EE5609 Assignment 15

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***Abstract*—This document solves problem based on Matrix Theory.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_15

1 Problem

Let Q be the field of rational numbers. Determine which of the following subsets of $Q[x]$ are ideals. When the set is an ideal, find its monic generator. All f such that $f(2) = f(4) = 0$.

2 Definition

Definition: Let F be a field. An ideal in $F[x]$ is a subspace M of $F[x]$ such that fg belongs to M whenever f is in $F[x]$ and g is in M.

3 solution

Given	Q be the field of rational numbers. Subset: All f such that $f(2) = f(4) = 0$
To prove	Given subset is an Ideal. If Ideal then find its monotic generator.
Proof	Let, $M = \{f \in F[x] \quad \vert \quad f(2) = f(4) = 0\}$. $g(2) = g(4) = 0$ and $f(2) = f(4) = 0$ $(df + g)(2) = df(2) + g(2) = 0$ $(df + g)(4) = df(4) + g(4) = 0$ $\implies M$ is a subspace of $F[x]$ Let suppose $f \in M$ and $g \in F[x]$. $(fg)(2) = f(2)g(2) = 0$ $(fg)(4) = f(4)g(4) = 0$ $\therefore fg \in M$ Hence, M is an ideal.
Representation of polynomial	$f(x) = (x - c)q(x) + r$ When $f(c) = 0$ then $r = 0$. $\implies f(x) = (x - 2)(x - 4)q(x)$ $\therefore f(x) = g(x)q(x)$ Hence, $f(x)$ is generated by $g(x)$.