EE5609 Assignment 10

Abhishek Thakur

***Abstract*—This document solves problem based on solution of vector space.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_10

1 Problem

Consider the vectors in $\mathbb{R}^4$ defined by:
$\alpha_1 = \begin{pmatrix}-1\\0\\1\\2\end{pmatrix}$, $\alpha_2 = \begin{pmatrix}3\\4\\-2\\5\end{pmatrix}$ and $\alpha_3 = \begin{pmatrix}1\\4\\0\\9\end{pmatrix}$.
Find a system of homogeneous linear equations for which the space of solutions is exactly the subspace of $\mathbb{R}^4$ spanned by the given three vectors.

2 Solution

A system of linear equations is homogeneous if all of the constant terms are zero. It can be represented as,

$$\mathbf{AX} = 0 \tag{2.0.1}$$

Let $\mathbf{R}$ be a echelon matrix which is reduced to A. Then the systems $\mathbf{AX} = 0$ and $\mathbf{RX} = 0$ have the same solutions. Here,

$$\begin{pmatrix}-1 & 3 & 1\\0 & 4 & 4\\1 & -2 & 0\\2 & 5 & 9\end{pmatrix} \tag{2.0.2}$$

By operating column operation on $\mathbf{A}$, we get:

$$\begin{pmatrix}-1 & 3 & 1\\0 & 4 & 4\\1 & -2 & 0\\2 & 5 & 9\end{pmatrix} \xleftrightarrow{C_3=C_3-2C_1-C_2} \begin{pmatrix}-1 & 3 & 0\\0 & 4 & 0\\1 & -2 & 0\\2 & 5 & 0\end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{C_1=-C_1} \begin{pmatrix}1 & 3 & 0\\0 & 4 & 0\\-1 & -2 & 0\\-2 & 5 & 0\end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{C_2=C_2-3C_1} \begin{pmatrix}1 & 0 & 0\\0 & 4 & 0\\-1 & 1 & 0\\-2 & 11 & 0\end{pmatrix} \tag{2.0.5}$$

The bais vector is non zero vector which are given from 2.0.5,

$$\rho_1 = \begin{pmatrix}1\\0\\-1\\-2\end{pmatrix}, \rho_2 = \begin{pmatrix}0\\4\\1\\11\end{pmatrix} \tag{2.0.6}$$

ρ_1, ρ_2 forms the basis of the solution space. The subspace spanned by b_1 and b_2 is given as:

$$\begin{pmatrix}\rho_1 & \rho_2\end{pmatrix}\begin{pmatrix}b_1\\b_2\end{pmatrix} = \mathbf{X} \tag{2.0.7}$$

Using 2.0.7, we can write the augmented matrix as:

$$\begin{pmatrix}1 & 0 & x_1\\0 & 4 & x_2\\-1 & 1 & x_3\\-2 & 11 & x_4\end{pmatrix} \xleftrightarrow{R_3=R_3+R_1-\frac{1}{4}R_2} \tag{2.0.8}$$

$$\begin{pmatrix}1 & 0 & x_1\\0 & 4 & x_2\\0 & 0 & x_1-\frac{1}{4}x_2+x_3\\-2 & 11 & x_4\end{pmatrix} \xleftrightarrow{R_4=R_4+2R_1-\frac{11}{4}R_2} \tag{2.0.9}$$

$$\begin{pmatrix}1 & 0 & x_1\\0 & 4 & x_2\\0 & 0 & x_1-\frac{1}{4}x_2+x_3\\0 & 0 & 2x_1-\frac{11}{4}x_2+x_4\end{pmatrix} \tag{2.0.10}$$

Using 2.0.10, The required homogeneous equation is given as:

$$\begin{pmatrix} 1 & -\frac{1}{4} & 1 & 0 \\ 2 & -\frac{11}{4} & 0 & 1 \end{pmatrix} \mathbf{X} = 0 \tag{2.0.11}$$