Subject Name: Machine Learning, Mid-Sem Exam, MM:40

Question 1 : [10 Points] : Consider a standard observation model in additive noise.

$$\mathbf{d}(i) = \mathbf{U}_i^H \mathbf{W} + \mathbf{n}(i)$$

$\mathbf{d}(i)$...noisy measurement linearly related to $\mathbf{W}$
$\mathbf{W}$…Is the unknown vector to be estimated
$\mathbf{U}_i$…Given column vector
$\mathbf{n}(i)$…the noise vector

In a practical scenario, the $\mathbf{W}$ can be the weight vector, $\mathbf{U}_i$ The data vector, $\mathbf{d}(i)$ the observed output using a Series of Arrays and $\mathbf{n}(i)$ is the noise vector added at each sensor. In matrix form, it is represented as $\mathbf{d} = \mathbf{U}\mathbf{W} + \mathbf{n}$.

Find the weights that minimizes the distance between $\mathbf{d}$ and $\mathbf{U}\hat{\mathbf{W}}$, here $\hat{\mathbf{W}}$ is the estimated weight vector.

Question 2 : [5 Points]: Computes the gradient of the quadratic function of x given the starting points x=[1,2,3] and then uses the result of the gradient to feed the next iterations, with new points. Prints out the result of the function at each iteration till 3rd run. Use Python script to print the results.

Question 3 : [10 Points] With given input and output relation

x = [-1, -0.8, -0.6, -0.4, -0.2, 0, 0.2, 0.4, 0.6, 0.8, 1]

t = [-4.9, -3.5, -2.8, 0.8, 0.3, -1.6, -1.3, 0.5, 2.1, 2.9, 5.6]

Please fit a curve with M=4 Gaussian basis functions having unity variance.

Question 4 : a) [5 Points] Show that the matrix $\phi(\phi^T\phi)^{-1}\phi^T$ takes any vector **v** and projects it onto the space spanned by the columns of Φ. Use this result to show that the least-squares solution $w_{ML} = \phi(\phi^T\phi)^{-1}\phi^T t$ corresponds to an orthogonal projection of the vector **t** onto the manifold S (space span by column of Φ)

b) [5 Points] In the below joint probability density, the three data points are observed. Kindly find out the values of μ and σ that results in giving the maximum value of the below expression.

$$P(9, 9.5, 11; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(9-\mu)^2}{2\sigma^2}\right) \times \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(9.5-\mu)^2}{2\sigma^2}\right) \times \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(11-\mu)^2}{2\sigma^2}\right)$$

c) [5 Points] Determine the conditions under when the least square solution is equivalent to the maximum likelihood estimates.