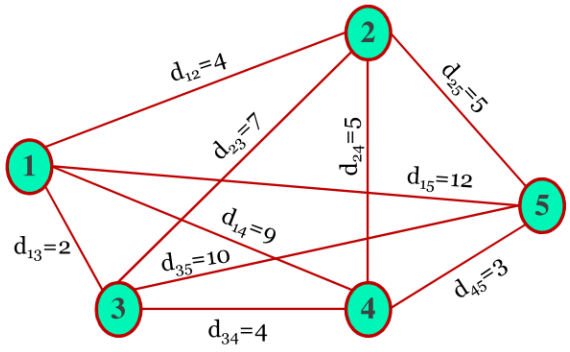
2)



The number of ants $m=2$.
Initially each ant is put on node 1 as the problem asks.
At each construction step, ant k applies a probabilistic action choice rule.
In particular the probability with which ant k at city i, chooses to go to city j at the tth iteration is:

$$p_{ij}^k = \begin{cases} \dfrac{\tau_{ij}^{\alpha}\, \eta_{ij}^{\beta}}{\sum_{u \in N_i^k} \tau_{iu}^{\alpha}\, \eta_{iu}^{\beta}} & \text{if } j \in N_i^k \\ 0 & \text{if } j \notin N_i^k \end{cases}$$

N_i^k is set of feasible nodes connected to node i, with respect to ant k

$\eta_{ij} = 1/d_{ij}$

Initially we chose $\alpha = \beta = 1$, as this provides a good balance between the influence of τ_{ij} and influence of η_{ij}.
We chose $\rho = 0.7$, to match the factor taken into account in lecture.

Pheromone update is performed using:

$$\tau_{ij}(t+1) = (1-\rho)\,\tau_{ij}(t) + \Delta\tau_{ij}(t)$$

with

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{N_k} \Delta\tau_{ij}^k(t)$$

$\Delta\tau_{ij}^k$ is chosen to be calculated using Ant-cycle AS:

$$\Delta\tau_{ij}(t) = \frac{1}{L^k(t)}$$

Iteration 1:

1) First Ant: $k=1$
initial parameters: $\alpha=\beta=1$, $\rho=0.7$, $\tau_{ij}(t)=0.5$
starting at node 1, neighbours are: $\{2,3,4,5\}$

2) Construct Ant Solutions

$$\sum_{n \in N_1'} \tau_{1n}^{\alpha} / d_{1n}^{\beta} = \frac{0.5}{4} + \frac{0.5}{12} + \frac{0.5}{2} + \frac{0.5}{9} = 0.47$$

$$p_{12}' = \frac{0.5/4}{0.47} = 0.267 \qquad p_{14}' = \frac{0.5/9}{0.47} = 0.118$$

$$p_{13}' = \frac{0.5/2}{0.47} = 0.532 \qquad p_{15}' = \frac{0.5/12}{0.47} = 0.088$$

so chose node 3 as highest probability
& also add to tabu list of ant: $\{1,3\}$ nodes visited by ant

Then at node 3, neighbours are: $\{2, 4, 5\}$

$$\sum_{n \in N_3'} \tau_{3n}^{\alpha} / d_{3n}^{\beta} = \frac{0.5}{7} + \frac{0.5}{4} + \frac{0.5}{10} \cong 0.25$$

$$p_{34}' = \frac{0.50/4}{0.25} = 0.50 \qquad p_{35}' = \frac{0.50/10}{0.25} = 0.2$$

$$p_{32}' = \frac{0.50/7}{0.25} = 0.285$$

chose node 4 as highest probability & add to tabu list: $\{1, 3, 4\}$

Now neighbours of 4: $\{2, 5\}$

$$\sum_{n \in N_4'} \tau_{4n}^{\alpha} / d_{4n}^{\beta} = \frac{0.5}{3} + \frac{0.5}{5} \cong 0.27$$

$$p_{42}' = \frac{0.5/5}{0.27} = 0.37$$

$$p_{45}' = \frac{0.5/3}{0.27} = 0.617$$

chose node 5 as highest probability
so then only node left is 2
& path becomes: $\{1, 3, 4, 5, 2, 1\}$
Cost: $2 + 4 + 3 + 5 + 4$
$= 18$

Repeating process for ant 2 gives the same path.

3) Update Pheromones

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t) \qquad \rho = 0.7$$

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{N_K} \Delta\tau_{ij}^{k}(t)$$

$$\Delta\tau_{ij}^{k}(t) = \frac{1}{18} + \frac{1}{18} = \frac{2}{18} \frac{1}{9}$$ as 2 ants take the best path

$$\tau_{ij}^{(new)} = (1-0.7)\tau_{ij}^{(old)} + \Delta\tau_{ij}(t)$$
$$= 0.3(0.5) + \frac{1}{9} = 0.26$$ for paths selected in iteration 1

$$\tau_{ij}^{(new)} = 0.3(0.5) = 0.15$$ for all other paths

Iteration 2:

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Iteration 2:

1) First Ant: $k=1$

initial parameters: $\alpha = \beta = 1$, $\rho = 0.7$

Starting at node 1, neighbours are: $\{2,3,4,5\}$

2) Construct Ant Solutions

$$\sum_{n \in N_1'} \tau_{1n}^{\alpha} / d_{1n}^{\beta} = \frac{0.26}{2} + \frac{0.15}{9} + \frac{0.15}{12} + \frac{0.15}{4} \cong 0.20$$

$$p_{12}' = \frac{0.15/4}{0.20} \cong 0.188 \qquad p_{14}' = \frac{0.15/9}{0.20} = 0.083$$

$$p_{13}' = \frac{0.26/2}{0.20} \cong 0.65 \qquad p_{15}' = \frac{0.15/12}{0.20} = 0.0625$$

so chose node 3 as highest probability & also add to tabu list of ant: $\{1,3\}$ nodes visited by ant

Then at node 3, neighbours are: $\{2,4,5\}$

$$\sum_{n \in N_3'} \tau_{3n}^{\alpha} / d_{3n}^{\beta} = \frac{0.15}{7} + \frac{0.26}{4} + \frac{0.15}{10} \cong 0.1014$$

$$p_{34}' = \frac{0.26/4}{0.1014} = 0.64 \qquad p_{35}' = \frac{0.15/10}{0.1014} = 0.15$$

$$p_{32}' = \frac{0.15/7}{0.1014} = 0.2113$$

chose node 4 as highest probability & add to tabu list: $\{1,3,4\}$

Now neighbours of 4: $\{2,5\}$

$$\sum_{n \in N_4'} \tau_{4n}^{\alpha} / d_{4n}^{\beta} = \frac{0.26}{3} + \frac{0.15}{5} \cong 0.116$$

$$p_{42}' = \frac{0.15/5}{0.116} = 0.258$$

$$p_{45}' = \frac{0.26/3}{0.116} = 0.747$$

chose node 5 as highest probability

so then only node left is 2

& path becomes: $\{1,3,4,5,2,1\}$

Cost: $2 + 4 + 3 + 5 + 4$

$= 18$