Subject Name: Statistical Foundation of Machine Learning, Quiz Exam, 40 Marks, Time : 2 hrs exam. (All questions are compulsory), Date : 8/9/2024

Q1 : a) [10 Marks]
Prove that for a real auto correlation matrix R all the eigenvalues must be real and the eigenvectors corresponding to distinct eigenvalues of R are mutually orthogonal.
b) [5 Marks] : Prove that n × n matrix A is diagonalizable if and only if it has n linearly independent eigenvectors. In this case A is similar to a matrix D whose diagonal elements are the eigenvalues of A.

Q2 : [10 Marks] Consider a 2x2 matrix as shown below. Kindly perform SVD decomposition and compute eigenvectors and eigenvalues of the same.
[4 0]
[3 -5]

Q3 : [10 Marks] A box contains 3 coins; Two regular and one fake coin with heads on both sides

1. Pick a coin at random and toss it. What is the probability that it will land with a head?
2. Pick a coin at random and toss it and get head. What is the probability that it is the two headed coin?

Q4 : [10 Marks] Compute $||A||_\infty$, $||A||_1$ for the matrix A defined as

$$\begin{bmatrix} 100 & 300 & -50 \\ -30 & 20 & -70 \\ -100 & 50 & 10 \end{bmatrix}$$