

Assignment 10 Solutions

1. Define the Bayesian interpretation of probability.

Ans: Bayesian probability is the study of subjective probabilities or belief in an outcome, compared to the frequentist approach where probabilities are based purely on the past occurrence of the event. A Bayesian Network captures the joint probabilities of the events represented by the model.

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$$\boxed{P(A|B)}_{\text{posterior}} = \boxed{P(A)}_{\text{prior}} \times \frac{\boxed{P(B|A)}_{\text{likelihood}}}{\boxed{P(B)}_{\text{marginal}}}$$

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2. Define probability of a union of two events with equation.

Ans: The general probability addition rule for the union of two events states that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where $A \cap B$ is the intersection of the two sets.

3. What is joint probability? What is its formula?

Ans: Probabilities are combined using multiplication, therefore the joint probability of independent events is calculated as the probability of event A multiplied by the probability of event B. This can be stated formally as follows:

Joint Probability: $P(A \text{ and } B) = P(A) * P(B)$

4. What is chain rule of probability?

Ans: The chain rule, or general product rule, calculates any component of the joint distribution of a set of random variables using only conditional probabilities. This probability theory is used as a foundation for backpropagation and in creating Bayesian networks.

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Product rule

- Definition of conditional probability: $P(A|B) = \frac{P(A, B)}{P(B)}$
- Sometimes we have the conditional probability and want to obtain the joint:

$$P(A, B) = P(A|B)P(B) = P(B|A)P(A)$$
- The chain rule:

$$P(A_1, A_2, \dots, A_n) = P(A_1)P(A_2|A_1)P(A_3|A_1, A_2) \dots P(A_n|A_1, A_2, \dots, A_{n-1})$$

5. What is conditional probability means? What is the formula of it?

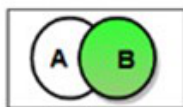
Ans: Conditional probability is defined as the likelihood of an event or outcome occurring, based on the occurrence of a previous event or outcome.

Conditional probability is calculated by multiplying the probability of the preceding event by the updated probability of the succeeding, or conditional, event.

Let's take a real-life example. Probability of selling a TV on a given normal day may be only 30%. But if we consider that given day is Diwali, then there are much more chances of selling a TV. The conditional Probability of selling a TV on a day given that Day is Diwali might be 70%.

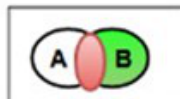
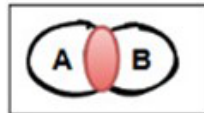
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$P(A|B) = P(A \text{ given } B \text{ has occurred})$



If B has already occurred then our sample space must be somewhere within B

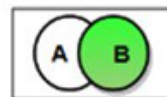
Now A can occur only within sample space B



$P(A|B)$ is the ratio of Red space divided by Green space

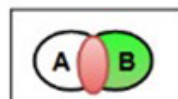
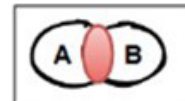
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$P(B|A) = P(B \text{ given } A \text{ has occurred})$



If A has already occurred then our sample space must be somewhere within A

Now B can occur only within sample space A



$P(B|A)$ is the ratio of Red space divided by White space

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

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6. What are continuous random variables?

Ans: A continuous random variable X takes all values in a given interval of numbers. ▪ The probability distribution of a continuous random variable is shown by a density curve. ▪ The probability that X is between an interval of numbers is the area under the density curve between the interval endpoints.

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- A continuous random variable is one which takes an **infinite** number of possible values.
- Continuous random variables are usually measurements.
- Examples:
 - height
 - weight

7. What are Bernoulli distributions? What is the formula of it?

Ans: A Bernoulli distribution is a discrete probability distribution for a Bernoulli trial — a random experiment that has only two outcomes (usually called a *Success* or a *Failure*). The expected value for a random variable, X .

For a Bernoulli distribution is: $E[X] = p$. For example, if $p = 0.04$, then $E[X] = 0.4$

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Properties of the Bernoulli distribution

The Bernoulli distribution is a distribution of the discrete type satisfying:

| Parameter | $p \in [0, 1]$: success probability |
|-----------|--|
| Support | $\{0, 1\}$ |
| PMF | $p^x(1 - p)^{1-x}$ |
| Mean | p |
| Variance | $pq = p(1 - p)$ |
| MGF | $(1 - p) + pe^t, \quad (t \in \mathbb{R})$ |

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8. What is binomial distribution? What is the formula?

Ans: The binomial is a type of distribution that has two possible outcomes (the prefix “bi” means two, or twice). For example, a coin toss has only two possible outcomes: heads or tails and taking a test could have two possible outcomes: pass or fail.

A Binomial Distribution shows either (S)uccess or (F)ailure.

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Binomial Distribution Formula

$$P(x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{(n-x)!x!} p^x q^{n-x}$$

where

n = the number of trials (or the number being sampled)

x = the number of successes desired

p = probability of getting a success in one trial

$q = 1 - p$ = the probability of getting a failure in one trial

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9. What is Poisson distribution? What is the formula?

Ans: A Poisson distribution is defined as a discrete frequency distribution that gives the probability of the number of independent events that occur in the fixed time.

In statistics, a Poisson distribution is a probability distribution that is used to show how many times an event is likely to occur over a specified period. ... The Poisson distribution is a discrete function, meaning that the variable can only take specific values in a (potentially infinite) list.

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Poisson Probability Distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

λ - mean number of successes over a given interval

$$\text{Var}(X) = \lambda$$

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10. Define covariance ?

Ans: Covariance is a measure of how much two random variables vary together. It's similar to variance, but where variance tells you how a single variable varies, co variance tells you how two variables vary together.

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Covariance Formula

For Population

$$\text{Cov}(x,y) = \frac{\sum (x_i - \bar{x}) * (y_i - \bar{y})}{N}$$

For Sample

11. Define correlation ?

Ans: Correlation explains how one or more variables are related to each other. These variables can be input data features which have been used to forecast our target variable. It's a bi-variate analysis measure which describes the association between different variables.

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$$r_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}$$

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12. Define sampling with replacement. Give example.

Ans: If you sample with replacement, you would choose one person's name, put that person's name back in the hat, and then choose another name. The possibilities for your two-name sample are: John, John. John, Jack.

13. What is sampling without replacement? Give example.

Ans: In sampling without replacement, each sample unit of the population has only one chance to be selected in the sample. For example, if one draws a simple random sample such that no unit occurs more than one time in the sample, the sample is drawn without replacement.

14. What is hypothesis? Give example.

Ans: A hypothesis (plural hypotheses) is a proposed explanation for a phenomenon. For a hypothesis to be a scientific hypothesis, the scientific method requires that one can test it. ... Even though the words "hypothesis" and "theory" are often used synonymously, a scientific hypothesis is not the same as a scientific theory.

Examples of hypothesis statements: If garlic repels fleas, then a dog that is given garlic every day will not get fleas. Bacterial growth may be affected by moisture levels in the air. If sugar causes cavities, then