

EE5609 Assignment 14

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Abstract—This document solves problem based on solution of vector space.

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_14

Using (2.0.7) in (2.0.9) we get

$$f(\mathbf{T}) = \mathbf{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \quad (2.0.10)$$

1 PROBLEM

Let T be the linear operator on R^3 defined by

$$T(x_1, x_2, x_3) = (x_1, x_3, -2x_2 - x_3)$$

Let f be the polynomial over R defined by $f = -x^3 + 2$. Find $f(T)$.

2 SOLUTION

The transformation is given as:

$$T(\mathbf{x}) = \begin{pmatrix} x_1 \\ x_3 \\ -2x_2 - x_3 \end{pmatrix} \quad (2.0.1)$$

$$T(\mathbf{x}) = \mathbf{TX} \quad (2.0.2)$$

$$T(\mathbf{x}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -1 \end{pmatrix} \mathbf{X} \quad (2.0.3)$$

Characteristic equation of \mathbf{T} can be written as:

$$|\mathbf{T} - \lambda \mathbf{I}| = 0 \quad (2.0.4)$$

$$\begin{pmatrix} 1 - \lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & -2 & -1 - \lambda \end{pmatrix} = 0 \quad (2.0.5)$$

The characteristics equation of the matrix will be,

$$-\lambda^3 - \lambda + 2 = 0 \quad (2.0.6)$$

The characteristics equation will satisfy its own matrix

$$-\mathbf{T}^3 - \mathbf{T} + 2\mathbf{I} = \mathbf{0} \quad (2.0.7)$$

The given polynomial

$$f(x) = -x^3 + 2 \quad (2.0.8)$$

$$f(\mathbf{T}) = -\mathbf{T}^3 + 2\mathbf{I} \quad (2.0.9)$$