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PRACTICAL NO: 1

TOPIC : Limits & continuity.

① $\lim\limits_{x\to a} \dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

$= \lim\limits_{x\to a} \dfrac{\sqrt{a+2x}-3\sqrt{x}\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$= \dfrac{a+\cancel{2x}-3x}{3a+\cancel{x}-4x} \lim\limits_{x\to a} \dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}$

$= \lim\limits_{x\to a} \dfrac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{3a+x})^2-(2\sqrt{x})^2}$

$= \lim\limits_{x\to a} \dfrac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{3a+x}+2\sqrt{x})}{3a+x-4x}$

$= \lim\limits_{x\to a} \dfrac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{3a+x}+\sqrt{2x})}{3a-3x}$

$= \dfrac{1}{3} \lim\limits_{x\to a} \dfrac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{3a+x}+\sqrt{2x})}{a-x}$

$= \dfrac{1}{3} \lim\limits_{x\to a} \dfrac{(\sqrt{a+2x}-\sqrt{3x})(\sqrt{3a+x}+2\sqrt{x})}{a-x} \times \dfrac{(\sqrt{a+2x}+\sqrt{3x})}{\sqrt{a+2x}+\sqrt{3x}}$

$= \dfrac{1}{3} \lim\limits_{x\to a} \dfrac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$

$= \dfrac{1}{3} \lim\limits_{x\to a} \dfrac{\cancel{(a-x)}(\sqrt{3a+x}+2\sqrt{a})}{\cancel{(a-x)}(\sqrt{a+2x}+\sqrt{3x})}$

$= \dfrac{1}{3} \lim\limits_{x\to a} (\sqrt{3a+x}+2\sqrt{x}) / (\sqrt{a+2x}+\sqrt{3x})$

$$= \frac{1}{3} \times \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}} = \frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}} = \frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{4\sqrt{a}}{3 \cdot 2\sqrt{3a}} = \frac{2\sqrt{a}}{3\sqrt{3}}$$

② $$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$$

$$= \lim_{y \to 0} \frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}}$$

$$= \lim_{y \to 0} \frac{\cancel{a} + y - \cancel{a}}{y\sqrt{a+y} \times \sqrt{a+y} + \sqrt{a}}$$

$$= \lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y} \times \sqrt{a+y} + \sqrt{a}}$$

$$= \lim_{y \to 0} \frac{1}{\sqrt{a} \times \sqrt{a} + \sqrt{a}}$$

$$= \frac{1}{\sqrt{a} \times 2\sqrt{a}} = \frac{1}{2a}$$

③ $$\lim_{x \to \frac{\pi}{6}} \left[\frac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$$

put $h = x - \frac{\pi}{6}$ since $x \to \frac{\pi}{6} \Rightarrow h \to 0$

$x = h + \frac{\pi}{6}$

$$= \lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{6}\right) - \sqrt{3}\sin\left(h + \frac{\pi}{6}\right)}{\pi - 6\left(h + \frac{\pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\left(\cos h \cdot \cos\frac{\pi}{6} - \sin h \cdot \sin\frac{\pi}{6}\right) - \sqrt{3}\left(\sin h \cdot \cos\frac{\pi}{6} + \cos h \cdot \sin\frac{\pi}{6}\right)}{\pi - \cancel{6}\left(\frac{6h + \pi}{\cancel{6}}\right)}$$

$$= \lim_{h \to 0} \frac{\left(\cosh \cdot \cos\frac{\pi}{6} - \sinh \times \sin\frac{\pi}{6}\right) - \sqrt{3}\left(\sinh \times \cos\frac{\pi}{6}\right.}{\pi - 6h - \pi}$$

$$= \lim_{h \to 0} \frac{\left(\cosh \cdot \cos\frac{\pi}{6} - \sinh \times \sin\frac{\pi}{6}\right) - \sqrt{3}\left(\sinh \times \cos\frac{\pi}{6}\right.}{-6h}$$

$$= \lim_{h \to 0} \frac{\frac{\sqrt{3}}{2} \times \cancel{\cosh} - \sinh \times \frac{1}{2} - \sqrt{3}\sin h \times \frac{\sqrt{3}}{2} - \cancel{\cosh} \times \frac{\sqrt{3}}{2}}{-6h}$$

$\cos\frac{\pi}{6} =$

$\sin\frac{\pi}{6} =$

$$= \lim_{h \to 0} \frac{\frac{-\sinh}{2} - \sinh \times \frac{3}{2}}{-6h}$$

$$= \lim_{h \to 0} \frac{-\frac{\cancel{4}^{2}\sinh}{\cancel{2}}}{-6h} = \lim_{h \to 0} \frac{-2\sinh}{-6h}$$

$$= \lim_{h \to 0} \frac{\sinh}{3h} = \frac{1}{3} \lim_{h \to 0} \frac{\sinh}{h}$$

$$= \frac{1}{3}$$

(9) $\lim_{h \to \infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}$

By rationalizing numerator & denominator

$$\lim_{x \to \infty} \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2+3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{\ }}{\sqrt{x^2+3} + \sqrt{\ }}$$

$$= \lim_{x \to \infty} \frac{x^2+5 - x^2+3}{x^2+3 - x^2-1} = \lim_{x \to \infty} \frac{x^2+5 - x^2+3}{x^2+3 - x^2-1}$$

$$= \lim_{x\to\infty} \frac{8}{2} \frac{(\sqrt{x^2+3} + \sqrt{x^2-1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$= \frac{8}{2} \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{x^2\left(1+\frac{5}{x^2}\right) + x^2\left(\sqrt{\frac{1}{x}} + \frac{3}{x^2}\right)}$$

$$= 4 \lim_{x\to\infty} \frac{x}{x} \times$$

(5) Examine the continuity of the following function at given point.

(i) $F(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ for $0 < x \le \frac{\pi}{2}$ $\Big\}$ at $x = \frac{\pi}{2}$

for $\frac{\pi}{2} < x < \pi$

$$F\left(\frac{\pi}{2}\right) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2\left(\frac{\pi}{2}\right)}} = \frac{\sin\pi}{\sqrt{1-\cos\pi}} = 0$$

$$L.H.L = \lim_{x\to\frac{\pi}{2}} f(x)$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\pi - 2x}$$

Let $x - \frac{\pi}{2} = h$ $\qquad x \to \frac{\pi}{2}$

$x = \frac{\pi}{2} + h$ $\qquad h \to 0$

$$\therefore \lim_{h\to 0} \frac{\cos x}{\pi - 2x}$$

$\therefore \lim\limits_{h \to 0} \dfrac{\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$

$= \lim\limits_{h \to 0} \dfrac{-\sin h}{\pi - \pi + 2h}$

$= -\frac{1}{2} \lim\limits_{h \to 0} \dfrac{\sin h}{h}$

$= -\frac{1}{2}$

$R.H.L = \lim\limits_{x \to \frac{\pi}{2}^+} f(x)$

$= \lim\limits_{x \to \frac{\pi}{2}^+} \dfrac{\sin 2x}{\sqrt{1 - \cos 2x}}$

$= \lim\limits_{x \to \frac{\pi}{2}^+} \dfrac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$= \lim\limits_{x \to \frac{\pi}{2}^+} \dfrac{2\cancel{\sin x} \cdot \cos x}{\cancel{\sin x} \cdot \sqrt{2}}$

$= \frac{2}{\sqrt{2}} \lim\limits_{x \to \frac{\pi}{2}} \cos x$

$= 0$

$\therefore L.H.L \neq R.H.L$

ii) $f(x) = \dfrac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \leq x < 6$

$= \dfrac{x^2-9}{x+3}$ $\quad 6 \leq x < 9$

at $x = 3$ or $x = 6$

At $x = 3$

$f(3) = x+3 = 3+3 = 6$

$L.H.L = \lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3} x+3$

$= 3+3$

$= \underline{\underline{6}}$

$R.H.L = \lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3} \dfrac{x^2-9}{x-3}$

$= \lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$

$= \lim\limits_{x \to 3} (x+3)$

$= 3+3 = \underline{\underline{6}}$

$\therefore$ L.H.L = R.H.L = $f(3)$

$\therefore f$ is continuous at $x = \underline{\underline{3}}$

At $x = 6$

$f(6) = \dfrac{x^2-9}{x+3} = \dfrac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}} = \cancel{6}-3 = \underline{\underline{3}}$

$L.H.L = \lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$= \lim_{x \to 6} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$= \lim_{x \to 6} x-3$

$= 6-3 = 3$

$R.H.L = \lim_{x \to 6^+} f(x) = \lim_{x \to 6} x+3$

$= 6+3$

$= 9$

$\therefore L.H.L \neq R.H.L$

$\therefore f$ is not continuous at $x=6$

Q6) (i) $f(x) = \frac{1-\cos 4x}{x^2}$ $\quad x \neq 0$

(4) $= k$ $\quad x = 0$ $\Big\}$ at $x=0$

$f(x)$ is continuous at $x=0$

$\therefore \lim_{x \to 0} f(x) = k$

$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k$

$= 2 \times \frac{1}{4} = k$

$k = \frac{1}{2}$

(ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$ $= k$ $x = 0$ } at $x = 0$

f is continuous at $x = 0$

$$\lim_{x \to 0} f(x) = k$$

$$\lim_{x \to 0} (\sec^2 x)^{\cot^2 x} = k$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}} = k$$

compairing with, $\lim_{x \to 0} (1+x)^{\frac{1}{x}}$

$\therefore k = e$

(iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \frac{\pi}{3}$ $= k$ $x = \frac{\pi}{3}$ } at $x = \frac{\pi}{3}$

f is continuous at $x = \frac{\pi}{3}$

$$\lim_{x \to \frac{\pi}{2}} f(x) = k$$

$$= \lim_{x \to \frac{\pi}{3}} \frac{\sqrt{3} - \tan x}{\pi - 3x} = k$$

let $x - \frac{\pi}{3} = h$ As $x = \frac{\pi}{3}$

$\therefore x = h + \frac{\pi}{3} \qquad h \to 0$

$$\therefore \lim_{x \to 0} \frac{\sqrt{3} - \tan\left(h + \frac{\pi}{3}\right)}{\pi - 3\left(h + \frac{\pi}{3}\right)}$$

$$\therefore \frac{\lim_{h \to 0} \sqrt{3} - \frac{\tan h + \tan \frac{\pi}{3}}{1 - \tan h \cdot \tan \frac{\pi}{3}}}{\lim_{h \to 0} \pi - 3h + 3\pi}$$

$$= \lim_{h \to 0} \frac{\sqrt{3}\left(1 - \tan h \cdot \tan \frac{\pi}{3}\right) - \tan h - \tan \frac{\pi}{3}}{-3h\left(1 - \tan h \cdot \tan \frac{\pi}{3}\right)}$$

$$= -\frac{\sqrt{3}}{3} \lim_{h \to 0} \frac{(1 - \sqrt{3}\tan h) - \tan h - \sqrt{3}}{h(1 - \sqrt{3}\tan h)}$$

$$= \frac{1}{-3} \lim_{h \to 0} \frac{\sqrt{3} - 3\tan h - \tan h - \sqrt{3}}{h(1 - \sqrt{3}\tan h)}$$

$$= -\frac{1}{3} \times (-4) \lim_{h \to 0} \frac{\tan h}{h} \times \lim_{h \to 0} \frac{1}{1 - \sqrt{3}\tan h}$$

$$= \frac{4}{3} \times 1 \times 1$$

$$= \frac{4}{3}$$

Q.7) (i) $f(x) = \dfrac{1_0 - \cos x}{x \tan x}$ $x \neq 0$, $x = 0$ $\Big\{$ at $x = 0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{2\sin^2\left(\frac{x}{2}\right)}{\frac{x\cdot\tan x}{x}\cdot x}$$

$$= \frac{2\lim\limits_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 \times \frac{x^2}{4}}{\lim\limits_{x\to 0} x^2 \cdot \frac{\tan x}{x}}$$

$$= \frac{2 \times x^2}{x^2 \times 4} = \frac{2}{4} = \frac{1}{2}$$

for f to be continuous

$$\lim_{h\to 0} f(x) = \frac{1}{2}$$

$\therefore$ It is removable discontinuous at $x = 0$

$\therefore$ redefining the limit,

$f(x) = \dfrac{1 - \cos x}{x \tan x}$ $x \neq 0$

$= \dfrac{1}{2}$ $x = 0$

(ii) $f(x) = \dfrac{(e^{7x} - 1)\sin x^{\circ}}{x^2}$ $x \neq 0$ $\Big\{$ at $x = 0$

$= \dfrac{\pi}{60}$ $x = 0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{(e^{3x}-1)\sin x^{\circ}}{x^2}$$

$$= \lim_{x\to 0} \frac{(e^{3x}-1)}{3x} \cdot \frac{\sin\left(\frac{\pi x}{180}\right)}{\frac{\pi x}{180}} \cdot \frac{\times \frac{\pi x}{180}}{x}$$

$$= \cancel{3} \cdot 3 \times 1 \cdot \frac{\pi \cancel{x}}{\cancel{180}_{60}\, \cancel{x}}$$

$$= \frac{\pi}{60}$$

$$\therefore \lim_{x\to 0} f(x) = \frac{\pi}{60}$$

$\therefore$ function is continuous

Q.9) $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$

$\therefore$ f is continuous

$$\therefore \lim_{x\to 0} (e^{x^2}-1)+(1-\cos x) = f(0)$$

$$\lim_{x\to 0} \frac{(e^{x^2}-1)}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2} = f(0)$$

$$1+\frac{1}{2} = f(0)$$

$$\therefore f(0) = \frac{3}{2}$$

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1) $F(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$

F is continuous

$\therefore \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} = f\left(\frac{\pi}{2}\right)$

$\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sqrt{2} - \sqrt{1+\sin x} \cdot \sqrt{2} + \sqrt{1+\sin x}}{1 - \sin^2 x \left(\sqrt{2} + \sqrt{1+\sin x}\right)} = f\left(\alpha \frac{\pi}{2}\right)$

$\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sqrt{2} - \sqrt{2} - 1 - \sin x}{(1-\sin x)\ (1+\sin x)(\sqrt{2} + \sqrt{1+\sin x}} = f\left(\frac{\pi}{2}\right)$

$= \lim\limits_{x \to \frac{\pi}{2}} \dfrac{1}{(1+\sin x)\left(\sqrt{2} + \sqrt{1+\sin x}\right.} = f\left(\frac{\pi}{2}\right)$

$\therefore \dfrac{1}{(1+1)(\sqrt{2}+\sqrt{2})} = f\left(\frac{\pi}{2}\right)$

$= \dfrac{1}{2(2\sqrt{3})} = f\left(\frac{\pi}{2}\right)$

$\therefore f\left(\frac{\pi}{2}\right) = \dfrac{1}{4\sqrt{2}}$

PRACTICAL:3

Q.1) Show that the following function defined from are differentiable.

(i) $\cot x$ (ii) $\operatorname{cosec} x$ (iii) $\sec x$

Q.2) if $f(x) = 4x+1$, $x \le 2$
$= x^2+5$, $x > 0$ at $x=2$

then find f is differentiable or not?

Q.3) If $f(x) = 4x+7$, $x<3$
$= x^2+3x+1$, $x \ge 3$ at $x=3$

Then find f is differentiable or not?

Q.4) If $f(x) = 8x-5$, $x \le 2$
$= 3x^2 - 4x + 7$, $x > 2$ at $x=2$

Then find f is differentiable or not

ANSWER

(1) $f(x) = \cot x$

consider,

$$D\,f(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$

put $(x-a) = h$

$\therefore x = a+h$

As, $x \to a$; $h \to 0$

$$= \lim_{x \to 0} \frac{\cot(a+h) - \cot a}{(\cancel{a}+h) - \cancel{a}}$$

$$= \frac{\lim_{h \to 0} \frac{\cos(a+h)}{\sin(a+h)} - \frac{\cos a}{\sin a}}{\lim_{h \to 0} h}$$

$$= \frac{1}{h} \lim_{h \to 0} \frac{\cancel{\sin}\ \cos(a+h)\sin a - \cos a \cdot \sin(a+h)}{\sin(a+h)\sin a}$$

$$= \frac{1}{h} \lim_{h \to 0} \frac{\sin(a-(a+h))}{\sin(a+h)\sin a}$$

$$= -\lim_{h \to 0} \frac{\sin h}{h} \cdot \lim_{h \to 0} \operatorname{cosec}(a+h) \operatorname{cosec} a$$

$= -1 \cdot \operatorname{cosec} a\,(a+0).$

$= -\operatorname{cosec}^2 a.$

$\therefore Df(a) = -\operatorname{cosec}^2 a$

$\therefore f$ is differentiable $\forall a \in R$

(ii) $f(x) = \operatorname{cosec} x$

consider,

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

let,

$x - a = h$; $x = a+h$

$$= \lim_{h \to 0} \frac{\operatorname{cosec}(a+h) - \operatorname{cosec} a}{a+h-a}$$

$$= \frac{\lim_{h \to 0} \frac{1}{\sin(a+h)} - \frac{1}{\sin a}}{\lim_{h \to 0} h}$$

$$= \frac{1}{h} \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{\sin(a+h)\sin a}$$

$$= \frac{1}{h} \frac{\lim_{h \to 0} \cancel{\sin a} \cdot 2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{\lim_{h \to 0} \sin^2 a \cdot \cos h + \cos a \cdot \sin a \cdot \sin h}$$

$$= \frac{\frac{1}{h} \lim_{h \to 0} 2\cos\left(\frac{2a+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\sin^2 a}$$

$$= -\lim_{h\to 0} \frac{\sin(\frac{h}{2})}{\frac{h}{2}} \cdot \lim_{h\to 0} \frac{\cos\left(\frac{2a+h}{2}\right)}{\sin 2a}$$

$$= -1 \cdot \frac{\cos\left(\frac{2a}{2}\right)}{\sin^2 a}$$

$$= -1 \cdot \frac{\cos a}{\sin^2 a} = -1 \cdot \frac{\cos a}{\sin a} \cdot \frac{1}{\sin a}$$

$= -\cot a \cdot \operatorname{cosec} a$

$\therefore Df(a) = -\cot a \cdot \operatorname{cosec} a$

$\therefore f$ is differentiable at $\forall x \in R$

(iii) $f(x) = \sec x$

consider,

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

Let,

$x - a = h$; $x = a+h$

as , $x \to a$, $h \to 0$

$$= \lim_{h\to 0} \frac{\sec(a+h) - \sec a}{a+h-a}$$

$$= \frac{1}{h} \lim_{h\to 0} \frac{1}{\cos(a+h)} - \frac{1}{\cos a}$$

$$= \frac{1}{h} \cdot \lim_{h\to 0} \frac{\cos a - \cos(a+h)}{\cos(a+h)\cos a}$$

$$= \frac{\frac{1}{h}\lim_{h\to 0} -2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{\lim_{h\to 0}\left(\cos a \cdot \cos h - \frac{\sin a \cdot \sin h}{\cos a}\right)}$$

$$= \frac{\frac{-2}{h}\lim_{h\to 0}\sin\left(\frac{2a+h}{2}\right)\sin\left(\frac{h}{2}\right)}{\cos^2 a}$$

$$= \frac{2}{\cos^2 a}\lim_{h\to 0}\sin\left(\frac{2a+h}{2}\right)\cdot\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{h}$$

$$= \frac{\cancel{2}}{\cos^2 a}\sin\left(\frac{\cancel{2}a+0}{\cancel{2}}\right)\cdot\frac{1}{\cancel{2}}$$

$$= \frac{\sin a}{\cos^2 a}$$

$= \tan a \cdot \sec a$

$\therefore Df(a) = \tan a \cdot \sec a$

f is differentiable at $\forall x \in R$.

Q.2) $F(x) = 4x+1 \quad x \le 2$

$\quad = x^2+5 \quad x<2 \quad$ at $x=2$

→ R.H.D

$$Df(2^+) = \lim_{x\to 2^+}\frac{F(x)-F(a)}{x-a}$$

$$= \lim_{x\to 2}\frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= \lim_{x \to 2} (x+2)$$

$$= 2+2 = 4$$

$\therefore \boxed{R.H.D = 4}$

L.H.D

$$Df(2^-) = \lim_{h \to 2^-} \frac{F(x) - F(a)}{x-a}$$

$$= \lim_{h \to 2^-} \frac{4x+1-9}{x-9}$$

$$= \lim_{x \to 2} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2} \frac{4\cancel{(x-2)}}{\cancel{x-2}}$$

$$= \cancel{\lim_{x \to 2}}\ 4 = 4$$

$\therefore L.H.D = 4$

$\therefore L.H.D = R.H.D$

$\therefore$ F is differentiable at $x = 2$

Q.3) $f(x) = 4x + 7, \quad x < 3$ at $x = 3$

$= x^2 + 3x + 1, \quad x \geq 3$

→ R.H.D

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(a)}{x - 9}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$$

$$= \lim_{x \to 3} \frac{x^2 + 3x - 18}{x - 3}$$

$$= \lim_{x \to 3} \frac{\cancel{(x-3)}(x+6)}{\cancel{(x-3)}}$$

$$= \lim_{x \to 3} (x + 6)$$

R.H.D $= 3 + 6 = 9$

$\boxed{R.H.D = 9}$

L.H.D

$$Df(3^-) = \lim_{x \to 3^-} \frac{f(x) - f(a)}{x - 9}$$

$$= \lim_{x \to 3} \frac{4x + 7 - 19}{x - 3}$$

$$= \lim_{x \to 3} \frac{4x - 12}{x - 3}$$

$$= 4 \lim_{x \to 3} \frac{\cancel{x-3}}{\cancel{x-3}}$$

L.H.D = 4

$\therefore$ R.H.D $\neq$ L.H.D

f is not differentiable at $x=3$

Q.4) $f(x) = 8x-5 \quad x \leq 2$ at $x=2$

$= 3x^2-4x+7 \quad x>2$

R.H.D

$$Df(2^+) = \lim_{x\to2^+} \frac{f(x)-f(4)}{x-4}$$

$$= \lim_{x\to2} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x\to2} \frac{3x^2-4x-4}{x-2}$$

$$= \lim_{x\to2} \frac{3x^2-6x+2x-4}{x-2}$$

$$= \lim_{x\to2} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x\to2} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= \lim_{x\to2} (3x+2)$$

$$= 3(2)+2$$

$$= 6+2 = 8$$

L.H.D

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to 2} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2} \frac{8x - 16}{x - 2}$$

$$= 8 \lim_{x \to 2} \frac{(x - \cancel{2})}{(\cancel{x-2})}$$

L.H.D = 8

$\therefore$ L.H.D = R.H.D

$\therefore$ It is differentiable at $x = 2$

(4)

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PRACTICAL – 3

TOPIC : Application of Derivative.

Q1) Find the intervals in which function is increasing or decreasing

(i) $f(x) = x^3 - 5x - 11$ (ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$ (iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q2) Find the intervals in which function is concave upwards and concave downwards.

(i) $y = 3x^2 - 2x^3$ (ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

(iii) $y = x^3 - 27x + 5$ (iv) $y = 69 - 24x - 9x^2 + 2x^3$

(v) $y = 2x^3 + x^2 - 20x + 4$

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Q.1) $F(x) = x^3 - 5x - 11$

$\rightarrow F'(x) = 3x^2 - 5$

$\therefore f$ is increasing iff $F'(x) > 0$

$\therefore 3x^2 - 5 > 0$

$3x^2 > 5$

$x^2 > \frac{5}{3}$

$x > \pm \sqrt{\frac{5}{3}}$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -\sqrt{\frac{5}{3}} & & \sqrt{\frac{5}{3}} & \end{array}$$

$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

f is decreasing iff $F'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3x^2 < 5$

$x^2 < \frac{5}{3}$

$x < \pm \sqrt{\frac{5}{3}}$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -\sqrt{\frac{5}{3}} & , & \sqrt{\frac{5}{3}} & \end{array}$$

$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

(ii) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x-2 > 0$

$x > 2$

$\therefore x \in (2, \infty)$

$\therefore f$ is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x-2) < 0$

$x-2 < 0$

$x < 2$

$\therefore x \in (-\infty, 2)$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\therefore f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$6x^2 + 12x - 10x - 20 > 0$

$6x(x+2) - 10(x+2) > 0$

$(6x-10)(x+2) > 0$

SA.

$$\underset{-2}{+} \quad - \quad \underset{\frac{10}{6}}{+}$$

$\therefore x \in (-\infty, -2) \cup \left(\frac{10}{6}, \infty\right)$

f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$6x^2 + 12x - 10x - 20 < 0$

$6x(x+2) - 10(x-2) < 0$

$(6x-10)(x+2) < 0$ | $x = \frac{10}{6}$, $x = -2$

$$\underset{-2}{+} \quad - \quad \underset{\frac{10}{6}}{+}$$

$\therefore x \in \left(-2, \frac{10}{6}\right)$

(4) iv.) $f(x) = x^3 - 27 + 5$

$f'(x) = 3x^2 - 27$

$= 3(x^2 - 9)$

f is increasing iff $f'(x) > 0$

$\therefore \cancel{3}(x^2 - 9) > 0$

$x^2 - 9 > 0$

$(x-3)(x+3) > 0$

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$+ \quad - \quad +$

$-3 \qquad 3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

is decreasing iff $f'(x) < 0$

$3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \qquad 3$

$\therefore x \in (-3, 3)$

v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = -24 - 18x + 6x^2$

i.e $6x^2 - 18x - 24$

$6(x^2 - 3x - 4)$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x+1)(x-4) > 0$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -1 & & 4 & \end{array}$$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

f is decreasing iff $f'(x) < 0$

$\therefore \cancel{f6}\ 6(x^2 - 3x - 4) < 0$

$x^2 - 3x - 4 < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x+1)(x-4) < 0$

$$\begin{array}{ccccc} + & & - & & + \\ \hline & -1 & & 4 & \end{array}$$

$\therefore x \in (1, 4)$

Q.2) i) $y = 3x^2 - 2x3$

let,

$f(x) = y = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\cancel{6x}\ ($

$\cancel{f''}(x) = 6 - 12x$

$= 6(1 - 2x)$

$f''(x)$ is concave upwards iff,

$f''(x) > 0$

$6(1-2x) > 0$

$1-2x > 0$

$-2x > -1$

$x < \frac{1}{2}$

$\therefore x \in (-\infty, \frac{1}{2})$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$6(1-2x) < 0$

$1-2x < 0$

$-2x < -1$

$2x > 1$

$x > \frac{1}{2}$

$\therefore x \in (\frac{1}{2}, \infty)$

(11) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

let,

$F(x) = y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

$= 12(x^2 - 3x + 2)$

$f''(x)$ is concave upwards iff

$f''(x) > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$x^2 - x - 2x + 2 > 0$

$x(x-1) - 2(x-1) > 0$

$(x-2)(x-1) > 0$

$+ \quad - \quad +$

$1 \quad\quad 2$

$\therefore x \in (-\infty, 1) \cup (2, \infty)$

$\therefore$ ~~$x \in (-\infty,$~~

$f''(x)$ is concave downwards iff

$f''(x) < 0$

$12(x^2 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$x^2 - x - 2x + 2 < 0$

$x(x-1) - 2(x-1) < 0$

$(x-1)(x-2) < 0$

$+ \quad - \quad +$

$1 \quad\quad 2$

$\therefore x \in (1, 2)$

(18) $y = x^3 - 27x + 5$

let,

$f(x) = y = x^3 - 27x + 5$

$f'(x) = y = 3x^2 - 27$

$f''(x) = 6x$

$f''(x)$ is concave ~~iff~~ upwards iff,

$f''(x) = 6x \, 0$

$f''(x) = 6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

$f''(x)$ is concave downwards iff,

$f''(x) < 0$

$6x < 0$

$x < 0$

$\therefore x \in (\infty, 0)$

(19) $y = 69 - 24x - 9x^2 + 2x^3$

let

$f(x) = y = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x$

$f'(x)$ is concave upward iff,

$f''(x) > 0$

$-18+12x>0$

$12x>18$

$x>\frac{18}{12}$

$\therefore x\in(\frac{3}{2},\infty)$

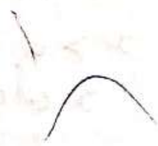
$f''(x)$ is concave downward iff

$f''(x)<0$

$-18+12x<0$

$12x<18$

$x<\frac{18}{12}$



$\therefore x\in(-\infty,\frac{3}{2})$

(iv) $y=2x^3+x^2-20x+4$

Let,

$f(x)=y=2x^3+x^2-20x+4$

$f'(x)=6x^2+2x-20$

$f''(x)=12x+2$

$=2(6x+1)$

$\therefore f''(x)$ is concave upward iff,

$f''(x)>0$

$2(6x-1)>0$

$6x-1>0$

$6x>1$

$x > \frac{1}{6}$

$\therefore x \in \left(\frac{1}{6}, \infty\right)$

$\therefore f''(x)$ is concave downwards iff,

$f''(x)$ ~~is~~ < 0

$2(6x-1) < 0$

$6x - 1 < 0$

$6x < 1$

$x < \frac{1}{6}$

$\therefore x \in \left(-\infty, \frac{1}{6}\right)$

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PRACTICAL NO: 4

Q.1) Find maximum and minimum value of following function.

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ in $[-\frac{1}{2}, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ in $[2, 3]$

Q.2) Find root of following equation by newtons method

[Take four itration only] & correct upto 4 dicimal

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $(x_0 = 0)$

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1, 2]$

Answer.

(1) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 2 \times \frac{16}{x^3}$

$f'(x) = 2x - \frac{32}{x^3}$

$\therefore f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x^4 - 32 = 0$

$x^4 - 16 = 0$

$(x^2 - 4)(x^2 + 4) = 0$

$x^2 - (4) = 0, \quad x^2 + 4 = 0$

$x^2 = 4, \quad x^2 = -4$

$x = \pm 2 \qquad x = 2$

$f''(x) = 2 + \frac{32}{x^4} \times 3$

$= 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{x^4} = 2 + \frac{\cancel{96}}{\cancel{16}}\, 6$

$= 2 + 6$

$= 8$

$= 8 > 0$

has minimumvalue at $x = 2$

$f(2) = 2^2 + 16/2^2 = 4 + 4 = 8$

The maximum value is 8

$f''(2) = 2 + 96/2^4 = 2 + \frac{96}{16} 6$

There is no minimum value at $x = 8 > 0$ because f is an even function at $f''(x)$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

$f'(x) = 15x^4 - 15x^2$

$f(x) = 0$

$15x^4 - 15x^2 = 0$

$15x^2(x^2 - 1) = 0$

$x = \pm 1$

$f''(x) = 60x^3 - 30x$

$f''(1) = 60 - 30 = 30 > 0$

$f''(1)$ has maximum value at $x = 1$

$f(1) = 3 - 5 + 3 = 6 - 5 = 1$

maximum value is 1

$f''(1)\ \cancel{=x^3} - 30x = 60(1)^3 - 30(1)$

f has minimum value at $x = -1$

$f(1) = 3 - 5(-1)^3 + 3(-1)^5 = 3 + 5 - 3$

$= 5$

iii) $F(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

consider $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x-2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6\times(0) - 6 = -6 < 0$

f has maximum value at $x = 0$

$f(0) = (0)^3 - 3(0)^3 + 1$

$f(0) = 1$

The value of f at maxima is 1

$f''(2) = 6(2) - 6 = 12 - 6 = 6 > 0$

f has minimum value at $x = 2$

$f(2) = 2^3 - 3(2)^2 + 1 = 8 - 12 + 1 = 9 - 12$

$= -3$

f has minimum value at $x = 2$

The ~~value of~~ f is -3 at ~~minimum~~ minima.

iv) $F(x) = 2x^3 - 3x^2 - 12x + 1$

$f(x) = 6x^2 - 6x - 12$

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

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$x(x+1)-2(x+1)=0$

$(x+1)(x-2)=0$

$x=-1$ & $x=2$

$f''(x) = 12x-6$

$f''(x) = 12x-6$

$f''(-1) = 12(-1)-6$

$= -12-6$

$= -18 < 0$

$f''(2) = 12(2)-6$

$= 24-6$

$= 18 > 0$

F as maximum value at $x=-1$

$= 2(-1)^3 - 3(-1)^2 - 12(-1)+1$

$= -2-3+12+1$

$= 13-5 = 8$

F ~~has~~ as minimum value at $x=2$

$F(2) = 2(2)^3 - 3(2)^2 - 12(2)+1$

$= 16-12-24+1$

$= -19$

(Q.2) i) $F(x) = x^3 - 3x^2 - 55x + 9.5$

$F'(x) = 3x^2 - 6x - 55$

by Newton' method.

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)} = 0$$

$$x_{0+1} = x_0 - \frac{F(x_n)}{F'(x_n)} = 0 - \frac{9.5}{-55} = 0.1727$$

$$x_1 = 0.1727$$

$$F(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$
$$= 0.0051 - 0.0895 - 9.4985 + 9.5$$
$$= 0.0829$$

$$F'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$
$$= 0.0895 - 1.0362 - 55$$
$$= -55.9467$$

$$x_2 = x_1 - \frac{F(x_1)}{F'(x_1)} = 0.1727 - \frac{0.0829}{-55.9467} = 0.1727 + \frac{0.0829}{55.9467}$$

$$x_2 = 0.1712$$

$$F(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$
$$= 0.0050 - 0.0879 - 9.416 + 9.5$$
$$= 0.0011$$

$$F'(x_2) = 3 \times (0.1712)^2 - 6(0.1712) - 55$$
$$= 0.0879 - 1.0272 - 55$$
$$= -55.9393$$

$$x_3 = x_2 - \frac{F(x_2)}{F'(x_2)} = 0.1712 + \frac{0.0011}{-55.9393}$$
$$= 0.1712$$

The root of the equation is 0.1712 in [2, 3]

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$\therefore f(2) = 2^3 - 4(2) - 9 = 8 - 8 - 9$

$f(2) = -9$

$\therefore f(3) = 3^3 - 4(3) - 9 = 27 - 12 - 9 = 6$

$f(3) = 6$

Let $x_0 = 3$ be initial approximation

$\therefore$ by newton method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{6}{23} = 2.7392$$

$x_1 = 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$f(x_1) = 0.596$

$f'(x) = 3 \times (2.7392)^2 - 9 = 22.5096 - 4$

$= 18.5096$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.7392 - \frac{0.5966}{18.5096}$$

$x_2 = 2.7071$

$F(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$= 19.8386 - 10.8284 - 9$

$F(x_2) = 0.0102$

$F'(x_2) = 3(2.7071)^2 - 4 = 17.9851$

$x_3 = x_2 - \frac{F(x_2)}{F'(x_2)} = 2.7071 - \frac{0.0102}{17.9851} = 2.7071 - 0.0056$

$\boxed{x_3 = 2.7015}$

$F(x_3) = (2.7015)^3 - 4(2.7015) - 9 = 19.7158 - 10.806 - 9$

$F(x_3) = -0.0901$

$F'(x_0) = 3(2.7015) - 4 = 17.8943$

$x_4 = 2.7015 + \frac{0.0901}{17.8943} = 2.7015 + 0.0050$

$x_4 = 2.7065$

(iii) $F(x) = x^3 - 1.8x^2 - 10x + 17$ in $[1,2]$

$F'(x) = 3x^2 - 3.6x - 10$

$F(1) = (1)^3 - 1.8(1)^2 - 10 \times 1 + 17 = 1 - 1.8 - 10 + 17 = 6.2$

$\therefore F(1) = 6.2$

$\therefore F(2) = (2)^3 - (1.8)(2)^2 - 10(2) + 17 = 8 - 7.2 - 20 + 17$
$= -2.2$

$\therefore F(2) = -2.2$

Let $x_0 = 2$ be initial approximation

$x_{n+1} = x_n + \frac{F(x_n)}{F'x_n} \quad \therefore x_1 = x_0 - \frac{F(x_0)}{F'(x_0)} = 2 - \frac{-2.2}{5.2}$

$x_1 = 2 - 0.4230$
$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(7.577) - 10(1.577) - 1$

$= 3.9219 - 4.4769 - 15.77\cancel{+}17$

$f(x_1) = 0.6755$

$f'(x_1) = 3 \times (1.577)^2 - 5.6772 - 10 = -8.2614$

$f'(x_1) = -8.2614$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.577 + \frac{0.6753}{8.2164}$$

$= 1.577 + 0.0822$

$x_2 = 1.6592$

$f(x_2) = (1.6592)^3 - (1.8)(1.6592) - 10(1.6592) + 17$

$= 4.5672 - 4.9553 - 16.592 + 17$

$f(x_2) = 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10 = 8.2588 - 5.9731$

$f'(x_2) = -7.7143$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.6592 + \frac{0.6204}{7.7143} = 1.6592 + 0.026$$

$x_3 = 1.6618$

$f(x_3) = [1.6618]^3 - (1.8)(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$f(x_3) = 0.0604$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10 = 8.2847 - 5.9$

$f'(x_3) = -7.6477$

$$x_4 = x_3 - \frac{F(x_3)}{F'(x_3)} = 1.8618 + \frac{0.0004}{7.6977}$$

$$x_4 = 1.6618$$

root of equation 1.6618

10/01/2020

PRACTICAL 5

Q.1) Solve the following integration

i) $\int \frac{dx}{\sqrt{x^3+2x-3}}$ ii) $\int (4e^{3x}+1)\,dx$

iii) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$ iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int t^4 \cdot \sin(2t^4)\,dt$ vi) $\sqrt{x}\cdot(x^2-1)\,dx$

vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)\cdot dx$ viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

ix) $\int e^{\cos^2 x}\cdot \sin 2x \cdot dx$

x) $\int \frac{(x^2-2x)\,dx}{x^3-3x^2+1}$

ANSWER

(1) $$\int \frac{dx}{\sqrt{x^2+2x-3}} = \int \frac{dx}{\sqrt{x^2+2x-3}}$$

$$= \int \frac{dx}{\sqrt{x^2+2x+1-4}}$$

$$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$$

$$\therefore \int \frac{dx}{x^2+a^2} = \log(x+\sqrt{x^2+a^2} + c$$

$$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$$

$$= \log\left| x+1+\sqrt{(x+1)^2-(2)^2} \right| + c$$

$$\int (4e^{3x}+1)\,dx$$

$$= \int 4e^{3x}\cdot dx +)\,dx$$

$$= 4\int e^{3x}\cdot dx + \int dx$$

$$= \frac{4}{3} e^{3x} + x + c$$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx = \int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$

$= \int 2x^2\,dx - \int 3\sin x\,dx + \int 5\sqrt{x}\,dx.$

$= 2\frac{x^3}{5} - 3[-\cos x] + \int 5x^{\frac{1}{2}}\,dx.$

$= \frac{2}{3}x^3 + 3\cos x + 5 \times \frac{x^{3/2}}{3/2}$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10x^{3/2}}{3} + c$

iv) $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\,dx$

$= \int \frac{x^3}{\sqrt{x}}\,dx + \int \frac{3x}{\sqrt{x}}\,dx + 4\int \frac{1}{\sqrt{x}}\,dx$

$= \int x^{3-\frac{1}{2}}\,dx + 3\int x^{1-\frac{1}{2}}\,dx + 4\int x^{-\frac{1}{2}}\,dx$

$= \int x^{5/2}\,dx + 3\int x^{\frac{1}{2}}\,dx + 4\int x^{-1/2}\,dx$

$= \left[\frac{x^{\frac{7}{2}}}{7/2}\right] + 3\left[\frac{x^{3/2}}{3/2}\right] + 4 \times \left[\frac{x^{1/2}}{1/2}\right]$

$= \frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{2} + 8x^{1/2} = \frac{2x^{\frac{7}{2}}}{7} + 2x^{3/2} + 8$

$= \frac{2x^{7/2}}{7} + 2x^{\frac{3}{2}} + 8x^{\frac{1}{2}}$

PRACTICAL NO. 6

Topic: Application of integration & Numerical integration.

1) Find the length of the following curve.

1) $2 = t\sin t$, $y = 1-\cos t$ $F[0, 2\pi]$
for t belong to $[0, 2\pi]$

Sol^n: $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

$x = t - \sin t$

$\frac{dx}{dt} = t - \cos t.$

$y = 1 - \cos t$

$\frac{dy}{dt} = 0 - (-\sin t)$

$\frac{dy}{dt} = \sin t$

$L = \int_0^{2\pi} \sqrt{\cancel{= 2\cot + 1}} \cancel{dt} \sqrt{(t - \cos t + \sin t)^2 + (\sin t)^2} \, dt$

$\int_0^{2\pi} \sqrt{1 - 2\cos t + 1} \, dt$

$\int_0^{2\pi} \sqrt{\cancel{2-6}\ 2 - 2\cos t} \, dt.$

$\int_0^{2\pi} 2 \left| \sin \frac{t}{2} \right| dt \cdot \sin^2 \frac{t}{2} - \frac{1 - \cos t}{2}$

$\int_0^{2\pi} 2 \sin \frac{t}{2} \, dt$

$$\int_0^{2\pi} \left(-4\cos\left(\frac{t}{2}\right)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$$= 4 + 4 = 8$$

Q.2) $y = \sqrt{4 - x^2}$ $\quad x \in [-2, 2]$

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$\frac{dy}{dx} = 2\int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$$

$$= 2\pi$$

3) $y = x^{3/2}$ in $[0, 4]$

$$f'(x) = \frac{3}{2} x^{/2}$$

$$[f'(x)]^2 = \frac{9}{4} x$$

$$\lambda = \int_0^b \sqrt{1 + [f'(x)]^2}\, dx$$

$$= \int_0^x \sqrt{1 + \frac{9}{4}x}\, dx$$

put $u = 1 + \frac{9}{4}x$, $du = \frac{9}{4}dx$

$$L = \int_{1}^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u}\,du = \left[\frac{4}{9} \cdot \frac{2}{3}\left(u^{\frac{3}{2}}\right) \right]^{1+\frac{9}{4}x}$$

$$= \frac{8}{27}\left[\left(1 + \frac{9x}{4}\right) - 1\right]$$

$x = 3\sin t$ $Ty = 3\cos t$.

$\underline{\underline{\text{sol}^n}}$ $\frac{dx}{dt} = 3\cos t$

$\frac{dy}{dt} = -3\sin t$

$$l = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\,dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\,dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\,dt$$

$$= 3\int_0^{2\pi} x\,dt$$

$$= 3\,[x]_0^{2\pi}$$

$$= 3\,[2\pi - 0]$$

03

5.) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = [1,2]$

Soln. $\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{\left(\frac{y^4+1}{(2y)}\right)^2}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2}\int_1^2 y^2 dy + \frac{1}{2}\int_1^2 y^{-2} dy$

$= \frac{1}{2}\left(\frac{y^3}{3} - \frac{y^2}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= \frac{17}{12}$ units.

2) i) $\int_0^2 e^{x^2} dx$ with $n=4$

Soln $\int_0^2 e^{x^2} dx = 16.4526$

In each case the width of the sub interval be $\Delta x = \frac{2-0}{4} = \frac{1}{2}$ and so the sub interval will be $[0, 0.5]$ $[0.5, 1]$ $[1, 1.5]$, $[1.5, 2]$

~~and so the sub interval will~~

By simpson rule.

$$\int_{0}^{2} e^{x^2} dx = \frac{1/2}{3}\left(\frac{1}{0} + 4y_1 + 2y^2 + 4y_3^3 + 4y\, y_4\right)$$

$$\simeq \frac{1}{6}\ \frac{1/2}{3}\left(e^{0^2} + 4e(0.5)^2 + 2e(1)^2 + 4e(1.5)^2 + e^{(2)^2\cdot 1}\right.$$

$\simeq$ 17.3536.

2) $\int_{1}^{4} x^2 dx \quad n = 4$

$$\Delta x = \frac{4-0}{4} = 1$$

$$\int_{a}^{b} f(x)dx = \frac{\Delta x}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right]$$

$$= \frac{1}{3}\left[y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{1}{3}\left[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2\right]$$

$$= \frac{1}{3}\ \cancel{\frac{64}{3}}$$

3) $\int_{0}^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6$

Soln $\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$\pi 5/18$
y	0	0.4167	0.584	0.707	0.801	0.87

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx \simeq \frac{\Delta x}{3} \left(y_0 + 4\left(y_1 + y_3 + \frac{1}{5}\right) + 2\left(y_2 + y_4\right) + \right.$$

$$= \frac{\pi/18}{3} \left(0 + 4\left(0.4167 + 0.707 + 0.875\right) + 0. \right.$$

$$\simeq 2\left(0.584 + 0.801\right) + 0.930$$

$$\simeq 0.681$$

PRACTICAL NO. 7

Topic Differential equation

i) $x \cdot \frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^x}{x}$

$I.F = e^{\int P(x)\, dx}$

$= e^{\int 1/x \, dx}$

$= e^{\ln x}$

$I.F = x$

$y(I.F) = \int Q(x)(I.F)\, dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x \, dx + c$

$xy = e^x + c$

ii) $x \cdot \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \quad Q(x) = \frac{\cos x}{x^2}$

$I.F = e^{\int P(x) dx}$

$= e^{\int 2/x \, dx}$

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$= e^{2/x}\, x$

$= \ln x^2$

$I.F = x^2$

$y(I.F) = \int Q(x)(I.F)\,dx + C$

$= \int \cos x + c$

$x^2 y = \sin x + c$

iv) $x \cdot \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both side)

~~$P(x) = \frac{dQ}{dx}$~~

$P(x) = 3/x \quad Q(x) = \sin x / x^3$

$= e^{\int P(x)\,dx}$

$= e^{\int 3/x\, dx}$

$= e^{3/x}\, dx.$

$= e^{\ln x^3}$

$I.F = x^3$

$= \cancel{\frac{3v+3}{v+2}}$

$= \frac{3(v+1)}{v+2}$

$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$

$v + \log|x| = 3x + c$

$2x + 3y + \log|2x + 3y + 1| = 3x + c$

$3y = x - \log|2x + 3y + 1| + c$

$= \int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3x$

$v + \log|x| = 3x + c$

$2x + 3y + \log|2x + 3y + 1| = 3x + c$

$3y = x - \log|2x + 3y + 1| + c$

(4) $e^x \frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x} y = \frac{1}{e^x}$ $\quad$ $(\div \text{ by } e^x)$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \quad Q(x) = e^{-x}$

$\int P(x)\,dx$

$IF = e^{\int 2\,dx}$

$= e^{2x}$

$y = (I.F) = \int Q(x)\,(I.F)\,dx + c$

$y \cdot e^{2x} = \int e^{-x + 2x}\,dx + c$

$= \int e^x\,dx + c$

$= e^x + c$

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iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \qquad n=6$

$h = \frac{\frac{\pi}{3} - 0}{6} = \frac{\pi}{18}$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$	(
y	0	0.4167	0.4585	0.7071	0.8017	0.8752	0.9306	

$\int_0^{\pi/3} \sqrt{\sin x\, dx} \cdot \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4\right.$

$= \frac{\frac{\pi}{18}}{3}\left[0.4167 + 0.9306 + 4(0.4167 + 0.7071 + 0.97\right.$
$2(0.5845 + 0.801$

$= \frac{\pi}{54}\left[1.3473 + 4(1.999) + 2(1.3865)\right)$

$= \frac{\pi}{54}\left[1.3473 + 7.996 + 2.7783\right]$

$= \frac{\pi}{54} \times 12.1163$

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049$

viii) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$

put $2x + 3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\cancel{\frac{dv}{dx} - 2}\ \frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

vi) $\sec^2 x \cdot \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\cancel{\sec^2 x \cdot \tan y + \sec^2 y \tan x\, dy}$

$\sec^2 x \cdot \tan y\, dx = \sec^2 y \cdot \tan x\, dy$

$$\frac{\sec^2 x\, dx}{\tan x} = \frac{-\sec^2 y\, dy}{\tan y}$$

$$\int \frac{\sec^2 x\, dx}{\tan x} = -\int \frac{\sec^2 y\, dy}{\tan y}$$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x - \tan y| = c$

$\tan x \cdot \tan y \cancel{= e^c}$

vii) $$\frac{dy}{dx} = \sin^2(x - y + 1)$$

put $x - y + 1 = v$

Differentiating on both side

$x \cancel{= y} - x - y + 1 = v$

$$1 \cdot \frac{dy}{dx} = \frac{dv}{dx}$$

$$1 - \frac{du}{dx} = \frac{dy}{dx}$$

$$1 - \frac{du}{dx} = \sin^2 v$$

$$\frac{du}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{du}{\cos^2 v} = dx$$

$$\int \sec^2 v \, du = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

viii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

put $2x + 3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{du}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{du}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{du}{dx} = \frac{v-1+2v+4}{v+2}$$

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$$= \frac{3v+3}{v+2}$$

$$= 3\,\frac{(v+1)}{v+2}$$

$$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$$

$$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv = 3x$$

$$v + \log|x| = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c.$$

10/01/202

PRACTICAL: NO 8

TOPIC:- Using Euler's method find the following

1. $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$ Find $y(2)$

2. $\frac{dy}{dx} = 1 + y^2$ $y(0) = 0$, $h = 0.2$ find $y(1)$

3. $\frac{dy}{dx} = \frac{\sqrt{x}}{y}$, $y(0) = 1$ $h = 0.2$ Find $y(1)$

4. $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ Find $y(2)$

5. $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1)$ Find $y(1.2)$ with $h = 0.2$

① $\frac{dy}{dx} = y + e^x - 2$

$F(x, y) = y + e^x - 2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$F(x_n + y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.87	3.57435
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h \; F(x_n, y_n)$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28303
4	2	9.2831		

$\therefore$ By Euler's formula,

$y(2) = 9.2831$

① $\frac{dy}{dx} = 1 + y^2$

$F(x, y) \cancel{= 1 + y^2}$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Eulers e'itration formula,

aa

$y_{n+1} = y_n + h\, f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1665	0.6413
3	0.6	0.6413	1.4113	0.9236
4	0.8	0.9236	1.8530	1.2942

$\therefore$ By Eulers formula,

$y(1) = 1.2942$

(3) $\dfrac{dy}{dx} = \sqrt{\dfrac{x}{y}}$ $\quad y(0) = 1, \quad x_0 = 0, \quad h = 0.2$

using Euler's iteration formula.

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

$\frac{dy}{dx} = 3x^2 + 1$ $\quad y_0 = 2$, $x_0 = 1$, $h = 6$

for $h = 0.5$

using Euler's iteration formula,

$y_{n+1} = y_n + h f(x_n, y_n)$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	49	28.5
2	2	28.5		

By Eulers formula.

$y(2) = 28.5$

for $h = 0.25$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.3594
3	1.75	6.3594	10.1515	8.9048
4	2	8.9048		

By Euler's Formula.

$y(2) = 8.9048$

$$\frac{dy}{dx} = \sqrt{xy} + 2 \qquad y_0 = 1, \quad x_0 = 1, \quad h = 0.2$$

using Euler's iteration formula,

$$y_{n+1} = y_n + hf(x_n, y_n)$$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

$\therefore$ By Euler's formula,

$$y(1.2) = 1.6$$

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PRACTICAL 4

Q.1) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4,-1)$, Dinominator $\neq 0$

~~At (-4,-1)~~ By applying limit

$= \dfrac{(-4)^3 - 3(-1) + (-1)^2 - 1}{4(-1) + 5}$

$= \cancel{-66}\ \dfrac{-61}{9}$

ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

At $(2,0)$, Denominator $\neq 0$

$\therefore$ By applying limit,

$= \dfrac{(0+1)((2)^2 + 0 - 4(2))}{2 + 0}$

$= \dfrac{1(4 + 0 - 8)}{2}$

$= -\dfrac{4}{2}$

$= -2$

Ad

$$\lim_{(x,y,z) \to (1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 yz}$$

At $(1,1,1)$, Denominator $= 0$

$$\lim_{(x,y,z) \to (1,1,1)} \frac{x^2 - y^2 z^2}{x^3 - x^2 yz}$$

$$= \lim_{(x,y,z) \to (1,1,1)} \frac{(x - \cancel{yz})(x + yz)}{x^2 (x - \cancel{yz})}$$

$$= \lim_{(x,y,z) \to (1,1,1)} \frac{x + yz}{x^2}$$

on applying limit

$$= \frac{1 + 1(1)}{(1)^2}$$

$$= 2$$

(Q.2) 1) $f(x,y) = xye^{x^2+y^2}$

$\therefore f_x = \frac{\partial}{\partial x} (f(x,y))$

$= \frac{\partial}{\partial x} (xy e^{x^2+y^2})$

$ye^{x^2+y^2} (2x)$

$\therefore\ F_x = 2xye^{x^2+y^2}$

$F_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(xye^{x^2+y^2})$

$= xe^{x^2+y^2}(2y)$

$\therefore\ f_y = 2yxe^{x^2+y^2}$

ii) $f_x = e^x \cos y$

$F_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$F_y = -e^x \sin y$

iii) $F(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$Fx = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore\ \cancel{Ft}\ f_x = 3x^2y^2 - 6xy$

$f_y = \frac{\partial}{\partial y}(F(x,y))$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2 + y^3 + 1)$

$\therefore\ F_y = 2x^3y - 3x^2 + 3y^2$

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Q.3) i) $F(x,y) = \frac{2x}{1+y^2}$

$$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2 \ (2) \quad (1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2}{(1+y^2)^2}$$

$$= \frac{2(\cancel{1+y^2})}{(\cancel{1+y^2})(1+y^2)^2}$$

$$= \frac{2}{1+y^2}$$

At $(0,0)$

$$= \frac{2}{1+0}$$

$$= 2$$

$F_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)}$$

$$= \frac{1+y^2 (0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{\cancel{-4y} - 4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= -\frac{4(0)(0)}{(1+0)^2}$$

$$= \underline{\underline{0}}$$

y) i) $F(x,y) = \frac{y^2 - xy}{x^2}$

$$F_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= -\frac{x^2 y - 2x(y^2 - xy)}{x^4}$$

$$F_y = \frac{2y - x}{x^2}$$

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$$f_{xx} = \frac{\partial}{\partial x}\left(-x^2y - 2x\left(y^2 - xy\right)\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}\left(-x^2y - 2xy^2 + 2x^2y\right)\right) - \left(-x^2y - 2xy + 2x^2y\right)\frac{\partial}{\partial x}\left(x^4\right)}{\left(x^4\right)^2}$$

$$= \frac{x^4\left(-2xy - 2y^2 + 4xy\right) - 4x^3\left(-x^2y - 2xy + 2x^2y\right)}{x^6}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2\frac{\partial}{\partial x}\left(2y - x\right) - \left(2y - x\right)\frac{\partial}{\partial x}\left(x^2\right)}{\left(x^2\right)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4}$$

$\therefore$ From (3) & (4)

$\therefore f_{xy} = f_{yx}$

(iii) $f(x,y) = x^3 + 3x^2 y^2 - \log(x^2+1)$

$$f_x = \frac{\partial}{\partial x}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f_{xx} = 6x + 6y^2 - \left(x^2+1\,\frac{\partial(2x)}{\partial x} - 2x\,\frac{\partial}{\partial x}(x^2+1)\right)$$

$$= 6x + 6y^2 - \frac{\left(2(x+1) - 4x^2\right)}{(x^2+1)^2} \quad - (1)$$

$$f_{yy} = \frac{\partial}{\partial y}(6x^2y) \quad - (2)$$

$$= 6x$$

$$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 6 + 12xy - 0$$

$$= 12xy$$

$$F_{yx} = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12$$

$$F_{yx} = \frac{\partial}{\partial x}(6x^2y)$$

$$= 12xy$$

From (3) & (4)

$$\therefore F_{xy} = F_{yx} \quad - (4)$$

$f(x,y) = \sin(x,y) + e^{x+y}$

$f_y = x\cos(xy) + e^{x+y}$

$= x\cos(xy) + e^{x+y}$

$f_x = y\cos(xy) + e^{x+y}(1)$

$= y\cos(xy) + e^{x+y}$

$\therefore\ f_{xx} = \frac{\partial}{\partial x}\left(y\cos(xy) + e^{x+y}\right)$

$= -y\sin(xy)\cdot(y) + e^{x+y}(1)$

$= -y^2\sin(xy) + e^{x+y}$ — ①

$f_{yy} = \frac{\partial}{\partial y}\left(x\cos(xy) + e^{x+y}\right)$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ ①

$f_{yy} = \frac{\partial}{\partial y}\left(x\cos(xy) + e^{x+y}\right)$

$= -x\sin((xy)(x) + e^{x+y}$

$= -x^2\sin(xy) + e^{x+y}$ — ②

$f_{xy} = \frac{\partial}{\partial y}\left(y\cos(xy) + e^{x+y}\right)$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — ③

$f_{yx} = \frac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$

$f_{yx} = \frac{\partial}{\partial x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — (4)

$\therefore$ from (3) & (4)

$f_{xy} \neq f_{yx}$

5) i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \frac{x+y}{\sqrt{2}}$

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ii) $f(x,y) = 1 - x + y\sin x$ at $(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{2}, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$

$f_x = 0 - 1 + y\cos x$

$f_y = 0 - 0 + \sin x$

f_x at $(\frac{\pi}{2}, 0) = -1 + 0 = -1$

f_y at $(\frac{\pi}{2}, 0) = \sin$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} + (-1)(x - \frac{\pi}{2}) + 1(y-0)$

$= 1 - \cancel{\frac{\pi}{2}} - x + \cancel{\frac{\pi}{2}} + y$

$= 1 - x + y$

iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$

$f_y = 0 + \frac{1}{y}$

F_x at $(1,1) = 1$

F_y at $(1,1) = 1$

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$\therefore L(x,y) = F(a,b) + F_x(a,b)(x-a) + F_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2$

1) i) $f(x,y) = x + 2y - 3$

Here

$u = 3i \cdot j$ is not a unit vector

$\bar{u} = 3\bar{i} - \bar{j}$

$|u| = \sqrt{10}$

$\therefore$ unit vector along u is $\frac{\bar{u}}{|u|} \frac{-1}{\sqrt{10}} (3\bar{i} - \bar{j})$

$= \frac{1}{\sqrt{10}} (3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

Now,

$f(a+hu) = f((1-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right)$

$= 1 + \frac{3h}{\sqrt{10}} + 2\left(-1, \frac{-1}{\sqrt{10}}\right) - 3$

$= \cancel{\text{o}} 1 - 2 - 3 + \frac{3h}{\sqrt{10}} - \frac{2h}{\sqrt{10}}$

$= -4 + \frac{h}{\sqrt{10}}$

$=$

$\therefore f'(u) = \lim_{h \to 0} \dfrac{f(a+hu) - f(a)}{h}$

$\lim_{h \to 0} \dfrac{-4 + \frac{h}{\sqrt{10}} - (-4)}{h}$

$= \lim_{h \to 0} \dfrac{h}{h\sqrt{10}}$

$= \dfrac{1}{\sqrt{10}}$

ii.) $f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

Here
$u = i + 5j$ is not a unit vector
$\bar{u} = \bar{i} + 5\bar{j}$
$|\bar{u}| = \sqrt{26}$

$\therefore$ unit vector along u is $\dfrac{\bar{u}}{|\bar{u}|} = \dfrac{1}{\sqrt{26}}(i+5$

$= \dfrac{1}{\sqrt{26}} (1,5)$

$= \left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)$

Now,
$f(a+hu) = f\left((3,4) + h\left(\frac{1}{\sqrt{26}} + \frac{5}{\sqrt{26}}\right)\right.$

$$= f\left(3+\frac{h}{\sqrt{26}},\ 4+\frac{5h}{\sqrt{26}}\right)$$

$$= \left(4+\frac{5h}{\sqrt{26}}\right)^2 - 4\left(3+\frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{26h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{25h^2}{26} + \frac{36}{\sqrt{26}} + 5 - 5}{h}$$

$$= \lim_{h\to 0} \frac{25h^2}{26} + \frac{36h}{\sqrt{26}}$$

$$= \lim_{h\to 0} \frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$= \frac{25(0)}{26} + \frac{36}{\sqrt{26}}$$

$$= \frac{36}{\sqrt{26}}$$

iii) $f(x,y) = 2x + 3y$ $a(1,2)$, $u = 3i + 4j$

Here,

$u = 3i + 4j$ is not a unit vector

$\bar{u} = 3\bar{i} + 4\bar{j}$

$= |\bar{u}| = \sqrt{25} = 5$

unit vector along $u = \frac{\bar{u}}{|\bar{u}|} = \frac{1}{5}(3i + 4j)$.

$= \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

Now,

$f(a + hu) = f((1,2) + (\frac{3}{5}, \frac{4}{5})$

$= f(1 + \frac{3h}{5} + 3(2 + \frac{4h}{5})$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= 8 + \frac{18h}{5}$

$D_u = f(u) = \lim_{h \to 0} \frac{f(a + hu) - f(a)}{h}$

$$\lim_{h\to 0} \frac{8 + \frac{18h}{5} - 8}{h}$$

$$= \lim_{h\to 0} \frac{18h}{5h}$$

$$= \frac{18}{5}$$

ii) $f(x,y) = x^y + y^x \qquad a = (1,1)$

$F(x) = y(x^{y-1}) + y^x \log y$

$F_y = x(y^{x-1}) + x^y \log x.$

$\nabla F(x,y) = (F_x, F_y)$

$= (yx^{y-1} + y^x \log y,\ xy^{x-1} + x^y \log x)$

$\nabla F(x,y)$ at $(1,1)$

$= (1,(1)^0 + 1^1 \log(1),\ (1)^{(1-1)} + \log(1))$

$= (1,1)$

ii) $F(x,y) = (\tan^{-1} x) \cdot y \qquad a = (1,-1)$

$F_x = y^2 \left(\frac{1}{1+x^2}\right) = \frac{y^2}{1+x^2}$

$F_y = 2y + \tan^{-1} x$

$\nabla F(x,y) = (F_x, F_y)$

$= \left(\frac{y^2}{1+x^2}, - F_y \tan^{-1} x\right)$

$\nabla F(x,y)$ at $(1,-1)$

$= \left(\frac{(-1)^2}{1+(-1)^2}, 2(-1) \tan^{-1}(-1)\right)$

$= \left(\frac{1}{2}, -\frac{2\pi}{4}\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

iii) $F(x,y,z) = xyz - e^{x-y+z}$

$F_x = yz - e^{x+y+z}$

$F_y = xz - e^{x-y+z}$

$F_z = xy - e^{x+y+z}$

$\nabla F(x,y,z) = (F_x, F_y, F_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy$

$\nabla f(x,y,z)$ at $(1,-1,0)$

$= (-1(0) \cdot e^{1-1+0}, 1(0) - e^{1-1+0}, 1(1) - e^{1-1+0})$

$= (0-1, 0-1, -1-1)$

$= (-1, -1, -2)$

Q3) i) $x^2 \cos y + e^{xy}$ at $(1,0)$

$F(x,y) = x^2 \cos y + e^{xy} - 2$

$F_x = 2x \cos y + y e^{xy}$

$(x_0, y_0) = (1,0)$

F_x at $(1,0) = 2(1) \cos 0 + 0$

F_y at $(1,0) = -(1)^2 \sin(0) + 1(e)^{1(0)}$

$= 1$

$F_x(x-x_0) + F_y(y-y_0) = 0$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ equation of tangent

Now,

For equation of Normal

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$bx + ay + d = 0$

$x + 2y + d = 0$

$(1) + 2(0) + d = 0$ At $(1, 0)$

$1 + d = 0$

$d = -1$

$\therefore x + 2y - 1 = 0$ — equation of Normal

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$F(x, y) = x^2 + y^2 - 2x + 3y + 2$

$F_x - 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$\therefore F_x$ at $(2, -2) = 2(2) - 2$

$= 2$

$F_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

F_y at $(2, -2) = 2(-2) + 3$

$= 2$

$\therefore$ equation of tangent

$F_x (x - x_0) + F_y (y - y_0) = 0$

$2(x - 2) + (-1)(y + 2) = 0$

$2x - 4 - y - 2 = 0$

$2x - y - 6 = 0$ — equation of tangent

equation of Normal.

$x + ay + d = 0$

$-x + 2y + d = 0$

$(-2) + 2(-2) + d = 0$ at $(2,-2)$

$-2 - 4 + d = 0$

$d = 6$

$\therefore -x + 2y + 6 = 0$ — equation of Normal

ex) i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f(x,y,z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x - 0 + 0 + z - 0$

$= 2x + z$

$\therefore f_x$ at $(2,1,0) = 2(2) + 0$

$f_y = -2z + 3 + 0 - 0$

$= -2z + 3$

f_y at $(2,1,0) = -2(0) + 3$

$f_z = 0 - 2y + 0 + x - 0$

$= -2y + x$

f_z at $(2,1,0) = -2(1) + 2$

$= 0$

equation of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$4(x-2) + 3(y-1) + 0(z-0) = 0$

$4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0$ — equation of tangent

equation of Normal

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z} \quad \text{(equation of Normal}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z-0}{0}$$

ii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$f(x,y,z) = 3xyz - x - y + z + 4$

$f_x = 3yz - 1 - 0 + 0 + 0$ $\quad$ f_x at $(1,-1,2)$

$= 3yz - 1$ $\quad$ $= 3(1)(2) - 1 =$

$f_y = 3xz - 0 - 1 + 0 + 0$ $\quad$ f_y at $(1,-1,2) = 3(1)(2)(-1)$

$= 3xz + 1$ $\quad$ $= -5$

$f_z = 3xy - 0 - 0 + 1 - 0$

$= 3xy + 1$ $\quad$ f_z at $(1,-1,2) = 3(1)(-1)$

$= -2$

equation of tangent

$f_x(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$

$-7(x-1) + 5(y+1) + (-2)(z-2) = 0$

$= \cancel{-7x+} -7x + 7 + 5y + 5 - 2z + 4 = 0$

$y - 2z + 11 = 0$ — equation of tangent

equation of Normal.

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$\frac{x-1}{-7} = \frac{y+1}{2} = \frac{z-2}{-2}$$ — equation of normal

Q.5) i) $f(x,y) = 3x^2+y^2-3xy+6x-4y$

$\therefore f_x = 6x+0-3y+6-0$ — ①

$= 6x-3y+6$

$f_y = 2y-3x+0-4$

$= 2y-3x-4$ — ②

$f_x = 0$

$6x-3y+6=0$

$3(2x-y+2)=0$

$\cancel{2x}(2x-y+2=0$

$2x-y=-2$ ③

$f_y=0$

$2y-3x-4=0$

$2y-3x=4$ ④

multiplying ③ by 2 & subtracting ④ from ③

$$\begin{array}{rrr} 4x & -2y & =-4 \\ -2y & -3x & -4 \\ \hline 7x & & =0 \end{array}$$

Subt. substituting value of in (2)

$x(0) - y = -2$

$-y = -2$

$y = 2$

$\therefore$ critical points are $(0,2)$

Now,

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = 3$

$rt - s^2 = 12 - 9$

$3 > 0$

Here, $r > 0$ and $rt - s^2 > 0$

f has minimum at $(0,2)$

$F(0,2) = \cancel{3\,ces}\ 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 - 8$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy - 0$

$= 8x^3 + 6xy$

$f_y = 0 + 3x^2 - 2y$

$= 3x^2 - 2y$

Now,

$f_x = 0$

$4x^3 + 6xy = 0$

$x(4x^2 + 6xy) = 0$

$4x^2 + 6y = 0$ (1)

$f_y = 0$

$3x^2 - 2y = 0$

$3x^2 - 2y = 0$ (2)

multiply in (1) by (3) & (2) by (4)

subtracting (2) from (1)

$$\begin{aligned} 12x^2 + 18y &= 0 \\ -12x^2 - 8y &= 0 \\ \hline 24y &= 0 \end{aligned}$$

$y = 0$ - (3)

Substituting (3) in (2)

$x^2 - 2(0) = 0$

$3x^2 = 0$

$x^2 = 0$

$x = 0$ (4)

critical points are $(0, 0)$.

Now,

$r = f_{xx} = 12x^2 + 6y$

$t = f_{yy} = -2$

$s = f_{xy} = 6x$

$rt - s^2 = (24x^2 + 6y)(-2) - (6x)^2$

$= -48x^2 - 12y - 36x^2$

$= \cancel{-8x} \; -84x^2 - 12y$

at $(0,0)$

$r = 24(0)^2 + 6(0)$
$= 0$

$s = 6(0) = 0$

$rt - s^2 = -84(0)^2 - 12(0) = 0$

$r = 0$ & $rt - s^2 = 0$

$\therefore$ Nothing can be said