# EE5609 Assignment 9

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***Abstract*—This document solves problem based on solution of vector space.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_9

## 1 Problem

let $\alpha = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $\beta = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$ be vectors in $\mathbb{R}^2$ such that
$x_1 y_1 + x_2 y_2 = 0; \quad x_1^2 + x_2^2 = y_1^2 + y_2^2 = 1$.
Proove that ß = $\{\alpha, \beta\}$ is a basis of $\mathbb{R}^2$. Find the coordinates of the vector $\begin{pmatrix} a, b \end{pmatrix}$ in the ordered basis ß = $\{\alpha, \beta\}$. (The conditions on $\alpha$ and $\beta$ say, geometrically, that $\alpha$ and $\beta$ are perpendicular and each has length 1).

## 2 Solution

we need to show that $\alpha$ and $\beta$ are linearly independent in order to proove that ß = $\{\alpha, \beta\}$ is a basis of $\mathbb{R}^2$.

Given in the question are:

$$\alpha^T \beta = 0 \tag{2.0.1}$$

$$||\alpha||^2 = ||\beta||^2 = 1 \tag{2.0.2}$$

Let,

$$\mathbf{A} = \begin{pmatrix} \alpha & \beta \end{pmatrix} = \begin{pmatrix} x_1 & y_1 \\ x_2 & y_2 \end{pmatrix} \tag{2.0.3}$$

then,

$$\mathbf{A}^T \mathbf{A} = \begin{pmatrix} ||\alpha||^2 & \alpha^T \beta \\ \alpha^T \beta & ||\beta||^2 \end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.5}$$

$$\therefore \mathbf{A}^T \mathbf{A} = \mathbf{I} \tag{2.0.6}$$

Inverse of $\mathbf{A}$ exist. $\mathbf{A}^T$ is the inverse of $\mathbf{A}$. Thus, the columns of $\mathbf{A}$ are linearly independent i.e, $\alpha$ and $\beta$ are linearly independent.

Hence, ß = $\{\alpha, \beta\}$ is a basis of $\mathbb{R}^2$.

To, find the coordinates of the vector $\begin{pmatrix} a, b \end{pmatrix}$ in the ordered basis ß = $\{\alpha, \beta\}$.

$$\begin{pmatrix} \alpha & \beta \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.7}$$

$$\begin{pmatrix} A \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.8}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} A \end{pmatrix}^{-1} \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} A \end{pmatrix}^{T} \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.11}$$

$$\therefore c_1 = a x_1 + b x_2 \tag{2.0.12}$$

$$\therefore c_2 = a y_1 + b y_2 \tag{2.0.13}$$

Hence,

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \alpha & \beta \end{pmatrix} \begin{pmatrix} a x_1 + b x_2 \\ a y_1 + b y_2 \end{pmatrix} \tag{2.0.14}$$