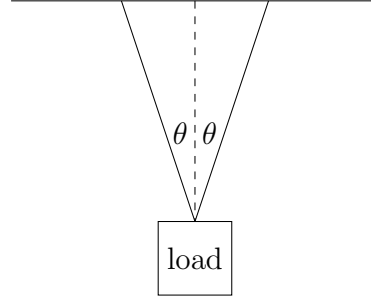


3. *Minimum time maneuver for a crane.* A crane manipulates a load with mass  $m > 0$  in two dimensions using two cables attached to the load. The cables maintain angles  $\pm\theta$  with respect to vertical, as shown below.



The (scalar) tensions  $T^{\text{left}}$  and  $T^{\text{right}}$  in the two cables are independently controllable, from 0 up to a given maximum tension  $T^{\text{max}}$ . The total force on the load is

$$F = T^{\text{left}} \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix} + T^{\text{right}} \begin{bmatrix} \sin \theta \\ \cos \theta \end{bmatrix} + mg,$$

where  $g = (0, -9.8)$  is the acceleration due to gravity. The acceleration of the load is then  $F/m$ .

We approximate the motion of the load using

$$p_{i+1} = p_i + hv_i, \quad v_{i+1} = v_i + (h/m)F_i, \quad i = 1, 2, \dots,$$

where  $p_i \in \mathbf{R}^2$  is the position of the load,  $v_i \in \mathbf{R}^2$  is the velocity of the load, and  $F_i \in \mathbf{R}^2$  is the force on the load, at time  $t = ih$ . Here  $h > 0$  is a small (given) time step.

The goal is to move the load, which is initially at rest at position  $p^{\text{init}}$  to the position  $p^{\text{des}}$ , also at rest, in minimum time. In other words, we seek the smallest  $k$  for which

$$p_1 = p^{\text{init}}, \quad p_k = p^{\text{des}}, \quad v_1 = v_k = (0, 0)$$

is possible, subject to the constraints described above.

- Explain how to solve this problem using convex (or quasiconvex) optimization.
- Carry out the method of part (a) for the problem instance with

$$m = 0.1, \quad \theta = 15^\circ, \quad T^{\text{max}} = 2, \quad p^{\text{init}} = (0, 0), \quad p^{\text{des}} = (10, 2),$$

with time step  $h = 0.1$ . Report the minimum time  $k^*$ . Plot the tensions versus time, and the load trajectory, *i.e.*, the points  $p_1, \dots, p_k$  in  $\mathbf{R}^2$ . Does the load move along the line segment between  $p^{\text{init}}$  and  $p^{\text{des}}$  (*i.e.*, the shortest path from  $p^{\text{init}}$  and  $p^{\text{des}}$ )? Comment briefly.