Algebraic solution

Let X be the random variable denoting the number of throws required to observe consecutive sixes.

When we throw the coin for the first time we observe (NOT SIX) with probability $\frac{5}{6}$ and (SIX) with probability $\frac{1}{6}$. In the former case the whole game is as good as starting anew except the new random variable measuring the (number of throws to get consecutive sixes) is $X + 1$.

In the latter case i.e. we observe a (SIX) at the first throw, when we throw the dice for a second time one of the following happens. We either get a (SIX) with probability $\frac{1}{6}$ and game ends. In this case the random variable X takes the value 2. Or we get a (NOT SIX) with probability $\frac{5}{6}$ and the rest of the game is as good as starting new with the new random variable having value $X + 2$.

To conclude we arrive at the following relation:

$$E(X) = \frac{5}{6}.E(X+1) + \frac{1}{6}\left(\frac{5}{6}.E(X+2) + \frac{1}{6}.2\right)$$

Solving for $E(X)$ we arrive at $E(X) = 42$