EE5600 Assignment 4

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***Abstract*—This document contains the solution of Quadratic equation using gradient descent.**

Download latex and python codes from

https://github.com/abhishekt711/EE5600/tree/master/Assignment_4

1 Problem

Find the maximum and minimum values, if any, of the follwing function given by
$f(x) = (2x-1)^2 + 3$

2 Explanation

Given Equation can be written as:

$$f(x) = (2x-1)^2 + 3 = 4x^2 - 4x + 4 \tag{2.0.1}$$

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 4 = 0 \tag{2.0.2}$$

For a function to be a convex, It should follow this condition:

$$f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1) + (1-\lambda) f(x_2) \tag{2.0.3}$$

When, we take $\lambda = 0.5, x_1 = 2, x_2 = 0$, It satisfy the equation 2.0.3. Thus, It is a convex function.

Using gradient descent method,

$$x_1 = x_0 - lr\frac{df(x)}{dx} \tag{2.0.4}$$

Here, lr is learning rate.

$$\frac{df(x)}{dx} = 8x - 4 \tag{2.0.5}$$

Taking $lr = 0.1$, $x_0 = 2$ and substiuting in equation 2.0.4 . After 4 iteration, the value of x where minima occurs is:

$$x = 0.50048 \tag{2.0.6}$$

At maxima / minima slope of $f(x)$ will be zero.

$$\frac{df(x)}{dx} = 0 \tag{2.0.7}$$

$$\implies 8x - 4 = 0 \tag{2.0.8}$$

$$\therefore x = 0.5 \tag{2.0.9}$$

Now, to know whether x=0.5 is the point of maxima or minima, we need to calculate $f''(x)$.

$$\frac{d''f(x)}{dx} = 8 \tag{2.0.10}$$

$\because \frac{d''f(x)}{dx} > 0$, therefore, we can conclude that x=0.5 is the point of minima.
At $f(x)$ at x=0.5,

$$f(0.5) = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}^T \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} + \begin{pmatrix} -4 & 0 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} + 4 = 3 \tag{2.0.11}$$

The following python code computes the minimum value of the given polynomial as plotted in Fig. 0.

./codes/Assignment_4.py

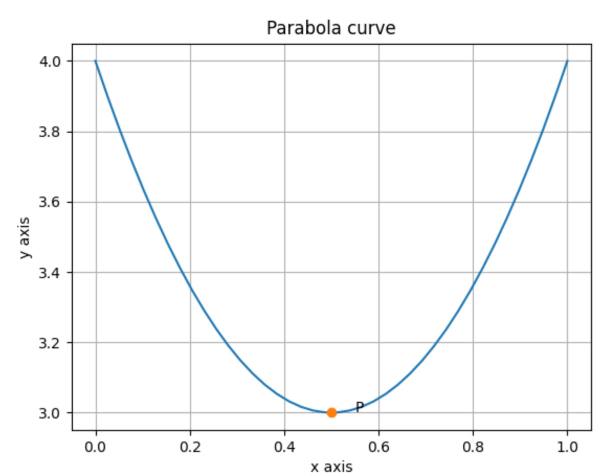

Fig. 0: Plot of the given polynomial

Hence, the minimum value of $f(x)$ at $x = 0.5$ is 3.