Total No. of Questions : 8] [Total No. of Printed Pages : 3

Paper Code : 21308

F-408

B.C.A. (Second Semester) Examination, 2022

(New Course)

Paper No. BCA-N-203

MATHEMATICS-II

Time : Three Hours] *[Maximum Marks : 70*

Note : Attempt any **five** questions. **All** questions carry equal marks. Symbols used are as usual.

1. (a) Prove that the system Q of rational numbers has the Archimedean property i.e. is an Archimedean ordered field.

   (b) Show that any open interval is a neighbourhood of each of its points.

2. (a) Find the limit points of the set

$$S = \left\{ \frac{n}{n+1} ; n \in \mathbb{N} \right\}$$

   (b) Prove that the intersection of an arbitrary collection of closed sets is closed.

3. (a) Evaluate : $\lim\limits_{x \to 0} \dfrac{2^x - 1}{(1+x)^{1/2} - 1}$

   (b) Examine the following function for continuity at x=0 and x=1.

$$f(x) = \begin{cases} x^2 ; & x \le 0 \\ 1 ; & 0 < x \le 1 \\ \dfrac{1}{x} ; & x > 1 \end{cases}$$

4. (a) If $<s_n>$ is a Cauchy sequence of real numbers which has a *subsequence* converging to ℓ, prove that $<s_n>$ itself converges to ℓ.

   (b) Prove that the set of limit points of every sequence is a closed set.

5. (a) Expand tan x by Maclaurin's theorem.

   (b) Verify Cauchy's Mean Value Theorem for the functions x^2 and x^3 in the interval [1, 2].

6. (a) Let $f:[0, 1]\rightarrow R$ be defined by

$f(x)=(x-1)^2+2, \forall x\in[0, 1]$.

Find the equation of the tangent to the graph of this curve which is parallel to the chord joining the points (0,3) and (1,2) of the curve.

(b) Test for convergence of the series whose n^{th} term is given by $\sqrt{n^2+1}+\sqrt{n^2-1}$

7. (a) Evaluate : $\lim_{x\to 0}\left[\frac{(1+x)^{1/x}-e}{x}\right]$

(b) Show that if the perimeter of a triangle is constant, its area is maximum when it is equilateral.

8. (a) Find the maxima and minima of $(4-3x)^2e^x$.

(b) Show that the sequence $<s_n>$ where $s_n=\sin n\pi\theta$ and θ is a rational number such that $0<\theta<1$, is not convergent.