

# EE5600 Assignment 4

Abhishek Thakur

**Abstract**—This document contains the solution of Quadratic equation using gradient descent.

Download latex and python codes from

[https://github.com/abhishekt711/EE5600/tree/master/Assignment\\_4](https://github.com/abhishekt711/EE5600/tree/master/Assignment_4)

## 1 PROBLEM

Find the maximum and minimum values, if any, of the following function given by

$$f(x) = (2x - 1)^2 + 3$$

## 2 EXPLANATION

Given Equation can be written as:

$$f(x) = (2x - 1)^2 + 3 = 4x^2 - 4x + 4 \quad (2.0.1)$$

A function is said to be convex if following inequality is true:

$$\lambda f(x_1) + (1 - \lambda)f(x_2) \geq f(\lambda x_1 + (1 - \lambda)x_2) \quad (2.0.2)$$

and for  $\lambda \in [0, 1]$

$$\begin{aligned} \lambda(4x_1^2 - 4x_1 + 4) + (1 - \lambda)(4x_2^2 - 4x_2 + 4) &\geq \\ 4(\lambda x_1 + (1 - \lambda)x_2)^2 - 4(\lambda x_1 + (1 - \lambda)x_2) + 4 & \end{aligned} \quad (2.0.3)$$

$$x_1^2(4\lambda - 4\lambda^2) + x_2^2(4\lambda - 4\lambda^2) - 2x_1x_2(4\lambda - 4\lambda^2) \geq 0 \quad (2.0.4)$$

$$4\lambda(1 - \lambda)(x_1 - x_2)^2 \geq 0 \quad (2.0.5)$$

The above inequality is always true for all values from the domain. Hence the given function  $f(x)$  is convex.

Using gradient descent method,

$$x_1 = x_0 - \mu \frac{df(x)}{dx} \quad (2.0.6)$$

Here,  $\mu$  is learning rate.

$$\frac{df(x)}{dx} = 8x - 4 \quad (2.0.7)$$

Taking  $\mu = 0.1$ ,  $x_0 = 2$  and substituting in equation 2.0.6. After 4 iteration, the value of  $x$  where minima occurs is:

$$x = 0.50048 \quad (2.0.8)$$

At maxima / minima slope of  $f(x)$  will be zero.

$$\frac{df(x)}{dx} = 0 \quad (2.0.9)$$

$$\implies 8x - 4 = 0 \quad (2.0.10)$$

$$\therefore x = 0.5 \quad (2.0.11)$$

Now, to know whether  $x=0.5$  is the point of maxima or minima, we need to calculate  $f''(x)$ .

$$\frac{d^2f(x)}{dx^2} = 8 \quad (2.0.12)$$

$\because \frac{d^2f(x)}{dx^2} > 0$ , therefore, we can conclude that  $x=0.5$  is the point of minima.

At  $f(x)$  at  $x=0.5$ ,

$$f(0.5) = 3 \quad (2.0.13)$$

`./codes/Assignment_4.py`

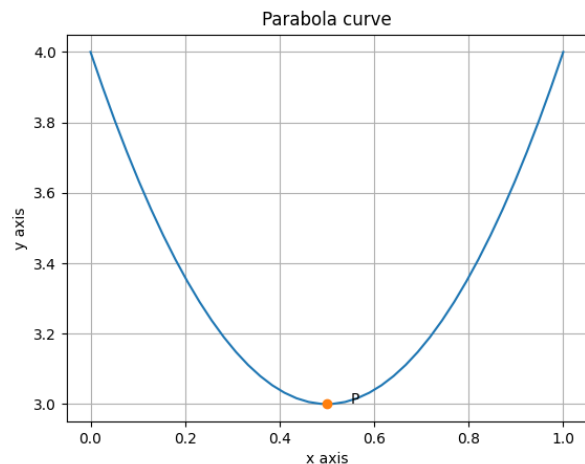


Fig. 0: Plot of the given polynomial

The following python code computes the minimum value of the given polynomial as plotted in Fig. 0.

Hence, The minimum value of  $f(x)$  at  $x = 0.5$  is 3.