EE5600 Assignment 4

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***Abstract*—This document contains the solution of Quadratic equation using gradient descent.**

Download latex and python codes from

https://github.com/abhishekt711/EE5600/tree/master/Assignment_4

1 Problem

Find the maximum and minimum values, if any, of the follwing function given by
$f(x) = (2x-1)^2 + 3$

2 Explanation

Given Equation can be written as:

$$f(x) = (2x-1)^2 + 3 = 4x^2 - 4x + 4 \tag{2.0.1}$$

A function is said to be convex if following inequality is true:

$$\lambda f(x_1) + (1-\lambda) f(x_2) \geq f(\lambda x_1 + (1-\lambda)x_2) \tag{2.0.2}$$

and for $\lambda \in [0,1]$

$$\lambda(4x_1^2 - 4x_1 + 4) + (1-\lambda)((4x_2^2 - 4x_2 + 4) \geq 4(\lambda x_1 + (1-\lambda)x_2)^2 - 4(\lambda x_1 + (1-\lambda)x_2) + 4 \tag{2.0.3}$$

$$x_1^2(4\lambda - 4\lambda^2) + x_2^2(4\lambda - 4\lambda^2) - 2x_1x_2(4\lambda - 4\lambda^2) \geq 0 \tag{2.0.4}$$

$$4\lambda(1-\lambda)(x_1 - x_2)^2 \geq 0 \tag{2.0.5}$$

The above inequality is always true for all values from the domain. Hence the given function f(x) is convex.

Using gradient descent method,

$$x_n = x_{n-1} - \mu \frac{df(x)}{dx} \tag{2.0.6}$$

Here, μ is learning rate.

$$\frac{df(x)}{dx} = 8x - 4 \tag{2.0.7}$$

After substituting 2.0.7 in 2.0.6 we get:

$$x_n = x_{n-1} - \mu(8x_{n-1} - 4) \tag{2.0.8}$$

In equation (2.0.8), μ is a variable parameter known as step size. x_{n+1} is the next position. The minus sign refers to the minimization part of gradient descent. Assume, $\mu = 0.01$, $x_0 = 2$ and following the above method, we keep doing iterations until $x_{n+1} - x_n$ becomes less than the value of precision we have chosen.

$$x_n = 0.5 \tag{2.0.9}$$

The following python code computes the minimum value as plotted in Fig. 0. Hence, The minimum value of $f(x)$ at $x = 0.5$ is 3.

./codes/Assignment_4.py

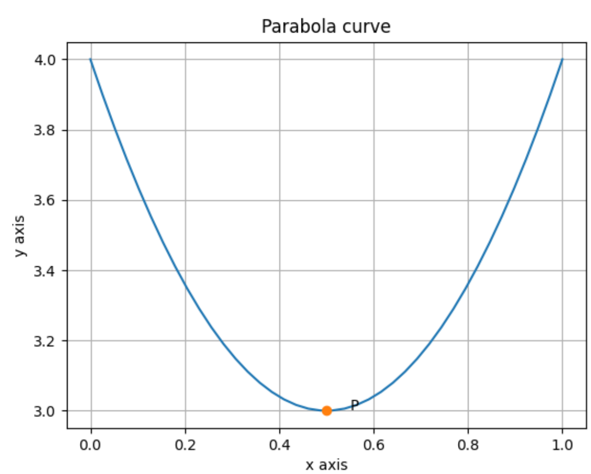

Fig. 0: Plot of the given polynomial