EE5609 Assignment 10

Abhishek Thakur

***Abstract*—This document solves problem based on solution of vector space.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_10

1 Problem

Consider the vectors in $\mathbb{R}^4$ defined by:
$\alpha_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix}$ and $\alpha_3 = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 9 \end{pmatrix}$.
Find a system of homogeneous linear equations for which the space of solutions is exactly the subspace of $\mathbb{R}^4$ spanned by the given three vectors.

2 Solution

A system of linear equations is homogeneous if all of the constant terms are zero. It can be represented as,

$$\mathbf{AX} = 0 \tag{2.0.1}$$

Let $\mathbf{R}$ be a echelon matrix which is reduced to A. Then the systems $\mathbf{AX} = 0$ and $\mathbf{RX} = 0$ have the same solutions. Here,

$$\mathbf{A} = \begin{pmatrix} -1 & 0 & 1 & 2 \\ 3 & 4 & -2 & 5 \\ 1 & 4 & 0 & 9 \end{pmatrix} \tag{2.0.2}$$

By row reducing on $\mathbf{A}$, we get:

$$\mathbf{A} = \begin{pmatrix} -1 & 0 & 1 & 2 \\ 3 & 4 & -2 & 5 \\ 1 & 4 & 0 & 9 \end{pmatrix} \xleftrightarrow{R_3=R_3-2R_1-R_2} \begin{pmatrix} -1 & 0 & 1 & 2 \\ 3 & 4 & -2 & 5 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_2=R_2+3R_1} \begin{pmatrix} -1 & 0 & 1 & 2 \\ 0 & 4 & 1 & 11 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_1=-R_1} \begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 4 & 1 & 11 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.5}$$

The bais vector is non zero vector which are given from 2.0.5,

$$\rho_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \end{pmatrix}, \rho_2 = \begin{pmatrix} 0 \\ 4 \\ 1 \\ 11 \end{pmatrix} \tag{2.0.6}$$

ρ_1, ρ_2 forms the basis of the solution space. The subspace spanned by b_1 and b_2 is given as:

$$\begin{pmatrix} \rho_1 & \rho_2 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \mathbf{X} \tag{2.0.7}$$

Using 2.0.7, we can write the augmented matrix as:

$$\begin{pmatrix} 1 & 0 & x_1 \\ 0 & 4 & x_2 \\ -1 & 1 & x_3 \\ -2 & 11 & x_4 \end{pmatrix} \xleftrightarrow{R_3=4R_3+4R_1-R_2} \tag{2.0.8}$$

$$\begin{pmatrix} 1 & 0 & x_1 \\ 0 & 4 & x_2 \\ 0 & 0 & 4x_1 - x_2 + 4x_3 \\ -2 & 11 & x_4 \end{pmatrix} \xleftrightarrow{R_4=4R_4+8R_1-11R_2} \tag{2.0.9}$$

$$\begin{pmatrix} 1 & 0 & x_1 \\ 0 & 4 & x_2 \\ 0 & 0 & 4x_1 - x_2 + 4x_3 \\ 0 & 0 & 8x_1 - 11x_2 + 4x_4 \end{pmatrix} \tag{2.0.10}$$

Using 2.0.10, The required homogeneous equation is given as:

$$\begin{pmatrix} 4 & -1 & 4 & 0 \\ 8 & -11 & 0 & 4 \end{pmatrix} \mathbf{X} = 0 \tag{2.0.11}$$