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## Detailed Course 2.0 on Sequence and Series For IIT JAM' 23

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# SEQUENCE OF REAL NUMBER

Some important theorem on Limit :

(1) If  $\lim_{n \rightarrow \infty} a_n = l$  then  $\lim_{n \rightarrow \infty} |a_n| = |l|$  But converse may not true

$$\langle a_n \rangle = \langle (-1)^n \rangle$$

$\langle (-1)^n \rangle$

(2) **Cauchy's First Theorem** : Let  $\langle a_n \rangle$  be a sequence of real numbers

and  $\lim_{n \rightarrow \infty} a_n = l$  then  $\lim_{n \rightarrow \infty} \frac{a_1 + a_2 + \dots + a_n}{n} = l$

$$\lim_{n \rightarrow \infty} \frac{1 + \frac{1}{2} + \dots + \frac{1}{n}}{n} = 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$



Q1. Find the Limit of  $\frac{1 + \sqrt{2} + \sqrt[3]{3} + \dots + \sqrt[n]{n}}{n} = 1$  **CSIR NET 2022**

(a) 0

~~(b) 1~~

(c) 2

(d)  $\infty$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 1$$

**Note :** The converse of this theorem may not be true

$$\langle a_n \rangle = \langle f(n) \rangle$$

$$\frac{a_1 + a_2 + \dots + a_n}{n} = \frac{(1+1) + \dots + f(n)}{n}$$

$$\begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{1}{n} & \text{if } n \text{ is odd} \end{cases}$$

$$\lim_{n \rightarrow \infty} \frac{a_1 + a_2 + \dots + a_n}{n} = 0$$

$$\lim_{n \rightarrow \infty} a_n = 0$$



**(2) Cauchy's Second theorem :** Let  $\langle a_n \rangle$  be a sequence of real number

and  $\lim_{n \rightarrow \infty} a_n = l$  Then  $\lim_{n \rightarrow \infty} (a_1 \cdot a_2 \cdots a_n)^{\frac{1}{n}} = l$

$$\lim_{n \rightarrow \infty} \left( \frac{2}{1} \left( \frac{3}{2} \right)^2 \left( \frac{4}{3} \right)^3 \cdots \left( \frac{n+1}{n} \right)^n \right)^{\frac{1}{n}} = e$$

$$\lim_{n \rightarrow \infty} \left( \frac{n+1}{n} \right)^n = \lim_{n \rightarrow \infty} \left( 1 + \frac{1}{n} \right)^n = e$$



**Q2.** Find the limit of  $\left[ (1)(2)^{\frac{1}{2}}(3)^{\frac{1}{3}} \dots \dots \dots (n)^{\frac{1}{n}} \right]^{\frac{1}{n}}$

(a) 0

~~(b) 1~~

(c) -1

(d) 2

$$\lim_{n \rightarrow \infty} (n)^{\frac{1}{n}} = 1$$



(3) Let  $\langle a_n \rangle$  be sequence of real number and  $a_n > 0 ; \forall n \in \mathbb{N}$

Then  $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} \Rightarrow \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}} = l ; l > 0$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left[ (n+1)(n+2) \dots (n+n) \right] = 4$$

$$\lim_{n \rightarrow \infty} \left[ \frac{(n+1)(n+2) \dots (n+n)}{n^n} \right]$$

$$\lim_{n \rightarrow \infty} \frac{(n+2)(n+3) \dots (2n+2)}{(n+1)^{n+1}}$$

$$\frac{n^n}{(n+1)(n+2) \dots (n+n)}$$

$$a_1 + a_2 + \dots + a_n$$

$$\frac{(2n+1)(2n+2) \dots n^n}{(n+1)^n (n+1)(n+2)}$$

$$\frac{4 \times (1 + \frac{1}{2}) (1 + \frac{1}{3})}{(1 + \frac{1}{4})^2 (1 + \frac{1}{5})^2}$$

$$= \frac{4}{e}$$



**NOTE :** The converse of this theorem is not true

$$\langle a_n \rangle = \frac{3 + (-1)^n}{2}$$

$$a_n = \begin{cases} 2 & n - \text{even} \\ 1 & n - \text{odd} \end{cases}$$

$$(a_n)^{1/n} = \begin{cases} 2^{1/n} & n - \text{even} \\ 1^{1/n} & n - \text{odd} \end{cases}$$

$$\lim_{n \rightarrow \infty} (a_n)^{1/n} = 1$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = 1$$

$$\frac{a_{n+1}}{a_n} = \left( \frac{3 + (-1)^{n+1}}{2} \right) \left( \frac{2}{3 + (-1)^n} \right)$$
$$= 1$$



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Q3.  $\lim_{n \rightarrow \infty} \frac{(n!)^{\frac{1}{n}}}{n}$  Which of following is true

(a) e

(c)  $e^2$

~~(b)  $1/e$~~

(d)  $1/e^2$

$$\lim_{n \rightarrow \infty} \left( \frac{n!}{n^n} \right)^{\frac{1}{n}}$$

$$a_n = \frac{n!}{n^n}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} &= \lim_{n \rightarrow \infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \lim_{n \rightarrow \infty} \frac{\cancel{(n+1)} n^n}{\cancel{(n+1)} (n+1)^{n+1}} \\ &= \lim_{n \rightarrow \infty} \frac{\cancel{n}^{\cancel{n}}}{\cancel{n}^{\cancel{n}} (1 + \frac{1}{n})^n} = \frac{1}{e} \end{aligned}$$



Q4.  $L = \lim_{n \rightarrow \infty} \frac{1}{\sqrt[n]{n!}}$  Then which of the following is true

- (a)  $L = 0$   
(b)  $0 < L < \infty$

- (b)  $L = 1$   
(d)  $L = \infty$

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$$\lim_{n \rightarrow \infty} \left( \frac{1}{n!} \right)^{\frac{1}{n}}$$

$$a_n = \frac{1}{n!}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} &= \lim_{n \rightarrow \infty} \frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} \\ &= \lim_{n \rightarrow \infty} \frac{n!}{(n+1)n!} = 0 \end{aligned}$$

Q

$$\langle S_k \rangle$$

$$S_k = k^{\frac{\alpha}{k}}, \quad k \geq 1, \alpha > 0$$

$$\lim_{n \rightarrow \infty} (S_1 S_2 \cdots S_n)^{\frac{1}{n}} = \lim_{n \rightarrow \infty} S_n$$

$$\lim_{n \rightarrow \infty} n^{\alpha_n} = \lim_{n \rightarrow \infty} \left( n^{\frac{1}{n}} \right)^{\alpha}$$

$$\textcircled{a} \quad 0 \quad \textcircled{4} \quad 1$$

$$\textcircled{c} \quad -1 \quad \textcircled{4} \quad 2$$

$$= 1^{\alpha}$$



$\langle x_n \rangle$  is monotonic and bounded

- ~~(a)~~  $\exists$  a subseq. of  $\langle x_n \rangle$  which diverges.
- ~~(b)~~  $\exists$  a subseq of  $\langle x_n \rangle$  which is not monotonic
- (c) All subseq. of  $\langle x_n \rangle$  are convergent to some limit
- ~~(d)~~  $\exists$  at least two subseq. of  $\langle x_n \rangle$  which diverge to different limits.



$\langle x_n \rangle$  be an unbounded seq in  $\mathbb{R}$

~~(a)~~  $\langle x_n \rangle$  has convergent subseq.

~~(b)~~  $\langle x_n \rangle$  has subseq.  $\langle \underline{x_{n_k}} \rangle$  s.t.  $\underline{x_{n_k}} \rightarrow 0$

(c)  $\langle x_n \rangle$  has a subseq.  $\langle \underline{x_{n_k}} \rangle$  s.t.  $\underline{\frac{1}{x_{n_k}}} \rightarrow 0$

~~(d)~~ Every subseq. of  $\langle x_n \rangle$  is unbounded

$\langle n \rangle$

$\langle \frac{1}{x_{n_k}} \rangle$

$\langle n \rangle$

$\langle 0, 1, 0, 2, 0, 3, 0, 4, \dots \rangle$



Q5.  $L = \lim_{n \rightarrow \infty} \left\{ \frac{(3n)!}{(n!)^3} \right\}^{\frac{1}{n}}$  Then which of the following is true

$$a_n = \frac{(3n)!}{(n!)^3}$$

(a)  $L = 0$

(b)  $L = 3$

(c)  $L = 27$

(d)  $L = 30$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \lim_{n \rightarrow \infty} \frac{(3n+3)!}{((n+1)!)^3} \cdot \frac{(n!)^3}{(3n)!}$$

$$= \lim_{n \rightarrow \infty} \frac{(3n+3)(3n+2)(3n+1)(3n)!}{(n+1)^3 (n!)^3} \cdot \frac{(n!)^3}{(3n)!}$$

$$= \lim_{n \rightarrow \infty} \frac{27n^3 \left(1 + \frac{1}{n}\right) \left(1 + \frac{2}{3n}\right) \left(1 + \frac{1}{3n}\right)}{n^3 \left(1 + \frac{1}{n}\right)^3}$$

$$= \frac{27}{1} = 27$$

(4) Let  $\langle a_n \rangle$  be a sequence of real number such that

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = l \quad \text{Where } |l| < 1 \text{ then } \lim_{n \rightarrow \infty} a_n = 0$$

(5) Let  $\langle a_n \rangle$  be a sequence of real number such that

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = l \quad \text{Where } |l| > 1 \text{ Then } \lim_{n \rightarrow \infty} a_n = \infty$$



**Q7 :**  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left[ \frac{1}{\sqrt{1} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{5}} + \dots + \frac{1}{\sqrt{2n-1} + \sqrt{2n+1}} \right]$

equals **CSIR NET JUNE 2014**

(a)  $\sqrt{2}$

(b)  $\frac{1}{\sqrt{2}}$

(c)  $\sqrt{2} + 1$

(d)  $\frac{1}{\sqrt{2} + 1}$

**Q8 :**  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left[ \frac{1}{\sqrt{2} + \sqrt{4}} + \frac{1}{\sqrt{4} + \sqrt{6}} + \dots + \frac{1}{\sqrt{2n} + \sqrt{2n+2}} \right]$

**CSIR NET DEC 2015**

- (a)  $\sqrt{2}$       (b)  $\frac{1}{\sqrt{2}}$
- (c)  $\sqrt{2} + 1$       (d)  $\frac{1}{\sqrt{2} + 1}$





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## Educator highlights

- Works at Pacific Science College
- Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in Udaipur, Rajasthan, India
- Unacademy Educator since



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