

$$C_2 \text{ norm} = \sqrt{1^2 + 2^2} = \sqrt{5}$$

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$$V_1 = \begin{bmatrix} -1/\sqrt{5} \\ +2/\sqrt{5} \end{bmatrix} \begin{bmatrix} -2\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$$

for $\lambda = 40$

$$\begin{bmatrix} 16 & 12 \\ 12 & 34 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 40V_1 \\ 40V_2 \end{bmatrix}$$

$$16V_1 + 12V_2 = 40V_1$$

$$12V_1 + 34V_2 = 40V_2$$

$$12V_2 = +24V_1$$

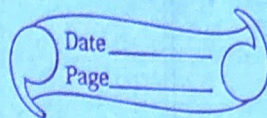
$$V_2 = +2V_1$$

$$V_1 = 1, V_2 = +2$$

$$V_2 = \begin{bmatrix} 1/\sqrt{5} \\ -2/\sqrt{5} \end{bmatrix} \begin{bmatrix} 1/\sqrt{5} \\ +2/\sqrt{5} \end{bmatrix}$$

$$V = \begin{bmatrix} +1/\sqrt{5} & -2/\sqrt{5} \\ +2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}$$

Application of SVD



① Moore Penrose Pseudo Inverse

↳ most useful feature of SVD is that it can be used to generalize matrix inversion to non square matrices

$$A = U \Sigma V^T$$

$$A^{-1} = (U \Sigma V^T)^{-1}$$

$$= (V^T)^{-1} \Sigma^{-1} U^{-1}$$

$$\left[(U \cdot U)^{-1} = V^{-1} \cdot U^{-1} \right]$$

U and V are orthogonal

$$\therefore U \cdot U^T = I \rightarrow U^T = U^{-1}$$

$$\therefore V \cdot V^T = I \rightarrow V^T = V^{-1}$$

Σ is diagonal matrix.

$$\boxed{A^{-1} = V \Sigma^{-1} U^T}$$

$$\Sigma = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$\Sigma^{-1} = \begin{bmatrix} \frac{1}{\lambda_1} & 0 \\ 0 & \frac{1}{\lambda_2} \end{bmatrix}$$

② PCA → dimension reduction.