**Z-TEST / Z-STATISTIC:** used to test hypotheses about μ when the population standard deviation is known

- and population distribution is normal or sample size is large

**T-TEST / T-STATISTIC:** used to test hypotheses about μ when the population standard deviation is unknown

- Technically, requires population distributions to be normal, but is robust with departures from normality
- Sample size can be small

The only difference between the *z*- and *t*-tests is that the *t*-statistic estimates standard error by using the sample standard deviation, while the *z*-statistic utilizes the population standard deviation

# One Sample T-test

Formula:

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}} \quad \text{where} \quad s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

- $S_{\bar{x}}$ = estimated standard error of the mean
- Because we're using sample data, we have to correct for sampling error. The method for doing this is by using what's called degrees of freedom

# Degrees of Freedom

- degrees of freedom ($df$) are defined as the number of scores in a sample that are free to vary
- we know that in order to calculate variance we must know the mean ($\bar{X}$)

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

- this limits the number of scores that are free to vary

- $df = n - 1$ where $n$ is the number of scores in the sample

# ***Degrees of Freedom Cont. Picture Example***

•There are five balloons: one blue, one red, one yellow, one pink, & one green.

•If 5 students (n=5) are each to select one balloon only 4 will have a choice of color (df=4). The last person will get whatever color is left.

- The particular t-distribution to use depends on the number of degrees of freedom(df) there are in the calculation
- Degrees of freedom (df)
  - df for the t-test are related to sample size
  - For single-sample t-tests, df= n-1
  - df count how many observations are free to vary in calculating the statistic of interest
- For the single-sample t-test, the limit is determined by how many observations can vary in calculating **s** in

$$t_{obt} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

# Z-test vs. T-test

$$z_{obt} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

The z-test assumes that:

- the numerator varies from one sample to another
- the denominator is constant

**Thus, the sampling distribution of z derives from the sampling distribution of the mean**

$$t_{obt} = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

The t-test assumes that:

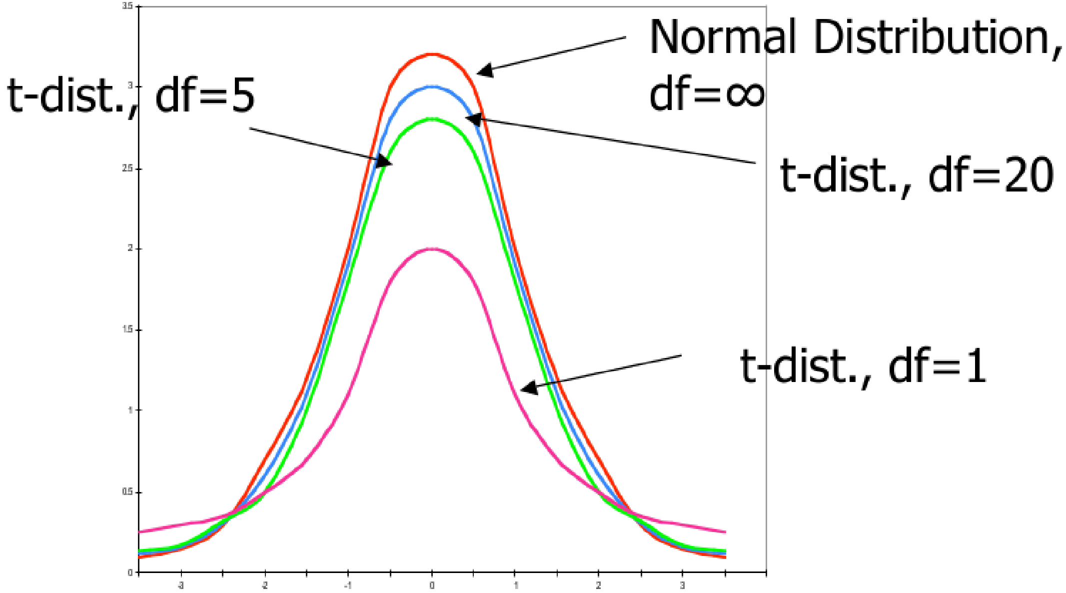
- the numerator varies from one sample to another
- the denominator varies from one sample to another

- **Therefore the sampling distribution is broader than it otherwise would be**
- **The sampling distribution changes with *n***
- **It approaches normal as *n* increases**

## **Characteristics of the t-distribution:**

- The t-distribution is a family of distributions -- a slightly different distribution for each sample size (degrees of freedom)
- It is flatter and more spread out than the normal z-distribution
- As sample size increases, the t-distribution approaches a normal distribution

# Introduction to the *t*-statistic




When df are large the curve approximates the normal distribution. This is because as n is increased the estimated standard error will not fluctuate as much between samples.

- Note that the t-statistic is analogous to the z-statistic, except that both the sample mean and the sample s.d. must be calculated
- Because there is a different distribution for each df, we need a different table for each df
  - Rather than actually having separate tables for each t-distribution, Table D in the text provides the critical values from the tables for df= 1 to df= 120
  - As df increases, the t-distribution becomes increasingly normal
  - For df=∞, the t-distribution is

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Example:

A population of heights has a $\mu=68$. What is the probability of selecting a sample of size $n=25$ that has a mean of 70 or greater and a $s=4$?

- We hypothesized about a population of heights with a mean of 68 inches. However, we do not know the population standard deviation. This tells us we must use a t-test instead of a z-test

**<u>Step 1</u>: State the hypotheses**

$H_0$: $\mu=68$

$H_1$: $\mu\geq68$

## <u>Step 2</u>: Set the criterion

- one-tail test or two-tail test?
- $\alpha$=?
- df = n-1 = ?
- See table for critical t-value

## <u>Step 3</u>: Collect sample data, calculate *x* and s

From the example we know the sample mean is 70, with a standard deviation (s) of 4.

## Step 4: Calculate the test statistic

- Calculate the estimated standard error of the mean

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{4}{\sqrt{25}} = 0.8$$

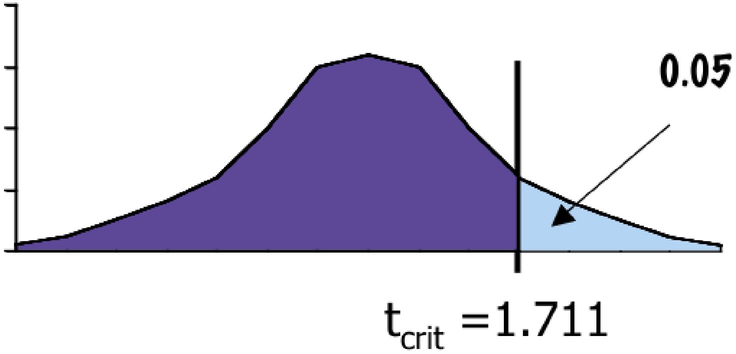
- Calculate the t-statistic for the sample

$$t = \frac{\bar{x} - \mu}{s_{\bar{x}}}$$

$$t = \frac{70 - 68}{0.8} = 2.5$$

## <u>Step 5</u>: Reject $H_0$ if $t_{obt}$ is more extreme than $t_{crit}$

- The critical value for a one-tailed t-test with df=24 and $\alpha$=.05 is 1.711
- Will we reject or fail to reject the null hypothesis?



Example:

A researcher is interested in determining whether or not review sessions affect exam performance.

The independent variable, a review session, is administered to a sample of students ($n=9$) in an attempt to determine if this has an effect on the dependent variable, exam performance.

Based on information gathered in previous semesters, I know that the population mean for a given exam is 24.

The sample mean is 25, with a standard deviation ($s$) of 4.