

Matrix Theory Assignment 1

Abhishek Thakur

Abstract—This document contains the solution to a Lines and planes problem.

Download all python codes from

<https://github.com/abhishekt711/EE5609/codes>

1 PROBLEM

60. A person standing at the junction of two straight paths represented by the equations

$$(2 \ -3)\mathbf{x} = 4$$

$$(3 \ 4)\mathbf{x} = 5$$

want to reach the path whose equation is

$$(6 \ -7)\mathbf{x} = 8$$

in the least time. Find the equation of the path that he should follow.

2 SOLUTION

Step1: we need to find the solution of equation:

$$(2 \ -3)\mathbf{x} = 4$$

$$(3 \ 4)\mathbf{x} = 5$$

(2.0.1)

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

(2.0.2)

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R1 \leftarrow \frac{4}{17} * (R1 + \frac{3}{4} R2)}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix}$$

(2.0.3)

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R2 \leftarrow -\frac{1}{4}(R2 - 3R1)} \begin{pmatrix} 1 & 0 & 31/17 \\ 0 & 1 & -2/17 \end{pmatrix} \quad (2.0.4)$$

After solving this two equation we will get the junction point, which is intersection of this line segments. Thus, Junction Point is $(31/17, -2/17)$. To reach in the least time, he should follow the shortest path, i.e, perpendicular from the junction point to the line give by this equation:

$$(6 \ -7)\mathbf{x} = 8$$

(2.0.5)

is in the direction of $(7 \ 6)$ as

$$(6 \ -7)(7 \ 6)^T = 0$$

(2.0.6)

The equation of the line in terms of normal vector

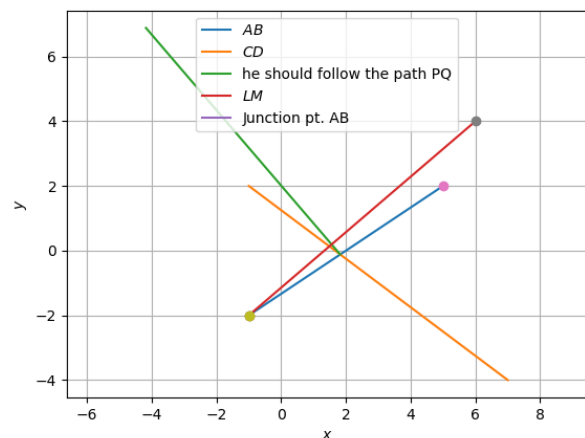


Fig. 1: The Required path is PQ.

passing through a given point is obtained as

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0$$

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} \quad (2.0.7)$$

$$(2.0.8)$$

Hence, he should follow this path PQ:

$$(7 \ 6) \mathbf{x} = \frac{205}{17} \quad (2.0.9)$$