Calculating a covariance matrix involves several steps. Let's break it down with a simple example.

### Step 1: Collect Data

Assume we have a dataset with two variables, $X$ and $Y$. Here's a small dataset:

| Observation | $X$ | $Y$ |
|---|---|---|
| 1 | 2 | 3 |
| 2 | 4 | 5 |
| 3 | 6 | 7 |
| 4 | 8 | 9 |

### Step 2: Calculate the Means

Calculate the mean of each variable.

$$\text{Mean of } X = \frac{2+4+6+8}{4} = 5$$

$$\text{Mean of } Y = \frac{3+5+7+9}{4} = 6$$

### Step 3: Calculate Deviations from the Mean

For each observation, calculate the deviation from the mean for $X$ and $Y$.

| Observation | $X$ | $Y$ | $X - \text{Mean of } X$ | $Y - \text{Mean of } Y$ |
|---|---|---|---|---|
| 1 | 2 | 3 | $2 - 5 = -3$ | $3 - 6 = -3$ |
| 2 | 4 | 5 | $4 - 5 = -1$ | $5 - 6 = -1$ |
| 3 | 6 | 7 | $6 - 5 = 1$ | $7 - 6 = 1$ |
| 4 | 8 | 9 | $8 - 5 = 3$ | $9 - 6 = 3$ |

### Step 4: Calculate the Covariance

Now, we can calculate the covariance for $X$ and $Y$.

$$\text{Cov}(X, Y) = \frac{1}{n-1}\sum(X_i - \bar{X})(Y_i - \bar{Y})$$

Where $n$ is the number of observations.

1. Calculate the products of the deviations:

| Observation | $(X - \bar{X})(Y - \bar{Y})$ |
|---|---|
| 1 | $(-3) \cdot (-3) = 9$ |
| 2 | $(-1) \cdot (-1) = 1$ |
| 3 | $1 \cdot 1 = 1$ |
| 4 | $3 \cdot 3 = 9$ |

2. Sum the products:

$$\sum(X_i - \bar{X})(Y_i - \bar{Y}) = 9 + 1 + 1 + 9 = 20$$

3. Divide by $n - 1$:

$$\text{Cov}(X, Y) = \frac{20}{4-1} = \frac{20}{3} \approx 6.67$$

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### Step 5: Calculate Variances

Now, calculate the variances for $X$ and $Y$:

**Variance of $X$:**

$$\text{Var}(X) = \frac{1}{n-1}\sum(X_i - \bar{X})^2$$

| Observation | $(X - \bar{X})^2$ |
|---|---|
| 1 | $(-3)^2 = 9$ |
| 2 | $(-1)^2 = 1$ |
| 3 | $1^2 = 1$ |
| 4 | $3^2 = 9$ |

$$\sum(X_i - \bar{X})^2 = 9 + 1 + 1 + 9 = 20$$

$$\text{Var}(X) = \frac{20}{3} \approx 6.67$$

Variance of $Y$:

$$\text{Var}(Y) = \frac{1}{n-1}\sum(Y_i - \bar{Y})^2$$

| Observation | $(Y - \bar{Y})^2$ |
|---|---|
| 1 | $(-3)^2 = 9$ |
| 2 | $(-1)^2 = 1$ |
| 3 | $1^2 = 1$ |
| 4 | $3^2 = 9$ |

$$\sum(Y_i - \bar{Y})^2 = 9 + 1 + 1 + 9 = 20$$

$$\text{Var}(Y) = \frac{20}{3} \approx 6.67$$

### Step 6: Construct the Covariance Matrix

The covariance matrix is structured as follows:

$$\begin{bmatrix} \text{Var}(X) & \text{Cov}(X, Y) \\ \text{Cov}(Y, X) & \text{Var}(Y) \end{bmatrix}$$

Substituting our results:

$$\text{Covariance Matrix} = \begin{bmatrix} \frac{20}{3} & \frac{20}{3} \\ \frac{20}{3} & \frac{20}{3} \end{bmatrix} \approx \begin{bmatrix} 6.67 & 6.67 \\ 6.67 & 6.67 \end{bmatrix}$$

### Conclusion

The covariance matrix for the dataset is:

$$\begin{bmatrix} 6.67 & 6.67 \\ 6.67 & 6.67 \end{bmatrix}$$

This matrix indicates the variance of each variable along the diagonal and the covariance between the variables off the diagonal.