

1. Write a MATLAB function to solve the diffusion equation $u_t = u_{xx}$ for $x \in [0, 1]$ and $t > 0$ subject to the initial and boundary conditions $u(x, 0) = \cos(\pi x) - 1$, $u(0, t) = 0$, $u(1, t) = -2$ by Crank-Nicolson method.
2. Write a MATLAB function to find a root of $x^3 - 2x + 5 \cos(x) = 0$ using Newton-Raphson method.
3. Write a MATLAB function to solve a system $Ax = b$ using Conjugate Gradient algorithm and compare your solution with the one with Gauss-Seidel method, where

$$A = \begin{pmatrix} 7 & -6 & 9 \\ 4 & 5 & -4 \\ -7 & -3 & 8 \end{pmatrix}, \quad b = \begin{pmatrix} 10 \\ 5 \\ -2 \end{pmatrix}.$$