

# EE5609 Assignment 16

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**Abstract**—This document solves problem based on Matrix Theory.

Download all solutions from

[https://github.com/abhishekt711/EE5609/tree/master/Assignment\\_16](https://github.com/abhishekt711/EE5609/tree/master/Assignment_16)

## 1 PROBLEM

Let  $A$  be an  $n \times n$  self-adjoint matrix with eigenvalues  $\lambda_1, \dots, \lambda_n$ .

Let  $\|X\|_2 = \sqrt{|X_1|^2 + \dots + |X_n|^2}$  for  $X = (x_1, \dots, x_n) \in \mathbb{C}^n$ . If  $\rho(A) = a_0I + a_1A + \dots + a_nA^n$  then  $\sup_{\|X\|_2=1} \|\rho(A)X\|_2$  is equal to.

## 2 SOLUTION

Given,  $A$  be an  $n \times n$  self-adjoint matrix with eigenvalues  $\lambda_1, \dots, \lambda_n$ .

$$\implies AX = \lambda X \quad (2.0.1)$$

$$\implies A^n X = \lambda^n X \quad (2.0.2)$$

Also given,

$$\rho(A) = a_0I + a_1A + \dots + a_nA^n \quad (2.0.3)$$

$$\implies (\rho(A))X = (a_0I + a_1A + \dots + a_nA^n)X \quad (2.0.4)$$

$$(\rho(A))X = a_0X + a_1AX + \dots + a_nA^nX \quad (2.0.5)$$

Using 2.0.2 in 2.0.5,

$$(\rho(A))X = a_0X + a_1\lambda X + \dots + a_n\lambda^n X \quad (2.0.6)$$

$$\implies \|\rho(A)X\|_2 = \|(a_0 + a_1\lambda + \dots + a_n\lambda^n)X\|_2 \quad (2.0.7)$$

$$= |(a_0 + a_1\lambda + \dots + a_n\lambda^n)| \|X\|_2 \quad (2.0.8)$$

Hence,

$$\sup_{\|X\|_2=1} \|\rho(A)X\|_2 = \max\{|(a_0 + a_1\lambda_j + \dots + a_n\lambda_j^n)| : 1 \leq j \leq n\} \quad (2.0.9)$$