

EE5609 Assignment 17

Abhishek Thakur

Abstract—This document solves problem based on Matrix Theory.

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_17

1 PROBLEM

Usig the result of Exercise 8 to prove that , If A and B are $n \times n$ matrices over the field F , then AB and BA have precisely the same characteristic values.

2 SOLUTION

Given	A and B are $n \times n$ matrices over the field F . In Exercise 8, If $(I - AB)$ is invertible then $(I - BA)$ is invertible.
To prove	AB and BA have precisely the same characteristic values.
Proof	Let suppose c is the characteristic value of AB . Then, $ cI - AB = 0$ Here, I , A and B are $n \times n$ matrix. $\implies c^n I - \frac{1}{c}AB = 0$ Using the result of Exercise 8, $(I - \frac{1}{c}AB)$ non-invertible $\implies (I - \frac{1}{c}BA)$ is also non-invertible. $\implies c^n I - \frac{1}{c}BA = 0$ $\implies cI - BA = 0$ AB and BA have precisely the same characteristic value.

Given	A and B are $n \times n$ matrices over the field F . In Exercise 8, If $(I - AB)$ is invertible then $(I - BA)$ is invertible.
To prove	AB and BA have precisely the same characteristic values.
Observation	We have to show that if c is a characteristic value for AB then c is a characteristic value for BA . Conversely, This is equivalent to the statement if c is not a characteristic value for AB then it is not a characteristic value for BA .
Proof	Suppose that c is not a characteristic value for BA , this means that $ cI - BA \neq 0$. $\implies c^n I - \frac{1}{c}AB \neq 0$ $\therefore (I - \frac{1}{c}AB)$ is invertible $\implies (I - \frac{1}{c}BA)$ is invertible. $\implies I - \frac{1}{c}AB \neq 0$ $\implies c^n I - \frac{1}{c}AB = c^n I - \frac{1}{c}BA \neq 0$ Hence, If c is not a characteristic value for AB then it is not a characteristic value for BA .

Alternative Way,