Electrical Engineering and Measurement (EE1101) (Errors)

Presented

by

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Measurement Error

❖ No measurement can be made with perfection and accuracy, but it is important to find out what the accuracy actually is and how different errors have entered into the measurement.

❖ Error occurs due to several sources like human carelessness in taking reading, calculating and in using instrument etc.

❖ Some of the time error is due to instrument and environment effects. Errors come from different sources and are classified in three types:

1. Gross Error

2. Systematic Errors

3. Random Errors

Measurement Error

Gross Error

- The gross error occurs due to the human mistakes in reading or using the instruments. These errors cover human mistakes like in reading, calculating and recordings etc. In sometimes it occurs due to incorrect adjustments of instruments.
- The complete elimination of gross errors is impossible, but we can minimize them by the following ways:

1. It can be avoided by taking care while reading and recording the measurement data.
2. Taking more than one reading of same quantity. At least three or more reading must be taken by different persons.

Measurement Error

Systematic Errors

A systematic error is divided in three different categories: instrumental errors, environmental errors and observational errors.

Measurement Error

Instrumental Errors

The instrument error generate due to instrument itself. It is due to the inherent shortcomings in the instruments, misuse of the instruments, loading effects of instruments.

For example in the D' Arsonval movement friction in bearings of various moving components may cause incorrect readings.

There are so many kinds of instrument errors, depending on the type of instrument used.

Instrumental errors may be avoided by

(a) Selecting a suitable instrument for the particular measurement application

(b) Applying correction factors after determining the amount of instrumental error

(c) Calibrating the instruments against a standard.

Measurement Error

Environmental Errors

Environmental errors arise as a result of environmental effects on instrument. It includes conditions in the area surrounding the instrument, such as the effects of changes in temperature, humidity, barometric pressure or of magnetic or electrostatic fields.

For example when making measurements with a steel rule, the temperature when the measurement is made might not be the same as that for which the rule was calibrated.

Environmental errors may be avoided by

(a) Using the proper correction factor and information supplied by the manufacturer of the instrument.

(b) Using the arrangement which will keep the surrounding condition constant like use of air condition, temperature controlled enclosures etc.

(c) Making the new calibration under the local conditions.

Measurement Error

Observational Errors

These errors occur due to carelessness of operators while taking the reading. There are many sources of observational errors such as parallax error while reading a meter, wrong scale selection, the habits of individual observers etc.

To eliminate such observational errors, one should use the instruments with mirrors, knife edged pointers, etc. Now a day's digital display instruments are available, which are much more versatile.

Measurement Error

Random Errors

- These errors are due to unknown causes and occur even when all systematic errors have been accounted for. In some experiments some random errors usually occur, but they become important in high-accuracy work.
- These errors are due to friction in instrument movement, parallax errors between pointer and scale, mechanical vibrations, hysteresis in elastic members etc.
- These errors are of variable magnitude and sign and do not obey any known law. The presences of random errors become evident when different results are obtained on repeated measurements of one and the same quantity

Measurement Error

Absolute Error

Measurement is the process of comparing an unknown quantity with an accepted standard quantity.

Absolute error may be defined as the difference between the measured value of the variable and the true value of the variable.

$$\delta A = A_m - A$$

δA=absolute error

A_m= expected value

A=measured value

Measurement Error

Relative Error

The relative error is the ratio of absolute error to the true value of the quantity to be measured. Mathematically, the relative error can be expresses as

$$\epsilon_r = \frac{Absolute\ error}{true\ value} = \frac{\delta A}{A}$$

$$\%\text{Error} = \frac{\delta A}{A} \times 100 = \frac{A_m - A}{A} \times 100$$

$$\text{Relative accuracy} = 1 - \text{relative error} = 1 - \frac{A_m - A}{A}$$

Measurement Error

Example 1. *A voltage has a true value of 1.50 V. An analog indicating instrument with a scale range of 0-2.50 V shows a voltage of 1.46 V. What is the value of absolute error?*

Measurement Error

Solution. Given:

$A_m = 1.46, A = 1.5$, we know that the absolute error,

$$\delta A = 1.46 - 1.50 = -0.04 \text{ V}$$

Measurement Error

Example 2. *The expected value of the voltage across a resistor is 80 V. However, the measurement gives a value of 79 V. Calculate (i) absolute error, (ii) % error, (iii) relative accuracy, and (iv)% of accuracy.*

Measurement Error

Answer

(i) absolute error= 1V

(ii) % error= 1.25%

(iii) relative accuracy=0.9875

(iv) % of accuracy=98.75%.

Measurement Error

Limiting error:

The manufacturer has to specify the deviations from the nominal value, the limit of these deviations from the specified value are defined as limiting error or guarantee errors.

$$\delta A = A_m - A$$

Measurement Error

The value of capacitance of a capacitor is specified as 1μF±5% by the manufacturer. Find the limits between which the value of the capacitance is guaranteed.

Measurement Error

Solution : The guaranteed value of the capacitance lie within the limits :

$$A_a = A_s(1 \pm \epsilon_r) = 1 \times (1 \pm 0{\cdot}05) = 0{\cdot}95 \text{ to } 1{\cdot}05 \ \mu F.$$

Measurement Error

Solution : The guaranteed value of the capacitance lie within the limits :

$$A_a = A_s(1 \pm \epsilon_r) = 1 \times (1 \pm 0{\cdot}05) = 0{\cdot}95 \text{ to } 1{\cdot}05\ \mu F.$$

Measurement Error

Example A 0–150 V voltmeter has a guaranteed accuracy of 1 percent of full scale reading. The voltage measured by this instrument is 75 V. Calculate the limiting error in percent. Comment upon the result.

Measurement Error

Solution : The magnitude of limiting error of instrument,

$$\delta A = \epsilon_r A_s$$

$\therefore$ $$\delta A = 0{\cdot}01 \times 150 = 1{\cdot}5 \text{ V}.$$

The magnitude of the voltage being measured is 75 V.

The relative error at this voltage is $\epsilon_r = \dfrac{\delta A}{A_s} = \dfrac{1{\cdot}5}{75} = 0{\cdot}02$

Therefore, the voltage being measured is between the limits of

$$A_a = A_s(1 \pm \epsilon_r)$$
$$= 75(1 \pm 0{\cdot}02) \text{ V} = 75 \pm 1{\cdot}5 \text{ V}.$$

The percentage limiting error is : $\% \, \epsilon_r = \dfrac{1{\cdot}5}{75} \times 100 = 2$ percent.

Measurement Error

Example A wattmeter having a range 1000 W has an error of $\pm 1\%$ of full scale deflection. If the true power is 100 W, what would be the range of readings ?

Suppose the error is specified as percentage of true value, what would be the range of the readings.

Measurement Error

Solution : When the error is specified as a percentage of full scale deflection, the magnitude of limiting error at full scale

$$=\pm\frac{1}{100}\times 1000=\pm 10\ \text{W}.$$

Thus the wattmeter reading when the true reading is 100 W may be 100 ± 10 W *i.e.*, between 90 to 110 W.

The relative error $=\frac{\pm 10}{100}\times 100=\pm 10\%$.

Now suppose the error is specified as percentage of true value.

The magnitude of error $=\pm\frac{1}{100}\times 100=\pm 1$ W.

Therefore the meter may read 100 ± 1 W or between 99 to 101 W.

Measurement Error

Combination of quantities with limiting error:

(*i*) Sum of two Quantities. Let y be the final result which is the sum of measured quantities u and v.

$\therefore \qquad y = u + v.$

The relative increment of the function is given by

$$\frac{dy}{y} = \frac{d(u+v)}{y} = \frac{du}{y} + \frac{dv}{y}.$$

Expressing the result in terms of relative increment of the component quantities

$$\frac{dy}{y} = \frac{u}{y} \cdot \frac{du}{u} + \frac{v}{y} \cdot \frac{dv}{v}.$$

If the errors in the component quantities are represented by $\pm\delta u$ and $\pm\delta v$ then corresponding limiting error δy in y is given by :

$$\frac{\delta y}{y} = \pm\left(\frac{u}{y} \cdot \frac{\delta u}{u} + \frac{v}{y} \cdot \frac{\delta v}{v}\right)$$

The above equation shows that the resultant relative error is equal to the sum of the products formed by multiplying the individual relative errors by the ratio of each term to the function.

Measurement Error

(*ii*) **Difference of two Quantities.**

Let $$y = u - v$$

$$\therefore \quad \frac{dy}{y} = \frac{du}{y} - \frac{dv}{y}.$$

Expressing the result in terms of relative increments of component quantities

$$\frac{dy}{y} = \frac{u}{y} \cdot \frac{du}{u} - \frac{v}{y} \cdot \frac{dv}{v}.$$

If the errors in u, v are $\pm\delta u$ and $\pm\delta v$ respectively, the signs may be interpreted to give the worst possible discrepancy *i.e.* when the error in u is $+\delta u$, the error in v is $-\delta v$ and vice versa, then the corresponding relative limiting error δy in y is given by

$$\frac{\delta y}{y} = \pm\left(\frac{u}{y} \cdot \frac{\delta u}{u} + \frac{v}{y} \cdot \frac{\delta v}{v}\right)$$

(*iii*) **Sum or Difference of more than two Quantities.** The sum or difference of more than two quantities may be treated in a similar way.

If we have $$y = \pm u \pm v \pm w$$

Then the limiting error is given by:

$$\frac{\delta y}{y} = \pm\left(\frac{u}{y} \cdot \frac{\delta u}{u} + \frac{v}{y} \cdot \frac{\delta v}{v} + \frac{w}{y} \cdot \frac{\delta w}{w}\right)$$

Measurement Error

(*iv*) **Product of two Components.**

Let $y = uv$

$\therefore \quad \log_e y = \log_e u + \log_e v$

Differentiating with respect to y

$$\frac{1}{y} = \frac{1}{u} \cdot \frac{du}{dy} + \frac{1}{v} \cdot \frac{dv}{dy} \qquad \text{or} \qquad \frac{dy}{y} = \frac{du}{u} + \frac{dv}{v}.$$

Representing the errors in u and v as $\pm\delta u$ and $\pm\delta v$ respectively, the error δy in y is given by :

$$\frac{\delta y}{y} = \pm\left(\frac{\delta u}{u} + \frac{\delta v}{v}\right)$$

Thus from above we conclude that the relative limiting error of product of terms is equal to the sum of the relative errors of terms.

Measurement Error

(v) **Quotient.** Let $y = \frac{u}{v}$

$\therefore$ $\log_e y = \log_e u - \log_e v.$

Differentiating with respect to y, we have

$$\frac{1}{y} = \frac{1}{u} \cdot \frac{du}{dy} - \frac{1}{v} \cdot \frac{dv}{dy} \qquad \text{or} \qquad \frac{dy}{y} = \frac{du}{u} - \frac{dv}{v}.$$

Representing the errors in u and v as $\pm\delta u$ and $\pm\delta v$ respectively, the relative error in y is

$$\frac{\delta y}{y} = \pm\frac{\delta u}{u} \mp \frac{\delta v}{v}.$$

Thus maximum possible error occurs when $\frac{\delta u}{u}$ is + ve and $\frac{\delta v}{v}$ is $-$ve or vice versa.

$\therefore$ Relative limiting error in is $\frac{\delta y}{y} = \pm\left(\frac{\delta u}{u} + \frac{\delta v}{v}\right)$

The above result is the same as the corresponding result for the product of two quantities.

Measurement Error

(*vi*) **Product or Quotient of more than two Quantities.**

Let $$y=uvw \quad \text{or} \quad y=\frac{u}{vw} \quad \text{or} \quad y=\frac{1}{uvw}.$$

Considering Eqns. 3·12 and 3·14, we have relative limiting error for y

$$\frac{\delta y}{y}=\pm\left(\frac{\delta u}{u}+\frac{\delta v}{v}+\frac{\delta w}{w}\right)$$

(*vii*) **Power of a Factor**

Let $y=u^n$ $\quad\therefore\quad \log_e y=n\log_e u.$

Differentiating with respect to y,

$$\frac{1}{y}=n.\frac{1}{u}.\frac{du}{dy} \quad \text{or} \quad \frac{dy}{y}=n\frac{du}{u}.$$

Hence, the relative limiting error of y is $\dfrac{\delta y}{y}=\pm n\dfrac{\delta u}{u}$

Measurement Error

(*viii*) **Composite Factors**

Let $$y = u^n \cdot v^m$$

$\therefore$ $$\log_e y = n \log_e u + m \log_e v$$

or $$\frac{1}{y} = \frac{n}{u} \cdot \frac{du}{dy} + \frac{m}{v} \frac{dv}{dy}$$

or $$\frac{dy}{y} = n \frac{du}{u} + m \frac{dv}{v}$$

$\therefore$ Relative limiting error of y is $\dfrac{\delta y}{y} = \pm\left(n \dfrac{\delta u}{u} + m \dfrac{\delta v}{v} \right)$

Measurement Error

Example Three resistors have the following ratings :

$$R_1 = 37\ \Omega \pm 5\%, \qquad R_2 = 75\ \Omega \pm 5\%, \qquad R_3 = 50 \pm 5\%.$$

Determine the magnitude and limiting error in ohm and in percent of the resistance of these resistances connected in series.

Measurement Error

Solution : The values of resistances are :

$$R_1 = 37 \pm \frac{5}{100} \times 37 = 37 \pm 1{\cdot}85\ \Omega$$

$$R_2 = 75 \pm \frac{5}{100} \times 75 = 75 \pm 3{\cdot}75\ \Omega$$

$$R_3 = 50 \pm \frac{5}{100} \times 50 = 50 \pm 2{\cdot}5\ \Omega$$

The limiting value of resultant resistance.

$$R = (37 + 75 + 50) \pm (1{\cdot}85 + 3{\cdot}75 + 2{\cdot}5) = 162 \pm 8{\cdot}1\ \Omega.$$

$\therefore$ Magnitude of resistance $= 162\ \Omega$

Error in ohm $= \pm 8{\cdot}1\ \Omega$

Percent limiting error $= \pm \frac{8{\cdot}1}{162} \times 100 = \pm 5\%$.

Measurement Error

Three resistors have values $100\Omega \pm 5\%$, $200\Omega \pm 5\%$ and $700\Omega \pm 5\%$ respectively. Determine the magnitude and limiting error in ohm and in the percent of the resistance of these resistances connected in series.

Measurement Error

Answer:

Magnitude of res=1000Ω,

Error in ohm=±50

Percent limiting error =50/1000*100=5%

Measurement Error

Example The resistance of a circuit is found by measuring current flowing and the power fed into the circuit. Find the limiting error in the measurement of resistance when the limiting errors in the measurement of power and current are respectively $\pm 1.5\%$ and $\pm 1.0\%$.

Measurement Error

Solution : Resistance $R = \frac{\text{(power)}}{\text{(current)}^2} = \frac{P}{I^2} = PI^{-2}$.

From Eqn. 3·16, relative limiting error in meaurement of resistance is

$$\frac{\delta R}{R} = \pm\left(\frac{\delta P}{P} + 2\ \frac{\delta I}{I}\right)$$

$$= \pm(1{\cdot}5 + 2\times 1{\cdot}0) = \pm 3{\cdot}5\%.$$

Measurement Error

Example The solution for the unknown resistance for a Wheatstone bridge is

$$R_x = \frac{R_2 R_3}{R_1}$$

where $R_1 = 100 \pm 0.5\%\ \Omega$, $R_2 = 1000 \pm 0.5\%\ \Omega$, $R_3 = 842 \pm 0.5\%\ \Omega$

Determine the magnitude of the unknown resistance and the limiting error in percent and in ohm for the unknown resistance R_x.

Measurement Error

Solution : Unknown resistance

$$R_x = \frac{R_2 R_3}{R_1} = \frac{1000 \times 842}{100} = 8420 \ \Omega$$

Relative limiting error of unknown resistance is :

$$\frac{\delta R_x}{R_x} = \pm\left(\frac{\delta R_2}{R_2} + \frac{\delta R_3}{R_3} + \frac{\delta R_1}{R_1}\right) = \pm(0{\cdot}5 + 0{\cdot}5 + 0{\cdot}5) = \pm 1{\cdot}5\%$$

Limiting error in ohm is :

$$= \pm\frac{1{\cdot}5}{100} \times 8420 = \pm 126{\cdot}3 \ \Omega$$

Guaranteed values of resistance are between.

$$8420 - 126{\cdot}3 = 8293{\cdot}7 \ \Omega, \qquad 8420 + 126{\cdot}3 = 8546{\cdot}3 \ \Omega$$

Measurement Error

Example A 4-dial decade box has

decade a of $10 \times 1000\Omega \pm 0{\cdot}1\%$

decade b of $10 \times 100\Omega \pm 0{\cdot}1\%$

decade c of $10 \times 10\Omega \pm 0{\cdot}5\%$

decade d of $10 \times 1\Omega \pm 1{\cdot}0\%$

It is set at 4639 Ω. Find the percentage limiting error and the range of resistance value.

Measurement Error

Solution : Decade a is set at 4000 Ω

Therefore, $\text{error} = \pm 4000 \times \frac{0.1}{100} = \pm 4\ \Omega$

Decade b is set at 600 Ω.

Therefore, $\text{error} = \pm 600 \times \frac{0.1}{100} = \pm 0.6\ \Omega$

Similarly

error in decade $c = \pm 30 \times \frac{0.5}{100} = \pm 0.15\ \Omega$

error in decade $d = \pm 9 \times \frac{1}{100} = \pm 0.09\ \Omega$

Total error $= \pm(4 + 0.6 + 0.15 + 0.09) = 4.84\ \Omega$

Relative limiting error $\epsilon_r = \pm \frac{4.84}{4639} = \pm 0.00104$

Percentage limiting error $\%\ \epsilon_r = \pm(0.00104 \times 100) = \pm 0.104\%$

Limiting values of resistance $A_a = 4639(1 \pm 0.00104)$

$= 4639 \pm 5\ \Omega = 4634\ \Omega\ 4644\ \Omega.$

Measurement Error

Known Errors. When the error of a quantity or an instrument is known the effect of this error, when combined with other errors, can be computed in a manner similar to the combinations of limiting errors. But the difference is that in case of known errors the signs of relative errors are given and must be preserved in the calculations.

Example A resistance is rated at 3200 Ω and the current flowing through this is 64 mA. (*a*) Compute the power loss in the resistor. (*b*) It was later found that the resistance of the resistor was 0·2 percent greater than the specified resistance and the ammeter read 0·75 percent more than the true current. Determine the known error in the computed power in part (*a*).

Measurement Error

Solution : (*a*) Power consumed $P = I^2R = (64 \times 10^{-3})^2 \times 3200 = 13{\cdot}1$ W.

(*b*) Relative error in power

$$\frac{\delta P}{P} = \left(\frac{2\delta I}{I} + \frac{\delta R}{R}\right) = (2 \times 0{\cdot}75 + 0{\cdot}2) = 1{\cdot}7\% \text{ more.}$$

Measurement Error

Example Current was measured during a test as 30·4 A, flowing in a resistor of 0·105 Ω. It was discovered later that the ammeter reading was low by 1·2 percent and the marked resistance was high by 0·3 percent. Find the true power as a percentage of the power that was originally calculated.

Measurement Error

Solution : True value of $I = 30{\cdot}4(1-0{\cdot}012) = 30{\cdot}035$ A

True value of $R = 0{\cdot}105(1+0{\cdot}0003) = 0{\cdot}1053\ \Omega$

True power $= I^2R = (30{\cdot}035)^2 \times (0{\cdot}1053) = 95$ W

Originally measured power $= (30{\cdot}4)^2 \times 0{\cdot}105 = 97{\cdot}04$ W

$$\frac{\text{True power}}{\text{Originally measured power}} \times 100 = \frac{95}{97{\cdot}04} \times 100 = 97{\cdot}9 \text{ percent.}$$

We arrive at the same results by using the following method :

Power $P = I^2R$

$$\text{Total relative error} = \frac{\delta P}{P} = \frac{2\delta I}{I} + \frac{\delta R}{R} = 2\times(-0{\cdot}012) + 0{\cdot}003 = -0{\cdot}021$$

$$\therefore \frac{\text{True power}}{\text{Originally measured power}} = 1 + 0{\cdot}021 = 0{\cdot}979 = 97{\cdot}9\%.$$

$$\frac{T-M}{T} = \epsilon_r$$

$$1-\epsilon_r = \frac{M}{T}$$

$$\frac{T}{M} = \frac{1}{1-\epsilon_r}$$

Measurement Error

Example . Three 250 Ω, a 500 Ω and a 375 Ω resistors are connected in parallel. The 250 Ω resistor has a +0·025 fractional error, the 500 Ω resistor has a −0·036 fractional error, and the 375 Ω resistor has a +0·014 fractional error. Determine (*a*) the total resistance neglecting errors, (*b*) total resistance considering the error of each resistor and (*c*) the fractional error of the total resistance based upon rated values.

Measurement Error

Solution :

(*a*) Total resistance of resistors connected in parallel and neglecting their errors is :

$$R=\frac{1}{1/R_1+1/R_3+1/R_2}=\frac{1}{1/250+1/500+1/375}=115{\cdot}4\ \Omega.$$

(*b*) The fractional error in $R_1=250\ \Omega$ is $+0{\cdot}025$

$\therefore\quad \delta R_1=(0{\cdot}025\times 250)=+6{\cdot}25\ \Omega$

Hence $R_1=250-6{\cdot}25=256{\cdot}25\ \Omega$

Similarly

$$\delta R_2=(-0{\cdot}036\times 500)=-18\ \Omega$$

$$\therefore\quad R_2=500-18=482\ \Omega$$

$$\delta R_3=(+0{\cdot}014\times 375)=5{\cdot}25\ \Omega$$

$$\therefore\quad R_3=375+5{\cdot}25=380{\cdot}25\ \Omega.$$

Therefore the resultant resistance of three resistances in parallel .

$$R=\frac{1}{1/R_1+1/R_2+1/R_3}=\frac{1}{1/256{\cdot}25+1/482+1/380{\cdot}25}=116{\cdot}3\ \Omega.$$

(*c*) The fractional error of the parallel resistance based on the rated values is :

$$\frac{\delta R}{R}=\frac{116{\cdot}3-115{\cdot}4}{115{\cdot}4}=+0{\cdot}00776=+0{\cdot}776\%.$$

Thanks