EE5609 Assignment 10

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***Abstract*—This document solves problem based on solution of vector space.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_10

1 Problem

Consider the vectors in $\mathbb{R}^4$ defined by:
$\alpha_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix}$ and $\alpha_3 = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 9 \end{pmatrix}$.
Find a system of homogeneous linear equations for which the space of solutions is exactly the subspace of $\mathbb{R}^4$ spanned by the given three vectors.

2 Solution

$$\alpha_3 = 2\alpha_1 + \alpha_2 \tag{2.0.1}$$

$$\begin{pmatrix} 1 \\ 4 \\ 0 \\ 9 \end{pmatrix} = 2\begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix} \tag{2.0.2}$$

As α_3 is dependent on α_1 and α_2. The system of homogeneous linear equation is spanned by α_1 and α_2 only.
Let,

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \tag{2.0.3}$$

By operating column operation on $\mathbf{A}$, we get:

$$\begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \xleftrightarrow{C_3=C_3-2C_1-C_2} \begin{pmatrix} -1 & 3 & 0 \\ 0 & 4 & 0 \\ 1 & -2 & 0 \\ 2 & 5 & 0 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{C_1=-C_1} \begin{pmatrix} 1 & 3 & 0 \\ 0 & 4 & 0 \\ -1 & -2 & 0 \\ -2 & 5 & 0 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{C_2=C_2-3C_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ -1 & 1 & 0 \\ -2 & 11 & 0 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{C_2=\frac{1}{4}C_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{1}{4} & 0 \\ -2 & \frac{11}{4} & 0 \end{pmatrix} \tag{2.0.7}$$

The required system of homogeneous linear equation are given as:

$$\mathbf{Y} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{1}{4} & 0 \\ -2 & \frac{11}{4} & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \tag{2.0.8}$$