EE5609 Assignment 20

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***Abstract*—This document solves problem based on Matrix Theory.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_20

1 Problem

Consider a Markov Chain with state space $S = \{1, 2, 3\}$ and transition matrix

$$P = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \end{array}$$

Let π be a stationary distribution of the Markov chain and $d(1)$ denote the period of state 1. Which of the following statements are correct?

a) $d(1) = 1$
b) $d(1) = 2$
c) $\pi_1 = \frac{1}{2}$
d) $\pi_1 = \frac{1}{3}$

2 solution

a) The period of state 1 i.e, $d(1)$ is given as:

$$d(1) = GCD\{n : p_{11}^n > 0\} \tag{2.0.1}$$
$$d(1) = GCD\{2, 3, 4, \cdots\} \tag{2.0.2}$$
$$\therefore d(1) = 1 \tag{2.0.3}$$

Thus statement a is correct

b) As calucalted above in 2.0.3, $d(1) = 1$
Thus statement b is incorrect.

c) For stationary distribution,

$$\sum_{i=1}^{i=n} \pi_i = 1 \tag{2.0.4}$$
$$\pi\mathbf{P} = \pi \tag{2.0.5}$$

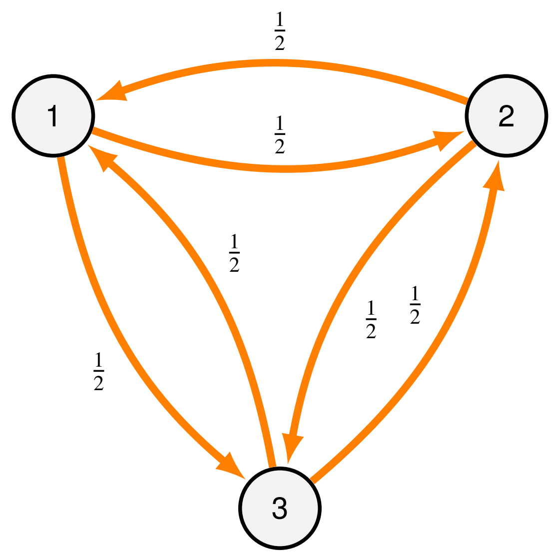

Fig. 1: State transition diagram

Solving 2.0.4 and 2.0.5, we get:

$$\pi_1 = \pi_2 = \pi_3 = \frac{1}{3} \tag{2.0.6}$$

Thus statement c is incorrect

d) As, calculated in 2.0.6, $\pi_1 = \frac{1}{3}$
Thus statement d is correct

Hence, statements a and d are correct.