

EE5609 Assignment 17

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Abstract—This document solves problem based on Matrix Theory.

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_17

1 PROBLEM

Use the result of Exercise 8 to prove that, If A and B are $n \times n$ matrices over the field F , then AB and BA have precisely the same characteristic values.

2 SOLUTION

Given	<p>A and B are $n \times n$ matrices over the field F.</p> <p>In Exercise 8, If $(I - AB)$ is invertible then $(I - BA)$ is invertible.</p>
To prove	AB and BA have precisely the same characteristic values.
Observation	<p>We have to show that if c is a characteristic value for AB then c is a characteristic value for BA. Conversely, This is equivalent to the statement if c is not a characteristic value for AB then it is not a characteristic value for BA.</p>
Proof	<p>Suppose that c is not a characteristic value for BA, this means that $cI - BA \neq 0$.</p> $\implies c^n I - \frac{1}{c}BA \neq 0$ <p>$\therefore (I - \frac{1}{c}BA)$ is invertible $\implies (I - \frac{1}{c}AB)$ is invertible. $\implies I - \frac{1}{c}AB \neq 0$</p> $\implies c^n I - \frac{1}{c}AB = c^n I - \frac{1}{c}BA \neq 0$ <p>Hence, If c is not a characteristic value for AB then it is not a characteristic value for BA.</p> <p>Hence, AB and BA have precisely the same characteristic value.</p>