EE5609 Assignment 3

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***Abstract*—This document contains the solution of geometry through linear algebra.**

Download latex and python codes from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_3

1 Problem

ABCE is a Quadrilateral and D is a midpoint on BC such that, AC=AE, AB=AD and $\angle BAD$=$\angle EAC$. Show that BC=DE.

2 Explanation

In, $\triangle ABC$ and $\triangle ADE$

$$\angle BAD = \angle EAC \quad (given) \tag{2.0.1}$$

Adding $\angle DAC$ on both side, We get:

$$\angle BAD + \angle DAC = \angle EAC + \angle DAC \tag{2.0.2}$$

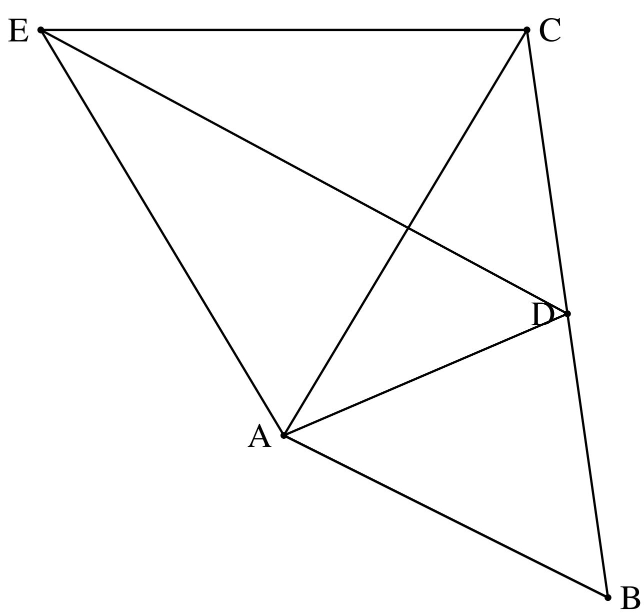

Fig. 0: Quadrilateral ABCE

We have,

$$\angle BAC = \angle DAE \tag{2.0.3}$$

$$\implies \cos\angle DAE = \cos\angle BAC \tag{2.0.4}$$

Using the formula of dot product, i.e.,

$$\mathbf{a}.\mathbf{b} = ||\mathbf{a}||\,.\,||\mathbf{b}||\,.\cos\theta \tag{2.0.5}$$

$$\implies \cos\theta = \frac{\mathbf{a}.\mathbf{b}}{||\mathbf{a}||\,.\,||\mathbf{b}||} \tag{2.0.6}$$

$$\frac{(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E})}{||\mathbf{A}-\mathbf{D}||\,||\mathbf{A}-\mathbf{E}||} = \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{||\mathbf{A}-\mathbf{B}||\,||\mathbf{A}-\mathbf{C}||} \tag{2.0.7}$$

We are given AE=AC and we know AD=AB always. Thus,

$$||\mathbf{A}-\mathbf{E}|| = ||\mathbf{A}-\mathbf{C}|| \tag{2.0.8}$$

$$||\mathbf{A}-\mathbf{D}|| = ||\mathbf{A}-\mathbf{B}|| \tag{2.0.9}$$

Then, from (2.0.6), we have,

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E}) = (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \tag{2.0.10}$$

$$\implies ||\mathbf{A}-\mathbf{D}||^2 - (\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{E}) = ||\mathbf{A}-\mathbf{D}||^2 - (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) \tag{2.0.11}$$

$$\implies (\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{E}) = (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) \tag{2.0.12}$$

$$\implies ||\mathbf{D}-\mathbf{A}||\,||\mathbf{D}-\mathbf{E}||\cos\angle ADE = ||\mathbf{B}-\mathbf{A}||\,||\mathbf{D}-\mathbf{E}||\cos\angle ABC \tag{2.0.13}$$

$$\implies ||\mathbf{D}-\mathbf{E}||\cos\angle ADE = ||\mathbf{B}-\mathbf{C}||\cos\angle ABC \tag{2.0.14}$$

We need to prove: $||\mathbf{B}-\mathbf{C}|| = ||\mathbf{D}-\mathbf{E}||$.
From (2.0.11),

$$(\mathbf{D}-\mathbf{A})^T(\mathbf{D}-\mathbf{E}) = (\mathbf{B}-\mathbf{A})^T(\mathbf{B}-\mathbf{C}) \tag{2.0.15}$$

$$\implies ||\mathbf{D}-\mathbf{E}||^2 - (\mathbf{E}-\mathbf{D})^T(\mathbf{E}-\mathbf{A}) = ||\mathbf{B}-\mathbf{C}||^2 - (\mathbf{C}-\mathbf{B})^T(\mathbf{C}-\mathbf{A}) \tag{2.0.16}$$

$$\implies ||\mathbf{D}-\mathbf{E}||^2 - (||\mathbf{A}-\mathbf{E}||^2 - (\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E})) = ||\mathbf{B}-\mathbf{C}||^2 - (||\mathbf{A}-\mathbf{C}||^2 - (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})) \tag{2.0.17}$$

We are given that AE=AC, AD=AB. Using, (2.0.7),(2.0.8)

$$\therefore \|\mathbf{D}-\mathbf{E}\|^2 + (\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E}) = \|\mathbf{B}-\mathbf{C}\|^2 + (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) \quad (2.0.18)$$

$$\implies \|\mathbf{D}-\mathbf{E}\|^2 + \|\mathbf{A}-\mathbf{D}\|\,\|\mathbf{A}-\mathbf{E}\|\cos\angle DAE = \|\mathbf{B}-\mathbf{C}\|^2 + \|\mathbf{A}-\mathbf{B}\|\,\|\mathbf{A}-\mathbf{C}\|\cos\angle BAC \quad (2.0.19)$$

From the question, $\angle DAE = \angle BAC$ and $\|\mathbf{A}-\mathbf{E}\| = \|\mathbf{A}-\mathbf{C}\|$. We also know $\|\mathbf{A}-\mathbf{D}\| = \|\mathbf{A}-\mathbf{B}\|$. Thus, from (2.0.18), we get,

$$\|\mathbf{D}-\mathbf{E}\|^2 = \|\mathbf{B}-\mathbf{C}\|^2 \quad (2.0.20)$$

$$\implies \|\mathbf{D}-\mathbf{E}\| = \|\mathbf{B}-\mathbf{C}\| \quad (2.0.21)$$

$$\therefore BC = DE \quad (2.0.22)$$