

# EE5609 Assignment 10

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**Abstract**—This document solves problem based on solution of vector space.

Download all solutions from

[https://github.com/abhishekt711/EE5609/tree/master/Assignment\\_10](https://github.com/abhishekt711/EE5609/tree/master/Assignment_10)

## 1 PROBLEM

Consider the vectors in  $\mathbb{R}^4$  defined by:

$$\alpha_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix} \text{ and } \alpha_3 = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 9 \end{pmatrix}.$$

Find a system of homogeneous linear equations for which the space of solutions is exactly the subspace of  $\mathbb{R}^4$  spanned by the given three vectors.

## 2 SOLUTION

A system of linear equations is homogeneous if all of the constant terms are zero. It can be represented as,

$$\mathbf{A}\mathbf{X} = 0 \quad (2.0.1)$$

Let  $\mathbf{R}$  be a echelon matrix which is reduced to  $\mathbf{A}$ . Then the systems  $\mathbf{A}\mathbf{X} = 0$  and  $\mathbf{R}\mathbf{X} = 0$  have the same solutions. Here,

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \quad (2.0.2)$$

By operating column operation on  $\mathbf{A}$ , we get:

$$\begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \xleftrightarrow{C_3=C_3-2C_1-C_2} \begin{pmatrix} -1 & 3 & 0 \\ 0 & 4 & 0 \\ 1 & -2 & 0 \\ 2 & 5 & 0 \end{pmatrix} \quad (2.0.3)$$

$$\xleftrightarrow{C_1=-C_1} \begin{pmatrix} 1 & 3 & 0 \\ 0 & 4 & 0 \\ -1 & -2 & 0 \\ -2 & 5 & 0 \end{pmatrix} \quad (2.0.4)$$

$$\xleftrightarrow{C_2=C_2-3C_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ -1 & 1 & 0 \\ -2 & 11 & 0 \end{pmatrix} \quad (2.0.5)$$

$$\xleftrightarrow{C_2=\frac{1}{4}C_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{1}{4} & 0 \\ -2 & \frac{11}{4} & 0 \end{pmatrix} \quad (2.0.6)$$

The required system of homogeneous linear equations for which the space of solution is exactly the subspace of  $\mathbb{R}^4$  spanned by the given three vectors are given as:

$$\mathbf{Y} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{1}{4} & 0 \\ -2 & \frac{11}{4} & 0 \end{pmatrix} \mathbf{X} \quad (2.0.7)$$