Curve tracing of polar form :

(1) Symmetry :

(i) If $f(r, \theta) = f(r, -\theta)$

Then this curve is symmetric about initially line (i.e. $\theta = 0$ line)

(ii) If $f(r, \theta) = f(r, \pi - \theta)$

Then this curve is symmetric about $\theta = \frac{\pi}{2}$ line.

(iii) **Pole :** Put $r = 0$, then find value of θ.

Hence (r, θ) is a pole.

(iv) **Tangent at pole :** Put $r = 0$, then value of θ is tangent at pole.

(v) **Table :**

r									
θ	0	30	45	60	90	120	135	150	180

(vi) **Asymtotes :**

For any value of θ if r become ∞, then a curve has asymtotes.

Transformation of Variables :

Sometime, it is convenient to solve the double integral by transforming the variables.

(A) Transformation in polar form :

1. Let $\iint f(x,y)dxdy$ is a integration in cartisian form, then put x = r cosθ, y = r sin θ in givenintegration.

2. $$dxdy = \frac{\partial(x,y)}{\partial(r,\theta)} drd\theta$$

$$dxdy = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} d\theta dr$$

dxdy = r dθdr

Putting this value, then we get $\iint f(r,\theta) r d\theta dr$.

Q.1. The value of $\iint_G \frac{\log(x^2+y^2)}{x^2+y^2}\,dxdy$ where $G = \{(x, y) \in R^2;\ 1 \le x^2 + y^2 \le e^2\}$ is **IIT JAM 2010**

(a) π (b) 2π

(c) 3π (d) 4π

Q.2. The value of $\iint \sqrt{x^2 + y^2}\, dxdy$ over the region lying in xy-plane and bounded by $x^2 + y^2 = 4$, $x^2 + y^2 = 9$

(a) $\frac{37\pi}{3}$ (b) $\frac{38}{3}\pi$

(c) $\frac{19}{2}\pi$ (d) $\frac{19}{3}\pi$

Q.3. Let D be the region in the first quadrant lying between $x^2 + y^2 = 1$ & $x^2 + y^2 = 4$, value of integral $\iint_D \sin(x^2 + y^2)\,dx\,dy$. **IIT JAM – 2007**

(a) $\frac{\pi}{4}(\cos 1 - \cos 2)$

(b) $\frac{\pi}{4}(\cos 1 - \cos 4)$

(c) $\frac{\pi}{2}(\cos 1 - \cos 2)$

(d) $\frac{\pi}{2}(\cos 1 - \cos 4)$

Q.4 If G is the region in R^2 given by

$$G=\left\{(x,y)\in R^2 : x^2+y^2<1, \frac{x}{\sqrt{3}}<y<\sqrt{3}x, x>0, y>0\right\}$$

then the value of $\frac{200}{\pi}\iint_G x^2\,dx\,dy$ is **IIT JAM 2022**

(a) 2.5 (b) 4.16

(c) 3 (d) 5.5

Q.8. The integral $\iint_R e^{x^2+y^2}\,dydx$, where R is the semicircle region bounded by the x – axis and the curve $y=\sqrt{1-x^2}$ equals **SAU 2017**

(a) $\frac{\pi}{2}(e+1)$

(b) $\frac{\pi}{2}(e-1)$

(c) $\frac{\pi}{2}(e^2)$

(d) $\frac{\pi}{2}e$