

EE5609 Assignment 10

Abhishek Thakur

Abstract—This document solves problem based on solution of vector space.

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_10

1 PROBLEM

Consider the vectors in \mathbb{R}^4 defined by:

$$\alpha_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix} \text{ and } \alpha_3 = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 9 \end{pmatrix}.$$

Find a system of homogeneous linear equations for which the space of solutions is exactly the subspace of \mathbb{R}^4 spanned by the given three vectors.

2 SOLUTION

Let,

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \quad (2.0.1)$$

By operating column operation on \mathbf{A} , we get:

$$\begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \xleftrightarrow{C_3 = C_3 - 2C_1 - C_2} \begin{pmatrix} -1 & 3 & 0 \\ 0 & 4 & 0 \\ 1 & -2 & 0 \\ 2 & 5 & 0 \end{pmatrix} \quad (2.0.2)$$

$$\xleftrightarrow{C_1 = -C_1} \begin{pmatrix} 1 & 3 & 0 \\ 0 & 4 & 0 \\ -1 & -2 & 0 \\ -2 & 5 & 0 \end{pmatrix} \quad (2.0.3)$$

$$\xleftrightarrow{C_2 = C_2 - 3C_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ -1 & 1 & 0 \\ -2 & 11 & 0 \end{pmatrix} \quad (2.0.4)$$

$$\xleftrightarrow{C_2 = \frac{1}{4}C_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{1}{4} & 0 \\ -2 & \frac{11}{4} & 0 \end{pmatrix} \quad (2.0.5)$$

As α_3 is dependent of α_1 and α_2 . The system of homogeneous linear equation is spanned by α_1 and α_2 .

The required system of homogeneous linear equation are given as:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{1}{4} & 0 \\ -2 & \frac{11}{4} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad (2.0.6)$$