Parallel Sorting

Merge Sort & Bubble Sort

CS370 Parallel Processing

Spring 2014

Sorting

- Arrange elements of a list into certain order
- Make data become easier to access
- Speed up other operations such as searching and merging
- Many sorting algorithms with different time and space complexities

Parallel Sorting

Design methodology

- Based on an existing sequential sort algorithm
  - Try to utilize all resources available
  - Possible to turn a poor sequential algorithm into a reasonable parallel algorithm (bubble sort and parallel bubble sort)
- Completely new approach
  - New algorithm from scratch
  - Harder to develop
  - Sometimes yield better solution

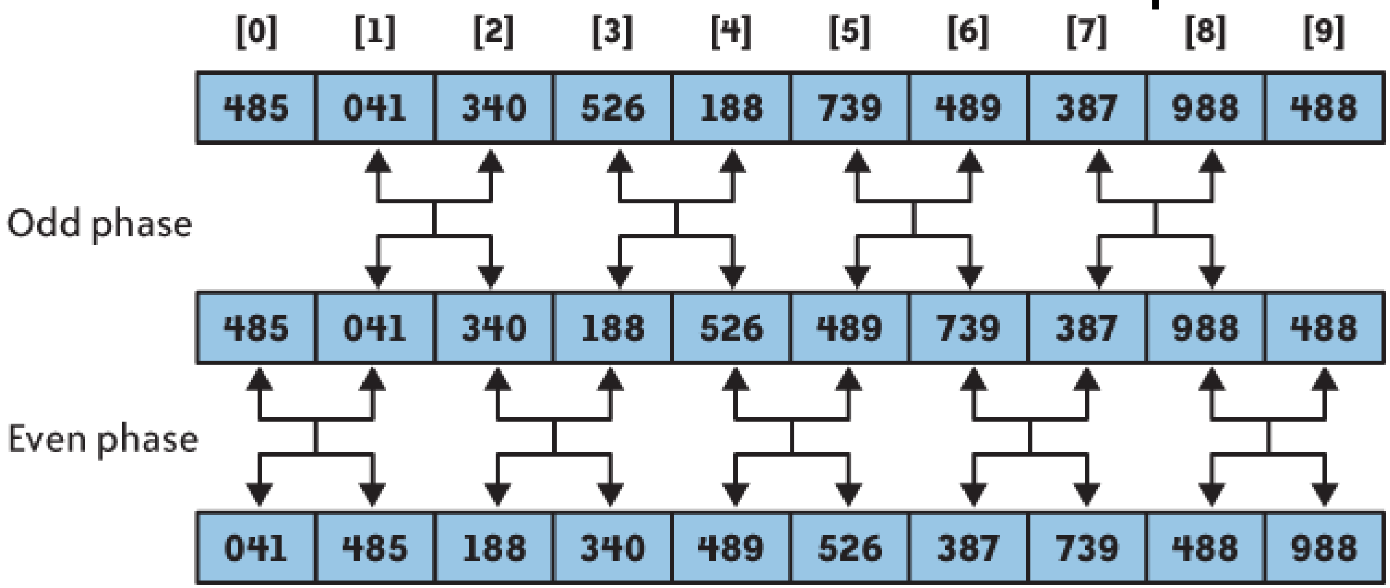
Bubble Sort

6 5 3 1 8 7 2 4

- One of the straight-forward sorting methods
  - Cycles through the list
  - Compares consecutive elements and swaps them if necessary
  - Stops when no more out of order pair
- Slow & inefficient
- Average performance is $O(n^2)$

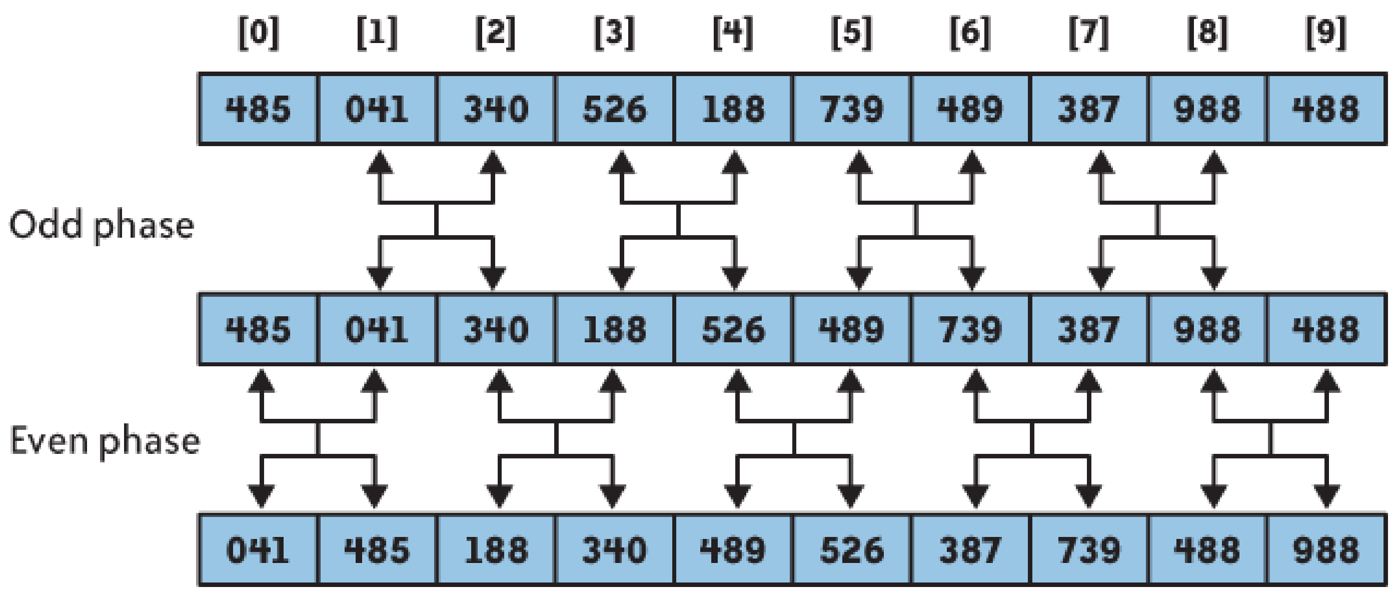
Parallel Bubble Sort
Odd-even Transposition Sort

- Compare all pairs in the list in parallel
- Alternate between odd and even phases



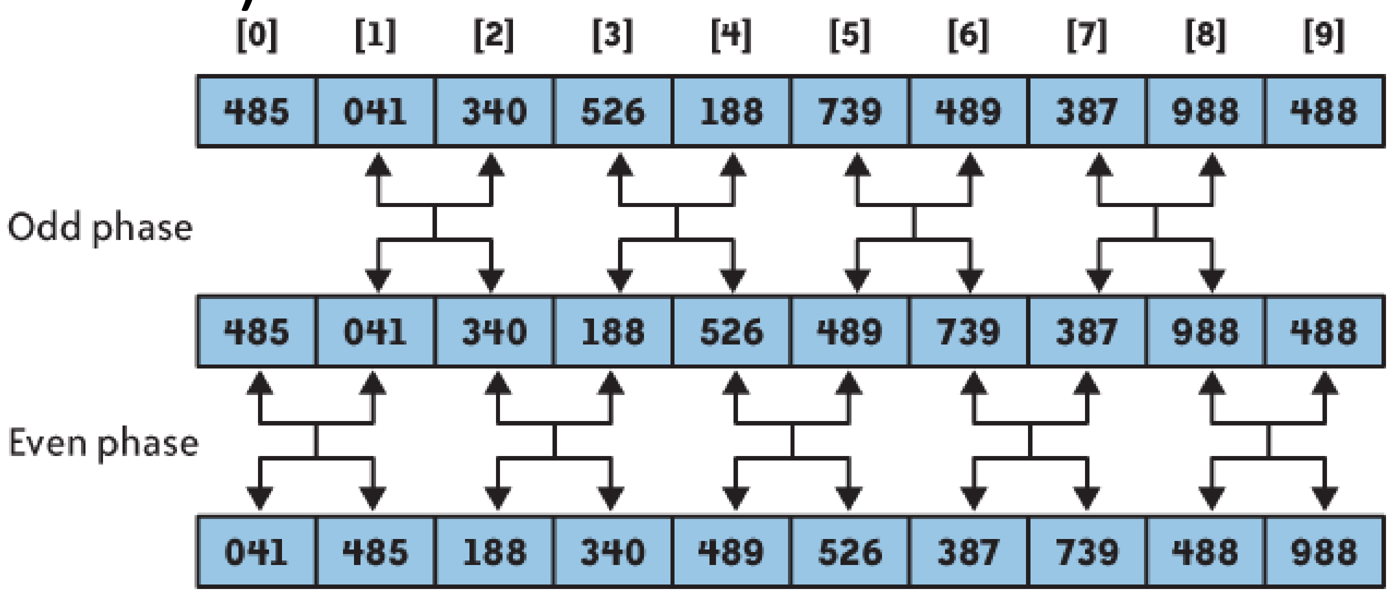
Parallel Bubble Sort

- When to stop?
- Shared flag, **sorted**, initialized to true at beginning of each iteration (2 phases), if any processor perform swap, **sorted** = false



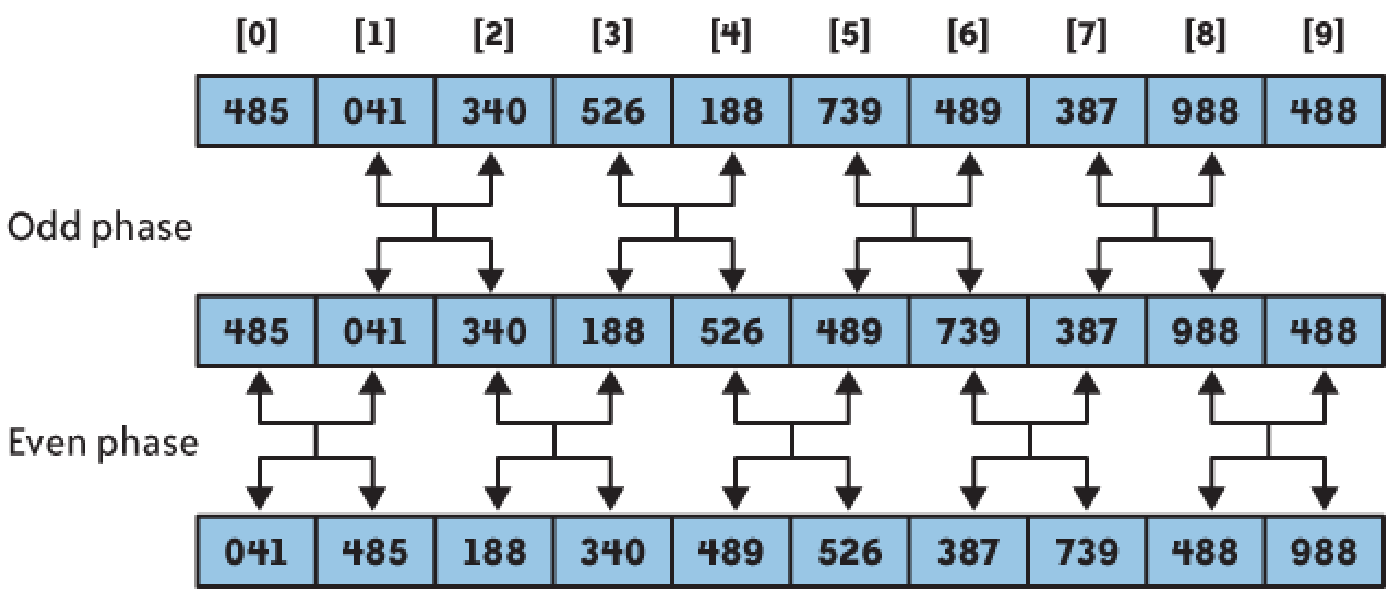
Parallel Bubble Sort Complexity

- Sequential bubble sort, $O(n^2)$
- Parallel bubble sort? (if we have unlimited # of processors)



Parallel Bubble Sort Complexity

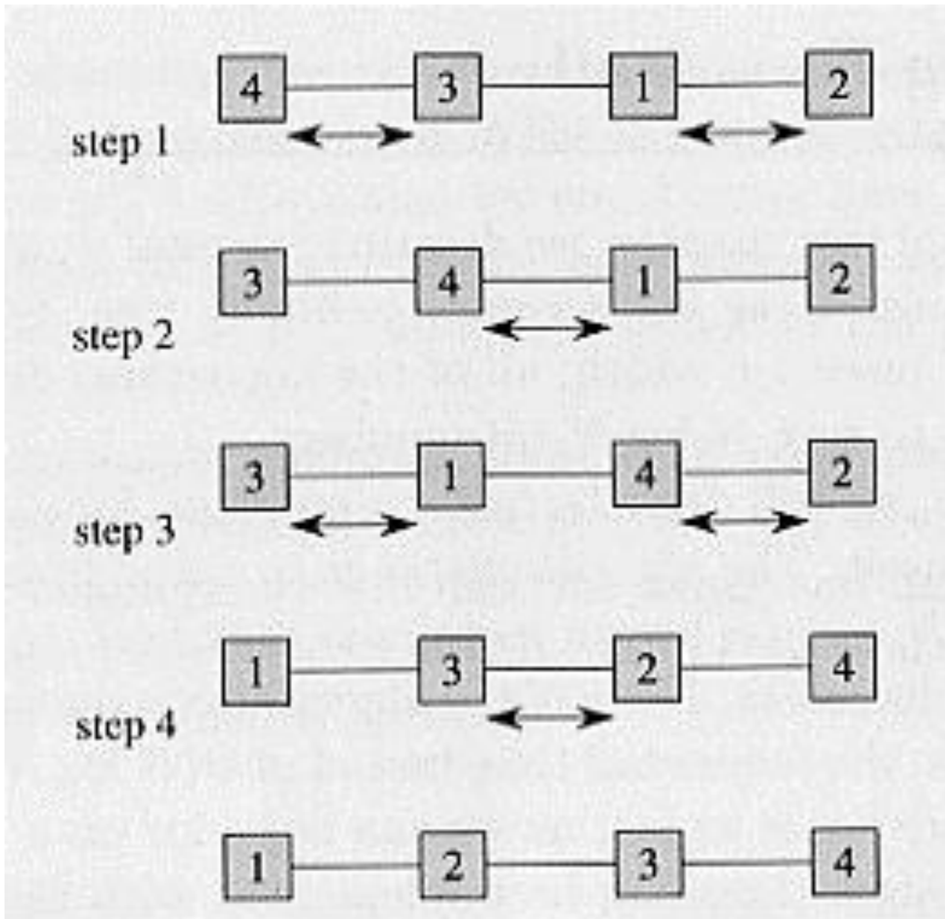
- Sequential bubble sort, $O(n^2)$
- Parallel bubble sort?
- Do n-1 comparisons for each iteration => O(n)
- Seq. Quicksort is O(nlogn)



Parallel Bubble Sort Example

- How many steps does it take to sort the following sequence from least to greatest using the Parallel Bubble Sort? How does the sequence look like after 2 cycles?
- 4,3,1,2

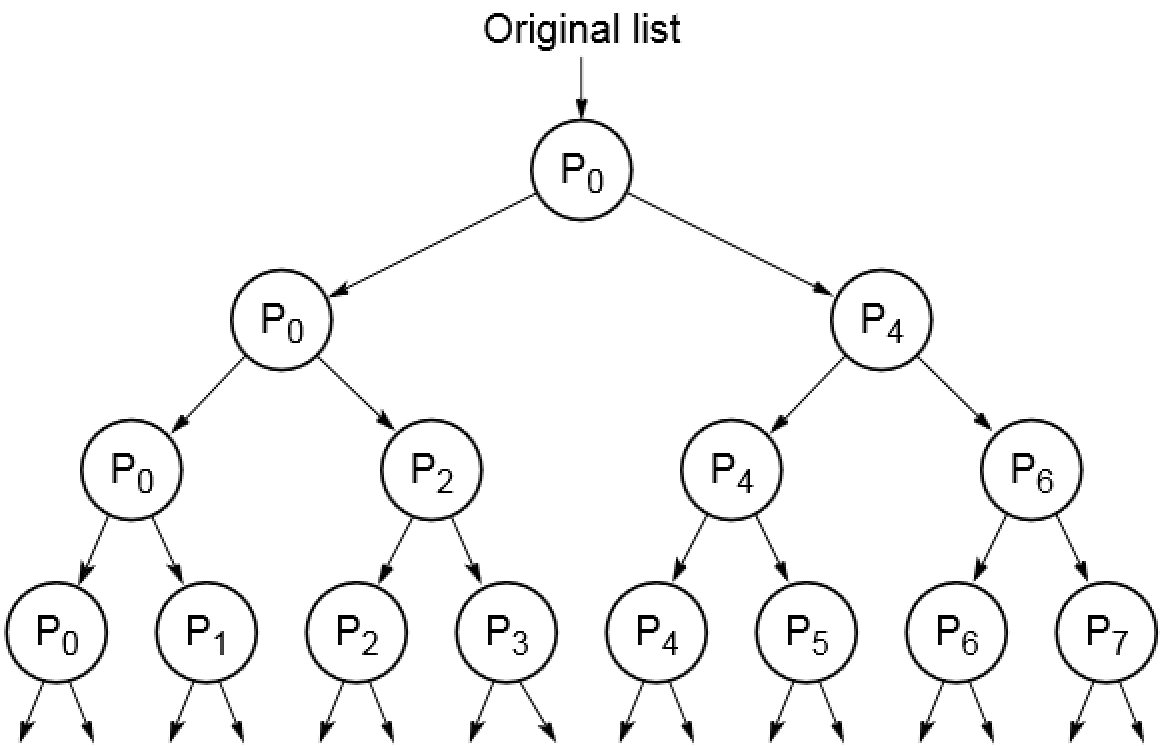
Parallel Bubble Sort Example



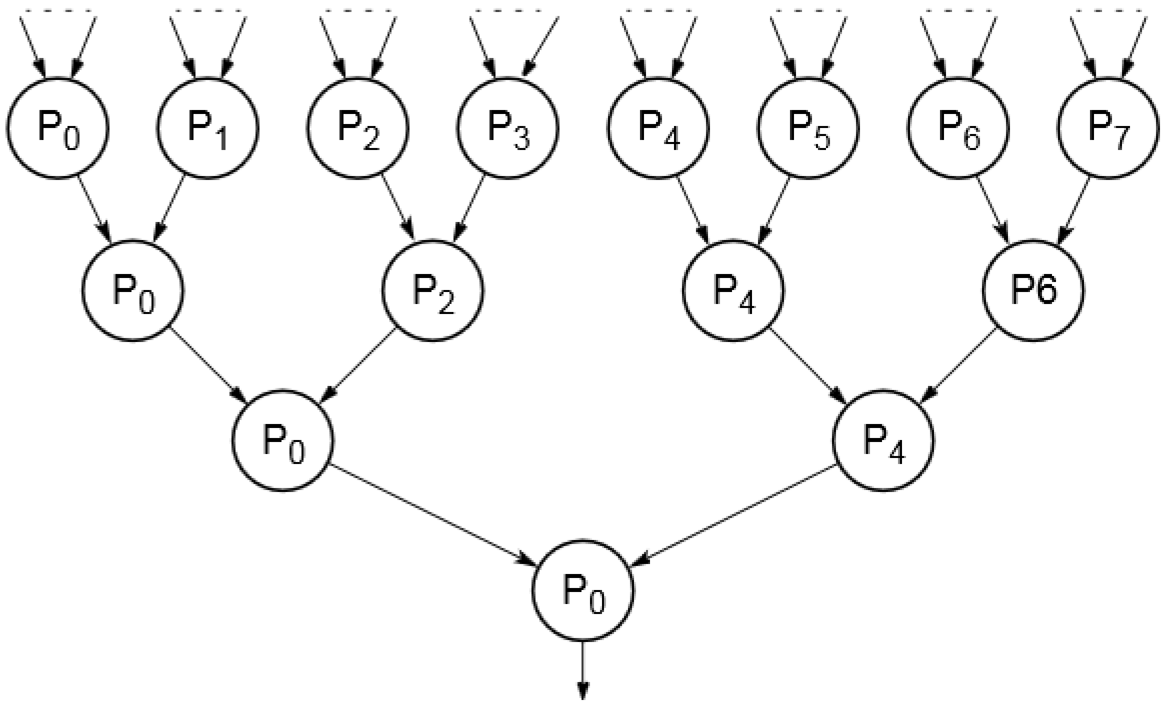
Divide and Conquer

- Dividing problem into sub-problems
- Division usually done through recursion
- Solutions from sub-problems then combined to give solution to the original problem

“Divide”

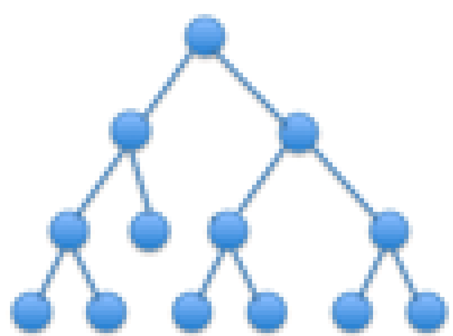


“Conquer”

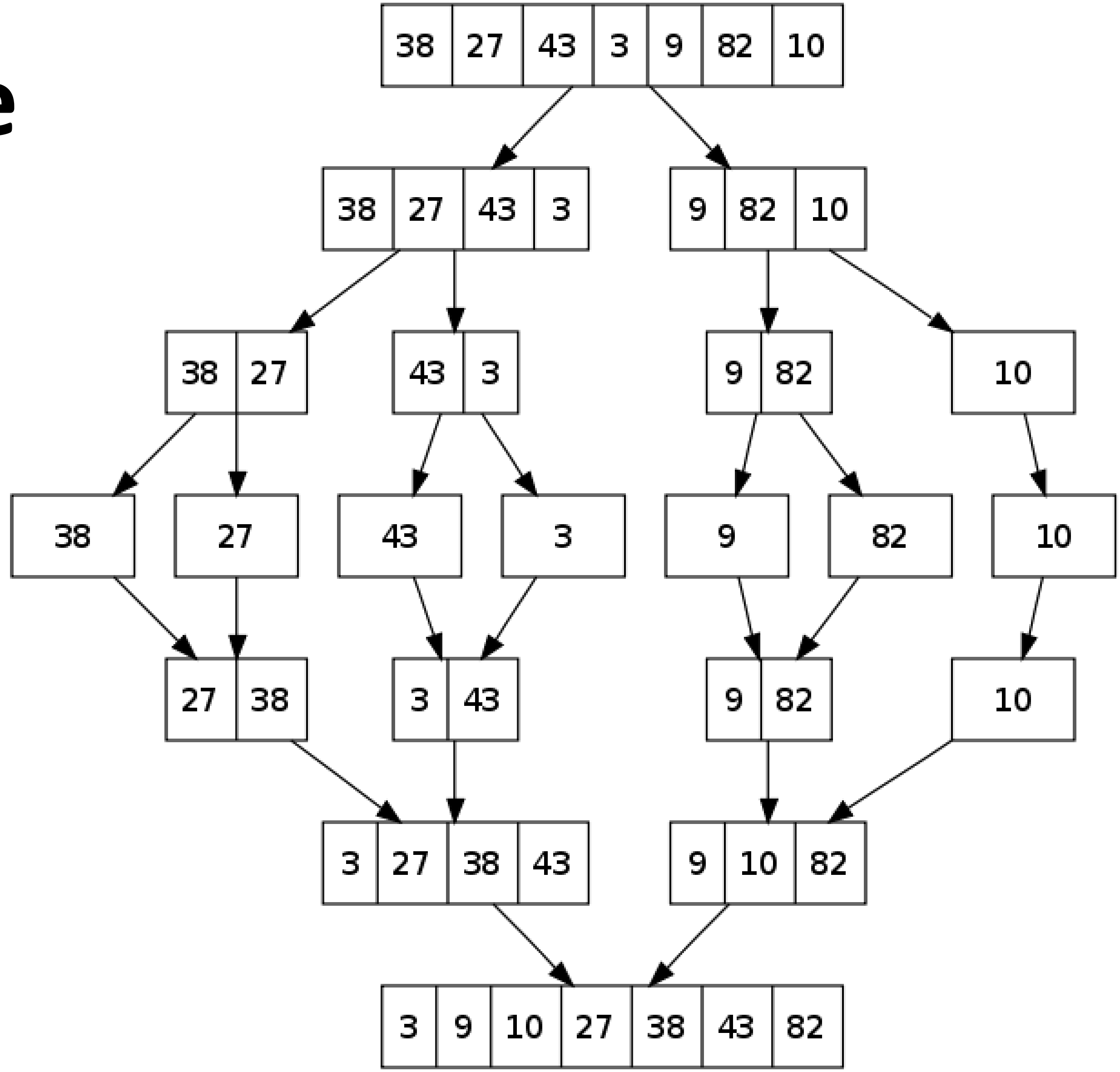


Merge Sort

- Collects sorted list onto one processor
- Merges elements as they come together
- Simple tree structure
- Parallelism is limited when near the root



Example



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3 27 38 43
9 10 82
3 9 10 27 38 43 82

Merge Sort Complexity

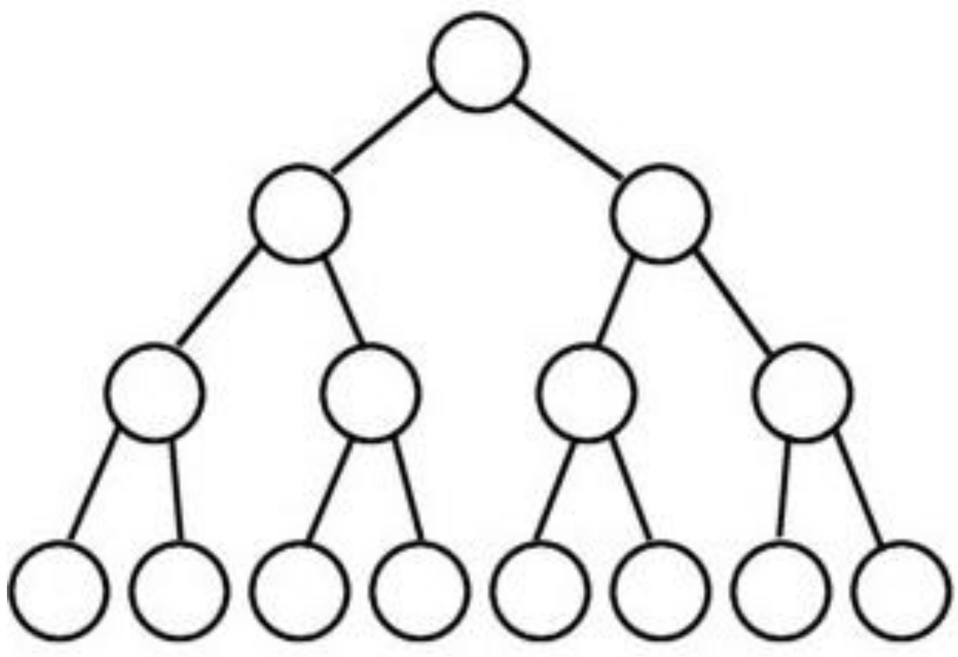
$$T(n) = \begin{cases} b & n = 1 \\ 2T\left(\frac{n}{2}\right) + bn & n > 1 \end{cases}$$

Solve the recurrence relation, (CS331)

T(n) = O(nlogn)

Parallel Merge Sort

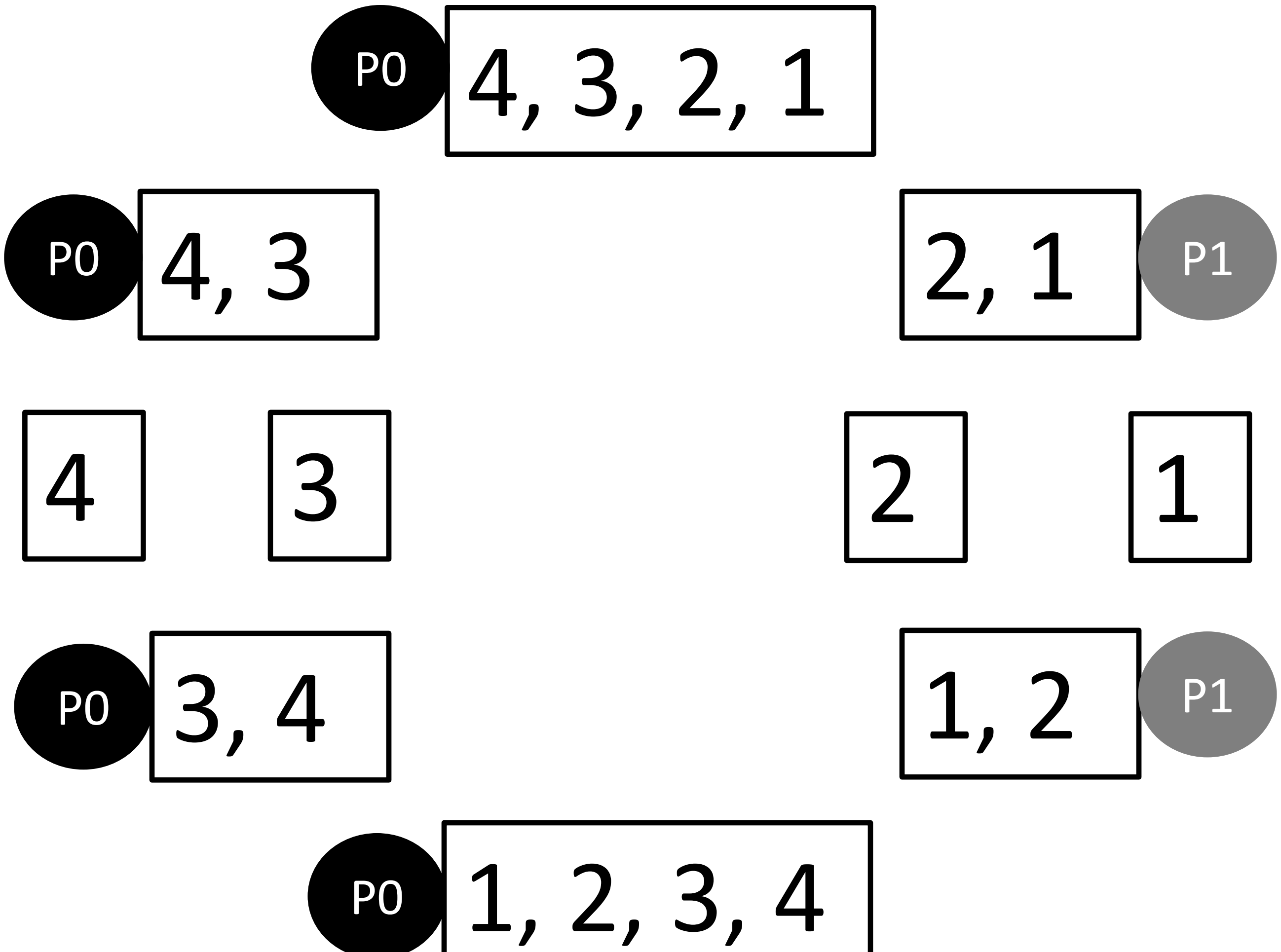
- Parallelize processing of sub-problems
- Max parallelization achived with one processor per node (at each layer/height)



Parallel Merge Sort Example

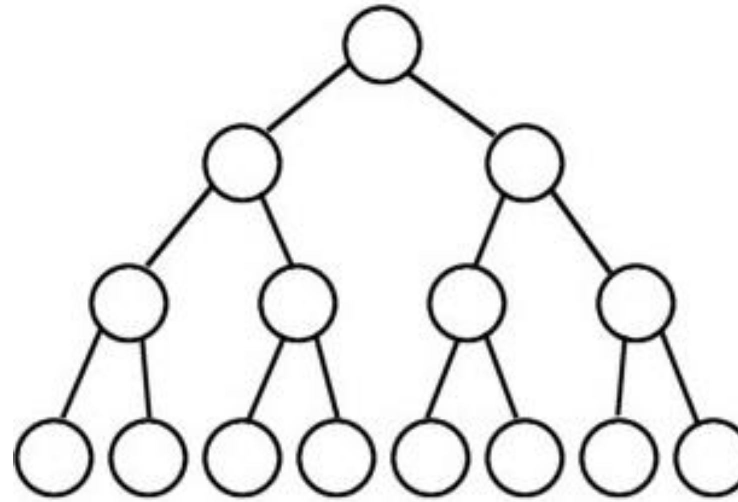
- Perform Merge Sort on the following list of elements. Given 2 processors, P0 & P1, which processor is reponsible for which comparison?
- 4,3,2,1

Parallel Merge Sort Example



Parallel Merge Sort Complexity

- Merge sort, O(nlogn)
- Easy way to remember complexity, n (elements) x logn (tree depth)
- If we have n processors, O(logn)



Parallel Bubble Sort KLA

- Sort a list of #s from smallest to largest
- Each person represent a processor
- Only has to do 1 comparison per phase
- Compare and swap if necessary

Parallel Merge Sort KLA

- Sort a list of #s from smallest to largest
- Each person represent a processor
- Merge sort in parallel

Post-test

- Try your best!