

EE5600 Assignment 4

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Abstract—This document contains the solution of linear algebra through the concept of Matrix Theory.

Download latex and python codes from

https://github.com/abhishekt711/EE5600/tree/master/Assignment_4

1 PROBLEM

Find the maximum and minimum values, if any, of the following function given by

$$f(x) = (2x - 1)^2 + 3$$

2 EXPLANATION

Given Equation can be written as:

$$f(x) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4 \quad (2.0.1)$$

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 & 0 \end{pmatrix} \mathbf{x} + 4 = 0 \quad (2.0.2)$$

For a function to be a convex, It should follow this condition:

$$f(\lambda x_1 + (1 - \lambda)x_2) \leq \lambda f(x_1) + (1 - \lambda)f(x_2) \quad (2.0.3)$$

When, we take $\lambda = 0.5$, $x_1 = 2$, $x_2 = 0$, It satisfy the equation 2.0.3. Thus, It is a convex function.

Using gradient descent method,

$$x_1 = x_0 - lr \frac{df(x)}{dx} \quad (2.0.4)$$

Here, lr is learning rate.

$$\frac{df(x)}{dx} = 8x - 4 \quad (2.0.5)$$

Taking $lr = 0.1$, $x_0 = 2$ and substituting in equation 2.0.4. After 4 iteration, the value of x where minima occurs is:

$$x = 0.50048 \quad (2.0.6)$$

At maxima / minima slope of $f(x)$ will be zero.

$$\frac{df(x)}{dx} = 0 \quad (2.0.7)$$

$$\Rightarrow 8x - 4 = 0 \quad (2.0.8)$$

$$\therefore x = 0.5 \quad (2.0.9)$$

Now, to know whether $x=0.5$ is the point of maxima or minima, we need to calculate $f''(x)$.

$$\frac{d''f(x)}{dx} = 8 \quad (2.0.10)$$

$\therefore \frac{d''f(x)}{dx} > 0$, therefore, we can conclude that $x=0.5$ is the point of minima.

At $f(x)$ at $x=0.5$,

$$f(0.5) = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}^T \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} + \begin{pmatrix} -4 & 0 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} + 4 = 3 \quad (2.0.11)$$

The following python code computes the minimum value of the given polynomial as plotted in Fig. 0.

`./codes/Assignment_4.py`

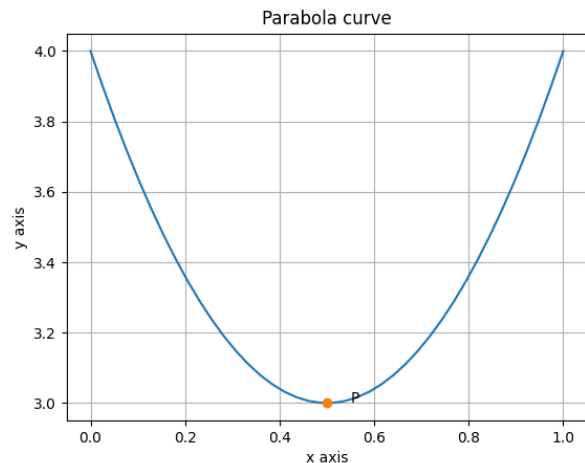


Fig. 0: Plot of the given polynomial

Hence, the minimum value of $f(x)$ at $x = 0.5$ is 3.