

# EE5600 Assignment 3

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**Abstract**—This document contains the solution of geometry through linear algebra through the concept of optimization.

Download latex and python codes from

[https://github.com/abhishekt711/EE5600/tree/master/Assignment\\_3](https://github.com/abhishekt711/EE5600/tree/master/Assignment_3)

## 1 PROBLEM

Maximize  $Z = 5x + 3y$  subject to  $3x + 5y \leq 15$ ,  
 $5x + 2y \leq 10$ ,  $x \geq 0$ ,  $y \geq 0$ .

## 2 EXPLANATION

$$Z - 5x - 3y = 0 \quad (2.0.1)$$

$$3x + 5y + s_1 = 15 \quad (2.0.2)$$

$$5x + 2y + s_2 = 10 \quad (2.0.3)$$

We will write the simplex tableau

$$\begin{pmatrix} x & y & s_1 & s_2 & c \\ 3 & 5 & 1 & 0 & 15 \\ \boxed{5} & 2 & 0 & 1 & 10 \\ -5 & -3 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.4)$$

Keeping the pivot element as 5, we will use gauss-jordan elimination.

$$\begin{pmatrix} x & y & s_1 & s_2 & c \\ 0 & \boxed{\frac{19}{5}} & 1 & \frac{-3}{5} & 9 \\ 1 & \frac{2}{5} & 0 & \frac{1}{5} & 2 \\ 0 & -1 & 0 & 1 & 10 \end{pmatrix} \quad (2.0.5)$$

Keeping the pivot element as  $\frac{19}{5}$ , we will use gauss-jordan elimination.

$$\begin{pmatrix} x & y & s_1 & s_2 & c \\ 0 & 1 & \frac{5}{19} & \frac{-3}{19} & \frac{45}{19} \\ 1 & 0 & \frac{-2}{19} & \frac{1}{19} & \frac{20}{19} \\ 0 & 0 & \frac{5}{19} & \frac{16}{19} & \frac{235}{19} \end{pmatrix} \quad (2.0.6)$$

In this tableau, there are no negative elements in the bottom row. We have therefore determined the optimal solution to be:

$$(x, y, s_1, s_2) = \left(\frac{20}{19}, \frac{45}{19}, 0, 0\right) \quad (2.0.7)$$

$$Z = 5x + 3y \quad (2.0.8)$$

$$Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} \quad (2.0.9)$$

$$Z = \frac{235}{19} \quad (2.0.10)$$

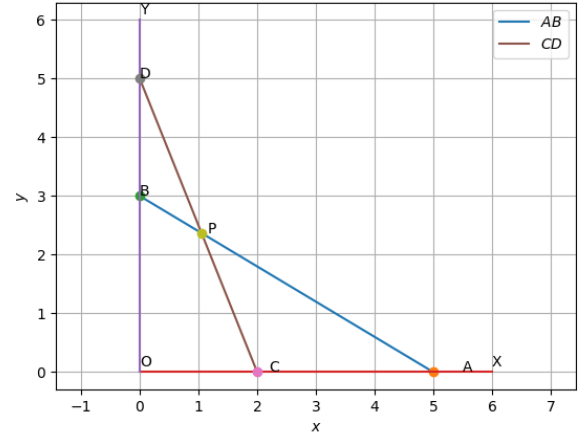


Fig. 0: optimal point through the intersection of various lines

The given problem can be expressed in general as matrix inequality as:

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 5 & 3 \end{pmatrix} \mathbf{x} \quad (2.0.11)$$

$$s.t. \quad \begin{pmatrix} 3 & 5 \\ 5 & 2 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 15 \\ 10 \end{pmatrix} \quad (2.0.12)$$

$$\mathbf{x} \geq \mathbf{0} \quad (2.0.13)$$

$$\mathbf{y} \geq \mathbf{0} \quad (2.0.14)$$

$$\max_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \quad (2.0.15)$$

$$s.t. \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}, \quad (2.0.16)$$

$$\mathbf{x} \geq \mathbf{0} \quad (2.0.17)$$

$$\mathbf{y} \geq \mathbf{0} \quad (2.0.18)$$

where

$$\mathbf{c} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad (2.0.19)$$

$$\mathbf{A} = \begin{pmatrix} 3 & 5 \\ 5 & 2 \end{pmatrix} \quad (2.0.20)$$

$$\mathbf{b} = \begin{pmatrix} 15 \\ 10 \end{pmatrix} \quad (2.0.21)$$

and can be solved using *cvxpy* through the following code

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code/Assignment_3.py
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Hence,

$$\mathbf{x} = \begin{pmatrix} 1.05263158 \\ 2.36842105 \end{pmatrix}, Z = 12.36842102 \quad (2.0.22)$$