Projective & Image Geometry.

3D transf.

2D

scaling

Rot

Trans

affine

shear

Camera Model and Calibration

matrix.

Image 2D ⟷ 3D Real

① 3D WC → 2D Image coordinates

⇓

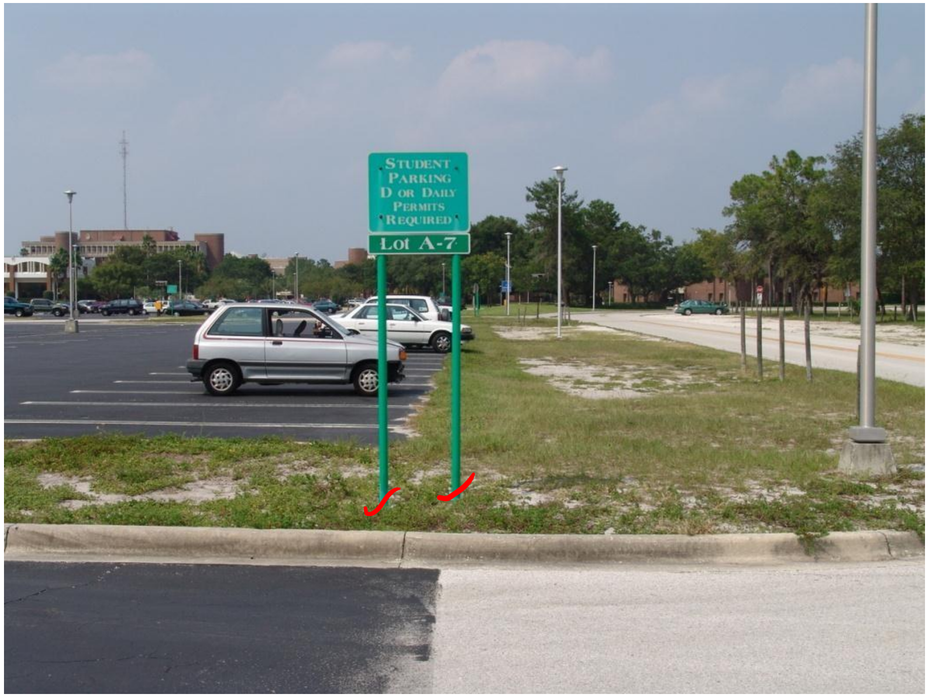
camera projection

Camera Calibration

[2D] = [Camera matrix] [3D]

- Determine extrinsic and intrinsic parameters of camera
  - Extrinsic parameters
    - 3D location and orientation of camera
  - Intrinsic
    - Focal length
    - The size of the pixels

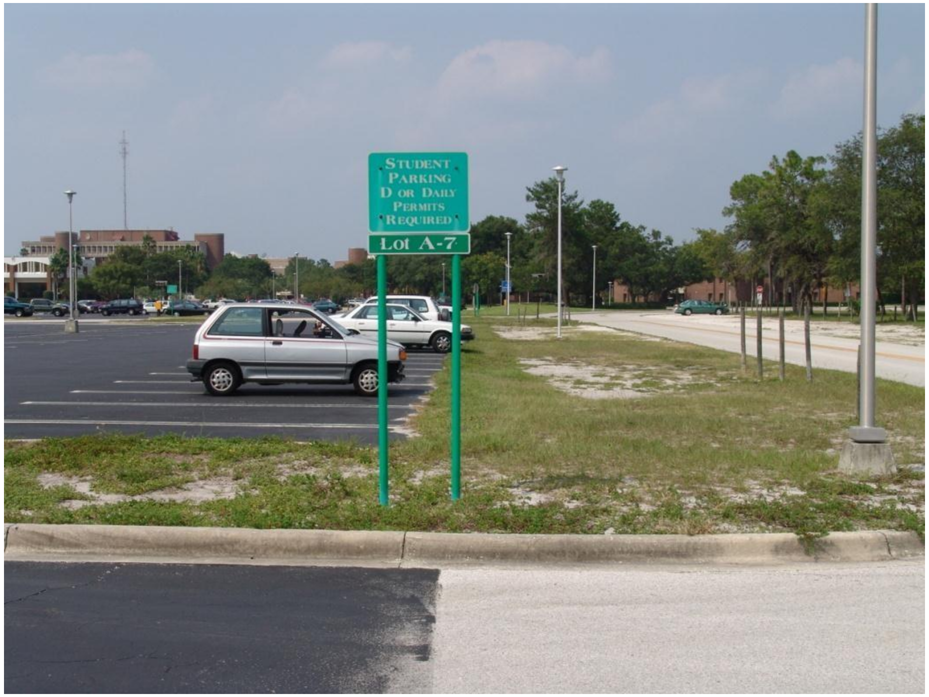
Application: Object Transfer



Source Image

Target Image

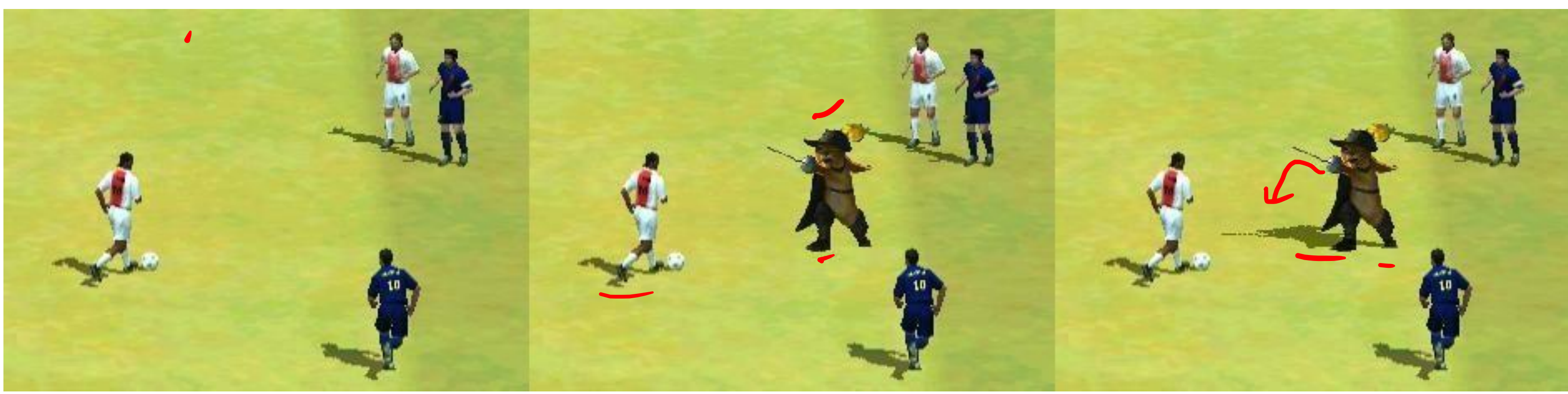
Application: Object Transfer



Source Image

Target Image

More Results



Application in Film Industry

Pose Estimation

- Given 3D model of object, and its image (2D projection) determine the location and orientation (translation & rotation) of object such that when projected on the image plane it will match with the image.

http://www.youtube.com/watch?v=ZNHRH00UMvk

Transformations

in the 3D world.

3-D Translation [x, y, z]

upper case → 3D world cordinate

$x_2 = x_1 + dx$
$y_2 = y_1 + dy$
$z_2 = z_1 + dz$

$y_1 + dy = y_2$
$z_1 + dz = z_2$

3D cord

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}$$

Homo

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = T \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Translation Matrix}$$

3D (x, y, z) Homo (x, y, z, 1) scaling

$$T^{-1} = \begin{bmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$TT^{-1} = T^{-1}T = I$$

$RR^{-1} = I$

$TT^{-1} = I$

$$\begin{bmatrix} 1 & 0 & 0 & d_x \\ 0 & 1 & 0 & d_y \\ 0 & 0 & 1 & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -d_x \\ 0 & 1 & 0 & -d_y \\ 0 & 0 & 1 & -d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scaling

$SS^{-1} = I.$
$TT^{-1} = I$
$RR^{-1} = I$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} X_1 \times S_x \\ Y_1 \times S_y \\ Z_1 \times S_z \end{bmatrix}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$x_2 = S_x X_1$
$y_2 = S_y \cdot y_1$
$z_2 = S_z z_1$

$$S^{-1} = \begin{bmatrix} 1/S_x & 0 & 0 & 0 \\ 0 & 1/S_y & 0 & 0 \\ 0 & 0 & 1/S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$SS^{-1} = S^{-1}S = I$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \\ 1 \end{bmatrix} = S \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/S_x & 0 & 0 & 0 \\ 0 & 1/S_y & 0 & 0 \\ 0 & 0 & 1/S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \text{Scaling Matrix}$$

Rotation

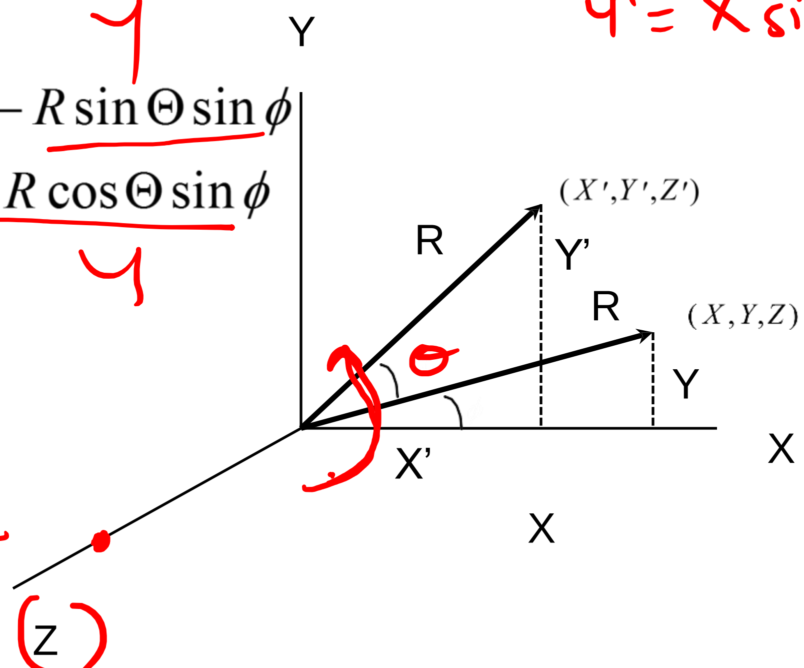
$$X \xrightarrow{R_z\ \theta} X'$$
$$Y \qquad Y'$$
$$Z \qquad Z$$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$$X = R\cos\phi$$
$$Y = R\sin\phi$$

$$X' = R\cos(\Theta+\phi) = \overbrace{R\cos\Theta\cos\phi}^{X} - \overbrace{R\sin\Theta\sin\phi}^{Y}$$
$$Y' = R\sin(\Theta+\phi) = \underbrace{R\sin\Theta\cos\phi}_{Y} + \underbrace{R\cos\Theta\sin\phi}_{Y}$$



$$X' = X\cos\Theta - Y\sin\Theta$$
$$Y' = X\sin\Theta + Y\cos\Theta$$

R

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Rotation matrices are orthonormal matrices

$$(R_\theta^Z)^{-1} = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_z\theta$ $\quad$ $R_z\theta^{-1}$ $\quad$ I

$$(R_\theta^Z)^{-1} = (R_\theta^Z)^T$$

$$(R_\theta^Z)(R_\theta^Z)^T = I$$

$$r_i . r_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Euler Angles

Rotation around an arbitrary axis:

$$R = R_Z^{\alpha} R_Y^{\beta} R_X^{\gamma} = \begin{bmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta\cos\gamma + \sin\alpha\sin\gamma \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta\cos\gamma - \cos\alpha\sin\gamma \\ -\sin\beta & \cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

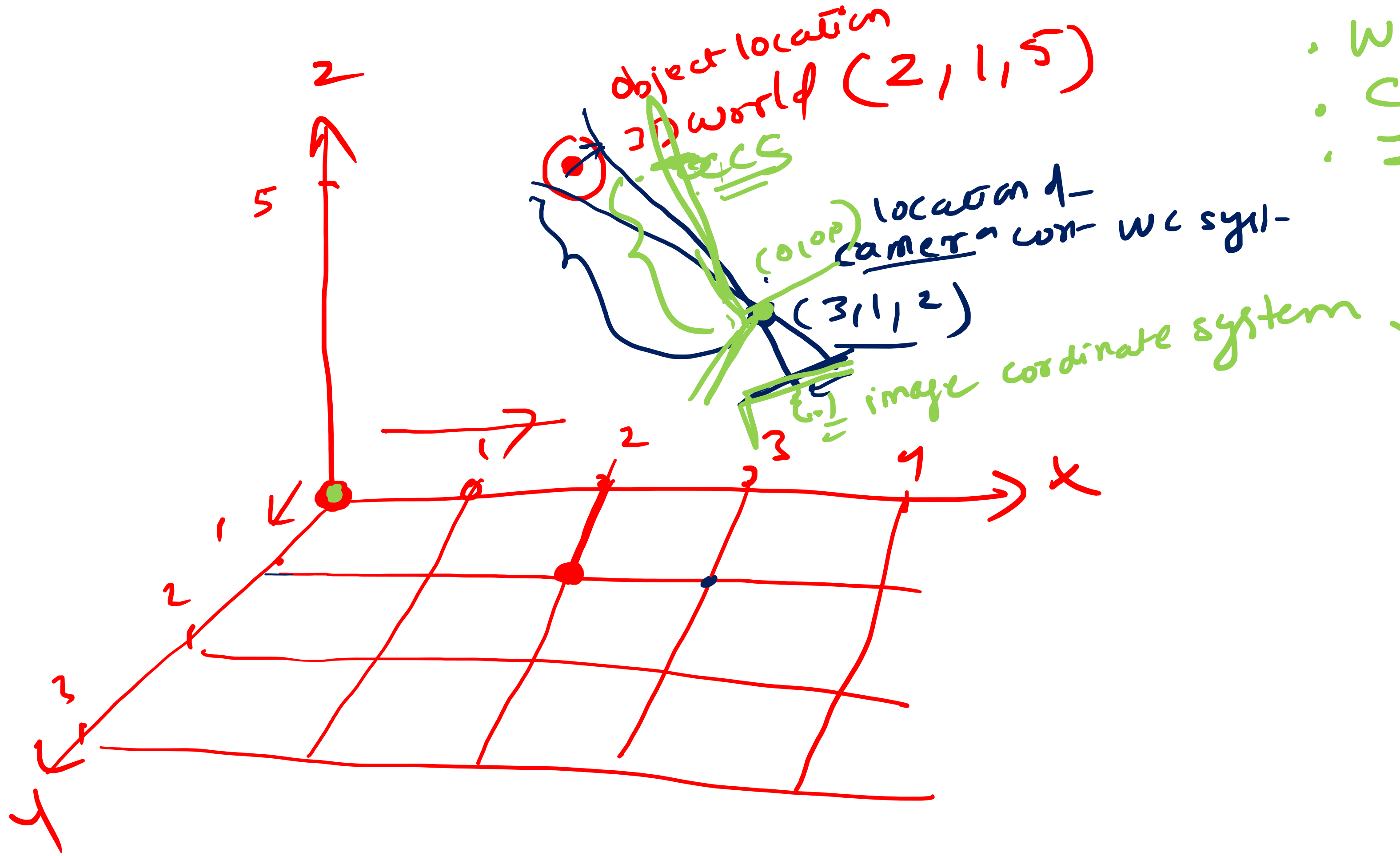
if angles are small $\cos\Theta \approx 1$ $\sin\Theta \approx \Theta$

$$R = \begin{bmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{bmatrix}$$

Similarity ← • 3D scaling
• 3D Translation
• 3D Rotation.

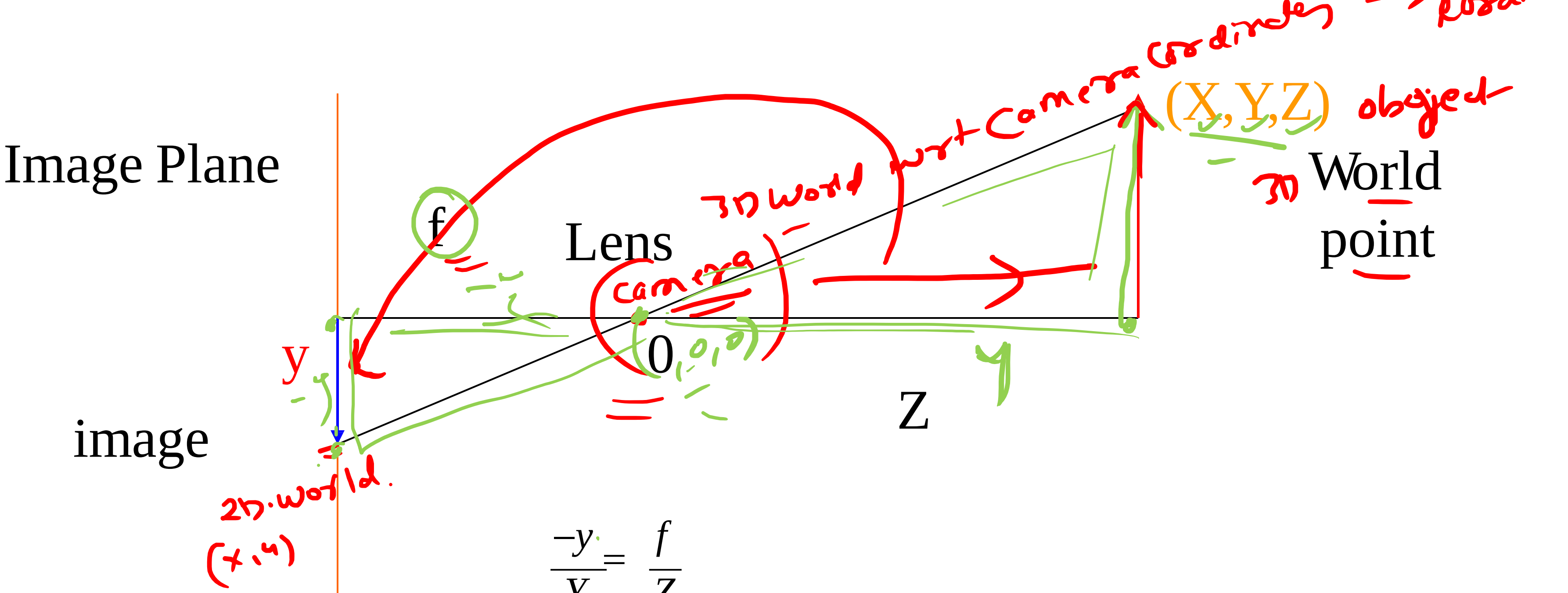
Rigid body Trans
Euclidean → (Trans + Rot)

- WCS - World
- CCS - Camera
- ICS - Image



WCS → CCS.

Perspective Projection (origin at the lens center)



$$\frac{-y}{Y} = \frac{f}{Z}$$

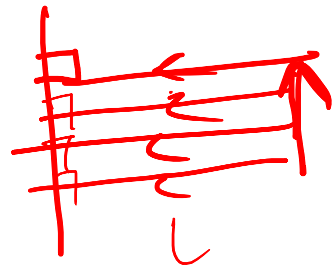
$$y = -\frac{fY}{Z} \qquad x = -\frac{fX}{Z}$$

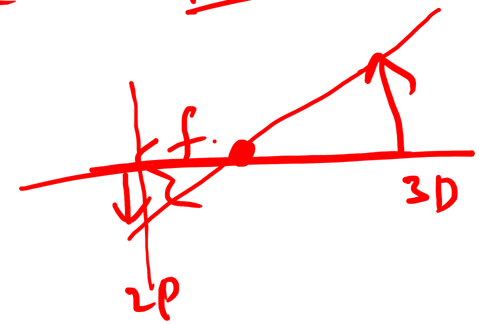
Image transformation

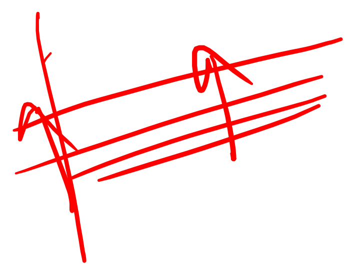
- Rot
- scaled
- trans
- shear
- affine

3D $\xrightarrow{\text{projection}}$ 2D

- orthographic
- perspective

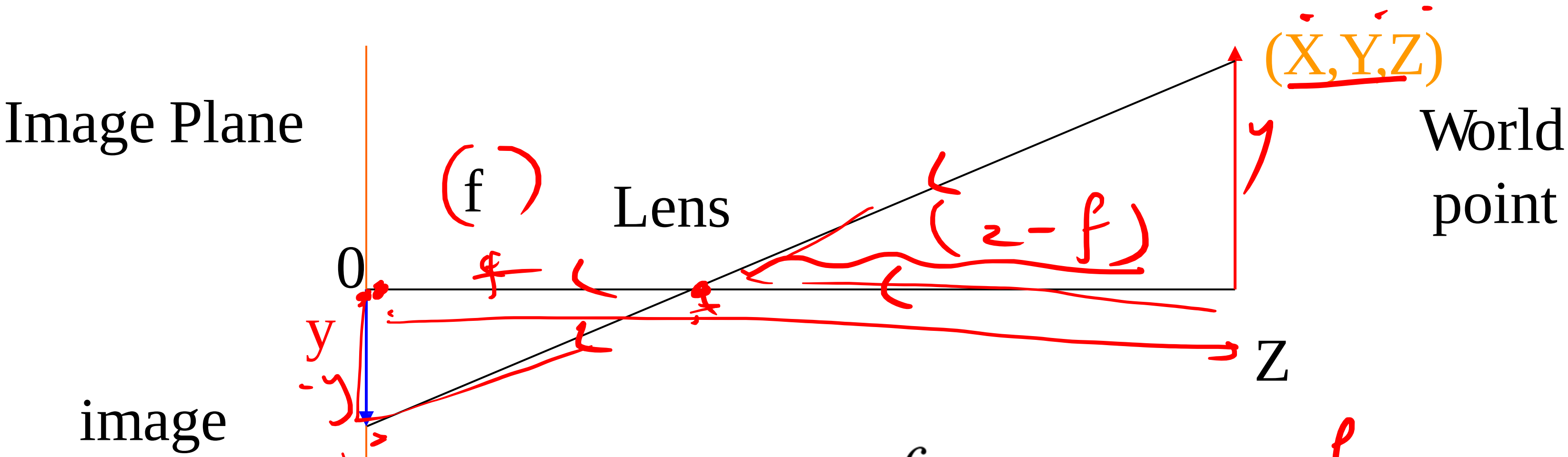






WCO Image

Perspective Projection (origin at image center)



$$\frac{-y}{Y} = \frac{f}{Z-f}$$

$$\frac{-x}{X} = \frac{f}{Z-f}$$

$$y = -\frac{fY}{Z-f} \qquad x = -\frac{fX}{Z-f}$$

$$y = \frac{fY}{f-Z} \qquad x = \frac{fX}{f-Z}$$

Perspective

$$y = \frac{fY}{f-Z} \qquad x = \frac{fX}{f-Z}$$

Trans 3D.

$$\begin{bmatrix} x \\ y \\ \\ \\ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0\,? & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Image coordinates

World coordinates

$$(X, Y, Z) \rightarrow \quad \rightarrow\rightarrow \qquad (kX, kY, kZ, k), \text{ Homogenous transformation}$$

k=1

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix}$$

Perspective

$$y = \frac{fY}{f-Z} \qquad x = \frac{fX}{f-Z}$$

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix}$$

$$x = \frac{C_{h1}}{C_{h4}} = \frac{kX}{k - \frac{kZ}{f}} = \frac{fX}{f-Z}$$

$$y = \frac{C_{h2}}{C_{h4}} = \frac{kY}{k - \frac{kZ}{f}} = \frac{fY}{f-Z}$$

$$\begin{bmatrix} C_{h1} \\ C_{h2} \\ C_{h3} \\ C_{h4} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix} \begin{bmatrix} kX \\ kY \\ kZ \\ k \end{bmatrix} = \begin{bmatrix} kX \\ kY \\ kZ \\ k - \frac{kZ}{f} \end{bmatrix}$$

$(x, y, z, 1)$

$(x, y, z) \rightarrow (kX, kY, kZ, k)$

$\frac{kX}{k}, \frac{kY}{k}, \frac{kZ}{k}$

Important Definitions

Frame of reference: a measurements are made with respect to a particular coordinate system called the frame of reference.

1 **World Frame:** a fixed coordinate system for representing objects (points, lines, surfaces, etc.) in the world.

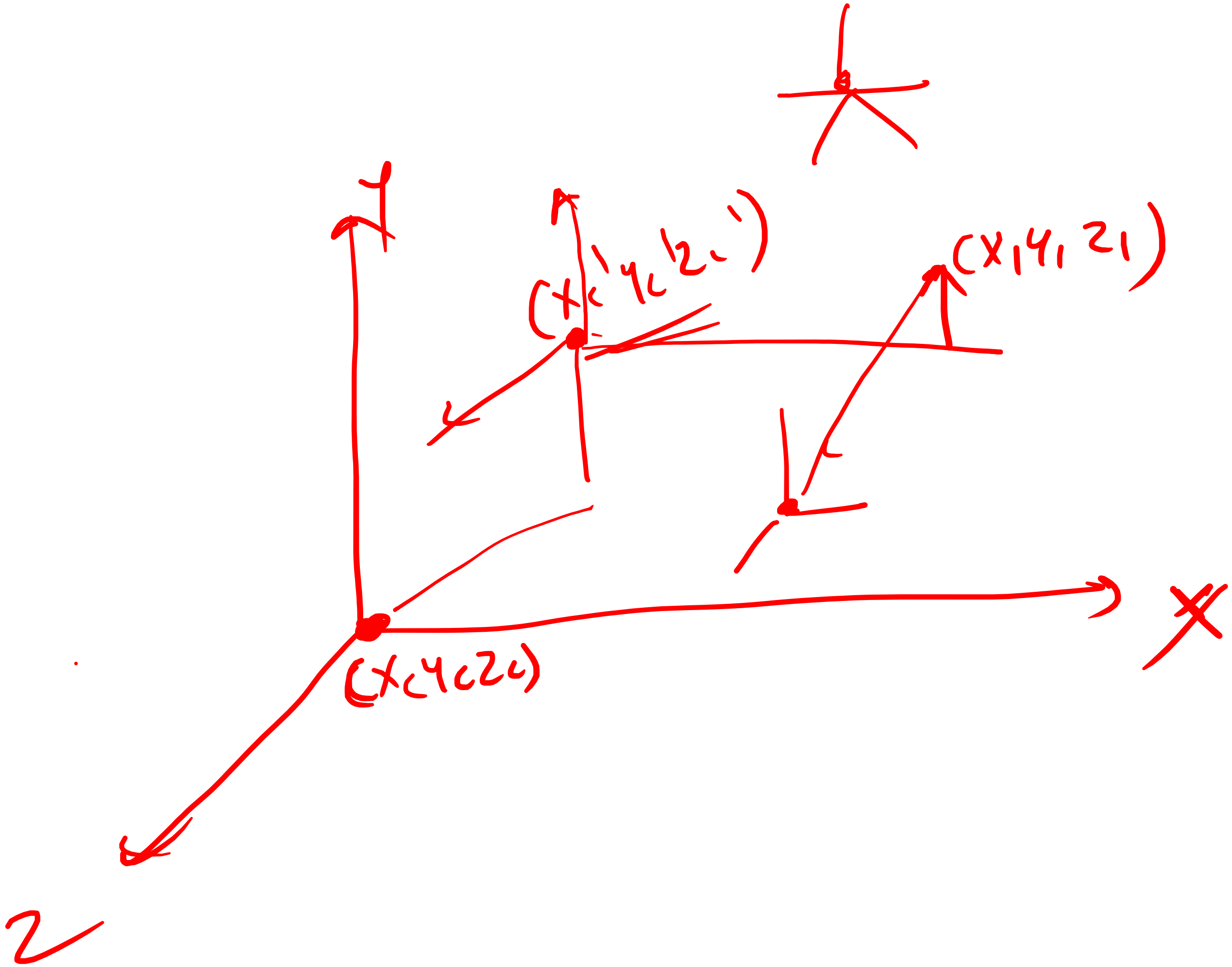
2 **Camera Frame:** coordinate system that uses the camera center as its origin (and the optic axis as the Z-axis)

Image or retinal plane: plane on which the image is formed, note that the image plane is measured in camera frame coordinates (mm)

3 **Image Frame:** coordinate system that measures pixel locations in the image plane.

Intrinsic Parameters: Camera parameters that are internal and fixed to a particular camera/digitization setup

Extrinsic Parameters: Camera parameters that are external to the camera and may change with respect to the world frame.



y
x
z
(xc yc zc)
(xc', yc', zc')
(x1 y1 z1)

Camera Model

Effect on image points (x,y) if ………

- Camera is at the origin of the world coordinates first W_h
- Then translated by some amount(G),
- Then rotated around Z axis in counter clockwise direction,
- Then rotated again around X in counter clockwise direction, and
- Then translated by C.

$$C_h = PCR^{X}_{-\phi}R^{Z}_{-\theta}GW_h$$

Since we are moving the camera instead of object we need to use inverse transformations

Camera Model

$$C_h = PCR^{X}_{-\phi}R^{Z}_{-\theta}GW_h$$

$$G = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera Model

$$C_h = PCR^X_{-\phi}R^Z_{-\theta}GW_h$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-1}{f} & 1 \end{bmatrix}, R^Z_{-\theta} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R^X_{-\phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 \\ 0 & -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 & 0 & -X_0 \\ 0 & 1 & 0 & -Y_0 \\ 0 & 0 & 1 & -Z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & -r_1 \\ 0 & 1 & 0 & -r_2 \\ 0 & 0 & 1 & -r_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Camera Model

$$C_h = PCR^X_{-\phi} R^Z_{-\theta} G W_h$$

$$x = f \frac{(X - X_0)\cos\theta + (Y - Y_0)\sin\theta - r_1}{-(X - X_0)\sin\theta\sin\phi + (Y - Y_0)\cos\theta\sin\phi - (Z - Z_0)\cos\phi + r_3 + f}$$

$$y = f \frac{(X - X_0)\sin\theta\cos\phi + (Y - Y_0)\cos\theta\cos\phi + (Z - Z_0)\sin\phi - r_2}{-(X - X_0)\sin\theta\sin\phi + (Y - Y_0)\cos\theta\sin\phi - (Z - Z_0)\cos\phi + r_3 + f}$$

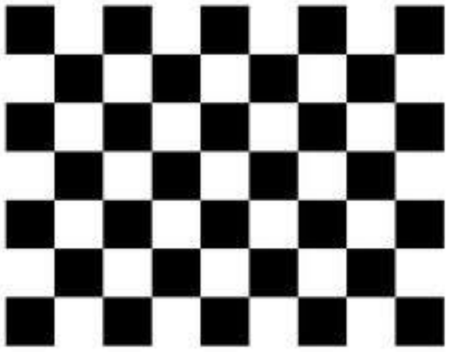
Camera Calibration

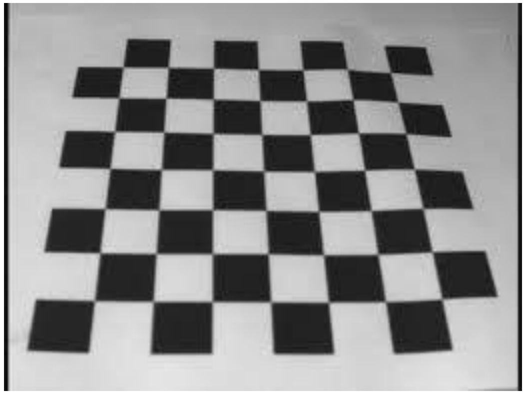
Camera Model: Relates world coordinates to image coordinates

- For that it assume the focal length, rotation angles, translation parameters are known.
- However, Camera can also be used as a measuring device.
- Knowledge of some set of point mappings can also be used to find the MODEL parameters.
- This process is known as CAMERA CALIBRATION

Camera Model

- How to determine camera matrix?
- Select some known 3D points (*X*,*Y*,*Z*), and find their corresponding image points (*x*,*y*).
- Solve for camera matrix elements using least squares fit.





Camera Model

$$C_h = PCR^X_{-\phi} R^Z_{-\theta} G W_h$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$C_h = AW_h$$

$$y = \frac{C_{h2}}{C_{h4}}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

Ch_3 is not needed, we have 12 unknowns.

Camera Model

$$C_h = PCR^X_{-\phi} R^Z_{-\theta} G W_h$$

$$x = \frac{C_{h1}}{C_{h4}}$$

$$y = \frac{C_{h2}}{C_{h4}}$$

$$C_h = AW_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} Ch_1 &= a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x \\ Ch_2 &= a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y \\ Ch_4 &= a_{41}X + a_{42}Y + a_{43}Z + a_{44} \end{aligned}}$$

Ch_3 is not needed, we have 12 unknowns.

Camera Model

$$Ch_1 = a_{11}X + a_{12}Y + a_{13}Z + a_{14} = Ch_4 x$$

$$Ch_2 = a_{21}X + a_{22}Y + a_{23}Z + a_{24} = Ch_4 y$$

$$Ch_4 = a_{41}X + a_{42}Y + a_{43}Z + a_{44}$$

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$

$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

Camera Model

$$a_{11}X + a_{12}Y + a_{13}Z + a_{14} - a_{41}Xx - a_{42}Yx - a_{43}Zx - a_{44}x = 0$$

$$a_{21}X + a_{22}Y + a_{23}Z + a_{24} - a_{41}Xy - a_{42}Yy - a_{43}Zy - a_{44}y = 0$$

One point

$$a_{11}X_1 + a_{12}Y_1 + a_{13}Z_1 + a_{14} - a_{41}X_1x_1 - a_{42}Y_1x_1 - a_{43}Z_1x_1 - a_{44}x_1 = 0$$

$$a_{11}X_2 + a_{12}Y_2 + a_{13}Z_2 + a_{14} - a_{41}X_2x_2 - a_{42}Y_2x_2 - a_{43}Z_2x_2 - a_{44}x_2 = 0$$

$$\vdots$$

$$a_{11}X_n + a_{12}Y_n + a_{13}Z_n + a_{14} - a_{41}X_nx_n - a_{42}Y_nx_n - a_{43}Z_nx_n - a_{44}x_n = 0$$

$$a_{21}X_1 + a_{22}Y_1 + a_{23}Z_1 + a_{24} - a_{41}X_1y_1 - a_{42}Y_1y_1 - a_{43}Z_1y_1 - a_{44}y_1 = 0$$

$$a_{21}X_2 + a_{22}Y_2 + a_{23}Z_2 + a_{24} - a_{41}X_2y_2 - a_{42}Y_2y_2 - a_{43}Z_2y_2 - a_{44}y_2 = 0$$

$$\vdots$$

$$a_{21}X_n + a_{22}Y_n + a_{23}Z_n + a_{24} - a_{41}X_ny_n - a_{42}Y_ny_n - a_{43}Z_ny_n - a_{44}y_n = 0$$

n points
2n equations,
12 unknowns

Camera Model

$$\begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 & -x_1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2X_2 & -x_2X_2 & -x_2 \\ & & & & & & \vdots & & & & & \\ X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n & -x_n \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 & -y_1 \\ 0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 & -y_2 \\ & & & & & & \vdots & & & & & \\ 0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nX_n & -y_n \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ a_{44} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

This is a homogenous system, no unique solution $\qquad CP = 0$

Select $a_{44} = 1$

Camera Model

$$
\begin{bmatrix}
X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 & -x_1X_1 & -x_1Y_1 & -x_1Z_1 \\
X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 & -x_2X_2 & -x_2X_2 & -x_2X_2 \\
 & & & & & & \vdots & & & & \\
X_n & Y_n & Z_n & 1 & 0 & 0 & 0 & 0 & -x_nX_n & -x_nY_n & -x_nZ_n \\
0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 & -y_1X_1 & -y_1X_1 & -y_1Z_1 \\
0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 & -y_2X_2 & -y_2X_2 & -y_2Z_2 \\
 & & & & & & \vdots & & & & \\
0 & 0 & 0 & 0 & X_n & Y_n & Z_n & 1 & -y_nX_n & -y_nX_n & -y_nX_n
\end{bmatrix}
\begin{bmatrix}
a_{11} \\ a_{12} \\ a_{13} \\ a_{14} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{24} \\ a_{41} \\ a_{42} \\ a_{43} \\ \\
\end{bmatrix}
=
\begin{bmatrix}
x_1 \\ x_2 \\ \\ \\ \\ x_n \\ y_1 \\ y_2 \\ \\ \\ \\ y_n
\end{bmatrix}
$$

Pseudo inverse

$$DQ = R$$

$$D^T DQ = D^T R$$

$$Q = (D^T D)^{-1} D^T R$$

Recovering Camera Parameters

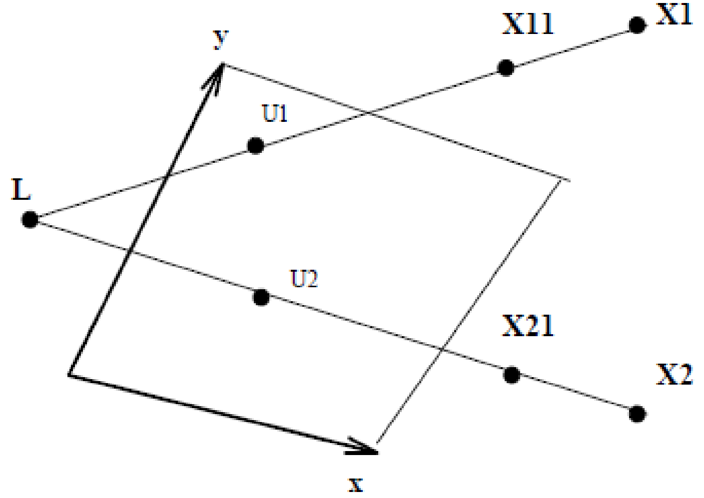
Given a photograph taken from an unknown camera, unknown location….

How can one determine:

- Camera Location??
- Camera Orientation??
- Extent to which picture was croppped or enlarged.??

Finding Camera Location

- Take one 3D point X_1 and find its image homogenous coordinates.
- Set the third component of homogenous coordinates to zero, find corresponding World coordinates of that point, X_{11}
- Connect X_1 and X_{11} to get a line in 3D.
- Repeat this for another 3D point X_2 and find another line
- Two lines will intersect at the location of camera.



Camera Location

$$C_h = AW_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$U_1 = AX_1$$

$$U_1{}' = \begin{pmatrix} Ch_1 & Ch_2 & 0 & Ch_4 \end{pmatrix}$$

$$X_{11} = A^{-1}U_1{}'$$

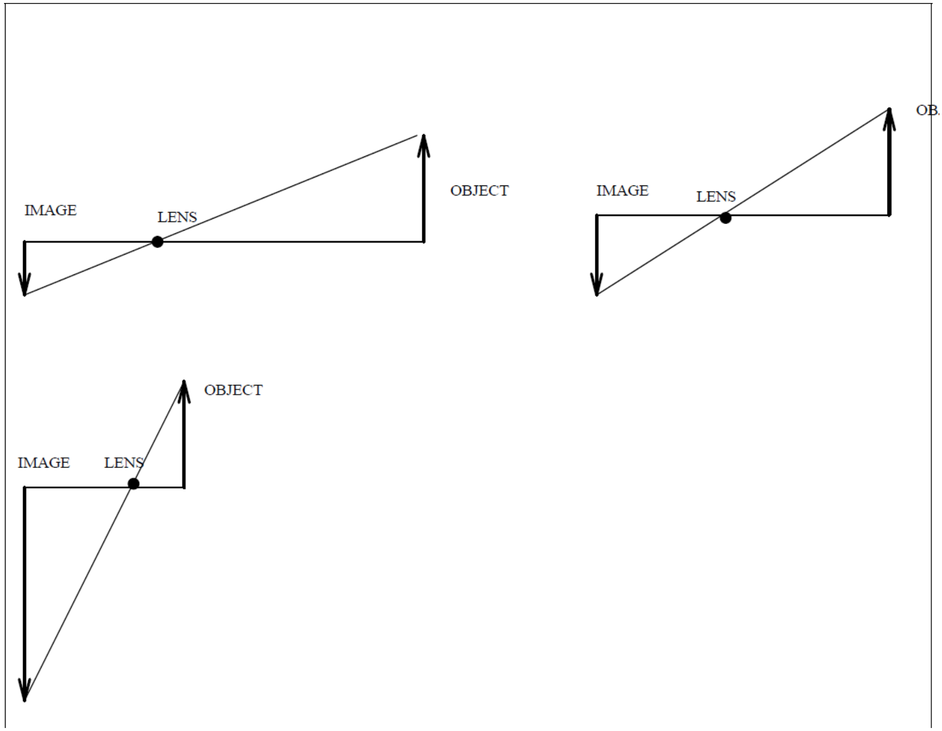
$$U_2 = AX_2$$

$$U_2{}' = (\begin{pmatrix} Ch_1 & Ch_2 & 0 & Ch_4 \end{pmatrix}$$

$$X_{22} = A^{-1}U_2{}'$$

Camera Orientation

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ Ch_4 \end{bmatrix} = A \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \qquad x = \frac{C_{h1}}{C_{h4}} \qquad y = \frac{C_{h2}}{C_{h4}}$$



•Only time the image will be formed at infinity if $Ch_4=0$.

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_3 \\ 0 \end{bmatrix} = A \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$a_{41}X + a_{42}Y + a_{43}Z + a_{44} = 0$$

- This is equation of a plane, going through the lens, and parallel to image plane.
- (a41, a42, a43) is normal to image plane and parallel to the camera axis.
- Clockwise rotation $R_x(\theta)$, $R_y(\emptyset)$, … θ and $\emptyset$ can be computed

$$\theta = \arctan \frac{a_{42}}{-a_{43}}, \qquad \phi = \arcsin \frac{-a_{41}}{\sqrt{{a_{41}}^2 + {a_{42}}^2 + {a_{43}}^2}}.$$

Application

$$M = \begin{bmatrix} .17237 & -.15879 & .01879 & 274.943 \\ .131132 & .112747 & .2914 & 258.686 \\ .000346 & .0003 & .00006 & 1 \end{bmatrix}$$

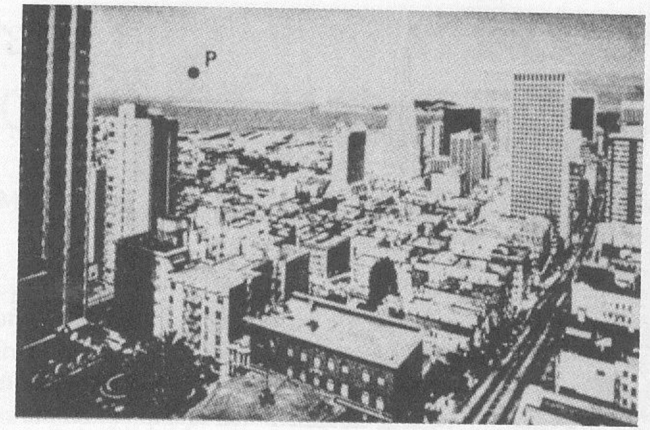


FIGURE 8 PHOTOGRAPH OF SAN FRANCISCO

Camera location: intersection of California and Mason streets, at an elevation of 435 feet above sea level. The camera was oriented at an angle of 8^0 above the horizon. $fs_x=495$, $fs_y=560$.

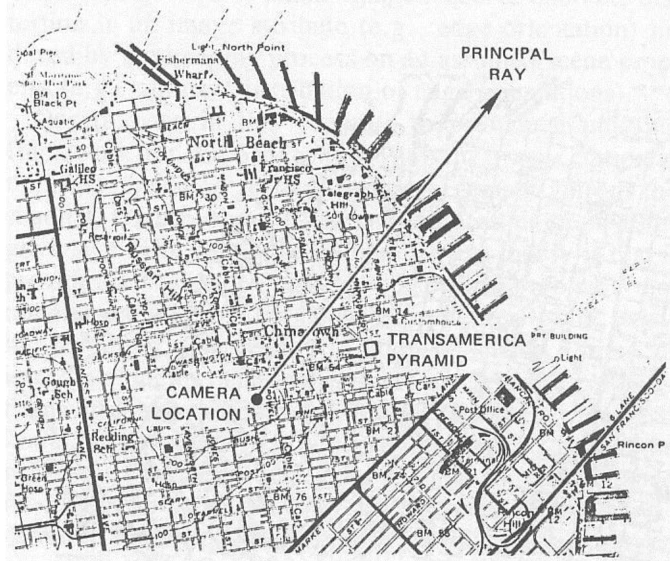


FIGURE 9 MAP OF SAN FRANCISCO

Application

$$M = \begin{bmatrix} -.175451 & -.10520 & .00435 & 297.83 \\ .02698 & -.09635 & .2303 & 249.574 \\ .00015 & -.00016 & .00001 & 1.0 \end{bmatrix}$$

Camera location: at an elevation of 1200 feet above sea level. The camera was oriented at an angle of 4^0 above the horizon. $fs_x=876$, $fs_y=999$.

Recovering the camera parameters from a transformation matrix
TM **Strat -** Readings in Computer Vision, 1987

FIGURE 10 ANOTHER PHOTOGRAPH OF SAN FRANCISCO

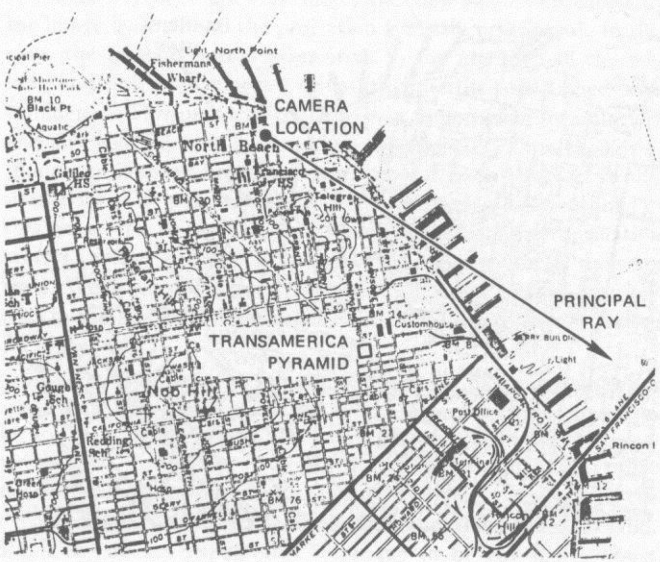


FIGURE 11 MAP OF SAN FRANCISCO

Camera Parameters

- Extrinsic parameters
  - Parameters that define the location and orientation of the camera reference frame with respect to a known world reference frame
    - 3-D translation vector
    - A 3 by 3 rotation matrix

$$= \left[A \right] = \underbrace{\left[I_n \right] \left[E_x \right]}$$

- Intrinsic parameters
  - Parameters necessary to link the pixel coordinates of an image point with the corresponding coordinates in the camera reference frame
    - Perspective projection (focal length)
    - Transformation between camera frame coordinates and pixel coordinates

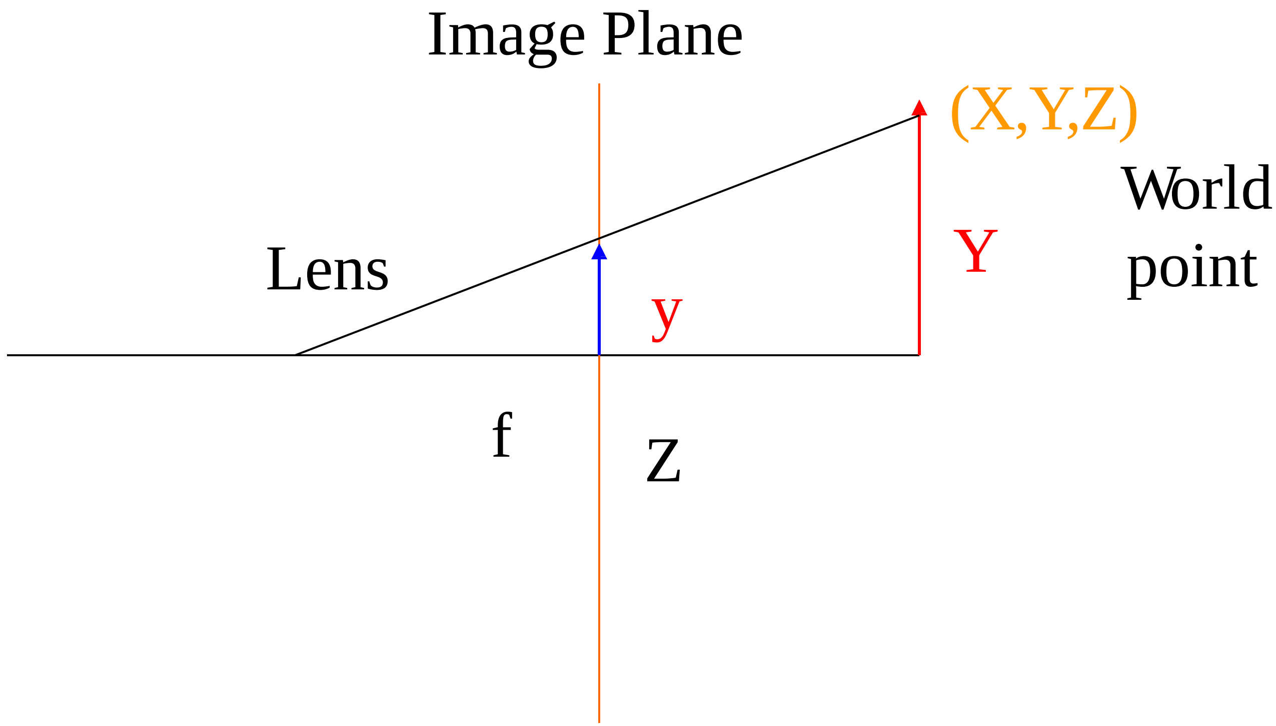
Camera Model Revisited: Rotation & Translation

$$P_c = TRP_w = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$P_c = M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Perspective Projection: Revisited



$$\frac{y}{Y} = \frac{f}{Z}$$

$$y = \frac{fY}{Z} \qquad x = \frac{fX}{Z}$$

Origin at the lens
Image plane in front of the lens

Camera Model Revisited: Perspective

$$C_h = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$x = \frac{fX}{Z}$$

$$y = \frac{fY}{Z}$$

Origin at the lens

Image plane in front of the lens

Camera Model Revisited: Image and Camera coordinates

$$x = -(x_{im} - o_x)s_x$$

$$y = -(y_{im} - o_y)s_y$$

$$x_{im} = -\frac{x}{s_x} + o_x$$

$$y_{im} = -\frac{y}{s_y} + o_y$$

$$\begin{bmatrix} x_{im} \\ y_{im} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(x_{im}, y_{im}) image coordinates

(x, y) camera coordinates

(o_x, o_y) image center (in pixels)

(s_x, s_y) effective size of pixels (in millimeters) in the horizontal and vertical directions.

Camera Model Revisited

$$C_h = C'P'T'R'W_h$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{s_x} & 0 & o_x \\ 0 & -\frac{1}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M_{\text{int}} M_{ext} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & T_x \\ r_{21} & r_{22} & r_{23} & T_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} r_{11} + r_{31} o_x & -\frac{f}{s_x} r_{12} + r_{32} o_x & -\frac{f}{s_x} r_{13} + r_{33} o_x & -\frac{f}{s_x} T_x + T_z o_x \\ -\frac{f}{s_y} r_{21} + r_{31} o_y & -\frac{f}{s_y} r_{22} + r_{32} o_y & -\frac{f}{s_y} r_{23} + r_{33} o_y & -\frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Camera Model Revisited

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -\frac{f}{s_x} r_{11} + r_{31} o_x & -\frac{f}{s_x} r_{12} + r_{32} o_x & -\frac{f}{s_x} r_{13} + r_{33} o_x & -\frac{f}{s_x} T_x + T_z o_x \\ -\frac{f}{s_y} r_{21} + r_{31} o_y & -\frac{f}{s_y} r_{22} + r_{32} o_y & -\frac{f}{s_y} r_{23} + r_{33} o_y & -\frac{f}{s_y} T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

f_x effective focal length expressed in effective horizontal pixel size

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Ch_1 \\ Ch_2 \\ Ch_4 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Computing Camera Parameters

- Using known 3-D points and corresponding image points, estimate camera matrix employing pseudo inverse method of section 1.6 (Fundamental of Computer Vision).
- Compute camera parameters by relating camera matrix with estimated camera matrix.
  - Extrinsic
    - Translation
    - Rotation
  - Intrinsic
    - Horizontal f_x and f_y vertical focal lengths
    - Translation o_x and o_y

Comparison

$$\hat{M} = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix}$$

$$M = \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Computing Camera Parameters

estimated

$$\hat{M} = \gamma M$$

Since M is defined up to a scale factor

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$\sqrt{\hat{m}_{31}^2 + \hat{m}_{32}^2 + \hat{m}_{33}^2} = |\gamma| \sqrt{r_{31}^2 + r_{32}^2 + r_{33}^2} = |\gamma|$$

Because rotation matrix is orthonormal

Divide each entry of $\hat{M}$ by $|\gamma|$.

Computing Camera Parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

- Compute T_z and third row r_{3i} $(i=1,2,3)$
- Compute o_x and o_y
- Compute f_x and f_y
- Compute r_{1i} and r_{2i} $i=1,2,3$
- Computer T_x and T_y

Computing Camera Parameters: estimating third row of rotation matrix and translation in depth

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$T_z = \sigma \hat{m}_{34}, \quad \sigma = \pm 1$$

$$r_{3i} = \sigma \hat{m}_{3i}, \qquad i = 1,2,3$$

Since we can determine T_z*>0 (origin of world reference is in front)*
Or T_z*<0(origin of world reference is in back)*
we can determine sign.

Computing Camera Parameters: origin of image

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$q_1^T q_3 = \hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33}$$

$$\begin{aligned} \hat{m}_{11}\hat{m}_{31} + \hat{m}_{12}\hat{m}_{32} + \hat{m}_{13}\hat{m}_{33} = & \begin{pmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x \end{pmatrix} \cdot \begin{pmatrix} r_{31} & r_{32} & r_{33} \end{pmatrix} \\ & = \begin{pmatrix} -f_x r_{11} & -f_x r_{12} & -f_x r_{13} \end{pmatrix} \cdot \begin{pmatrix} r_{31} & r_{32} & r_{33} \end{pmatrix} + \\ & \begin{pmatrix} r_{31} o_x & r_{32} o_x & r_{32} o_x \end{pmatrix} \cdot \begin{pmatrix} r_{31} & r_{32} & r_{33} \end{pmatrix} \\ & = \begin{pmatrix} r_{31} o_x & r_{32} o_x & r_{32} o_x \end{pmatrix} \cdot \begin{pmatrix} r_{31} & r_{32} & r_{33} \end{pmatrix} \\ & = (r_{31}{}^2 o_x + r_{32}{}^2 o_x + r_{33}{}^2 o_x) \\ & = o_x (r_{31}{}^2 + r_{32}{}^2 + r_{33}{}^2) \\ q_1^T q_3 \quad & = o_x \end{aligned}$$

Therefore:

$$o_x = q_1^T q_3$$

$$o_y = q_2^T q_3$$

Computing Camera Parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

Let

$$q_1 = \begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} \end{bmatrix}$$
$$q_2 = \begin{bmatrix} \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} \end{bmatrix}$$
$$q_3 = \begin{bmatrix} \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} \end{bmatrix}$$

Computing Camera Parameters: vertical and horizontal focal lengths

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$q_1^T q_1 = \hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13}$$

$$\hat{m}_{11}\hat{m}_{11} + \hat{m}_{12}\hat{m}_{12} + \hat{m}_{13}\hat{m}_{13} =$$

$$(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x).(-f_x r_{11} + r_{31} o_x \quad -f_x r_{12} + r_{32} o_x \quad -f_x r_{13} + r_{33} o_x)$$

$$= (-f_x r_{11} + r_{31} o_x)^2 + (-f_x r_{12} + r_{32} o_x)^2 + (-f_x r_{13} + r_{33} o_x)^2$$

$$= (f_x{}^2 r_{11}{}^2 + r_{31}{}^2 o_x{}^2) + (f_x{}^2 r_{12}{}^2 + r_{32}{}^2 o_x{}^2) + (f_x{}^2 r_{13}{}^2 + r_{33}{}^2 o_x{}^2)$$

$$= f_x{}^2 + o_x{}^2$$

$$q_1^T q_1 \qquad = f_x{}^2 + o_x{}^2$$

$$\sqrt{q_1^T q_1 - o_x^2} \qquad = f_x$$

$$f_x = \sqrt{q_1^T q_1 - o_x^2}$$

Therefore:

$$f_y = \sqrt{q_2^T q_2 - o_y^2}$$

Computing Camera Parameters: remaining rotation and translation parameters

$$\begin{bmatrix} \hat{m}_{11} & \hat{m}_{12} & \hat{m}_{13} & \hat{m}_{14} \\ \hat{m}_{21} & \hat{m}_{22} & \hat{m}_{23} & \hat{m}_{24} \\ \hat{m}_{31} & \hat{m}_{32} & \hat{m}_{33} & \hat{m}_{34} \end{bmatrix} = \gamma \begin{bmatrix} -f_x r_{11} + r_{31} o_x & -f_x r_{12} + r_{32} o_x & -f_x r_{13} + r_{33} o_x & -f_x T_x + T_z o_x \\ -f_y r_{21} + r_{31} o_y & -f_y r_{22} + r_{32} o_y & -f_y r_{23} + r_{33} o_y & -f_y T_y + T_z o_y \\ r_{31} & r_{32} & r_{33} & T_z \end{bmatrix}$$

$$0_x r_{31} + f_x r_{11} - r_{31} 0_x = \sigma(0_x \hat{m}_{31} - \hat{m}_{11}),$$

$$r_{11} = \sigma(0_x \hat{m}_{31} - \hat{m}_{11}) / f_x,$$

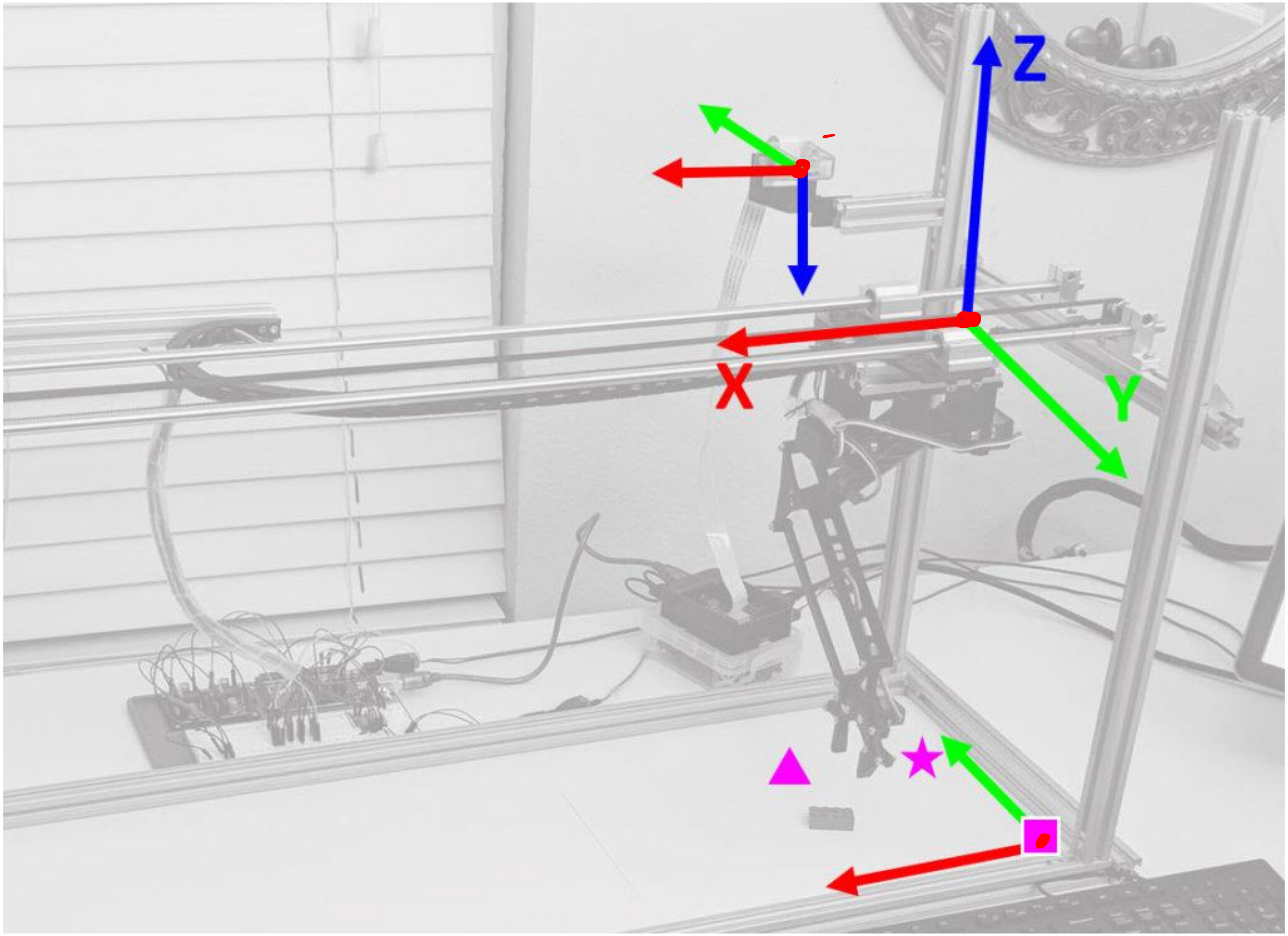
$$r_{1i} = \sigma(0_x \hat{m}_{3i} - \hat{m}_{1i}) / f_x, \quad i = 1,2,3$$

$$r_{2i} = \sigma(0_y \hat{m}_{3i} - \hat{m}_{2i}) / f_y, \quad i = 1,2,3$$

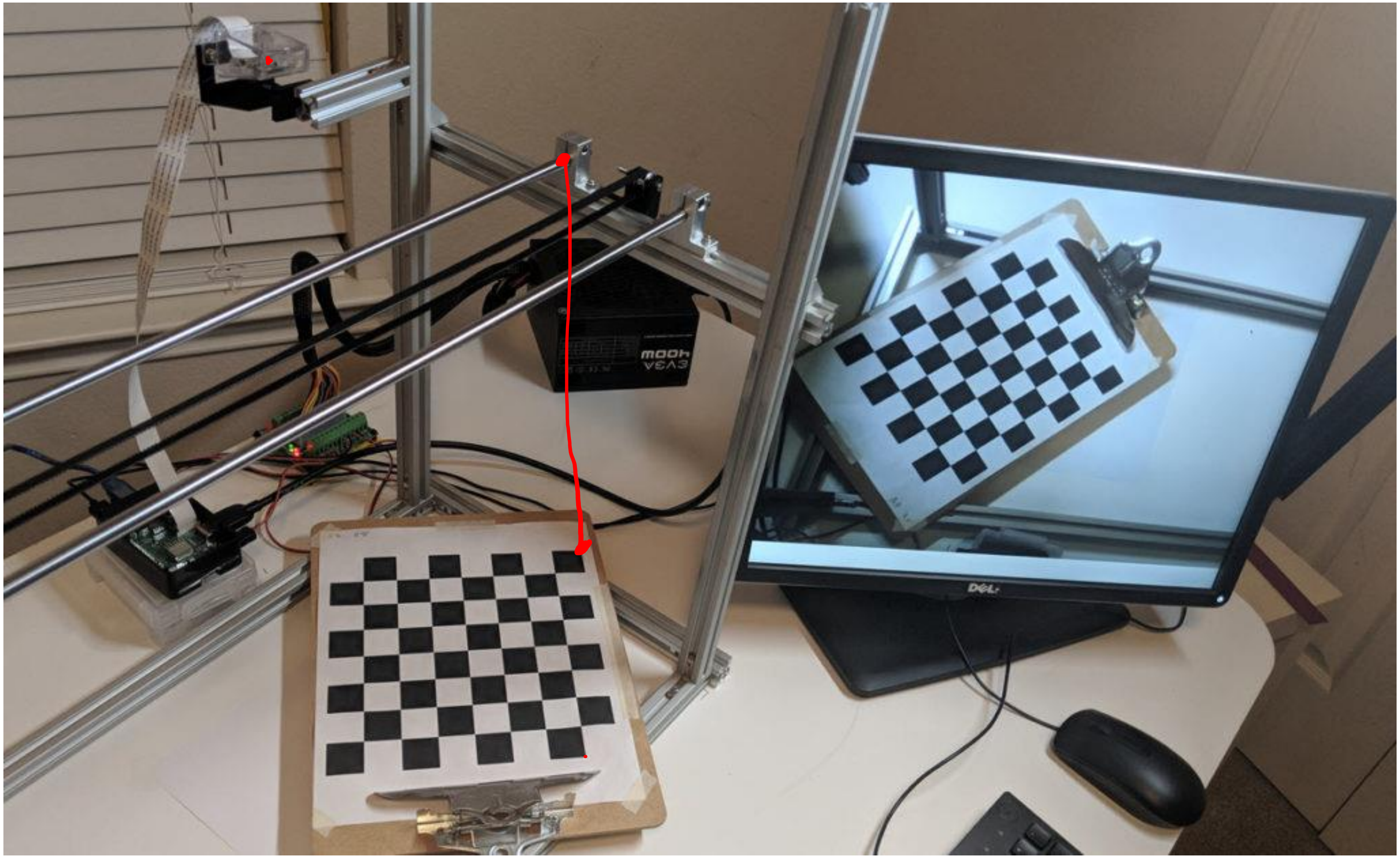
$$T_x = \sigma(0_x \hat{m}_{34} - \hat{m}_{14}) / f_x$$

$$T_y = \sigma(0_y \hat{m}_{34} - \hat{m}_{24}) / f_y$$

Assignment



Chessboard Calibration



Chess board calibaration

for **single fixed camera, a matrix is available say:**

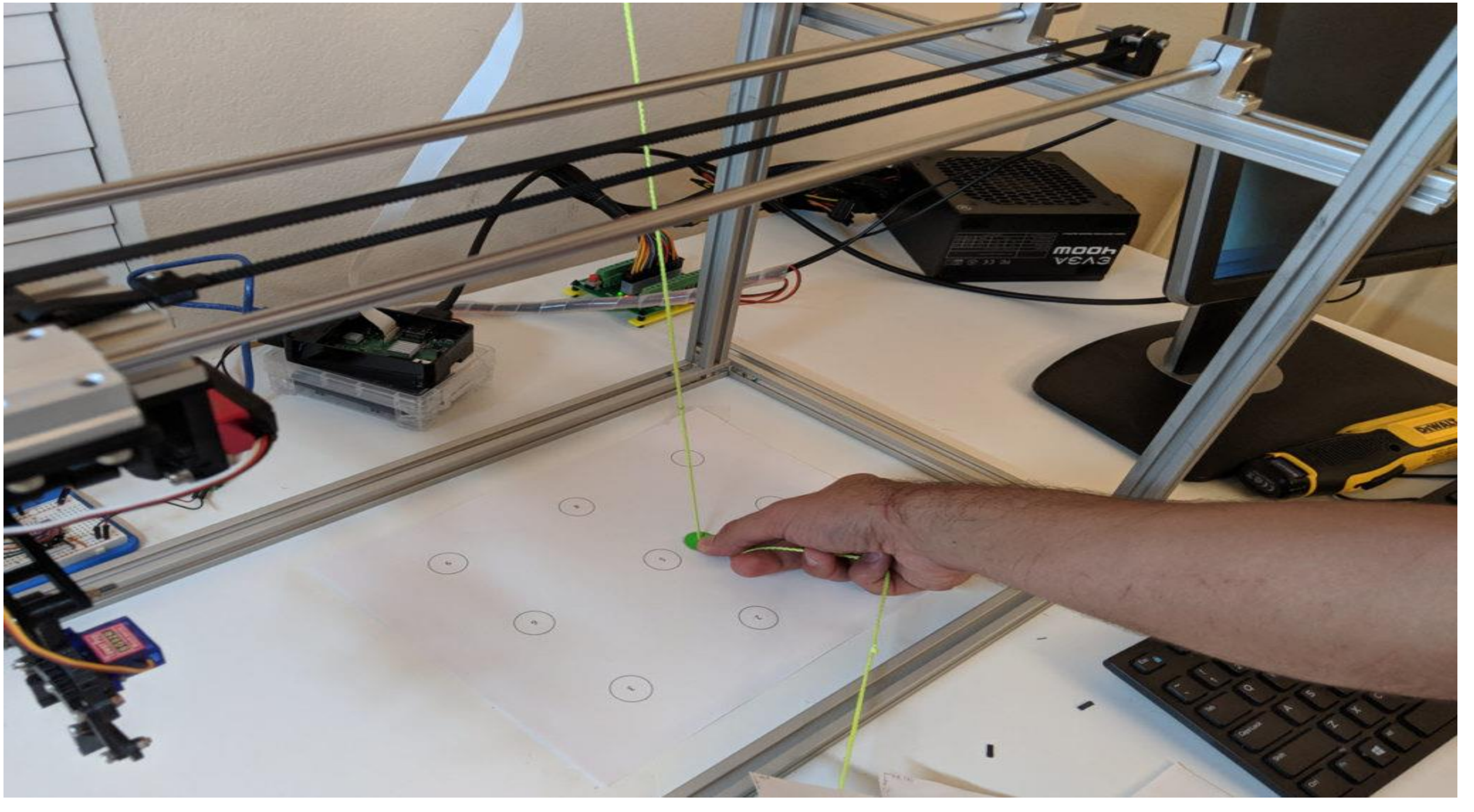
$$s\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}f_x & 0 & c_x\\0 & f_y & c_y\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}r_{11} & r_{12} & r_{13} & t_1\\r_{21} & r_{22} & r_{23} & t_2\\r_{31} & r_{32} & r_{33} & t_3\end{bmatrix}\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

where:

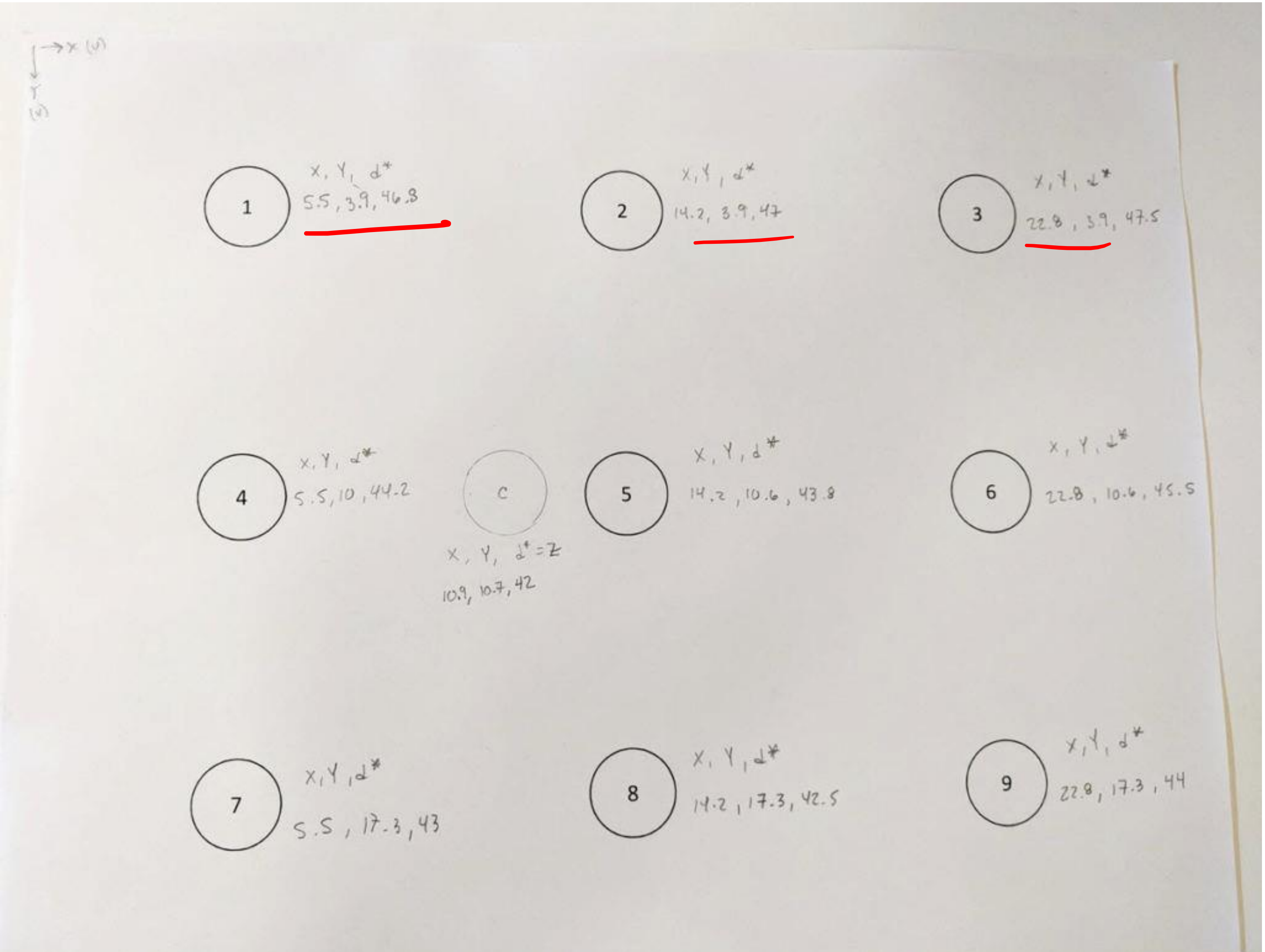
- (X, Y, Z) are the coordinates of a 3D point in the world coordinate space
- (u, v) are the coordinates of the projection point in pixels
- A is a camera matrix, or a matrix of intrinsic parameters
- (cx, cy) is a principal point that is usually at the image center
- fx, fy are the focal lengths expressed in pixel units.

The pinhole camera model

We locate X and Y points and we measure with a string the Z value:



Real World Points for our perspective calibration



Given these image points and real world cordinates, Find camera parameters

- X_center=10.9 Y_center=10.7 Z_center=42

- WorldPoints=([[X_center,Y_center,Z_center], [5.5,3.9,46.8], [14.2,3.9,47.0], [22.8,3.9,47.4], [5.5,10.6,44.2], [14.2,10.6,43.8], [22.8,10.6,44.8], [5.5,17.3,43], [14.2,17.3,42.5], [22.8,17.3,44.4]])

- ImagePoints=([[cx=6.28,cy=3.42], [502,185], [700,197], [894,208], [491,331], [695,342], [896,353], [478,487], [691,497], [900,508]])

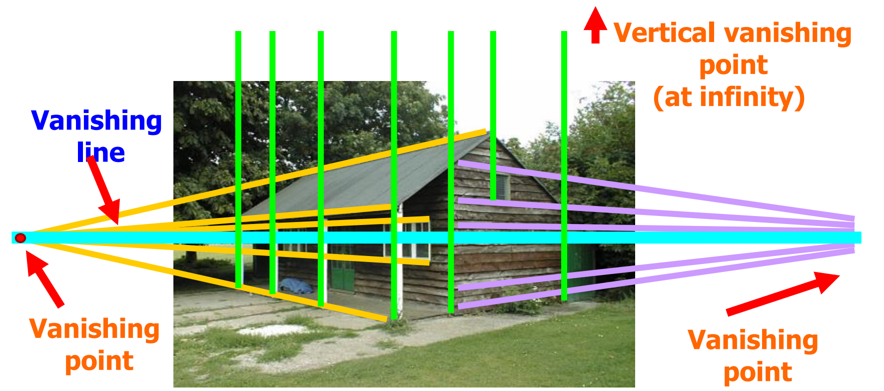
Given these image points and real world coordinates, Find camera parameters

- **HW Assignment:**

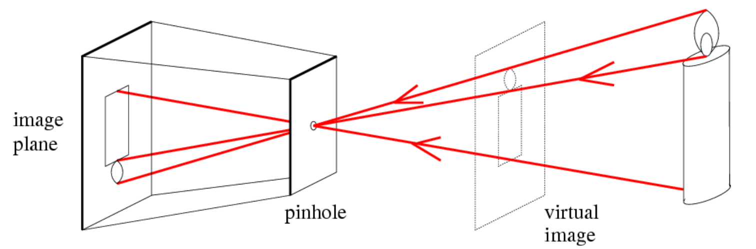
1. Solve to find the intrinsic and extensive camera parameters…
2. Minimum how many points are required to solve this problem. Explain
3. Find the location and orientation of camera object

Things to remember

Vanishing points and vanishing lines



Pinhole camera model and camera projection matrix



$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{R} & \mathbf{t}\end{bmatrix}\mathbf{X}$$

Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

James Hays

Demo – Kyle Simek

"Dissecting the Camera Matrix"
Three-part blog series
http://ksimek.github.io/2012/08/14/decompose/
http://ksimek.github.io/2012/08/22/extrinsic/
http://ksimek.github.io/2013/08/13/intrinsic/

"Perspective toy"
http://ksimek.github.io/perspective_camera_toy.html

Reading Material

- Chapter 1, Fundamental Of Computer Vision, Mubarak Shah
- Chapter 6, Introductory Techniques, E. Trucco and A. Verri, Prentice Hall, 1998.
- **Recovering the camera parameters from a transformation matrix,** TM **Strat** - Readings in Computer Vision, 1987

Slide Credits

1. Fundamentals of computer vision, by Mubarak shah.