

Matrix Theory Assignment 1

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Abstract—This document contains the solution to a Lines and planes problem.

Download all python codes from

<https://github.com/abhishekt711/EE5609/codes>

1 PROBLEM

60. A person standing at the junction of two straight paths represented by the equations

$$(2 - 3)x = 4$$

$$(3 \ 4)x = 5$$

want to reach the path whose equation is

$$(6 - 7)x = 8$$

in the least time. Find the equation of the path that he should follow.

2 SOLUTION

Step1: we need to find the solution of equation:

$$(2 - 3)x = 4$$

$$(3 \ 4)x = 5$$

(2.0.1)

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix}$$

(2.0.2)

Transforming the matrix into row-echelon form

$$\begin{pmatrix} 2 & -3 & 4 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R1 \leftarrow \frac{4}{17}(R1 + \frac{3}{4}R2)}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix}$$

(2.0.3)

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 3 & 4 & 5 \end{pmatrix} \xrightarrow{R2 \leftarrow \frac{1}{4}(R2 - 3R1)}$$

$$\begin{pmatrix} 1 & 0 & 31/17 \\ 0 & 1 & -2/17 \end{pmatrix}$$

(2.0.4)

After solving this two equation we will get the junction point, which is intersection of this line segments.

Thus, Junction Point is $(31/17, -2/17)$. To reach in the least time, he should follow the shortest path, i.e, perpendicular from the junction point to the line give by this equation:

$$(6 - 7)x = 8$$

(2.0.5)

is in the direction of $(7,6)$ as

$$(6, -7)^T (7, 6) = 0$$

(2.0.6)

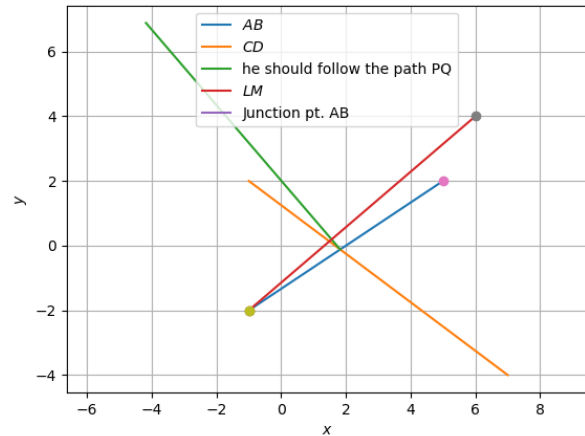


Fig. 1: The Required path is PQ.

The equation of the line in terms of normal vector passing through a given point is obtained as

$$\mathbf{n}^T(\mathbf{x} - \mathbf{A}) = 0$$

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A} \quad (2.0.7)$$

$$(2.0.8)$$

Hence, he should follow this path PQ:

$$(7, 6)\mathbf{x} = 12.05$$

$$(2.0.9)$$