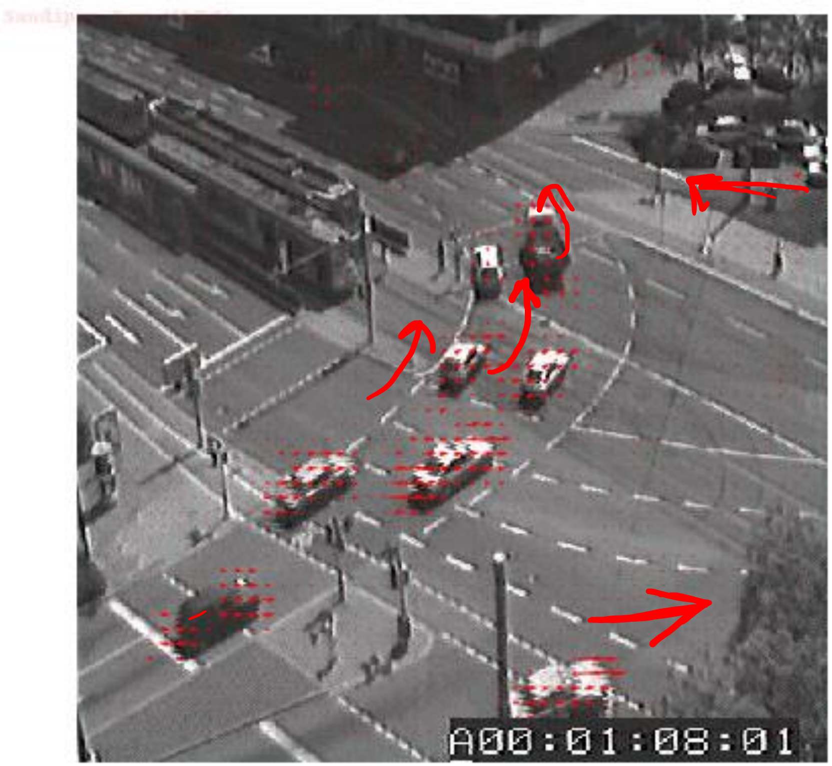
Optical Flow

Optical Flow

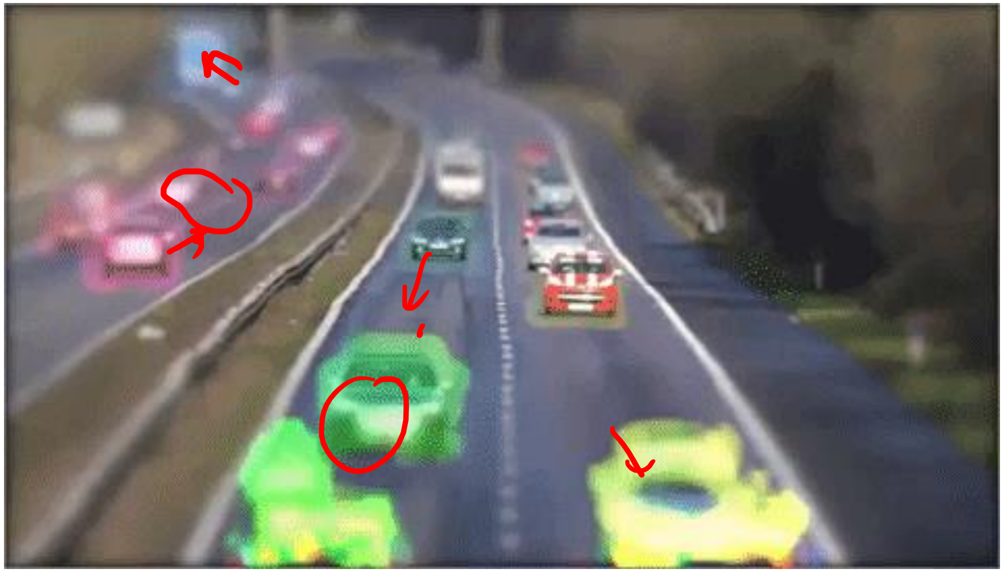
Optical Flow

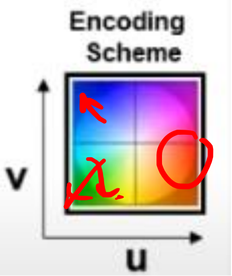
- Motion of objects in 3D induces motion in 2D plane-> optical flow
- We have seen static image and moving camera
- Here sequence of images taken at different time intervals. (from same camera)
- Can be used to compute 3D motion ., i.e., translation, rotation and 3D shape from motion
- Motion based Segmenation
- Alignment, speed up, slow down…

Optical Flow



Optical Flow



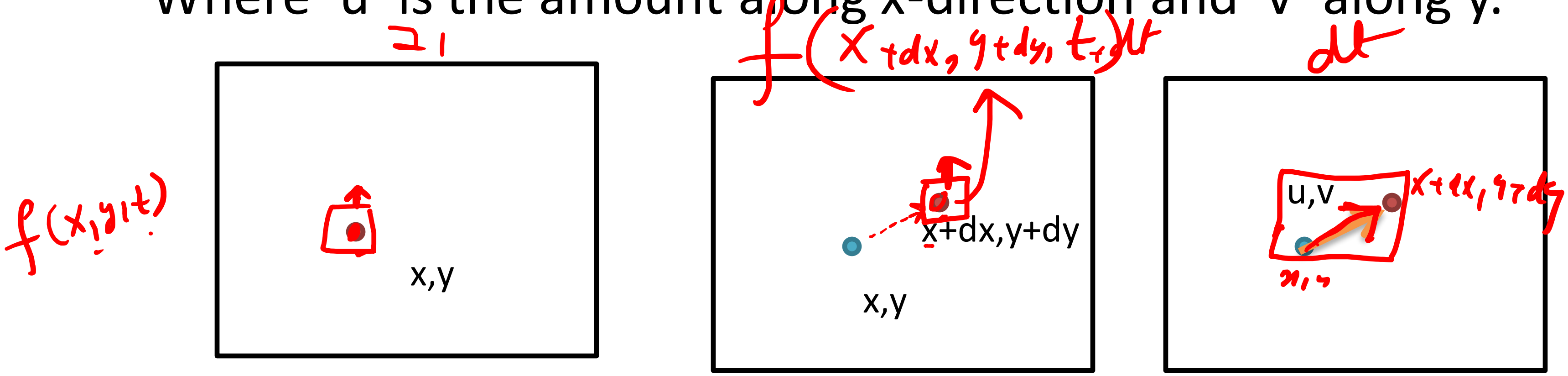


$I = f(x,y)$

Optical flow

- Let 3 D function $f(x,y,t)$ where x,y are spatial coordinates and t is time denote image sequence.
- Assume that with a small change dx, dy, dt in x,y,t, there is "no change in intensity."
- We know some pixel is moving along direction (u,v)

Where ‘u’ is the amount along x-direction and ‘v’ along y.



Time=t

Time=t+dt

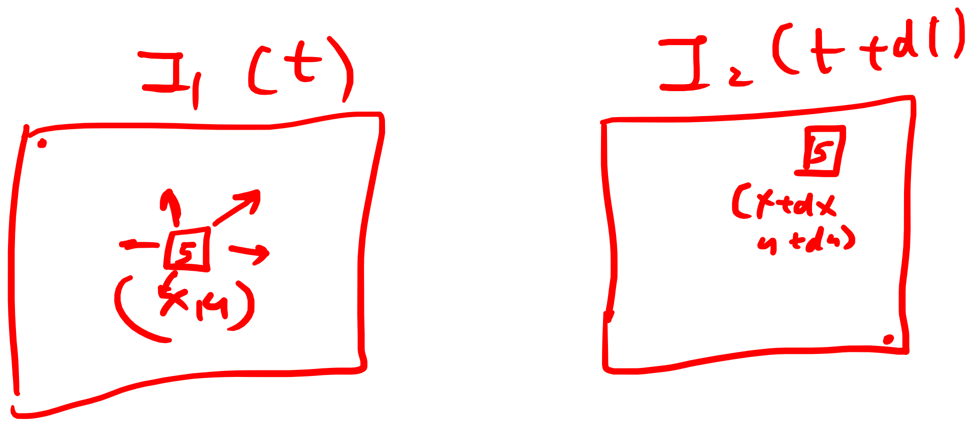
Time=t+dt

Optical Flow

(1) motion is small
(2) pixel intensity remains same.

(u,v)

- Let us assume that the brightness (intensity) does not changes when the point moves.
- Also assume that the motion is very small…
- i.e (u,v) are very small, typically less than 1 pixel….



Optical Flow Constraint

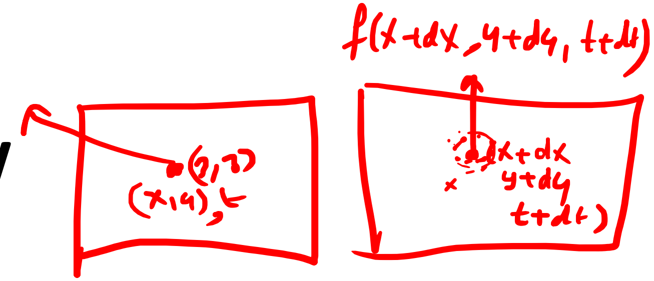
(u, v)

- We will learn about two constraints:
- 1. Brightness Constraint Assumption
- 2. Smoothness constraint Assumption

1. Brightness Constraint Assumption

Optical Flow

f(x,y,t)



- Let us assume that the brightness (intensity) does not changes when the point moves.
- Also assume that the motion is very small…
- i.e (u,v) are very small, typically less than 1 pixel…
- Then,

brightness constancy constraint…

$$\boxed{f(x,y,t) = f(x+dx, y+dy, t+dt)}$$

  – Where u –> dx, amount in which point has moved along x-direction,
  – y –> dy, amount in which point has moved along y-direction

Optical Flow Constraint

- Brightness Consistency Assumption

$$f(x, y, t) - f(x + dx, y + dy, t + dt) = 0$$

- i.e., if we look at a point (x,y) in one frame, then *exactly in the same point, but just little bit around it* in the next sequence frame, the intensity will not change

Horn and Schunck Optical Flow

- B. K. Horn and B. G. Schunck. Determining optical flow. *Artificial Intelligence*, 17(1-3):185–203, Aug. 1981.

Horn and Schunck Optical Flow

RHS.

$f(x,y,t) = f(x+dx, y+dy, t+dt)$

Assumed:

$$f(x,y,t) = \left(f(x+dx, y+dy, t+dt)\right)$$

- Finding Taylors series expansion of the right term at point $a = (x, y, t)$.

- Taylor Series:

$f(x) = f(a) + (x-a) f_x$.

$\boxed{\text{f(x)}}$ Can be represented at point 'a' in terms of its derivatives

$$f(x) = \left(f(a) + (x-a) f_x + \frac{(x-a)^2}{2!} f_{xx} + \frac{(x-a)^3}{3!} f_{xxx} + \ldots\right)$$

Horn and Schunck Optical Flow

$$f(x) = f(a) + f_x(x-a)$$

Assumed:

$$f(x,y,t) = f(\underbrace{x+dx}_{x},\ \underbrace{y+dy}_{y},\ \underbrace{t+dt}_{t})$$

- Finding Taylors series expansion of the right term at point $a = (x, y, t)$
- $\boxed{f(x+dx, y+dy, t+dt)} = \underbrace{f(x, y, t)}_{f(a)} + \frac{\partial f}{\partial x}(\cancel{x} + dx - \cancel{x}) + \frac{\partial f}{\partial y}(\cancel{y} + dy - \cancel{y}) + \frac{\partial f}{\partial t}(\cancel{t} + dt - \cancel{t})$

$$f(x+dx, y+dy, t+dt) = \cancel{f(x,y,t)} + \frac{\partial f}{\partial x}(dx) + \frac{\partial f}{\partial y}(dy) + \frac{\partial f}{\partial t}(dt)$$

$$\boxed{\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} \cdot dy + \frac{\partial f}{\partial t} dt = 0}$$

Horn and Schunck Optical Flow

Assumed:

$$f(x, y, t) = f(x + dx, y + dy, t + dt)$$

- Finding Taylors series expansion of the right term at point $a = (x, y, t)$
- $f(x + dx, y + dy, t + dt)$=$f(x, y, t) + \frac{\partial f}{\partial x}(x + dx - x)$+$\frac{\partial f}{\partial y}(y + dy - y) + \frac{\partial f}{\partial t}(t + dt - t)$
- $f(x, y, t) = f(x, y, t) + \frac{\partial f}{\partial x}\mathrm{dx}$+ $\frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt$

Horn and Schunck Optical Flow

- $f(x,y,t) = f(x,y,t) + \frac{\partial f}{\partial x}\mathrm{dx} + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt$
- $\boxed{0 = \frac{\partial f}{\partial x}\mathrm{dx} + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial t}dt}$

$\frac{\partial f}{\partial x} = f_x$ $\qquad$ $\frac{\partial f}{\partial t} = f_t$

- $\boxed{0 = f_x \mathrm{dx}^{u} + f_y \mathrm{dy}^{v} + f_t \mathrm{dt}}$ — basic optical flow eq.

$\frac{\partial f}{\partial y} = f_y$

Dividing by dt throughout

- $f_x \underbrace{\frac{dx}{dt}}_{u} + f_y \underbrace{\frac{dy}{dt}}_{v} + f_t = 0$

$f_x = \frac{\partial f}{\partial x}$, $f_y = \frac{\partial f}{\partial y}$, and $f_t = \frac{\partial f}{\partial t}$.

$$\boxed{f_x u + f_y v + f_t = 0}$$

Horn and Schunck Optical Flow

$\Rightarrow u$ $\Rightarrow v$

- $f_x \dfrac{dx}{dt} + f_y \dfrac{dy}{dt} + f_t = 0$
- Let $\dfrac{dx}{dt} = u$ => change along x-direction in dt
- Let $\dfrac{dy}{dt} = v$ => change along y-direction in dt

$f_x = I_x$ $I_t = I_t$

$f_y = I_y$

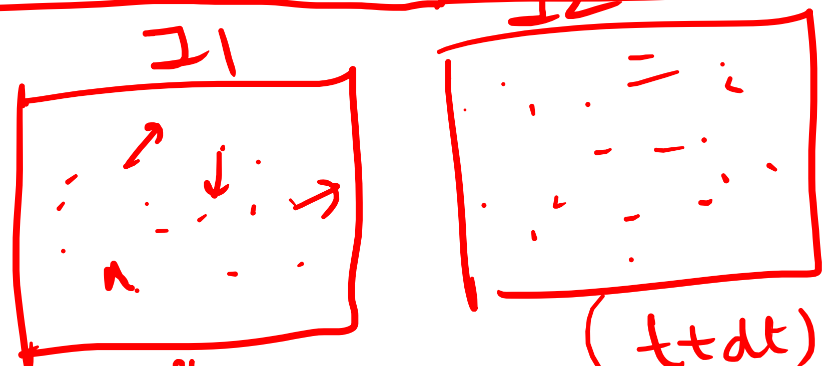
$f_x = ✓$

$f_y = ✓$

$f_t = ✓$

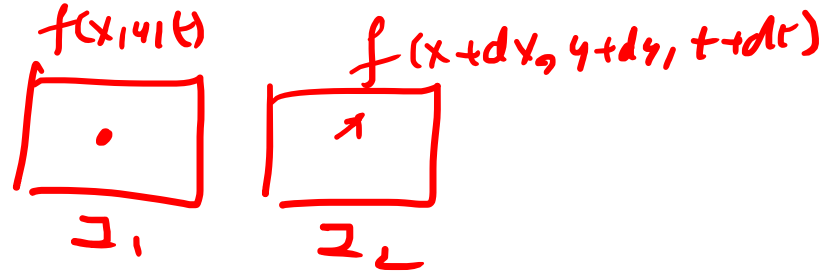
estimate (u,v)? for every pixel in the image

I_1 I_2



t $(t+dt)$

Horn and Schunck Optical Flow



- $f_x \frac{dx}{dt} + f_y \frac{dy}{dt} + f_t = 0$
- Let $\frac{dx}{dt} = u$ => change along x-direction in dt
- Let $\frac{dy}{dt} = v$ => change along y-direction in dt
- Then……

$$\boxed{f_x \overset{?}{u} + f_y \overset{?}{v} + f_t = 0}$$

↳ basic optical flow eq derived using brightness constancy assumption.

Horn and Schunck Optical Flow

- The partial derivatives of the spatiotemporal function *I* are approximated using finite differences between the two given images. That is, at pixel *(x,y)*

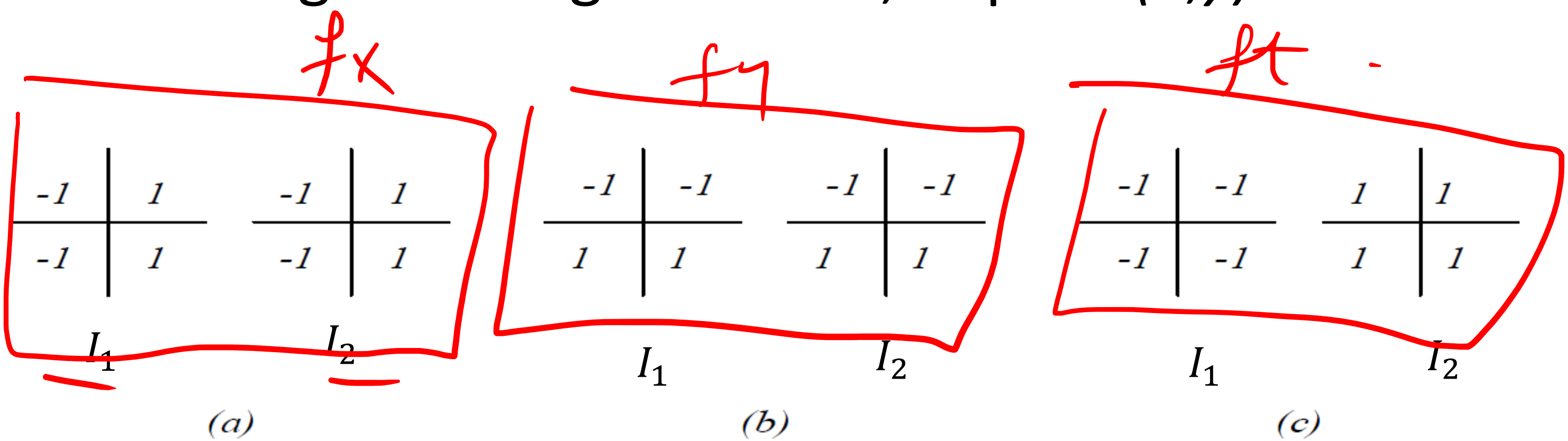
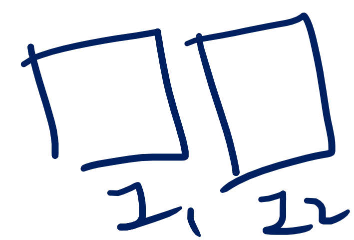


Figure 5.1: Masks for computing spatial and temporal derivatives. Note that the center of mask is at the lowere right pixel. (a) Masks for f_x, (b) Masks for f_y, and (c) Masks for f_t.

Horn and Schunck Optical Flow

- The partial derivatives of the spatiotemporal function *I* are approximated using finite differences between the two given images. That is, at pixel *(x,y)*

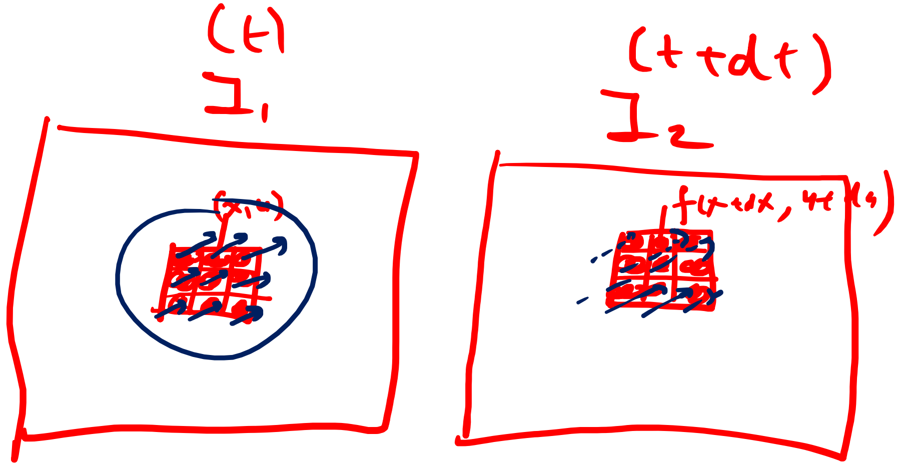


$$f_x = \frac{\partial I}{\partial x} \approx \frac{1}{4}\big(I_1(x+1,y) - I_1(x,y) + I_1(x+1,y+1) - I_1(x,y+1)$$
$$+ I_2(x+1,y) - I_2(x,y) + I_2(x+1,y+1) - I_2(x,y+1)\big)$$

$$f_y = \frac{\partial I}{\partial y} \approx \frac{1}{4}\big(I_1(x,y+1) - I_1(x,y) + I_1(x+1,y+1) - I_1(x+1,y)$$
$$+ I_2(x,y+1) - I_2(x,y) + I_2(x+1,y+1) - I_2(x+1,y)\big)$$

$$f_t = \frac{\partial I}{\partial t} \approx \frac{1}{4}\big(I_2(x,y) - I_1(x,y) + I_2(x+1,y) - I_1(x+1,y)$$
$$+ I_2(x,y+1) - I_1(x,y+1) + I_2(x+1,y+1) - I_1(x+1,y+1)\big)$$

$f(x,y,t) = f(x+dx, y+dy, t+dt)$



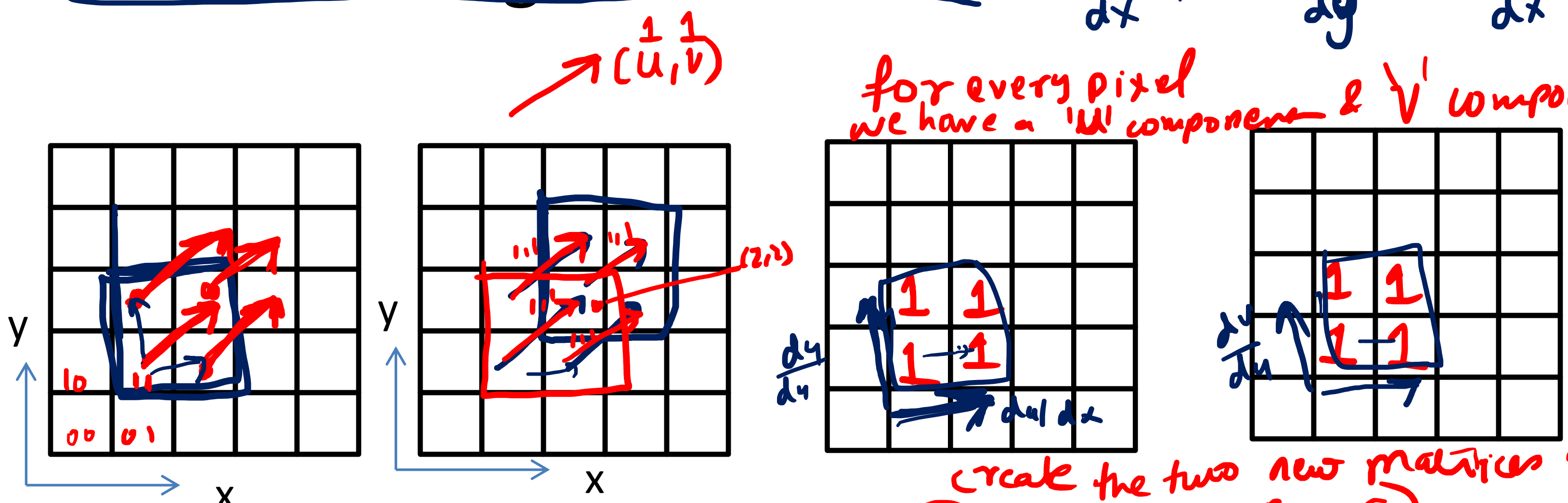
f_x
f_y
f_t

2. Smoothness Constraint Asumption

Horn and Schunck Optical Flow

- Remember we said the change $\frac{dx}{dt}$->u and $\frac{dy}{dt}$->v is very small.
- Also possible is that nearby pixels will tend to have flow along same direction $\frac{du}{dx} \simeq 0$ $\frac{du}{dy} \simeq 0$ $\frac{dv}{dx} = 0$ $\frac{dv}{dy} = 0$

for every pixel we have a 'u' component & 'v' component



I(x,y,t)

I(x,y,t+dt)

create the two new matrices of 'u' same size as that of image 'v'

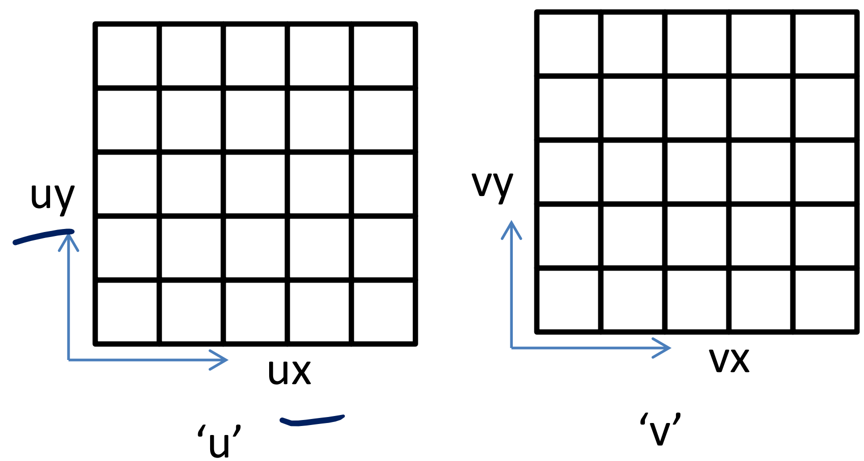
Horn and Schunck Optical Flow

- Horn and Schunck phrased this constraint by requiring that the ***gradient magnitude of the flow*** field should be small:

du/dx du/dy dv/dx dv/dy

(except for edge pixels)

$$(u,v) = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \approx 0$$



Horn and Schunck Optical Flow

- Remember we said the change dx->u and dy->v is very small.
- Also possible is that nearby pixels will tend to have flow along same direction
- Then the error term e_s

$$e_s = (u_x^2 + u_y^2 + v_x^2 + v_y^2)$$

$u_x = \frac{du}{dx}$ $u_y = \frac{du}{dy}$

$v_x = \frac{dv}{dx}$ $v_y = \frac{dv}{dy}$

1. $\left(f_x u + f_y v + f_t\right)^2 = 0$

2. $\left(u_x^2 + u_y^2 + v_x^2 + v_y^2 = 0.\right.$

3. Merging the two constraints

Horn and Schunck Optical Flow

- We have derived that:
- $f(x, y, t) - f(x + dx, y + dy, t + dt) =$ $f_x u + f_y v + f_t$

- 1. $e_a = (f_x u + f_y v + f_t)^2$ (Error from brightnes)
- 2. $e_s = (u_x^2 + u_y^2 + v_x^2 + v_y^2)$ (Error from smootheness)

Horn and Schunck Optical Flow

= for every pixel in the image

- Estimate u,v, such that the error function E is mininized:

$$\boxed{E(x,y) = \underbrace{(f_x u + f_y v + f_t)^2}_{E_d} + \underbrace{\lambda(u_x{}^2 + u_y{}^2 + v_x{}^2 + v_y{}^2)}_{E_s}}$$

- The above term is summed over all the pixels
- For correct optical flow the first term should be close to zero
- Also the second term should also be very less as nearby pixels will tend to have same flow.
- Error can be positive or negative that is why squared error is taken

Horn and Schunck Optical Flow

- Estimate u,v , such that the error function E is mininized:

$$E(x,y) = (f_x u + f_y v + f_t)^2 + \lambda({u_x}^2 + {u_y}^2 + {v_x}^2 + {v_y}^2)$$

- where λ is a parameter that specifies the influence of the smoothness term, also known as a **regularization** parameter.
- The larger the value of λ, the smoother the Optical flow field.

Horn and Schunck Optical Flow

overall E(xu) summed over all pixel is minized.

Ea   Es

$$E(x,y)=\iint\{(f_x u+f_y v+f_t)^2+\lambda(u_x^2+u_y^2+v_x^2+v_y^2)\}dxdy$$

??

ok.??

Brightness constancy   Smoothness constraint

- Differentiating $\boxed{E(x,y)}$ wrt to u,v and equating it to zero we get

$$\frac{\partial E}{\partial u}=\left[(f_x u+f_y v+f_t)f_x+\lambda(\overset{u_x''}{u_{xx}}+\overset{u_y''}{u_{yy}})\right]=0,$$

$$\frac{\partial E}{\partial v}=(f_x u+f_y v+f_t)f_y+\lambda(\overset{v_x''}{v_{xx}}+\overset{v_y''}{v_{yy}})=0.$$

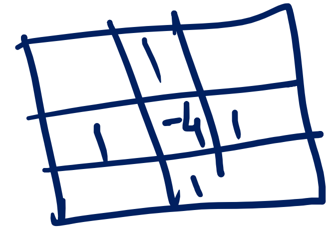
Horn and Schunck Optical Flow

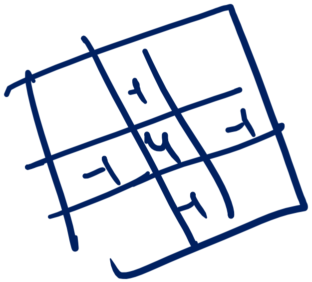
- Let $\Delta^2 u = u_{xx} + u_{yy}$, and $\Delta^2 v = v_{xx} + v_{yy}$

$$(f_x u + f_y v + f_t) f_x + \lambda(\Delta^2 u) = 0,$$
$$(f_x u + f_y v + f_t) f_y + \lambda(\Delta^2 v) = 0.$$

Horn and Schunck Optical Flow

- Let $\Delta^2 u = u_{xx} + u_{yy}$, and $\Delta^2 v = v_{xx} + v_{yy}$





$$(f_x u + f_y v + f_t) f_x + \lambda(\boxed{\Delta^2 u}) = 0,$$
$$(f_x u + f_y v + f_t) f_y + \lambda(\Delta^2 v) = 0.$$

$$\begin{matrix} 0 & -\frac{1}{4} & 0 \\ -\frac{1}{4} & 1 & -\frac{1}{4} \\ 0 & -\frac{1}{4} & 0 \end{matrix}$$

- Also let $\Delta^2 u = u - u_{av}$ where u_{av} is the average of optical flow over u component for 4 nearest neigbor pixels, similarly $\Delta^2 v = v - v_{av}$

Horn and Schunck Optical Flow

- Let $\Delta^2 u = u_{xx} + u_{yy}$, and $\Delta^2 v = v_{xx} + v_{yy}$

$$(f_x u + f_y v + f_t) f_x + \lambda(\Delta^2 u) = 0,$$
$$(f_x u + f_y v + f_t) f_y + \lambda(\Delta^2 v) = 0.$$

- Also let $\Delta^2 u = u - u_{av}$, where u_{av} is the average of optical flow over u component for 4 nearest neigbor pixels, similarly $\Delta^2 v = v - v_{av}$
- Then

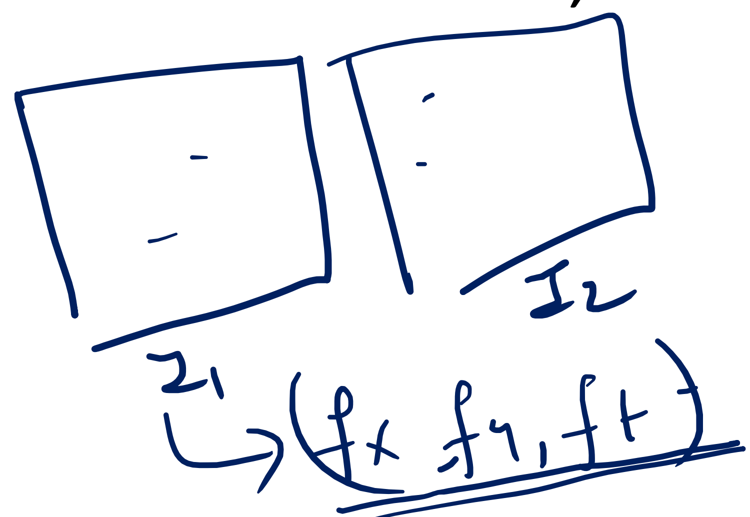
$$(f_x u + f_y v + f_t) f_x + \lambda(u - u_{av}) = 0,$$
$$(f_x u + f_y v + f_t) f_y + \lambda(v - v_{av}) = 0.$$

Horn and Schunck Optical Flow

- The two equations can now be solved for u,v as

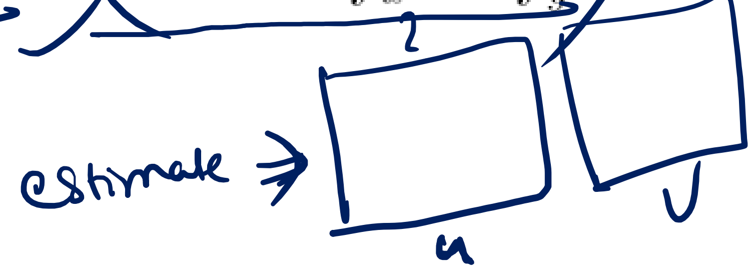
$$\boxed{u = u_{av} - f_x \frac{P}{D},}$$

$$\boxed{v = v_{av} - f_y \frac{P}{D},}$$



- where

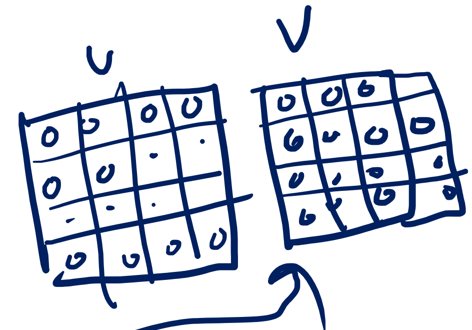
$$P = f_x u_{av} + f_y v_{av} + f_t, \text{ and } D = \lambda + f_x{}^2 + f_y{}^2$$



Horn and Schunck Optical Flow

- We know to compute f_x, f_y, f_t,
- Intialize u,v to zero
- Compute U_{av}, $v_{av.}$ ← 4 Nav → 4 NgV
- Iteratively approximate 'u' and 'v' now using previous set of equations
- For entire image pixels
- And minimize the error

K=0



$$u = u_{av} - f_x \frac{P}{D},$$

$$v = v_{av} - f_y \frac{P}{D},$$

$E = E_d + E_s$

Horn and Schunck Optical Flow

- The iterative algorithm can be given as

1. $k = 0$.
2. Initialize u^k and v^k to zero.
3. Until some error measure is satisfied, do:

$$u^k = u_{av}^{k-1} - f_x \frac{P}{D},$$

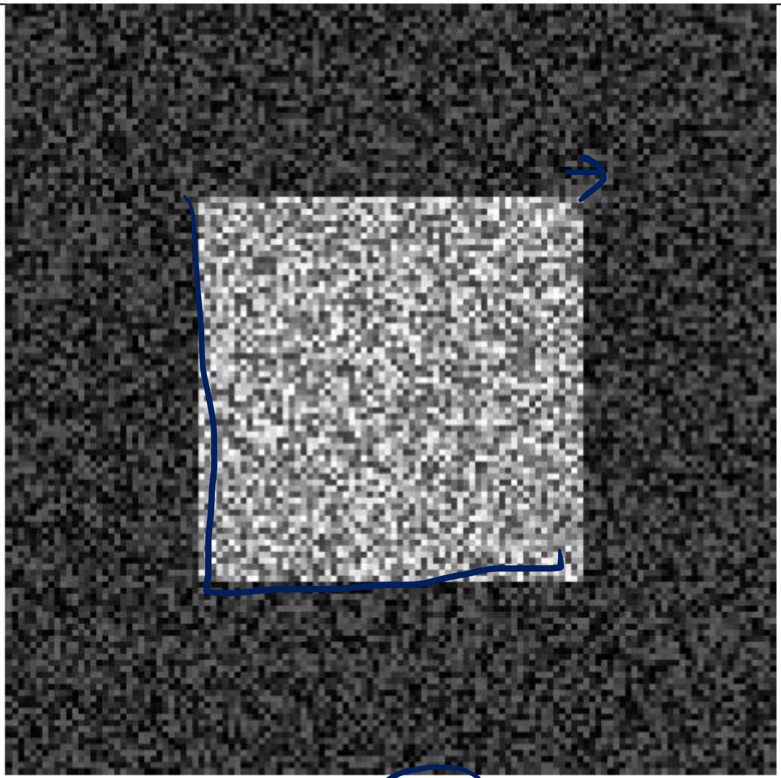
$$v^k = v_{av}^{k-1} - f_y \frac{P}{D}.$$

- Solve for E(x,y) and try to minimize

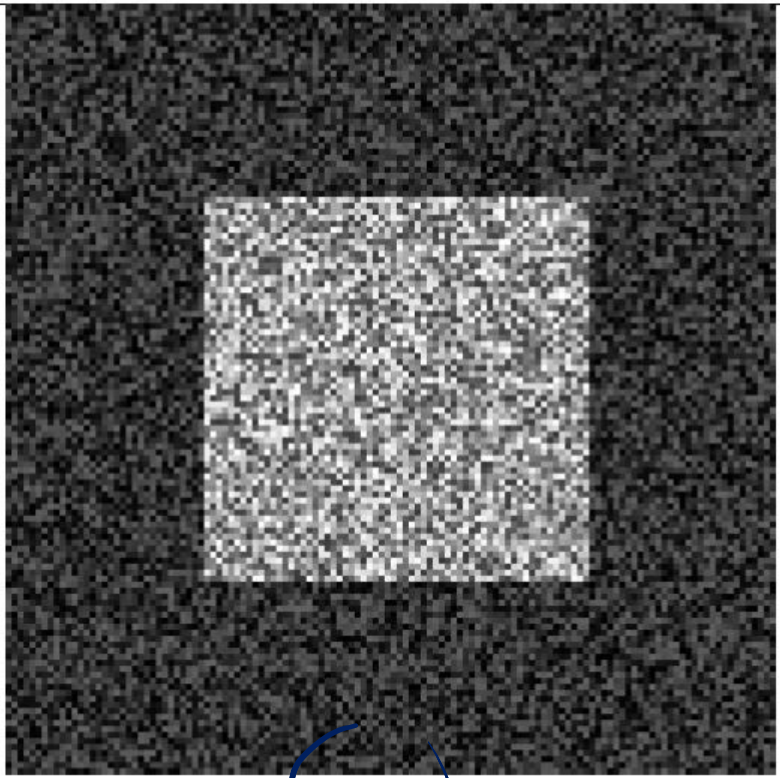
$$E(x,y) = \iint \{(f_x u + f_y v + f_t)^2 + \lambda(u_x^2 + u_y^2 + v_x^2 + v_y^2)\} dxdy$$

Horn and Schunck Optical Flow Example

- Object displaced right in frame 2

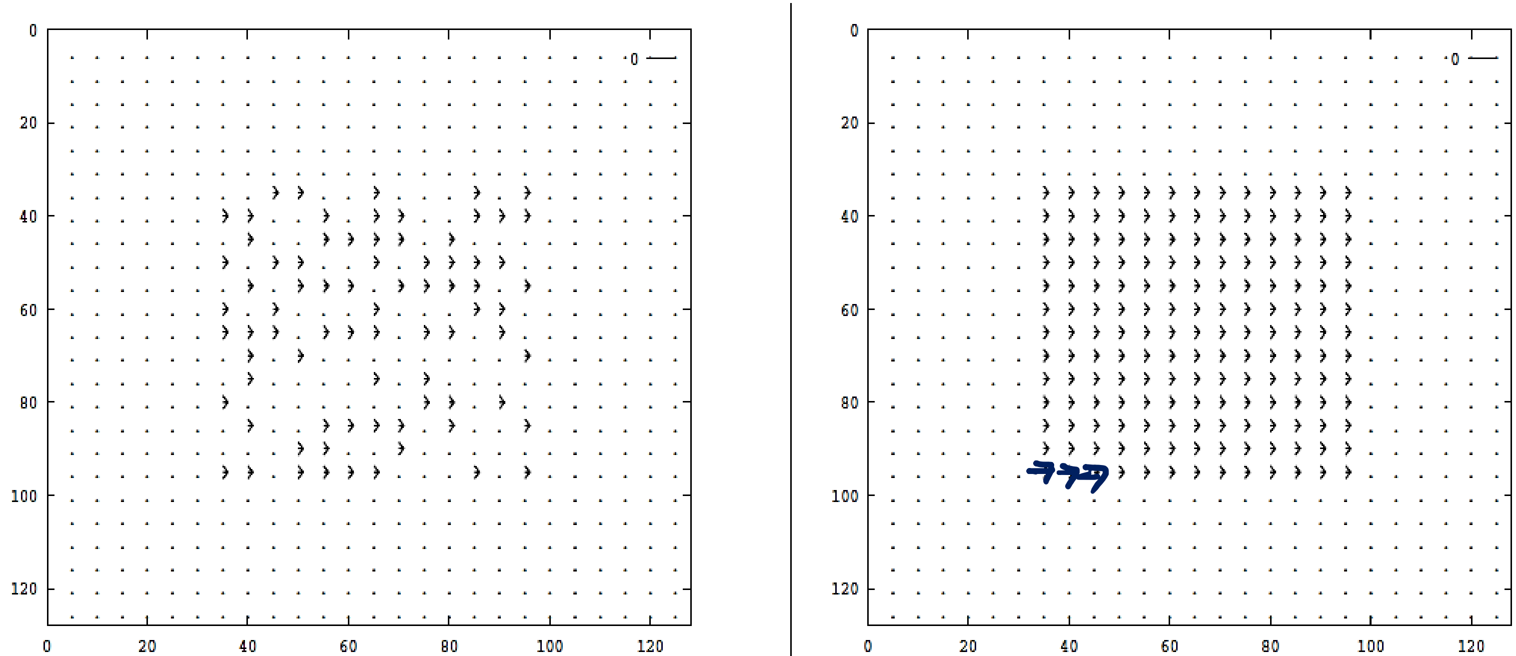


(a)Frame 1



(b) Frame 2

Horn and Schunck Optical Flow Example



(c) 1 iteration

(d) 10 iterations

converge

Figure 5.4: Results for Horn and Schunck algorithm for displacement of 1 pixel and $\lambda = 4$.

Reference Book

- Ebook: Mubarak shah, fundamentals of CV
- (present in the ebook folder of google classroom)