EE5609 Assignment 4

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***Abstract*—This document contains the solution of geometry through linear algebra.**

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_4

1 Problem

Find the smaller area enclosed by the circle $\mathbf{x}\mathbf{x}^T = 4$ and the line $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 2$.

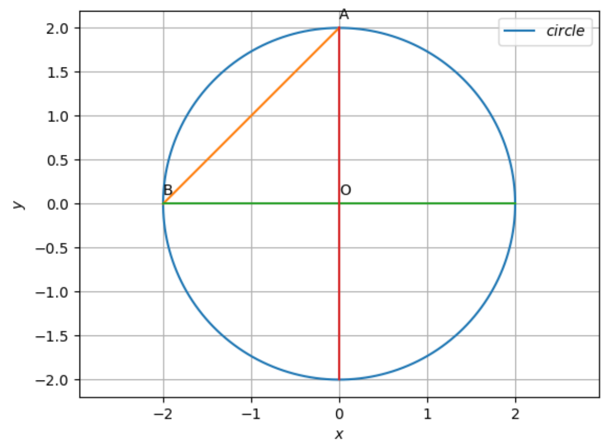

Fig. 0: Smaller area enclosed by line and circle

2 Explanation

General equation of circle is

$$\mathbf{x}^T\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.1}$$
$$\|\mathbf{x}\|^2 + 2\mathbf{u}_1^T\mathbf{x} + f_1 = 0 \tag{2.0.2}$$
$$\mathbf{x}^T\mathbf{x} - 4 = 0 \tag{2.0.3}$$
$$\mathbf{u_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$
$$f_1 = -4 \tag{2.0.5}$$
$$\mathbf{O_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.6}$$
$$r = \sqrt{\mathbf{c}^T\mathbf{c} - f} = \sqrt{4} \tag{2.0.7}$$
$$\implies \boxed{r = 2} \tag{2.0.8}$$

From equation (2.0.8), the point at which circle touches x-axis is $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

The direction vector of the given line $\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 2$ is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

To find point $\mathbf{A}$ and $\mathbf{B}$, The parametric form of line is,

$$\mathbf{A} = \mathbf{q} + \lambda\mathbf{m} \tag{2.0.9}$$
$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2.0.10}$$
$$\lambda^2 = \frac{-f_1 - \|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.11}$$
$$= \frac{4-2}{2} = 1 \tag{2.0.12}$$
$$\implies \lambda = \pm 1 \tag{2.0.13}$$
$$\mathbf{A} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{2.0.14}$$
$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.15}$$
$$\langle \mathbf{AO}, \mathbf{BO} \rangle = (0 \times (-2)) + (2 \times 0) = 0 \tag{2.0.16}$$

Inner product of $\mathbf{AO}$ and $\mathbf{BO}$ = 0. Therefore $\mathbf{AO} \perp \mathbf{BO}$

Smaller area enclosed by circle and line $\mathbf{AB}$ is:

Area = (Area of circle in 2nd Quadrant) - (Area of right triangle formed by line AB, X and Y axis)

$$Area = \frac{\pi\theta_1}{360}r^2 - \frac{1}{2} \times 2 \times 2 \tag{2.0.17}$$

$$= \frac{90}{360}\pi \times 2^2 - 2 \tag{2.0.18}$$

$$= \pi - 2 \tag{2.0.19}$$

Hence, the smaller area enclosed by the circle $\mathbf{x}\mathbf{x}^T = 4$ and the line $\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 2$ is $\pi - 2$