

Translation

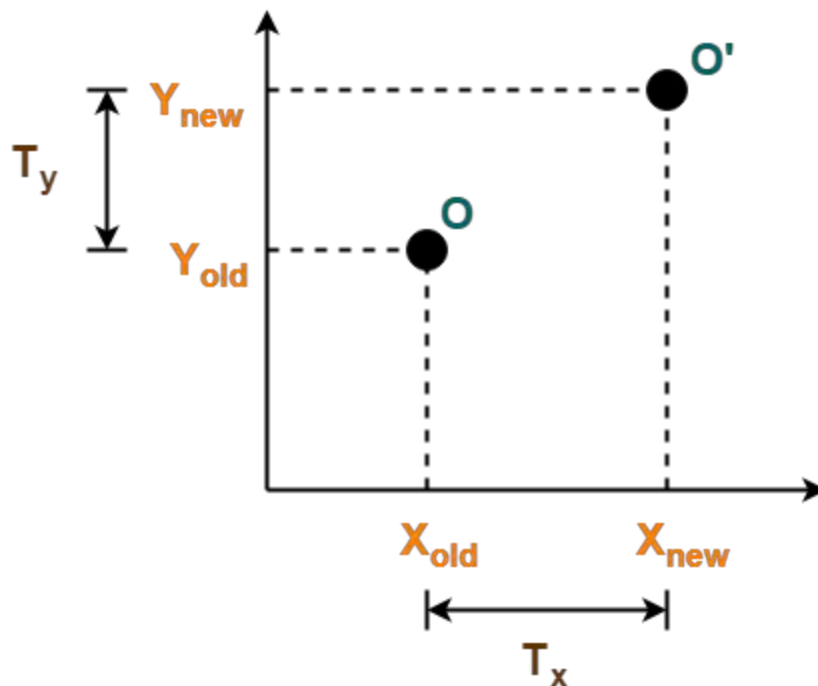
Consider a point object O that has to be moved from one position to another in a plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the object O after translation = (X_{new}, Y_{new})
- Translation vector or Shift vector = (T_x, T_y)

Given a Translation vector (T_x, T_y) -

- T_x defines the distance the X_{old} coordinate has to be moved.
- T_y defines the distance the Y_{old} coordinate has to be moved.



This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

- $X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)
- $Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)

In Matrix form, the above translation equations may be represented as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

Translation Matrix

- The homogeneous coordinate representation of (X, Y) is (X, Y, 1).
- Through this representation, all the transformations can be performed using matrix / vector multiplications.

The above translation matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \\ 1 \end{bmatrix}$$

Translation Matrix
(Homogeneous Coordinates Representation)

PRACTICE PROBLEMS BASED ON TRANSLATION

Problem-01:

Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Solution-

Given-

- Old center coordinates of C = $(X_{\text{old}}, Y_{\text{old}}) = (1, 4)$
- Translation vector = $(T_x, T_y) = (5, 1)$

Let the new center coordinates of C = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 1 + 5 = 6$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 4 + 1 = 5$

Thus, New center coordinates of C = (6, 5).

Alternatively,

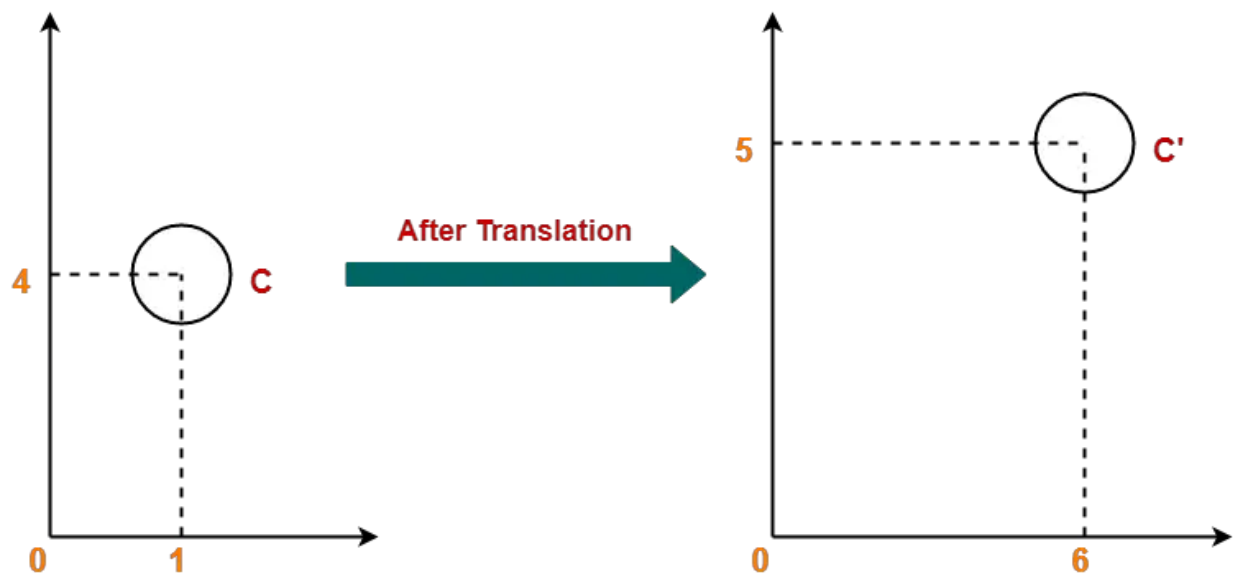
In matrix form, the new center coordinates of C after translation may be obtained as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Thus, New center coordinates of C = (6, 5).



Problem-02:

Given a square with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the translation with distance 1 towards X axis and 1 towards Y axis. Obtain the new coordinates of the square.

Solution-

Given-

- Old coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
- Translation vector = $(T_x, T_y) = (1, 1)$

For Coordinates A(0, 3)

Let the new coordinates of corner A = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 0 + 1 = 1$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 3 + 1 = 4$

Thus, New coordinates of corner A = (1, 4).

For Coordinates B(3, 3)

Let the new coordinates of corner B = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 3 + 1 = 4$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 3 + 1 = 4$

Thus, New coordinates of corner B = (4, 4).

For Coordinates C(3, 0)

Let the new coordinates of corner C = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 3 + 1 = 4$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 0 + 1 = 1$

Thus, New coordinates of corner C = $(4, 1)$.

For Coordinates D(0, 0)

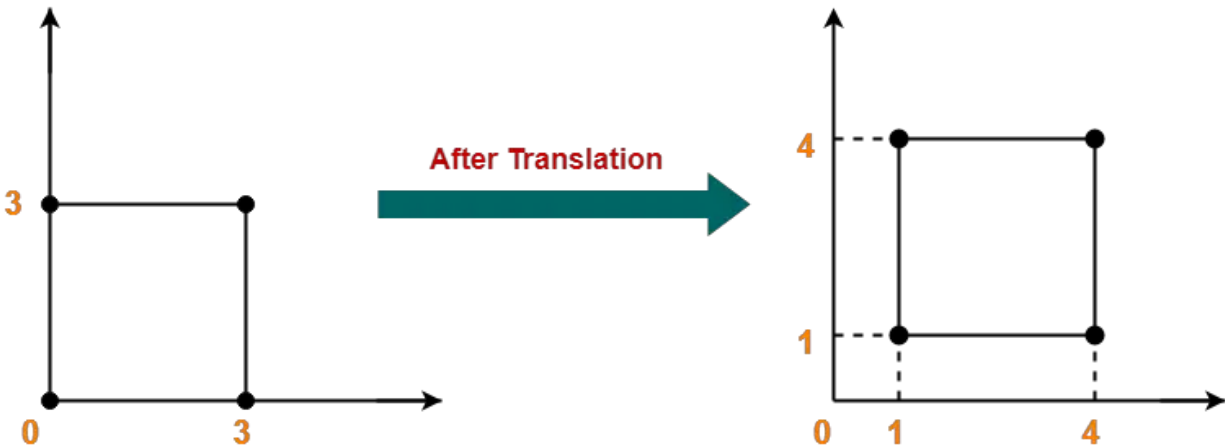
Let the new coordinates of corner D = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the translation equations, we have-

- $X_{\text{new}} = X_{\text{old}} + T_x = 0 + 1 = 1$
- $Y_{\text{new}} = Y_{\text{old}} + T_y = 0 + 1 = 1$

Thus, New coordinates of corner D = $(1, 1)$.

Thus, New coordinates of the square = A $(1, 4)$, B $(4, 4)$, C $(4, 1)$, D $(1, 1)$.

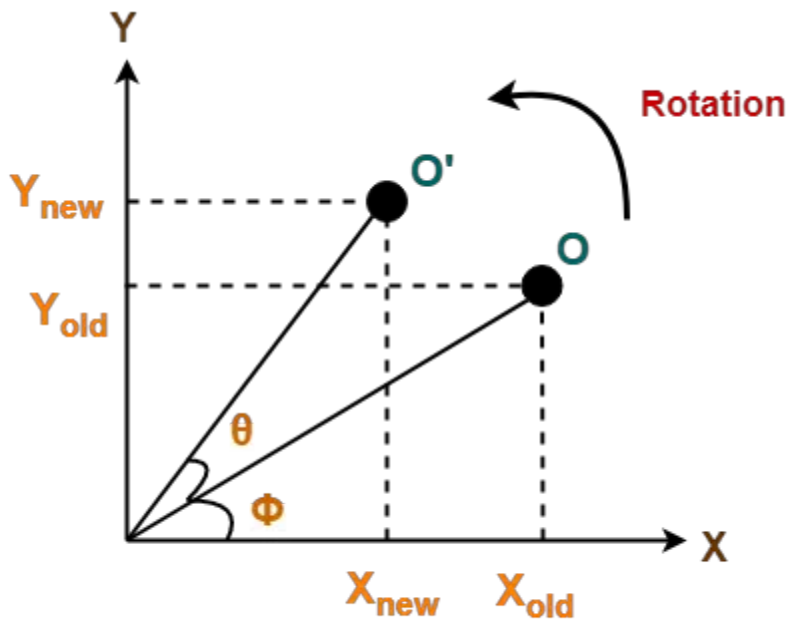


Rotation

Consider a point object O has to be rotated from one angle to another in a plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = (X_{new}, Y_{new})



This rotation is achieved by using the following rotation equations-

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix}$$

Rotation Matrix

For homogeneous coordinates, the above rotation matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \\ 1 \end{bmatrix}$$

Rotation Matrix
(Homogeneous Coordinates Representation)

PRACTICE PROBLEMS BASED ON ROTATION

Problem-01:

Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Solution-

We rotate a straight line by its end points with the same angle. Then, we re-draw a line between the new end points.

Given-

- Old ending coordinates of the line = $(X_{\text{old}}, Y_{\text{old}}) = (4, 4)$
- Rotation angle = $\theta = 30^\circ$

Let new ending coordinates of the line after rotation = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the rotation equations, we have-

$$X_{\text{new}}$$

$$= X_{\text{old}} \times \cos\theta - Y_{\text{old}} \times \sin\theta$$

$$= 4 \times \cos 30^\circ - 4 \times \sin 30^\circ$$

$$= 4 \times (\sqrt{3} / 2) - 4 \times (1 / 2)$$

$$= 2\sqrt{3} - 2$$

$$= 2(\sqrt{3} - 1)$$

$$= 2(1.73 - 1)$$

$$= 1.46$$

$$Y_{\text{new}}$$

$$= X_{\text{old}} \times \sin\theta + Y_{\text{old}} \times \cos\theta$$

$$= 4 \times \sin 30^\circ + 4 \times \cos 30^\circ$$

$$= 4 \times (1 / 2) + 4 \times (\sqrt{3} / 2)$$

$$= 2 + 2\sqrt{3}$$

$$= 2(1 + \sqrt{3})$$

$$= 2(1 + 1.73)$$

$$= 5.46$$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

Alternatively,

In matrix form, the new ending coordinates of the line after rotation may be obtained as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix}$$

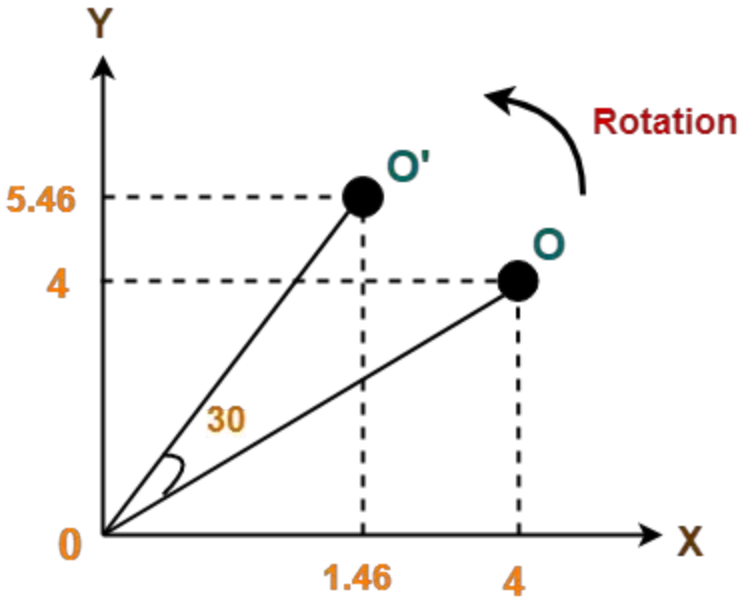
$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).



Problem-02:

Given a triangle with corner coordinates $(0, 0)$, $(1, 0)$ and $(1, 1)$. Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

Solution-

We rotate a polygon by rotating each vertex of it with the same rotation angle.

Given-

- Old corner coordinates of the triangle = A $(0, 0)$, B $(1, 0)$, C $(1, 1)$
- Rotation angle = $\theta = 90^\circ$

For Coordinates A(0, 0)

Let the new coordinates of corner A after rotation = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the rotation equations, we have-

$$\begin{aligned}X_{\text{new}} &= X_{\text{old}} \times \cos\theta - Y_{\text{old}} \times \sin\theta \\&= 0 \times \cos 90^\circ - 0 \times \sin 90^\circ \\&= 0\end{aligned}$$

$$\begin{aligned}Y_{\text{new}} &= X_{\text{old}} \times \sin\theta + Y_{\text{old}} \times \cos\theta \\&= 0 \times \sin 90^\circ + 0 \times \cos 90^\circ \\&= 0\end{aligned}$$

Thus, New coordinates of corner A after rotation = (0, 0).

For Coordinates B(1, 0)

Let the new coordinates of corner B after rotation = (X_{new} , Y_{new}).

$$\begin{aligned}X_{\text{new}} &= X_{\text{old}} \times \cos\theta - Y_{\text{old}} \times \sin\theta \\&= 1 \times \cos 90^\circ - 0 \times \sin 90^\circ \\&= 0\end{aligned}$$

$$\begin{aligned}Y_{\text{new}} &= X_{\text{old}} \times \sin\theta + Y_{\text{old}} \times \cos\theta \\&= 1 \times \sin 90^\circ + 0 \times \cos 90^\circ \\&= 1 + 0 \\&= 1\end{aligned}$$

Thus, New coordinates of corner B after rotation = (0, 1).

For Coordinates C(1, 1)

Let the new coordinates of corner C after rotation = (X_{new} , Y_{new}).

$$X_{\text{new}}$$

$$= X_{\text{old}} \times \cos\theta - Y_{\text{old}} \times \sin\theta$$

$$= 1 \times \cos 90^\circ - 1 \times \sin 90^\circ$$

$$= 0 - 1$$

$$= -1$$

$$Y_{\text{new}}$$

$$= X_{\text{old}} \times \sin\theta + Y_{\text{old}} \times \cos\theta$$

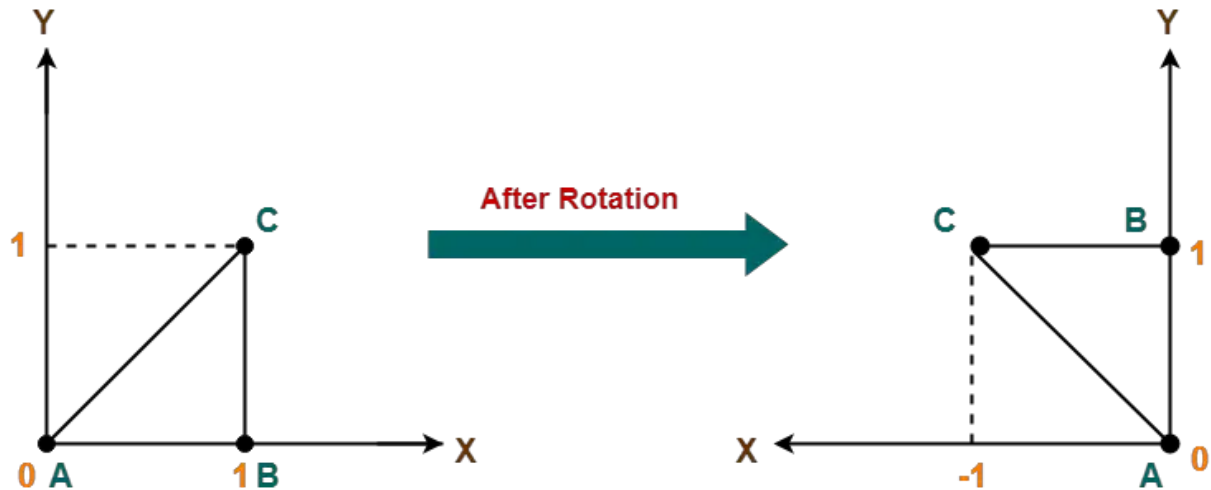
$$= 1 \times \sin 90^\circ + 1 \times \cos 90^\circ$$

$$= 1 + 0$$

$$= 1$$

Thus, New coordinates of corner C after rotation = (-1, 1).

Thus, New coordinates of the triangle after rotation = A (0, 0), B(0, 1), C(-1, 1).



Reflection

- Reflection is a kind of rotation where the angle of rotation is 180 degree.
- The reflected object is always formed on the other side of the mirror.
- The size of the reflected object is the same as the size of the original object.

Consider a point object O that has to be reflected in a plane.

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the reflected object O after reflection = (X_{new}, Y_{new})

Reflection On X-Axis:

This reflection is achieved by using the following reflection equations-

- $X_{new} = X_{old}$
- $Y_{new} = -Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix}$$

Reflection Matrix
(Reflection Along X Axis)

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \\ 1 \end{bmatrix}$$

Reflection Matrix
(Reflection Along X Axis)
(Homogeneous Coordinates Representation)

Reflection On Y-Axis:

This reflection is achieved by using the following reflection equations-

- $X_{\text{new}} = -X_{\text{old}}$
- $Y_{\text{new}} = Y_{\text{old}}$

In Matrix form, the above reflection equations may be represented as-

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \\ 1 \end{bmatrix}$$

Reflection Matrix
(Reflection Along Y Axis)
(Homogeneous Coordinates Representation)

PRACTICE PROBLEMS BASED ON REFLECTION

Problem-01:

Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the X axis

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = X_{\text{old}} = 3$
- $Y_{\text{new}} = -Y_{\text{old}} = -4$

Thus, New coordinates of corner A after reflection = $(3, -4)$.

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = X_{\text{old}} = 6$
- $Y_{\text{new}} = -Y_{\text{old}} = -4$

Thus, New coordinates of corner B after reflection = $(6, -4)$.

For Coordinates C(5, 6)

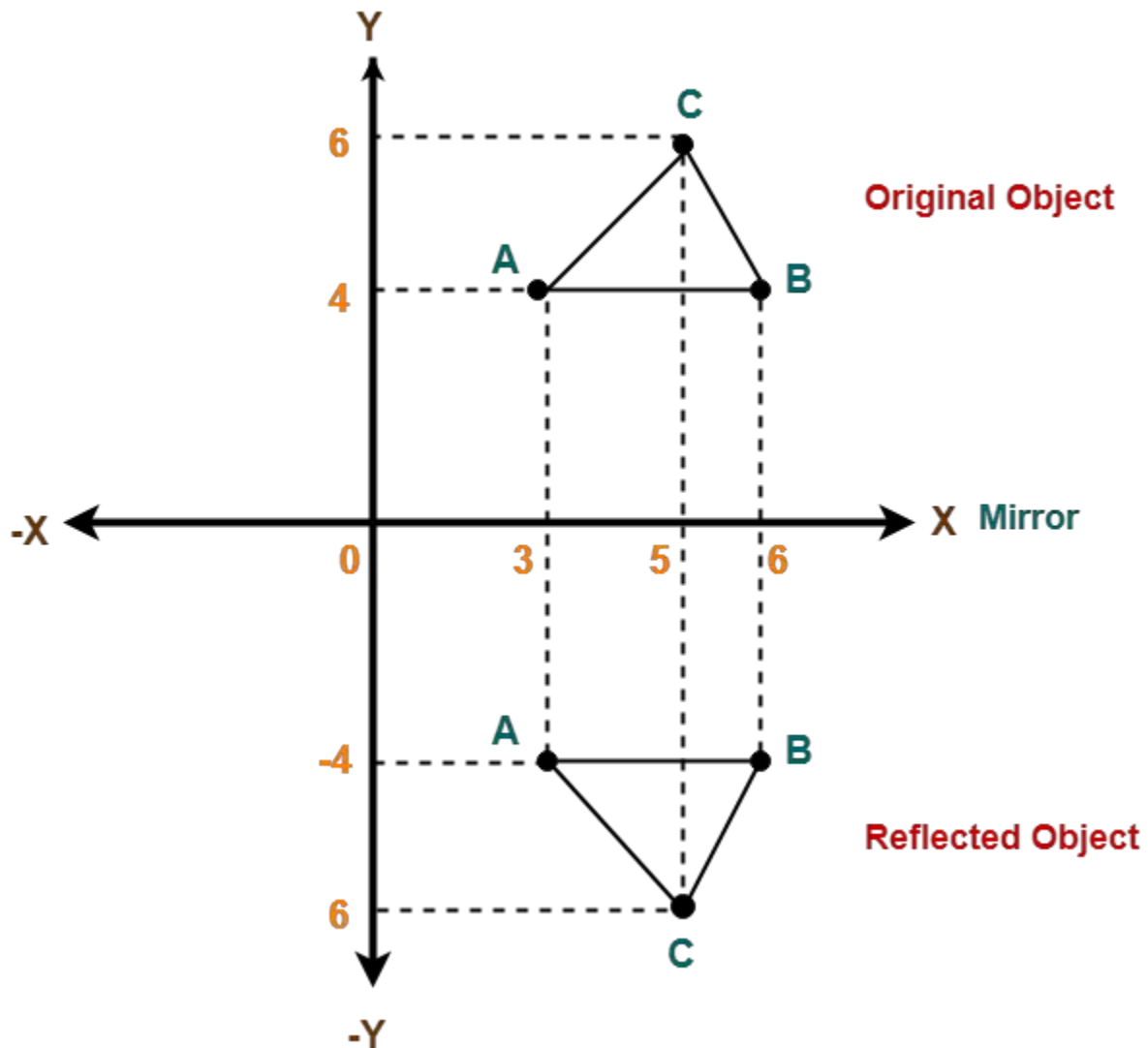
Let the new coordinates of corner C after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = X_{\text{old}} = 5$
- $Y_{\text{new}} = -Y_{\text{old}} = -6$

Thus, New coordinates of corner C after reflection = $(5, -6)$.

Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).



Problem-02:

Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the Y axis and obtain the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)

- Reflection has to be taken on the Y axis

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = -X_{\text{old}} = -3$
- $Y_{\text{new}} = Y_{\text{old}} = 4$

Thus, New coordinates of corner A after reflection = $(-3, 4)$.

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = -X_{\text{old}} = -6$
- $Y_{\text{new}} = Y_{\text{old}} = 4$

Thus, New coordinates of corner B after reflection = $(-6, 4)$.

For Coordinates C(5, 6)

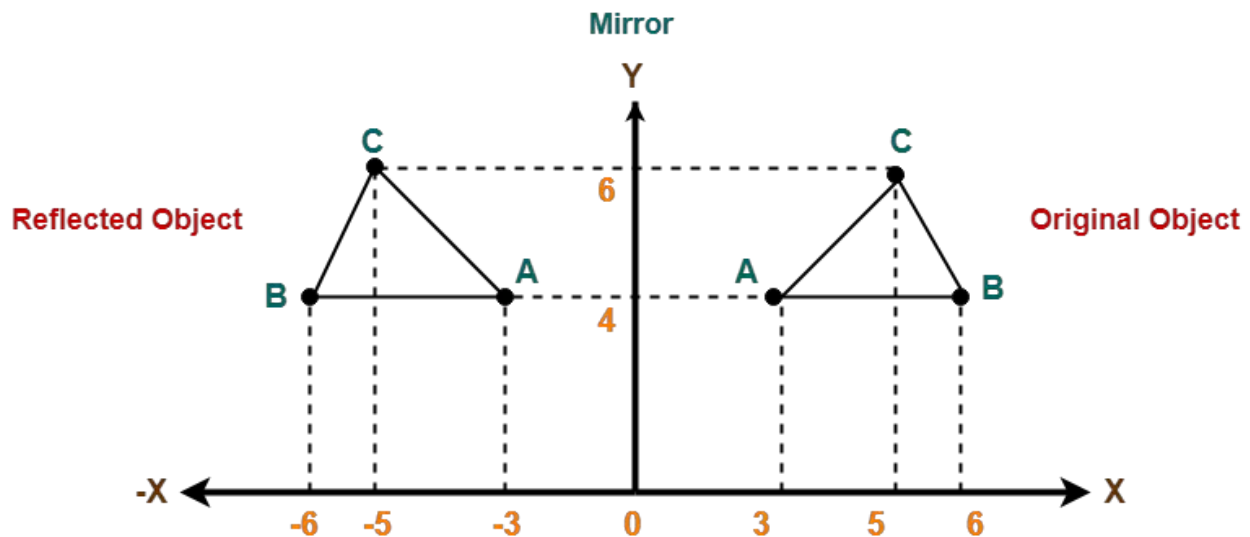
Let the new coordinates of corner C after reflection = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the reflection equations, we have-

- $X_{\text{new}} = -X_{\text{old}} = -5$
- $Y_{\text{new}} = Y_{\text{old}} = 6$

Thus, New coordinates of corner C after reflection = (-5, 6).

Thus, New coordinates of the triangle after reflection = A (-3, 4), B(-6, 4), C(-5, 6).



Shearing

In a two dimensional plane, the object size can be changed along X direction as well as Y direction.

So, there are two versions of shearing-

1. Shearing in X direction
2. Shearing in Y direction

Consider a point object O has to be sheared in a plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Shearing parameter towards X direction = Sh_x
- Shearing parameter towards Y direction = Sh_y

- New coordinates of the object O after shearing = $(X_{\text{new}}, Y_{\text{new}})$

Shearing in X Axis-

Shearing in X axis is achieved by using the following shearing equations-

- $X_{\text{new}} = X_{\text{old}} + Sh_x \times Y_{\text{old}}$
- $Y_{\text{new}} = Y_{\text{old}}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \end{bmatrix}$$

Shearing Matrix
(In X axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{\text{new}} \\ Y_{\text{new}} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{\text{old}} \\ Y_{\text{old}} \\ 1 \end{bmatrix}$$

Shearing Matrix
(In X axis)
(Homogeneous Coordinates Representation)

Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

- $X_{\text{new}} = X_{\text{old}}$
- $Y_{\text{new}} = Y_{\text{old}} + Sh_y \times X_{\text{old}}$

In Matrix form, the above shearing equations may be represented as-

PRACTICE PROBLEMS BASED ON SHEARING

Problem-01:

Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)
- Shearing parameter towards X direction (Sh_x) = 2
- Shearing parameter towards Y direction (Sh_y) = 2

Shearing in X Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new} , Y_{new}).

Applying the shearing equations, we have-

- $X_{\text{new}} = X_{\text{old}} + Sh_x \times Y_{\text{old}} = 1 + 2 \times 1 = 3$

- $Y_{\text{new}} = Y_{\text{old}} = 1$

Thus, New coordinates of corner A after shearing = (3, 1).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the shearing equations, we have-

- $X_{\text{new}} = X_{\text{old}} + Sh_x \times Y_{\text{old}} = 0 + 2 \times 0 = 0$
- $Y_{\text{new}} = Y_{\text{old}} = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the shearing equations, we have-

- $X_{\text{new}} = X_{\text{old}} + Sh_x \times Y_{\text{old}} = 1 + 2 \times 0 = 1$
- $Y_{\text{new}} = Y_{\text{old}} = 0$

Thus, New coordinates of corner C after shearing = (1, 0).

Thus, New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).

Shearing in Y Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the shearing equations, we have-

- $X_{\text{new}} = X_{\text{old}} = 1$
- $Y_{\text{new}} = Y_{\text{old}} + Sh_y \times X_{\text{old}} = 1 + 2 \times 1 = 3$

Thus, New coordinates of corner A after shearing = (1, 3).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the shearing equations, we have-

- $X_{\text{new}} = X_{\text{old}} = 0$
- $Y_{\text{new}} = Y_{\text{old}} + Sh_y \times X_{\text{old}} = 0 + 2 \times 0 = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = $(X_{\text{new}}, Y_{\text{new}})$.

Applying the shearing equations, we have-

- $X_{\text{new}} = X_{\text{old}} = 1$
- $Y_{\text{new}} = Y_{\text{old}} + Sh_y \times X_{\text{old}} = 0 + 2 \times 1 = 2$

Thus, New coordinates of corner C after shearing = (1, 2).

Thus, New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).

