Reg. No.															

B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, JULY 2024

First Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA

(For the candidates admitted from the academic year 2021 – 2022 onwards)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.

(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours

Max. Marks: 75

PART – A (20 × 1 = 20 Marks)

Answer **ALL** Questions

	Marks	BL	CO	PO

1. The eigen values of A^3 if $A=\begin{pmatrix}1 & 2 & 3\\ 0 & 2 & -7\\ 0 & 0 & 3\end{pmatrix}$ — 1 | 1 | 1 | 1,2

(A) $1^3, 2^3, 3^3$ (B) $1, 2, 3$

(C) $1, \frac{1}{2^3}, \frac{1}{3^3}$ (D) $1, \frac{1}{2}, \frac{1}{3}$

2. The eigen vectors corresponding to distinct eigen values of real symmetric matrix are — 1 | 2 | 1 | 1,2

(A) Real and equal (B) Distinct and complex

(C) Orthogonal (D) Equal and non-real

3. The characteristic equation of the matrix $A=\begin{pmatrix}1 & 2\\ 4 & 3\end{pmatrix}$ is — 1 | 1 | 1 | 1,2

(A) $\lambda^2-4\lambda+5=0$ (B) $\lambda^2-4\lambda-5=0$

(C) $\lambda^2+4\lambda-5=0$ (D) $\lambda^2+4\lambda+5=0$

4. Write the quadratic form defined by the matrix $A=\begin{pmatrix}6 & 1 & -7\\ 1 & 2 & 0\\ -7 & 0 & 1\end{pmatrix}$ — 1 | 1 | 1 | 1,2

(A) $6x_1^2+2x_2^2+x_3^2+2x_1x_2-14x_1x_3$

(B) $6x_1^2+x_2^2+6x_3^2+x_1x_2-7x_1x_3$

(C) $6x_1^2+2x_2^2+x_3^2+2x_1x_2+14x_1x_3$

(D) $6x_1^2+x_2^2+6x_3^2+x_1x_2-14x_1x_3$

5. If $x=r\cos\theta, y=r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}=$ — 1 | 1 | 2 | 1,2

(A) 1/r (B) r^2

(C) 1 (D) r

6. The stationary points of the function $3x^2 - y^2 + x^3$ are 1 2 2 1,2

(A) (0, 0) and (2, 0) (B) (0, 0) and (–2, 0)
(C) (1, 1) and (–2, 0) (D) (1, 1) and (2, 0)

7. If $f(x,y) = e^x \cos y$ then find $f_{xy}(0,0)$ 1 2 2 1,2

(A) 1 (B) 0
(C) 2 (D) –1

8. If $z = x^2 + y^2 + 3xy$ then $\frac{\partial z}{\partial x}$ is 1 1 2 1,2

(A) $2y+3x$ (B) $3y$
(C) $2x+3y$ (D) $2x$

9. Solution of $\left(D^2 + 4\right)y = 0$ 1 2 3 1,2

(A) $y = Ae^{2x} + Be^{-2x}$ (B) $y = A\cos 2x + B\sin 2x$
(C) $y = A\cos\sqrt{2}x + B\sin\sqrt{2}x$ (D) $y = A(x+B)e^{2x}$

10. The particular integral of $\left(D^2 + 4\right)y = \sin(2x+5)$ is 1 1 3 1,2

(A) $-\frac{x}{4}\cos(2x+5)$ (B) $\frac{x}{4}\sin(2x+5)$
(C) $-\frac{x}{2}\cos(2x+5)$ (D) $-\frac{x}{2}\sin(2x+5)$

11. If $y_1 = \cos x, y_2 = \sin x$ then the value of $y_1 y_2' - y_2 y_1'$ is 1 2 3 1,2

(A) –1 (B) 0
(C) 1 (D) 1/2

12. If two roots of the auxiliary equation become equal to the real number 'a' then the corresponding complementary function is 1 2 3 1,2

(A) $A\cos ax + B\sin ax$ (B) $Ax + B$
(C) $Ae^{ax} + Be^{-ax}$ (D) $(Ax+B)e^{ax}$

13. What is the curvature of a circle of radius 3 1 1 4 1,2

(A) 1/3 (B) –1/3
(C) 3 (D) 2

14. The locus of center of curvature is called 1 2 4 1,2

(A) Radius of curvature (B) Envelope
(C) Evolute (D) Involute

15. The curve which touches every member of the family of curve is 1 1 4 1,2

(A) Evolute (B) Involute
(C) Circle (D) Envelope

16. $\beta(m,n)=$ _____ 1 1 4 1,2

(A) $\frac{\lceil m \lceil n}{\lceil (m+n)}$ (B) $\frac{\lceil (m+n)}{\lceil (m-n)}$

(C) $\frac{\lceil (m+n)}{\lceil m \lceil n}$ (D) $\frac{\lceil mn}{\lceil (m+n)}$

17. The value of $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is 1 1 5 1,2

(A) $1/e$ (B) e^n

(C) e (D) $1/e^n$

18. The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots\infty$ is 1 1 5 1,2

(A) Oscillates (B) Divergent

(C) Absolutely convergent (D) Conditionally convergent

19. If $\sum u_n$ is a series of positive terms such that $\lim_{n\to\infty} u_n^{1/n} = l$ then the series $\sum u_n$ is convergent if 1 1 5 1,2

(A) $l=0$ (B) $l=1$

(C) $l<1$ (D) $l>1$

20. The series $\sum \frac{1}{(\log n)^n}$ is 1 2 5 1,2

(A) Convergent (B) Divergent

(C) Monotonic (D) Oscillates

PART – B (5 × 8 = 40 Marks)

Marks BL CO PO

Answer **ALL** Questions

21. a. Find the eigen values and eigen vectors of $A=\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$. 8 3 1 1,2

(OR)

b. Find the inverse of $A=\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$ using Cayley-Hamilton theorem. 8 2 1 1,2

22. a. Expand $e^x \sin y$ in powers of x and y as far as terms of the second degree. 8 4 2 1,2

(OR)

b.	If $z = z(x, y)$ where $x = e^u + e^{-v}, y = e^{-u} - e^v$. Prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.	8	3	2	1,2
23. a.	Solve $\left(x^2D^2 + xD - 9\right)y = \frac{5}{x^2}$.	8	2	3	1,2
	(OR)				
b.	Solve $y'' + y = \tan x$ by the method of variation of parameters.	8	4	3	1,2
24. a.	Find the radius of curvature for the curve $y^2 = x^3 + 8$ *at* $(-2, 0)$.	8	3	4	1,2
	(OR)				
b.	Find the envelope of the family of straight lines $y = mx \pm \sqrt{a^2m^2 + b^2}$.	8	2	4	1,2
25. a.	Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots$	8	4	5	1,2
	(OR)				
b.	Test the convergence of the series $\sum \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots(2n)} x^n$.	8	2	5	1,2

		Marks	BL	CO	PO
	PART – C (1 × 15 = 15 Marks) Answer **ANY ONE** Questions				
26.	Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to canonical form by orthogonal reduction and find the rank, index, signature and the nature of the quadratic form.	15	4	1	1,2
27.	Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	15	4	2	1,2

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