Practice Question

Q1 The random process $\mathbf{X}(t) = 2e^{-At} sin(\omega t + B)u(t)$ where $u(t)$ is the unit step function and random variables A and B are independent. A is distributed uniformly in (0,2) and B is distributed uniformly in $(-\pi, \pi)$. Verify whether the process is wide sense stationary?

Q2. Apply Gram-Schmidt orthogonalization to find the orthogonal vectors

v_1=[1, -1, 1]^T; v_2=[1, 0, 1]^T; v_3=[1,1, 2]^T

Q3. Let $X(t)$ be a random process with mean function $\mu_X(t)$ and autocorrelation function $R_X(s,t)$ (not necessary WSS). Let $Y(t)$ be given by $Y(t) = h(t) * X(t)$ where $h(t)$ is the impulse response of the system. Show that

a) $\mu_Y(t) = \mu_X(t) * h(t)$

b) $R_{XY}(t_1, t_2) = h(t_2) * R_X(t_1, t_2 = \int_{-\infty}^{\infty} h(\alpha) R_X(t_1, t_2 - \alpha) d\alpha$

Q4. Suppose that the chance of rain tomorrow depends on previous weather conditions only through whether or not it is raining today and not on past weather conditions. Suppose also that if it rains today, then it will rain tomorrow with probability 0.7; and if it does not rain today, then it will rain tomorrow with probability 0.4. If we say that the process is in state 0 when it rains and state 1 when it does not rain, then the above is a two-state Markov chain . Then calculate the probability that it will rain four days from today given that it is raining today.

Q5. Compute $||A||_\infty$, $||A||_1$ for the matrix A defined as

$$\begin{bmatrix} 100 & 300 & -50 \\ -30 & 20 & -70 \\ -100 & 50 & 10 \end{bmatrix}$$

Q6: Prove that n × n matrix A is diagonalizable if and only if it has n linearly independent eigenvectors. In this case A is similar to a matrix D whose diagonal elements are the eigenvalues of A.

Q7 Let u = (2, −1, 3) and v = (4, −1, 2) be vectors in $\mathcal{R}^3$. Find the orthogonal projection of u on v and the component of u orthogonal to v.

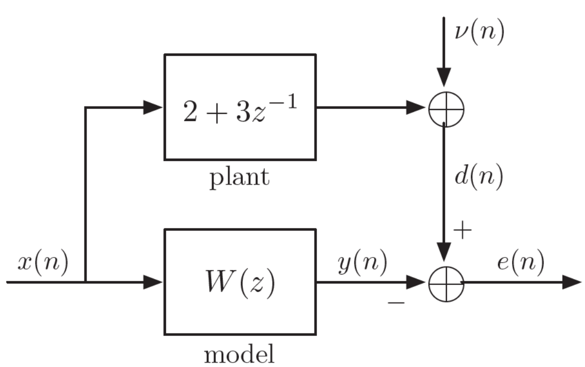
Q8. A box contains 3 coins;Two regular and one fake coin with heads on both sides $(P(H) = 1)$

1. Pick a coin at random and toss it. What is the probability that it will land with a head?
2. Pick a coin at random and toss it and get head. What is the probability that it is the two headed coin?

Q9. Let $X_1, X_2, \cdots$ be identically distributed random variables with mean μ, and variance σ^2. Let N be a random variable taking values in the non-negative integers and independent of the $X_{i's}$. Let $S = X_1 + X_2 + .. + X_N$

1. Show that E[S] = μE[N]
2. The variance of S can be written as var[S] = σ^2E [N] + σ^2 var[N]. Suppose now that the random variable N obeys the Poisson distribution with the parameter lambda. Then, compute this variance var[S].

Q10. Consider the modeling problem shown in Figure below. The plant is a two-tap filter with an additive noise, $v(n)$, added to its output. A two-tap Wiener filter with tap weights w_0 and w_1 is used to model the plant parameters. The same input is applied to both the plant and Wiener filter. The input, $x(n)$, is a stationary white process with variance of unity. The additive noise $v(n)$, is zero-mean and uncorrelated with $x(n)$, and its variance is σ_v^2 = 0.1. We want to compute the optimum values of w_0 and w_1, which minimizes E[e2(n)].



Q11. Show that in a Markov chain there is at least one recurrent class?

Q12. Calculate the row and column rank of the following matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 5 & -8 \\ 3 & 2 & -2 \\ 8 & 2 & 0 \end{bmatrix}$$

Q13. Can (3, -1, 4) be written as a linear combination of (1, -1, 0) , (0, 1, 1) and (3, -5, 2)?