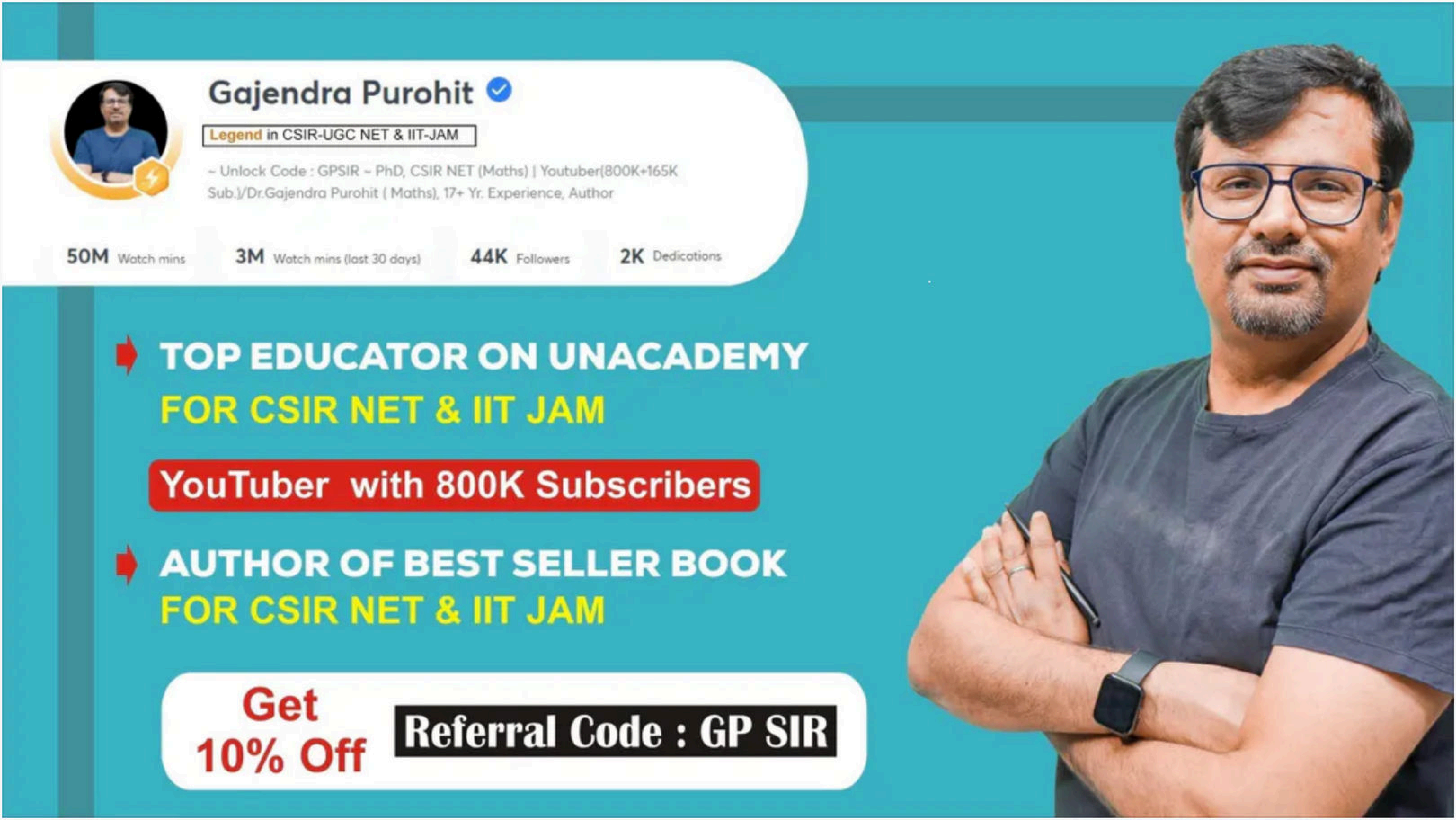
Group Homomorphism

Detail Course 2.0 on Group Theory for IIT JAM '23

Gajendra Purohit • Lesson 10 • Oct 18, 2022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Conjugate Class and Class Equation

Conjugate elements in a group :

Let G be a group, $a \in G$ and $b \in G$. Then b is said to be conjugate to a.

If $b = xax^{-1}$ for some $x \in G$.

Conclusion : Two elements of S_n are said to be conjugate to each other if they have same cycle decomposition

$Q_8 = \langle \pm 1, \pm i, \pm j, \pm k \rangle$

$a = 1$ $\quad 8 = 1 + 1 + 2 + 2 + 2$

$x \cdot 1 \cdot x^{-1} = 1$

$x(-1)x^{-1} = -1$

$x(i)x^{-1} = \quad \langle i, -i \rangle$

$x(j)x^{-1} \quad \langle j, -j \rangle$

$x(k)x^{-1} \quad \langle k, -k \rangle$

$1 \cdot i \cdot 1 = i$

$(-1)\, i\, (-1) = i$

$i\,(i)\,(-i) = i$

$(-i)\,(i)\,(i) = i$

$j\,(i)\,(-j) =$

$-jk = -i$

$k\,(i)\,(-k) = -k(-j)$

$= kj = -i$

$S_3 = \{ e, (12), (13), (23), (123), (132) \}$

$x(e)x^{-1} = e$

$x(12)x^{-1} = \{ (12), (13), (23) \}$

$x(123)x^{-1} = \{ (23), (132) \}$

$1 + 3 + 2$

$(12)(12)(12) = (12)$

$(13)(12)(13)$

$(123)(13) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = (23)$

$(23)(12)(23) = (23)(123)$

$= \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = (13)$

$(123)(12)(132) = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$

$= (23)$

$K_4 = \langle e, a, b, ab \rangle$

$x(a)x = a$

$Cl(a) = \langle a \rangle$

$Cl(b) = \langle b \rangle$

$Cl(ab) = \langle ab \rangle$

$1 + 1 + 1 + 1$

$a \cdot a \cdot a = a$

$bab = bba = a^2$

$ab\,a\,ab$

$ab\,a^2\,b$

abb^2

$ab^{-1} = a$

$Z_4 = \langle 0, \underbrace{1, 2, 3}\rangle$, +4

$x a x^{-1} = x + a + x^{-1}$

$cl(1) = 1$

$cl(2) = 2$

$cl(3) = 3$

$= 1+1+1+1$

$1+2+3 = 2$

$2+2+2 = 2$

$3+2+1 = 2$

Note : In abelian group, every element is conjugate to itself.

Self Conjugate element :

Let G be a group of an element a $\in$ G is called self conjugate element of G if a is conjugate to itself only.

$xax^{-1} = a$

Conjugacy classes : The set of all conjugate elements of 'a' is called conjugacy classes of a in G and it is denoted by Cl(a) i.e. $Cl(a) = \{xax^{-1} ; x \in G\}$

Result : Number of distinct conjugate classes in S_n are partion of S_n.

$Cl(12) = \{(12), (23), (13)\}$

$Cl(12) = \langle(12)\rangle$

S_3

1

2

3

S4

S3

1+1+1

1+2

3

1+3+2

1+1+1+1

1+1+2

2+2

1+3

4

S_4

$1+1+1+1$ 1

$1+1+2$ 6

$2+2$ 3

$1+3$ 8

4 6

$24 = 1 + 6 + 3 + 8 + 6$

A_4

$(a\,b)$

$(a\,b)(c\,d)$

$1+1+1+1$ 1

$2+2$ 3

$1+3$ 8

$12 = 1 + 3 + 8$

$12 = 1 + 3 + 4 + 4$

Result :

(1) If a is self conjugate element then $Cl(a) = \{a\}$

(2) $Cl(e) = \{e\}$

(3) $Z(G)$, the centre of the group is the collection of all self conjugate elements.

i.e. if $a \in Z(G)$ then $Cl(a) = \{a\}$

(4) If G be a group a and b are conjugate element then $O(a) = O(b)$

i.e. if $O(a) \neq O(b)$ then a and b not conjugate to each other.

Example : In S_3, $(1\ 2)$ and $(1\ 2\ 3)$ can not be conjugate because order are different.

(5) If $O(a) = O(b)$ then we cannot say that a and b are conjugate.

Example : Let $G = Z_4$

Here 1 & 3 have same order but both are not conjugate to each other, they are self conjugate element.

$Z_4 = \langle 0, 1, 2, 3 \rangle$

$Cl(0) = 0$

$Cl(1) = 1$

$Cl(2) = 2$

$Cl(3) = 3$

$K_4 = \langle e, a, b, ab \rangle$

$Z(K_4) = K_4$

$Q_8 = \langle \pm 1, \pm i, \pm j, \pm k \rangle$

$Z(Q_8) = \langle 1, -1 \rangle$

$Z(S_2) = S_2$

$C_\ell(i) = \langle i, -i \rangle$

$S_3 = \langle e, (12), (13), (23), (123), (132) \rangle$

$Z(S_3) = e$

$Z(S_n) = \langle e \rangle$

$n \geq 3$

$C_\ell(123) = \langle (123), (132) \rangle$

Q.1. Let G be a group of order p^2 then $|Cl(a)|$ is, $a \in G$.

(a) 0 (b) 1

(c) p (d) p^2

$Cl(a) = \{a\}$

$1 < 4$

Q.1. Total number of distinct conjugate classes S_4 are

(a) 2 (b) 3

(c) 5 (d) 7

$1+1+1+1$

$1+1+2$

$2+$

$1+3$

4

Class equation : Let G be a finite group of order n and $c_1, c_2, \ldots. c_k$ be k-distinct conjugate classes of cardinality $n_1, n_2, \ldots., n_k$ then the expression is called class equation of G.

Result :

(1) Let G be a finite group then $O(G) = O[Z(G)] + \sum_{a \notin Z(G)} O[Cl(a)]$

Here $O[Cl(a)] = \dfrac{O(G)}{O[N(a)]}$

$\Rightarrow \quad O[Z(G)] + \sum_{a \notin Z(G)} \dfrac{O(G)}{O[N(a)]}$

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Normalizer or Centralizer of an elements :

Let G be a group and $a \in G$ then $N(a) = \{ x \in G \mid ax = xa$, for all $x \in G$

Note :

(i) N(a) is a subgroup of G

(ii) $a \in Z(G) \Leftrightarrow N(a) = G \Leftrightarrow cl(a) = \{a\}$

(iii) Let G be a finite group and $a \in G$ then $O(cl(a)) = \dfrac{O(G)}{O(N(a))}$

(iv) Number of elements of G which commute to a is $O(N(a)) = \dfrac{O(G)}{O(cl(a))}$

$$Cl(g) = \frac{o(G)}{o N(g)}$$

$$o(N(i)) = \frac{o(G)}{cl(i)}$$

$$= \frac{8}{2} = 4$$

$$Q_8 = \langle \pm 1, \pm i, \pm j, \pm k \rangle$$

$$cl(i) = \langle i, -i \rangle$$

$$N_G(i) = \langle 1, -1, i, -i \rangle$$

$$o(N(i)) = 4$$

$$S_3 = \{e, \underline{(12)}, \underline{(13)}, (23), (123), (132)\}$$

$$Cl(12) = \{(12), (13), (23)\}$$

$$N(12) = \{e, (12)\}$$

$$Cl(12) = \frac{O(S)}{ON(12)}$$

$$ON(12) = \frac{O(S)}{Cl(M)} = \frac{6}{3} = 2$$

S_{10}

$5+1+1+1+1+1$

$(abcde)$

$$OL(S) = \frac{O(S)}{ON(S)}$$

$$\frac{10!}{1^5 \cdot 5! \, 5} = \frac{10!}{ON(S)}$$

$$ON(S) = 5! \times 5$$

$$= 120 \times 5 = 600$$

Q.2. Determine which of the following cannot be the class equation of a group. **CSIR NET DEC 2013**

(a) $10 = 1 + 1 + 1 + 2 + 5$

(b) $4 = 1 + 1 + 2$

(c) $8 = 1 + 1 + 3 + 3$

(d) $6 = 1 + 2 + 3$

Q.3. Which of the following cannot be class equation of group of order 10. **CSIR NET JUNE 2015**

(a) $10 = 1 + 1 + 1 + 2 + 5$ (b) $10 = 1 + 2 + 3 + 4$

(c) $10 = 1 + 2 + 2 + 5$ (d) $10 = 1 + 1 + 2 + 2 + 2 + 2$

Q.4. Let G be a group of order 9, then class equation is

(a) $9 = 3 + 2 + 3 + 1$

(b) $9 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$

(c) $9 = 1 + 1 + 2 + 2 + 3$

(d) $9 = 2 + 2 + 2 + 2 + 1$

Q.5. Which of the following class equation for group?

(a) $3 = 1 + 2$

(b) $6 = 1 + 2 + 3$

(c) $14 = 1 + 7 + 2 + 2 + 2$

(d) $7 = 1 + 1 + 3 + 2$

2×7

(2,7) 7

7

6

(a) a, b

(b) b, c

(c) c, d

(d) a, d

Q.6. Let G be a group of all non – singular matrices of order n under multiplication i.e. $G = GL(n, Z_8)$, then number of self conjugate elements of G are

(a) Infinite

(b) Only one

(c) Finite but more than 10

(d) Finite but less than 10

Q.7. Number of elements of S_4 which commute to $\sigma = (1\ 2\ 3)$ are

(a) 1 (b) 2

(c) 3 (d) 5

$3(-1)$

$$o(cl(\sigma)) = \frac{o(S)}{o(N(\sigma))} = 4!$$

$$o(N(\sigma)) = 3 \qquad \frac{4!}{3 \times 1} = \frac{4!}{o(N(\sigma))}$$

Q.8. Number of elements of S_5 which commute to $\sigma = (1\ 2\ 3)(4\ 5)$ are

(a) 1 (b) 10

(c) 15 (d) 6

$$\frac{5!}{3 \times 2} = \frac{\cancel{5!}}{o(\sigma)}$$

$o(\sigma) = 6$

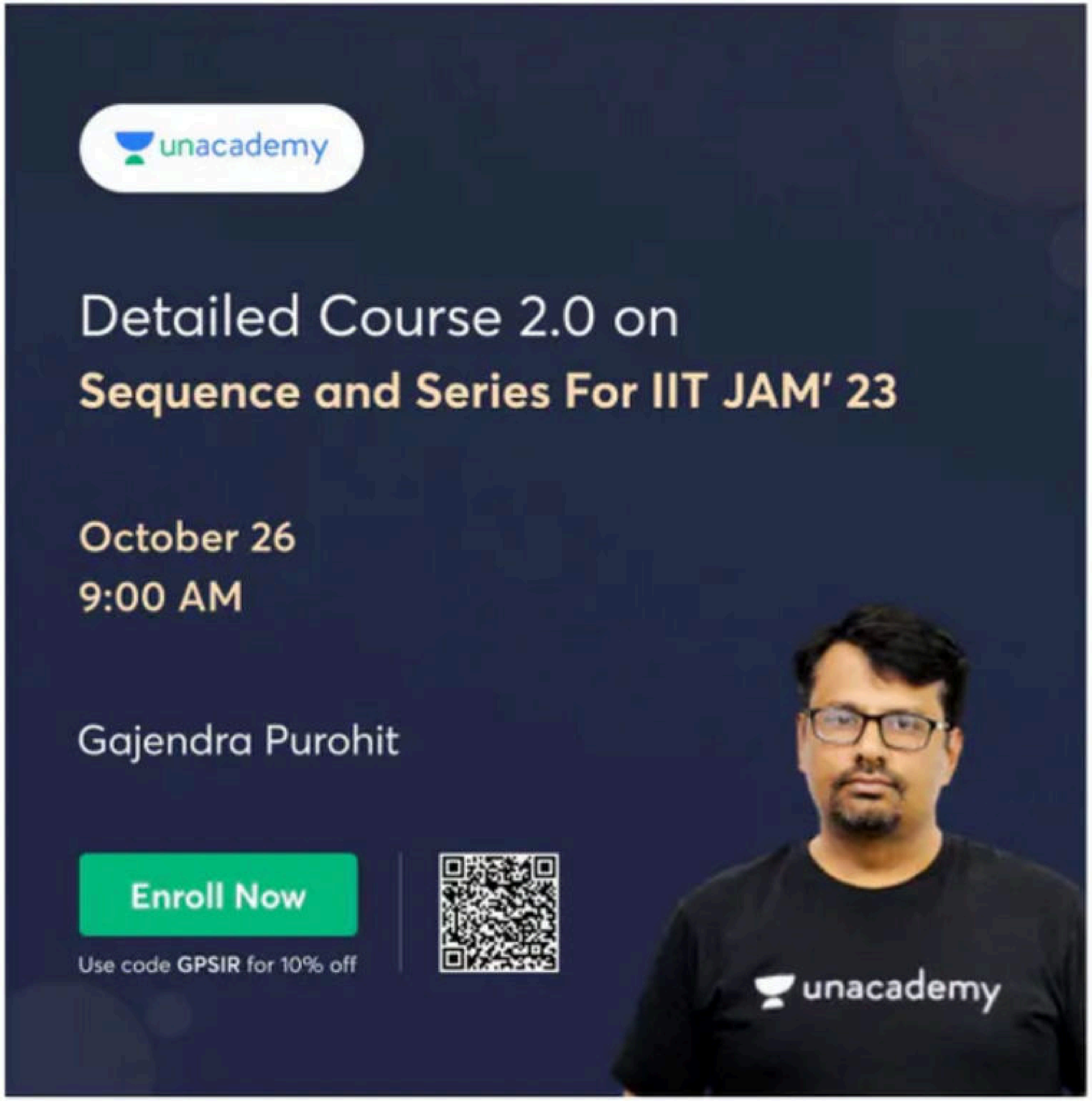
Q.9. which of the following are possible class equation of given order

(A) $39 = 1 + 3 + 3 + 3 + 3 + 13 + 13$

(b) $14 = 1 + 1 + 2 + 2 + 2 + 2 + 2 + 2$

(c) $21 = 1 + 3 + 3 + 7 + 7$

(d) $15 = 1 + 1 + 5 + 5 + 3$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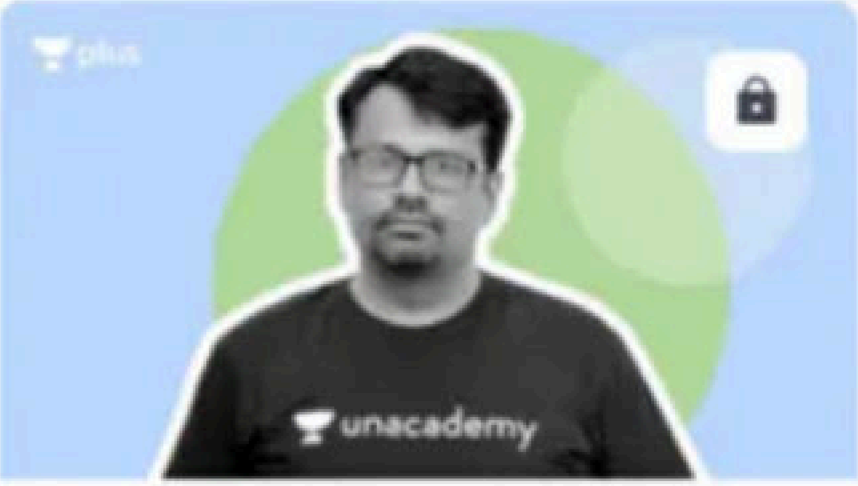

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