

Matrix Theory Assignment 1

Abhishek Thakur

Abstract—This document contains the solution to a Lines and planes problem.

Download all python codes from

<https://github.com/abhishekt711/EE5609/codes>

1 PROBLEM

60. A person standing at the junction of two straight paths represented by the equations

$$\begin{aligned}(2 \quad -3)\mathbf{x} &= 4 \\ (3 \quad 4)\mathbf{x} &= 5\end{aligned}$$

want to reach the path whose equation is

$$(6 \quad -7)\mathbf{x} = 8$$

in the least time. Find the equation of the path that he should follow.

2 SOLUTION

Step1: we need to find the solution of equation:

$$\begin{aligned}(2 \quad -3)\mathbf{x} &= 4 \\ (3 \quad 4)\mathbf{x} &= 5\end{aligned} \quad (2.0.1)$$

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \quad (2.0.2)$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R1 \leftarrow \frac{4}{17}*(R1 + \frac{3}{4}R2)} \begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \quad (2.0.3)$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R2 \leftarrow -\frac{1}{4}(R2 - 3*R1)} \begin{pmatrix} 1 & 0 & 31/17 \\ 0 & 1 & -2/17 \end{pmatrix} \quad (2.0.4)$$

After solving this two equation we will get the junction point, which is intersection of this line segments. Thus, Junction Point is $(31/17 \quad -2/17)$. To reach in the least time, he should follow the shortest path, i.e, perpendicular from the junction point to the line give by this equation:

$$(6 \quad -7)\mathbf{x} = 8 \quad (2.0.5)$$

normal vector to the given line is:

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 6 \\ -7 \end{pmatrix} \quad (2.0.6)$$

$$\mathbf{n} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \quad (2.0.7)$$

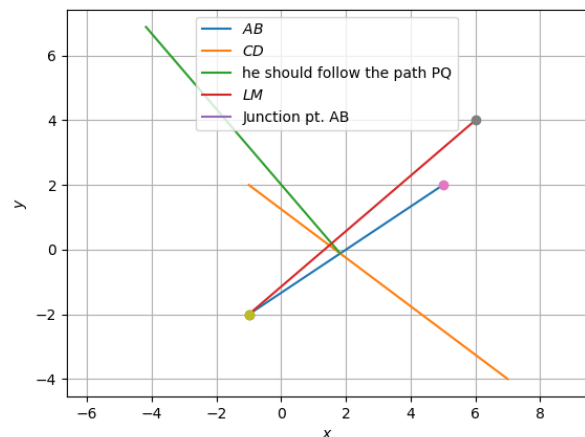


Fig. 1: The Required path is PQ.

The equation of the line in terms of normal vector passing through a given point is obtained as

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \quad (2.0.8)$$

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} \quad (2.0.9)$$

Hence, he should follow this path PQ:

$$\begin{pmatrix} 7 & 6 \end{pmatrix} \mathbf{x} = \frac{205}{17} \quad (2.0.10)$$