EE5600 Assignment 4

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Abstract—This document contains the solution of Quadratic equation using gradient descent.

Download latex and python codes from

1 Problem

Find the maximum and minimum values, if any, of the follwing function given by
$f(x) = (2x - 1)^2 + 3$

2 Explanation

Given Equation can be written as:

$$f(x) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4 \qquad (2.0.1)$$

A function is said to be convex if following inequality is true:

$$\lambda f(x_1) + (1 - \lambda)f(x_2) \geq f(\lambda x_1 + (1 - \lambda)x_2) \quad (2.0.2)$$

and for $\lambda \in [0, 1]$

$$\lambda(4x_1^2 - 4x_1 + 4) + (1 - \lambda)((4x_2^2 - 4x_2 + 4) \geq$$
$$4(\lambda x_1 + (1 - \lambda)x_2)^2 - 4(\lambda x_1 + (1 - \lambda)x_2) + 4$$
$$(2.0.3)$$
$$x_1^2(4\lambda - 4\lambda^2) + x_2^2(4\lambda - 4\lambda^2) - 2x_1 x_2(4\lambda - 4\lambda^2) \geq 0$$
$$(2.0.4)$$
$$4\lambda(1 - \lambda)(x_1 - x_2)^2 \geq 0 \qquad (2.0.5)$$

The above inequality is always true for all values from the domain. Hence the given function f(x) is convex.

Using gradient descent method,

$$x_n = x_{n-1} - \mu \frac{df(x)}{dx} \qquad (2.0.6)$$

Here, μ is learning rate.

$$\frac{df(x)}{dx} = 8x - 4 \qquad (2.0.7)$$

After substituting 2.0.7 in 2.0.6 we get:

$$x_n = x_{n-1} - \mu(8x - 4) \qquad (2.0.8)$$

taking $\mu = 0.1$, $x_0 = 2$, we get:

$$x_1 = 2 - 0.1(8 \times 2 - 4) \qquad (2.0.9)$$
$$x_1 = 0.8 \qquad (2.0.10)$$
$$x_2 = 0.8 - 0.1(8 \times 0.8 - 4) \qquad (2.0.11)$$
$$x_2 = 0.56 \qquad (2.0.12)$$
$$x_3 = 0.56 - 0.1(8 \times 0.56 - 4) \qquad (2.0.13)$$
$$x_3 = 0.512 \qquad (2.0.14)$$
$$x_4 = 0.512 - 0.1(8 \times 0.512 - 4) \qquad (2.0.15)$$
$$x_4 = 0.5024 \qquad (2.0.16)$$
$$x_5 = 0.5024 - 0.1(8 \times 0.5024 - 4) \qquad (2.0.17)$$
$$x_5 = 0.50048 \qquad (2.0.18)$$

after iteriationg, finally:

$$x_n = 0.5 \qquad (2.0.19)$$

The following python code computes the minimum value as plotted in Fig. 0. Hence, The minimum value of $f(x)$ at $x = 0.5$ is 3.
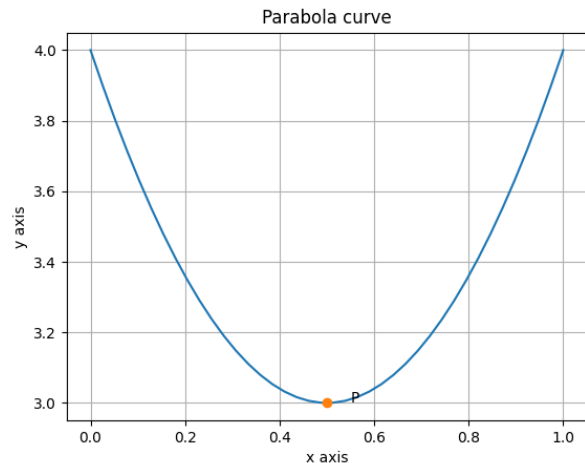
./codes/Assignment_4.py

Fig. 0: Plot of the given polynomial