Subject Name: Statistical Foundation for Machine LearningMachine Learning, End-Sem, MM:40 Marks, 3 hrs exam

Question 1 : [9 Points] Answer the following :

a) **[5 Points]:** An unbiased dice is rolled and for each number on the dice a bag is chosen:

Numbers on the Dice	Bag chosen
1	Bag A
2 or 3	Bag B
4 or 5 or 6	Bag C

Bag A contains 3 white ball and 2 black ball, bag B contains 3 white ball and 4 black ball and bag C contains 4 white ball and 5 black ball. Dice is rolled and bag is chosen, if a white ball is chosen find the probability that it is chosen from bag B.

b) [**4 Points**] If A and {B,C} are conditionally independent given D (e.g p(A,B,C|D) = p(A|D)p(B,C|D)), are A and B conditionally independent given D? Hint: you can use the fact $P(X) = \sum_{Y} P(X,Y)$.

Question 2 [5 Marks] Consider a single sigmoid threshold unit with three inputs, x1, x2, and x3.

y =g(w0+w1x1 +w2x2 +w3x3) where g(z) = 1 / 1 +exp(−z)

We input values of either 0 or 1 for each of these inputs. Assign values to weights w0, w1, w2 and w3 so that the output of the sigmoid unit is greater than 0.5 if an only if (x1 AND x2) OR x3.

Question 3 : [7 Marks] Decompose a Matrix

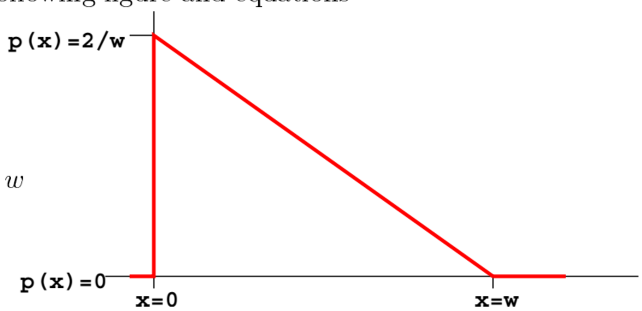
$$A = \begin{bmatrix} 4 & 0 \\ 3 & -5 \end{bmatrix}$$

via SVD and prove it is correctly decomposed into its orthogonal matrices and diagonal matrix.

Question 4 : [19 Marks]

Consider the PDF shown in the following figure and equations

$$\begin{aligned} p(x) &= 0 && \text{if } x < 0 \\ p(x) &= \frac{2}{w} - \frac{2x}{w^2} && \text{if } 0 \le x \le w \\ p(x) &= 0 && \text{if } w < x \end{aligned}$$



(a) *(3 points)* Which one of the following expressions is true? (note—exactly one is true). Write your answer (a choice between 1 to 12) here:

(1) $E[X] = \int_{x=-\infty}^{\infty} (\frac{2}{w} - \frac{2x}{w^2})dx$ (2) $E[X] = \int_{x=-\infty}^{\infty} x(\frac{2}{w} - \frac{2x}{w^2})dx$ (3) $E[X] = \int_{x=-\infty}^{\infty} w(\frac{2}{w} - \frac{2x}{w^2})dx$

(4) $E[X] = \int_{x=0}^{w} (\frac{2}{w} - \frac{2x}{w^2})dx$ (5) $E[X] = \int_{x=0}^{w} x(\frac{2}{w} - \frac{2x}{w^2})dx$ (6) $E[X] = \int_{x=0}^{w} w(\frac{2}{w} - \frac{2x}{w^2})dx$

(7) $E[X] = \int_{w=-\infty}^{\infty} (\frac{2}{w} - \frac{2x}{w^2})dw$ (8) $E[X] = \int_{w=-\infty}^{\infty} x(\frac{2}{w} - \frac{2x}{w^2})dw$ (9) $E[X] = \int_{w=-\infty}^{\infty} w(\frac{2}{w} - \frac{2x}{w^2})dw$

(10) $E[X] = \int_{w=0}^{x} (\frac{2}{w} - \frac{2x}{w^2})dw$ (11) $E[X] = \int_{w=0}^{x} x(\frac{2}{w} - \frac{2x}{w^2})dw$ (12) $E[X] = \int_{w=0}^{x} w(\frac{2}{w} - \frac{2x}{w^2})dw$

(b) *(4 points)* What is $P(x = 1|w = 2)$?

(c) *(4 points)* What is $p(x = 1|w = 2)$?

(d) *(4 points)* What is $p(x = 0|w = 1)$?

(e) *(4 points)* Suppose you don't know the value of w but you observe one sample from the distribution: $x = 3$. What is the maximum likelihood estimate of w?