Reg. No.															

B.Tech. / M.Tech (Integrated) DEGREE EXAMINATION, MAY 2024
First Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA
(For the candidates admitted during the academic year2021-2022, 2022-2023 & 2023-2024)

Note:
(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40th minute.
(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours   Max. Marks: 75

PART – A (20 × 1 = 20Marks)
Answer **ALL** Questions

	Question	Marks	BL	CO	PO
1.	The inverse of the eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is (A) 1, 1/2 (B) 1, 2 (C) 1, 1/3 (D) 1, 3	1	2	1	2
2.	The index of the canonical form $-x^2+y^2+4z^2$ is (A) 3 (B) 2 (C) 1 (D) 0	1	2	1	2
3.	The sum of eigen values of $A=\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ is (A) 2 (B) 4 (C) –3 (D) 0	1	1	1	2
4.	The eigen values of A–3I, where $A=\begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$ are (A) –1, 6 (B) 1, –6 (C) 0, –3 (D) –4, 3	1	2	1	2
5.	If $u=x^2, v=y^2$ then $\frac{\partial(u,v)}{\partial(x,y)}=$ (A) $4xy$ (B) $2xy$ (C) $6xy$ (D) xy	1	2	2	1,2
6.	If u and v are functionally dependent, then their Jacobian value is (A) Zero (B) One (C) Two (D) Three	1	1	2	2
7.	If $f(x,y)=e^{x+2y}$ then the value of $f_{xy}(0,0)$ is (A) 0 (B) 1 (C) 2 (D) 3	1	2	2	1,2

8. If $rt-s^2>0$ *and* $r<0$ at (a,b) then (a,b) is 1 2 2 1,2

(A) A maximum point (B) A minimum point

(C) A saddle point (D) A point of discontinuity

9. Particular integral of the differential equation $\left(D^2+a^2\right)y=\cos ax$ is 1 2 3 1,2

(A) $\frac{x}{2a}\sin ax$ (B) $-\frac{x}{2a}\sin ax$

(C) $\frac{x}{2a}\cos ax$ (D) $-\frac{x}{2a}\cos ax$

10. The general solution of an ordinary differential equation represents 1 1 3 1,2

(A) A single curve (B) A family of curves

(C) A straight line passing through (0,0) (D) A family of circles

11. The number of arbitrary constants present in the general solution of $3\frac{d^3y}{dx^3}-4\frac{d^2y}{dx^2}+6\frac{dy}{dx}-3y=0$ are 1 1 3 1,2

(A) 1 (B) 2

(C) 3 (D) 4

12. The roots of the auxiliary equation of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$ are 1 2 3 1,2

(A) 1,2 (B) 2,3

(C) 1±i (D) 1±2i

13. A curve which touches each member of a family of the curves is called 1 1 4 2

(A) Evolute (B) Envelope

(C) Circle of curvature (D) Radius of curvature

14. The radius of curvature of a curve in polar co-ordinate is 1 2 4 1,2

(A) $\left(r^2+r'^2\right)^{3/2}\Big/ r^2-rr''+2r'^2$ (B) $\left(r^2-r'^2\right)^{3/2}\Big/ r^2-rr''+2r'^2$

(C) $\left(r^2-r'^2\right)^{2/3}\Big/ r^2+rr''+2r'^2$ (D) $\left(r^2+r'^2\right)^{2/3}\Big/ r^2-rr''+2r'^2$

15. The value of $\Gamma(1/2)$ is 1 1 4 1,2

(A) π (B) 2π

(C) $\pi/2$ (D) $\sqrt{\pi}$

16. The relation between the Gamma function and the Beta function is 1 1 4 1,2

(A) $\beta(m,n)=\Gamma(m)\Gamma(n)$ (B) $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m-n)}$

(C) $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (D) $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(mn)}$

17. A sequence of real numbers $\{x_n\}$ is monotonically increasing if — 1 1 5 1,2

(A) $x_{n+1} \geq x_n$ (B) $x_{n+1} \leq x_n$
(C) $x_{n+1} = x_n$ (D) $x_{n+1} = 1/x_n$

18. In D'Alembert's ratio test if $\lim_{n\to\infty}\left(\frac{u_{n+1}}{u_n}\right) = l$, then the test fails if — 1 2 5 1,2

(A) $l>1$ (B) $l<1$
(C) $l\neq 1$ (D) $l=1$

19. The sequence $\{x_n\}$ where $x_n = 1+\frac{1}{n}, \forall n \in N$ is — 1 1 5 1

(A) Convergent and converges to 1 (B) Convergent and converges to 0
(C) Oscillates (D) Divergent

20. By Cauchy's root test $\lim_{n\to\infty} u_n^{1/n} = l$ is convergent if — 1 2 5 1,2

(A) $l<1$ (B) $l>1$
(C) $l=1$ (D) $l\leq 1$

PART – B (5 × 8 = 40 Marks)

Answer **ALL** Questions

	Marks	BL	CO	PO
21. a. Find the eigen values and eigen vectors of $A=\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.	8	3	1	1,2
(OR)				
b. Verify Cayley-Hamilton theorem and hence find A^{-1} where $A=\begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$.	8	3	1	1,2
22. a. If $u=x^2-y^2, v=2xy, f(x,y)=\varphi(u,v)$ then prove that $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=4\left(x^2+y^2\right)\left(\frac{\partial^2\varphi}{\partial u^2}+\frac{\partial^2\varphi}{\partial v^2}\right)$.	8	3	2	1,2
(OR)				
b. Expand $e^x \cos y$ in powers of x and y as far as the terms of the third degree.	8	2	2	1,2
23. a. Solve the differential equation $\left(D^2+2D+1\right)y = e^{-x}+3$ where $D=\frac{d}{dx}$.	8	4	3	1,2
(OR)				

		Marks	BL	CO	PO
b.	Solve the differential equation $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}=12\log x$.	8	4	3	1,2
24. a.	Find the radius of curvature at the point (1/4, 1/4) on the curve $\sqrt{x}+\sqrt{y}=1$.	8	2	4	1,2
	(OR)				
b.	Using gamma function, prove that $\int_0^{\infty} e^{-4x}x^{3/2}dx=\dfrac{3}{128}\sqrt{\pi}$.	8	2	4	1,2
25. a.	Examine the convergence of the series $\left(\dfrac{2^2}{1^2}-\dfrac{2}{1}\right)^{-1}+\left(\dfrac{3^3}{2^3}-\dfrac{3}{2}\right)^{-2}+\left(\dfrac{4^4}{3^4}-\dfrac{4}{3}\right)^{-3}+\ldots\infty$.	8	3	5	1,2
	(OR)				
b.	Examine the convergence of the series $\dfrac{1}{2}+\dfrac{1.3}{2.4}+\dfrac{1.3.5}{2.4.6}+\ldots.\infty$.	8	4	5	1,2

PART – C (1 × 15 = 15 Marks)
Answer **ANY ONE** Questions

		Marks	BL	CO	PO
26.	Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$.	15	3	2	2
27.	Diagonalize the matrix $A=\begin{pmatrix}3 & 1 & 1\\ 1 & 3 & -1\\ 1 & -1 & 3\end{pmatrix}$ using orthogonal transformation.	15	3	1	1,2

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