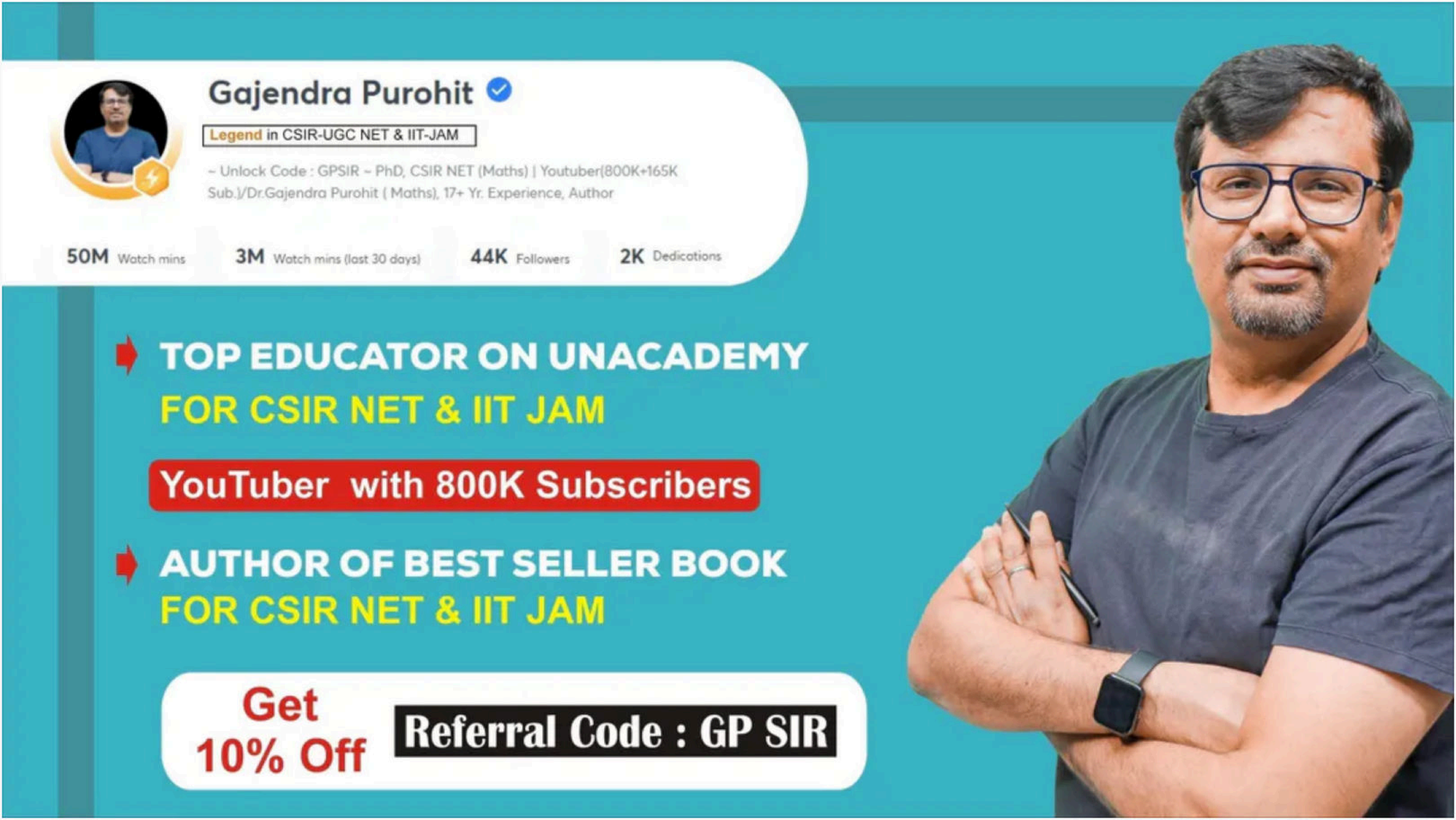
Linear Transformations - Part I

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 4 • Sept 14, 2022


unacademy
Ace your preparation
with Top Educators
Get 20% off* on IIT JAM subscriptions
Limited period offer
Unlimited access to Live and Recorded Classes
PYQs, Live Test and Quizzes
In class doubt solving by Top Educators
Structured Batches & courses
Subscribe Now
Use code GPSIR
For more details, contact: 8585858585
*T&C apply, as available on the platform.
20


unacademy
Ace your preparation
with Top Educators
Get 20% off* on CSIR-UGC NET subscriptions
Limited period offer
Unlimited access to Live and Recorded Classes
Learn from Top Educators
Comprehensive Notes and PDFs
Prepare with Unacademy Lite Subscription
Subscribe Now
Use code
For more details, contact: 8585858585
*T&C apply, as available on the platform.
20


Gajendra Purohit
Legend in CSIR-UGC NET & IIT-JAM
~ Unlock Code : GPSIR ~ PhD, CSIR NET (Maths) | Youtuber(800K+165K Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author
50M Watch mins
3M Watch mins (last 30 days)
44K Followers
2K Dedications
TOP EDUCATOR ON UNACADEMY
FOR CSIR NET & IIT JAM
YouTuber with 800K Subscribers
AUTHOR OF BEST SELLER BOOK
FOR CSIR NET & IIT JAM
Get 10% Off
Referral Code : GP SIR


unacademy
RANK BOOSTER COURSE UNIT 2
CSIR NET 2022
23rd AUGUST
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy


unacademy
DETAILED COURSE 2.0
LINEAR ALGEBRA FOR IIT JAM 2023
8th SEPTEMBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780	You get 6 months extra for free — Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974	You get 6 months extra for free — Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475	
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252	
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807	
1 month		₹ 2,723 / mo	Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code? Proceed to pay

Get 10% Off

After Using My Referral Code

GPSIR

Awesome! You get 10% off

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	₹ ~~21,780~~ ₹ 19,602	You get 6 months extra for free — Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo	₹ ~~14,974~~ ₹ 13,477	You get 6 months extra for free — Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo	₹ ~~13,475~~ ₹ 12,128	
6 months	Save 25%	₹ 1,838 / mo	₹ ~~12,252~~ ₹ 11,027	
3 months	Save 17%	₹ 2,042 / mo	₹ ~~6,807~~ ₹ 6,126	

Proceed to pay

Idempotent matrix : A square matrix A is said to be idempotent if $A^2 = A$.

Note :

(1) $A^2 - A = 0 \Rightarrow A(A - I) = 0$

$\Rightarrow A = 0$ (Null) or $A = I$

Identity and null matrix are trivial example.

(2) If A is idempotent and $|A| = \alpha$

Then $|A|^2 = |A| = \alpha^2 = \alpha$

$\Rightarrow \alpha = 0$ or $\alpha = 1$

Determinant of A is either 0 or 1.

(3) Identity is the only idempotent matrix where determinant is 1 and all other idempotent matrix have determinant zero.

Property :

(1) A is idempotent matrix then $I - A$ is also idempotent.

$(I - A)^2 = I + A^2 - 2A = I + A - 2A$

$= I - A$

(2) If A & B are idempotent matrix then AB is idempotent if

$AB = BA$ and $A + B$ is

idempotent if $AB = -BA$

Q.1. A & B are square matrix such that $AB = A$ & $BA = B$, then

(a) $A^2 = A, B^2 = B$ **(b)** $A^2 = A, B^2 \neq B$

(c) $A^2 \neq A, B^2 = B$ (d) $A^2 \neq A, B^2 \neq B$

(3) If A is idempotent then kA is also idempotent iff $k = 0$ or $k = 1$

$(kA)^2 = k^2A^2 = k^2A = kA$

$\Rightarrow k^2 = k$

$\Rightarrow k = 0$ or $k = 1$

(4) If A is idempotent then A^k is idempotent for positive integer k.

Involutory matrix : A square matrix A is involutory if $A^2 = I$.

Note :

(1) If A is involutary and $|A| = \alpha$ then $|A|^2 = |I| = 1$

$\Rightarrow \alpha^2 = 1$

$\Rightarrow \alpha = \pm 1$

Determinant of an involutory matrix is ± 1.

(2) If A is involutory matrix then kA is involutory iff $k = \pm 1$

(3) If A is involutory matrix then A^k is involutory.

$(A^k)^2 = (A^2)^k = I$

For every positive integers.

(4) If A & B are two involutory matrix then AB is involutory if $AB = BA$ and $A + B$ is involutory if $AB + BA = -I$

Nilpotent matrix : A square matrix A is said to be nilpotent if $\exists$ $m \in N$ such that $A^m = 0$

Index of nilpotent matrix : The smallest positive integer K s.t. $A^k = 0$ is known as index of nilpotent matrix where $A^{k-1} \neq 0$ and $A^{k+1} = A^{k+2} = \ldots\ldots = 0$

Property :

(1) If A & B are nilpotent then AB is nilpotent if $AB = BA$.

(2) If A is nilpotent matrix of index P then A^k is also nilpotent of index [P/k], where [.] is ceilling function.

Note :

If A & B are nilpotent matrix then $A + B$ may or may not be nilpotent.

Q.2. Let A & B are two nilpotent matrix of index 15 & 13 then which of the following are true?

(a) Index of A^2 is 9.

(b) Index of A^4 is 4

(c) Index of B^3 is 6

(d) Index of B^5 is 3

Note :

(1) $AA^T = I$

$|AA^T| = 1 \Rightarrow |A|\,|A^T| = 1$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

Determinant of an othrogonal matrix is ± 1 and $A^T = A^{-1}$.

(2) Sum of square of elements of each row or column are 1 and sum of the product of element of any row or column with corresponding elements of any other (column) is always zero.

COMPLETE COURSE ON
MATHEMATICS
FOR IIT-JAM 2022

TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINAER ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo, Total ₹ 21,780
	You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo, Total ₹ 14,974
	You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo, Total ₹ 13,475
6 months	Save 25%	₹ 2,042 / mo, Total ₹ 12,252
3 months	Save 17%	₹ 2,269 / mo, Total ₹ 6,807
1 month		₹ 2,723 / mo, Total ₹ 2,723

To be paid as a one-time payment

Have a referral code? | Proceed to pay

Get 10% Off

After Using My Referral Code

GPSIR

Awesome! You get 10% off

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo, ~~₹ 21,780~~ ₹ 19,602
	You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo, ~~₹ 14,974~~ ₹ 13,477
	You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo, ~~₹ 13,475~~ ₹ 12,128
6 months	Save 25%	₹ 1,838 / mo, ~~₹ 12,252~~ ₹ 11,027
3 months	Save 17%	₹ 2,042 / mo, ~~₹ 6,807~~ ₹ 6,126

Proceed to pay

FOUNDATION COURSE OF

MATHEMATICS

FOR CSIR-NET

Q.3. Number of orthogonal matrix of order n whose entries are 0 & 1 only

(a) n (b) n!

(c) n – 1 (d) None of these

Property :

(1) If A is orthogonal then kA is orthogonal if $k = \pm 1$

Example : If A is orthogonal then 3A is not orthogonal.

(2) If A & B are orthogonal then A + B cannot orthogonal but AB is always orthogonal.

(3) If A is orthogonal then A^n is orthogonal.

Unitary matrix : A matrix A is said to be unitory if

$AA^{\theta} = A^{\theta}A = I$

$$A = \frac{1}{2}\begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}$$

Now $\quad A^{\theta} = (\overline{A})^{T} = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$

$$AA^{\theta} = \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

and $A^{\theta}A = I$

So, A is unitory matrix.

Q.4. If A is orthogonal matrix then which of the following are true?

(a) 2A is orthogonal

(b) A^2 is orthogonal

(c) $-A$ is orthogonal

(d) None of these

Q.5. The number of orthogonal matrix of order 5 whose entries are 0 & 1 only

(a) 5^2 (b) $5!$

(c) 120 (d) 0

Q.6. The matrix $M = \begin{bmatrix} \cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha \end{bmatrix}$ is a unitary matrix when α is

(a) $(2n+1)\frac{\pi}{2}, n \in Z$ (b) $(3n+1)\frac{\pi}{3}, n \in Z$

(c) $(4n+1)\frac{\pi}{4}, n \in Z$ (d) $(5n+1)\frac{\pi}{5}, n \in Z$

Q.7 Suppose A is idempotent matrix of order n , then which of the following is true?

(a) $Tr(A) > n$ (b) $|A| > n$

(c) $Tr(A) \in N$ (d) $Tr(A) \in Z$

Q.8. Suppose A is involutory matrix of order n , then which of the following is true?

(a) $I - A$ is involutory (b) $3A$ is involutory

(c) $Tr(A)$ may be $n/2$ (d) $Tr(A) \in Z$

Q.9. If A is nilpotent matrix of index 2022, then matrix A^{2011} is

(a) nilpotent matrix of index 1

(b) nilpotent matrix of index 2

(c) nilpotent matrix of index 2022

(d) nilpotent matrix of index 2011

Q.10. If A and B are orthogonal matrix then which of the following is true?

(a) $A + B$ is orthogonal (b) AB is orthogonal

(c) $2A$ is orthogonal (d) B^2 is orthogonal


unacademy
RANK BOOSTER COURSE UNIT 2
CSIR NET 2022
23rd AUGUST
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy


unacademy
DETAILED COURSE 2.0
LINEAR ALGEBRA FOR IIT JAM 2023
8th SEPTEMBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

unacademy CSIR-UGC NET Search Login

Educator Profile

Gajendra Purohit

#5 Educator in CSIR-UGC NET

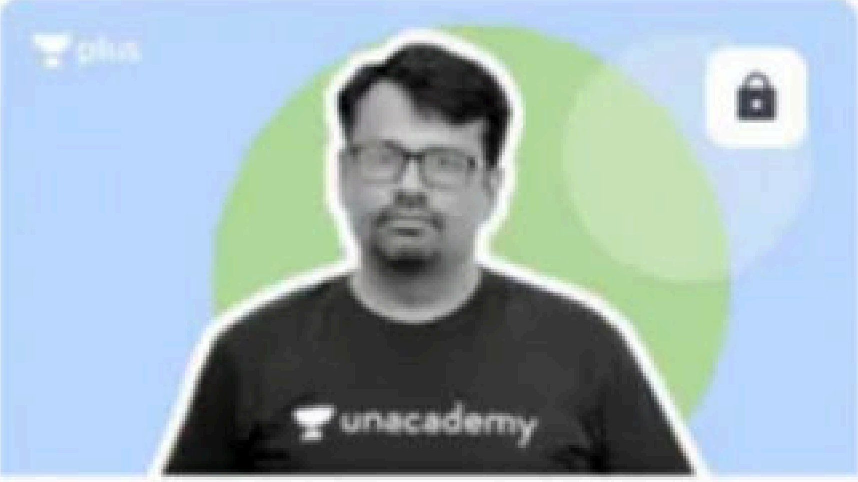
Dr.Gajendra Purohit PhD, CSIR NET (Maths) | Youtuber(330K+30k Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author of Bestseller

Follow

11M Watch mins | **1M** Watch mins (last 30 days) | **22k** Followers | **1k** Dedications

CSIR-UGC NET

SEE ALL



HINDI MATHEMATICAL SCIENCES

Course on Linear Algebra, Partial Diff. Equation & Calculus

Starts on Mar 1, 2021 • 24 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Course on Complex Analysis & Integral Equation

Starts on Jan 14, 2021 • 16 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Foundation Course on Mathematics for CSIR 2021

Starts on Dec 7, 2020 • 20 lessons

Gajendra Purohit

Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780	You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974	You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475		
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252		
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807		
1 month		₹ 2,723 / mo	Total ₹ 2,723		

To be paid as a one-time payment

Have a referral code? | Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	₹ ~~21,700~~ ₹ 19,602	You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo	₹ ~~14,974~~ ₹ 13,477	You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo	₹ ~~13,475~~ ₹ 12,128		
6 months	Save 25%	₹ 1,838 / mo	₹ ~~12,252~~ ₹ 11,027		
3 months	Save 17%	₹ 2,042 / mo	₹ ~~6,807~~ ₹ 6,126		

GPSIR

Awesome! You get 10% off

Proceed to pay