On the Achievable Rate of MIMO Cognitive Radio Network with Multiple Secondary Users

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Abstract—**This paper investigates the achievable rate of MIMO cognitive radio network when one primary user (PU) and multiple secondary users (SU) are present, where the latter adopt dirty paper coding (DPC) to cancel the interference of PU's transmission at their receivers. We formulate an optimization problem to maximize the achievable rate of the system under the constraints of power limits of each transmitter, where the requirement of not affecting PU's transmission rate is also incorporated. An algorithm is proposed to jointly determine the inflation factors in DPC method and the input covariance matrix of each SU. Simulations show that the proposed problem achieves better achievable rate when compared with the existing results without compromising PU's transmission rate.**

Index Terms—**Cognitive radio, MIMO, Achievable rate, Dirty paper coding**

I. INTRODUCTION

Currently, there are of great interest on cognitive radio network (CRN) because of the high spectral efficiency it can achieve. The CRN should design CR techniques to accommodate cognitive devices without disrupting the communications of the primary users (PU). Generally, CR techniques falls into three types, i.e., underlay, overlay and interweave [1]. Among these techniques, the secondary user (SU) in the overlay approach uses part of its power to relay primary transmissions in order to compensate its interference to PU. In addition, to remove the interference of the PU's data at its receiver, it employs dirty paper coding (DPC) [2] method at the transmitter.

Multiple-input multiple-output (MIMO) has been well-known for its multiplexing and diversity gain, and many papers have investigated MIMO CRN to exploit the benefits of MIMO [3]-[8]. When perfect transmitter-channel-knowledge (CSIT) is available [3]-[4], [3] derives the optimal beamforming and power allocation to maximize the secondary rate while satisfying the primary rate requirement. A practical MIMO-DPC scheme is proposed in [4], and the optimal inflation factor is derived. On the other hand, [5]-[8] focus on the design of various schemes under imperfect CSIT. [5] studies the impact of imperfect channel state information (CSI) on a MIMO system with interference and compares the performance of DPC with that of a scheme where interference is decoded at the receiver, referred to as beamforming with joint decoding. [6] deals with the fading dirty paper channel (FDPC) with positive semi-definite input covariance matrix and develops an iterative algorithm to jointly optimize the input covariance matrix and

the inflation factor. [7] considers FDPC under imperfect CSIT and develops two iterative algorithms to determine the inflation factor used in DPC. In contrast to the channel model in [6] and [7] that the signal X and interference S experience the same fading channel, [8] generalizes this model to the one that X and S experience different fading channels. However, the algorithms developed in these papers only apply for the network with one SU.

There have been recently some researches on MIMO CRN with multiple SUs [9]-[11]. [9] considers multiuser MIMO CRN with limited feedback, where zero-forcing beamforming is performed under imperfect CSI at each SU, and an adaptive resource allocation is designed to provide a feedback-efficient and delay-guaranteed service. [10] studies the sum rate maximization problem for spectrum sharing MIMO broadcast channels under Rayleigh fading with partial CSI, which maximizes the sum capacity under the power limits. [11] considers a spectrum sharing scenario in a MIMO CRN where the overall objective is to maximize the total throughput of SU by jointly optimizing the detection operation and the power allocation, under a interference constraint bound to PUs. However, these papers mainly focus on the optimization of resource allocation among SUs, and they fail to provide the optimization of parameters in DPC method. Therefore, the overlay scenario when multiple SUs exploiting DPC needs further investigation.

In this paper, we analyze the achievable rate performance in Gaussian MIMO CRN in order to obtain preferable transmission parameters for each SU. The network consists of one PU and K SUs ($K > 1$). Each SU is cognitive such that it knows the messages of the PU transmitter non-causally. However, the interactions among different SUs are not allowed since it means heavy transmission costs in the network, especially when the number of SUs is large. Under the transmit power constraints, we propose design methods to optimize the rate performance of the system while keeping the PU's rate unaffected. The considered problem is almost analytical intractable. Therefore, we provide an iterative algorithm to find the suboptimal inflator factor and transmit covariance of each SU. Simulations show that the proposed algorithm achieve better performance when compared with the existing results.

The remainder of this paper is organized as follows. In Section II, we present the system model. Section III describes the formulated optimization problem and corresponding algorithm. Simulation results are given in Section IV. Finally, we

conclude this paper in Section V.

II. SYSTEM MODEL

Suppose there are one PU and K SUs in the network, where their transmitters are denoted as S_p and $\{S_k\}$, and the receivers are D_p and $\{D_k\}$, respectively ($k = 1, \cdots, K$). The numbers of the transmit and receive antennas of PU are t_p and r_p, respectively, while those of the kth CR are t_k and r_k, respectively. Fig. 1 depicts the transmission model of PU and the kth CR for example, where each SU not only transmits its own signals by adopting the DPC method to cancel the PU's interference, but also relays PU's signals. The transmitted signals by S_p and S_k are denoted as $X_p \sim \mathcal{CN}(0, \Sigma_p)$ and $X_k \sim \mathcal{CN}(0, \Sigma_k)$, respectively, where X_k consists of S_k's own data $X_{kk} \sim \mathcal{CN}(0, \Sigma_{kk})$ and the relayed information of PU's data $X_{kp} \sim \mathcal{CN}(0, \Sigma_{kp})$. Assume $|\Sigma_p| > 0$, and $|\Sigma_k| > 0$. Denote the channels from S_p to D_p and D_k as H_{pp} and H_{pk}, respectively. The channels from S_k to D_p and D_k are H_{kp} and H_{kk}, respectively. Assume the receiver of the kth SU also suffers from the interference from other SUs, and the channel from S_k to D_j are defined as H_{kj}.
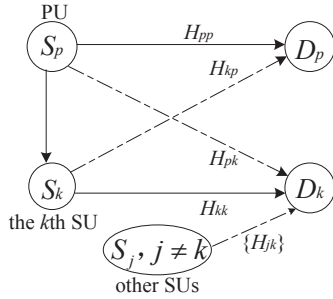

Fig. 1. Transmission model of PU and the kth SU

The received signal at D_p is

$$Y_p = H_p X_p + \sum_{k=1}^{K} H_{kp} X_k + Z_p \qquad (1)$$

where $Z_p \sim \mathcal{CN}(0, \Sigma_{Z_p})$ is the noise at D_p with $|\Sigma_{Z_p}| > 0$. By defining $\overline{H}_p = [H_p, H_{1p}, \cdots, H_{Kp}]$, and $\overline{X}_p = [X_p^H, X_{1p}^H, \cdots, X_{Kp}^H]^H$, where H means the conjugate transpose, (1) becomes

$$Y_p = \overline{H}_p \overline{X}_p + \sum_{k=1}^{K} H_{kp} X_{kk} + Z_p \qquad (2)$$

In (2), the equivalent noise is $\hat{Z}_p = \sum_{k=1}^{K} H_{kp} X_{kk} + Z_p$ with covariance matrix being $\Sigma_{\hat{Z}_p} = \Sigma_{Z_p} + \sum_{k=1}^{K} H_{kp} \Sigma_{kk} H_{kp}^H$. In addition, denote the distribution of $\overline{X}_p$ as $\overline{X}_p \sim \mathcal{CN}(0, \Sigma)$, where $\Sigma = \begin{bmatrix} \Sigma_p & & & \\ & \Sigma_{1p} & Q & \\ & Q^H & \ddots & \\ & & & \Sigma_{Kp} \end{bmatrix}$ with Q being the off-diagonal matrix.

The received signal at D_k is

$$Y_k = H_{kk} X_k + H_{pk} X_p + \sum_{j=1, j \neq k}^{K} H_{jk} X_j + Z_k$$

$$= H_{kk} X_{kk} + H_{pk} X_p + \sum_{j=1}^{K} H_{jk} X_{jp} + \sum_{j=1, j \neq k}^{K} H_{jk} X_{jj} + Z_k \qquad (3)$$

where $Z_k \sim \mathcal{CN}(0, \Sigma_{Z_k})$ is the noise at D_k with $|\Sigma_{Z_k}| > 0$. Define $\overline{H}_k = [H_{pk}, H_{1k}, \cdots, H_{Kk}]$, and (3) becomes

$$Y_k = H_{kk} X_{kk} + \overline{H}_k \overline{X}_p + \sum_{j=1, j \neq k}^{K} H_{jk} X_{jj} + Z_k \qquad (4)$$

As a result, the equivalent noise in (4) is $\hat{Z}_k = \sum_{j=1, j \neq k}^{K} H_{jk} X_{jj} + Z_k$ with covariance matrix being $\Sigma_{\hat{Z}_k} = \Sigma_{Z_k} + \sum_{j=1, j \neq k}^{K} H_{jk} \Sigma_{jj} H_{jk}^H$. Without loss of generality, suppose the covariance matrices of noises all equal to identity matrix, i.e., $\Sigma_{Z_p} = I_{r_p}$, and $\Sigma_{Z_k} = I_{r_k}$.

III. JOINT OPTIMIZATION OVER THE TRANSMIT COVARIANCE AND INFLATOR FACTOR

Suppose each SU adopts the same relaying method, then it also knows X_{kp} based on X_p with its cognitive ability. As a result, the channel between S_k and D_k is a generalized fading DPC channel with $\overline{X}_p$ acting as the interference, where the DPC coding method in [2] can be adopted. Let the rank of Σ_{kk} be $m_k \leq t_k$. Define $\Sigma_{kk} = T_{2k} T_{2k}^H$ for some $T_{2k} \in \mathcal{C}^{t_k \times m_k}$, and $X_{kk} = T_{2k} X'_{kk}$ for some $X'_{kk} \sim \mathcal{CN}(0, I_{m_k})$. According to [9], the auxiliary random variable of the kth CR is set to be $U_k = X'_{kk} + W_k \overline{X}_p$ with W_k as the inflation factor.

Let $\Sigma = T_1 T_1^H$ for some T_1. Based on (2), the achievable rate of PU is [8]

$$R_p = \max_{\{W_k\}} E_{\tilde{H}} \log \frac{\left| \overline{H}_p T_1 T_1^H \overline{H}_p^H + \sum_{k=1}^{K} H_{kp} T_{2k} T_{2k}^H H_{kp}^H + I_{r_p} \right|}{\left| \sum_{k=1}^{K} H_{kp} T_{2k} T_{2k}^H H_{kp}^H + I_{r_p} \right|}$$

$$= \max_{\{W_k\}} E_{\tilde{H}} \log \frac{|A|}{|B|} \qquad (5)$$

with $\tilde{H} = \{\{H_{pp}\}, \{H_{kk}\}, \{H_{kp}\}, \{H_{pk}\}, \{H_{jk}\}\}$, $A = \overline{H}_p T_1 T_1^H \overline{H}_p^H + \sum_{k=1}^{K} H_{kp} T_{2k} T_{2k}^H H_{kp}^H + I_{r_p}$ and $B = \sum_{k=1}^{K} H_{kp} T_{2k} T_{2k}^H H_{kp}^H + I_{r_p}$.

By using (4) and extending the result in [8], the achievable rate of the kth SU is

$$R_k = \max_{W_k} E_{\tilde{H}} \log \frac{\left| \Sigma_{\hat{Z}_k} + H_{kk}T_{2k}T_{2k}^H H_{kk}^H + \overline{H}_k T_1 T_1^H \overline{H}_k^H \right|}{|D_k|}$$

$$= \max_{W_k} E_{\tilde{H}} \log \frac{|C_k|}{|D_k|} \tag{6}$$

where $C_k = I_{r_k} + \sum_{j=1, j \neq k}^{K} H_{jk}T_{2j}T_{2j}^H H_{jk}^H + H_{kk}T_{2k}T_{2k}^H H_{kk}^H + \overline{H}_k T_1 T_1^H \overline{H}_k^H$, and D_k is given in (7) on the top of next page. Note that [8] has propoposed two algorithms to determine the optimal W_k in (6) when T_1 and T_{2k} are fixed.

In CRN, the transmission of primary user cannot be affected by the secondary users. In the simulations of Fig. 3 in [8], the authors mentioned that the optimal Σ_p is $\Sigma_p = 0$, i.e., the PU needs to turn off its power. In other words, the optimization in [8] cannot guarantee the PU's performance, which is unacceptable in practical scenario. In this paper, we try to optimize the achievable sum-rate of the system without compromising the achievable rate of PU. Thus, considering the power constraint at each transmitter, the optimization problem will be

$$\max_{\{W_k\}, \{T_{2k}\}, T_1} \sum_{k=1}^{K} R_k + R_p \tag{8a}$$

$$\text{s.t.} \quad R_p \geq R'_p \tag{8b}$$

$$tr(\Sigma_p) \leq P_p \tag{8c}$$

$$tr(\Sigma_{kp} + \Sigma_{kk}) \leq P_k, \forall k \tag{8d}$$

where $R'_p = E_{\tilde{H}} \log \left| I_{r_p} + H_p \Sigma_p H_p^H \right|$ is the rate of PU when there is no CR in the network, and $tr(\cdot)$ stands for the trace of a matrix. The constraint in (8b) guarantees that the introduction of SU would not affect PU's rate. The constraints in (8c) and (8d) are the power limits of PU and the kth SU, respectively. To solve the optimization problem in (8), we form the lagrangian function as $J = \sum_{k=1}^{K} R_k + R_p - \lambda_p(R_p - R'_p) - \sum_{k=1}^{K} \lambda_k^{-1} \left[tr(\sum_{kp} + \sum_{kk}) - P_k \right] - \lambda_{pp}^{-1} \left[tr(\sum_p) - P_p \right]$ with λ_p, λ_k^{-1} and λ_{pp}^{-1} being the lagrange multipliers. By simple manipulations, it can be written in the following form

$$J = \sum_{k=1}^{K} R_k + R_p - \lambda_p(R_p - R'_p) -$$
$$tr \left[diag(\lambda_{pp}^{-1} I_{t_p}, \lambda_1^{-1} I_{t_1}, ..., \lambda_K^{-1} I_{t_K})\Sigma \right] - \tag{9}$$
$$\sum_{k=1}^{K} \lambda_k^{-1} \left[tr(\Sigma_{kk}) - P_k \right] + \lambda_{pp}^{-1} P_p$$

where $diag(x_1, x_2, \cdots, x_N)$ is a diagonal matrix with x_i being the diagonal entries. To obtain the necessary conditions, set $\frac{\partial J}{\partial T_1} = 0$ and $\frac{\partial J}{\partial T_{2k}} = 0$. After some calculations, we arrive at (10) and (11) on the top of next page.

The optimization involves the determination of λ_p, λ_k and λ_{pp}, which is complicated. To facilitate this procedure, we consider a suboptimal solution of solving the power constraints in (8c) and (8d) as strict equalities, and reduce the number of lagrange multipliers to one by expressing both λ_{pp} and λ_k with λ_p.

Based on (10), we obtain

$$T_1 T_1^H = diag(\lambda_{pp} I_{t_p}, \lambda_1 I_{t_1}, ..., \lambda_K I_{t_K}) E_{\tilde{H}} \{M - N - \lambda_p Q\}$$
$$T_1 T_1^H E_{\tilde{H}} \{M^H - N^H - \lambda_p Q^H\}$$
$$diag(\lambda_{pp} I_{t_p}, \lambda_1 I_{t_1}, ..., \lambda_K I_{t_K})$$
$$= diag(\lambda_{pp} I_{t_p}, \lambda_1 I_{t_1}, ..., \lambda_K I_{t_K}) g_1(T_1, T_{21}, ..., T_{2K}, \lambda_p, W)$$
$$diag(\lambda_{pp} I_{t_p}, \lambda_1 I_{t_1}, ..., \lambda_K I_{t_K}) \tag{12}$$

where $M = \sum_{k=1}^{K} \overline{H}_k^H C_k^{-1} \overline{H}_k + \overline{H}_P^H A^{-1} \overline{H}_P$, $N = \sum_{k=1}^{K} \left[W_k^H \ \overline{H}_k^H \right] D_k^{-1} \left[\begin{array}{c} W_k \\ \overline{H}_k \end{array} \right]$, $Q = \overline{H}_P^H A^{-1} \overline{H}_P$, and $g_1(T_1, T_{21}, ..., T_{2K}, \lambda_p, W) = f_1 f_1^H$. Write the obtained g_1 in the form of

$$g_1(T_1, T_{21}, ..., T_{2K}, \lambda_p, W)$$
$$= \left[\begin{array}{cccc} \Psi_p(\lambda_p) & & & \\ & \Psi_{1p}(\lambda_p) & U & \\ & V & \ddots & \\ & & & \Psi_{Kp}(\lambda_p) \end{array} \right] \tag{13}$$

where $\Psi_p(\lambda_p) \in \mathcal{C}^{t_p \times t_p}$, $\Psi_{kp}(\lambda_p) \in \mathcal{C}^{t_k \times t_k}$ are the diagonal elements of $g_1(T_1, T_{21}, ..., T_{2K}, \lambda_p, W)$, U and V stand for the off-diagonal elements. By replacing $g_1(T_1, T_{21}, ..., T_{2K}\lambda_p, W)$ in (12) with (13), we obtain

$$T_1 T_1^H = diag(\lambda_{pp} I_{t_p}, \lambda_1 I_{t_1}, ..., \lambda_K I_{t_K}) g_1(T_1, T_{2k}, \lambda_p, W)$$
$$diag(\lambda_{pp} I_{t_p}, \lambda_1 I_{t_1}, ..., \lambda_K I_{t_K})$$
$$= \left[\begin{array}{cccc} \lambda_{pp}^2 \Psi_p(\lambda_p) & & & \\ & \lambda_1^2 \Psi_{1p}(\lambda_p) & U' & \\ & V' & \ddots & \\ & & & \lambda_K^2 \Psi_{Kp}(\lambda_p) \end{array} \right] \tag{14}$$

for some U' and V'. Considering that $\Sigma = T_1 T_1^H$ and $tr(\Sigma_p) = \lambda_{pp}^2 tr[\Psi_p(\lambda_p)] = P_p$, we get

$$\lambda_{pp} = \sqrt{\frac{P_p}{tr[\Psi_p(\lambda_p)]}} \tag{15}$$

On the other hand, since $\Sigma_{kk} = T_{2k}T_{2k}^H$ and $tr(\Sigma_{kp} + \Sigma_{kk}) = tr[\lambda_k^2 \Psi_{kp}(\lambda_p) + \lambda_k^2 f_{2k} f_{2k}^H] = P_k$, we have

$$\lambda_k = \sqrt{\frac{P_k}{tr[\Psi_p(\lambda_p) + f_{2k} f_{2k}^H]}} \tag{16}$$

As a result, λ_{pp} and λ_k are expressed by λ_p as in (15) and (16), respectively.

With the above discussions, we give the algorithm for joint optimization of W_k, T_{2k} and T_1 in Algorithm 1. Since our optimization problem is non-convex, the algorithm does not necessarily yield the optimum solution. However, the results are shown to be good in the simulations afterwards.

$$D_k = \left[\begin{array}{cc} I_{t_k} + W_k T_1 T_1^H W_k^H & T_{2k}^H H_{kk}^H + W_k T_1 T_1^H \overline{H}_k^H \\ H_{kk} T_{2k} + \overline{H}_k T_1 T_1^H W_k^H & \Sigma_{\hat{Z}_k} + H_{kk} T_{2k} T_{2k}^H H_{kk}^H + \overline{H}_k T_1 T_1^H \overline{H}_k^H \end{array} \right] \tag{7}$$

$$diag(\lambda_{pp}^{-1} I_{t_p}, \lambda_1^{-1} I_{t_1}, ..., \lambda_K^{-1} I_{t_K}) T_1 = E_{\tilde{H}} \left\{ \sum_{k=1}^{K} \overline{H}_k^H C_k^{-1} \overline{H}_k - \sum_{k=1}^{K} \left[W_k^H \ \overline{H}_k^H \right] D_k^{-1} \left[\begin{array}{c} W_k \\ \overline{H}_k \end{array} \right] + (1-\lambda_p) \overline{H}_p^H A^{-1} \overline{H}_p \right\} T_1$$
$$\triangleq f_1 \tag{10}$$

$$\lambda_k^{-1} T_{2k} = E_{\tilde{H}} \left\{ H_{kk}^H C_k^{-1} H_{kk} T_{2k} - \left[0 \ H_{kk}^H \right] D_k^{-1} \left[\begin{array}{c} I_{t_k} \\ H_{kk} T_{2k} \end{array} \right] + (1-\lambda_p)(H_{kp}^H A^{-1} H_{kp} T_{2k} - H_{kp}^H B^{-1} H_{kp} T_{2k}) \right\} \triangleq f_{2k} \tag{11}$$

Algorithm 1

1: Set the iteration number $n = 1$. Set some initial values of $T_1^{(0)}$ and $T_{2k}^{(0)}$. With these values and the rates in (5) and (6), derive $W_k^{(0)}$ according to the algorithms in [8].

2: At the nth iteration:
   ▶ With the derived $W_k^{(n-1)}$, determine the transmit covariance $T_1^{(n)}$ and $T_{2k}^{(n)}$ $(k = 1, 2, \cdots, K)$ as in (11) and (12), where the expectations with respect to $\tilde{H}$ are evaluated numerically. λ_p, λ_{pp} and λ_k are determined as following. We first use (15) and (16) to express λ_{pp} and λ_k with λ_p. Then, the derived expressions are used to solve (8b) with equality, where the Discrete Newton Method is used to get a feasible solution λ_p.
   ▶ With the derived $T_1^{(n)}$ and $T_{2k}^{(n)}$, calculate $W_k^{(n)}$ according to the algorithms in [8].

3: Repeat Step 2 until the improvement in the achievable rate of $\sum_{k=1}^{K} R_k + R_p$ is negligible.

Fig. 2.   Achievable rates vs. P_p when $P_k = P_p/2$

IV. SIMULATIONS

In this section, we carry out some simulations to show the achievable rates vs. the power constraint P_p at PU's transmitter in different scenarios. The numbers of transmitting and receiving antennas of PU are $t_p = r_p = 1$, while those of the kth SU are $t_k = 2$ and $r_k = 3$. Suppose no CSI is available at the transmitter. Define $R_{sum} = \sum_{k=1}^{K} R_k + R_p$ and $R_c = \frac{1}{K} \sum_{k=1}^{K} R_k$. All results are based on an average of 100 trials.

We first consider that $\{H_{ij}\}$ are independent and obey uniform distribution ranging from 0 to 1. Fig. 2 displays the results of achievable rate when the power of each SU equals to half of the PU's, i.e., $P_k = \frac{1}{2} P_p$, which is also the case considered in Fig. 4 in [8]. Note that no matter how many SU exists in the network, the achievable rate of PU is identical to the one when SU is absent. In addition, the average achievable rate of SU (i.e., R_c) generally decreases with the increase of
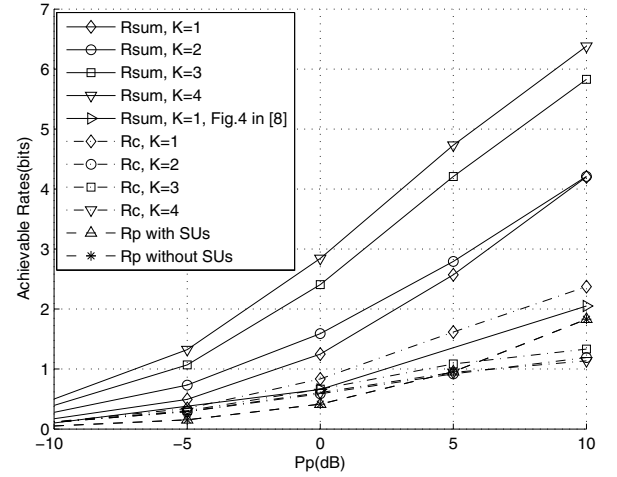
the number of SU. It is due to the fact that the case of more SU introduces more interference. However, the figure shows that the sum of achievable rate R_{sum} increases when K is larger, and the result of $K = 1$ is better than that in Fig. 4 in [8]. Therefore, our proposed scheme achieves preferable system performance without compromising PU's performance.

Fig. 3 shows the achievable rates when $P_k = P_p/2K$. That is, the total power of SUs is fixed. The achievable rate of PU is still not affected by the introduction of SU. However, we observe that the sum of achievable rate in the system not necessarily increases with the increase of K. The reason is as follows. The benefit of having more transmitting SU cannot always compensate the negative effect produced due to the interference it introduces.

When all channels follow Rayleigh distributions such that $\{H_{ij}\} \sim$ i.i.d. $\mathcal{CN}(0, 1)$, Fig. 4 and Fig. 5 present the corresponding results. In this scenario, R_c always decreases when K increases, while R_{sum} is shown to increase when K becomes larger.
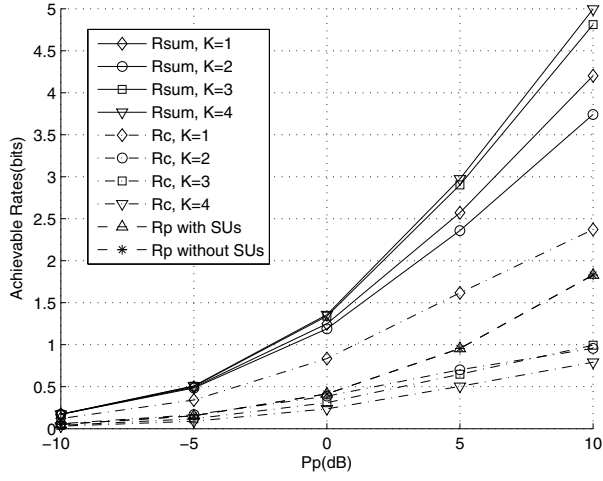
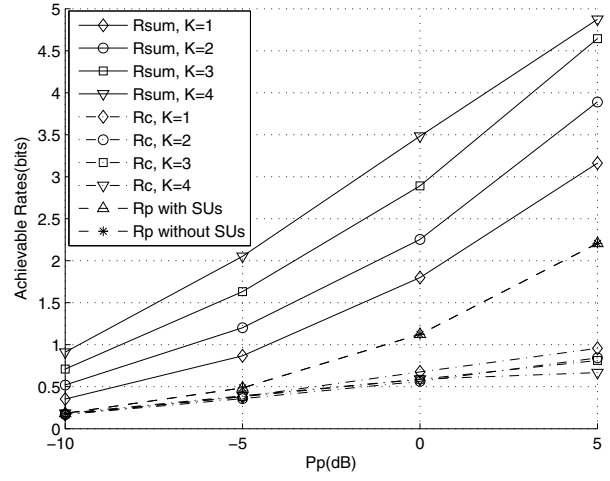Fig. 3. Achievable rates vs. P_p when $P_k = P_p/2K$

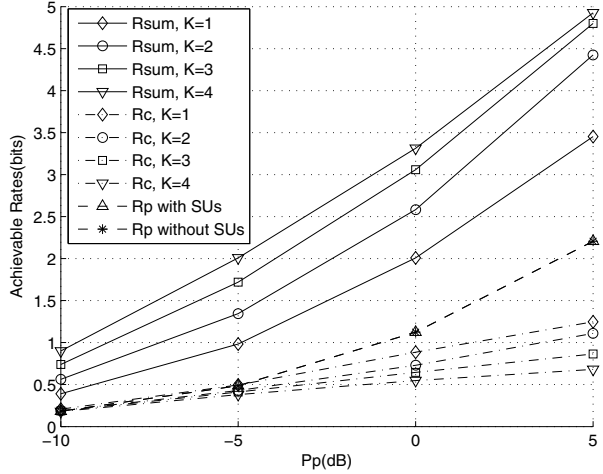Fig. 5. Achievable rates vs. P_p when $P_k = P_p/2K$

Fig. 4. Achievable rates vs. P_p when $P_k = P_p/2$

V. CONCLUSION

When multiple secondary users are present in the MIMO cognitive radio network, the joint optimization of inflator factor and input covariance matrix of each SU is studied. This problem is formulated as the maximization of the system achievable rate with the guarantee of not affecting PU's transmission. We propose one algorithm to solve the problem iteratively and it is shown to work well by simulations. In this paper, we give simulations when the transmitters only have channel distribution information. However, once there exists the feedbacks of CSI from the receivers, they can be easily utilized during the optimization as the same way in [8]. That is, the results in this paper also apply for the network with imperfect CSIT.

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