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Interleaving dynamics in autonomous vehicle storage and retrieval systems

CHARLES J. MALMBORG†

A state equation model is proposed for predicting the proportion of dual command cycles in autonomous vehicle storage and retrieval systems that use interleaving. This proportion is used to estimate storage and retrieval cycle times, system utilization and throughput capacity for alternative system design profiles defined by the number of storage aisles, storage rack height and depth, the vehicle fleet size and the number of lifts used for vertical movement. The state equation model enables the user to explicitly trade off model accuracy and computational complexity in the pre-engineering or 'concepting' stage of system development. A sample problem is used to demonstrate the model.

1. Introduction

In traditional automated storage and retrieval systems (AS/RSs), unit loads (ULs) are handled using aisle-captive storage cranes that move simultaneously in the vertical and horizontal dimensions (Tompkins *et al*. 1996). In autonomous vehicle storage and retrieval systems (AVS/RSs), ULs are handled by vehicles moving horizontally along rails within storage racks with vertical movement provided by lifts mounted along the rack periphery (Zizzi 2000). Analytical pre-engineering or 'concepting' tools have been proposed for AVS/RSs that estimate vehicle utilization and throughput capacity as a function of system design parameters including the number of storage aisles, the storage rack height and depth, the vehicle fleet size and the number of lifts (Malmborg 2002). Like traditional AS/RSs, AVS/RSs can operate more efficiently by combining or 'interleaving' storage and retrieval transactions on S/R cycles to reduce the average vehicle time per transaction. Interleaving is typically performed on an opportunistic basis where a retrieval transaction is combined with a storage transaction at the start of the S/R cycle when one or more of both types are pending in the active queue. Cycles that include both a storage and retrieval transaction are referred to as dual command (DC) cycles (Bozer and White 1984). Opportunistic interleaving results in a dynamic process where random fluctuations in the rate of transaction arrivals can provide more opportunities for interleaving. However, as more DC cycles are executed, queue sizes are reduced and single command (SC) cycles become more likely. The steady state proportion of DC versus SC cycles depends on the system loading and the average DC and SC cycle times (Malmborg 2001). This proportion is frequently denoted as α in the AS/RS modelling literature.

Until recently, a significant limitation with AS/RS concepting tools was the need to specify α in order to generate estimates of system performance, including crane

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utilization and throughput capacity. This limitation forced designers to apply concepting tools in an imprecise sensitivity mode. For example, using concepting tools to find system designs with robust performance across a range of plausible α values. For AS/RSs that use random storage and first-come first-serve (FCFS) dispatching of transactions, two alternative analytical tools have been introduced recently that estimate α directly as a function of storage/retrieval (S/R) transactions demand and the estimated SC and DC cycle times (Bozer and Cho 1998, Malmborg 2001). These models do not extend directly to AVS/RSs since the queuing properties of S/R transactions in AS/RSs are generated by aisle-captive cranes that form parallel queuing systems associated with storage aisles. In AVS/RSs, queuing characteristics are analogous to a multi-channel system with a single queue since vehicles select transactions from a common buffer. However, like an AS/RS, opportunistic interleaving in an AVS/RS produces a dynamic, queue state-dependent service process that cannot be accurately modelled with standard queuing models. In this study, analytical tools are introduced for estimating α based on the level of S/R transactions demand and the estimated SC and DC cycles times in AVS/RSs that use opportunistic interleaving. In a manner analogous to the recent models proposed for AS/RSs, these models eliminate the need for rough estimates of α to bootstrap AVS/RS concepting models.

The next section provides background information on analytical concepting tools for AVS/RSs, including the estimation of SC and DC cycle times. The α bootstrapping problem in AVS/RSs is explained in detail. The third section presents an analytical tool for estimating α as a function of transactions demand levels and estimated SC/DC cycle times. The application of this tool is illustrated in the fourth section, including a simulation-based validation of its accuracy and estimation of AVS/RS performance for alternative system configurations. The final section offers a summary and conclusions.

2. Background

Figure 1 illustrates three key elements of an AVS/RS including the storage rack, the vehicle, and the lift mechanism. In operating this system, vehicles independently serve randomly arriving S/R transactions from a buffer area similar to the one illustrated in figure 1. They move horizontally in a rectilinear pattern on rails running within and along the ends of storage aisles. Vertical vehicle movement is provided by lifts installed on the rack periphery. The lifts represent an embedded queuing system with waiting time for lifts a component of total SC and DC cycle times. SC cycles involve vehicle movement from the storage buffer to the lift position, movement on the lift to the tier containing the storage position of the UL, and return travel to the buffer area. DC cycles include similar movement to the position of the stored load, direct movement from the position of the stored load to the position of the retrieved load, and return travel to the buffer area. SC and DC cycles require a maximum of two and three lift movements respectively.

From an operational perspective, a design advantage of AVS/RS technology is the flexibility to allocate variable numbers of vehicles and lifts based on the level of transactions demand and the storage rack configuration in a system. A second advantage is that any vehicle can access any storage position in the system. (Apart from time losses due to blocking, it is also possible for multiple vehicles to access the same storage aisle simultaneously.) The disadvantages of AVS/RS technology include longer flow paths from sequential vertical and horizontal travel, vehicle

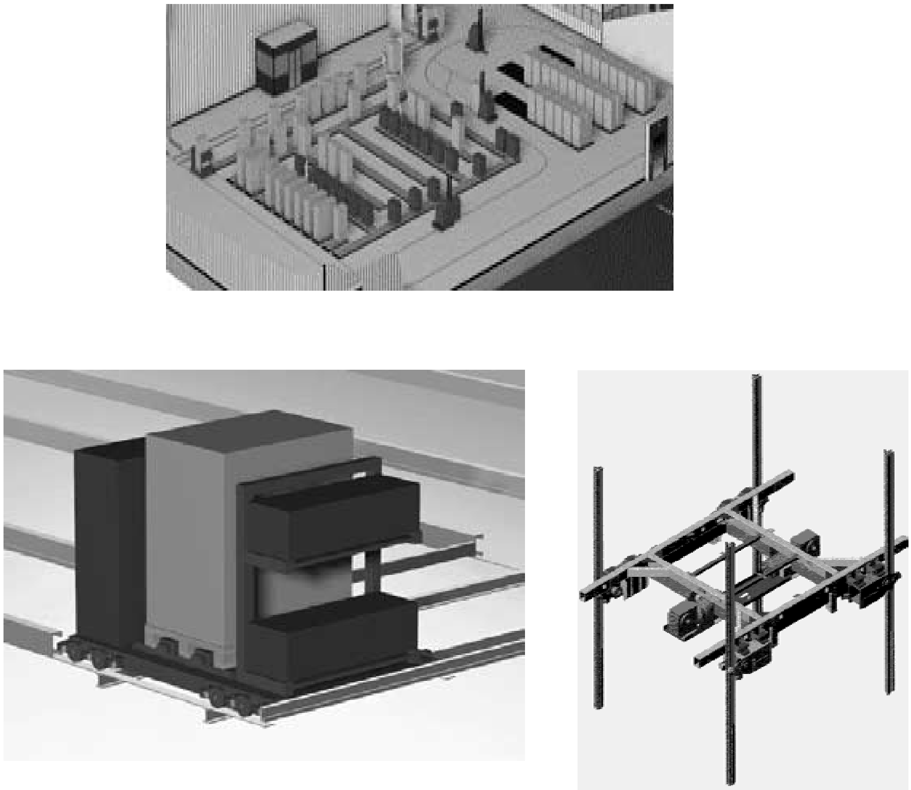

Figure 1. Illustration of AVSRS system components including the load buffer area (top), vehicle (bottom left) and lift mechanism (bottom right).

waiting time for lifts, and vehicle-to-lift transfer time. AVS/RSs also require more complex control systems to facilitate traffic management and collision avoidance.

Two key differences between AVS/RS and AS/RS technologies with respect to interleaving should be noted. First, the relative efficiency of DC versus SC cycles in AVS/RSs is apt to be less than in AS/RSs since random storage and FCFS dispatching causes the majority of paired S/R transactions on DC cycles to include storage positions on different tiers. The need to access lifts for movement between tiers and the sequential rectilinear movement patterns of AVS/RS vehicles will tend to reduce travel time efficiencies available through DC cycles relative to the analogous travel time efficiencies in AS/RSs. However, the pooling of transactions in a single buffer, as opposed to the use of aisle segregated buffers, tends to create more opportunities for interleaving in AVS/RSs relative to AS/RSs for a given level of utilization. The net effect of these differences is difficult to measure without the support of concepting tools such as those described below.

Basic concepting tools for AVS/RS design (Malmborg 2001) focus on the definition of expected SC and DC cycle times (denoted as τ_{SC} and τ_{DC} respectively). For systems using random storage and FCFS transactions dispatching, these values are based on computing travel times between any pair of storage positions j and k (for $j, k = 1, \ldots, n$), that are either co-located in the same aisle and tier, located in different aisles on the same tier, or located on different storage tiers using:

$$d_{jk} = u_d|a_j - a_k|/\nu_h + u_w|c_j - c_k|/\nu_h,$$

$$d_{jk} = u_d|a_j - a_k|/\nu_h + u_w(c_j + c_k)/\nu_h$$

and

$$d_{jk} = 2u_d(a_j + a_k)/\nu_h + u_w(c_j + c_k)/\nu_h + u_h|t_j - t_k|/\nu_v + (W_1 + \varepsilon_2),$$

respectively, where ν_h and ν_v denote horizontal and vertical travel velocity (fpm), u_d, u_w and u_h denote storage position clearances for depth, width and height (feet), ε_2 denotes the vehicle transfer time between rails and lifts (minutes), and W_1 denotes the expected vehicle waiting time for a lift (minutes). Movement times between the buffer (location 0) and individual storage position j are estimated using:

$$d_{0j} = d_{j0} = y(W_1 + \varepsilon_2) + u_h(t_j - 1)/\nu_v + u_d a_j/\nu_h + u_w c_j/\nu_h,$$

where t_j, a_j and c_j denote the tier, aisle and column of storage position j with $y = 1$ if $t_j \neq 1$. Using these models, expected values of travel times among storage positions, and between storage positions and the I/O point are given as:

$$d_3 = \Sigma_{j=1\ldots n}\,\Sigma_{k=1\ldots n}\, f_{jk} d_{jk},$$

$$d_1 = \Sigma_{j=1\ldots n}\, f_{0j} d_{0j}$$

and

$$d_2 = \Sigma_{j=1\ldots n}\, f_{j0} d_{j0},$$

where f_{jk}, f_{0j} and f_{j0} denote the expected volume of vehicle movements per unit time between storage positions, between the buffer and storage position j, and between storage position j and the buffer (see Malmborg 2002 for details). SC and DC cycle times are then computed using:

$$\tau_{SC} = d_1 + d_2 + 2\varepsilon_1 \qquad \text{and} \qquad \tau_{DC} = d_1 + d_2 + d_3 + 4\varepsilon_1,$$

where ε_1 denotes the load transfer time.

As illustrated above, the SC and DC cycle times are a direct function of the configuration of a storage rack as defined by the number of aisles (A), the number of storage columns per (two-sided) aisle (C), and the number of storage tiers (T). The cycle time values are also influenced by the number of lifts in a system (L) through the effect of vehicle waiting time. The number of vehicles used in a system (V) is the last component in the five-variable vector summarizing an AVS/RS design profile, (A,C,T,L,V). This vector is used in conjunction with the estimated vehicle time per transaction to compute the two key AVS/RS performance measures of vehicle utilization and hourly throughput capacity.

With opportunistic interleaving, the AVS/RS bootstrapping issue can be illustrated through the definitions of utilization (U) and throughout capacity (θ):

$$U = (\lambda_r + \lambda_s)\tau/60V \qquad \text{and} \qquad \theta = 60V/\tau,$$

where τ denotes the expected vehicle time per transaction, (average S/R cycle time), and is defined as:

$$\tau = (1 - \alpha)\tau_{SC} + \alpha\tau_{DC}/2.$$

Since α is a function of system utilization, i.e. more DC cycle opportunities occur with higher utilization, the designer at the concepting stage has the problem of needing an implicit estimate of utilization (in the form of the α value), prior to computing the expected utilization. The analogous problem in the case of AS/RSs has been solved using the two different analytical approaches presented in Bozer and Cho (1998) and (Malmborg 2001). In the next section, analytical tools extending the second of these approaches are proposed for solving the AVS/RS bootstrapping problem.

3. State equation model for AVS/R systems

To model opportunistic interleaving dynamics in an AVS/RS, it is possible to define system states based on the number of vehicles in the system and the transactions queue. System states are defined using a vector of the form: $\{v_1, v_2, \ldots, v_V, q_s, q_r\}$ where v_i denotes the state of vehicle i, for $i = 1, \ldots, V$, q_s denotes the number of pending storage transactions in the active queue, and q_r denotes the number of pending retrieval transactions in the active queue. For each vehicle, the values in the state vector can take on values equal to $v_i = 0, 1, 2$ or 3 indicating that vehicle i is idle, performing a SC storage cycle, performing a SC retrieval cycle or performing a DC cycle, respectively. q_s and q_r can take on the values:

$$q_s = 0, 1, \ldots, Q - q_r \qquad \text{and} \qquad q_r = 0, 1, \ldots, Q - q_s,$$

where Q denotes a positive integer greater than or equal to the maximum expected number of transactions observed in the active queue during normal system operation.

Using this representation of system states, it is possible to describe random variation in AVS/RS states using equations describing fluctuations associated with state changing events, i.e. service completions and transaction arrivals to the active queue. Using this approach, we can obtain α as a function of λ_r, λ_s, τ_{SC} and τ_{DC} by using these values to solve for the state probability distribution of an AVS/RS, and then applying this distribution to compute α. This involves formulating equations describing rates of flow between system states and then solving them for state probabilities of the form:

$$P_{v1,v1,v2,\ldots,vV,qs,qr,} = \text{probability of state } \{v_1, v_2, \ldots, v_V, q_s, q_r\},$$

where

$$v_i = 0, 1, 2, 3, i = 1, \ldots V, q_s = 0, 1, \ldots, Q - q_r \qquad \text{and} \qquad q_r = 0, 1, \ldots, Q - q_s.$$

Once the state probability distribution is obtained, the probability that vehicles are either performing SC cycles or DC cycles at randomly selected intervals can be estimated using:

$$P_{SC} = 1 - [\Sigma_{v1=0,3}\, \Sigma_{v2=0,3}, \ldots, \Sigma_{vv=0,3}\, \Sigma_{qs=0,\ldots,Q-qr}\, \Sigma_{qr=0,\ldots,Q-qs} P_{v1,v2,\ldots vv,qs,qr}]$$

and

$$P_{DC} = 1 - [\Sigma_{v1=0,1,2}\, \Sigma_{v2=0,1,2}, \ldots, \Sigma_{vv=0,1,2}\, \Sigma_{qs=0,\ldots,Q-qr}\, \Sigma_{qr=0,\ldots,Q-qs} P_{v1,v2,\ldots vv,qs,qr}].$$

The corresponding value of α, i.e. the proportion of transactions served on DC cycles is then obtainable as:

$$\alpha = 2P_{DC}/(2P_{DC} + P_{SC}).$$

As an example, let $V = 2$ and $Q = 3$. The state equations associated with $q_s = q_r = 0$ for this case are summarized below.

State	State equation		
{0,0,0,0}	$(\lambda_r + \lambda_s)P_{0000}$	$=$	$(\Sigma_{i=1,\ldots,3} P_{i000} + P_{0i00})\tau_{SC}^{-1}$
{1,0,0,0}	$(\tau_{SC}^{-1} + \lambda_r + \lambda_s)P_{1000}$	$=$	$(\lambda_s/2)P_{0000}$
{2,0,0,0}	$(\tau_{SC}^{-1} + \lambda_r + \lambda_s)P_{2000}$	$=$	$(\lambda_r/2)P_{0000}$
{3,0,0,0}	$(\tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3000}$	$=$	$\tau_{SC}^{-1}(P_{3100} + P_{3200}) + (\tau_{DC}/2)^{-1} P_{3300}$
{0,1,0,0}	$(\tau_{SC}^{-1} + \lambda_r + \lambda_s)P_{0100}$	$=$	$(\lambda_s/2)P_{0000}$
{0,2,0,0}	$(\tau_{SC}^{-1} + \lambda_r + \lambda_s)P_{0200}$	$=$	$(\lambda_r/2)P_{0000}$
{0,3,0,0}	$(\tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{0300}$	$=$	$\tau_{SC}^{-1}(P_{1300} + P_{2300}) + (\tau_{DC}/2)^{-1} P_{3300}$
{1,1,0,0}	$(2\tau_{SC}^{-1} + \lambda_r + \lambda_s)P_{1100}$	$=$	$\lambda_s(P_{1000} + P_{0100})$ $+ \tau_{SC}^{-1}(2P_{1110} + P_{1210} + P_{2110})$ $+ \tau_{DC}^{-1}(P_{3110} + P_{1301})$
{2,1,0,0}	$(2\tau_{SC}^{-1} + \lambda_r + \lambda_s)P_{2100}$	$=$	$\lambda_r P_{0100} + \lambda_s P_{2000}$ $+ \tau_{SC}^{-1}(P_{1101} + P_{2110} + P_{2101} + P_{2210})$ $+ \tau_{DC}^{-1}(P_{2310} + P_{3101})$
{3,1,0,0}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3100}$	$=$	$\lambda_r P_{3000} + \tau_{SC}^{-1}(P_{3110} + P_{3210} + P_{2111})$ $+ \tau_{DC}^{-1}(P_{3111} + P_{3310})$
{1,2,0,0}	$(2\tau_{SC}^{-1} + \lambda_r + \lambda_s)P_{1200}$	$=$	$\lambda_r P_{1000} + \lambda_s P_{0200}$ $+ \tau_{SC}^{-1}(P_{1101} + P_{1210} + P_{1201} + P_{2210})$ $+ \tau_{DC}^{-1}(P_{1301} + P_{3210})$
{2,2,0,0}	$(2\tau_{SC}^{-1} + \lambda_r + \lambda_s)P_{2200}$	$=$	$\lambda_r(P_{2000} + P_{0200})$ $+ \tau_{SC}^{-1}(P_{1201} + P_{2101} + 2P_{2201})$ $+ \tau_{DC}^{-1}(P_{3110} + P_{1301})$
{3,2,0,0}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3200}$	$=$	$\lambda_r P_{3000} + \tau_{SC}^{-1}(P_{3101} + P_{3201} + P_{2211})$ $+ \tau_{DC}^{-1}(P_{3111} + P_{3301})$
{1,3,0,0}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{1300}$	$=$	$\lambda_s P_{0300} + \tau_{SC}^{-1}(P_{1111} + P_{1211} + P_{1310})$ $+ \tau_{DC}^{-1}(P_{1311} + P_{3310})$
{2,3,0,0}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{2300}$	$=$	$\lambda_r P_{0300} + \tau_{SC}^{-1}(P_{1301} + P_{2301} + P_{2211})$ $+ \tau_{DC}^{-1}(P_{2311} + P_{3301})$
{3,3,0,0}	$(2\tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3300}$	$=$	$\tau_{SC}^{-1}(P_{1311} + P_{2311} + P_{3111} + P_{3211})$ $+ \tau_{DC}^{-1}(2P_{3311})$

Each of the above vehicle state combinations is also associated with queue states where $q_s \neq 0$ and $q_r \neq 0$. For example, consider the vehicle combination $v_1 = 3$, $v_2 = 1$. The corresponding state equations for $q_s = 0, 1, \ldots, Q - q_r$, $q_r = 0, 1, \ldots, Q - q_s$, are given by:

State	State equation		
{3,1,0,1}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3101}$	$=$	$\lambda_r P_{3100} + \tau_{DC}^{-1} P_{3112}$
{3,1,0,2}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3102}$	$=$	$\lambda_r P_{3101}$
{3,1,0,3}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1})P_{3103}$	$=$	$\lambda_r P_{3102}$
{3,1,1,0}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3110}$	$=$	$\lambda_s P_{3100} + \tau_{SC}^{-1} P_{3120}$
{3,1,1,1}	$(\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3111}$	$=$	$\lambda_s P_{3101} + \lambda_r P_{3110}$

$$
\begin{array}{lll}
\{3,1,1,2\} & (\tau_{SC}^{-1} + \tau_{DC}^{-1})P_{3112} & = \lambda_s P_{3102} + \lambda_r P_{3111} \\
\{3,1,2,0\} & (\tau_{SC}^{-1} + \tau_{DC}^{-1} + \lambda_r + \lambda_s)P_{3120} & = \lambda_s P_{3110} + \tau_{SC}^{-1}(P_{3130} + P_{3230}) \\
\{3,1,2,1\} & (\tau_{SC}^{-1} + \tau_{DC}^{-1})P_{3121} & = \lambda_s P_{3111} + \lambda_r P_{3120} \\
\{3,1,3,0\} & (\tau_{SC}^{-1} + \tau_{DC}^{-1})P_{3130} & = \lambda_s P_{3120}
\end{array}
$$

As the above equations suggest, the implementation difficulty of the state equation approach to estimating α lies in dimensionality since the number of states grows rapidly with the number of vehicles and the maximum queue size. Specifically, the total number of vehicle states is given by 4^V with the total number of queue states given as:

$$\Sigma_{i=0,\dots,Q}(Q + 1 - i).$$

With opportunistic interleaving, non-empty queue states can only exist for the 3^V vehicle states where there are no idle vehicles. The total number of feasible AVS/RS states is therefore the product of the number of vehicle states with no idle vehicles and the total number queue states, plus the number of vehicle states with at least one idle vehicle (in these states it must be that $q_s = q_r = 0$). Thus, the total number of AVS/RS system states as a function of V and Q can be written as:

$$S = \Sigma_{i=0,\dots,Q}(Q + 1 - i)3^V + (4^V - 3^V).$$

For example, $V = 2$ and $Q = 3$ yields $S = 97$. Although S increases rapidly with V and Q, the generation of state equations is easily automated since there are only four logically differentiated cases based on the transaction queue states defined *as* $(q_s = q_r = 0)$, $(q_s = 0,\ q_r \geq 1)$, $(q_s \geq 1,\ q_r = 0)$ and $(q_s \geq 1,\ q_r \geq 1)$. Each of these cases requires a marginally different programming logic to associate flow rates between system states that result from service completions and transaction arrivals. Apart from this, the programming logic does not change with the values of Q and V, making it possible to implement easily the method for reasonably dimensioned problems.

It should also be noted that the state equation model assumes that it is generally easier for designers at the concepting stage to specify the Q parameter rather than the α parameter for an AVS/RS application. This assumption is justified by the fact that over-estimation of the Q value does not adversely impact the accuracy of the model results, (although it imposes unnecessary computation to obtain results). In contrast, errors of both underestimation and overestimation in the specification of α can significantly influence model accuracy. Therefore, the state equation model allows the user to explicitly trade off model accuracy and computational cost.

4. Illustration of the AVS/RS state equation model

To illustrate the AVS/RS state equation model, a total of 120 AVS/RS profiles are examined. Each case represents an AVS/RS with approximately 3000 storage positions, where the number of storage columns is computed as a function of the number of aisles and tiers to be an integer value using $C = \lceil 3000/2AT \rceil$. The resultant 120 system profiles are summarized below:

$(A = 5 - 10, C = \lceil 3000/2AT \rceil, T = 5 - 8, V = 2, L = 1)$ – 24 Rack Configurations
$(A = 5 - 10, C = \lceil 3000/2AT \rceil, T = 5 - 8, V = 2, L = 2)$ – 24 Rack Configurations

$(A = 5 - 10, C = \lceil 3000/2AT \rceil, T = 5 - 8, V = 3, L = 1) - 24$ Rack Configurations
$(A = 5 - 10, C = \lceil 3000/2AT \rceil, T = 5 - 8, V = 3, L = 2) - 24$ Rack Configurations
$(A = 5 - 10, C = \lceil 3000/2AT \rceil, T = 5 - 8, V = 3, L = 3) - 24$ Rack Configurations

Other parameter values used in the analysis include:

$\nu_\mathrm{h} = 400\,\mathrm{fpm}$, $\nu_\mathrm{v} = 200\,\mathrm{fpm}$, $u_\mathrm{d} = 5\,\mathrm{feet}$, $u_\mathrm{w} = 5\,\mathrm{feet}$, $u_\mathrm{h} = 6\,\mathrm{feet}$,
$\varepsilon_1 = 0.05$ minutes, $\varepsilon_2 = 0.08$ minutes, $Q = 12$,
$\lambda_\mathrm{s} = 50$ storage transactions per hour, $\lambda_\mathrm{r} = 50$ retrieval transactions per hour.

To validate the accuracy of the state equation model, a simulation of an AVS/RS was run for each of the 120 system profiles and coded to collect statistics on the number of S/R transactions served on SC versus DC cycles. For each rack scenario, 100 twenty-four hour periods of system operation were simulated to obtain observations of the frequency of SC and DC cycles. In all simulations, a Poisson arrival process for storage and retrieval transactions was programmed (i.e. exponential inter-arrival times). Vehicles completing transactions were assumed to use the load buffer area as the dwell point regardless of where in the system they were released by the last completed transaction. The simulation logic was to generate the storage address of the next transaction based on a uniform distribution of transactions demand across the set of rack locations, i.e. a random storage policy. All arriving transactions were first checked for vehicle availability. FCFS seizures of vehicles by arriving or queued transactions were followed by an examination of the transactions queue to check for availability of interleaving opportunities. When feasible without waiting, DC cycles were performed, otherwise SC cycles were implemented. Horizontal travel times to the lift area, (as well as horizontal and vertical travel time components of the remainder of the cycle) were computed using velocity values with acceleration and deceleration effects ignored. Both rack and vehicle load transfer times were assumed to be constant. The number of S/R transactions served on SC versus DC cycles obtained from the simulations were used to compute the corresponding value of α with the average value from the 100 simulation runs used for comparison with the analytical estimate. The results are summarized in table 1 and figure 2. In both cases, the results are organized into five groups of storage rack configurations associated with the five combinations of V and L. The results presented in figure 2 are in the same order that they appear in table 1.

The results in table 1 demonstrate the impact of Q on the accuracy of the model results. For example, for the system profile described as ($A = 5$, $C = 60$, $T = 5$, $V = 2$, $L = 1$), there is considerable discrepancy between the simulation and analytical model results with $\alpha_\mathrm{Simulation} = 0.6193$ versus $\alpha_\mathrm{Analytical} = 0.5612$ yielding an error of 10.4%. This error is significantly greater than the average error of approximately 0.5% that was observed over the 124 cases studied. It can be explained by observing that the estimated utilization for this scenario as predicted by the state equation model is 91.3%. At this high utilization level (which actually underestimates the true utilization value due to the capacitated queue assumption), the active queue has a non-negligible probability of exceeding a total of $Q = 12$ storage and retrieval transactions. Subsequently, the state equation model fails to recognize the full transactions demand rate since it assumes that transactions arriving when $q_\mathrm{s} + q_\mathrm{r} = Q$ are lost to the system. The same effect can be observed to a lesser degree with other system profiles:

T	A	$V = 2$	$L = 1$	$V = 2$	$L = 2$	$V = 3$	$L = 1$	$V = 3$	$L = 2$	$V = 3$	$L = 3$
5	5	0.619	0.561	0.569	0.551	0.280	0.279	0.263	0.261	0.260	0.259
5	6	0.531	0.527	0.518	0.518	0.266	0.265	0.249	0.248	0.246	0.246
5	7	0.507	0.504	0.496	0.495	0.256	0.256	0.240	0.239	0.236	0.237
5	8	0.488	0.489	0.478	0.480	0.251	0.251	0.232	0.233	0.231	0.231
5	9	0.478	0.477	0.469	0.467	0.247	0.246	0.229	0.228	0.225	0.226
5	10	0.468	0.465	0.456	0.456	0.243	0.241	0.223	0.223	0.221	0.221
6	5	0.543	0.536	0.526	0.525	0.273	0.271	0.256	0.251	0.248	0.249
6	6	0.510	0.509	0.498	0.499	0.260	0.261	0.240	0.241	0.238	0.238
6	7	0.491	0.489	0.480	0.479	0.254	0.253	0.233	0.233	0.229	0.231
6	8	0.476	0.478	0.465	0.467	0.249	0.249	0.229	0.228	0.227	0.226
6	9	0.466	0.466	0.453	0.455	0.245	0.244	0.224	0.223	0.225	0.221
6	10	0.456	0.458	0.449	0.447	0.241	0.242	0.219	0.220	0.218	0.217
7	5	0.524	0.521	0.510	0.509	0.269	0.268	0.246	0.245	0.243	0.242
7	6	0.497	0.498	0.485	0.486	0.263	0.259	0.238	0.236	0.232	0.233
7	7	0.483	0.482	0.469	0.470	0.254	0.253	0.228	0.230	0.227	0.227
7	8	0.472	0.471	0.461	0.458	0.249	0.249	0.229	0.225	0.222	0.222
7	9	0.461	0.463	0.450	0.450	0.246	0.242	0.221	0.222	0.219	0.219
7	10	0.459	0.459	0.448	0.446	0.245	0.245	0.221	0.220	0.217	0.217
8	5	0.518	0.514	0.503	0.500	0.269	0.268	0.241	0.242	0.236	0.239
8	6	0.493	0.495	0.482	0.480	0.260	0.261	0.235	0.234	0.231	0.231
8	7	0.477	0.479	0.463	0.465	0.256	0.255	0.229	0.228	0.224	0.225
8	8	0.472	0.471	0.456	0.457	0.253	0.252	0.225	0.225	0.222	0.221
8	9	0.463	0.463	0.447	0.449	0.249	0.249	0.227	0.221	0.219	0.218
8	10	0.461	0.459	0.444	0.447	0.248	0.248	0.219	0.220	0.219	0.216

Table 1. Simulation generated and model generated α values for alternative vehicle, lift and rack configuration parameters. System scenarios one through five.

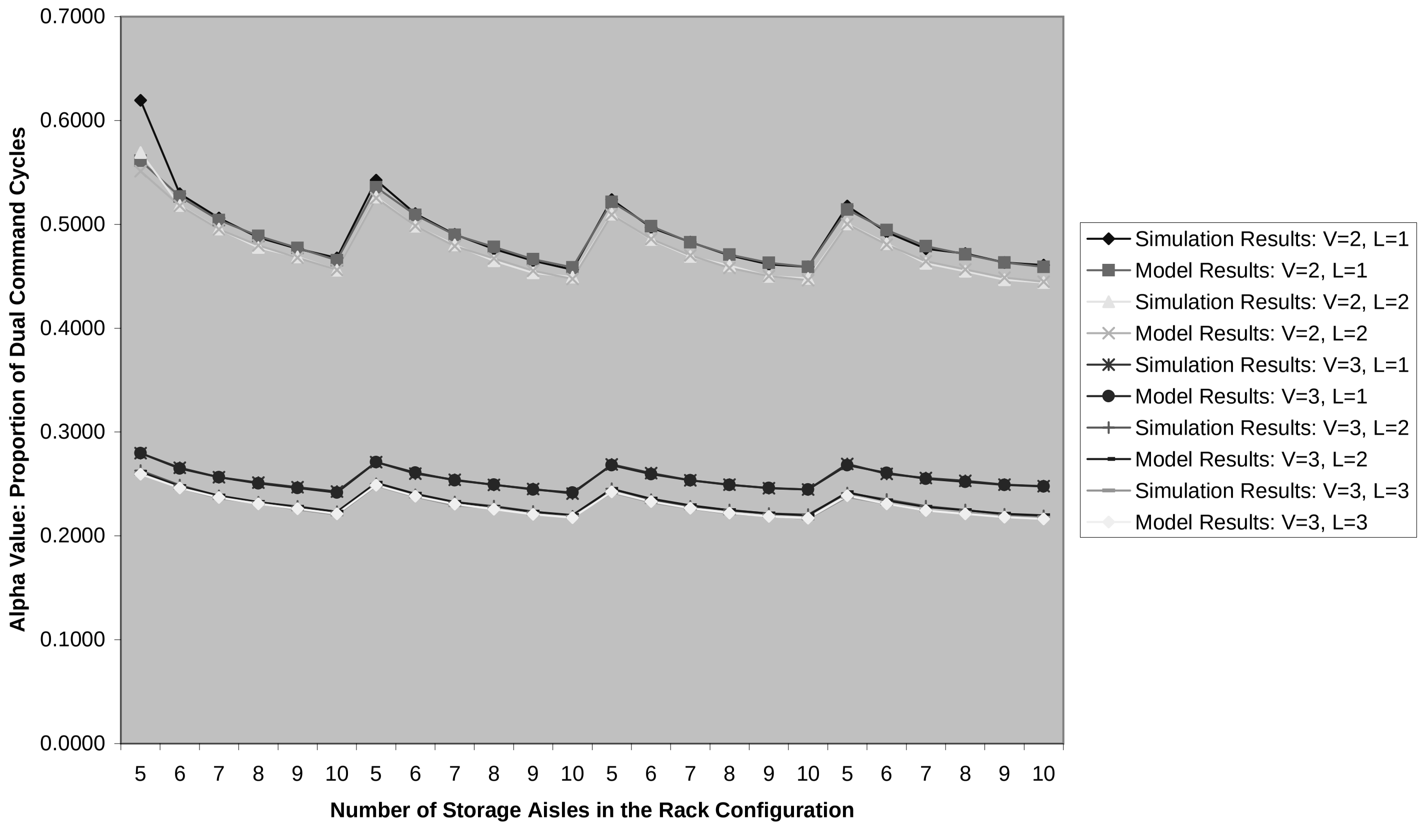

Figure 2. Comparison of simulation and model generated alpha values for $T = 5$, $T = 6$, $T = 7$ and $T = 8$: maximum queue size of 12 transactions.

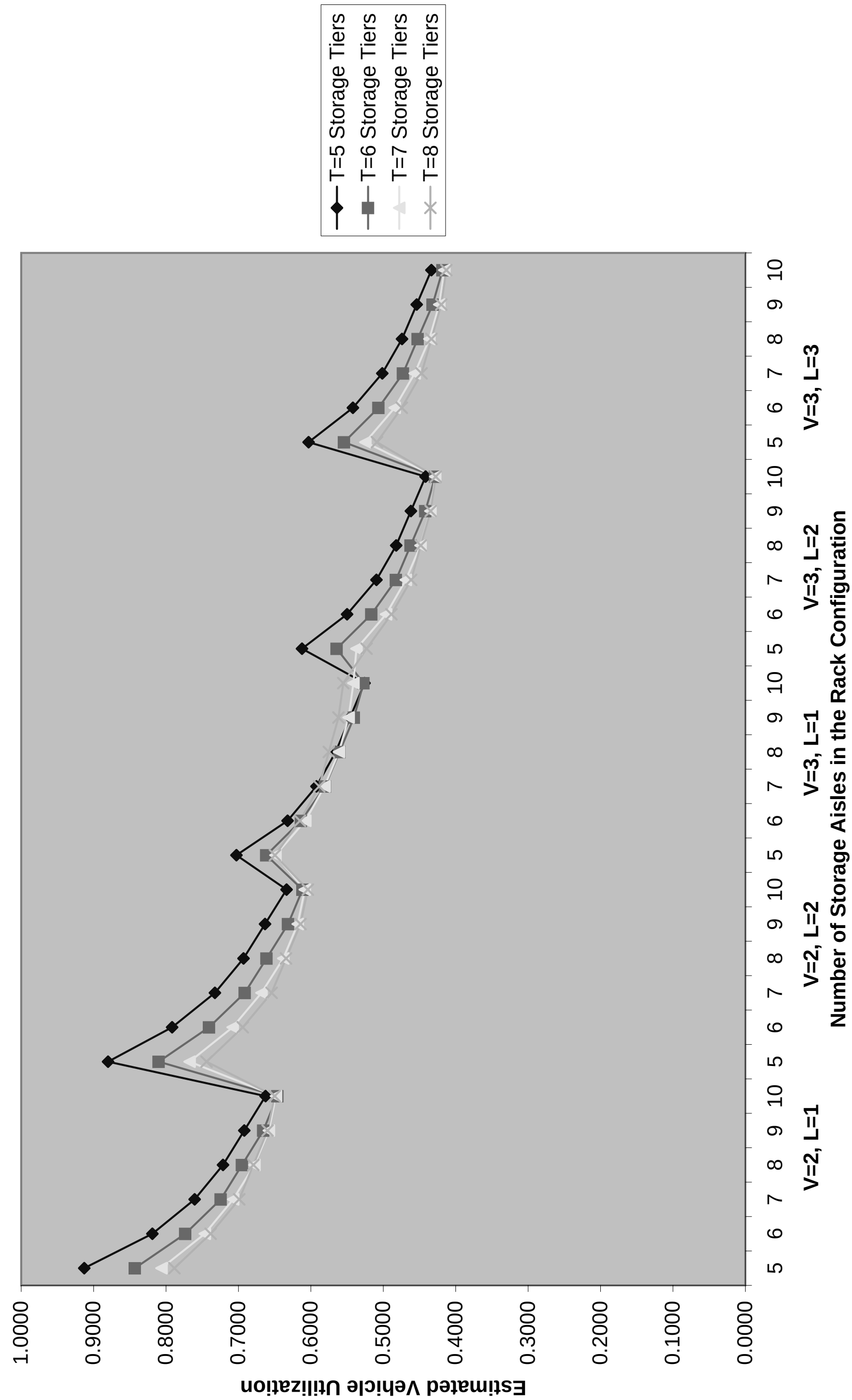

Figure 3. Estimated vehicle utilization values for the five system scenarios.

(V, L)	A	$T=5$	$T=6$	$T=7$	$T=8$
(2,1)	5	0.9130	0.8430	0.8063	0.7883
(2,1)	6	0.8189	0.7736	0.7465	0.7384
(2,1)	7	0.7604	0.7246	0.7073	0.6991
(2,1)	8	0.7213	0.6952	0.6778	0.6795
(2,1)	9	0.6920	0.6658	0.6582	0.6599
(2,1)	10	0.6626	0.6462	0.6484	0.6501
(2,2)	5	0.8802	0.8101	0.7676	0.7441
(2,2)	6	0.7917	0.7407	0.7079	0.6942
(2,2)	7	0.7325	0.6911	0.6681	0.6542
(2,2)	8	0.6930	0.6613	0.6382	0.6342
(2,2)	9	0.6634	0.6315	0.6183	0.6143
(2,2)	10	0.6337	0.6116	0.6083	0.6043
(3,1)	5	0.7027	0.6617	0.6491	0.6495
(3,1)	6	0.6321	0.6136	0.6078	0.6150
(3,1)	7	0.5923	0.5804	0.5812	0.5884
(3,1)	8	0.5657	0.5605	0.5613	0.5753
(3,1)	9	0.5457	0.5405	0.5480	0.5620
(3,1)	10	0.5257	0.5272	0.5414	0.5554
(3,2)	5	0.6124	0.5642	0.5370	0.5230
(3,2)	6	0.5499	0.5165	0.4960	0.4888
(3,2)	7	0.5093	0.4826	0.4689	0.4616
(3,2)	8	0.4822	0.4623	0.4485	0.4481
(3,2)	9	0.4619	0.4419	0.4350	0.4345
(3,2)	10	0.4416	0.4284	0.4282	0.4277
(3,3)	5	0.6031	0.5543	0.5249	0.5086
(3,3)	6	0.5421	0.5067	0.4841	0.4745
(j,3)	7	0.5014	0.4727	0.4568	0.4473
(3,3)	8	0.4742	0.4523	0.4364	0.4336
(3,3)	9	0.4539	0.4319	0.4228	0.4200
(3,3)	10	0.4335	0.4183	0.4159	0.4131

Table 2. Model estimated vehicle utilization values for alternative vehicle, lift and rack configuration parameters.

$(A = 5,\ C = 60,\ T = 5,\ V = 2,\ L = 2) - 3.3\%$ error at 88% utilization, and
$(A = 5,\ C = 50,\ T = 6,\ V = 2,\ L = 1) - 1.3\%$ error at 84% utilization.

As these results suggest, the probability of system states with $q\text{s} + q\text{r} = Q$ becomes negligible with utilization levels below about 80% where the state equation model based estimates of α closely reflect those obtained from the simulation model.

Table 2 presents values of AVS/RS utilization as computed by the state equation model for each of the 124 scenarios. Figure 3 presents these results graphically where each V and L combination exhibits a similar pattern of utilization. Specifically, utilization is maximized as the number of aisles is minimized. This reflects the impact of long horizontal flow paths associated with the rack configurations having the deepest aisles, i.e. when A and T are at their minimum values. This observation emphasizes one of the design advantages of AVS/RS. Specifically, the addition of storage aisles may present a more attractive means for increasing capacity in an AVS/RS since, unlike an AS/RS, adding aisles does not necessitate the acquisition of additional vehicles. It can also be noted from figure 3 that the slightly lower utilization values observed with $V = 2$, $L = 1$ versus $V = 2$, $L = 2$ demonstrates a reduced utilization associated with eliminating vehicle waiting time for lifts. A similar pattern is observed for the system profiles with $V = 3$ where the diminishing returns on a third lift are clearly evident from the small differences in utilization associated with profiles with $V = 3$, $L = 2$ and $V = 3$, $L = 3$.

5. Summary and conclusions

A state equation model is proposed to estimate the proportion of dual command S/R cycles for autonomous vehicle storage systems using opportunistic interleaving. The model extends AVS/RS concepting tools that require an estimate of this proportion to predict system utilization and throughout capacity. The model substitutes a prediction of the maximum practical queue size in place of the proportion of DC cycles to bootstrap the estimation of utilization and throughput. This enables the user to trade off model accuracy versus computational resources by using high-end or low-end estimates of the maximum queue size. The impact of this decision for $Q = 12$ is illustrated through a series of sample problems with estimated utilization varying from 91.3% to 41.3% and demonstrates losses in model accuracy at higher utilization values for a given estimate of the maximum queue size. Despite this limitation, the model provides a potentially useful addition to the tools available for AVS/RS concepting when opportunistic interleaving is used. This could be through direct search of the AVS/RS design space for identifying a reduced set of system profiles prior to simulation validation. Alternatively, the state equation model could be used to validate selectively estimates of AVS/RS performance measures based on user estimated proportions of DC cycles during the pre-simulation concepting phase.

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