EE5609 Assignment 10

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***Abstract*—This document solves problem based on solution of vector space.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_10

1 Problem

Consider the vectors in $\mathbb{R}^4$ defined by:
$\alpha_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix}$ and $\alpha_3 = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 9 \end{pmatrix}$.
Find a system of homogeneous linear equations for which the space of solutions is exactly the subspace of $\mathbb{R}^4$ spanned by the given three vectors.

2 Solution

A system of linear equations is homogeneous if all of the constant terms are zero. It can be represented as,

$$\mathbf{AX} = 0 \tag{2.0.1}$$

Let $\mathbf{R}$ be a echelon matrix which is reduced to A. Then the systems $\mathbf{AX} = 0$ and $\mathbf{RX} = 0$ have the same solutions. Here,

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \tag{2.0.2}$$

By operating column operation on $\mathbf{A}$, we get:

$$\begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \xleftrightarrow{C_3 = C_3 - 2C_1 - C_2} \begin{pmatrix} -1 & 3 & 0 \\ 0 & 4 & 0 \\ 1 & -2 & 0 \\ 2 & 5 & 0 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{C_1 = -C_1} \begin{pmatrix} 1 & 3 & 0 \\ 0 & 4 & 0 \\ -1 & -2 & 0 \\ -2 & 5 & 0 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{C_2 = C_2 - 3C_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ -1 & 1 & 0 \\ -2 & 11 & 0 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{C_2 = \frac{1}{4}C_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{1}{4} & 0 \\ -2 & \frac{11}{4} & 0 \end{pmatrix} \tag{2.0.6}$$

The bais vector is non zero vector which are given from 2.0.6,

$$\rho_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \end{pmatrix}, \rho_2 = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{4} \\ \frac{11}{4} \end{pmatrix} \tag{2.0.7}$$

Vector $\mathbf{V}$ formed using the above basis vectors are given as,

$$c_1\rho_1 + c_2\rho_2 = \begin{pmatrix} c_1 \\ c_2 \\ -c_1 + \frac{1}{4}c_2 \\ -2c_2 + \frac{11}{4} \end{pmatrix} \tag{2.0.8}$$

If $\mathbf{X}$ is in $\mathbf{V}$, so it can be written in this form.

$$\implies \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & \frac{1}{4} \\ -2 & \frac{11}{4} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \mathbf{X} \tag{2.0.9}$$

The augmented matrix using 2.0.11 can be written as,

$$\begin{pmatrix} 1 & 0 & x_1 \\ 0 & 1 & x_2 \\ -1 & \frac{1}{4} & x_3 \\ -2 & \frac{11}{4} & x_4 \end{pmatrix} \tag{2.0.10}$$

Converting the above augmented matrix into row reduced echelon form:

$$\begin{pmatrix} 1 & 0 & x_1 \\ 0 & 1 & x_2 \\ -1 & \frac{1}{4} & x_3 \\ -2 & \frac{11}{4} & x_4 \end{pmatrix} \xleftrightarrow{R_3=R_3+R_1-\frac{1}{4}R_2, R_4=R_4+2R_1-\frac{11}{4}R_2}$$

$$\begin{pmatrix} 1 & 0 & x_1 \\ 0 & 1 & x_2 \\ 0 & 0 & x_1 - \frac{1}{4}x_2 + x_3 \\ 0 & 0 & 2x_1 - \frac{11}{4}x_2 + x_4 \end{pmatrix} \tag{2.0.11}$$

From 2.0.11, the basis vector for the solution space is $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$.