Back-on-Speed is a medical rehabilitation facility which can treat up to P_1 "mobile" patients and P_2 "immobile" patients. Each of the patients has a private room, and there is also a common room with sufficient space to seat all $P = P_1 + P_2$ patients. Due to its patient-friendly atmosphere and excellent medical care, the rehabilitation services at Back-on-Speed are in high demand, and therefore one can assume that at any time, P clients are present in the facility.

Model description

Mobile patients spend generally distributed time periods with mean β_1 in their private rooms. After this time, they either visit the common room (with probability p) or get individual treatment in the gym (with probability $1 - p$). When the patient goes to the gym, the patient returns to her or his private room after a generally distributed period with mean τ. Immobile patients spend generally distributed time periods with mean β_2 in their private rooms. After this time, they would like to bring a visit to the common room. Mobile and immobile patients reside generally distributed time durations in the common room with mean values δ_1 and δ_2, respectively, before they would like to retreat to their private rooms. The immobile patients require assistance to move from their private room to the common room and back. There are two persons on staff to provide such assistance, one dedicated to assist patients in getting from their private room to the common room, and one assigned to help patients in returning from the common room to their private room. The time periods involved are generally distributed with mean μ, in either direction (also for the return). When the nurse is occupied, the patients wait (in first-come-first-served order) for the nurse to become available. Note that the *actual* time spent by immobile patients in their private room is the sum of *two* random variables: the residing time (with mean β_2) plus the waiting time for the nurse (which might be zero if the nurse is immediately available). Similarly, the *actual* time that immobile patients spend in the common room consists of the residing time (with mean δ_2) plus the waiting time for the nurse to bring them to their private room. For simplicity, we assume that a nurse always returns to his/her "base" (which is the common room for one nurse, and the private rooms for the other nurse) without a patient. We also assume that the duration of this return trip has the same distribution as the trip *with* a patient. The gym and the common room have sufficient space for all patients.

Some notation

Denote by N_{c1} and N_{c2} the number of mobile and immobile patients residing in the common room, respectively (including those receiving or waiting for assistance to move from the common room to their private room). Denote by N_{p1} and N_{p2} the number of mobile and immobile patients residing in their private rooms, respectively (where N_{p2} includes the patients receiving or waiting for assistance to move from their private room to the common room). Let $N_c := N_{c1} + N_{c2}$ and $N_p := N_{p1} + N_{p2}$. Denote by N_w the number of mobile patients in the gym. Denote by W_a the waiting time for an immobile patient before receiving assistance to move from her or his private room to the common room. Lastly, denote by W_b the waiting time for an immobile patient before receiving assistance to move from the common room to her or his private room.

Assignment: Write a stochastic simulation to simulate the flow of patients, by modelling the rehabilitation facility as a closed queueing network with two customer classes (mobile and immobile). Each of the two nurses, moving the patients between private room and common room, should be modelled as a single server queue.

Performance measures: We would like you to study the performance of the system. In particular, we are interested in the mean and standard deviation of the number of patients in each of the areas (private room, common room, gym), for each patient type.

Question 1: Discrete-event simulation with event scheduling. For this question, we request that you write a discrete-event simulation, using event scheduling, as described in Section 8.3 of the lecture notes. Use this simulation to determine all the performance measures described above, for each of the following three sets of model parameters:

	P_1	P_2	p	β_1	β_2	δ_1	δ_2	τ	μ
Set 1:	4	2	2/3	2	4	2	4	1	1/2
Set 2:	6	1	1/2	1	3	2	6	1	1/2
Set 3:	2	4	1/3	1	3	2	4	2	1/3

For the generic service times, we ask you to compare the results when all distributions are exponential and another simulation with a distribution of your choice, preserving the first moment. Describe the implementation of your simulation in detail.

We kindly ask you to provide a table as the one below, containing simulation results for each of the three sets of parameters ("std" stands for standard deviation). This table will be used to verify correctness of your simulation. Create this table for the case where all distributions are exponential, and for the simulation with a distribution of your choice.

	$\mathbb{E}[N_c]$	std$[N_c]$	$\mathbb{E}[N_p]$	std$[N_p]$	$\mathbb{E}[N_w]$	std$[N_w]$
Set 1						
Set 2						
Set 3						

Question 2: Discrete-event simulation without event scheduling. For this question, we assume that *all* distributions are exponential: the residing time in the private rooms, the time in the gym, the time spent in the common room, and a *single* trip by the nurse from common room to private room (or vice versa). Now we request that you write a discrete-event simulation *without* event scheduling, using a continuous-time Markov chain as described in Section 7.5 of the lecture notes. Again, we are only interested mean queue lengths. Describe the model in detail: what are the states and the transition rates of the Markov chain? Also describe the implementation of your simulation in detail.

Obviously, the results should agree with those from the exponential case of Question 1. If this is indeed the case, you do not need to include any additional results for this question – the model and simulation description are sufficient. If, however, the results do not agree, there must be a mistake in one of the two simulations. In this case, we recommend including *both* sets of results, so we can check which results are correct.

More details

The assignment will be 20% of the final grade of the course 2DI66. Each group should hand in a well-written report and the source code of their simulation programs. In contrast to other assignments, the focus in this assignment will *not at all* be on the report! In fact, your report only should contain:

- A simulation description, for each of the two implementations:
  - The source code of your project, which should be well annotated.
  - A description of all classes that you created (if applicable).
  - A description of all the events and an explanation of how they are handled (for the discrete-event simulation).
- Results:
  - A description of the probability distributions you chose as "service time distributions" in the model.
  - The aforementioned tables with simulation results.
- Confidence intervals for (some of) the simulated values and a discussion on the number of runs you chose.
- A page where you discuss how the work was divided among the group members. Who contributed to each part of the assignment?
- Optional: bonus material. You are always allowed to include extra work, which *might* lead to bonus points. We cannot give any guarantees about the number of awarded bonus points, but you are welcome to make suggestions to the lecturer (before actually starting the extra work) so he can advise you on whether your planned extra work would indeed lead to extra bonus points. For example: you can add results for queue length *distributions* or waiting times.

More detailed guidelines can be found in Canvas, so check the rubric for more information. Urgent questions can be sent to m.a.a.boon@tue.nl. Upload your report in PDF format before the deadline specified in Canvas. **This is a hard deadline**! Please include your source code in a separate `.txt` file.