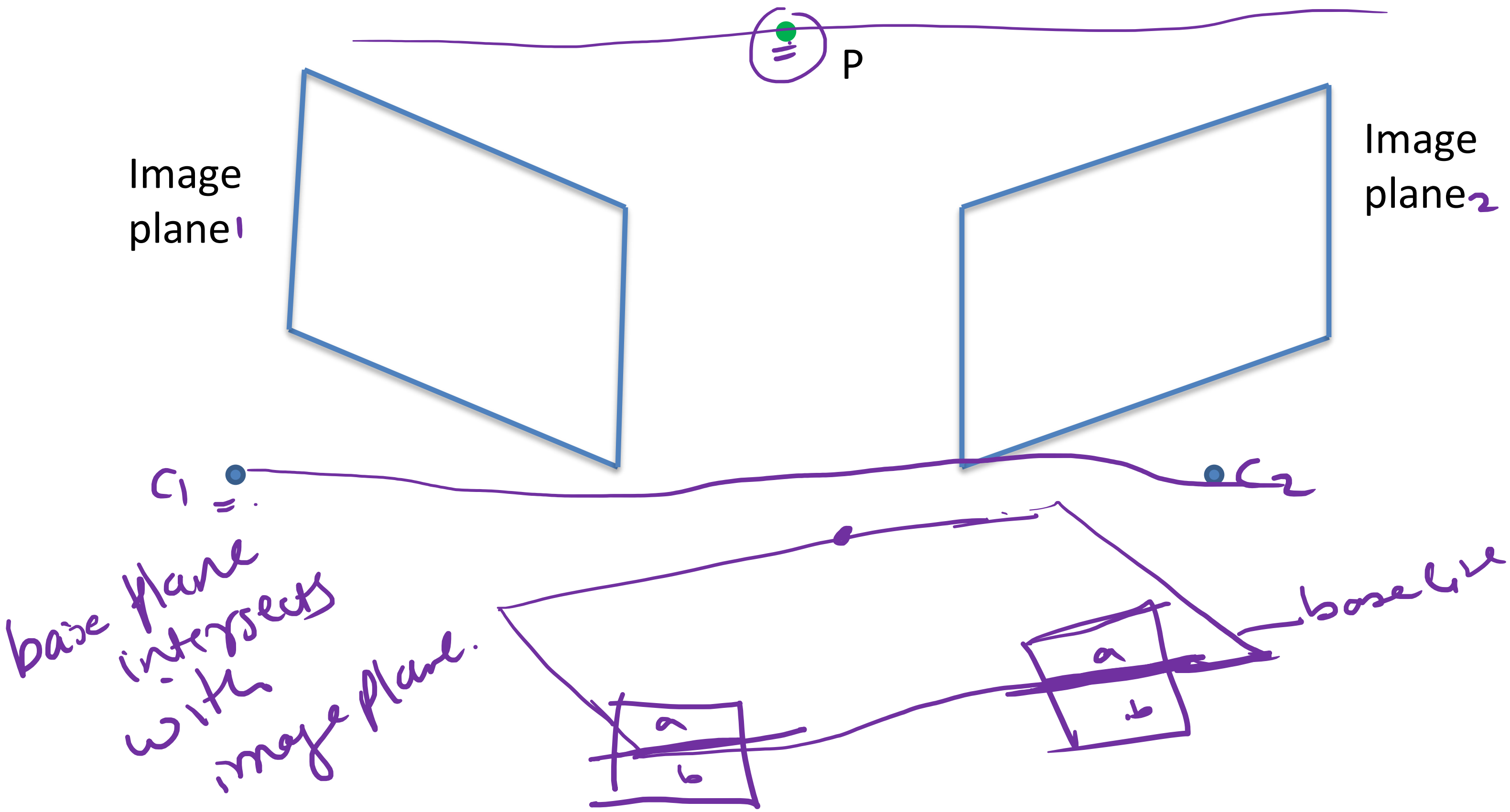
I have only the two Images I_1 & I_2

- where is the correspondance of (x,y) located in Img I_2

- search for $(x,y) \in I_1$ in I_2?
- look along epipolar line l_1 in I_2
- correspondance of $\overset{\in I_1}{(x,y)}$ $\longrightarrow$ somewhere along the line l_1 in I_2

EPIPOLAR LINES

- How are these lines traced up for a 3D point 'P'



EPIPOLAR LINES

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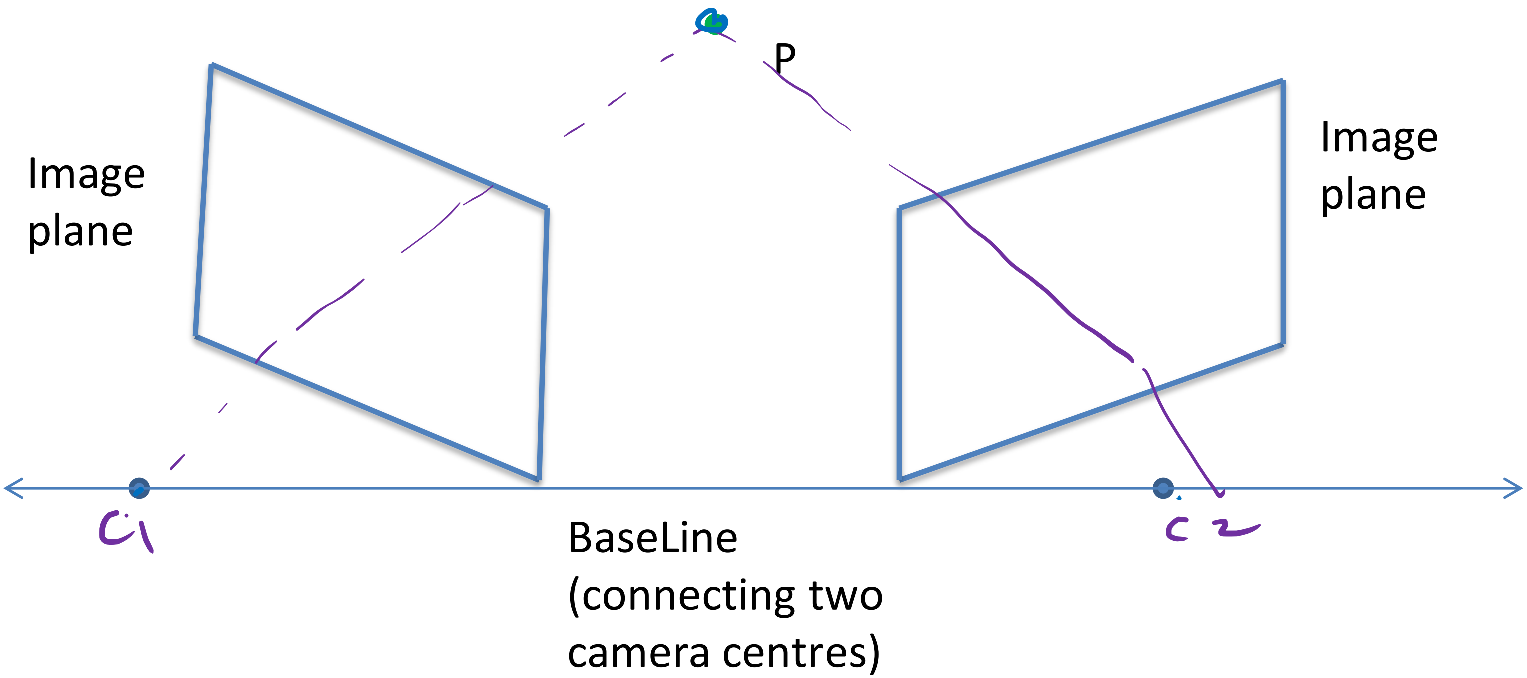
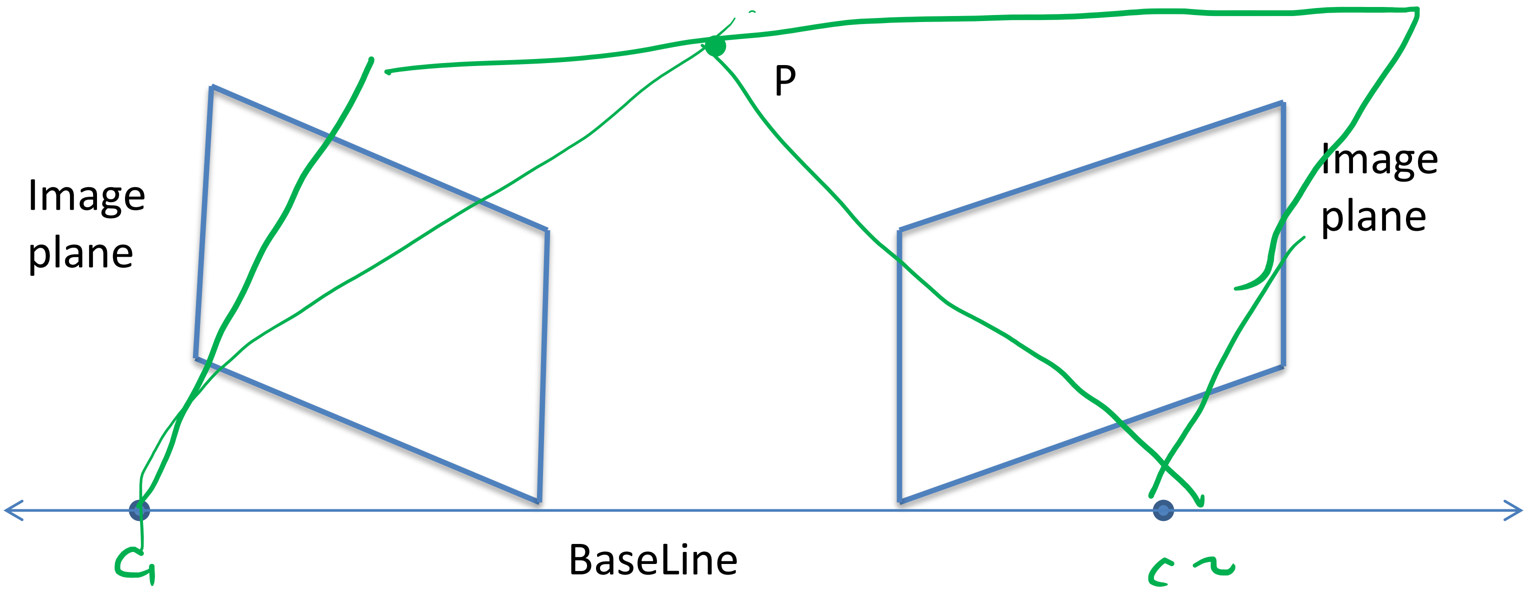


EPIPOLAR LINES

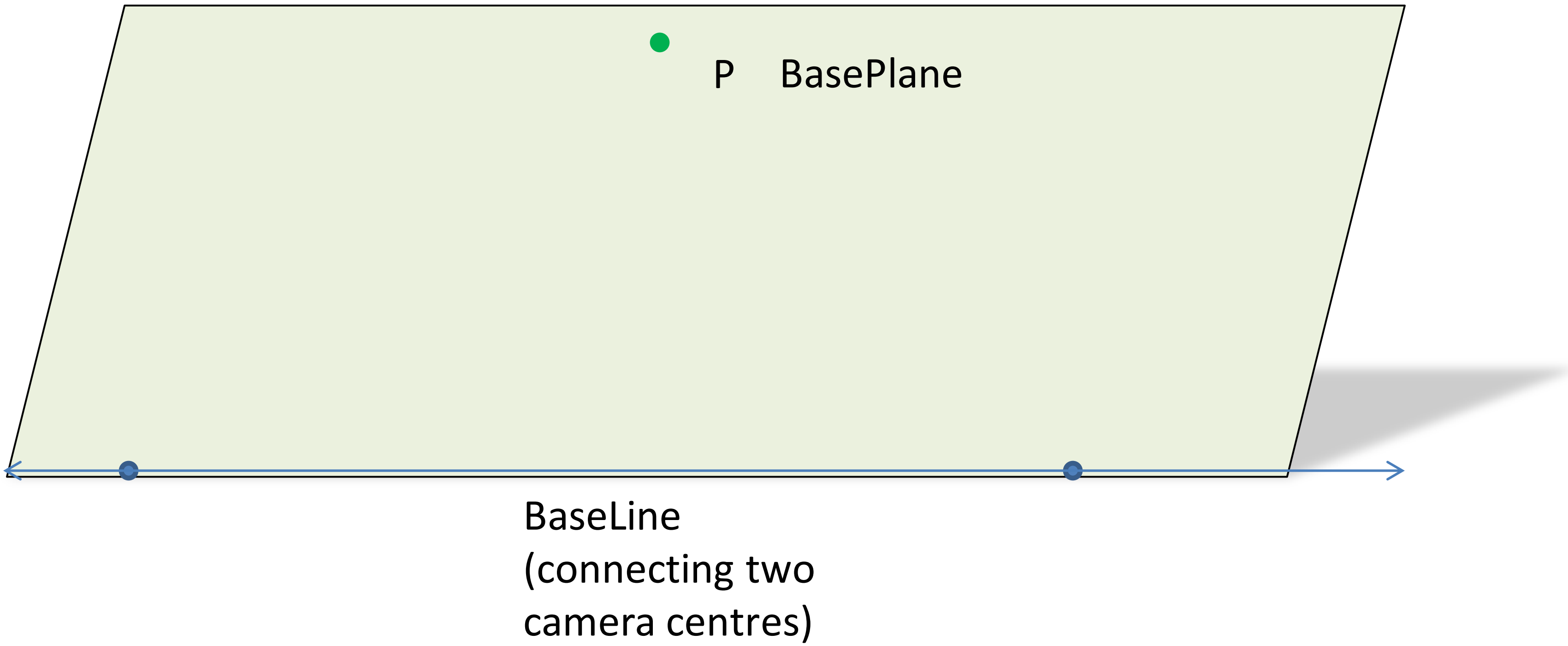
- How are these lines traced up for a 3D point ‘P’



- Find the BaseLine (connecting two camera centres)

EPIPOLAR LINES

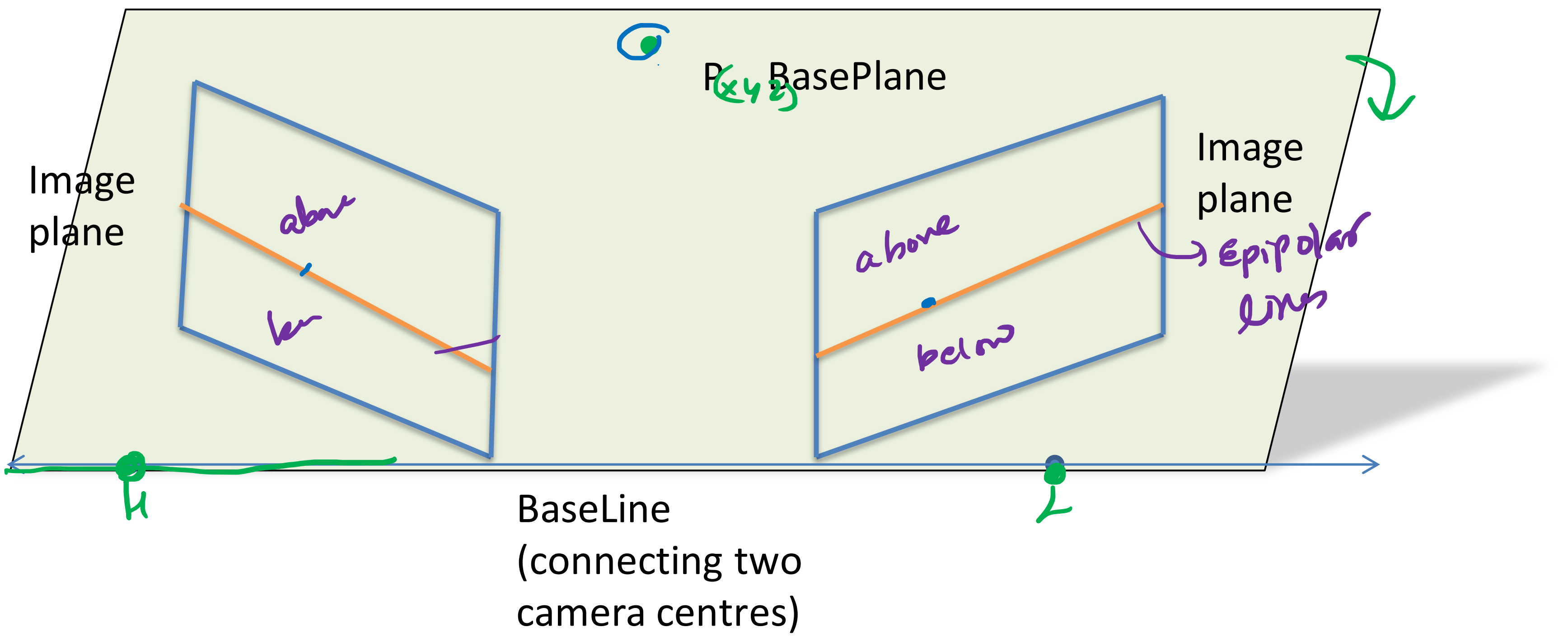
- How are these lines traced up for a 3D point ‘P’



- Find the plane passing through base line and point P

EPIPOLAR LINES

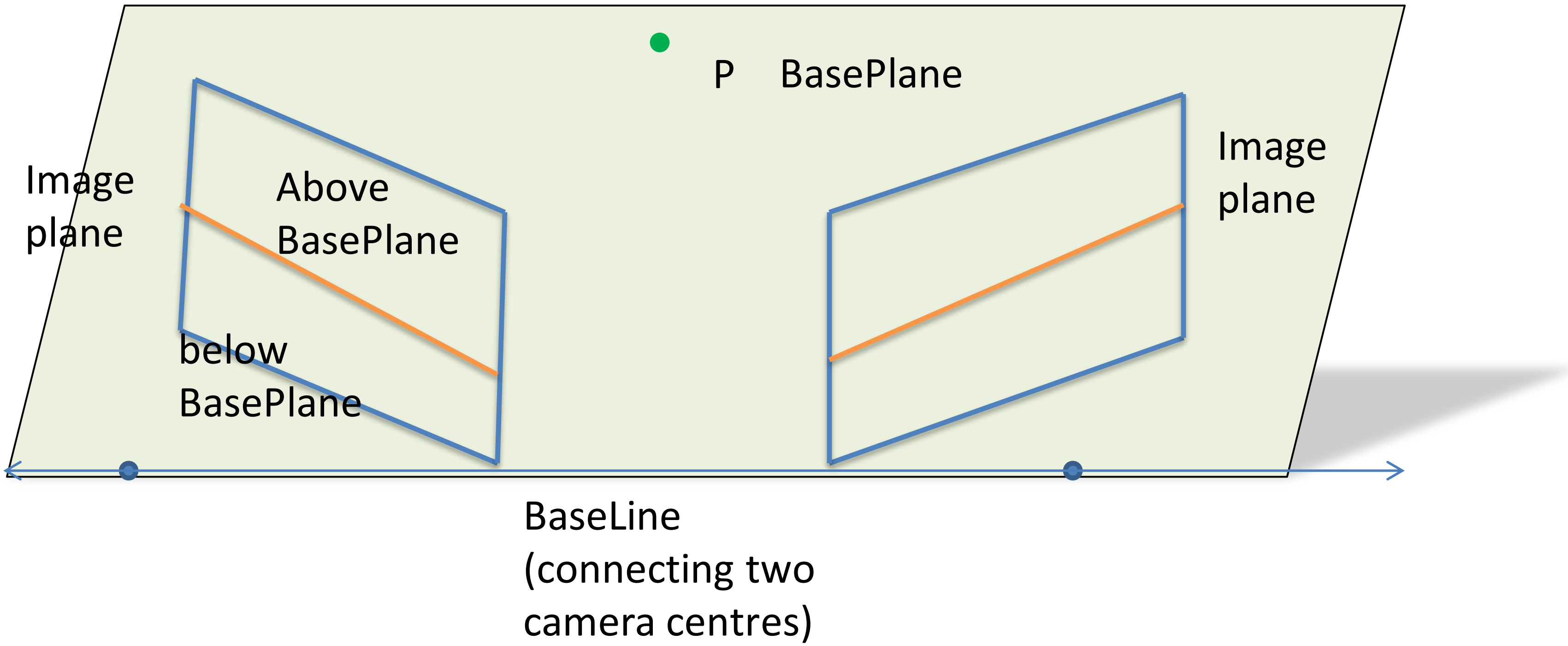
- How are these lines traced up for a 3D point 'P'



- Find the intersection of image planes with base plane.The lines of intersection are epipolar lines

EPIPOLAR LINES

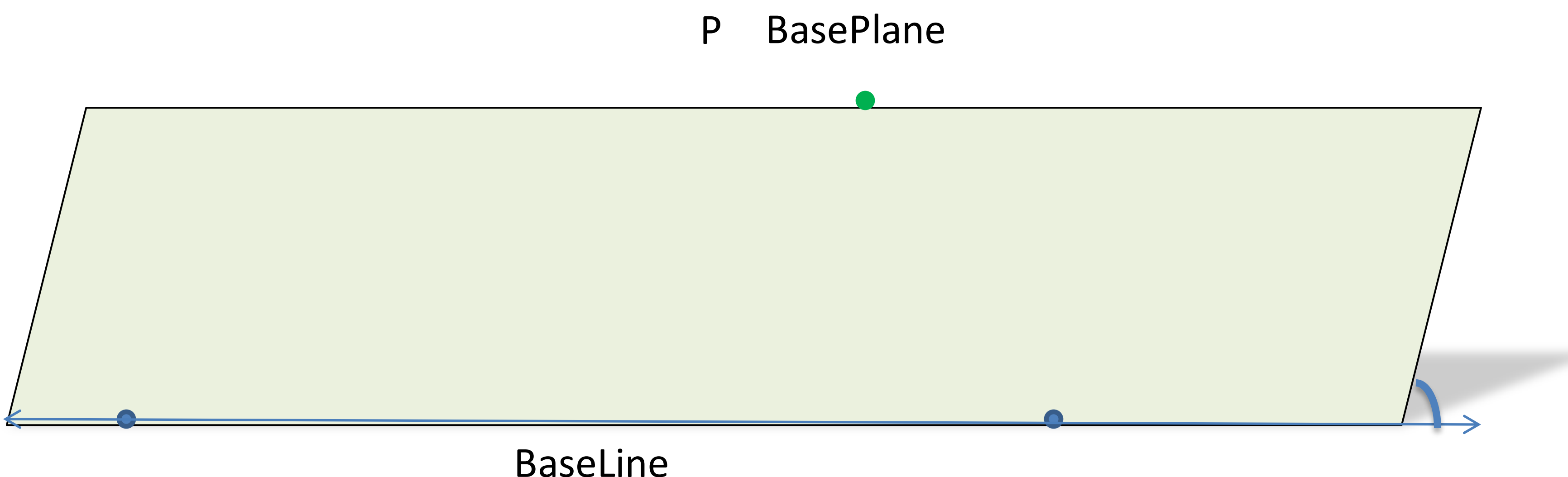
- How are these lines traced up for a 3D point ‘P’

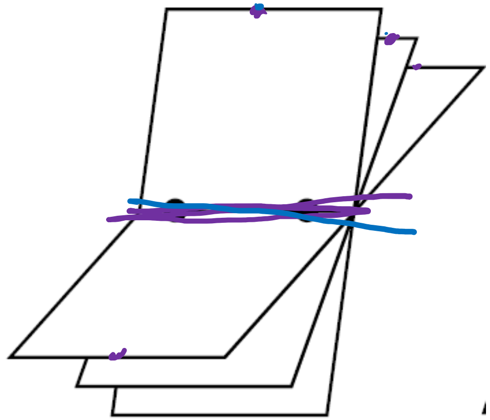


- Find the intersection of image planes with base plane.The lines of intersection are epipolar lines

EPIPOLAR LINES for Different Points

- Base plane will be oriented differently for each a 3D point 'P'





EPIPOLAR LINES for Different Points

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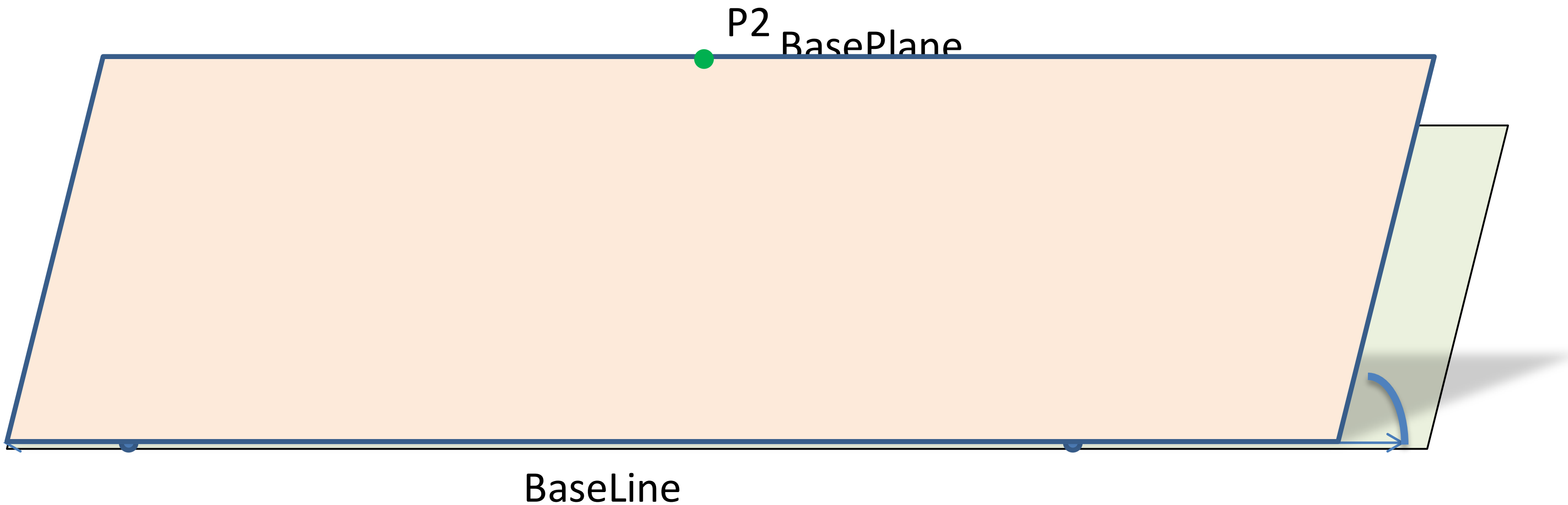
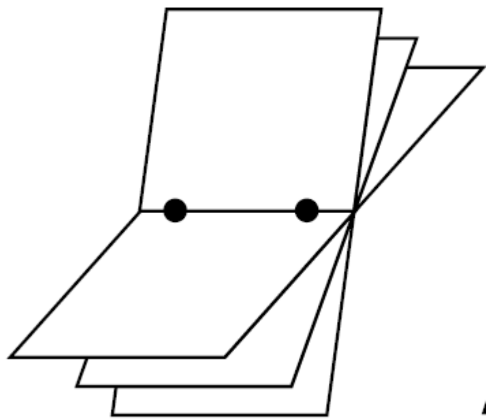
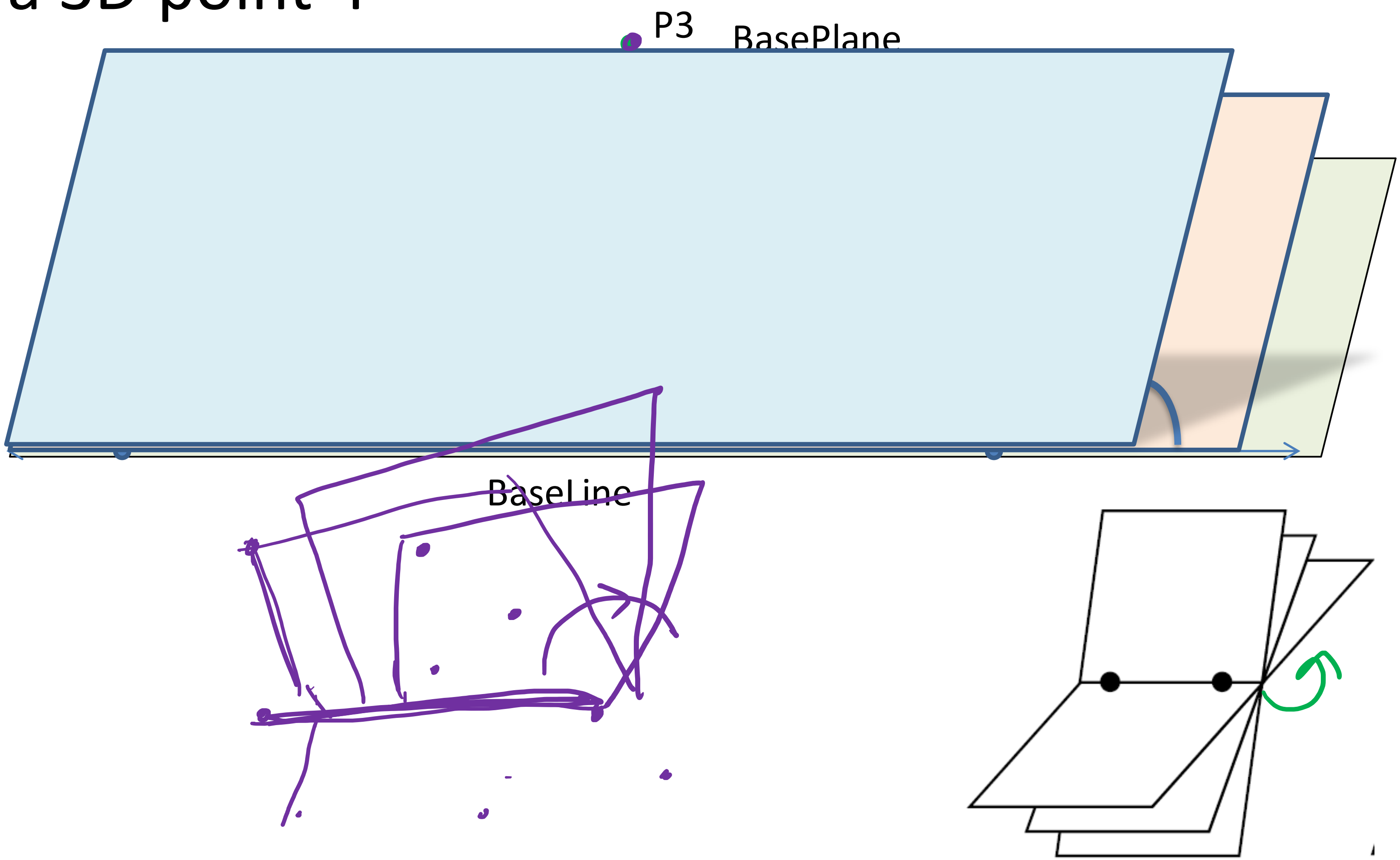


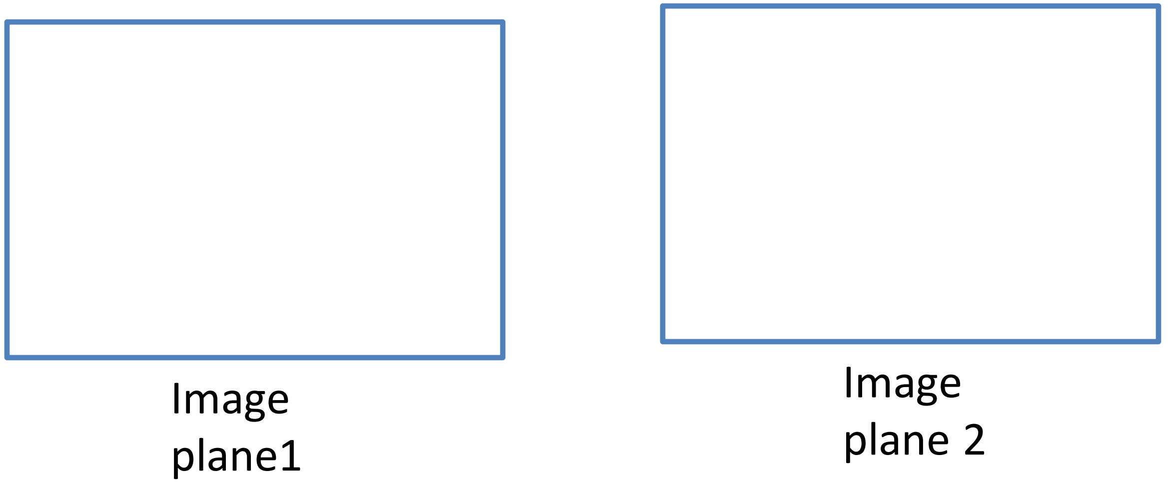


EPIPOLAR LINES for Different Points

- Base plane will be oriented differently for each a 3D point 'P'



Mutual Correspondance



Mutual Correspondance

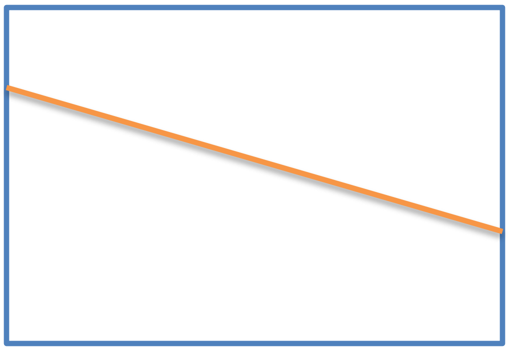


Image
plane1

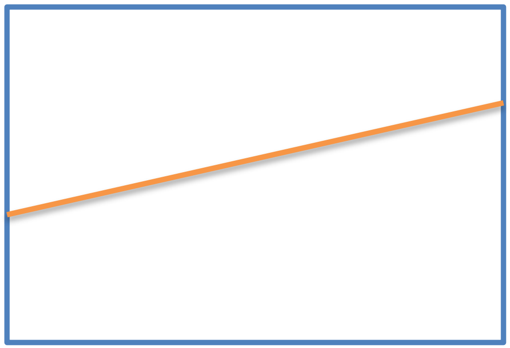
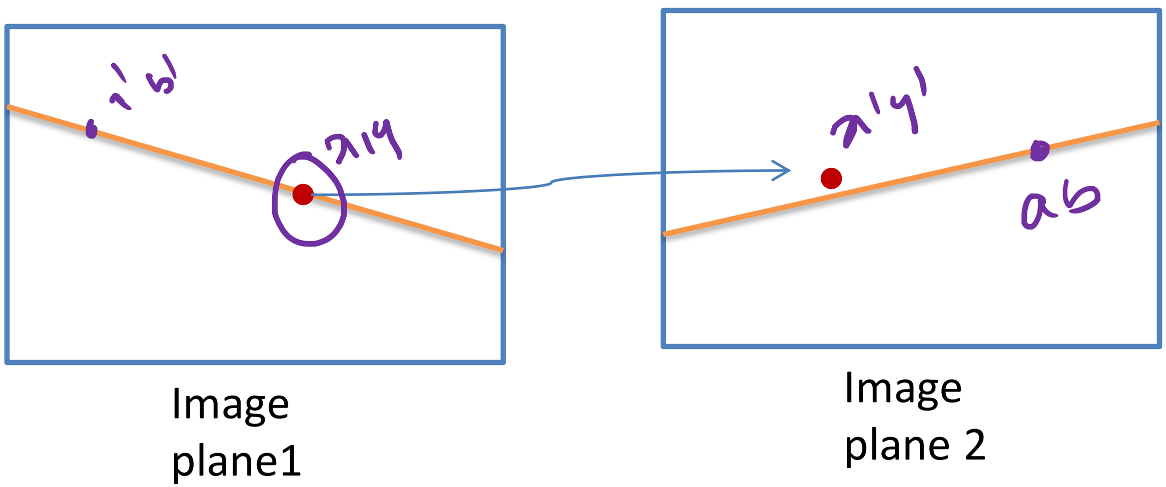
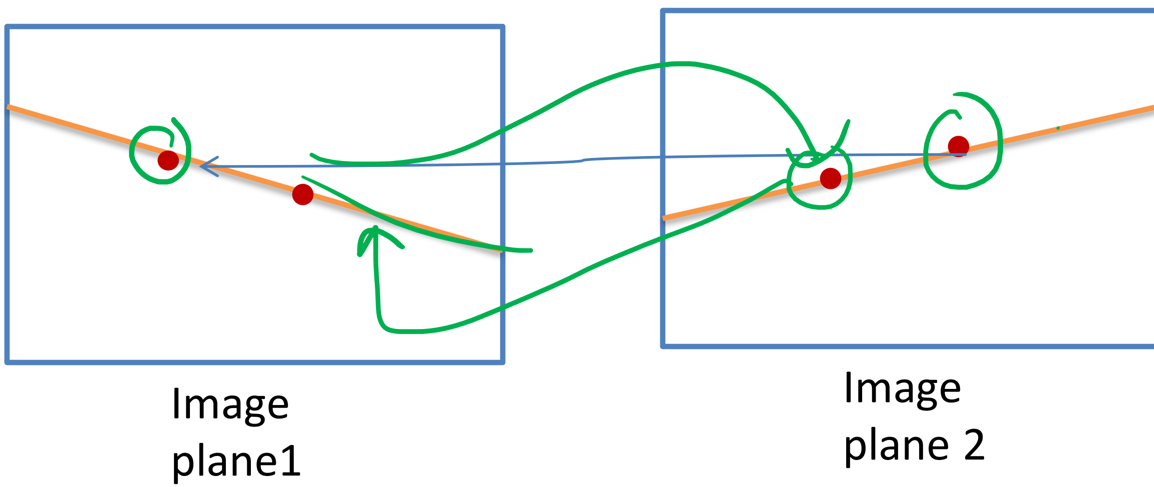


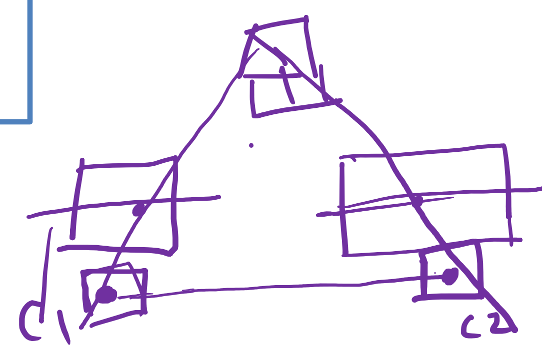
Image
plane 2

Mutual Correspondance



Mutual Correspondance





- Conjugate Epipolar Lines
- Search for correspondence is constrained and efficient

Mutual Correspondance

Homogeneous Coordinate System

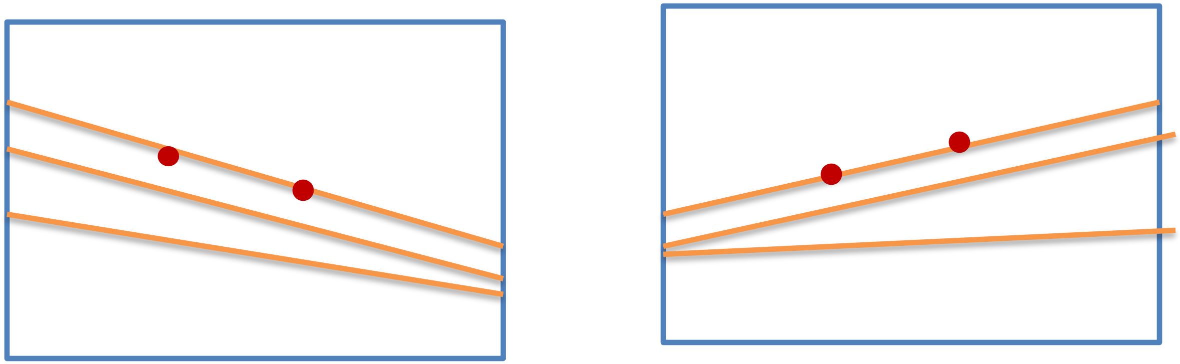
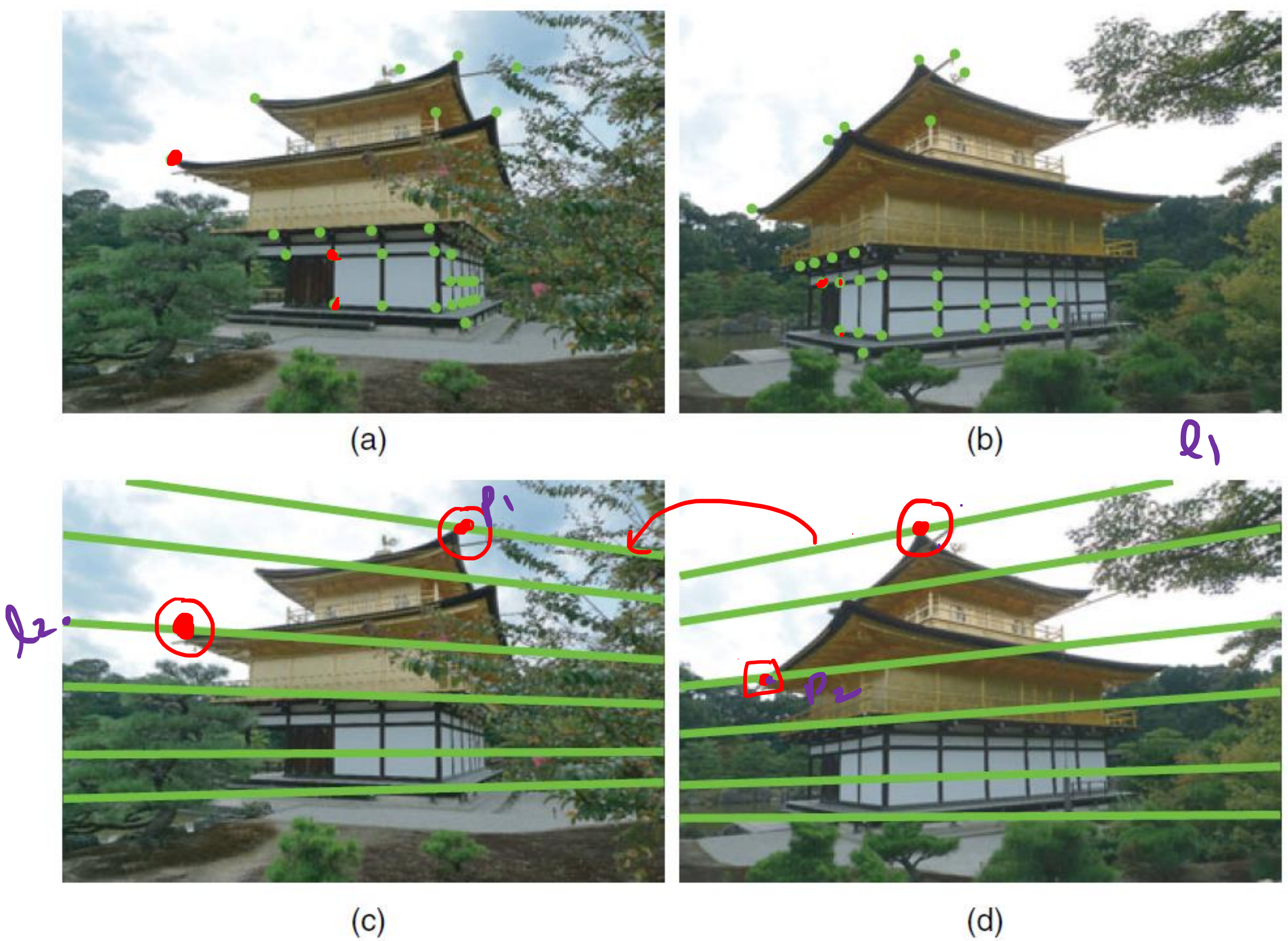


Image
plane1

Image
plane 2

- Conjugate Epipolar Lines
- Search for correspondence is constrained and efficient

Mutual Correspondance



(a) (b)

(c) (d)

Note

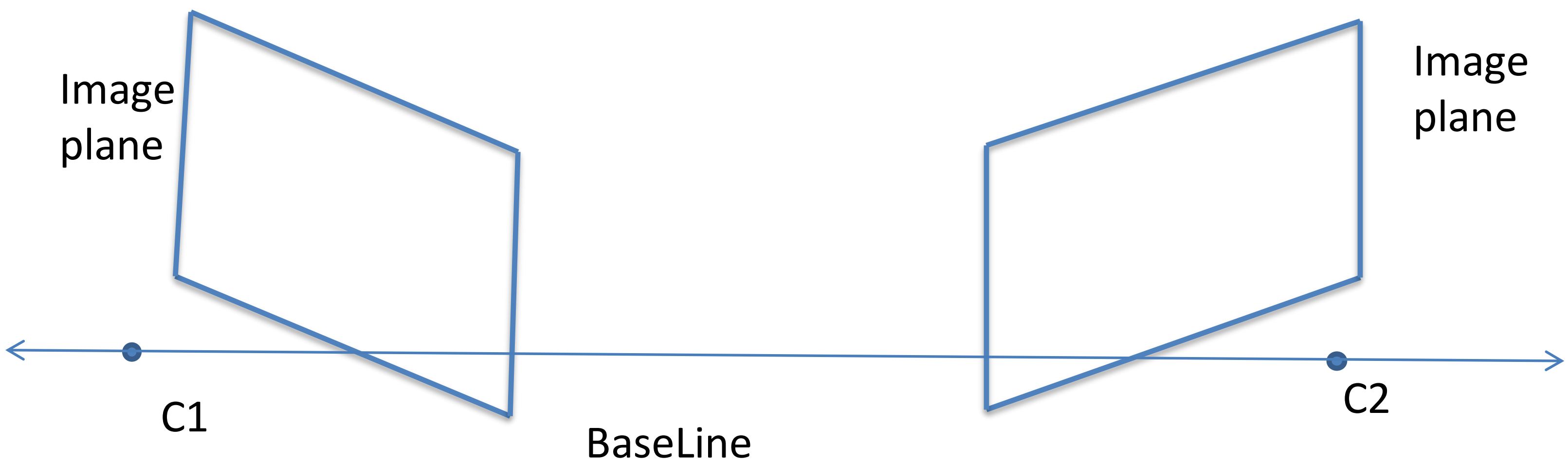
- Why are epipolar line slanted?

EPIPOLAR LINES

- If the cameras are in-field of view of each other

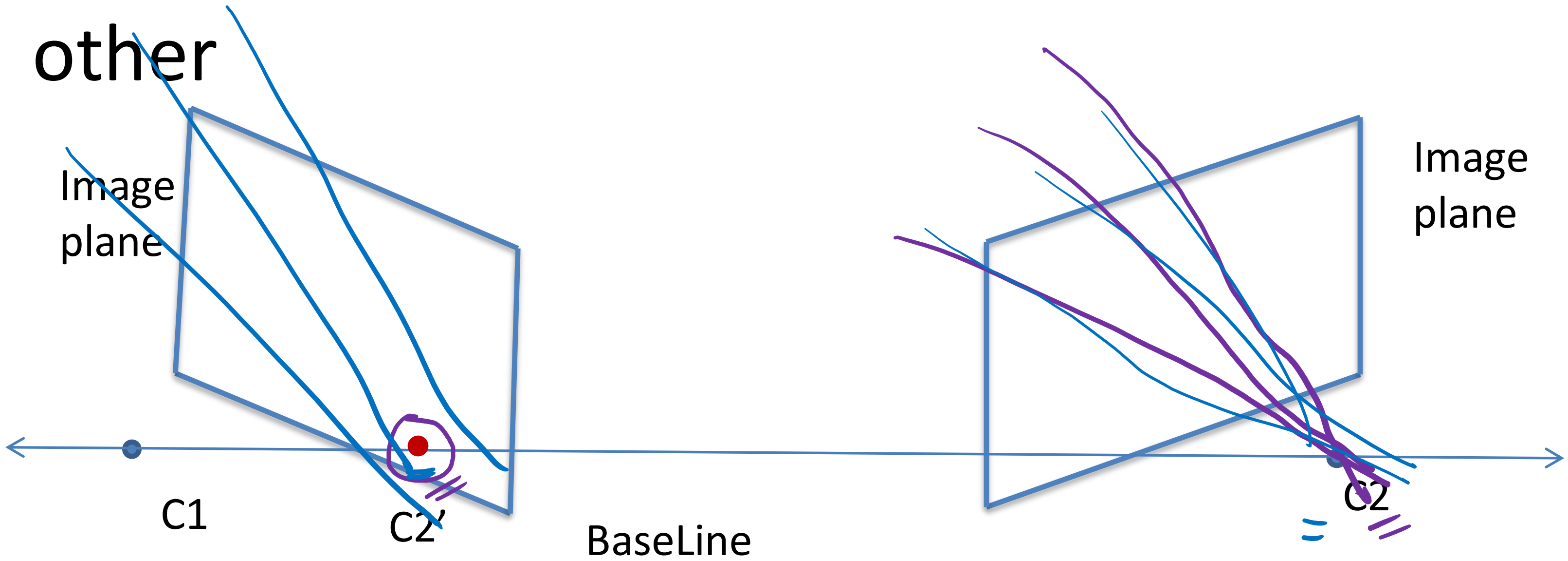
EPIPOLAR LINES

- If the cameras are in-field of view of each other



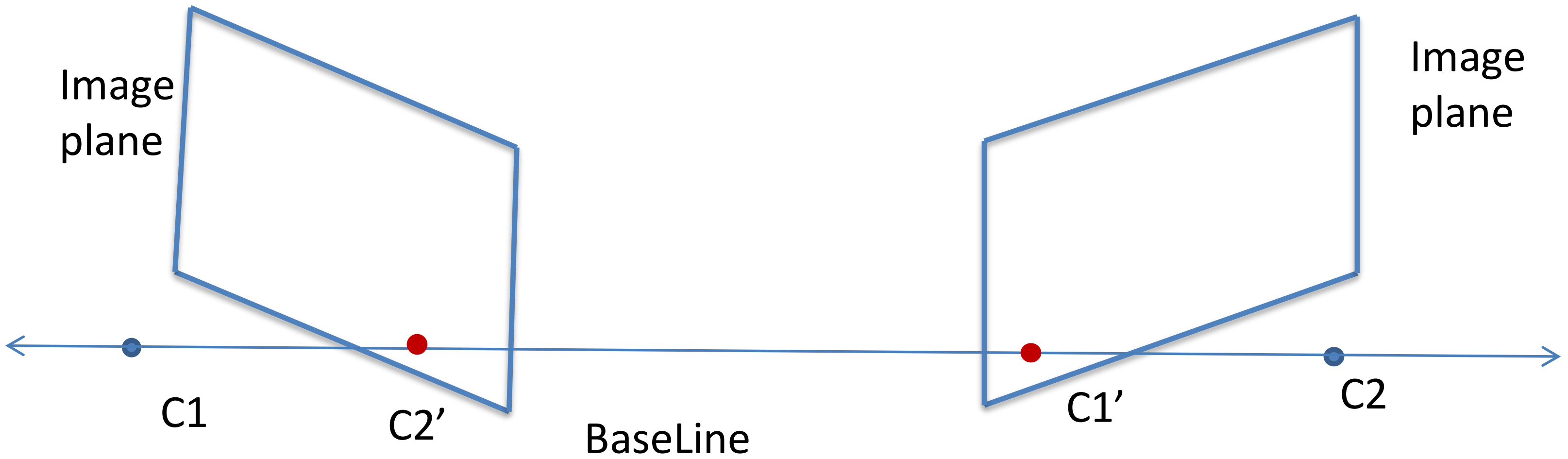
EPIPOLES & EPIPOLAR LINES

- If the cameras are in-field of view of each other



EPIPOLES & EPIPOLAR LINES

- If the cameras are in-field of view of each other



EPIPOLES & EPIPOLAR LINES

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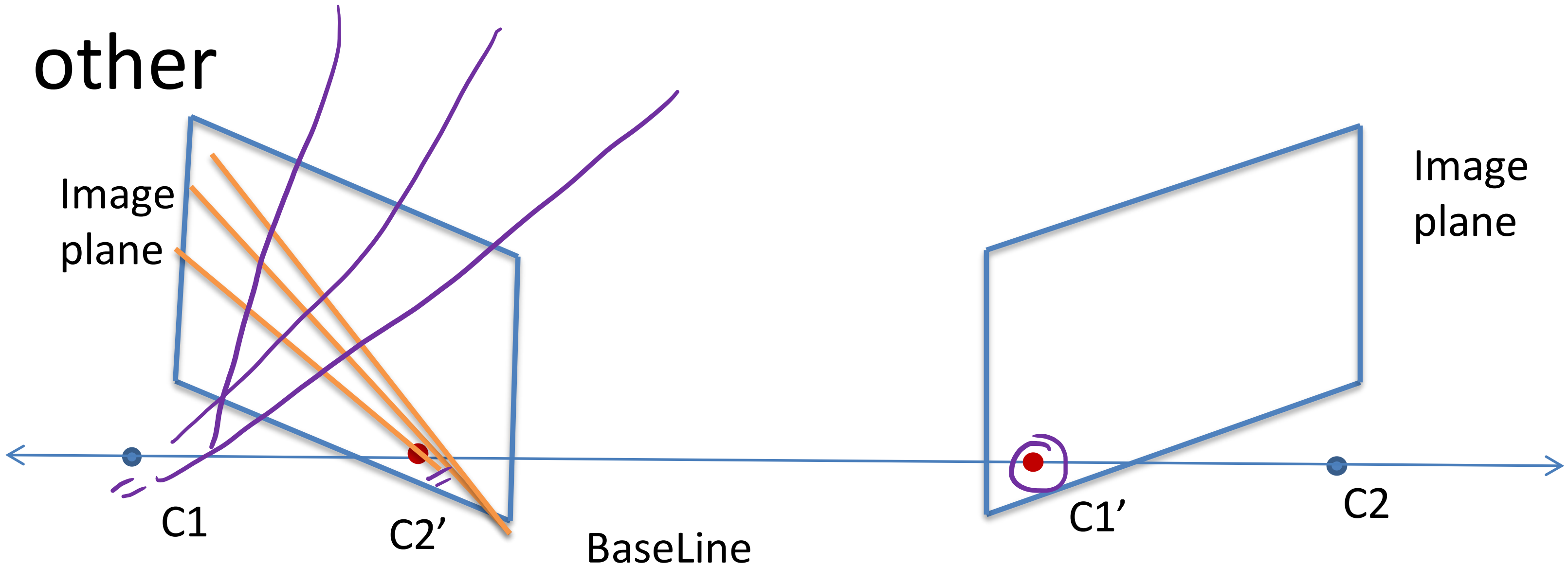
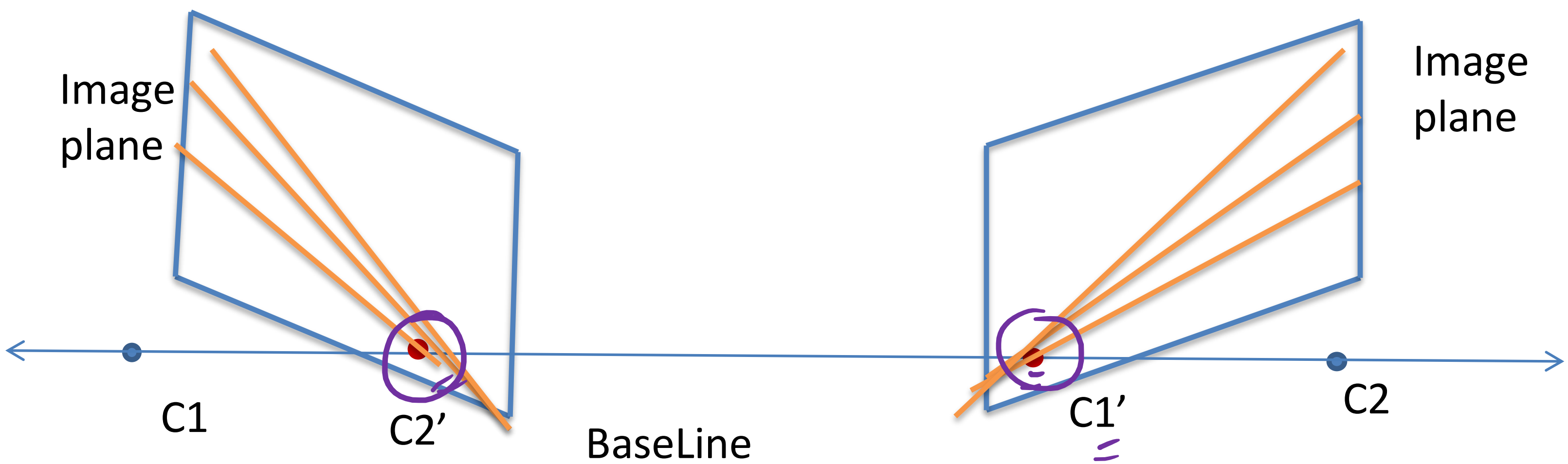


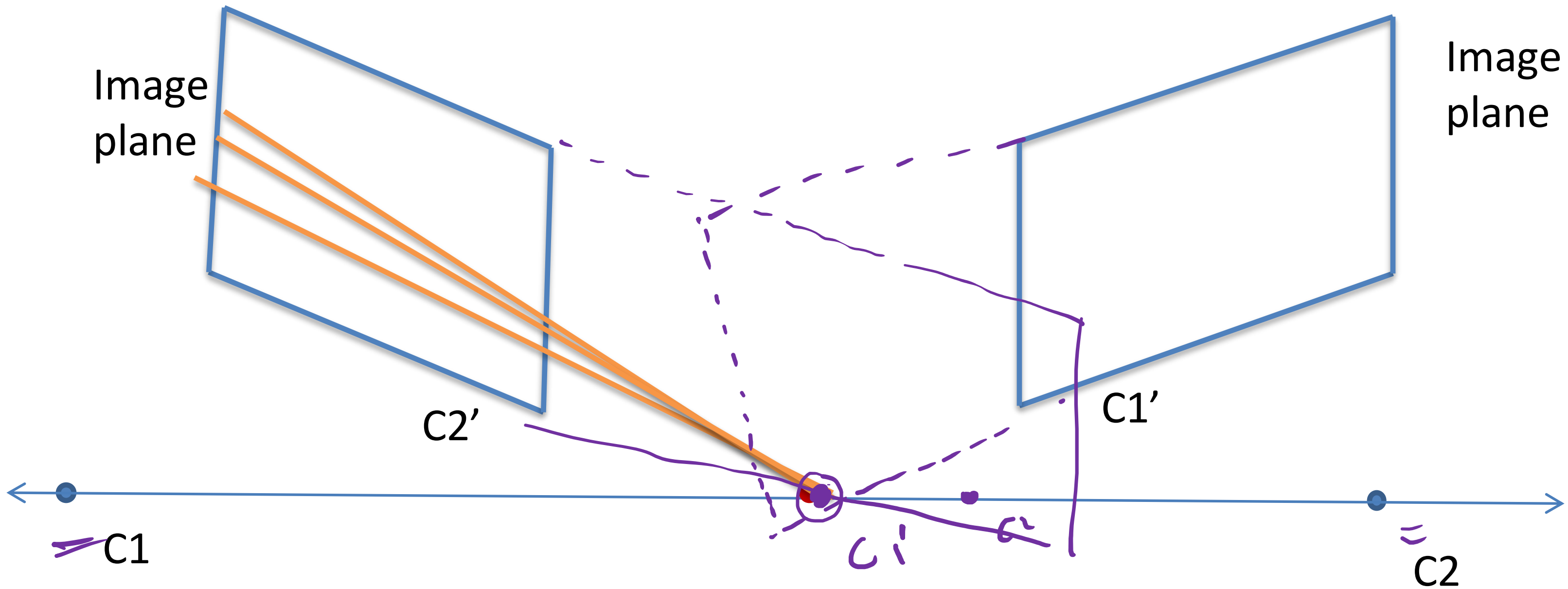
EPIPOLES & EPIPOLAR LINES

- If the cameras are in-field of view of each other



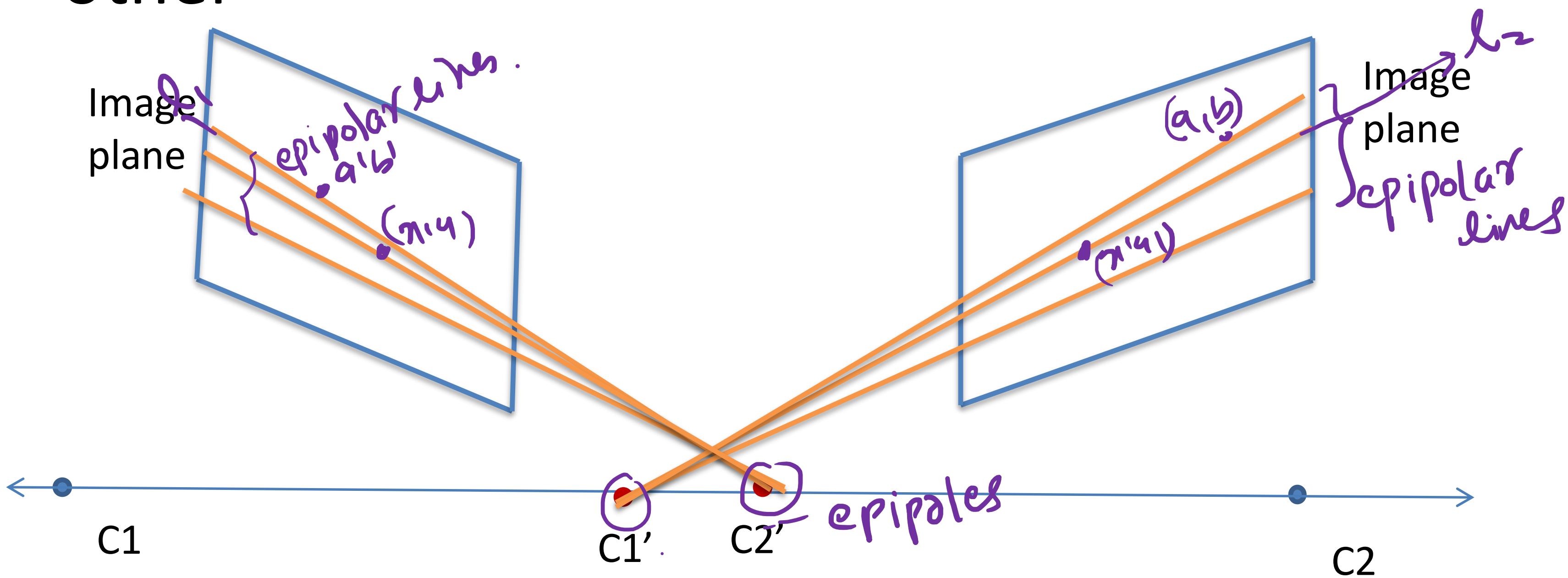
EPIPOLES & EPIPOLAR LINES

- If the cameras are **not** in-field of view of each other



EPIPOLES & EPIPOLAR LINES

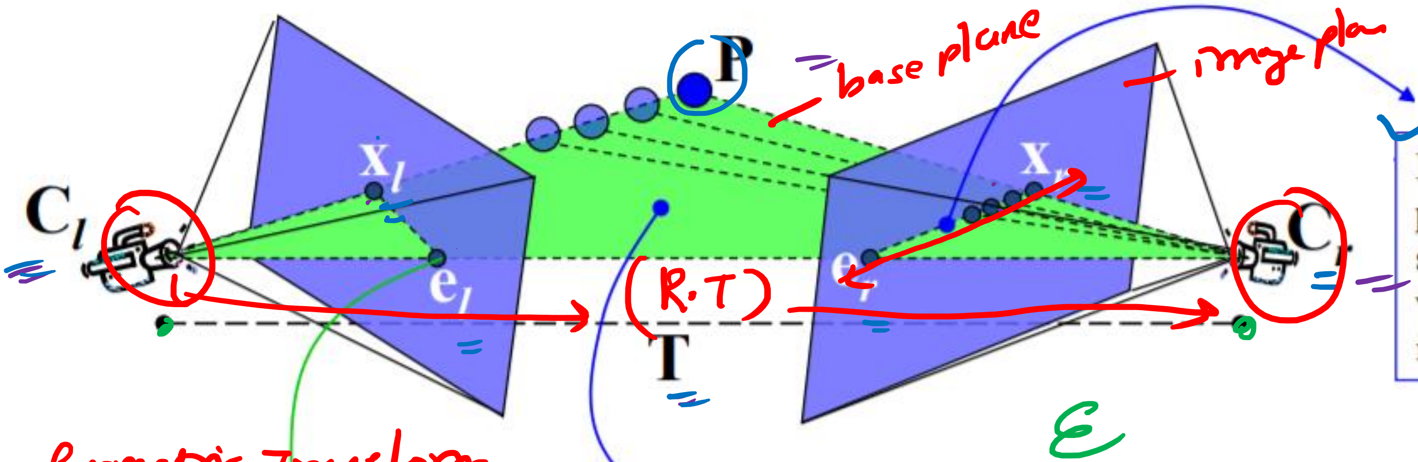
- If the cameras are **not** in-field of view of each other



- Camera matrix
- fundamental matrix.

- Geometric Transf.

- Camera matrix.
- Essential
- fundamental.



① **Epipolar line:** set of world points that project to the same point in left image, when projected to right image forms a line.

- Geometric Transform
  - 3D
  - 2D

② **Epipole:** intersection of image plane with line connecting camera centers. Image of a left camera center in the right, and vice versa.

③ **Epipolar plane:** plane defined by the camera centers and world point.

- RT
- Trans
- Scale

Camera = [Intrinsic] [Extrinsic] (R, T, ...)

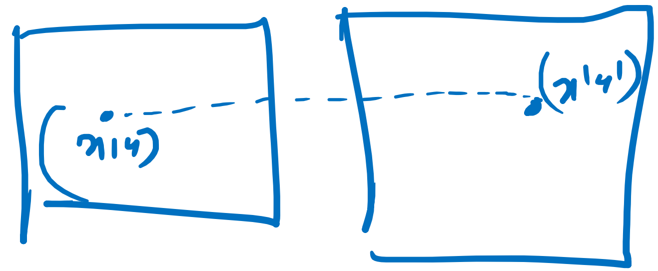
⇓

Essential matrix = [RT] b/w 2 cameras (C_l, C_r)

Relation Between Correspondence

- ***Essential Matrix:*** Measures Rotation and Translation

$$E = R\,S$$ Transl



- ***Fundamental Matrix:*** Relation btw image point co-ordinates

$$\boxed{[x' \quad y' \quad 1]\, F \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0}$$

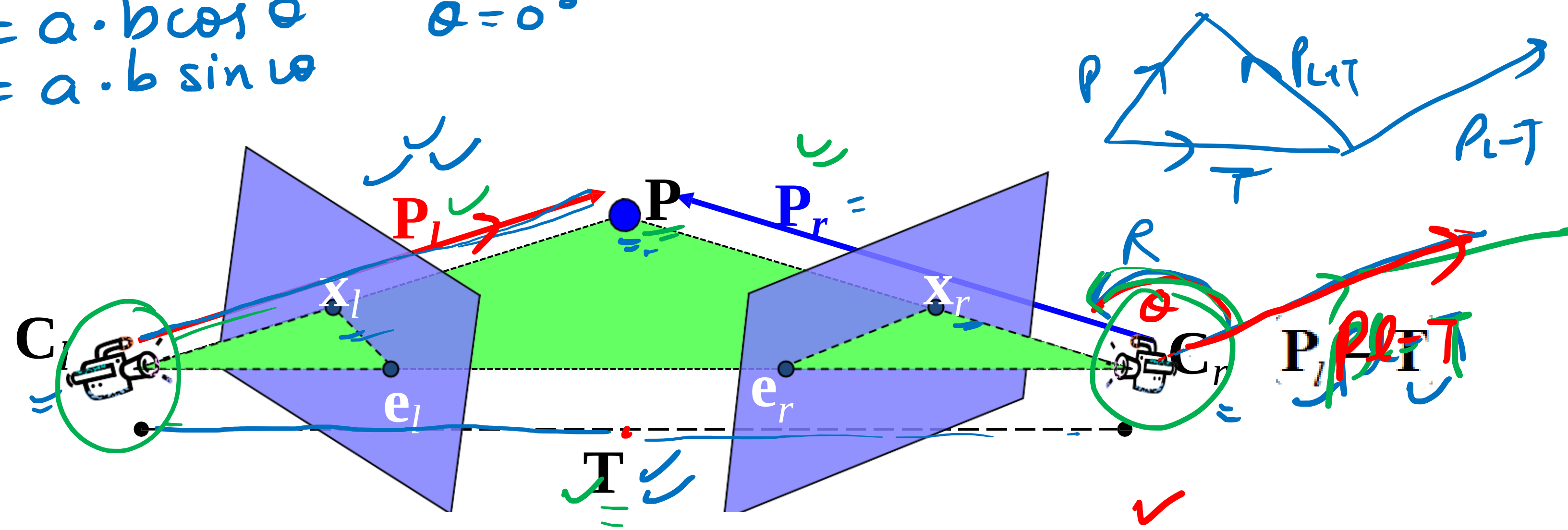
image point in right planes

image point in left image

Essential Matrix

dot $a \cdot b = a \cdot b \cos\theta$ $\quad \theta = 0°$

or $a \cdot b = a \cdot b \sin\theta$



$R^T = R^{-1}$

(1) Coplanarity constraint between vectors $(\mathbf{P}_l - \mathbf{T})$, $\mathbf{T}$, $\mathbf{P}_l$.

$(P_r^T R)$

$$(\mathbf{P}_l - \mathbf{T})^T \mathbf{T} \times \mathbf{P}_l = 0$$

$$\mathbf{P}_r = \mathbf{R}(\mathbf{P}_l - \mathbf{T})$$

$$\mathbf{P}_r^T \mathbf{R} \mathbf{T} \times \mathbf{P}_l = 0$$

cross product matrix

$$\mathbf{R}^T \mathbf{P}_r = (\mathbf{P}_l - \mathbf{T})$$

$$\mathbf{P}_r^T \mathbf{R} = (\mathbf{P}_l - \mathbf{T})^T$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

R_θ

Vector Cross Product to Matrix- vector multiplication

cross product

$$A \times B = \begin{bmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} = \begin{bmatrix} -A_zB_y + A_yB_z & A_zB_x - A_xB_z & -B_xA_y + B_yA_x \end{bmatrix}$$

$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$

i j k

$-A_zB_y + A_yB_z \rightarrow i$

$B_xA_z - A_xB_z \rightarrow j$

$-A_yB_x + B_yA_x \rightarrow k$

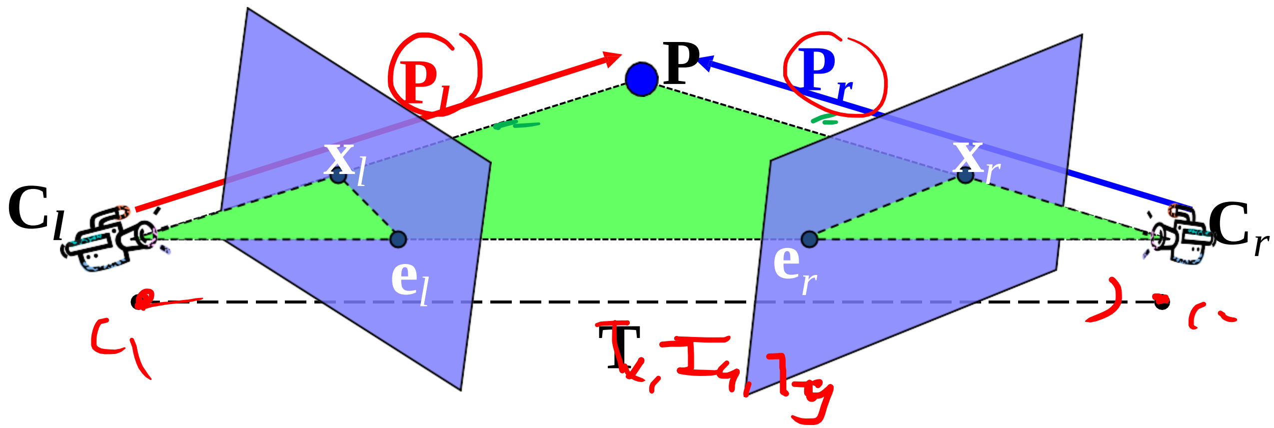
S

$$A \times B = S.B = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

element of first vector

$$= \begin{bmatrix} -A_zB_y + A_yB_z & A_zB_x - A_xB_z & -B_xA_y + B_yA_x \end{bmatrix}$$

Essential Matrix



$$\mathbf{P}_r^{\mathrm{T}} \mathbf{R} \mathbf{T} \times \mathbf{P}_l = 0 \quad \Big\} \quad \mathbf{P}_r^{\mathrm{T}} \mathbf{R} \, \boxed{\mathbf{S}} \, \mathbf{P}_l = 0 \quad \Rightarrow \quad \boxed{\mathbf{P}_r^{\mathrm{T}} E \mathbf{P}_l = 0}$$

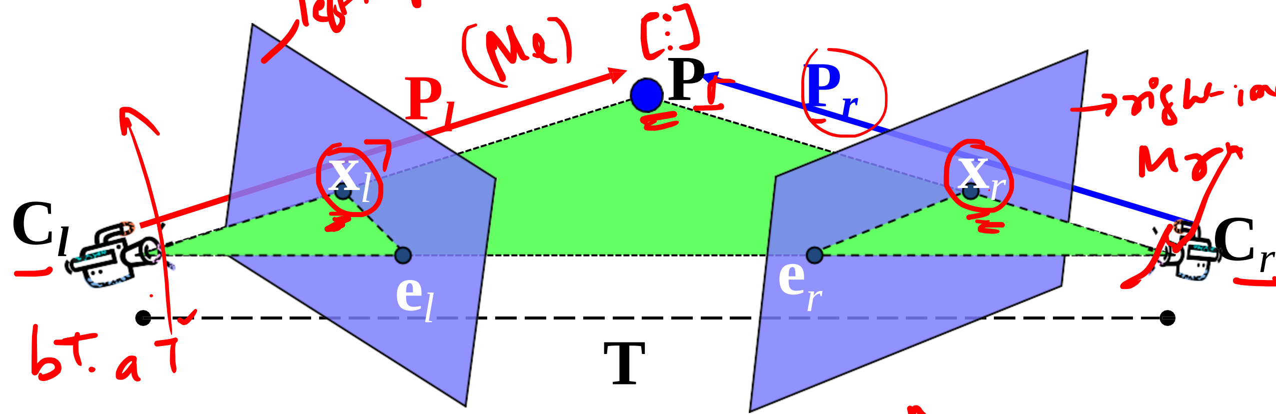
essential matrix

Cross product can be expressed as matrix mutliplication

$$\boxed{\begin{pmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{pmatrix}}$$

$$E = \mathbf{R} \, \mathbf{S}$$

Fundamental Matrix



$(a \cdot b)^T = b^T \cdot a^T$

using CM rem

$$\mathbf{x}_l = M_l \mathbf{P}_l \quad (1)$$
$$\mathbf{x}_r = M_r \mathbf{P}_r \quad (2)$$
$$\mathbf{P}_r^{\mathrm{T}} E \mathbf{P}_l = 0 \quad (3)$$

Apply Camera model

$$M_l^{-1}\mathbf{x}_l = \mathbf{P}_l$$
$$M_r^{-1}\mathbf{x}_r = \mathbf{P}_r$$
$$\mathbf{x}_r^{T} M_r^{-T} = \mathbf{P}_r^{T}$$

$P_r^T \, E \, M_l^{-1} x_l = 0$

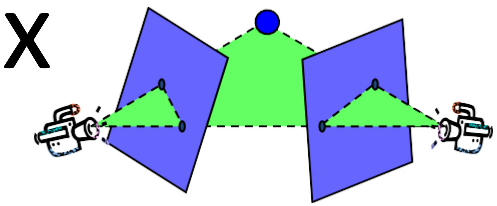
$$\mathbf{x}_r^{\mathrm{T}} M_r^{-\mathrm{T}} E M_l^{-1} \mathbf{x}_l = 0$$
$$\mathbf{x}_r^{\mathrm{T}} \left(M_r^{-\mathrm{T}} E M_l^{-1} \right) \mathbf{x}_l = 0$$

$$\boxed{\mathbf{x}_r^{\mathrm{T}} F \mathbf{x}_l = 0}$$

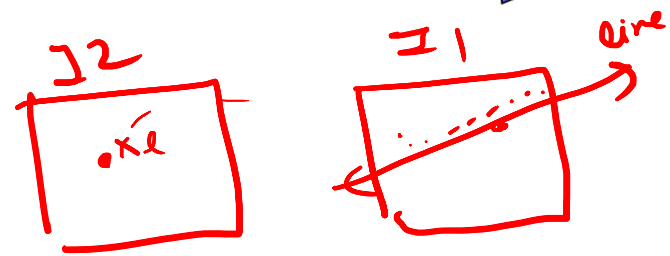
fundamental matrix

$F = M_r^{-T} E M_l^{-1}$

Fundamental Matrix



$$\underset{x_r}{\mathbf{x}'^{\mathrm{T}}} F \underset{x_l}{\mathbf{x}} = \mathbf{x}'^{\mathrm{T}} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & m \end{pmatrix} \mathbf{x} = 0$$

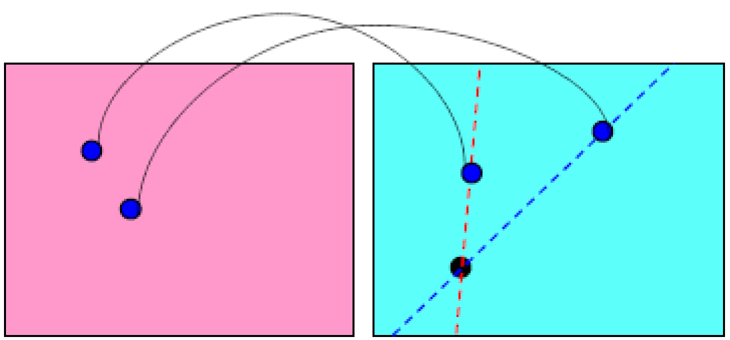


$Fix\ (x, y)\ in\ \ Image\ 1$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}^T F \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}^T \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

Given a point in left camera $\mathbf{x}$, epipolar line in right camera is: $\mathbf{u}_r = F\mathbf{x}$

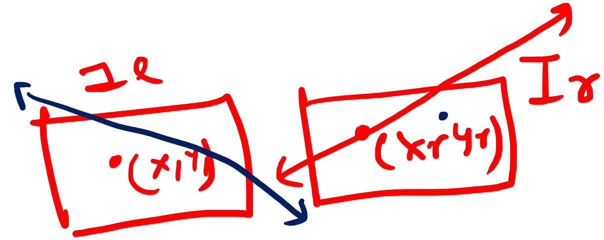


px'+qy'+r=0 *(eq of line-> epipolar line in image 2*)

$$X'^T F X = 0$$

$$X_r^T F X_l = 0$$

$$X_r^T \left[F\right]_{3\times3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3\times1} = 0$$



$$\left[F\right]_{3\times3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_{3\times1}$$

$$X_r^T \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix}_{3\times1} = 0$$

$$\begin{bmatrix} x_r \\ y_r \\ 1 \end{bmatrix}^T \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

$$\begin{bmatrix} x_r & y_r & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

$$p \cdot x_r + q\, y_r + r = 0$$ (eq^n of line)

$$X_r^T F X_l = 0$$

$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}^T_{1\times3} \left[F\right]_{3\times3} \left[x_l\right] = 0$$

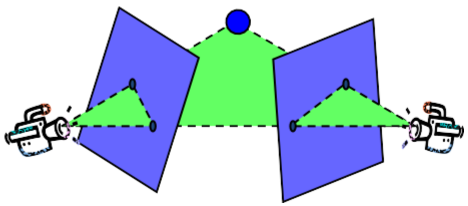
$$\left[\quad\right]_{3\times3} \left[x_l\right] = 0$$

single view geo

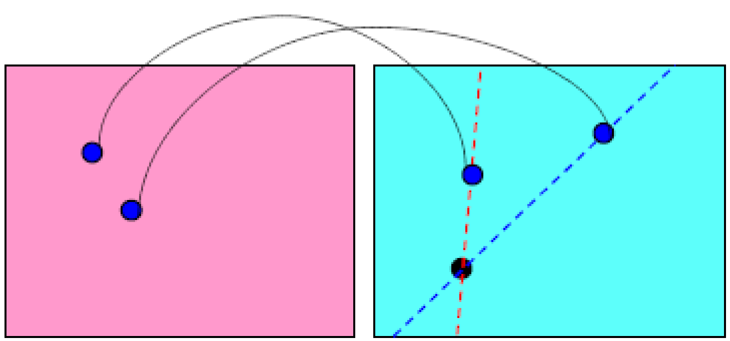
1. $CM = 3D - 2D$
2. $EM = [R\ S]$
3. $FM = \left[M_r^{-T}[EM]M_l^{-1}\right]$

Fundamental Matrix

$$\boxed{\mathbf{x}'^{\mathrm{T}} F \mathbf{x} = \mathbf{x}'^{\mathrm{T}} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & m \end{pmatrix} \mathbf{x} = 0}$$



Given a point in left camera $\mathbf{x}$, epipolar line in right camera is: $\mathbf{u}_r = F\mathbf{x}$



- 3x3 matrix with 9 components
- Rank 2 matrix (due to S) rank of matrix
- 7 degrees of freedom (3-rot-3 traslation-1 -scaling)

$$S = \begin{pmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{pmatrix}$$

Homework

- For the two epipoles $e = (x_e, y_e)$ and $e' = (x_e', y_e')$.
- Show that $[x_e, y_e, 1]^T$ is an eigenvector of F with eigenvalue 0.
- And similarly, $[x_e', y_e', 1]^T$ is an eigenvector of F^T with eigenvalue 0.

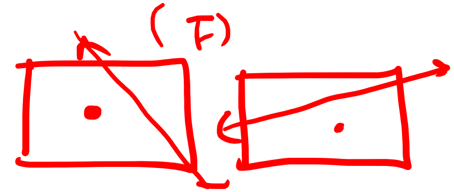
Richard Radke

Hint: Refer the reading material shared with you.

$E = R \cdot S$

$S = [T]$ modified

$F = [M_r^{-1} E M_l^{-1}]$



Fundamental Matrix

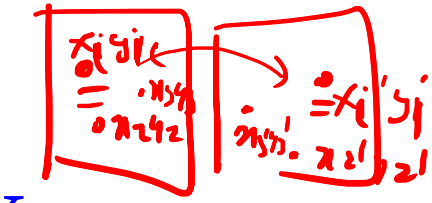
Il Ir

- Fundamental matrix captures the relationship between the corresponding points in two views.

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^T \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = 0,$$

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^T \begin{bmatrix} f_{11}x' + f_{12}y'_i + f_{13} \\ f_{21}x' + f_{22}y'_i + f_{23} \\ f_{31}x' + f_{32}y'_i + f_{33} \end{bmatrix} = 0,$$

T → s = [ ]



Fundamental Matrix

$$x_i(f_{11}x' + f_{12}y'_i + f_{13}) + y_i(f_{21}x' + f_{22}y'_i + f_{23}) + (f_{31}x' + f_{32}y'_i + f_{33}) = 0$$

$$x_i x' f_{11} + x_i y'_i f_{12} + x_i f_{13} + y_i x' f_{21} + x' y'_i f_{22} + y'_i f_{23} + x' f_{31} + y'_i f_{32} + f_{33} = 0$$ — for one point

One equation for one point correspondence

(known) uk = 0

$$n\quad Mf = \begin{bmatrix} x'_1x_1 & x'_1y_1 & x'_1 & y'_1x_1 & y'_1y_1 & y'_1 & x_1 & y_1 & 1 \\ x'_2x_2 & x'_2y_2 & x'_2 & y'_2x_2 & y'_2y_2 & y'_2 & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x'_nx_n & x'_ny_n & x'_1 & y'_nx_n & y'_ny_n & y'_n & x_n & y_n & 1 \end{bmatrix} f = 0$$

Homogenous system, no unique solution, fix one unknown to be=1 arbitrarily

M is 9 by n matrix $\quad f = [f_{11} \quad f_{12} \quad f_{13} \quad f_{21} \quad f_{22} \quad f_{23} \quad f_{31} \quad f_{32} \quad f_{33}]$

To solve the equation, the rank(M) must be 8.

Normalized 8-point algorithm (Hartley)

Objective:

Compute fundamental matrix F such that $\mathbf{x}_i' F \mathbf{x}_i = 0$

Algorithm

1. Normalize the image $\hat{\mathbf{x}}_i = T\mathbf{x}_i$ $\hat{\mathbf{x}}_i' = T'\mathbf{x}_i'$

$$T = \begin{bmatrix} a_x & 0 & d_x \\ 0 & a_y & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

Find centroid of points in each image, substract with remaining points determine the range, and normalize all points between 0 and 1

Linear solution

$(\hat{x}, y, 1)$ $(\hat{x}, y, 1)$

determining the eigen vector (9x1) corresponding to the smallest eigen value of A => gives the approximation of fundamental matrix

Point correspondence

$$Af = \begin{bmatrix} \hat{x}_1'\hat{x}_1 & \hat{x}_1'\hat{y}_1 & \hat{x}_1' & \hat{y}_1'\hat{x}_1 & \hat{y}_1'\hat{y}_1 & \hat{y}_1' & \hat{x}_1 & \hat{y}_1 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \hat{x}_8'\hat{x}_8 & \hat{x}_8'\hat{y}_8 & \hat{x}_8' & \hat{y}_8'\hat{x}_8 & \hat{y}_8'\hat{y}_8 & \hat{y}_8' & \hat{x}_8' & \hat{y}_8 & 1 \end{bmatrix} f = 0$$

Eigenvector of the matrix A = Smallest Eigen value

Normalized 8-point algorithm (Hartley)

eigen vector (9x1)->reshaped $\hat{F} = \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix}$

L1 matrix norm is maximum of absolute column sum.

$$\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{m} |a_{ij}|,$$

Normalize $\hat{F} = \hat{F} / \| \hat{F} \|$

Constraint enforcement of rank=2 by using SVD decomposition

L infinity norm is maximum of sum of absolute of row sum.

$$\hat{F} = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \boxed{\sigma_3} \end{bmatrix} V' \qquad (\sigma_1 \ge \sigma_2 \ge \sigma_3)$$

$=0$

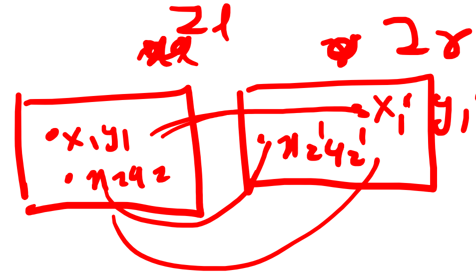
Rank enforcement by setting smallest singular value=0

$$\widetilde{F} = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \boxed{0} \end{bmatrix} V' \qquad (\sigma_3 = 0)$$

$$T = \begin{bmatrix} a_x & 0 & d_x \\ 0 & a_y & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

De-normalization: $F = T'^T \widetilde{F} T$

Normalized 8 point Algo



1) Step1 → we normalized the I_l & I_r → $\hat{x}_l$ $\hat{x}_r$

2) Step2 → we take pt corresp in the normalized image

$$\hat{x}_{l_i} \longleftrightarrow \hat{x}_{r_i}$$

3) $$\boxed{[A][f] = 0}$$

4) we take the E.V decomp of [A], the Eigenvector smallest E.V are → first estimate of Fm. f.

4) (F) → (9x1 → reshaping 3x3) first estimation fn. matrix

5) (F) has a rank = 2. $(R \times T = R.S]$ $S = [\;]$

(6) Enforce rank-2 on F.

(7) first we take $F \longrightarrow$ normalize $\hat{F}$

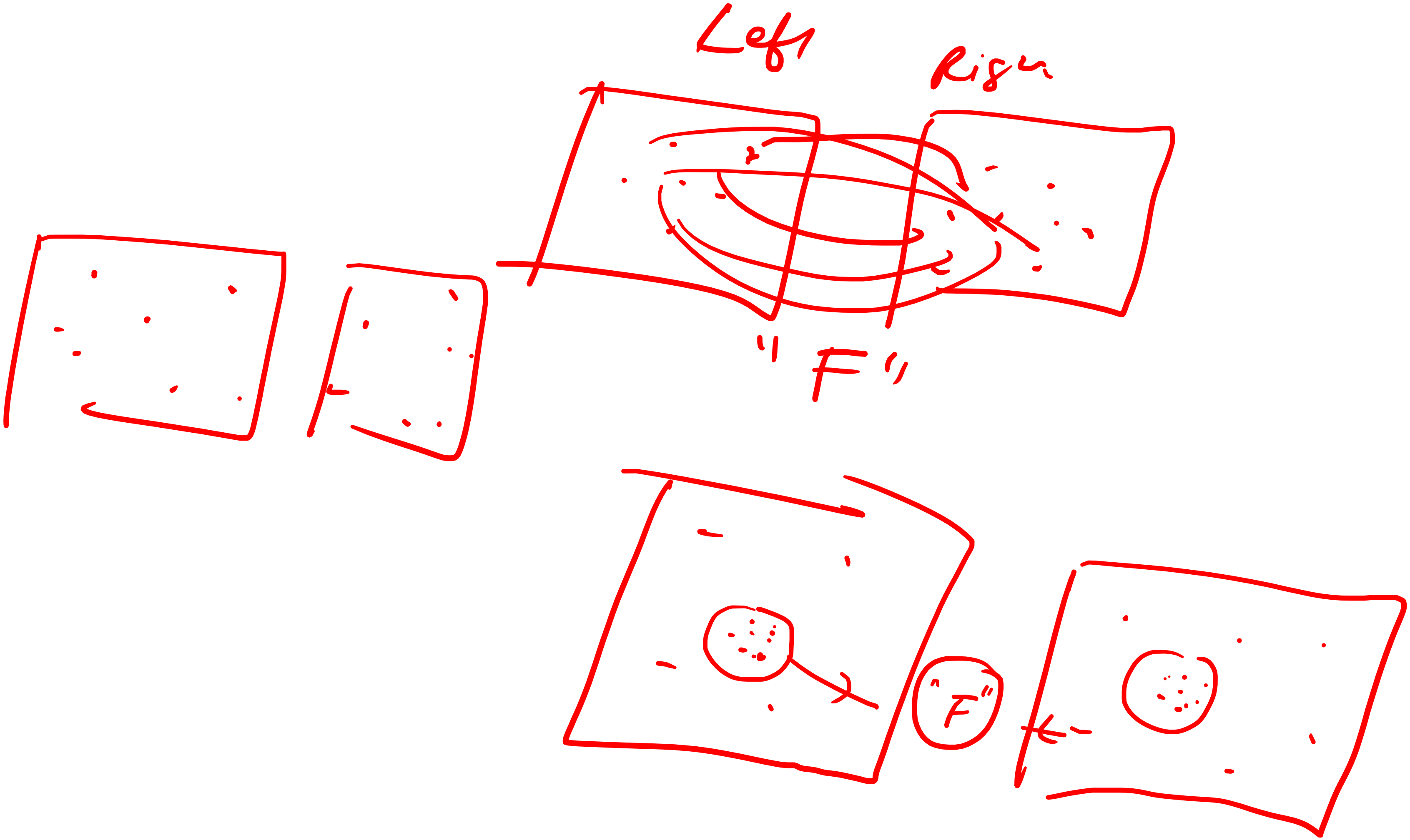
(8) SVD of $\hat{F}$

$$\hat{F} = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V$$

(9) rank-enforcement by setting $\sigma_3 = 0$

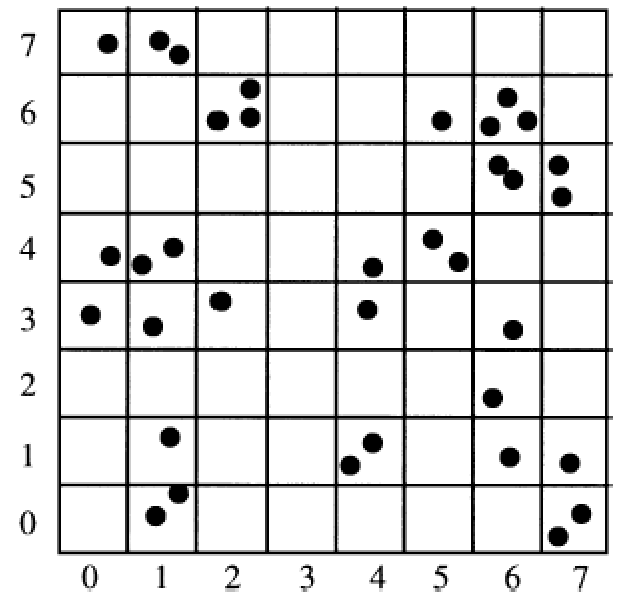
$$\hat{\underline{\underline{F}}} = \underline{U} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} V.$$

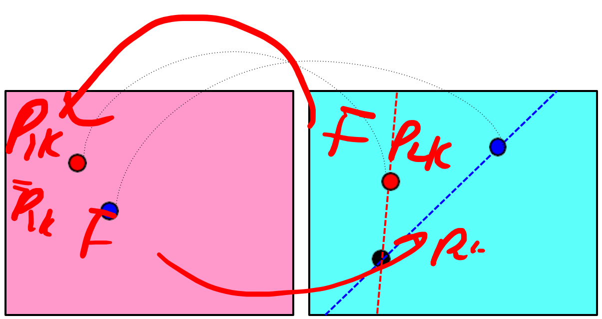
(10) $\underset{\text{(rank 2)}}{\hat{\underline{\underline{F}}}} \longrightarrow$ denormalize $\left(F = \underline{T}\, \hat{F}\, \hat{T} \right)$


Left
Right
"F"
"F"

Robust Fundamental Matrix Estimation (by Zhang)

- Uniformly divide the image into 8×8 grid.
- Randomly select 8 grid cells and pick one pair of corresponding points from each grid cell, then use Hartley's 8-point algorithm to compute Fundamental Matrix F_i.
- For each F_i, compute the median of the squared residuals R_i.
  - $R_i = \text{median}_k[d(p_{1k},F_i p_{2k}) + d(p_{2k},F'_i p_{1k})]$
- Select the best F_i according to R_i.
- Determine outliers if $R_k>Th$.
- Using the remaining points compute the fundamental Matrix F by weighted least square method.

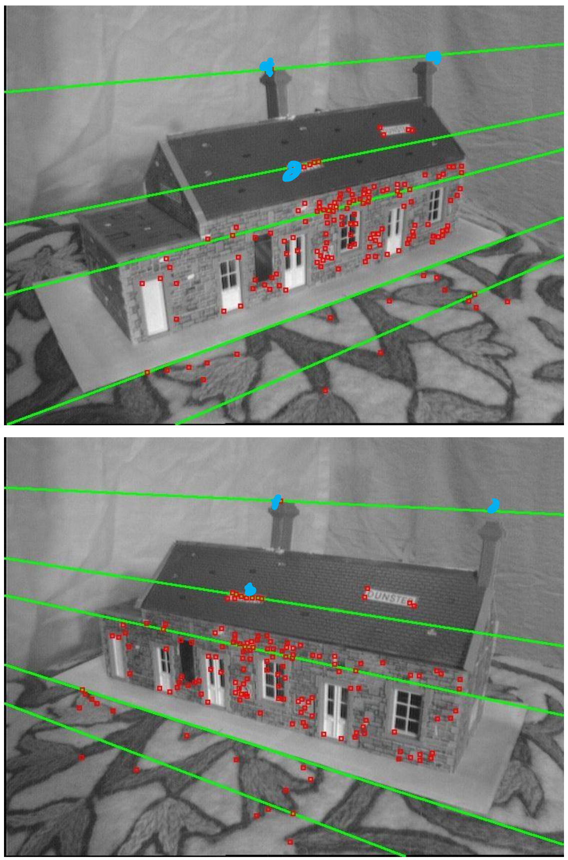




Epi-polar Lines

x_1

x_2



x_1

x_2

Epi-polar lines