7. *Decomposing a PV array output time series.* We are given a time series $p \in \mathbf{R}^T_+$ that gives the output power of a photo-voltaic (PV) array in 5-minute intervals, over $T = 2016$ periods (one week), given in `pv_output_data.*`. In this problem you will use convex optimization to decompose the time series into three components:

   - The *clear sky output* $c \in \mathbf{R}^T_+$, a smooth daily-periodic component, which gives what the PV output would have been without clouds. This signal is 24-hour-periodic, *i.e.*, $c_{t+288} = c_t$ for $t = 1, \ldots, T - 288$. (The clear sky output is zero at night, but we will not use this prior information in our decomposition method.)
   - A *weather shading loss* component $s \in \mathbf{R}^T_+$, which gives the loss of power due to clouds. This component satisfies $0 \preceq s \preceq c$, can change rapidly, and is not periodic.
   - A *residual* $r \in \mathbf{R}^T$, which accounts for measurement error, anomalies, and other errors.

   These components satisfy $p = c - s + r$.

   We will assume that the average absolute value of the residual is no more than 4 (which is less than 1% of the average of $p$).

   Smoothness of $c$ is measured by its Laplacian,

   $$\mathcal{L}(c) = (c_1 - c_2)^2 + \cdots + (c_{287} - c_{288})^2 + (c_{288} - c_1)^2.$$

   (Note that the term involves $c_1$ and $c_{288}$.)

   We will choose $c$, $s$, and $r$ by minimizing $\mathcal{L}(c) + \lambda \mathbf{1}^T s$ subject to the constraints described above, where $\lambda$ is a positive parameter, that we take to be one.

   Solve this problem, and plot the resulting $c$, $s$, $r$, and $p$ (which is given), on separate plots. Give the average values of $c$, $s$, and $p$, and the average absolute value of $r$ (which should be 4).