NUMBERS

1. **Some Basic Formulae:**
   i. $(a + b)(a - b) = (a^2 - b^2)$
   ii. $(a + b)^2 = (a^2 + b^2 + 2ab)$
   iii. $(a - b)^2 = (a^2 + b^2 - 2ab)$
   iv. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
   v. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
   vi. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
   vii. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
   viii. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

LCM and HCF

1. **Factors and Multiples:**

   If number *a* divided another number *b* exactly, we say that *a* is a **factor** of *b*.

   In this case, *b* is called a **multiple** of *a*.

2. **Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.):**

   The H.C.F. of two or more than two numbers is the greatest number that divided each of them exactly.

   There are two methods of finding the H.C.F. of a given set of numbers:

   I. **Factorization Method:** Express the each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.
   II. **Division Method:** Suppose we have to find the H.C.F. of two given numbers, divide the larger by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is required H.C.F.

   Finding the H.C.F. of more than two numbers: Suppose we have to find the H.C.F. of three numbers, then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given number.

   Similarly, the H.C.F. of more than three numbers may be obtained.

3. **Least Common Multiple (L.C.M.):**

   The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

   There are two methods of finding the L.C.M. of a given set of numbers:

   I. **Factorization Method:** Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.
   II. **Division Method (short-cut):** Arrange the given numbers in a rwo in any order. Divide by a number which divided exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.
4. **Product of two numbers = Product of their H.C.F. and L.C.M.**
5. **Co-primes:** Two numbers are said to be co-primes if their H.C.F. is 1.
6. **H.C.F. and L.C.M. of Fractions:**

   1. $\text{H.C.F.} = \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$
   2. $\text{L.C.M.} = \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

8. **H.C.F. and L.C.M. of Decimal Fractions:**

   In a given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

9. **Comparison of Fractions:**

   Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

AVERAGE

1. **Average:**

   $$\text{Average} = \left(\frac{\text{Sum of observations}}{\text{Number of observations}}\right)$$

2. **Average Speed:**

   Suppose a man covers a certain distance at *x* kmph and an equal distance at *y*kmph.

   Then, the average speed druing the whole journey is $\left(\frac{2xy}{x + y}\right)$ kmph.

Problems on Ages and Calenders

1. **Odd Days:**

   We are supposed to find the day of the week on a given date.

   For this, we use the concept of 'odd days'.

   In a given period, the number of days more than the complete weeks are called**odd days**.

2. **Leap Year:**

   (i). Every year divisible by 4 is a leap year, if it is not a century.

   (ii). Every 4^{th} century is a leap year and no other century is a leap year.

   Note: **A leap year has 366 days.**

   Examples:

   i. Each of the years 1948, 2004, 1676 etc. is a leap year.
   ii. Each of the years 400, 800, 1200, 1600, 2000 etc. is a leap year.
   iii. None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

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3. **Ordinary Year:**

   The year which is not a leap year is called an **ordinary years**. An ordinary year has 365 days.

4. **Counting of Odd Days:**
   1. 1 ordinary year = 365 days = (52 weeks + 1 day.)

      ∴ 1 ordinary year has 1 odd day.

   2. 1 leap year = 366 days = (52 weeks + 2 days)

      ∴ 1 leap year has 2 odd days.

   3. 100 years = 76 ordinary years + 24 leap years

      = (76 x 1 + 24 x 2) odd days = 124 odd days.

      = (17 weeks + days) ≡ 5 odd days.

      ∴ Number of odd days in 100 years = 5.

      Number of odd days in 200 years = (5 x 2) ≡ 3 odd days.

      Number of odd days in 300 years = (5 x 3) ≡ 1 odd day.

      Number of odd days in 400 years = (5 x 4 + 1) ≡ 0 odd day.

      Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd days.

5. **Day of the Week Related to Odd Days:**

No. of days:	0	1	2	3	4	5	6
Day:	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

PERCENTAGE

1. **Concept of Percentage:**

   By a certain **percent**, we mean that many hundredths.

   Thus, x percent means x hundredths, written as x%.

   To express x% as a fraction: We have, $x\% = \frac{x}{100}$.

   Thus, $20\% = \frac{20}{100} = \frac{1}{5}$.

   To express $\frac{a}{b}$ as a percent: We have, $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$.

   Thus, $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%$.

2. **Percentage Increase/Decrease:**

   If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is:

$$\left[\frac{R}{(100 + R)} \times 100\right]\%$$

   If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is:

$$\left[\frac{R}{(100 - R)} \times 100\right]\%$$

3. **Results on Population:**

   Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

   1. Population after n years $= P\left(1 + \frac{R}{100}\right)^n$

   2. Population n years ago $= \frac{P}{\left(1 + \frac{R}{\ }\right)^N}$

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100

4. **Results on Depreciation:**

   Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:

   1. Value of the machine after *n* years = $P\left(1 - \frac{R}{100}\right)^n$
   2. Value of the machine *n* years ago = $\frac{P}{\left(1 - \frac{R}{100}\right)^n}$
   3. If A is R% more than B, then B is less than A by $\left[\frac{R}{(100 + R)} \times 100\right]\%$.
   4. If A is R% less than B, then B is more than A by $\left[\frac{R}{(100 - R)} \times 100\right]\%$.

Profit and Loss

IMPORTANT FACTS

Cost Price:

The price, at which an article is purchased, is called its **cost price**, abbreviated as **C.P.**

Selling Price:

The price, at which an article is sold, is called its **selling prices**, abbreviated as **S.P.**

Profit or Gain:

If S.P. is greater than C.P., the seller is said to have a **profit** or **gain**.

Loss:

If S.P. is less than C.P., the seller is said to have incurred a **loss**.

IMPORTANT FORMULAE

1. Gain = (S.P.) - (C.P.)
2. Loss = (C.P.) - (S.P.)

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3. Loss or gain is always reckoned on C.P.
4. Gain Percentage: (Gain %)

5. Gain % = $\left(\dfrac{\text{Gain x 100}}{\text{C.P.}}\right)$

6. Loss Percentage: (Loss %)

$$\text{Loss \%} = \left(\frac{\text{Loss x 100}}{\text{C.P.}}\right)$$

7. Selling Price: (S.P.)

$$\text{SP} = \left[\frac{(100 + \text{Gain \%})}{100} \text{ x C.P}\right]$$

8. Selling Price: (S.P.)

$$\text{SP} = \left[\frac{(100 - \text{Loss \%})}{100} \text{ x C.P.}\right]$$

9. Cost Price: (C.P.)

$$\text{C.P.} = \left[\frac{100}{(100 + \text{Gain \%})} \text{ x S.P.}\right]$$

10. Cost Price: (C.P.)

$$\text{C.P.} = \left[\frac{100}{(100 - \text{Loss \%})} \text{ x S.P.}\right]$$

11. If an article is sold at a gain of say 35%, then S.P. = 135% of C.P.
12. If an article is sold at a loss of say, 35% then S.P. = 65% of C.P.
13. When a person sells two similar items, one at a gain of say *x*%, and the other at a loss of *x*%, then the seller always incurs a loss given by:

$$\text{Loss \%} = \left(\frac{\text{Common Loss and Gain \%}}{10}\right)^2 = \left(\frac{x}{10}\right)^2.$$

14. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain \%} = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100\right]\%.$$

Ratio and Proportion

1. **Ratio:**

   The ratio of two quantities *a* and *b* in the same units, is the fraction $\frac{a}{b}$ and we write it as *a* : *b*.

   In the ratio *a* : *b*, we call *a* as the first term or **antecedent** and b, the second term or **consequent**.

   Eg. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

   Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

   Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

2. **Proportion:**

   The equality of two ratios is called proportion.

   If *a* : *b* = *c* : *d*, we write *a* : *b* **::** *c* : *d* and we say that *a, b, c, d* are in proportion.

   Here *a* and *d* are called **extremes**, while *b* and *c* are called **mean terms**.

   Product of means = Product of extremes.

   Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.

3. **Fourth Proportional:**

If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

Third Proportional:

$a : b = c : d$, then c is called the third proportion to a and b.

Mean Proportional:

Mean proportional between a and b is ab.

4. **Comparison of Ratios:**

   We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.

5. **Compounded Ratio:**
6. The compounded ratio of the ratios: $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.
7. **Duplicate Ratios:**

   Duplicate ratio of $(a : b)$ is $(a^2 : b^2)$.

   Sub-duplicate ratio of $(a : b)$ is $(a : b)$.

   Triplicate ratio of $(a : b)$ is $(a^3 : b^3)$.

   Sub-triplicate ratio of $(a : b)$ is $(a^{1/3} : b^{1/3})$.

   If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a + b}{a - b} = \frac{c + d}{c - d}$. [componendo and dividendo]

8. **Variations:**

   We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

   We say that x is inversely proportional to y, if $xy = k$ for some constant k *and* we write, $x \propto \frac{1}{y}$.

Time and Work

1. **Work from Days:**

   If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.

2. **Days from Work:**

   If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.

3. **Ratio:**

   If A is thrice as good a workman as B, then:

   Ratio of work done by A and B = 3 : 1.

   Ratio of times taken by A and B to finish a work = 1 : 3.

Pipes and Cistern

1. **Inlet:**

   A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

   Outlet:

   A pipe connected with a tank or cistern or reservoir, emptying it, is known as an outlet.

2. If a pipe can fill a tank in x hours, then:

   part filled in 1 hour = $\frac{1}{x}$.

3. If a pipe can empty a tank in y hours, then:

part emptied in 1 hour = $\frac{1}{y}$.

4. If a pipe can fill a tank in x hours and another pipe can empty the full tank in $y$hours (where $y > x$), then on opening both the pipes, then

   the net part filled in 1 hour = $\left(\frac{1}{x} - \frac{1}{y}\right)$.

5. If a pipe can fill a tank in x hours and another pipe can empty the full tank in $y$hours (where $y > x$), then on opening both the pipes, then

   the net part emptied in 1 hour = $\left(\frac{1}{y} - \frac{1}{x}\right)$.

Time and Distance

1. **Speed, Time and Distance:**

   Speed = $\left(\frac{\text{Distance}}{\text{Time}}\right)$, Time = $\left(\frac{\text{Distance}}{\text{Speed}}\right)$, Distance = (Speed x Time).

2. **km/hr to m/sec conversion:**

   x km/hr = $\left(x \times \frac{5}{18}\right)$ m/sec.

3. **m/sec to km/hr conversion:**

   x m/sec = $\left(x \times \frac{18}{5}\right)$ km/hr.

4. If the ratio of the speeds of A and B is $a : b$, then the ratio of the

   the times taken by then to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.

5. Suppose a man covers a certain distance at x km/hr and an equal distance at $y$km/hr. Then,

   the average speed during the whole journey is $\left(\frac{2xy}{\quad}\right)$ km/hr.

x + *y*

Boats and Streams

1. **Downstream/Upstream:**

   In water, the direction along the stream is called **downstream**. And, the direction against the stream is called **upstream**.

2. If the speed of a boat in still water is u km/hr and the speed of the stream is $v$km/hr, then:

   Speed downstream = $(u + v)$ km/hr.

   Speed upstream = $(u - v)$ km/hr.

3. If the speed downstream is a km/hr and the speed upstream is b km/hr, then:

   Speed in still water = $\frac{1}{2}(a + b)$ km/hr.

   Rate of stream = $\frac{1}{2}(a - b)$ km/hr.

Problems on Trains

1. **km/hr to m/s conversion:**

   a km/hr = $\left(a \times \frac{5}{18}\right)$m/s.

2. **m/s to km/hr conversion:**

   a m/s = $\left(a \times \frac{18}{5}\right)$ km/hr.

3. **Formulas for finding Speed, Time and Distance**
4. Time taken by a train of length l metres to pass a pole or standing man or a signal post is equal to the time taken by the train to cover l metres.
5. Time taken by a train of length l metres to pass a stationery object of length $b$metres is the time taken by the train to cover $(l + b)$ metres.
6. Suppose two trains or two objects bodies are moving in the same direction at um/s and v m/s, where $u > v$, then their relative speed is = $(u - v)$ m/s.
7. Suppose two trains or two objects bodies are moving in opposite directions at um/s and v m/s, then their relative speed is = $(u + v)$ m/s.

8. If two trains of length *a* metres and *b* metres are moving in opposite directions at*u* m/s and *v* m/s, then:

   The time taken by the trains to cross each other = $\frac{(a + b)}{(u + v)}$ sec.

9. If two trains of length *a* metres and *b* metres are moving in the same direction at*u* m/s and *v* m/s, then:

   The time taken by the faster train to cross the slower train = $\frac{(a + b)}{(u - v)}$ sec.

10. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take *a* and *b* sec in reaching B and A respectively, then:

    (A's speed) : (B's speed) = (b : a)

Simple Interest

1. **Principal:**

   The money borrowed or lent out for a certain period is called the **principal** or the**sum**.

2. **Interest:**

   Extra money paid for using other's money is called **interest**.

3. Simple Interest (S.I.):

   If the interest on a sum borrowed for certain period is reckoned uniformly, then it is called **simple interest**.

   Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then

   (i). Simple Intereest = $\left(\frac{P \times R \times T}{100}\right)$

   (ii). P = $\left(\frac{100 \times S.I.}{R \times T}\right)$; R = $\left(\frac{100 \times S.I.}{P \times T}\right)$ and T = $\left(\frac{100 \times S.I.}{P \times R}\right)$.

Compound Interest

1. Let Principal = P, Rate = R% per annum, Time = *n* years.
2. **When interest is compound Annually:**

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n$$

3. **When interest is compounded Half-yearly:**

$$\text{Amount} = P\left[1 + \frac{(R/2)}{100}\right]^{2n}$$

4. **When interest is compounded Quarterly:**

$$\text{Amount} = P\left[1 + \frac{(R/4)}{100}\right]^{4n}$$

5. **When interest is compounded Annually but time is in fraction, say 3 $\frac{2}{5}$ years.**

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100}\right)$$

6. **When Rates are different for different years**, say R_1%, R_2%, R_3% for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right).$$

7. **Present worth of Rs. *x* due *n* years hence is given by:**

$$\text{Present Worth} = \frac{x}{\left(1 + \frac{R}{100}\right)}.$$