Reg. No.														

B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, DECEMBER 2023
First Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted from the academic year 2023 - 2024)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours | Max. Marks: 75

PART – A (20 × 1 = 20Marks)
Answer **ALL** Questions

	Marks	BL	CO
1. If 1 and 3 are eigen values of $A=\begin{bmatrix}1&0&-1\\1&2&1\\2&2&3\end{bmatrix}$, then the eigen values of A^3 are (A) 1, 2, 3 (B) 1, 4, 9 (C) 1, 8, 27 (D) 1, 8, 9	1	1	1
2. The characteristic equation of $A=\begin{bmatrix}4&3&1\\2&1&-2\\1&2&1\end{bmatrix}$ is (A) $\lambda^3-6\lambda^2+6\lambda-11=0$ (B) $\lambda^3+6\lambda^2+6\lambda+11=0$ (C) $\lambda^3-6\lambda^2+6\lambda+6=0$ (D) $\lambda^3-5\lambda^2+6\lambda-10=0$	1	2	1
3. The quadratic form of $A=\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$ is (A) $3x_1^2+3x_2^2+3x_3^2-2x_1x_2+2x_2x_3-x_1x_3$ (B) $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_1x_3$ (C) $6x_1^2+3x_2^2+3x_3^2+2x_1x_2+2x_2x_3+x_1x_3$ (D) $3x_1^2+3x_2^2+3x_3^2+4x_1x_2+2x_2x_3+4x_1x_3$	1	3	1
4. If 6 is one of the eigen values of $A=\begin{pmatrix}3&-1&1\\-1&5&-1\\1&-1&3\end{pmatrix}$, then other eigen values are (A) 1, 3 (B) 3, 6 (C) 2, 6 (D) 2, 3	1	3	1
5. If $u=x^3+y^3+z^3-3xyz$, then $\frac{\partial^2 u}{\partial x^2}$ is (A) $3x^2-3yz$ (B) $6x$ (C) $6x-3yz$ (D) $3x^2+3y^2+3z^2-3yz$	1	2	2

6. If u and v are functionally dependent then their Jacobian value is 1 3 2

(A) 0 (B) 1
(C) −1 (D) 2

7. If $w=f\left(y-z,z-x,x-y\right)$, then $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$ *is* 1 2 2

(A) 0 (B) 1
(C) −1 (D) $x+y+z$

8. The condition for a function f(x, y) to have a maximum value at the stationary points if $r=\frac{\partial^2 f}{\partial x^2};s=\frac{\partial^2 f}{\partial x\partial y};t=\frac{\partial^2 f}{\partial y^2}$ is 1 1 2

(A) $rt-s^2<0\,\&\,r<0$ (B) $rt-s^2>0\,\&\,r<0$
(C) $rt-s^2>0\,\&\,r>0$ (D) $rt-s^2<0\,\&\,r>0$

9. The solution of $(D^2-6D+9)y=0$ is 1 1 3

(A) $Ae^{x}+Be^{3x}$ (B) $Ae^{3x}+Be^{3x}$
(C) $Ae^{6x}+Be^{9x}$ (D) $\left(Ax+B\right)e^{3x}$

10. The particular integral of $\left(D^2-2D+1\right)y=e^x$ is 1 1 3

(A) $\frac{xe^x}{2}$ (B) $\frac{x^2e^x}{2}$
(C) xe^x (D) x^2e^x

11. The solution of $\left(D^2-2D+2\right)y=0$ is 1 1 3

(A) $A\cos x+B\sin x$ (B) $Ae^{x}+Be^{ix}$
(C) $e^x\left(A\cos x+B\sin x\right)$ (D) $\left(Ax+B\right)e^x$

12. The auxiliary equation of $\left(x^2D^2-7xD+12\right)y=x\log x$ is 1 1 3

(A) $m^2-7m+12=0$ (B) $m^2-8m+12=0$
(C) $m^2+7m+12=0$ (D) $m^2+8m-12=0$

13. The locus of the centre of curvature is called 1 1 4

(A) Involute (B) Evolute
(C) Radius of curvature (D) Envelope

14. A curve which touches each member of a family of the curves is called _________ of that family. 1 1 4

(A) Evolute (B) Envelope
(C) Circle of curvature (D) Radius of curvature

15. The radius of curvature of $r=a\cos\theta$ is 1 1 4

(A) a (B) 1/2
(C) $a^2/2$ (D) a/2

16. The curvature of a circle of radius 5 is 1 1 4

(A) 1/5 (B) 5
(C) 25 (D) 1/25

17.	The series $\sum_{n=1}^{\infty}\frac{1}{n!}$ is (A) Convergent (B) Divergent (C) Oscillating (D) Monotonic	1	1	5
18.	The series $\sum_{n=1}^{\infty}\frac{1}{n}$ is (A) Convergent (B) Divergent (C) Oscillating (D) Monotonic	1	1	5
19.	The series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\ldots$ is (A) Convergent (B) Divergent (C) Conditionally convergent (D) Absolutely convergent	1	1	5
20.	By D'Alembert's ratio test $\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=l$ the series is convergent when (A) $l<1$ (B) $l=0$ (C) $l>1$ (D) $l=1$	1	1	5

PART – B (5 × 8 = 40 Marks)

Answer **ALL** Questions

		Marks	BL	CO
21. a.	Find Eigen values and Eigen vectors of $A=\begin{pmatrix}11 & -4 & -7\\ 7 & -2 & -5\\ 10 & -4 & -6\end{pmatrix}$.	8	3	1
	(OR)			
b.	Find A^{-1} using Cayley Hamilton theorem for the matrix $A=\begin{pmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{pmatrix}$.	8	3	1
22. a.	Using Taylor's series expand $e^x\cos y$ in powers of x and y as far as terms of second degree.	8	4	2
	(OR)			
b.	If $x=r\cos\theta, y=r\sin\theta$, then prove that $\frac{\partial(x,y)\partial(r,\theta)}{\partial(r,\theta)\partial(x,y)}=1$.	8	4	2
23. a.	Solve $\left(D^2-2D+1\right)y=x^2+1+\sin 2x$.	8	3	3
	(OR)			
b.	Solve $\left(D^2+1\right)y=\cot x$ by the method of variation of parameter.	8	3	3
24. a.	Find the evolute of the parabola $x^2=4ay$.	8	3	4
	(OR)			

b.	Find the envelope of the family of straight line $\frac{x}{a}+\frac{y}{b}=1$ where a and b are connected by the relation $ab=c^2$ where c is a constant.	8	3	4
25. a.	Test the convergence of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+....\infty$.	8	3	5
	(OR)			
b.	Discuss the convergence of the series $1+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+...\infty$.	8	3	5

PART – C (1 × 15 = 15 Marks)
Answer **ANY ONE** Questions

		Marks	BL	CO
26.	Reduce the quadratic form $3x_1^2+2x_2^2+3x_3^2-2x_1x_2-2x_2x_3$ to canonical form and find the rank, index signature and nature of the quadratic form.	15	3	1
27.	Find the extreme values of $f(x,y)=x^3y^2(1-x-y)$.	15	4	2

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