

EE5609 Assignment 4

Abhishek Thakur

Abstract—This document contains the solution of geometry through linear algebra.

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_4

1 PROBLEM

Find the smaller area enclosed by the circle $\mathbf{x}\mathbf{x}^T = 4$ and the line $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 2$.

2 EXPLANATION

General equation of circle is

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

$$\|\mathbf{x}\|^2 + 2\mathbf{u}_1^T \mathbf{x} + f_1 = 0 \quad (2.0.2)$$

$$\mathbf{x}^T \mathbf{x} - 4 = 0 \quad (2.0.3)$$

$$\mathbf{u}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.4)$$

$$f_1 = -4 \quad (2.0.5)$$

$$\mathbf{O}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.6)$$

$$r = \sqrt{\mathbf{c}^T \mathbf{c} - f} = \sqrt{4} \quad (2.0.7)$$

$$\Rightarrow \boxed{r = 2} \quad (2.0.8)$$

From equation (2.0.8), the point at which circle touches x -axis is $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

The direction vector of the given line $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 2$ is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

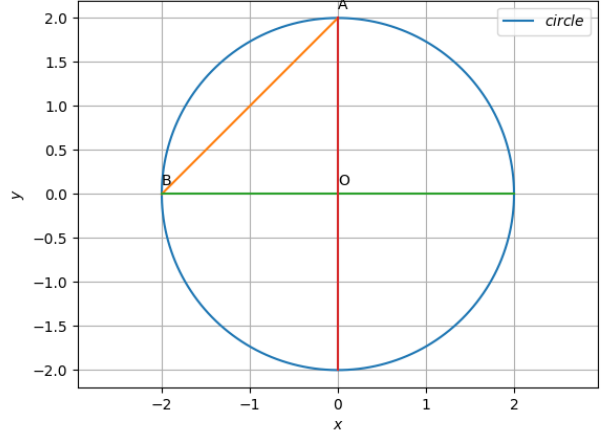


Fig. 0: Smaller area enclosed by line and circle

To find point **A** and **B**, The parametric form of line is,

$$\mathbf{A} = \mathbf{q} + \lambda \mathbf{m} \quad (2.0.9)$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (2.0.10)$$

$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \quad (2.0.11)$$

$$= \frac{4 - 2}{2} = 1 \quad (2.0.12)$$

$$\Rightarrow \lambda = \pm 1 \quad (2.0.13)$$

$$\mathbf{A} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.15)$$

$$(\mathbf{A} - \mathbf{O}) = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (2.0.16)$$

$$(\mathbf{B} - \mathbf{O}) = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \quad (2.0.17)$$

Inner product of $(\mathbf{A} - \mathbf{O})$ and $(\mathbf{B} - \mathbf{O})$ is given as:

$$(\mathbf{A} - \mathbf{O})^T (\mathbf{B} - \mathbf{O}) = 0 \quad (2.0.18)$$

Therefore, $(\mathbf{A} - \mathbf{O}) \perp (\mathbf{B} - \mathbf{O})$

Smaller area enclosed by circle and line **AB** is:

Area = (Area of circle in 2nd Quadrant) - (Area of right triangle formed by line AB, X and Y axis)

$$Area = \frac{\pi\theta_1}{360}r^2 - \frac{1}{2} \times 2 \times 2 \quad (2.0.19)$$

$$= \frac{90}{360}\pi \times 2^2 - 2 \quad (2.0.20)$$

$$= \pi - 2 \quad (2.0.21)$$

Hence, the smaller area enclosed by the circle $\mathbf{xx}^T = 4$ and the line $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 2$ is $(\pi - 2)$