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EE5609 Assignment 13

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Abstract—This document solves problem based on solution of vector space.

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/ master/Assignment 13

1 Problem

Let F be a sub field of the complex numbers , and let A be the following 2×2 matrix over F.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

compute $f(\mathbf{A})$ for the polynomial $f(x) = x^2 - 5x + 7$

2 Solution

Characteristic equation of A can be written as:

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \tag{2.0.1}$$

$$\begin{pmatrix} 2 - \lambda & 1 \\ -1 & 3 - \lambda \end{pmatrix} = 0 \tag{2.0.2}$$

The characteristics equation of the matrix will be,

$$\lambda^2 - 5\lambda + 7 = 0 \tag{2.0.3}$$

The characteristics equation will satisfy its own matrix

$$\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} = 0 \tag{2.0.4}$$

The given polynomial

$$f(x) = x^2 - 5x + 7 (2.0.5)$$

$$f(\mathbf{A}) = \mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I}$$
 (2.0.6)

substituting (2.0.4) in (2.0.6) we get

$$f(\mathbf{A}) = 0 \tag{2.0.7}$$