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EE5609 Assignment 3

Abhishek Thakur

Abstract—This document contains the solution of geometry through linear algebra.

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_3

1 Problem

ABCE is a Quadrilateral and D is a midpoint on BC such that, AC=AE, AB=AD and $\angle BAD = \angle EAC$. Show that BC=DE.

2 EXPLANATION

In, $\triangle ABC$ and $\triangle ADE$

$$\angle BAD = \angle EAC$$
 (given) (2.0.1)

Adding $\angle DAC$ on both side, We get:

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC$$
 (2.0.2)

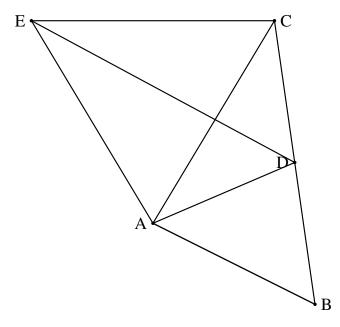


Fig. 0: Quadrilateral ABCE

We have,

$$\angle BAC = \angle DAE$$
 (2.0.3)

$$\implies \cos \angle DAE = \cos \angle BAC$$
 (2.0.4)

In triangle ABC, Using law of cosine, Using the formula of dot product, i.e.,

$$\mathbf{a}.\mathbf{b} = \|\mathbf{a}\| \cdot \|\mathbf{b}\| \cdot \cos \theta \tag{2.0.5}$$

$$\implies \cos \theta = \frac{\mathbf{a.b}}{\|\mathbf{a}\| \cdot \|\mathbf{b}\|} \tag{2.0.6}$$

$$\frac{(\mathbf{A} - \mathbf{D})^T (\mathbf{A} - \mathbf{E})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{A} - \mathbf{E}\|} = \frac{(\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{C}\|}$$
(2.0.7)

We are given AE=AC and we know AD=AB always. Thus,

$$\|\mathbf{A} - \mathbf{E}\| = \|\mathbf{A} - \mathbf{C}\| \tag{2.0.8}$$

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{A} - \mathbf{B}\| \tag{2.0.9}$$

Then, from (2.0.7), we have,

$$(\mathbf{A} - \mathbf{D})^T (\mathbf{A} - \mathbf{E}) = (\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}) \qquad (2.0.10)$$

We need to prove: $\|\mathbf{B} - \mathbf{C}\| = \|\mathbf{D} - \mathbf{E}\|$

$$\implies \|\mathbf{B} - \mathbf{C}\|^2 - \|\mathbf{D} - \mathbf{E}\|^2 =$$

$$(\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{C}) - (\mathbf{D} - \mathbf{E})^T (\mathbf{D} - \mathbf{E}) \quad (2.0.11)$$

$$= ((\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{B}))^{T} ((\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{B}))$$
$$- ((\mathbf{A} - \mathbf{E}) - (\mathbf{A} - \mathbf{D}))^{T} ((\mathbf{A} - \mathbf{E}) - (\mathbf{A} - \mathbf{D}))$$
$$(2.0.12)$$

$$= ||\mathbf{A} - \mathbf{C}||^2 + ||\mathbf{A} - \mathbf{B}||^2 - (\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{C})(\mathbf{A} - \mathbf{B})$$

$$-||\mathbf{A} - \mathbf{E}||^2 - ||\mathbf{A} - \mathbf{D}||^2 + (\mathbf{A} - \mathbf{D})^T (\mathbf{A} + \mathbf{E}) - (\mathbf{A} - \mathbf{E})(\mathbf{A} - \mathbf{D})$$
(2.0.13)

Thus, from (2.0.8), (2.0.9) and (2.0.10)

$$\|\mathbf{B} - \mathbf{C}\|^2 - \|\mathbf{D} - \mathbf{E}\|^2 = 0$$
 (2.0.14)

$$\implies \|\mathbf{B} - \mathbf{C}\|^2 = \|\mathbf{D} - \mathbf{E}\|^2 \qquad (2.0.15)$$

$$\therefore \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{D} - \mathbf{E}\| \tag{2.0.16}$$

Hence, BC=DE