Jacobian Matrix

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if u and v are two independent variable of x & y the Jacobian of u and v w.r. to x & y is→

$$\frac{\partial(u,v)}{\partial(x,y)} = J\left[\frac{u,v}{x,y}\right] = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$$

Ex → $x = r\cos\theta$, $y = r\sin\theta$

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{bmatrix}$$

$\frac{\partial x}{\partial r} = \cos\theta$ $\quad \frac{\partial x}{\partial \theta} = -r\sin\theta$

$\frac{\partial y}{\partial r} = \sin\theta$ $\quad \frac{\partial y}{\partial \theta} = r\cos\theta$

$$= \begin{bmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{bmatrix}$$

$$= r\cos^2\theta + r\sin^2\theta$$

$$= r \times 1$$

$$= r$$