1

Matrix Theory Assignment 1

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Abstract—This documnet contains the solution to a Lines and planes problem.

Download all python codes from

https://github.com/abhishekt711/EE5609/codes

1 Problem

60. A person standing at the junction of two straight paths represented by the equations

$$(2 -3)\mathbf{x} = 4$$

 $(3 \ 4)\mathbf{x} = 5$

want to reach the path whose equation is

$$(6 - 7)\mathbf{x} = 8$$

in the least time. Find the equation of the path that he should follow.

2 Solution

Step1: we need to find the solution of equation:

$$(2-3)\mathbf{x} = 4$$
$$(34)\mathbf{x} = 5$$
$$(2.0.1)$$

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \tag{2.0.2}$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R1 \leftarrow \frac{4}{17} * (R1 + \frac{3}{4}R2)}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix}$$
(2.0.3)

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R2 \leftarrow \frac{1}{4}(R2 - 3 * R1)}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 0 & 1 & -2/17 \end{pmatrix}$$
(2.0.4)

After solving this two equation we will get the junction point, which is intersection of this line segments.

Thus, Junction Point is (31/17, -2/17). To reach in the least time, he should follow the shortest path, i.e, perpendicular from the junction point to the line give by this equation:

$$(6-7)\mathbf{x} = 8 \tag{2.0.5}$$

is in the direction of (7,6) as

$$(6, -7)^{T}(7, 6) = 0$$
(2.0.6)

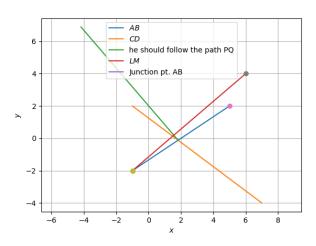


Fig. 1: The Required path is PQ.

The equation of the line in terms of normal vector passing through a given point is obtained as

$$\mathbf{n}^{T}(\mathbf{x} - \mathbf{A}) = 0$$

$$\mathbf{n}^{T}\mathbf{x} = \mathbf{n}^{T}\mathbf{A}$$
(2.0.7)
(2.0.8)

Hence, he should follow this path PQ:

$$(7,6)\mathbf{x} = \frac{205}{17} \tag{2.0.9}$$