Eigenvalues and Eigenvector

Saturday, 26 July 2025 3:20 PM

$$\boxed{(A = -\lambda I)x = 0}$$

A = given matrix

λ = Constant

I = Identity Matrix

x = unknown Matrix

0 = Null Matrix

$$A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

$$\underline{(A - \lambda I)}x = 0$$

$$\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$(A - \lambda I) = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} (8-\lambda) & -8 & -2 \end{bmatrix}$$

$$= \begin{vmatrix} 4 & (-3-\lambda) & -2 \\ 3 & -4 & (1-\lambda) \end{vmatrix} = 0$$

$$\lambda^3 - \begin{bmatrix} \text{Sum of} \\ \text{Dignol Elem} \end{bmatrix} \lambda^2 + \begin{bmatrix} \text{Sum of} \\ \text{Dignol} \\ \text{Minor} \end{bmatrix} - |A| = 0$$

$$\begin{bmatrix} \text{Sum of} \\ \text{Dignol Elem} \end{bmatrix} = 8 + (-3) + 1 = 6$$

$$\begin{bmatrix} \text{Sum of} \\ \text{Dignol} \\ \text{Minor} \end{bmatrix} = Det(A_1) + Det(b_2) + Det(C_3)$$

$$= (-3-8) + (8+6) + (-24+32)$$

$$= -11 + 14 + 8$$

$$= 11$$

$$|A| = 8(-3-8) - (-8)(4+6) - 2(-16+9)$$

$$= -88 + 80 + 14$$

$$= 6$$

$$\lambda^3 - \begin{bmatrix} \text{Sum of} \\ \text{Dignol Elem} \end{bmatrix} \lambda^2 + \begin{bmatrix} \text{Sum of} \\ \text{Dignol} \\ \text{Minor} \end{bmatrix} - |A| = 0$$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

Put $\lambda = 0$ $(\times)$

$-6 \neq 0$

Put $\lambda = 1$ $(\checkmark)$

$1 - 6 + 11 - 6 = 0$

$-12 + 12 = 0$

$0 = 0$

Put $\lambda = 2$ $(\checkmark)$

$0 = 0$

Put $\lambda = 3$ $(\checkmark)$

$0 = 0$

$\lambda = (1, 2, 3)$

$(A - \lambda I)x = 0$

$$\begin{bmatrix} (8-\lambda) & -8 & -2 \\ 4 & (-3-\lambda) & -2 \\ 3 & -4 & (1-\lambda) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Putting $\lambda = 1$

$$\begin{bmatrix} 7 & -8 & -2 \\ 4 & -4 & -2 \\ 3 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$7x_1 - 8x_2 - 2x_3 = 0$

$$4x_1 - 4x_2 - 2x_3 = 0$$

$$3x_1 - 4x_2 = 0$$

Grammer's Rule →

$$\frac{x_1}{\begin{vmatrix} -8 & -2 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 7 & -2 \\ 4 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 7 & -8 \\ 4 & -4 \end{vmatrix}}$$

$$\frac{x_1}{(16-8)} = \frac{-x_2}{(-14+8)} = \frac{x_3}{(-28+32)}$$

$$\frac{x_1}{\cancel{8}\,4} = \frac{\cancel{-}x_2}{\cancel{-}\cancel{6}\,3} = \frac{x_3}{\cancel{4}\,2}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

$\lambda = 1$

Putting $\lambda = 2$

Putting $\lambda = 3$