EE5609 Assignment 18

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Abstract

This document solves problem based on Matrix Theory.

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_18

1 Problem

Let T be the linear operator on R^2, the matrix of which in the standard ordered basis is
$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$
Let W_1 be the subspace of R^2 spanned by the vector $\epsilon_1 = (1, 0)$

a) Proove that W_1 is invariant under T.
b) Prove that there is no subspace W_2 which is invariant under T and is complementary to W_1: $R^2 = W_1 \oplus W_2$
(Compare with exercise 1 of section 6.5.)

2 solution

Statement	**Solution**
Given	T be the linear operator on R^2 the matrix of which in the standard ordered basis is $\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$ W_1 be the subspace of R^2 spanned by the vector $\epsilon_1 = (1, 0)$.
To Proove	a) W_1 is invariant under T. b) There is no subspace W_2 which is invariant under T and is complementary to W_1: $R^2 = W_1 \oplus W_2$ c) Compare with exercise 1 of section 6.5.

Proof (a)	$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix} \quad (2.0.1)$ $\lvert A - \lambda I \rvert = 0 \quad (2.0.2)$ $\implies \begin{pmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{pmatrix} \quad (2.0.3)$ $= (2-\lambda)^2 = 0 \quad (2.0.4)$ $\therefore \lambda = 2 \quad (2.0.5)$ for $\lambda = 2$, the corresponding vector is $(\mathbf{A} - \lambda I)X = 0 \quad (2.0.6)$ $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} X = 0 \quad (2.0.7)$ $\therefore X = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.0.8)$ Hence, W_1 be the subspace of R^2 spanned by the vector $\epsilon_1 = (1,0)$ is invariant under T.
Proof (b)	Corresponding to $\lambda = 2$, Among, two eigen vectors only one is independent and other one is dependent. Thus, $P^{-}1$ does not exist and A can not be diagonalized. Hence, there is no subspace W_2 which is invariant under T and is complementary to W_1: $R^2 = W_1 \oplus W_2$
Observation	In exercise 1 of section 6.5, for 2×2 matrix there is 2 distinct characteristic value, corresponding to which there is a eigen vector. Hence, P^{-1} exists. $\therefore$ the given matrix is diagonalizable.

Table1:Solution