Double Integrals

Let f(x, y) is a function of two variable then double integral of f(x, y) is denoted by $\iint f(x,y)\,dx\,dy$ or $\iint f(x,y)\,dy\,dx$.

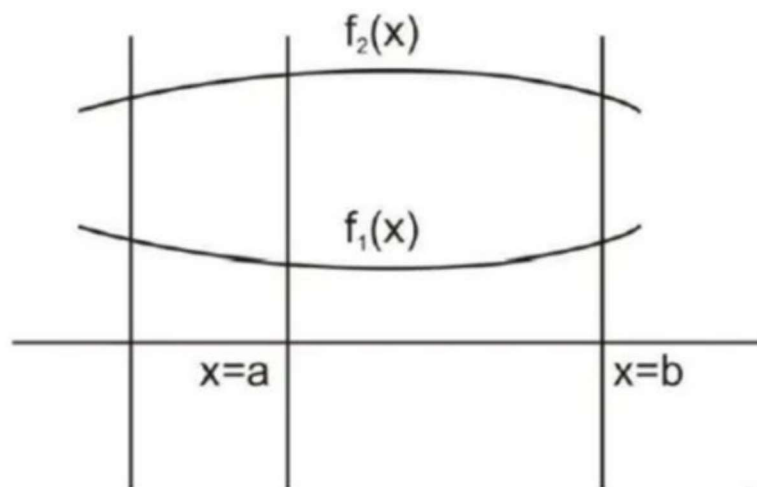**Note :**

(1) For $\iint f(x, y)\,dx\,dy$, f(x,y) is first integrated w.r.t. x and then it is integrated w.r.t. y.
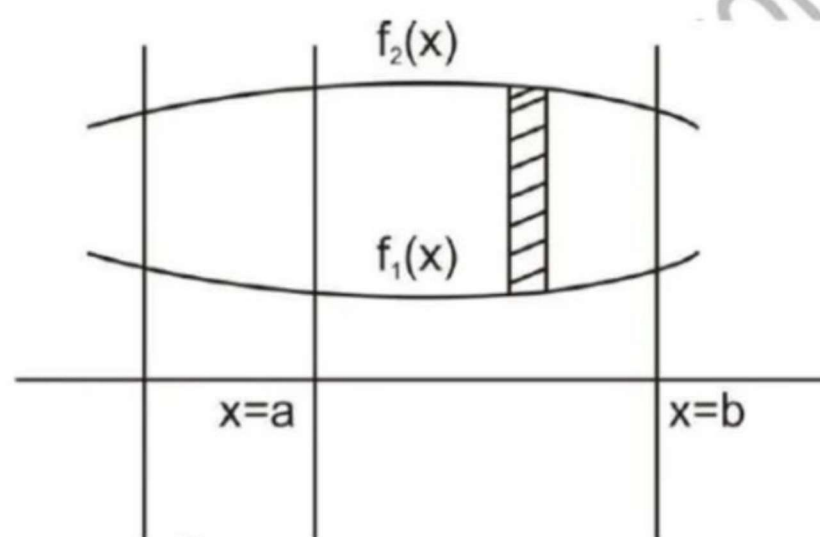
(2) For $\iint f(x, y)\,dy\,dx$, f(x, y) is first integrated w.r.t. y and then it is integrated w.r.t. x.

Find limit by a given curve :

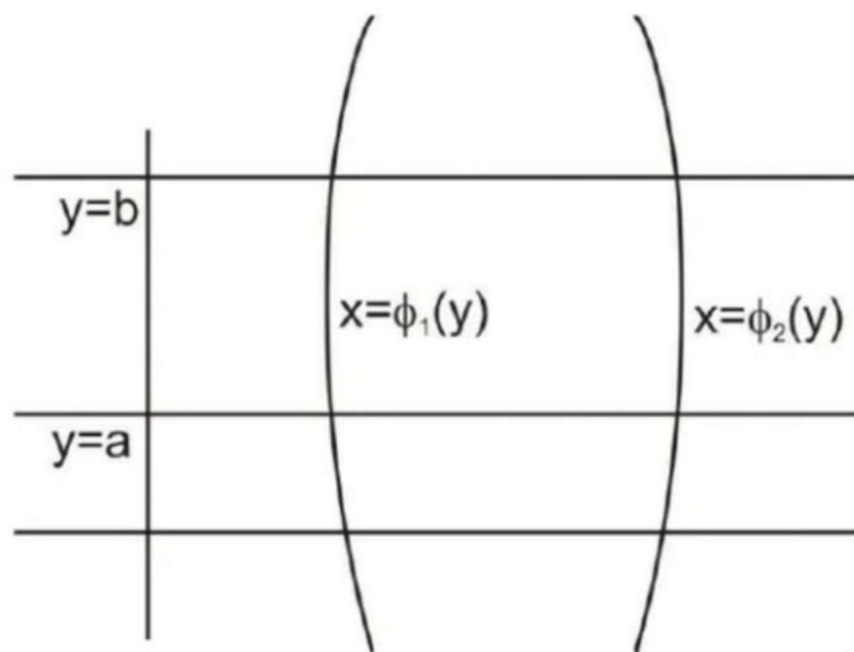(a)   If the region A is bounded by the curves $y = f_1(x)$ & $= f_2(x)$ and the coordinate $x = a$ and $x = b$.
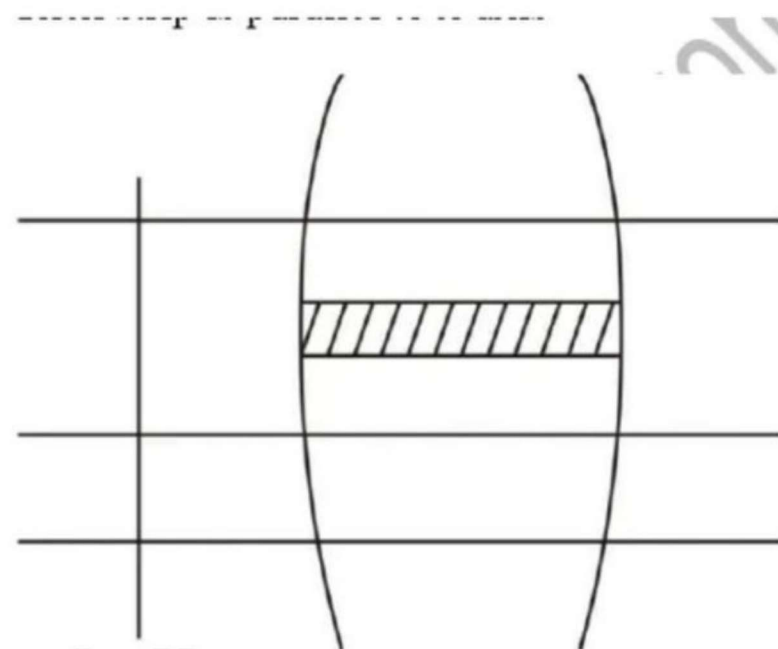

Then strip is parallel to y-axis

Then $\displaystyle\iint_A f(x, y)\,dA = \int_{x=a}^{b} \int_{y=f_1(x)}^{y=f_2(x)} f(x, y)\,dy\,dx$

(b) If the region A is the area bounded by the curve $x = \phi_1(y)$ & $x = \phi_2(y)$ and coodinate $y = a$ and $y = b$.

Then strip is parallel to x-axis

So, $\iint\limits_A f(x, y)\,dA = \int\limits_{y=a}^{y=b} \int\limits_{x=\phi_1(y)}^{x=\phi_2(y)} f(x, y)\,dx\,dy$

Q.1. The value of $\iint\limits_R x e^{y^2} \, dx \, dy$, where R is the region bounded by the line $x = 0$, $y = 1$ and the parabola $y = x^2$. **IIT JAM-2006**

(a) $-\dfrac{1}{4}[e-1]$

(b) $\dfrac{1}{4}[e-1]$

(c) $\dfrac{1}{4}[e+1]$

(d) None

Q.2. The value of $\iint xy(x+y)\,dx\,dy$ over the area between $y^2 = x$ and $y = x$

(a) $\dfrac{1}{56}$

(b) $\dfrac{3}{56}$

(c) $\dfrac{5}{56}$

(d) $\dfrac{3}{55}$

Q.3. The value of $\iint\limits_{R} xy\,dx\,dy$, where R is the quadrant of the

circle $x^2 + y^2 = a^2$

(a) $a^4/8$ (b) $a^2/8$

(c) $a/8$ (d) $3a/2$

Q.4. The value of $\iint\limits_{A} xy\,dy\,dx$ where A is domain bounded by x-axis ordinate $x = 2a$ and the curve $x^2 = 4ay$.

(a) $a/7$

(b) $a^4/3$

(c) $a^4/5$

(d) $a/5$

Q.5 The value of integral $\displaystyle\int_0^3 \int_0^{\sqrt{3}x} \frac{dy\,dx}{\sqrt{x^2+y^2}}$ .JAM-2008

(a) $3\log(\sqrt{3}-2)$ (b) $\log(\sqrt{3}+2)$

(c) $3\log(\sqrt{3}+2)$ (d) $-3\log(\sqrt{3}+2)$

Q.6. The value of the integral $\iint\limits_{D} \dfrac{\sin(2x)}{x}\,dx\,dy$ .where D denotes the region bounded by the x – axis and the lines y = x and x = 1. **IIT JAM 2007**

(a) $-\dfrac{\cos 2}{2}+\dfrac{1}{2}$

(b) $\dfrac{\cos 2}{2}$

(c) $\cos 2$

(d) $\sin 2$