Function of Several Variables - Part III

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 11 • Nov 22, 2022

Gajendra Purohit

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Important result :

(1) Every continuous function in [a,b] is integrable in [a,b]

(2) A monotonic function in [a,b] is integrable in [a,b].

(3) If f is bounded and discontinuous at only countable point, then it is integrable.

(4) If f(x) is bounded in [a, b] and limit point of set of discontinuous point is finite then it will be integrable.

Q.1. Let $f(x) = \begin{cases} \sin 1/x & x \neq 0 \\ 7; & x = 0 \end{cases}$

Then which of the following is/are true?

(a) f(x) is continuous on [0, 1]

(b) f(x) is differentiable on [0, 1]

(c) f(x) is Riemann integrable.

(d) None of these

Q.2. A function defined on [0, 4] by f(x) = [x] where [x] denotes the greatest integer function t then

(a) f is not integrable

(b) f is integrable

(c) $\int_0^4 f(x)dx = 5$

(d) $\int_0^4 f(x)dx = 6$

$\int_0^1 [x]dx + \int_1^2 [x]dx + \int_2^3 [x]dx + \int_3^4 [x]dx$

$\int_0^2 dx + 2\int_2^3 dx + 3\int_3^4 dx$

$(2-1) + 2(3-2) + 3(4-3)$

$1 + 2 + 3 = 6$

Q.3. Define $f : [0,1] \to [0,1]$ by

$$f(x) = \begin{cases} 1 & \text{if } x = 0 \\ \frac{1}{n} & \text{if } x = \frac{m}{n} \text{ for some } m,n \in N \text{ with } m \le n \text{ and } \gcd(m,n)=1 \\ 0 & \text{if } x \in [0,1] \text{ is irrational} \end{cases}$$

and define $g : [0,1] \to [0,1]$ by

$$g(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x \in (0,1] \end{cases}$$

Then which of the following is true **JAM 2022**

(a) f is Riemann integrable in [0,1]

(b) g is Riemann integrable in [0,1]

(c) fog is Riemann integrable in [0,1]

(d) gof is Riemann integrable in [0,1]

$gof = g[f(x)] = \begin{cases} 0 & x \in \mathbb{Q}^c \\ 1 & x \in \mathbb{Q} \cup \{0\} \end{cases}$

$g(1) = 1$

Q.4. The function defined by

$$f(x) = \begin{cases} 0; & x = 0 \\ \frac{1}{2^n}; & \frac{1}{2^{n+1}} < x < \frac{1}{2^n}, \ n = 0, 1, \ldots \end{cases} \text{ then}$$

(a) f is integrable (b) f is not integrable

(c) $\int_0^1 f(x)dx = \frac{2}{3}$ (d) $\int_0^1 f(x)dx = \frac{3}{2}$

$f(x) = \begin{cases} 0 & n=0 \\ 1 & \frac{1}{2} < x < 1 \\ \frac{1}{2} & \frac{1}{2^2} < x < \frac{1}{2} \\ \frac{1}{4} & \frac{1}{2^3} < x < \frac{1}{4} \\ \vdots & \vdots \\ \frac{1}{2^n} & \frac{1}{2^{n+1}} < x < \frac{1}{2^n} \end{cases}$

$$\int_0^1 f(x)dx = \int_{1/2}^{1} 1\,dx + \int_{1/2^2}^{1/2} \frac{1}{2}dx + \int_{1/2^3}^{1/2^2} \frac{1}{4}dx + \cdots + \int_{1/2^{n+1}}^{1/2^n} \frac{1}{2^n}dx$$

$$= \left(1 - \frac{1}{2}\right) + \frac{1}{2}\left[\frac{1}{2} - \frac{1}{2^2}\right] + \frac{1}{2^2}\left(\frac{1}{2^2} - \frac{1}{2^3}\right) + \cdots$$

$$= \left(\frac{1}{2}\right)\left[1 + \frac{1}{2^2} + \frac{1}{2^4} + \cdots \frac{1}{2^{2n}}\right] = \frac{1}{2}\left(\frac{1}{1 - \frac{1}{4}}\right) = \frac{1}{2}\left(\frac{4}{3}\right)$$

$$= \frac{2}{3}$$

Q.5. Define $f : [0, 1] \to [0, 1]$ by $f(x) = \frac{2^k-1}{2^k}$ for $x \in \left[\frac{2^{k-1}-1}{2^{k-1}}, \frac{2^k-1}{2^k}\right]$, $k \geq 1$. Then f is a Riemann integrable function such that **CSIR NET DEC 2011**

(a) $\int_0^1 f(x)dx = \frac{2}{3}$ (b) $\frac{1}{2} < \int_0^1 f(x)dx < \frac{2}{3}$

(c) $\int_0^1 f(x)dx = 1$ (d) $\frac{2}{3} < \int_0^1 f(x)dx < 1$

$f(x) = \begin{cases} \frac{1}{2} & [0, \frac{1}{2}] \\ \frac{3}{4} & [\frac{1}{2}, \frac{3}{4}] \\ \frac{7}{8} & (\frac{3}{4}, \frac{7}{8}] \end{cases}$

$\int_0^1 f(x)dx = \int_0^{1/2} \frac{1}{2}dx + \int_{1/2}^{3/4} \frac{3}{4}dx + \int_{3/4}^{7/8} \frac{7}{8}dx + \cdots$

$= \frac{1}{2}\left(\frac{1}{2}-0\right) + \frac{3}{4}\left(\frac{3}{4}-\frac{1}{2}\right) + \frac{7}{8}\left(\frac{7}{8}-\frac{3}{4}\right)$

$= \frac{1}{2^2} + \frac{3}{2^4} + \frac{7}{2^6} + \cdots$

$= \sum_{n=1}^{\infty} \frac{2^n-1}{2^{2n}} = \sum_{n=1}^{\infty} \frac{1}{2^n} - \sum_{n=1}^{\infty} \frac{1}{2^{2n}}$

$\left(\frac{1}{2} + \frac{1}{2^2} + \cdots\right) - \left(\frac{1}{2^2} + \frac{1}{2^4} + \cdots\right)$

$\frac{1}{2}\left(\frac{1}{1-\frac{1}{2}}\right) - \frac{1}{4}\left(\frac{1}{1-\frac{1}{4}}\right)$

$1 - \frac{1}{3} = \frac{2}{3}$

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Q.6. Let f(x) be real valued function defined by

$$f(x)=\begin{cases}0 & x=0\\ \frac{1}{n} & \frac{1}{n+1}<x\le\frac{1}{n}\end{cases} \text{ for } n\in N$$

which of the following is true **D.U. 2020**

(a) f is monotonically decreasing function on [0,1] and $f\notin R[0,1]$

(b) f is monotonically decreasing function on [0,1] and $f\in R[0,1]$

(a) f is monotonically increasing function on [0,1] and $f\in R[0,1]$

(a) f is discontinuous at infinitely many points in on [0,1] and $f\in R[0,1]$

Q.7. Let $= \int_0^{\infty} \frac{1}{1+t^2} dt$. which of the following are true? **CSIR NET JUNE 2018**

(a) $\frac{d\alpha}{dt} = \frac{1}{1+t^2}$

(b) α is a rational number

(c) $\log(\alpha) = 1$

(d) $\sin(\alpha) = 1$

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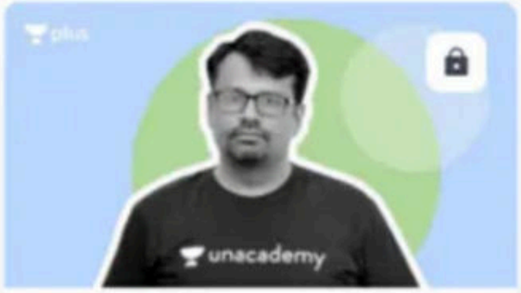

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Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
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