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Q2) Prove that for a real auto correlation matrix R all the eigen values must be real and the eigenvectors corresponding to distinct eigen values of R are mutually orthogonal.

Soln

Auto correlation matrix

$$R = E[\underline{x}\,\underline{x}^H] \quad - (1)$$

we know,

→ Auto correlation matrix is always Hermition

$$R = R^H \quad - (2)$$

→ Eigen vector cannot be zero

(1) To prove real auto correlation matrix R, all the eigen values must be real.

→ Eigen value decomposition of auto correlation matrix

$$\underline{R}\,\underline{x} = \lambda\,\underline{x} \quad - (3)$$

| R = auto corretion
| x = eigen vector
| λ = eigen value.

$$(\underline{R}\,\underline{x})^H = (\lambda\,\underline{x})^H$$ | Take Hermition

$$\underline{x}^H \underline{R}^H = \lambda^* \underline{x}^H$$ | λ is scalar.

$$\underline{x}^H \underline{R} = \lambda^* \underline{x}^H$$ | from (1)

$$\underline{X}^H \underline{R} = \lambda^* \underline{X}^H$$

$$\underline{X}^H \underline{R}\,\underline{X} = \lambda^* \underline{X}^H \underline{X}$$ multiply by $\underline{X}$ both Side

$$\underline{X}^H \underline{\lambda}\,\underline{X} = \lambda^* \underline{X}^H \underline{X}$$ from 3

$$\lambda \underline{X}^H \underline{X} = \lambda^* \underline{X}^H \underline{X}$$ λ is Scalar eigen value

$$(\lambda - \lambda^*)\,\underline{X}^H \underline{X} = 0$$

Eigen vector cannot be zero

$$\therefore \quad \lambda - \lambda^* = 0$$

$$\lambda = \lambda^* \quad \text{---} \quad (4)$$

$\therefore$ all eigen value in ~~Rsap~~ auto correlation matrix ~~cannot be~~ are always real.

(2) Eigen vectors corresponding to distinct eigen value of R are mutually orthogonal.

~~Given~~ let X_1 and X_2 are eigen vectors and corresponding eigen values λ_1 and λ_2, $\underline{R}$ is auto correlation matrix

Given $\quad \lambda_1 \neq \lambda_2$

$R X_1 = \lambda_1 X_1 \quad - (5)$

$R X_2 = \lambda_2 X_2 \quad - (6)$

Apply Hermition on (5)

$(R X_1)^H = (\lambda_1 X_1)^H$

$X_1^H R^H = \lambda_1^* X_1^H$

$X_1^H R^H = \lambda_1 X_1^H$ | from (4)

$X_1^H R^H X_2 = \lambda_1 X_1^H X_2$ | multply X_2

$X_1^H R X_2 = \lambda_1 X_1^H X^2$ | $R = R^H$

$X_1^H \lambda_2 X_2 = \lambda_1 X_1^H X_2$

$\lambda_2 X_1^H X_2 = \lambda_1 \lambda_1^H X_2$

$(\lambda_2 - \lambda_1) X_1^H X_2 = 0$

$\lambda_1 \neq \lambda_2$, cannot be zero

$\therefore X_1^H X_2 = 0$

X_1 and X_2 are orthogonal to each other if, eigen values are not same.

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Q3) if X and Y are independent random variable, then prove that

$$H(XY) = H(X) + H(Y)$$

a) we know

Joint entropy.

$$H(X,Y) = \sum_{x,y} p(x,y) \log_2 \frac{1}{p(x,y)}$$

$$= -\sum_{x,y} P(x) \cdot P(y) \log_2 P(x) \cdot P(y)$$

$\because$ x and y are independent

$$p(x,y) = p(x) \, p(y)$$

$$= -\sum_{x,y} P(x) \cdot P(y) \left[\log_2 P(x) + \log_2 P(y)\right]$$

$$= -\sum_{x,y} P(x) \cdot P(y) \log_2 P(x) - \sum_{x,y} P(x) \cdot P(y) \log_2 P(y) \quad \text{— (1)}$$

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Now

$$H(x) = -\sum_{x} P(x) \log_2 P(x) \quad - ②$$

$$H(y) = -\sum_{y} P(y) \log_2 P(y) \quad - ③$$

$$H(x) + H(y) = -\left[\sum_{x} P(x) \log_2 P(x) + \sum_{y} P(y) \log_2 P(y) \right]$$

$$= -\sum_{y} P(y) \cdot \sum_{x} P(x) \log_2 P(x)$$

$$- \sum_{x} P(x) \sum_{y} P(y) \log_2 P(y)$$

— from ①

$$\because \sum_{y} P(y) = 1 \quad \text{and} \quad \sum_{x} P(x) = 1$$

$$= \cancel{-\sum_{x} P(x)} \sum_{y}$$

$$= -\sum_{x} P(x) \log_2 P(x) - \sum_{y} P(y) \log_2 P(y)$$

$$H(x,y) = H(x) + H(y) \quad \Big| \text{ from 2 and 3}$$

(b) $P_{X \neq Y}$ =

$$H(X,Y) = -\sum_{xy} P(x,y) \log_2 P(x,y)$$

$$= -\Big[\tfrac{1}{2}\log_2\tfrac{1}{2} + \tfrac{1}{16}\log_2\tfrac{1}{16} + \tfrac{1}{16}\log_2\tfrac{1}{16}$$

$$+ \tfrac{1}{16}\log_2\tfrac{1}{16} + \tfrac{1}{16}\log_2\tfrac{1}{16} + \tfrac{1}{16}\log_2\tfrac{1}{16}$$

$$+ \tfrac{1}{16}\log_2\tfrac{1}{16} + \tfrac{1}{16}\log_2\tfrac{1}{16} + \tfrac{1}{16}\log_2\tfrac{1}{16}$$

$$+ \tfrac{1}{16}\log_2\tfrac{1}{16}\Big]$$

$$= -\Big[-\tfrac{1}{2} - \overset{2}{\cancel{8}}\cdot\tfrac{1}{\cancel{4}}\Big]$$

$$= \Big[\tfrac{1}{2} + 2\Big] = 2.5 \text{ bits}$$

$$H(X) = -\sum_{x} P(x)\log_2 P(x)$$

=

$P_X(0) = \frac{4}{9}$ $P_X(1) = \frac{5}{9}$

$$-0.3219 - 0.848$$

$$0.444 \times 0.321 - 0.555 \times 0.848$$

$$= 0.14 + 0.65$$

=

$$P_{x=1} = \frac{1}{2} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = 0.5$$

$$P_{x=0} = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = 0.25$$

$$H(X) = -\left[0.5 \log_2 0.5 + 0.25 \log_2 0.25\right]$$

$$\cancel{= -[-1/2 - 0.\ldots}$$

$$= \cancel{0.811}$$

$$= -\left[0.5 \times (-1) + 0.25 \times (-2)\right]$$

$$= \left[0.5 + 0.25 \times 2\right]$$

$$= \left[0.5 + 0.5\right] = 1$$

$$H(y) = H(x,y) - H(x)$$

$$= 2.5 - 1$$

$$H(y) = 1$$

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Q1

$$P = \begin{bmatrix} 0.8 \\ 0.3 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$$\mu = 1$$

$$W(i+1) = W(n) + \mu[P - RW(n)]$$

$$= W(n) + P - \underline{R}\,\underline{W}(n)$$

$$W_{opt} = R^{-1} P$$

$$=$$

$$|R| = (1 \cdot 1 - (0.5)^2)$$
$$= 1 - 0.25$$
$$= 0.75$$

$$R^{-1} = \begin{bmatrix} \frac{1}{0.75} & -\frac{0.5}{0.75} \\ -\frac{0.5}{0.75} & \frac{1}{0.75} \end{bmatrix}$$

$$\begin{bmatrix} 1.333 & -0.666 \\ -0.666 & 1.333 \end{bmatrix}$$

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$$R^{-1}p = \begin{bmatrix} 1.333 & -0.666 \\ -0.666 & 1.333 \end{bmatrix} \cdot \begin{bmatrix} 0.8 \\ 0.3 \end{bmatrix}$$

$= (1.333 \times 0.8) + (-0.666 \times 0.3)$

$= 0.866$

$= (-0.666 \times 0.8) + (1.333 \times 0.3)$

$= -0.133$

$$R^{-1}p = \begin{bmatrix} 0.866 \\ -0.133 \end{bmatrix}$$