ECE 457a Assignment 3 – Group 1

Question 1

Part (a)

Hooke's law in physics states that the **length** x of a uniform spring is a linear function of the **force** y applied to it. The function can be represented as shown below where a is a constant and b represents stiffness of the spring.

$$y = a + bx$$

We would like to find an optimal roots (solution) to the above polynomial. This is a real-valued problem and the solution can be represented by a vector of parameters a, b where a and b are real values. Thus the solution can be represent as:

$$Sol = [a\ b]$$

Part (b)

Crossover operators for real numbers can be:

I. Single Arithmetic
II. Simple Arithmetic
III. Whole Arithmetic

The most commonly selected one is whole arithmetic, so we will use this operator. The operator is of the following form:

$$\begin{array}{ll} Parents: & < p_1, \dots, p_n > and < q_1, \dots, q_n >, \\ Child\ 1\ is: & \alpha . p + (1 + \alpha) q \\ Child\ 2\ is: & \alpha . q + (1 + \alpha) p \end{array}$$

Where α is an arbitrary floating point number between 0 and 1 (exclusive). For this problem, we will use a value of α = 0.2.

Part (c)

For real-valued problems a mutation scheme is to add random noise, for each gene:

$$x_i' = x_i + N(0, \sigma)$$

Where N(0,σ) is a random Gaussian number with zero mean and standard deviation σ.

Part (d)

Objective/fitness function can be least squares of errors. Given (x, y) that exist on the polynomial, a potential solution vector can be evaluated with the following objective function

$$f(a, b) = \sum_{i=1}^{all(x,y)} (y_i - y_{desired})^2$$

Where y_i is the value calculated from the corresponding x_i using the equation $y = a + bx$.

Part (e)

Iteration 1

The parents are $p_1 = [1.08 \quad -11.47]$
$q_1 = [1.75 \quad -5.8]$

Using a and b from p_1 $\quad y = 1.08 - 11.47x$

If the fitness function is applied for p_1, we get the following results.

$y_{desired}$	x_i	y_i	**Square of Errors**
0	6.1	-68.9	4747.21
2	7.6	-86.1	7761.61
4	8.7	-98.7	10547.29
6	10.4	-118.2	15425.64

$$f_1 = 38481.78$$

Using a and b from q_1 $\quad y = 1.75 - 5.8x$

If the fitness function is applied for q_1, we get the following results.

$y_{desired}$	x_i	y_i	**Square of Errors**
0	6.1	-33.6	1128.96
2	7.6	-42.3	1962.49
4	8.7	-48.7	2777.29
6	10.4	-58.6	4173.16

$$f_2 = 10041.9$$

Using α = 0.2 in the crossover operator the children are:

$Child\ 1\ is: \quad \alpha.p_1 + (1+\alpha)q_1 \quad = \quad [0.216 - 2.294] + [1.4 \quad -4.68] = [1.616 \quad -6.974]$

$Child\ 2\ is: \quad \alpha.q_1 + (1+\alpha)p_1 \quad = \quad [0.35 \quad -1.17] + [0.864 \ -9.176]$

$= [1.214 \quad -10.346]$

Using a and b from $Child\ 1$ $\quad y = 1.616 - 6.974x$

If the fitness function is applied for $Child\ 1$, we get the following results.

$y_{desired}$	x_i	y_i	**Square of Errors**
0	6.1	-40.9	1672.81
2	7.6	-51.4	2851.56
4	8.7	-59.1	3981.61
6	10.4	-70.9	5913.61

$$f_3 = 14419.59$$

Using a and b from $Child\ 2$ $y = 1.214 - 10.345x$

If the fitness function is applied for $Child\ 2$, we get the following results.

$y_{desired}$	x_i	y_i	**Square of Errors**
0	6.1	-61.9	3831.61
2	7.6	-77.4	6304.36
4	8.7	-88.8	8611.84
6	10.4	-106.4	12633.76

$$f_4 = 31381.57$$

Iteration 2

From the calculations we see that f_2 and f_3 have the smallest values for the fitness function. Thus, the parent solution for the next generation will be $Child\ 1$ from Iteration 1 and q_1. Hence:

The second generation parents are $p_2 = [1.616 \quad -6.974]$ $f_1 = 14419.59$

$q_2 = [1.75 \quad -5.85]$ $f_2 = 10041.9$

Using α = 0.2 in the crossover operator the children are:

$Child\ 1\ is:$ $\alpha.p_1 + (1+\alpha)q_1 = [0.323\ -1.39] + [1.4\ -4.68] = [1.723\ -6.07]$

$Child\ 2\ is:$ $\alpha.q_1 + (1+\alpha)p_1 = [0.35\ -1.17] + [1.293\ -5.58] = [1.643\ -6.75]$

Using a and b from $Child\ 1$ $y = 1.723 - 6.07x$

If the fitness function is applied for $Child\ 1$, we get the following results.

$y_{desired}$	x_i	y_i	**Square of Errors**
0	6.1	-35.3	1246.09
2	7.6	-44.4	2152.96
4	8.7	-51.1	3036.01
6	10.4	-61.4	4542.76

$$f_3 = 10977.82$$

Using a and b from $Child\ 2$ $y = 1.643 - 6.75x$

If the fitness function is applied for $Child\ 2$, we get the following results.

$y_{desired}$	x_i	y_i	**Square of Errors**
0	6.1	-39.5	1560.25
2	7.6	-49.7	2672.89
4	8.7	-57.1	3733.21
6	10.4	-68.6	5565.16

$$f_4 = 13531.51$$

From the calculations we see that f_2 and f_3 have the smallest values for the fitness function. Thus, the parent solution for the next generation will be $Child\ 1$ from Iteration 2 and q_2.

Question 2

2) a) The length of the binary encoding chromosome for a variable x_i with a range, L_i and a precision m_i is given by

$$n_i = \lceil \log_2(L_i \cdot 10^{m_i}) \rceil$$

This problem specifies two variables, x and y, each with a range of [0, 10), so $L_i = 10$ and precision $m_i = 4$. Therefore, the length of chromosomes should be

$$n_x + n_y = 2 \cdot \lceil \log_2(10 \cdot 10^4) \rceil = 34$$

Four individuals of this length are

A 0011111111111111101110000111100001

B 0101000101101110010000110110111101

C 1000110111011101101011100111101111

D 1001001000100010000000000000000000

b) In terms of the crossover operator, we can use 1-point crossover over two parents to produce two children. A and B crossover to produce A1 and B1. Similarly, C and D crossover to produce C1 and D1. The point of cross over is after 10 bits. After 10 bits, the tails of the parents get exchanged to produce the two new children.

Following are the resultant chromosomes after crossover

A1 0011111111 101110010000110110111101

B1 0101000101 111111101110000111100001

C1 1000110111 100010000000000000000000

D1 1001001000 011101101011100111101111

c) The mutation operator takes in 1 chromosome and produces 1 child. Assume a mutation threshold r in the range [0,1]. For every bit in the chromosome, generate a random probability, p, in the range [0, 1]. If p > r, then flip the bit. Here, chromosome A2 is a mutation of chromosome A, B2 of B and so on.

A2 0001111111101111100111000000101000010

B2 0101000000110101001100001101011111010

C2 1001101111011101110101110010110111110

D2 100100100011001001000100000000110001

2d) **Repeat a, b, c for Decimal Real Number Floating Point Encoding**

a) The length of the chromosome is the sum of the number of bits needed to represent the integer portion and the number of bits needed to represent the decimal portion. For a real number with integer part range of α_i and decimal part range of β_i is calculated as

$$n_i = \lfloor \log_2(\alpha_i) + 1 \rfloor + \lfloor \log_2(\beta_i) + 1 \rfloor$$

In the case described by this question, we have two variables, x and y, both with $\alpha_i = 10$ and $\beta_i = 9999$. Therefore the number of bits required to represent our chromosome is

$$n_x + n_y = 2 \cdot \lfloor \log_2(10)+1 \rfloor + \lfloor \log_2(9999)+1 \rfloor = 36$$

Four individuals of this length are

A 0011 11111111111111 0111 00001111000011

B 0101 00010110111001 1000 01101101111011

C 1000 11011101110111 0101 11001111011111

D 1001 00100010001001 0000 00000000000001

b) In terms of the crossover operator, we can use 1-point crossover over two parents to produce two children. A and B crossover to produce A1 and B1. Similarly, C and D crossover to produce C1 and D1. The point of cross over is after 18 bits. After 18 bits, the tails of the parents get exchanged to produce the two new children.

A1 0011 11111111111111 1000 01101101111011

B1 0101 00010110111001 0111 00001111000011

C1 1000 00100010001001 0000 00000000000001

D1 1001 11011101110111 0101 11001111011111

c) The mutation operator takes in 1 chromosome and produces 1 child. Assume a mutation threshold r in the range [0,1]. For every bit in the chromosome, generate a random probability, p, in the range [0, 1]. If p > r, then flip the bit. Here, chromosome A2 is a mutation of chromosome A, B2 of B and so on.

A2 0001 11111110111110 0111 00000101000010

B2 0101 00000110101001 1000 01101011111010

C2 1001 10111101110111 0101 11001011011110

D2 1001 00100011001001 0001 00000000110001