

EE5600 Assignment 4

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Abstract—This document contains the solution of Quadratic equation using gradient descent.

Download latex and python codes from

https://github.com/abhishekt711/EE5600/tree/master/Assignment_4

1 PROBLEM

Find the maximum and minimum values, if any, of the following function given by

$$f(x) = (2x - 1)^2 + 3$$

2 EXPLANATION

Given Equation can be written as:

$$f(x) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4 \quad (2.0.1)$$

A function is said to be convex if following inequality is true:

$$\lambda f(x_1) + (1 - \lambda)f(x_2) \geq f(\lambda x_1 + (1 - \lambda)x_2) \quad (2.0.2)$$

and for $\lambda \in [0, 1]$

$$\begin{aligned} \lambda(4x_1^2 - 4x_1 + 4) + (1 - \lambda)(4x_2^2 - 4x_2 + 4) &\geq \\ 4(\lambda x_1 + (1 - \lambda)x_2)^2 - 4(\lambda x_1 + (1 - \lambda)x_2) + 4 & \end{aligned} \quad (2.0.3)$$

$$x_1^2(4\lambda - 4\lambda^2) + x_2^2(4\lambda - 4\lambda^2) - 2x_1x_2(4\lambda - 4\lambda^2) \geq 0 \quad (2.0.4)$$

$$4\lambda(1 - \lambda)(x_1 - x_2)^2 \geq 0 \quad (2.0.5)$$

The above inequality is always true for all values from the domain. Hence the given function $f(x)$ is convex.

Using gradient descent method,

$$x_1 = x_0 - \mu \frac{df(x)}{dx} \quad (2.0.6)$$

Here, μ is learning rate.

$$\frac{df(x)}{dx} = 8x - 4 \quad (2.0.7)$$

Taking $\mu = 0.1$, $x_0 = 2$ and substituting in equation 2.0.6. After 4 iteration, the value of x where minima occurs is:

$$x = 0.50048 \quad (2.0.8)$$

After each iteration starting from $x=2$, when we come leftward on x -axis the slope increases i.e., slope becoming less negative. Thus, it is a convex function.

At maxima / minima slope of $f(x)$ will be zero.

$$\frac{df(x)}{dx} = 0 \quad (2.0.9)$$

$$\implies 8x - 4 = 0 \quad (2.0.10)$$

$$\therefore x = 0.5 \quad (2.0.11)$$

Now, to know whether $x=0.5$ is the point of maxima or minima, we need to calculate $f''(x)$.

$$\frac{d''f(x)}{dx} = 8 \quad (2.0.12)$$

$\because \frac{d''f(x)}{dx} > 0$, therefore, we can conclude that $x=0.5$ is the point of minima.

At $f(x)$ at $x=0.5$,

$$f(0.5) = 3 \quad (2.0.13)$$

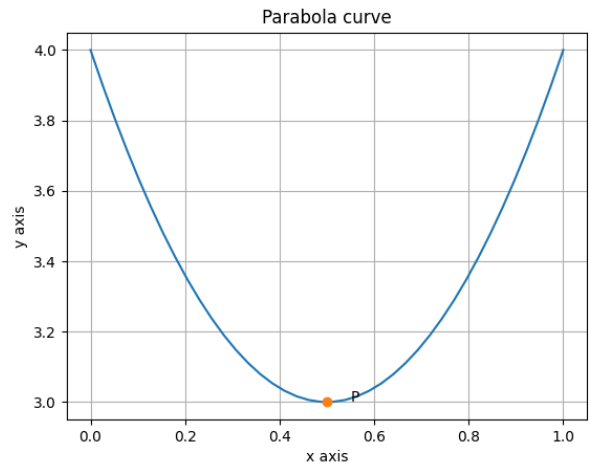


Fig. 0: Plot of the given polynomial

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./codes/Assignment_4.py
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The following python code computes the minimum value of the given polynomial as plotted in Fig. 0. Hence, The minimum value of $f(x)$ at $x = 0.5$ is 3.