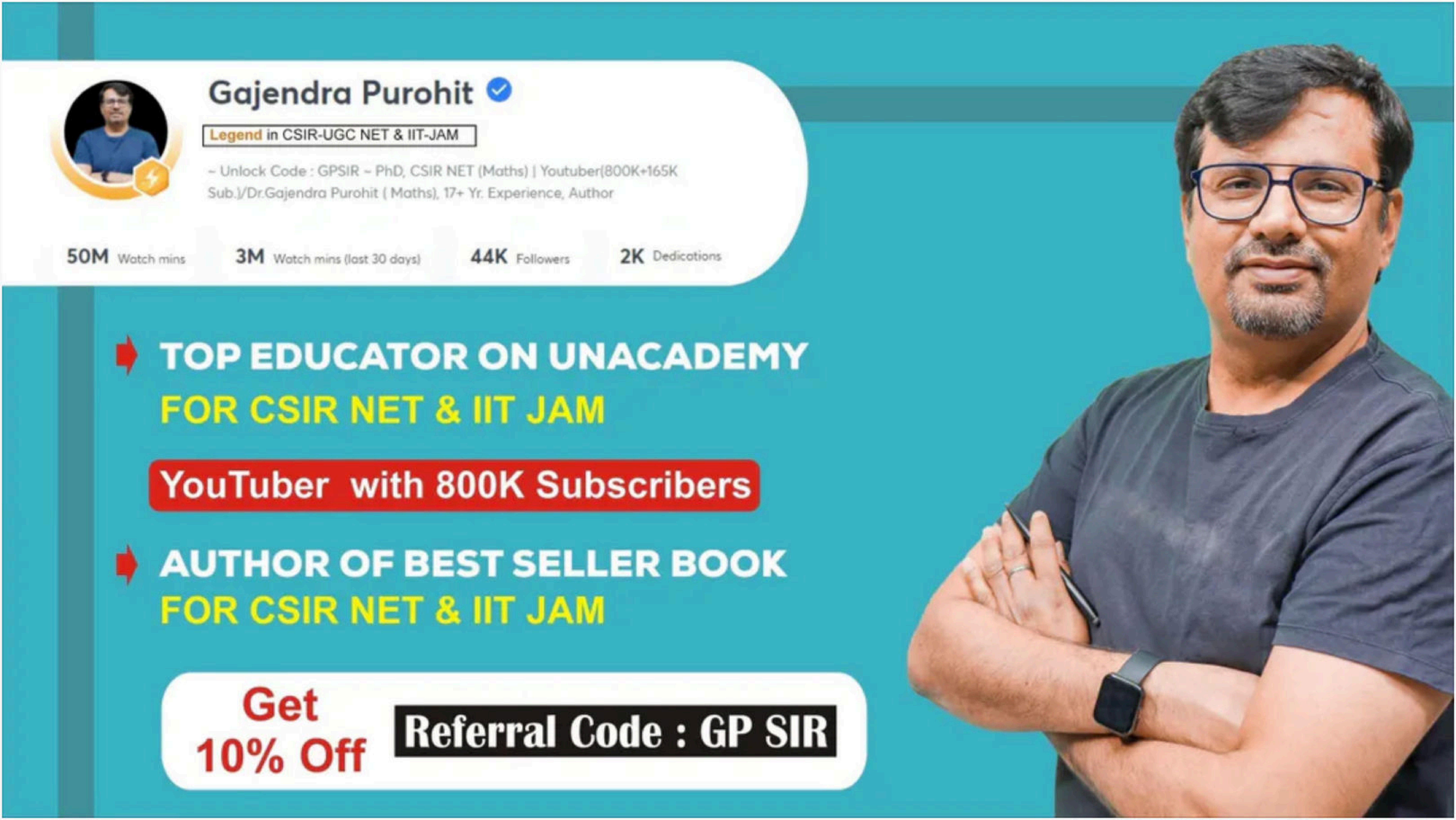
Doubt Clearing Session

Detailed Course on Differential Equation for IIT JAM' 23 - II

Gajendra Purohit • Lesson 7 • Aug 16, 2022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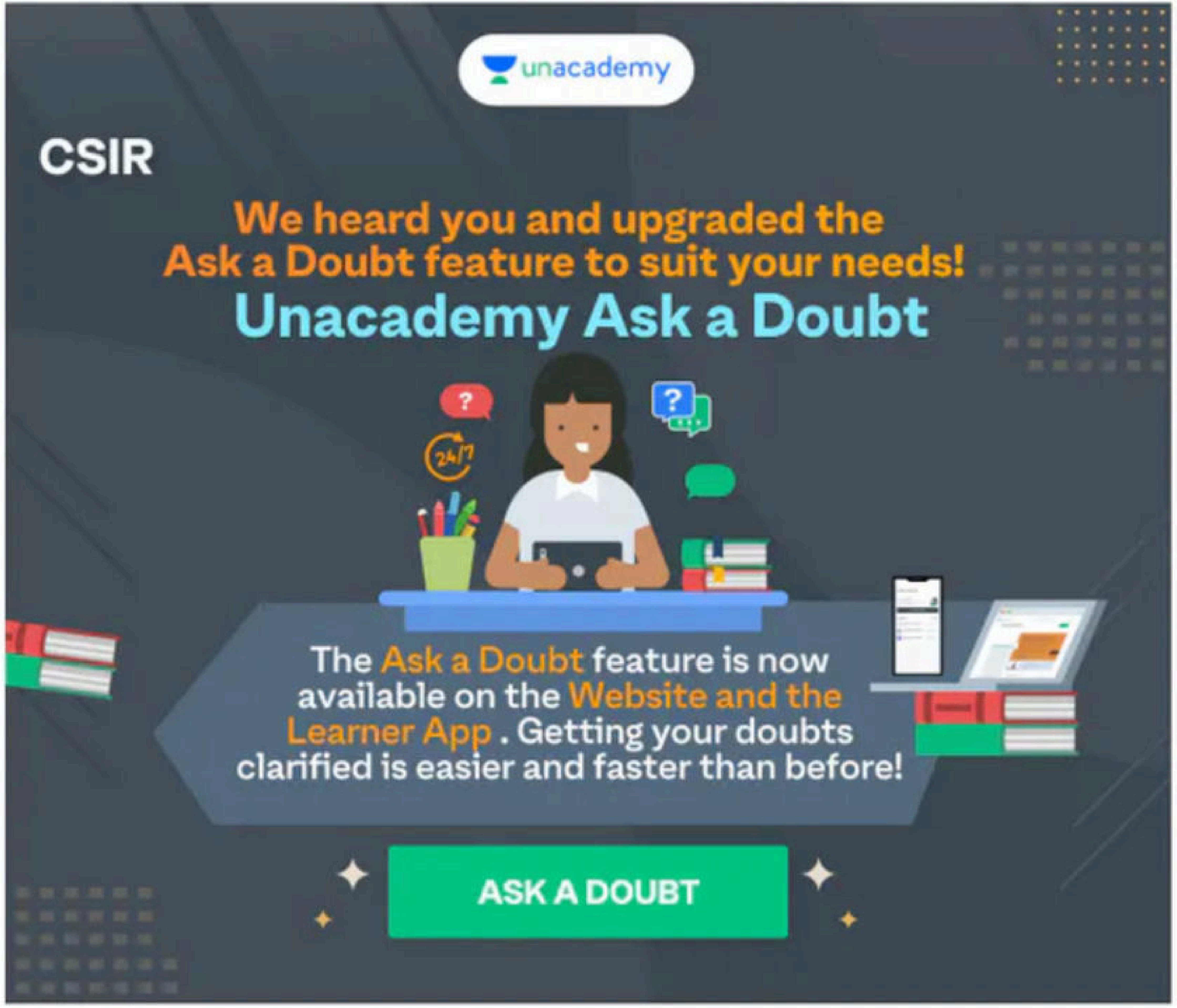
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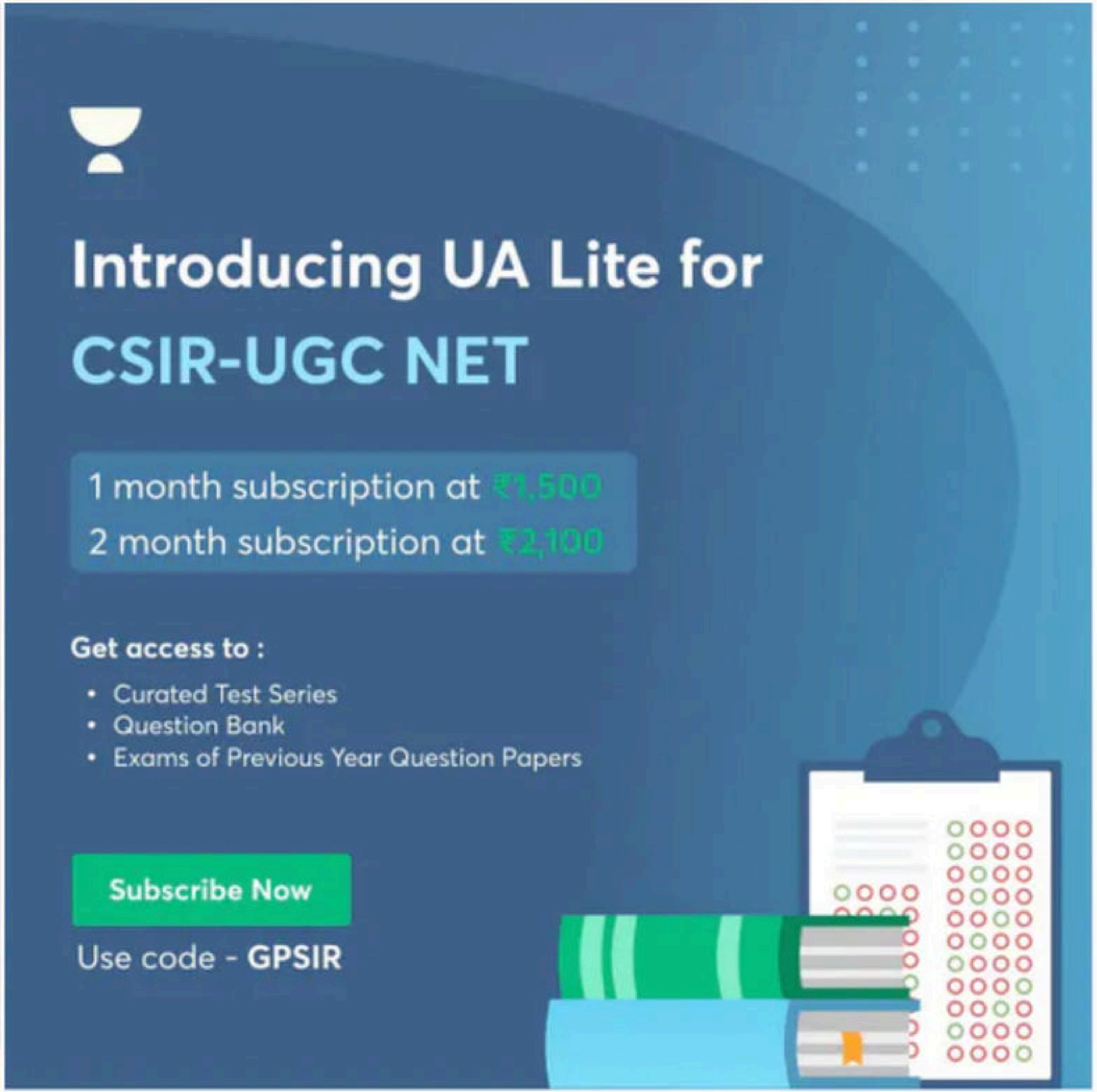
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Integrating factor : If an equation of the form Mdx + Ndy = 0 is not exact, it can always be made exact by multiplying by some function of x and y such a multiplier is called an integrating factor.

Note : The differential equation Mdx + Ndy = 0 posses an infinite number of integrating factor.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\int M\,dx + \int N\,dy = c$$

Not containing x term

$\frac{\partial m}{\partial y} \neq \frac{\partial w}{\partial x}$

If =

$$(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0$$

$$M = x^2y - 2xy^2$$

$$N = -(x^3 - 3x^2y)$$

$$\frac{\partial M}{\partial y} = x^2 - 4xy$$

$$\frac{\partial N}{\partial x} = -(3x^2 - 6xy)$$

$$\mu = \frac{1}{x^2y^2}$$

$$M' = \frac{1}{y} - \frac{2}{x}$$

$$\frac{\partial M'}{\partial y} = -\frac{1}{y^2}$$

$$N' = -\left(\frac{x}{y^2} - \frac{3}{y}\right)$$

$$\frac{\partial N'}{\partial x} = -\frac{1}{y^2}$$

$$\frac{1}{x^2y^2}\left((x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy\right) = 0$$

$$\left(\frac{1}{y} - \frac{2}{x}\right)dx - \left(\cancel{\frac{x}{y^2}} - \frac{3}{y}\right)dy = 0$$

$$\int\left(\frac{1}{y} - \frac{2}{x}\right)dx - \int -\frac{3}{y}\,dy = C$$

$$\frac{x}{y} - 2\log x + 3\log y = C$$

Rule – 1 : If $Mdx + Ndy = 0$ is homogeneous and $(Mx + Ny) \neq 0$

then $\dfrac{1}{Mx+Ny}$ is an integrating factor.

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

$$M = x^2y - 2xy^2 \qquad N = -(x^3 - 3x^2y)$$

$$Mx + Ny = \cancel{x^3y} - 2x^2y^2 - \cancel{x^3y} + 3x^2y^2 = x^2y^2$$

$$\frac{1}{Mx+Ny} = \frac{1}{x^2y^2}$$

$$(x^2y - 2xy^3)\,dx - (x^3 - 3x^2y)\,dy = 0$$

Let $If = x^h y^k$

$$\therefore If = x^{-2}y^{-2} = \frac{1}{x^2y^2}$$

$$x^h y^k \left[(x^2y - 2xy^3)\,dx - (x^3 - 3x^2y)\,dy\right] = 0$$

$$(x^{h+2}y^{k+1} - 2x^{h+1}y^{k+2})\,dx - (x^{h+3}y^k - 3x^{h+2}y^{k+1})\,dy = 0$$

$$\frac{\partial M}{\partial y} = \boxed{(k+1)}\, x^{h+2}y^k - 2(k+2)\, x^{h+1}y^{k+1}$$

$$\frac{\partial N}{\partial x} = \boxed{-(h+3)}\, x^{h+2}y^k + 3(h+2)\, x^{h+1}y^{k+1}$$

$$k+1 = -(h+3)$$

$$h+k = -4$$

$$-2(k+2) = 3(h+2)$$

$$-2k - 4 = 3h + 6$$

$$3h + 2k = -10$$

$$h = k = -2$$

Rule – 2 : If Mdx + Ndy = 0 is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$

$= 0$ then $\dfrac{1}{Mx - Ny}$ is an integrating factor provided $Mx - Ny \neq 0$

$(1+xy)\,y\,dx + (1-xy)\,x\,dy = 0$

$M = y + xy^2 \quad | \quad N = x - x^2y$

$\frac{\partial M}{\partial y} = 1 + 2xy \neq \frac{\partial N}{\partial x} = 1 - 2xy$

$Mx - Ny = \cancel{xy} + x^2y^2 - \cancel{xy} + x^2y^2$

$= 2x^2y^2$

$IF = \frac{1}{x^2y^2}$

$\frac{1}{x^2y^2}\left((y + xy^2)dx + (x - x^2y)dy\right) = 0$

$\left(\frac{1}{x^2y} + \frac{1}{x}\right)dx + \left(\cancel{\frac{x}{x^2y^2}} - \frac{1}{y}\right)dy = 0$

$\frac{\partial M}{\partial y} = -\frac{1}{x^2y^2} = \frac{\partial N}{\partial x}$

$\int\left(\frac{1}{x^2y} + \frac{1}{x}\right)dx + \int -\frac{1}{y}\,dy = C$

$-\frac{1}{xy} + \log x - \log y = C$

$(xy\sin xy + \cos xy)\,y\,dx + (xy\sin xy - \cos xy)\,x\,dy = 0$ $\quad \frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

$Mx - Ny = \cancel{x^2y^2\sin xy} + xy\cos xy - \cancel{x^2y^2\sin xy} + xy\cos xy = 2xy\cos xy$

$\frac{x}{y}\sec xy = c \;\Big|\; \frac{x}{\cos xy} = yc$

$IF = \frac{1}{Mx-Ny} = \frac{1}{xy\cos xy}$

$\frac{1}{xy\cos xy}\left[(xy^2\sin xy + y\cos xy)\,dx + (x^2y\sin xy - x\cos xy)\right] = 0$ $\quad x = yc\cos xy$

$\left(y\tan xy + \frac{1}{x}\right)dx + \left(x\tan xy - \frac{1}{y}\right)dy = 0$

$\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x} \;\Big|\; \int \left(y\tan xy + \frac{1}{x}\right)dx + \int -\frac{1}{y}\,dy = c$

$\frac{\cancel{y}\log\sec xy}{\cancel{y}} + \log x - \log y = \log c$

$\log \frac{x}{y}\sec xy = \log c$

Rule – 3 : If $\dfrac{1}{N}\left(\dfrac{\partial M}{\partial y}-\dfrac{\partial N}{\partial x}\right)$ is a function of x alone say f(x) then

$e^{\int f(x)dx}$ is an integrating facor.

$(x^2+y^2+x)\,dx + xy\,dy = 0$

M $\qquad$ N

$\text{I.F} = x^h y^k = x^1 y^0 = x$

$x^h y^k \left[(x^2+y^2+x)\,dx + xy\,dy \right] = 0$

$M = x^{h+2}y^k + x^h y^{k+2} + x^{h+1}y^k$ $\qquad$ $N = x^{h+1}y^{k+1}$

$\dfrac{\partial M}{\partial y} = k\,x^{h+2}y^{k-1} + (k+2)\,x^h y^{k+1} + (k)\,x^{h+1}y^{k-1}$ $\qquad$ $\dfrac{\partial N}{\partial x} = (h+1)\,x^h y^{k+1}$

$k+2 = h+1$ $\qquad$ $k = 0$

$h+1 = 2$

$h = 1$

Rule – 4 : If $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$ is function of y-alone say f(y) then $e^{\int f(y)dy}$ is an integrating factor.

$(xy^2 - x^2)dx + (3x^2y^2 + x^2y - 2x^3 + y^2)dy =$

$\frac{\partial M}{\partial y} = 2xy \qquad \frac{\partial N}{\partial x} = 6xy^2 + 2xy - 6x^2$

$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{6xy^2 - 6x^2}{xy^2 - x^2} = 6$

$e^{6y}((xy^2 - x^2)dx + (\qquad)dy) = 0 \qquad IF = e^{\int 6\,dy} = e^{6y}$

$\frac{\partial M'}{\partial y} = \frac{\partial N'}{\partial x}$

$\int e^{6y}(xy^2 - x^2)dx + \int e^{6y}y^2\,dy = C$

Q.1 An integrating factor of the differential equation $\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)dy=0$ is

(a) x^2 (b) $3\log_e x$

(c) x^3 (d) $2\log_e x$

$\frac{\partial M}{\partial y} = 1+y^2 \qquad \frac{\partial N}{\partial x} = \frac{1}{4}(1+y^2)$

$\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right) = \frac{\frac{3}{4}\cancel{(1+y^2)}}{\frac{1}{4}x\cancel{(1+y^2)}} = \frac{3}{x}$

$IF = e^{\int \frac{3}{x}dx} = e^{3\log x}$

$= e^{\log x^3} = x^3$

Q.2 If $x^h y^k$ is an integrating factor of the differential equation $y(1 + xy)dx + x(1 - xy)dy = 0$, then the ordered pair (h, k) is equal to

$$\frac{1}{Mx - Ny} = \frac{1}{2x^2y^2} = x^{-2}y^{-2}$$

(a) (-2, -2)   (b) (-2, -1)

(c) (-1, -2)   (d) (-1, -1)

$$Mx - Ny = \cancel{xy} + x^2y^2 - \cancel{xy} + x^2y^2 = 2x^2y^2$$

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Q.3. The solution of differential equation

$$(1+y^2)\,dx + \left(x - e^{\tan^{-1} y}\right) dy = 0 \text{ is}$$

(a) $(x-2) = ke^{-\tan^{-1} y} + k$

(b) $xe^{\tan^{-1} y} - 2e^{2\tan^{-1} y} = k$

(c) $xe^{-\tan x} = \tan^{-1} y + k$

(d) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

$\frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = 1$

$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1-2y}{1+y^2}$

$IF = e^{\int \frac{1-2y}{1+y^2} dy}$

$= e^{\int \frac{1}{1+y^2} dy - \int \frac{2y\,dy}{1+y^2}}$

$= e^{\tan^{-1} y - \log(1+y^2)}$

$= \frac{e^{\tan^{-1} y}}{1+y^2}.$

$e^{\tan^{-1} y} dx + \left(\cancel{\frac{x e^{\tan^{-1} y}}{1+y^2}} - \frac{e^{2\tan^{-1} y}}{1+y^2}\right) dy = 0$

$\int e^{\tan^{-1} y} dx - \int \frac{e^{2\tan^{-1} y}}{1+y^2} dy = c$

$x e^{\tan^{-1} y} - e^{2\tan^{-1} y}/2 = c$

Q.4 Let y(x) be the solution of the differential equation $(xy + y + e^{-x})dx + (x + e^{-x})\, dy = 0$ satisfying $y(0) = 1$. Then $y(-1)$ is equal to

(a) $\frac{e}{e-1}$ (b) $\frac{2e}{e-1}$

(c) $\frac{e}{1-e}$ (d) 0

$y = \frac{2}{1-e^{-1}}$

$y = \frac{2e}{e-1}$

$\frac{\partial M}{\partial y} = x+1 \quad \frac{\partial N}{\partial x} = 1 - e^{-x}$

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{x + e^{-x}}{x + e^{-x}} = 1$

$\text{I.F.} = e^{\int dx} = e^{x}$

$(e^{x}xy + ye^{x} + 1)dx + (xe^{x} + 1)dx = 0$

$\int (xe^{x}y + ye^{x} + 1)dx + \int dy = c = y(xe^{x} - \cancel{e^{x}}) + \cancel{1e^{x}} + x + y = c$

$yxe^{x} + x + y = c$

$0 + 0 + 1 = c$

$yxe^{x} + x + y = 1$

$-ye^{-1} - 1 + y = 1$

$y(1 - e^{-1}) = 2$

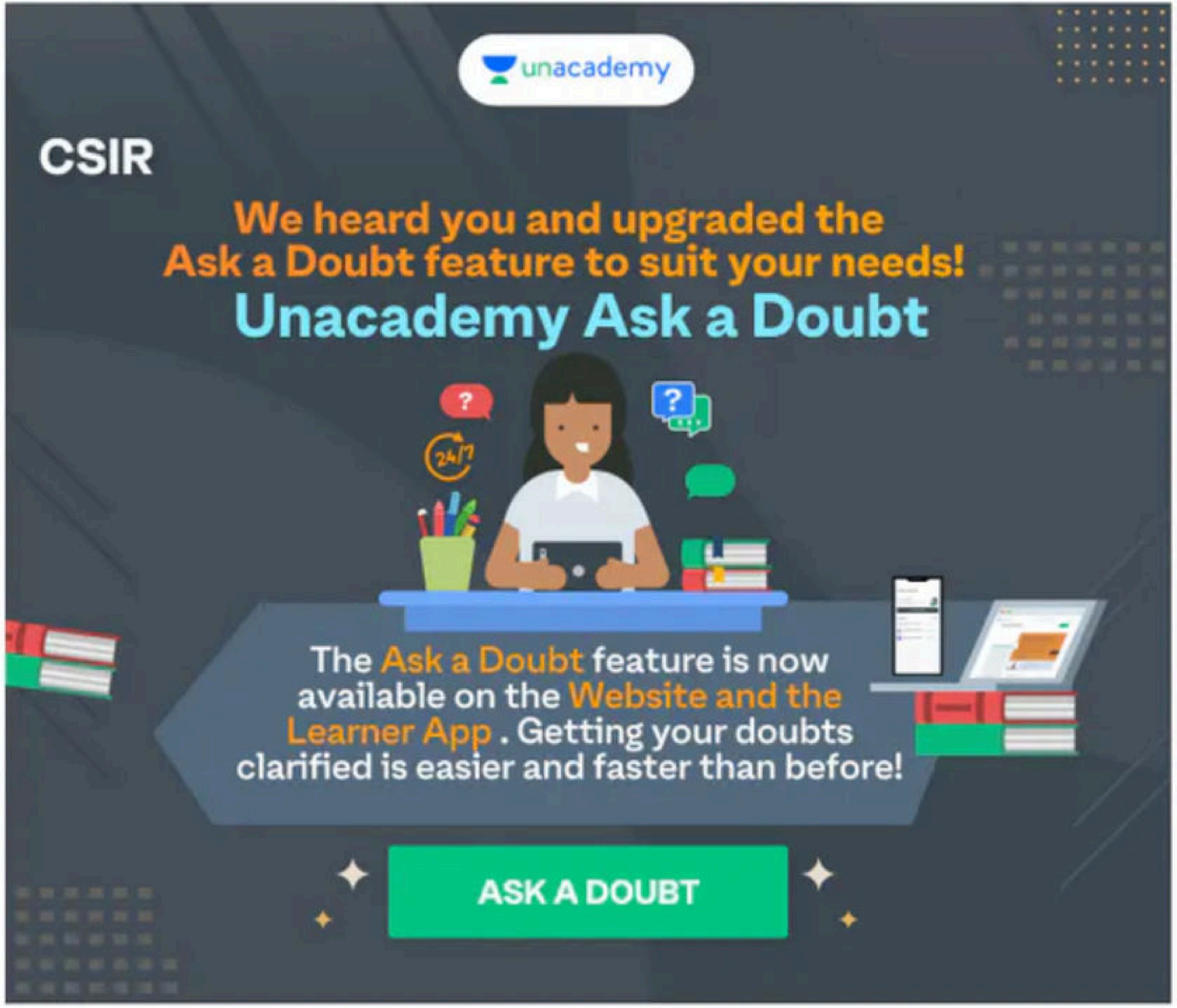
Q5. Consider the differential equation $\left(x-\frac{1}{y}\right)dy + y^2dx = 0$; $y(1) = 1$ then as $y \to \infty$, x equals

(a) 0

(b) $\frac{1}{e}$

(c) $1+\frac{1}{e}$

(d) $1-\frac{1}{e}$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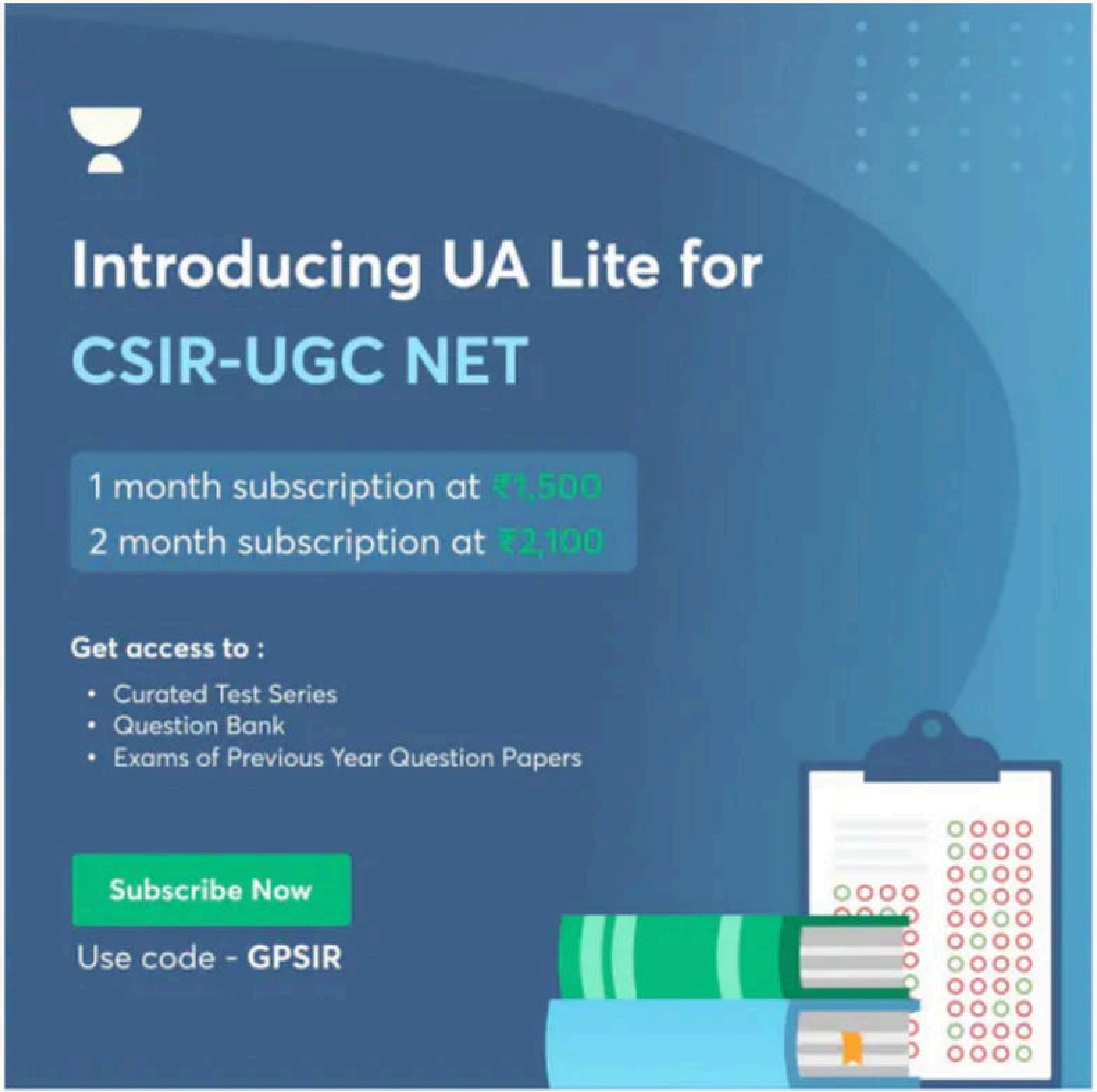
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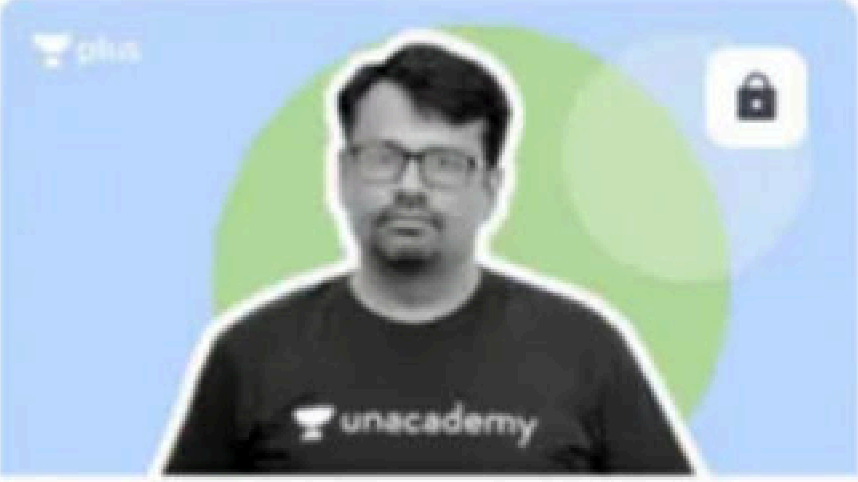

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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