

1. Use MATLAB in-built `rref` function (Gauss-Jordan Method) to find the inverse of the following matrices, if exist:

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 3 & 3 \\ 2 & 3 & 1 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 3 & 3 \\ 2 & 5 & 6 \end{pmatrix}$$

2. Test whether Jacobi and Gauss-Seidel iteration converge for the following matrices:

$$\begin{pmatrix} 1 & 0 & 0 \\ 22 & 12 & 0 \\ 3 & 2 & 10 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 1/2 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

Write a common testing algorithm.

3. Test the convergence rates of Jacobi and Gauss-Seidel iteration, if they converge for the following matrices:

$$\begin{pmatrix} 3 & 0 & 4 \\ 7 & 4 & 2 \\ -1 & 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} -3 & 3 & -6 \\ -4 & 7 & -8 \\ 5 & 7 & -9 \end{pmatrix}, \quad \begin{pmatrix} 4 & 1 & 1 \\ 2 & -9 & 0 \\ 0 & -8 & -6 \end{pmatrix}, \quad \begin{pmatrix} 7 & 6 & 9 \\ 4 & 5 & -4 \\ -7 & -3 & 8 \end{pmatrix}.$$

Write a common testing algorithm.

4. Write MATLAB functions to solve a system  $Ax = b$  using Jacobi and Gauss-Seidel algorithms and plot the error  $e_k = \|Ax^{(k)} - b\|$  vs iteration number to compare the two algorithms. Use above matrices and `randi(10,n,1)` as test cases.