EE5609 Assignment 12

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***Abstract*—This document solves problem based on solution of vector space.**

Download all solutions from

https://github.com/abhishekt711/EE5609/tree/master/Assignment_12

1 Problem

In R^3, let $\alpha_1 = \begin{pmatrix}1\\0\\1\end{pmatrix}$, $\alpha_2 = \begin{pmatrix}0\\1\\-2\end{pmatrix}$ and $\alpha_3 = \begin{pmatrix}-1\\-1\\0\end{pmatrix}$. if f is a linear functional on R^3 such that $f(\alpha_1) = 1,\ f(\alpha_2) = -1,\ f(\alpha_3) = 3,$

And if α=$\begin{pmatrix}a\\b\\c\end{pmatrix}$, find $f(\alpha)$.

2 Solution

Given, $\alpha = \begin{pmatrix}a\\b\\c\end{pmatrix}$. Let,

$$\mathbf{A} = \begin{pmatrix}\alpha_1 & \alpha_2 & \alpha_3\end{pmatrix} \tag{2.0.1}$$

$$\mathbf{AX} = \alpha \tag{2.0.2}$$

$$\begin{pmatrix}1 & 0 & -1\\0 & 1 & -1\\1 & -2 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix} \tag{2.0.3}$$

$\mathbf{X} = A^{-1}\alpha$ will give solution of the equation.

$$\begin{pmatrix}1 & 0 & -1 & 1 & 0 & 0\\0 & 1 & -1 & 0 & 1 & 0\\1 & -2 & 0 & 0 & 0 & 1\end{pmatrix} \xleftrightarrow{R_3\leftarrow R_3-R_1} \tag{2.0.4}$$

$$\begin{pmatrix}1 & 0 & -1 & 1 & 0 & 0\\0 & 1 & -1 & 0 & 1 & 0\\0 & -2 & 1 & -1 & 0 & 1\end{pmatrix} \xleftrightarrow{R_3\leftarrow R_3+2R_2} \tag{2.0.5}$$

$$\begin{pmatrix}1 & 0 & -1 & 1 & 0 & 0\\0 & 1 & -1 & 0 & 1 & 0\\0 & 0 & -1 & -1 & 2 & 1\end{pmatrix} \xleftrightarrow{R_3\leftarrow R_3/(-1)} \tag{2.0.6}$$

$$\begin{pmatrix}1 & 0 & -1 & 1 & 0 & 0\\0 & 1 & -1 & 0 & 1 & 0\\0 & 0 & 1 & 1 & -2 & -1\end{pmatrix} \xleftrightarrow{R_1\leftarrow R_1+R_3} \tag{2.0.7}$$

$$\begin{pmatrix}1 & 0 & 0 & 2 & -2 & -1\\0 & 1 & -1 & 0 & 1 & 0\\0 & 0 & 1 & 1 & -2 & -1\end{pmatrix} \xleftrightarrow{R_2\leftarrow R_2+R_3} \tag{2.0.8}$$

$$\begin{pmatrix}1 & 0 & 0 & 2 & -2 & -1\\0 & 1 & 0 & 1 & -1 & -1\\0 & 0 & 1 & 1 & -2 & -1\end{pmatrix} \tag{2.0.9}$$

Thus,

$$A^{-1} = \begin{pmatrix}2 & -2 & -1\\1 & -1 & -1\\1 & -2 & -1\end{pmatrix} \tag{2.0.10}$$

$$\mathbf{X} = A^{-1}\alpha = \begin{pmatrix}2 & -2 & -1\\1 & -1 & -1\\1 & -2 & -1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix} \tag{2.0.11}$$

Given, f is a linear functional on R^3,

$$\alpha = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 \tag{2.0.12}$$

$$\implies f(\alpha) = \mathbf{X}^T\begin{pmatrix}f(\alpha_1)\\f(\alpha_2)\\f(\alpha_3)\end{pmatrix} \tag{2.0.13}$$

Given, $f(\alpha_1) = 1$, $f(\alpha_2) = -1$ and $f(\alpha_3) = 3$.

$$f(\alpha) = \mathbf{X}^T \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \tag{2.0.14}$$

$$\implies f(\alpha) = \begin{pmatrix} a \\ b \\ c \end{pmatrix}^T \begin{pmatrix} 2 & 1 & 1 \\ -2 & -1 & -2 \\ -1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \tag{2.0.15}$$

$$f(\alpha) = \begin{pmatrix} 2a - 2b - c \\ a - b - c \\ a - 2b - c \end{pmatrix}^T \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \tag{2.0.16}$$

Hence,

$$f(\alpha) = 4a - 7b - 3c \tag{2.0.17}$$