EE5609 Assignment 2

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***Abstract*—This document contains the solution of complex number using matrix method.**

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_2

1 Problem

Let $\mathbf{Z_1} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \mathbf{Z_2} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$. Find

a)Re $\left(\frac{\mathbf{z_1 z_2}}{\mathbf{z_1}^*}\right)$

b)Im $\left(\frac{1}{\mathbf{z_1 z_1}^*}\right)$

2 Explanation

Any complex number can be expressed in matrix representation as follows:

$$\begin{pmatrix} a1 \\ a2 \end{pmatrix} = \begin{pmatrix} a1 & -a2 \\ a2 & a1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.1}$$

and,

$$\mathbf{a^{-1}} = \begin{pmatrix} a1 & -a2 \\ a2 & a1 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{a^{-1}} = \frac{1}{\left\|\mathbf{a}^2\right\|} \begin{pmatrix} a_1 \\ -a_2 \end{pmatrix} \tag{2.0.3}$$

$$\left(\frac{\mathbf{z_1 z_2}}{\mathbf{z_1}^*}\right) = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -2 & -1 \\ 1 & -2 \end{pmatrix} \left[\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\right]^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.4}$$

$$\left(\frac{\mathbf{z_1 z_2}}{\mathbf{z_1}^*}\right) = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} -2 & -1 \\ 1 & -2 \end{pmatrix} \left[\frac{1}{5}\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}\right] \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.5}$$

$$\left(\frac{\mathbf{z_1 z_2}}{\mathbf{z_1}^*}\right) = \frac{1}{5}\begin{pmatrix} -2 & -11 \\ 11 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.6}$$

$$\left(\frac{\mathbf{z_1 z_2}}{\mathbf{z_1}^*}\right) = \frac{1}{5}\begin{pmatrix} -2 \\ 11 \end{pmatrix} \tag{2.0.7}$$

Hence, the real part of $\left(\frac{\mathbf{z_1 z_2}}{\mathbf{z_1}^*}\right) = -\frac{2}{5}$

$$\left(\frac{1}{\mathbf{z_1 z_1}^*}\right) = (\mathbf{z_1 z_1}^*)^{-1} \tag{2.0.8}$$

$$\left(\frac{1}{\mathbf{z_1 z_1}^*}\right) = \left[\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}\right]^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.9}$$

$$\left(\frac{1}{\mathbf{z_1 z_1}^*}\right) = \left[\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\right]^{-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.10}$$

$$\left(\frac{1}{\mathbf{z_1 z_1}^*}\right) = \frac{1}{25}\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.11}$$

$$\left(\frac{1}{\mathbf{z_1 z_1}^*}\right) = \frac{1}{25}\begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{2.0.12}$$

Hence, the imaginary part of $\left(\frac{1}{\mathbf{z_1 z_1}^*}\right) = 0$.