25/11/2019

PRACTICAL-1

Topic:- Limit and continuity.

Q1.

i.) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

ii.) $\lim_{x \to 0} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

iii.) $\lim_{x \to \pi/6} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

iv.) $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2+3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

Q2.) Examine the function and find whether it is continuous or not.

i.) $f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & 0 \leq x \leq \pi/2 \\ = \dfrac{\cos x}{\pi - 2x}, & \pi/2 < x < \pi \end{cases}$ at $x = \pi/2$

25) ii.)
$$f(x) = \left\{\begin{array}{ll} \dfrac{x^2-9}{x-3}, & 0 < x < 3 \\ = x+3 & 3 \le x < 6 \\ = \dfrac{x^2-9}{x+3} & 6 \le x < 9 \end{array}\right\} \text{ at } x = 3 \text{ and } x = 6$$

5.) Find the value of k - so that function f(x) is continuous at the indicated point.

i.)
$$f(x) = \left\{\begin{array}{ll} \dfrac{1-\cos 4x}{x^2} & x < 0 \\ = k & x = 0 \end{array}\right\} \text{ at } x = 0$$

ii.)
$$f(x) = \left\{\begin{array}{ll} (\sec^2 x)^{\cot^2 x} & , x \neq 0 \\ = k & , x = 0 \end{array}\right\} \text{ at } x = 0$$

iii.)
$$f(x) = \left\{\begin{array}{ll} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} & , x \neq \pi/2 \\ = k & x = \pi/3 \end{array}\right\} \text{ at } x = \pi/3$$

7.) Discuss the continuity of the following function if not continuous remove the discontinuity? Redefine the function so as to remove the discontinuity.

i.)
$$f(x) = \left\{\begin{array}{ll} \dfrac{1-\cos 3x}{x \tan x} & ; x \neq 0 \\ = 9 & \end{array}\right\} \text{ at } x = 0$$

ii.)
$$f(x) = \left\{\begin{array}{ll} \dfrac{(e^{3x}-1) \cdot \sin x^{\circ}}{x^2} & , x \neq 0 \\ = \pi/60 & , x = 0 \end{array}\right\} \text{ at } x = 0$$

8.) If, $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ for $x \neq 0$ is continuous at $x = 0$, find $f(0)$

9.) If $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ for $x \neq \pi/2$ is continuous at $x = \pi/2$. Find $f(\pi/2)$

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Q1.]

i.) $\lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$=\lim_{x\to a}\left\{\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\dfrac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right.$

$=\lim_{x\to a}\dfrac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$

$=\lim_{x\to a}\dfrac{1}{3}\dfrac{(\sqrt{3a+x})+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$

$=\dfrac{1}{3}\ \dfrac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$=\dfrac{1}{3}\times\dfrac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$=\dfrac{2}{3\sqrt{3}}$

2.] $\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$=\lim_{y\to 0}\left[\dfrac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\dfrac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$=\lim_{y\to 0}\left[\dfrac{y}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}\right]$

$=\dfrac{1}{\sqrt{a+0}(\sqrt{a+0}+\sqrt{a})}$

$=\dfrac{1}{\sqrt{a}(2\sqrt{a})}$

$=\dfrac{1}{2a}$

3.] $\lim_{x\to\pi/6}\left[\dfrac{\cos x-\sqrt{3}\sin x}{\pi-6x}\right]$

By substituting $x-\frac{\pi}{6}=h$

$x=h+\pi/6$

where $h\to 0$

$=\lim_{h\to 0}\left[\dfrac{\cos h+\pi/6-\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\pi/6)}\right]$

$=\cos\lim_{h\to 0}\left[\dfrac{\cos h.\frac{\sqrt{3}}{2}\cdot\sin h\frac{1}{\sqrt{2}}-\sqrt{3}(\sin h\frac{\sqrt{3}}{2}+\cos h\frac{1}{\sqrt{2}}}{\pi-6h+\pi}\right.$

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$$= \lim_{h \to 0} \frac{-\sin 4h}{6h}$$

$$= \frac{1}{3} \cdot \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{3}$$

4) $$\lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2+3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$$

$$= \lim_{x \to \infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2+3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2+3}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2+3}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right]$$

$$= \lim_{x \to \infty} \left[\frac{(x^2+5-x^2+3)}{(x^2+3-x^2-1)} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2+3})} \right]$$

$$= \lim_{x \to \infty} \left[\frac{8}{2} \frac{(\sqrt{x^2+3}) + (\sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})} \right]$$

$$= 4 \lim_{x \to \infty} \left[\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}} \right]$$

$$= 4$$

Q2) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$

$= \frac{\cos x}{\pi - 2x}$ for $\pi/2 < x < \pi$

at $x = \pi/2$

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

$\therefore f$ at $x = \pi/2$ is defined

$$\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} \frac{\cos x}{\pi - 2x}$$

By Substituition Method:-

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

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$$\lim_{h\to 0} \frac{\cos\left(h+\pi/2\right)}{\pi-2\left(h+\pi/2\right)}$$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi-2\left(\frac{2h+\pi}{2}\right)}$$

$$= \lim_{h\to 0} \frac{\cosh\cdot\cos\frac{\pi}{2} - \sinh\cdot\sin\frac{\pi}{2}}{-2h}$$

$$= \lim_{h\to 0} \frac{\cosh\cdot 0 - \sinh}{-2h}$$

$$= \frac{1}{2} \lim_{h\to 0} \frac{\sinh}{h}$$

$$= \frac{1}{2}$$ //

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b) $\lim_{x\to \pi/2^-} f(x) = \lim_{x\to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$$\lim_{x\to \pi/2^-} \frac{2\sin x\cdot\cos x}{\sqrt{2}\cdot\sin x}$$

$$= \lim_{x\to \pi/2^-} \frac{2\cos x}{\sqrt{2}} = \frac{2}{\sqrt{2}} \lim_{x\to \pi/2^-} \cos x$$

$\Rightarrow$ ∴ L.H.L ≠ R.H.L

∴ f is not continuous at $x=\pi/2$

Q 5)

i) $f(x) = \frac{x^2-9}{x-3}$ $0 < x \le 3$

$= x+3$ $3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $6 \le x < 9$

at $x=3$ & $x=6$

at $x=3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

∴ f ~~at x=3~~ is defined.

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3)$

$f(3) = 6$ //

251. SD

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} \cdot (x+3)$$

$$f(3) = 3+3 = 6$$

$\therefore$ L.H.L = R.H.L

$\therefore$ f is continuous at $x = 3$

for $x = 6$

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$$

$$\lim_{x \to 6^+} \frac{(x-3)(x+3)}{(x+3)}$$

$$\lim_{x \to 6^+} (x-3) = 6-3 = 3$$

$$\lim_{x \to 6^-} (x+3) = 3+6 = 9$$

$\therefore$ f is not continous.

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iii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$
$= K$ $x = 0$ } at $x = 0$

Sol^n:
$f(x) = (\sec^2 x)^{\cot^2 x}$

$\therefore \lim_{x \to 0} f(x) = f(0)$

$\therefore \lim_{x \to 0} = (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}$.

we know that,
$\lim_{x \to 0} (1 + px)^{1/px} = e$

$= e$
$\therefore K = e$

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$$= \lim_{h\to 0} \frac{\frac{\sqrt{3}(1-\tan\pi/3 \cdot \tan h) - (\tan\pi/3 + \tan h)}{1 - \tan\pi/3 \cdot \tan h}}{-3h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{\sqrt{3} - 3 \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3} \cdot \tan h}\right.}{-3h}$$

$$= \lim_{h\to 0} \frac{-4 \tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \cdot \lim_{h\to 0} \left(\frac{1}{1-\sqrt{3}\tan h}\right)$$

$$= \frac{4}{3} \frac{(1)}{1(1-\sqrt{3}(0))}$$

$$= \frac{4}{3}$$ "

7) $$\left.\begin{array}{ll} f(x) = \frac{1-\cos 3x}{x \cdot \tan x} & x \neq 0 \\ = \frac{9}{2} & x = 0 \end{array}\right\} \text{at } x = 0$$

$$f(x) = \frac{1-\cos 3x}{x \cdot \tan x}$$

$\therefore f$ has removable discontinuity at $x=0$

$$\therefore f(x) = \frac{1-\cos 3x}{x \cdot \tan x}$$

$$\lim_{x\to 0} = \frac{2\sin^2 3/2\, x}{x \tan x}$$

$$\lim_{x\to 0} \frac{\frac{2\sin^2 \frac{3x}{2}}{x^2}}{\frac{x \cdot \tan x}{x^2}} \times x^2$$

$$= \cancel{2 \lim_{x\to 0}} \frac{\left(\frac{3}{2}\right)^2}{1} = 2 \times \frac{9}{4} = \frac{9}{2}$$

7) $f(x) = \dfrac{(e^{3x}-1)\cdot \sin x^\circ}{x^2}$, $x \neq 0$
$= \dfrac{\pi}{6}$, $x = 0$ } at $x=0$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \cdot \lim_{x\to 0} \frac{\sin \frac{\pi x}{180}}{x}$$

$$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \quad \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3\log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

∴ f is continuous at $x=0$.

8) $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$, $x = 0$
is continuous at $x=0$

∴ Given,
f is continuous at $x=0$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x\to 0} \frac{(e^{x^2}-1)}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

$$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

∴ The value of $f(0)$ is $3/2$

351 Q.] $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$f(x)$ is continuous at $x = \pi/2$

$$\lim_{x\to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x\to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x\to \pi/2} \frac{1+\sin x}{1-\sin^2 x(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x\to \pi/2} \frac{1+\sin x}{(1+\sin x)(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x\to \pi/2} \frac{1}{2(\sqrt{2}+\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

02/12/19 $\quad f(\pi/2) = \dfrac{1}{4\sqrt{2}}$ //

2/12/19

PRACTICAL-02

TOPIC :- DERIVATIVE

Q1] Show that the following function obtained from R to R are differentiable

i) cot x :

$f(x) = \cot x$

$$Df(x) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x\to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

Put,

$x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$\therefore Df(x) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2}{\sin^2 a}$$

$Df(a) = -\operatorname{cosec}^2 a$

$\therefore f$ is differentiable $\forall a \in R$

ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$$

$$= \lim_{x \to a} \frac{\sin a - \sin x}{(x - a)\sin a \cdot \sin x}$$

Put,

$x - a = h$

$x = a + h$

as $x \to a,\ h \to 0$

25 $Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$

Formula: $-2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \cdot \sin\frac{-h}{2}}{\cos a \cdot \cos(a+h) \times -h/2} \times -\frac{1}{2}$

$= -\frac{1}{2} \times \frac{-2\sin a}{\cos a \times \cos a}$

$= \tan a \cdot \sec a$

$\therefore$ $f(x)$ is differentiable for $\forall a \in R$

R·H·D :-

$$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$= 2+2 = 4$

$\therefore Df(2^+) = 4$

$\therefore$ L·H·D = R·H·D

$\therefore$ f is differentiable at $x=2$

Q3) If $f(x) = 4x+7$, $x < 3$

$= x^2+3x+1$, $x \geq 3$ at $x=3$

Soln :-

R·H·D

$$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+9+1)}{x-3}$$

Q4] If $f(x) = 8x - 5$, $x \le 2$

OR

251 $= 3x^2 - 4x + 7$, $x > 2$ at $x = 2$

Find f is differentiable or not.

Soln:-

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD:-

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{(x-2)}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3 \times 2 + 2 = 8$$

$\therefore Df(2^+) = 8$

PRACTICAL-03

TOPIC:- Application of Derivative

Q1.] Find the intervals in which function is increasing or decreasing.

i.) $f(x) = x^3 - 5x - 11$
ii.) $f(x) = x^2 - 4x$
iii.) $f(x) = 2x^3 + x^2 - 20x + 4$
iv.) $f(x) = x^3 - 27x + 5$
v.) $f(x) = 69 - 24x - 9x^2 + 2x^3$

Q2.] Find the interval in which function is concave upwards.

i.) $y = 3x^2 - 2x^3$
ii.) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$
iii.) $y = x^3 - 27x + 5$
iv.) $y = 69 - 24x - 9x^2 + 2x^3$
v.) $y = 2x^3 + x^2 - 20x + 4$

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2.) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$

$$2x - 4 > 0$$
$$2(x-2) > 0$$
$$x > 2$$

$\cancel{x \in (-\infty, 2)}$ $x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$$2x - 4 < 0$$
$$x < 2$$

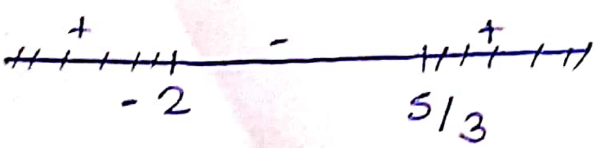
$$x \in (-\infty, 2)$$

3.) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$$\therefore 6x^2 + 2x - 20 > 0$$
$$3x^2 + x - 10 > 0$$
$$3x^2 + 6x - 5x - 10 > 0$$
$$(x+2)(3x-5) > 0$$



$x \in (-\infty, -2) \cup (5/3,$

5.) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because$ f is increasing iff $f'(x) > 0$

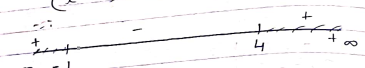
$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore x^2 - 3x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

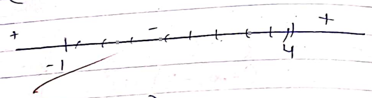
$(x-4)\ (x+1) > 0$



$x \in (-\infty, -1) \cup (4, \infty)$

and f(x) is decreasing iff $f'(x) < 0$

$\therefore (x-4)\ (x+1) < 0$



$x \in (-1, 4)$

Q2.)

i.) $y = 3x^2 - 2x^3$

$f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward iff $f''(x) > 0$

$(6 - 12x) > 0$

$12(6/12 - 1) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\therefore x \in (1/2, \infty)$

ii.) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$12x^2 - 36x + 24 > 0$

$(x-2)\ (x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

iii.) $y = x^3 \cancel{- 27x} + 5$

~~f(x)~~

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$6x > 0$

$x > 0$

$\therefore x \in (0, \infty)$

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5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$f''(x) > 0$

$12x + 2 > 0$

$12(x + 1/6) > 0$

$f''(x) \not> 0$

$\therefore$ There is no interval.

AK 16/12/19

Q1.]

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

consider,

$f(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(2) = 2 + \frac{96}{(2)^2}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has min value at $x = 2$

$\therefore f(2) = \cancel{2^2 + \frac{16}{2^2}}$

$= 8$

$f''(2) = 2 + \frac{96}{-24}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has min value at -2

$\therefore$ The function reaches min value at $x = 2, -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider,

$f'(x) = 0$

$-16x^2 + 15x^4 = 0$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$

$\therefore$ f has min value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)$

$= 6 - 5$

$= 1$

$f(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore$ f has max value at $x = -1$

$f(-1) = 3 - 5(-1)^3 + (3)(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore$ f has max value at $x = -1, 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

consider

$f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has max value at $x = 0$

$\therefore f(0) = (0)^3 - 3(0)^3 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 6 > 0$

$\therefore$ f has min value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has max value at $x = 0, 2$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

consider,

$f'(x) = 0$

$\therefore x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 18 > 0$

$\therefore$ f has min value at $x = 2$

$f(2) = \cancel{12(2)}\ 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 16 - 12 - 24 + 1$

$= -19$

f has max value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^3 - 12(-1) + 1$

$= 2 - 3 + 12 + 1$

$= 8$

$\therefore$ f has max value at $x = -1, 2$

2:

Q2):

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x): 3x^2 - 6x - 55$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= -55.9467$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$= 0.1712$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0011$

$$x_3 = 0.1712 + \frac{0.0011}{55.9393}$$

$= 0.1712$

$\therefore$ The root is 0.1712

25 58.

$$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$$
$$= 0.0102$$
$$f'(x_2) = 3(2.7071)^2 - 4$$
$$= 17.9851$$

$$x_3 = 2.7071 - \frac{0.0102}{17.9851}$$

$$= 2.7015$$

$$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$$
$$= -0.0901$$
$$f'(x_3) = 3(2.7015)^2 - 4$$
$$= 17.8943$$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$

$$= 2.7065$$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$
$$f'(x) = 3x^2 - 3.6x - 16$$

$$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$$
$$= 6.2$$
$$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$$
$$= -2.2$$
$$x_0 = 2$$

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PRATICAL-5

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INTEGRATION:-

Q1-) Solve the following integration:-

i.) $\int \frac{dx}{\sqrt{x^2+2x-3}}$ ii.) $\int [4(e^{3x})+1]\,dx$ iii.) $\int (2x^2 - 3\sin x + 5\sqrt{x})$

iv.) $\int \frac{x^3+3x+4}{\sqrt{x}}.dx$ v.) $\int t^7 . \sin(2t^4)\,dt$

vi.) $\int \sqrt{x}\,(x^2-1)\,dx$ vii.) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

viii.) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$ ix.) $\int e^{\cos^2 x} . \sin 2x .dx$

x.) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

$= \ln(|x+1 + \sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|) + C$,

2.] $\int 4e^{3x} + 1\, dx$

$I = \int 4e^{3x} \cdot dx + \int 1\, dx$

$= 4\int e^{3x} \cdot dx + \int 1\, dx$

$= \frac{4e^{3x}}{3} + x + C$

3.] $\int (2x^2 - 3\sin x + 5\sqrt{x})dx$

$= \int 2x^2 \cdot dx - \int 3\sin x\, dx\ \int 5\sqrt{x}\, dx$

$= 2\int x^2 \cdot dx - 3\int \sin x \cdot dx + 5\int \sqrt{x}\, dx$

$= 2\int x^2 \cdot dx - 3\int \sin x \cdot dx + 5\int x^{1/2}\, dx$

$= \frac{2x^3}{3} - 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10\sqrt{x}}{3} + 3\cos x$

4.] $\int \frac{x^3+3x+4}{\sqrt{x}}\, dx$

$I = \int \frac{x^3+3x+4}{\frac{1}{x^2}}\, dx$

Splitting the denominator:-

$\int \frac{x^3}{\frac{1}{x^2}} + \frac{3x}{\frac{1}{x^2}} + \frac{4}{\frac{1}{x^2}}\, dx$

Simplifying the expression.

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{\frac{1}{x^2}}\, dx$

$= \int x^{5/2} \cdot dx + \int 3x^{1/2}\, dx + 4\int \sqrt{x^7}\, dx$

$= \frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

5. $\int t^7 (\sin 2t^4) \cdot dx$

$I = \int t^7 \cdot \sin(2t^4)\, dx$

Let, $t^4 = x$

$4t^3\, dt = \cancel{dx}$

$= \frac{1}{4}\int 4t^3 \cdot t^4 \sin(2t^4)$

$= \frac{1}{4}\int x \cdot \sin(2x)\, dx$

$= -\frac{1}{8}x\cos 2x + \frac{1}{16}\sin x + c$

$= -\frac{1}{8}t^4\cos(2t^4) + \frac{1}{16}\sin(2t^4) + c$

6.) $\int \sqrt{x}\,(x^2-1)\,dx$

$I = \int \sqrt{x}\,(x^2-1)\,dx$

$= \int (x^{5/2} - x^{1/2})\,dx$

$= \frac{2}{7}x^{7/2} - \frac{2}{3}x^{3/2} + C$

7.) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

Let, $\frac{1}{x^2} = t$

$x^{-2} = t$

$-\frac{2}{x^3}\,dx = dt$

$I = -\frac{1}{2}\int -\frac{2}{x^3}\sin\left(\frac{1}{x^2}\right)\cdot dx$

$= -\frac{1}{2}\int \sin t$

$= -\frac{1}{2}(-\cos t) + C$

$= \frac{1}{2}\cos t + C$

Resubstituting the value $t = 1/x^2$

$I = \frac{1}{2}\cos\left(\frac{1}{x^2}\right) + C$

8.) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}$

Let, $\sin x = t$

$\therefore \cos x \cdot dx = dt$

$I = \int \frac{dt}{\sqrt[3]{t^2}}$

$= \int \frac{dt}{t^{2/3}}$

$= \int t^{-2/3}\cdot dt$

$= 3t^{1/3} + C$

$= 3(\sin x)^{1/3} + C$

$= 3\sqrt[3]{\sin x}$ ~~+C~~

2a

9) $\int e \cos^2 x - \sin^2 x \, dx$

$I = \int e \cos^2 x \cdot \sin^2 x \, dx$

$I \int e^{\cos^2 x} \cdot \sin^2 x \, dx$

Let,

$\cos^2 x = t$

$-2 \cos x \cdot \sin x \cdot dx = dt$

$-\sin 2x \, dx = dt$

$I = -\int e^t \cdot dt$

$= e^t + C$

$I = -e^{\cos 2x} + C$

10) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$

$I = \int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} dx$

Let, $x^3 - 3x^2 + 1 = t$

$3(x^2 - 2x) dx = dt$

$(x^2 - 2x) dx = dt/3$

06/01/2020

$I = \int \frac{1}{t} dt$

$= \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \log t + C = \frac{1}{3} \log(x^3 - 3x^2 \ldots$

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1) $x = t - \sin t$ $\quad t \in [0, 2\pi]$

$y = 1 - \cos t$

for t belongs to [0, 2π]

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \cdot dt$$

$x = t - \sin t$

$\frac{dx}{dt} = \cancel{t - \cos} \; 1 - \cos t$

$\frac{dy}{dx} = 0 - (-\sin t)$

$\frac{dy}{dx} = \sin t$

$$L = \int_0^{2\pi} \sqrt{1 - 2\cos t + 1} \; dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t} \; dt$$

$$= \int_0^{2\pi} 2\sin\frac{t}{2} \; dt$$

$$= \left(-4\cos(t/2)\right)_0^{2\pi} = (-4\cos\pi) - (-4\cos 0)$$

$$= 4 + 4 = 8$$

2) $y = \sqrt{4 - x^2}$ $\quad x \in [-2, 2]$

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$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)} \, dt$$

$$= 2\int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4 - x^2}}\right)^2} \, dx$$

$\frac{dy}{dx}$

$$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}} \, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4 - x^2}} \, dx$$

$$= 4\left(\sin^{-1}(x/2)\right)_0^2$$

$\frac{dy}{dx} = 2\pi$

3) $y = x^{3/2}$ $\quad$ in [0, 4]

$f'(x) = \frac{9}{4}x$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} \, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4}x} \, dx$$

Put, $u = 1 + \frac{9}{4}x$, $du = \frac{9}{4}dx$

$$L = \int_1^{1+\frac{9}{4}x} \frac{4}{9}\sqrt{u} \, du = \left[\frac{4}{9} \cdot \frac{2}{3}(u^{3/2})\right]^{1+\frac{9}{4}x}$$

$$= \frac{8}{27}\left[\left(1 + \frac{9x}{4}\right) - 1\right]$$

4) $x = 3\sin t$, $y = 3\cos t$

$\frac{dx}{dt} = 3\cos t$

$\frac{dy}{dt} = -3\sin t$

$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} 3\sqrt{1}\, dt$

$= 3\int_0^{2\pi} t\, dt$

$= 3[x]_0^{2\pi}$

$= 3[2\pi - 0]$

$= 6\pi$ units

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in [1,2]$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2}\int_1^2 \left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]$

$= \frac{1}{2}\left[\frac{8}{3} - \cancel{\frac{1}{2}} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= \frac{17}{12}$ units

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Q2.) $\int_0^2 e^{x^2}\,dx$ with $n=4$

i.) $\int_0^2 e^{x^2}\,dx = 16.4526$

In each case the width of the sub interval be $\Delta x = \frac{2-0}{4} = \frac{1}{e}$ and so the sub intervals will be $[0, 0.5]$ $[0.5, 1]$ $[1, 1.5]$, $[1.5, 2]$

$$\int_0^2 e^{x^2}\,dx = \frac{1/2}{3}\left(\frac{1}{0} + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$= 17.3536$

ii.) $\int_0^4 x^2\,dx$, $n=4$

$\Delta x = \frac{4-0}{4} = 1$

$$= \int_a^b f(x)\,dx = \frac{\Delta x}{3}\left[y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + (4)^2\right]$$

$= \frac{1}{3}\{0^2 + 4(1)^2 + (2)^2 + 4(3)^2 + (4)^2]$

$= \frac{64}{3}$

PRACTICAL- 7

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Q1.] Solve the following differential equations.

1) $x \cdot \frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $\qquad Q(x) = \frac{e^x}{x}$

$IF = e^{\int 1/x \, dx} = x$

$y(IF) = \int Q(x)\,(IF)\,dx + C$

$= \int \frac{e^x}{x} \cdot x \, dx + C$

$xy = e^x + C$

2) $e^x \frac{dy}{dx} + 2e^x y = 1$

Soln:

$\frac{dy}{dx} + \frac{2e^x}{e^x} = \frac{1}{e^x}$

$\therefore \frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\therefore \frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $\qquad Q(x) = e^{-x}$

$\therefore \int P(x)\,dx$

$IF = e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \cancel{e^x + c}\ y \cdot e^{2x} + C$

4.) $x \cdot \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Sol^n:

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$P(x) = 3/x$, $Q(x) = \frac{\sin x}{x^3}$

$IF = e^{\int P(x) \cdot dx}$

$= e^{\int 3/x \, dx}$

$= e^{3\int \frac{1}{x} dx}$

$= e^{3\log x}$

$= x^3$

$y(IF) = \int Q(x) \cdot (IF)\, dx + C$

$= \int \frac{\sin x}{\cancel{x^3}} \cdot \cancel{x^3} \cdot dx + C$

$= -\cos x + C$

$=$

5.) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

Sol^n:

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

8) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put, $2x+3y = u$

$2 + 3\frac{dy}{dx} = \frac{du}{dx}$

$\frac{dy}{dx} - \frac{1}{3}\left(\frac{du}{dx} - 2\right)$

$= \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right).$

$\therefore \frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$= \int \frac{(v+2)}{v+1}\, dv$

$= 3dx$

$= \cancel{\int \frac{v+1}{v}\, dx} + \int \frac{1}{v+1}\, dv$

$= \int 3dx$

$v \log v = 3x + c$

$2x+3y+\log|2x+3y+1| = 3x+c$

$3y = x - \log|2x+3y+1| + c$

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③ $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h \cdot 0.2$, find $y(1)$

$x_0 = 0$, $y(0) = 1$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
			0.4472	1.0894
1	0.2	1	0.6059	1.2105
2	0.4	1.0894	0.7040	1.3513
3	0.6	1.2105	0.7696	1.5051
4	0.8	1.3513		
5	1	1.5051		

Q) $\frac{dy}{dx} = 3x^2 + 1$; $y(1) = 2$ find $y(2)$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.25	7.875
2	2	7.875		

$y(2) = 7.875$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2yz}$

$$\lim_{(x,y,z)\to(1,1,1)} = \frac{(x-yz)(x+yz)}{x^2(x-yz)}$$

$$= \frac{x+yz}{x^2}$$

$\therefore$ By applying limit

$$= \frac{1+(1)1}{(1)^2}$$

$$= 2$$

Q2) $f(x,y) = xye^{x^2+y^2}$ ~~$+2y$~~

$$f(x) = \frac{\delta}{\delta x} f(x,y)$$

$$= \frac{\delta}{\delta x}(xy\, e^{x^2+y^2}\cancel{+2y})$$

$$= y\,e^{x^2+y^2}(2x)$$

$$f_y = x\cdot e^{x^2+y^2}(2y)$$

Q3.) $f(x,y) = \frac{2x}{1+y^2}$

$$f_x = \frac{\delta}{\delta x}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{(1+y^2)\cdot\frac{d}{dx}2x - 2x\frac{d}{dx}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2-0}{(1+y^2)^2}$$

$$= \frac{2}{1+y^2}$$

At $(0,0) = 2$

$$f_y = \frac{\delta}{\delta y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0) = \frac{-4(0)(0)}{(1+0)^2}$

$= 0$

4) $f(x,y) = \frac{y^2-xy}{x^2}$

$$f(x) = \frac{x^2\cdot\frac{d}{dx}(y^2-xy) - (y^2-xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2-xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2-xy)}{x^4}$$

$$f(y) = \frac{2y-x}{x^2}$$

$$f_{xx} = \frac{d}{dx}\left(\frac{-x^2y-2x(y^2-xy)}{x^4}\right)$$

$$= \frac{x^4(-2xy-2y^2+4xy) - 4x^3(-x^2y-2xy^2+2x^2y)\cdot\frac{d}{dx}(x^4)}{x^6}$$

$$f(xy) = \frac{d}{dy}\left(\frac{-x^2-4xy+2x^2}{x^4}\right)$$

$$f(yx) = \frac{-x^2-4xy+2x^2}{x^4}$$

$$f_{yy} = \frac{d}{dy}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$$

$\therefore f(xy) = f(yx)$

iii) $f(x,y) = x^3 + 3x^2 y^2 - \log(x^2+1)$

$$f_x = \frac{d}{dx}\left(x^3 + 3x^2 y^2 - \log(x^2+1)\right)$$

$$f_x = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f_y = 0 + 6x^2 y - 0$$
$$= 6x^2 y$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \rightarrow (1)$$

$$f_{yy} = \frac{d}{dy}(6x^2 y)$$

$$= 6x^2$$

2. $f(x,y) = \cancel{1+y}$ at $(\pi/2, 0)$

$= 1 - x + y \sin x$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 = 1 - \pi/2$

$f_x = 0 - 1 + y \cos x$

f_x at $(\pi/2, 0) = -1 + 0$

$= -1$

$f_y = 0 - 0 + \sin x$

$f(y)$ at $(\frac{\pi}{2}, 0) = 1$

$L(x,y) = f(a,b) + f(x)(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1 - \frac{\pi}{2} - x + \pi/2 + y$

$= 1 - x + y$

PRACTICAL - 10

TOPIC :- Directional Derivative, Gradient vector, Maxima, Minima, Tangent and Normal eqn.

1) Find the directional derivative of the following function at given points and in the direction of given vector.

i) $f(x,y) = x + 2y + 3$ at $(1, -1)$ $u = 3i - j$

Here,

$u = 3i - j$ is not a unit vector

$|u| = \sqrt{(3)^2 + (1)^2} = \sqrt{10}$

unit vector along $u = \frac{u}{|u|} = \frac{1}{\sqrt{10}}(3, -1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1, -1) = 1 + 2(-1) - 3 = -4$

$f(a + hu) = f(1, -1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{1}{\sqrt{10}}\right)$

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$$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$$

$$= 1+\frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$$

$$f(a+hu) = -4+\frac{h}{\sqrt{10}}$$

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{-4+\frac{h}{\sqrt{10}}+4}{h}$$

$$\therefore D_u f(a) = \frac{1}{\sqrt{10}}$$

(ii) $2x+3y$ $\quad a=(1,2)$; $u=(3i+4j)$

Here, $u = 3i+4j$ is not a unit vector.

$\therefore$ Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

35.

iii) $f(x,y,z) = xyz - e^{x+y+z}$, $a = (1,-1,0)$

$f(x) = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{1-1+0}, 1(0) - e^0, (1)(-1) - e^0)$

$= (-1, -1, -2)$

Q3) Find the equation of tangent and normal to each of curves using curves at given points

i) $x^2 \cos y + e^{xy} = 2$ at $(1, 0)$

$f(x) = \cos y \cdot 2x + e^{xy} \cdot y$

$f(y) = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0)$ $x_0 = 1$, $y_0 = 0$

Eqn of tangent

$f_x(x - x_0) + f(y)(y - y_0) = 0$

$f(x_0, y_0) = \cos 0 \cdot 2(1) + e^0(0)$

$= 1(2) + 0$

$= 2$

Q4) Find the eq^n of tangent and normal using the following surface:

i) $x^2 - 2yx + 3y + xz = 7$ at $(2,1,0)$

$f(x) = 2x - 0 + 0 + z$
$f(x) = 2x + z$

$f(y) = 0 - 2x + 3 + 0$
$= 2x + 3$

$f(z) = -2y + x$

$(x_0, y_0, z_0) = (2,1,0)$

$f_x(x_0, y_0, z_0) = 4$

$f_y(x_0, y_0, z_0) = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

Eq^n of tangent:

$F(x)(x - x_0) + f_y(y - y_0) + f_z(z - z_0) = 0$
$= 4x - 8 + 3y - 8 = 0$
$\therefore 4x + 3y - 11 = 0$

Q5:) Find local maxima and minima for the following

i:) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6$
$= 6x - 3y + 6$

$f_y = 2y - 3x - 4$

$f_x = 0$
$6x - 3y + 6 = 0$
$2y - 3x = -2 \rightarrow (1)$

$f_y = 0$
$2y - 3x = 4 \rightarrow (2)$

On solving eqn (1) & (2) simaltaneously, critical points are $(0,2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$

Here, $r > 0$

$= rt - s^2$
$= 6(2) - (-3)^2$
$= 12 - 9$
$= 3 > 0$

$\therefore f$ has maximum at $(0,2)$.

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$= 3(0) + (2)^2 - 3(0)(2) + 6(0) - 4(2)$
$= 0 + 4 - 8$
$= -4$

ii:) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f(x) = 8x^3 + 6xy$
$f(y) = 3x^2 - 2y$

$f_x = 0$
$4x^2 + 3y = 0 \rightarrow (1)$

$f_y = 0$
$3x^2 - 2y = 0 \rightarrow (2)$

On solving eqn (1) & (2) simaltaneously we get,
$x = 0, y = 0$
critical points is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 - 2 = -2$
$s = f_{xy} = 6x = 0 - 6x = 6(0) = 0$

$f(x,y) = at (0,0) = 0,$

$rt - s^2 = 0 - 0 = 0$
$r = 0$ & $rt - s^2 = 0$