Perceptron

Saturday, May 17, 2025 7:34 PM

And Gate

	A	B	Actual Y
1	0	0	0
2	0	1	0
3	1	0	0
4	1	1	1

$w_1 = 1.2$ $w_2 = 0.6$

Threshold = 1

learning rate = $\eta = 0.5$

$y = f(w^T x)$

1st instance predictions $y = w_1 x_1 + w_2 x_2$

$= 1.2 \times 0 + 0.6 \times 0$

$y = 0 < 1$

$= 0$

2nd instance

$y = w_1 x_1 + w_2 x_2$

$= 1.2 \times 0 + 0.6 \times 1 = 0.6 < 1$

$y = 0$

3rd instance

$y = w_1 x_1 + w_2 x_2$

$= 1.2 \times 1 + 0.6 \times 0 = 1.2 > 1$

$= 1$

$w_{i+1} = w_i + \eta \times e \times x_i$ → steepest descent

e → (Actual - predicted)

$w_1 = 1.2 + 0.5 (-1) \times 1$

$= 1.2 - 0.5 = 0.7$

$w_2 = 0.6 + 0.5 (-1) \times 0 = 0.6$

Instance 4

Instance 4

$$y = w_1 x_1 + w_2 x_2$$
$$= 0.7 \times 1 + 0.6 \times 1 = 1.3 > 1$$
$$= 1 \checkmark$$

Step 2 =

1st instance

$$y = w_1 x_1 + w_2 x_2$$
$$= 0.7 \times 0 + 0.6 \times 0 = 0 < 1$$
$$= 0$$

So find weights

$w_1 = 0.7$, $w_2 = 0.6$

Fisher LDA

Q: Factory produces very expensive & high quality chips rings that their qualities are measured in terms of Curvature (C) & diameter (D). Results shown below:

Curv (C)	D	Result (O/P)	
2.95	6.63	P	g_1
2.53	7.79	P	
3.57	5.65	P	
3.16	5.47	P	
2.58	4.46	NP	g_2
2.16	6.22	NP	
3.27	3.52	NP	

$Y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

New chip ring width

C = 2.81

D = 5.46

is passed / Not passed.

3.27

X = (7×2)

y = (7×1) = 1, 1, 1, 2, 2, 2

X_1 = (4×2) = 2.95, 6.63; 2.53, 7.79; 3.57, 5.65; 3.16, 5.47

X_2 = 2.58, 4.46; 2.16, 6.22; 3.27, 3.52

μ_1 = [3.05, 6.38]

μ_2 = [2.67, 4.73]

μ = [2.88, 5.67]

mean corrected data $(x_1 - \mu)$

X_1^0 = 0.07, 0.96; −0.357, ... (2×2)

X_2^0 = $(x_2 - \mu)$

2×4 × 4×2

Covariance matrix for group 1 & group 2

$C_i = \frac{(X_i^0)^T (X_i^0)}{n_i}$ → no. of samples

C_1 = [...] (2×2) & C_2 (2×2)

$g = 2$ (groups)

$$C(r,s) = \frac{1}{n} \sum_{i=1}^{g} n_i \times c_i(r,s)$$

C = pooled covariance matrix = 0.206, −0.233; −0.233, 1.689

pooled covariance inverse = 5.745, 0.791; 0.791, 0.701

C^{-1} → pooled covariance inverse = $\begin{bmatrix} 5.745 & 0.791 \\ 0.791 & 0.701 \end{bmatrix}$

$p_i = \frac{n_i}{N}$ → prior probabilities

$P = \begin{bmatrix} 4/7 \\ 3/7 \end{bmatrix}$

$\underline{w} \propto (\underline{m}_2 - \underline{m}_1)$

$y = \underline{w}^T \underline{x}$

f → Discriminant function

$f_i = \underset{1\times2}{\mu_i} \underset{2\times2}{C^{-1}} \underset{2\times1}{x^T} - \frac{1}{2} \underset{1\times2}{\mu_i} \underset{2\times2}{C^{-1}} \underset{2\times1}{\mu_i^T} + \ln(p_i)$ — 1×1

$f_1 = \begin{bmatrix} 3.05 & 6.08 \end{bmatrix} \begin{bmatrix} 5.745 & 0.791 \\ 0.791 & 0.701 \end{bmatrix} \begin{bmatrix} 2.95 \\ 6.63 \end{bmatrix}$

$- \frac{1}{2} \begin{bmatrix} 3.05 & 6.08 \end{bmatrix} \begin{bmatrix} C^{-1} \end{bmatrix} \begin{bmatrix} 3.05 \\ 6.08 \end{bmatrix} + \ln\left(\frac{4}{7}\right)$

$f_1 - 7\times1$

$f_2 - 7\times1$

$\underline{x}_1 \rightarrow 1$ if $f_1 > f_2$ $(\underline{x}_1)$

$\underline{x}_2 \rightarrow 2$ if $f_2 > f_1$ $(\underline{x}_2)$

10 marks

Submit 4 June

check for new chip ring

$x_{new} = (2.81 \quad 5.46)$

f_1 ↑ ↑

if $f_1 > f_2$ passed

8 June

f_1 / f_2

$f_1 > f_2$
otherwise
NP