Reg. No.													

B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, JANUARY 2024
First Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted from the academic year 2023 - 2024)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours   Max. Marks: 75

PART – A (20 × 1 = 20Marks)
Answer ALL Questions

				Marks	BL	CO

1. The number of positive terms in the canonical form is called — Marks 1, BL 1, CO 1
   (A) Signature
   (B) Index
   (C) Rank
   (D) Positive definite

2. Find the eigen values of A^2 if $A = \begin{bmatrix} 3 & 2 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ — Marks 1, BL 3, CO 1
   (A) 6,2,5
   (B) 9,4,25
   (C) 9,4,5
   (D) 3,2,5

3. Two of the eigen values of 3×3 matrix A are 2, 1 and |A|=12. Find the third eigen value — Marks 1, BL 3, CO 1
   (A) 6
   (B) 3
   (C) 2
   (D) 1

4. If A is an orthogonal matrix then — Marks 1, BL 1, CO 1
   (A) |A|=0
   (B) A^2=I
   (C) $A^T = A^{-1}$
   (D) A is non-singular

5. If $rt - s^2 < 0$ at (a,b) then the point is — Marks 1, BL 1, CO 2
   (A) Maximum point
   (B) Minimum point
   (C) Saddle point
   (D) Discontinuous point

6. If u and v are functionally dependent then their Jacobian value is — Marks 1, BL 1, CO 2
   (A) 0
   (B) 1
   (C) −1
   (D) 2

7. If $f(x,y) = e^x \cos y$ then what is $f_{xy}(0,0)$? — Marks 1, BL 2, CO 2
   (A) 1
   (B) −1
   (C) 0
   (D) 2

8. If $f(x,y)$ is an implicit function then $\dfrac{dy}{dx} = ?$ 1 1 2

(A) $-\dfrac{(\partial f / \partial x)}{(\partial f / \partial y)}$

(B) $\dfrac{(\partial f / \partial x)}{(\partial f / \partial y)}$

(C) $\dfrac{(\partial f / \partial y)}{(\partial f / \partial x)}$

(D) $-\dfrac{(\partial f / \partial y)}{(\partial f / \partial x)}$

9. The complementary function of $(D^2 - 4D + 5)y = 4e^x$ is   1 3 3

(A) $Ae^x + Be^{-5x}$

(B) $Ae^{-x} + Be^{5x}$

(C) $e^{-2x}(A\cos x + B\sin x)$

(D) $e^{2x}(A\cos x + B\sin x)$

10. The particular integral of $(D^2 + 1)y = \cos(2x)$ is   1 3 2

(A) $\dfrac{x\sin x}{2}$

(B) $\dfrac{\cos 2x}{3}$

(C) $\dfrac{\cos 2x}{-3}$

(D) $\dfrac{x\cos x}{2}$

11. Convert the equation $(5+2x)^2 y'' - 6(5+2x)y' + 8y = 0$ to an equation with   1 2 2
constant coefficient by using the transformation $z = \log(5+2x)$

(A) $(\theta^2 + 4\theta + 2)y = 0$

(B) $(\theta^2 - 4\theta + 2)y = 0$

(C) $(\theta^2 + 4\theta + 4)y = 0$

(D) $(\theta^2 + 4\theta - 2)y = 0$

12. If $1 \pm 2i$ are the roots of A.E of a differential equation $f(D)y = 0$ then the   1 2 3
general solution is

(A) $e^{-2x}(A\cos x - B\sin x)$

(B) $Ae^x + Be^{-2x}$

(C) $e^x(A\cos 2x + B\sin 2x)$

(D) $Ae^{2x} + Be^{5x}$

13. The locus of center of curvature is called   1 1 4

(A) Involute

(B) Evolute

(C) Radius of curvature

(D) Envelope

14. The radius of curvature in polar coordinates is   1 1 4

(A) $\rho = \dfrac{\left(r^2 + (r')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$

(B) $\rho = \dfrac{\left(r^2 - (r')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$

(C) $\rho = \dfrac{\left(r^2 - (r'')^2\right)^{3/2}}{r^2 - rr' + 2(r')^2}$

(D) $\rho = \dfrac{\left(r^2 + (r')^2\right)^{3/2}}{r^2 - rr'' + 2(r')^2}$

15. A curve which touches each member of a family of the curves is called   1 1 4
_____ of that family.

(A) Evolute

(B) Envelope

(C) Circle of curvature

(D) Radius of curvature

16. The value of $\Gamma\left(\dfrac{1}{2}\right)$ is _____

(A) $\sqrt{\pi}$ (B) π^2
(C) π (D) 2π

17. The series $\sum_{n=1}^{\infty}\dfrac{1}{n^p}$ is convergent if

(A) p=1   (B) p>1
(C) p=0   (D) p<1

18. The sequence $4(-1)^n$ is

(A) Oscillating   (B) Convergent
(C) Divergent   (D) Monotonic

19. $\sum(-1)^n \sin\left(\dfrac{1}{n}\right)$ converges by the following test

(A) Leibnitz's test   (B) Ratio test
(C) Root test   (D) Raabe's test

20. The series $1-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}+...\infty$ is

(A) Divergent   (B) Convergent
(C) Absolutely convergent (D) Conditionally convergent

PART – B (5 × 8 = 40 Marks)
Answer ALL Questions

21. a.

Find Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$.

(OR)

b.

Using the Cayley-Hamilton theorem find A^{-1} when $A=\begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$

22. a. If $u=x+y+z, uv=y+z, uvw=z$, then find $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$.

(OR)

b. Expand the function $f(x,y)=e^{xy}$ in the Taylor series at $(1,1)$ upto second degree.

23. a. Solve $(D^2-4D-5)y=e^{2x}+3\sin 4x$.

(OR)

b. Solve $(D^2 + 4)y = \tan 2x$ by the method of variation of parameter. 8 4 3

24. a. Find the equation of the evolute of the parabola $y^2 = 4ax$. 8 3 4

(OR)

b. Find the equation of the circle of curvature at (c,c) on $xy = c^2$. 8 3 4

25. a. Test the convergence or divergence of the series 8 3 5
$\dfrac{x}{1.2} + \dfrac{x^2}{2.3} + \dfrac{x^3}{3.4} + \dots \infty, x > 0$.

(OR)

b.i.
Test the convergence of $\displaystyle\sum_{n=1}^{\infty}\left(1 + \dfrac{1}{n}\right)^{-n^2}$. 4 3 5

ii.
Test the convergence of the series $1 - \dfrac{1}{2\sqrt{2}} + \dfrac{1}{3\sqrt{3}} - \dfrac{1}{4\sqrt{4}} + \dots$ 4 3 5

PART – C (1 × 15 = 15 Marks)
Answer ANY ONE Questions   Marks BL CO

26. Reduce the quadratic form $8x^2 + 7y^2 + 3z^2 - 12xy - 8yz + 4xz$ to canonical 15 3 1
form by an orthogonal reduction. Also find the rank, index signature and
nature of the quadratic form.

27. The temperature T at any point (x,y,z) in space is T=400 xyz^2. Find the 15 1 5
highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

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