EE5609 Assignment 17

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***Abstract*—This document solves problem based on Matrix Theory.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_17

1 Problem

Usig the result of Exercise 8 to proove that , If A and B are $n \times n$ matrices over the field F, then AB and BA have precisely the same characteristic values.

2 solution

Given	A and B are $n \times n$ matrices over the field F.
To prove	AB and BA have precisely the same characteristic values. $$\implies det\left(cI - AB\right) = det\left(cI - BA\right)$$
Proof	$$\begin{aligned} det\left(cI - AB\right) &= det\begin{pmatrix} I & 0 \\ A & cI - AB \end{pmatrix} \\ &= det\left(\begin{pmatrix} I & B \\ A & cI \end{pmatrix}\begin{pmatrix} I & -B \\ 0 & I \end{pmatrix}\right) \\ &= det\begin{pmatrix} I & B \\ A & cI \end{pmatrix} det\begin{pmatrix} I & -B \\ 0 & I \end{pmatrix} \end{aligned}$$ In the above equation $det\begin{pmatrix} I & -B \\ 0 & I \end{pmatrix} = 1$ $$\begin{aligned} \implies det\begin{pmatrix} I & B \\ A & cI \end{pmatrix} &= det\begin{pmatrix} cI - BA & 0 \\ A & I \end{pmatrix} \\ &= det\left(cI - BA\right) \end{aligned}$$ Thus, $det\left(cI - AB\right) = det\left(cI - BA\right)$ Hence, AB and BA have precisely the same characteristic values.