EE5609 Assignment 5

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Abstract—**This document contains the solution of geometry through linear algebra.**

Download latex and python codes from

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https://github.com/abhishekt711/EE5609/tree/
    master/Assignment_5
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1 Problem

Prove that the following equations represent two straight lines, find also their point of intersection and the angle between them.
$6y^2 - xy - x^2 + 30y + 36 = 0$.

2 Explanation

The given equation can be written as:

$$-x^2 - xy + 6y^2 + 30y + 36 = 0 \tag{2.0.1}$$

$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix}$ of (2.0.1) becomes

$$\begin{vmatrix} -1 & -\frac{1}{2} & 0 \\ \frac{-1}{2} & 6 & 15 \\ 0 & 15 & 36 \end{vmatrix} = 0 \tag{2.0.2}$$

Expanding equation (2.0.2), we get zero.
Hence given equation represents a pair of straight lines.

The general equation second degree is given by

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{2.0.3}$$

Let (α, β) be their point of intersection, then

$$\begin{pmatrix} a & h \\ h & b \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} -d \\ -e \end{pmatrix} \tag{2.0.4}$$

Given equation is

$$-x^2 - xy + 6y^2 + 30y + 36 = 0 \tag{2.0.5}$$

Substituting in (2.0.4)

$$\begin{pmatrix} -1 & \frac{-1}{2} \\ \frac{-1}{2} & 6 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ -15 \end{pmatrix} \tag{2.0.6}$$

$$\implies \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \frac{6}{5} \\ \frac{-12}{5} \end{pmatrix} \tag{2.0.7}$$

Hence, the intersection point is $\begin{pmatrix} \frac{6}{5} \\ -\frac{12}{5} \end{pmatrix}$
Also, Verified using python code from

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codes/Assignment_5.py
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From, Spectral decomposition,

$$\mathbf{V} = \mathbf{PDP}^T \tag{2.0.8}$$

$$\mathbf{V} = \begin{pmatrix} -1 & \frac{-1}{2} \\ \frac{-1}{2} & 6 \end{pmatrix} \tag{2.0.9}$$

$$\mathbf{P} = \begin{pmatrix} 7 - 5\sqrt{2} & 7 + 5\sqrt{2} \\ 1 & 1 \end{pmatrix} \tag{2.0.10}$$

$$\mathbf{D} = \begin{pmatrix} \frac{5+5\sqrt{2}}{2} & 0 \\ 0 & \frac{5-5\sqrt{2}}{2} \end{pmatrix} \tag{2.0.11}$$

P and D are also verified using python code from

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codes/diagonalize1.py
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Using, (2.0.7), (2.0.10) and (2.0.11) in,

$$u_1(x-\alpha) + u_2(y-\beta) = \pm\sqrt{-\frac{\lambda_2}{\lambda_1}}(v_1(x-\alpha) + v_2(y-\beta)) \tag{2.0.12}$$

$$\implies \left(7 - 5\sqrt{2}\right)\left(x - \frac{30}{23}\right) + \left(y + \frac{60}{23}\right)$$
$$= \pm\sqrt{-\frac{\frac{5-5\sqrt{2}}{2}}{\frac{5+5\sqrt{2}}{2}}}\left(\left(7 - 5\sqrt{2}\right)\left(x - \frac{6}{5}\right) + \left(y + \frac{12}{5}\right)\right) \tag{2.0.13}$$

simplifying 2.0.13, we get:

$$-x + 2y + 6 = 0 \text{ and } x + 3y + 6 = 0 \tag{2.0.14}$$

$$\implies (-x + 2y + 6)(x + 3y + 6) = 0 \tag{2.0.15}$$

$$\therefore -x + 2y = -6 \quad , \quad x + 3y = -6 \tag{2.0.16}$$

Angle between two lines, θ can be given by

$$n_1 = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{2.0.17}$$

$$n_2 = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \tag{2.0.18}$$

$$\cos\theta = \frac{\mathbf{n_1}^T \mathbf{n_2}}{\|\mathbf{n_1}\| \, \|\mathbf{n_2}\|} \tag{2.0.19}$$

$$\cos\theta = \frac{\begin{pmatrix} -2 & -1 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix}}{\sqrt{(-2)^2 + (-1)^2} \times \sqrt{+(-3)^2 + 1}} = \frac{1}{\sqrt{2}} \tag{2.0.20}$$

$$\implies \theta = 45^\circ \tag{2.0.21}$$

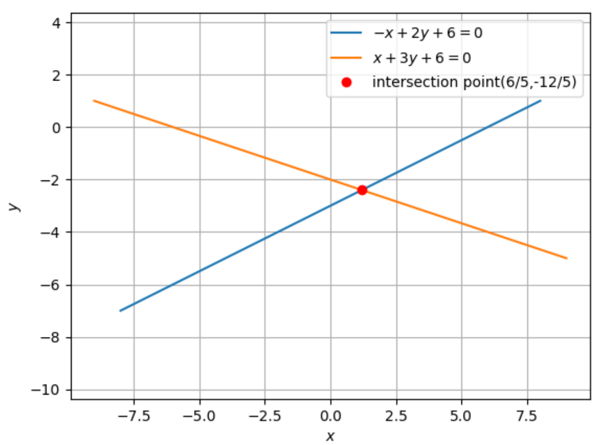

Fig. 0: plot showing intersection of lines.