

# EE5609 Assignment 11

Abhishek Thakur

**Abstract**—This document solves problem based on solution of vector space.

Download all solutions from

[https://github.com/abhishekt711/EE5609/tree/master/Assignment\\_11](https://github.com/abhishekt711/EE5609/tree/master/Assignment_11)

The matrix of T in the ordered basis  $\{\alpha_2, \alpha_1\}$  is given as:

$$[\mathbf{T}]_{\alpha} = [\mathbf{T}]_{\beta}[\alpha]_{\beta} = \begin{pmatrix} -i & 1 \\ 2 & i \end{pmatrix} \begin{pmatrix} -i & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -i \\ -2i & 2 \end{pmatrix} \quad (2.0.10)$$

## 1 PROBLEM

Let T be the linear operator on  $\mathbb{C}^2$  defined by  $T(x_1, x_2) = (x_1, 0)$ . Let  $\beta$  be the standard ordered basis for  $\mathbb{C}^2$  and  $\beta' = \{\alpha_1, \alpha_2\}$  be the ordered basis defined by  $\alpha_1 = (1, i)$ ,  $\alpha_2 = (-i, 2)$ . What is the matrix of T in the ordered basis  $\{\alpha_2, \alpha_1\}$ ?

## 2 SOLUTION

Transformation T from  $\mathbb{C}^2$  to  $\mathbb{C}^2$ . Let

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (2.0.1)$$

$$\beta = (\mathbf{e}_1 \quad \mathbf{e}_2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (2.0.2)$$

$$\beta' = (\alpha_2 \quad \alpha_1) = \begin{pmatrix} -i & 1 \\ 2 & i \end{pmatrix} \quad (2.0.3)$$

T in the ordered basis  $\beta$  is:

$$[\mathbf{T}]_{\beta} = \begin{pmatrix} -i & 1 \\ 2 & i \end{pmatrix} \quad (2.0.4)$$

T is defined by

$$T(\mathbf{x}) = \mathbf{Ax} \quad (2.0.5)$$

$$T(\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} \quad (2.0.6)$$

$$T(\alpha_2) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -i \\ 2 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} \quad (2.0.7)$$

$$T(\alpha_1) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.8)$$

$$[\alpha]_{\beta} = \begin{pmatrix} -i & 1 \\ 0 & 0 \end{pmatrix} \quad (2.0.9)$$