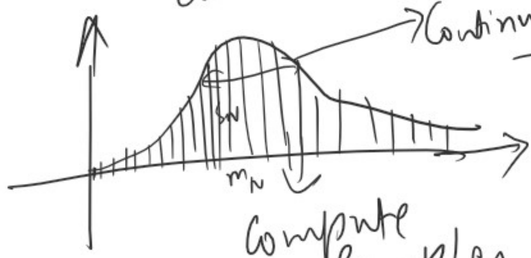
Gibbs

Sunday, April 6, 2025 8:25 AM

Gibbs Sampling — MC MC → chain $p(w/t)$

Monte Carlo markov



$f(x,w) = w_0 + w_1 x$

sample

$p(w_0/t) \rightarrow$ Gaussian

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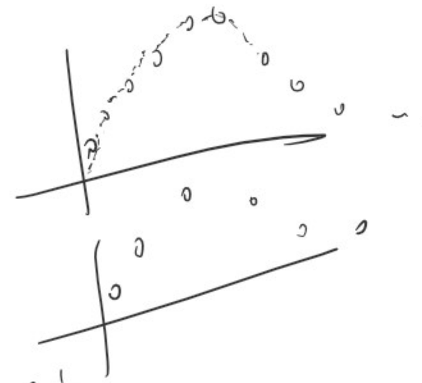
$\propto \underbrace{p(t|w_0)}_{\text{Gaussian}} \underbrace{p(w_0)}_{\text{~~Uniform~~ Gaussian}}$

100 samples

1000



Metropolis Hasting

Gibbs Sampling → Sampling from joint distribution is very difficult.

$p(w_0, w_1, \cdots w_{N-1}/t) \rightarrow$ extract samples

$p(w_0 | w_1, \cdots w_{N-1}) \rightarrow p(w_0 | w_1) \rightarrow$ Conditional distribution

$p(\underline{w}/t) = p(w_0, w_1 | t)$ — $p(w_0/w_1, t)$
$p(w_1/w_0, t)$

Concept → ① we update the values of variable w_0 keeping w_1 fixed.

② keep the w_0 from previous step fixed, update w_1.

③ update w_0 keeping the w_1 fixed obtained ... step.

③ update w_0 keeping the w_1 fixed ... from previous step.

→ x $X w_0, w_1$ → initialise

① w_0 $\quad p(w_0 | w_1)$ → w_0 sampler kept w_1 fixed

② sample w_1 $\quad p(w_1 / w_0)$ → w_1 sampler keept w_0 fixed

③ sample w_0 $\quad p(w_0 | w_1)$ → w_0 sampler keep w_1 fixed

$$\rightarrow \sum p(w_0)\, p(w_1 / w_0) = \sum p(w_1)\, p(w_0 / w_1)$$

Example Given $(w_0 \,\&\, w_1) \rightarrow (0 \,\&\, 1)$. we want to sample from joint distribution $p(w_0, w_1)$

$$\left\{ \begin{array}{l} p(w_0 = 0, w_1 = 0) = 0.2 \\ p(w_0 = 1, w_1 = 0) = 0.3 \\ p(w_0 = 0, w_1 = 1) = 0.1 \\ p(w_0 = 1, w_0 = 1) = 0.4 \end{array} \right\}$$

$$\left\{ \begin{array}{l} w_0 - 0 \,\&\, 1 \\ w_1 - 0 \,\&\, 1 \end{array} \right.$$

Initialise $w_0 = w_1 = 0$ fixed w_1

$$\left\{ \begin{array}{l} p(w_0 = 0 | w_1 = 0) - (0.4) \\ p(w_0 = 1 / w_1 = 0) - (0.6) \end{array} \right.$$

$$= \frac{p(w_0 = 0, w_1 = 0)}{p(w_1 = 0)} = \frac{0.2}{0.5} = 0.4$$

fix → $w_0 = 1$, sample w_1

0.429

fix → $\underline{w_0 = 1}$, sample

$p(w_1 = 0 \mid w_0 = 1) = 0.429$

$p(w_1 = 1 \mid w_0 = 1) = 0.571$

$w_1 = 1$, fix sample w_0

$p(w_0 = 0 \mid w_1 = 1) =$

$p(w_0 = 1 \mid w_1 = 1) =$

$\overset{w_0 \;\; w_1}{(0, 0)} \rightarrow (1, 1) \rightarrow (1, 1) \rightarrow (1, 1) \text{---}$

$f(x, w) = w_0 + w_1 x$

$t = f + \epsilon \rightarrow N(0, \alpha^{-1} I)$