EE5609 Assignment 19

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***Abstract*—This document solves problem based on Matrix Theory.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_19

1 Problem

Let $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 4 \\ -2 & 1 & -4 \end{pmatrix}$, Given that 1 is an eigenvalue of M, then which among the following are correct?

a) The minimal polynomial of M is $(X-1)(X+4)$.
b) The minimal polynomial of M is $(X-1)^2(X+4)$.
c) M is not diagonalizable
d) $M^{-1} = \frac{1}{4}(M+3I)$.

2 solution

Given,

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 4 \\ -2 & 1 & -4 \end{pmatrix} \tag{2.0.1}$$

$$\lambda_1 = 1 \tag{2.0.2}$$

We need to find the other eigenvalues,

$$|M - \lambda I| = 0 \tag{2.0.3}$$

$$\begin{pmatrix} 1-\lambda & -1 & 1 \\ 2 & 1-\lambda & 4 \\ -2 & 1 & -4-\lambda \end{pmatrix} = 0 \tag{2.0.4}$$

$$\lambda^3 + 2\lambda^2 - 7\lambda + 4 = 0 \tag{2.0.5}$$

$$\implies (\lambda-1)^2(\lambda+4) = 0 \tag{2.0.6}$$

$$\therefore \lambda = 1, -4 \tag{2.0.7}$$

For $\lambda = 1$, corresponding eigenvector is $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$.

For $\lambda = -4$, corresponding eigenvector is $\begin{pmatrix} -\frac{1}{3} \\ -\frac{2}{3} \\ 1 \end{pmatrix}$.

For 3×3 matrix only 2 independent eigenvectors are there. Hence, P^{-1} does not exist and M can't be diagonalizable.
Thus, option c is correct.
Characteristic equation is given as,

$$p(x) = (X-1)^2(X+4) \tag{2.0.8}$$

Minimal polynomial will be,

$$p(x) = (X-1)^a(X+4)^b : a \le 2, b \le 1 \tag{2.0.9}$$

For $a = 1$, $b = 1$,

$$p(\mathbf{M}) = (\mathbf{M}-1)(\mathbf{M}+4) \neq 0 \tag{2.0.10}$$

$\therefore (X-1)(X+4)$ is not a minimal polynomial.
Thus option a is not correct.
For $a = 2$, $b = 1$,

$$p(\mathbf{M}) = (\mathbf{M}-1)^2(\mathbf{M}+4) = 0 \tag{2.0.11}$$

$\therefore (X-1)^2(X+4)$ is minimal polynomial.
Thus option b is correct

$$p(\mathbf{M}) = (\mathbf{M}-1)^2(\mathbf{M}+4) = 0 \tag{2.0.12}$$

$$\implies \mathbf{M}^3 + 2\mathbf{M}^2 - 7\mathbf{M} + 4\mathbf{I} = 0 \tag{2.0.13}$$

$$\mathbf{I} = -\frac{1}{4}(\mathbf{M}^3 + 2\mathbf{M}^2 - 7\mathbf{M}) \tag{2.0.14}$$

$$\mathbf{M}^{-1} = -\frac{1}{4}(\mathbf{M}^2 + 2\mathbf{M} - 7\mathbf{I}) \tag{2.0.15}$$

Thus option d is not correct.
Hence, the correct options are b and c.