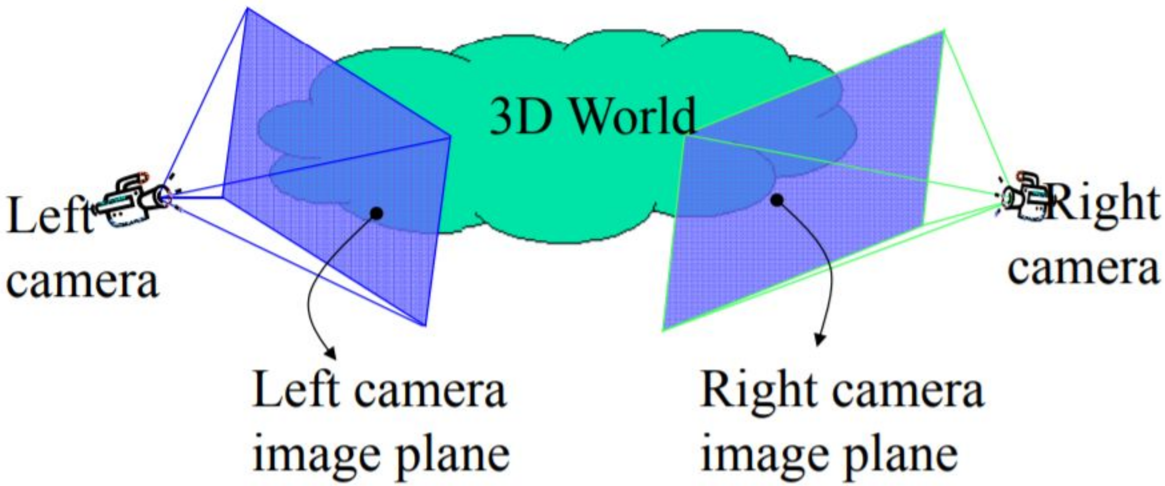
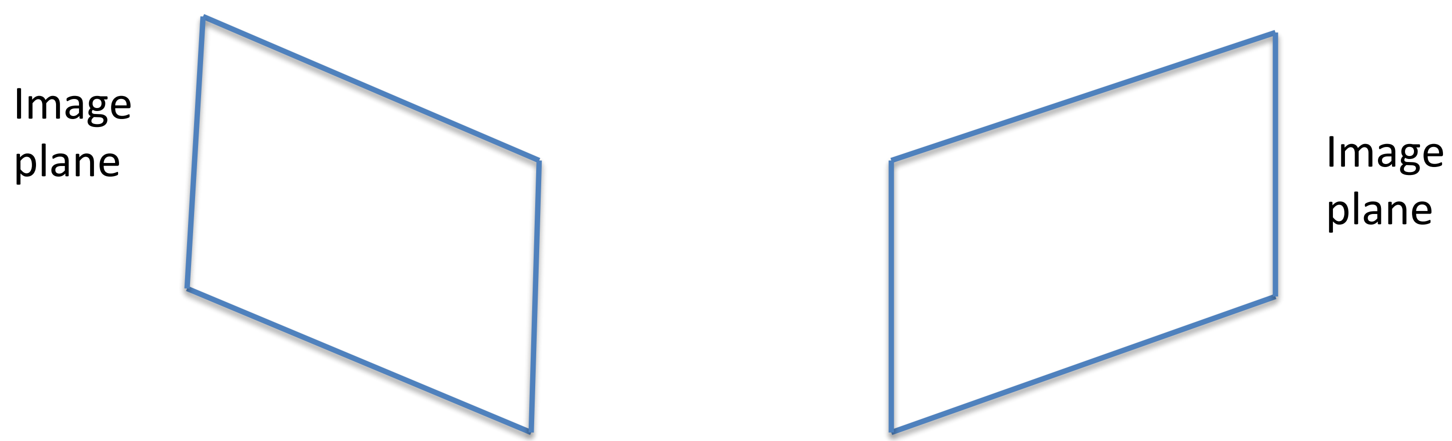
EPIPOLAR GEOMETRY FOR PAIR OF IMAGES

- Defined for two static cameras



SET UP

- Static Scene (Camera Moves)
- Two pictures of the same scene, taken at the same time, from two different cameras



(a)

(b)

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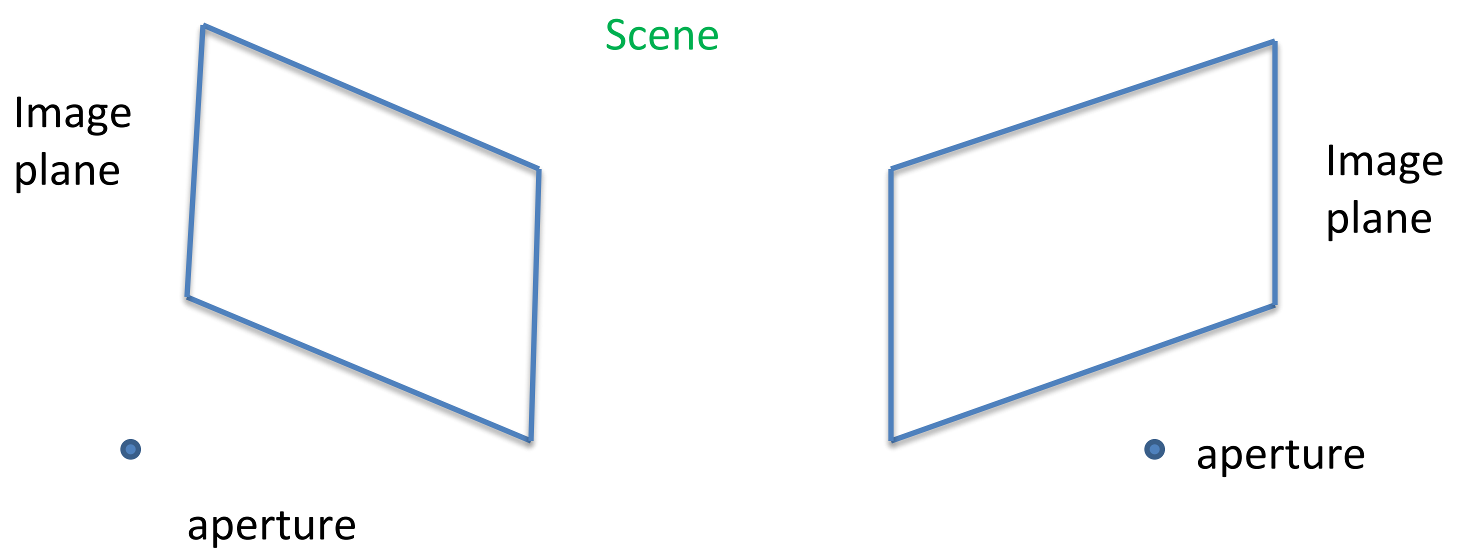

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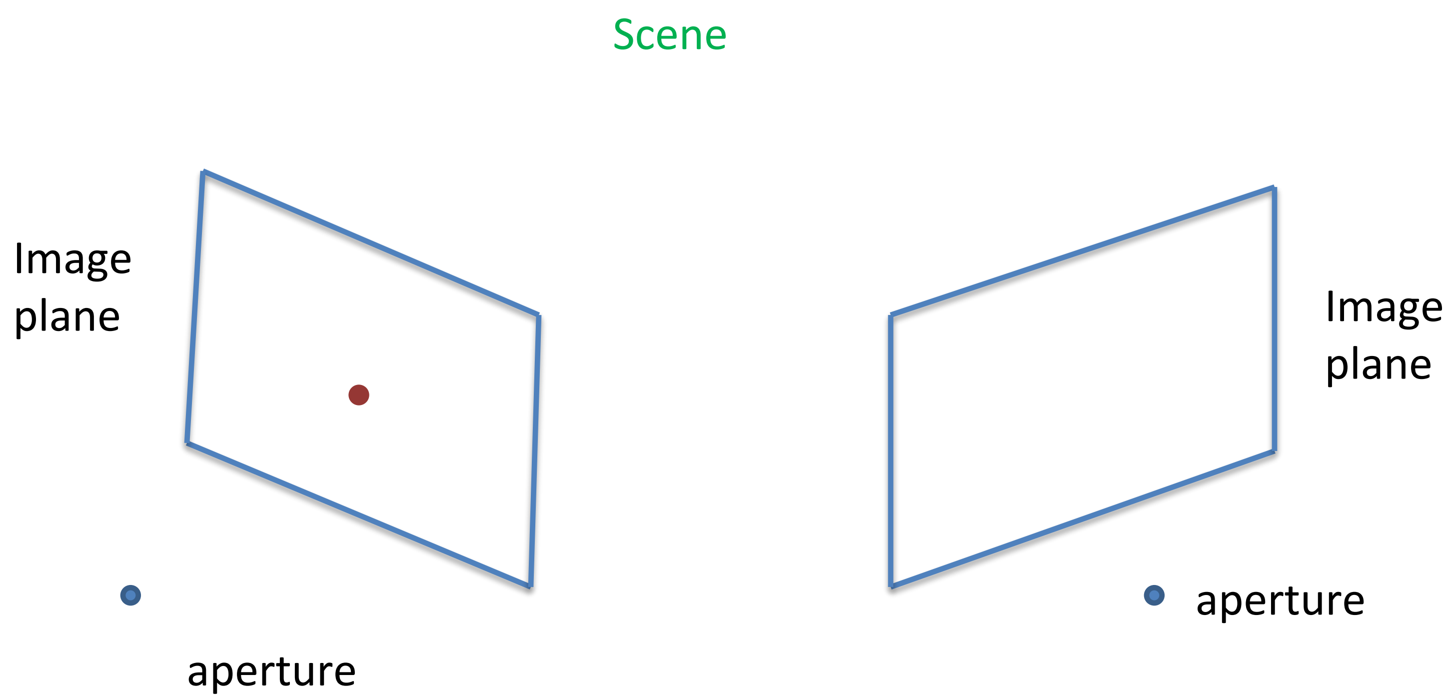
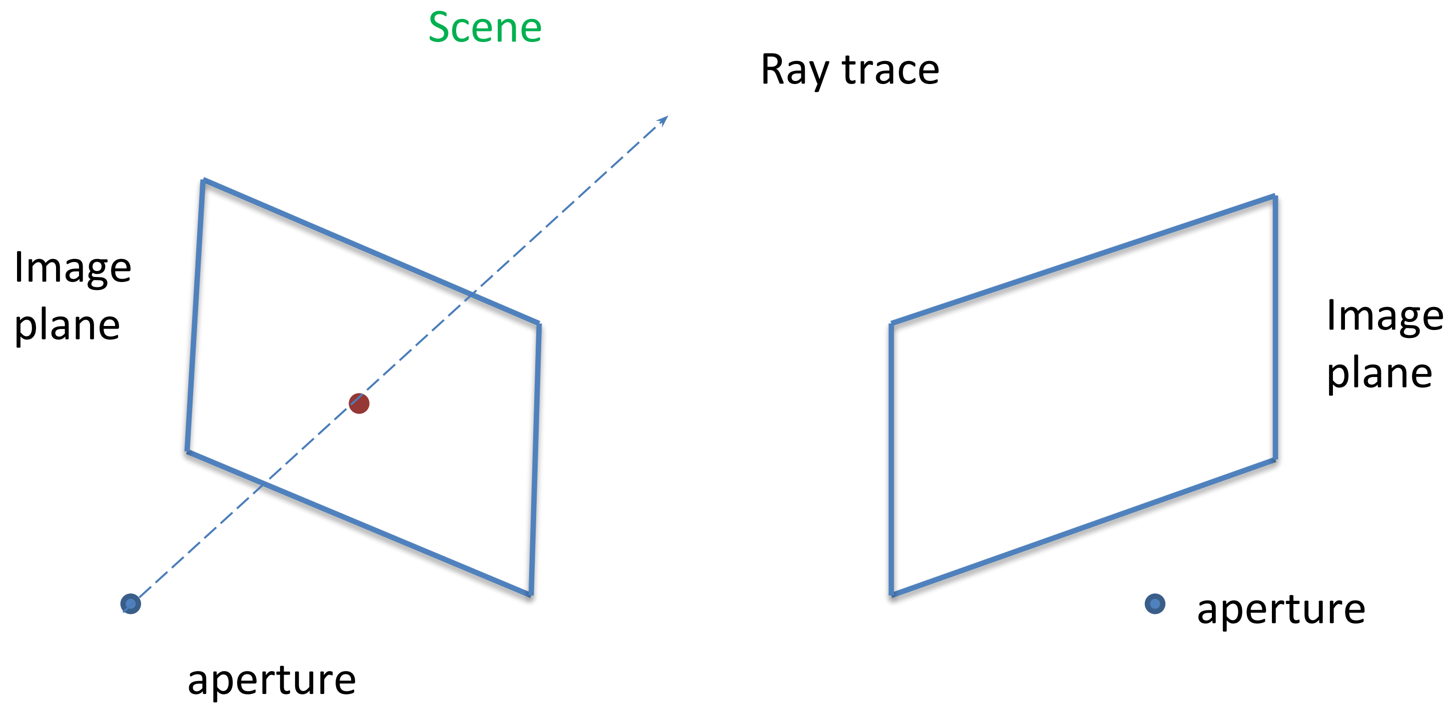


SET UP

- Finding correspondence for image plane 1



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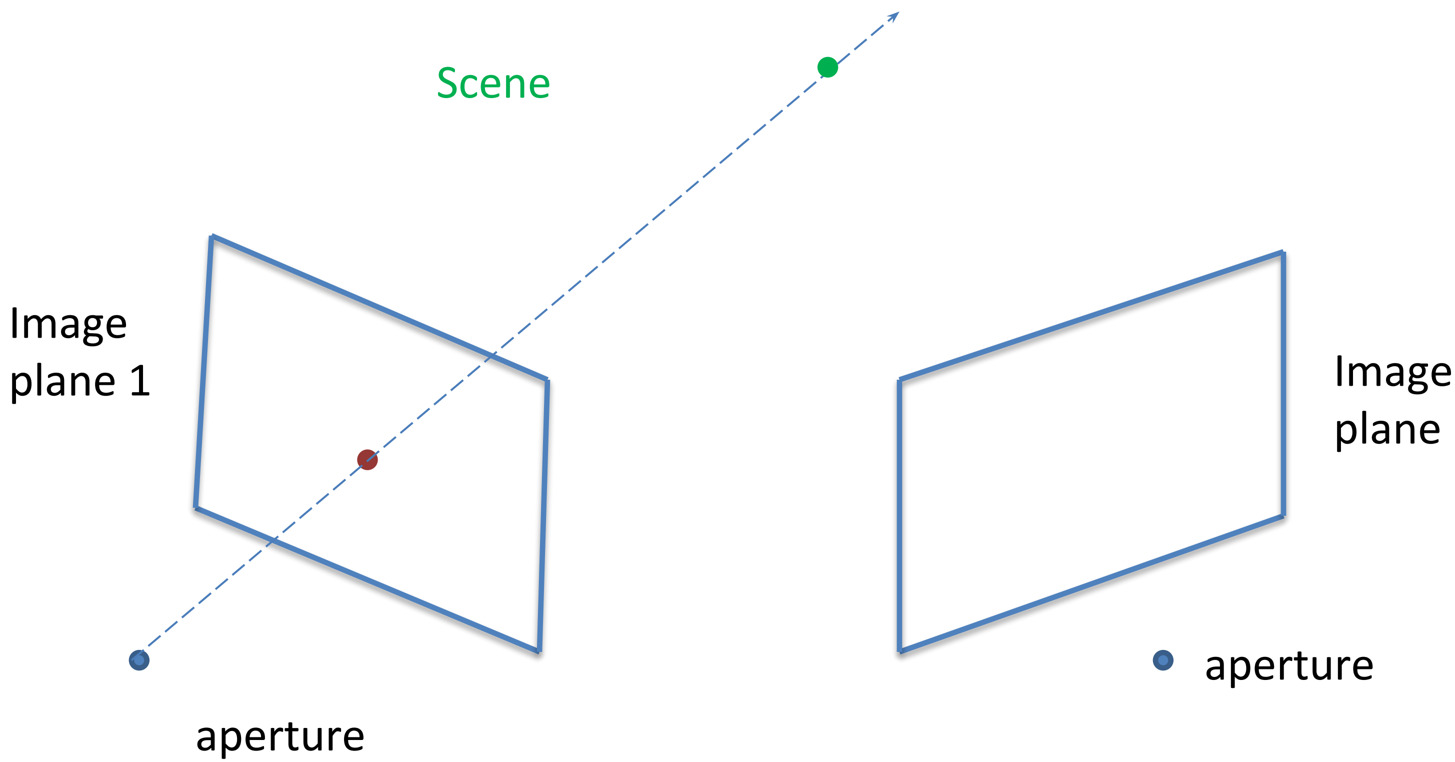

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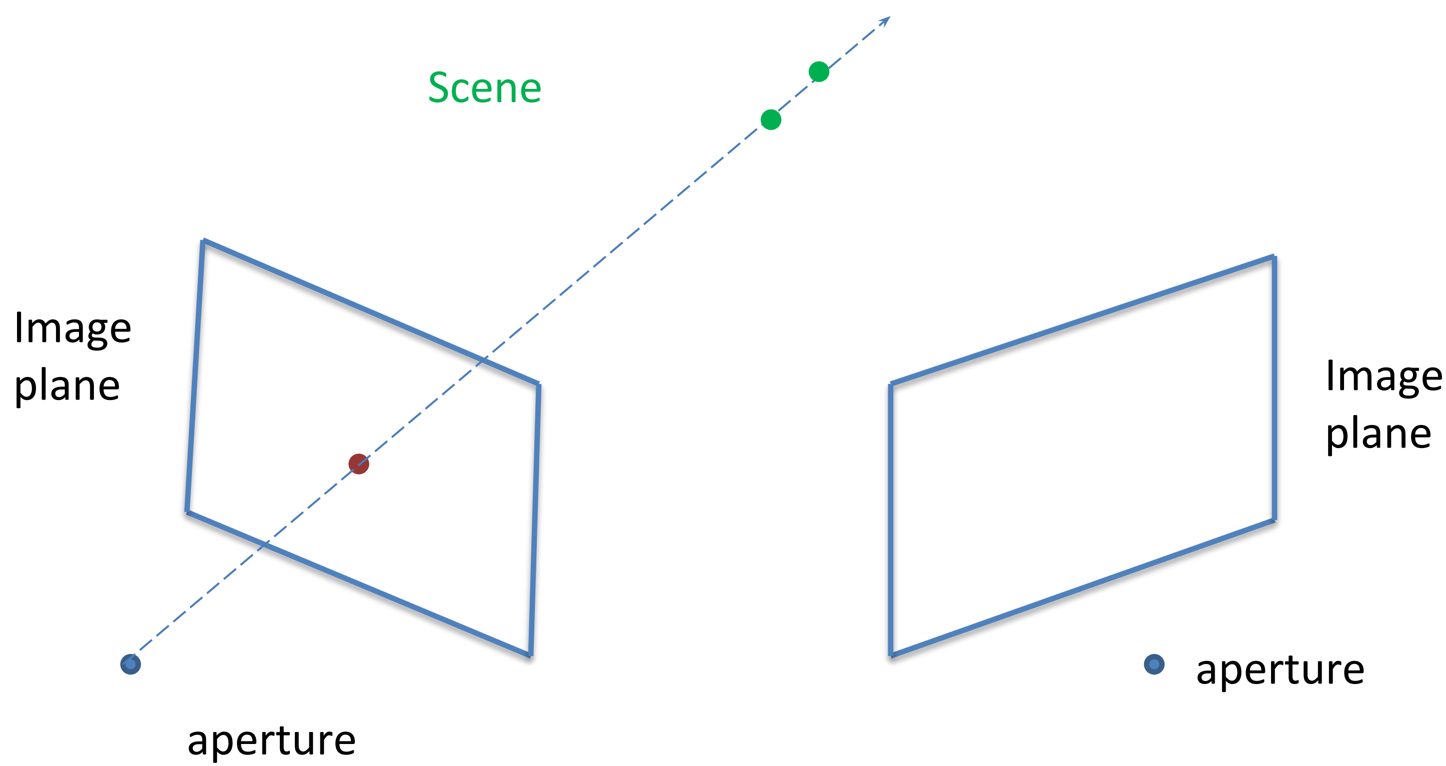

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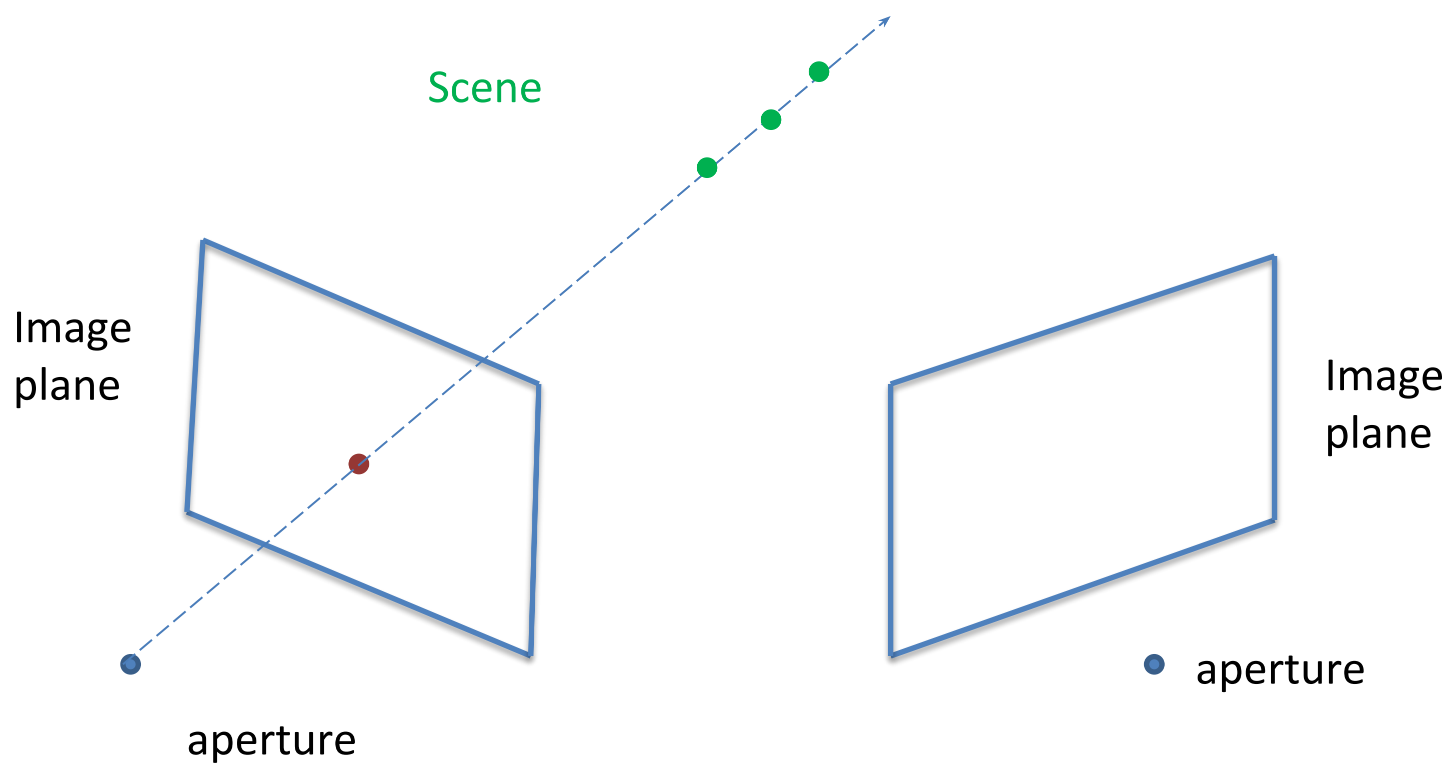

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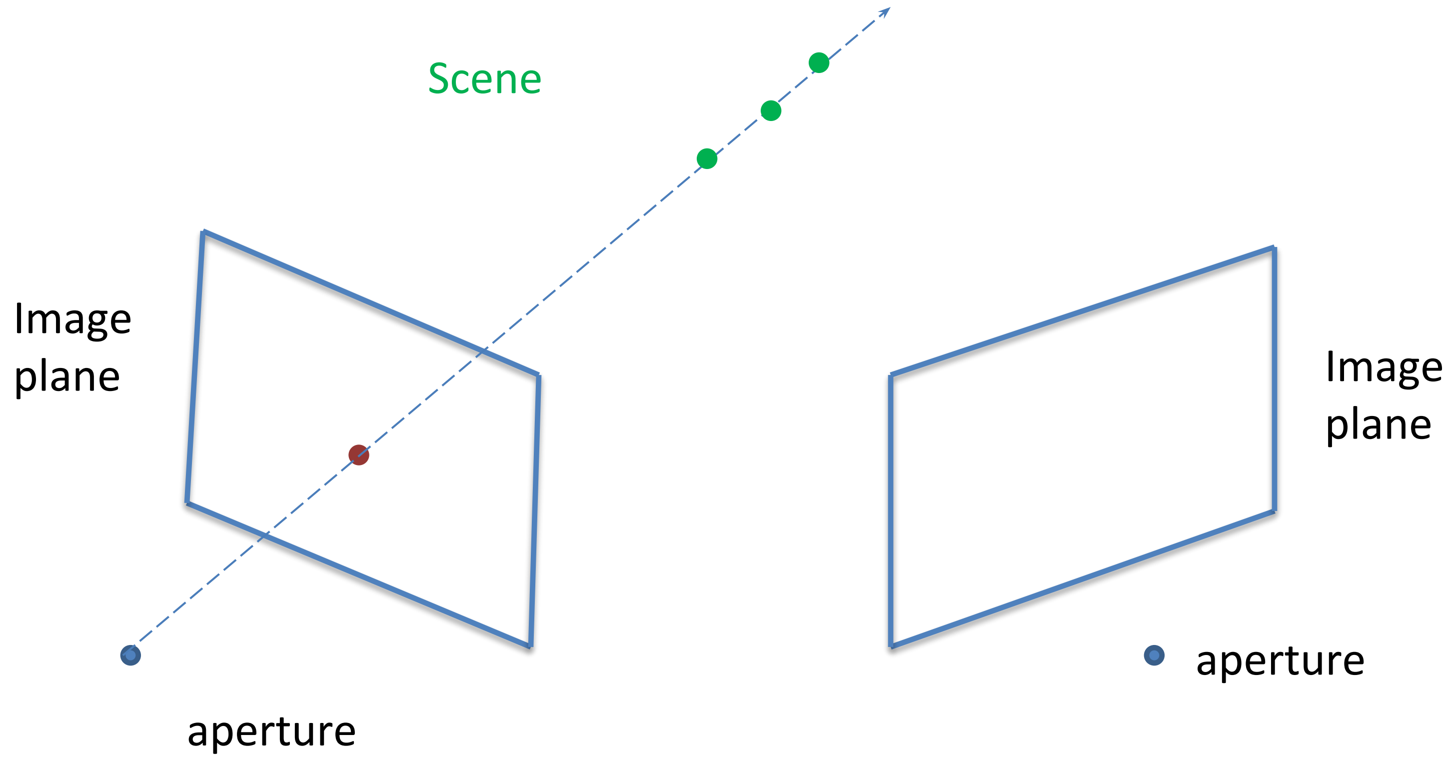
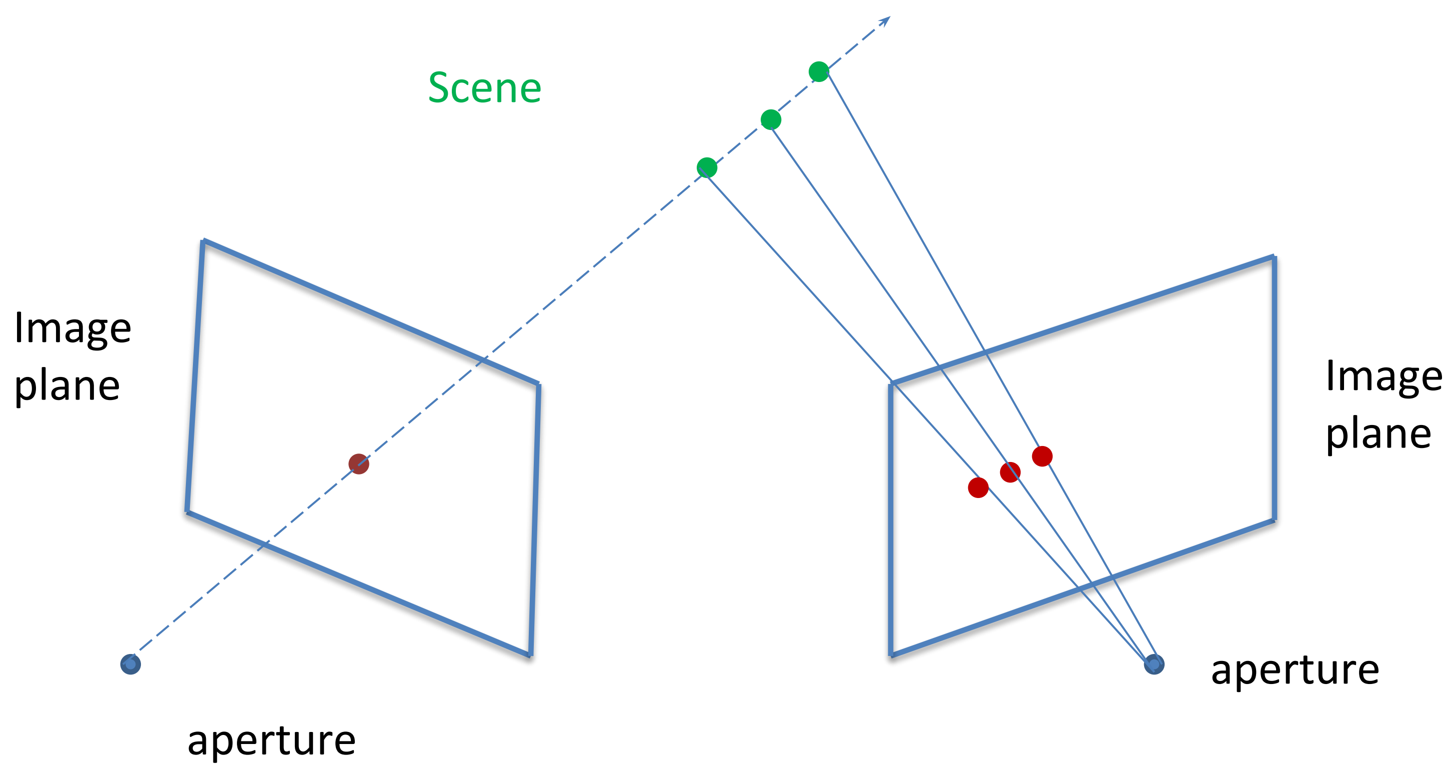


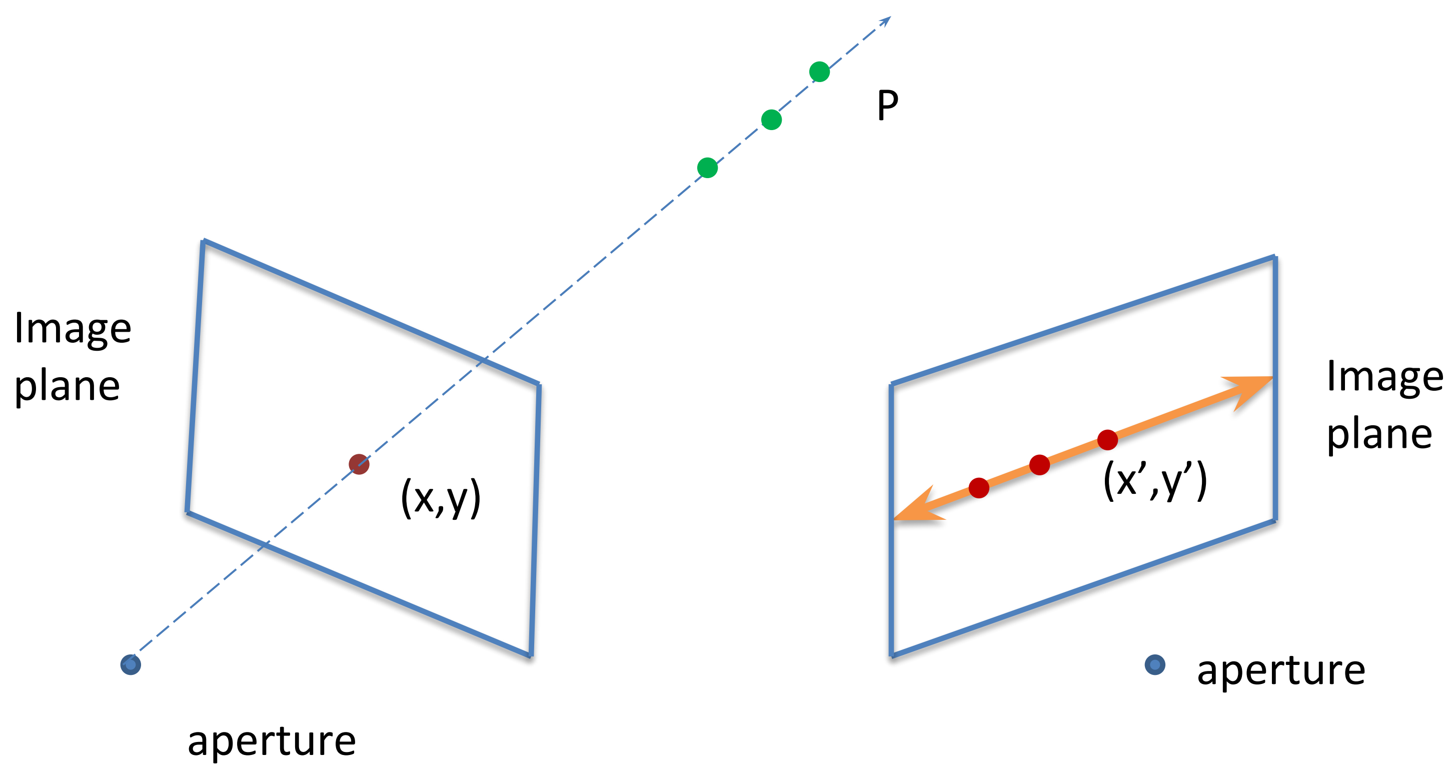
SET UP

- Finding correspondence for same points in by projecting back on image plane 2



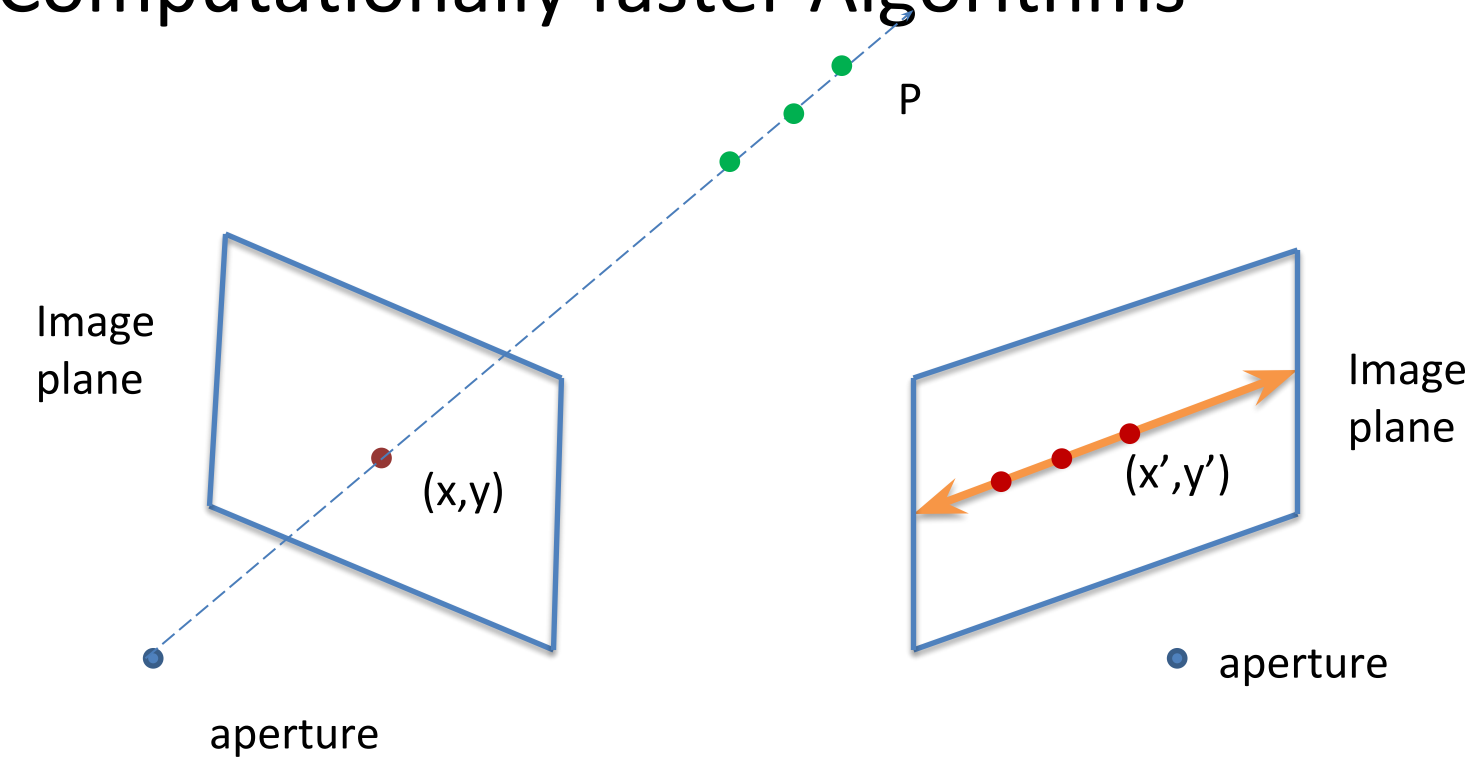
SET UP

- Point in image plane 1 will have a matching correspondence on the line in image plane 2.



SET UP

- Don’t need to search the entire plane space for match…only the line……
- Computationally faster Algorithms



Note

- There exits a parallel and mutual relationship between the two images

EPIPOLAR LINES

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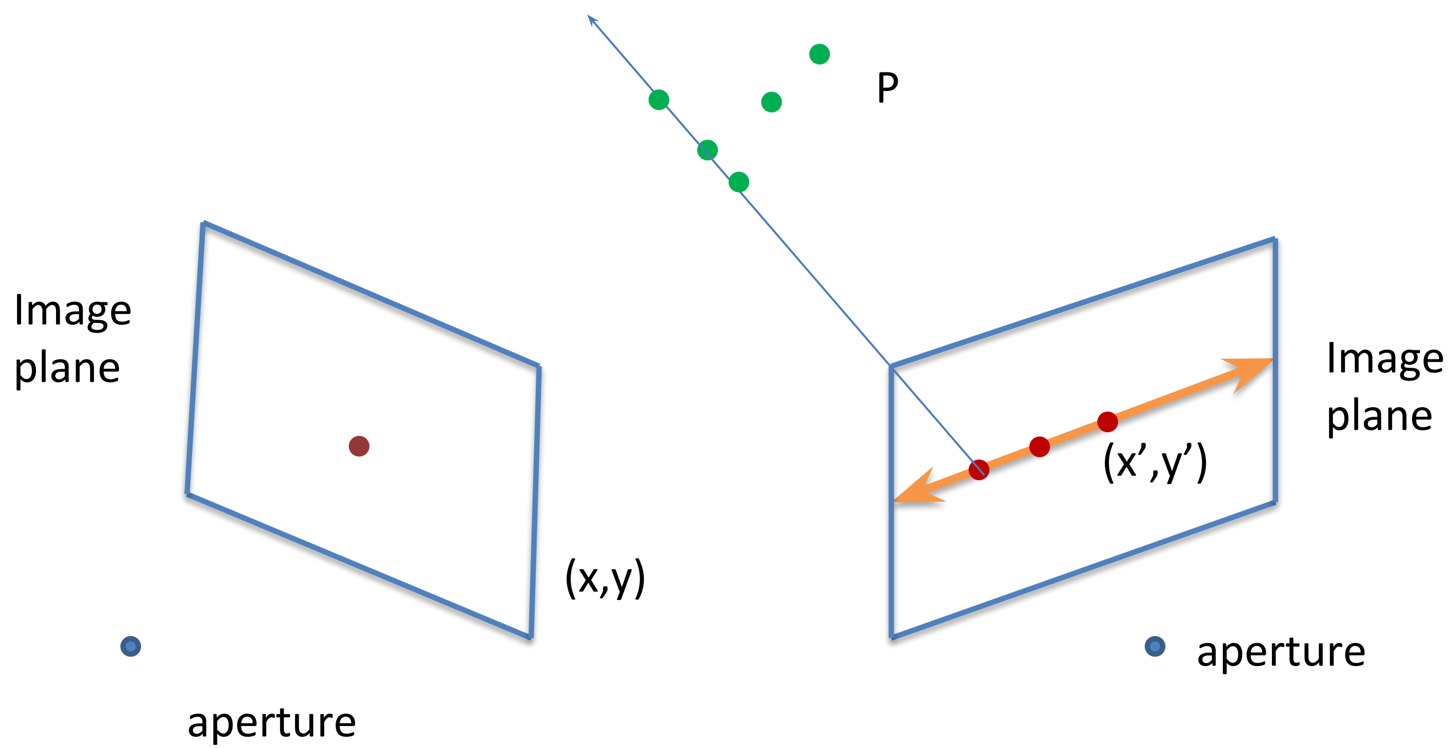

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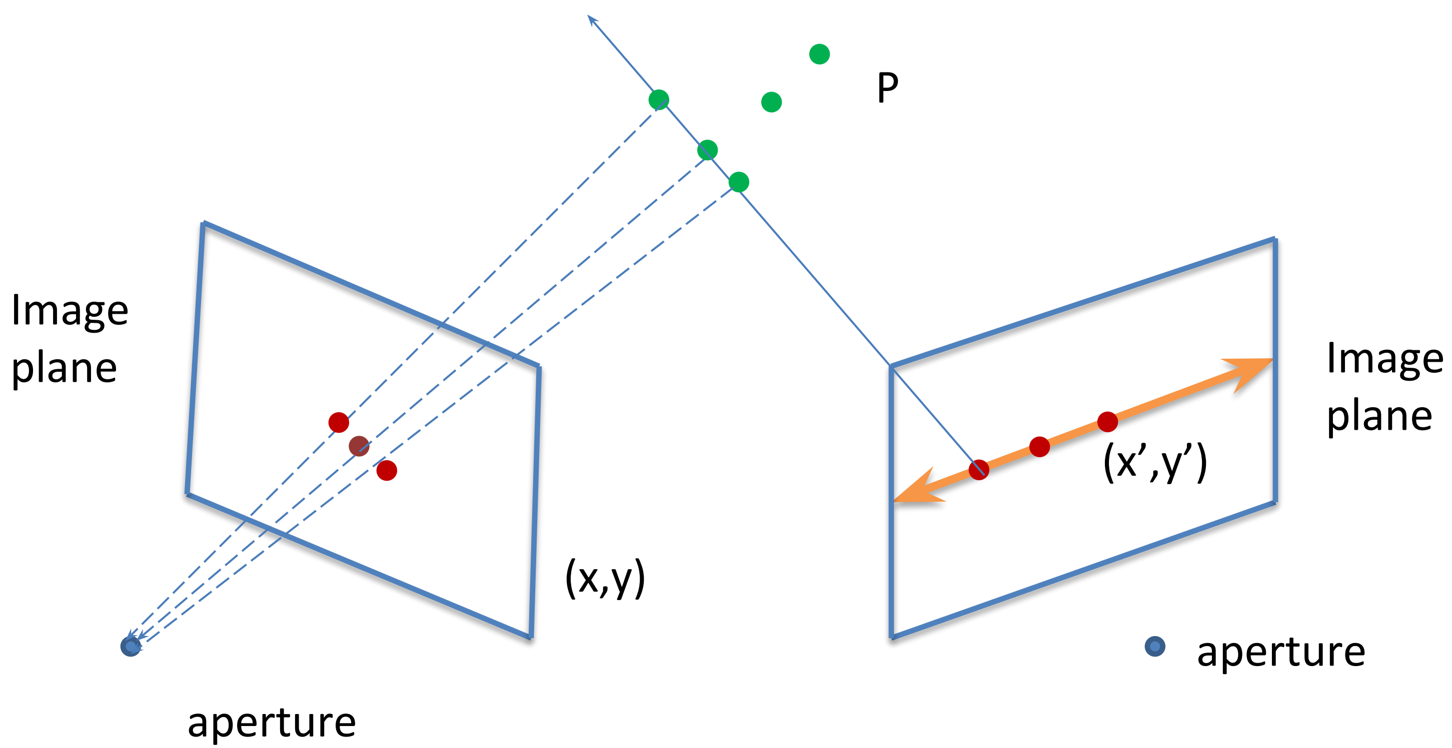

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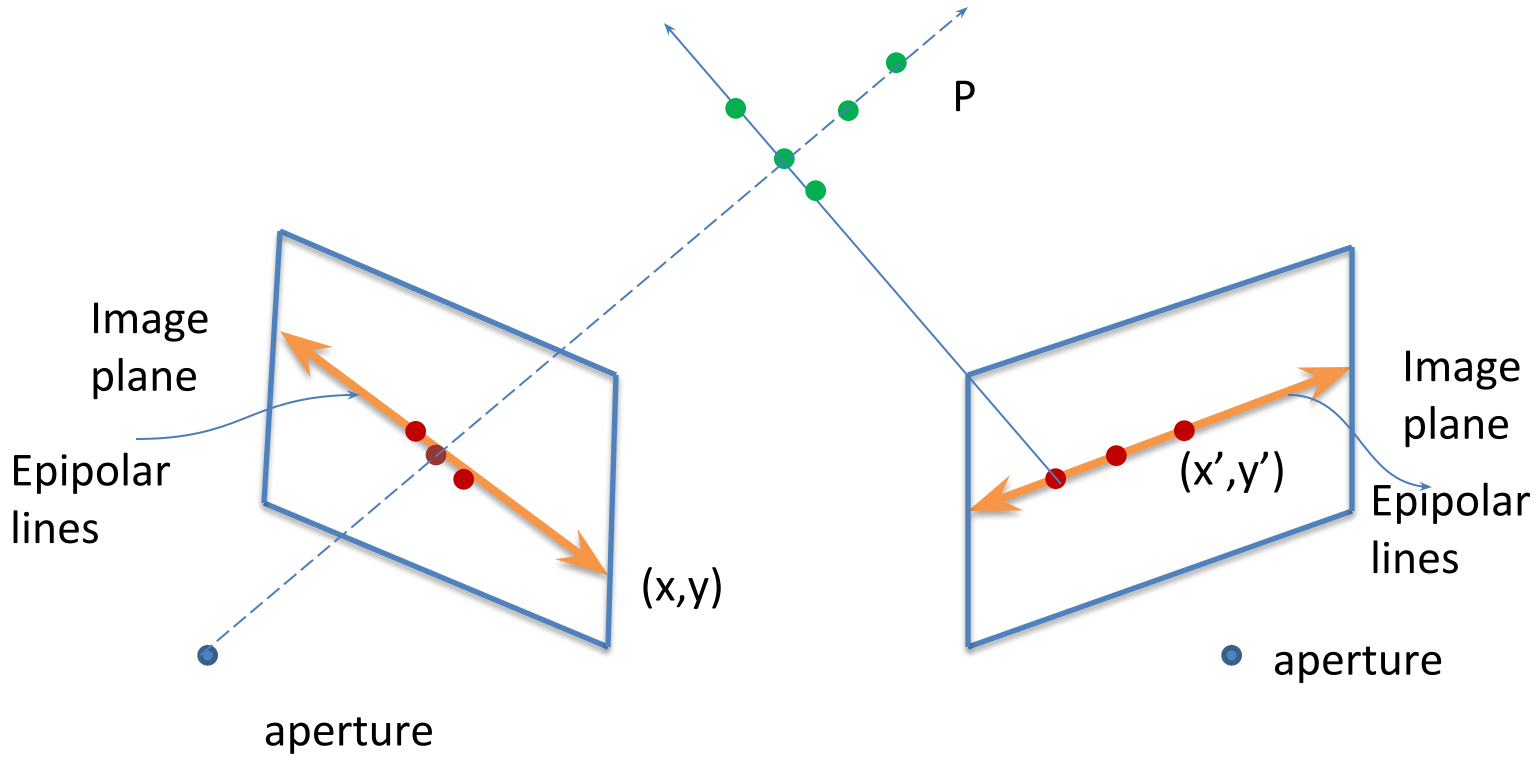
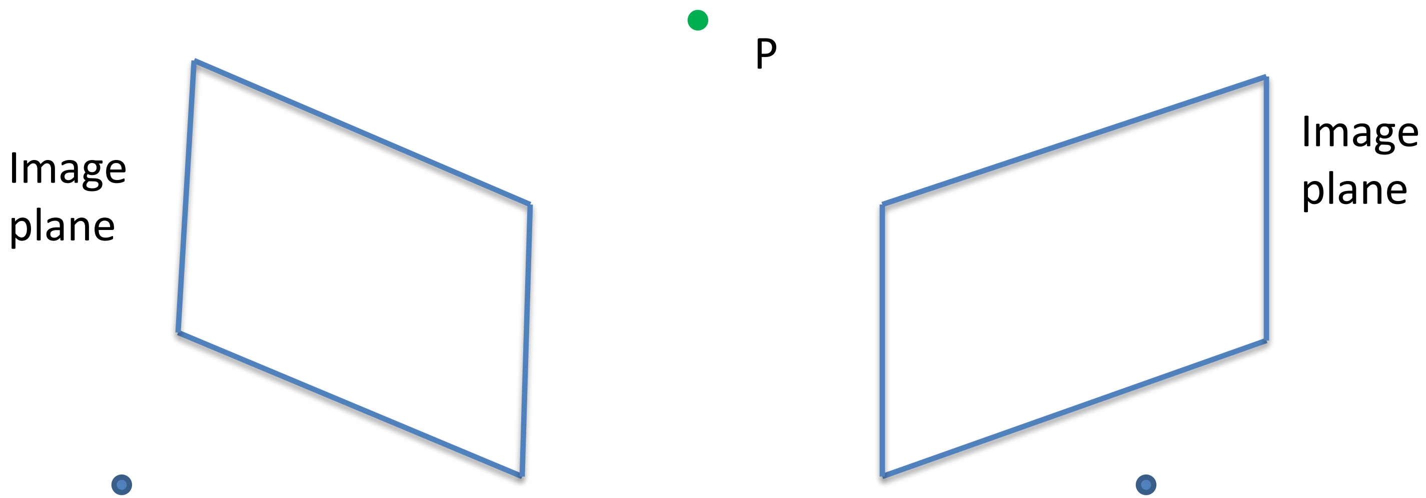


EPIPOLAR LINES

- How are these lines traced up for a 3D point ‘P’



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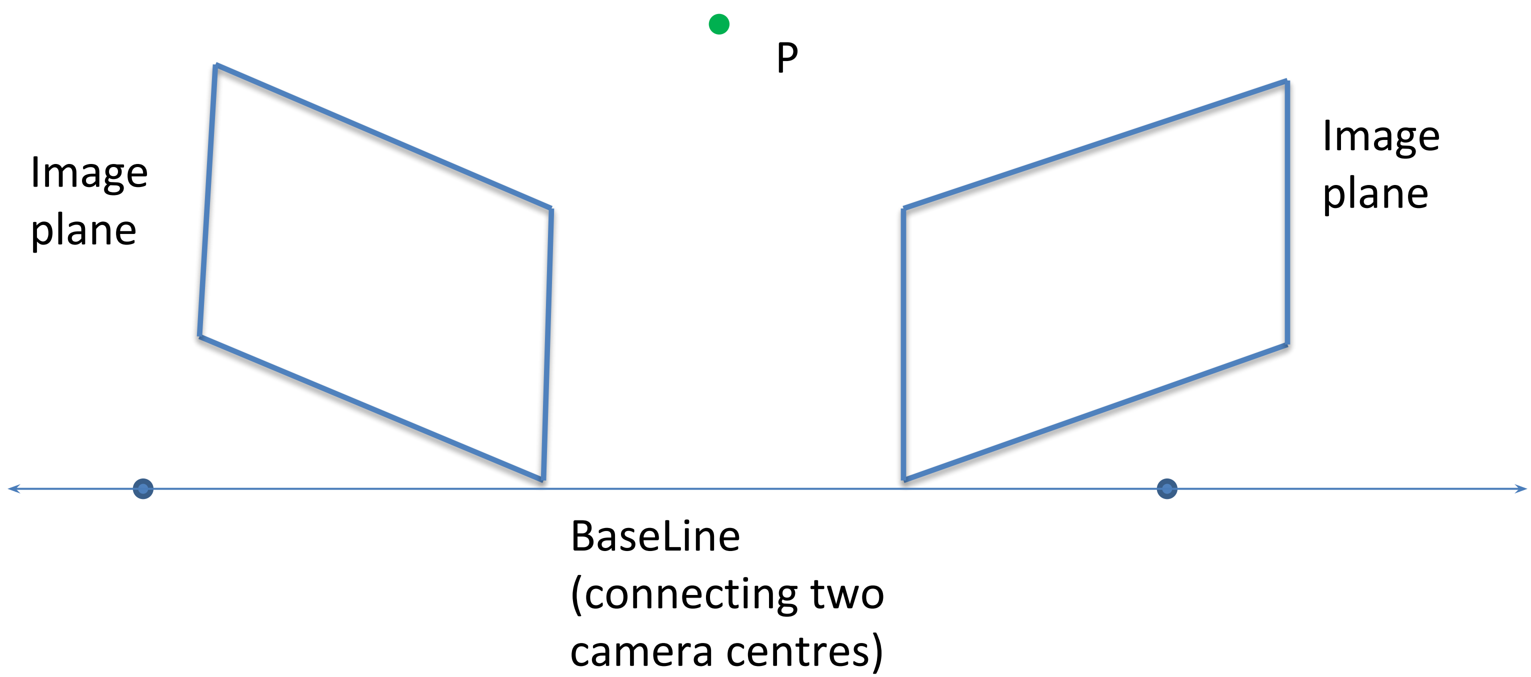

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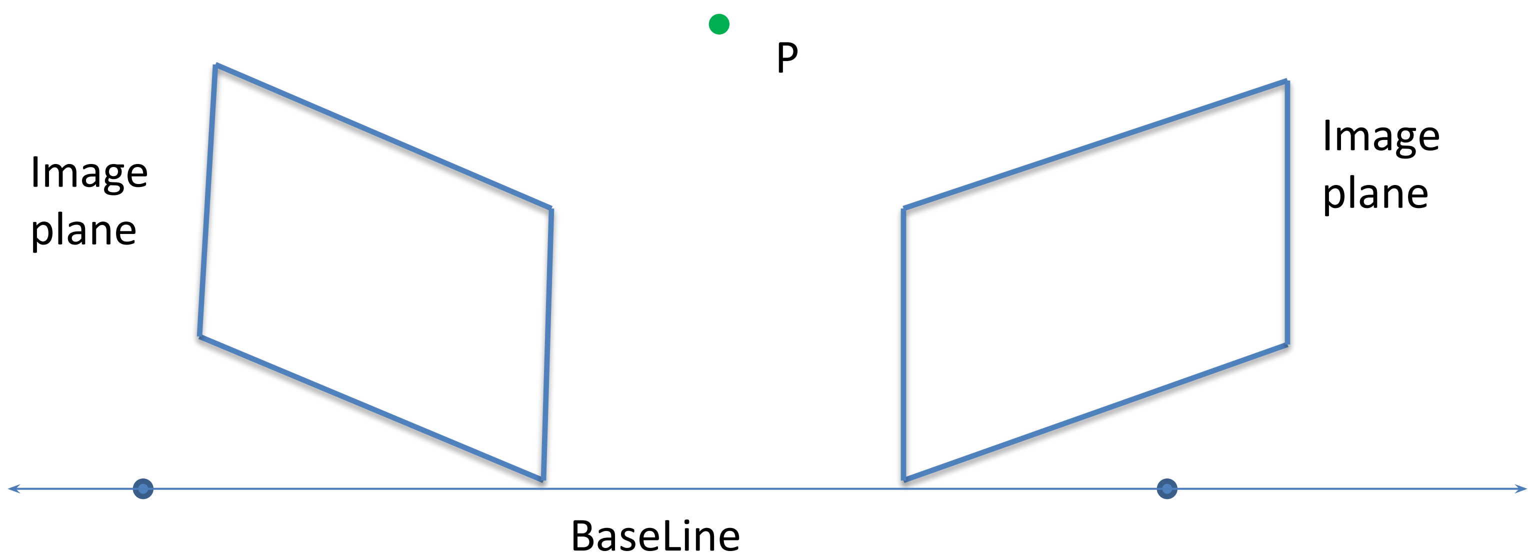

- Find the BaseLine (connecting two camera centres)

EPIPOLAR LINES

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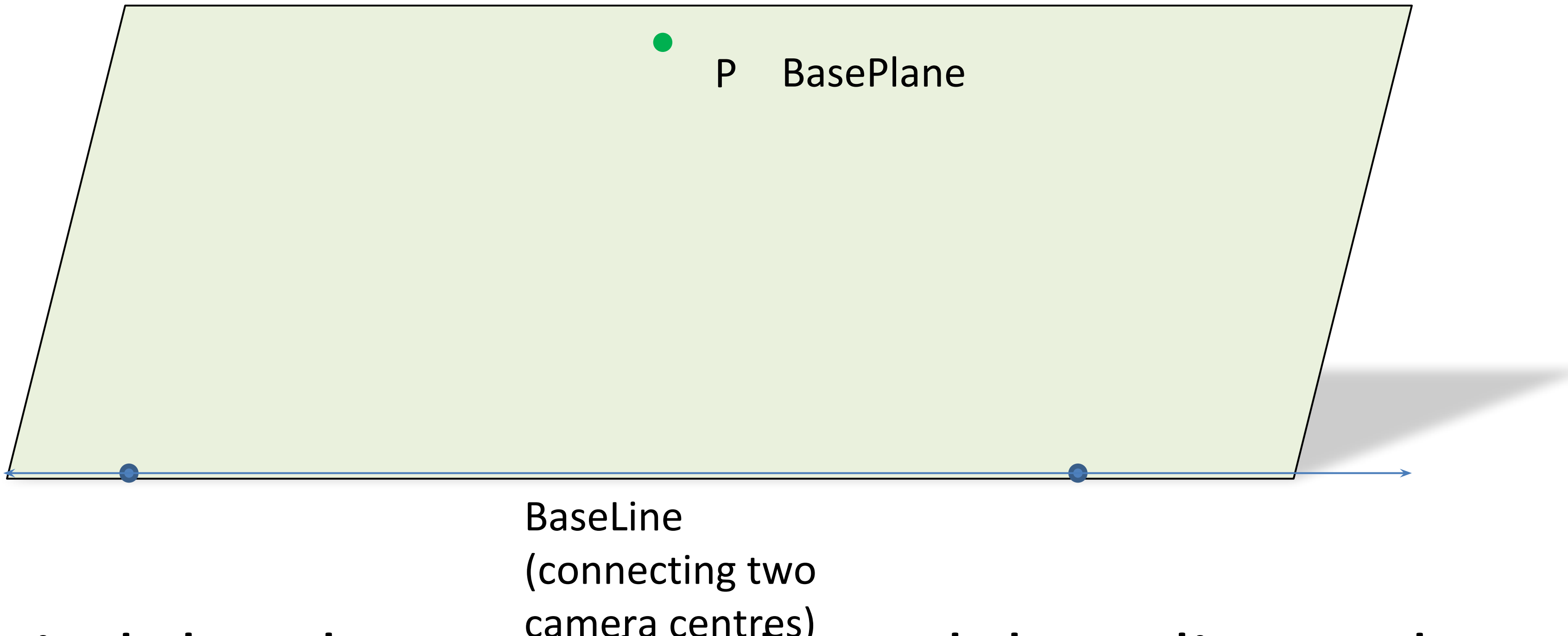

- Find the plane passing through base line and point P

EPIPOLAR LINES

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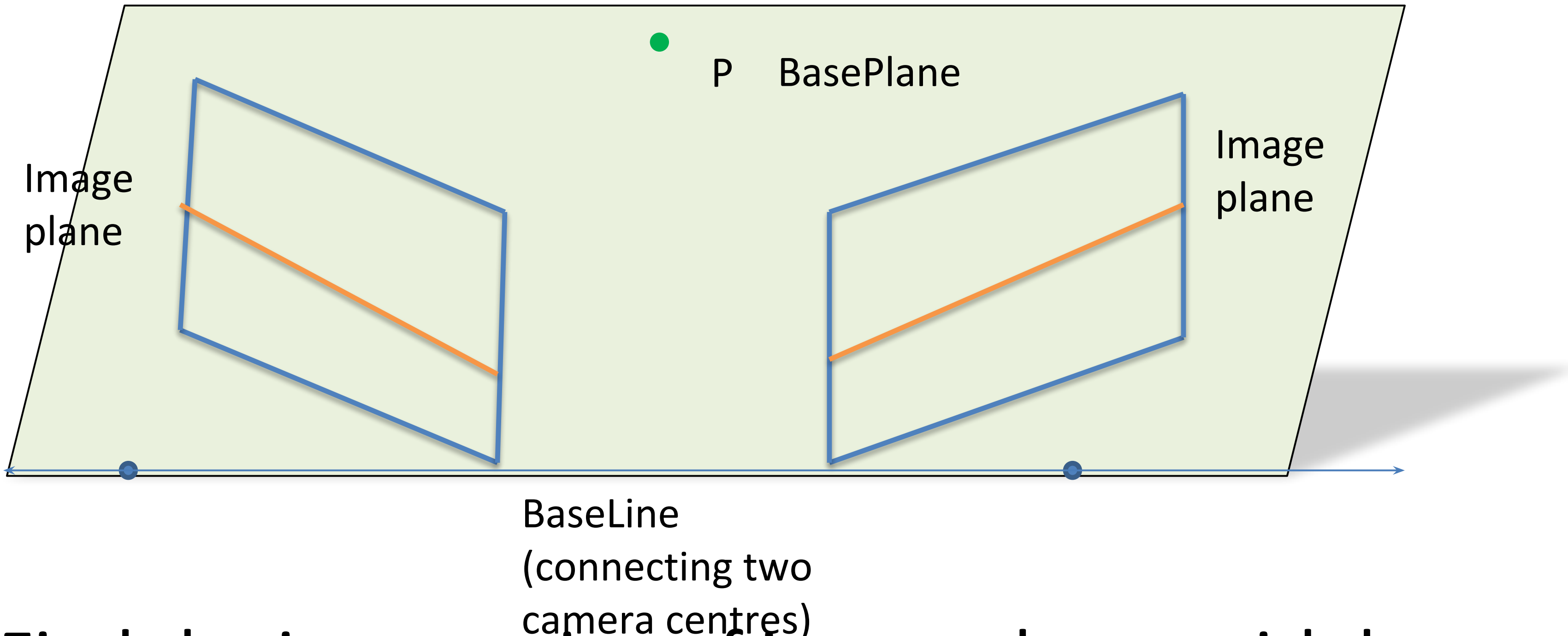
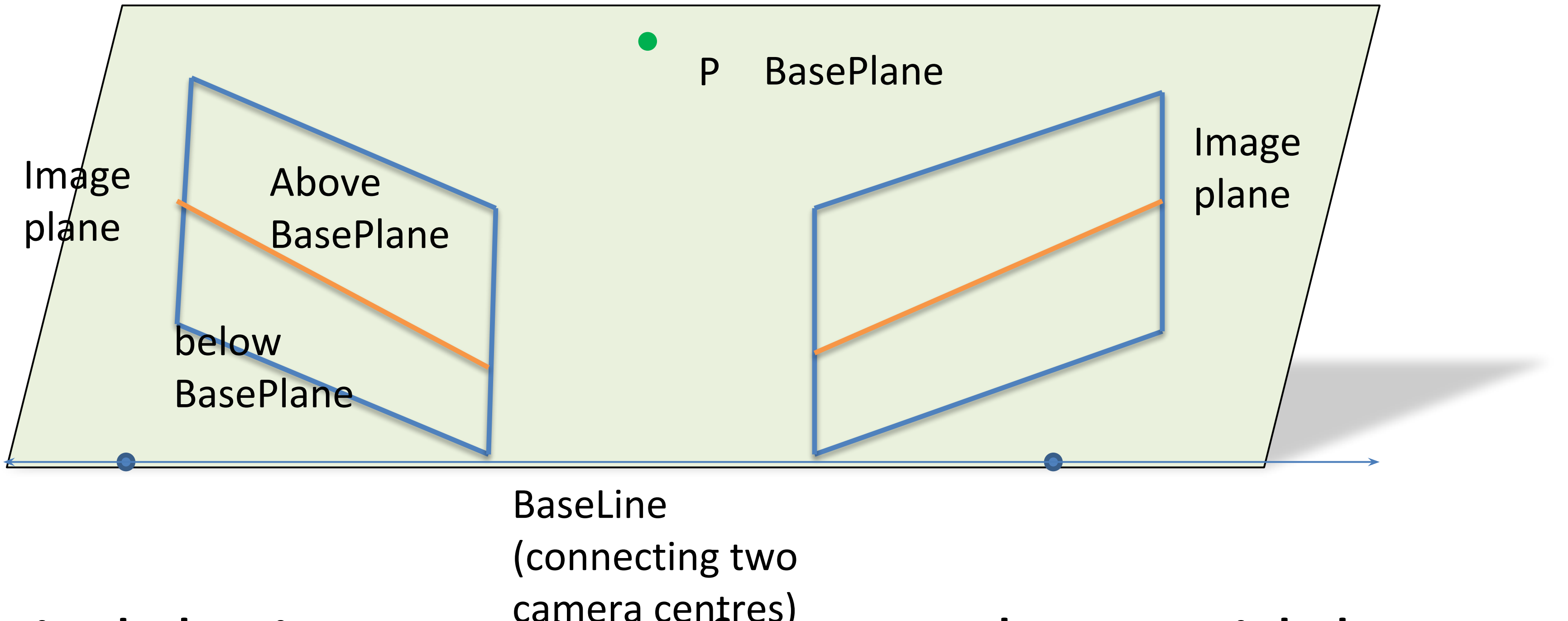


- Find the intersection of image planes with base plane.The lines of intersection are epipolar lines

EPIPOLAR LINES

- How are these lines traced up for a 3D point ‘P’



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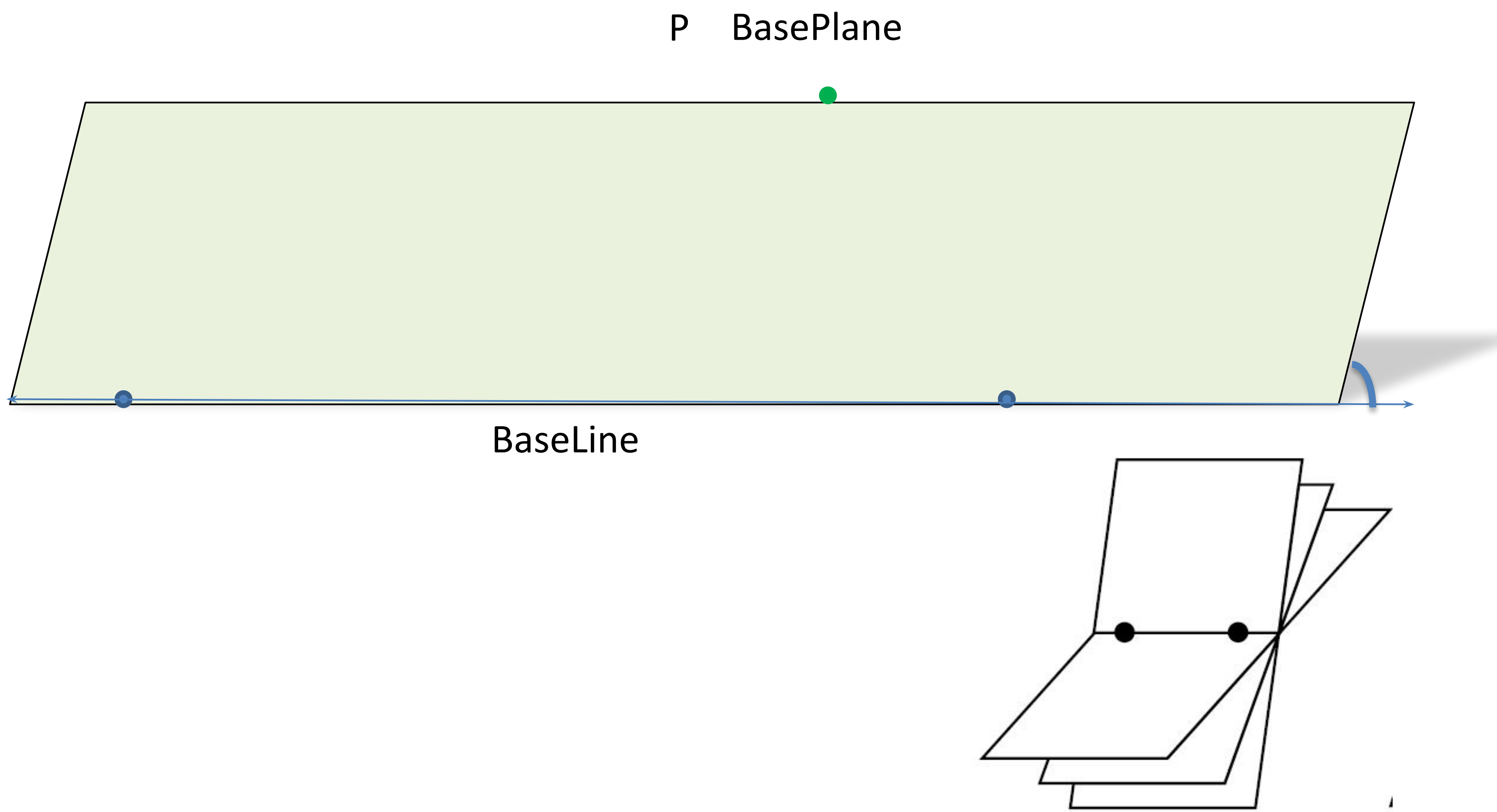
EPIPOLAR LINES for Different Points

- Base plane will be oriented differently for each a 3D point 'P'



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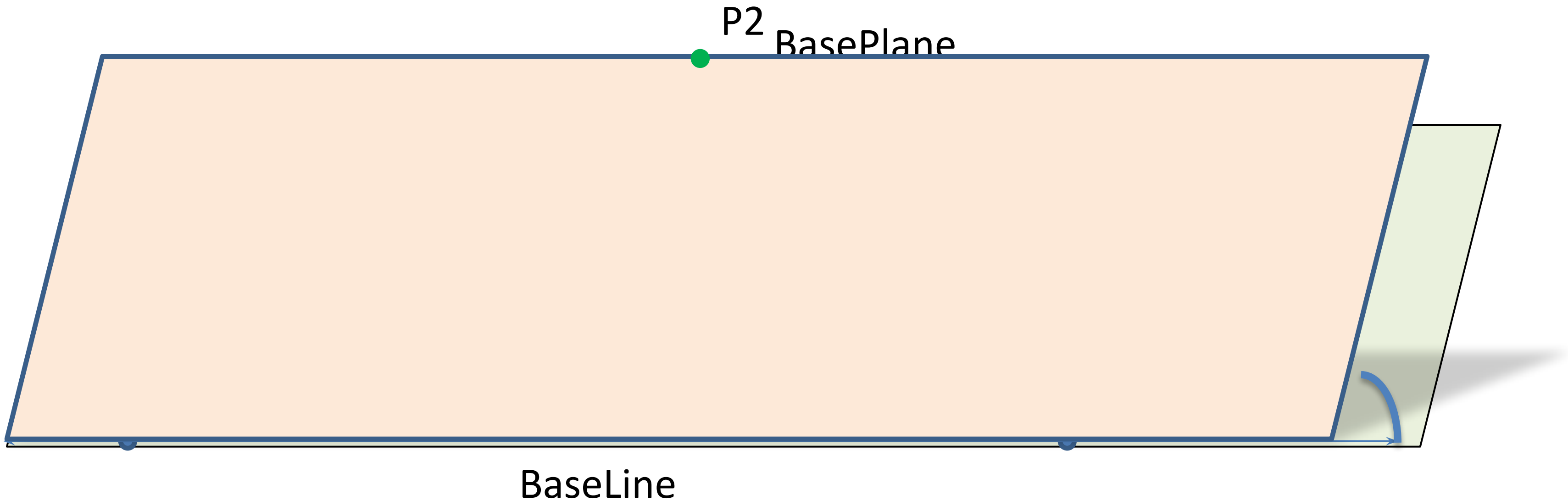

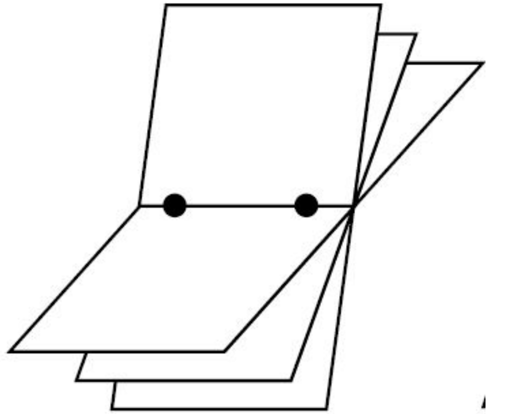

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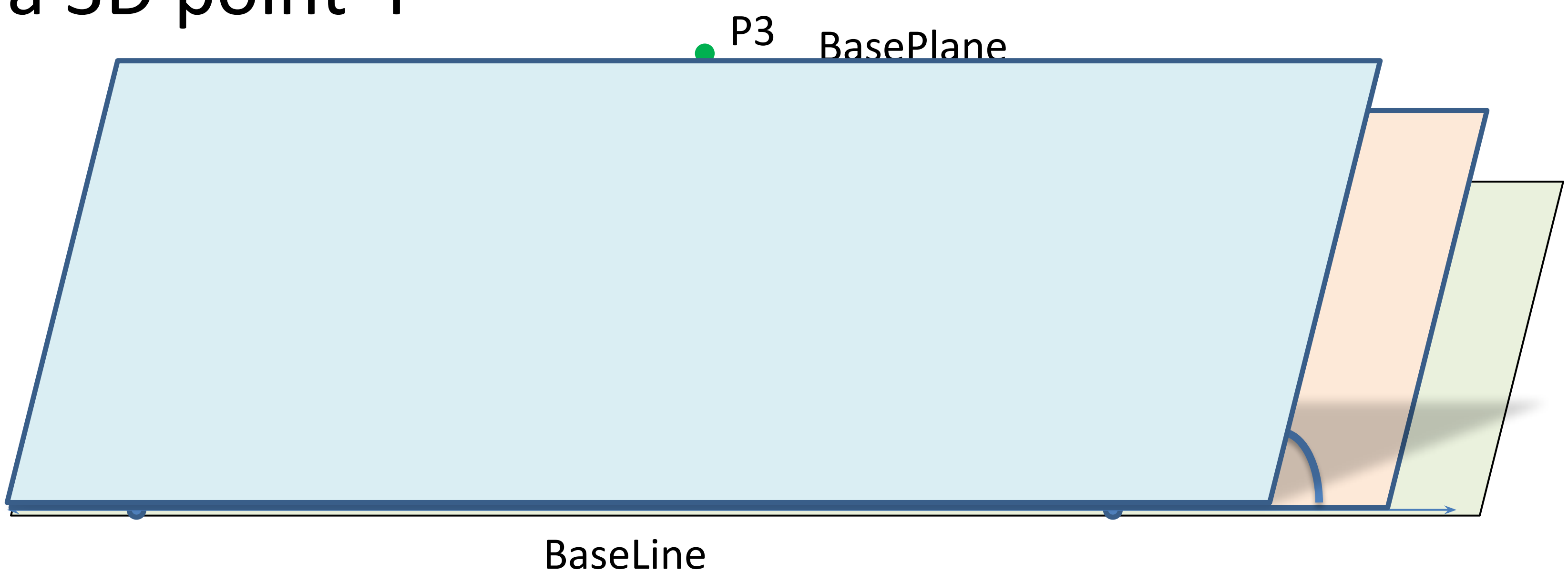
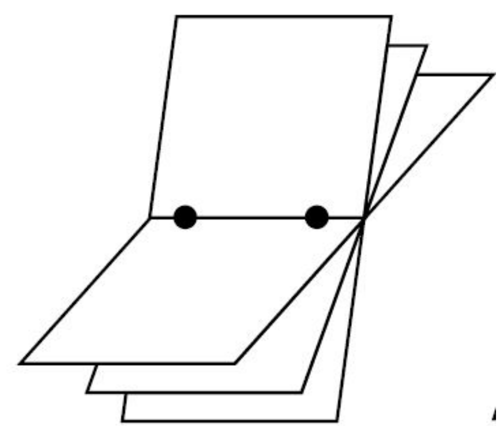
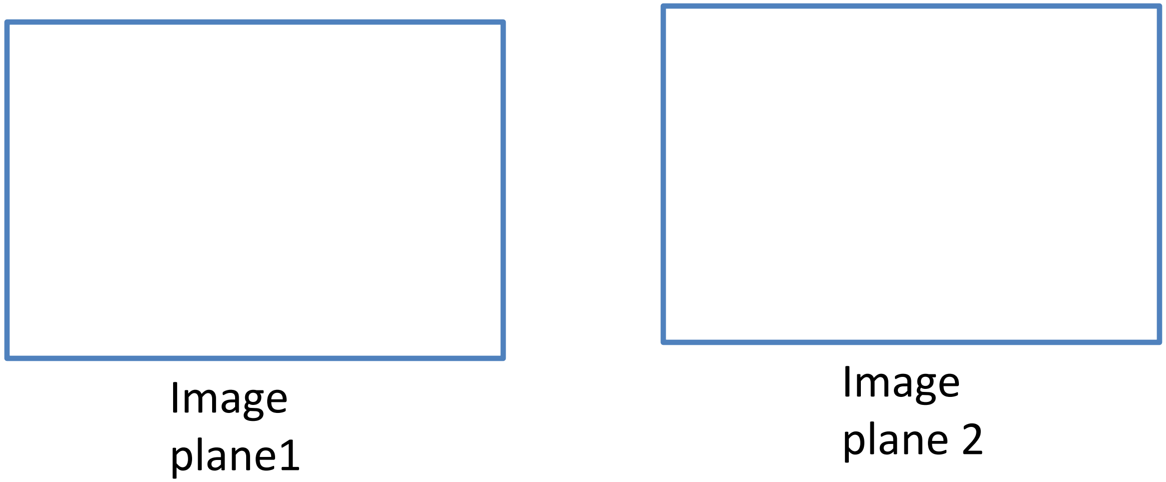




Mutual Correspondance



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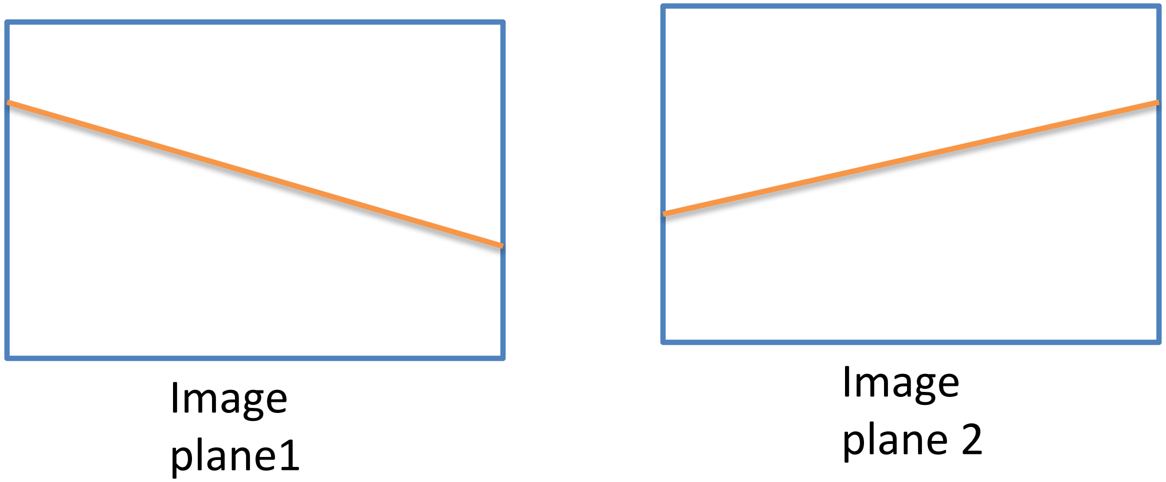

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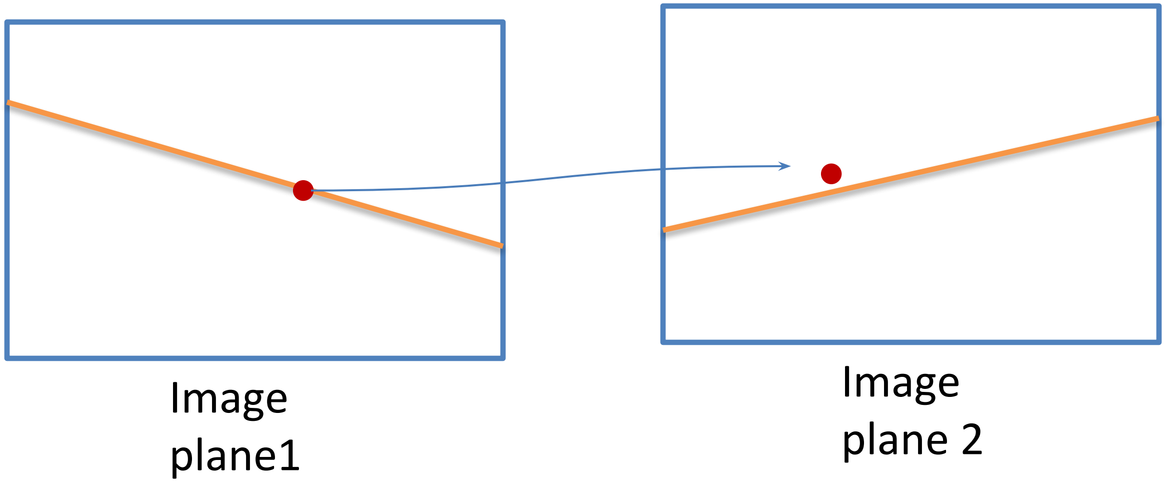

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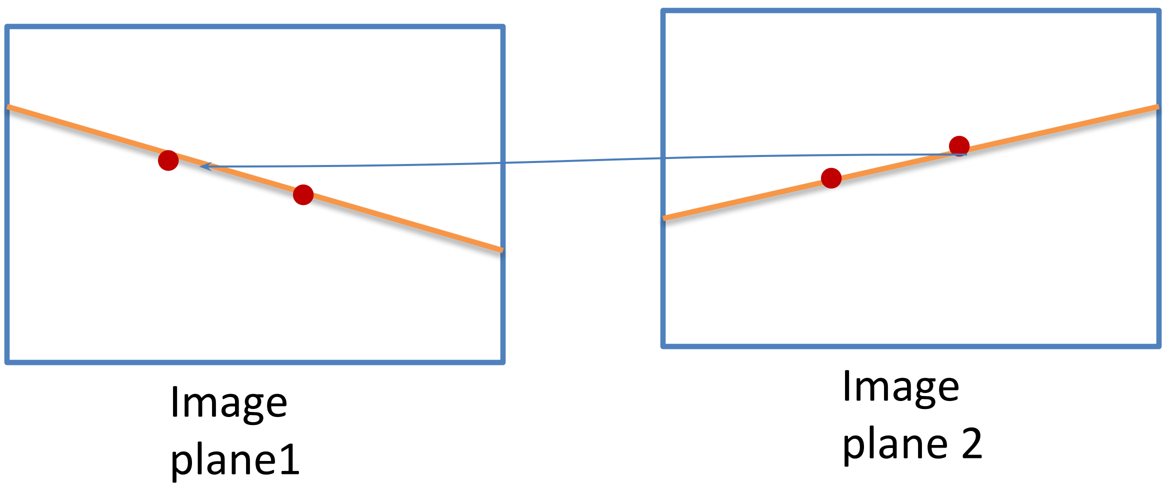


- Conjugate Epipolar Lines
- Search for correspondence is constrained and efficient

Mutual Correspondance

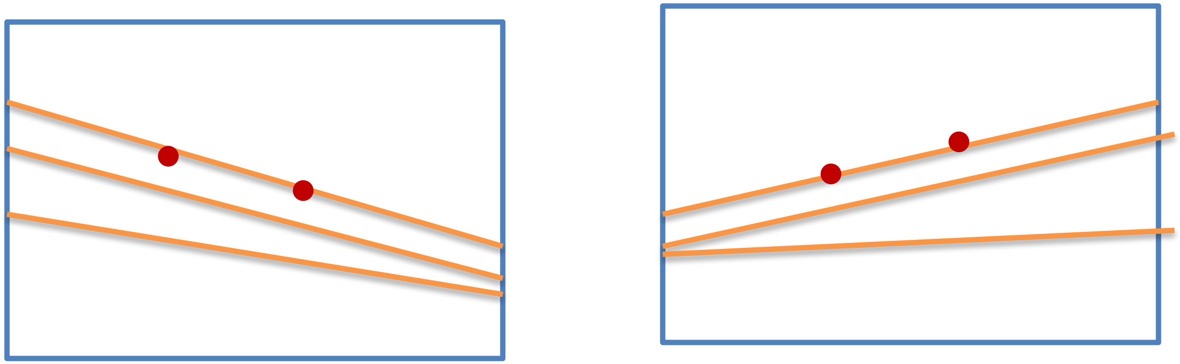
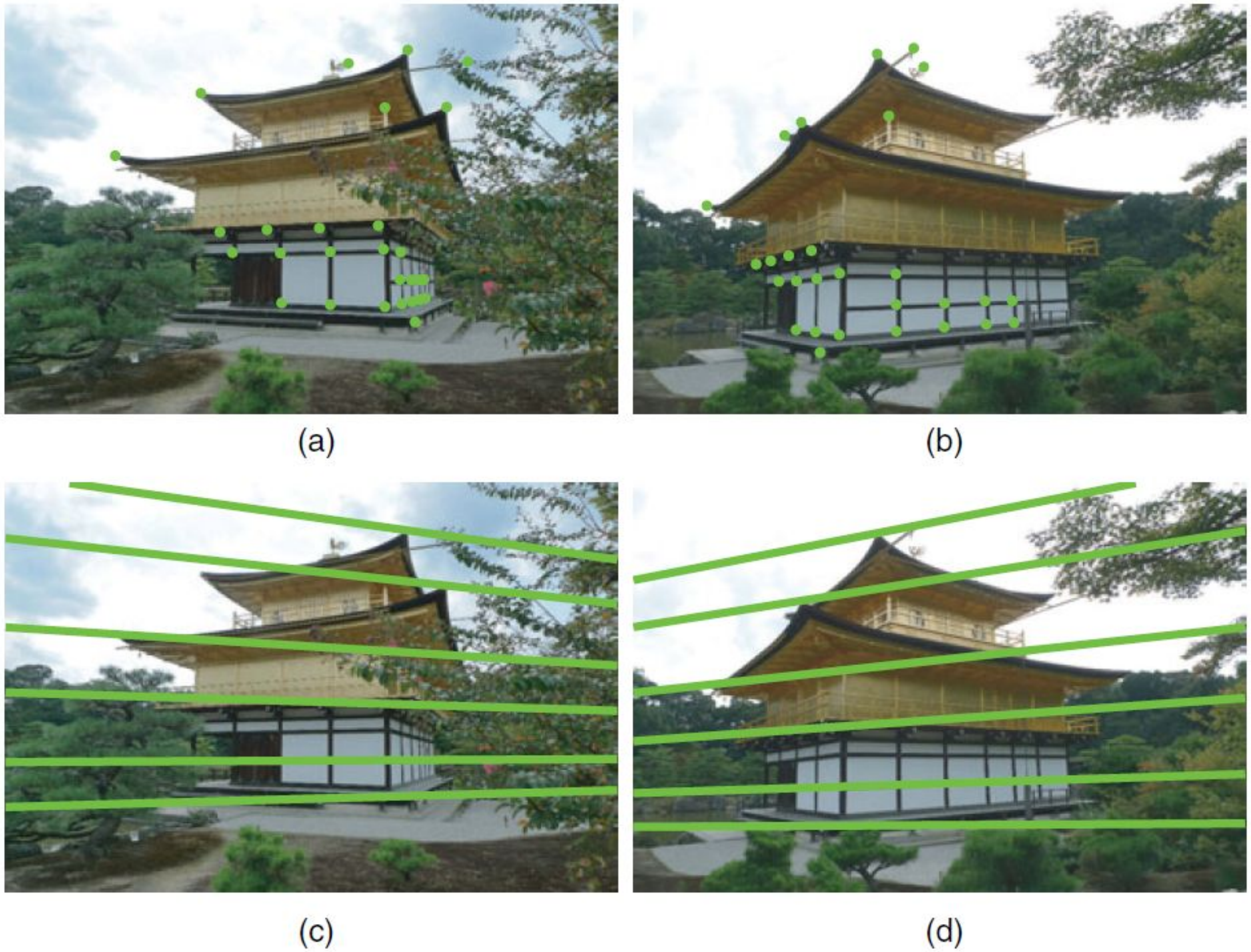


Image plane1

Image plane 2

- Conjugate Epipolar Lines
- Search for correspondence is constrained and efficient

Mutual Correspondance



(a) (b) (c) (d)

Note

- Why are epipolar line slanted?

EPIPOLAR LINES

- If the cameras are in-field of view of each other

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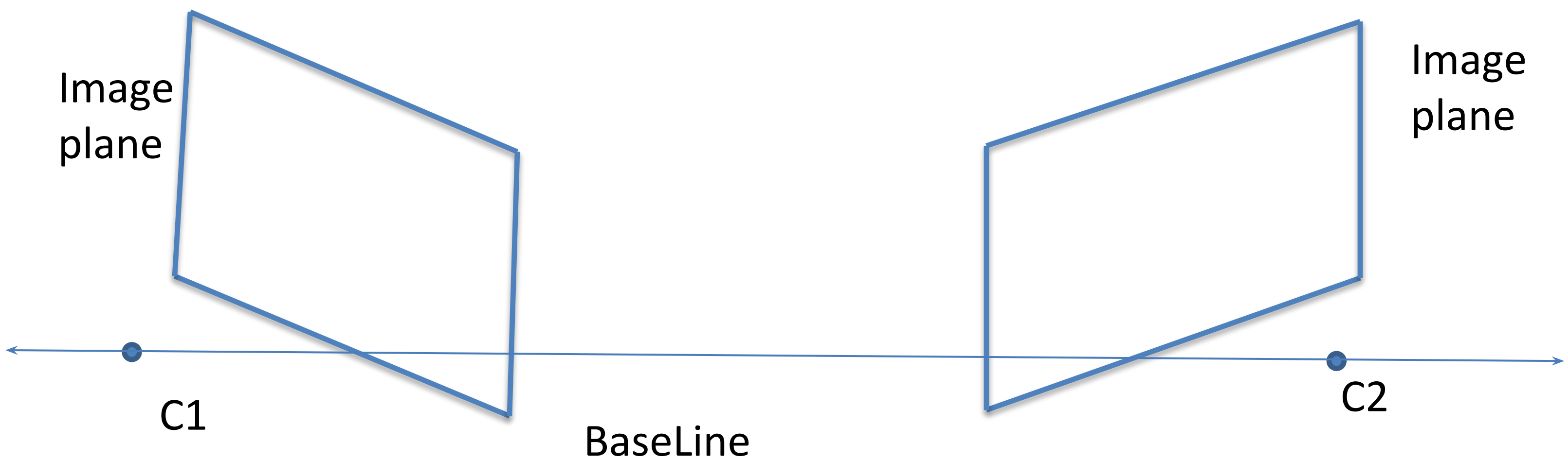
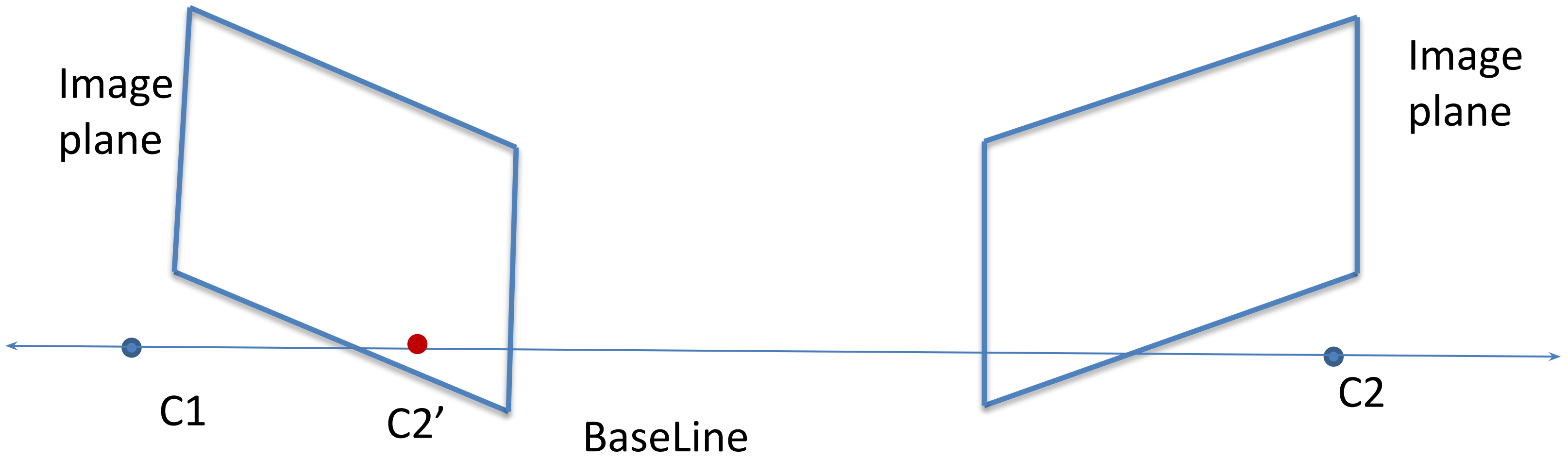


EPIPOLES & EPIPOLAR LINES

- If the cameras are in-field of view of each other



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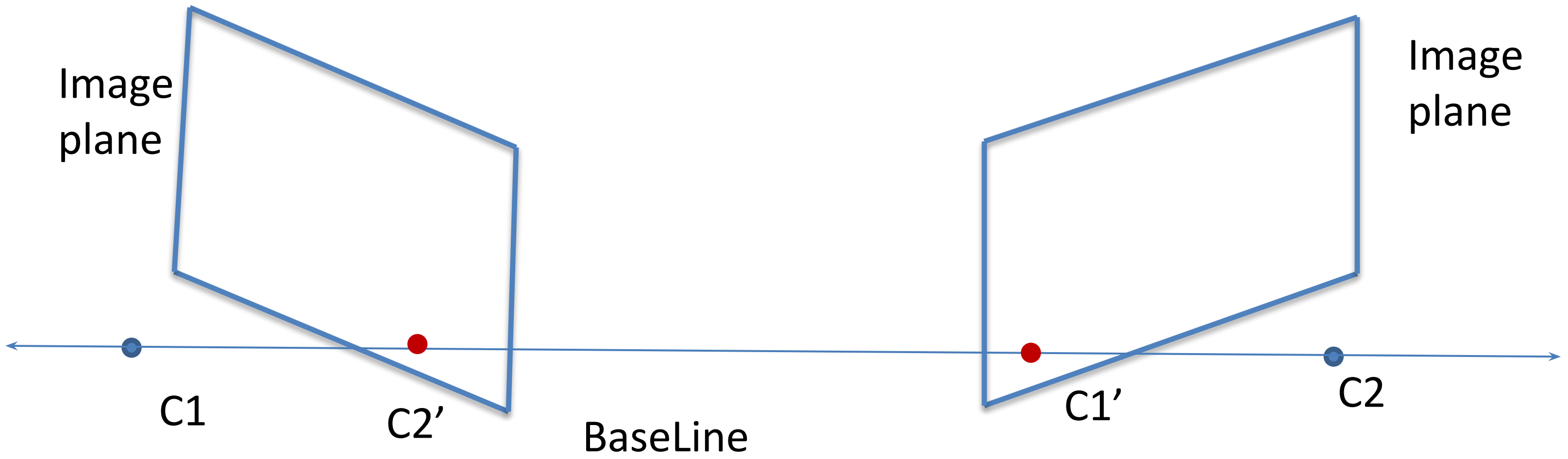

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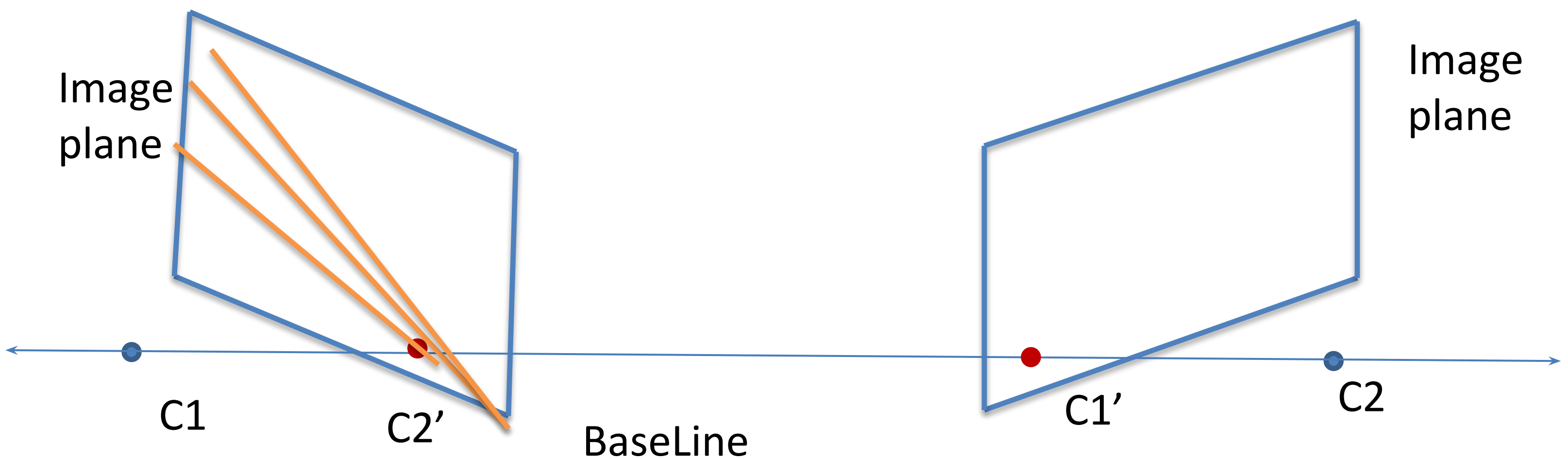

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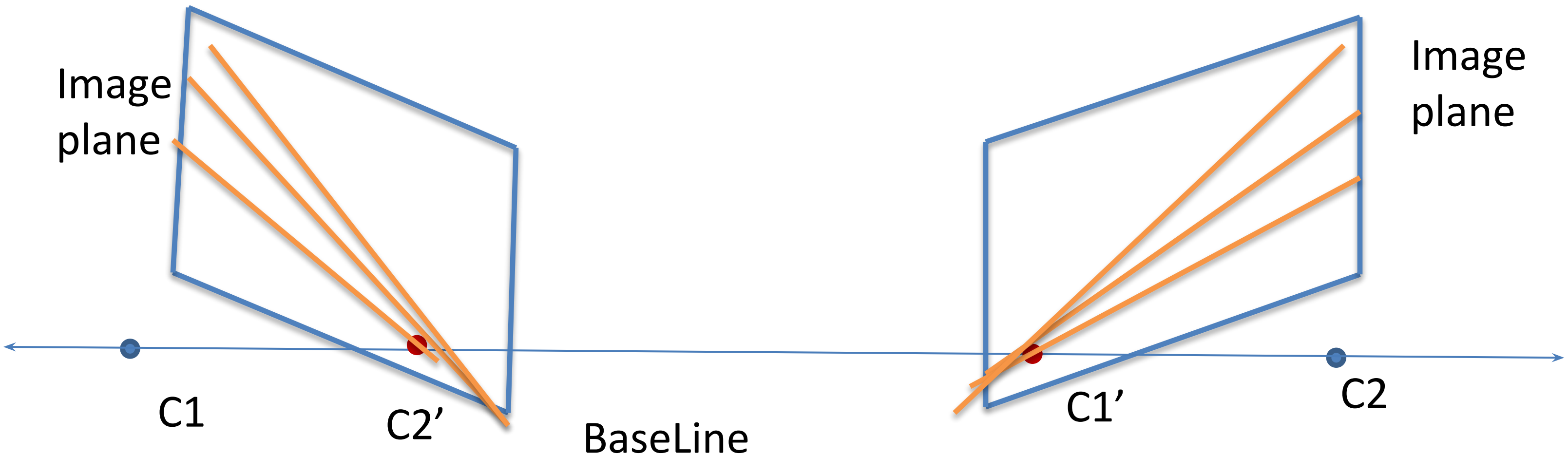
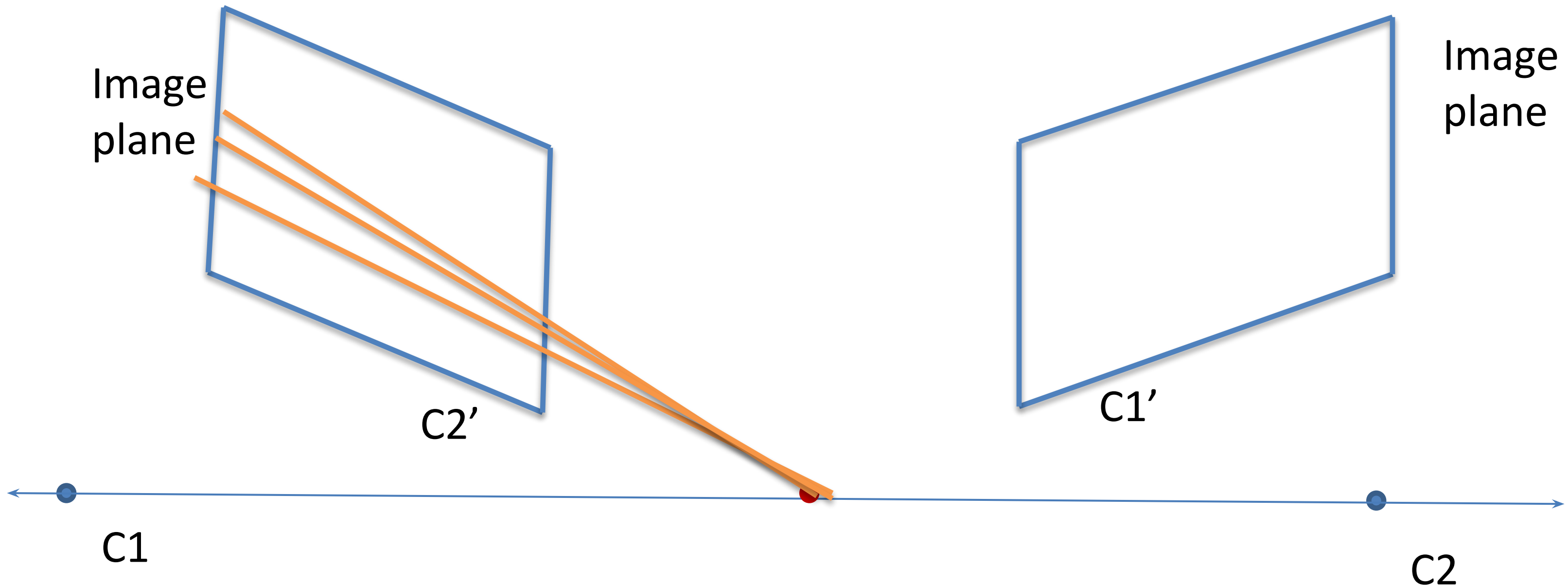


EPIPOLES & EPIPOLAR LINES

- If the cameras are **not** in-field of view of each other



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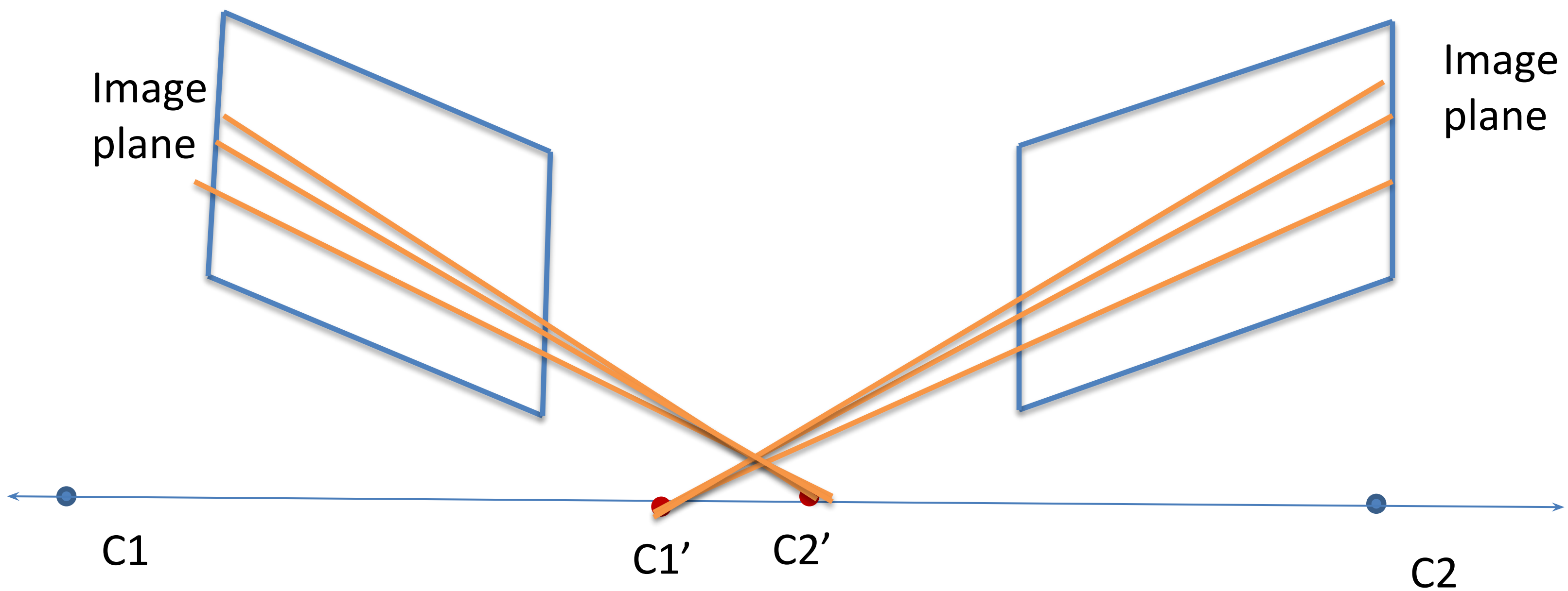
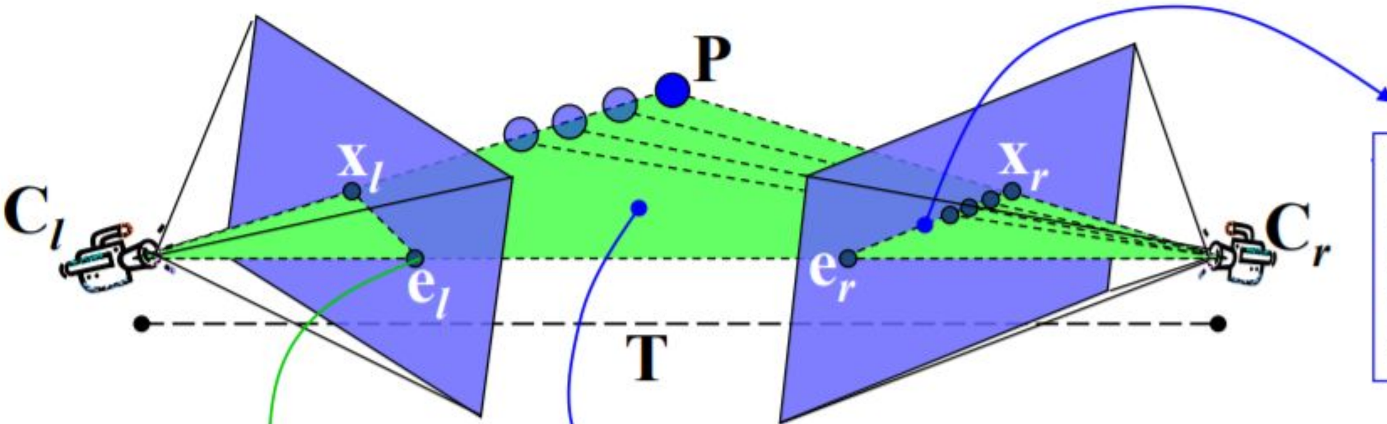




Epipolar line: set of world points that project to the same point in left image, when projected to right image forms a line.

Epipolar plane: plane defined by the camera centers and world point.

Epipole: intersection of image plane with line connecting camera centers. Image of a left camera center in the right, and vice versa.

Relation Between Correspondence

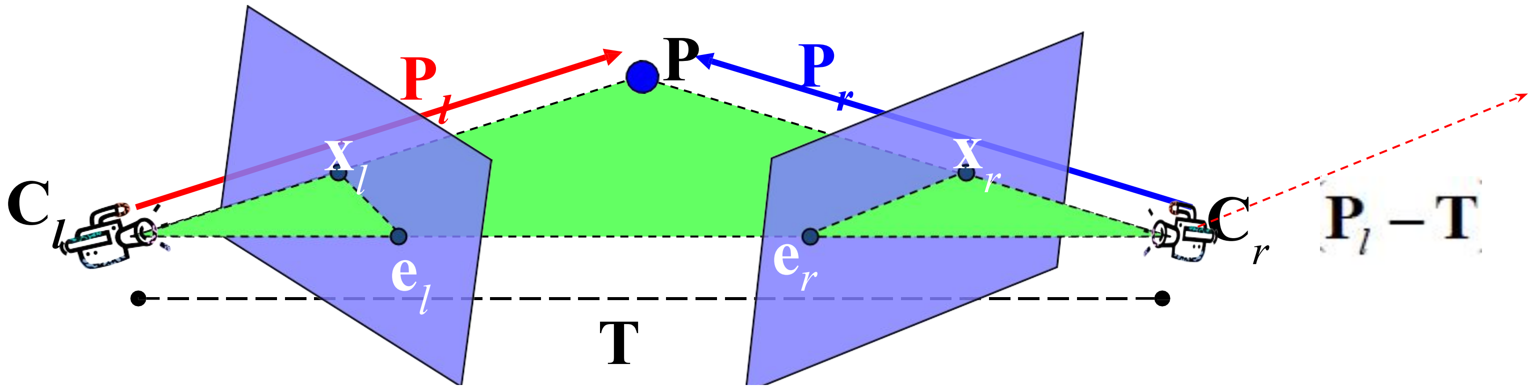
- ***Essential Matrix:*** Measures Rotation and Translation

$$E = R\ S$$

- ***Fundamental Matrix:*** Relation btw image point co-ordinates

$$\begin{bmatrix} x' & y' & 1 \end{bmatrix} \mathrm{F} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

Essential Matrix



Coplanarity constraint between vectors ($\mathbf{P}_l$-$\mathbf{T}$), $\mathbf{T}$, $\mathbf{P}_l$.

$$\left.\begin{aligned} (\mathbf{P}_l - \mathbf{T})^{\mathrm{T}}\mathbf{T} \times \mathbf{P}_l = 0 \\ \mathbf{P}_r = \mathbf{R}(\mathbf{P}_l - \mathbf{T}) \end{aligned}\right\} \quad \mathbf{P}_r^{\mathrm{T}}\mathbf{R}\mathbf{T} \times \mathbf{P}_l = 0$$

$$\mathbf{R}^T\mathbf{P}_r = (\mathbf{P}_l - \mathbf{T})$$

$$\mathbf{P}_{\mathbf{r}}^{\ T}\mathbf{R} = (\mathbf{P}_l - \mathbf{T})^T$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

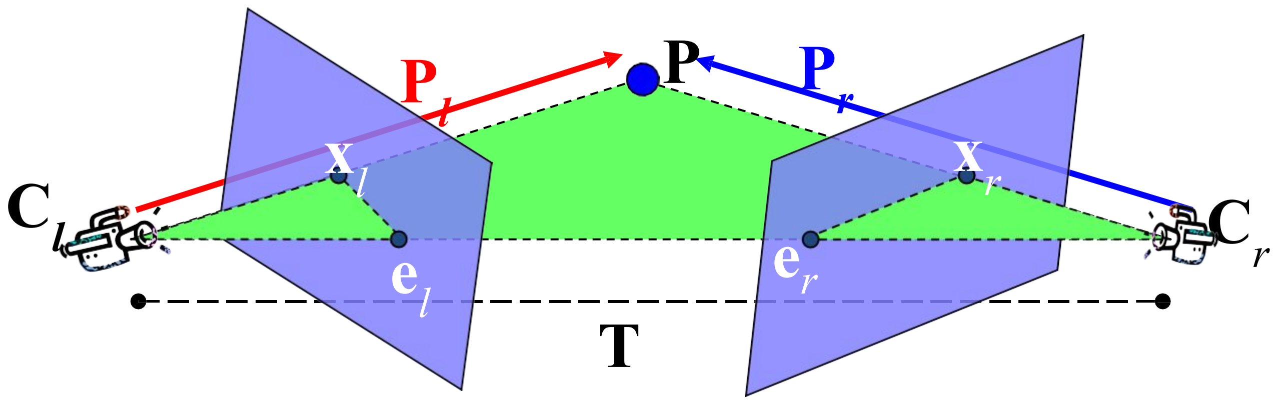
Vector Cross Product to Matrix- vector multiplication

$$A \times B = \begin{bmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} = \begin{bmatrix} -A_z B_y + A_y B_z & A_z B_x - A_x B_z & -B_x A_y + B_y A_x \end{bmatrix}$$

$$A \times B = S.B = \begin{bmatrix} 0 & -A_z & A_y \\ A_z & 0 & -A_x \\ -A_y & A_x & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

$$= \begin{bmatrix} -A_z B_y + A_y B_z & A_z B_x - A_x B_z & -B_x A_y + B_y A_x \end{bmatrix}$$

Essential Matrix



$$\mathbf{P}_r^{\mathrm{T}} \mathbf{R} \mathbf{T} \times \mathbf{P}_l = 0$$

$$\mathbf{P}_r^{\mathrm{T}} \mathbf{R} \boxed{\mathbf{S}} \mathbf{P}_l = 0 \quad \Rightarrow \quad \boxed{\mathbf{P}_r^{\mathrm{T}} E \mathbf{P}_l = 0}$$

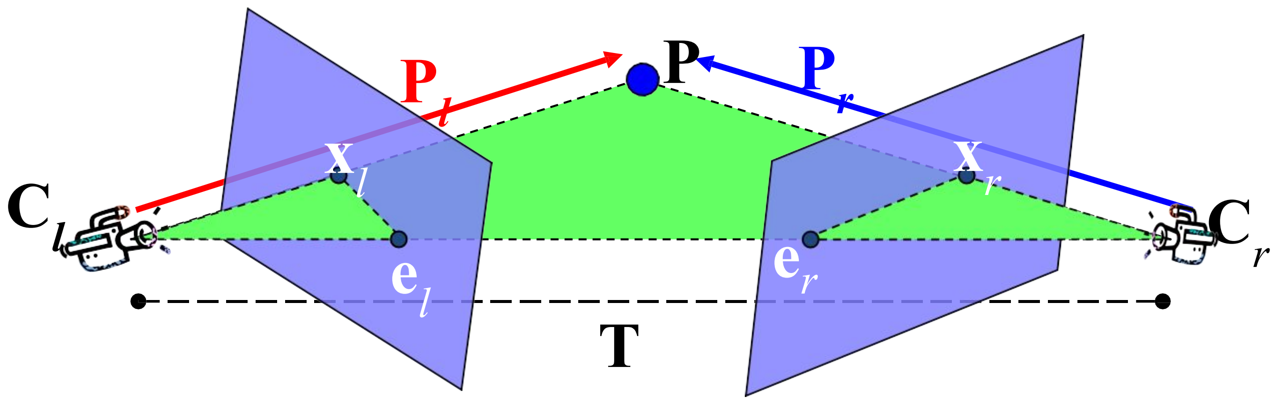
essential matrix

Cross product can be expressed as matrix mutliplication

$$\mathbf{S} = \begin{pmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{pmatrix}$$

$$E = \mathbf{R}\,\mathbf{S}$$

Fundamental Matrix



Apply Camera
mo

$$M_l^{-1}\mathbf{x}_l = \mathbf{P}_l$$

$$M_r^{-1}\mathbf{x}_r = \mathbf{P}_r$$

$$\mathbf{x}_r^{T} M_r^{-T} = \mathbf{P}_r^{T}$$

$$\left.\begin{aligned} \mathbf{x}_l &= M_l \mathbf{P}_l \\ \mathbf{x}_r &= M_r \mathbf{P}_r \\ \mathbf{P}_r^{\mathrm{T}} E \mathbf{P}_l &= 0 \end{aligned}\right\}$$

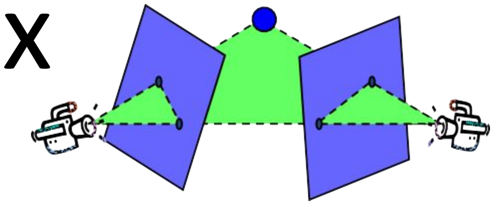
$$\mathbf{x}_r^{\mathrm{T}} M_r^{-\mathrm{T}} E M_l^{-1} \mathbf{x}_l = 0$$

$$\mathbf{x}_r^{\mathrm{T}} \left(M_r^{-\mathrm{T}} E M_l^{-1} \right) \mathbf{x}_l = 0$$

$$\boxed{\mathbf{x}_r^{\mathrm{T}} F \mathbf{x}_l = 0}$$

fundamental matrix

Fundamental Matrix



$$\mathbf{x}'^{\mathrm{T}} F\mathbf{x} = \mathbf{x}'^{\mathrm{T}} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & m \end{pmatrix} \mathbf{x} = 0$$

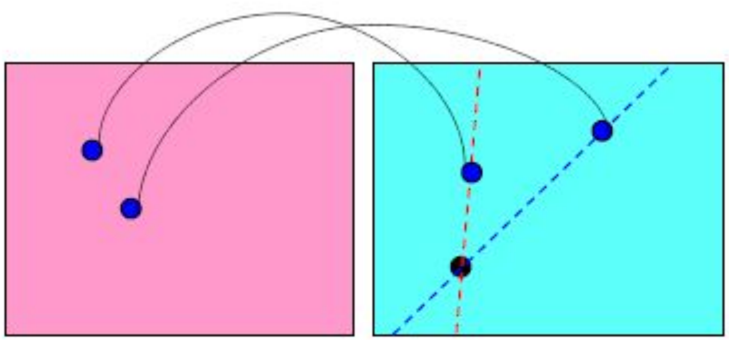
$Fix\ (x, y) in\ \ Image\ 1$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}^{T} \mathrm{F} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}^{T} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 0$$

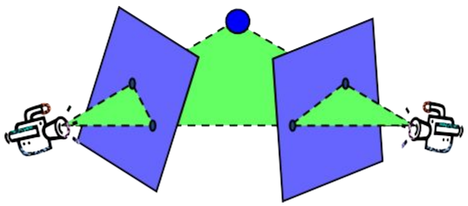
px'+qy'+r=0 *(eq of line-> epipolar line in image 2*

Given a point in left camera $\mathbf{x}$, epipolar line in right camera is: $\mathbf{u}_r = F\mathbf{x}$



Fundamental Matrix

$$\mathbf{x}'^{\mathrm{T}} F\mathbf{x} = \mathbf{x}'^{\mathrm{T}} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & m \end{pmatrix} \mathbf{x} = 0$$



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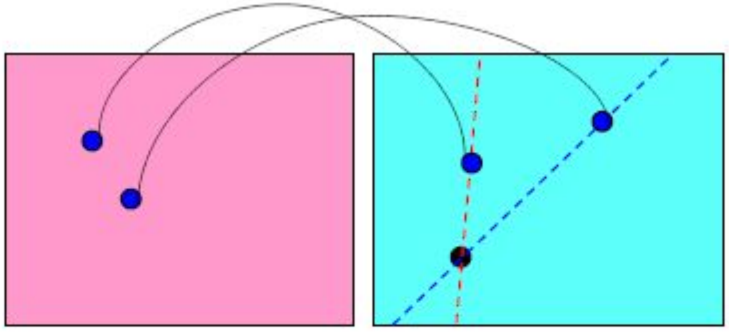


- 3x3 matrix with 9 components
- Rank 2 matrix (due to S)
- 7 degrees of freedom (3-rot-3 traslation-1 -scaling)

$$S = \begin{pmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{pmatrix}$$

Homework

- For the two epipoles $e = (x_e, y_e)$ *and* $e' = (x_e', y_e')$.
- Show that $[x_e, y_e, 1]^T$ is an eigenvector of F with eigenvalue 0.
- And similarly, $[x_e', y_e', 1]^T$ is an eigenvector of F^T with eigenvalue 0.

Hint: Refer the reading material shared with you.

Fundamental Matrix

- Fundamental matrix captures the relationship between the corresponding points in two views.

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^T \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = 0,$$

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}^T \begin{bmatrix} f_{11}x' + f_{12}y_i' + f_{13} \\ f_{21}x' + f_{22}y_i' + f_{23} \\ f_{31}x' + f_{32}y_i' + f_{33} \end{bmatrix} = 0,$$

Fundamental Matrix

$$x_i(f_{11}x' + f_{12}y_i' + f_{13}) + y_i(f_{21}x' + f_{22}y_i' + f_{23}) + (f_{31}x' + f_{32}y_i' + f_{33}) = 0$$

$$x_i x' f_{11} + x_i y_i' f_{12} + x_i f_{13} + y_i x' f_{21} + x' y_i' f_{22} + y_i' f_{23} + x' f_{31} + y_i' f_{32} + f_{33} = 0$$

One equation for one point correspondence

$$Mf = \begin{bmatrix} x_1'x_1 & x_1'y_1 & x_1' & y_1'x_1 & y_1'y_1 & y_1' & x_1 & y_1 & 1 \\ x_2'x_2 & x_2'y_2 & x_2' & y_2'x_2 & y_2'y_2 & y_2' & x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n'x_n & x_n'y_n & x_1' & y_n'x_n & y_n'y_n & y_n' & x_n & y_n & 1 \end{bmatrix} f = 0$$

Homogenous system, no unique solution, fix one unknown to be=1 arbitrarily

M is 9 by n matrix $\quad f = [f_{11} \quad f_{12} \quad f_{13} \quad f_{21} \quad f_{22} \quad f_{23} \quad f_{31} \quad f_{32} \quad f_{33}]$

To solve the equation, the rank(M) must be 8.

Normalized 8-point algorithm (Hartley)

Objective:

Compute fundamental matrix F such that $\mathbf{x}_i' F \mathbf{x}_i = 0$

Algorithm

Normalize the image $\hat{\mathbf{x}}_i = T\mathbf{x}_i$ $\hat{\mathbf{x}}_i' = T'\mathbf{x}_i'$

$$T = \begin{bmatrix} a_x & 0 & d_x \\ 0 & a_y & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

Find centroid of points in each image, substract with remaining points determine the range, and normalize all points between 0 and 1

Linear solution

determining the eigen vector (9x1) corresponding to the smallest eigen value of A => gives the approximation of fundamental matrix

$$Af = \begin{bmatrix} \hat{x}_1'\hat{x}_1 & \hat{x}_1'\hat{y}_1 & \hat{x}_1' & \hat{y}_1'\hat{x}_1 & \hat{y}_1'\hat{y}_1 & \hat{y}_1' & \hat{x}_1 & \hat{y}_1 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \hat{x}_8'\hat{x}_8 & \hat{x}_8'\hat{y}_8 & \hat{x}_8' & \hat{y}_8'\hat{x}_8 & \hat{y}_8'\hat{y}_8 & \hat{y}_8' & \hat{x}_8 & \hat{y}_8 & 1 \end{bmatrix} f = 0$$

Normalized 8-point algorithm (Hartley)

eigen vector (9x1)->reshaped

$$\hat{F} = \begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix}$$

Normalize $\hat{F} = \hat{F} / \| \hat{F} \|$

L1 matrix norm is maximum of absolute column sum.

$$\|A\|_1 = \max_{1 \le j \le n} \sum_{i=1}^{m} |a_{ij}|,$$

Constraint enforcement of rank=2 by using SVD decomposition

L infinity norm is maximum of
sum of absolute of row sum.

$$\hat{F} = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} V' \qquad (\sigma_1 \ge \sigma_2 \ge \sigma_3)$$

Rank enforcement by setting smallest singular value=0

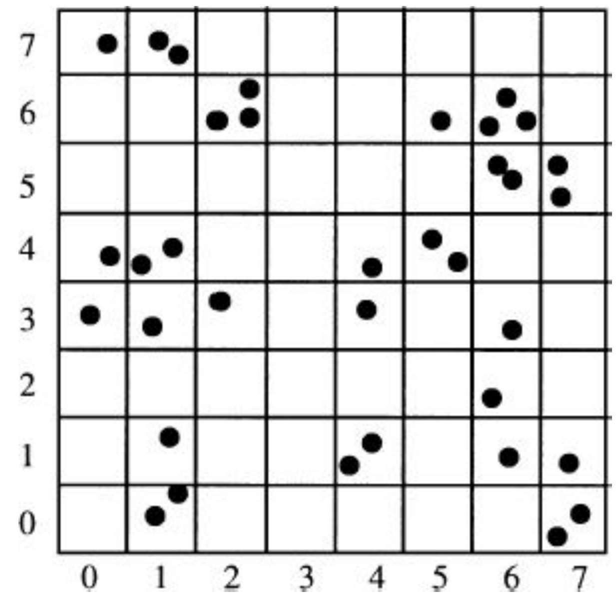
$$\widetilde{F} = U \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} V' \qquad (\sigma_3 = 0)$$

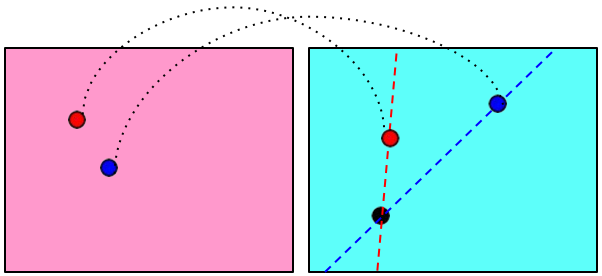
$$T = \begin{bmatrix} a_x & 0 & d_x \\ 0 & a_y & d_y \\ 0 & 0 & 1 \end{bmatrix}$$

De-normalization: $F = T'^T \widetilde{F} T$

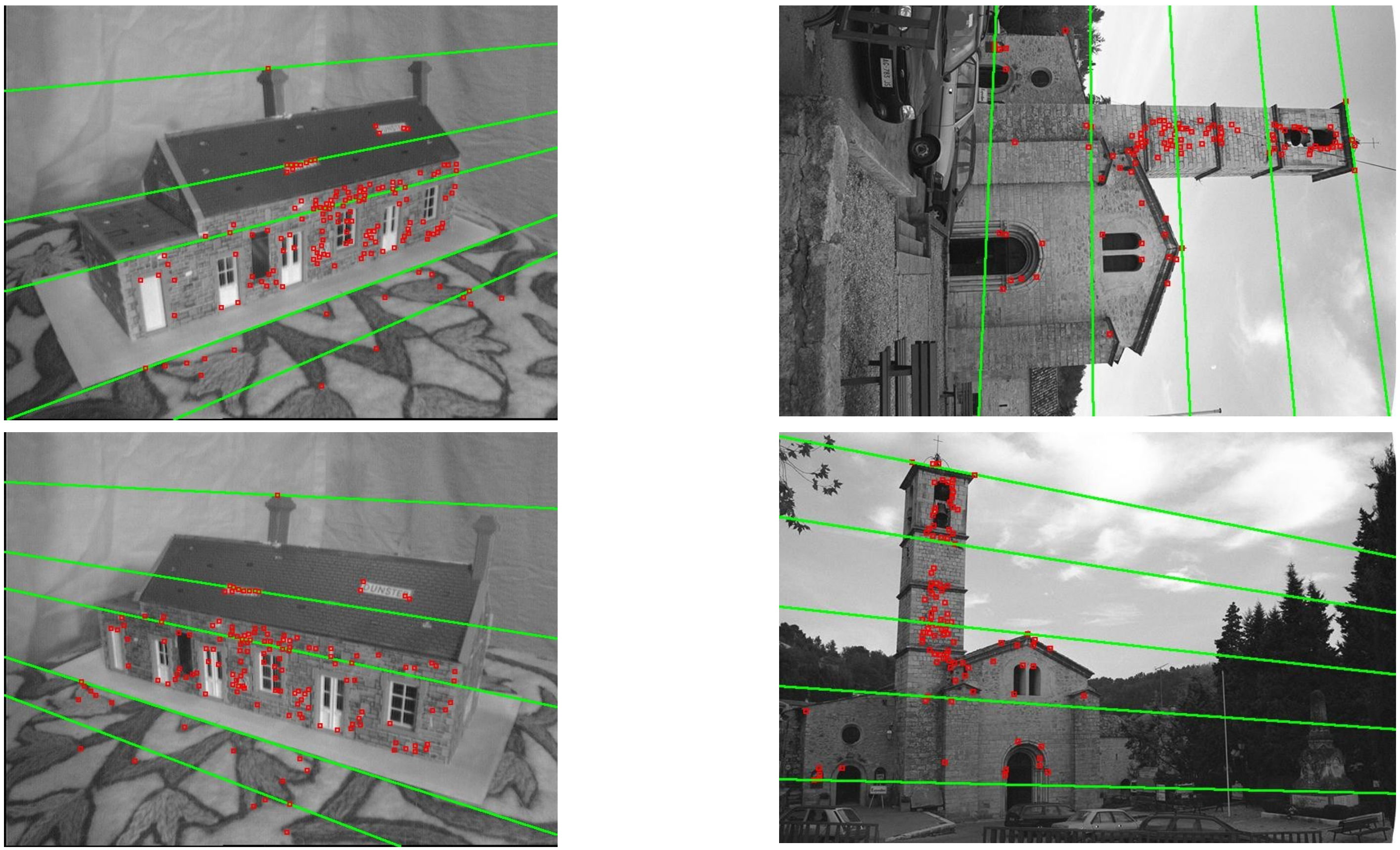
Robust Fundamental Matrix Estimation (by Zhang)

- Uniformly divide the image into 8×8 grid.
- Randomly select 8 grid cells and pick one pair of corresponding points from each grid cell, then use Hartley's 8-point algorithm to compute Fundamental Matrix F_i.
- For each F_i, compute the median of the squared residuals R_i.
  - $R_i = \text{median}_k[d(p_{1k}, F_i p_{2k}) + d(p_{2k}, F'_i p_{1k})]$
- Select the best F_i according to R_i.
- Determine outliers if $R_k > Th$.
- Using the remaining points compute the fundamental Matrix F by weighted least square method.





Epi-polar Lines



Epi-polar lines