EE5609 Assignment 13

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Abstract—**This document solves problem based on so-lution of vector space.**

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1 PROBLEM

Let F be a sub field of the complex numbers , and let A be the following 2×2 matrix over F.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

compute f($\mathbf{A}$) for the polynomial $f(x) = x^2 - 5x + 7$

2 SOLUTION

Characteristic equation of $\mathbf{A}$ can be written as:

$$\left| \mathbf{A} - \lambda \mathbf{I} \right| = 0 \qquad (2.0.1)$$

$$\begin{pmatrix} 2 - \lambda & 1 \\ -1 & 3 - \lambda \end{pmatrix} = 0 \qquad (2.0.2)$$

The characteristics equation of the matrix will be,

$$\lambda^2 - 5\lambda + 7 = 0 \qquad (2.0.3)$$

The characteristics equation will satisfy its own matrix

$$\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} = \mathbf{0} \qquad (2.0.4)$$

The given polynomial

$$f(x) = x^2 - 5x + 7 \qquad (2.0.5)$$
$$f(\mathbf{A}) = \mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} \qquad (2.0.6)$$

substituting (2.0.4) in (2.0.6) we get

$$f(\mathbf{A}) = \mathbf{0} \qquad (2.0.7)$$