Knapsack Assignment

Capacity = 10

Item	Weight	Profit
1	3	30
2	4	45
3	2	25
4	3	36

$$v[i,j] = \begin{cases} 0 & \text{if } i=0 \text{ or } j=0 \\ v[i-1,j] & \text{if } w_i > j \end{cases}$$

$$\max\left[v[i-1,j],\ v[i-1,j-w_i]+P_i\right] \quad \text{if } w_i \le j$$

n \ m →	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	30	30	30	30	30	30	30	30
2	0	0	0	30	45	45	45	75	75	75	75
3	0	0	25	25	25	55	70	75	75	100	100
4	0	0	25	36	36	61	70	75	91	106	111

Step 1

When $i=1, w_1=3, P_1=30$

$j=1$ $v[1,1] = v[0,1] = 0$

$j=2$ $v[1,2] = \max\{v[0,2], v[0,3]+30\} = 30$ ~~= 0~~

$j=3$ $v[1,3] = \max\{v[0,3], v[0,0]+30\} = 30$

$j=4$ $v[1,4] = \max\{v[0,4], v[0,1]+30\} = 30$

$j=5$ $v[1,5] = \max\{v[0,5], v[0,2]+30\} = 30$

$j=6$ $v[1,6] = \max\{v[0,6], v[0,3]+30\} = 30$

$j=7$ $v[1,7] = \max\{v[0,7], v[0,4]+30\} = 30$

$j=8$ $v[1,8] = \max\{v[0,8], v[0,5]+30\} = 30$

$j=9$ $v[1,9] = \max\{v[0,9], v[0,6]+30\} = 30$

$j=10$ $v[1,10] = \max\{v[0,10], v[0,7]+30\} = 30$

When $i=2$, $w_2=4$, $P_2=45$

$j=1$ $V[2,1]=V[1,1]=0$

$j=2$ $V[2,2]=V[1,2]=0$

$j=3$ $V[2,3]=V[1,3]=30$

$j=4$ $V[2,4]=\max\{V[1,4], V[1,0]+45\}=45$

$j=5$ $V[2,5]=\max\{V[1,5], V[1,1]+45\}=45$

$j=6$ $V[2,6]=\max\{V[1,6], V[1,2]+45\}=45$

$j=7$ $V[2,7]=\max\{V[1,7], V[1,3]+45\}=45$

$j=8$ $V[2,8]=\max\{V[1,8], V[1,4]+45\}=45$

$j=9$ $V[2,9]=\max\{V[1,9], V[1,5]+45\}=45$

$j=10$ $V[2,10]=\max\{V[1,10], V[1,6]+45\}=45$

Step 3 When $i=3$, $w_2=2$, $P_3=25$

$j=1$ $V[3,1]=V[2,1]=0$

$j=2$ $V[3,2]=\max\{V[2,2], V[2,0]+25\}=25$

$j=3$ $V[3,3]=\max\{V[2,3], V[2,1]+25\}=25$

$j=4$ $V[3,4]=\max\{V[2,4], V[2,2]+25\}=25$

$j=5$ $V[3,5]=\max\{V[2,5], V[2,3]+25\}=55$

$j=6$ $V[3,6]=\max\{V[2,6], V[2,4]+25\}=70$

$j=7$ $V[3,7]=\max\{V[2,7], V[2,5]+25\}=75$

$j=8$ $V[3,8]=\max\{V[2,8], V[2,6]+25\}=75$

$j=9$ $V[3,9]=\max\{V[2,9], V[2,7]+25\}=100$

$j=10$ $V[3,10]=\max\{V[2,10], V[2,8]+25\}=100$

Step 4 When $i=4$, $w_4=3$, $P_4=36$

$j=1$, $V[4,1]=\max\{V[3,1]\}=0$

$V[4,2]=\max\{V[3,2]\}=25$

$V[4,3]=\max\{V[3,3], V[3,0]+36\}=36$

$V[4,4]=\max\{V[3,4], V[3,1]+36\}=36$

$V[4,5]=\max\{V[3,5], V[3,2]+36\}=61$

$V[4,6]=\max\{V[3,6], V[3,3]+36\}=70$

$V[4,7]=\max\{V[3,7], V[3,4]+36\}=75$

$V[4,8]=\max\{V[3,8], V[3,5]+36\}=91$

$V[4,9]=\max\{V[3,9], V[3,6]+36\}=106$

$V[4,10]=\max\{V[3,10], V[3,7]+36\}=111$

Optimal solution is $V[n,m] = V[4,10] = 111$

If ith object has been selected then $V[i,j] \neq V[i-1,j]$

- 4^{th} object is selected as max profit is only in 4^{th} row

∴ So, 111 − Profit = 111 − 36 = 75

So, $V[3,7] = V[2,7] = 75$

So, 3^{rd} obj is not selected

Since, $V[2,7] \neq V[1,7]$

$75 \neq 36$

So. 2^{nd} obj is selected

- So, $75 - R = 75 - 45 = 30$

$V[1,3] \neq V[0,3]$

$30 \neq 0$

So, 1^{st} obj is selected

x_1	x_2	x_3	x_4
1	1	0	1