

24. a. Find the evolute of the parabola $y^2=4ax$. 8 4 4 1,2

(OR)

b. Find the envelope of the straight line $x\cos\alpha + y\sin\alpha = a\sin\alpha\cos\alpha$. 8 3 4 1,2

25. a. Test the convergence of $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \dots\infty$. 8 3 5 1,2

(OR)

b. Prove that the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ is conditionally convergent, but not absolutely convergent. 8 3 5 1,2

PART - C (1 × 15 = 15 Marks)

Answer ANY ONE Question

26. If $\phi(v^2 - x^2, v^2 - y^2, v^2 - z^2) = 0$ where $v = v(x, y, z)$ then prove that $\frac{1}{x} \frac{\partial v}{\partial x} + \frac{1}{y} \frac{\partial v}{\partial y} + \frac{1}{z} \frac{\partial v}{\partial z} = \frac{1}{v}$. 15 4 2 1,2

27. Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$. 15 3 4 1,2

Reg. No.

B.Tech. / M.Tech (Integrated) DEGREE EXAMINATION, JUNE 2023

First Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA

(For the candidates admitted from the academic year 2021 - 2022 & 2022 - 2023)

Note:

- (i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
- (ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours

Max. Marks: 75

PART - A (20 × 1 = 20 Marks)

Answer ALL Questions

- | | Marks | BL | CO | PO |
|---|-------|----|----|-----|
| 1. The quadratic form $x^2 + y^2 + z^2$ is
(A) Positive definite (B) Negative definite
(C) Indefinite (D) Positive semi definite | 1 | 2 | 1 | 1,2 |
| 2. What are the eigen values of A^3 , if $A = \begin{pmatrix} 1 & 3 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{pmatrix}$
(A) 1, -2, -1 (B) 1, -8, 1
(C) 1, -8, -1 (D) -1, -8, -1 | 1 | 1 | 1 | 1,2 |
| 3. The homogeneous polynomial of the _____ degree in any number of variables is called a quadratic form
(A) First (B) Second
(C) Third (D) Fourth | 1 | 1 | 1 | 1,2 |
| 4. The values of a and b such that $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3 and -2 as its eigen values
(A) $a=1, b=1$ (B) $a=1, b=2$
(C) $a=-1, b=-2$ (D) $a=2, b=-1$ | 1 | 2 | 1 | 1,2 |
| 5. If $u = \frac{x^3 + y^3}{x^2 + y^2}$ then the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ is
(A) u (B) 3u
(C) 4u (D) 6u | 1 | 2 | 2 | 1,2 |
| 6. The total derivative of xy is
(A) $x dx + y dy$ (B) $x dx - y dy$
(C) $x dy - y dx$ (D) $x dy + y dx$ | 1 | 1 | 2 | 1,2 |
| 7. The stationary point of $x^2 + y^2 + 6x + 12 = 0$ is
(A) (-3, 0) (B) (0, 3)
(C) (0, -3) (D) (3, 0) | 1 | 2 | 2 | 2 |

8. If $rt - s^2 > 0$ and $r > 0$ at (a, b) then (a, b) is
 (A) A maximum point (B) A minimum point
 (C) A saddle point (D) A point of discontinuity
9. The roots of the auxiliary equation of $(D^2 - 4)y = 0$ are
 (A) ± 2 (B) $1 \pm 2i$
 (C) $\pm \sqrt{2}$ (D) $\pm i\sqrt{3}$
10. If $1 \pm 3i$ are the roots of a differential equation $f(D)y = 0$ then the complementary function is
 (A) $Ae^t + Be^{-2t}$ (B) $e^{-2t}(A \cos t + B \sin t)$
 (C) $e^t(A \cos 3t + B \sin 3t)$ (D) $Ae^t + Be^{3t}$
11. If y_1 and y_2 are two linearly independent solution of an ordinary differential equation, then the Wronskian $W(y_1, y_2) =$
 (A) $W=0$ (B) $W \neq 0$
 (C) $W=1$ (D) $W=2$
12. The particular integral of $(D^2 + 9)y = e^{-2x}$ is
 (A) $\frac{e^{-2x}}{13}$ (B) $\frac{e^{-2x}}{14}$
 (C) $\frac{e^{-2x}}{15}$ (D) $\frac{e^{-2x}}{16}$
13. The value of $\Gamma(n+1)$ is
 (A) n (B) $\Gamma(n)$
 (C) $n\Gamma(n)$ (D) $n/2$
14. The envelope of the family of curves $A\alpha^2 + B\alpha + C = 0$ is
 (A) $B^2 + 4AC = 0$ (B) $B^2 - 4AC < 0$
 (C) $B^2 + 4AC > 0$ (D) $B^2 - 4AC = 0$
15. The radius of curvature of $y = e^x$ at $x=0$ is
 (A) $2\sqrt{2}$ (B) $\frac{2}{\sqrt{2}}$
 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$
16. The value of $\beta(2, 3)$ is
 (A) $1/10$ (B) $1/11$
 (C) $1/12$ (D) $1/13$

17. The sequence $\left\{\frac{1}{n^2}\right\}$ is
 (A) Convergent (B) Divergent
 (C) Oscillates (D) Convergent for finite values of n
18. The series $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ is
 (A) Convergent (B) Divergent
 (C) Oscillates (D) Conditionally convergent
19. A sequence of real numbers $\{a_n\}$ is monotonically decreasing if
 (A) $a_{n+1} \geq a_n$ (B) $a_{n+1} \leq a_n$
 (C) $a_{n+1} = a_n$ (D) $a_{n+1} \neq a_n$
20. The series $\sum_{n=0}^{\infty} x^n$ is divergent if
 (A) $|x| < 1$ (B) $|x| \geq 1$
 (C) $x \geq -1$ (D) $x = 0$

PART – B (5 × 8 = 40 Marks)
 Answer ALL Questions

21. a. Find the eigen values and the eigen vectors of $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$.
 (OR)
 b. Write down the quadratic form corresponding to the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.
22. a. Are the functions $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1} x + \tan^{-1} y$ functionally dependent? If so, find the relation between them.
 (OR)
 b. Find the Taylor series expansion of e^{xy} at $(1, 1)$ upto third degree terms.
23. a. Solve the differential equation $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 4 \sin(\log x)$.
 (OR)
 b. Solve the differential equation $(D^2 + 3D + 2)y = \sin x$.