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1) Let $g(x) = W^T x + b$

2) we want to maximize k such that

- $W^T x_i + b \geq k$ for $d_i = 1$
- $W^T x_i + b \leq k$ for $d_i = -1$

So we can conclude that values of $g(x)$ depends on $||W||$,

let $g(x) \geq 1$ minimize $||W||$

we have

$W^T x + b + 1 = 0$ $w^T x + b - 1 = 0$

$W^T X_{pos} + b = 1$

$W^T X_{neg} + b = -1$

$W^T X_{pos} + b = 1$

$W^T X_{neg} + b = -1$

$W^T X_{pos} - W^T X_{neg} = 2$

$W^T (X_{pos} - X_{neg}) = 2$

$\frac{W^T}{||W||} (X_{pos} - X_{neg}) = 2$

$\frac{W^T}{||W||} (X_{pos} - X_{neg}) = \frac{2}{||W||}$

To maximize margine $\frac{2}{||W||}$ minimize $\frac{1}{2} W W^T$

Minimize $\frac{1}{2} W^T W$ ie $||W||$ subject to

given $d_i(w^T x_i + b) \geq 1$ $\forall i = 1 \sim N$ $d_i = +1$, $d_i = -1$

for all classes

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Minimal $\frac{1}{2} W^T W$ (ie $||W||$ subject to

given $d_i (W^T x_i + b) \geq 1 \quad \forall i = 1 \dots N$

where $d_i = +1$ or -1 for all classes

We formulate the above example to the following laragian form with the constraint lagrangian from with the constraint $\alpha_i \geq 0$ for all i

$$J(w, b, \alpha) = \frac{1}{2} w^T w - \sum_{i=1}^{N} \alpha_i y_i (w^T x_i + b) + \sum_{i=1}^{N} \alpha_i$$

We know that at optimum

(1) $\frac{dJ}{dw} = 0$ (2) $\frac{dJ}{db} = 0$

Solving both eq^n

$$W_0 = \sum_{i=1}^{N} \alpha_i y_i \; x_i$$

$$\sum_{i=1}^{N} \alpha_i y_i = 0$$

$$\alpha_i (y (w_0^T x_i + b_0) - 1) = 0$$

Converting to dual form

$$Q(\alpha) = \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_i \alpha_j y_i y_j x_i^T x_j$$

This can be solved by QP solver