Rollno :- 2024OAI1002

name :- ABHISHEK JANGID

Date 27/10/24

Page 1

STATS AND FOUNDATION MIDTERM

Q1) Proove two similar matrices A and B have the same characterstics polynomial hence same eigen values.

we know,

Two matrix are similar if

$$B = P^{-1} \cdot A \cdot P \quad -(1)$$

characterstics equation

$$|A - \lambda I|$$

now

$$|B - \lambda I| = |P^{-1} A P - \lambda I| \quad \text{- From (1)}$$

$$= |P^{-1} A P - \lambda I P P^{-1}|$$

$$= |P P^{-1} (A - \lambda I)|$$

$$= |P| |P^{-1}| |A - \lambda I|$$

$$|B - \lambda I| = |A - \lambda I| \quad \because |P| |P^{-1}| = 1$$

$\therefore$ characterstics equation are same

hence eigen values are also going to be same

Q2) a) if A and B are conditionally independent given C, are A and B independent?

if A and B are conditionally independent given C, A and B are independent

False, Not necessarily true

$$P(A \cap B) = P(A) \cdot P(B)$$

if A and B are independent

Given

$$P(A \cap B | C) = P(A|C) \cdot P(B|C)$$

P(C) = person from Italy
P(A) = person like pizza.
P(B) = person like football.

Given person from Italy, there may be chances that person likes pizza and person like football.

but there may be population in Italy which do not like pizza and football. Which implies conditional independence is not absolute independence

b) if A and {B C} are conditionally independent given D, are A and B conditionally independent given D.

False, if A and {B, C} are conditionally independent given D, it does not means A and B are conditional independent given D

Ex

D : a student studies in quiet library
A : student gets a high score on exam
B : student submit all homework
C : student complete practise problems

now given D, chances of student scoring high marks doesn't effect probability of submitting home work and complete practise problem. or not submit
ie. student submit all home work or submit pratice problem doesn't effect probability of getting high score.

∴ A and {B, C} are conditional independent given D now, given student studies in quiet library, submit all homework likely to perform better.

So just by knowing D alone, it is not always true that A and B are conditionally independent.

Q3) Give Random variable X_1 and X_2

their Covariance matrix

$$\Sigma = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} Var(X_1) & Cov(X_1, X_2) \\ Cov(X_2, X_1) & Var(X_2) \end{bmatrix}$$

from Covariance matrix ~~defination~~ defination

$Cov(X, Y_1) = 0$

Since Covariance of X_1 and Y_1 are zero
hence X_1 and Y_1 are independent

~~Q4)~~ b)

Q5)

numbers of dice	Bag Chosen
1	Bag A
2 or 3	Bag B
4 or 5 or 6	Bag C

Bag A contains 3 white and 2 black ball
Bag B contains 3 white and 4 black ball
Bag C contain 4 white and 5 black ball.

To find

~~P(Bag~~

$P(\text{Bag}_{=B} \mid \text{White Ball})$

$$P(\text{Bag}=B \mid \text{White}B) = \frac{P(\text{White Ball} \mid \text{Bag}_B)\, P(\text{Bag}_B)}{P(\text{White Ball})}$$

$P(\text{white Ball} \mid \text{Bag}_A) = 3/5$

$P(\text{White Ball} \mid \text{Bag}_B) = 3/7$

$P(\text{white Ball} \mid \text{Bag}_C) = 4/9$

$P(A) = 1/6$

$P(B) = 2/6 = 1/3$

$P(C) = 3/6 = 1/2$

$$P(\text{white Ball}) = P(\text{white Ball} \mid \text{Bag}_A) \cdot P(\text{Bag}_A)$$
$$+ P(\text{white Ball} \mid \text{Bag}_B) \cdot P(\text{Bag}_B)$$
$$+ P(\text{white Ball} \mid \text{Bag}_C) \cdot P(\text{Bag}_C)$$

$$= \left(\frac{\cancel{3}}{5} \times \frac{1}{\cancel{6}_{2}}\right) + \left(\frac{\cancel{3}}{7} \times \frac{1}{\cancel{3}}\right) + \left(\frac{\cancel{4}^{2}}{9} \times \frac{1}{\cancel{2}}\right)$$

$$= \frac{1}{10} + \frac{1}{7} + \frac{2}{9}$$

$$= \cancel{0.3558} \; 0.465$$

$$P(\text{Bag}_B \mid \text{white Ball}) = \frac{P(\text{white Ball} \mid \text{Bag}_B)\, P(\text{Bag}_B)}{P(\text{white Ball})}$$

$$= \frac{\left(\frac{3}{7} \times \frac{1}{3}\right)}{0.353}$$

$$= \frac{1/7}{0.465}$$

$$\boxed{P(Bag_B \mid \text{white Ball}) = 0.307 = 30.7\ \%}$$

Q6 i) $H(X) = -\sum_i P(x_i) \log_2(P(x_i))$

Pixel value 0 to 255

$P(x_i) = \frac{1}{256}$

$$H(X) = -\sum_{i=0}^{255} \frac{1}{256} \log_2\left(\frac{1}{256}\right)$$

$$= -\cancel{256} \times \left(\frac{1}{\cancel{256}} \log_2\left(\frac{1}{256}\right)\right)$$

$$= -\log_2\left(\frac{1}{256}\right) \qquad 2^8 = 256$$

$H(X) = 8$ bits

ii) Two Category mammal and not mammal.

$P(\text{mammal}) = \frac{1}{2}$ $\quad P(\text{non mammal}) = \frac{1}{2}$

$$H(X) = -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{2}\log_2\frac{1}{2}\right]$$

$$= -\left[\frac{1}{2}(-1) + \frac{1}{2}(-1)\right]$$

$H(X) = 1$ bit.

iii) three genders

$P(G_1) = \frac{1}{4}$ $\quad P(G_2) = \frac{1}{4}$ $\quad P(G_3) = \frac{1}{2}$

$$H(X) = -\left[\left[\frac{1}{4}\log_2\left(\frac{1}{4}\right)\right] + \left[\frac{1}{4}\log_2\left(\frac{1}{4}\right)\right] + \left[\frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]\right]$$

$$= -\left[\left(\frac{1}{4}\times -2\right) + \left(\frac{1}{4}\times -2\right) + \left(\frac{1}{2}(-1)\right)\right]$$

$$= -\left[-\frac{1}{2} - \frac{1}{2} - \frac{1}{2}\right]$$

$H(X) = \frac{3}{2}$ or 1.5 bits

IV) $P(\text{older}) = \frac{1}{2}$ $P(\text{not older}) = \frac{1}{2}$

$$H(x) = -\left[\frac{1}{2}\log_2\left(\frac{1}{2}\right) + \frac{1}{2}\log_2\left(\frac{1}{2}\right)\right]$$

$$= -\left[\frac{1}{2}\times(-1) + \frac{1}{2}(-1)\right]$$

$$H(x) = \underline{\underline{1}} \text{ bit}$$

Q4) a

$\cancel{R_x(t) = E[x(t)}$

<u>autocorrelation</u>

$$R_x(t_1, t_2) = E[X(t_1)\,X(t_2)]$$

<u>Given</u>

$E[X(t)] = 0$ mean

$\cancel{E}$ $Var(X(t)) = E[x(t)^2] = 1$

when $t_1 = t_2$

$$R_x(t, t) = E[x(t)^2]$$
$$= Var(X(t))$$
$$= 1$$

when $t_1 \neq t_2$

$$R_X(t_1, t_2) = E[X(t_1)\, X(t_2)]$$
$$= E[X(t_1)] \cdot E[X(t_2)]$$
$$= 0 \qquad \text{Given } E[X(t)] = 0$$

$\therefore$ auto corelation :-

$$R_X(t_1, t_2) = \begin{cases} 1 & \text{if } t_1 = t_2 \\ 0 & \text{if } t_1 \neq t_2 \end{cases}$$