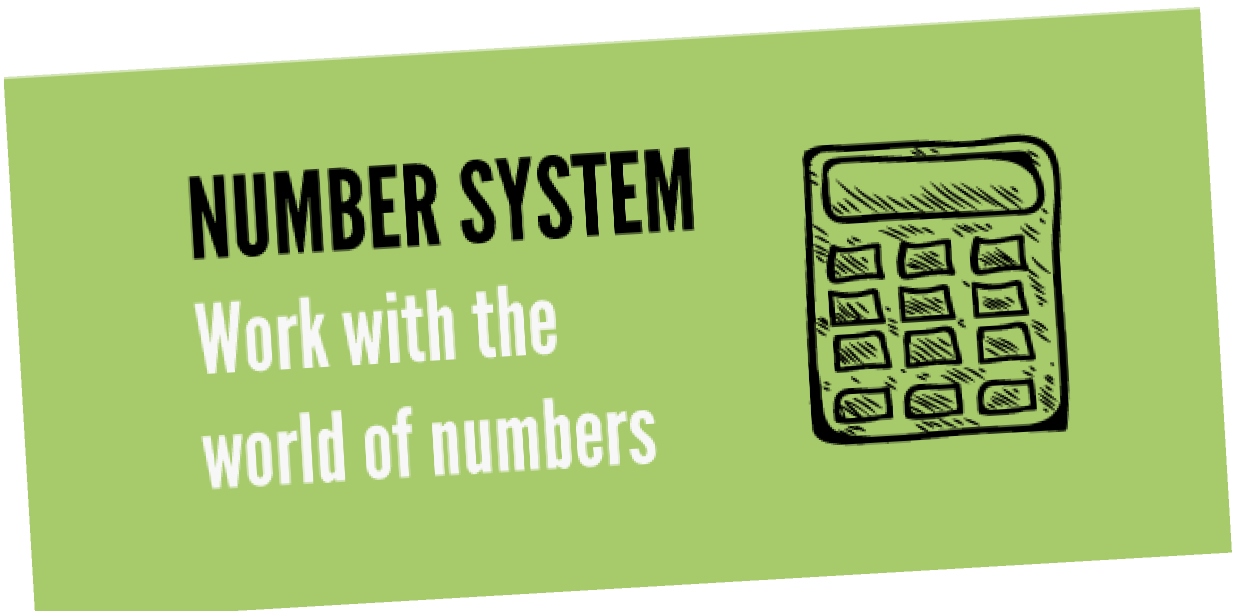
Number Systems



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- *HCF & LCM*
- *AP & GP*

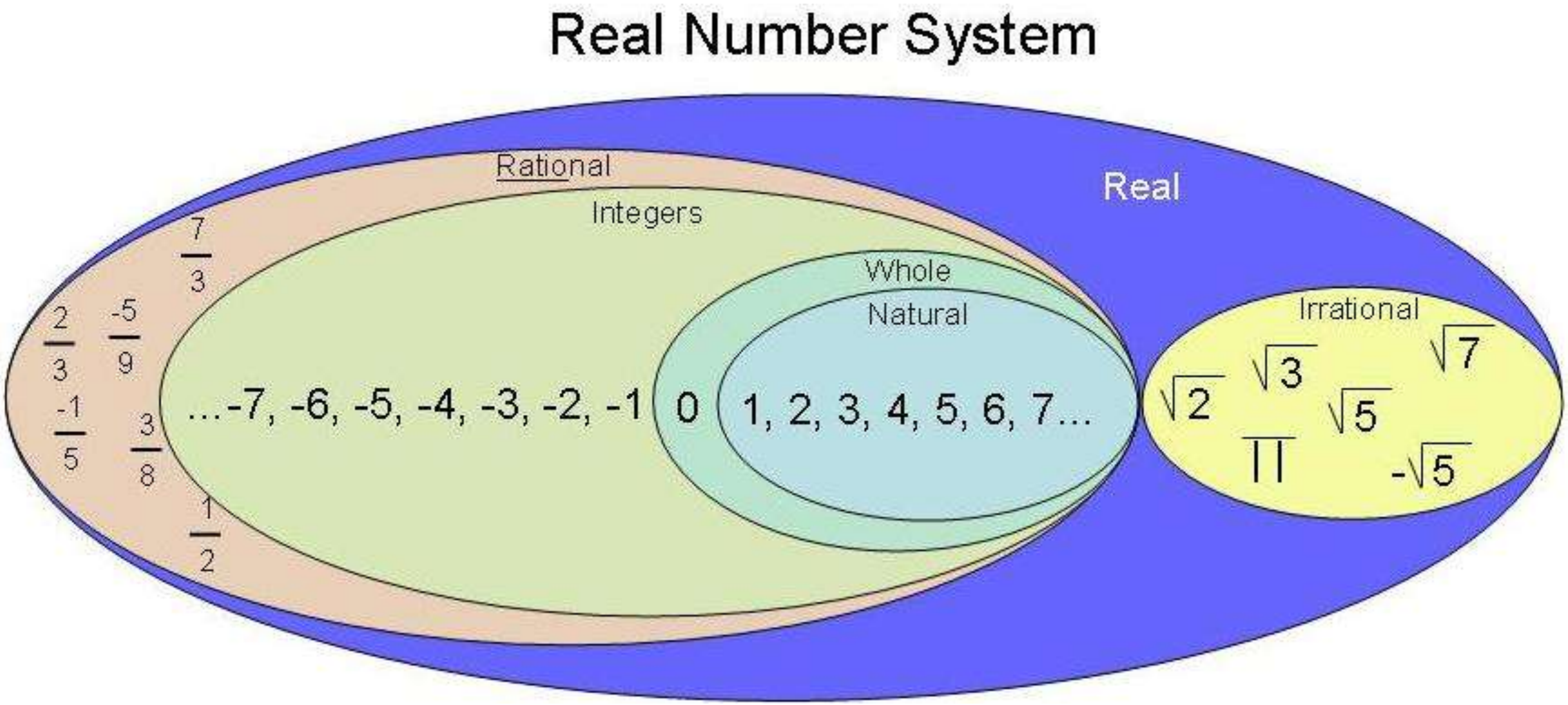
Face Value and Place value

4567

Face Value of 4 = 4
Face Value of 5 = 5
Face Value of 6 = 6
Face Value of 7 = 7

Place Value of 4 = 4000
Place Value of 5 = 500
Place Value of 6 = 60
Place Value of 7 = 7

1. Types of numbers



What is a rational number?

A **rational number** is a number which can be expressed in the form of **p/q** where **p & q are integers** and **$q \neq 0$.**
0.5, 0.3, 22/7

Numbers other than rational numbers are called **irrational numbers** which is **non-terminating and non-repeating.**
Pie, $\sqrt{3}$, 0.234387597................................

Rational no. b/w a and b = $(ak+b)/(k+1)$

irrational no. b/w a and b = $\sqrt{ab}$, …..

What are prime numbers?

Prime number is a number which has **exactly two factors** which is **1 and itself**.

Numbers other than prime is called **composite numbers** which has **more than two factors**. First positive composite number is 4.

What are Even numbers?

number which is divisible by 2.

What are Odd numbers?

number which is not divisible by 2.

What about 0 ?

Important rules related to Even and Odd numbers:

odd ± odd = even;
even ± even = even;
even ± odd = odd

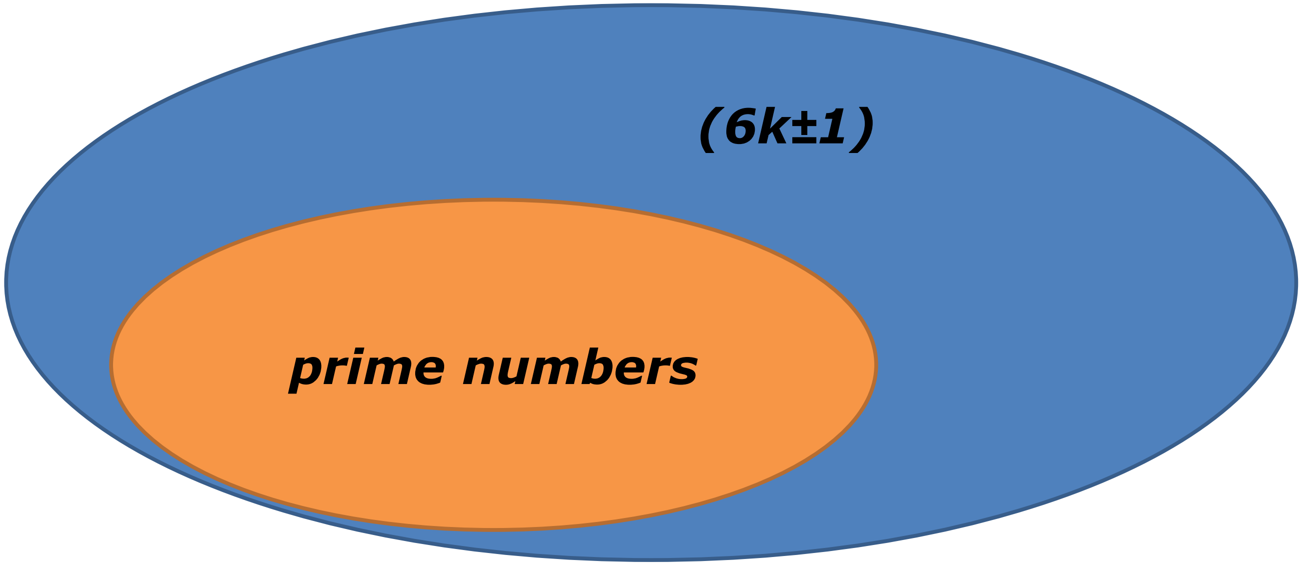
odd × odd = odd;
even × even = even;
even × odd = even.

$\text{odd}^{(\text{any number})} = \text{odd}$
$\text{even}^{(\text{any number})} = \text{even}$

1.i) How to find if a number is prime or not?

N is a prime number if it is not divisible by numbers lesser than √N.
Example: 191 is a prime number since it is not divisible by 2, 3, 5, 7, 11 and 13 [numbers less than √191 (≈14)].

Note: Prime numbers *will always be in the form* ***(6k±1)*** *where k= 1, 2, 3....*
But not all (6k±1) will be a prime number.



1.ii) Conversion of a decimal number to fraction:

1.ii) Conversion of a decimal number to fraction:

Example:

$3.\overline{713} =$

Solution:

$$3.\overline{713} = 3 + \frac{713}{999} = \frac{2997 + 713}{999} = \frac{3710}{999}$$

Example:

$12.3\overline{45}=$

Solution:

Here only 45 are recurring.

Therefore, $12.3\overline{45} = 12 + \frac{345 - 3}{990} = 12 + \frac{342}{990} = 12 + \frac{38}{110} = 12 + \frac{19}{55} = \frac{679}{55}$

Q. Convert 0.7222222…………….

2. Divisibility Rules

A number is divisible by

- **2** If the last digit is even.
- **3** If the sum of the digits is divisible by 3.
- **4** If the last two digits of the number divisible by 4.
- **5** If the last digit is a 5 or a 0.
- **6** If the number is divisible by both 3 and 2.
- **7** If the number formed by subtracting twice the last digit with the number formed by; rest of the digits is divisible by 7.

  Example: 343. 34-(3x2) = 28 is divisible by 7.

  We'll try 161623
- **8** If the last three digits form a number divisible by 8.
- **9** If the sum of the digits is divisible by 9.
- **10** If the last digit of number is 0.
- **11** If the difference between sum of digits in even places and the sum of the digits in odd places is 0 or divisible by 11.
  Example: 365167484

  (3+5+6+4+4) – (6+1+7+8) = 0

  365167484 is divisible by 11.
- **12** If the number is divisible by both 3 and 4.

Any other numbers can be written in terms of the numbers whose divisibility is already known.

Example: 15 = 3 x 5
18 = 2 x 9
33 = 3 x 11

Note: The numbers expressed should be co-prime (i.e., the HCF of the two numbers should be 1)

Example: 40 = 4 x 10 is wrong because HCF(4,10) is 2.
∴ 40 = 5 x 8 because HCF(5,8) is 1.

Q. What should come in place of x if 563x5 is divisible by 9?

Q. What should come in place of x if 4857x is divisible by 88?

Unit Digit Concept

	Power			
Base	1	2	3	4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6
4	4	6		
9	9	1		

Number	Cyclicity
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

Choose the nth value in the cycle if the remainder is n except for the last value whose remainder should be 0.

Note: *The last digit of an expression will always depend on the unit digit of the values.*

Example: *The unit digit of 123 x 456 x 789 = 3 x 6 x 9*

= 18 x 9

= 8 x 9

= 2

Example 2: *What is the unit digit of (123)^42?*

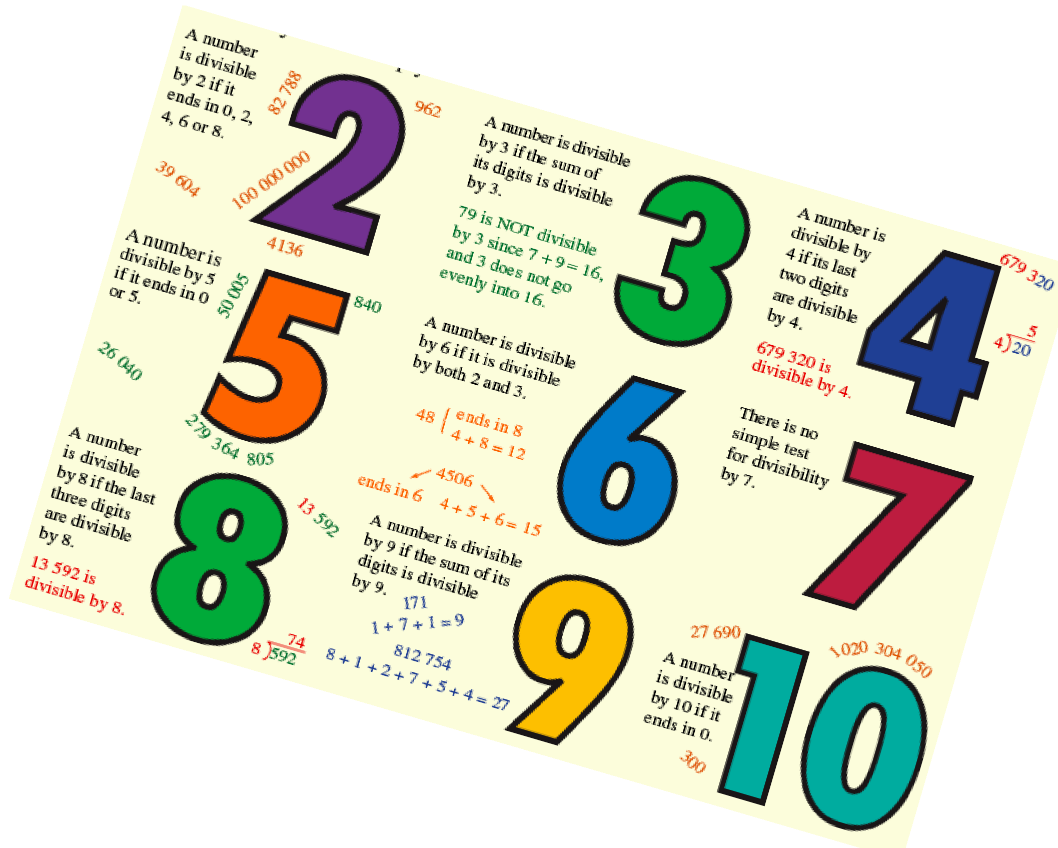
The unit digit pattern of 3 repeats four times. So find the remainder when the power value is divided by 4.

*42/4= R(**2**)*

2^{nd} value in 3 cycle is 9.

∴ Unit digit of (123)^42 is 9

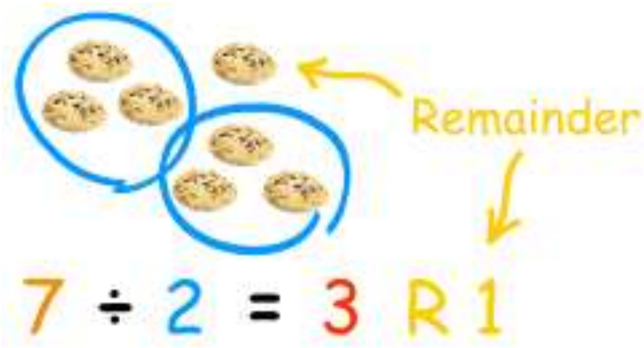
Q) What is the unit digit of (127)^223



4. Remainder theorem

Type 1: *Numerator in terms of powers*

The remainder pattern should be found starting from the power of 1. The same procedure should be followed as done in the unit digit concept.



Example: What is the remainder when 2^202 is divided by 7?

2^1/7 = R(2)
2^2/7= R(4)
2^3/7 = R(1)

The next three remainder values will be the same. i.e., The remainder pattern is 2,4,1, 2,4,1, 2,4,1.....

The size of the pattern is 3.

Now divide the power by number of repeating values (3) to choose the remainder.

Choose the nth value in the cycle if the remainder is n except for the last value whose remainder should be 0.

202/3 = R(**1**).

The 1^{st} value in the cycle is 2.

Note: *While finding the remainder pattern if the remainder becomes 1, then the process can be stopped as the it will always repeat after 1.*

∴ 2^202/7 = R(2)

Note: *While finding the remainder pattern if the remainder becomes 1, then the process can be stopped as the it will always repeat after 1.*

Type 2: *Different numerator values*

Replace each of the values of the numerator by its remainder when divided by the denominator and simplify.

Example: What is the remainder when 13 x 14 x 16 is divided by 6.

13/6 = R(1) ∴ replace 13 by 1

Similarly replace 14 and 16 by 2 and 4 respectively.

∴ (13 x 14 x 16)/6 = (1 x 2 x 4)/6

= 8/6

= R(2)

Note: *Do not cancel any numerator value with the denominator value as the remainder will differ.*

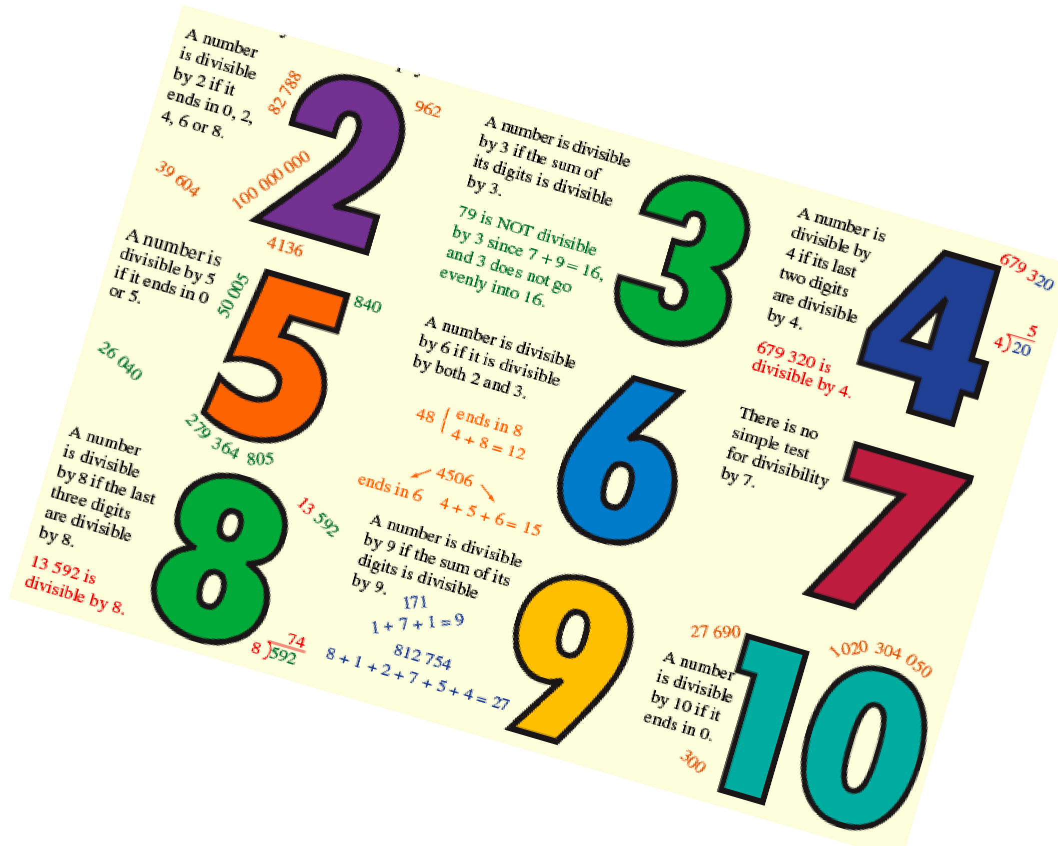
R(6/4) ≠ R(3/2)

6/4 = R(2)

But 3/2 = R(1)

Q) What is the remainder when 3 to the power 7 is divided by 8?

A)3
B)4
C)5
D)7
E)none



Q) Remainder when 17^23 is divided by 16?

A)1

B)2

C)3

D)4

5. Factors

Factors of a number are the values that divides the number completely.

Example: Factors of 10 are 1, 2, 5 and 10.

Multiple of a number is the product of that number and any other whole number.

Example: multiples of 10 are 10, 20, 30,…..

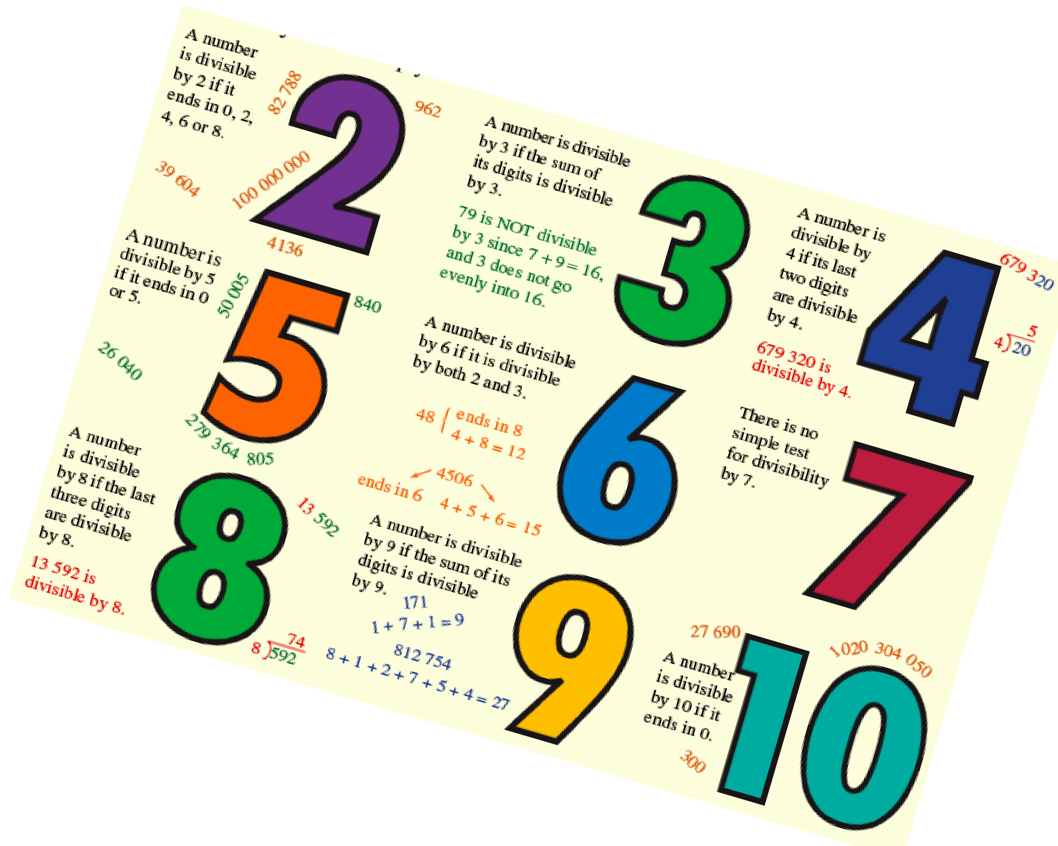
- ***5.i) Number of factors:***

- **Example:** 3600

- **Step 1:** Prime factorize the given number
- $3600 = 36 \times 100$
- $= 6^2 \times 10^2$
- $= 2^2 \times 3^2 \times 2^2 \times 5^2$
- $= 2^4 \times 3^2 \times 5^2$

- **Step 2:** Add 1 to the powers and multiply.
- $(4+1) \times (2+1) \times (2+1)$
- $= 5 \times 3 \times 3$
- $= 45$
- ∴ Number of factors of 3600 is 45.

Q) Find the number of factors of 14400?



5.ii) Sum of factors:

Example: 45

Step 1: Prime factorize the given number

$45 = 3^2 \times 5^1$

Step 2: Split each prime factor as sum of every distinct factors.

$(3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$

The following result will be the sum of the factors

$= 78$

- **The number of ways of writing a number as a product of two number** = $[(p+1).(q+1).(r+1)...]/2$

- **Example:**
  Find no. of ways of writing 140 as a product of two numbers
  The prime factorization of 140 = $2^2 \times 5 \times 7$
  number of ways = $(3*2*2)/2 = 6$

- **The number of ways of writing a number N as a product of two co-prime numbers** = $2^{(n-1)}$
  where n=the number of prime factors of a number.

- **Example:**
  The prime factorization of 60 = $2^2 \times 3 \times 5$
  The no of ways of writing 60 as a product of two co - primes
  = $2^{(3-1)} = 4$

Factors will occur in pairs for the numbers except perfect squares.

Example 1: A non perfect square number- 10

1 x 10 = 10

2 x 5 = 10

‵∴ Factors of 10 are 1, 2, 5 and 10.

Non perfect squares *will have* ***even number of factors***

Example 2: A perfect square number- 16

1 x 16 = 16
2 x 8 = 16
4^2 = 16
∴ Factors of 16 are 1, 2, 4, 8 and 16.

*Every **perfect square** will have **odd number of factors** because its square root number will pair with itself.*

This has odd number of factors because 4 will pair with itself.

*Every **perfect square** will have **odd number of factors** because its square root number will pair with itself.*

Example 3: A prime square number- 49

The factors of 49 are 1, 7 and 49.

Prime square number *will have exactly* ***3 factors*** *(1, that number itself and square root of that number).*

If ***N*** *is a* ***prime square number*** *then the* ***factors are 1, N and √N.***

Q) If 11^2, 3^4 and 2^5 are the factors of $a \times 12^7 \times 17^6 \times 21^5$ then what is the minimum possible value of a?

Q.How many zeros are there in 100!?

6. HCF & LCM

- The greatest number that will exactly divide a, b and c is **HCF(a, b, c)**.

- The greatest number that will divide a, b and c leaving remainder of x, y and z respectively is **HCF(a-x, b-y, c-z)**.

- The greatest remainder which when it divides a, b and c will leave the same remainder in each case is **HCF(a-b, b-c, c-a)**.

- The least number which is exactly divisible by a, b and c is **LCM(a, b, c)**.

- The least number which when divided by a, b and c leaves the same reminder r in each case is **LCM(a, b, c) + r**.

- The least number which when divided by a, b and c leaves the remainder x, y and z respectively is **LCM(a, b, c) – K**.
This is possible only if a-x = b-y = c-z = K.

FINDING THE H.C.F. OF BIG NUMBERS

For larger numbers you can use the following method:

Step 1 Find all prime factors of both numbers.

Step 2 Write both numbers as a multiplication of prime numbers.

Step 3 Find which factors are repeating in both numbers and multiply them to get H.C.F

FINDING L.C.M. OF BIG NUMBERS

Step 1 Find all the prime factors of both numbers.

Step 2 Multiply all the prime factors of the larger number by those prime factors of the smaller number that are not already included

Important formulae:

$$\boxed{LCM(a,b) = \frac{a \times b}{HCF(a,b)}}$$

- Product of Two numbers = LCM X HCF
- HCF of fractions = $\frac{HCF\ OF\ numerators}{LCM\ OF\ denominators}$
- LCM of fractions = $\frac{LCM\ of\ numerators}{HCF\ of\ denominators}$

Q) Four bells ring at an interval 3min, 4min, 5min and 6 minutes respectively. If all the four bells ring at 9am first, when will it ring again?

Q) The H.C.F. of two numbers is 11 and their L.C.M. is 7700. If one of the numbers is 275, then the other is:

A)308
B)310
C)312
D)None

Q) The H.C.F of 9/10, 12/25, 18/35, and 21/40 is?

A)3/1400
B)5/1400
C)7/1400
D)None

Q) Which of the following fraction is the largest? 7/8, 13/16, 31/40, 63/80

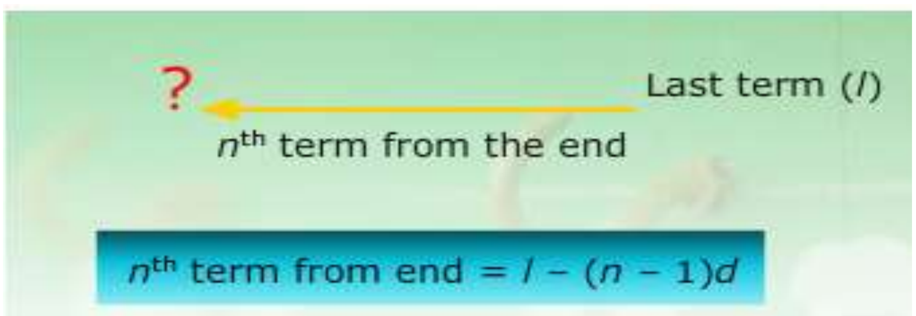
A)7/8
B)13/16
C)31/40
D)6380

ARITHMETIC PROGRESSION

An Arithmetic Progression (A.P.) is a sequence in which the difference between any two consecutive terms is constant.

Let a = first term, d = common difference

- Then nth term $a_n = a + (n-1)d$



Sum of an A.P

The sum of n terms of an A.P. whose first term is a and common difference is d, is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

The sum of n terms of an A.P. whose first term is a and last term is l is given by the formula:

$$S_n = \frac{n}{2}\left[a + l\right]$$

AM (Arithmetic mean)

If a, b, c are in AP then the arithmetic mean is given by

$b = (a+c)/2$

Inserting AM

To insert k means between a and b the formula for common difference is given by

d = (b-a) / (k+1)

For example: Insert 4 AM's between 4 and 34

d= (34 - 4) / (4+1)

= 30/5

= 6

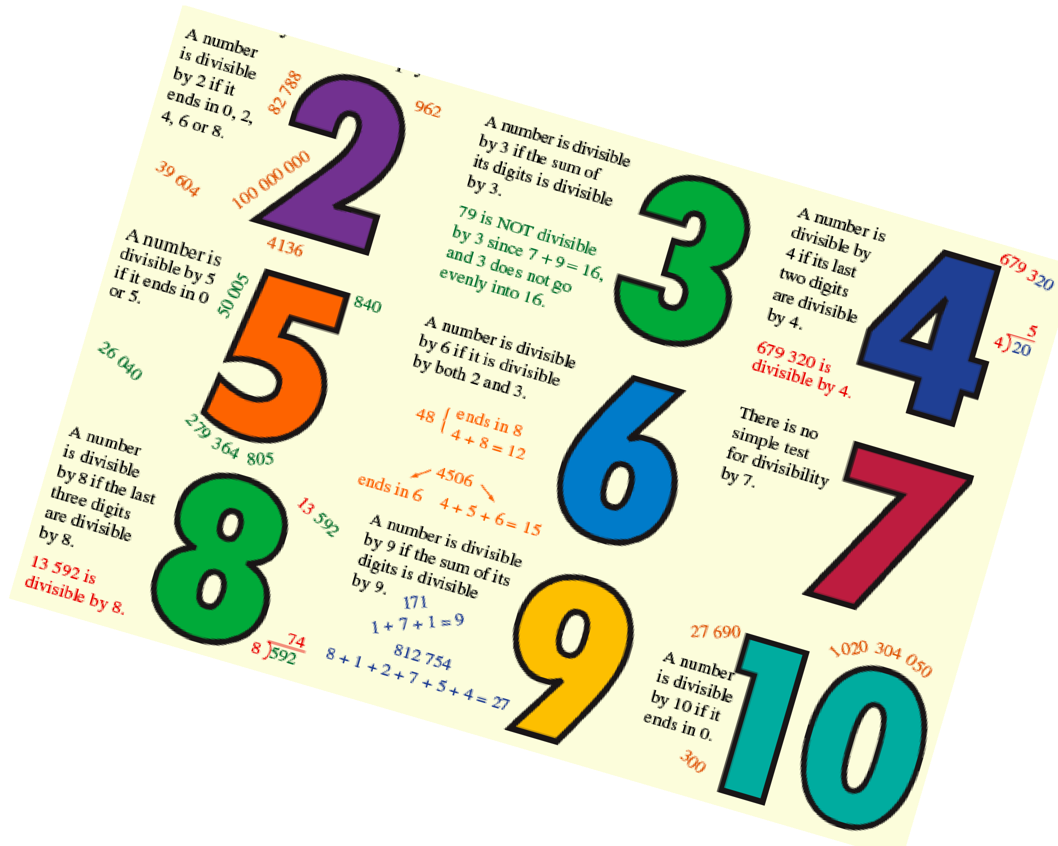
∴ The means are 4+6=10

10+6=16

16+6=22

22+6=28

Q) Find the sum of the series 5,8,11,…….. 221



GEOMETRIC PROGRESSION

A geometric sequence are powers r^k of a fixed number r, such as 2^k and 3^k. The general form of a geometric sequence is

The n-th term of a geometric sequence with initial value a and common ratio r is given by

$$a_n = a\,r^{n-1}.$$

Such a geometric sequence also follows the recursive relation

$a_n = r\,a_{n-1}$ for every integer $n \geq 1.$

Sum= $\dfrac{a(1-r^m)}{1-r}$

GM (Geometric mean)

If a, b, c are in GP Then the GM is given by

$b = \sqrt{ac}$

Inserting GM

To insert k means between a and b the formula for common ratio is given by

$r = (b/a)^{(1/(k+1))}$

For example: Insert 4 GM's between 2 and 486

$r = (486/2)^{(1/(4+1))}$

$= (243)^{(1/5)}$

$= 3$

$\therefore$ the means are $2 \times 3 = 6$

$6 \times 3 = 18$

$18 \times 3 = 54$

$54 \times 3 = 162$

Q) Find the sum of the series 2, 4, 8, 16…. 256.

Next Class Average