Sharpening Filters, Edge Detectors

Sharpening Spatial Filters

Sharpening spatial filters seek to highlight fine detail

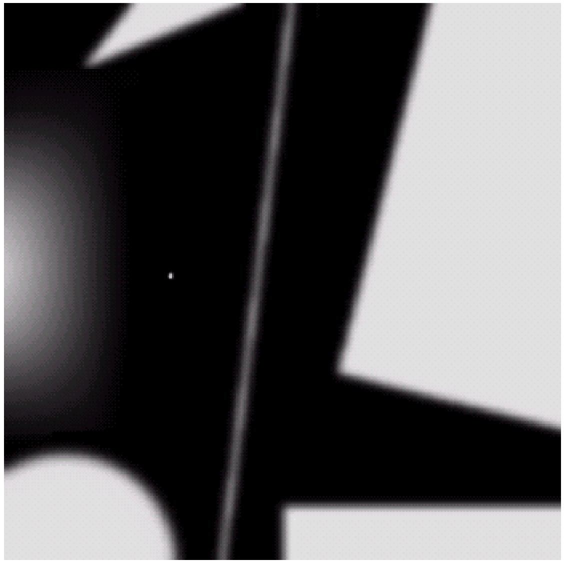
- Remove blurring from images
- Highlight edges

Sharpening filters are based on *spatial differentiation*

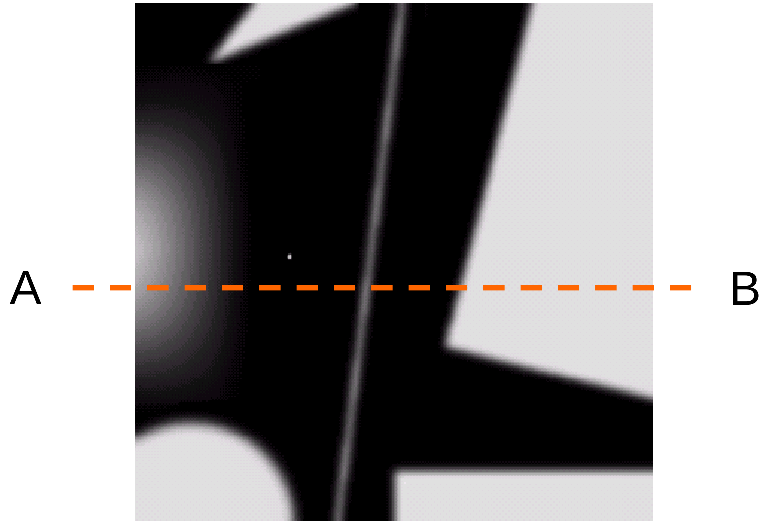
Spatial Differentiation

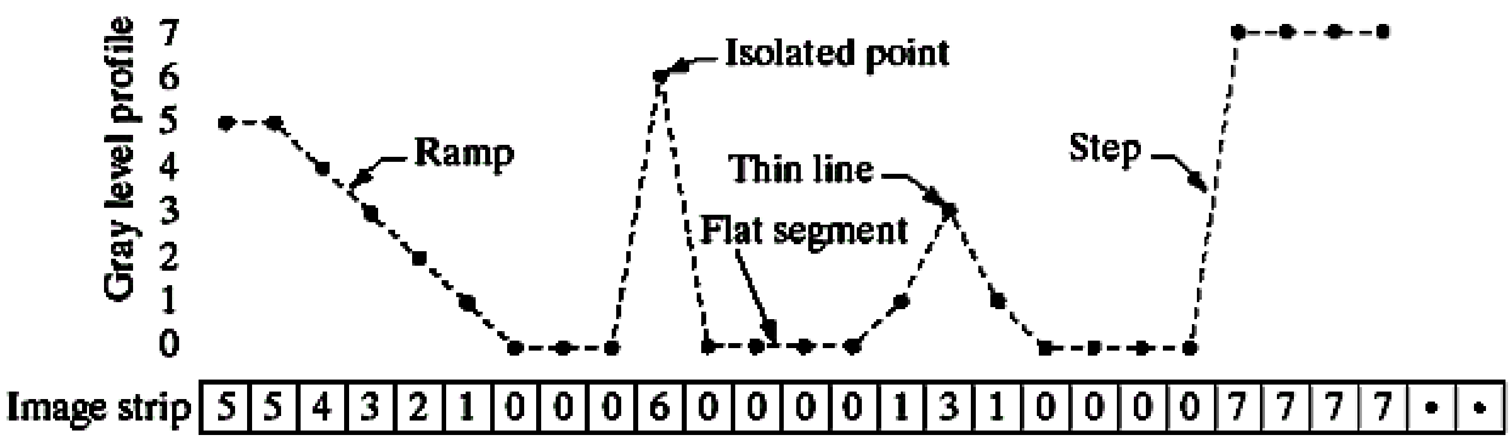
Differentiation measures the *rate of change* of a function

Let's consider a simple 1 dimensional example



Spatial Differentiation





Derivative Filters Requirements

First derivative filter output

- Zero at constant intensities
- Non zero at the onset of a step or ramp
- Non zero along ramps

- Second derivative filter output
  - Zero at constant intensities
  - Non zero at the onset and end of a step or ramp
  - Zero along ramps of constant slope

1st Derivative

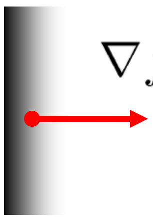
The formula for the 1st derivative of a function is as follows:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

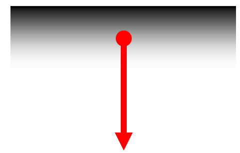
It's just the difference between subsequent values and measures the rate of change of the function

1st Derivative (cont.)

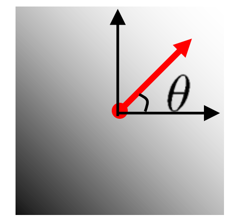
- The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

- 

$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The gradient points in the direction of most rapid increase in intensity.

Gradient direction $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

The *edge strength* is given by the gradient magnitude

$$||\nabla f|| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seitz

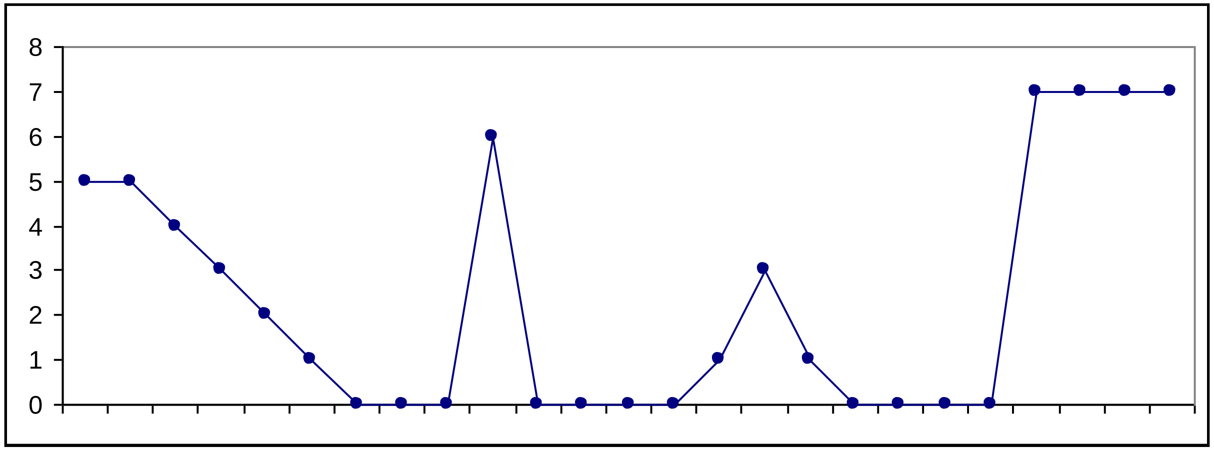
Edge Detection

a b
c d

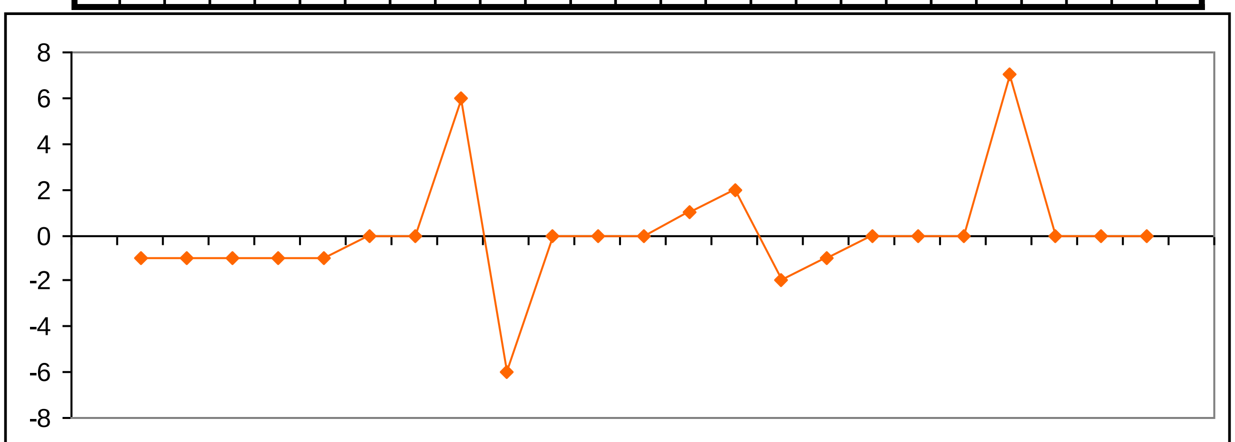
FIGURE 10.10 (a) Original image. (b) $|G_x|$, component of the gradient in the x-direction. (c) $|G_y|$, component in the y-direction. (d) Gradient image, $|G_x| + |G_y|$.

$$\nabla f \approx |G_x| + |G_y|$$

1^{st} Derivative (cont…)



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7
	0	-1	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0



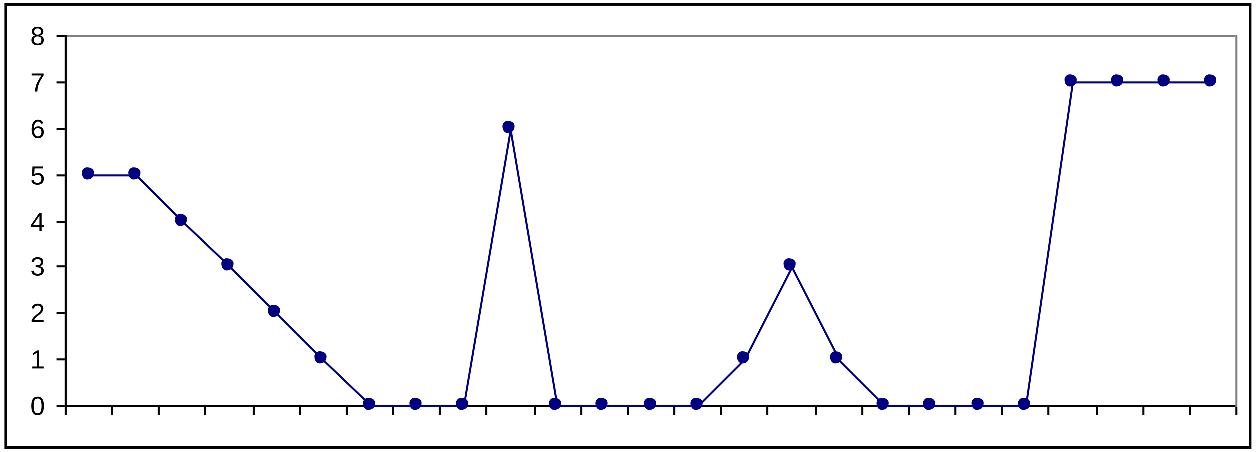
2^{nd} Derivative

The formula for the 2^{nd} derivative of a function is as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

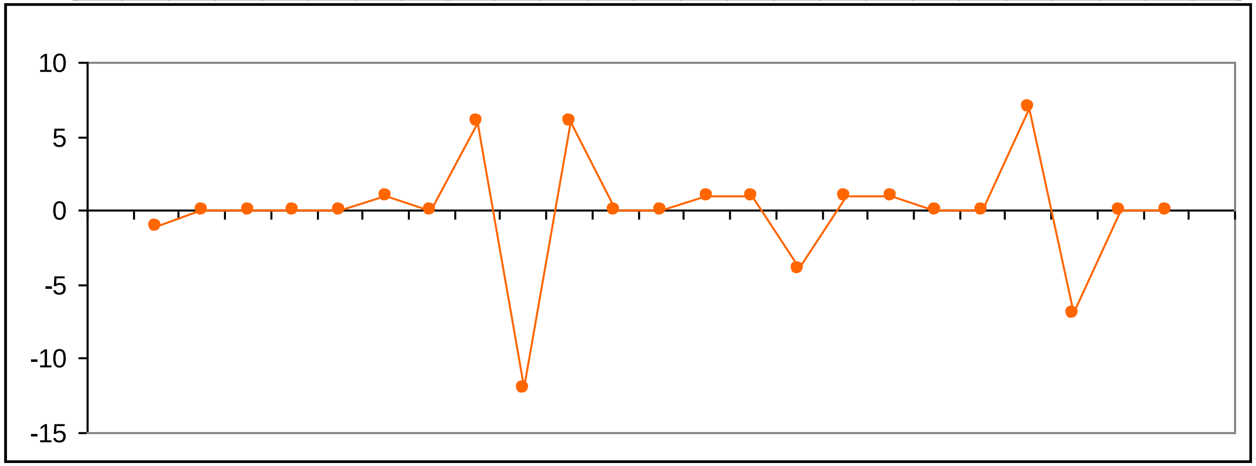
Simply takes into account the values both before and after the current value

2^{nd} Derivative (cont…)



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	



Using Second Derivatives For Image Enhancement

Edges in images are often ramp-like transitions

- 1st derivative is constant and produces thick edges
- 2nd derivative zero crosses the edge (double response at the onset and end with opposite signs)

- 2nd derivative gives a very high response to fine details and noise.

1^{st} & 2^{nd} Derivatives

Comparing the 1^{st} and 2^{nd} derivatives we can conclude the following:

- 1^{st} order derivatives generally produce thicker edges (if thresholded at ramp edges)
- 2^{nd} order derivatives have a stronger response to fine detail e.g. thin lines
- 1^{st} order derivatives have stronger response to grey level step
- 2^{nd} order derivatives produce a double response at step changes in grey level (which helps in detecting zero crossings)

The Laplacian

The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

where the partial 1st order derivative in the x direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

and in the y direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Laplacian filter

A common sharpening filter is the *Laplacian*

- Isotropic
- One of the simplest sharpening filters

The Laplacian (cont…)

So, the Laplacian can be given as follows:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

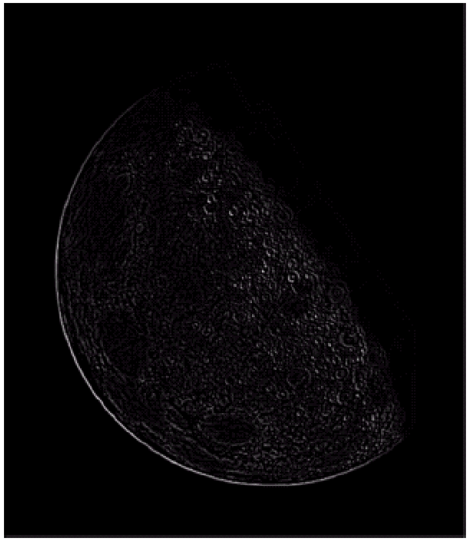
We can easily build a filter based on this

0	1	0
1	-4	1
0	1	0

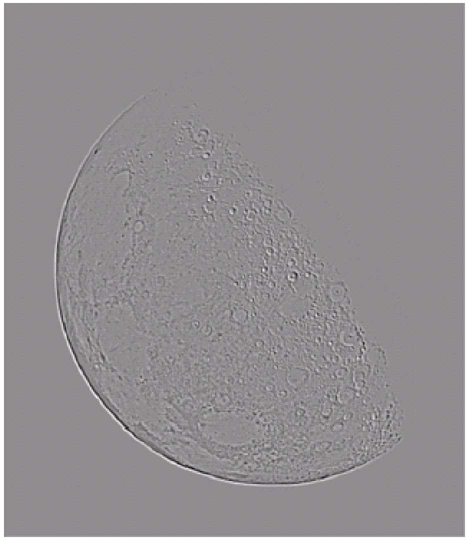
The Laplacian (cont…)

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Original Image



Laplacian Filtered Image



Laplacian Filtered Image Scaled for Display

In order to preserve the background, along with the fine details, we add / subtract the Laplacian to / from the image.

$$g(x, y) = f(x, y) + c\nabla^2 f$$

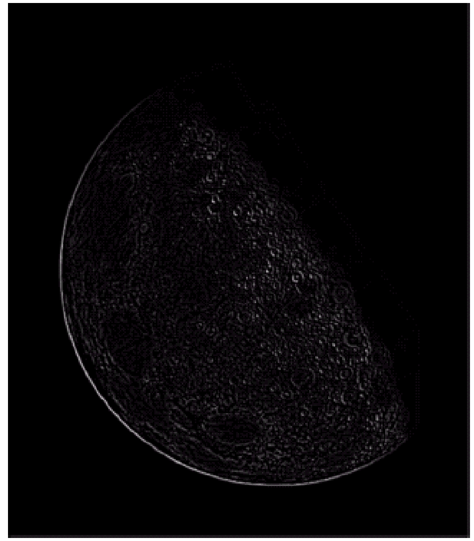
$c = 1$ if centre of mask is +ve

$c = -1$ if centre of mask is -ve

Laplacian Image Enhancement

Original Image

-



Laplacian Filtered Image

=

Sharpened Image

In the final sharpened image edges and fine detail are much more obvious

Laplacian Image Enhancement

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

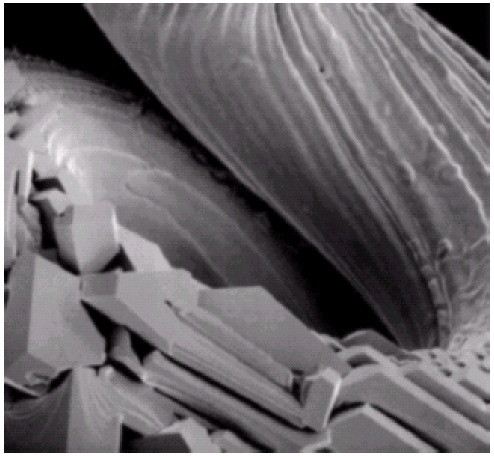
Image Enhancement

The entire enhancement can be combined into a single filtering operation

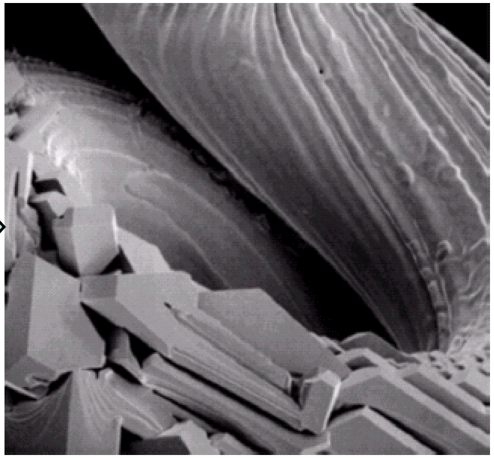
$$
\begin{aligned}
g(x,y) &= f(x,y) - \nabla^2 f \\
&= f(x,y) - [f(x+1,y) + f(x-1,y) \\
&\qquad + f(x,y+1) + f(x,y-1) \\
&\qquad - 4f(x,y)] \\
&= 5f(x,y) - f(x+1,y) - f(x-1,y) \\
&\qquad - f(x,y+1) - f(x,y-1)
\end{aligned}
$$

Simplified Image Enhancement (cont…)

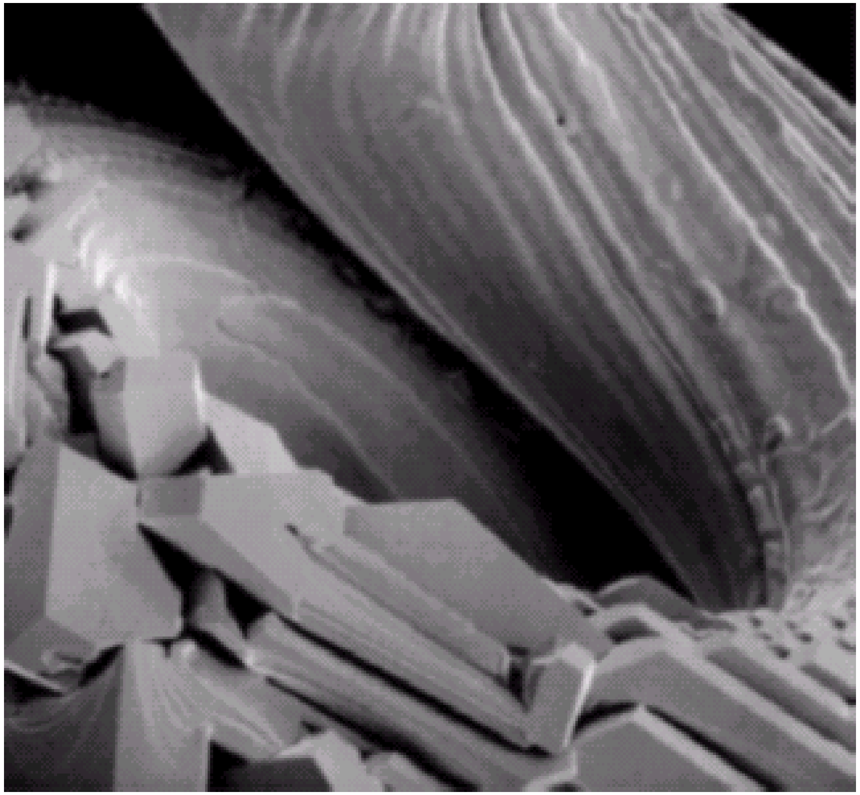
This gives us a new filter which does the whole job for us in one step

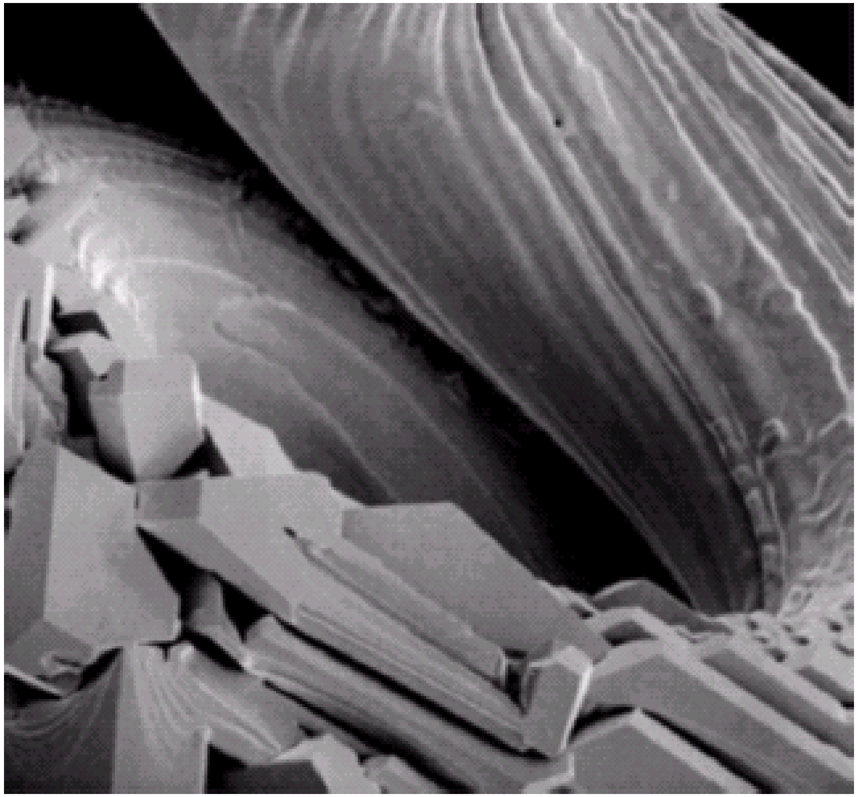


0	-1	0
-1	5	-1
0	-1	0



Simplified Image Enhancement (cont…)





Images taken from Gonzalez & Woods, Digital Image Processing (2002)

Variants On The Simple Laplacian

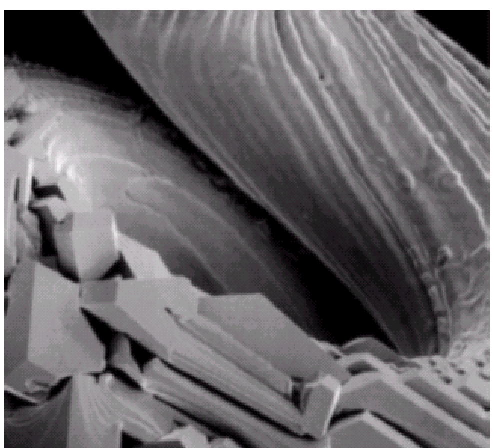
There are lots of slightly different versions of the Laplacian that can be used:

0	1	0
1	-4	1
0	1	0

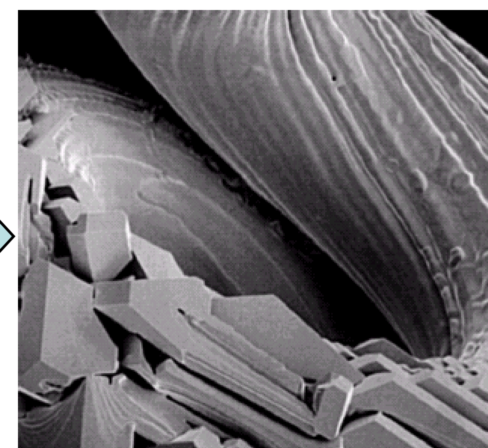
Simple Laplacian

1	1	1
1	-8	1
1	1	1

Variant of Laplacian



-1	-1	-1
-1	9	-1
-1	-1	-1



Unsharp masking

Used by the printing industry

Subtracts an unsharped (smooth) image from the original image $f(x,y)$.

- Blur the image

  $b(x,y)=Blur\{f(x,y)\}$

- Subtract the blurred image from the original (the result is called the *mask*)

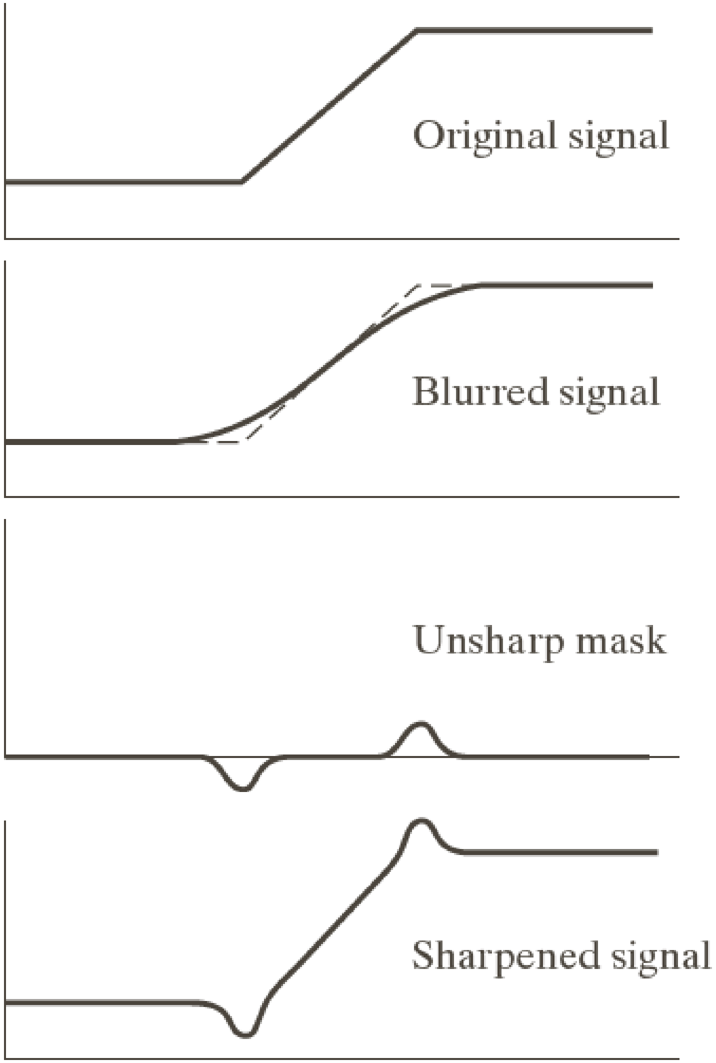
  $g_{mask}(x,y)=f(x,y)-b(x,y)$

- Add the mask to the original

  $g(x,y)=f(x,y)+k\ g_{mask}(x,y)$ with k non negative

Unsharp masking (cont...)

Sharping mechanism



When $k>1$ the process is referred to as highboost filtering

Unsharp masking (cont...)

Original image

Blurred image

Mask

Unsharp masking

Highboost filtering *(k=4.5)*

Images taken from Gonzalez & Woods, Digital Image Processing (2002)

1^{st} Derivative Filtering

For a function $f(x, y)$ the gradient of f at coordinates (x, y) is given as the column vector:

$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

1st Derivative Filtering (cont…)

The magnitude of this vector is given by:

$$\begin{aligned}\nabla f &= mag(\nabla \mathrm{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}\end{aligned}$$

For easier implementation, this can be simplified as: $\nabla f \approx |G_x| + |G_y|$

1st Derivative Filtering (cont…)

There is some debate as to how best to calculate these gradients but we will use:

$$\nabla f \approx \left|(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)\right| + \left|(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)\right|$$

which is based on these coordinates

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Sobel Operators

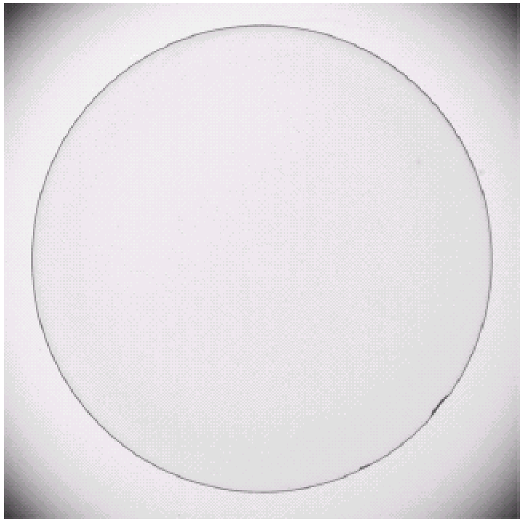
Based on the previous equations we can derive the *Sobel Operators*

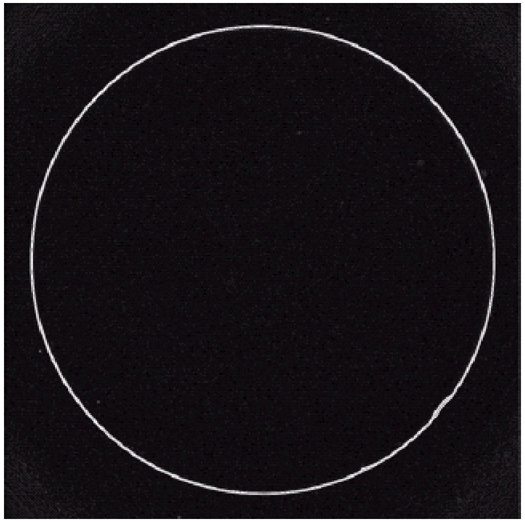
-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

To filter an image it is filtered using both operators the results of which are added together

Sobel Example





Sobel filters are typically used for edge detection

1st & 2nd Derivatives

Comparing the 1st and 2nd derivatives we can conclude the following:

- 1st order derivatives generally produce thicker edges (if thresholded at ramp edges)
- 2nd order derivatives have a stronger response to fine detail e.g. thin lines
- 1st order derivatives have stronger response to grey level step
- 2nd order derivatives produce a double response at step changes in grey level (which helps in detecting zero crossings)

Detection of discontinuities

- There are three kinds of discontinuities of intensity: points, lines and edges.
- The most common way to look for discontinuities is to scan a small mask over the image. The mask determines which kind of discontinuity to look for.

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 = \sum_{i=1}^{9} w_i z_i$$

FIGURE 10.1 A general 3 × 3 mask.

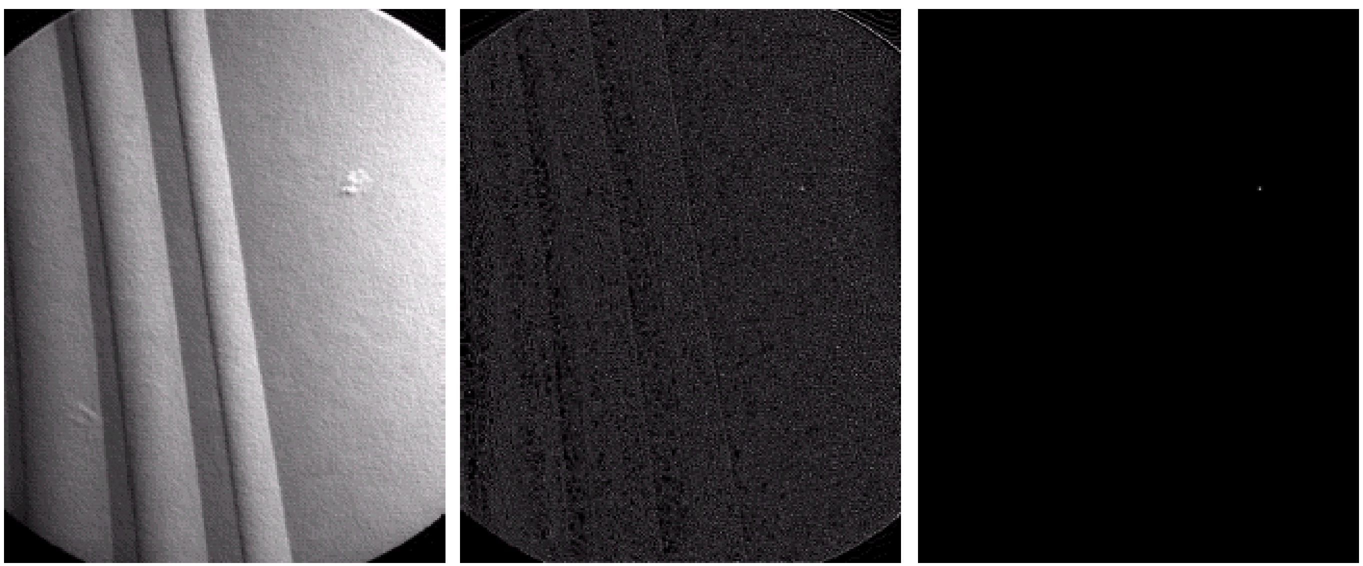
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Point Detection

$$|R| \geq T$$

$$\text{where } T: \text{a nonnegative threshold}$$

−1	−1	−1
−1	8	−1
−1	−1	−1



a
b c d

FIGURE 10.2 (a) Point detection mask. (b) X-ray image of a turbine blade with a porosity. (c) Result of point detection. (d) Result of using Eq. (10.1-2). (Original image courtesy of X-TEK Systems Ltd.)

Line detection

- Only slightly more common than point detection is to find a one pixel wide line in an image.
- For digital images the only three point straight lines are only horizontal, vertical, or diagonal (+ or –45°).

FIGURE 10.3 Line masks.

Horizontal

−1	−1	−1
2	2	2
−1	−1	−1

+45°

−1	−1	2
−1	2	−1
2	−1	−1

Vertical

−1	2	−1
−1	2	−1
−1	2	−1

−45°

2	−1	−1
−1	2	−1
−1	−1	2

Line detection

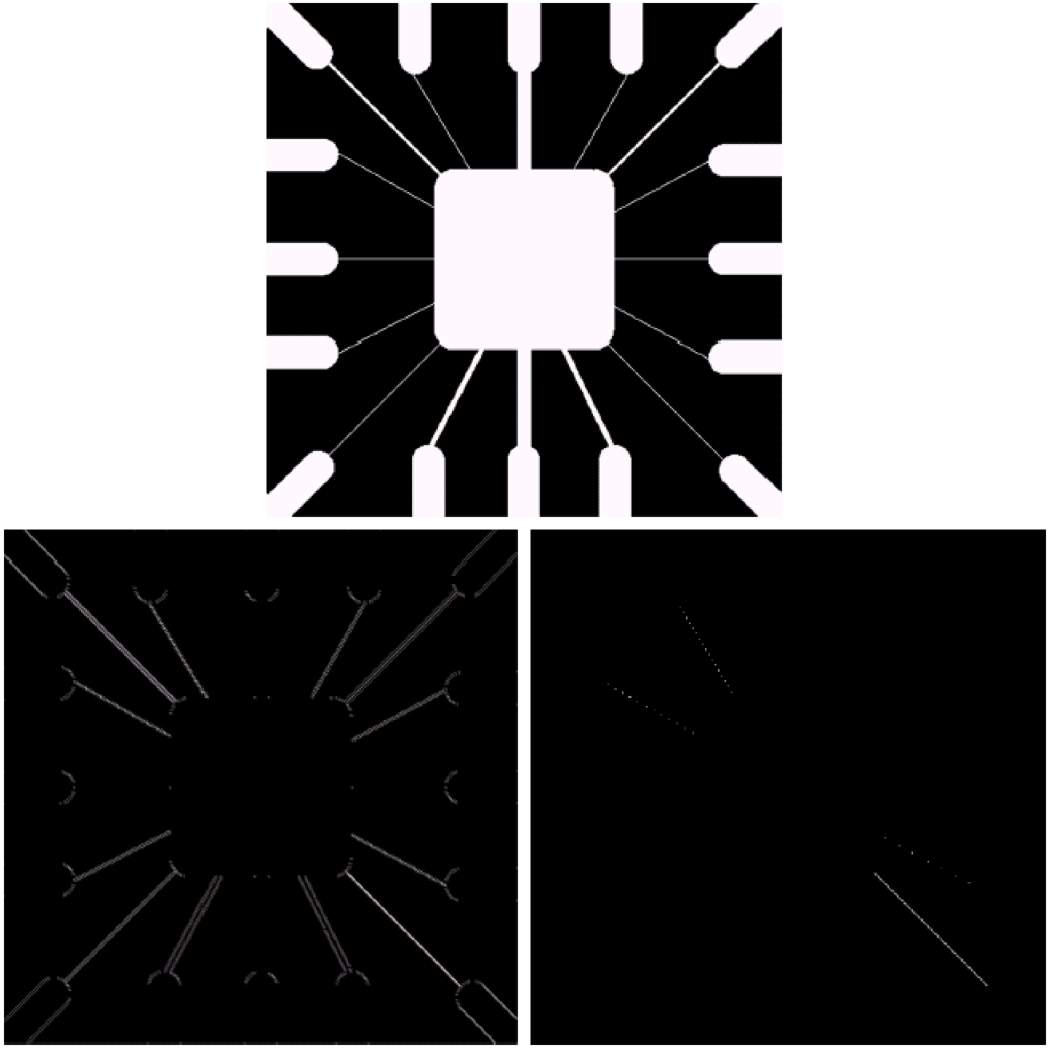


FIGURE 10.4 Illustration of line detection. (a) Binary wire-bond mask. (b) Absolute value of result after processing with −45° line detector. (c) Result of thresholding image (b).

Edge Detection

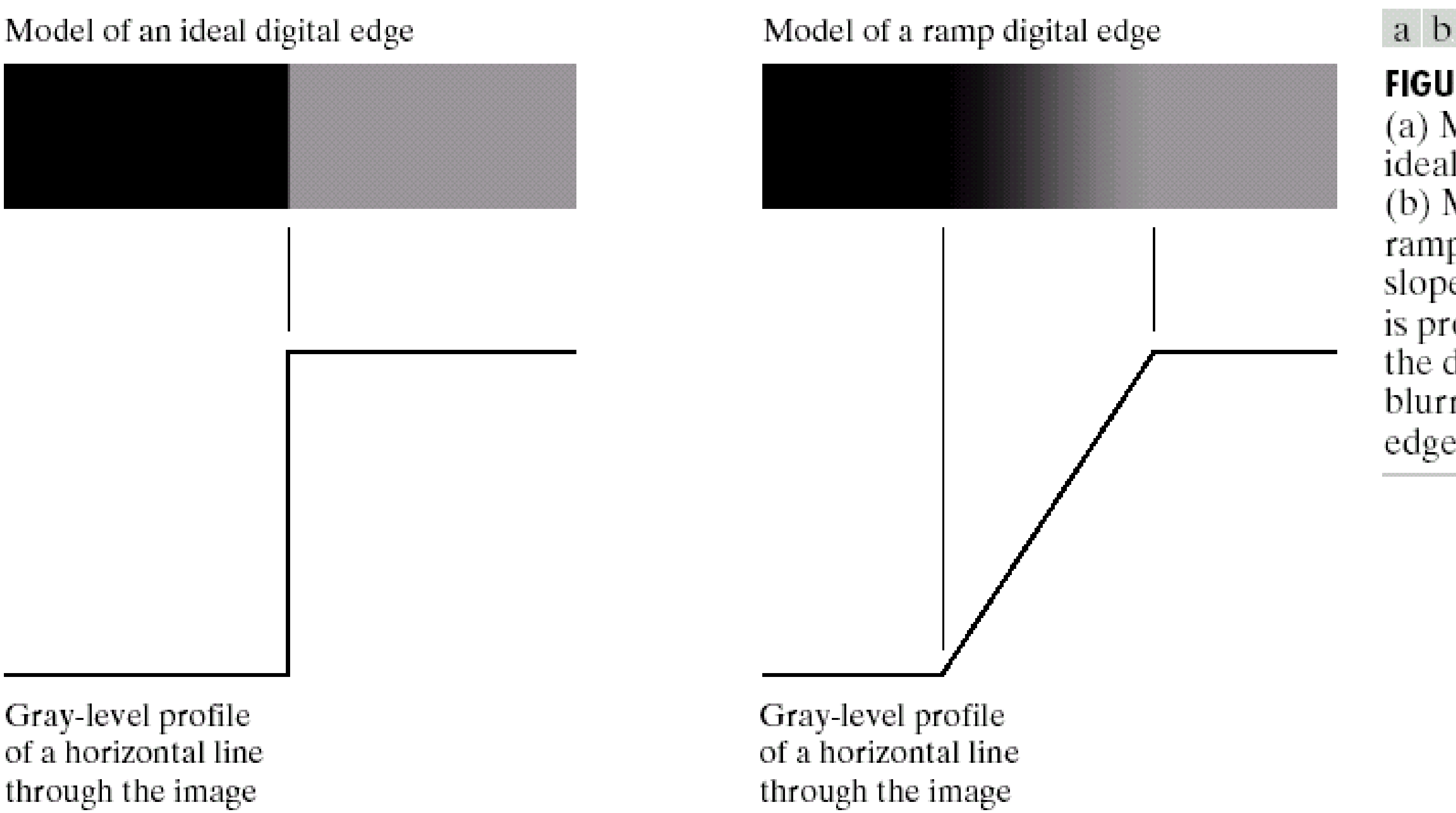
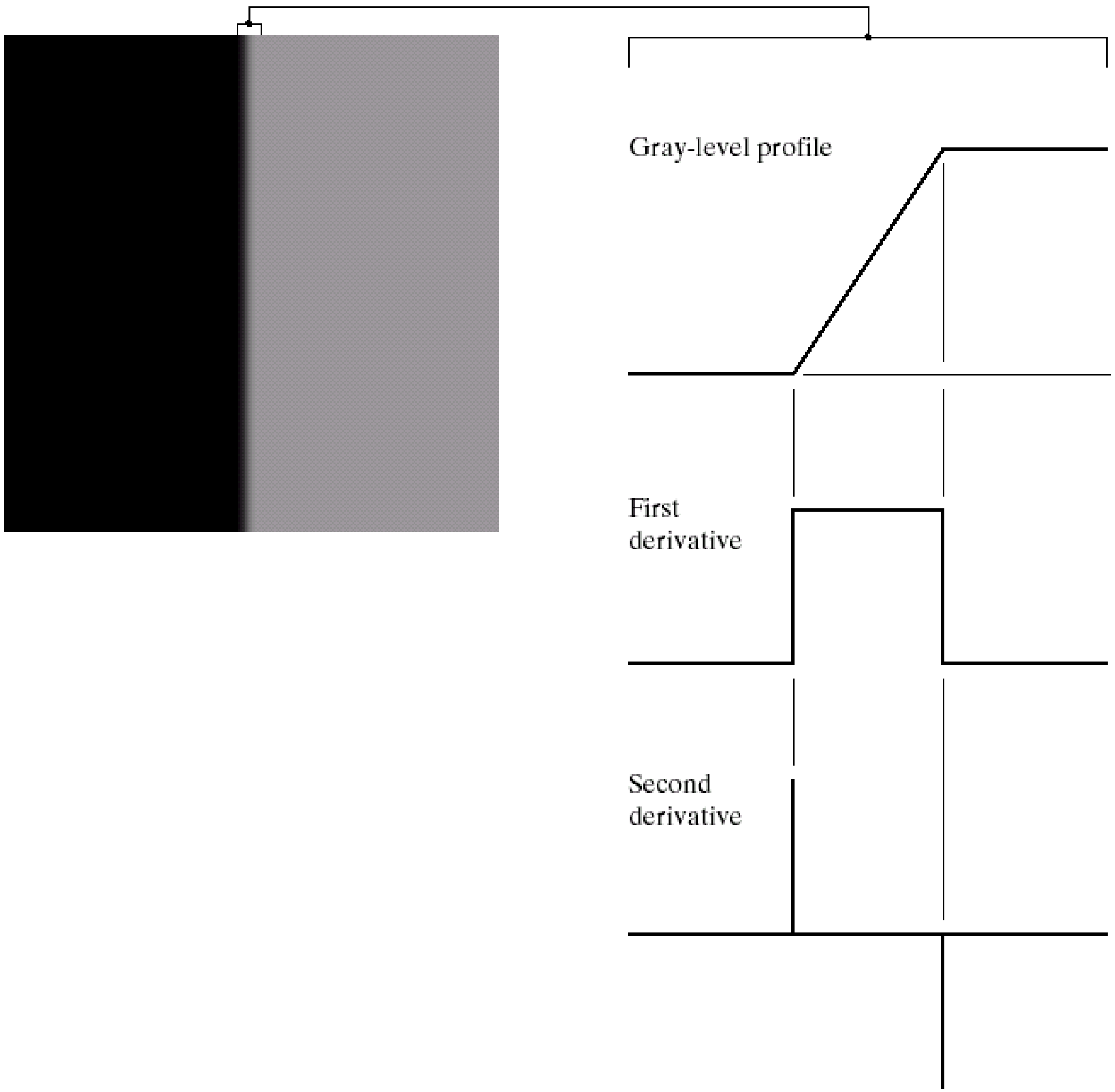


FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.

Edge Detection

a b

FIGURE 10.6
(a) Two regions separated by a vertical edge.
(b) Detail near the edge, showing a gray-level profile, and the first and second derivatives of the profile.



Edge Detection

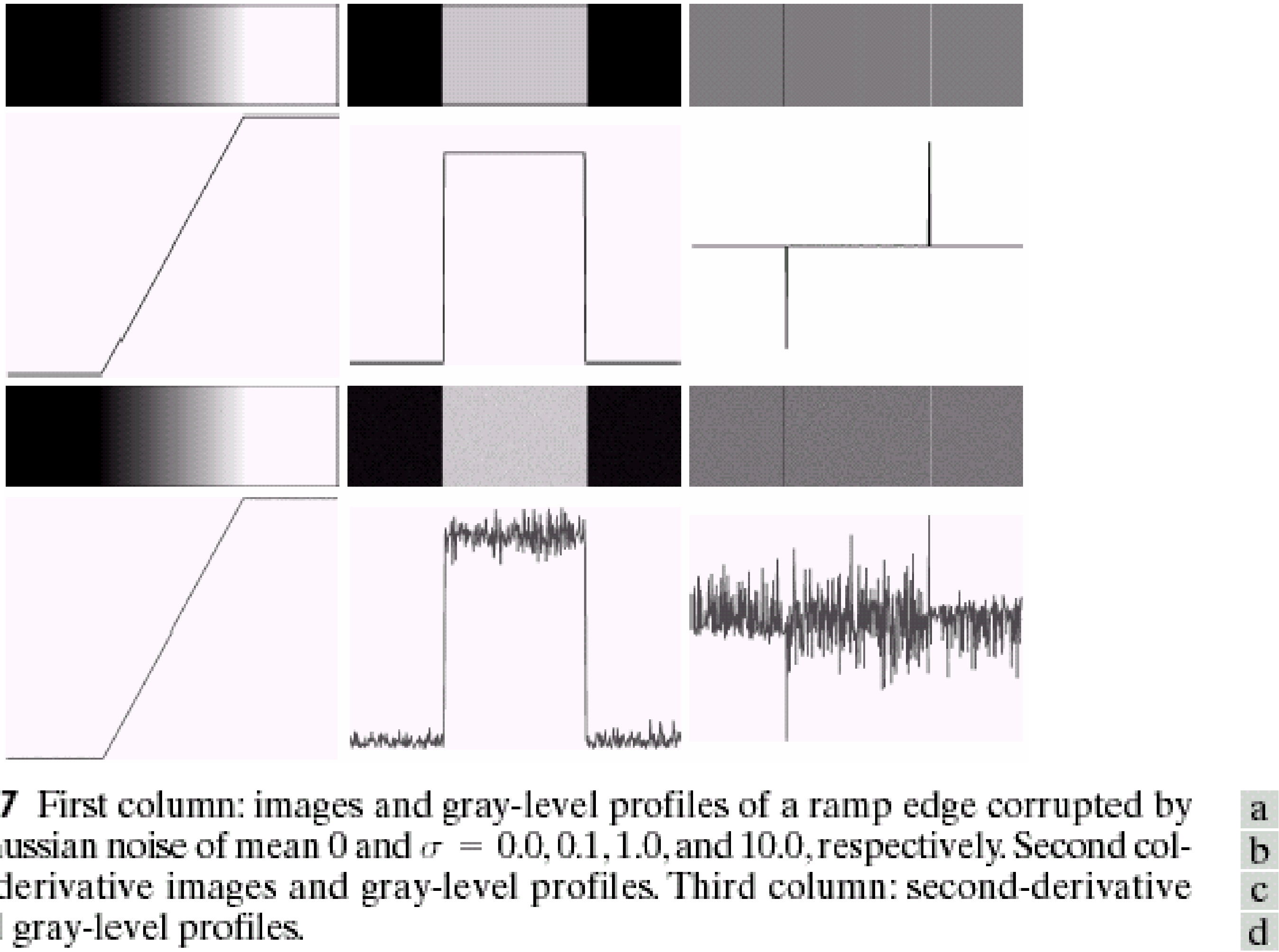


FIGURE 10.7 First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and $\sigma = 0.0, 0.1, 1.0$, and 10.0, respectively. Second column: first-derivative images and gray-level profiles. Third column: second-derivative images and gray-level profiles.

Edge Detection

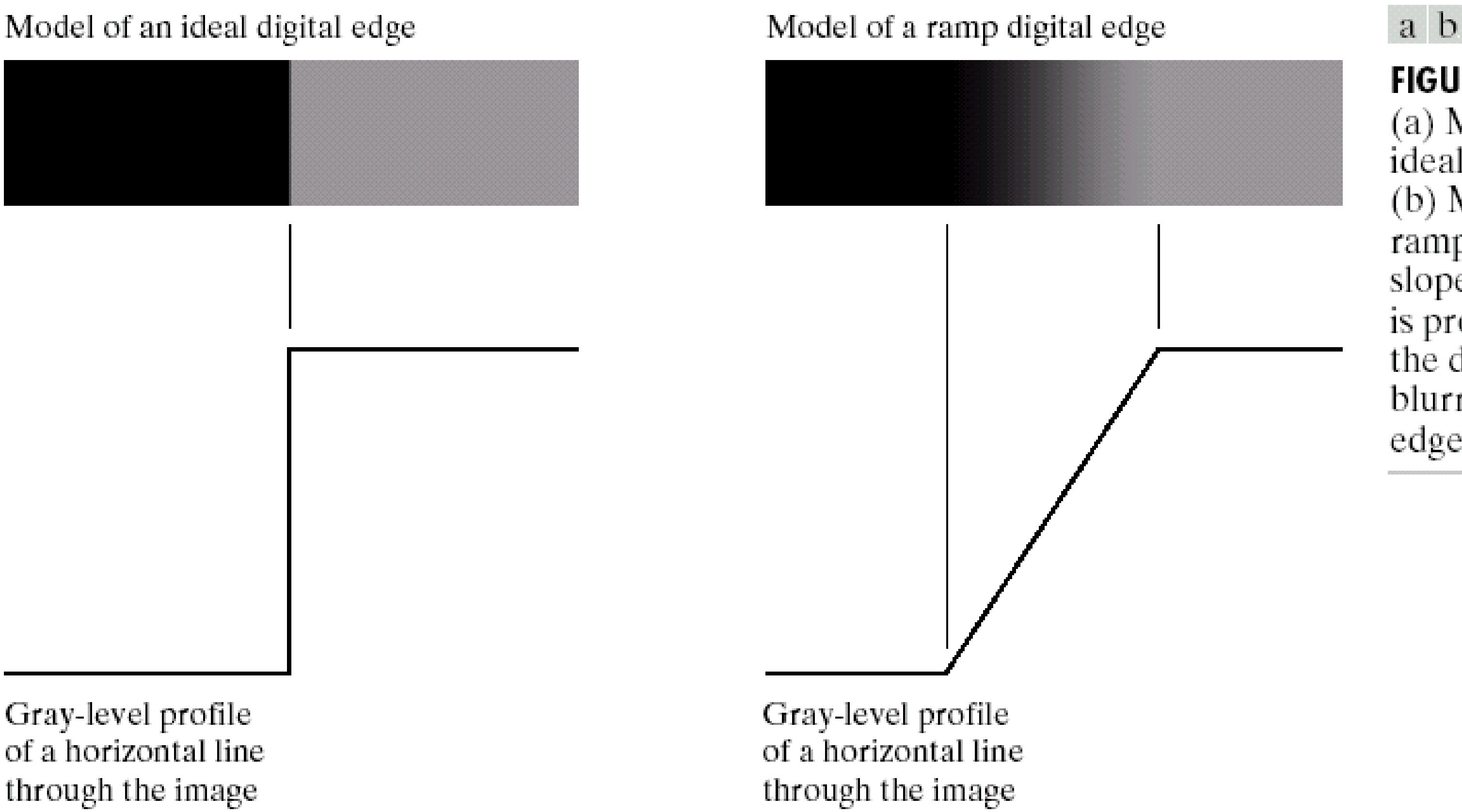


FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.

Edge Detection

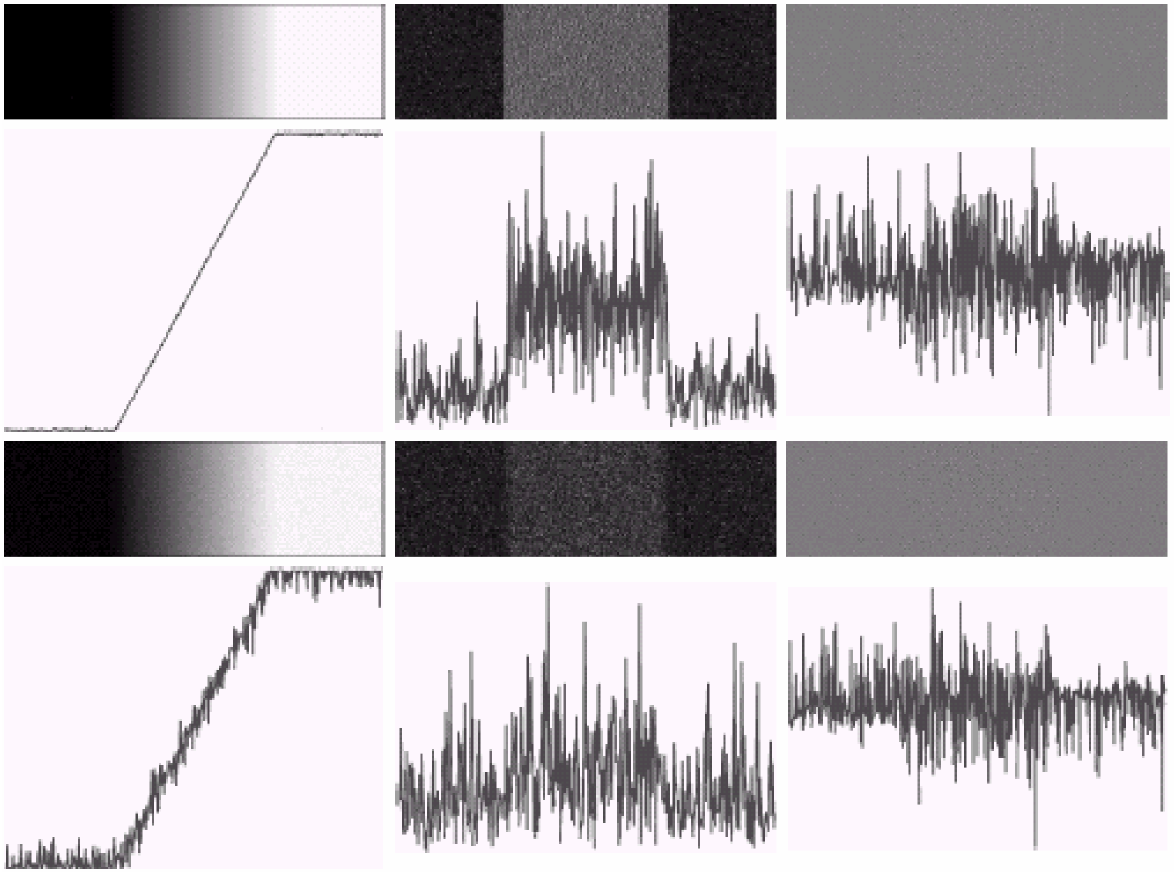


FIGURE 10.7 First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and $\sigma = 0.0, 0.1, 1.0$, and 10.0, respectively. Second column: first-derivative images and gray-level profiles. Third column: second-derivative images and gray-level profiles.

a
b
c
d

Edge Detection

- First-order derivatives:
  - The gradient of an image $f(x,y)$ at location (x,y) is defined as the vector:

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

  - The magnitude of this vector:

$$\nabla f = \mathrm{mag}(\nabla \mathbf{f}) = \left[G_x^2 + G_y^2 \right]^{1/2}$$

  - The direction of this vector:

$$\alpha(x, y) = \tan^{-1}\left(\frac{G_x}{G_y} \right)$$

Edge Detection

Roberts cross-gradient operators

−1	0		0	−1
0	1		1	0

Roberts

Prewitt operators

−1	−1	−1		−1	0	1
0	0	0		−1	0	1
1	1	1		−1	0	1

Prewitt

Sobel operators

−1	−2	−1		−1	0	1
0	0	0		−2	0	2
1	2	1		−1	0	1

Sobel

Edge Detection

Prewitt masks for detecting diagonal edges

Sobel masks for detecting diagonal edges

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

Prewitt

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Sobel

a	b
c	d

FIGURE 10.9 Prewitt and Sobel masks for detecting diagonal edges.

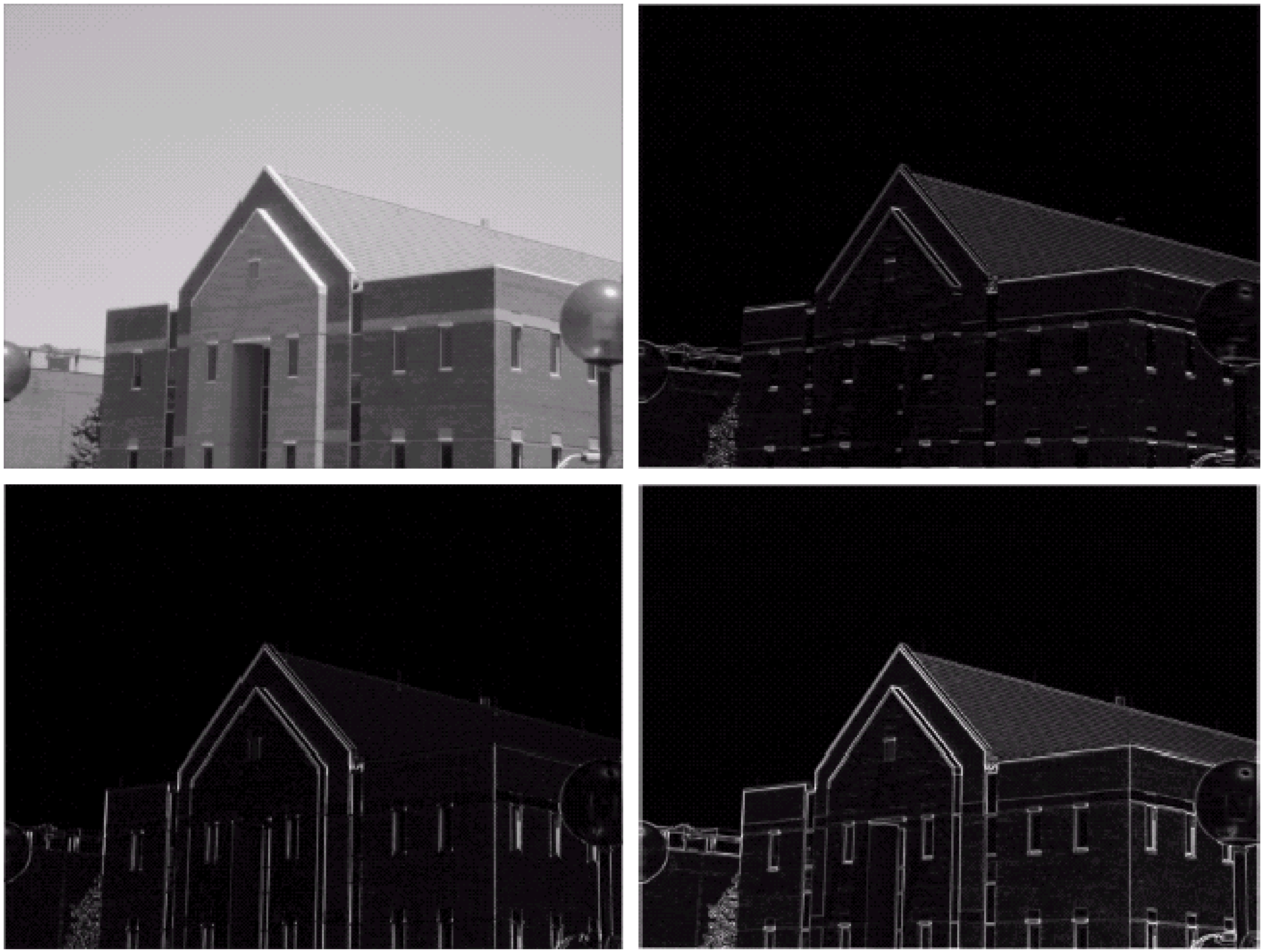
Edge Detection

a b
c d

FIGURE 10.10 (a) Original image. (b) $|G_x|$, component of the gradient in the x-direction. (c) $|G_y|$, component in the y-direction. (d) Gradient image, $|G_x| + |G_y|$.

$$\nabla f \approx |G_x| + |G_y|$$

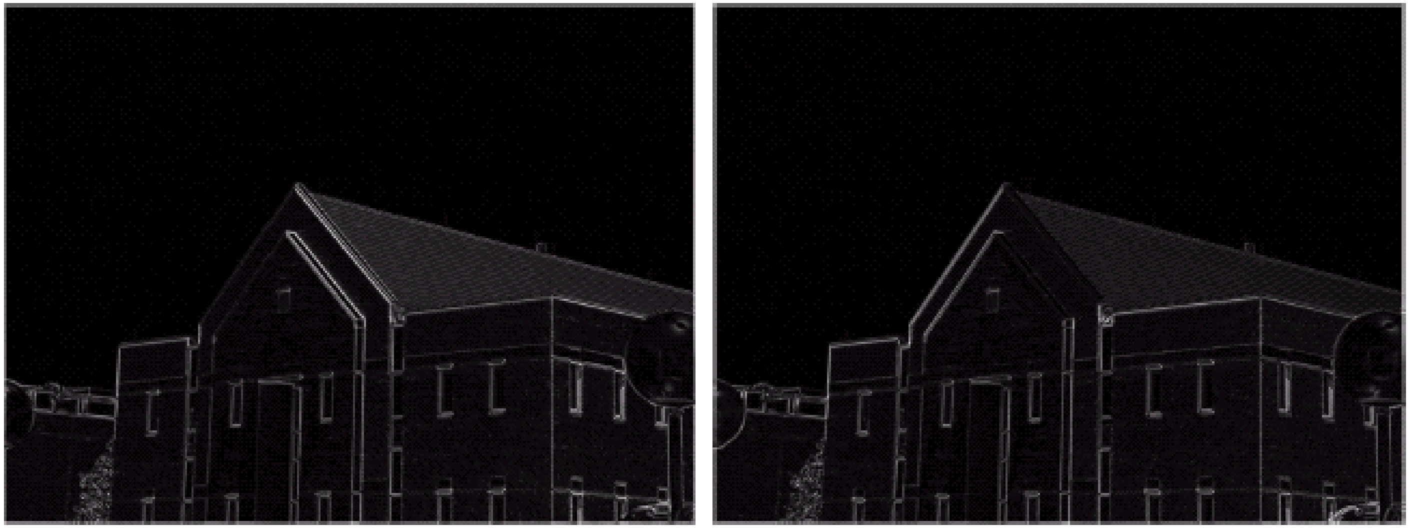
Edge Detection



a	b
c	d

FIGURE 10.11 Same sequence as in Fig. 10.10, but with the original image smoothed with a 5×5 averaging filter.

Edge Detection



a b

FIGURE 10.12 Diagonal edge detection. (a) Result of using the mask in Fig. 10.9(c). (b) Result of using the mask in Fig. 10.9(d). The input in both cases was Fig. 10.11(a).

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

Edge Detection

- Second-order derivatives: (The Laplacian)
  - The Laplacian of an 2D function *f*(*x*,*y*) is defined as

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

  - Two forms in practice:

FIGURE 10.13 Laplacian masks used to implement Eqs. (10.1-14) and (10.1-15), respectively.

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Edge Detection

- Consider the function:

A Gaussian function

$$h(r) = -e^{-\frac{r^2}{2\sigma^2}} \quad \text{where} \quad r^2 = x^2 + y^2$$

$$\text{and} \quad \sigma: \text{the standard deviation}$$

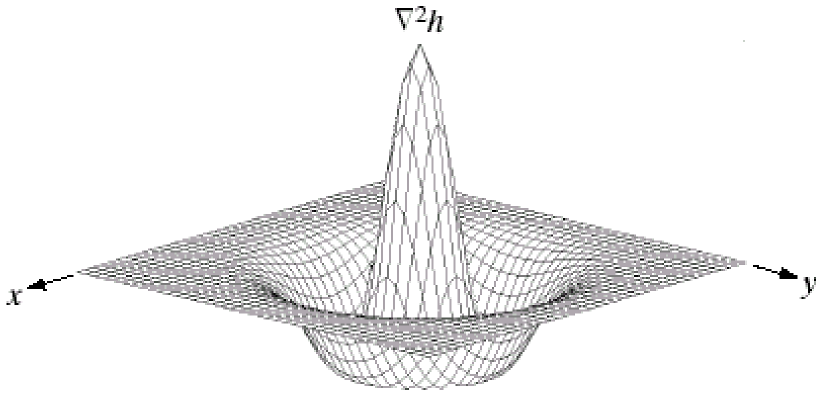
- The Laplacian of *h* is

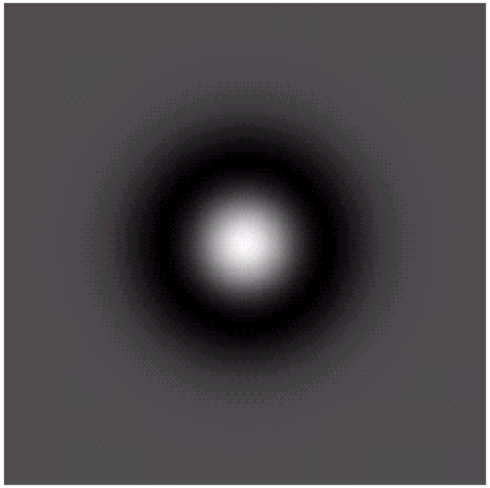
$$\nabla^2 h(r) = -\left[\frac{r^2 - \sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$

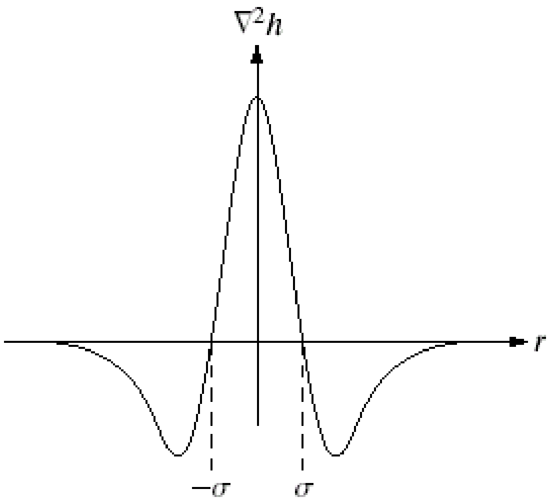
The Laplacian of a Gaussian (LoG)

- The Laplacian of a Gaussian sometimes is called the Mexican hat function. It also can be computed by smoothing the image with the Gaussian smoothing mask, followed by application of the Laplacian mask.

Edge Detection







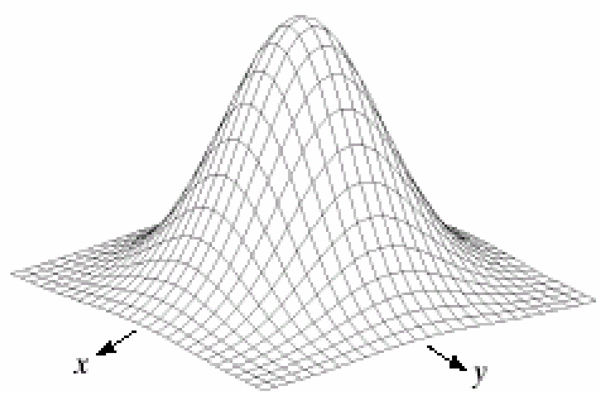
0	0	–1	0	0
0	–1	–2	–1	0
–1	–2	16	–2	–1
0	–1	–2	–1	0
0	0	–1	0	0

a	b
c	d

FIGURE 10.14 Laplacian of a Gaussian (LoG). (a) 3-D plot. (b) Image (black is negative, gray is the zero plane, and white is positive). (c) Cross section showing zero crossings. (d) 5 × 5 mask approximation to the shape of (a).

Edge Detection

Sobel gradient

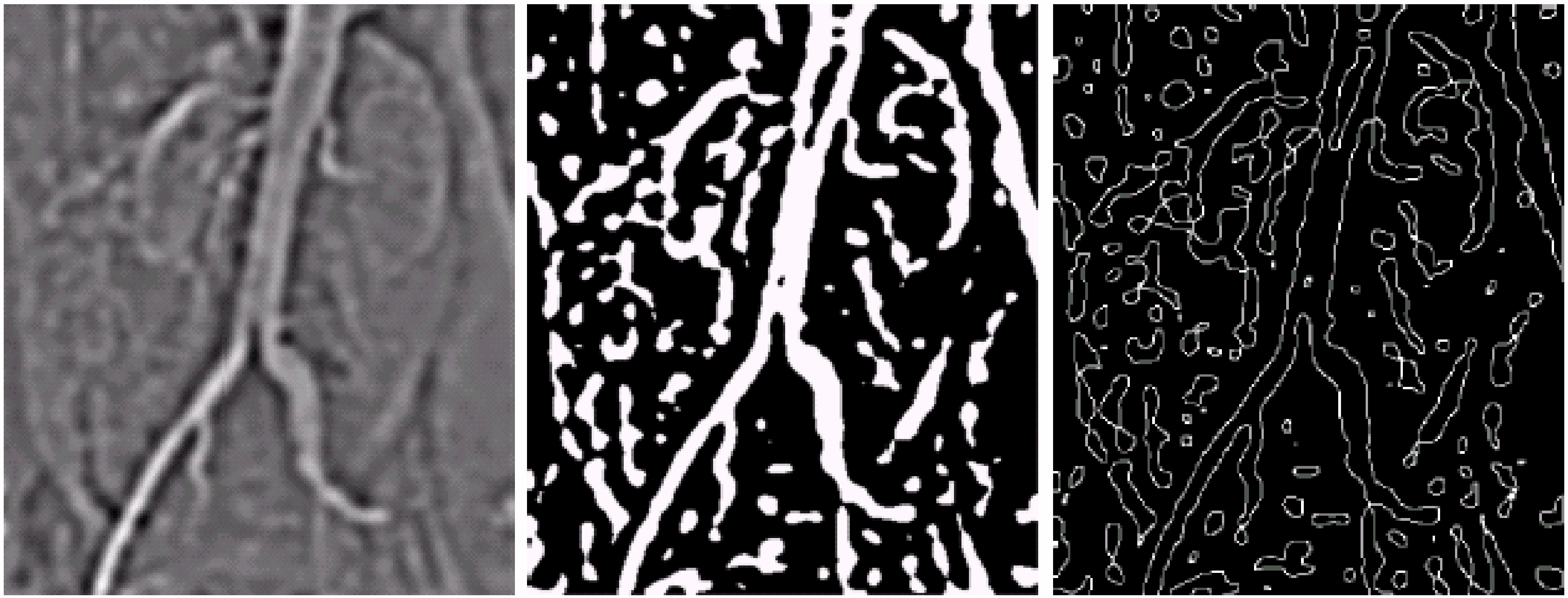


Gaussian smooth function

−1	−1	−1
−1	8	−1
−1	−1	−1

Laplacian mask

Edge Detection



a b
c d
e f g

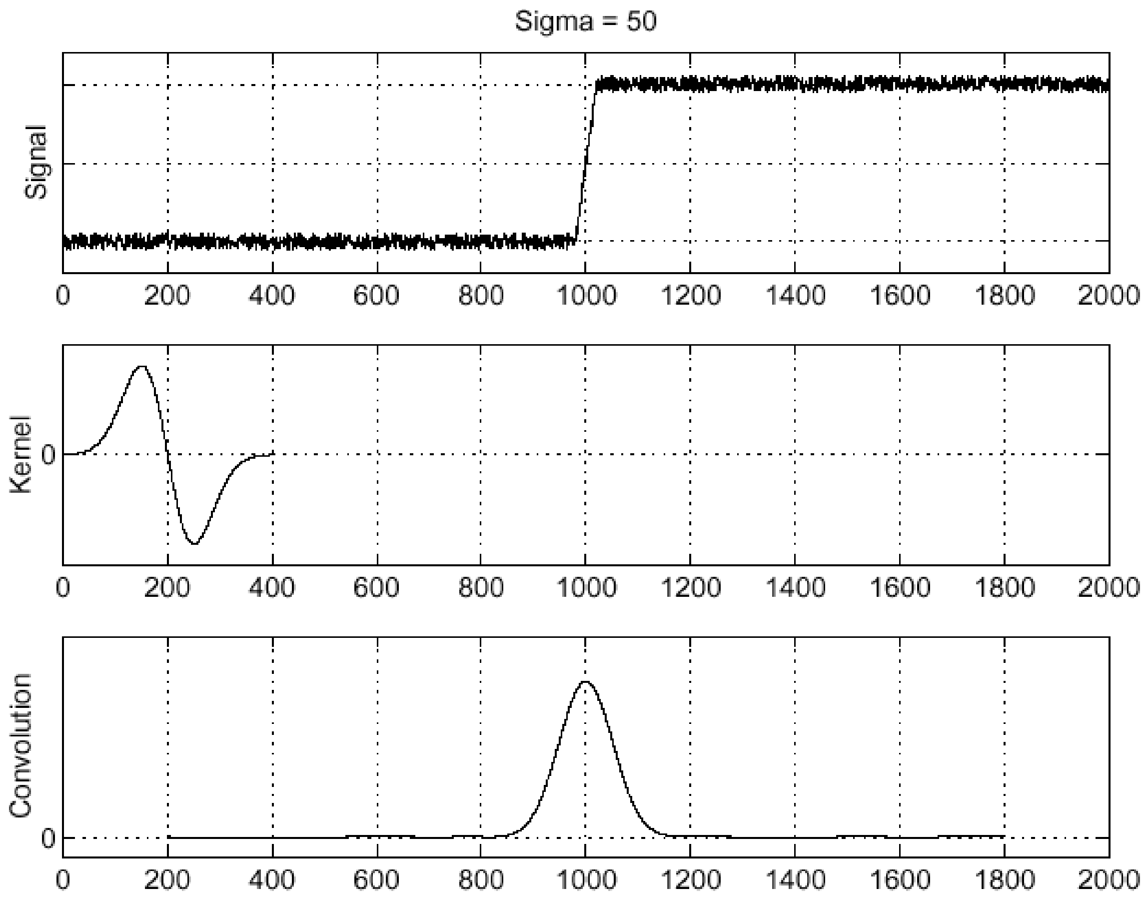
FIGURE 10.15 (a) Original image. (b) Sobel gradient (shown for comparison). (c) Spatial Gaussian smoothing function. (d) Laplacian mask. (e) LoG. (f) Thresholded LoG. (g) Zero crossings. (Original image courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Summary

$$f$$

$$\frac{d}{dx}g$$

$$f * \frac{d}{dx}g$$



Summary

f

$\frac{d^2}{dx^2} g$

$f * \frac{d^2}{dx^2} g$