4

The value (S) of the integral $\int^{\pi}|x| \cos nx \, dx$,

$n \geq 1$ is (are)

(a) 0 when n is even

(b) 0 When n is odd

(c) $-\dfrac{4}{n^2}$ when n is even

(d) $-\dfrac{4}{n^2}$ when n is odd

[A, D]

1) Here $I = \int_{-n}^{\pi} |x| \cos x,\quad n \geq 1$

$$= 2\int_{0}^{\pi} |x| \cos nx,\, dx \qquad \text{[even function].}$$

$$= 2\int_{0}^{\pi} x \cos nx \, dx.$$

$$= 2\left[\frac{x}{n}\sin nx + \frac{1}{n^2}\cos nx\right]_0^{\pi}$$

$$= \frac{2}{n^2}\left[nx \sin nx + \cos nx\right]_0^{\pi}$$

$$= \frac{2}{n^2}\left[\cos n\pi - 1\right]$$

So, $I = \begin{cases} 0 & \text{if } n \text{ is even.} \\ -\frac{4}{n^2} & \text{if } n \text{ is odd.} \end{cases}$

5

Let $f: (0, \infty) \to \mathbb{R}$ be a continuous function such that $\int_0^x f(t)\,dt = 2 + \frac{x^2}{2} + 4x \sin 2x + 2 \cos 2x$.

Then the value of $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$ is __________.

Subjective.

2) Here

$$\int_0^x f(t)\,dt = -2+\frac{x^2}{2}+4x\sin 2x+2\cos^2 x \text{ ----- (1)}$$

Differentiating (1) w.r.t x

we obtain

$$f(x) = x+4\sin 2x+8x\cos 2x-4\sin 2x \quad \text{[Leibnitz's Rule]}.$$

$$f(x) = x+8x\cos 2x$$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4}+8\cdot\frac{\pi}{4}\cos\frac{\pi}{2} = \frac{\pi}{4}.$$

So, $\frac{1}{\pi}f\left(\frac{\pi}{4}\right) = \frac{1}{\pi}\cdot\frac{\pi}{4} = \underline{\underline{\frac{1}{4}}}.$

6

Let R be the region enclosed by $x^2 + 4y^2 \geq 1$ and $x^2 + y^2 \leq$ Then the value of $\iint_R |xy|\, dx\, dy$

is ___________.

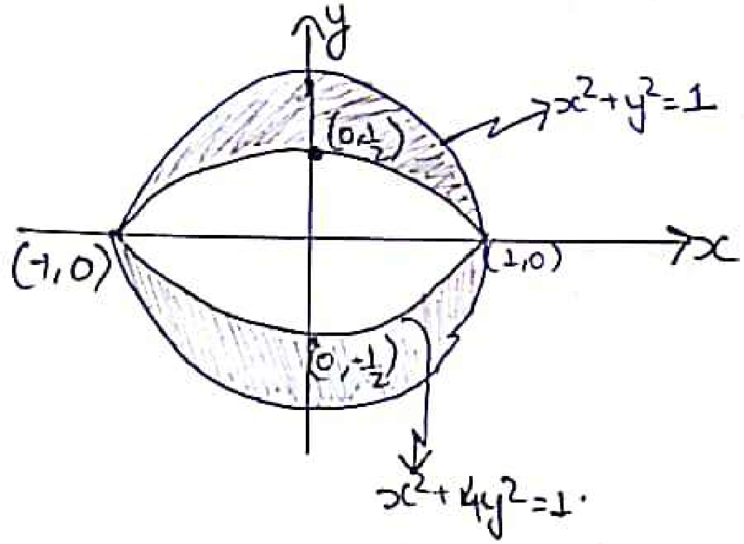
Subjective.

3) Let $I = \iint |xy| \, dx \, dy$.

where R is the region enclosed by $x^2+4y^2 \geq 1$ and $x^2+y^2 \leq 1$.

$$I = \int_{x=-1}^{1} \int_{y=\frac{1}{2}\sqrt{1-x^2}}^{\sqrt{1-x^2}} |xy| \, dx \, dy.$$

$$\Rightarrow 4 \times \int_{x=0}^{1} \int_{y=\frac{1}{2}\sqrt{1-x^2}}^{\sqrt{1-x^2}} |xy| \, dx \, dy.$$



$$= 4\int_{x=0}^{1}\int_{y=\frac{1}{2}\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy\,dx\,dy \quad (x,y\geq 0)$$

$$= 4\int_{x=0}^{1}\frac{x}{2}\left[\frac{y^2}{2}\right]_{\frac{1}{2}\sqrt{1-x^2}}^{\sqrt{1-x^2}} dx.$$

$$\Rightarrow 2\cdot\int_{0}^{1} x\left[\frac{3}{4}-\frac{3}{4}x^2\right]dx.$$

$$\Rightarrow \frac{3}{2}\int_{0}^{1}\left[x-x^3\right]dx = \frac{3}{2}\left[\frac{x^2}{2}-\frac{x^4}{4}\right]_{0}^{1}$$

$$\Rightarrow \frac{3}{2}\times\frac{1}{4} = \underline{\frac{3}{8}}$$