Reg. No.												

B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, JULY 2023
First / Second Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted from the academic year 2021-2022 & 2022-2023)

Note:
(i)   **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours   Max. Marks: 75

PART – A (20 × 1 = 20Marks)
Answer **ALL** Questions

	Marks	BL	CO	PO

1. If $A = \begin{bmatrix} 2 & 5 & 1 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then the eigen values of A^{-1} are _____
                                                         1   2   1   2

    (A) $\dfrac{1}{2}, \dfrac{1}{4}, 1$ (B) $1, 3, 4$

    (C) $2, 4, 1$ (D) $\dfrac{1}{3}, \dfrac{1}{4}, 1$

2. The signature of the quadratic form whose canonical form is $2y_1^2 - y_2^2 - y_3^2$ _____   1   2   1   2

    (A) 1 (B) -1
    (C) 0 (D) 6

3. The eigen values of an orthogonal matrix are _____   1   1   1   1
    (A) 0 (B) ± 1
    (C) ± 2 (D) $\pm \dfrac{1}{2}$

4. Cayley Hamilton Theorem states that "Every _____ matrix satisfies its own characteristic equation".   1   1   1   1
    (A) Row   (B) Column
    (C) Square   (D) Zero

5. If $f_{x, y} = 0$ and y is an implicit function of x, then $\dfrac{dy}{dx}$ is _____   1   1   2   2

    (A) $-\dfrac{\frac{\partial f}{\partial x}}{\frac{\partial f}{dy}}$ (B) $\dfrac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial x}}$

    (C) $-\dfrac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$ (D) $\dfrac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$

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6. If $x = r\cos\theta, y = r\sin\theta$, then $\dfrac{\partial(x,y)}{\partial(r,\theta)} =$ _____   1 2 2 1

(A) $\dfrac{1}{r}$ (B) r^2

(C) $2r$ (D) r

7. If u and v are functionally dependent then their Jacobian value   1 1 2 1
is _____ .
(A) 0   (B) 1
(C) Non-zero   (D) Greater than zero

8. If $rt - s^2 < 0$ at (a,b) where $r = \dfrac{\partial^2 f}{\partial x^2}, s = \dfrac{\partial^2 f}{\partial x \partial y}, t = \dfrac{\partial^2 f}{\partial y^2}$, then the point (a,b) is   1 1 2 2

_____

(A) Maximum point   (B) Minimum point
(C) Saddle point   (D) Unique point

9. The complementary function of $D^2 - 2D + 1 y = 0$ is _____   1 2 3 2
(A) $C_1 e^x + C_2 e^{-x}$ (B) $C_1 + C_2 x e^x$
(C) $C_1 e^{2x} + C_2 e^{-2x}$ (D) $C_1 + C_2 x e^{-x}$

10. The particular integral of $D^2 + 9y = e^{-2x}$ is _____   1 2 3 2
(A) $\dfrac{e^{-2x}}{15}$ (B) $\dfrac{e^{2x}}{15}$

(C) $\dfrac{e^{-2x}}{13}$ (D) $\dfrac{e^{2x}}{13}$

11. The equation $a_0 x^2 D^2 + a_1 x D + a_2 y = Q(y)$ is called _____ , where a_0, a_1, a_2 1 1 3 1
are constants.
(A) Taylor's equation   (B) Cauchy's Euler equation
(C) Legendre's equation   (D) Clairaut's equation

12. If $y_1 = \cos ax, y_2 = \sin ax$, then the value of $y_1 y_2' - y_2 y_1'$ is _____   1 1 3 1
(A) $-a$ (B) 0
(C) 1   (D) a

13. The radius of curvature of the curve $y = e^x$ at (0,0) is _____   1 2 4 2
(A) 1   (B) 2
(C) $2\sqrt{2}$ (D) $\sqrt{2}$

14. A curve which touches each member of a family of curves is called   1 1 4 1

(A) Evolute   (B) Envelope
(C) Circle of curvature   (D) Radius of curvature

15. The value of $\Gamma\dfrac{1}{2}$ is _____   1 1 4 2

    (A) π (B) $\pi/2$
    (C) $\sqrt{\pi}$ (D) $\sqrt{\dfrac{\pi}{2}}$

16. The curvature of straight line is _____   1 1 4 1
    (A) 1   (B) 2
    (C) -1 (D) 0

17. The series $\dfrac{1}{1^p}+\dfrac{1}{2^p}+\dfrac{1}{3^p}+...$ diverges if _____   1 1 5 1
    (A) $p>0$ (B) $p<1$
    (C) $p>1$ (D) $p\leq1$

18.   1 1 5 2
If $\sum u_n$ is a series of positive terms and $\lim\limits_{n\to\infty} u_n^{\frac{1}{n}}=l$, then $\sum u_n$ converges if _____

    (A) $l<1$ (B) $l>1$
    (C) $l=1$ (D) $-1<l<1$

19. A series $\sum u_n$ is said to be absolutely convergent if the series is _____   1 1 5 1
    (A) $\sum u_n$ is convergent   (B) $\sum u_n$ is divergent
    (C) $\sum u_n$ is oscillating   (D) $\sum u_n$ is monotonic

20. The series $1+\dfrac{1}{2!}+\dfrac{1}{3!}+\dfrac{1}{4!}+....$ is _____   1 2 5 2
    (A) Divergent   (B) Convergent
    (C) Absolutely convergent   (D) Conditionally convergent

PART – B (5 × 8 = 40 Marks)
Answer ALL Questions

	Marks	BL	CO	PO

21. a.   8 3 1 1

Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

(OR)

   b.   8 3 1 1

Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and hence

find A^{-1}.

22. a. Expand $e^x\cos y$ as Taylor's series in powers of x and y as far as the terms of the second degree.   8 4 2 2

(OR)

   b. A rectangular box open at the top is to have a volume of 32cc. Find the dimensions of the box that requires the least material for its construction.   8 4 2 2

23. a. Solve $D^2 - 3D + 2y = e^{2x}\sin x$.   8 3 3 1

(OR)

b. Solve $\dfrac{d^2 y}{dx^2} + y = \tan x$ by the method of variation of parameters.   8 3 3 1

24. a. Find the equation of the circle of curvature of the curve $y^2 = 12x$ at (3,6).   8 3 4 2

(OR)

b. Find the envelope of $\dfrac{x}{a} + \dfrac{y}{b} = 1$ where the parameters a and b are connected by $a + b = c$.   8 3 4 2

25. a. test the convergence of the series $\dfrac{1}{1.2.3} + \dfrac{3}{2.3.4} + \dfrac{5}{3.4.5} + ...$ 8 4 5 2

(OR)

b. Test the convergence of the series $\dfrac{3}{4} + \dfrac{3.4}{4.6} + \dfrac{3.4.5}{4.6.8} + ...$ 8 4 5 2

PART – C (1 × 15 = 15 Marks)
Answer **ANY ONE** Questions

                                                   Marks BL CO PO

26. Reduce quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1 x_2 + 2x_2 x_3$ to canonical form through an orthogonal transformation. Also find the rank, index, signature and nature of the quadratic form.   15 3 1 2

27. Find the Evolute of the parabola $y^2 = 4ax$.   15 3 4 2

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