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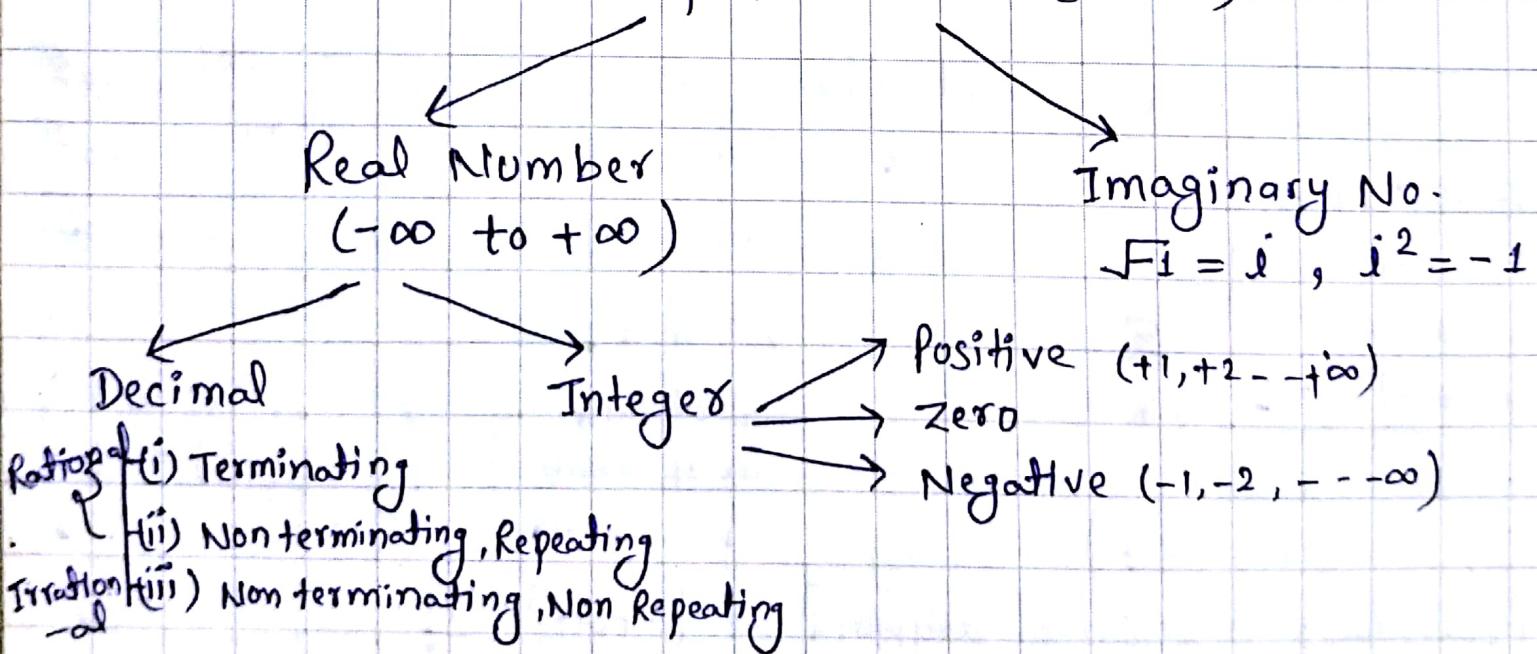
"Number System"

Face value: 6783 ; Face value of 6 = 6

Place value: 6783 , Place value of 6 = 6000

Classification of Numbers: All numbers in this world fall under the category of complex numbers.

Complex Number ($a+ib$)



$0.5 = 5/10$ (Terminating Decimal) \rightarrow Rational

$0.\overline{3} = 3/9$ (Non terminating, Repeating) \rightarrow Rational

~~π, √2, √3~~ all are Irrational numbers.

$\sqrt{5}, \sqrt{7}$

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Integers

Even (no. divisible by 2) (-∞, -4, -2, 0, 2, 4)

Odd (no. not divisible by 2)

Positive Integers

Prime no.

(No. having only two factors 1 & itself)

Composite

(No. having more than two factors)
e.g. 4, 6, 8

Neither prime nor composite

(e.g. = 1)

**
3

Every prime no. is written in the form $6N+1, 6N-1$

3 2 is the smallest prime no.

3 2 is the only even prime no.

3 101 is the smallest 3 digit prime no.

Relatively Prime: Set of any two no. having $HCF = 1$

e.g. $\{1, 3\}, \{9, 25\}$

Co-prime No.: Set of two prime No. having $HCF = 1$

e.g. $\{2, 3\}, \{5, 7\}$

Every co-prime is relatively prime no.

Twin-Prime No.: Set of two prime no. having difference is 2

e.g. $(5, 7), (17, 19), (11, 13)$

Perfect No.: If sum of all factors of a number (excluding that no.) is equal to that number.

e.g. $6 = 1, 2, 3, 6 = 1+2+3 = 6$

$28 = 1, 2, 4, 7, 14, 28 = 1+2+4+7+14 = 28$

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Extra Numerical

Q. Rational numbers b/w a & B = $\frac{aK+B}{K+1}$ where $K=1,2,3 \dots \infty$

Q. If P and Q are two relatively prime no. such that $P+Q=10$ $P < Q$. How many pairs are possible for P, Q

ans:

$$P+Q=10$$

$$\boxed{1+9=10} \checkmark$$

$$\boxed{2+8=10} \times$$

$$\boxed{3+7=10} \checkmark$$

$$4+6=10 \times$$

$\{1,9\}, \{3,7\}$

[2 pairs] are possible

DIVISIBILITY Rule :

- 0 is divisible by all numbers except 0, but no number is divisible by 0.
- 1 is not divisible by any number except 1, but all numbers are divisible by 1.
- [2] → (A no. is divisible by 2, if its last digit is 0 or Even)
- [3] → (A no. is divisible by 3, if sum of all digits is divisible by 3 for e.g. 4689)
- [4] → (A no. is divisible by 4 if the no. formed by the last 2 digits is divisible by 4)
- [5] → (A no. is divisible by 5 if the last digit is 0 or 5)
- [6] → (A no. is divisible by 6 if it is divisible by both 2 and 3)

- [7] (Take the unit's digit of the number, multiply it by 2 and subtract from the remaining number, if this is divisible by 7, the original number is also divisible by 7)
- [8] (A no. is divisible by 8 if the no. formed by last three digits is divisible by 8)
- [9] (sum of digits should be divisible by 9)
- [10] (A no. is divisible by 10 if the last digit is zero)
- [11] (If the difference b/w sum of all digits at odd places & sum of all digits at even places is divisible by 11 or becomes zero)
- [12] (A no. is divisible by 12, if it is divisible by both 3 and 4)

Divisibility rule of 3 & 9 can't be expanded for 27 as well.

e.g. Let take 54, sum of digit = 9, which is not divisible by 27.
but 54 is divisible by 27.

How the Rule goes for 3, & 9

$$xy = 10x + y = \underbrace{9x}_{\substack{\text{div. by} \\ 3 \& 9}} + x + y \quad \text{Now we have to check } x + y$$

if $(x+y)$ is divisible by 3 then it will be divisible by 3
if $(x+y)$ is divisible by 9 then it is div. by 9.

Similarly $xyz = 100x + 10y + z$
 $= \underbrace{99x + 9y}_{\Leftarrow} + x + y + z \quad \text{we have to check for } x + y + z$



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Ques: Find remainder when 857873 $\boxed{025}$ divided by 8.

ans: Rem = 1

Ques: Find rem. when no. 53762 is divided by 11.

ans Odd = $(5+7+2) = 14$, even = $(3+6) = 9$

$$\text{Difference} = 14 - 9 = 5$$

So remainder will be 5.

Q. If a no. 9467 x 893 is divisible by 3. Find sum of all possible values of x .

ans: $4+x$ So, $x = 2, 5, 8$

$$\text{So, } \text{sum} = 2+5+8 = \boxed{15}$$

Q. If a no. 875 x 321 is divisible by 9, then find value of x .

ans: $8+x$ $\boxed{x=1}$

* Q. If 876A43B is divisible by 9. & B is an even no. Find the sum of all values of A.

ans: $1+A+8$ it should be divisible by 9.

$$1+6+2$$

$$1+4+4$$

$$1+2+6$$

$$1+\boxed{0,9}+8$$

$$\text{Sum of all values of } A = 6+4+2+0+9$$

$$= \boxed{21}$$

Question: If a no. $857x32$ is divisible by 11. Find value of x .

ans: $\begin{array}{r} 857x32 \\ \uparrow \uparrow \uparrow \\ 3+7-x+1 = \text{difference} \\ 11-x \quad \text{so, } \boxed{x=0} \end{array}$

Combined divisibility Rule of 7, 11, 13

$$\begin{array}{r} 659\ 624 \\ \text{odd} \quad \text{even} \\ \text{difference} = 35 \rightarrow \boxed{\text{divisible by 7}} \\ \text{not divisible by 11} \\ \text{not divisible by 13} \end{array}$$

so, given no. also divisible by 7

not divisible by 11

not divisible by 13.

If difference = 77, \rightarrow No. will be divisible by 7 & 11
but not by 13.

Question: If a six digit no. is formed by repeating a three digit no. for e.g. 214214, 656656
So, this resultant no. will be divisible by
(a) Only 7 (b) Only 11 (c) Only 13 (d) ~~1001~~ 1001

ans: $\begin{array}{r} 656, 656 \\ \text{diff} = 0 \end{array} \rightarrow \begin{array}{l} \text{div by 7} \\ \text{div by 11} \\ \text{div by 13} \end{array}$

so, no. also divisible by $= 7 \times 11 \times 13$
 $= \boxed{1001}$

OR $656656 = 656000 + 656$
 $= 656(1000+1) = 656 \times 1001$

It will be divisible by 1001.

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Question

If A, B, C, D, E are digits and $ABC \times DEED = ABCABC$

Find $DEED$

ans : 1001

$$\overline{3} \quad AB \times 101 = ABAB$$

$$\overline{3} \quad ABC \times 1011 = ABCABC$$

$$\overline{3} \quad AB \times 10101 = ABABAB$$

How to make divisibility Rule :

Let's take 13

(Last digit) $\alpha + 1$
 ↓
 any no.

(Last digit) any no. + Rem

for $\alpha = 4$ it will be divisible by 13
 ↓ odd factor

Q.

$$\begin{array}{r} 10\textcircled{4} \rightarrow 4 \\ + 16 \\ \hline 26 \end{array} \qquad \begin{array}{r} 16\textcircled{9} \rightarrow 4 \\ - 36 \\ \hline 52 \end{array}$$

Odd factors :

for 11 $\rightarrow -1$

for 17 $\rightarrow -5$

for 29 $\rightarrow 3$

for 7 $\rightarrow -2$

<u>Unit digit concept:</u>	$2^1 = \boxed{2}$	$2^5 = \boxed{32}$	cyclicity = 4
	$2^2 = \boxed{4}$	$2^6 = \boxed{64}$	
	$2^3 = \boxed{8}$	$2^7 = \boxed{128}$	
	$2^4 = \boxed{16}$	$2^8 = \boxed{256}$	

Cyclicity 1: $\{0, 1, 5, 6\}^n = \{0, 1, 5, 6\}$

Question: Find unit digit of $5^{444} + 6^{666} = 5+6 = \boxed{1}$

Cyclicity 2: $\{4, 9\}$

$$\begin{array}{l|l} 4^1 = \boxed{4} & 4^3 = 6 \boxed{4} \\ 4^2 = 1 \boxed{6} & 4^4 = 25 \boxed{6} \end{array}$$

it means

$$\begin{cases} 4^{\text{odd}} = 4 \\ 4^{\text{even}} = 6 \end{cases}$$

Similarly for 9, $9^1 = \boxed{9}$ | $9^3 = 72 \boxed{9}$
 $9^2 = 8 \boxed{1}$ | $9^4 = 656 \boxed{1}$

it means $\begin{cases} 9^{\text{odd}} = 9 \\ 9^{\text{even}} = 1 \end{cases}$

Question: Find unit digit $9^{124} + 4^{123}$
 $= 1 + 4 = \boxed{5}$

Cyclicity 4: $\{2, 3, 7, 8\}$

$$\begin{array}{l|l} 3^1 = \boxed{3} & 3^5 = 24 \boxed{3} \\ 3^2 = \boxed{9} & 3^6 = 72 \boxed{9} \\ 3^3 = 2 \boxed{7} & 3^7 = 218 \boxed{7} \\ 3^4 = 8 \boxed{1} & 3^8 = 656 \boxed{1} \end{array} \quad \begin{array}{ll} 7^1 = 7 & 8^1 = 8 \\ 7^2 = 9 & 8^2 = 4 \\ 7^3 = 3 & 8^3 = 2 \\ 7^4 = 1 & 8^4 = 6 \end{array}$$

Question: $2^{1037} = 2^1 = \boxed{2}$

Question: $7^{5398} = 7^2 = \boxed{9}$

Question: $8^{1000} = 8^4 = 64 \times 64 = \boxed{6}$

Question: $3^{41} \times 7^{42} \times 8^{43}$

$$= 3^1 \times 7^2 \times 8^3$$

$$= 3 \times 9 \times 2$$

$$= \boxed{4} \text{ unit digit}$$

Q. Find unit place of 7^{44}^{43}
 $\frac{44^{43}}{4} = 4K$ completely
 div. by 4.
 ∴ ans = 1

Question: $(121)^{111} \times (1287)^{53}$

$$= 1 \times 7^1 = \boxed{7}$$

Q. Find unit place of 3^{42}^{44}
 $\frac{42^{44}}{4} = \frac{2^{44} \times 21^{44}}{4} \rightarrow \text{Rem} = 0$
 ∴ ans = 1

Question: $(259)^{148} - (123)^{43}$ Find unit digit

ans: $9^{148} - 3^{43}$

$$= 1 - 7 = 4 \quad (\text{unit digit can't be negative})$$

Question: Find unit digit of $1! + 2! + 3! + 4! + 5! + \dots + 100!$

→ Unit digit of $5!$ & above that are having 0 at its unit place.

ans:

$$1! + 2! + 3! + 4!$$

$$= 1 + 2 + 6 + 24$$

$$= \boxed{3} \text{ unit digit}$$

Q. Find unit place of 8^{43}^{43}
 $\frac{43^{43}}{4} = \frac{(-1)^{43}}{4} \Rightarrow \text{Rem} = 3$
 ∴ ans = 2

Q. Find Unit digit of $2^{11!}$

$$11! = 1 \times 2 \times 3 \times 4 \cdots \times 11$$

\Rightarrow Any factorial greater than $3!$ is divisible by 4 completely.

$$2^{11!} = 2^4 = \boxed{6} \text{ unit digit}$$

Q. Find unit digit of $(823)^{123!} \times (237)^{234!}$

$$= 3^4 \times 7^4$$

$$= 1 \times 1 = \boxed{1} \text{ unit digit}$$

Concept: 1, 1, 1, 1, 1, 1, 1, 1 \rightarrow cyclicity 1

so, cyclicity of 1 can be 4

$\{0, 1, 5, 6\} \rightarrow$ can be 4

Similarly $4, 6, 4, 6, 4, 6, 4, 6$ $\{4, 9\}$ cyclicity 4

$\{2, 3, 7, 8\}$ cyclicity 4

All no. has cyclicity = 4

Question: If x is a real no. $x^7 - x^3 = 1232$, how many value are possible for x ?

ans: $x^7 - x^3 = 1232$

$$\begin{matrix} \downarrow \\ \text{unit same} \end{matrix} x^3 - x^3 = 1232 \rightarrow \text{zero}$$

it means there is no such value of x is possible.

NOTES

Conversion of a Number from Non-terminating & Repeating decimal to the form p/q

Question Convert $0.\overline{233333\dots}$ into p/q .

ans: $N = 0.\overline{23}$

$$10N = 2.\overline{33333\dots} \quad \text{(i)}$$

$$100N = 23.\overline{333\dots} \quad \text{(ii)}$$

$$\text{eq. } \textcircled{2} - \textcircled{1} = 90N = 21$$

$$N = \frac{21}{90} = \boxed{\frac{7}{30}}$$

Short cut: $N = 0.\overline{23}$

$$\text{Fraction} = \frac{23 - 2}{90} \quad \begin{matrix} \nearrow \text{Non Repeating part} \\ \searrow \end{matrix}$$

$\begin{matrix} \nwarrow \\ \text{Repeating part} \end{matrix} \quad \begin{matrix} \nearrow \\ \text{Non Repeating} \end{matrix}$

$$= \frac{21}{90} = \frac{7}{30}$$

Question: Convert $37.\overline{565656\dots}$ into the form p/q

ans: Let, $N = 37.\overline{56} \quad \textcircled{1}$

$$100N = 3756.\overline{56} \quad \textcircled{2}$$

$$99N = 3756 - 37$$

$$N = \frac{3719}{99}$$

Factors: A number which is able to completely divide a number greater than or equal to it.

Total Number of factors of a number:

If a number can be expressed as a power of its prime components as

$$N = a^p \times b^q \times c^r \text{ & so on}$$

where a, b, c etc. are prime numbers and p, q, r etc are positive integers, then, number of factors of a number = $(p+1) \times (q+1) \times (r+1)$ **

Ex. Find no. of factors of 24

ans $24 = 2^3 \times 3^1$

$$\text{No. of factors} = (3+1) \times (1+1) = 4 \times 2 = 8$$

We know those are, 1, 2, 3, 4, 6, 8, 12, 24

Ex. Find no. of factors of 3600

ans $3600 = 6^2 \times 10^2$

$$= 2^2 \times 3^2 \times 2^2 \times 5^2$$

$$= 2^4 \times 3^2 \times 5^2$$

$$\therefore \text{Total no. of factors} = (4+1) \times (3) \times (2+1) \\ = 5 \times 3 \times 3 \\ = 45$$

Sum of factorsEx. 45

$$45 = 3^2 \times 5^1$$

Now, split each prime factor as sum of every distinct factors.

$$= (3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$$

$$= (1+3+9) \times (1+5) = 13 \times 6 = \boxed{78}$$

3 Non perfect squares will have even number of factors
 Every perfect square will have odd number of factors because its square root number will pair with itself.

Product of factors: As we know, factors will occur in pairs for the numbers except perfect squares.

So, product of factors = $\frac{\text{Total factors}}{2}$

for eg. 24 has 8 factors. So, product = $\boxed{4}$

which are $(1, 24), (2 \times 12), (3 \times 8), (4, 6)$

Even factors: Let $N = 2^3 \times 3^2 \times 5^1$

even factors = $3 \times (2+1) \times (1+1) = 18$ factors

Odd factors:

Total factors - even factors

Conceptually, For finding out the number of odd factors, the contribution of the even prime number, that is, 2 will not be considered except for its power 0, that is, for an odd factor, only 2^0 will be taken into consideration.

Number of Zeros : How to find no. of zeros in $n!$

For 1 zero we need a $\frac{10}{5} \rightarrow 2$
So, we will try to find no. of 5 in $n!$

$$= \left[\frac{n}{5} \right] + \left[\frac{n}{5^2} \right] + \left[\frac{n}{5^3} \right] + \dots$$

Question Find no. of zeros in $100!$

ans

$$\begin{array}{r} 5 | 100 \\ 5 | 20 \\ \hline 4 \end{array} = 20 + 4 = 24 \text{ ans}$$

Question Find no. of zeros in $154!$

ans

$$\begin{array}{r} 5 | 154 \\ 5 | 30 \\ \hline 6 \\ | \end{array} = 30 + 6 + 1 = 37 \text{ ans}$$

Question Find trailing zeros in $56!$

ans:

$$\begin{array}{r} 5 | 56 \\ 5 | 11 \\ \hline 2 \end{array} = 11 + 2 = 13 \text{ ans}$$

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Question: $2 \times 4 \times 6 \times 8 \times \dots \times 250$. Find no. of zeros.

ans: $2^{125} (1 \times 2 \times 3 \times 4 \times \dots \times 125)$

$$= 2^{125} \times 125!$$

Now we will find how many 5 in 125!

$$\begin{array}{r} 125 \\ \hline 5 | 25 \\ 5 | 5 \\ \hline 5 | 1 \\ \end{array}$$

Total = (31)

Question: $5 \times 10 \times 15 \times 20 \times \dots \times 500$

$$5^{100} (1 \times 2 \times 3 \times \dots \times 100)$$

$$5^{100} \times 100!$$

$$\begin{aligned} \text{Total zeros} &= 50 + 25 + 12 + 6 + 3 + 1 \\ &= (97) \end{aligned}$$

$$\begin{array}{r} 100 \\ \hline 2 | 50 \\ 2 | 25 \\ 2 | 12 \\ 2 | 6 \\ 2 | 3 \\ \hline 1 \end{array}$$

Question: $10 \times 20 \times 30 \times 40 \times \dots \times 1000$ Find no. of zeros

ans: $10^{100} (1 \times 2 \times 3 \times 4 \times \dots \times 100)$

$$= 10^{100} \times 100! \quad \text{we know in } 100! = 24$$

No. of zeros = $100 + 24 = (124)$

Question: Find no. of zeros $126! - 125!$. Find zeros.

ans: $126 \times 125! - 125!$

$$125! \times 125$$

$$= 31 + 3 = [34] \text{ ans}$$

$$\begin{array}{r} 125 \\ \hline 5 | 25 \\ 5 | 5 \\ \hline 1 \end{array} = 31$$

Question: Find no. of zeros in $18! + 19! = 18! \times (20)$

$\frac{5}{\cancel{3}} \quad 3+1 = \boxed{4}$ zeros.

Question: Find no. of zeros in

$$(3^{123} - 3^{122} - 3^{121}) (2^{122} - 2^{121} - 2^{120})$$

$$= 3^{121}(3^2 - 3^1 - 1) 2^{120}(2^2 - 2 - 1)$$

$$= 3^{121} \times 5 \times 2^{120} \times 1$$

$$= \boxed{1} \text{ zero.}$$

Question: Find no. of zeros in

$$1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times \dots \times 100^{100}$$

ans: Try to find 5 here,

$5^5 = 1$ five, $10^{10} = 10$ fives, $15^{15} = 15$ fives, $20^{20} = 20$ fives $25^{25} = 50$ fives
we will not disturb A.P. & write double terms alone.

$$5 + 10 + 15 + 20 + 25 + \dots + 100$$

$$= \frac{n}{2} (a+l) = (100+5) \times \frac{20}{2} = 10 \times 105$$

$$= \boxed{1050} \text{ zeros}$$

Now for double terms $25 + 50 + 75 + 100 = \boxed{250}$ zeros

$$\text{so, Total zeros} = 1050 + 250$$

$$= \boxed{1300}$$

Remainder Theorem :

$$\begin{array}{r} 3 \\ \text{Quotient} \\ \hline 5 \sqrt{17} \\ \text{divisor} \quad | \\ 15 \\ \hline 2 \end{array} \rightarrow \text{dividend}$$

(2) → Remainder

5 $\boxed{\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}}$

Question 1 : In a division process divisor is 5 times of quotient & remainder is 3 times of quotient. If remainder is 15. Find dividend.

ans

$$\text{Dividend} = (5 \times 25) + 15 = 125 + 15 = \boxed{140}$$

$$25 \sqrt{140} (5)$$

$$\underline{15}$$

Question : When a no. is divided by 102 leaves remainder 87. If same no. is divided by 17. Find Remainder

ans

$$102 \sqrt{m} (x)$$

$$\underline{87}$$

$$m = 102x + 87$$

Now when divided by 17

$$\cancel{102} \overline{102x + 87} \\ \cancel{102} \quad |$$

$$= \boxed{2} \quad \underline{\text{ans}}$$

$$\begin{array}{ccc} 102 & \longrightarrow & 87 \\ 17 & \longrightarrow & 85 \end{array}$$

$\boxed{2}$ Remainder

Q. When a no. is divided by 361 leaves remainder 47
if the same no. is divided by 19. Find remainder.

ans: $361 \rightarrow 47$

$$19 \rightarrow 38$$

Rem. $\boxed{9}$ ans

Note \rightarrow

$$5 \sqrt{26} \boxed{5}$$

$\frac{25}{R_1 = 1}$

$$5 \sqrt{27} \boxed{5}$$

$\frac{25}{R_2 = 2}$

$$\text{Now } 26+27=53$$

$$5 \sqrt{53} \boxed{10}$$

$\frac{50}{3}$

$$5 \sqrt{28} \boxed{5} + 5 \sqrt{29} \boxed{5} = 5 \sqrt{57} \boxed{11}$$

$\frac{25}{R = 3}$ $\frac{25}{R = 4}$ $\frac{55}{R = 2}$

it means,

$$\text{divisor} = R_1 + R_2 - R_3$$

Question: When a certain no. is divided by a certain divisor leaves remainder 43 and another no. is divided by the same divisor leaves remainder 37. If the sum of both no. is divided by same divisor leaves remainder 13.

Find divisor.

ans $\text{Divisor} = 43 + 37 - 13$

$$= \boxed{67}$$

Important concept: $\frac{x}{D} \rightarrow \boxed{\text{Rem} = R}$, $\frac{2x}{D} \rightarrow \text{Rem} = \boxed{\frac{2R}{D}}$

e.g. $x/13 \rightarrow \text{Remainder} = 10$

$$2x/13 \rightarrow \text{Rem} = 20/13 = \boxed{7}$$

Question: $x/m \rightarrow \text{Rem} \rightarrow 20$, $2x/m \rightarrow 9$

divisor $m = ?$

ans $m = 40 - 9 = \boxed{31}$

* $\frac{x}{D} \rightarrow \text{Rem} = R$, $\frac{x^2}{D} \rightarrow \text{Rem} = \boxed{R^2/D}$
 $\frac{x^3}{D} \rightarrow \text{Rem} = \boxed{R^3/D}$

e.g. $x/11 \rightarrow \text{Rem} = 7$, $x^2/11 \rightarrow \text{Rem} = 49/11 = \boxed{R=5}$

3 $\frac{A \times B}{m}$ $\boxed{R = A_R \times B_R}$

3 $\frac{A+B}{m}$ $\boxed{R = A_R + B_R}$

Negative Remainder

$$11 \sqrt{98} \quad 9$$

$$\begin{array}{r} \\ 99 \\ \hline \end{array}$$

$$R = \boxed{-1} + 11 = \boxed{10}$$

Concept of modularity:

$$\frac{14}{4} \quad \boxed{R=2}$$

We can't cut down
the ratios

$$\frac{7}{2} \quad \boxed{R=1}$$

if done then multiply
New R with 2, $1 \times 2 = \boxed{2} R$

$$\frac{(x+a)^n}{x} \Rightarrow R = a^n$$

$$\frac{(-x+a)^n}{x} \Rightarrow R = a^n$$

$$\frac{(x+1)^n}{x} \Rightarrow \text{Rem} = 1^n = 1$$

$$\frac{(x-1)^n}{x} \Rightarrow \text{Rem} = (-1)^n$$

↓

1 when n is even
-1 when n is odd

$$-1+x$$

$$\text{Q. } \frac{25^{26}}{26} = \frac{(26-1)^{26}}{26} = (-1)^{26} = 1$$

$$\text{Q. } \frac{27^{29}}{28} = \frac{(28-1)^{29}}{28} = (-1)^{29} = -1 + 28 \Rightarrow \boxed{\text{Rem} = 27}$$

$$\text{Q. } \frac{37^{39}}{36} = \frac{(36+1)^{39}}{36} = (1)^{39} = 1$$

$$\frac{4^{567-12}}{6}$$

$$\boxed{\text{Rem} = 4}$$

$$\frac{4^1}{6} \quad \text{Rem} = 4$$

$$\frac{4^2}{6} \quad \text{Rem} = 4$$

$$\frac{4^3}{6} \quad \text{Rem} = 4$$

Every power of 4 when divided by 6 will give us 4 as remainder.

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Question: $\frac{2^{111}}{5}$ Find remainder.

ans: $\frac{2 \cdot 2^{110}}{5} = \frac{2 \cdot 4^{55}}{5} = \frac{2(5-1)^{55}}{5} = \frac{(-2)^{55}}{5} = \boxed{-2} + 5 = \boxed{3}$ final remainder

Question: $\frac{3^{162}}{162}$ → Find remainder.

ans $\frac{3^{162}}{162} = \frac{3^4 \cdot 3^{158}}{2 \times 81} = \frac{3^{158}}{2} = \frac{(2+1)^{158}}{2}$

Rem = $(1)^{158} = 1$ (which is not correct by concept of modularity)

$$\boxed{\text{Rem} = 81}$$

Question: $\frac{65 \times 78}{9} = \frac{2 \times 6}{9} = \boxed{3}$ Remainder

Question: $\frac{1! + 2! + 3! + 4! + 5! + 6! + \dots + 1000!}{10}$

When any no. is divided by 10, the remainder will be last digit of that no.

ans $\frac{1 + 2 + 6 + 24 + 120 + \dots}{10} = \frac{13}{10}$

$\boxed{\text{Rem} = 3}$

Similarly, when you divide by 100 then the remainder is the last two digits.

$$\# \frac{103 \times 1298 \times 13702 \times 1197}{100} = \frac{36}{100} \text{ Rem} = [36]$$

$$\text{Q. } \frac{(37)^{27}}{9} = \frac{(9 \times 4 + 1)^{27}}{9} = \frac{(+1)^{27}}{9} \Rightarrow \boxed{\text{Rem} = 1}$$

$$\text{Q. } \frac{2^{63}}{9} = \frac{(2^3)^{21}}{9} = \frac{(9-1)^{21}}{9} = \frac{(-1)^{21}}{9} \Rightarrow \text{Rem} = -1 + 9 = [8]$$

$$\text{Q. } \frac{2^{64}}{9} = \frac{(2^3)^{21} \times 2}{9} = \frac{(9-1)^{21} \times 2}{9} \Rightarrow \text{Rem} = -2 + 9 = [7]$$

$$\text{Q. } \frac{3^{68}}{82} = \frac{(3^4)^{17}}{82} = \frac{(82-1)^{17}}{82} = \frac{(-1)^{17}}{82} = -1 + 82 \Rightarrow [81]$$

$$\text{Q. } \frac{7^{400}}{400} = \frac{(7^4)^{100}}{400} = \frac{(2401)^{100}}{400} = \frac{(1)^{100}}{400} \quad \boxed{\text{Rem} = 1}$$

$$\text{Q. } \frac{3^{432}}{80} = \frac{(3^4)^{1080} \times 3}{80} = \frac{1 \times 3}{80} = [3] \text{ Rem}$$

$$\text{Q. } \frac{2^{76}}{96} = \frac{2^5 \times 2^{71}}{2^5 \times 3} = \frac{2^{71}}{3} = \frac{(-1)^{71}}{3} = +2$$

$$\text{Final remainder} = 2 \times 2^5 = 64$$

$$\text{Q. } \frac{5^{500}}{500} = \frac{5^{500}}{5^3 \times 4} = \frac{5^{497}}{4} = \frac{(1)^{497}}{4} \quad \boxed{\text{Rem} = 1}$$

$$\text{Final remainder} = 1 \times 125 = [125]$$

$$\text{Q. } \frac{2^{35}}{10} = \frac{2^{35}}{2 \times 5} = \frac{2^{34}}{5} = \frac{(4)^{17}}{5} = \frac{(-1)^{17}}{5} \Rightarrow \text{Rem} = -1 + 5 = 4$$

$$\text{Final remainder} = 4 \times 2 = [8]$$



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$$\text{Q. } \frac{15^{40}}{100} = \frac{5^{40} \times 3^{40}}{5^2 \times 4} = \frac{5^{38} \times 3^{40}}{4} = \frac{1 \times 1}{4} = \boxed{1}$$

$$\text{Rem} = 1 \times 25 = 25$$

Euler's Theorem

$$\boxed{\frac{n^{\Theta(P)}}{P} = 1}$$

where n & P are co-prime no.

where, $\Theta(P)$ = Euler's totient for P (cyclicity)

if $P = a^b \times c^d \times e^f$ where a, c, e are prime factors.

$$\text{then } \Theta(P) = P \left(1 - \frac{1}{a}\right) \times \left(1 - \frac{1}{c}\right) \times \left(1 - \frac{1}{e}\right)$$

$$\text{Q. } \frac{2^{111}}{5} = \frac{2^3}{5} = \frac{8}{5} = \boxed{\text{Rem} = 3}$$

$$\text{How to find totient of } 5 = 5 \times \left(1 - \frac{1}{5}\right) = 4$$

for any prime no. Euler's totient is 1 less than the number.

$$\text{Q. } \frac{3^{2146}}{17} \rightarrow \text{totient} = 16 = \frac{3^2}{17} = \frac{9}{17} \rightarrow \boxed{\text{Rem} = 9}$$

$$\text{Q. } \frac{2^{5557}}{13} \rightarrow \text{totient} = 12 = \frac{2^1}{13} \rightarrow \boxed{\text{Rem} = 2}$$

$$\text{Q. } \frac{32^{32^{32}}}{7} \rightarrow \text{totient} = 6 = \frac{32^4}{7} = \frac{4^4}{7} = \frac{16 \times 16}{7} = \boxed{4} \text{ Rem} = 4$$

Fermat Theorem

$$\boxed{\frac{a^{p-1}}{p} = 1}$$

Rem.

where p is prime

$$\text{Q. } \frac{2^{100}}{101} = 1$$

Lcm & HCF :

Lcm : (Least common multiple)

e.g. $12 = 12, 24, 36, 48, 60, 72, 84, 96 \dots$

$16 = 16, 32, 48, 64, 80, 96 \dots$

common multiple = $48, 96$

Least common multiple = $\boxed{48}$

Lcm of x, y, z is the smallest no. which is exactly divisible by x, y, z .

How to Find LCM : (i) Prime Factorization method

$$12 = 2 \times 2 \times 3$$

$$16 = 2 \times 2 \times 2 \times 2$$

$$\text{Lcm} = 2 \times 2 \times 2 \times 2 \times 3 = \boxed{48}$$

Division method :

2	12	16
2	6	8
2	3	4
2	3	2
3	3	1
	1	1

$$\begin{aligned}\text{ans} &= 2 \times 2 \times 2 \times 2 \times 3 \\ &= \boxed{48}\end{aligned}$$

Third method : Let us suppose 16 is LCM.

$$\frac{3}{4} > 12$$

$$16 \times 3 = \boxed{48}$$

Q. Lcm of 9, 12, 15

ans $15 \times 3 \times 4 = \boxed{180}$

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Question

Answer

Q. Find smallest no. which is exactly divisible by x, y, z

Lcm of x, y, z

Q. Find the smallest no. which when divided by x, y, z and leaves remainder R in each case

Lcm of x, y, z + R

↓
exactly div. by x, y, z

Q. Find the smallest no. which when divided by x, y, z leaves remainder a, b, c respectively.

Lcm of $x, y, z - K$

where $K = (x-a) = (y-b) = (z-c)$

Question: Find the smallest no. which when divided by 25, 15, 30 and leaves remainder 21, 11, 26 respectively.

ans: $25-21 = 15-11 = 30-26 = 4$

Now, Lcm of 25, 15, 30 = $30 \times 5 = 150$

ans will be, $150 - 4 = 146$

$$\text{Lcm} = \frac{150}{9} = 25 + 25 = 125 + 21 + 4$$

$$150 = 135 + 15 = 135 + 11 + 4$$

$$150 = 120 + 30 = 120 + 26 + 4$$

146

Question: Find the smallest no. which when divided by 2, 3, 4, 5, 6 and leaves remainder 1 in each case & no. is completely divisible by 7.

ans: Lcm of (2, 3, 4, 5, 6) + 1
= 60 + 1 = 61 but it is not divisible by 7.

so, we will take $60K + 1 = 56K + \boxed{4K + 1}$
 \therefore No. will be $(60 \times 5 + 1)$
= 300 + 1
= $\boxed{301}$

Question: Find the sum of digit of a smallest no. which is divided by 5, 6, 7, 8 leaves remainder 3 but when divided by 9 leaves no remainder.

- (a) 12 (b) 16 ~~(c) 18~~ (d) 21

ans Lcm of (5, 6, 7, 8) = 840
 \therefore No. can be $840K + 1 = 840K + 3$
= $810K + 30K + 3$
= $837K + \boxed{3K + 3}$, for $K = 2$
it is divisible by 9.

\therefore No. will be $= 840 \times 2 + 3$

$$= 1680 + 3 = \boxed{1683}$$

\therefore sum of digits = $1 + 6 + 8 + 3$
= $\boxed{18}$ ans

Question: Let x be a smallest no. which when added to 2000 and resultant no. is exactly divisible by 12, 16, 18, 21. Find the sum of x 's digits.

ans LCM of (12, 16, 18, 21) = $21 \times 6 \times 2 \times 2 \times 2 = 1008$

Now as no. is added to 2000 the LCM can be doubled $2016 = 2000 + \boxed{16}$

↓ added no.

and sum of digits of 16 = $1+6 = \boxed{7}$

- (a) 6 (b) 7 (c) 8 (d) 4

we can go with options. As no. divisible by 18 it is also divisible by 9. $\therefore 2000 + x$

$$x + 2000 \Rightarrow x + \boxed{7} = 9 \\ (\text{sum of } 2000) =$$

Question: Find the largest four digit no. which is completely divisible by 12, 15, 18, 27

- (a) 9690 (b) 9930 (c) 9960 (d) 9720

$$\begin{array}{r} 540 \sqrt{9999} [18] \\ \underline{540} \\ 4599 \\ \underline{4320} \\ \text{Rem} = \boxed{279} \end{array}$$

$$\therefore \text{Largest No.} = 9999 - 279$$

$$= \boxed{9720}$$

HCF (Highest common factor)

$$12 = 1, 2, 3, 4, 6, 12$$

common factors = 1, 2, 4

$$16 = 1, 2, 4, 8, 16$$

Highest common factor = 4

HCF of x, y, z is the largest no. which can divide x, y, z exactly.

How to Find HCF: ① Prime factorization method :

$$12 = \boxed{2} \times \boxed{2} \times 3$$

$$16 = \boxed{2} \times \boxed{2} \times 2 \times 2$$

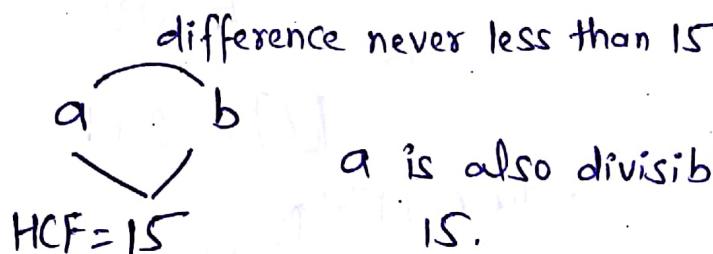
$$\text{HCF} = 2 \times 2 = \textcircled{4}$$

② Division method: For e.g. 12 & 16

$$\begin{array}{r} 12 \sqrt{16} (1) \\ \underline{12} \\ 4 \sqrt{12} (3) \\ \underline{12} \\ 0 \end{array}$$

when remainder becomes zero then final divisor is called HCF.

③ Third method:



a is also divisible by 15.

b is also divisible by 15.

→ HCF can never be greater than difference of two numbers. HCF may be difference or factor of difference.

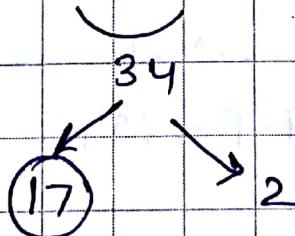
e.g. $12, 16$

$\boxed{HCF = 4}$

<u>Q.</u> Find the largest no. which can divide x, y, z exactly	HCF of (x, y, z)
<u>Q.</u> Find the largest no. which can divide x, y, z and leaves remainder R in each case	HCF of $(x-R), (y-R), (z-R)$ "OR" HCF of $ x-y , y-z , z-x $
<u>Q.</u> Find the largest no. which can divide x, y, z & leaves remainder a, b, c respectively.	HCF of $(x-a), (y-b), (z-c)$

Q. Find the largest no. which can divide 306, 340, 187 exactly.

ans: HCF of 306, 340, 187



ans = 17

Q. Find the largest no. which can divide 1305, 4665, 6905 & leaves remainder equal in each case.

ans hcf of $|6905 - 1305|, |6905 - 4665|, |4665 - 1305|$

hcf of 5600, 2240, 3360

$$\text{HCF} = 1120$$

ans

we can divide the values to get HCF.

2240, 3360, 5600

$$\begin{array}{r} 3360 \overline{)5600} & | \\ 3360 & \\ \hline 2240 \overline{)3360} & | \\ 2240 & \\ \hline (1120) \overline{)2240} & | \\ 2240 & \\ \hline & x \end{array}$$

$$\therefore \text{HCF} = 1120$$

If two no. are divided by their difference or factor of difference then leaves same remainder.

$$\begin{array}{c} 29 \quad 39 \\ \swarrow \quad \searrow \\ (10) \rightarrow 2 \\ \quad \quad \quad \rightarrow 5 \\ \quad \quad \quad \rightarrow 10 \end{array}$$

Question: Two no. 225 and 147 are divided by a two digit no. then leaves same remainder. How many two digit no. are possible?

ans:

$$\begin{array}{c} 225 \quad 147 \\ \swarrow \quad \searrow \\ (78) \rightarrow 39 \\ \quad \quad \quad \rightarrow 26 \\ \quad \quad \quad \rightarrow 13 \end{array}$$

ans is 4 no. are possible.

Question: Two no. 875 and 2272 are divided by a 3 digit no. & leaves same remainder in each case. Find the sum of digit of that no.

ans:

$$\begin{array}{c} 875 \quad 2272 \\ \swarrow \quad \searrow \\ 1397 = 11 \times 127 \end{array}$$

$$\text{sum of digit} = 1+2+7 = 10$$

Relation b/w HCF & LCM : Product of two no. = HCF x LCM

$$a \times b = \text{HCF} \times \text{LCM}$$

Q. HCF & LCM of two no. are 144 & 864, if one no. is 288.
Find second no.

ans:

$$\frac{288}{2} \times b = 144 \times 864$$

$$b = 432$$

Q. If LCM of two no. is twice of bigger no. & difference of HCF & smaller no. is 4. Find smaller no.

ans: Let larger no. = a, smaller = b

$$\frac{2a \times (b-4)}{\text{LCM}} = a \times b$$

$$2b - 8 = b \Rightarrow b = 8$$

3

$$\begin{array}{c} 20 \\ 25 \\ \swarrow \nearrow \end{array} \quad \boxed{\text{HCF} = 5}$$

(5x4)
(5x5)

Nothing common in 4 & 5
first no = 5a
Second no. = 5b

If HCF of two no. is H then no. can be written as Ha, Hb where (a & b) are relatively prime no.

Question: If hcf & LCM of two no. are 13 & 455 if one no. is b/w 75 & 125. Find other no.

ans: No. will be 13a, 13b

$$13a \times 13b = 13 \times 455$$

$$13^2 ab = 455 \Rightarrow ab = 35$$

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x & y can't be 1, 35

$$7, 5 \Rightarrow 13x = 13 \times 7 = 91$$

$$13y = 13 \times 5 = \boxed{65} \text{ ans}$$

Question If HCF & Lcm of two no. are 17. & 408
How many numbers are possible for such type.

ans first = $17x$, $17y$ = second

$$17xy = 408 \Rightarrow xy = 24$$

(1, 24) ✓

(3, 8) ✓

(4, 6) ✗

(12, 2) ✗

only 2 pair
are possible.

Question If HCF & Lcm of two no. are 29 & 4147. Find
the sum of both no., if both numbers are greater
than 29.

ans

$$\text{Let } 29x, 29y \Rightarrow 29xy = 4147$$

$$xy = 143$$

(11) (13)

$$29x = 29 \times 11 =$$

$$29y = 29 \times 13$$

$$\therefore \text{sum} = 29(11+13) = 29 \times 24$$

$$= \boxed{696}$$

Last 2 places :

$$\textcircled{1} \quad 2^{10 \times \text{Even}} = \dots \dots 76$$

$$\textcircled{2} \quad 2^{10 \times \text{Odd}} = \dots \dots 24$$

$$\textcircled{3} \quad (\dots \dots A1)^{\dots \dots B} = \dots \dots (AB)1$$

$$\textcircled{4} \quad (\dots \dots \cancel{N}5)^{\text{odd}} = \dots \dots 75 \\ \dots \dots 25, \text{ otherwise}$$

$$\textcircled{5} \quad 2^{240} = 2^{24 \times 10} = \dots \dots 76$$

$$\textcircled{6} \quad 2^{170} = 2^{17 \times 10} = \dots \dots 24$$

$$\textcircled{7} \quad (71)^{123} = \dots \dots 11$$

$$\textcircled{8} \quad (65)^{125} = \dots \dots 25$$

↓
Even

Q. Find last two places of $(88)^{76}$

$$88^{76} = (8 \times 11)^{76} = 8^{76} \times 11^{76} = 2^{3 \times 76} \times 11^{76}$$

$$= 2^{228} \times 11^{76}$$

$$= 2^{220} \times 2^8 \times 11^{76}$$

$$= \dots \dots 76 \times \dots \dots 56 \times \dots \dots 61$$

$$= \boxed{16 \text{ ans}}$$

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Q. Find last two places of $(59)^{27}$.

$$\begin{aligned}
 \underline{\text{ans}} \quad 59^{27} &= (59)^1 \times (59)^{26} \\
 &= 59^1 \times (59^2)^{13} \\
 &= 59 \times (81)^{13} \\
 &= 59 \times 41 \\
 &= \boxed{19} \quad \underline{\text{ans}}
 \end{aligned}$$

Q. Find last two places of $(75)^{24}$

$$\underline{\text{ans}} \quad \boxed{25}$$

Q. Find last two places of 2^{2222}

$$\begin{aligned}
 \underline{\text{ans}} \quad 2^{2222} &= 2^{2220} \times 2^2 \\
 &= 2^{222 \times 10} \times 2^2 \\
 &= \dots 76 \times 4 \\
 &= \dots 04 \\
 2^{2222} &= 2^{-04} = 2^{A04} \quad \uparrow \text{even} \\
 &= 2^{A00} \times 2^4 = 2^{A0 \times 10} \times 16 \\
 &= 76 \times 16 \\
 &= \boxed{16} \quad \underline{\text{ans}}
 \end{aligned}$$

Q. Find last two places of $21^{50} - 8$

$$\begin{aligned}
 \underline{\text{ans}} \quad 21^{50} &= \dots \underline{\underline{01}} \\
 &\quad - 08
 \end{aligned}$$

$$\boxed{93} \quad \underline{\text{ans}}$$

Q Find unit place of 89^{43} ans 9

Q Find unit place of $2^{88^{18}}$ ans 6

Q Find last two places of 13^{48} ans 21

Q Find last two places of $2^{3^{100}}$ ans 52