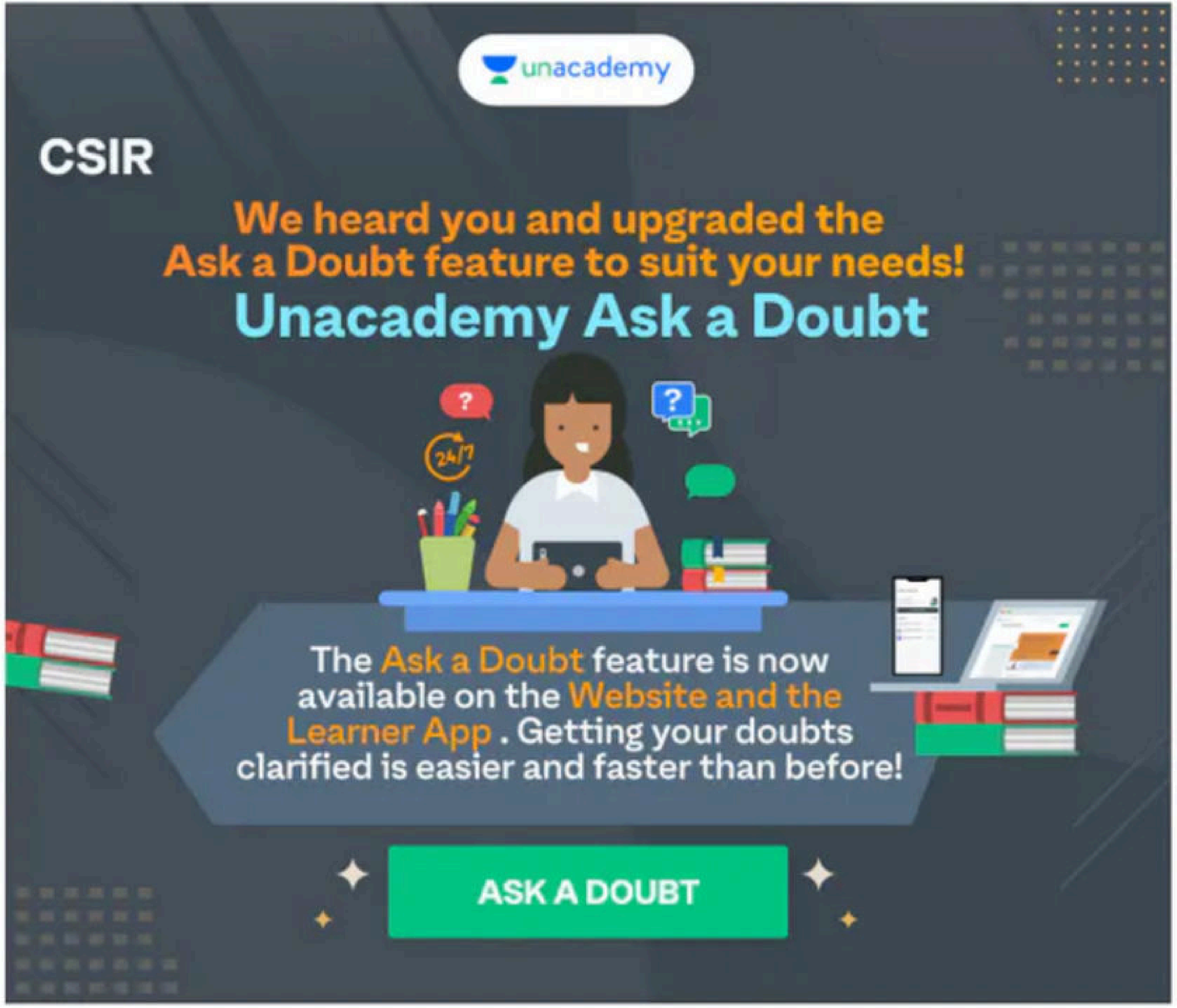

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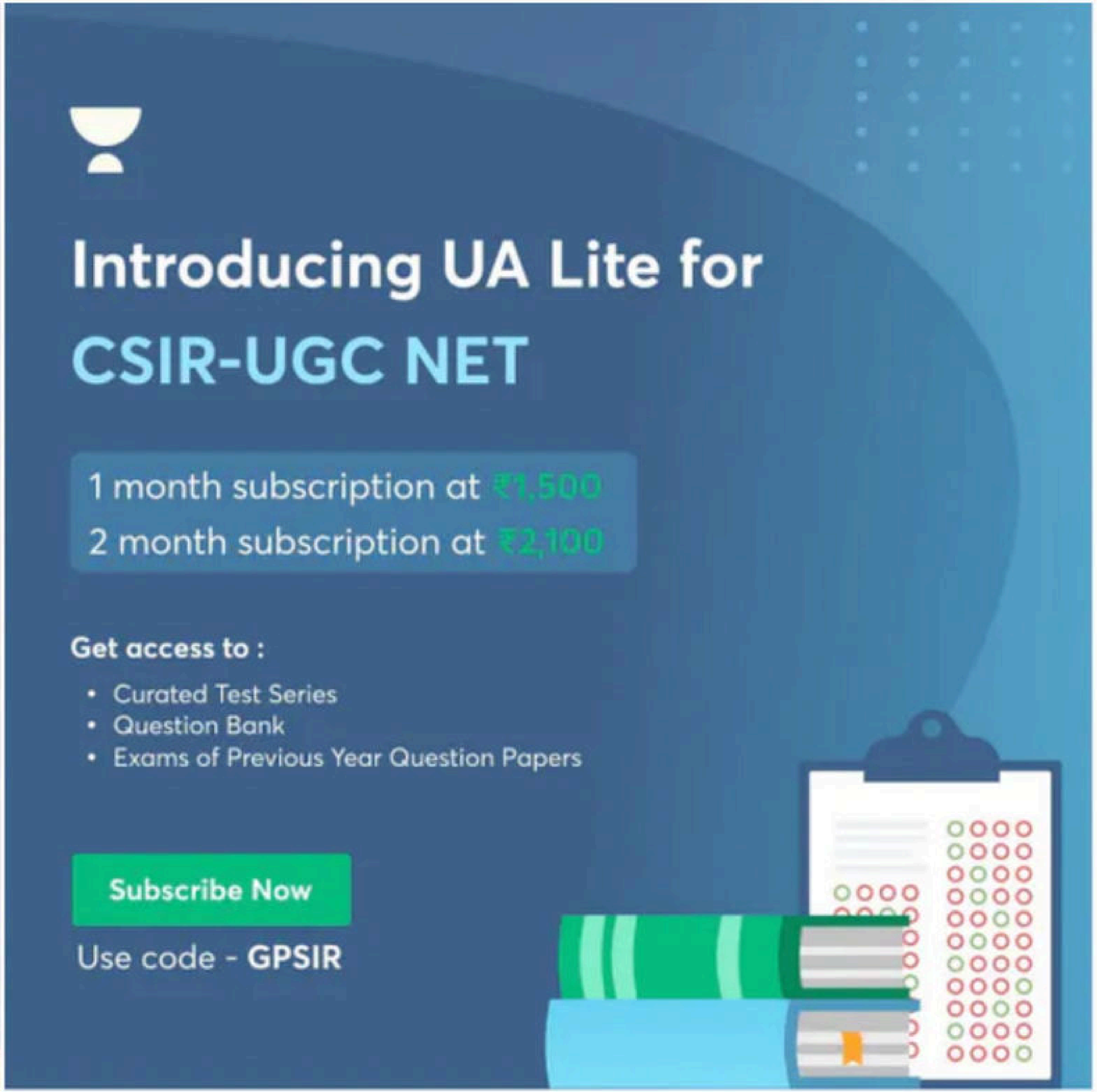
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5. The Wronskian :

Definition :The Wronskian of n functions $y_1(x)$, $y_2(x)$ ...... $y_n(x)$ is denoted by $w(x)$ or $w(y_1, y_2, \ldots\ldots, y_n)$

$$= \begin{vmatrix} y_1 & y_2 \ldots\ldots y_n \\ y'_1 & y_2' \ldots\ldots y'_n \\ \vdots & \\ y_1^{(n-1)} & y_2^{(n-1)} \ldots\ldots y_n^{(n-1)} \end{vmatrix}$$

$$\frac{d^2y}{dx^2} + y = 0$$

$$W = 1$$

$$1\frac{d^2y}{dx^2} + 0\frac{dy}{dx} + y = 0$$

$$\cancel{W = Ae^{-\int P dx}}$$

$$W = A$$

$$W = 1$$

Wronskian of second order DE

Let $a_0(x)\, y'' + a_1(x)\, y' + a_2(x)\, y = 0$

Where $a_0(x)$, $a_1(x)$, $a_2(x)$ are continuous and $a_0(x) \neq 0 \;\; \forall\; x$

If $y_1(x)$ and $y_2(x)$ are solution

Then $w(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

Able's Formula:

Let $a_0(x)\, y'' + a_1(x)\, y' + a_2(x)\, y = 0$

Where $a_0(x)$, $a_1(x)$, $a_2(x)$ are continuous and $a_0(x) \neq 0\ \forall$ x

$w(y_1, y_2) = Ae^{-\int \frac{a_1(x)}{a_0(x)}dx}$ is called Able's formula.

~~y_1 y_2~~

2D

$w(x) = 0$

$w(x) \neq 0$

$w(x) = 0$

RESULTS:

(1) If $w(y_1, y_2, ...y_n) \neq 0$ then $y_1, y_2 ...... y_n$ are L.I. solution.

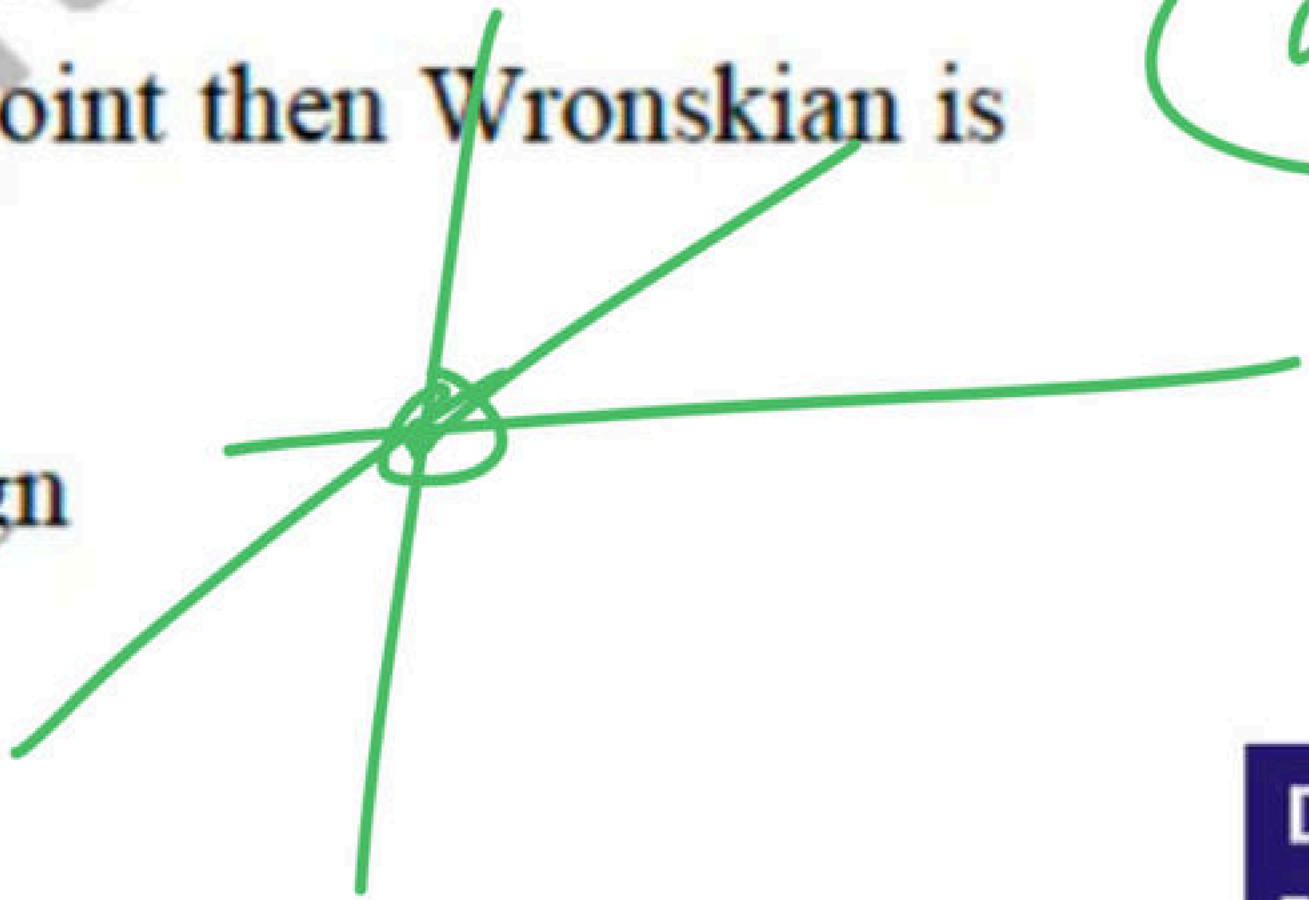
(2) If $w(y_1, y_2, ... y_n) = 0$ then $y_1, y_2, ...y_n$ are LD solution.

(3) Wronskian is either identically zero or non-zero.

(4) If Wronskian is non– zero at least one point then Wronskian is identically non– zero

(5) If Wronskian is zero at least one point then Wronskian is identically zero

(6) Wronskian can never change its sign



[-1, 1]

[-1, 0]

−

0

w(x) = x

+

[0, 1]

w(x) = 0

Q2. Consider two solution $x(t)=x_1(t)$ and $x(t)=x_2(t)$ of differential equation $\frac{d^2x(t)}{dt^2}+x(t)=0,\ t>0$ such that $x_1(0)=1, \left.\frac{dx_1(t)}{dt}\right|_{t=0}=0, x_2(0)=0, \left.\frac{dx_2(t)}{dt}\right|_{t=0}=1$ the Wronskian $W(t)=\begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_1(t)}{dt} \end{vmatrix}$ and at $t=\pi/2$ is

(a) 1 (b) – 1
(c) 0 (d) $\pi/2$

Q3. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of the differential equation $x^2 y''(x) - 2xy'(x) - 4y(x) = 0$ for $x \in [1,10]$. Considered the wronskian $W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x)$. If $W(1) = 1$, then $W(3) - W(2)$ equals

(a) 1 (b) 2

(c) 3 (d) 5

$w(x) = Ae^{-\int -\frac{2}{x} dx}$

$w(x) = Ae^{\int \frac{2}{x} dx}$

$w(x) = Ae^{2\log x}$

$= Ae^{\log x^2}$

$w(x) = Ax^2$

$w(1) = A$

$A = 1$

$w(x) = x^2$

$w(3) = 9$

$-w(2) = -4$

5

Q4. Let $y_1(x)$ and $y_2(x)$ be the linearly independent solutions of $xy''+2y'+xe^x y=0$ If $W(x)=y_1(x)y_2'(x)-y_2(x)y_1'(x)$ with $W(1)=2$ find W (5)

$w(x) = Ae^{-\int \frac{2}{x}dx}$

(a) $\frac{2}{25}$

$w(x) = Ae^{-2\log x}$

(b) $\frac{1}{25}$

$w(x) = \frac{A}{x^2}$

(c) $\frac{2}{5}$

(d) None of the above

$2 = A$

$w(x) = \frac{2}{x^2}$

$w(5) = \frac{2}{25}$

Q.5. Consider the ODE

$$u''(t)+P(t)u'(t)+Q(t)u(t)=R(t), t\in[0,1]$$

There exist continuous function P, Q and R defined on $[0,1]$ and two solutions u_1 and u_2 of the ODE such that the Wronskian W of u_1 and u_2 is

(a) $W(t)=2t-1, 0\le t\le 1$ $t=\frac{1}{2}$

(b) $W(t)=\sin 2\pi t, 0\le t\le 1$ $t=1$

(c) $W(t)=\cos 2\pi t, 0\le t\le 1$ $t=\frac{1}{4}$

(d) $W(t)=1, 0\le t\le 1$

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Q.6. Consider the ordinary DE $y'' + P(x)\,y' + Q(x)\,y = 0$, Where P and Q are smooth functions. Let y_1 and y_2 be any two solution of the ODE. Let w(x) be the Wronskian. Then which of the following is always true.

(a) If y_1 & y_2 are LD then $\exists\, x_1, x_2$ s.t. $w(x_1) = 0$ and $w(x_2) \neq 0$

(b) If y_1 & y_2 are LI then $w(x) = 0\ \forall\, x$

(c) If y_1 & y_2 are LD then $w(x) \neq 0\ \forall\, x$

(d) If y_1 & y_2 are LI then $w(x) \neq 0\ \forall\, x$

Q.7. Let P,Q be continuous real valued functions defined on $[-1,1]$ and $u_i:[-1,1]\to R, i=1,2$ be solutions of the ODE: $\frac{d^2u}{dx^2}+P(x)\frac{du}{dx}+Q(x)d=0, x\in[-1,1]$ satisfying $u_1\geq 0, u_2\leq 0$ and $u_1(0)=u_2(0)=0$. Let w denote the Wronskian of u_1 and u_2, then

(a) u_1 and u_2, are linearly independent

(b) u_1 and u_2, are linearly dependent

(c) $w(x)=0$ for all $x\in[-1,1]$

(d) $w(x)\neq 0$ for some $x\in[-1,1]$

Q.8. Let $Y_1(x)$ and $Y_2(x)$ defined on $[0,1]$ be twice continuously differentiable functions satisfying $Y''(x)+Y'(x)+Y(x)=0$. Let $W(x)$ be the Wronskian of Y_1 and Y_2 and satisfy $W\left(\frac{1}{2}\right)=0$. Then

(a) $W(x)=0$ for $x\in[0,1]$

(b) $W(x)\neq 0$ for $x\in[0,1/2)\cup(1/2,1]$

(c) $W(x)>0$ for $x\in(1/2,1]$

(d) $W(x)<0$ for $x\in[0,1/2)$

Q.9. Let $y_1(x)$ and $y_2(x)$ be two solutions of $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \sec x . y = 0$ with Wronskian W(x). If $y_1(0)=1, \left(\frac{dy_1}{dx}\right)_{x=0} = 0$ and $W\left(\frac{1}{2}\right) = \frac{1}{3}$, then $\left(\frac{dy_2}{dx}\right)_{x=0}$ equals

(a) 1/4 (b) 1

(c) 3/4 (d) 4/3

$W(x) = A e^{-\int -\frac{2x}{1-x^2}dx}$

$= A e^{-\log(1-x^2)}$

$\boxed{W(x) = \frac{A}{1-x^2}}$

$W(0) = \begin{vmatrix} y_1(0) & y_2(0) \\ y_1'(0) & y_2'(0) \end{vmatrix}$

$\frac{1}{3} = \frac{A}{1-1/4}$

$\frac{1}{3} = \frac{4}{3}A$

$A = \frac{1}{4}$

$W(0) = \begin{vmatrix} 1 & y_2(0) \\ 0 & y_2'(0) \end{vmatrix} = y_2'(0)$

$W(x) = \frac{1}{4(1-x^2)}$

$W(0) = \frac{1}{4} = y_2'(0)$

Q.10. Let $y = \phi(x)$ and $y = \psi(x)$ be solutions of $y'' - 2xy' + (\sin x^2)y = 0$ such that $\phi(0) = 1, \phi'(0) = 1$ and $\psi(0) = 1, \psi'(0) = 2$. Then the value of the Wronskian $W(\phi, \psi)$ at $x = 1$ is

$$W(0) = \begin{vmatrix} \phi(0) & \psi(0) \\ \phi'(0) & \psi'(0) \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1$$

(a) 0 (b) 1

(c) e (d) e^2

$$W(x) = A e^{\int 2x\,dx} = A e^{x^2}$$

$$W(0) = A$$

$$W(x) = e^{x^2}$$

$$W(1) = e$$

Q.11. Consider the ordinary DE $y'' + P(x)\, y' + Q(x)\, y = 0$

Where P and Q are smooth function. Let y1 and y2 be any two solution of the ODE. Let w(x) be the Wronskian. Then which of the following is always true.

(a) If y_1 & y_2 are LD then $\exists\ x_1, x_2$ s.t. $w(x_1)=0$ & $w(x_2) \neq 0$

(b) If y_1 & y_2 are LI then $w(x) = 0\ \forall\ x$

(c) If y_1 & y_2 are LD then $w(x) \neq 0\ \forall\ x$

(d) If y_1 & y_2 are LI then $w(x) \neq 0\ \forall\ x$

Q.12 The wronskian of two solutions of the differential equation $t^2y'' - t(t+2)y' + (t+2)y = 0$ satisfies $W(1) = 1$ is

(a) t^2e^t

(b) ~~t^2e^{t-1}~~

(c) $t\,e^t$

(d) $t\,e^{t-1}$

$$W = Ae^{+\int \frac{t(t+2)}{t^2}dt}$$

$$= Ae^{\int\left(1+\frac{2}{t}\right)dt}$$

$$= Ae^{t+2\log t}$$

$$W(t) = At^2e^t$$

$$1 = Ae$$

$$A = e^{-1}$$

$$W(t) = e^{-1}e^t t^2$$

$$= t^2e^{t-1}$$

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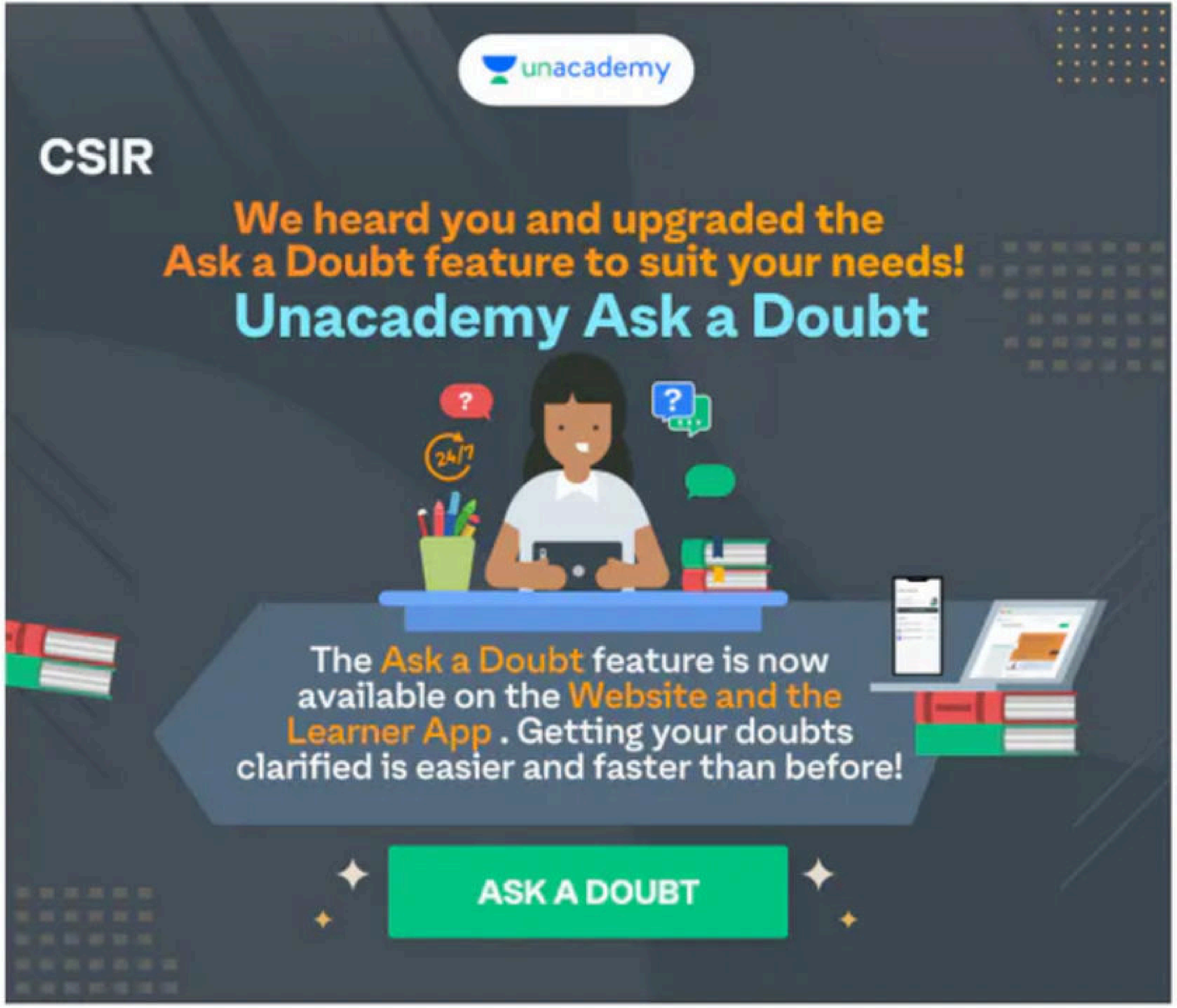
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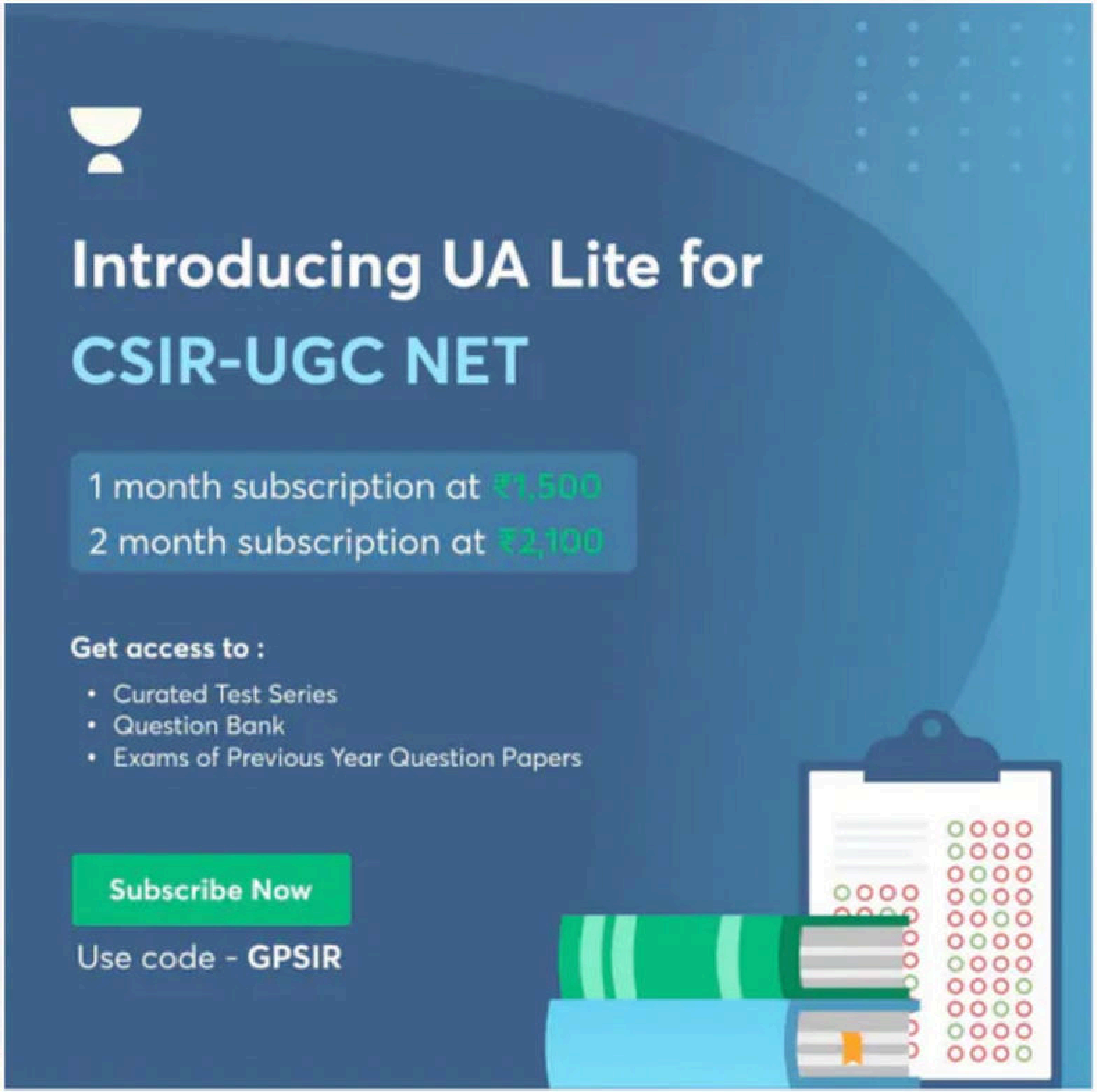
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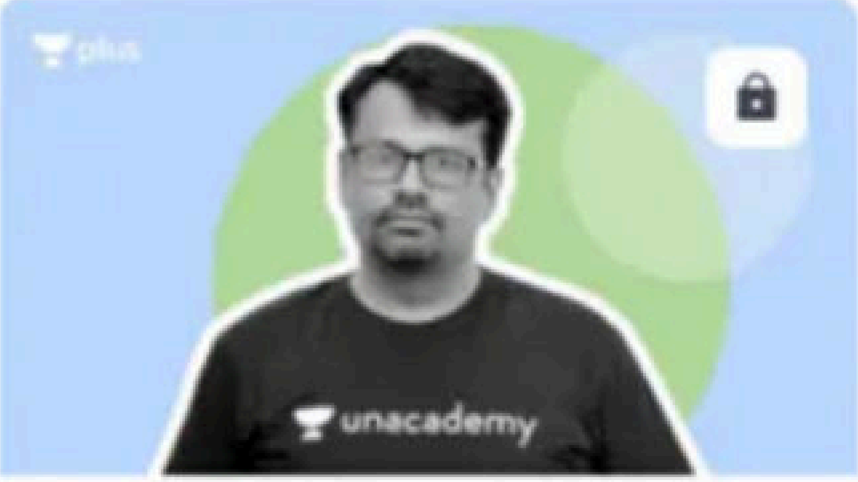

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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