Normal Subgroup

Detail Course 2.0 on Group Theory for IIT JAM '23

Gajendra Purohit • Lesson 7 • Oct 14, 2022

Gajendra Purohit

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Subgroup

Subgroup : Let (G, *) be a group a non-empty subset H of G is said to be subgroup of G if H itself a group under composition *.

One step test : Let (G, *) be a group. A non-empty subset H of G is called a subgroup of G if

$a * b^{-1} \in H$; for all $a \in H$ & for all $b \in H$ where b^{-1} is inverse of b.

Q.1. Consider the alternating group A_4 .Which of the following is FALSE?

(a) A_4 has 12 elements

(b) A_4 has exactly one subgroup of order 4

(c) A_4 has a subgroup of order 6

(d) Number of 3 cycles in A_4 is 8

Result :

(1) Let G be a cyclic group of order n, then number of subgroup are $\tau(n)$ where $\tau(n)$ are number of +ve divisor of n.

(2) Number of cyclic subgroup of order d

$$= \frac{\text{number of elements of order d}}{\phi(d)}$$

(3) Intersection of two subgroup is always a subgroup.

(4) Union of two subgroup is a subgroup iff one is contained in other.

(5) Let H & K are two subgroup of group G, then

$$O(H,K)=\frac{O(H).O(K)}{O(H\cap K)}.$$

(6) $H \cap K$ is a subgroup of H & K both.

Lagrange's theorem

Order of group is divisible by its every subgroup but converse need not be true

Q.2. Let H & K be subgroups of Z_{144}. If the order of H is 24 and the order of K is 36 then the order of the subgroup $H \cap K$ is

(a) 3 (b) 4

(c) 12 (d) None of these

Q.3. Let S_5 be the symmetric group on five symbols. Then which of the following statements is false? **CSIR NET DEC. 2019**

(a) S_5 contains a cyclic subgroup of order 6

(b) S_5 contains a non-abelian subgroup of order 8

(c) S_5 does not contain a subgroup isomorphic to $Z/2Z \times Z/2Z$

(d) S_5 does not contain a subgroup of order 7

Q.4. Which of the following is true? **JAM 2018**

(a) Z_n is cyclic iff n is prime

(b) Every proper subgroup of Z_n is cyclic.

(c) Every proper subgroup of S_n is cyclic

(d) If every proper subgroup of a group is cyclic then the group is cyclic.

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Centre of group : Let G be a group. A collection $Z(G) = \{x \in G \mid xa = ax;$ for all $a \in G\}$ is also a subgroup of G, which is called centre of group.

Q.5. $G=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a,b,c,d \in Z\right\}$ under addition.

(a) $H=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G \mid a+b+c+d=0\right\}$ then H is a subgroup of G.

(b) $H=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G \mid a+b+c+d=1\right\}$ then H is a subgroup of G.

(c) Both (a) and (b) are correct.

(d) None of above.

Q.6. Pick out the true statements

(a) $H_1 = \{A \in GL(2, R) \mid \det A \text{ is an integer power of } 2\}$ then H_1 is a subgroup.

(b) $H_2 = \{A \in GL(2, R) \mid \det A \text{ is an integer power of } 6\}$ then H_2 is a subgroup.

(c) Both (a) and (b) are correct.

(d) None of the above

Q.7. Given a group G, its center is $\{h \in G : hg = gh \text{ for all } g \in G\}$.

Let G be the group $G = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} : a,b,c \in Z \right\}$. Then the center of G

(a) is G itself

(b) is $\left\{ \begin{pmatrix} 1 & a & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : x \in Z \right\}$.

(c) consists only of the 3×3 identity matrix.

(d) $\left\{ \begin{pmatrix} 1 & x & x \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix} : x \in Z \right\}$

Q.8. : If H_1 and H_2 are two subgroups of G, then following is also a subgroup of G

(a) $H_1 \cap H_2$ (b) $H_1 \cup H_2$

(c) H_1H_2 (d) None of these

Q.9 If $G = \{1, \omega, \omega^2\}$, then there are

(a) 2 subgroups

(b) 3 subgroups

(c) 4 subgroups

(d) No subgroup possible

Q.10 Which of the following statement(s) is/are true?

(a) Any group of prime order can have no proper subgroups.

(b) Any group of prime order can have proper subgroup.

(c) The order of a subgroup divides the order of a group.

(d) The order of a subgroup does not divides the order of a group.

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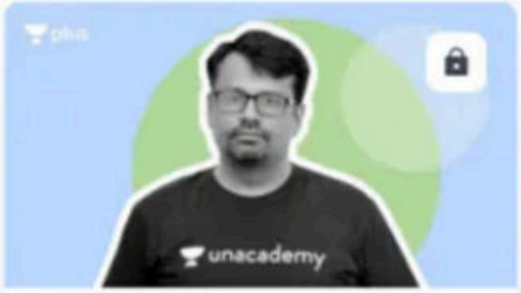

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
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