Understanding Risk Aversion: A Study of Quantitative vs. Non-Quantitative Profiles

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1 Introduction

Risk aversion describes a preference for certainty over uncertainty when outcomes involve potential gains or losses. It is a fundamental concept in decision-making under uncertainty, explaining why individuals or organizations often forego potentially higher but uncertain rewards in favor of safer, more predictable outcomes.

Some other terminology used in our experiment and inferences:-
Utility Function : A mathematical representation of an individual's preferences. It assigns a numerical value to each possible outcome, with higher values indicating greater satisfaction or preference. It is represented as

$$U(x)$$

For a concave utility function (risk aversion):

$$U''(x) < 0$$

Certainty Equivalent : The guaranteed amount of value an individual would accept instead of relying on uncertain outcomes. It represents the point where the individual is indifferent between the certain amount and the risky prospect.

$$U(CE) = \mathbb{E}[U(X)]$$

Where $\mathbb{E}[U(X)]$ is the expected utility of the risky prospect.
Risk Premium : The difference between the expected value of a gamble and the certainty equivalent. It represents how much an individual is willing to forego to avoid the risk.

$$RP = \mathbb{E}[X] - CE$$

In this project, our group aims to focus on risk aversion. Through a randomized reward-based experiment that will be conducted across different groups within a controlled environment, we portray the difference in choices and ultimately the difference in final outcome that varying levels of risk aversion can lead to.

2 Experimental Setup

The subjects partaking in the experiment are drawn from 2 groups, one having a quantitative background and the other lacking one. Both these groups will have their members individually participate in a game. We hypothesize that this difference will lead to starkly contrasting attitude regarding risk appetite and therefore a difference in average final reward among other variables.
The game involves numbers being drawn from a random distribution known to the subjects. In the first iteration of the experiment this will be a Discrete Uniform Distribution between 0 & 100 whereas in the latter half we switch it out for a Discrete Binomial Distribution with n=100 and p=0.5.

The rules of the game are as follows. Each subject has the opportunity to either:-
A. Pass on the current number presented to them (thereby foregoing the current number and any past numbers they had "passed on") and wait for the next number.
B. Keep the latest number, i.e. the number currently displayed to them and end the game.
C. A subject can pass at most 19 times. That is, the 20th number displayed to them (after presumed 19 passes) will be their final number, after which the game ends automatically.
It is important to note once the subject has passed on a number and a new number is presented, the **previous number has no effect on the final reward.** The final reward therefore is the number the game ends with (ending either by (B) or (C)).

The objective of each subject is to obtain the **highest final reward possible.** Thus at each stage, they must decide whether to keep the current number or risk whatever remaining passes they have in hopes of receiving a higher number within the limited trials. Each trial or time-step being the act of drawing a number from the aforementioned distribution and presenting it to the subject. The final score a subject obtains will be their compensation at the end of the game.

3 Ideal Strategy

Since the numbers are drawn from a random distribution, it is impossible for a single trial to determine what the ideal action (passing or keeping) should be. But since the distribution for each phase remains the same, what can be determined at each stage is the expected value of the number drawn over the remaining time-steps.

For the ease and brevity of explanation, we introduce the following notation:-
The random variable representing the number that is drawn at time-step t : X_t
The number actually drawn at t : x_t
The random variable that represents the max number one can obtain from time-step $t+1$ to 20 : $X_t^{\mathbf{max}}$
From the above definition it is clear that $X_t^{\max} = \max\{X_{t+1}, X_{t+2}, \ldots, X_{20}\}$

Thus, at each time-step t, a rational subject must weigh x_t and $\mathbb{E}[X_t^{\max}]$. In fact, said rational subject should continue to pass on numbers until a time-step t where $x_t \geq \mathbb{E}[X_t^{\max}]$.

4 Assumptions of Risk Aversion

We assume that Group 1,consisting of subjects with a quantitative background will act as perfect rational players with no overall risk aversion therefore their utility function with respect to this number will simply be the identity function.

$$U(x_t) = x_t$$

On the other hand, we assume the non-quantitative background operates with risk averse, i.e. concave utility functions. For our experiment there are assumed to be **CARA (Constant Absolute Risk Aversion)** and **CRRA (Constant Relative Risk Aversion)**.
The **CARA utility function** models risk preferences where the level of risk aversion remains **constant** regardless of the wealth or payoff level.

Mathematical Form

$$U(x) = \begin{cases} \frac{1-e^{-a\cdot x}}{a}, & \text{if } a \neq 0 \\ x, & \text{if } a = 0 \end{cases}$$

- x: Input, typically representing **wealth** or **payoff**.
- a: **Risk aversion parameter** (larger a implies higher risk aversion).

The **CRRA utility function** models risk preferences where the level of risk aversion depends on the **relative wealth or payoff**. Individuals care more about **proportional changes** in wealth than absolute changes.

Mathematical Form

$$U(x) = \begin{cases} \frac{x^{1-\gamma}-1}{1-\gamma}, & \text{if } \gamma \neq 1 \\ \ln(x), & \text{if } \gamma = 1 \end{cases}$$

- x: Input, representing **wealth** or **payoff** ($x > 0$).
- γ: **Risk aversion parameter** (higher γ implies higher risk aversion).

Since all groups operate using some utility function, we rewrite our comparison equation (from the previous section) to incorporate the same. Thus the stopping at a time-step t occurs when $U(x_t) \geq \mathbb{E}[U(X_t^{\max})]$ is satisfied. Thus it is apparent from this equation a player will stop passing and end the game as soon the **certainty equivalent** is met or just passed at any time-step t.

5 Statistical Tests

To compare the distribution of our results across the 2 groups we propose the following tests :-

5.1 Comparing Means

We propose a Hotelling's T^2 Test for comparing the mean between our 2 groups. Hotelling's T^2 Test can be used for analyzing the means of multivariate data, especially when variables are interdependent. It extends the t-test to handle multidimensional mean comparisons while considering the relationships between variables. The test statistic is computed as:-

$$T^2 = n(\bar{\mathbf{x}} - \boldsymbol{\mu}_0)^T \mathbf{S}^{-1} (\bar{\mathbf{x}} - \boldsymbol{\mu}_0),$$

where:

- $\bar{\mathbf{x}}$: Sample mean vector.
- $\boldsymbol{\mu}_0$: Hypothesized mean vector.
- $\mathbf{S}$: Sample covariance matrix.
- n: Sample size.

5.2 Comparing Variances

For similar reasons as mentioned earlier we would also like to compare the standard deviations (σ) of the 2 distributions. For this we conduct an F-Test. The test statistic is computed as:-

$$F = \frac{s_1^2}{s_2^2},$$

where

- s_1^2: Variance of the first sample.
- s_2^2: Variance of the second sample.

Sample variance is calculated as:

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1},$$

where:

- x_i: Observations in sample.
- $\bar{x}$: Mean of the sample.
- n: Sample size of the group.

5.3 Independence of Distributions

Most independence tests do not easily extend to multivariate data thus we propose subdividing this hypothesis test into 2 parts, one for each marginal. Holding the value of t and keep x as our variable and vice-versa. For each of the 2 cases we conduct a Kolmogorov-Smirnov Test to compare these marginal distributions i.e. 2 1 dimensional distributions instead. Thus for $X \mid T = t$ and $T \mid X = x$. This statistic is computed as:-

$$D = \sup_x |F_{1,n}(x) - F_{2,m}(x)|,$$

where:

- D: The KS test statistic, representing the maximum absolute difference between the two empirical cumulative distribution functions (ECDFs).
- $F_{1,n}(x)$: The ECDF of the first sample with size n.
- $F_{2,m}(x)$: The ECDF of the second sample with size m.
- $\sup_x$: The supremum (maximum) over all values of x.

Once we have proven the independence of the marginals across both groups, we can thus trivially conclude the joint distributions will be independent as well.

6 Certainty Equivalents across time

In this section we display graphs regarding the certainty equivalents as our time-step or trial variable t varies from 1 to 20.

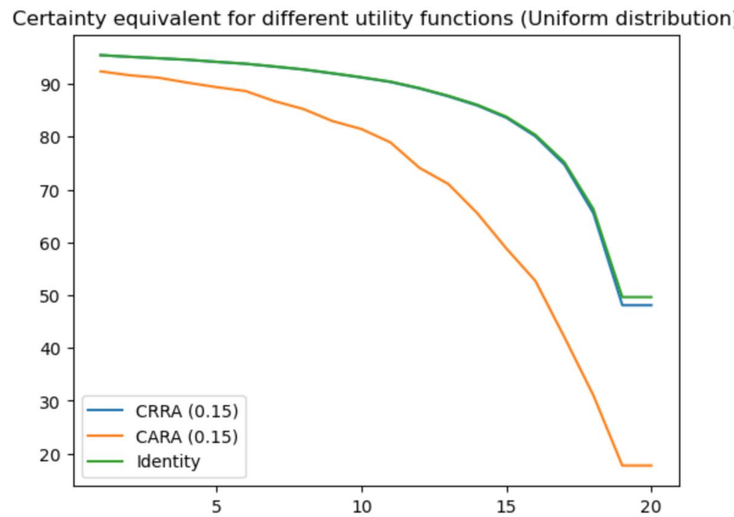

Figure 1: Certainty equivalent for Uniform distribution displaying various utility functions across time.

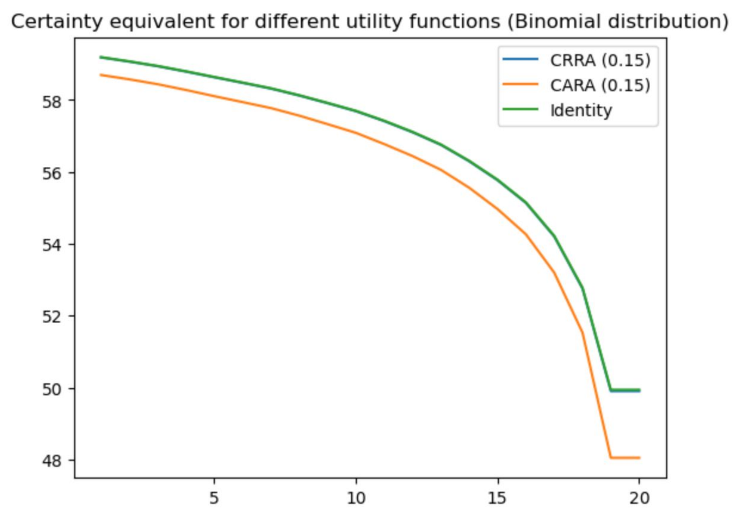

Figure 2: Certainty equivalent for Uniform distribution displaying various utility functions across time.

7 oTree Implementation

Using oTree, we developed an experiment to simulate decision-making under uncertainty. The implementation is structured as follows:

7.1 Implementation Details

- **Random Number Generation:** The random integers are generated and stored using oTree's `settings.py`, ensuring reproducibility and consistency.
- **User Interaction:** The interface for each round is handled in `pages.py`, displaying the current number and offering the stop/continue choices.
- **Score Calculation:** The score is automatically saved in the database when the game ends.
- **Data Analysis:** oTree's built-in data export tools facilitate easy collection and analysis of participant behavior.

7.2 Advantages of oTree

oTree provides a seamless framework for implementing behavioral experiments by:

- Automating data storage and management.
- Simplifying interface design for repeated decision-making tasks.
- Supporting randomized number generation and real-time feedback.
- Enabling quick deployment and scalability for online experiments.

8 Conclusion

Thus we have proposed an experiment to analyze risk aversion in 2 groups: quantitative vs non-quantitative