EE5609 Assignment 17

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***Abstract*—This document solves problem based on Matrix Theory.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_17

1 Problem

Usig the result of Exercise 8 to proove that , If A and B are $n \times n$ matrices over the field F, then AB and BA have precisely the same characteristic values.

2 solution

Given	A and B are $n \times n$ matrices over the field F. In Exercise 8, If $(I - AB)$ is invertible then $(I - BA)$ is invertible.
To prove	AB and BA have precisely the same characteristic values.
Proof	Let suppose c is the characteristic value of AB. Then, $\lvert cI - AB \rvert = 0$ Here, I, A and B are $n \times n$ matrix. $\implies c^n \lvert I - \frac{1}{c}AB \rvert = 0$ Using the result of Exercise 8, $(I - \frac{1}{c}AB)$ is non-invertible $\implies (I - \frac{1}{c}BA)$ is also non-invertible. $\implies c^n \lvert I - \frac{1}{c}BA \rvert = 0$ $\implies \lvert cI - BA \rvert = 0$ Hence, AB and BA have precisely the same characteristic value.

Alternative Way,

Given	A and B are $n \times n$ matrices over the field F. In Exercise 8, If $(I - AB)$ is invertible then $(I - BA)$ is invertible.
To prove	AB and BA have precisely the same characteristic values.
Observation	We have to show that if c is a characteristic value for AB then c is a characteristic value for BA. Conversely, This is equivalent to the statement if c is not a characteristic value for AB then it is not a characteristic value for BA.
Proof	Suppose that c is not a characteristic value for BA, this means that $\lvert cI - AB \rvert \neq 0$. $\implies c^n \lvert I - \frac{1}{c}AB \rvert \neq 0$ $\therefore (I - \frac{1}{c}AB)$ is invertible $\implies (I - \frac{1}{c}BA)$ is invertible. $\implies \lvert I - \frac{1}{c}AB \rvert \neq 0$ $\implies c^n \lvert I - \frac{1}{c}AB \rvert = c^n \lvert I - \frac{1}{c}AB \rvert \neq 0$ Hence, If c is not a characteristic value for AB then it is not a characteristic value for BA. Hence, AB and BA have precisely the same characteristic value.