Finding the keys for a Relation R, from FDs

* Any combination that uniquely identify all attributes of a relation is a key or Superkey (SK)
* Minimal Super key (SK), whose no proper subset can't be used for unique identification of all attributes is a candidate key (CK).

Process:—

1. From given FDs, findout which attribute is not identified by any other attributes or we can say the attribute which doesn't appear on the R.H.S of any FD. If found, they are essentials for CK.

2. If any such attribute(X) exist, then it must be included in CK. say such attributes are X. (X can be single or multiple)
   - check whether X^+ can include all the attributes of R. If yes then it means that we can identify all attributes of R with X. and its a candidate key (CK). Any other combination containing X as subset can be a SK but can't be CK.
   - If X alone can't identify all attributes. Next step is to try all possible combinations containing X as subset, starting from the minimum length. If any such combination found include it in set of CK, and try other combinations.

3. If no such attributes found from step 1, then try all combinations of attributes that can identify all attributes of relation R. starting from length one.

NOTE: If any attribute or set of attributes is selected as CK, then no other CK can include existing CKs as subset.

8 Finding the keys for a Relation R

R (A, B, C, D, E, H)   [Navathe Exercise]

FDs = {A → BC, CD → E, E → C, D → AEH, ABH → BD, DH → BC}

* As all the attributes are identifiable, so try combinations :-

$A^+ = ABC$
$B^+ = B$
$C^+ = C$
] $\neq R$

$\boxed{D^+ = DAEHBC = R}$ ✓

$E^+ = EC$
$H^+ = H$
] $\neq R$

CK = {D, AH}

* Try other combinations with rest of attributes :-
(without including D as its a CK)

$(AB)^+ = ABC$
$(AC)^+ = ABC$
$(AE)^+ = AEBC$
] $\neq R$

$\boxed{(AH)^+ = AHBCDE = R}$

$(BC)^+ = BC$
$(BE)^+ = BEC$
$(BH)^+ = BH$
$(CE)^+ = CE$
$(CH)^+ = CH$
$(EH)^+ = EHC$
] $\neq R$

* Try another combinations not containing D, AH as subsets :-

$(ABC)^+ = ABC$
$(ACE)^+ = ACEB$
$(HBC)^+ = HBC$
$(HCE)^+ = HCE$
$(BCE)^+ = BCE$
$(ABE)^+ = ABEC$
] $\neq R$

* Try another combinations :-

$(ABCE)^+ = ABCE$
$(HBCE)^+ = HBCE$
] $\neq R$

Finding the keys from given FDs for relation R

1. R(A, B, C, D, E, F)

$A \to C$	A and E are not identified by any other
$C \to D$	attributes. So they must be included in
$D \to B$	every CK.
$E \to F$	$(AE)^+ = AECFDB = R$

So (AE) alone can identify all other attributes.
So its the only CK for R. Any other key combination must include AE as subset and they can be SK but can't be CK.

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2. R(A, B, C, D, E, F, G, H) [GATE Question]

FDs = { $CH \to G$, $A \to BC$, $B \to CFH$, $E \to A$, $F \to EG$ }

$D^+ = D$ // So D alone can't identify R

Try combinations :- DA, DB, DC, DE, DF, DG, DH

$(DA)^+ = (DE)^+ = (DB)^+ = (DF)^+ = R$

[redacted], $(DC)^+$, [redacted], $(DG)^+$, $(DH)^+ \neq R$

So set of Ck includes :- {DA, DE}

As any other combination can't be formed without including DA or DE.

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3. R(W, X, Y, Z) [GATE Question]

FDs = { $Z \to W$, $Y \to XZ$, $WX \to Y$ }

As all attributes are identifiable. So try combinations :-

$X^+ = X \quad \neq R$

$\boxed{Y^+ = YXZW = R}$ ✓

$W^+ = W \quad \neq R$

$Z^+ = ZW \quad \neq R$

Any other combination of length 3 or more can't be formed without including Y, WX or XZ.

$\boxed{\text{Candidate keys} = \{Y, WX, XZ\}}$

Try other combinations not including Y :- Options of length 2 :-

(WX, WZ, XZ)

$(WX)^+ = WXYZ = R$ ✓

$(WZ)^+ = WZ \quad \neq R$

$(XZ)^+ = XZWY = R$ ✓

Finding keys from given FDs for R

(4) R(A, B, C, D, E)

FDs = {AB → CD, D → A, BC → DE}

As B is not identified by any attribute:-

$B^+ = B \neq R$, B alone can't be a CK

Try combinations:- BA, BC, BD, BE

$(AB)^+ = (BC)^+ = (BD)^+ = ABCDE = R$

But $(BE)^+ = BE \neq R$

$\boxed{\text{So, CK} = \{AB, BC, BD\}}$

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(5) R(A, B, C, D, E, F, G, H)

FDs = {A → C, A → DE, B → F, F → GH}

Taking A and B as they are unidentified:-

$(AB)^+ = ABCDEFGH = R$

So (AB) is the only CK.

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(6) R(A, B, C, D, E, F, G, H)

FD = {AB → C, BD → EF, AD → G, A → H}

Taking A, B and D:-

$(ABD)^+ = ABDCEFGH = R$

So (ABD) is the only CK

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(7) R(A, B, C, D, E) [Gate Question]

CK = {BC, CD}

FDs = {BC → ADE, D → B}

Taking C, $C^+ = C \neq R$

So try combinations:- (AC, BC, CD, CE)

$(BC)^+ = (CD)^+ = BCADE = R$ ✓

$(AC)^+, (CE)^+ \neq R$

Try combination including C, but not BC or CD:-

only one combination is possible → (ACE)

$(ACE)^+ = ACE \neq R$

Equivalence of Sets of Functional dependencies:-

definition:- For two sets of FDs:- E and F.

1. There is a FD $X \to Y$ in E. as $E = \{X \to Y\}$
2. Calculate X^+ with respect to F
3. check whether this X^+ includes the attributes in Y
4. If this is the case for every FD in E, then F covers E. or $E \subseteq F$
5. E and F set are equivalnt if $E \subseteq F$ and $F \subseteq E$

Example 1:- R(ACDEH)

$F = \{A \to C, AC \to D, E \to AD, E \to H\}$

$E = \{A \to CD, E \to AH\}$

Check whether $F = E$ or not

Step 1:- Check whether all the dependencies in E Can be inferred from F or not:-

*→ $A \to CD$ // FD from E

$A^+ = ACD$ // with respect to F

As it CD or all the attributes of R.H.S of $A \to CD$, So this FD is coved in F

*→ $E \to AH$

$E^+ = EADHC$ // with respect to F

as $AH \subseteq E^+$, So $E \to AH$ is coved in F

*→ So we can say $E \subseteq F$ or E is Covered in F.

step 2:- Check whether $F \subseteq E$ or not?

*→ $A \to C$

$A^+ = \{CDA\}$ // with respect to E

$C \subseteq A^+$ ✓

*→ $AC \to D$

$(AC)^+ = ACD$ // from E

$D \subseteq (AC)^+$ ✓

*→ $E \to AD$

$E^+ = EAHCD$ // from E

$AD \subseteq E^+$ ✓

*→ $E \to H$

$E^+ = EAHCD$ // from E

$H \subseteq E^+$ ✓

So $F \subseteq E$

Step 3:- As $E \subseteq F$ and $F \subseteq E$ So $E = F$

Example 2:- Equivalence of Set of Functional Dependencies (6)

$F = \{A \to C, C \to DI, E \to AB\}$

$E = \{A \to C, C \to DI, EC \to AB, E \to C\}$

As $A \to C$, $C \to DI$ are common for both, so needn't to check for them:

Step 1:- check for $E \subseteq F$ or not?

* $EC \to AB$

  $(EC)^+ = EABCDI$ // inferred from F

  As $AB \subseteq (EC)^+$, so $EC \to AB$ is covered in F

* $E \to C$

  $E^+ = EABCDI$ // inferred from F

  As $C \in E^+$, then $E \to C$ covered in F

* This means that E is covered in F or $E \subseteq F$

Step 2:- check for $F \subseteq E$ or not?

* $E \to AB$

  $E^+ = ECABDI$ // inferred from E

* Now $F \subseteq E$

Step 3:- As $E \subseteq F$

and $F \subseteq E$

then $F = E$

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Example 3 :- Given below are two sets of FDs for a Relation $R(A,B,C,D,E)$. Are they equivalent?

$F = \{A \to B, AB \to C, D \to AC, D \to E\}$
$E = \{A \to BC, D \to AE\}$

Step 1 :- check for $E \subseteq F$ or not ?

* $A \to BC$
  $A^+ = ABC$ // inferred from F
  $BC \subseteq A^+$ ✓
  ∴ $A \to BC$ Covered in F

* $D \to AE$
  $D^+ = DACE$ // inferred from F
  $AE \subseteq D^+$ ✓
  ∴ $D \to AE$ Covered in F

* $E \subseteq F$

Step 2 :- Check for $F \subseteq E$ or not :-?

* $A \to B$
  // This dependency is directly Covered in $E = \{A \to BC\}$

* $AB \to C$
  $(AB)^+ = ABC$ // inferred from E
  $C \in (AB)^+$, i.e. $AB \to C$ covered in E

* $D \to AC$
  $D^+ = DAEBC$ // inferred from E
  $AC \subseteq D^+$, i.e. $D \to AC$ covered in E

* $D \to E$
  // This dependency is directly Covered in $E = \{D \to AE\}$

* all FDs of F are Covered in E, $F \subseteq E$

Step 3 :- $E \subseteq F$ and $F \subseteq E$
then $F = E$

Minimal Set / Minimum Cover / Canonical Cover of FDs :-

R(A, B, D) (1)

B → A

D → A

AB → D

Step 1 :- one element on R.H.S

Step 2 → Check for extra attributes on L.H.S

AB → D

may be replaced with A → D or B → D

if so remove AB → D and Add A → D

$(AB)^+ = ABD$

with A → D : $A^+ = AD$

$(AB)^+ \neq A^+$, So A → D can't replace AB → D

with B → D : $B^+ = BDA$

$(AB)^+ = B^+$, So B → D can replace AB → D

So :- after Step 2 :-

B → A

D → A

B → D

Step 3 :- Check for redundant FDs :-

~~B → A~~ X

D → A → essential

B → D → essential

(a) is B → A redundant or not :-

$B^+ = BAD$

without B → A, $B^+ = BDA$, so B → A is redundant

(b) is D → A redundant :-

$D^+ = DA$

without D → A, $D^+ = D$, so essential

(c) is B → D redundant

$B^+ = BD$

without B → D, $B^+ = B$, so essential

Final :- D → A and B → D

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~~RE~~ Finding Minimal Cover / Canonical Cover

R(A, B, C)

FD = { A→BC, B→C, A→B, AB→C }

Step 1 :-

A → B
A → C
B → C
A → B
AB → C

Step 2:- Check for extra attributes on L.H.S.

For AB → C
taking $(AB)^+ = ABC$
taking A → C ↳ $A^+ = ABC$] so AB→C = A→C

Taking B → C ↳ $B^+ = BC$] so AB→C ≠ B→C

So we can replace AB → C with A → C

Final Set after step 2 :-

A→B
A→C
B→C
~~A→B~~ duplicate
~~A→C~~ duplicate

A→B
~~A→C~~ redundant
B→C

Step 3:- Check for redundant FDs

(a) is A→B redundant :-
with A→B, $A^+ = ABC$
without A→B, $A^+ = AC$] ≠
So A→B is essential

(b) is A→C redundant :-
with A→C, $A^+ = ABC$
without A→C, $A^+ = ABC$] =
So A→C is redundant

(c) is B→C redundant :-
with B→C, $B^+ = BC$
without B→C, $B^+ = B$] ≠
So B→C is essential

Resultant Set :-

A→B
B→C

(3) Finding Minimal Set of FDs :-

$R(A, B, C, D, E)$

FDs = $\{A \to BC, CD \to E, B \to D, E \to A\}$

Step1:- FDs = $\{A \to B, A \to C, CD \to E, B \to D, E \to A\}$

Step2 :- Check extra attributes on L.H.S :-

with $CD \to E$, $(CD)^+ = CDEAB$

with $C \to E$, $C^+ = CEABD$

with $D \to E$, $D^+ = DEABC$

= so we can replace $CD \to E$ with either $C \to E$ Or $D \to E$

FDs = $\{A \to B, A \to C, C \to E, B \to D, E \to A\}$

Step3:- Check for redundant FDs :-

(a) is $A \to B$ redundant

with $A \to B$, $A^+ = ABCDE$

without $A \to B$, $A^+ = ACE$ $\Big] \neq$

So $A \to B$ is essential

(b) is $A \to C$ redundant

with $A \to C$, $A^+ = ABCDE$

without $A \to C$, $A^+ = ABD$ $\Big] \neq$

So; $A \to C$ is essential

(c) is $C \to E$ redundant

with $C \to E$, $C^+ = CE$

without $C \to E$, $C^+ = C$ $\Big] \neq$

So, $C \to E$ is essential

(d) is $B \to D$ redundant

with $B \to D$, $B^+ = BD$

without $B \to D$, $B^+ = B$ $\Big] \neq$

So $B \to D$ is essential

(e) is $E \to A$ redundant

with $E \to A$, $E^+ = EABCD$

without $E \to A$, $E^+ = E$ $\Big] \neq$

So $E \to A$ is essential

$A \to B$ ✓
$A \to C$ ✓
$C \to E$ ✓
$B \to D$ ✓
$E \to A$ ✓

so final FDs Set :-

$A \to B$
$A \to C$
$C \to E$
$B \to D$
$E \to A$

OR

$A \to B$
$A \to C$
$D \to E$
$B \to D$
$E \to A$

(4) Finding Minimal Cover:-

$R(A, B, C, D, E)$

$FD = \{A \to B, AB \to C, D \to AC, D \to E\}$

Step 1:- $\{A \to B, \underline{AB \to C}, D \to A, D \to C, D \to E\}$

Step 2:- check extra symbols at L.H.S.

with $AB \to C$, $(AB)^+ = (ABC)$

with $A \to C$, $A^+ = ACB$, So, $AB \to C = A \to C$

with $B \to C$, $B^+ = B$, so $AB \to C \neq B \to C$

Replace, $\underline{AB \to C \text{ with } A \to C}$

$FD = \{A \to B, A \to C, D \to A, \cancel{D \to C}, D \to E\}$

Step 3:- Check for redundant FDs:-

(a) with $A \to B$, $A^+ = ABC$] $\neq$
without $A \to B$, $A^+ = AC$
$A \to B$ = essential

(b) with $A \to C$, $A^+ = ABC$] $\neq$
without $A \to C$, $A^+ = AB$
$A \to C$ = essential

(c) with $D \to A$, $D^+ = DABCE$] $\neq$
without $D \to A$, $D^+ = DCE$
$D \to A$ essential

(d) with $D \to C$, $D^+ = DABCE$] $=$
without $D \to C$, $D^+ = DAEBC$
So $D \to C$ is redundant

(e) with $D \to E$, $D^+ = DAEBC$] $\neq$
without $D \to E$, $D^+ = DABC$
So $D \to E$ is essential.

Final Set:-

$$\boxed{\begin{array}{c} A \to B \\ A \to C \\ D \to A \\ D \to E \end{array}} \quad \text{OR} \quad \boxed{\begin{array}{c} A \to BC \\ D \to AE \end{array}}$$

5 $R(W, X, Y, Z)$ (Finding Minimal Cover)

$FD = \{X \to W, WZ \to XY, Y \to WXZ\}$

Step 1:- decompose R.H.S:-

$X \to W$
$WZ \to X$
$WZ \to Y$
$Y \to W$
$Y \to X$
$Y \to Z$

Step 2:- Check for extra elements on L.H.S:- $WZ \to XY$

$(WZ)^+ = (WZXY)$

* Removing W from LHS, $Z \to XY$

$Z^+ = (ZXYW)$

as $(WZ)^+ = Z^+$, so $Z \to XY$ can replace $WZ \to XY$

* Removing Z from R.H.S, $W \to XY$

$W^+ = XYWZ$

So $W \to XY$ can also replace $WZ \to XY$

After Step 3:-

$X \to W$		$X \to W$
$W \to X$		$Z \to X$
$W \to Y$	OR →	$Z \to Y$
$Y \to W$		$Y \to W$
$Y \to X$		$Y \to X$
$Y \to Z$ → taking this set		$Y \to Z$

Step 3:-

$X \to W$ ✓
~~$W \to X$~~
$W \to Y$ ✓
~~$Y \to W$~~
$Y \to X$ ✓
$Y \to Z$ ✓

Step 3:- Check for redundant FDs:-

(a) is $X \to W$ redundant:-
with $X^+ = XWYZ$
without $X \to W$, $X^+ = (X)$ — $\neq$
so $X \to W$ is essential

(b) is $W \to X$ redundant:-
with $W^+ = (WXZY)$
without $W \to X$, $W^+ = (WYXZ)$ — $=$
So $W \to X$ is redundant.

(c) is $W \to Y$ redundant:-
$W^+ = (WYXZ)$
without $W \to Y$, $W^+ = (W)$ — $\neq$
So essential.

(d) is $Y \to W$ redundant:-
with $Y^+ = (YWXZ)$
without $Y \to W$, $Y^+ = (YXZW)$ — $=$
So $Y \to W$ is redundant.

(e) is $Y \to X$ redundant:-
with $Y^+ = (YXZW)$
without $Y \to X$, $Y^+ = (YZ)$ — $\neq$
so essential

(f) is $Y \to Z$ redundant:-
with $Y^+ = YXZW$
without $Y \to Z$, $Y^+ = (YXW)$ — $\neq$
So $Y \to Z$ is essential

Final:- set of FD:-

$$\boxed{\begin{array}{l} X \to W \\ W \to Y \\ Y \to XZ \end{array}}$$

6 Finding Minimal Cover:– R(A,B,C,D,E,I)

FDs = {A→C, AB→C, C→DI, CD→I, EC→AB, EI→C}

Step 1:– FDs = {A→C, AB→C, C→D, C→I, $\underset{C\to I}{\underline{CD\to I}}$, $\underset{E\to A}{\underline{EC\to A}}$, $\underset{E\to B}{\underline{EC\to B}}$, $\underset{E\to C}{\underline{EI\to C}}$}

Step 2:– Check for extra attributes on L.H.S:–

(a) AB→C, $(AB)^+$ = ABCDI
with B→C, B^+ = B] X ≠
with A→C, A^+ = ACDI
So, AB→C is required.

(b) CD→I, $(CD)^+$ = CDI] =
with C→I, C^+ = CDI
with D→I, D^+ = DI] ≠
So CD→I replaced with C→I

(c) EC→AB, $(EC)^+$ = ECABDI] =
with E→AB, E^+ = EABCDI
with C→AB, C^+ = CABDI] ≠
So replace EC→AB with E→AB

(d) ~~EC→AB, $(EC)^+$ = ECABDI] =
with E→AB, E^+ = EABCDI
with C→AB, E^+ = CABDI ≠
So replace EC→AB with
E→AB~~

(e) EI→C, $(EI)^+$ = EICDAB] =
with E→C, E^+ = ECDIAB
with I→C, I^+ = ICD] ≠
So replace EI→C with E→C

Step 3 check for redundant attributes on right. H.S:–

FDs = { ~~AB→C~~ X
A→C ✓
C→D ✓
C→I ✓
~~C→I~~ redundant
E→A ✓
E→B ✓
~~E→C~~ }

(a) $(AB)^+$ = ABCDI
without $(AB)^+$ = ABCDI] = So AB→C is redundant

(b) A^+ = ACDI
without A^+ = A] ≠ so A→C is essential

(c) C^+ = CDI
without C^+ = CI] ≠ so C→D = essential

(d) without C→I, C^+ = CD, So essential

(e) E→A, E^+ = EABCDI, so =
without E→A, E^+ = (EBCDI)] ≠ So essential

(f) without E→B, E^+ = EACDI ≠, so essential

(g) without E→C, E^+ = EABCDI, so redundant

Final:–

A→C
C→DI
E→AB