

Subject Name: Statistical Foundation of Machine Learning
Mid-sem Exam, 40 Points, Time : 2 hrs exam. (All questions are compulsory)

Q1 : a) (5 points) : Prove two similar matrices A and B have the same characteristic polynomial hence same eigenvalues.

Q2 : (5 points). Which of the following are true ? Prove or give a counterexample (in terms of probability and conditional probability)

- a) (2.5 points) If A and B are conditionally independent given C, are A and B independent ?
- b) (2.5 points) If A and {B, C} are conditionally independent given D, are A and B conditionally independent given D ?

Q3 : (5 points) : $X=(X_1, X_2)$ is drawn from a two dimensional Gaussian distribution with a diagonal covariance matrix

$X=(X_1, X_2) \sim N(\mu, \Sigma)$ $\Sigma = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$. Here a and b are some real numbers.

Are X_1 and X_2 independent ? Explain as succinctly as possible.

Q4 : (10 points)

- a) (5 points) Consider a random process $X(t)$ such that for every t, $X(t)$ is an i.i.d. Gaussian random variable with zero mean and unit variance. Find $R_X(t)$.
- b) (5 points) With the help of data points given in the table below, fit a second-order polynomial to the following data using the least square method

I	1	2	3	4	5	6
x	0	0.5	1.0	1.5	2.0	2.5
y	0	0.25	1.0	2.25	4.0	6.25

Q5 : (5 points) : An unbiased dice is rolled and for each number on the dice a bag is chosen:

Numbers on the Dice	Bag chosen
---------------------	------------

1	Bag A
2 or 3	Bag B
4 or 5 or 6	Bag C

Bag A contains 3 white ball and 2 black ball, bag B contains 3 white ball and 4 black ball and bag C contains 4 white ball and 5 black ball. Dice is rolled and bag is chosen, if a white ball is chosen find the probability that it is chosen from bag B.

Q 6 (10 Points) : Calculate the entropy in bits for each of the following random variables:

- (i) Pixel values in an image whose possible grey values are all the integers from 0 to 255 with uniform probability.
- (ii) Humans classified according to whether they are, or are not, mammals.
- (iii) Gender in a tri-sexed insect population whose three genders occur with probabilities $\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{2}$.
- (iv) A population of persons classified by whether they are older, or not older, than the population's median age.