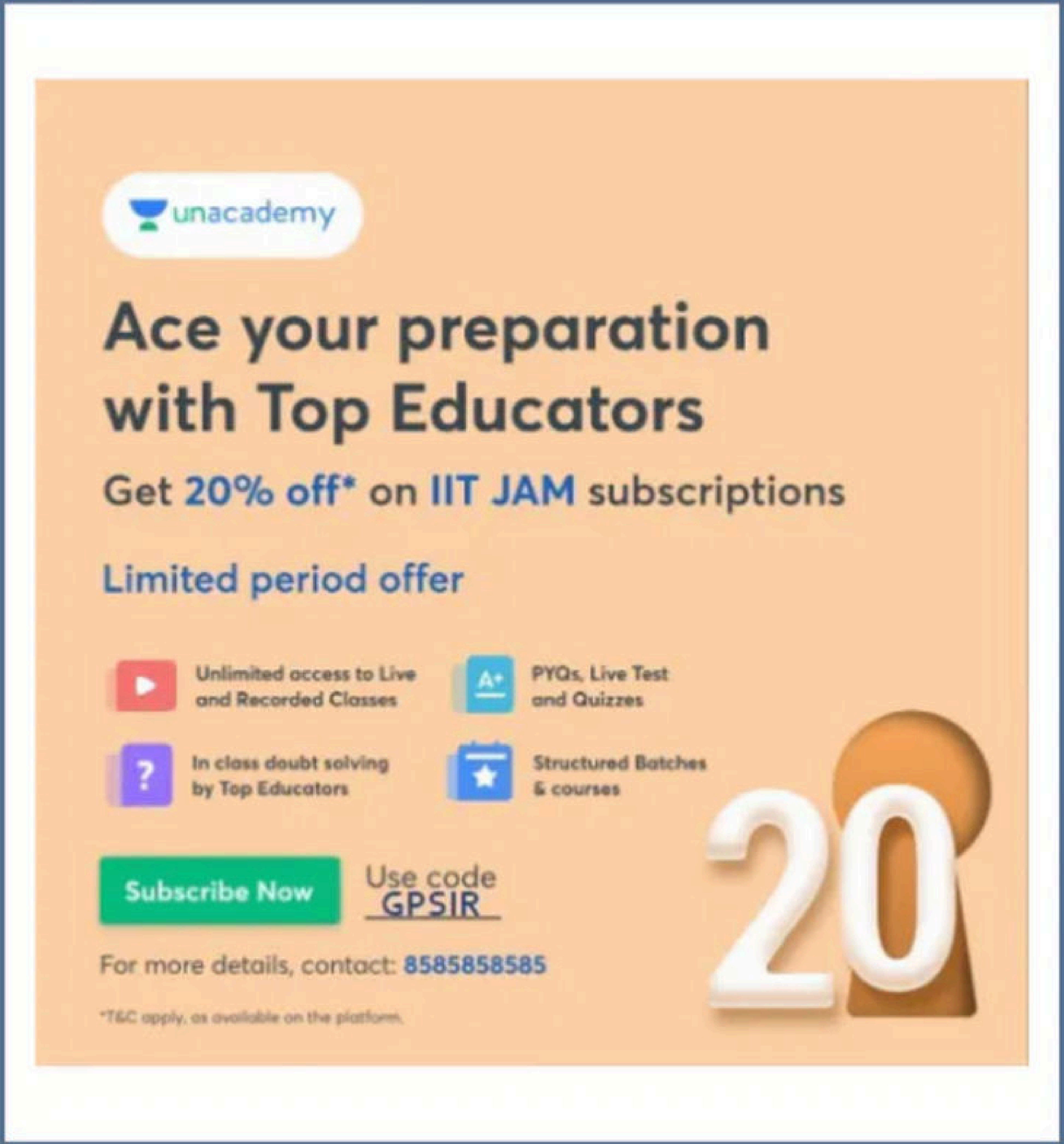
Doubt Clearing Session

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 6 • Sept 16, 2022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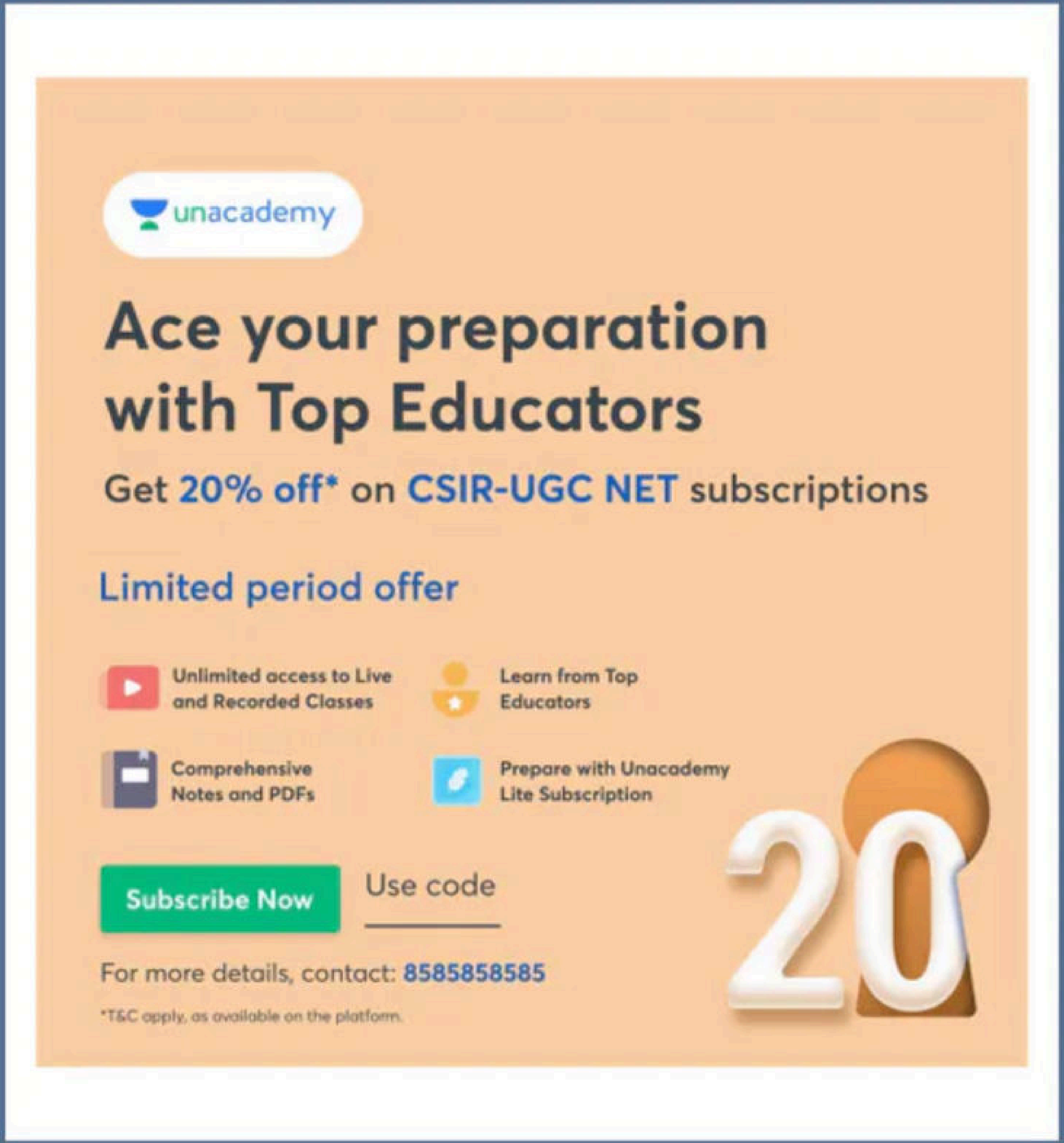
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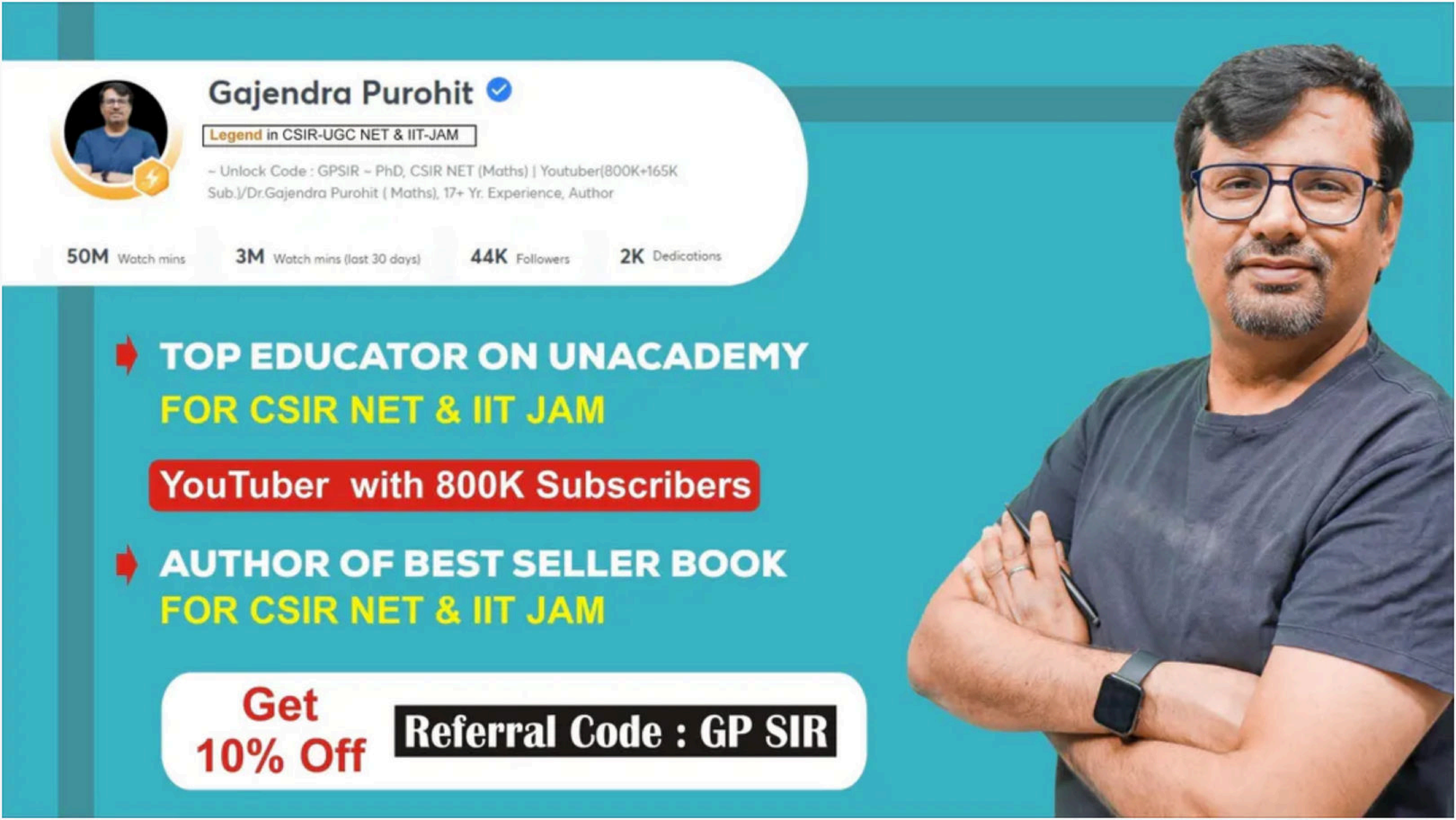
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$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$$

$$= \underline{(1\,2)}\,\underline{(3\,4)}$$

$$2$$

$$Tr(A) = 1$$

$$|A| = (-1)^{\underline{\text{No. of Transpositions}}}$$

$$= (-1)^2 = 1$$

$$\begin{array}{ccc|c} & & & \\ \end{array}$$

$$\begin{array}{c c}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{array}{l} =1 \\ \\ 1 \end{array} \\
\begin{array}{ccc} 1 & 1 & 1 \end{array} &
\end{array}$$

$$(1,1)\ (2,2)\ (3,3)$$

$$Tr(A) = 3$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix} = (1\,5)(2\,4)$$

$\underline{(-1)^{2}}$

$\mathrm{Tr}(A) = 1$

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 1 & 5 & 3 \end{pmatrix}$$

$$= (1\,2\,4\,5\,3)$$

$$(1\,3)(1\,5)(1\,4)(1\,2)$$

$$|A| = (-1)^4 = 1$$

$\cancel{|A|}$ -

$$Tr(A) = 0$$

$A = \begin{bmatrix} & \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} = (1\,3\,2)$$

$$= (1\,2)(1\,3)$$

$$|A| = (-1)^2 = 1$$

$$\mathrm{Tr}(A) = 0$$

Matrix and their properties

Block Matrix : Let $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{m\times p}$, $C = [c_{ij}]_{x\times n}$

& $D = [d_{ij}]_{x\times p}$ are matrix then a matrix

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_{(m+x)\times(n+p)}$$ is called block matrix.

$$A = \left[\begin{array}{cc|cc} 1 & 2 & 3 & 4 \\ 5 & \text{[illegible]} & 2 & 5 \\ \hline 3 & 5 & 7 & 2 \\ 2 & 1 & 6 & 1 \end{array}\right]$$

Block Diagonal Matrix : Let A & B are two matrix then a matrix of type $M = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ is called block diagonal matrix.

$\det M = \det(A)\det(B)$

$$\left|\begin{array}{cc|cc} 4 & 3 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ \hline 1 & 2 & 1 & 5 \\ 2 & 1 & 6 & 2 \end{array}\right|$$

$\det M = (4-6)(2-30)$

$= -2 \times -28$

$= 56$

Block Upper Triangular matrix : If all blocks are above the diagonal then this matrix is called block upper triangular matrix.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$= \underline{(12)}\,\underline{(3)}$$

$$= \underline{(x^2-1)(x-1)}$$

$$\begin{vmatrix} 0-\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

$$= (1)(2)(3)$$

$$(x-1)(x-1)(x-1)$$

$$(x-1)^2$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 1 & 3 & 5 & 2 \end{pmatrix} = (1\,4\,5\,2)(3)$$

$$= \underline{(x^4 - 1)(x - 1) = 0}$$

$$(x^2 - 1)(x^2 + 1)(x - 1) = 0$$

$\cancel{x = \pm 1,}\ x = \pm i, 1$

$1, 1, -1, i, -i$

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Which of following is correct -

(a) a, c

(b) b, c

(c) b, d

(d) a, d

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 1 & 2 & 3 \end{pmatrix}$$

$(14)(25)(36)$

(a) $\pm 1, \pm i, \pm i$

(b) $\pm 1, \pm 1, \pm 1$

$\det(A) = (-1)^3$

$= -1$

$(x^2-1)(x^2-1)(x^2-1) = 0$

$x = \pm 1$

$x = \pm 1$

$x = \pm 1$

(c) $\det(A) = 1$

(d) $\det(A) = -1$

$$\begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

4×4

(a) $x^4 + 3x^2 + 2x + 1$

(b) $x^4 - 3x^2 - 2x - 1$

(c) $x^4 - 3x^2 + 2x + 1$

(d) $x^4 + 3x^2 - 2x + 1$

$x^4 - 0x^3 - (-3)x^2 - 2x - (-1)$

$x^4 + 3x^2 - 2x + 1$

$$\begin{bmatrix} 0 & 0 & 0 & 3 \\ 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$x^4 + 2x^3 + 5x^2 - 7x - 3$$

$$= 0$$

Property :

(1) Trace of block matrix is sum of trace of all diagonal block of matrix.

(2) Determinant of block diagonal matrix is product of determinant of all diagonal block of matrix.

Companion matrix :

Let $P(x) = a_0 + a_1x + a_2x + \ldots .\ a_{n-1}x^{n-1} + a_nx^n$ be a polynomial then a matrix A corresponding to P(x) is called companion matrix if

$$A = \begin{bmatrix} 0 & 0 & \cdots & -a_0 \\ 1 & 0 & \cdots & -a_1 \\ 0 & 1 & \cdots & -a_2 \\ \vdots & & & \\ 0 & 0 & \cdots & 1-a_{n-1} \end{bmatrix}.$$

$\det(A) = (-1)^n a_0$

Property :

(1) Characteristic polynomial and minimal polynomial are same which equal to given polynomial.

(2) Suppose $p(x) = a_0 + a_1x + ... + a_{n-1}x^{n-1} + a_nx^n$, then $Trace(A) = -(a_{n-1})$ and $|A| = (-1)^m a_0$.

Similar Matrix : A & B are said to be similar matrix if $\exists$ a non-singular matrix P s.t. $A = P^{-1}BP$

Congruent Matrix : A & B are said to be congruent matrix if $\exists$ a matrix o s.t. $A = P^{T}BP$

Adjoint of matrix : Adjoint of a matrix A is the transpose of the cofactor matrix of A denoted as adj(A).

Property :

(1) $A(\text{adj } A) = |A|\ I_n$

(2) $|A(\text{adj}A)| = |\ |A|\ I_n| = |A|n$

$\Rightarrow |A|\ |\text{adj } A| = |A|n$

$\Rightarrow |\text{adj } A| = |A|n\text{-}1$

In general $|\text{adj}\ \ \text{adj}\ \ .....\ \text{adj } A| = |A|^{(n-1)^k}$.

(k-times)

(3) $(\text{adj } A)^T = \text{adj }(A^T)$

(4) $\text{adj}(AB) = \text{adj}(B)\ .\ \text{adj}(A)$

(5) $|\text{adj }(kA)| = |kA|^{n\text{-}1} = k^{n(n\text{-}1)}|A|^{n\text{-}1}$

Q.2. Suppose A be a 3×3 matrix of determinant 6, then determinant of (adj A)

(a) 36 (b) 9

(c) 25 (d) None of these

Q.3. Let A be a 5 × 5 skew-symmetric matrix with entries in R and B be the 5 × 5 symmetric matrix whose $(i, j)^{th}$ entry is the binomial coefficient $\binom{i}{j}$ for $1 \le i \le j \le 5$. Consider the 10 × 10 matrix, given in block form by $C = \begin{pmatrix} A & A+B \\ 0 & B \end{pmatrix}$. Then

$i<j$

$\det(C) = \det(A)\det(B)$

$= 0 \times \det B = 0$

(a) det C = 1 or -1

(b) det C = 0

(c) trace of C is 0

(d) trace of C is 5

$$A = \left[\begin{array}{cc|cc} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ \hline 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{array}\right]$$

$\det(A) = 100$

$Tr(A) = 14$

$|a-b|$

(a) 2 (b) 3

(c) 4 (d) 5

$\det(A) = (5a)(2b) = 100$

$10ab = 100$

$ab = 10$

$a + 5 + 2 + b = 14$

$a + b = 7$

$a = 5$

$b = 2$

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Q.4. Let $A = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, then which of the following is true?

(a) $|A| = 0$ (b) $|A| = 1$

(c) $|A| = 2$ (d) $|A| = -1$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & -5 \\ 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$$

5×5

5×5

$(-1)\ 5$

-5

Q5. Let $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & -2 \end{bmatrix}$ then characteristic polynomial.

(a) $x^3 - 2x^2 + 1$ (b) $x^2 - 2x + 1$

(c) $x^3 - 2x^2 - 1$ (d) None of these

3×3

$(-1)(-1)$

$-1 \times -1 = 1$

$x^3 + 2x^2 - 0x - 1$

$x^3 + 2x^2 - 1 = 0$

Q6. Let α, β, γ, δ be the eigenvalues of the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

. Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 =$ ________.

(a) 4 (b) 6

(c) 9 (d) 10

$x^4 - 2x^3 - x^2 + 2x = 0$

$x^3(x-2) - x(x-2) = 0$

$(x^3 - x)(x-2) = 0$

$x(x^2-1)(x-2) = 0$

$x(x-1)(x+1)(x-2)$

$x = 0, 1, -1, 2$

$0 + 1 + 1 + 4$

Rank of matrix : If A is matrix then a non-negative integer is said to be rank of A. if $\exists$ a non-singular submatrix of order r of A and all matrix of order greater than r are singular.

Note :

(1) A matrix $A_{n\times n}$ is a non-singular iff rank(A) = n

(2) Rank of matrix is denoted by $\rho(A)$

(3) If $|A| \neq 0$ then $\rho(A) = n$ where n is order of A

Elementary Transformation : Those transformation that does not effect on rank of matrix are called elementary transformation.

Elementary matrix : A matrix obtained by a single elementary operation over identity matrix is known as elementary matrix.

Q.1. Find elementary matrix

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 2 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

Note :

(1) Every elementary matrix is non-singular.

(2) Every permutation matrix is elementary matrix.

Q.2. Let $A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 151 & 262 & 373 & 484 \end{bmatrix}$.

Then Rank(A) is

(a) 1 (b) 2

(c) 3 (d) 4


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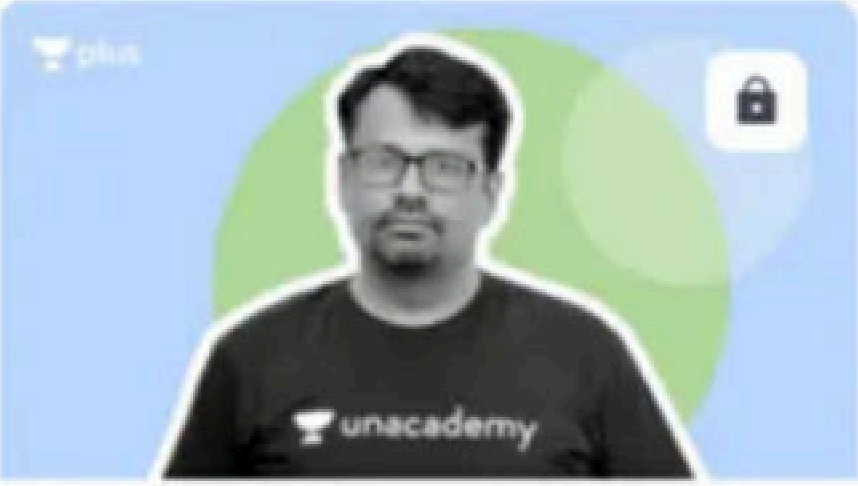

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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