

EE5609 Assignment 9

Abhishek Thakur

Abstract—This document solves problem based on solution of vector space.

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_9

1 PROBLEM

let $\alpha = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\beta = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ be vectors in \mathbb{R}^2 such that

$$x_1 y_1 + x_2 y_2 = 0; \quad x_1^2 + x_2^2 = y_1^2 + y_2^2 = 1.$$

Proove that $\beta = \{\alpha, \beta\}$ is a basis of \mathbb{R}^2 . Find the coordinates of the vector (a, b) in the ordered basis $\beta = \{\alpha, \beta\}$. (The conditions on α and β say, geometrically, that α and β are perpendicular and each has length 1).

2 SOLUTION

we need to show that α and β are linearly independent in order to prove that $\beta = \{\alpha, \beta\}$ is a basis of \mathbb{R}^2 .

Given in the question are:

$$\alpha^T \beta = 0 \quad (2.0.1)$$

$$\|\alpha\|^2 = \|\beta\|^2 = 1 \quad (2.0.2)$$

Let,

$$\mathbf{A} = (\alpha \quad \beta) = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} \quad (2.0.3)$$

then,

$$\mathbf{A}^T \mathbf{A} = \begin{pmatrix} \|\alpha\|^2 & \alpha^T \beta \\ \alpha^T \beta & \|\beta\|^2 \end{pmatrix} \quad (2.0.4)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.5)$$

$$\therefore \mathbf{A}^T \mathbf{A} = \mathbf{I} \quad (2.0.6)$$

Inverse of \mathbf{A} exist. \mathbf{A}^T is the inverse of \mathbf{A} . Thus, the columns of \mathbf{A} are linearly independent i.e, α and β are linearly independent.

Hence, $\beta = \{\alpha, \beta\}$ is a basis of \mathbb{R}^2 .

To, find the coordinates of the vector (a, b) in the ordered basis $\beta = \{\alpha, \beta\}$.

$$(\alpha \quad \beta) \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.7)$$

$$\mathbf{A}^T \mathbf{A} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \mathbf{A}^T \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.8)$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \mathbf{A}^T \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.9)$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad (2.0.10)$$

Hence,

$$\begin{pmatrix} a \\ b \end{pmatrix} = (\alpha \quad \beta) \begin{pmatrix} ax_1 + bx_2 \\ ay_1 + by_2 \end{pmatrix} \quad (2.0.11)$$