

The area of the planar region bounded by the curves $x = 6y^2 - 2$ and $x = 2y^2$ is

(a) $\frac{\sqrt{2}}{3}$

(b) $\frac{2\sqrt{2}}{3}$

(c) $\frac{4\sqrt{2}}{3}$

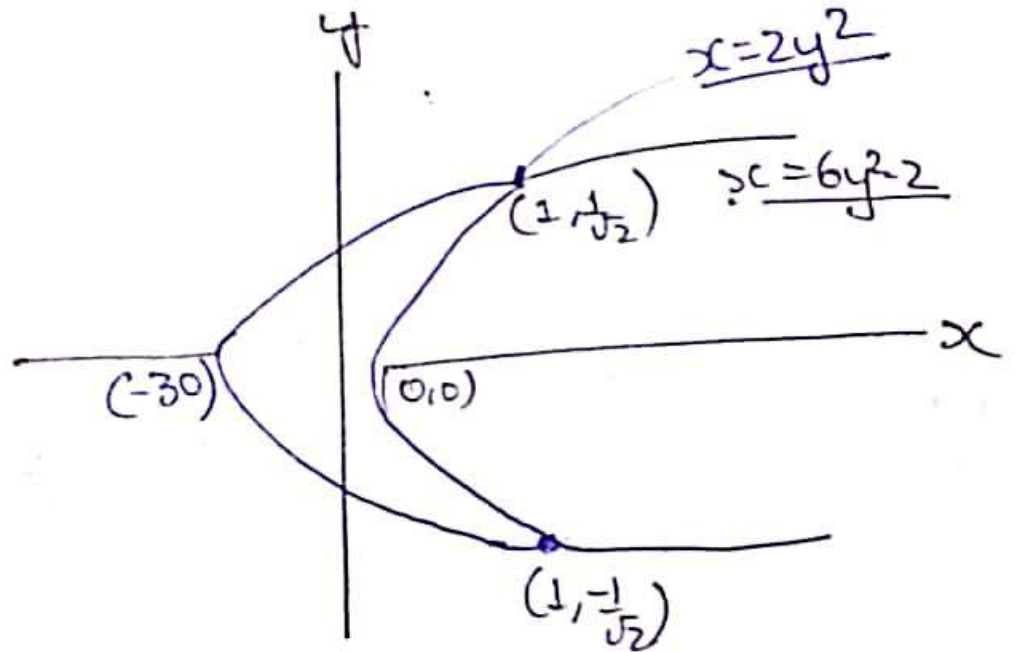
(d) $\sqrt{2}$

1) [C] Area = $\int_{y=-\frac{1}{\sqrt{2}}}^{\frac{1}{2}} \int_{x=6y^2-2}^{2y^2} dx dy$

$$= 2 \int_0^{\frac{1}{\sqrt{2}}} (2-4y^2) dy$$

$$= \underline{\underline{\frac{4\sqrt{2}}{3}}}$$

$$[\because \text{Area} = \iint_R dx dy]$$



Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a $f(x) = \int_{-5}^x (t-1)^3 dt$. In which

of the following interval(s), f takes the value 1?

- (a) If $f(x) \leq g(x)$ for all $x \in [0, 1]$, then $\sup R(f) \leq \inf R(g)$
- (b) If $f(x) \leq g(x)$ for some $x \in [0, 1]$, then $\inf R(f) \leq \sup R(g)$
- (c) If $f(x) \leq g(y)$ for some $x, y \in [0, 1]$, then $\inf R(f) \leq \sup R(g)$
- (d) If $f(x) \leq g(y)$ for all $x, y \in [0, 1]$, then $\sup R(f) \leq \inf R(g)$

[A, C, D]

2) Here $f(x) = \int_{-5}^x (t-1)^3 dt$

$$\Rightarrow \frac{(x-1)^4}{4} - 324$$

$$f(x) = 1$$

$$\Rightarrow (x-1)^4 = 1300$$

; $x \in 7, -5$

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Let C be the straight line segment from $P(0, \pi)$ to $Q\left(4, \frac{\pi}{2}\right)$, in the xy -plane. Then the value of

$$\int_C e^x (\cos y dx - \sin y dy) \text{ is } \underline{\hspace{2cm}}$$

3) Subjective Let $I = \int_C e^x (\cos y dx - \sin y dy)$.

[where C is straight line segment from $P(0, \pi)$ to $Q(4, \frac{\pi}{2})$ in x, y , plane].

Now, $I = \int_C d(e^x \cos y) = [e^x \cos y]_{(0, \pi)}^{(4, \frac{\pi}{2})}$

$$= e^4 \cos \frac{\pi}{2} - \cos(\pi)$$

$$\Rightarrow -(-1) = \underline{\underline{1}} \text{ Answer.}$$