Cumulative Distribution Function

Sunday, 27 July 2025 12:38 PM

$$\boxed{f_x(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx}$$

Example →

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ 2-x & 1 < x < 2 \\ 0 & o/w \end{cases}$$

(i)- find $P(x \ge 1.5)$

(ii)- find C·D·F

(i). $P(x \ge 1.5) = \int_{1.5}^{\infty} f(x)\,dx \Rightarrow \int_{1.5}^{2} (2-x)$ 2 (greater then 1.5)

$$= \left[(2x - x^2/2) \right]_{1.5}^{2}$$

$$= (4-3) - \left[\frac{4}{2} - \frac{2.25}{2} \right]$$

$$= 1 - \frac{1.75}{2}$$

$$= 0.125$$

(ii) CDF

$x \le 0$

$$P(x \le x) = \int_{-\infty}^{x} f(x)\,dx$$

$$f(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$$

$$= 0 + \int_{0}^{x} x\,dx$$

$$= \int_{0}^{x} x^2/2$$

take $x \le 2$ $\quad f(x) = \int_0^1 f(x)\,dx + \int_1^x f(x)\,dx$

$$f(x) = \int^{1} x\,dx + \int^{x} (2-x)\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left(2x - \frac{x^2}{2}\right)_1^x \Rightarrow \frac{1}{2} + \left(2x - \frac{x^2}{2}\right) - \left(2 - \frac{1}{2}\right)$$

$$= 1 - 2 + 2x - \frac{x^2}{2}$$

$$= -1 + 2x - x^2/2$$

$x > 2$

$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^{\infty} f(x)\,dx$$

$= 1$

final $\rightarrow$

$$F_x(x) = \begin{cases} 0 & -\infty < x < 0 \\ x^2/2 & x \le 1 \\ -1 + 2x - x^2/2 & x \le 2 \end{cases}$$