Reg. No.															

B.Tech. / M.Tech. (Integrated) DEGREE EXAMINATION, NOVEMBER 2023

First Semester

21MAB101T – CALCULUS AND LINEAR ALGEBRA

(For the candidates admitted from the academic year 2022-2023)

Note:

(i) **Part - A** should be answered in OMR sheet within first 40 minutes and OMR sheet should be handed over to hall invigilator at the end of 40^{th} minute.

(ii) **Part – B** and **Part - C** should be answered in answer booklet.

Time: 3 Hours | Max. Marks: 75

PART – A (20 × 1 = 20Marks)

Answer **ALL** Questions

	Question	Marks	BL	CO	PO
1.	If $A=\begin{pmatrix}3&1&4\\0&2&6\\0&0&5\end{pmatrix}$, then the eigen values of A^{-1} are (A) $3^2,2^2,5^2$ (B) $\frac{1}{3},\frac{1}{2},\frac{1}{5}$ (C) 3, 2, 5 (D) $\frac{1}{3^2},\frac{1}{2^2},\frac{1}{5^2}$	1	1	1	1,2
2.	Find the sum and product of the eigen values of $A=\begin{pmatrix}2&1&1\\1&2&1\\0&0&1\end{pmatrix}$ (A) 5, 2 (B) 2, 1 (C) 3, 2 (D) 5, 3	1	2	1	1,2
3.	If $A=\begin{pmatrix}1&5\\0&2\end{pmatrix}$, then (A) $A^2=3A-2$ (B) $A^2=3A+2$ (C) $A^2=3A-2I$ (D) $A^2=3A+2I$	1	1	1	1,2
4.	If A is an orthogonal matrix then (A) $\|A\|=0$ (B) A is singular (C) $A^2=I$ (D) $A^T=A^{-1}$	1	1	1	1,2
5.	If u, v, w are functionally dependent functions of three independent variables x, y and z then $\partial(u,v,w)/\partial(x,y,z)$ is (A) 3 (B) 2 (C) 1 (D) 0	1	1	2	1,2

6. If $v = \tan^{-1} x + \tan^{-1} y$ then $\frac{\partial v}{\partial x}$ is 1 2 2 1,2

(A) $\frac{1}{1+x^2}$ (B) $\frac{1}{x^2}$

(C) $\frac{1}{1+y^2}$ (D) $\frac{1}{\left(1+x^2y^2\right)}$

7. If $f\left(x,y\right)=x^2+y^2$ where $x=r\cos\theta$ and $y=r\sin\theta$ then $\frac{\partial f}{\partial \theta}$ is 1 2 2 1,2

(A) r (B) r^2

(C) 1 (D) 0

8. The stationary points of $x^2+y^2+6x+12$ are 1 1 2 1,2

(A) (3, 0) (B) (0, –3)

(C) (0, 3) (D) (–3, 0)

9. Which of the following is the general solution to $\frac{d^2y}{dx^2}+\frac{3dy}{dx}-10y=0$ 1 1 3 1,2

(A) $y = Ae^{2x}+Be^{5x}$ (B) $y = Ae^{-2x}+Be^{5x}$

(C) $y = Ae^{-2x}+Be^{-5x}$ (D) $y = Ae^{2x}+Be^{-5x}$

10. If $y_1=\cos ax, y_2=\sin ax$ then the value of $y_1y_2'-y_2y_1'$ is 1 1 3 1,2

(A) –a (B) 0

(C) a (D) 1

11. The particular integral of $\left(D^2+16\right)y=\cos 4x$ is 1 2 3 1,2

(A) $\frac{x}{2}\sin 2x$ (B) $\frac{x}{8}\sin 4x$

(C) $\frac{x}{2}\cos 2x$ (D) $\frac{x}{8}\cos 4x$

12. Complementary function of $\left(D^2-4D+4\right)y=8x^2$ is 1 2 3 1,2

(A) $\left(Ax+B\right)e^{2x}$ (B) $Ae^{2x}+Be^{-2x}$

(C) $\left(Ax+B\right)e^{-2x}$ (D) $\left(Ax+B\right)e^{-x}$

13. The curvature at any point of the circle is equal to ________ of its radius. 1 1 4 1,2

(A) Square (B) Same

(C) Reciprocal (D) Constant

14. The envelope of $at^2-ty+x=0, t$ is the parameter is 1 2 4 1,2

(A) $y^2=4ax$ (B) $x^2=4ay$

(C) $x^2=4y$ (D) $y^2=4x$

15.	The radius of curvature at any point on the curve $r = e^{\theta}$ is (A) $\frac{\sqrt{2}}{r}$ (B) $\frac{r}{\sqrt{2}}$ (C) r (D) $\sqrt{2}r$	1	1	4	1,2
16.	$\int_0^1 x^6 (1-x)^7 \, dx =$ (A) $\beta(9,8)$ (B) $\beta(6,7)$ (C) $\beta(7,6)$ (D) $\beta(7,8)$	1	1	4	1,2
17.	$\sum(-1)^n \sin\left(\frac{1}{n}\right)$ converges by the following test. (A) Leibnitz's test (B) Ratio test (C) Root test (D) Integral test	1	1	5	1,2
18.	$\lim\limits_{n\to\infty}\left(n^{1/n}\right) =$ (A) n (B) 0 (C) 2 (D) 1	1	1	5	1,2
19.	If $\sum\limits_{n=1}^{\infty} u_n$ is convergent then (A) $\lim\limits_{n\to\infty} u_n \neq 0$ (B) $\lim\limits_{n\to\infty} u_n = 0$ (C) $u_n = 0$ (D) $u_n = \infty$	1	1	5	1,2
20.	The series $\sum\limits_{n=1}^{\infty} \frac{2^n}{n!}$ is (A) Convergent (B) Divergent (C) Oscillating (D) Monotonic	1	2	5	1,2

PART – B (5 × 8 = 40 Marks)
Answer **ALL** Questions

		Marks	BL	CO	PO
21. a.	Find the eigen values and eigen vectors of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.	8	3	1	1,2
	(OR)				
b.	Find the inverse of $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 2 \end{pmatrix}$ using Cayley-Hamilton theorem.	8	2	1	1,2

22. a.	Expand $x^2y+3y-2$ in powers of *(x–1)* and *(y+2)* upto second degree terms.	8	4	2	1,2
	(OR)				
b.	If $u=f(x,y)$ where $x=e^r\cos\theta, y=e^r\sin\theta$. Show that $x\dfrac{\partial u}{\partial\theta}+y\dfrac{\partial u}{\partial r}=e^{2r}\dfrac{\partial u}{\partial y}$.	8	3	2	1,2
23. a.	Solve $\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+3y=5x^2$.	8	2	3	1,2
	(OR)				
b.	Solve $y''+y=\sec x$ by the method of variation of parameters.	8	4	3	1,2
24. a.	Find the radius of curvature for the curve $x=a(\cos t+t\sin t), y=a(\sin t-t\cos t)$.	8	3	4	1,2
	(OR)				
b.	Find the envelope of the family of lines $y=mx-am^3$.	8	2	4	1,2
25. a.	Test the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^2+1}-n\right)$.	8	4	5	1,2
	(OR)				
b.	Test the convergence of the series $\dfrac{1}{2}+\dfrac{4}{9}x+\dfrac{9}{28}x^2+....+\dfrac{n^2}{1+n^3}x^n+...to\ \infty, x>0$.	8	2	5	1,2

PART – C (1 × 15 = 15 Marks)

Answer **ANY ONE** Questions

		Marks	BL	CO	PO
26.	Reduce the quadratic form $-x^2+y^2+4yz+4zx$ to canonical form by orthogonal reduction and find the rank, index, signature and the nature of the quadratic form.	15	4	1	1,2
27.	Find the greatest and the least distances of the point (3,4,12) from the unit sphere whose centre is at the origin.	15	4	2	1,2

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