EE5600 Assignment 1

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Abstract—This documnet contains the solution to a Lines and planes problem.

Download all python codes from

https://github.com/abhishekt711/EE5600/codes

1 Problem

Quad 1) ABCD is a rectangle formed by the points

$$A(-1 \quad -1), B(-1 \quad 4), C(5 \quad 4), D(5 \quad -1), P, Q, R, S$$

are the midpoints of AB, BC, CD, DA respectively. Is the Quadrilateral PQRS a

- A) Square?
- B) Rectangle?
- C) Rhombus?

2 Solution

Step1: we will find the midpoint P, Q, R, S

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} -1 & \frac{3}{2} \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{Q} = \frac{\mathbf{B} + \mathbf{C}}{2} = \begin{pmatrix} 2 & 4 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{R} = \frac{\mathbf{C} + \mathbf{D}}{2} = \begin{pmatrix} 5 & \frac{3}{2} \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{S} = \frac{\mathbf{A} + \mathbf{D}}{2} = \begin{pmatrix} 2 & -1 \end{pmatrix} \tag{2.0.4}$$

Join P, Q, R, S to form a quadrilateral

Step2: We will check whether quadrilateral PQRS is parallelogram or not.

Diagonal Bisect Rule:

If,

$$\frac{\mathbf{P} + \mathbf{R}}{2} = \frac{\mathbf{Q} + \mathbf{S}}{2} = (2 \quad 1.5)$$
 (2.0.5)

Then, Quadrilateral PQRS is a parallelogram **Step3**: We will check whether Parallelogram is rhombus or not.

$$(\mathbf{P} - \mathbf{R}) = \begin{pmatrix} -6 & 0 \end{pmatrix} \tag{2.0.6}$$

$$(\mathbf{Q} - \mathbf{S}) = \begin{pmatrix} 0 & 5 \end{pmatrix} \tag{2.0.7}$$

If adjacent side of parallelogram are equal then it is a rhombus.

$$(\mathbf{P} - \mathbf{R})^T (\mathbf{Q} - \mathbf{S}) = \begin{pmatrix} -6 & 0 \\ 5 \end{pmatrix}$$
 (2.0.8)

$$(\mathbf{P} - \mathbf{R})^T (\mathbf{Q} - \mathbf{S}) = (0)$$

(2.0.9)

Diagonal bisect orthogonally. Therefore, it may be a rhombus or square.

Thus, PQRS is a rhombus.

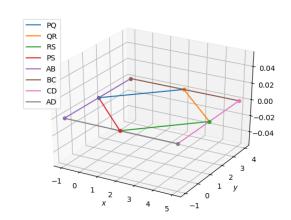


Fig. 0: Simulation of midpoint of ABCD forms PQRS.

Step4: We will check whether Parallelogram

PQRS is Square or not.

$$(\mathbf{P} - \mathbf{Q}) = \begin{pmatrix} -3 & -2.5 \end{pmatrix} \tag{2.0.10}$$

$$(\mathbf{P} - \mathbf{S}) = \begin{pmatrix} -3 & 2.5 \end{pmatrix} \tag{2.0.11}$$

If adjacent side of parallelogram are orthogonal to each other then PQRS is a Square.

$$(\mathbf{P} - \mathbf{Q})^T (\mathbf{P} - \mathbf{S}) = \begin{pmatrix} -3 & -2.5 \end{pmatrix} \begin{pmatrix} -3 \\ -2.5 \end{pmatrix}$$
 (2.0.12)

$$(\mathbf{P} - \mathbf{Q})^T (\mathbf{P} - \mathbf{S}) = (2.75) \qquad (2.0.13)$$

Here the angle between adjacent side is not 90 $^{\circ}$ Hence, PQRS is not a Square.

Step 5: We will check whether PQRS is rectangle or not.

If adjacent side of parallelogram and diagonal obeys pythagoras rule then only it is a rectangle. In (2.0.13) Rectangle condition not satisfying in this question.

Conclusion:PQRS is only a Rhombus. It is not a square and rectangle.