



## Binomial Heap

### Inserting a Node

Insert node  $x$  into binomial heap  $H$ , assuming Node  $x$  has already been allocated &  $key[x]$  has already been filled in.

### Binomial-heap-insert( $H, x$ )

$H' \leftarrow \text{Make-Binomial-Heap}()$

$p[x] \leftarrow \text{NULL}$

$\text{child}[x] \leftarrow \text{NULL}$

$\text{sibling}[x] \leftarrow \text{NULL}$

$\text{degree}[x] \leftarrow 0$

$\text{head}[H'] \leftarrow x$

$H \leftarrow \text{Binomial-heap-union}(H, H')$

// we make a one node binomial heap  $H'$  and unite it with  $H$ .

NodeGetMin( $H$ ) // Node with minimum key.

1. find the root  $x$  with the minimum key in the root list of  $H$ , and remove  $x$  from the root list of  $H$ .
2.  $H' \leftarrow \text{Make-BinomialHeap}()$
3. reverse the order of linked list of  $x$ 's children and set  $\text{head}[H']$  to point to the head of the resulting list.
4.  $H \leftarrow \text{Binomial-Heap-Union}(H, H')$
5. return  $x$



getMin (H) // Find Minimum Key

$y \leftarrow \text{NIL}$

$x \leftarrow \text{head}[H]$

$\text{min} \leftarrow \infty$

while  $x \neq \text{NIL}$

do if  $\text{Key}[x] < \text{min}$

then  $\text{min} \leftarrow \text{Key}[x]$

$y \leftarrow x$

$x \leftarrow \text{sibling}[x]$

return y.

Binomial-heap-Union ( $H_1, H_2$ )

1.  $H \leftarrow \text{MakeBinomialHeap}()$

$\text{head}[H] \leftarrow \text{Merge}(H_1, H_2)$

free the objects  $H_1$  and  $H_2$  but not the lists they point to

if  $\text{head}[H] = \text{NIL}$

then return H

$\text{prev} \leftarrow \text{NIL}$

$x \leftarrow \text{head}[H]$

$\text{next} \leftarrow \text{sibling}[x]$

while  $\text{next} \neq \text{NIL}$

do if  $(\text{degree}[x] \neq \text{degree}[\text{next}])$  or  
 $(\text{sibling}[\text{next}] \neq \text{NIL})$  and  $\text{degree}[\text{sibling}[\text{next}]]$   
 $= \text{degree}[x]$

then  $\text{prev} \leftarrow x$

Case 1 & 2

$x \leftarrow \text{next}$

Case 1 & 2

else if  $\text{Key}[x] \leq \text{Key}[\text{next}]$

~~Binomial link~~

then  $\text{sibling}[x] \leftarrow \text{sibling}[\text{next}]$

Case 3

Binomial link ( $\text{next}, x$ )

Case 3



else if $prev-x = NIL$	Case 4
then $Head[H] \leftarrow next-x$	Case 4
Binomial-Link ( $x, next-x$ )	Case 4
$x \leftarrow next-x$	Case 4
$next-x \leftarrow sibling[x]$	Case 4
return H	

// Case 1

//  $degree[x] \neq degree[next-x]$

// Case 2

//  $degree[x] = degree[next-x] = degree[sibling[next-x]]$

// Case 3 & 4

When  $x$  is first of the two roots of equal degree

~~not equal~~

Algo Merge( $y, z$ )

$P[y] \leftarrow z$

$sibling[y] \leftarrow child[z]$

$child[z] \leftarrow y$

$degree[z] \leftarrow degree[z] + 1$