Assignment 3 – Genetic Algorithms

Model Answer

I. Graded Exercises

1. **[Written Exercise - 5 Marks]** Hooke's law in physics states that the length *x* of a uniform spring is a linear function of the force *y* applied to it. If we write *y*=*a*+*bx*, where *a* is a constant and *b* represents stiffness of the spring. Suppose a particular unstretched spring has a measured length of 6.1 inches (i.e. *x*=6.1 when *y*=0). Forces of 2 pounds, 4 pounds, and 6 pounds are then applied to the spring, and the corresponding lengths are found to be 7.6 inches, 8.7 inches, and 10.4 inches respectively. Use GA to find the constant *a* and stiffness *b* of this spring.

a) Develop a solution representation,

b) Suggest a crossover operator that will be suitable for all instances of this problem,

c) Suggest a mutation operator,

d) Suggest a fitness function (objective function) for this problem,

e) Starting with two individuals (one with coefficients a=1.08 and b=-11.47 and other one with coefficients a=1.75 and b=-5.8), perform two iterations using the representation and crossover operator you provided. In these iterations, perform the crossover operation to produce two children, then for the next generation, select the two individuals that have the best fitness out of the four candidates.

Solution:

Same steps as curve-fitting problem in slides 45-57 in Lecture-13

2. **[Written Exercise - 5 Marks]** Maximizing the following nonlinear multimodal function (Fig. 1) can be achieved by evolving the set of *x* and *y* within [0, 10):

$$f(x,y) = \frac{6.452(x+0.125y)(\cos x - \cos 2y)^3}{\sqrt{0.8+(x-4.2)^2+2(y-7)^2}} + 3.226y$$

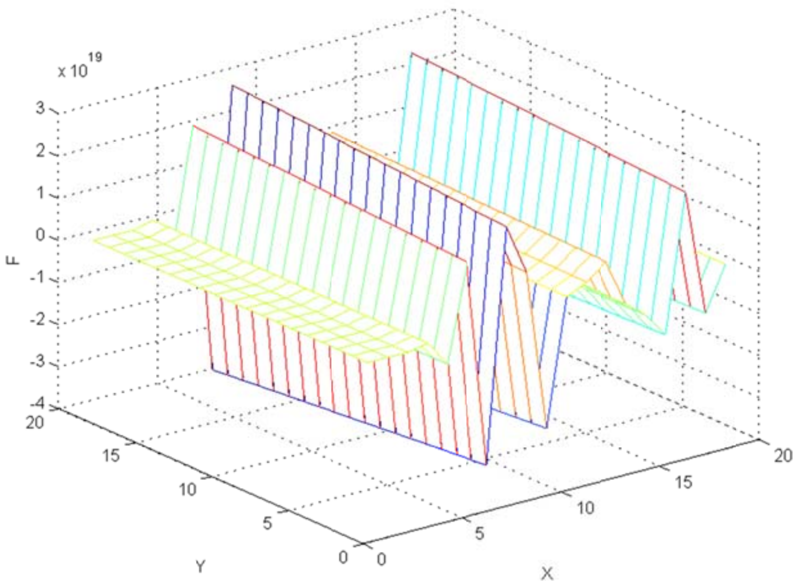

Fig. 1 Nonlinear Multimodal Function

Assume that the parameter set (x, y) is encoded into a chromosome string. Assuming a population of 4 individuals:

a) Using 2 decimal points precision, show the binary encoding of 4 initial individuals.

b) Suggest a crossover operator for this representation and show how it is applied to the examples.

c) Suggest a mutation operator and show how it is applied.

d) Repeat a, b and c for decimal real number encoding.

Solution:

a) We need ceil(log(10)+1) + ceil(log2(1/0.01)) =4+7 bits for each variable. Since we have 2 variables the length of chromosome is $2 \times 11 = 22$ bits. The first 11 bits are dedicated to x and the second 11 bits to y.

$x_1 = 5.00, y_1 = 2.25$	0101000000000100011001
$x_2 = 1.01, y_2 = 10.10$	0001000000110100001010

b) We can use 1-point crossover.

e.g. if we apply it on the two above chromosomes at index 14

01010000000001■ 00011001, 00010000001101 ■00001010 we have:

0101000000000100001010, 0001000000110100011001

c) For mutation we can flip each gene with probability p_m

e.g. 0101000000000100011001→ 0101000000000100111000

d)

$x_1 = 5.00, y_1 = 2.25$	(5.00 2.25)
$x_2 = 1.01, y_2 = 10.10$	(1.01 10.10)

We can use arithmetic cross over with $\alpha = 0.4$. e.g.

$0.4 \times 5.00 + 0.6 \times 1.01 = 2.60, 0.4 \times 2.25 + 0.6 \times 10.10 = 6.96 \rightarrow (2.60, 6.96)$

$0.6 \times 5.00 + 0.4 \times 1.01 = 3.40, 0.6 \times 2.25 + 0.4 \times 10.10 = 5.39 \rightarrow (3.40, 5.39)$

Mutation operator can be addition of random Gaussian noise to the solution with standard deviation equal to 0.5. e.g. (5.00, 2.25) + (0.12, 0.24) = (5.12, 2.49)

3. **[Programming Exercise - 5 Marks]** Given **GA-1**, a simple MATLAB program of genetic algorithms that tries to find the maximum of the well-known Easom function:

$$f(x) = -\cos(x)e^{-(x-\pi)^2}, \qquad x \in [-10,10]$$

As shown in Fig. 2, this function has a unique global maximum $f_{max}=1$, at $x^* = \pi$.

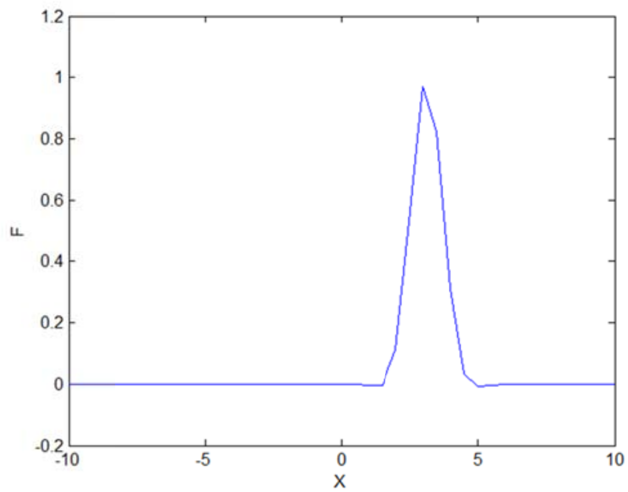

Fig. 2 Easom's Function

In the given MATLAB program, fixed-length 16-bit strings are used. The probabilities of crossover and mutation are respectively:

$$p_c = 0.95, \qquad p_m = 0.05$$

Modify the given program to find the global minimum of the following n-dimensional function

$$f(x) = \sum_{i=1}^{n} |x_i|^{i+1}, \qquad -1 \le x_i \le 1.$$

n is a user-defined variable, i.e. its value can be obtained from the user input or defined in the program.

Solution: Code is attached.