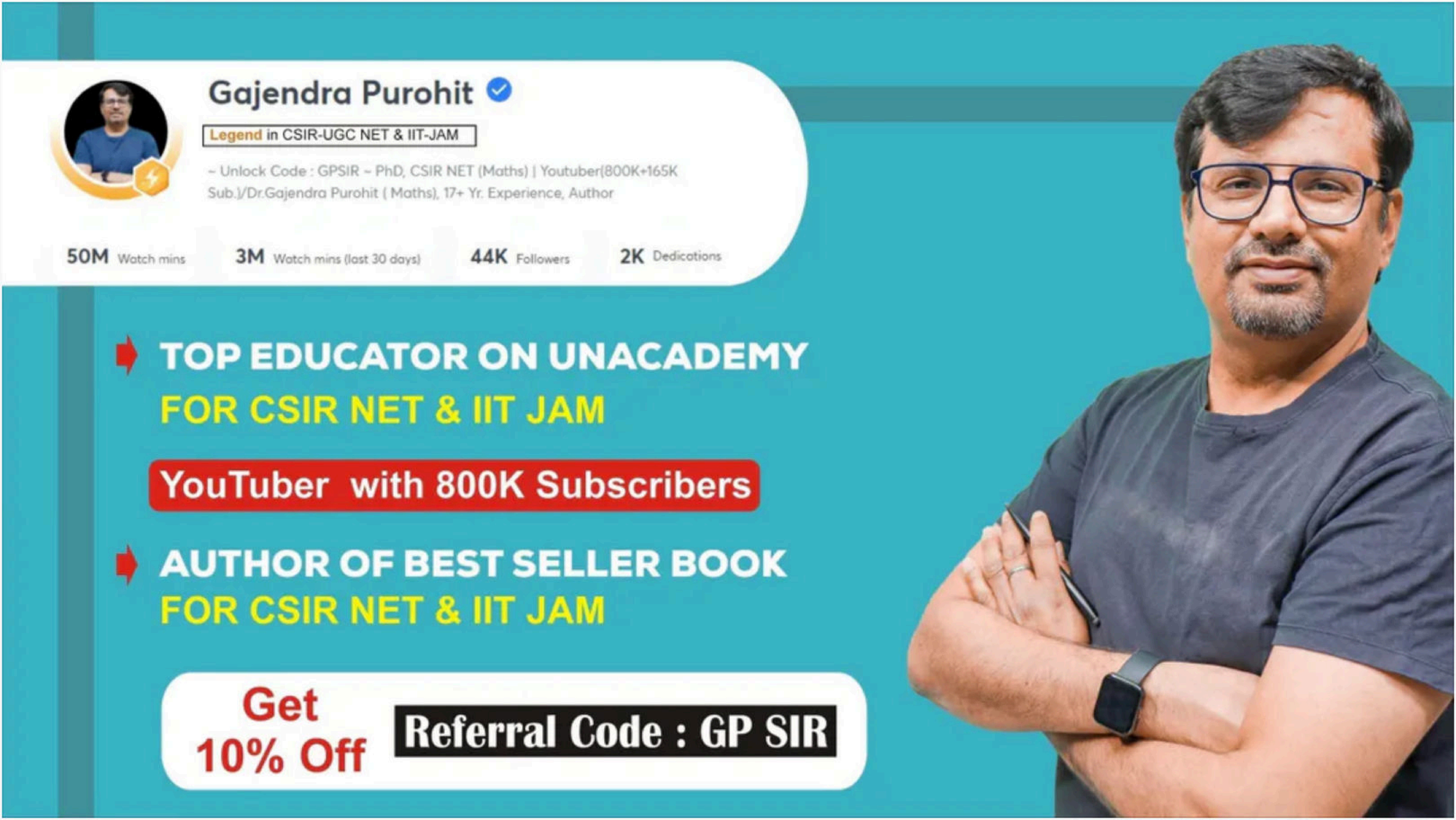
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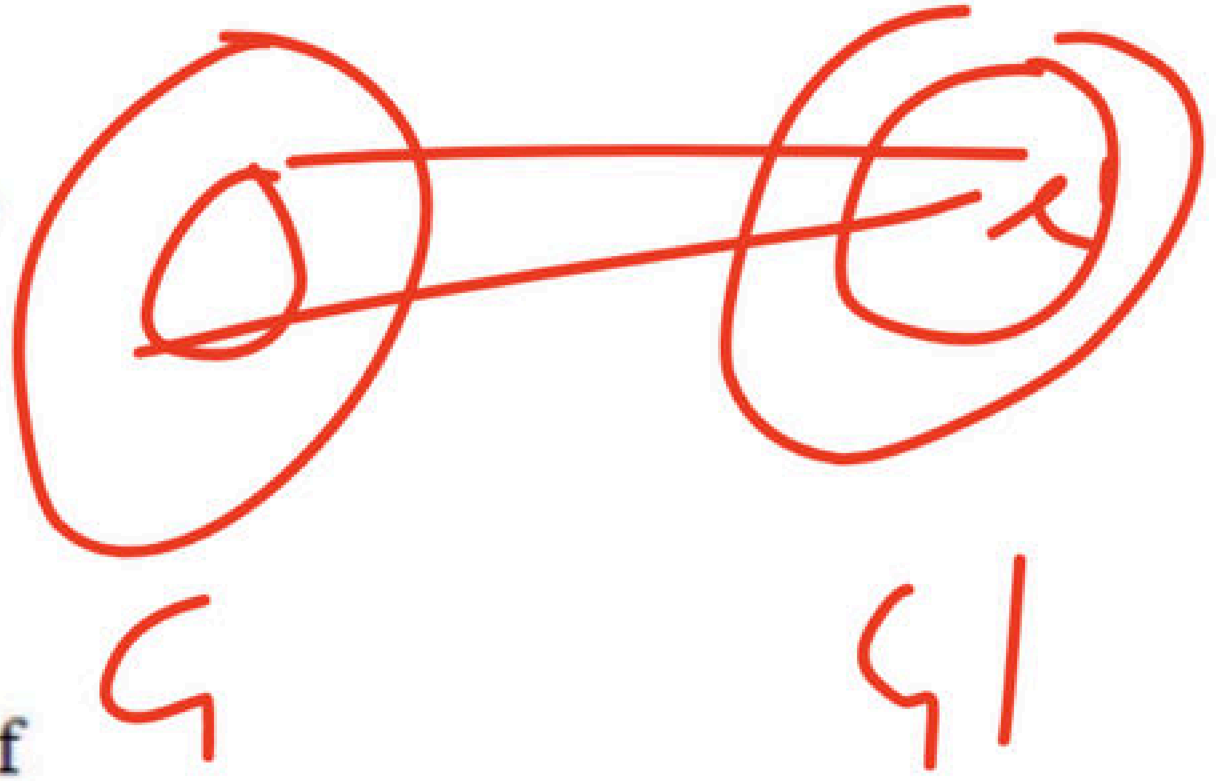
Basic Properties of Homomorphism

1. $f(e) = e`$, where e and e` are identity elements of G and G` respectively.

2. $f(x^{-1}) = (f(x))^{-1}$, for all $x \in G$

3. $f(x^n) = (f(x))^n$ for all $x \in G$, $n \in Z$

4. Let $f : G \to G`$ be a homomorphism, then ker f is a normal subgroup of G.



Fundamental Theorem of Homomorphism :

Let $f : G \to G`$ is a onto homomorphsim from G to G` if ker(f) is a kernel of f, then

$$\frac{G}{\ker(f)} \approx f(G) \approx G`$$

$$\Rightarrow \quad \frac{G}{\ker(f)} \approx G`$$

$$\frac{G}{Ker f} \cong f(G) \cong G'$$

$f : \mathbb{Z}_6 \longrightarrow \mathbb{Z}_4$

$f(x) = 2x$

$f(0) = 0$

$f(1) = 2$

$f(2) = 0$

$f(3) = 2$

$f(4) = 0$

$f(5) = 2$

$\ker f = \{0, 2, 4\} \cong \mathbb{Z}_3$

$\frac{\mathbb{Z}_6}{\ker f} = \frac{\mathbb{Z}_6}{\mathbb{Z}_3} \cong \mathbb{Z}_2 \subset \mathbb{Z}_4$

$f: \mathbb{Z}_6 \to \mathbb{Z}_4$

$f(x) = x$

$f(0) = 0$

$f(1) = 1$

$f(2) = 2$

$f(3) = 3$

$f(4) = 0$

$f(5) = 1$

$\underline{\ker f = \{0, 4\}}$

$\dfrac{\mathbb{Z}_6}{\ker f} = \dfrac{\mathbb{Z}_6}{\mathbb{Z}_2} \cong \mathbb{Z}_3 < \mathbb{Z}_4$

$f : Z_4 \to Z_4$

$f(x) = x$

$f(0) = 0$

$f(1) = 1$

$f(2) = 2$

$f(3) = 3$

$\ker f = \{0\}$

$$\frac{G}{\ker f} = \frac{Z_4}{\{0\}} \cong Z_4 = Z_4$$

Note : Let f : G → G\` is a group homomorphism then $\frac{G}{\ker(f)}$ is a subgroup of G\` because $\frac{G}{\ker(f)} \approx f(G)$ & f(G) is a subgroup of G\`, then

$$O\left[\frac{G}{\ker(f)}\right] \bigg| O(G`) .$$

Q.3. Let $f : Z_{14} \to Z_{10}$ be a homomorphism with $O(\ker f) = 7$ then order of range set of f is

(a) 1 (b) 2

(c) 5 (d) 10

$$\frac{O(G)}{O(\ker f)} = \frac{14}{7} = 2$$

Q.4. Let $f : Z_{10} \to Z_8$ be a homomorphism then which of the following possible order for Kernal of f

(a) 2 (b) 5

(c) 8 (d) 1

$\frac{o(G)}{o(\ker f)} =$

$\frac{10}{1} = 10$

$\frac{10}{2} = 5$

$\frac{10}{8}$

$\frac{10}{5} = 2$

Group Homomorphism

Group isomorphism : A mapping $f : (G, o) \to (G`, o`)$ is said to be a group isomorphism if

(a) f is group homomorphism.

(b) f is one-one

(c) f is onto

Note : If a mapping $f : G \to G`$ is a group isomorphism then G & G` are called isomorphic group.

$$f : \mathbb{Z}_4 \to \mathbb{Z}_4$$

$f(0) = 0$

$f(1) = 1$

$f(2) = 2$

$f(3) = 3$

$f(x) = x$

$f(x) = 2x$

$f(0) = 0$

$f(1) = 2$

$f(2) = 0$

$G = \{0, 1, 2, 3\}, +_4$

$+_4$	0	1	2	3
$e=0$	0	1	2	3
1	1	2	3	$\boxed{0}$
2	2	3	$\boxed{0}$	1
3	3	$\boxed{0}$	1	2

$1^{-1} = 3$

$2^{-1} = 2$

$3^{-1} = 1$

$o(0) = 1$

$o(1) = 4$

$o(2) = 2$

$o(3) = 4$

$\{1, -1, i, -i\}, \times$

$\times$	1	-1	i	$-i$
$e=1$	1	-1	i	$-i$
-1	-1	$\boxed{1}$	$-i$	i
i	i	$-i$	-1	$\boxed{1}$
$-i$	$-i$	i	$\boxed{1}$	-1

$(-1)^{-1} = -1$

$(i)^{-1} = (-i)$

$(-i)^{-1} = i$

$o(1) = 1$

$o(-1) = 2$

$o(i) = 4$

$o(-i) = 4$

$K_4 = \{e, a, b, ab\}$

$a^2 = b^2 = (ab)^2 = e$

$\times$	e	a	b	ab
e	e	a	b	ab
a	a	e	ab	b
b	b	ab	e	a
ab	ab	b	a	e

$o(e) = 1$

$o(a) = 2$

$o(b) = 2$

$o(ab) = 2$

$a^{-1} = a$

$b^{-1} = b$

$(ab)^{-1} = ab$

$U(8) = \{1, 3, 5, 7\}$

$\times_8$	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

$o(1) = 1$

$o(3) = 2$

$o(5) = 2$

$o(7) = 2$

$3^{-1} = 3$

$5^{-1} = 5$

$7^{-1} = 7$

$U(10) = \langle 1, 3, 7, 9 \rangle$

Z_4 $\qquad$ K_4

$Z_4 \cong U_{10}$

$\times_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

$3^{-1} = 7$

$7^{-1} = 3$

$9^{-1} = 9$

$o(9) = 2$

$o(3) = o(7) = 4$

$$U(p) \cong \mathbb{Z}_{p-1}$$

$$U(pq) \cong U(p)U(q)$$

$$\cong \mathbb{Z}_{p-1} \times \mathbb{Z}_{q-1}$$

$$U(p^n) \cong \mathbb{Z}_{p^n - p^{n-1}} \quad p \neq 2$$

$$U(2^n) \cong \mathbb{Z}_2 \times \mathbb{Z}_{2^{n-2}}$$

$$U(2^3) \cong \mathbb{Z}_2 \times \mathbb{Z}_{2^{3-2}} \cong \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$U(7) \cong \mathbb{Z}_6$$

$$U(10) \cong U(5)U(2)$$

$$\cong \mathbb{Z}_4 \times \mathbb{Z}_1$$

$$\cong \mathbb{Z}_4$$

$$U(25) = U(5^2)$$

$$\cong \mathbb{Z}_{5^2 - 5^1}$$

$$\cong \mathbb{Z}_{20}$$

General method :

To show that G & G` are isomorphism group.

Given that G = ({1, w, w^2}, .} & G` = (0, 1, 2}, $+_3$)

	1	w	w^2
1	1	w	w^2
w	w	w^2	1
w^2	w^2	1	w

&

$+_3$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

(1) O(G) = O(G`) = 3

(2) Number of elements of order 3 are 2 in both group.

So, both are isomorphic.

We can write G $\cong$ G`.

Note : If G & G` are two isomorphic group, then we can write G $\cong$ G`.

Conclusion : The additive group of integer $G = (Z, +)$ is isomorphic to the additive group $G` = (mZ, +)$; $m \neq 0$.

Important results :

1. Any two cyclic group of equal order are isomorphic.
2. Any cyclic group of order n is isomorphic to $(Z_n, +_n)$
3. An infinite cyclic group is isomorphic to the addditive group of integer i.e. isomorphic to $(Z, +)$
4. Every group of prime order is cyclic.
5. Let G be a finite group of order pq i.e. $O(G) = pq$ (where $p < q$ & p, q are prime)

   Case – 1 : If $p \nmid q-1$, then G is cyclic.

   Case – 2 : If $p \mid q-1$, then G need not be cyclic.

$O(G) = 1$

$(Z, +)$

3×7

z_{10}
z_1
z_2
z_5
z_{10}

Q.1. Let G be a group of order 15. Then total number of non-isomorphic subgroup of G are

(a) 1 (b) 2

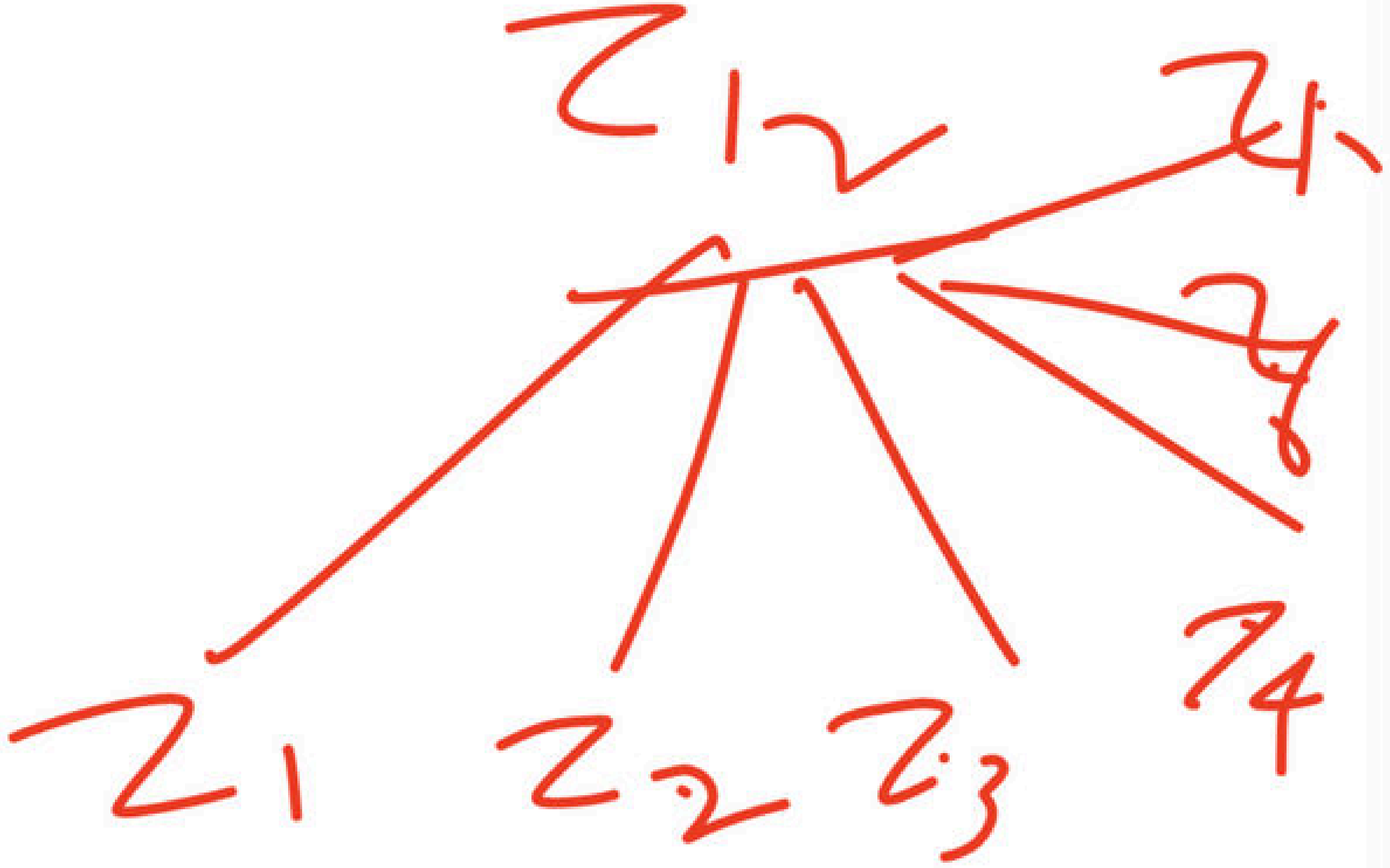
(c) 3 (d) 4

Q.2. Let G be a cyclic group of order 12, then the number of non-isomorphic subgroup of G is

2

(a) 2 (b) 4

(c) 6 (d) 12



Result : Let G be a abelian group of order n, then A mapping $f : G \to G$ s.t. $f(x) = x^m$ is isomorphism iff $\gcd(m, n) = 1$.

Example : Let G be a abelian group of orde 6, then mapping $\phi(x) = x^5$ is isomorphism.

Q.3. Let G be a group of order 7 and $\phi(x) = x^4$; $x \in G$. Then ϕ is

(a) Not one-one (b) Not a homomorphism

(c) Not onto (d) Isomorphism

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Q.4. G is a group of order 51. Then which of the following statement is false?

2×3

(a) All proper subgroup of G are cyclic.

(b) If G has only one subgroup of order 3 and only one subgroup of order 17 then G is cyclic.

Z_6

Z_3

(c) G must have an element of order 17.

(d) None of these

Cauchy Theorem : Let G be a finite group and p | O(G); where p is prime, then G has atleast one element of order p.

Result

(1) Let G be a group of order $n = p_1^{n_1} p_2^{n_2} \ldots\ldots p_m^{n_m}$, then number of non-isomorphic abelian group are $X(n_1) \cdot X(n_2) \ldots\ldots X(n_m)$, where X(n) is partition of n.

Example : Number of non-isomorphic abelian group of order $8 = 2^3$ are $X(3) = 3$.

$o(G) = 8 = 2^{3}$

$\mathbb{Z}_8$ $\qquad$ $\mathbb{Z}_2 \times \mathbb{Z}_4$ $\qquad$ $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$

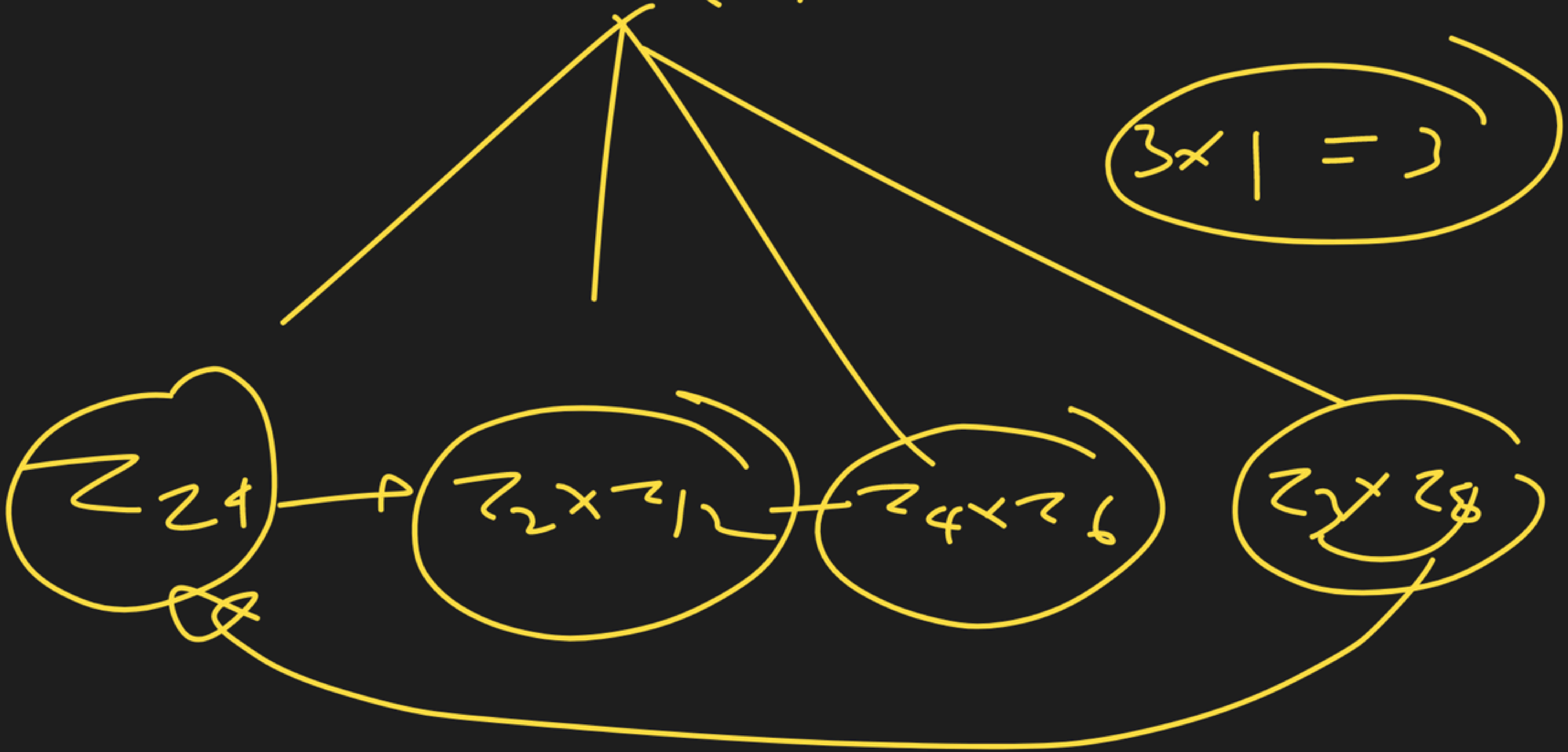
$1+1+1$

$1+2$

3

$d(4) = 24 = 2^{3} \cdot 3^{1}$

$3 \times 1 = 3$



$\frac{6}{2}$ | $\frac{6}{3}$ 6 | $\frac{48}{2}$

Q.5. Number of abelian group of order 72, which are non-isomorphic

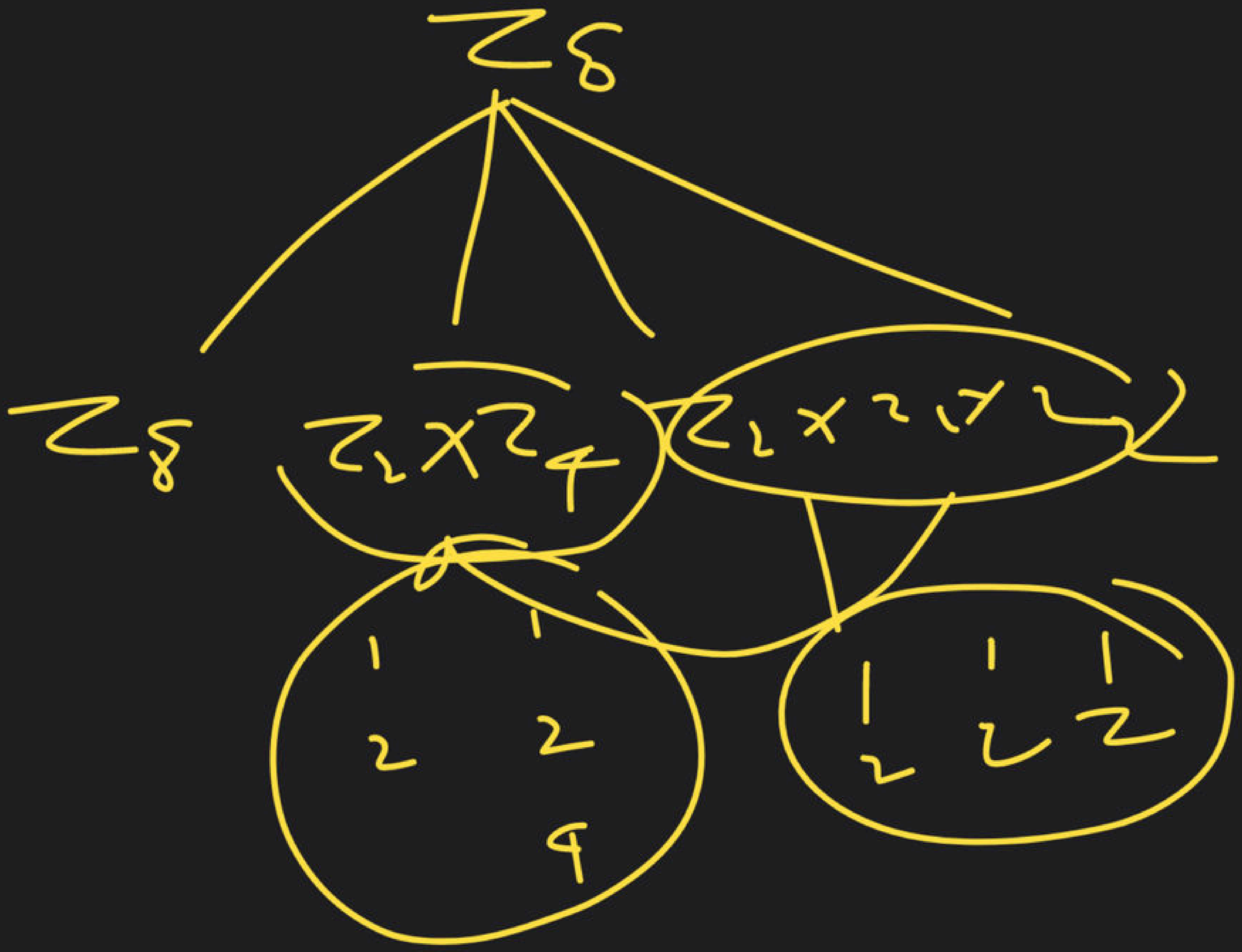
(a) 5 (b) 6 (c) 7

(d) 8

$O(G) = 2^3 \times 3^2$

$3 \times 2 = 6$

Z_{72} $Z_{36} \times Z_2$ $Z_{12} \times Z_6$ $Z_2 \times Z_2 \times Z_{12}$ $Z_3 \times Z_{24}$ $Z_{4} \times Z_{18}$



$\mathbb{Z}_8$
$\mathbb{Z}_8$
$\mathbb{Z}_2 \times \mathbb{Z}_4$
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
1 1
2 2
4
1 1 1
2 2 2

Automorphism : Let G be a group, then the mapping $f : G \to G$ is called automorphism if

(i) f is one-one

(ii) f is onto

(iii) f is homomorphism

i.e. A mapping $f : G \to G$ is called automorphism if it is isomorphism

Note : Let Z = group of integer under addition then $f : Z \to Z$ s.t. $f(x) = mx; m \neq \{1, -1\}$

Then it will not be onto mapping.

So, it will not be automorphism.

$\Rightarrow$ Z have only two automorphism.

Automorphism Group :

Let G be a group, then the set of all automorphism of G form a group under the composition of mapping and this is denoted by Aut G.

Example

(1) Let $G = Z$, then $\text{Aut } Z = \{f(x) = x \ \& \ f(x) = -x\}$

$\Rightarrow \text{Aut } Z \approx Z_2$.

(2) Let $G = Z_m$, then $\text{Aut } Z = (Z_m) \approx U(m)$

(i) Suppose $G = Z_4$, then $\text{Aut}(Z_4) \approx U(4) \approx Z_2$

(ii) Suppose $G = Z_5$ then $\text{Aut}(Z_5) \approx U(5) \approx Z_4$.

(3) $\text{Aut}(K_4) \approx S_3$

(4) $\text{Aut } (S_n) \approx S_n$; for all $n \geq 3$

(5) $\text{Aut}(Z_p \times Z_p \ ..... \times Z_p) \approx GL(n, Z_p)$

'n' times

Q6. For any group G, Aut(G) denote the group of automorphism of G. Which of the following are true?

(a) If G is finite, then Aut(G) is finite

(b) If G is cyclic, then Aut(G) is cyclic

(c) If G is infinite, then Aut(G) is infinite

(d) If Aut (G) is isomorphic to Aut(H) then G is isomorphic to H.

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Q7. Let $G = Z_3 \times Z_3$ be a group then order of $Aut(Z_3 \times Z_3)$ is

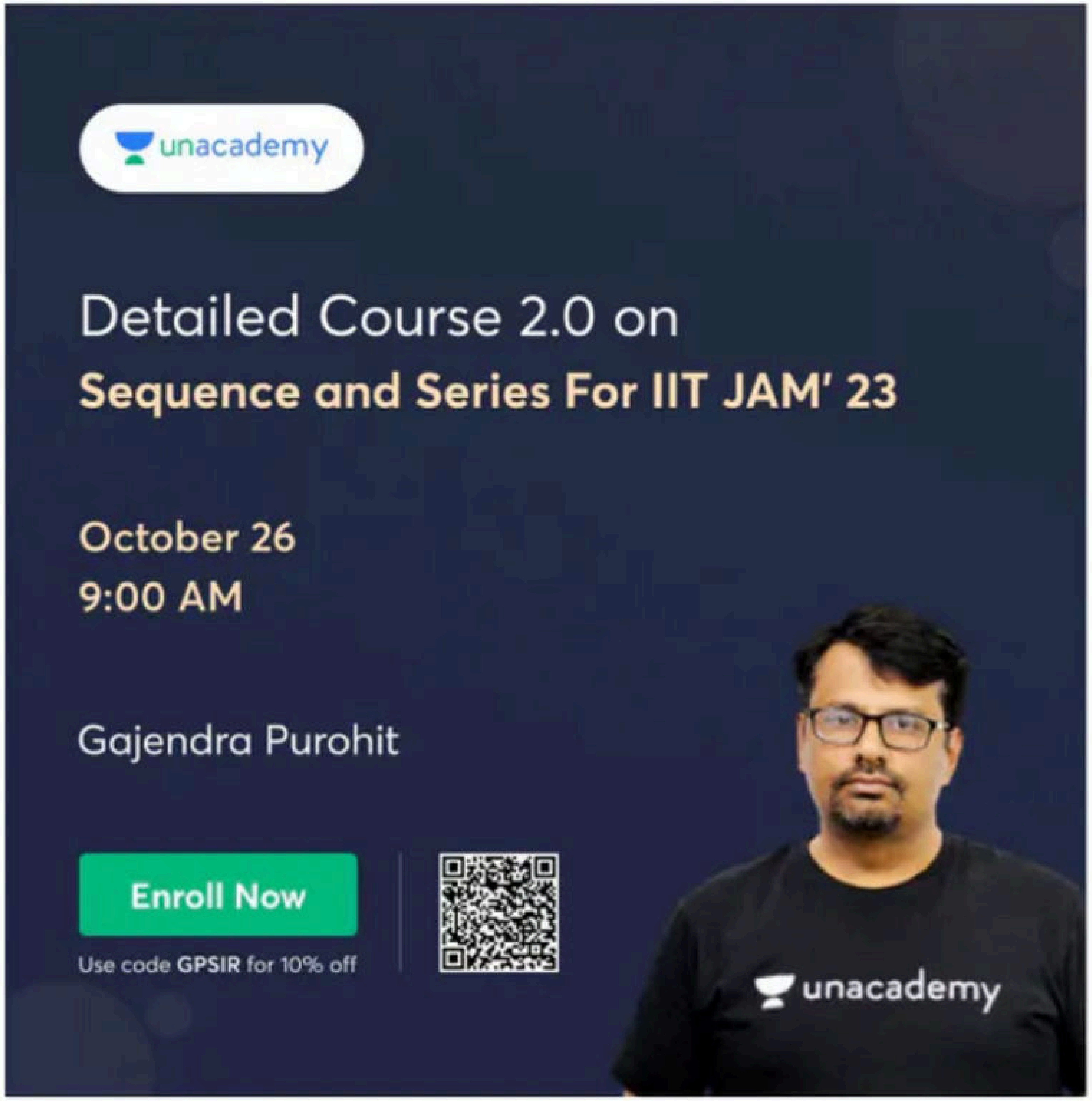
(a) 48 (b) 168

(c) 50 (d) 150

Q8. The order of $Aut(Aut(Aut(K_4)))$ is

(a) 4 (b) 5

(c) 6 (d) 8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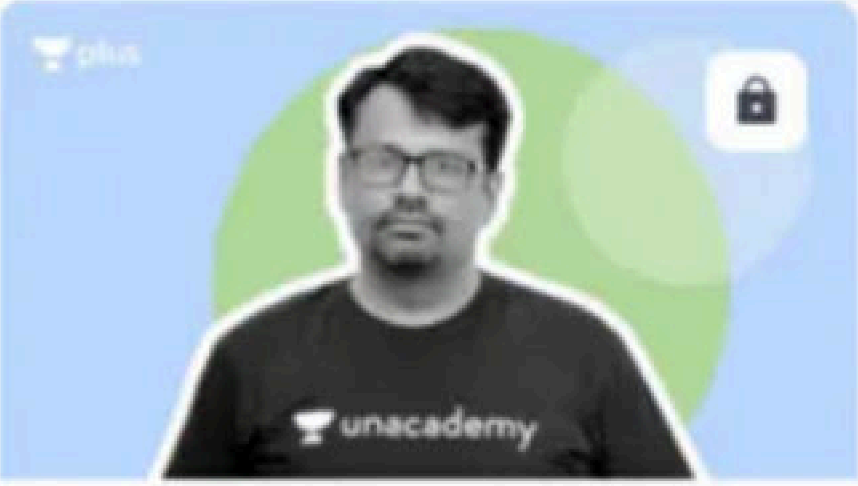

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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