Question 1

The following table shows the coordinates of each point included in the suboptimal solution calculated using Dijkstra algorithm.

Point	x	Y
S	1.5	0.5
v1	3	2
v2	3.5	3
v3	4	4
v4	4.5	5
T	4.5	6.5

The total length of the sub-optimal path, $l_{total} = l_1 + l_{2+}\ l_{3+}\ l_{4+}\ l_5$

$$l_1 = \sqrt{(x_{v1} - x_s)^2 + (y_{v1} - y_s)^2} = 2.12$$
$$l_2 = \sqrt{(x_{v2} - x_{v1})^2 + (y_{v2} - y_{v1})^2} = 1.12$$
$$l_3 = \sqrt{(x_{v3} - x_{v2})^2 + (y_{v3} - y_{v2})^2} = 1.12$$
$$l_4 = \sqrt{(x_{v4} - x_{v3})^2 + (y_{v4} - y_{v3})^2} = 1.12$$
$$l_5 = \sqrt{(x_T - x_{v4})^2 + (y_T - y_{v4})^2} = 1.5$$
$$l_{total} = fitness(particle) = 6.98\ units$$

Here l_{total} can be used as the fitness function for Particle Swarm Optimization (PSO).

To apply PSO to optimize the path, we need to move each point v_1 to v_2 along the MAKLINK lines to reduce the overall path length. Each point can be varied using a random number in the range [0,1] in order to vary where the point is along the MAKLINK line. In our implementation of Dijkstra algorithm we assumed the point to be the midpoint of each MAKLINK line. That is why it is a suboptimal solution.

Iteration 0

In this iteration we simply initialize the values. All the initialization steps are listed below.

Step 1

Choose the number of particles N as 4

Step 2

Initialize each of the particle's position with 4 random number in the range [0,1]. Here $p_o(0)$ indicates the position of the particle on the first MAKLINK line and so on.

Particle	$p_o(0)$	$p_o(1)$	$p_o(2)$	$p_o(3)$
1	0.425	0.690	0.556	0.444
2	0.528	0.234	0.179	0.682
3	0.989	0.524	0.123	0.253
4	0.629	0.690	0.554	0.888

Step 3

Set the initial velocities of each particle to zero.

Particle	$v_o(0)$	$v_o(1)$	$v_o(2)$	$v_o(3)$
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0

Iteration 1

The actual points of the paths these particles represent can be calculated using each of the MAKLINK lines' x and y coordinates of their endpoints shown as follows,

$x_{v1} = 2 + (4 - 2) \times p_i(0)$ and $y_{v1} = 2 + (2 - 2) \times p_i(0)$
$x_{v2} = 3 + (4 - 3) \times p_i(1)$ and $y_{v2} = 3 + (3 - 3) \times p_i(1)$
$x_{v3} = 3 + (5 - 3) \times p_i(2)$ and $y_{v3} = 4 + (4 - 4) \times p_i(2)$
$x_{v4} = 4 + (5 - 4) \times p_i(3)$ and $y_{v4} = 4 + (6 - 4) \times p_i(3)$

Calculating the fitness of each of the particles we have:

Particle	Fitness	p_best_o
1	7.868331	[0.425,0.690,0.556,0.444]
2	7.902492	[0.528,0.234,0.179,0.682]
3	8.463117	[0.989,0.524,0.123,0.253]
4	8.109887	[0.629,0.690,0.554,0.888]

$g_{best_0} = [0.425,0.690,0.556,0.444]$

Calculated the velocities with PSO velocity equation:

$$v_{t+1}^{id} = w * v_t^{id} + c_1 r_1^{id}\left(pbest_t^{id} - x_t^{id}\right) + c_2 r_2^{id}\left(gbest_t^{id} - x_t^{id}\right)$$

Let $w=1$ $C_1=C_2=1$. The r_1 and r_2 for this iteration are

$r_1 = [0.329,0.425,0.556,0.928]$

$r_2 = [0.123,0.552,0.326,0.825]$

Calculating the velocities

Particle	V_0	p_best_o	V_1
1	[0,0,0,0]	[0.425,0.690,0.556,0.444]	[0,0,0,0]
2	[0,0,0,0]	[0.528,0.234,0.179,0.682]	[-0.013,0.252,0.123,-0.196]
3	[0,0,0,0]	[0.989,0.524,0.123,0.253]	[-0.07,0.092,0.141,0.157]
4	[0,0,0,0]	[0.629,0.690,0.554,0.888]	[-0.025,0,0,-0.366]

Using the velocities, calculate the new positions for each particle:

Particle	P_0	V_1	P_1
1	[0.425,0.690,0.556,0.444]	[0,0,0,0]	[0.425,0.69,0.556,0.444]
2	[0.528,0.234,0.179,0.682]	[-0.013,0.252,0.123,-0.196]	[0.515,0.486,0.302,0.486]
3	[0.989,0.524,0.123,0.253]	[-0.07,0.092,0.141,0.157]	[0.92,0.616,0.264,0.411]
4	[0.629,0.690,0.554,0.888]	[-0.025,0,0,-0.366]	[0.604,0.69,0.555,0.522]

Iteration 2

Calculating the fitness of each of the particles we have:

Particle	Fitness	p_best_1
1	7.868331	[0.425,0.690,0.556,0.444]
2	7.78503	[0.515,0.486,0.302,0.486]
3	8.228337	[0.92,0.616,0.264,0.411]
4	7.914637	[0.604,0.69,0.555,0.522]

$g_{best_0} = [0.515,0.486,0.302,0.486]$

Calculated the velocities with PSO velocity equation:

$$v_{t+1}^{id} = w * v_t^{id} + c_1 r_1^{id} \left(pbest_t^{id} - x_t^{id} \right) + c_2 r_2^{id} \left(gbest_t^{id} - x_t^{id} \right)$$

Let $w=1$ $C_1=C_2=1$. The r_1 and r_2 for this iteration are
$r_1 = [0.123,0.456,0.789,0.458]$
$r_2 = [0.111,0.991,0.722,0.189]$

Calculating the velocities

Particle	V_1	p_best_1	V_2
1	[0,0,0,0]	[0.425,0.690,0.556,0.444]	[0.001,-0.202,-0.183,0.008]
2	[-0.013,0.252,0.123,-0.196]	[0.515,0.486,0.302,0.486]	[-0.015,0.616,0.308,-0.323]
3	[-0.07,0.092,0.141,0.157]	[0.92,0.616,0.264,0.411]	[-131,0.096,0.382,0.274]
4	[-0.025,0,0,-0.366]	[0.604,0.69,0.555,0.522]	[-0.041,-00.202,-0.182,-0.61]

Using the velocities, calculate the new positions for each particle:

Particle	P_1	V_2	P_2
1	[0.425,0.69,0.556,0.444]	[0.001,-0.202,-0.183,0.008]	[0.435,0.488,0.373,0.452]
2	[0.515,0.486,0.302,0.486]	[-0.015,0.616,0.308,-0.323]	[0.5,0.486,0.302,0.486]
3	[0.92,0.616,0.264,0.411]	[-131,0.096,0.382,0.274]	[0.92,0.616,0.264,0.411]
4	[0.604,0.69,0.555,0.522]	[-0.041,-00.202,-0.182,-0.61]	[0.563,0.488,0.374,0.522]