EE5600 Assignment 3

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***Abstract*—This document contains the solution of geometry through linear algebra through the concept of optimization.**

Download latex and python codes from

https://github.com/abhishekt711/EE5600/tree/master/Assignment_3

1 Problem

Maximize $Z = 5x + 3y$ subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

2 Explanation

$$Z - 5x - 3y = 0 \tag{2.0.1}$$
$$3x + 5y + s_1 = 15 \tag{2.0.2}$$
$$5x + 2y + s_2 = 10 \tag{2.0.3}$$

We will write the simplex tableau

$$\begin{array}{c} \begin{array}{ccccc} x & y & s_1 & s_2 & c \end{array} \\ \begin{bmatrix} 3 & 5 & 1 & 0 & 15 \\ \boxed{5} & 2 & 0 & 1 & 10 \\ \hline -5 & -3 & 0 & 0 & 0 \end{bmatrix} \end{array} \tag{2.0.4}$$

Keeping the pivot element as 5, we will use gauss-jordan elimination.

$$\begin{array}{c} \begin{array}{ccccc} x & y & s_1 & s_2 & c \end{array} \\ \begin{bmatrix} 0 & \boxed{\frac{19}{5}} & 1 & \frac{-3}{5} & 9 \\ 1 & \frac{2}{5} & 0 & \frac{1}{5} & 2 \\ \hline 0 & -1 & 0 & 1 & 10 \end{bmatrix} \end{array} \tag{2.0.5}$$

Keeping the pivot element as $\frac{19}{5}$, we will use gauss-jordan elimination.

$$\begin{array}{c} \begin{array}{ccccc} x & y & s_1 & s_2 & c \end{array} \\ \begin{bmatrix} 0 & 1 & \frac{5}{19} & \frac{-3}{19} & \frac{45}{19} \\ 1 & 0 & \frac{-2}{19} & \frac{5}{19} & \frac{20}{19} \\ \hline 0 & 0 & \frac{5}{19} & \frac{16}{19} & \frac{235}{19} \end{bmatrix} \end{array} \tag{2.0.6}$$

In this tableau, there are no negative elements in the bottom row. We have therefore determined the optimal solution to be:

$$(x, y, s_1, s_2) = \left(\frac{20}{19}, \frac{45}{19}, 0, 0\right) \tag{2.0.7}$$
$$Z = 5x + 3y \tag{2.0.8}$$
$$Z = 5 \times \frac{20}{19} + 3 \times \frac{45}{19} \tag{2.0.9}$$
$$Z = \frac{235}{19} \tag{2.0.10}$$

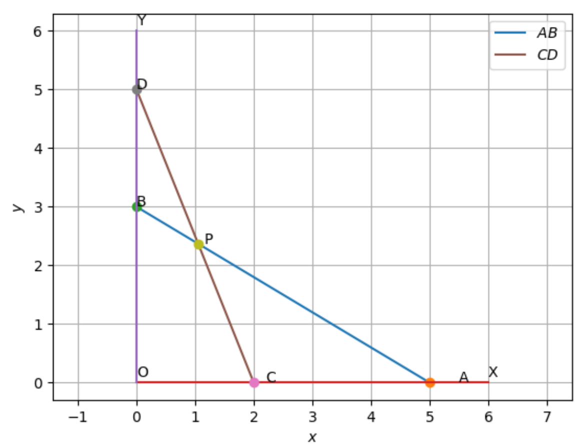

Fig. 0: optimal point through the intersection of various lines

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 5 & 3 \end{pmatrix} \mathbf{x} \tag{2.0.11}$$
$$s.t. \quad \begin{pmatrix} 3 & 5 \\ 5 & 2 \end{pmatrix} \mathbf{x} \preceq \begin{pmatrix} 15 \\ 10 \end{pmatrix} \tag{2.0.12}$$
$$\mathbf{x} \succeq \mathbf{0} \tag{2.0.13}$$
$$\mathbf{y} \succeq \mathbf{0} \tag{2.0.14}$$

using cvxpy.

The given problem can be expressed in general as matrix inequality as:

$$\max_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \tag{2.0.15}$$
$$s.t. \quad \mathbf{A}\mathbf{x} \leq \mathbf{b}, \tag{2.0.16}$$
$$\mathbf{x} \geq \mathbf{0} \tag{2.0.17}$$
$$\mathbf{y} \geq \mathbf{0} \tag{2.0.18}$$

where

$$\mathbf{c} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{2.0.19}$$
$$\mathbf{A} = \begin{pmatrix} 3 & 5 \\ 5 & 2 \end{pmatrix} \tag{2.0.20}$$
$$\mathbf{b} = \begin{pmatrix} 15 \\ 10 \end{pmatrix} \tag{2.0.21}$$

and can be solved using *cvxpy* through the following code

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code/Assignment_3.py
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Hence,

$$\mathbf{x} = \begin{pmatrix} 1.05263158 \\ 2.36842105 \end{pmatrix}, Z = 12.36842102 \tag{2.0.22}$$