EE5609 Assignment 16

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***Abstract*—This document solves problem based on Matrix Theory.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_16

1 Problem

Let A be an $n \times n$ self-adjoint matrix with eigenvalues $\lambda_1, \cdots, \lambda_n$.
Let $\|X\|_2 = \sqrt{|X_1|^2 + \cdots + |X_n|^2}$ for $X = (x_1, \cdots, x_n) \in \mathbb{C}^n$. If $\rho(A) = a_0 I + a_1 A + \cdots + a_n A^n$ then $sup_{\|X\|_2=1} \|\rho(A)X\|_2$ is equal to.

2 Definition

A matrix A for which $A^H = A$ is said to be a self-adjoint matrix, here conjugate transpose is denoted as A^H. If a matrix is self-adjoint, It is said to be hermitian.

3 solution

Given, A be an $n \times n$ self-adjoint matrix with eigenvalues $\lambda_1, \cdots, \lambda_n$.

$$\implies AX = \lambda X \tag{3.0.1}$$

$$\implies A^n X = \lambda^n X \tag{3.0.2}$$

Also given,

$$\rho(A) = a_0 I + a_1 A + \cdots + a_n A^n \tag{3.0.3}$$

$$\implies (\rho(A))X = (a_0 I + a_1 A + \cdots + a_n A^n)X \tag{3.0.4}$$

$$(\rho(A))X = a_0 X + a_1 AX + \cdots + a_n A^n X \tag{3.0.5}$$

Using 2.0.2 in 2.0.5,

$$(\rho(A))X = a_0 X + a_1 \lambda X + \cdots + a_n \lambda^n X \tag{3.0.6}$$

$$\implies \|\rho(A)X\|_2 = \|(a_0 + a_1 \lambda + \cdots + a_n \lambda^n)X\|_2 \tag{3.0.7}$$

$$= |(a_0 + a_1 \lambda + \cdots + a_n \lambda^n)| \, \|X\|_2 \tag{3.0.8}$$

Hence,

$$sup_{\|X\|_2=1} \|\rho(A)X\|_2 = \max\{|(a_0 + a_1 \lambda_j + \cdots + a_n \lambda_j^n)| : 1 \le j \le n\} \tag{3.0.9}$$