

EE5609 Assignment 3

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Abstract—This document contains the solution of geometry through linear algebra.

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_3

1 PROBLEM

ABCE is a Quadrilateral and D is a midpoint on BC such that, $AC=AE$, $AB=AD$ and $\angle BAD = \angle EAC$. Show that $BC=DE$.

2 EXPLANATION

In, $\triangle ABC$ and $\triangle ADE$

$$\angle BAD = \angle EAC \quad (\text{given}) \quad (2.0.1)$$

Adding $\angle DAC$ on both side, We get:

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC \quad (2.0.2)$$

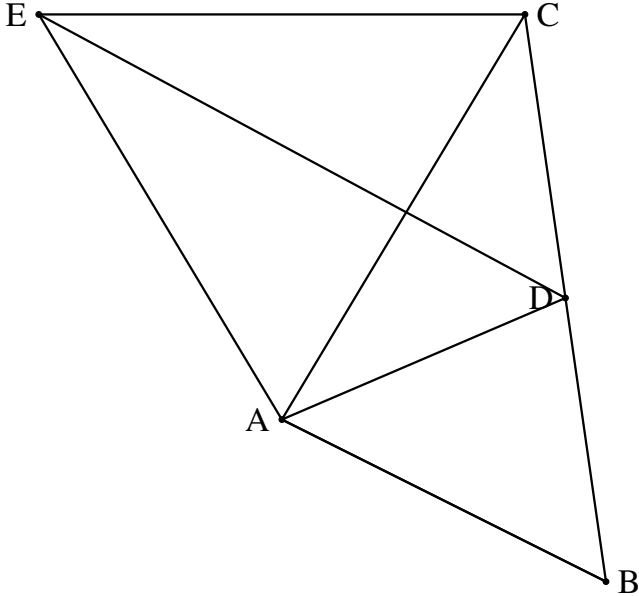


Fig. 0: Quadrilateral ABCE

We have,

$$\angle BAC = \angle DAE \quad (2.0.3)$$

$$\Rightarrow \cos \angle DAE = \cos \angle BAC \quad (2.0.4)$$

$$\frac{(\mathbf{A} - \mathbf{D})^T (\mathbf{A} - \mathbf{E})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{A} - \mathbf{E}\|} = \frac{(\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{C}\|} \quad (2.0.5)$$

We are given $AE=AC$ and we know $AD=AB$ always. Thus,

$$\|\mathbf{A} - \mathbf{E}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.6)$$

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{A} - \mathbf{B}\| \quad (2.0.7)$$

Then, from (2.0.5), we have,

$$(\mathbf{A} - \mathbf{D})^T (\mathbf{A} - \mathbf{E}) = (\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}) \quad (2.0.8)$$

Taking Transpose on both the side:

$$((\mathbf{A} - \mathbf{D})^T (\mathbf{A} - \mathbf{E}))^T = ((\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}))^T \quad (2.0.9)$$

$$(\mathbf{A} - \mathbf{E})^T (\mathbf{A} - \mathbf{D}) = (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B}) \quad (2.0.10)$$

We need to prove: $\|\mathbf{B} - \mathbf{C}\| = \|\mathbf{D} - \mathbf{E}\|$

$$\begin{aligned} \Rightarrow \|\mathbf{B} - \mathbf{C}\|^2 - \|\mathbf{D} - \mathbf{E}\|^2 &= \\ (\mathbf{B} - \mathbf{C})^T (\mathbf{B} - \mathbf{C}) - (\mathbf{D} - \mathbf{E})^T (\mathbf{D} - \mathbf{E}) &\quad (2.0.11) \end{aligned}$$

$$\begin{aligned} &= ((\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{B}))^T ((\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{B})) \\ &\quad - ((\mathbf{A} - \mathbf{E}) - (\mathbf{A} - \mathbf{D}))^T ((\mathbf{A} - \mathbf{E}) - (\mathbf{A} - \mathbf{D})) \\ &\quad (2.0.12) \\ &= \|\mathbf{A} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{B}\|^2 - (\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}) - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B}) \\ &\quad - \|\mathbf{A} - \mathbf{E}\|^2 - \|\mathbf{A} - \mathbf{D}\|^2 + (\mathbf{A} - \mathbf{D})^T (\mathbf{A} - \mathbf{E}) + (\mathbf{A} - \mathbf{E})^T (\mathbf{A} - \mathbf{D}) \\ &\quad (2.0.13) \end{aligned}$$

Thus, from (2.0.6), (2.0.7), (2.0.8) and (2.0.10)

$$\|\mathbf{B} - \mathbf{C}\|^2 - \|\mathbf{D} - \mathbf{E}\|^2 = 0 \quad (2.0.14)$$

$$\Rightarrow \|\mathbf{B} - \mathbf{C}\|^2 = \|\mathbf{D} - \mathbf{E}\|^2 \quad (2.0.15)$$

$$\therefore \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{D} - \mathbf{E}\| \quad (2.0.16)$$

Hence, $BC=DE$