Assignment 3

Numbers Made Dumber — Abhishek Shree

Project #13 — Roll: 200028

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1. Find the solution to the linear congruence

$$x \equiv 3 \mod 5$$

$$x \equiv 4 \mod 11$$

Sol.

$$N = 5 * 11 = 55 \implies N_1 = 11, N_2 = 5$$

Therefore by Chinese Remainder Theorem, a particular solution would be,

$$x_1 = 3 * 11 * 1 + 4 * 5 * 9 = 213$$

General solution would be,

$$x \equiv 213 \mod 55 \text{ which would be } \boxed{x \equiv 48 \mod 55}$$

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2. For a positive integer p, define the positive integer n to be p-safe if n differs in absolute-value by more than 2 from all multiples of p. For example, the set of 10-safe numbers is $3, 4, 5, 6, 7, 13, 14, 15, 16, 17, 23...$ Find the number of positive integers less than or equal to 10,000 which are simultaneously 7-safe, 11-safe, and 13-safe.

Sol. x is 7-safe if, $x \equiv 3 \mod 7$ or $x \equiv 4 \mod 7$ i.e. 2 residues
Similarly, 11-safe numbers will have 6 residues (3, 4, 5, 6, 7, 8) and 13-safe numbers will have 8 residues (3, 4, 5, 6, 7, 8, 9, 10).
By Chinese Remainder Theorem, we will have a total of 96 residues (2*6*8) $\mod 1001$
Total numbers ≤ 10010 satisfying this would be 960.
Removing 10006 and 10007 (values > 10000), we will have a total of **958** numbers.

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3. Consider a number line consisting of all positive integers greater than 7. A hole punch traverses the number line, starting from 7 and working its way up. It checks each positive integer n and punches it if and only if $\binom{n}{7}$ is divisible by 12. As the hole punch checks more and more numbers, the fraction of checked numbers that are punched approaches a limiting number ρ. If ρ can be written in the form $\frac{m}{n}$, where m and n are positive integers, find $m + n$.

Sol.

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4. Call a lattice point "visible" if the greatest common divisor of its coordinates is 1. Prove that there exists a 100×100 square on the board none of whose points are visible.

Sol.
