EE5600 Assignment 4

Abhishek Thakur

***Abstract*—This document contains the solution of Quadratic equation using gradient descent.**

Download latex and python codes from

https://github.com/abhishekt711/EE5600/tree/master/Assignment_4

1 Problem

Find the maximum and minimum values, if any, of the follwing function given by
$f(x) = (2x-1)^2 + 3$

2 Explanation

Given Equation can be written as:

$$f(x) = (2x-1)^2 + 3 = 4x^2 - 4x + 4 \tag{2.0.1}$$

A function is said to be convex if following inequality is true:

$$\lambda f(x_1) + (1-\lambda) f(x_2) \geq f(\lambda x_1 + (1-\lambda)x_2) \tag{2.0.2}$$

and for $\lambda \in [0,1]$

$$\lambda(4x_1^2 - 4x_1 + 4) + (1-\lambda)((4x_2^2 - 4x_2 + 4) \geq 4(\lambda x_1 + (1-\lambda)x_2)^2 - 4(\lambda x_1 + (1-\lambda)x_2) + 4 \tag{2.0.3}$$

$$x_1^2(4\lambda - 4\lambda^2) + x_2^2(4\lambda - 4\lambda^2) - 2x_1x_2(4\lambda - 4\lambda^2) \geq 0 \tag{2.0.4}$$

$$4\lambda(1-\lambda)(x_1 - x_2)^2 \geq 0 \tag{2.0.5}$$

The above inequality is always true for all values from the domain. Hence the given function f(x) is convex.

Using gradient descent method,

$$x_n = x_{n-1} - \mu \frac{df(x)}{dx} \tag{2.0.6}$$

Here, μ is learning rate.

$$\frac{df(x)}{dx} = 8x - 4 \tag{2.0.7}$$

After substituting 2.0.7 in 2.0.6 we get:

$$x_n = x_{n-1} - \mu(8x - 4) \tag{2.0.8}$$

taking $\mu = 0.1$, $x_0 = 2$, we get:

$$x_1 = 2 - 0.1(8 \times 2 - 4) \tag{2.0.9}$$
$$x_1 = 0.8 \tag{2.0.10}$$
$$x_2 = 0.8 - 0.1(8 \times 0.8 - 4) \tag{2.0.11}$$
$$x_2 = 0.56 \tag{2.0.12}$$
$$x_3 = 0.56 - 0.1(8 \times 0.56 - 4) \tag{2.0.13}$$
$$x_3 = 0.512 \tag{2.0.14}$$
$$x_4 = 0.512 - 0.1(8 \times 0.512 - 4) \tag{2.0.15}$$
$$x_4 = 0.5024 \tag{2.0.16}$$
$$x_5 = 0.5024 - 0.1(8 \times 0.5024 - 4) \tag{2.0.17}$$
$$x_5 = 0.50048 \tag{2.0.18}$$

after iterationg, finally:

$$x_n = 0.5 \tag{2.0.19}$$

At maxima / minima slope of $f(x)$ will be zero.

$$\frac{df(x)}{dx} = 0 \tag{2.0.20}$$
$$\implies 8x - 4 = 0 \tag{2.0.21}$$
$$\therefore x = 0.5 \tag{2.0.22}$$

Now, to know whether x=0.5 is the point of maxima or minima, we need to calculate $f''(x)$.

$$\frac{d'' f(x)}{dx} = 8 \tag{2.0.23}$$

$\because \frac{d'' f(x)}{dx} > 0$, therefore, we can conclude that x=0.5 is the point of minima.
At $f(x)$ at x=0.5,

$$f(0.5) = 3 \tag{2.0.24}$$

./codes/Assignment_4.py

The following python code computes the minimum value of the given polynomial as plotted in Fig. 0. Hence, The minimum value of $f(x)$ at $x = 0.5$ is 3.

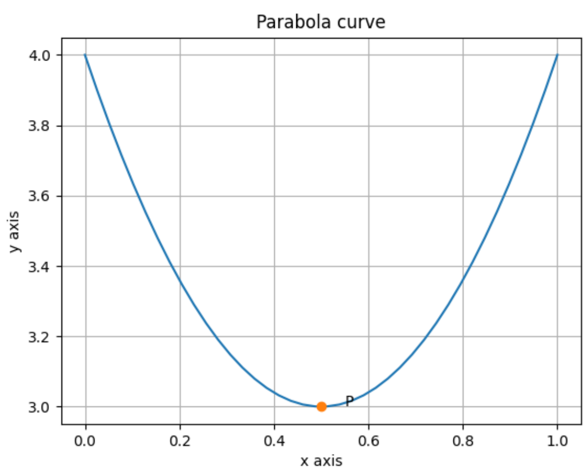

Fig. 0: Plot of the given polynomial