EE5609 Assignment 3

Abhishek Thakur

***Abstract*—This document contains the solution of geometry through linear algebra.**

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_3

1 Problem

ABCE is a Quadrilateral and D is a midpoint on BC such that, AC=AE, AB=AD and $\angle BAD$=$\angle EAC$. Show that BC=DE.

2 Explanation

In, $\triangle ABC$ and $\triangle ADE$

$$\angle BAD = \angle EAC \quad (given) \tag{2.0.1}$$

Adding $\angle DAC$ on both side, We get:

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC \tag{2.0.2}$$

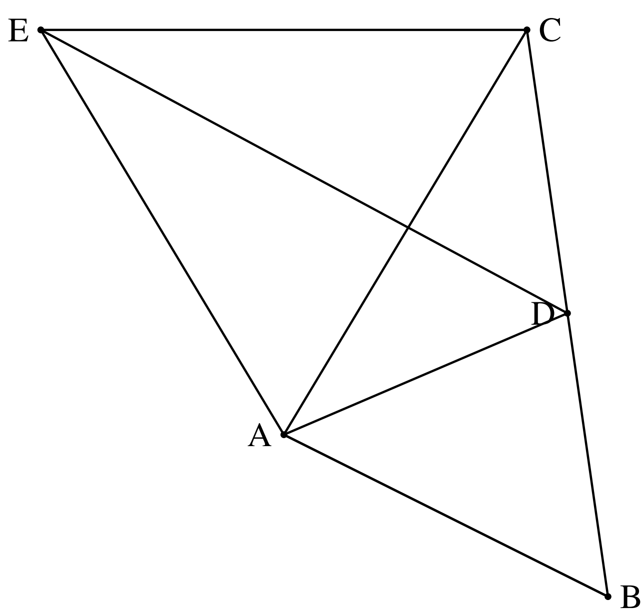

Fig. 0: Quadrilateral ABCE

We have,

$$\angle BAC = \angle DAE \tag{2.0.3}$$

$$\implies \cos\angle DAE = \cos\angle BAC \tag{2.0.4}$$

In triangle ABC, Using law of cosine, Using the formula of dot product, i.e.,

$$\mathbf{a}.\mathbf{b} = ||\mathbf{a}|| \,.\, ||\mathbf{b}|| \,.\cos\theta \tag{2.0.5}$$

$$\implies \cos\theta = \frac{\mathbf{a}.\mathbf{b}}{||\mathbf{a}|| \,.\, ||\mathbf{b}||} \tag{2.0.6}$$

$$\frac{(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E})}{||\mathbf{A}-\mathbf{D}||\,||\mathbf{A}-\mathbf{E}||} = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{||\mathbf{A}-\mathbf{B}||\,||\mathbf{A}-\mathbf{C}||} \tag{2.0.7}$$

We are given AE=AC and we know AD=AB always. Thus,

$$||\mathbf{A}-\mathbf{E}|| = ||\mathbf{A}-\mathbf{C}|| \tag{2.0.8}$$

$$||\mathbf{A}-\mathbf{D}|| = ||\mathbf{A}-\mathbf{B}|| \tag{2.0.9}$$

Then, from (2.0.7), we have,

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E}) = (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{2.0.10}$$

Taking Transpose on both the side:

$$((\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E}))^T = ((\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}))^T \tag{2.0.11}$$

$$(\mathbf{A}-\mathbf{E})^T(\mathbf{A}-\mathbf{D}) = (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) \tag{2.0.12}$$

We need to prove: $||\mathbf{B}-\mathbf{C}|| = ||\mathbf{D}-\mathbf{E}||$

$$\implies ||\mathbf{B}-\mathbf{C}||^2 - ||\mathbf{D}-\mathbf{E}||^2 = (\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) - (\mathbf{D}-\mathbf{E})^T(\mathbf{D}-\mathbf{E}) \tag{2.0.13}$$

$$= ((\mathbf{A}-\mathbf{C})-(\mathbf{A}-\mathbf{B}))^T((\mathbf{A}-\mathbf{C})-(\mathbf{A}-\mathbf{B})) - ((\mathbf{A}-\mathbf{E})-(\mathbf{A}-\mathbf{D}))^T((\mathbf{A}-\mathbf{E})-(\mathbf{A}-\mathbf{D})) \tag{2.0.14}$$

$$= ||\mathbf{A}-\mathbf{C}||^2 + ||\mathbf{A}-\mathbf{B}||^2 - (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) - (\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) - ||\mathbf{A}-\mathbf{E}||^2 - ||\mathbf{A}-\mathbf{D}||^2 + (\mathbf{A}-\mathbf{D})^T(\mathbf{A}+\mathbf{E}) + (\mathbf{A}-\mathbf{E})^T(\mathbf{A}-\mathbf{D}) \tag{2.0.15}$$

Thus, from (2.0.8), (2.0.9), (2.0.10) and (2.0.12)

$$\|\mathbf{B} - \mathbf{C}\|^2 - \|\mathbf{D} - \mathbf{E}\|^2 = 0 \tag{2.0.16}$$

$$\implies \|\mathbf{B} - \mathbf{C}\|^2 = \|\mathbf{D} - \mathbf{E}\|^2 \tag{2.0.17}$$

$$\therefore \|\mathbf{B} - \mathbf{C}\| = \|\mathbf{D} - \mathbf{E}\| \tag{2.0.18}$$

Hence, BC=DE