EE5609 Assignment 11

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***Abstract*—This document solves problem based on solution of vector space.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_11

1 Problem

Let T be the linear operator on $\mathbb{C}^2$ defined by $T(x_1, x_2) = \left(x_1, 0\right)$. Let β be the standard ordered basis for $\mathbb{C}^2$ and $\beta' = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by $\alpha_1 = \left(1, i\right), \alpha_2 = \left(-i, 2\right)$.
What is the matrix of T in the ordered basis $\{\alpha_2, \alpha_1\}$?

2 Solution

Transformation T from $\mathbb{C}^2$ to $\mathbb{C}^2$. Let

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.1}$$

$$\beta = \left(\mathbf{e_1}, \mathbf{e_2}\right) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.2}$$

$$\alpha = \left(\alpha_2, \alpha_1\right) = \begin{pmatrix} -i & 1 \\ 2 & i \end{pmatrix} \tag{2.0.3}$$

T in the ordered basis β is:

$$[\mathbf{T}]_\beta = \begin{pmatrix} -i & 1 \\ 2 & i \end{pmatrix} \tag{2.0.4}$$

T is defined by

$$T(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.5}$$

$$T(\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x} \tag{2.0.6}$$

$$T(\alpha_\mathbf{2}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -i \\ 2 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$T(\alpha_\mathbf{1}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.8}$$

$$[\alpha]_\beta = \begin{pmatrix} -i & 1 \\ 0 & 0 \end{pmatrix} \tag{2.0.9}$$

The matrix of T in the ordered basis $\{\alpha_2, \alpha_1\}$ is given as:

$$[\mathbf{T}_\alpha]_\beta = [\mathbf{T}]_\beta[\alpha]_\beta = \begin{pmatrix} -i & 1 \\ 2 & i \end{pmatrix}\begin{pmatrix} -i & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -i \\ -2i & 2 \end{pmatrix} \tag{2.0.10}$$