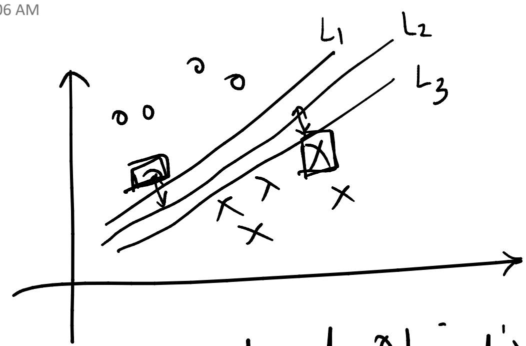
SVM 1

Sunday, June 29, 2025 8:06 AM



SVM

support vector

SVM Mathematical Objective function

$$\text{minize} \quad \frac{1}{2} \|w\|^2$$

Subject to the constraint

$$y_i (w^T x_i + b) \geq 1 \quad \text{for} \quad i = 1, 2 \ldots m$$

$y_i \rightarrow$ class labels $+1/-1$

$x_i \rightarrow$ feature vector for ith training instance

$m \rightarrow$ total number of training instance.

Softmargin SVM

$$\text{minimize} \quad \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{m} \alpha_i$$

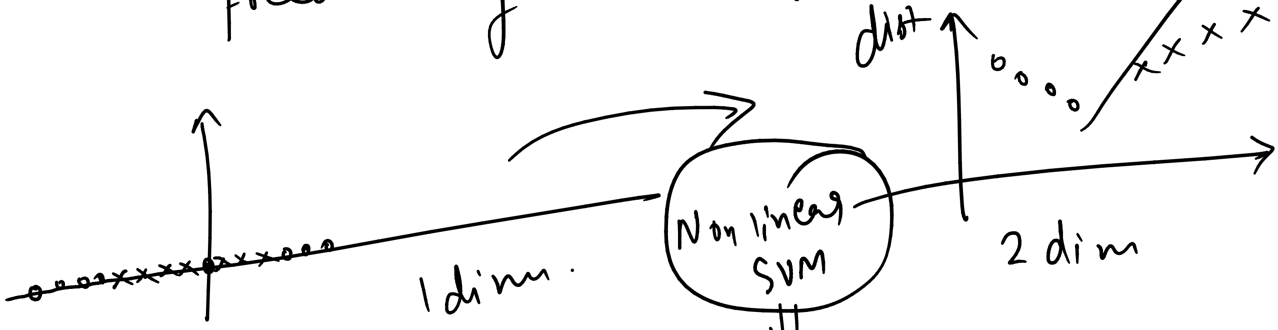
Subject to the Constraint

$$y_i (w^T x_i + b) \geq \underbrace{1 - \alpha_i} \quad \& \ \alpha_i \geq 0 \quad \text{for } i = 1, 2 \ldots m$$

C : regularization parameter that control trade off b/w margin maximation & penalty for margin

… variables that represent the degree of

α_i : Slack variables that represent the degree of freedom of the margin by each data point.



Kernel

Separability becomes linear but after non-linear transformation

Mathematical Expression for non-linear SVM

$$\begin{cases} \underset{\alpha}{\text{maximize}} \quad \sum_{i=1}^{m} \alpha_i - \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \alpha_j y_i y_j \underbrace{K(\underline{x_i}, \underline{x_j})} \\ \text{Subject to} \\ \sum_{i=1}^{m} \alpha_i y_i = 0 \end{cases}$$

$K(\underline{x_i}, \underline{x_j}) = (\underline{x_i} \cdot \underline{x_j})$

$\downarrow$

Kernel $\rightarrow$ Similarity b/w $\underline{x_i}$ & $\underline{x_j}$

Ex.

	x	r	y
✓ x_1	2	2	−1
✓ x_2	4	5	+1
✓ x_3	7	4	+1

$\underline{x_1} = (2, 2)$

$\underline{x_2} = (4, 5)$

$\underline{x_3} = (7, 4)$

$(2,2) \cdot (2,2)$

$-\alpha_1 + \alpha_2 + \alpha_3 = 0 \Rightarrow \boxed{\alpha_1 = \alpha_2 + \alpha_3}$ ✓

$(2,2)\cdot(2,2)$

$4+4=8$

$$-\alpha_1+\alpha_2+\alpha_3=0 \Rightarrow \boxed{\alpha_1=\alpha_2+\alpha_3}$$

$$\phi(\alpha)=\sum_{i=1}^{3}\alpha_i-\frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\alpha_i\alpha_j y_i y_j\underbrace{(\underline{x}_i\cdot\underline{x}_j)}$$

$(\underline{x}_1\cdot\underline{x}_1)=8$ $\quad(x_1\cdot x_2)=18$ $\quad(x_1\cdot x_3)=22$

$(x_2\cdot x_1)=18$ $\quad(\underline{x}_2\cdot\underline{x}_2)=41$ $\quad(x_2\cdot x_3)=48$

$(x_3\cdot x_1)=22$ $\quad(x_3\cdot x_2)=48$ $\quad(x_3\cdot x_3)=65$

$(\alpha_2+\alpha_3)$

$$\phi(\alpha)=(\alpha_1+\alpha_2+\alpha_3)-\frac{1}{2}\Big[8\alpha_1^2+41\alpha_2^2+65\alpha_3^2-36\alpha_1\alpha_2$$
$$-44\alpha_1\alpha_3+96\alpha_2\alpha_3\Big]$$

$$=2(\alpha_2+\alpha_3)-\frac{1}{2}\Big[8(\alpha_2+\alpha_3)^2+\ldots\ldots\Big]$$

$$\boxed{\phi(\alpha)=\underbrace{2(\alpha_2+\alpha_3)}-\frac{1}{2}\Big[13\alpha_2^2+29\alpha_3^2+32\alpha_2\alpha_3\Big]}$$

$$\frac{d\phi(\alpha)}{d\alpha_2}=0 \qquad\Bigg|\qquad \frac{d\phi(\alpha)}{d\alpha_3}=0$$

$$13\alpha_2+16\alpha_3=2 \quad — (1)$$

$$13\alpha_2 + 16\alpha_3 = 2 \quad \text{— (1)}$$

$$16\alpha_2 + 29\alpha_3 = 2$$

$$\alpha_2 = \frac{26}{121} \qquad \alpha_3 = \frac{-6}{121} \qquad \alpha_1 = \frac{20}{121}$$

$$\underline{w} = \sum_{i=1}^{m} \alpha_i y_i \underline{x}_i = \frac{20}{121}(-1)(2,2) + \frac{26}{121}(1)(4,5) - \frac{6}{121}(1)(7,4)$$

$$\left(-\frac{40}{121} + \frac{104}{121} - \frac{42}{121}\right)$$

Rough

$$= \left(\frac{2}{11}, \frac{6}{11}\right)$$

$$b = \frac{1}{2}\left[\min_{i:\,y_i=+1}(\underline{w}\cdot\underline{x}_i) + \max_{i:\,y_i=-1}(w\cdot\underline{x}_i)\right]$$

+1

−1

$$= \frac{1}{2}\left[\min\left((w\cdot x_2), (w\cdot x_3)\right) + \max(w\cdot x_1)\right]$$

$$= \frac{1}{2}\left[\min\left(\frac{38}{11}, \frac{38}{11}\right) + \max\left(\frac{16}{11}\right)\right]$$

$$b = \frac{1}{2}\left[\frac{38}{11} + \frac{16}{11}\right] = \frac{27}{11}$$

$$\begin{array}{cc} w & x_2 \\ \left(\frac{2}{11}, \frac{6}{11}\right) & (4, 5) \end{array}$$

$$\left(\frac{8}{11} + \frac{30}{11}\right) = \frac{38}{11}$$

$$\underline{x} = (u, r)$$

$$f(x) = \underline{w}\,\underline{x} - b$$

$$= \left(\frac{2}{11}, \frac{6}{11}\right)\cdot(u, r) - \frac{27}{11}$$

$$\boxed{f(x) = \frac{2}{11}u + \frac{6}{11}r - \frac{27}{11}}$$

on the hyperplane $f(x) = 0$

Hyperplane Expression

$$\boxed{\frac{2}{11}u + \frac{6}{11}r - \frac{27}{11} = 0}$$

$$\Rightarrow\ u + 3r - 27 = 0$$

$$u + 3r - \frac{27}{2} = 0$$

u r

(3, 3.5)