Transformation of Variables :

Sometime, it is convenient to solve the double integral by transforming the variables.

(A) Transformation in polar form :

1. Let $\iint f(x,y)dxdy$ is a integration in cartisian form, then put x = r cosθ, y = r sin θ in givenintegration.

2. $$dxdy = \frac{\partial(x,y)}{\partial(r,\theta)} drd\theta$$

$$dxdy = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} d\theta dr$$

dxdy = r dθdr

Putting this value, then we get $\iint f(r,\theta)rd\theta dr$.

Q.1. The integral $\iint_R e^{x^2+y^2}\,dydx$, where R is the semicircle region bounded by the x – axis and the curve $y=\sqrt{1-x^2}$ equals **SAU 2017**

(a) $\frac{\pi}{2}(e+1)$

(b) $\frac{\pi}{2}(e-1)$

(c) $\frac{\pi}{2}(e^2)$

(d) $\frac{\pi}{2}e$

Q.2. Let p and t be positive real numbers. Let D_t be the closed disc of radius t center (0,0) i.e. $D_t = \{(x,y) : x^2 + y^2 \leq t^2$.

Define $I(p,t) = \iint_{D_t} \frac{dx\,dy}{\left(p^2 + x^2 + y^2\right)^p}$

Then $\lim_{t\to\infty} I(p,t)$ is finite **IIT JAM 2021**

(a) only if $p > 1$

(b) only if $p < 1$

(c) only if $p = 1$

(d) for no value of p

Q.3. The value of the real number m in the following equation

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \left(x^2+y^2\right) dydx = \int_{m\pi}^{\pi/2} \int_0^{\sqrt{2}} r^3 drd\theta$$ is **IIT JAM 2016**

(a) 0 (b) 1

(c) 2 (d)1/4

Q.4. Let $I=\int_{0}^{2}\int_{\sqrt{4-y^2}}^{\sqrt{9-y^2}} 2xydxdy+\int_{2}^{3}\int_{2}^{\sqrt{9-y^2}} 2xydxdy$.**IIT-JAM 2010**

Then using the transformation $x = r\cos\theta$, $y = r\sin\theta$, integral I is equal to

(a) $\int_{0}^{\pi/2}\int_{0}^{3} r^2\sin 2\theta dr d\theta$

(b) $\int_{0}^{\pi/2}\int_{0}^{2} r^3\sin 2\theta dr d\theta$

(c) $\int_{0}^{\pi/2}\int_{0}^{3} r^3\sin 2\theta dr d\theta$

(d) $\int_{0}^{\pi/2}\int_{0}^{-3} r^2\sin 2\theta dr d\theta$

Q.5. The value of $\iint_D (x+2y)dxdy$, where D is region in the xy-plane bounded by the straight line $y = x + 3$, $y = x - 3$, $y = -2x + 4$ and $y = -2x + 2$. **IIT JAM – 2007**

(a) 10 (b) 11

(c) 12 (d) 13