EE5609 Assignment 14

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***Abstract*—This document solves problem based on solution of vector space.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_14

1 Problem

Let T be the linear operator on R^3 defined by $T(x_1, x_2, x_3) = (x_1, x_3, -2x_2 - x_3)$
Let f be the polynomial over R defined by $f = -x^3 + 2$. Find $f(T)$.

2 Solution

The transformation is given as:

$$T(\mathbf{x}) = \begin{pmatrix} x_1 \\ x_3 \\ -2x_2 - x_3 \end{pmatrix} \tag{2.0.1}$$

$$T(\mathbf{x}) = \mathbf{TX} \tag{2.0.2}$$

$$T(\mathbf{x}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \mathbf{X} \tag{2.0.3}$$

Characteristic equation of $\mathbf{T}$ can be written as:

$$\left| \mathbf{T} - \lambda \mathbf{I} \right| = 0 \tag{2.0.4}$$

$$\begin{pmatrix} 1-\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & -2 & -1-\lambda \end{pmatrix} = 0 \tag{2.0.5}$$

The characteristics equation of the matrix will be,

$$-\lambda^3 - \lambda + 2 = 0 \tag{2.0.6}$$

Now, Using the Cayley Hamilton Theorem, we get:

$$-\mathbf{T}^3 - \mathbf{T} + 2\mathbf{I} = \mathbf{0} \tag{2.0.7}$$

The polynomial $f(\mathbf{T})$ can be written using the chracteristic function of $\mathbf{T}$ as follows,

$$f(\mathbf{T}) = -\mathbf{T}^3 + 2\mathbf{I} \tag{2.0.8}$$

$$f(\mathbf{T}) = -\mathbf{T}^3 - \mathbf{T} + 2\mathbf{I} + \mathbf{T} \tag{2.0.9}$$

Using 2.0.7 in 2.0.9, We get:

$$f(\mathbf{T}) = \mathbf{T} \tag{2.0.10}$$

Hence,

$$f(\mathbf{T}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \tag{2.0.11}$$