EE5609 Assignment 15

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***Abstract*—This document solves problem based on Matrix Theory.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_15

1 Problem

Let Q be the field of rational numbers. Determine which of the following subsets of $Q[x]$ are ideals. When the set is an ideal, find its monic generator. All f such that $f(2) = f(4) = 0$.

2 Solution

Definition: Let F be a field. An ideal in $F[x]$ is a subspace M of $F[x]$ such that fg belongs to M whenever f is in $F[x]$ and g is in M.
Let,

$$M = \{f \in F[x] | f(2) = f(4) = 0\} \tag{2.0.1}$$

$$g(2) = g(4) = 0 \text{ and } f(2) = f(4) = 0 \tag{2.0.2}$$

then,

$$(df + g)(2) = df(2) + g(2) = 0 \tag{2.0.3}$$

$$(df + g)(4) = df(4) + g(4) = 0 \tag{2.0.4}$$

$\therefore M$ is a subspace of $F[x]$.
Let suppose $f \in M$ and $g \in F[x]$.

$$(fg)(2) = f(2)g(2) = 0 \tag{2.0.5}$$

$$(fg)(4) = f(4)g(4) = 0 \tag{2.0.6}$$

$$\therefore fg \in M \tag{2.0.7}$$

Hence, M is an ideal.
for a polynomial $f(x)$,

$$f(x) = (x - c)q(x) + r \tag{2.0.8}$$

When $f(c) = 0$ then $r = 0$.

$$f(x) = (x - 2)(x - 4)q(x) \tag{2.0.9}$$

$$f(x) = g(x)q(x) \tag{2.0.10}$$

Hence, $f(x)$ is generated by $g(x)$.