Surface of solid of revolution :

(a) **Revolution about x-axis :** Let curve is y = f(x), then the area between the ordinate x = a, x = b and x-axis is $\int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$.

(b) **Revolution about y-axis :** If the curve is x = f(y) then the area between the ordinate y = a, y = b, y-axis which is $\int_a^b 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$.

(c) **Parametric form :** Let x = f(t) and y = g(t) are two curve, then area of solid of revolution about x-axis $\int 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$.

(d) **Polar Form :** Let the equation of curve is r = f(θ), then the curve surface generated by revolution about the initial line and the arc intercept θ = α and θ = β is $\int_{\alpha}^{\beta} 2\pi(r\sin\theta)\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$.

Q.1 The surface area obtained by revolving $y = 2x$, for $x \in [0, 2]$, about y-axis is **IIT JAM 2009**

(a) $2\pi\sqrt{5}$ (b) $4\pi\sqrt{5}$

(c) $2\sqrt{5\pi}$ (d) $4\sqrt{5\pi}$

Area of the surface :

Let the equation of surface is z = f(x, y),

then the area of surface = $S = \iint_R \sqrt{1 + z_x^{\ 2} + z_y^{\ 2}}\, dx dy$.

Where R is projection of S on xy-plane.

Note :

(1) If the equation of surface is y = f(x, z), then surface area

$$S = \iint_R \sqrt{1 + {y_x}^2 + {y_z}^2}\, dxdz.$$

(2) If the equation of surface is x = f(y, z), then surface area

$$S = \iint_R \sqrt{1 + {x_y}^2 + {x_z}^2}\, dydz.$$

Q.2. The surface area of the portion of the plane $y + 2z = 2$ within the circle $x^2 + y^2 = 3$ is **IIT JAM – 2016**

(a) $\frac{3\sqrt{5}}{2}\pi$ (b) $\frac{5\sqrt{5}}{2}\pi$

(c) $\frac{7\sqrt{5}}{2}\pi$ (d) $\frac{9\sqrt{5}}{2}\pi$

Q.3. The area of the surface $z = \frac{xy}{3}$ intercepted by the cylinder $x^2 + y^2 \le 16$ lies in the interval **IIT JAM 2017**

(a) $(20\pi, 22\pi]$ (b) $(22\pi, 24\pi]$

(c) $(24\pi, 26\pi]$ (d) $(26\pi, 28\pi]$

Q.4. The area of the part of the surface of the paraboloid $x^2 + y^2 + z = 8$ lying inside the cylinder $x^2 + y^2 = 4$ is

IIT JAM – 2019

(a) $\frac{\pi}{2}(17^{3/2} - 1)$ (b) $\pi(17^{3/2} - 1)$

(c) $\frac{\pi}{6}(17^{3/2} - 1)$ (d) $\frac{\pi}{3}(17^{3/2} - 1)$

Q.5. Find the area of the portion of the surface $z = x^2 - y^2$ in R^3 which lies inside the solid cylinder $x^2 + y^2 \leq 1$.

IIT JAM – 2012

(a) $\frac{\pi}{6}[5^{3/2} - 1]$ (b) $\frac{\pi}{6}[13^{3/2} - 1]$

(c) $\frac{\pi}{3}[5^{3/2} - 1]$ (d) $\frac{\pi}{2}[5^{3/2} - 1]$

Q.6. Find the surface area generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the initial line.

(a) $\frac{72}{5}\pi a^2$ (b) $\frac{64}{5}\pi a^2$

(c) $\frac{32}{5}\pi a^2$ (d) None

Q.7. Find the surface area of the portion of the cone $z^2 = x^2 + y^2$ that is inside the cylinder $z^2 = 2y$. **IIT JAM – 2008**

(a) $2\sqrt{2}\pi$ (b) $4\sqrt{2}\pi$

(c) $6\sqrt{2}\pi$ (d) $8\sqrt{2}\pi$