1. Given the decision boundary defined by the equation $y = -\frac{3}{4}x + 2$, if a point $(8, y)$ is classified as class 0, what is the value of y?

Hint: Substitute $x = 8$ into the equation to solve for y.

2. A shape's boundary is represented by the chain codes $1, 2, 3, 0, 3, 4$. Starting from the origin $(0, 0)$, what are the final coordinates after executing these chain codes?

Hint: Map each code to a directional movement in a 2D grid.

3. Given $P(A, B) = 0.15$ and $P(B) = 0.5$, calculate $P(A|B)$.

Hint: Use the definition of conditional probability: $P(A|B) = \dfrac{P(A, B)}{P(B)}$.

4. A diagnostic test has a sensitivity of 85% and specificity of 90%. If the disease prevalence is 3%, what is the probability that a randomly selected individual has the disease given a positive test result?

Hint: Use Bayes' theorem: $P(\text{Disease}|\text{Positive}) = \dfrac{P(\text{Positive}|\text{Disease}) \cdot P(\text{Disease})}{P(\text{Positive})}$.

5. If the error probability of a classifier is $P(\text{error}) = 0.07$, how many errors would you expect out of 150 predictions?

Hint: Calculate expected errors using $E[\text{errors}] = P(\text{error}) \times \text{Total Predictions}$.

6. For a squared loss function, if the true label is 0 and the predicted probability is 0.4, what is the loss?

$$L(y, \hat{y}) = (y - \hat{y})^2 \text{ gives } L(0, 0.4).$$

Hint: Substitute y and $\hat{y}$ into the loss function.

7. Given two discriminant functions $g_0(x) = 4x - 8$ and $g_1(x) = -x + 3$, find the point of intersection where the decision boundary is located.

Hint: Set $g_0(x) = g_1(x)$ and solve for x.

8. If the minimum error rate of a classifier is 4% and it makes 250 predictions, how many errors would you expect?

Hint: Use the formula $E[\text{errors}] = \text{Minimum Error Rate} \times \text{Total Predictions}$.

9. Using the zero-one loss function, what is the loss incurred if the model predicts class 0 for a true class 1 instance? What about for a correct prediction?

Hint: Recall the definition of the zero-one loss function: $L(y, \hat{y}) = \begin{cases} 0 & \text{if } y = \hat{y} \\ 1 & \text{if } y \neq \hat{y} \end{cases}$.

10. Given the discriminant function $g(x) = 2x - 5$, what is the threshold value of x for classifying an observation into class 1?

Hint: Set $g(x) = 0$ and solve for x.

11. If the conditional risk for class 0 is $R(y|x) = 0.2$ and for class 1 is $R(y|x) = 0.4$, calculate the Bayes minimum risk using priors $P(0) = 0.5$ and $P(1) = 0.5$.

Hint: Use the formula $R = P(0) \cdot R(0|x) + P(1) \cdot R(1|x)$.

12. For a univariate normal distribution with mean $\mu = 10$ and variance $\sigma^2 = 16$, calculate the probability density function value at $x = 12$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

Hint: Substitute μ, σ^2, and x into the PDF formula.

13. Given a multivariate normal distribution with mean vector $\mu = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and covariance matrix $\Sigma = \begin{pmatrix} 2 & 0.5 \\ 0.5 & 3 \end{pmatrix}$, find the covariance between the two variables.

Hint: Look at the off-diagonal elements of the covariance matrix: $Cov(X, Y)$.

14. If the variance of a random variable X is 25 and its expected value $E[X] = 5$, what is the standard deviation? Also, if another variable Y has a variance of 36, what is the covariance $Cov(X, Y)$ if they are independent?

Hint: Standard deviation is $\sigma = \sqrt{\text{Variance}}$ and $Cov(X, Y) = 0$ for independent variables.