11. Wh

→ 19.DSC

Computer is a learn

Discrete Mathematical structures

My : Mathematical logic.

Proposition :- statement which can propogate True or false not both T or f.

variables → P, q, r, s, t, u, v

→ Conside the following Sentences

Ex:
1) Banglore is in karnataka (T)
2) 3 is a prime number (T)
3) 7 is divisible by 3 (F)
4) Every rectangle is a square. (F).

Each of the above Sentance is a Statement which can be said either True or false. In fact Sentatence 1 and 2 are true statements where as Sentance 3 and 4 are false Statement Sentance of this type are called Proposition.

A preposition is formaly defined as follows.
(A proposition is a statement declaration) which in given Condext can be said to be either true or false, but not whole.

Proposation are usually represented by small leathers such as p, q, r, s, t, u ....
The truth or falsity of a proposation is called it's truth value.

Logical Connectives and Truth tables:-

1. Negation :- $\sim p$
2. Conjuction $p \wedge q$
3. Disjunction
4. Conditional
5. Biconditional

New propotion's are obtained by starting with give proportions with the aid of word's or preosses like 'not', 'and', 'or', 'if-then', 'if and only if'. Such word's are prices are called logical Connectives. The new proposation obtained by the aid of Conectives are called Compound proposation.

Proposation which donot Contain any logical Connective are called Simple propositions.

1) Negation :- ('~', 'Not')

A proposation obtained by inserting the world Not at then appropriate place in given proposation is called the negation of given proposation.

The Nagation of a proposation is denoted by negation of p ~~and at~~ (not (read 'not p')

Ex :- 3 is a prime number.
3 is not a prime number

- 2 is an Even number.
- 2 is not an Even number

Rule for negation :- For any proposation, if is true then ~~neg~~ NP is false and if P is false Come up then NP is true. In other words of the truth value of a proporation truth value 0. If the truth value of P is 1

Truth Table :-

P	~P
0	1
1	0

2) Conjuction ('∧', 'and').

It works on two proposition variable.

Truth table :-

P	q	P∧q
0	0	0
0	1	0
1	0	0
1	1	1

rule :- P conjuction q is true then P∧q is true but when P is true, q is f then the conjuction is f

Ex :- P : 2 is an Even number

q : 3 is and add number

P∧q :- 2 is an Even numbe and 3 is an add number.

3) Disjunction ('v', 'or').

Rule:- $p \vee q$ is said to be false only when p and q are false. In other case it should be true.

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Ex:- p: 2 is an Even number.
q: 3 is an odd number
$p \vee q$: 2 is an Even number or 3 is an odd number.

4) Conditional. ('$\rightarrow$', 'if then')

Rule:- $p \rightarrow q$ is said to be false only when p is true & q is false. In other case it is true.

Ex:- p: 2 is an Even number
q: 3 is an odd number
$p \rightarrow q$: if 2 is an Even number then 3 is an odd number.

Truth table:-

p	q	$p \rightarrow q$
0	0	1
0	1	1
1	0	0
1	1	1

5) Biconditional. ('$\leftrightarrow$', 'if and only if').

Ex: p: 2 is an Even number
q: 3 is odd number
$p \leftrightarrow q$: 2 is an Even number if and only if 3 is an odd number.

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Rule:- $p \leftrightarrow q$ is said to be true and also $p \leftrightarrow q$ are also false then it is called true. In both case true. In other case it is false.

2). Conjuction:-

A compound proposation obtained by combining two given proposition by inserting the world and in between them is called the conjuction of the given proportiation. the conjuction of the two ~~points~~ proposation P and q is denoted by P ~~cos~~ $\wedge$ q the symbol '$\wedge$' denoting the world 'and'.

- The following rule is adapted the ~~asser~~ desiding the truth value of conjuction.

- The conjuction. $P \wedge q$ is true only when P is true and q is true, in all other cases it is false.

- In other words come up the truth value of conjuction $P \wedge q$ is 1 only when the truth value of P is 1 and the truth value of q is 1, in all other cases the truth value of P conjuction q is 0.

Truth table for conjuction.

P	q	$P \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Ex:- As illustration let' us conside the following

Propos: P: $\sqrt{2}$ is an irrational number.

q: 9 is prime number

$\sqrt{2}$ is an irrational number and 9 is prime number

3) Disjunction :

A compound proposation obtain by combaining to given propositions by inserting the word 'or' in between them is called the disjunction of given proposation.

The disjunction of the two proposation p & q is denoted by $p \vee q$. The symbol '$\vee$' denoting the word 'or'.

The following rule is adapted in deciding the truth value of a disjunction

The disjunction '$p \vee q$' is false only when p is false q is false, in all other cases it is true.

In other word's, the truth value of disjuction $p \vee q$ is 0 only when the truth value of p is 0 and the truth value of q is 0; in all other cases the truth value of $p \vee q$ is 1.

Truth table for disjuction.

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	0	1

4) Conditional ('→', 'if - then').

A Compound proposation obtained by combining two given proposation by using the world "if then" at appropriate ~~word~~ places is called a Conditional.

The following rule is adopted in deciding the truth value of conditional the conditional $p \to q$ is false only when p is true and q is false, in all the cases it is true. In other words, the truth value of the $p \to q$ is '0' only when the truth value of p is '1' and the truth value of q is '0', In all other cases $p \to q$ is 1.

Truth table :-

p	q	$p \to q$
0	0	1
0	1	1
1	0	0
1	1	1

as an illustration, let us consider the following proposition.

p : 2 is a Prime number

q : 3 is a Prime number

if 2 is a prime number the 3 is a prime number

5) Biconditional ('$\leftrightarrow$', 'if and only if').

Let P and q be two propositions. Then the conjuction of the conditionals $P \rightarrow q$ and $q \rightarrow P$ is called the biconditional of P and q.

Rule:- $P \leftrightarrow q$ is said to be true only when both P and q are true or when both P and q are false in other cases it is false.

$2^2 = 4$.

Truth table:-

P	q	$P \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

Ex:- r:- 6 is a perfect square.

s:- q is a multiple of 6.

$r \leftrightarrow s$:- 6 is a perfect square if and only if q is a multiple of 6

Construct the truth tables for the following Compound proposation.

1) $P \wedge (\sim q)$

$2^2 = 4$

P	q	~q	$P \wedge (\sim q)$
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

2) $(\sim P) \vee q$

P	q	~P	$(\sim P) \vee q$
0	0	1	1
0	1	0	1
1	0	1	0
1	1	0	1

3) $p \rightarrow (\sim q)$

P	q	~q	$P \rightarrow (\sim q)$
0	0	1	1
0	1	0	1
1	0	1	1
1	1	0	0

construct the truth table for the following compound proposation:

1) $(p \vee q) \wedge r$. $2^3 = 8$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

2) $p \vee (q \wedge r)$.

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Let P, q be premitive statements for which the Conditional $p \to q$ is false. Determain the truth value of the following Compound proposation.

1) $p \wedge q$

$1 \to 0$.

$p = 1$

$q = 0$

$1 \wedge 0$

$= 0$ (false)

2) $(\sim p) \vee q$

$p = 1, q = 0$

0 or

$\to$ (false)

3) $q \to p$

$0 \to 1$

$\to 1$ (true)

4) $(\sim q) \to (\sim p)$

$q \to 1$

$0 \to 1$

$= 0$ (false)

$\sim q = 1$ $\sim p = 0$

$1 \to 0$

Construct the truth tables for the following Compo-nd proposation.

1) $(p \wedge q) \to (\sim r)$

2) $q \wedge (\sim r) \to p)$.

If proposation q has the truth value 1, determa-
-ue assignments for the premitive proposation
truth value of

following compound proposation is I.

$\Rightarrow [q \to \{(\sim p \vee r) \wedge \sim s\}] \wedge \{\sim s \to (\sim r \wedge q)\}$

1)

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \to (\sim r)$
0 1	0 1	0 1	0 1	1 0	1 1 0
0 1	0 1	1 0	0 1	0 1	1 1 1
0 1	1 0	0 1	0 0	1 0	1 1 1
0 1	1 0	1 0	0 0	0 1	1 1 1
1 0	0 1	0 1	0 0	1 0	1 0 1
1 0	0 1	1 0	0 0	0 1	1 0 1
1 0	1 0	0 1	1 0	1 0	0 1 1
1 0	1 0	1 0	1 0	0 1	0 1 0 1

2)

p	q	r	$\sim r$	$(\sim r) \to p$	$q \wedge (\sim r) \to p)$
1 0	1 0	1 0	0 1	0 1 1	0
1 0	1 0	0 1	1 0	1 1 1	0
1 0	0 1	1 0	0 1	0 1 1	1
1 0	0 1	0 1	1 0	1 1 1	1
0 1	1 0	1 0	0 1	1 1 0	0
0 1	1 0	0 1	1 0	0 1	0
0 1	0 1	1 0	0 1	1 0	1
0 1	0 1	0 1	1 0	0 1	0

1) → q → true

$\sim s = 0$ (false)

$= 1$

$\sim p \vee r = 1$

$\sim r = 1 \Rightarrow r = 0$ (false

$\sim p \vee r = 1$

$s = 0$

$r = 0$

$\sim p \vee r = 1$ $\quad \sim p \vee 0 = 1$

$\sim p = 1 \Rightarrow p = 0$ false

$p = 0$

$r = 0$

$s = 0$

1) $(p \wedge q) \rightarrow (\sim r)$.

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \rightarrow (\sim r)$
0	0	0	0	1	1
0	0	1	0	0	1
0	1	0	0	1	1
0	1	1	0	0	1
1	0	0	0	1	1
1	0	1	0	0	1
1	1	0	1	1	1
1	1	1	1	0	0

3) $q \rightarrow \{(\sim p \vee r) \wedge \sim s\} \wedge \{\sim s \rightarrow (\sim r \wedge q)\}$

$\sim p \vee r = 1$
$1 \vee 0$.

$\sim p = 1$
$p = 0$

$q = 1$

$\sim r = 1$
$r = 0$

$\sim s = 1$
$s = 0$

$(\sim p \vee r) \wedge \sim s \equiv 1$
1 1

$q = 1$
$p = 0$
$s = 0$
$r = 0$

2)

Doubt

p : A circle is a conic

q : $\sqrt{5}$ is a real number

r : Exponantial Series is Convergent

Express the following Compound proposation in words

i) $p \wedge (\sim q)$

$\sim q$ is $\sqrt{5}$ is not a real number.

$p \wedge (\sim q)$ a circle is a conic $\sqrt{5}$ is not a real number

ii) $(\sim p) \vee q$

- A circle is not a conic.
- $(\sim p) \vee q$ a $\sqrt{5}$ is real numbe

iii) $p \vee (\sim q)$

$\sim p$:- $\sqrt{5}$ is not a real number

$p \vee (\sim q)$:- A circle is a conic or $\sqrt{5}$ is not a real number

iv) $q \rightarrow (\sim p)$

$\sim p$ - a circle is not a conic.

$q \rightarrow (\sim p) \rightarrow$ If $\sqrt{5}$ is a real number the a circle is not a conic

v) $p \rightarrow (q \vee r)$

$q \vee r \rightarrow \sqrt{5}$ is not real number or not Exponential Series

$p \rightarrow (q \vee r) \rightarrow$ If a circle is a conic then $\sqrt{5}$ is not real number or not Exponantial Series.

vi) $\sim p \leftrightarrow q$.

$(\sim p) \rightarrow$ A circle is not a conic

$(\sim p) \rightarrow q \rightarrow$ A circle is not a conic if and only is $\sqrt{5}$ is a real number

Let p, q and r be proposations having truth values 0, 0 and 1 respectively. find the truth values of the following Compound proposation. $p \equiv 0$, $q \equiv 0$, $r \equiv 1$

i) $(p \vee q) \vee r$.

$\rightarrow (0 \vee 0) \vee 1 \rightarrow 0 \vee 1 \Rightarrow 1$ (true

2) $(p \wedge q) \wedge r$

$(0 \wedge 0) \wedge 1$

$0 \wedge 1$

$= 0$

3) $(p \wedge q) \rightarrow r$

$(0 \wedge 0) \rightarrow 1$

$0 \rightarrow 1$

$= 1$

4) $p \rightarrow (q \wedge r)$

$0 \rightarrow (0 \wedge 1)$

$0 \rightarrow 0$

1

5) $p \wedge (r \rightarrow q)$

$0 \wedge (1 \rightarrow 0)$

$0 \wedge 0$

0

6) $p \rightarrow (q \rightarrow (\sim r))$

$0 \rightarrow (0 \rightarrow 0)$

$0 \rightarrow 1$

1

Find the possible truth values of p, q and r in the following cases.

1) $p \rightarrow (q \vee r)$ is false.

$1 \rightarrow 0 \equiv 0$

$q \vee r \equiv 0$

$0 \vee 0 \equiv 0$

$p \equiv 1$

$q \equiv 0$

$r \equiv 0$.

2) $p \wedge (q \rightarrow r)$ is true

$1 \wedge 1 \equiv 1$

$q \rightarrow r$

$0 \rightarrow 1$

1

$p \equiv 1$

$q \equiv 0$

$r \equiv 1$

p	q	r	$p \wedge (q \rightarrow$
1	0	0	1
1	0	1	1
1	1	1	1

i.e. indicate how many rows are needed in the truth table for the compound proposation

$(p \vee \sim q) \leftrightarrow [(\sim r \wedge s) \rightarrow t]$

find the truth value of this proposation if p & r are true and q, s, t are false

$2^5 = 32$ rows

$(p \vee \sim q) \leftrightarrow \{(\sim r \wedge s) \rightarrow t\}$

$(1 \vee 1) \leftrightarrow \{(0 \wedge 0) \rightarrow 0\}$

$1 \leftrightarrow \{0 \rightarrow 0\}$

$1 \leftrightarrow 1$

1 (true).

The give ~~prop~~ compound proposation contain 5 Premitues p, q, r, s, t therfore the number of ~~comb~~ posible combination of the truth values of this components which we have to consider this $2^5 = 32$ rows.

Hence 32 rows are needed in the truth table for the given compound proposation.

Toutology. Contradiction and contigency.

Toutology :-

A compound proposation which is true for all possible truth values of it's components is called a toutology (or a logical truth or a universaly valuested statement).

Contradiction:-

A Compound proposation which is possible for all posible ~~truth~~ false value of it's Components are called as Contradiction or an absurdity.

Contigency :-

A Compound proposation that can be true or false (depending upon the truth values of it's Components is called a contigency). In other word's, a contigency is the compound proposation which is neither a tautology nor a contradiction.

1) Prove that, for any proposation, p, the Compound Proposation $p \vee \sim p$ is a tautology and the compound Proposation $p \wedge \sim p$ Contradiction.

$p \vee \sim p \rightarrow 2^1 = 2.$

p	$\sim p$	$p \vee \sim p$
0	1	1
1	0	1

Since $p \vee \sim p$, contains only true values

$\therefore$ $p \vee \sim p$ is a tautology

$p \wedge \sim p \rightarrow 2^1 = 2$

p	$\sim p$	$p \wedge \sim p$
0	1	0
1	0	0

Since $p \wedge \sim p$ contains all false values

$\therefore$ $p \wedge \sim p$ is a contradiction

Hence proved.

2) Prove that, for any propositions p and q, the Compound Proposation $[(\sim q) \wedge (p \to q)] \to (\sim p)$ is a tautology.

$[(\sim q) \wedge (p \to q)] \to (\sim p)$. $\to 2^2 = 4$

p	q	$\sim q$	$\sim p$	$p \to q$	$(\sim q) \wedge (p \to q)$	$[(\sim q) \wedge (p \to q)] \to (\sim p)$
0	0	1	1	1	1	1
0	1	0	1	1	0	1
1	0	1	0	0	0	1
1	1	0	0	1	0	1

Since $[(\sim q) \wedge (p \to q)] \to (\sim p)$ contains only true values, Hence it is a Tautology.

3) Prove that for any proposation p, q, r the Compound Proposation $[(p \to q) \wedge (q \to r)] \to (p \to r)$ is a tautology.

p	q	r	$p \to q$	$q \to r$	$p \to r$	$[(p \to q) \wedge (q \to r)]$	$(p \to q) \wedge (q \to r) \to (p \to r)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	0	1
0	1	1	1	1	1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	1	1	0	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

Since $[(p \to q) \wedge (q \to r)] \to (p \to r)$ contains only true values, ~~[illegible]~~ is a tautology.

H.W.

1) Prove that, for any proposation p, q, r the Compound Proposation $[p \to (q \to r)] \to \{(p \to q) \to (p \to r)\}$ is a Tautology.

2) Prove that for any proposation p, q, r the compound Proposation $[(p \vee q) \wedge \{(p \to r) \wedge (q \to r)\}] \to r$ is a tautology.

$p \quad q \quad r \quad q \to r \quad p \to q \quad p \to r \quad p \to (q \to r) \quad (p \to q) \to (p \to r)$

p	q	r	$q \to r$	$p \to q$	$p \to r$	$p \to (q \to r)$	$(p \to q) \to (p \to r)$	$[p \to (q \to r] \to \{(p \to q) \to (p \to$
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	0	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	1	0	0	1	1	1
1	0	1	1	0	1	1	1	1
1	1	0	0	1	0	0	0	1
1	1	1	1	1	1	1	1	1

Since $[p \to (q \to r)] \to \{(p \to q) \to (p \to r)\}$ Contains only true values, hence it is tautology.

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge \{(p \to r) \wedge (q \to r)\}$	$(p \to r) \wedge (q \to r)$
0	0	0	0	1	1	0	1
0	0	1	1	1	1	1	1
0	1	0	1	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

$[(p \vee q) \wedge \{(p \to r) \wedge (q \to r)\}] \to r$
1
1
1
1
1
1
1
1

Since $[(p \vee q) \wedge \{(p \to r) \wedge (q \to r)\}] \to r$ contains only true values, Hence it is tautology.

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3. Find the possible truth values of p, q, r, s, t for which following are contradictions. $[(p \wedge q) \wedge r] \to (s \vee t)$

$$\underbrace{[(p \wedge q) \wedge r]}_{1} \to \underbrace{(s \vee t)}_{0} = 0 \text{ (false)}.$$

$[(p \wedge q) \wedge r] = 1$

$p \wedge q \equiv 1$

$r \equiv 1$

$p \wedge q \equiv 1$

$p \equiv 1, q \equiv 1, q \equiv 1$

$(s \vee t)$

$s \equiv 0$

$t \equiv 0$

$\therefore p \equiv 1, q \equiv 1, r \equiv 1, s \equiv 0, t \equiv 0.$

Logical Equatalance :-

Two proposation u and v are said to be logically Equivalent whenever you u and v have the same truth value, are Equivalently, the biconditional $u \Leftrightarrow v$ is tautology. Then, we write ~~u~~ $u \Leftrightarrow v$. Hear the symbol $\Leftrightarrow$ stand's for logicaly equasent to.

When the proposation u and v are not logical Equivalent, we write ~~u not~~ $u \not\Leftrightarrow v$.

Logicaly Equavalent proposation are treated as identical Proposation.

Prove that, for any three proposations p, q, are

$$[p \to (q \wedge r)] \Leftrightarrow [(p \to q) \wedge (p \to r)]$$

p	q	r	$q \wedge r$	$p \to (q \wedge r)$	$p \to q$	$p \to r$	$[(p \to q) \wedge (p \to r)]$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	0	0	1	0
1	1	0	0	0	1	0	0
1	1	1	1	1	1	1	1

Since $p \to (q \wedge r)$ & $(p \to q) \wedge (p \to r)$ contains same truth values

Hence $p \to (q \wedge r) \Leftrightarrow (p \to q) \wedge (p \to r)$.

Prove that, for any three Proposation. p, q, r.

$[(p \vee q) \to r] \Longleftrightarrow [(p \to r) \wedge (q \to r)]$

p	q	r	$p \vee q$	$(p \vee q) \to r$	$p \to r$	$q \to r$	$(p \to r) \wedge (q \to r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

Since $[(p \vee q) \to r] \Longleftrightarrow [(p \to r) \wedge (q \to r)]$ contains some truth values

Hence $[(p \vee q) \to r] \Longleftrightarrow [(p \to r) \wedge (q \to r)]$.

The laws of logic: The following results, known as the laws of logic, follow from the Definati of logical equivelence on this laws. To denotes a tatutology & To denotes a contradiction.

1) Laws of Double Negation:
For any proposation p.
$(\sim\sim p) \Leftrightarrow p$.

2) Idempotate law.
For any proposation P
a) $(P \vee P) \Leftrightarrow P$. b) $(P \wedge P) \Leftrightarrow P$

3) Indentity law.
For any proposation p.
a) $(P \vee F_0) \Leftrightarrow P$ b) $(P \wedge T_0) \Leftrightarrow P$.

4) Inverse law.
For any proposation P.
$(P \vee \sim P) \Leftrightarrow T_0$ b) $(P \wedge \sim P) \Leftrightarrow T_0$

5) ~~Com~~ Domination law.
For any proposation p
$(P \vee T_0) \Leftrightarrow T_0$ b) $(P \wedge F_0) \Leftrightarrow F_0$

6) commitative law.
For any two proposation.
a) $(p \wedge q) \Leftrightarrow (q \wedge p)$ b) $(p \vee q) \Leftrightarrow (q \vee p)$.

7) Absorption Laws
For any two proposation.
a) $[p \vee (p \wedge q)] \Leftrightarrow p$ b) $[p \wedge (p \vee q)] \Leftrightarrow p$

8) Demorgan laws:-

For any proposation

a) $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$ b) $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$

9) Associative laws.

For any Propolation p, q & r.

a) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$ b) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$

10) Distributive laws,

For any Proposition p & q.

a) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$.

b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

1) Prove the following logical equalenss without using truth table

i) $p \vee [p \wedge (p \vee q)] \Leftrightarrow p$.

$\therefore$ LHS $\equiv p \vee [p \wedge (p \vee q)]$

$\equiv p \vee p$ [$\because$ By appling absorption law]

$\equiv p$ [$\because$ By applying Idempotent law]

$\equiv$ RHS

$\therefore$ LHS = RHS

Hence proved.

ii) $[(p \vee q) \wedge (p \vee \sim q)] \vee q \Leftrightarrow p \vee q$.

LHS $\equiv [(p \vee q) \wedge (p \vee \sim q)] \vee q$.

$\equiv [p \vee (q \wedge \sim q)] \vee q$ [$\because$ By applying Distributive law]

$\equiv [p \vee F_0] \vee q$ [$\because$ By applying inverse law]

$\equiv p \vee q$ [$\because$ By applying identity law]

$\equiv$ RHS

$\therefore$ LHS $\equiv$ RHS

Hence proved.

iii) $[p \vee q \vee (\sim p \wedge \sim q \wedge r)] \Leftrightarrow (p \vee q \vee r)$

LHS $\equiv [p \vee q) \vee (\sim p \wedge \sim q \wedge r)]$

$\equiv [p \vee q \vee [\sim (p \vee q) \wedge r]$ [$\because$ By applying Demorgon's law]

$\equiv \{(p \vee q) \vee [\sim (p \vee q)]\} \wedge (p \vee q \vee r)$ [$\because$ By applying Distributive law]

$\equiv T_0 \wedge (p \vee q \vee r)$ [$\because$ By applying inverse law]

$\equiv p \vee q \vee r$ [$\because$ By applying identity law]

$\equiv$ RHS

LHS = RHS

Hence proved.

Simplify the following Compound proposations using the laws of logic.

1) $(p \vee q) \wedge [\sim \{(\sim p) \wedge q\}]$

$\equiv (p \vee q) \wedge [\sim (\sim p) \wedge \sim q]$ $\because$ By applying Demorgon's law.

$\equiv (p \vee q) \wedge (p \vee \sim q)$ $\because$ By applying Double negation

$\equiv p \vee (q \wedge \sim q)$ $\because$ By applying Distributive law.

$\equiv p \vee F_0$ $\because$ By applying inverse law

$\equiv p$ $\because$ By applying identity law

$\equiv$ RHS

LHS $\equiv$ RHS

Hence proved

ii) $(p \vee q) \wedge [\sim\{(\sim p) \vee q\}]$

$\equiv (p \vee q) \wedge [\sim(\sim p) \wedge \sim q]$ ∵ By applying Demorgan's law

$\equiv (p \vee q) \wedge (p \wedge \sim p)$ ∵ By applying Double negation.

$\equiv [(p \vee q) \wedge p] \wedge \sim q$ ∵ By applying Associative law

$\equiv [p \wedge (p \vee q)] \wedge \sim q$ ∵ By applying Commutative law.

$\equiv p \wedge \sim q$ ∵ By applying Absorption law.

iii) $\sim[\sim\{(p \vee q) \wedge r\} \vee \sim q]$

$\equiv \sim[\sim\{(p \vee q) \wedge r\}] \wedge \sim\sim q$ ∵ By applying Demorgan's law

$\equiv [(p \vee q) \wedge r] \wedge q$ ∵ By applying of Double negation.

$\equiv (p \vee q) \wedge (r \wedge q)$ ∵ By applying associative law.

$\equiv (p \vee q) \wedge (q \wedge r)$ ∵ By applying commutative law

$\equiv [(p \vee q) \wedge q] \wedge r$ ∵ " " associative "

$\equiv q \wedge r$ ∵ By applying absorption law.

<u>Duality</u>:- Suppose $\underline{u}$ is a compound proposation that's contains the conectives '$\wedge$' & '$\vee$'. Suppose $\underline{v}$ replace each occurrence of $\wedge$ and $\vee$ in u by $\vee$ and $\wedge$ are respectively also, if $\underline{u}$ contains T_0 and F_0 as components, $\underline{v}$ replace each occurrence of F_0 and T_0 by F_0 and T_0. Then, the resulting compound the proposation is called dual of $\underline{u}$ and is denoted by $\underline{u^d}$

Ex:- $u : p \wedge (q \vee \sim r) \vee (s \wedge T_0)$

$u^d : p \vee (q \wedge \sim r) \wedge (s \vee F_0)$

1) Write down the dual of the following proposation

i) $\sim(p \vee q) \wedge [p \vee \sim(q \wedge \sim s)]$

$\Rightarrow \sim(p \wedge q) \vee [p \wedge \sim(q \vee \sim s)]$

ii) $(p \wedge q) \vee [(\sim p \vee q) \wedge (\sim r \vee s)] \vee (r \wedge s)$

$\Rightarrow (p \vee q) \wedge [(\sim p \wedge q) \vee (\sim r \wedge s)] \wedge (r \vee s)$

iii) $(p \wedge \sim q) \vee (r \wedge T_0)$

$\Rightarrow (p \vee \sim q) \wedge (r \vee F_0)$

iv) $[(p \vee T_0) \wedge (q \vee F_0)] \vee [(r \wedge s) \wedge T_0]$

$\Rightarrow [(p \wedge F_0) \vee (q \wedge T_0)] \wedge [(r \vee s) \vee F_0]$

Converse, Inverse and Contropostive:-

Consider a conditional $p \to q$, then,

i) $q \to p$ is called the converse of $p \to q$.

ii) $\sim p \to \sim q$ is called the Inverse of $p \to q$.

iii) $\sim q \to \sim p$ is called the contropostive of $p \to q$.

Ex:- $p \to 2$ is an integer
$q \to 3$ is a Multiple of 3

$p \to q$:- If 2 is an integer then 3 is a multiple of 3.

$q \to p$ (converse):- If 3 is a multiple of 3 then 2 is an integer.

Inverse ($\sim p \to \sim q$):- If 2 is not an interge then 3 is not multiple of 3.

Contropositive ($\sim q \to \sim p$):- If 3 is not a multiple of 3 then 2 is not an integer.

i) Result:-

1) a Conditional and it's contrapositive are logicaly Equalent $p \to q \Leftrightarrow \sim q \to \sim p$

2) The converse and the inverse of a Conditional are logical equalent $q \to p \Leftrightarrow \sim p \to \sim q$

1) State the converse, Inverse, and contrapositive of the following Conditionals.

1) If a quadrilatral is a Parallelogram, then it's diagonals bisect Each other

p:- quadrilatral is a prallelogram

q:- Parallelogram diagonals bisect each other.

$q \to p$:- If a parallelogram diagnoals bisect Each other then quadrilatral is a parallelogram.

$\sim p \to \sim q$:- If a quadrilatral is not a parallelogram then the parallelogram diagnoles doesn't bisect Each other

$\sim q \to \sim p$:- If Parallelogram diagnoles doesn't bisect Each other then a quadrilatral is not a parallelogram

2) State the converse, Inverse, and contrapositive of the following conditionals.

2) If areal number x^2 is greater then zero, then x is not Equal to zero.

p:- areal number x^2 is greater then zero.

q:- x is not equal to zero.

$q \to p$:- x is not Equal to zero then areal number x^2 is greater then zero.

$\sim p \to \sim q$:- If areal number x^2 is not greater then zero then x is Equal to zero.

$\sim q \to \sim p$:- If x is Equal to zero then areal number x^2 is not greater then zero.

3) ∵ If a triangle is not isosceles; then it is no equilateral.

p: a triangle is not isosceles.

q: ~~isosceles~~ triangle is not a equilateral.

$q \to p$ → If ~~isosceles~~ triangle is not a equilateral then a triang is not isoscales.

$\sim p \to \sim q$: If a triangle is isosceles then ~~isosceles~~ triangle a equilateral.

$\sim q \to \sim p$:- If ~~isosceles~~ triangle is a equilateral then a triang is isosceles.

Rules of Inference:- Imp ***

Consider a set of proposation $P_1, P_2, \ldots P_n$ and a proposatio q. Then a compound proposation of the form $(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n)$ is called an the Argument. Hear $P_1, P_2, \ldots P_n$ are called the Premises of the argument and Q is called a conclution the argument.

It is a practice to write the above argument in the following tabuler coloum.

P_1
P_2
P_3
$\vdots$
P_n
$\overline{\therefore Q}$

Here, the three dot symbole stands fore (∴) Therefore. The preceding argument is said to be valid whenever each of premises $P_1, P_2, \ldots, P_n$ is true, then the conclution q is true. In other words the argument $(P_1 \wedge P_2 \wedge P_3 \wedge \ldots \wedge P_n) \to Q$

1) Rule of conjunctive Simplification.

This rule states that, for any two proposation p and q if $p \wedge q$ is true then p is true.

($\Rightarrow$) implies.

That is $p \wedge q \Rightarrow p$.

This rule follows from the defination of conjuction.

2) Rule of disjunctive Amplification.

This rule states that, for any two proposation p and q, if p is true then $p \vee q$ is also true.

That is $p \Rightarrow p \vee q$.

This rule follows from the defination of disjunction.

3) Rule of Syllogism

This rule states that, for any three proposation p, q, r if $p \to q$ is true and $q \to r$ is true, then $p \to r$ is true.

That is $\{(p \to q) \wedge (q \to r)\} \Rightarrow (p \to r)$.

This rule follows from the tautology and is expressed in the following tabuler form.

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

4) Modus Ponens (Rule of detachment):-

This rule states that, if p is true and $p \to q$ is true come up then q is true.

That is $\{p \wedge (p \to q)\} \Rightarrow q$

In tabuler form

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

Modues Tollen's :-

This rule state that, if $p \to q$ is true and q is false then p is false. That is $\{(p \to q) \wedge \sim q\} \Rightarrow \sim p$. This rule is expressed in the following tabular form.

$$\begin{array}{l} p \to q \\ \underline{\sim q} \\ \therefore \sim p. \end{array}$$

6) Rule of Disjunctive Syllogism.

This rule states that if $p \vee q$ is true and p is false then q is true. That is $[(p \vee q) \wedge \sim p] \Rightarrow q$

rule expressed in the following tabular form

$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ \therefore q \end{array}$$

7) Rule of Contradiction :-

This rule states that. If $\sim p \to F_0 \Rightarrow p$

The rule state that is $\sim p \to F_0$ is true, then p is true. That is mean F_0 is not contradiction (namely a proposation which is always false). The truth table given bellow from this rule

p	$\sim p$	F_0	$\sim p \to F_0$	$(\sim p \to F_0) \to p$
0	1	0	0	1
1	0	0	1	1

1) Test weather the following is a valid argument

If Sachin hits a century, then he get free car.

Sachin hits a Century

∴ Sachin gets a free car

⇒ p : sachin hits a century.

q : sachin get free car.

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

∴ In view of modus ponens rule it is a valid argument.

2) If sachin hits a century, then he gets free car.

Sachin does not get a free car.

∴ Sachin has not hit a century.

p : Sachin hits a century

q : sachin get free car.

$$\begin{array}{l} p \to q \\ \sim q \\ \hline \therefore \sim p. \end{array}$$

∴ In view of modus tollens rule it is a valid argument.

3) If a person is poor he is unhappy

if a person is unhappy, he dies young.

∴ Poor persons die young

p : If a person is poor

q : He is unhappy

r : he dies young.

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

∴ In view of rule of syllogism it is a valid argument

4) If there is strike by student, the examination will be postpones.
There was no strike by student
∴ The examination was not postponed

p :- There is strike by student
q :- There examination will be postpones

$p \to q$
$\underline{\sim p}$
$\therefore \sim q$

∴ Since this is not following any rules of inforance.

∴ ~~In view of modus tollens rule~~ ∴ Hence it is not valide orgunme

5) If I drive to work, then I will arrive tired
I do not drive to work.

∴ I will not arrive tired.

p : I drive to work.
q :- I will not arrive

$p \to q$
$\underline{\sim p}$
$\therefore \sim q$

∴ Since this is not following any rules of inforance
Hence it is not a valid argument.

6) If ravi goes out with friends, he will not study
If ravi does not study, his father becomes angry
If his father is not angry

∴ Ravi is not gone out with friends.

p :- Ravi goes out with friends.
q :- he will not study
r :- his father becomes angry.

$p \to q$
$q \to r$
$\underline{\sim r}$

$\Rightarrow$ $\left.\begin{array}{l} p \to q \\ \underline{\sim r} \\ \therefore \sim p \end{array}\right\}$ By rule of syllogism

∴ In view of Modus tollens rule, it is valid argument.

Quantifiers:-

Consider the following proposation

1) All squares are rectangles
2) for every integer x, x^2 is a non-negative integer.
3) Some determinates are equal to zero.
4) There exists a real number whoes square is equal to it'self.

In this proposation, the word "all", "Every", "Some", "there exists" are associated with idea of a quantity. Such word's are called quantifiers.

The symbole "$\forall$" for all has been used to denote the phrases "for all", "for every". In logic, this phrases are regarded as equavalent phrases. The phrases "for each" and "for any" are also taken to be the equalent to this. The symbole "$\forall$" is used to denote all of this phrases. Each of this equalent phrases is called the universal quantifier.

The symbole "$\exists$" has been used to denote the phrases "for some", "there exist". In logic, this two phrases are taken to be equalent to one another. There are also taken to be equalent to the phrase "for at least one". The symbole "$\exists$" is used to denote all of this equalent phrases. Each of this equalend phrases is called the existancesal quantifier.

1) For the universe of all integers let $p(x)$, $p(x): x > 0$

$q(x)$: x is Even. $r(x)$: x is a perfect Square.

$s(x)$: x is divisible 3. $t(x)$: x is divisible by 7.

Write down the following quantified Statement in Symbolic form.

i) Atleast one integer is Even.

$$\exists x, q(x)$$

ii) There Exist a positive integer that is Even.

$$\exists x \, [p(x) \wedge q(x)]$$

iii) Some Even integer are divisible by 3

$$\exists x \, [q(x) \wedge s(x)]$$

iv) Every integer is either Even or odd.

$$\forall x, [q(x) \vee (\sim q(x))]$$

v) If x is Even and Perfect Square, then x is not divisible by 3.

$$\forall x, [\{q(x) \wedge r(x)\} \rightarrow \sim s(x)]$$

vi) If x is odd or is not divisible by 7, then x is divisible by 3.

$$\forall x. [\{\sim q(x) \vee \sim t(x)\} \rightarrow s(x)]$$

Rule for Negation of a Quantified Statement:-

To construct the negation of a quantified statement, change the quantifier from universal to existancial and vice versa, and also replace the open statement by it's negation. That is

$$\sim\{\forall x, p(x)\} = \exists x, \{\sim p(x)\}$$

$$\& \sim\{\exists x, p(x)\} = \forall x, \{\sim p(x)\}$$

Negate and simplify each of the followings.

1). $\exists x, [p(x) \vee q(x)]$

$\sim\{\exists x [p(x) \vee q(x)]$

$\equiv \forall x, \sim[p(x) \vee q(x)]$

$\equiv \forall x, \{\sim p(x) \wedge \sim q(x)\}$ ∵ Demorgon's law.

2). $\forall x, [p(x) \wedge \sim q(x)]$

$\sim\{\forall x, [p(x) \wedge \sim q(x)]$

$\equiv \exists x, \sim[p(x) \wedge \sim q(x)]$

$\equiv \exists x, \{\sim p(x) \vee \sim\sim(q(x))\}$ ∵ By using demorgon's law.

$\equiv \exists x, \{\sim p(x) \vee q(x)\}$ ∵ By using double negation.

3/4 Marks Imp ***

3). Let the set Z of all integers be the universe. Consider the statements $p(x): 2x+1=5$ & $q(x): x^2=9$. obtained the negation of the quantified statement $\exists x \in Z, [p(x) \wedge q(x)]$ and express it in words

$p(x): 2x+1=5$

$q(x): x^2=9.$

$\exists x \in Z, [p(x) \wedge q(x)]$

$\sim\{\exists x \in Z, [p(x) \wedge q(x)]$

$\forall x \in Z, \sim[p(x) \wedge q(x)]$

$\forall x \in Z \{\sim p(x) \vee \sim q(x)\}$ ∵ By demorgon's law.

In words:

for all integers x, $2x+1$ is not equal to 5 or x^2 not equal to 9.

H.W

4) Let the universe compose of all integers given $p(x)$: x is odd and $q(x)$: x^2-1 is even, express the conditional for all any x, if x is odd then x^2-1 is even.

For any x, if x is odd

Let then x^2-1 is even.

Solution: $p(x) = x$ is odd

$q(x) = x^2-1$ is even. ∴ By Demorgan's law.

$\forall x\ [p(x) \to q(x)]$

$\equiv \sim\{\forall x\ \{p(x) \to q(x)\}$

$\equiv \exists x,\ \sim[p(x) \to q(x)]$

$\equiv \exists x \cdot [p(x) \wedge \sim q(x)]$

$[\sim(p \to q) \equiv p \wedge \sim q]$.

∴ There exists x, x is odd and x^2-1 is not even.

Methods of Proof.

Direct proof:-

1) Hypothesis
2) Analysis
3) Conclusion.

⇒ The direct method of proving a conditional $p \to q$ has the following lines of arguments.

1) hypothesis:- first assume that p is true
2) Analysis:- Starting with a hypothesis and employing the rules or loss of logic and other known facts, infer that q is true.
3) Conclusion:- $p \to q$ is true.

1) Give a direct proof of the statement "The square of an odd integer is an ~~an~~ odd integer".

⇒ "If n is odd integer, then n^2 is an odd integer".

p : n is odd integer

q : n^2 is an odd integer

Let $n = (2k+1)$ where k is any integer

1) Hypothesis $n = 2k+1$ is true

2) Analysis $n^2 = (2k+1)^2$

$= (2k)^2 + 2 \times 2k + 1 + (1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

n^2 is odd integer

3) Conclusion :. $p \rightarrow q$ is true.

---

2) Prove that, for all integers k and l if k and l are both odd then k+l is even and k×l is odd.

⇒ P : for all integer k & l are both odd.

q : k+l is even and k×l is odd.

$k = 2m+1$

$l = 2m+1$

where n and m are any integer.

1) Hypothesis :- ~~k = n+1 + km+1~~ Assume k and l are odd integers is true.

2) Analysis :- $k + l = 2n+1 + 2m+1$

$= 2n + 2m + 2$

$k + l = 2(n+m+1)$ which is an even integer.

$k \times l = (2n+1)(2m+1)$

$= 4nm + 2n + 2m + 1$

$= 2(2nm + n + m) + 1$ which is an odd

3) Conclusion : $p \to q$ is true.

Hence proved the direct proof.

---

3) Give a direct proof for the following statement of n: an odd integer than n+9 is an even integer.

=> p: n is an odd integer.

q: n+9 is an odd integer.

Hypothisis: an odd integer be $n = 2k+1$ where k is any integer.

Analysis:- $n+9 = (2k+1)+9$
$= 2k+10 = 2(k+5)$ which is an even integer.

conclusion:- $p \to q$ is true.

Hence proved the direct proof.

HW.

Give a direct proof for each of the following statement

1) If n is an even integer, than m+7 is an odd integer.

2) The square of an even integer is an even integer.

1) p = n is an even integer

q = m+7 is an odd integer.

~~$p \to q \Leftrightarrow \sim q \to \sim p$.~~

~~$\sim q \to$ m+7 is not odd integer~~

~~$\sim p \to$ n is not even integer~~

Hypothisis:- n is an even integer be $2k+1 = m$ where m is any integer.

Analysis:- $m+7 = (2k+1)+7$
$= 2k+8 = 2(k+4)$ which is an even integer.

Conclution :- $p \to q$ is true.

Hence proved the direct proof

---

2) p = ~~The square~~ of an even integer.

q = even integer.

Hypothisis:- n is an even integer be 2k is where k is any integer.

Analysis :- $n = 2k$
$n^2 = (2k)^2 = 2(4k^2) \Rightarrow 2(2k)$
which is an even integer.

conclution:- $p \to q$ is true Hence proved the direct proof

c) Indirect proof:-

A condition $p \to q$ and it's contrapositive $\sim q \to \sim p$ are logically equivalent. A given condition $p \to q$, a direct proof of the contrapositive $\sim q \to \sim p$ is easier. on the base of this proof, we effor the condition $p \to q$ is true this method of proving a conditional is called as indirect method of proof.

1) Let n be an integer prove that if n^2 is odd, then n is odd

p: n^2 is odd

q: n is odd

$p \to q \Leftrightarrow \sim q \to \sim p$.

$\sim q \to$ n is not odd.

$\sim p \to$ n^2 is not odd

Hypothisis:- $n^2 = 2k$ is even integer

Analysis:- $n^2 = (2k)^2 = 2(2k^2) \Rightarrow 2(2k)$.

which is an even integer.

Conclusion:- i.e. $\sim p$ is true

$\sim q \to \sim p$ is true. i.e $p \to q \Leftrightarrow \sim q \to \sim p$.

Hence $p \to q$ is true by indirect method proof.

---

2) ~~The product~~ Give an indirect proof of an ~~indirect~~ statement the product of two even integer is an even integer.

$\Rightarrow$ the product of

p: two even integer.

q: even integer.

$p \to q \Leftrightarrow \sim q \to \sim p$.

$\sim q \to$ is not even integer

$\sim p \to$ then not two even integer.

3. Conclusion : $p \to q$ is true.

Hence proved the direct proof.

---

3) Give a direct proof for the following Statement if n an odd integer than $n+9$ is an even integer.

$\Rightarrow$ p: n is an odd integer.

q : $n+9$ is an odd integer.

Hypothisis: an odd integer be $n = 2k+1$ where k is any integer.

Analysis:- $n+9 = (2k+1)+9$
$= 2k+10 = 2(k+5)$ which is an even integer.

conclusion:- $p \to q$ is true.

Hence proved the direct proof.

HW.

Give a direct proof for each of the following Statement

1) If n is an Even integer, than $m+7$ is an odd integer.

2) The square of an Even integer is an Even integer.

1) p = n is an Even integer

q = $m+7$ is an odd integer.

~~$p \to q \Leftrightarrow \sim q \to \sim p$.~~

~~$\sim q \to m+7$ is not odd integer~~

~~$\sim p \to$ n is not even integer~~

Hypothisis:- n is an even integer be $2k+1 = m$ where m is any integer.

Analysis:- $m+7 = (2k+1)+7$
$= 2k+8 = 2(k+4)$ which is an even integer.

Conclution :- $p \to q$ is true.

Hence proved the direct proof

---

2) p = ~~The square~~ n of an Even integer.

q = Even integer.

Hypothisis:- n is an even integer be $2k$ is where k is any integer.

Analysis :- $n = 2k$
$n^2 = (2k)^2 = 2(4k^2) \Rightarrow 2(2k)$
which is an even integer.

Conclution:- $p \to q$ is true Hence proved the direct proof

c) Indirect proof:-

A condition $p \to q$ and it's contrapositive $\sim q \to \sim p$. are logically equivalent. A given condition $p \to q$, a direct proof of the contrapositive $\sim q \to \sim p$ is easier. on the base of this proof, we effor that the condition $p \to q$ is true this method of proving a conditional is called as indirect method of proof.

1) Let n be an. integer prove that if n^2 is odd, then n is odd

p: n^2 is odd

q: n is odd

$p \to q \Leftrightarrow \sim q \to \sim p$.

$\sim q \to$ n is not odd.

$\sim p \to n^2$ is not odd

Hypothisis:- $n^2 = 2k$ is even integer

Analysis:- $n^2 = (2k)^2 = 2(2k)^2 \Rightarrow 2(2k)$.

which is an even integer.

Conclusion:- i.e. $\sim p$ is true

$\sim q \to \sim p$ is true. i.e $p \to q \Leftrightarrow \sim q \to \sim p$.

Hence $p \to q$ is true by indirect method proof.

---

2) ~~The product~~ Give an indirect proof of an ~~indirect~~ statement the product of two even integer is an even integer.

$\Rightarrow$ p: the product of two even integer.

q: even integer.

$p \to q \Leftrightarrow \sim q \to \sim p$.

$\sim q \to$ q is not even integer

$\sim p \to$ ~~the~~ Not two even integer.

The statement is equivalent to the following statement

A = If a and b are ~~even~~ Even integers".

& = If a & b are Even integers, then ab is an Even integer

p : a & b are Even integers

q : ab is an Even integer

we have to prove contrapositive $\sim q \Rightarrow \sim p$ form

assume that ~~a~~ $\sim q$ That is ab is not Even integer

This means that ab not divisible 2. Hence a ~~is~~ not ~~div~~ divisible and b is not divisible by 2. That is a is not integer b is a Even integer ab are Even integer which is false

∴ p is false Equivalent,

~~This proves the con~~

$\sim q \Rightarrow \sim p$

Consequently it's ~~false~~ given statements $p \Rightarrow q$ is true.

Proof contradiction:

The indirect of Method proof is equivalent ~~proof~~ by contradiction ~~after~~ ~~st~~

Hypothesis that = $p \Rightarrow q$ is false That is assume that p is true. q is false

Analysis :- Start by

Rule : the q is false ~~by~~ applying

assumes that : inform ~~that~~ false. This contradicts the

Conclusion :- p is true

p

p