1

Let S be the bounded surface of the cylinder $x^2 + y^2 = 1$ cut be the planes $z = 0$ and $z = 1 + x$. Then the value of the surface integral $\iint_S 3z^2\, d\sigma$ is equal to

(a) $\int_0^{2\pi} (1 + \cos\theta)^3\, d\theta$

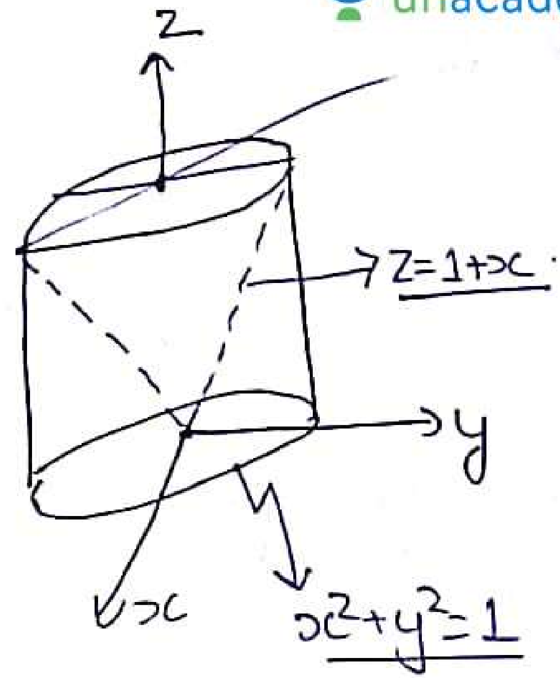
(b) $\int_0^{2\pi} \sin\theta \cos\theta\, (1 + \cos)^2\, d\theta$

(c) $\int_0^{2\pi} (1 + 2\cos\theta)^3\, d\theta$

(d) $\int_0^{2\pi} \sin\theta \cos\theta\, (1 + 2\cos)^2\, d\theta$

[A]

1) Let $I = \iint\limits_S 3z^2\, d\sigma$



Let $x = \cos\theta$.

$y = \sin\theta$.

$$= \int_{\theta=0}^{2\pi} \int_{z=0}^{1+x} 3z^2\, dz\, d\theta$$

$$= \int_{0}^{2\pi} \int_{z=0}^{1+\cos\theta} 3z^2\, dz\, d\theta.$$

$$= \int_0^{2\pi} (1+\cos\theta)^3\, d\theta$$

2

The limit $\lim_{x \to 0^+} \frac{1}{\sin^2 x} \int_{\frac{x}{2}}^{x} \sin^{-1} t \, dt$ is equal to

(a) 0

(b) $\frac{1}{8}$

(c) $\frac{1}{4}$

(d) $\frac{3}{8}$

[D]

2) Let $I = \lim_{x \to 0^+} \frac{1}{\sin^2 x} \int_{\frac{x}{2}}^{x} \sin^{-1} t \, dt$.

$$\Rightarrow \lim_{x \to 0^+} \frac{\int_{\frac{x}{2}}^{x} \sin^{-1} t \, dt}{\sin^2 x} \quad \left[\tfrac{0}{0} \text{ form}\right].$$

$$\Rightarrow \lim_{x \to 0^+} \frac{\sin^{-1} x - \sin^{-1}\left(\frac{x}{2}\right)\frac{1}{2}}{\sin 2x} \quad \left[\tfrac{0}{0} \text{ form}\right] \quad [\text{Leibnitz's Rules}]$$

$$\Rightarrow \lim_{x \to 0^+} \frac{\frac{1}{\sqrt{1-x^2}} - \frac{1}{4}\frac{1}{\sqrt{1-\frac{x^2}{4}}}}{2 \cdot \cos 2x} = \frac{1-\frac{1}{4}}{2} = \frac{3}{8}.$$

3

For what real values of x and y, does the integral

$\int_x^y (6 - t - t^2)dt$ attain

(a) $x = -3,\ y = 2$ (b) $x = 2,\ y = 3$

(c) $x = -2,\ y = 2$ (d) $x = -3,\ y = 4$

3) [A] Here $f(x,y) = \int_x^y (6-t-t^2)\,dt.$

For Critical points:-

$\frac{df}{dx} = 0$ & $\frac{df}{dy} = 0$

so, $\frac{df}{dx} = (6-y-y^2)\,\frac{d}{dx}\{y\} - (6-x-x^2) = 0$ [by Leibnitz's Rule].

$\Rightarrow 6-x-x^2 = 0$

$\Rightarrow x = -3, 2.$

for, $\frac{df}{dy} = (6-y-y^2) - 0 = 0$

$\Rightarrow y = -3, 2$

so, $r = \frac{d^2f}{dx^2} = 2x+1 < 0$ at point $(-3,2)$