$$P(X, Y) = \frac{n_{ij}}{N}$$

Marginal

$$P(X) = \sum_{Y} P(X, Y)$$

$$P(Y) = \sum_{X} P(X, Y)$$

$$P(X = x_i, Y = y_j) = \frac{n_{ij}}{N}$$

$$= \frac{n_{ij}}{c_i} \times \frac{c_i}{N}$$

$$P(X = x_i, Y = y_j) = P(Y = y_j \mid X = x_i)\, P(X = x_i)$$

$$P(Y = y_j \mid X = x_i) = \frac{P(X = x_i, Y = y_j)}{P(X = x_i)}$$

Bayes theorem

Conditional Distribution

Sum of probability

$$P(X = x_i \mid Y = y_j) = \frac{P(X = x_i, Y = y_j)}{P(Y = y_j)}$$

$$P(X, Y) = P(X/Y)\,P(Y)$$

$$= P(Y/X)\,P(X)$$

$$P(X/Y) = \frac{P(Y/X)\,P(X)}{P(Y)} = \frac{P(X, Y)}{P(Y)}$$

$$P(X) = \sum_{Y} P(X, Y)$$

$$= \sum_{Y} P(Y/X)\, P(X)$$

$$P(X) = \sum_{Y} P(X/Y)\, P(Y)$$

$$P(B=r \mid F=0) = \frac{P(F=0 \mid B=r)\,P(B=r)}{P(F=0)}$$

$$P(F=0) = P(F=0, B=r) + P(F=0, B=b)$$

$$P(F=0 \mid B=r) = 3/4$$

$$P(F=a \mid B=r) = 1/4$$

$$P(B=r) = 4/10$$

$$P(B=b) = 6/10$$

$$= \frac{3/4 \times 4/10}{9/20} = \frac{3/10}{9/20}$$

$$= 2/3$$

$$P(B = b \mid F = a) = \frac{P(F = a \mid B = b)\, P(B = b)}{P(F = a)}$$

$$= \frac{1/4 \times 6/10}{(\text{next page})} = 9/11$$

$$P(F=a) = P(F=a, B=r) + P(F=a, B=b)$$

$$= P(F=a \mid B=r)\, P(B=r) + P(F=a \mid B=b)\, P(B=b)$$

$$= 1/4 \times 4/10 + 3/4 \times 6/10$$

$$P(A \cap B) = \emptyset$$ mutually exclusive

$$P(A \mid P(B) = \{ \quad \}$$

$$P(A \cap B) = P(A)\,P(B)$$

Independent

$E[X] = \sum x\, p(x)$ ✓

$X \downarrow$ discr. R.V

X — 10 values from 0 to 9

Each of them are equiprobable

$p(0) = 1/3$

$p(1) = 2/3$: -

$p(0) = p(1) \ldots = p(9) = \frac{1}{10}$

$E(X) = \frac{0}{10} + \frac{1}{10} + \frac{2}{10} + \ldots \frac{9}{10}$

$= \frac{45}{10} = 4.5$

$$f(x) = x^2$$

$$E[f(x)] = \sum_{x} f(x)\,p(x)$$

$$E[f(x)] = \int f(x)\,p(x)\,dx$$