Matrix Theory Assignment 1

Abhishek Thakur

***Abstract*—This documnet contains the solution to a Lines and planes problem.**

Download all python codes from

https://github.com/abhishekt711/EE5609/codes

1 Problem

60. A person standing at the junction of two straight paths represented by the equations

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = 4$$
$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = 5$$

want to reach the path whose equation is

$$\begin{pmatrix} 6 & -7 \end{pmatrix} \mathbf{x} = 8$$

in the least time. Find the equation of the path that he should follow.

2 Solution

Step1: we need to find the solution of equation:

$$\begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = 4$$
$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = 5 \tag{2.0.1}$$

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \tag{2.0.2}$$

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \xleftrightarrow{R1 \leftarrow \frac{4}{17} * (R1 + \frac{3}{4} R2)} \begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \tag{2.0.3}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \xleftrightarrow{R2 \leftarrow \frac{1}{4}(R2 - 3 * R1)} \begin{pmatrix} 1 & 0 & 31/17 \\ 0 & 1 & -2/17 \end{pmatrix} \tag{2.0.4}$$

After solving this two equation we will get the junction point, which is intersection of this line segments. Thus, Junction Point is (31/17, -2/17). To reach in the least time, he should follow the shortest path, i.e, perpendicular from the junction point to the line give by this equation:

$$\begin{pmatrix} 6 & -7 \end{pmatrix} \mathbf{x} = 8 \tag{2.0.5}$$

normal vector to the given line is:

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 6 \\ -7 \end{pmatrix} \tag{2.0.6}$$

$$\mathbf{n} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \tag{2.0.7}$$

The equation of the line in terms of normal vector

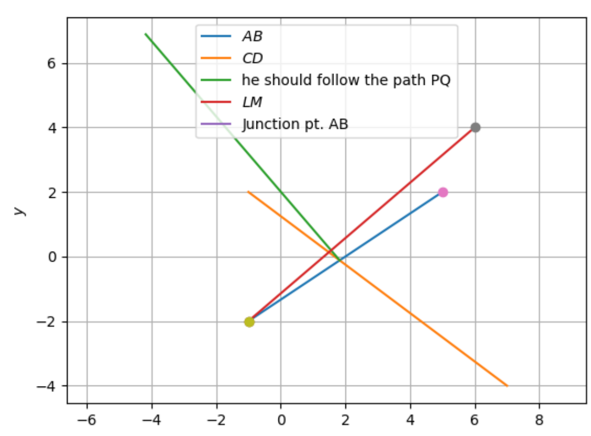

Fig. 1: The Required path is PQ.

passing through a given point is obtained as

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0 \tag{2.0.8}$$
$$\mathbf{n}^T\mathbf{x} = \mathbf{n}^T\mathbf{A} \tag{2.0.9}$$

Hence, he should follow this path PQ:

$$\begin{pmatrix} 7 & 6 \end{pmatrix}\mathbf{x} = \frac{205}{17} \tag{2.0.10}$$