L_2 norm $= \sqrt{1^2+2^2} = \sqrt{5}$

$$V_1 = \begin{bmatrix} \cancel{-1/\sqrt{5}} \\ \cancel{+2/\sqrt{5}} \end{bmatrix} \quad \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

for $\lambda = 40$

$$\begin{bmatrix} 16 & 12 \\ 12 & 34 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 40v_1 \\ 40v_2 \end{bmatrix}$$

$16v_1 + 12v_2 = 40v_1$

$12v_1 + 34v_2 = 40v_2$

$12v_2 = +24v_1$

$v_2 = +2v_1$

$v_1 = 1, \; v_2 = +2$

$$V_2 = \begin{bmatrix} \cancel{1/\sqrt{5}} \\ \cancel{-2/\sqrt{5}} \end{bmatrix} \quad \begin{bmatrix} 1/\sqrt{5} \\ +2/\sqrt{5} \end{bmatrix}$$

$$U = \begin{bmatrix} +1/\sqrt{5} & -2\sqrt{5} \\ +2\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$$

Application of SVD

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(1) Moore Penrose Pseudo inverse

↳ most useful feature of SVD is that it can be used to generalize matrix inversion to non square matrices

$$A = U \Sigma V^T$$

$$A^{-1} = (U \Sigma V^T)^{-1}$$

$$= (V^T)^{-1} \Sigma^{-1} U^{-1}$$

$$\{ (U \cdot V)^{-1} = V^{-1} \cdot U^{-1} \}$$

u and v are orthogonal

$\therefore \quad U \cdot U^T = I \rightarrow U^T = U^{-1}$

$\therefore \quad V \cdot V^T = I \rightarrow V^T = V^{-1}$

Σ is diagonal matrix.

$$\boxed{A^{-1} = V \Sigma^{-1} U^T}$$

$$\Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 20 \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} 1/1 & 0 \\ 0 & 1/20 \end{bmatrix}$$

(2) PCA → dimension reduction.