



**Gajendra Purohit** ✓

**Legend** in CSIR-UGC NET & IIT-JAM

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**Sum of two subspace :** Let  $W_1$  and  $W_2$  are two subspace then  
Sum of two subspace define as  $W_1 + W_2 = \{ x + y \mid x \in W_1, y \in W_2 \}$

**Example :**  $W_1 = \{ (x, 0) \mid x \in \mathbb{R} \}$  and  $W_2 = \{ (0, y) \mid y \in \mathbb{R} \}$

then  $W_1 + W_2 = \{ (x, y) \mid x, y \in \mathbb{R} \} = \mathbb{R}^2$

**Note :**

- (i) Sum of two subspace is also a subspace of vector space
- (ii) If  $V = W_1 + W_2$  then  $V$  is linear sum of  $W_1$  and  $W_2$
- (iii)  $W_1 + W_2$  is smallest subspace containing both  $W_1$  and  $W_2$   
i.e.  $W_1 \subseteq W_1 + W_2$  and  $W_2 \subseteq W_1 + W_2$



**Disjoint subspace :** Let  $W_1$  and  $W_2$  are two subspace s.t.  
 $W_1 \cap W_2 = \{0\}$  then Both subspace are called disjoint subspace

**Direct Product of subspace :**

Let  $V$  be a vector space then  $V$  is called direct product of  $W_1$  and  $W_2$  if

$$(i) V = W_1 + W_2$$

$$(ii) W_1 \cap W_2 = \{0\}$$

It is denoted by  $V = W_1 \oplus W_2$

## Result :

- (i) Intersection of any number of subspaces of a vector space  $V$  is always a subspace of  $V$ .
- (ii) Union of two subspaces is also a subspace iff one is contained in another.

**Note :**  $W_1 + W_2$  and  $W_1 \cup W_2$  are different terms



**Q.1.** Let  $W_1 = \{(a, 2a, 0) \mid a \in \mathbb{R}\}$ ,  $W_2 = \{(a, 0, -a) \mid a \in \mathbb{R}\}$ .  
Then

- (a)  $W_1 + W_2$  is a subspace of  $\mathbb{R}^3$  but  $W_1 \cup W_2$  is not
- (b)  $W_1 + W_2$ ,  $W_1 \cup W_2$  are both subspaces of  $\mathbb{R}^3$ .
- (c) Neither  $W_1 + W_2$  nor  $W_1 \cup W_2$  is a subspace of  $\mathbb{R}^3$ .
- (d)  $W_1 \cup W_2$  is a subspace of  $\mathbb{R}^3$  but  $W_1 + W_2$  is not.

**Q.2.** Let  $V = \{[a_{ij}]_{m \times n}; a_{ij} \in F\}$  be a vector space

$W_1 = \{A = [a_{ij}]_{m \times n} / A^k = 0; k \in N; A \text{ is diagonalizable matrix}\}$

and  $W_2 = \{A = [a_{ij}]_{m \times n} / A \text{ is diagonal matrix}\}$

then which of the following is true

(a)  $W_1$  is subspace of  $V$

(b)  $W_2$  is subspace of  $V$

(c)  $W_1 \cap W_2$  is non - subspace of  $V$

(d)  $W_1 \cup W_2$  is subspace of  $V$



**Q.3.** Let  $H_1 = \{(x, y) \mid y = x\}$  and  $H_2 = \{(x, y) \mid y = -x\}$  be subspaces of a vector space  $\mathbb{R}^2(\mathbb{R})$ .

Then which of the following statement is correct?

- (a)  $H_1 + H_2$  is an improper subspace of  $\mathbb{R}^2$
- (b)  $H_1 + H_2$  is a proper subspace of  $\mathbb{R}^2$
- (c)  $H_1 + H_2$  is not a subspace of  $\mathbb{R}^2$
- (d)  $H_1 + H_2$  is a trivial subgroup of  $\mathbb{R}^2$ .



# Linear Combination, LI & LD set of vectors

**Linear combination of a set of vectors :** Let  $v_1, v_2, \dots, v_n$  are vectors in a vector space  $V$ . A linear combination of vectors  $v_1, v_2, \dots, v_n$  in  $V$  is a vector of the form

$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$ , where  $\alpha_i \in F$  for all  $i = 1$  to  $n$ .

**Linear Span or Spanning set or Generating Set :**

Let  $S$  be a non-empty subset of vector space then the set of all possible linear combination of elements of  $S$  is called linear span or spanning set of  $S$  and denoted by  $L(S)$  or  $\langle S \rangle$  or  $\text{span}(S)$



## Result :

(1) Let  $V(F)$  be a vector space and  $S \subseteq V$  then  $L(S)$  is subspace of  $V$  called subspace spanned by  $S$ .

i.e.  $L(S)$  is subspace of  $V$  if  $S$  is subset of  $V$ .

(2) Let  $S$  be a subspace of a vector space  $V$  then  $L(S) = S$

We know that if  $S$  is subspace then  $S$  is closed i.e. all possible linear combination of elements of  $S$  belonging in  $S$ . So , Linear span of  $S$  is also  $S$

(3) Let  $A \subseteq B \subseteq V$  then  $L(A) \subseteq L(B)$

(4)  $L(\phi) = \{0\}$



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**Linear Dependent :** A subset  $S$  of a vector space  $V$  is said to be dependent if  $\exists x_1, x_2, \dots, x_n$  in  $S$  and scalar  $\alpha_1, \alpha_2, \dots, \alpha_n$  in  $F$ , not all zero s.t.  $\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = 0$

**Linear Independent :** Any set containing the vectors  $x_1, x_2, \dots, x_n$  defined over a field  $F$  is said to be LI if  $\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = 0 \Rightarrow \alpha_1 = \alpha_2 = \dots = \alpha_n = 0$



## Result :

- (1) Any set containing 0 vector is LD
- (2) The empty set is LI
- (3) Two vectors are LD, iff they are scalar multiple to each other.
- (4) Every subset of LI set is LI
- (5) Every superset of LD set is LD

**Note :** (i) If a matrix of order  $n$  and its rank is  $n$  then all columns/rows are LI

(ii) If  $|A| \neq 0$  then all columns/rows are LI



**Q.4.** Which one of the following is correct?

(a)  $S = \{(1, 0, 0), (0, -1, 0), (1, 1, 0)\}$  is a linearly independent set of vectors in  $\mathbb{R}^3$ .

(b)  $S = \{(1, 0, 0), (0, 2, 0), (1, 1, 0)\}$  is a linearly independent set of vectors in  $\mathbb{R}^3$ .

(c) A subset of a linearly dependent set of vectors is linearly independent.

(d) A subset of a linearly independent set of vectors is linearly independent.

**Q.5.** If  $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \alpha M_1 + \beta M_2 + \gamma M_3$  , where

$M_1 = I_{2 \times 2}, M_2 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$  &  $M_3 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  then

(a)  $\alpha = \beta = 1, \gamma = 2$

(b)  $\alpha = \beta = -1, \gamma = 2$

(c)  $\alpha = 1, \beta = -1, \gamma = 2$

(d)  $\alpha = -1, \beta = 1, \gamma = 2$



**Q.6.** If the set  $\left\{ \begin{bmatrix} x & -x \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ x & x \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \right\}$  is linearly dependent in the vector space of all  $2 \times 2$  matrices with real entries, then  $x$  is equal to

(a) 1

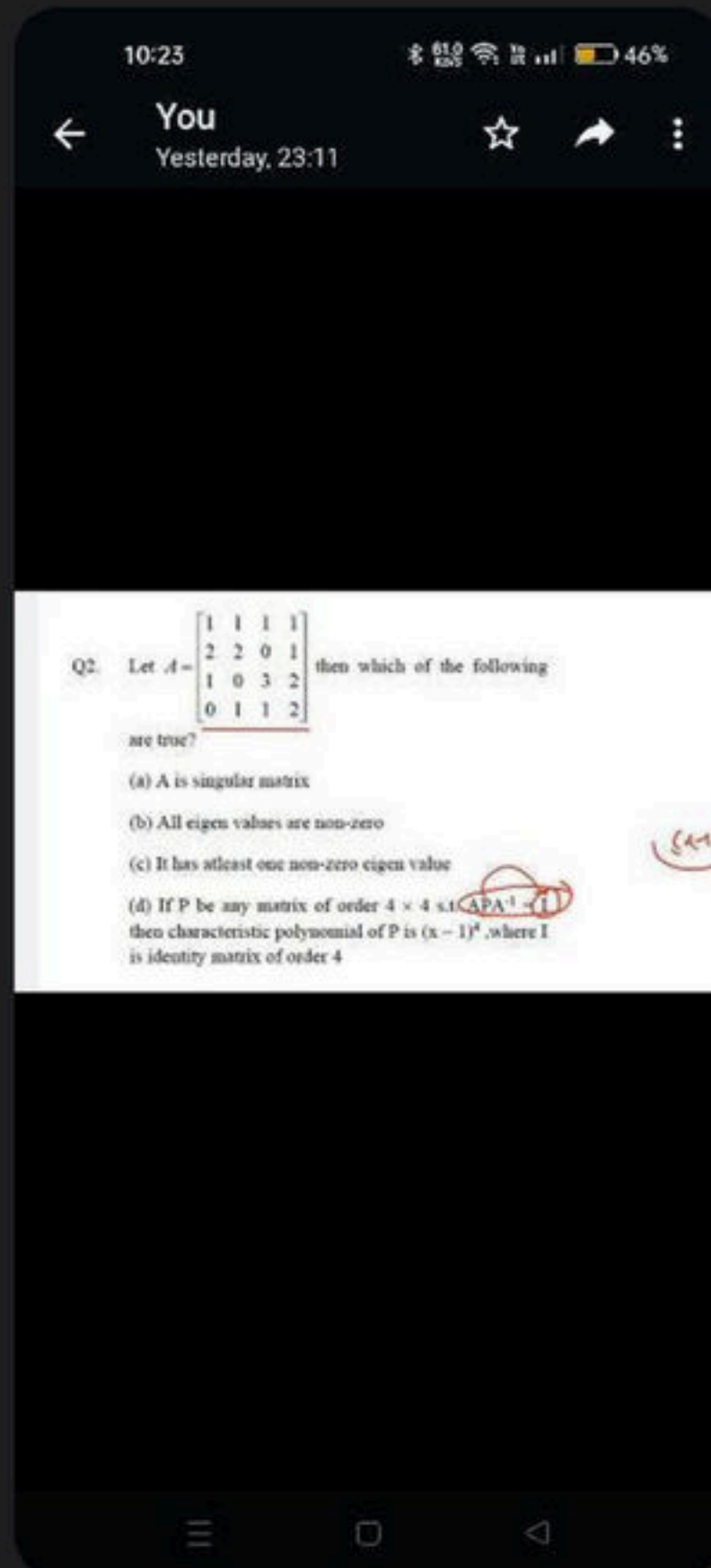
(b) -1

(c) 2

(d) -2

▲ 2 • Asked by Rishabh

Ye wala sir





### Result :

Let  $V_1, V_2, \dots, V_n$  are either column vector or row vector of a matrix  $A$  then  $V_1, V_2, \dots, V_n$  are LI or LD if  $|A| \neq 0$  or  $|A| = 0$ .

**Q.7.** In vector space  $\mathbb{R}^3(\mathbb{R})$  over the field of real numbers  $\mathbb{R}$  then the set  $S = \{(-1, 2, 1), (3, 0, -1), (-5, 4, 3)\}$  is

(a) LI

(b) LD

(c) Data is insufficient

(d) None of these



**Q.8.** If  $\alpha, \beta, \gamma$  are LI vector of  $V(F)$  then which of the following is LI.

(a)  $2\alpha, \beta, 2$

(b)  $\alpha + \beta, \alpha - \beta, \alpha - 2\beta + \gamma$

(c)  $\alpha - \beta, \beta + \gamma, \gamma + \alpha$

(d)  $\alpha + \beta, 2\alpha + \gamma, \alpha - \beta + \gamma$

**Q.9.** Let  $p_n(x) = x^n$  for  $x \in \mathbb{R}$  and let  $\wp = \text{span}\{p_0, p_1, p_2, \dots\}$ .  
Then

(a)  $\wp$  is the vector space of all real valued continuous function on  $\mathbb{R}$ .

(b)  $\wp$  is a subspace of all real valued continuous function on  $\mathbb{R}$ .

(c)  $\{p_0, p_1, p_3, \dots\}$  is a linearly independent set in the vector space of all continuous functions on  $\mathbb{R}$ .

(d) Trigonometric functions belong to  $\wp$





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## Educator highlights

- Works at Pacific Science College
- Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
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- Lives in Udaipur, Rajasthan, India
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