Gajendra Purohit

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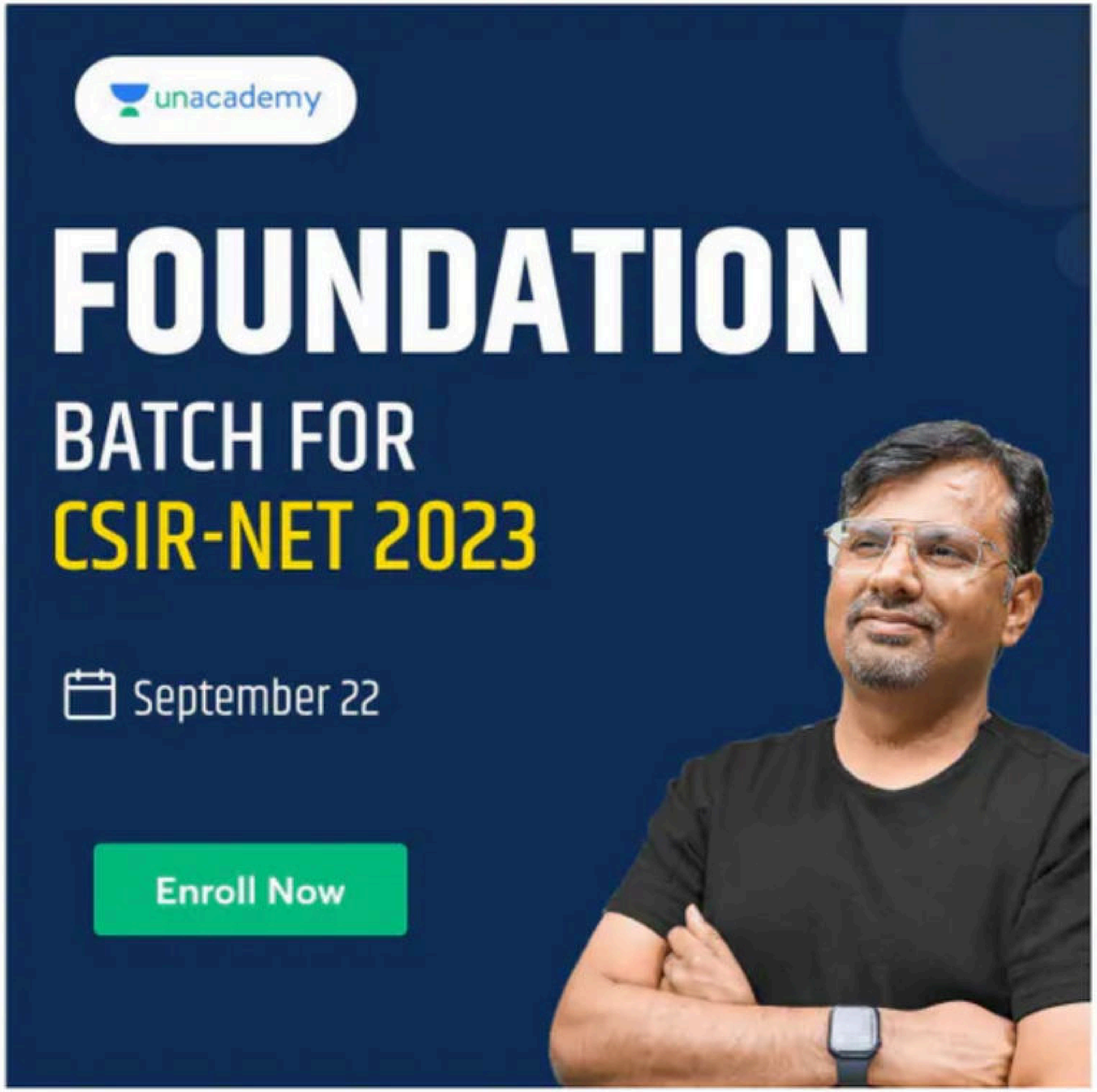
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TOPIC -: GROUP THEORY

LECTURE NO. 01

Basic definition related to group theory

Lecture Index:- i) Euler's function & Sum of relative Prime no.

ii) Congruent Modulo & its theorem

iii) No. of Positive Divisors

iv) Sum of positive divisors

1. <u>Euler's ϕ function</u> -:

$x \neq y \quad \phi(x) \neq \phi(y)$

A mapping $\phi : N \to N$ defined by $\phi(n) = |\{x \in \mathbb{N} ;\ 1 \leq x \leq n;\ \gcd(x,n) = 1\}|$

Example : $\phi(6) = |\{1,5\}| = 2$

$\phi(9) = |\{1,2,4,5,7,8\}| = 6$

- **Some shortcuts to find Euler's Phi Function**

➢ If p is a prime number and **a** is a positive integer then

$\phi(p^a) = (p^a - p^{a-1})$

$\phi(100) = \phi(2^2 \times 5^2)$

$= \phi(2^2)\,\phi(5^2)$

$= (2^2 - 2^1)(5^2 - 5^1)$

$= (4-2)(25-5)$

$= 2 \times 20 = 40$

➢ If $p_1, p_2, \dots p_n$ are prime numbers and a, b, …. z are the positive integers

$$\phi(p_1^a . p_2^b \dots p_n^z) = (p_1^a - p_1^{a-1})(p_2^b - p_2^{b-1}) \dots . (p_n^z - p_n^{z-1})$$

➢ $\phi(m.n) = \phi(m). \phi(n)$; iff $\gcd(m,n) = 1$

- **Sum of relative prime numbers -:** Sum of all positive integers (including unity) which are less than and co-prime to it.

or

$S = \{x \in \mathbb{N} ; 1 \leq x \leq n; \gcd(x,n) = 1\}$, Sum of all elements of set S

10

$1 + 3 + 7 + 9 = 20$

$S = \frac{n}{2} \phi(n)$

$= \frac{10}{2} \phi(10)$

$= 5 \phi(5 \times 2)$

$= 5 \phi(5) \phi(2)$

$= 5 \times 4 \times 1 = 20$

12

(a) 48 (b) 96

(c) 24 (c) 12

$$\frac{12}{2}\phi(12)$$

$$6\,\phi(2^2\times 3)$$

$$6\,(2^2-2^1)\,(3^1-3^0)$$

$$6\times 2\times 2 = 24$$

2. **Number of positive divisors, $\tau(n)$ -:** let $n>1$, n be a positive integer,

$n = p_1^a . p_2^b . p_3^c \dots . p_n^z$, where $p_1, p_2, p_3, \dots, p_n$ are prime number, then number of positive divisors of n is denoted by $\boldsymbol{\tau(n) = \tau(p_1^a . p_2^b . p_3^c \dots . p_n^z) = (a+1).(b+1).(c+1)\dots(z+1)}$.

$\tau(600) = \tau(2^3 \times 3^1 \times 5^2)$

$(3+1)(1+1)(2+1)$

$4 \times 2 \times 3$

24

10

1, 2, 5, 10

$\tau(10)$

$\tau(2^1 \times 5^1)$

$(1+1) \times (1+1) = 4$

$\tau(3000)$

(a) 27

(b) 12

(c) 16

(d) 32

$\tau(3^1 \times 2^3 \times 5^3)$

$(1+1)(3+1)(3+1)$

$2 \times 4 \times 4 = 32$

3. **Sum of positive divisors, σ(n)-:** let n>1, n be a positive integer, $n = p_1^a . p_2^b . p_3^c \dots . p_n^z$, where $p_1, p_2, p_3, \dots p_n$ are prime number, then sum of positive divisors of n is denoted by σ(n).

$$\boldsymbol{\sigma(n) = \left(\frac{p_1^{a+1}-1}{p_1-1}\right).\left(\frac{p_2^{b+1}-1}{p_2-1}\right).\left(\frac{p_3^{c+1}-1}{p_3-1}\right)\dots.\left(\frac{p_n^{z+1}-1}{p_n-1}\right).}$$

$2^1 \times 5^1$

$\sigma(n)$

10

1 + 2 + 5 + 10

18

$\sigma(10) = 18$

$$\sigma(10) = \frac{(2^2-1)}{2-1} \frac{(5^2-1)}{(5-1)} = \frac{3}{1} \times \frac{\cancel{24}^{6}}{\cancel{4}} = 18$$

$6(3000)$

(a) 2340 (b) 4680

(c) 9360 (d) 6870

$6(3^{1} 2^{3} 5^{3})$

$\frac{(3^{2}-1)}{(3-1)} \frac{(2^{4}-1)}{(2-1)} \frac{(5^{4}-1)}{(5-1)}$

$\frac{8}{2} \times \frac{15}{1} \times \frac{624}{4}$

4. **<u>Congruent modulo</u>** -: let n be a fixed positive integer, two integer **a** and **b** are congruent modulo n if $n \mid (a-b)$ and it is denoted by $a \equiv b \pmod{n}$

i.e. $a \equiv b \pmod{n}$ if $n \mid (a-b)$

$\Rightarrow a - b = nk \in \mathbb{Z} \quad \Rightarrow a = b + nk$

Note : $a \equiv b \pmod{n} \Leftrightarrow b \equiv a \pmod{n}$

Some special theorem

- **Fermat's Theorem-:** If p is a prime number, a is integer and $p \nmid a$ [p does not divide a], then $\mathbf{a^{p-1} \equiv 1 (mod\ p)}$.

  Example : $2^{10} (\text{mod } 11) = 1 \Leftrightarrow 2^{10} \equiv 1 (\text{mod } 11)$ $[\because 11 \nmid 2]$

- **Euler's theorem-:** If $n \geq 1$ and $\gcd(a,n) = 1$, then $\mathbf{a^{\phi(n)} = 1 (mod\ n)}$.

- **Wilson's Theorem-:** If p is a prime number, then **$(p - 1)! \equiv (-1) \pmod{p}$.**

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Q.1 Which of the following statements involving Euler's function ϕ is/are true?

(a) $\phi(n)$ is even as many times as it is odd

(b) $\phi(n)$ is odd for only two values of n

(c) $\phi(n)$ is even when $n > 2$

(d) $\phi(n)$ is odd when $n = 2$ or n is odd

$\phi(1) = 1$

$\phi(2) = 1$

$\phi(3) = 2$

$\phi(4) = 2$

$\phi(5) = 4$

Q.2. Find the total number of divisors of number 38808 excluding 1 and the number itself.

(a) 72 (b) 71

(c) 70 (d) 74

1

$38808 = 2^3\, 3^2\, 7^2\, 11$

$= (3+1)(2+1)(2+1)(1+1)$

$= 4 \times 3 \times 3 \times 2$

$= 72 - 2$

$= 70$

Q.3. The remainder when $\sum_{r=1}^{100} r!$ is divided by 12 is

(a) 5 (b) 7

(c) 9 (d) 11

Q.4. Remainder when the sum $1^5+2^5+3^5+4^5+\ldots\ldots+99^5+100^5$ is divided by 4 is

(a) 0 (b) 1

(c) 2 (d) 3

Q.5. For Euler's ϕ function (ϕ : **N→N**), $\phi(n)$ is

(a) Always even number

(b) Neither one-one nor onto

(c) $\phi(1000) = 400$

(d) None of the above

a) a, b

b) b, c

c) c, d

d) a, d

Q.6. Find the Sum of positive divisors of 50 is.

(a) 31 (b) 20

(c) 06 (d) 93

$\sigma(50) = \sigma(2^1 \times 5^2)$

$\frac{(2^2-1)}{(2-1)} \frac{(5^3-1)}{(5-1)}$

$(4-1) \frac{(125-1)}{(5-1)}$

$3 \times \frac{124}{4} = 3 \times 31 = \underline{93}$

Q.7. Find the number of divisors of N =2520 (excluding unity)

……………..

(a) 41 (b) 42

(c) 45 (d) 47

$\tau(N) = \tau(2^3 5^1 3^2 7^1)$

$(3+1)(1+1)(2+1)(1+1)$

487

48

Q.8 let S be the set of all positive integers (including unity) which are less than 3969 and co-prime to it. What is the sum of all the elements of S?

(a) ~~6001125~~ (b) ~~6001128~~

(c) ~~6001130~~ (d) ~~6001344~~

$n = 3969$

$S = \frac{3969}{2}\phi(3969)$

$= \frac{3969}{2} \times 78 \times 42$

(a) 6501125 (b) 6501128

(c) 6501222 (d) 6501130

$\phi(3969) = \phi(3^4 \times 7^2)$

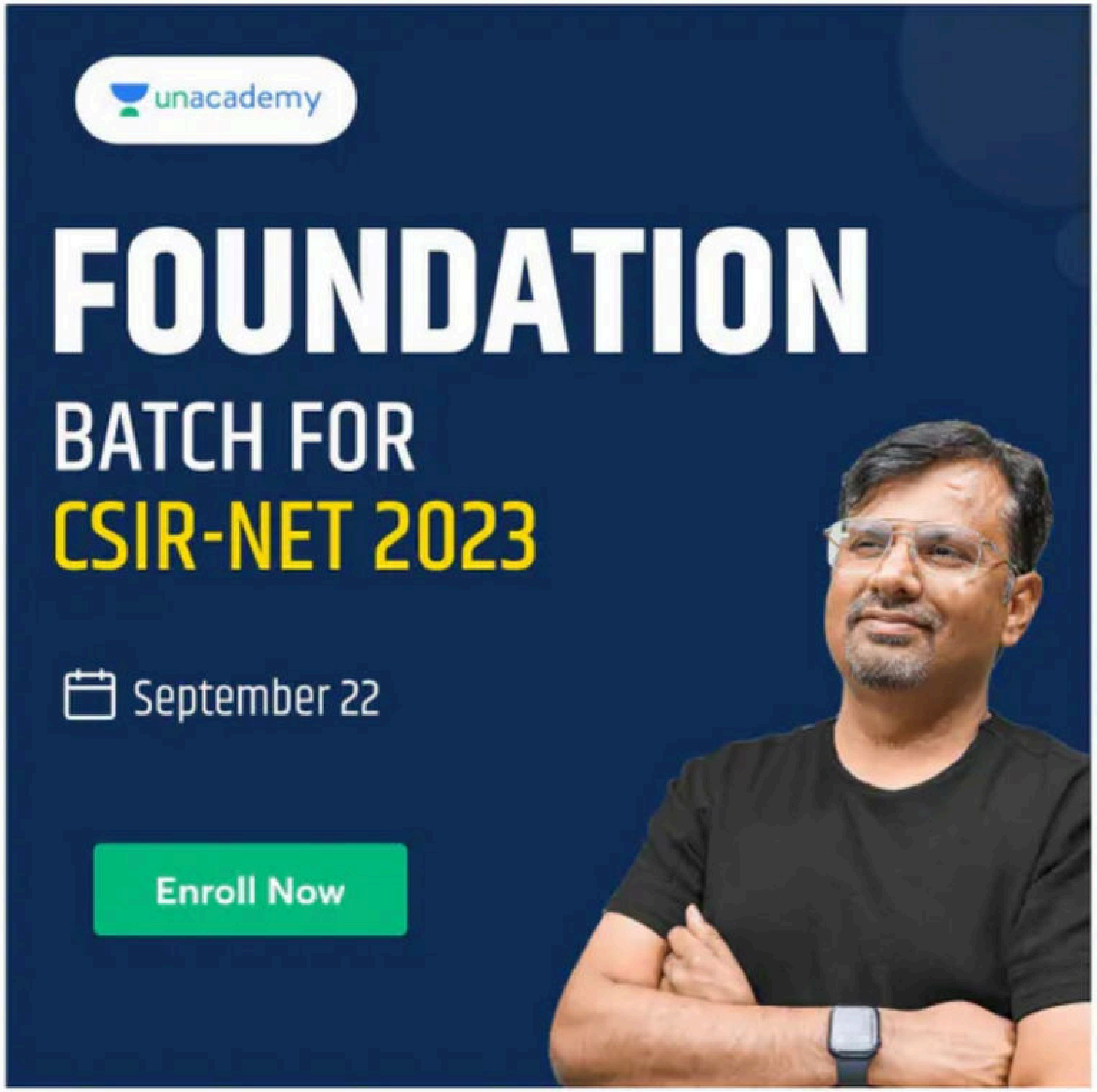
$= (3^4 - 3^3)(7^2 - 7)$

$= 78 \times 42$

Q.9. The number of positive divisors of 50,000 is

(a) 20 (b) 30

(c) 40 (d) 50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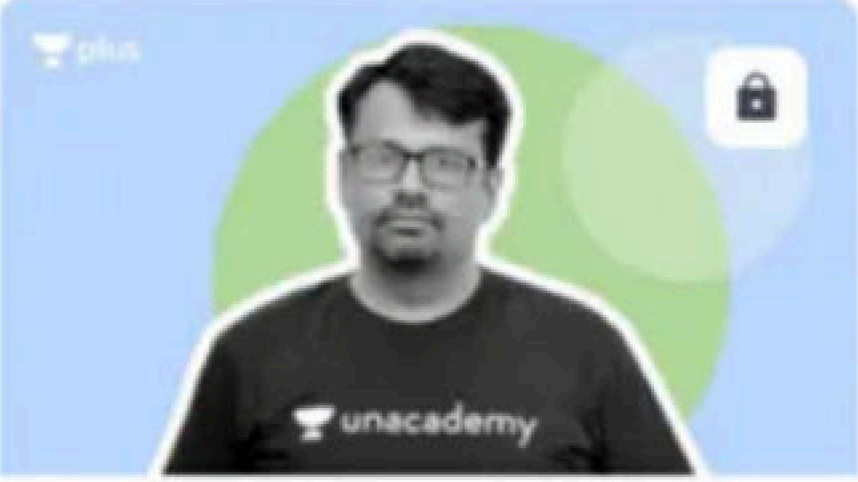

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Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

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