EE5609 Assignment 3

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***Abstract*—This document contains the solution of geometry through linear algebra.**

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_3

1 Problem

ABCE is a Quadrilateral and D is a midpoint on BC such that, AC=AE, AB=AD and $\angle BAD$=$\angle EAC$. Show that BC=DE.

2 Explanation

In, $\triangle ABC$ and $\triangle ADE$

$$\angle BAD = \angle EAC \quad (given) \tag{2.0.1}$$

Adding $\angle DAC$ on both side, We get:

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC \tag{2.0.2}$$

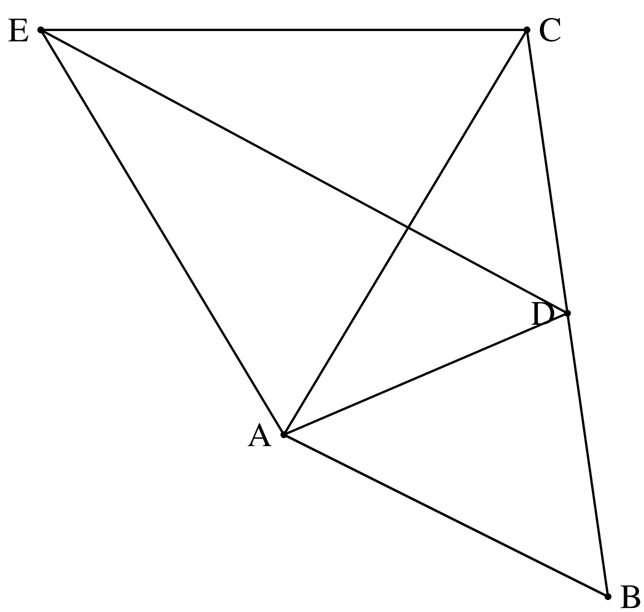

Fig. 0: Quadrilateral ABCE

We have,

$$\angle BAC = \angle DAE \tag{2.0.3}$$

$$\implies \cos\angle DAE = \cos\angle BAC \tag{2.0.4}$$

In triangle ABC, Using law of cosine, Using the formula of dot product, i.e.,

$$\mathbf{a}.\mathbf{b} = ||\mathbf{a}|| \,.\, ||\mathbf{b}|| \,.\cos\theta \tag{2.0.5}$$

$$\implies \cos\theta = \frac{\mathbf{a}.\mathbf{b}}{||\mathbf{a}|| \,.\, ||\mathbf{b}||} \tag{2.0.6}$$

$$\frac{(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E})}{||\mathbf{A}-\mathbf{D}||\,||\mathbf{A}-\mathbf{E}||} = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{||\mathbf{A}-\mathbf{B}||\,||\mathbf{A}-\mathbf{C}||} \tag{2.0.7}$$

We are given AE=AC and we know AD=AB always. Thus,

$$||\mathbf{A}-\mathbf{E}|| = ||\mathbf{A}-\mathbf{C}|| \tag{2.0.8}$$

$$||\mathbf{A}-\mathbf{D}|| = ||\mathbf{A}-\mathbf{B}|| \tag{2.0.9}$$

Then, from (2.0.7), we have,

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E}) = (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{2.0.10}$$

We need to prove: $||\mathbf{B}-\mathbf{C}|| = ||\mathbf{D}-\mathbf{E}||$

$$\implies ||\mathbf{B}-\mathbf{C}||^2 - ||\mathbf{D}-\mathbf{E}||^2 = (\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) - (\mathbf{D}-\mathbf{E})^T(\mathbf{D}-\mathbf{E}) \tag{2.0.11}$$

$$= ((\mathbf{A}-\mathbf{C})-(\mathbf{A}-\mathbf{B}))^T((\mathbf{A}-\mathbf{C})-(\mathbf{A}-\mathbf{B})) - ((\mathbf{A}-\mathbf{E})-(\mathbf{A}-\mathbf{D}))^T((\mathbf{A}-\mathbf{E})-(\mathbf{A}-\mathbf{D})) \tag{2.0.12}$$

$$= ||\mathbf{A}-\mathbf{C}||^2+||\mathbf{A}-\mathbf{B}||^2-(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})-(\mathbf{A}-\mathbf{C})^T(\mathbf{A}-\mathbf{B}) -||\mathbf{A}-\mathbf{E}||^2-||\mathbf{A}-\mathbf{D}||^2+(\mathbf{A}-\mathbf{D})^T(\mathbf{A}+\mathbf{E})-(\mathbf{A}-\mathbf{E})^T(\mathbf{A}-\mathbf{D}) \tag{2.0.13}$$

Thus, from (2.0.8), (2.0.9) and (2.0.10)

$$||\mathbf{B}-\mathbf{C}||^2 - ||\mathbf{D}-\mathbf{E}||^2 = 0 \tag{2.0.14}$$

$$\implies ||\mathbf{B}-\mathbf{C}||^2 = ||\mathbf{D}-\mathbf{E}||^2 \tag{2.0.15}$$

$$\therefore ||\mathbf{B}-\mathbf{C}|| = ||\mathbf{D}-\mathbf{E}|| \tag{2.0.16}$$

Hence, BC=DE