IMPROVING DISCOUNTING USING AVERAGE REWARD

Tea Time Talk
16 Aug 2023

Abhishek Naik

DISCOUNTING BECOMES UNSTABLE WHEN $\gamma \to 1$

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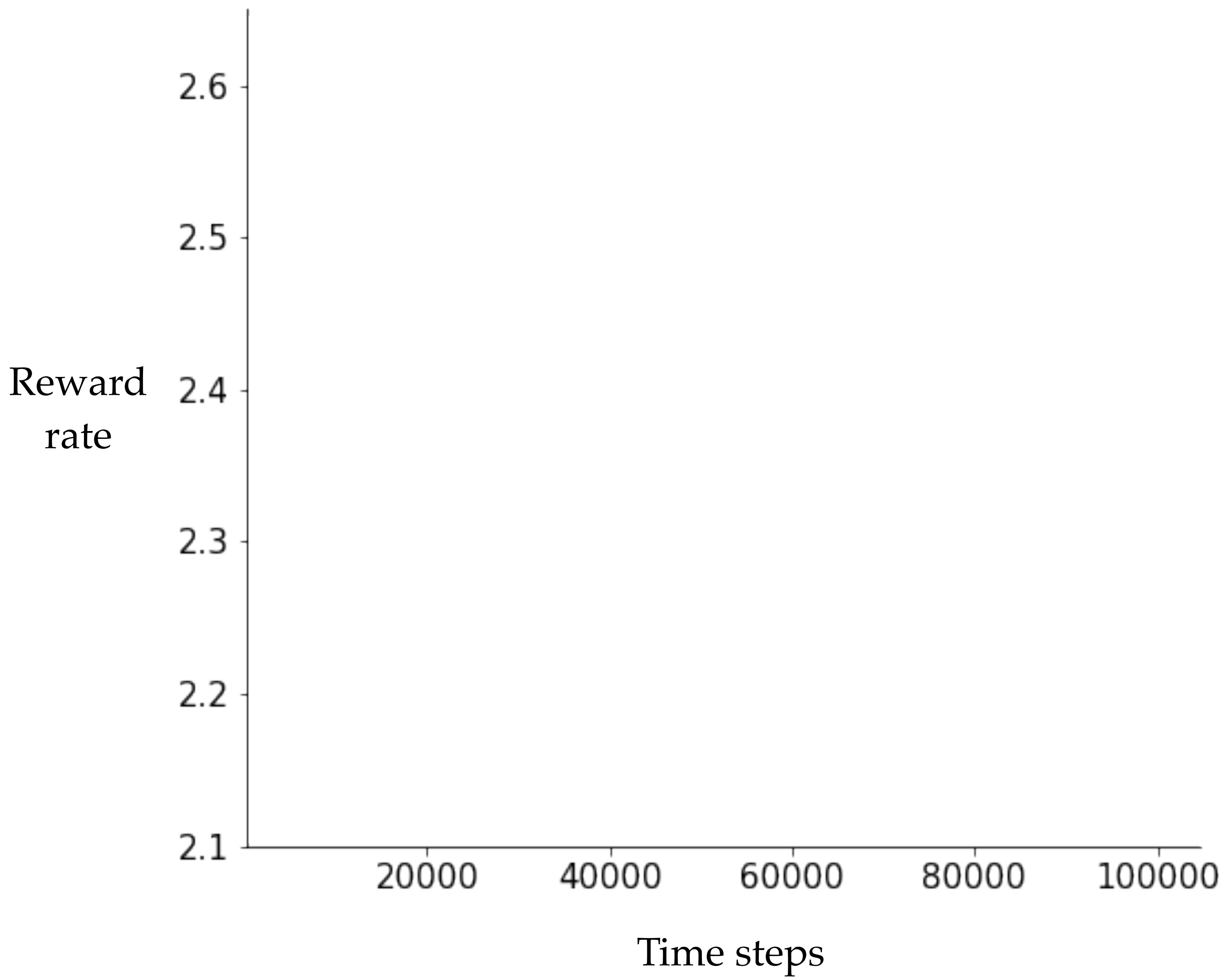

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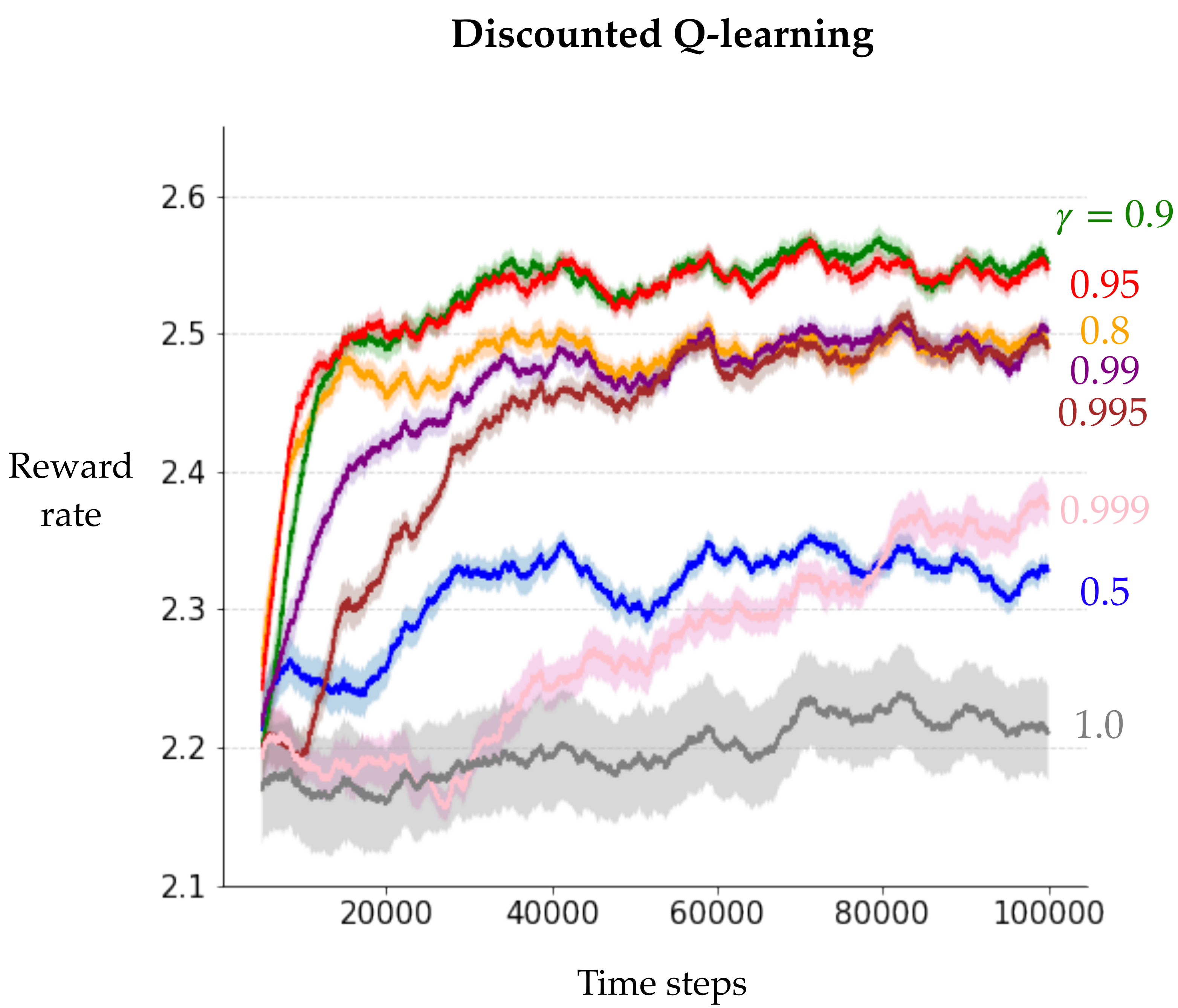
DISCOUNTING BECOMES UNSTABLE WHEN $\gamma \to 1$



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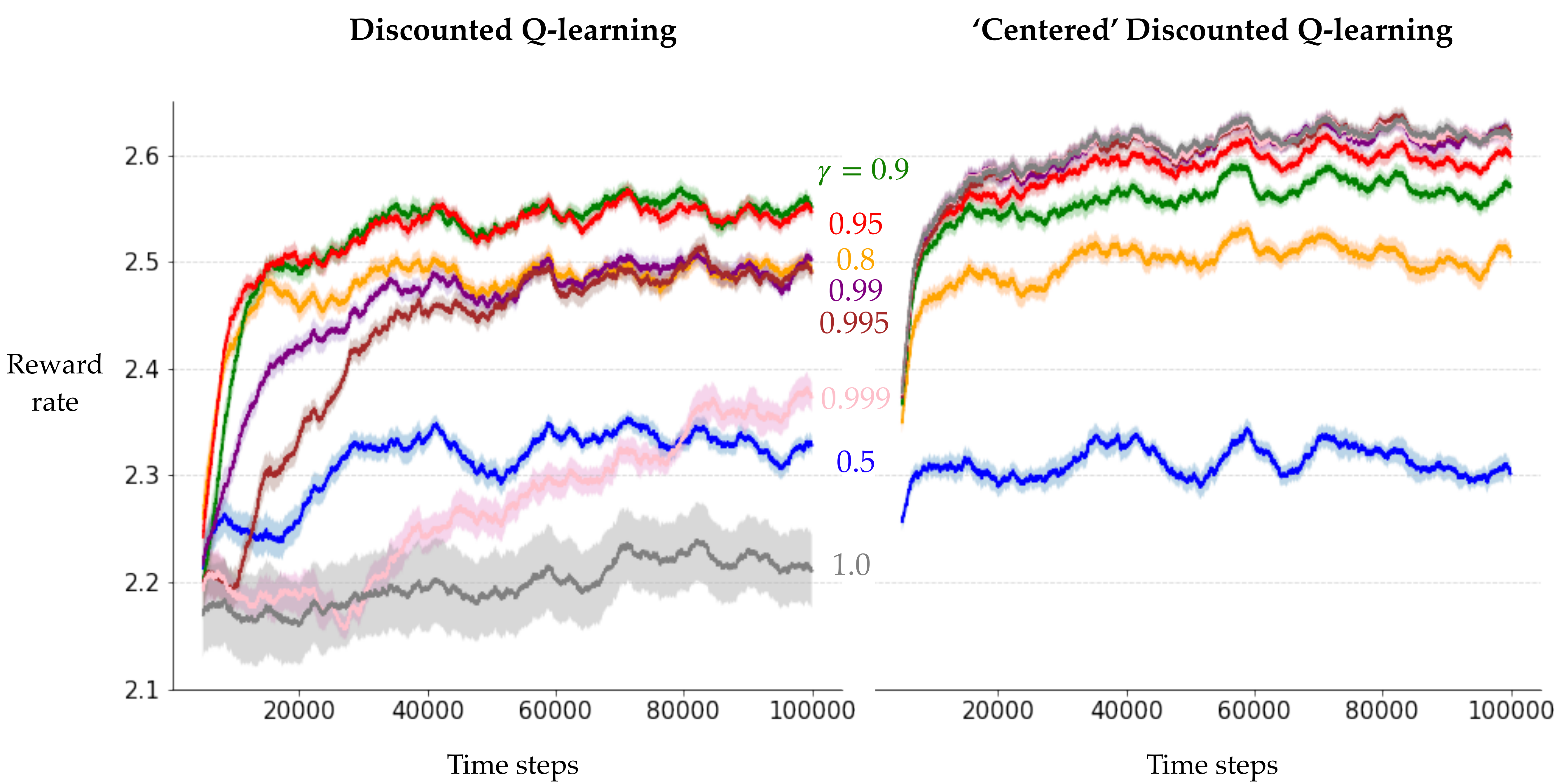
DISCOUNTING BECOMES UNSTABLE WHEN $\gamma \rightarrow 1$



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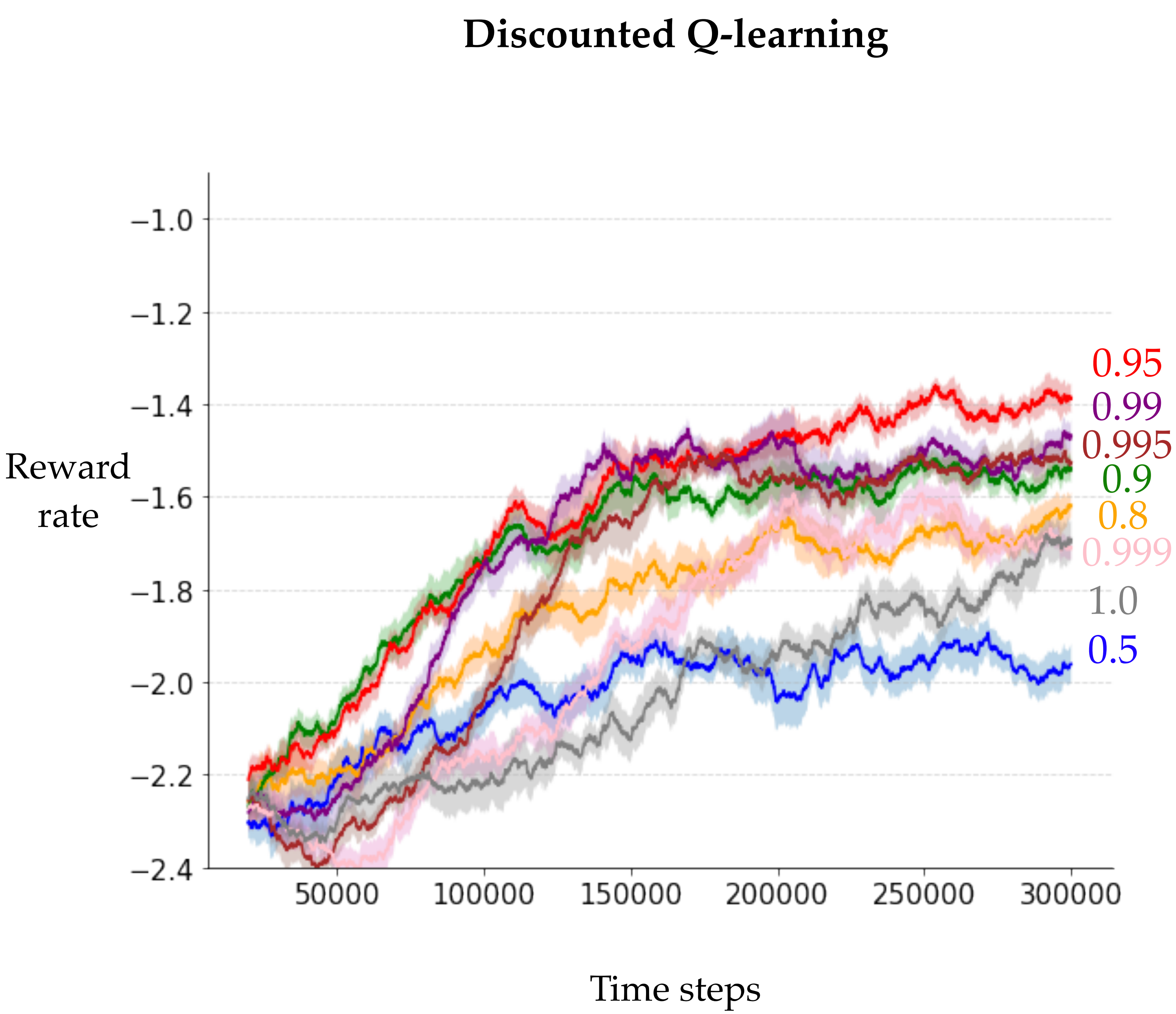
SIMILAR TRENDS FOR LINEAR AND NON-LINEAR FA

PuckWorld (linear FA)

SIMILAR TRENDS FOR LINEAR AND NON-LINEAR FA



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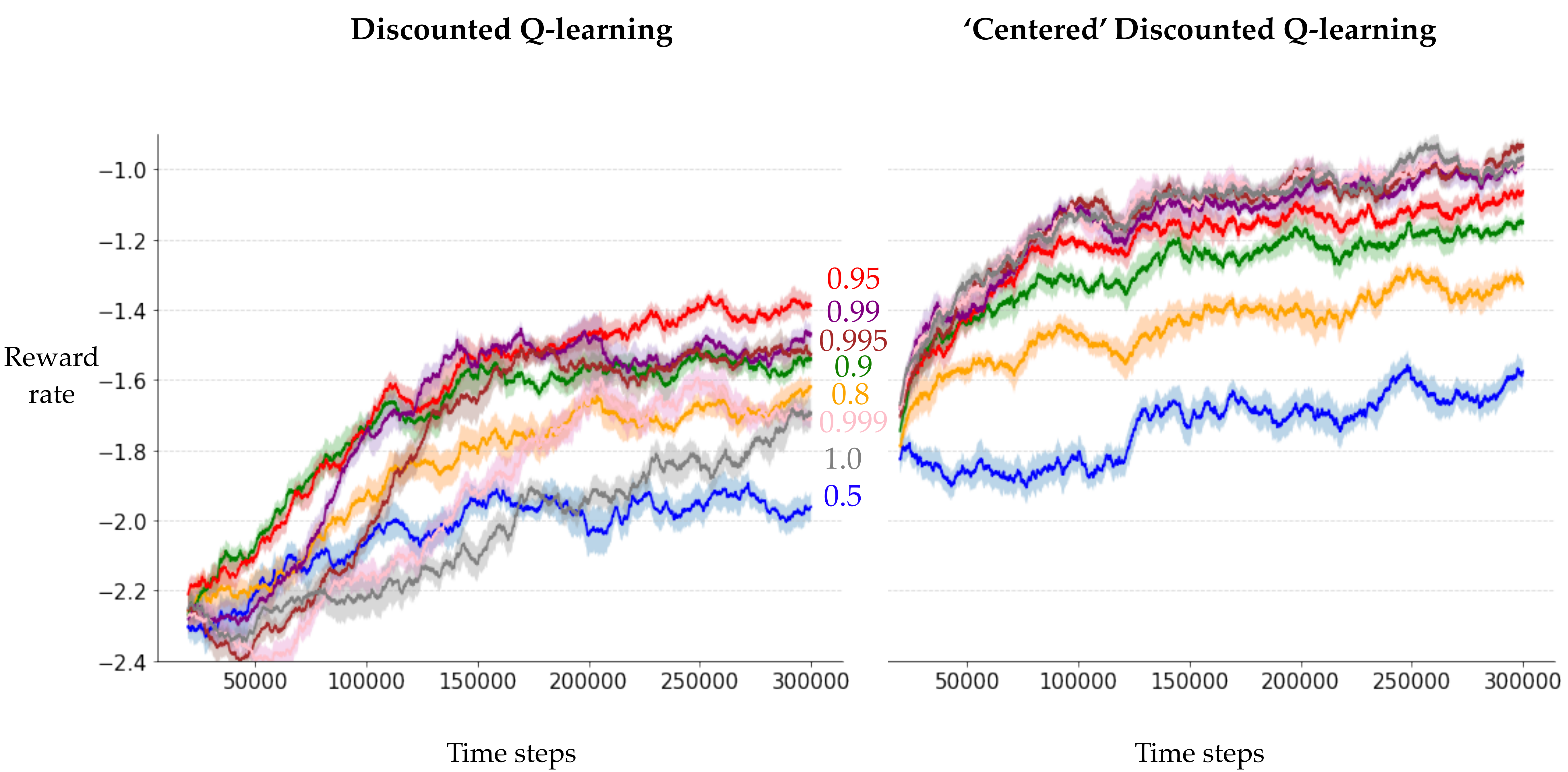

PuckWorld (linear FA)

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Pendulum (non-linear FA)

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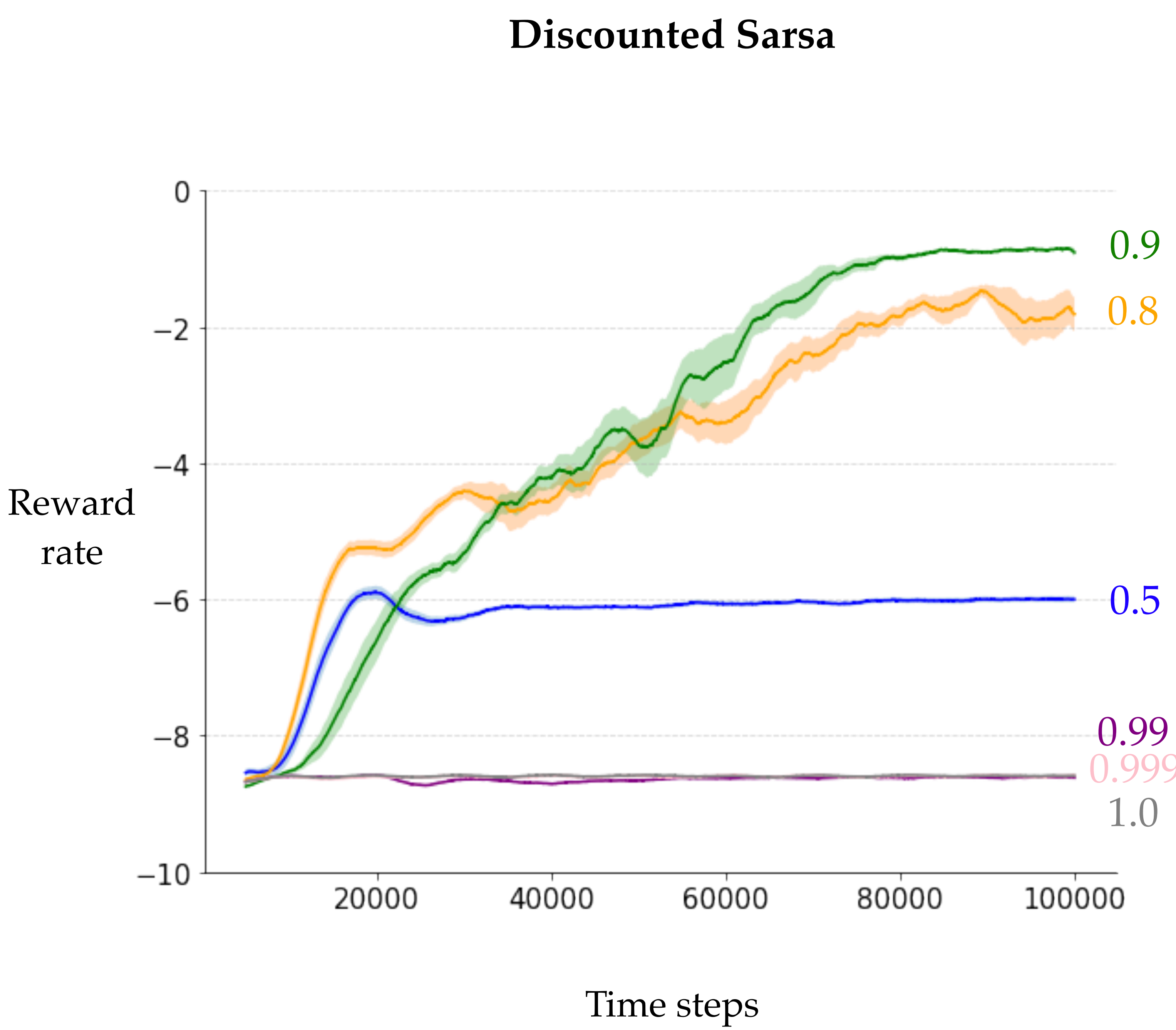

Pendulum (non-linear FA)

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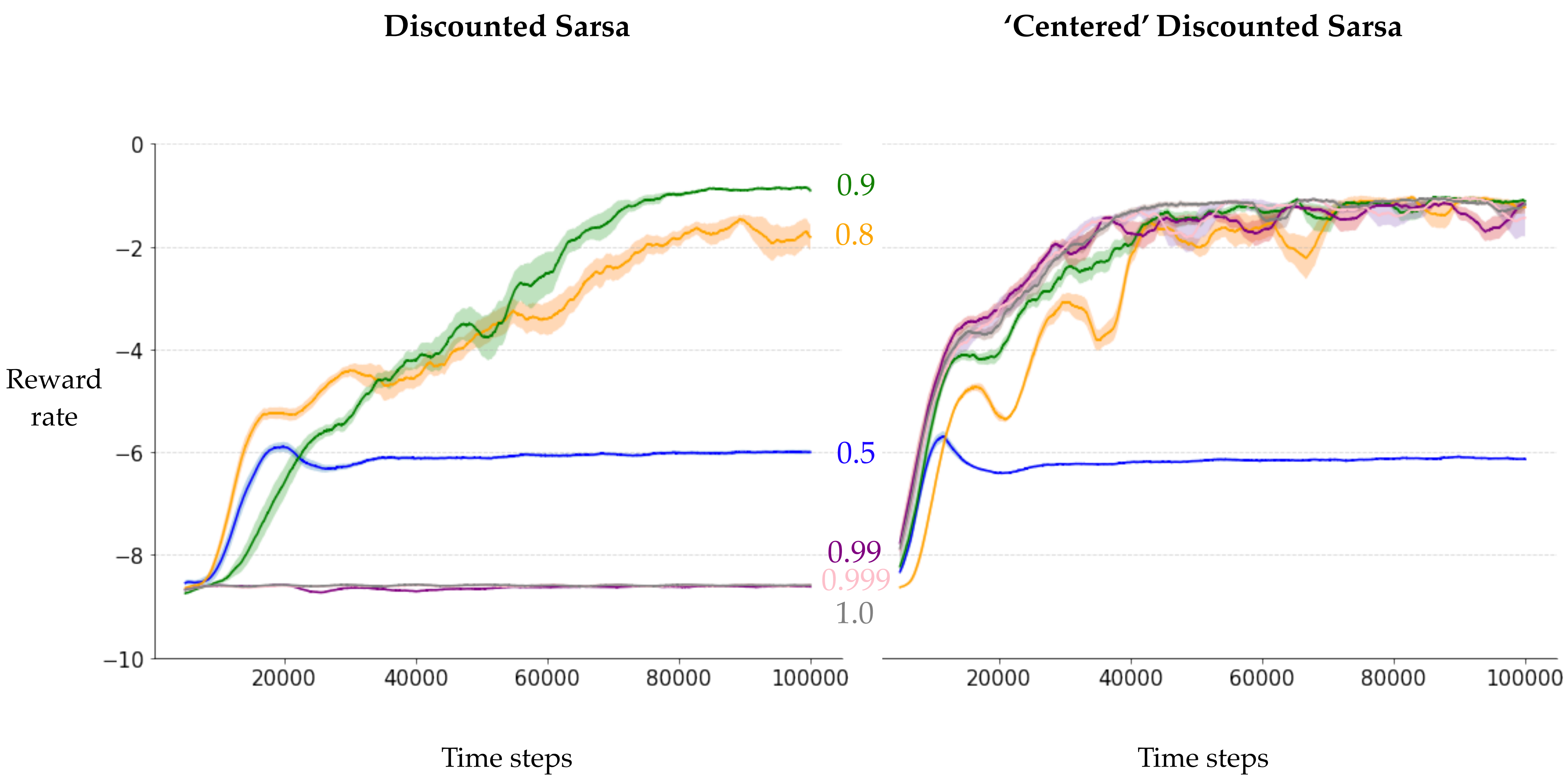

Pendulum (non-linear FA)

TRENDS ARE CONSISTENT ACROSS PARAMETERS

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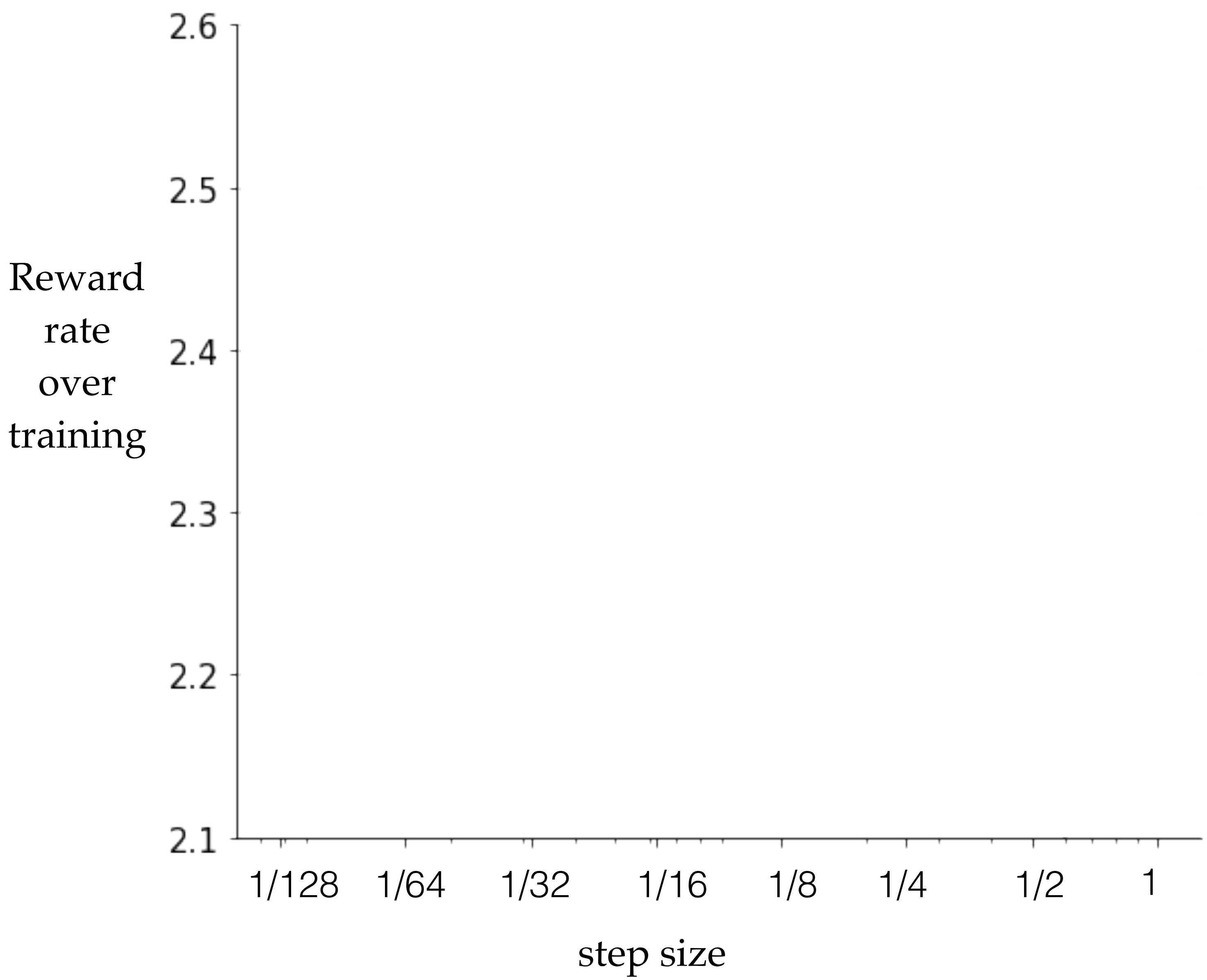

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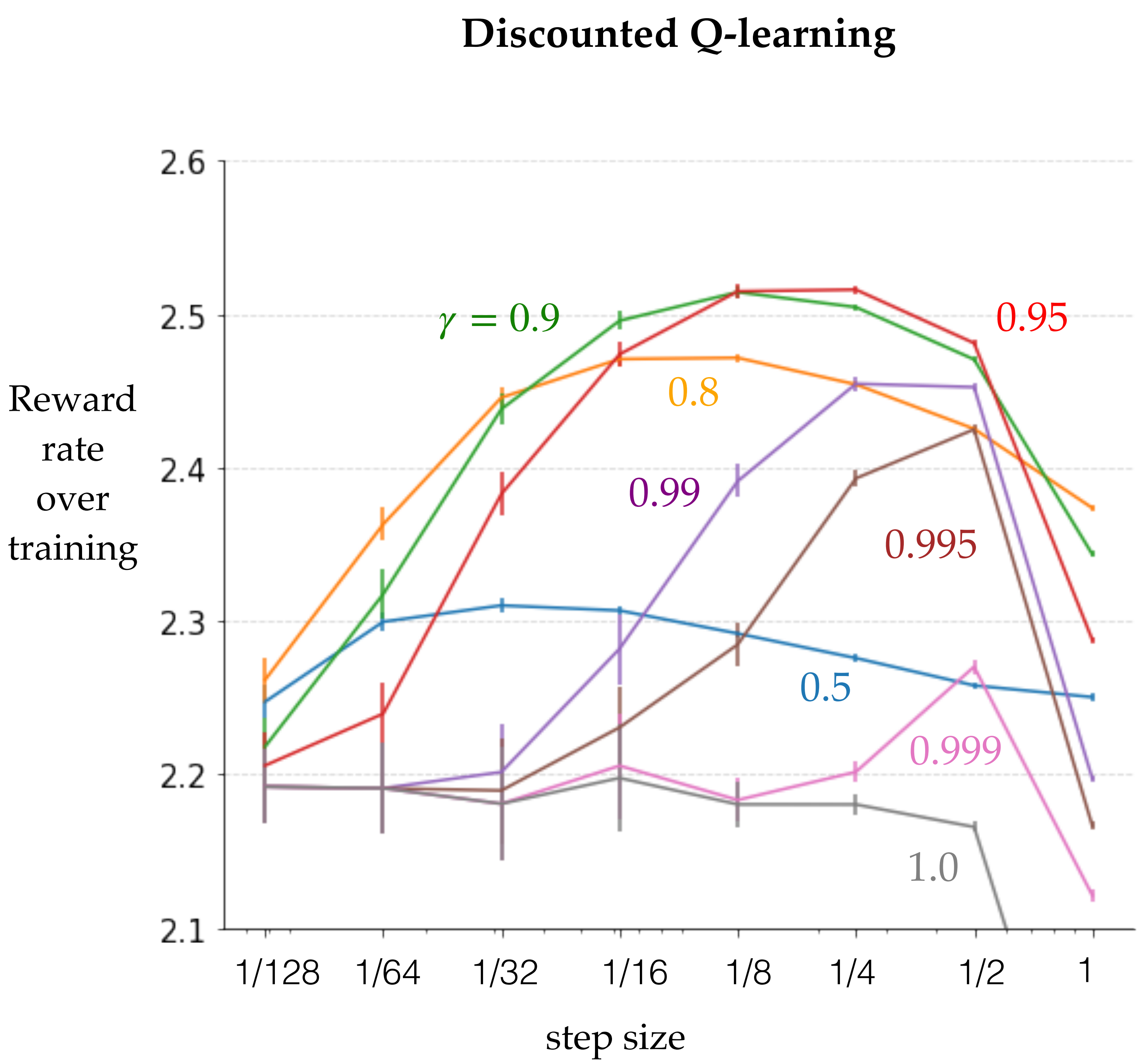

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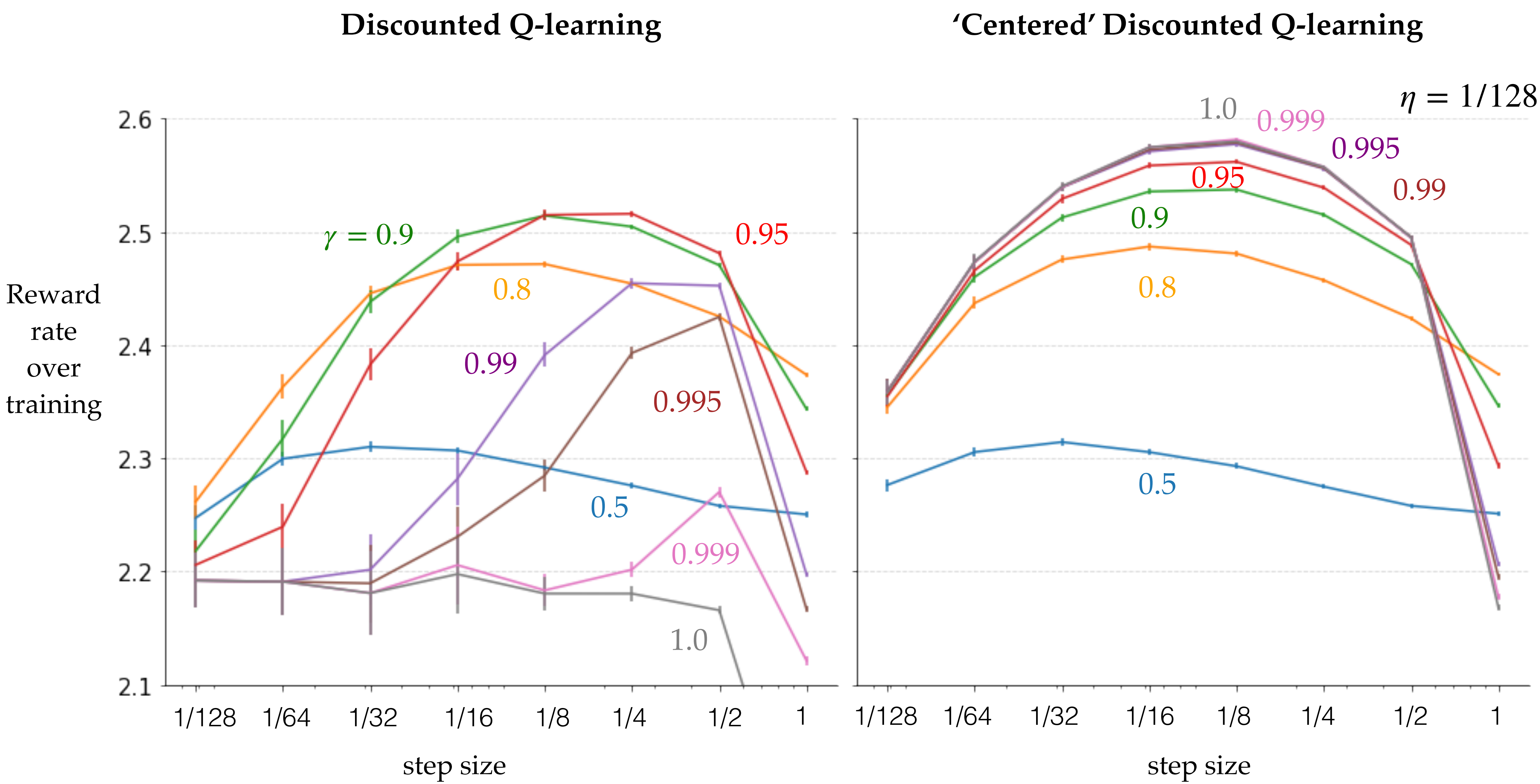

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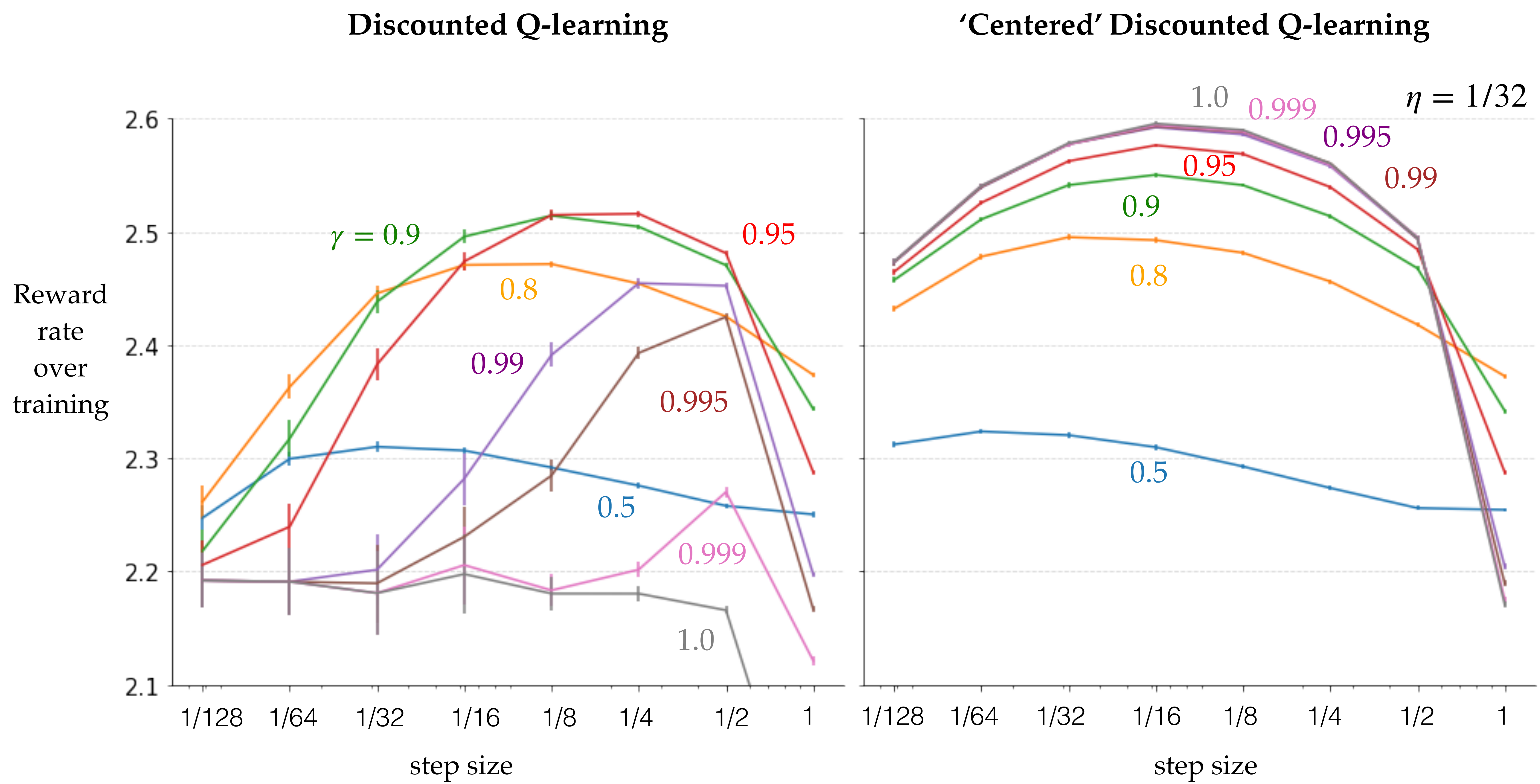

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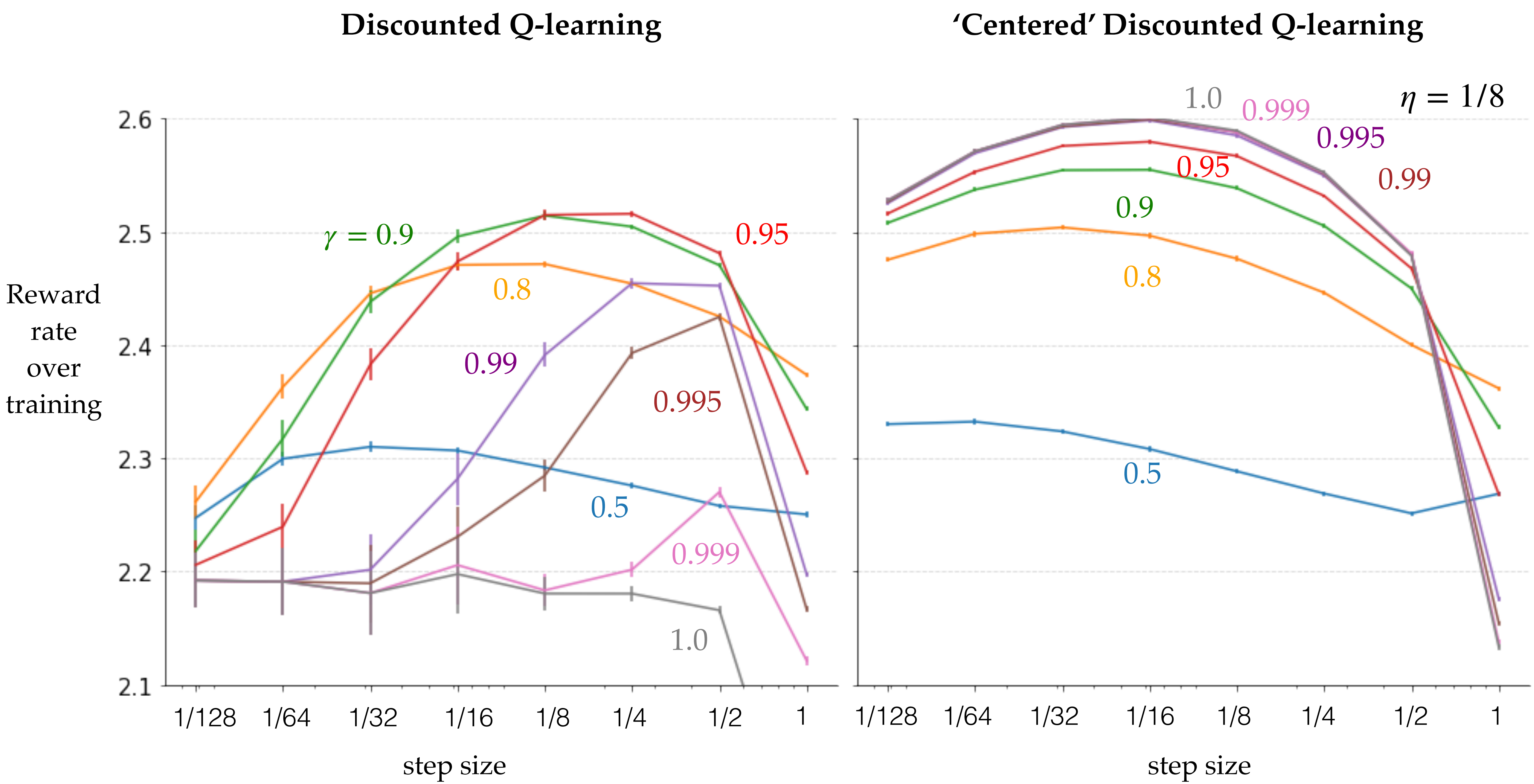

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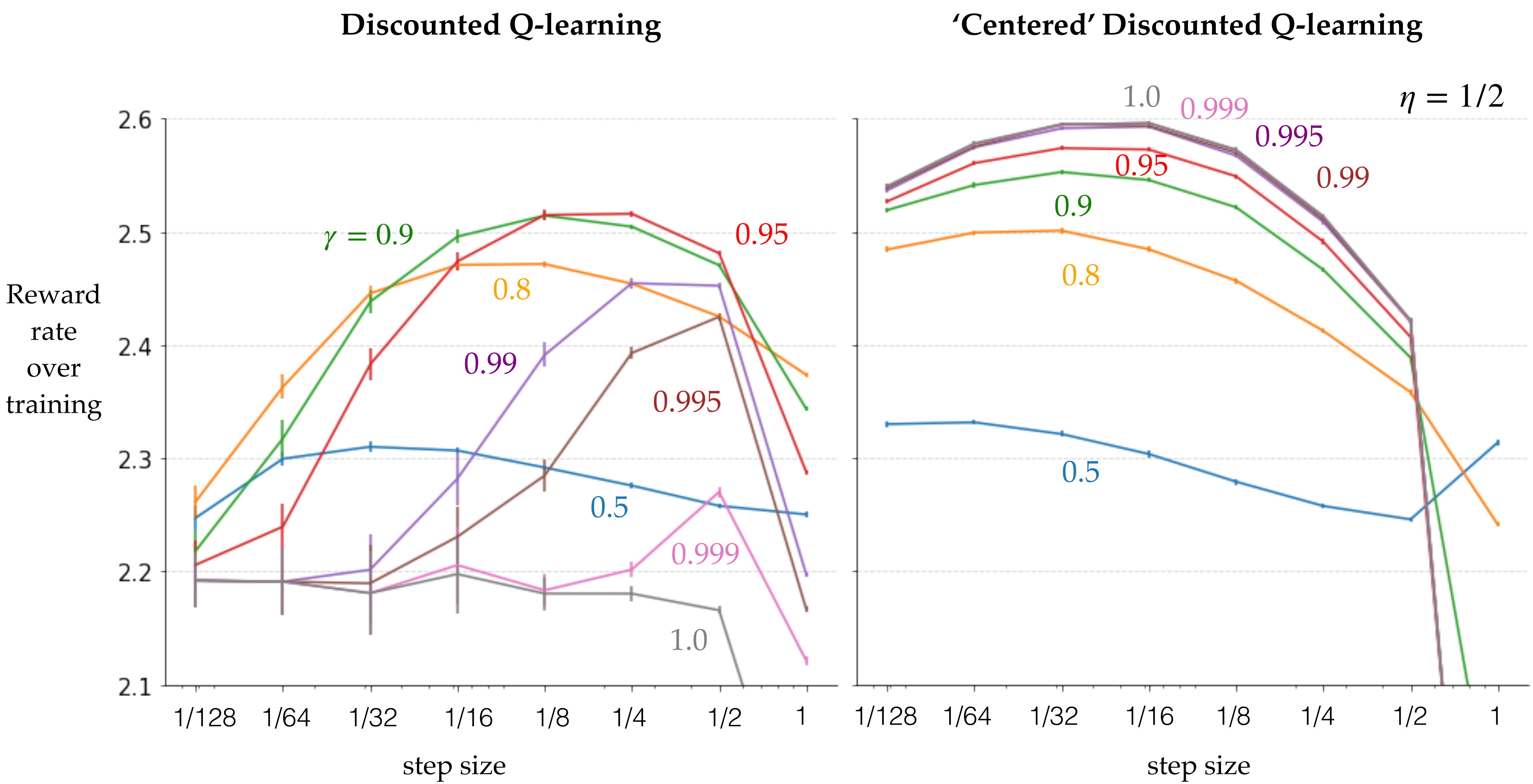

AccessControl (tabular)

KEY INSIGHT

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$$R_{t+1} \quad R_{t+2} \quad R_{t+3} \quad \cdots \quad R_{t+n} \quad \cdots$$

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$$R_{t+1} \quad R_{t+2} \quad R_{t+3} \quad \cdots \quad R_{t+n} \quad \cdots$$

$$v_\pi^\gamma(s) \doteq \mathbb{E}_\pi\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots \mid S_t = s\right] \longleftarrow$$

Discounted value function

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$$= \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s\Big]$$

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Discounted value function

Laurent-series expansion

$$v_\pi^\gamma(s) = \frac{r(\pi)}{1-\gamma} + \bar{v}_\pi(s) + e_\pi^\gamma(s)$$

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$$r(\pi) \doteq \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \mathbb{E}_\pi[R_t]$$

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← Discounted value function

$$= \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,|\, S_t = s\right]$$

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$$= v_\pi^\gamma(s) - \frac{r(\pi)}{1-\gamma}$$

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Centered discounted value function

ESTIMATING $r(\pi)$

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- On-policy:

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- On-policy:
  - sample average of rewards

ESTIMATING $r(\pi)$

$$R_{t+1} \quad R_{t+2} \quad R_{t+3} \quad \ldots \quad R_{t+n} \quad \ldots$$

- On-policy:
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$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta(R_{t+1} - \bar{R}_t)$$

ESTIMATING $r(\pi)$

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$$r(\pi) = \sum_s d_\pi(s) \sum_a \pi(a \mid s) \sum_r p(r \mid s, a)\ r$$

ESTIMATING $r(\pi)$

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$$\bar{R}_{t+1} \doteq \bar{R}_t + \beta\,\delta_t$$

ESTIMATING $r(\pi)$ IN THE AVG-REWARD FORMULATION

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$$\bar{v}_\pi(s) \doteq \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty}\Big(R_{t+k+1} - r(\pi)\Big) \,|\, S_t = s\Big]$$

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$$v_\pi^\gamma(s) = \sum_a \pi(a\,|\,s) \sum_{s',r} p(s', r\,|\,s, a)\Big[r + \gamma \bar{v}_\pi(s')\Big]$$

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$$\downarrow$$

$$V_{t+1}(S_t) \doteq V_t(S_t) + \alpha\Big(R_{t+1} + \gamma V_t(S_{t+1}) - V_t(S_t)\Big)$$

ESTIMATING $r(\pi)$ IN THE AVG-REWARD FORMULATION

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ESTIMATING $r(\pi)$ IN THE AVG-REWARD FORMULATION

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ESTIMATING $r(\pi)$ IN THE AVG-REWARD FORMULATION

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$$\bar{V}_{t+1}(S_t) \doteq \bar{V}_t(S_t) + \alpha\,\delta_t$$

$$v^\gamma_\pi(s) = \sum_a \pi(a\,|\,s) \sum_{s',r} p(s', r\,|\,s, a)\big[r + \gamma\bar{v}_\pi(s')\big]$$

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$$\bar{V}_{t+1}(S_t) \doteq \bar{V}_t(S_t) + \alpha\,\delta_t$$

$$\delta_t \doteq R_{t+1} - \bar{R}_t + \bar{V}_t(S_{t+1}) - \bar{V}_t(S_t)$$

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$$\bar{V}_{t+1}(S_t) \doteq \bar{V}_t(S_t) + \alpha\,\delta_t$$

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ESTIMATING $r(\pi)$ IN THE AVG-REWARD FORMULATION

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$$\bar{v}_\pi(s) \doteq \mathbb{E}_\pi\Big[\sum_{k=0}^{\infty} \big(R_{t+k+1} - r(\pi)\big) \,|\, S_t = s\Big]$$

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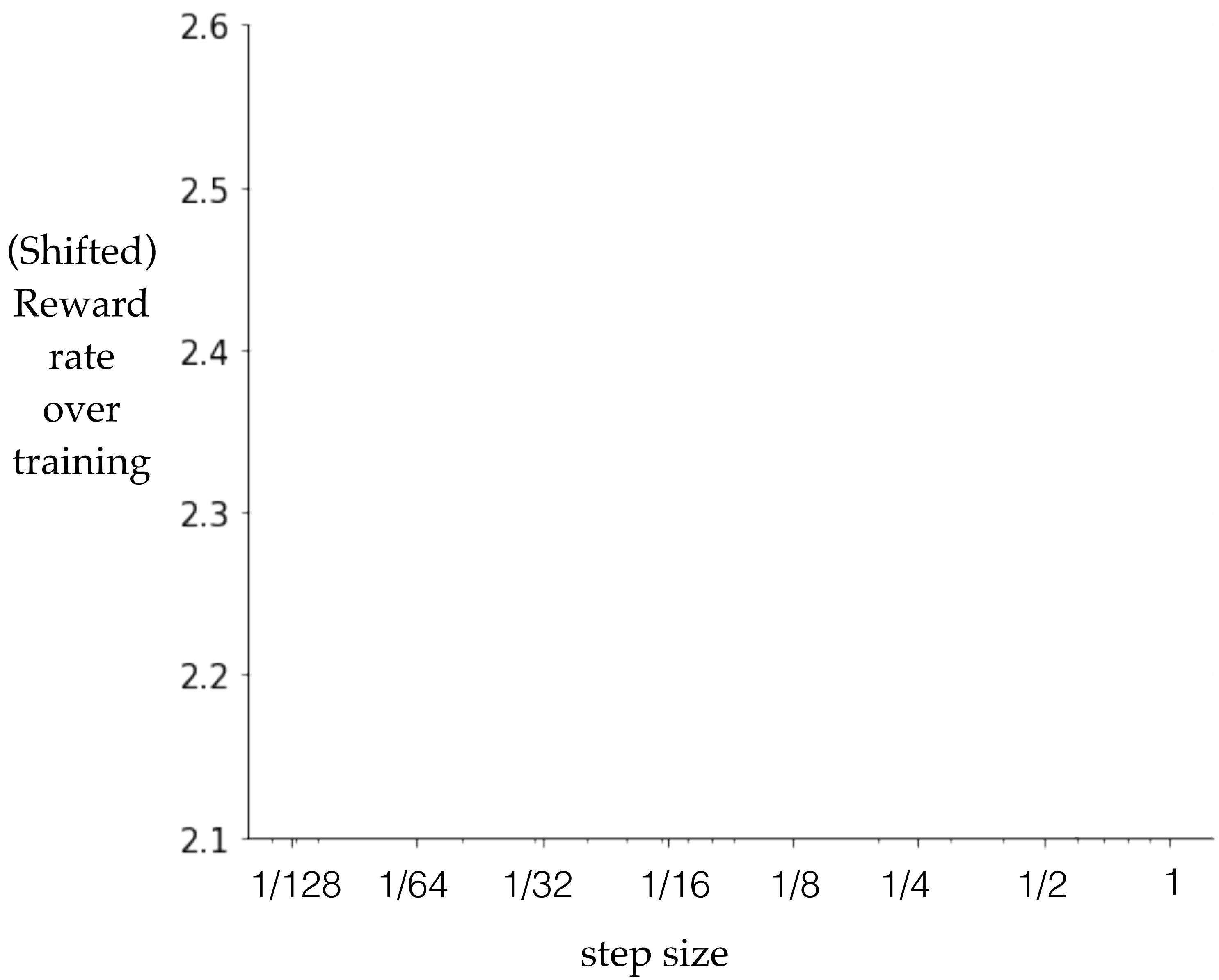

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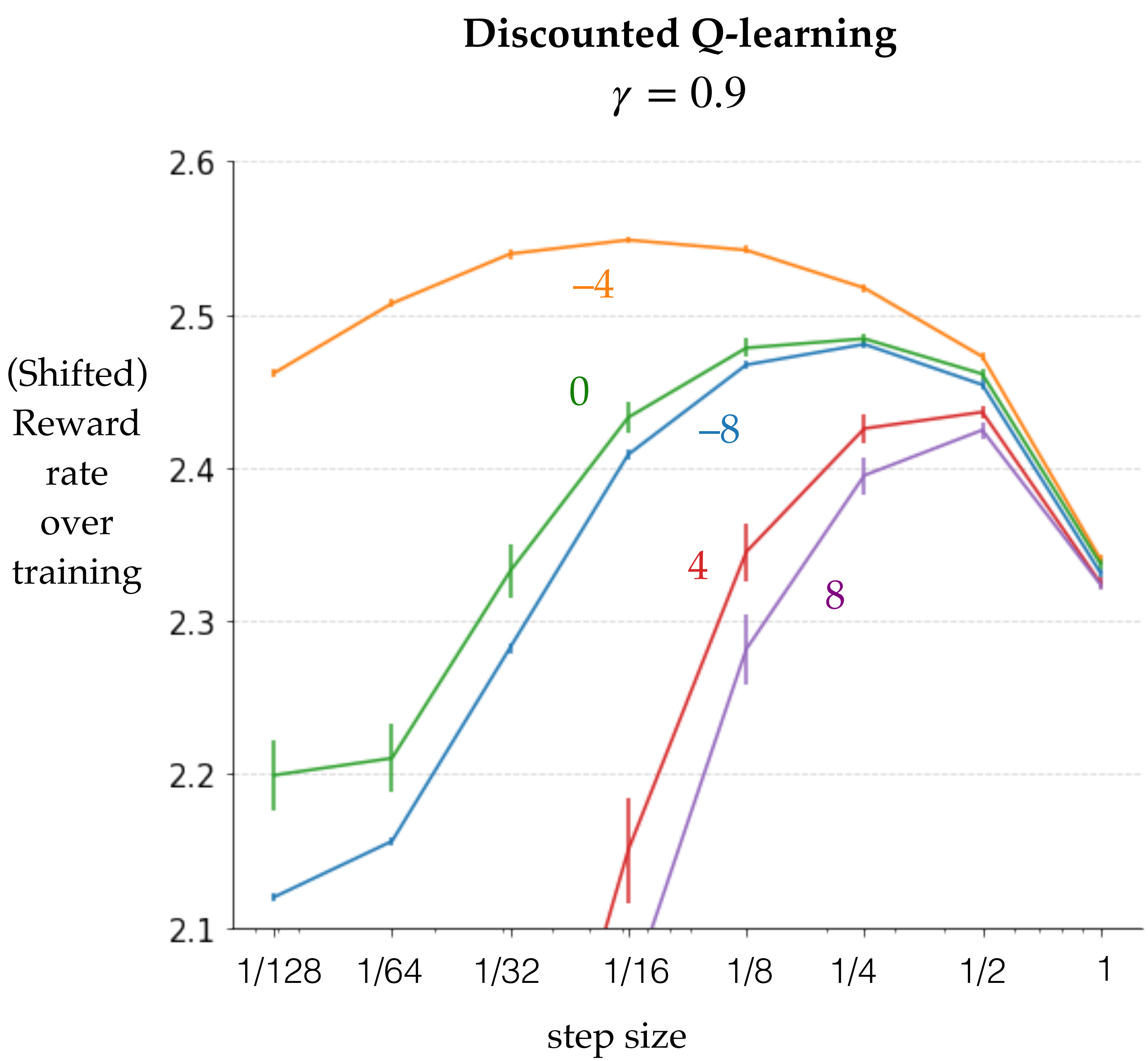

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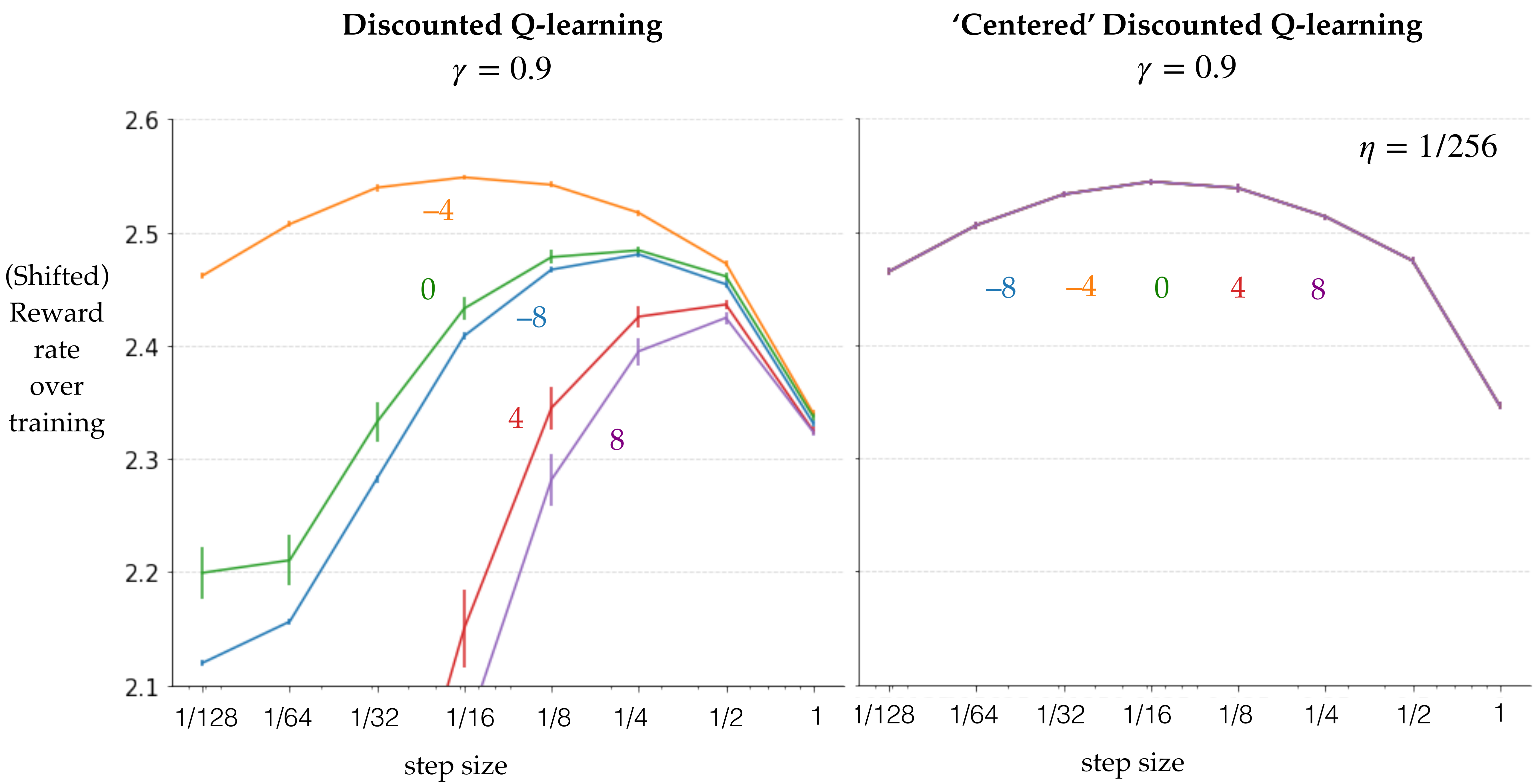

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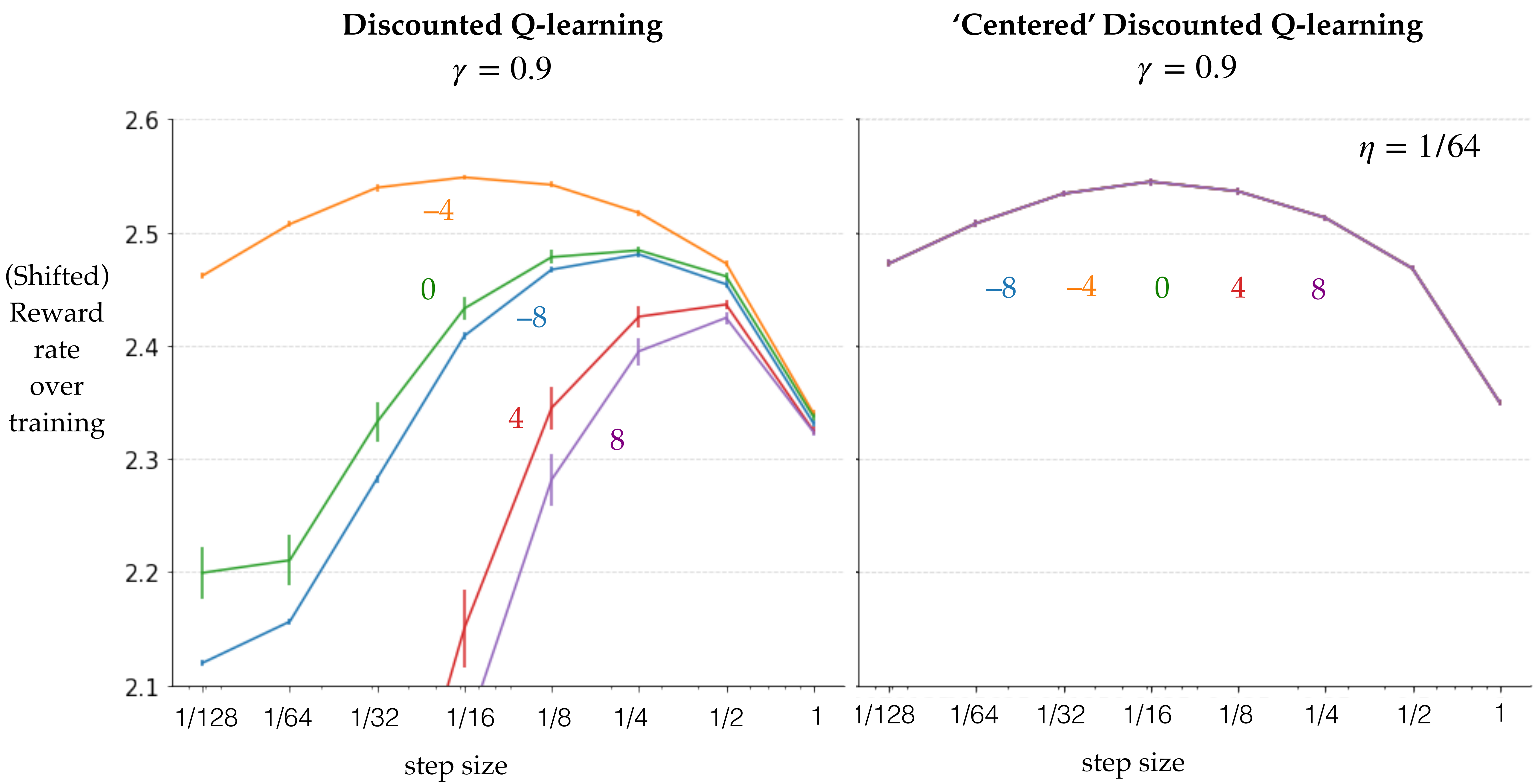

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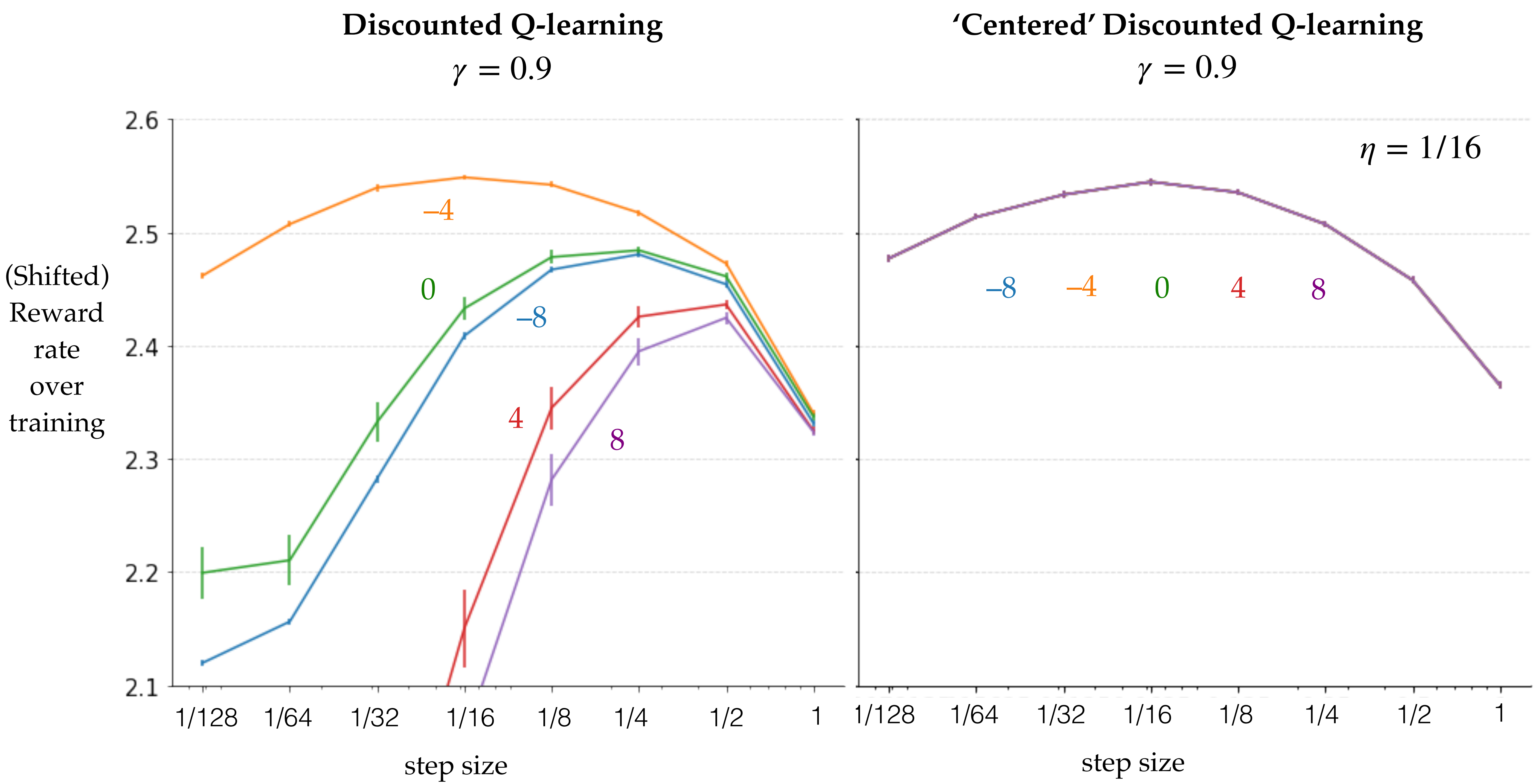

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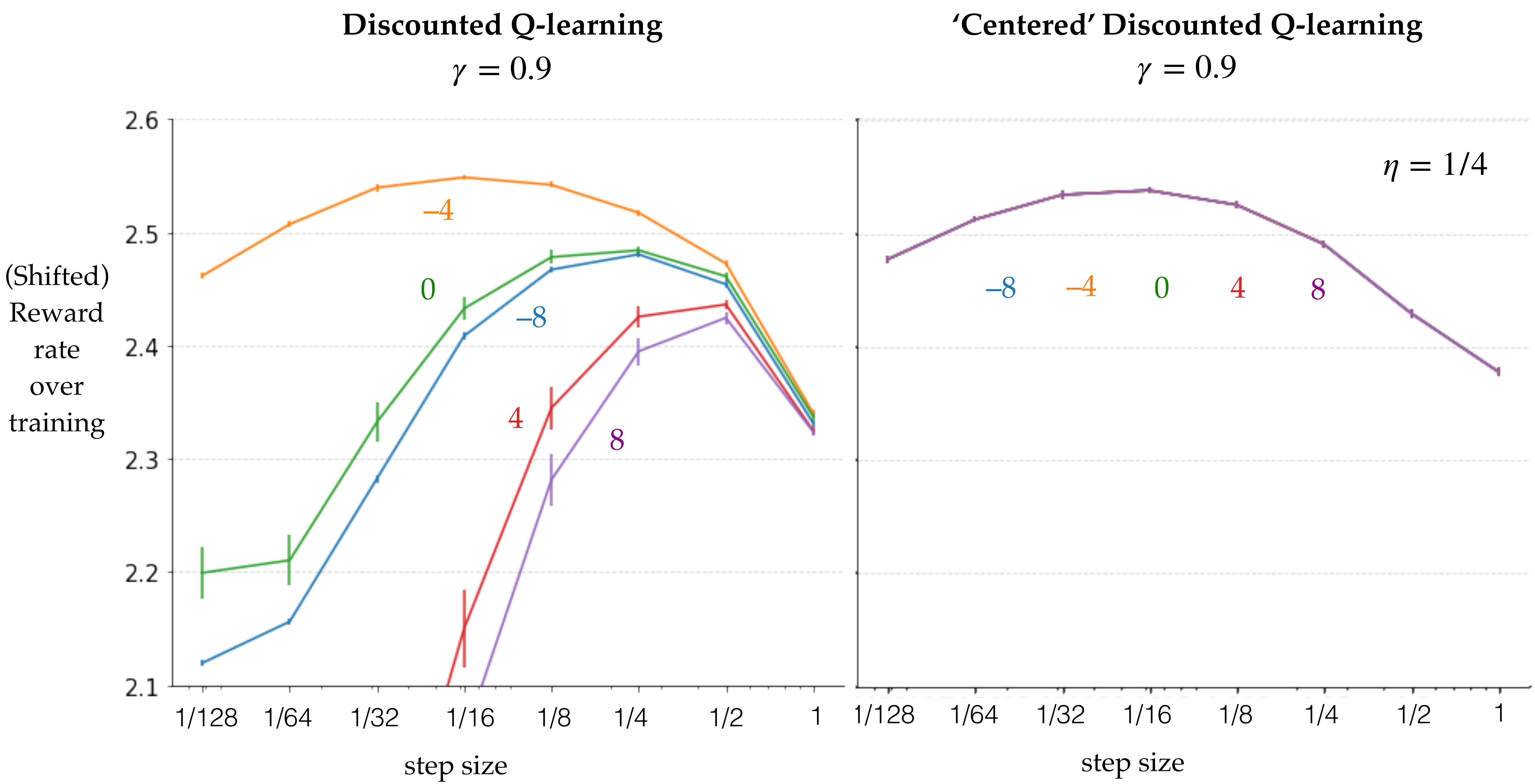

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maybe with $\gamma = 1$;)

THANK YOU

Questions?