

EE5609 Assignment 10

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Abstract—This document solves problem based on solution of vector space.

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_10

1 PROBLEM

Consider the vectors in \mathbb{R}^4 defined by:

$$\alpha_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 2 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 3 \\ 4 \\ -2 \\ 5 \end{pmatrix} \text{ and } \alpha_3 = \begin{pmatrix} 1 \\ 4 \\ 0 \\ 9 \end{pmatrix}.$$

Find a system of homogeneous linear equations for which the space of solutions is exactly the subspace of \mathbb{R}^4 spanned by the given three vectors.

2 SOLUTION

A system of linear equations is homogeneous if all of the constant terms are zero. It can be represented as,

$$\mathbf{A}\mathbf{X} = \mathbf{0} \quad (2.0.1)$$

Let \mathbf{R} be a echelon matrix which is reduced to \mathbf{A} . Then the systems $\mathbf{A}\mathbf{X} = \mathbf{0}$ and $\mathbf{R}\mathbf{X} = \mathbf{0}$ have the same solutions. Here,

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \quad (2.0.2)$$

By operating column operation on \mathbf{A} , we get:

$$\begin{pmatrix} -1 & 3 & 1 \\ 0 & 4 & 4 \\ 1 & -2 & 0 \\ 2 & 5 & 9 \end{pmatrix} \xleftrightarrow{C_3=C_3-2C_1-C_2} \begin{pmatrix} -1 & 3 & 0 \\ 0 & 4 & 0 \\ 1 & -2 & 0 \\ 2 & 5 & 0 \end{pmatrix} \quad (2.0.3)$$

$$\xleftrightarrow{C_1=-C_1} \begin{pmatrix} 1 & 3 & 0 \\ 0 & 4 & 0 \\ -1 & -2 & 0 \\ -2 & 5 & 0 \end{pmatrix} \quad (2.0.4)$$

$$\xleftrightarrow{C_2=C_2-3C_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ -1 & 1 & 0 \\ -2 & 11 & 0 \end{pmatrix} \quad (2.0.5)$$

$$\xleftrightarrow{C_2=\frac{1}{4}C_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & \frac{1}{4} & 0 \\ -2 & \frac{11}{4} & 0 \end{pmatrix} \quad (2.0.6)$$

The basis vector is non zero vector which are given from 2.0.6,

$$\rho_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ -2 \end{pmatrix}, \rho_2 = \begin{pmatrix} 0 \\ 1 \\ \frac{1}{4} \\ \frac{11}{4} \end{pmatrix} \quad (2.0.7)$$

Vector \mathbf{V} formed using the above basis vectors are given as,

$$c_1\rho_1 + c_2\rho_2 = \begin{pmatrix} c_1 \\ c_2 \\ -c_1 + \frac{1}{4}c_2 \\ -2c_1 + \frac{11}{4}c_2 \end{pmatrix} \quad (2.0.8)$$

If \mathbf{X} is in \mathbf{V} , so it can be written in this form.

$$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & \frac{1}{4} \\ -2 & \frac{11}{4} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \mathbf{X} \quad (2.0.9)$$

The augmented matrix using 2.0.9 can be written as,

$$\begin{pmatrix} 1 & 0 & x_1 \\ 0 & 1 & x_2 \\ -1 & \frac{1}{4} & x_3 \\ -2 & \frac{11}{4} & x_4 \end{pmatrix} \quad (2.0.10)$$

Converting the above augmented matrix into row reduced echelon form:

$$\begin{pmatrix} 1 & 0 & x_1 \\ 0 & 1 & x_2 \\ -1 & \frac{1}{4} & x_3 \\ -2 & \frac{11}{4} & x_4 \end{pmatrix} \xrightarrow{R_3=R_3+R_1-\frac{1}{4}R_2, R_4=R_4+2R_1-\frac{11}{4}R_2} \begin{pmatrix} 1 & 0 & x_1 \\ 0 & 1 & x_2 \\ 0 & 0 & x_1 - \frac{1}{4}x_2 + x_3 \\ 0 & 0 & 2x_1 - \frac{11}{4}x_2 + x_4 \end{pmatrix} \quad (2.0.11)$$

The required equation is given as:

$$\mathbf{Y} = \begin{pmatrix} x_1 \\ x_2 \\ x_1 - \frac{1}{4}x_2 \\ 2x_1 - \frac{11}{4}x_2 \end{pmatrix} \quad (2.0.12)$$