EE5609 Assignment 3

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***Abstract*—This document contains the solution of geometry through linear algebra.**

Download latex and python codes from

https://github.com/abhishekt711/EE5600/tree/master/Assignment_3

1 Problem

ABCE is a Quadrilateral and D is a midpoint on BC such that, AC=AE, AB=AD and $\angle BAD$=$\angle EAC$. Show that BC=DE.

2 Explanation

In, $\triangle ABC$ and $\triangle ADE$

$$\angle BAD = \angle EAC \quad (given) \tag{2.0.1}$$

Adding $\angle DAC$ on both side, We get:

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC \tag{2.0.2}$$
$$\angle BAC = \angle DAE \tag{2.0.3}$$
$$AB = AD \quad (given) \tag{2.0.4}$$
$$AC = AE \quad (given) \tag{2.0.5}$$

When two sides and the included angle of one triangle are equal to the corresponding sides and angle of another triangle, the triangles are congruent. Hence,

$$\triangle ABC \cong \triangle ADE \tag{2.0.6}$$

Hence, by SAS congruency, $\triangle ABC \cong \triangle ADE$.

3 Figure

We have,

$$\angle DAE = \angle BAC \tag{3.0.1}$$
$$\implies \cos\angle DAE = \cos\angle BAC \tag{3.0.2}$$

Using the formula of dot product, i.e.,

$$\mathbf{a}.\mathbf{b} = \|\mathbf{a}\| . \|\mathbf{b}\| . \cos\theta \tag{3.0.3}$$
$$\implies \cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\|\mathbf{a}\| . \|\mathbf{b}\|} \tag{3.0.4}$$

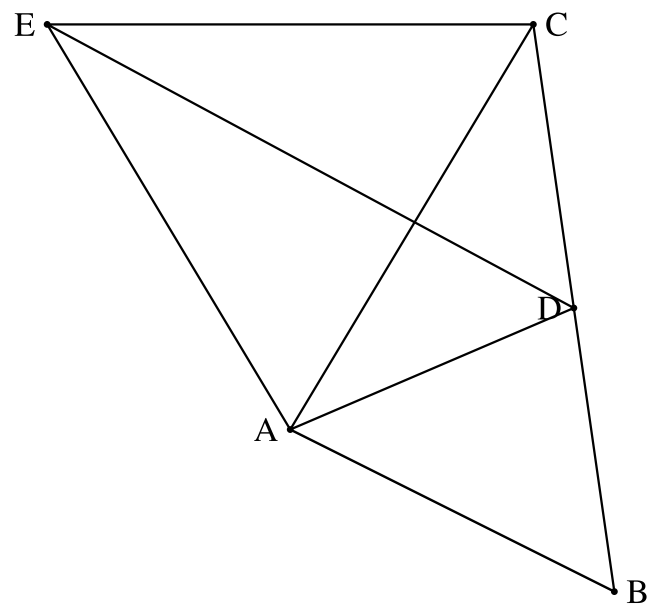

Fig. 0: Quadrilateral ABCE

$$\frac{(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E})}{\|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{A}-\mathbf{E}\|} = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{C}\|} \tag{3.0.5}$$

We are given AE=AC and we know AD=AB always. Thus,

$$\|\mathbf{A}-\mathbf{E}\| = \|\mathbf{A}-\mathbf{C}\| \tag{3.0.6}$$
$$\|\mathbf{A}-\mathbf{D}\| = \|\mathbf{A}-\mathbf{B}\| \tag{3.0.7}$$

Then, from (3.0.5), we have,

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E}) = (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{3.0.8}$$

$$\implies \|\mathbf{A}-\mathbf{D}\|^2 - (\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{E}) = \|\mathbf{A}-\mathbf{D}\|^2 - (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) \tag{3.0.9}$$

$$\implies (\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{E}) = (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) \tag{3.0.10}$$

$$\implies \|\mathbf{D}-\mathbf{A}\|\,\|\mathbf{D}-\mathbf{E}\|\cos\angle ADE = \|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{D}-\mathbf{E}\|\cos\angle ABC \tag{3.0.11}$$

$$\implies \|\mathbf{D}-\mathbf{E}\|\cos\angle ADE = \|\mathbf{B}-\mathbf{C}\|\cos\angle ABC \tag{3.0.12}$$

We need to prove: $\|\mathbf{B}-\mathbf{C}\| = \|\mathbf{D}-\mathbf{E}\|$.
From (3.0.10),

$$(\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{E}) = (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) \tag{3.0.13}$$

$$\implies \|\mathbf{D}-\mathbf{E}\|^2 - (\mathbf{E}-\mathbf{D})^T(\mathbf{E}-\mathbf{A}) = \|\mathbf{B}-\mathbf{C}\|^2 - (\mathbf{C}-\mathbf{B})^T(\mathbf{C}-\mathbf{A}) \tag{3.0.14}$$

$$\implies \|\mathbf{D}-\mathbf{E}\|^2 - (\|\mathbf{A}-\mathbf{E}\|^2 - (\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E})) = \|\mathbf{B}-\mathbf{C}\|^2 - (\|\mathbf{A}-\mathbf{C}\|^2 - (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})) \tag{3.0.15}$$

We are given that AE=AC, AD=AB. Using, (3.0.6),(3.0.7)

$$\therefore \|\mathbf{D}-\mathbf{E}\|^2 + (\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E}) = \|\mathbf{B}-\mathbf{C}\|^2 + (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{3.0.16}$$

$$\implies \|\mathbf{D}-\mathbf{E}\|^2 + \|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{A}-\mathbf{E}\|\cos\angle DAE = \|\mathbf{B}-\mathbf{C}\|^2 + \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{C}\|\cos\angle BAC \tag{3.0.17}$$

From the question, $\angle DAE = \angle BAC$ and $\|\mathbf{A}-\mathbf{E}\| = \|\mathbf{A}-\mathbf{C}\|$. We also know $\|\mathbf{A}-\mathbf{D}\| = \|\mathbf{A}-\mathbf{B}\|$. Thus, from (3.0.17), we get,

$$\|\mathbf{D}-\mathbf{E}\|^2 = \|\mathbf{B}-\mathbf{C}\|^2 \tag{3.0.18}$$

$$\implies \|\mathbf{D}-\mathbf{E}\| = \|\mathbf{B}-\mathbf{C}\| \tag{3.0.19}$$

$$\therefore BC = DE \tag{3.0.20}$$