Subject Name: Machine Learning
Class Test 1, 20 Marks, Time : 2 hr exam. (All questions are compulsory)

Q1 [5 Marks] X=(X1, X2) is drawn from a two dimensional Gaussian distribution with a diagonal covariance matrix X =(X1,X2) ~N(μ,R)

$$\mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where,

Here a and b are some real numbers. Are X1 and X2 independent ? Explain as succinctly as possible.

Q2 [10 Marks] a) [5 Marks] Define least square method with detailed mathematical expressions ?

b) [5 Marks] Write Pseudo-algorithm for PCA and apply PCA on the following data to reduce the dimensionality into one dimensional :

Features	Sample 1	Sample 2	Sample 3	Sample 4
x	4	8	15	7
y	11	4	5	14

Q3 [5 Marks] We are given a set of two dimensional inputs and their corresponding output pair : $(x_{i,1}, x_{i,2}, y_i)$. We would like to use the following regression model to predict y:

$$y_i = w_1^2 x_{i,1} + w_2^2 x_{i,2}.$$

Derive the optimal value for w_1 when using the least squares regression. NOTE : w_2 may appear in your resulting equation.