1

Matrix Theory Assignment 1

Abhishek Thakur

Abstract—This documnet contains the solution to a Lines and planes problem.

Download all python codes from

https://github.com/abhishekt711/EE5609/codes

1 Problem

60. A person standing at the junction of two straight paths represented by the equations

$$(2 -3)\mathbf{x} = 4$$
$$(3 4)\mathbf{x} = 5$$

want to reach the path whose equation is

$$(6 - 7)\mathbf{x} = 8$$

in the least time. Find the equation of the path that he should follow.

2 Solution

Step1: we need to find the solution of equation:

$$(2 -3)\mathbf{x} = 4$$
$$(3 \ 4)\mathbf{x} = 5$$

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \stackrel{R1 \leftarrow \frac{4}{17} * (R1 + \frac{3}{4}R2)}{\longleftrightarrow}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \tag{2.0.1}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R2 \leftarrow \frac{1}{4}(R2 - 3 * R1)}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 0 & 1 & -2/17 \end{pmatrix}$$
(2.0.2)

After solving this two equation we will get the junction point, which is intersection of this line segments.

Thus, Junction Point is (31/17, -2/17), i.e, (1.82,-0.11)

To reach in the least time, he should follow the shortest path, i.e, perpendicular from the junction point to the line give by this equation:

$$(6-7)\mathbf{x} = 8$$

is in the direction of (7,6) as

$$\begin{pmatrix} 6 \\ -7 \end{pmatrix} \cdot \begin{pmatrix} 7 & 6 \end{pmatrix} = 0$$

The equation of the line in terms of normal vector passing through a given point is obtained as

$$\mathbf{n}^{T}(\mathbf{x} - \mathbf{A}) = 0$$

$$\mathbf{n}^{T}\mathbf{x} = \mathbf{n}^{T}\mathbf{A}$$
(2.0.3)

Hence, he should follow this path PQ:

$$(7 \ 6)\mathbf{x} = 12.05$$

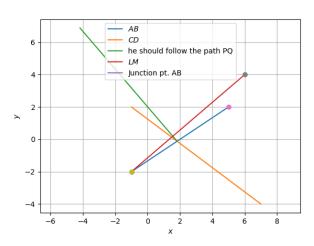


Fig. 1: The Required path is PQ.