

EE5609 Assignment 3

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Abstract—This document contains the solution of geometry through linear algebra.

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_3

1 PROBLEM

ABCE is a Quadrilateral and D is a midpoint on BC such that, $AC=AE$, $AB=AD$ and $\angle BAD = \angle EAC$. Show that $BC=DE$.

2 EXPLANATION

In, $\triangle ABC$ and $\triangle ADE$

$$\angle BAD = \angle EAC \quad (\text{given}) \quad (2.0.1)$$

Adding $\angle DAC$ on both side, We get:

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC \quad (2.0.2)$$

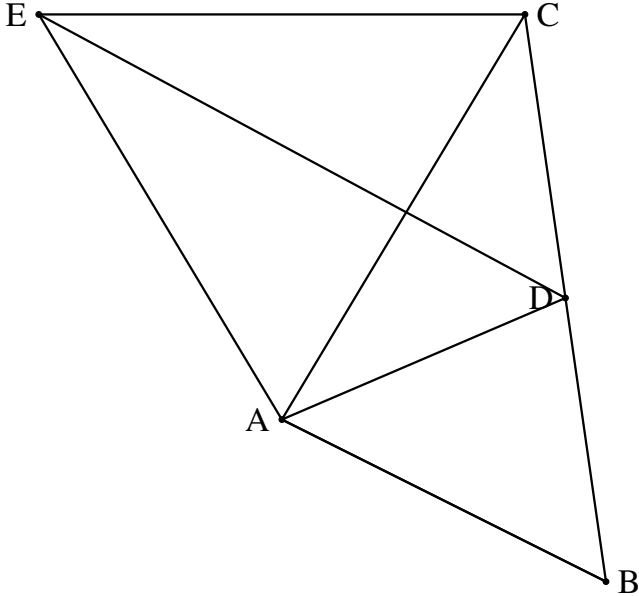


Fig. 0: Quadrilateral ABCE

We have,

$$\angle BAC = \angle DAE \quad (2.0.3)$$

$$\Rightarrow \cos \angle DAE = \cos \angle BAC \quad (2.0.4)$$

In triangle ABC, Using law of cosine,

$$a^2 = b^2 + c^2 - 2bc \cos \angle BCA \quad (2.0.5)$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{C}) = \|\mathbf{A} - \mathbf{B}\|^2 - (\mathbf{A} - \mathbf{C})^T (\mathbf{A} - \mathbf{B}) \quad (2.0.6)$$

Using the formula of dot product, i.e.,

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \cdot \|\mathbf{b}\| \cdot \cos \theta \quad (2.0.7)$$

$$\Rightarrow \cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \cdot \|\mathbf{b}\|} \quad (2.0.8)$$

$$\frac{(\mathbf{A} - \mathbf{D})^T (\mathbf{A} - \mathbf{E})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{A} - \mathbf{E}\|} = \frac{(\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C})}{\|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{C}\|} \quad (2.0.9)$$

We are given $AE=AC$ and we know $AD=AB$ always. Thus,

$$\|\mathbf{A} - \mathbf{E}\| = \|\mathbf{A} - \mathbf{C}\| \quad (2.0.10)$$

$$\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{A} - \mathbf{B}\| \quad (2.0.11)$$

Then, from (2.0.9), we have,

$$(\mathbf{A} - \mathbf{D})^T (\mathbf{A} - \mathbf{E}) = (\mathbf{A} - \mathbf{B})^T (\mathbf{A} - \mathbf{C}) \quad (2.0.12)$$

$$(2.0.13)$$

Using law of cosine (2.0.6)

$$\begin{aligned} \Rightarrow \|\mathbf{A} - \mathbf{D}\|^2 - (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{E}) \\ = \|\mathbf{A} - \mathbf{D}\|^2 - (\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{C}) \end{aligned} \quad (2.0.14)$$

$$\Rightarrow (\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{E}) = (\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{C}) \quad (2.0.15)$$

$$\begin{aligned} \Rightarrow \|\mathbf{D} - \mathbf{A}\| \|\mathbf{D} - \mathbf{E}\| \cos \angle ADE \\ = \|\mathbf{B} - \mathbf{A}\| \|\mathbf{D} - \mathbf{E}\| \cos \angle ABC \end{aligned} \quad (2.0.16)$$

$$\Rightarrow \|\mathbf{D} - \mathbf{E}\| \cos \angle ADE = \|\mathbf{B} - \mathbf{C}\| \cos \angle ABC \quad (2.0.17)$$

We need to prove: $\|\mathbf{B} - \mathbf{C}\| = \|\mathbf{D} - \mathbf{E}\|$. From (2.0.15),

$$(\mathbf{D} - \mathbf{A})^T (\mathbf{D} - \mathbf{E}) = (\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{C}) \quad (2.0.18)$$

Again, By using law of cosine (2.0.6)

$$\begin{aligned} \Rightarrow \|\mathbf{D} - \mathbf{E}\|^2 - (\mathbf{E} - \mathbf{D})^T(\mathbf{E} - \mathbf{A}) \\ = \|\mathbf{B} - \mathbf{C}\|^2 - (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{A}) \end{aligned} \quad (2.0.19)$$

$$\begin{aligned} \Rightarrow \|\mathbf{D} - \mathbf{E}\|^2 - (\|\mathbf{A} - \mathbf{E}\|^2 - (\mathbf{A} - \mathbf{D})^T(\mathbf{A} - \mathbf{E})) \\ = \|\mathbf{B} - \mathbf{C}\|^2 - (\|\mathbf{A} - \mathbf{C}\|^2 - (\mathbf{A} - \mathbf{B})^T(\mathbf{A} - \mathbf{C})) \end{aligned} \quad (2.0.20)$$

We are given that $AE=AC$, $AD=AB$. Using, (2.0.10),(2.0.11)

$$\begin{aligned} \therefore \|\mathbf{D} - \mathbf{E}\|^2 + (\mathbf{A} - \mathbf{D})^T(\mathbf{A} - \mathbf{E}) = \\ \|\mathbf{B} - \mathbf{C}\|^2 + (\mathbf{A} - \mathbf{B})^T(\mathbf{A} - \mathbf{C}) \end{aligned} \quad (2.0.21)$$

$$\begin{aligned} \Rightarrow \|\mathbf{D} - \mathbf{E}\|^2 + \|\mathbf{A} - \mathbf{D}\| \|\mathbf{A} - \mathbf{E}\| \cos \angle DAE = \\ \|\mathbf{B} - \mathbf{C}\|^2 + \|\mathbf{A} - \mathbf{B}\| \|\mathbf{A} - \mathbf{C}\| \cos \angle BAC \end{aligned} \quad (2.0.22)$$

From the question, $\angle DAE = \angle BAC$ and $\|\mathbf{A} - \mathbf{E}\| = \|\mathbf{A} - \mathbf{C}\|$. We also know $\|\mathbf{A} - \mathbf{D}\| = \|\mathbf{A} - \mathbf{B}\|$. Thus, from (2.0.22), we get,

$$\|\mathbf{D} - \mathbf{E}\|^2 = \|\mathbf{B} - \mathbf{C}\|^2 \quad (2.0.23)$$

$$\Rightarrow \|\mathbf{D} - \mathbf{E}\| = \|\mathbf{B} - \mathbf{C}\| \quad (2.0.24)$$

$$\therefore BC = DE \quad (2.0.25)$$