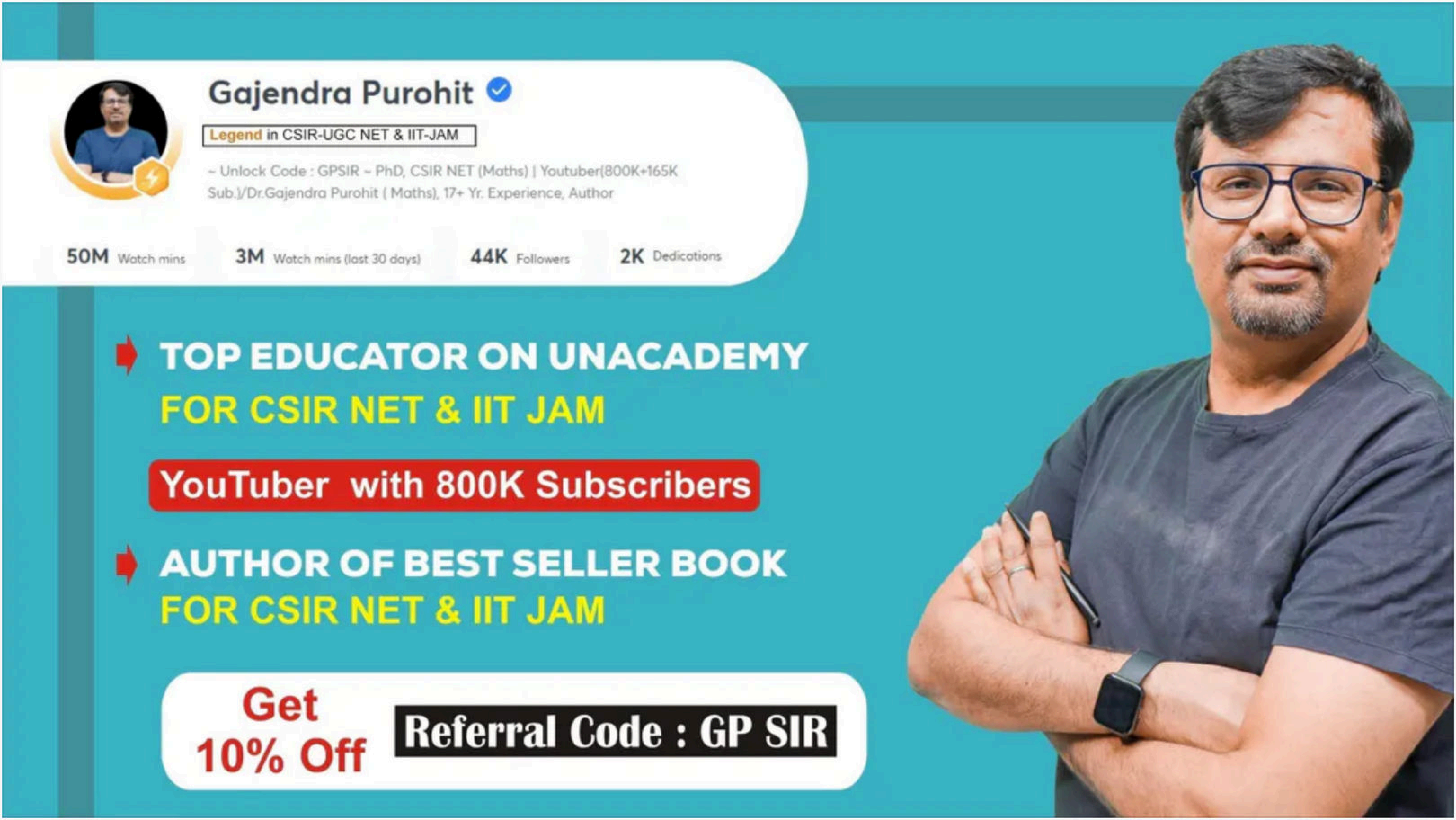
Doubt Clearing Session

Detailed Course on Integral Calculus - IIT JAM' 23

Gajendra Purohit • Lesson 17 • July 29, 2022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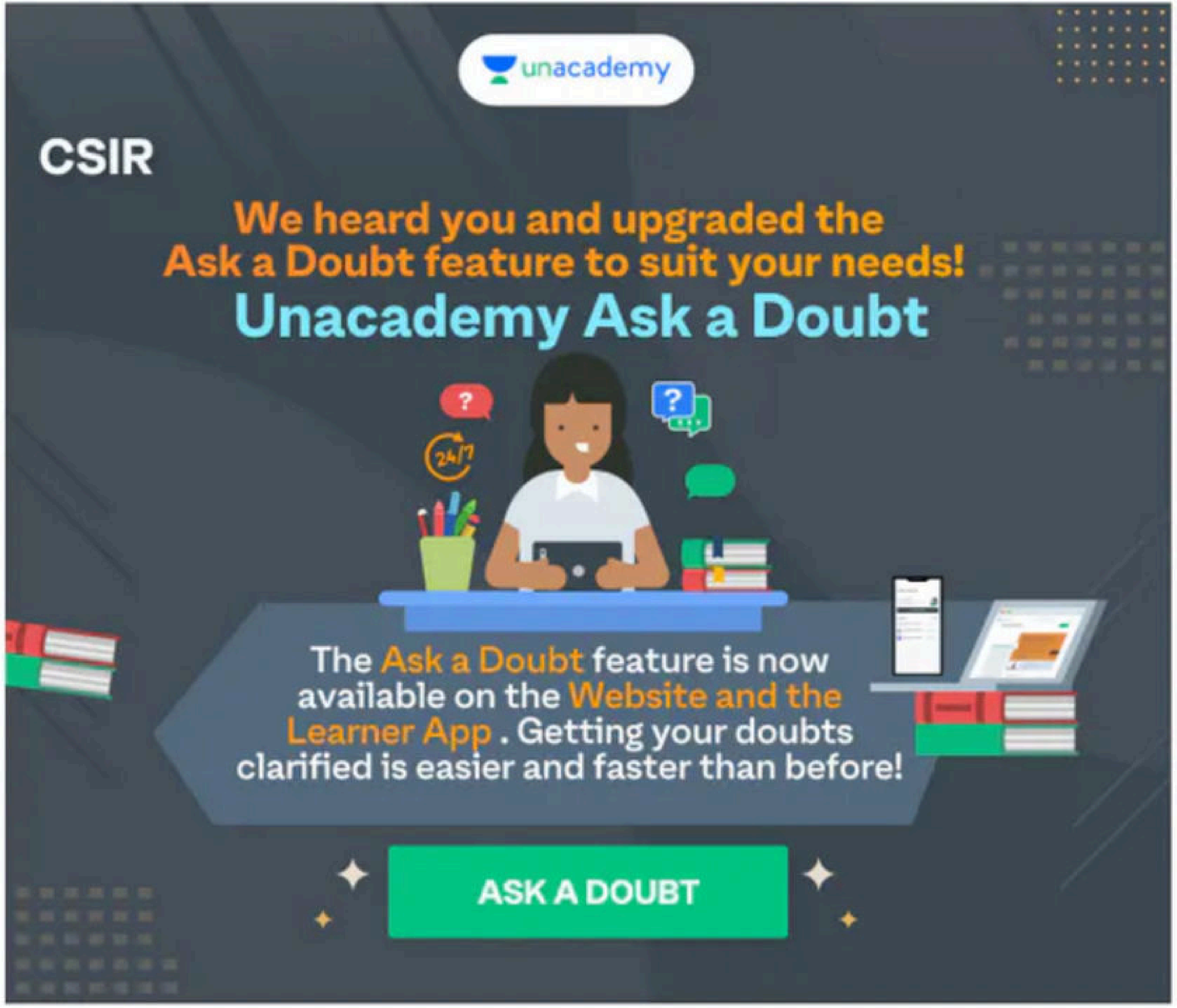

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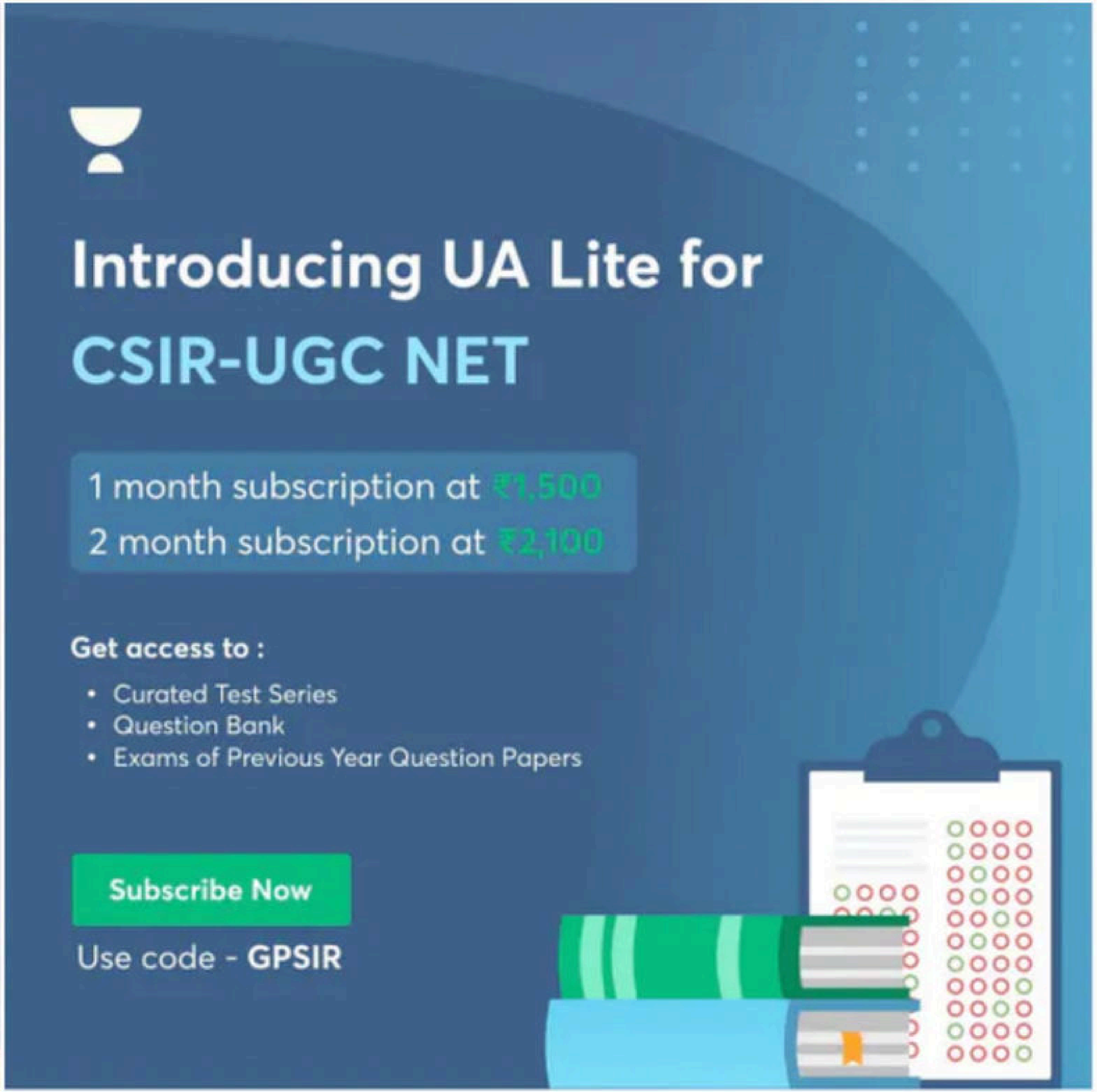
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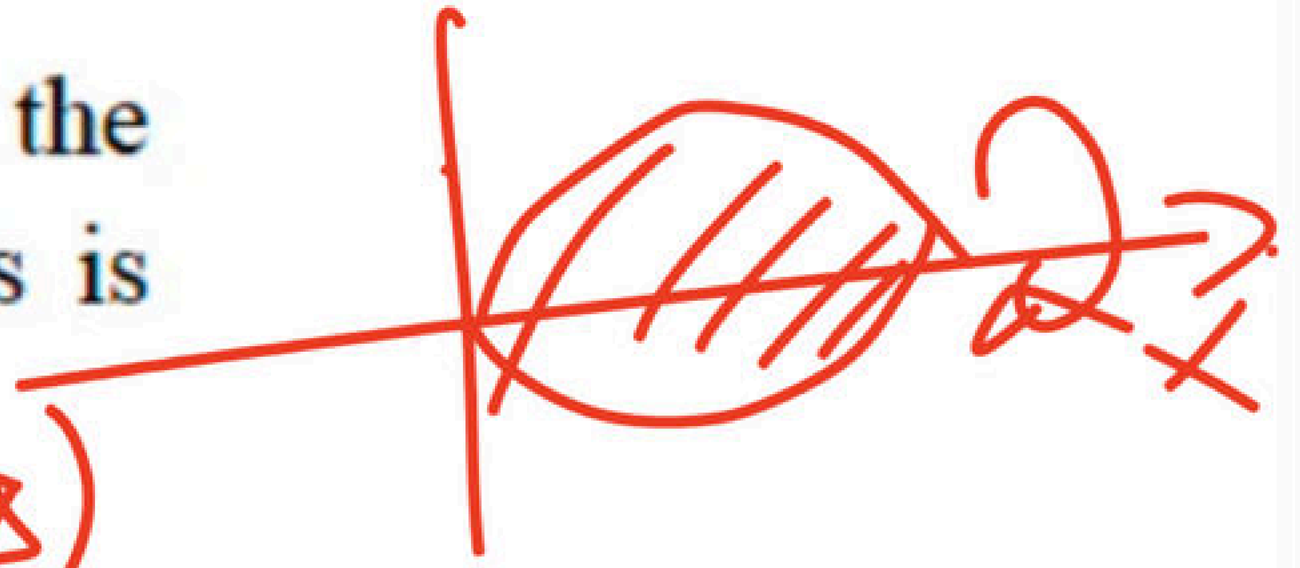
Surface of solid of revolution :

(a) Revolution about x-axis : Let curve is $y = f(x)$, then the area between the ordinate $x = a$, $x = b$ and x-axis is $\int_a^b 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$.



$S = \int 2\pi y \, ds$

(b) Revolution about y-axis : If the curve is $x = f(y)$ then the area between the ordinate $y = a$, $y = b$, y-axis which is $\int_a^b 2\pi x \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$.

$S = \int 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

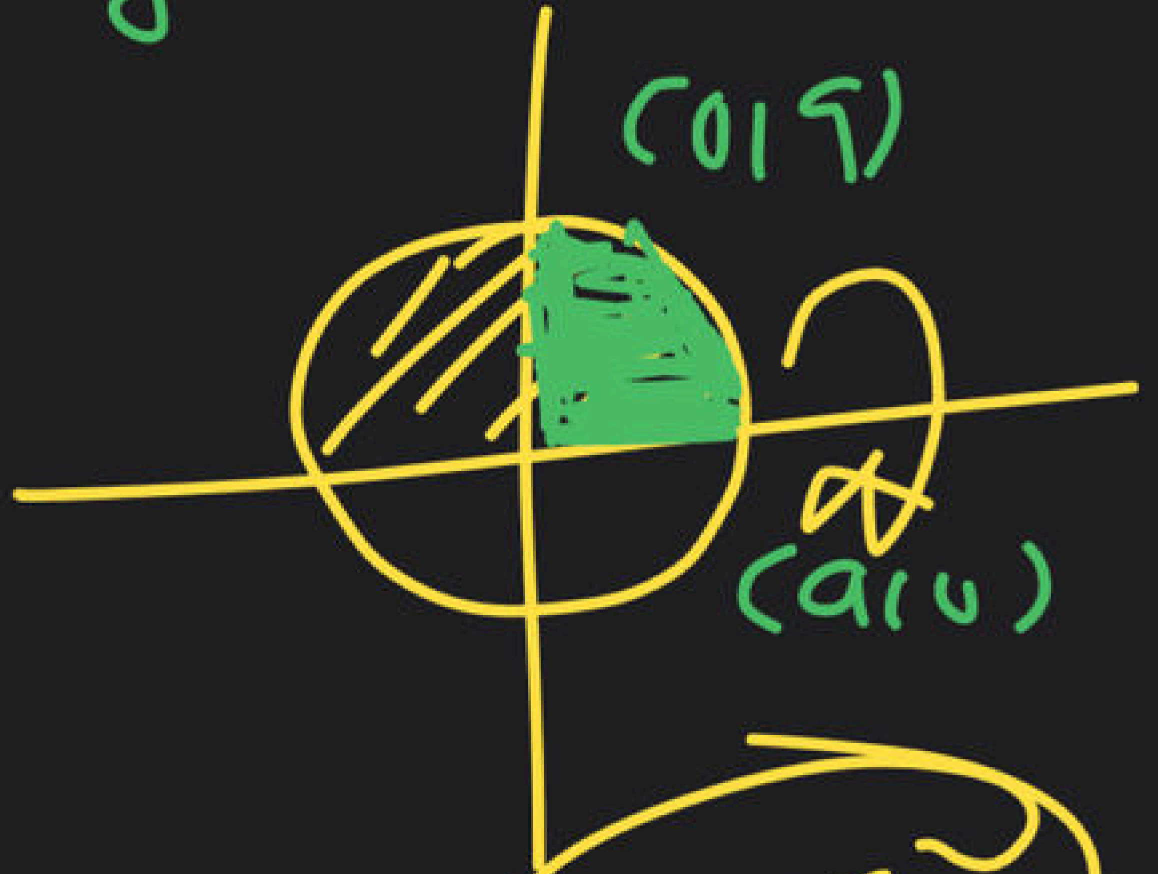
$S = \int 2\pi y \, ds$

$2x + 2y\frac{dy}{dx} = 0$

$x^2 + y^2 = a^2$

$\frac{dy}{dx} = -\frac{x}{y}$

$S = 2\pi \int_{x_1}^{x_2} y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$



$4\pi a^2$

$S = 4\pi\int_0^a y\sqrt{1+\frac{x^2}{y^2}}\,dx$

$S = 4\pi\int_0^a \cancel{y}\,\frac{\sqrt{y^2+x^2}}{\cancel{y}}\,dx$

$S = 4\pi\int_0^a a\,dx = 4\pi a\,(x)_0^a = \underline{4\pi a^2}$

$S = \int 2\pi x\,ds = \int 2\pi \cancel{x}\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,\cancel{dy}$

$$x = a\cos t$$
$$y = a\sin t$$

$$S = \int 2\pi y \, ds$$

$$S = \int 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$S = 2 \cdot 2\pi \int_0^{\pi/2} a\sin t \sqrt{(-a\sin t)^2 + (a\cos t)^2}\, dt$$

$$S = 4\pi a^2 \int_0^{\pi/2} \sin t \, dt = 4\pi a^2 \left(-\cos t\right)_0^{\pi/2}$$

$$= 4\pi a^2 \left(-\cos\frac{\pi}{2} + \cos 0\right) = \underline{4\pi a^2}$$

$$S = \int 2\pi y \, ds$$

$$= \int 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} \, d\theta$$

(c) **Parametric form :** Let x = f(t) and y = g(t) are two curve, then area of solid of revolution about x-axis $\int 2\pi y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$.

(d) **Polar Form :** Let the equation of curve is r = f(θ), then the curve surface generated by revolution about the initial line and the arc intercept θ = α and θ = β is $\int_{\alpha}^{\beta} 2\pi(r\sin\theta)\sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$.

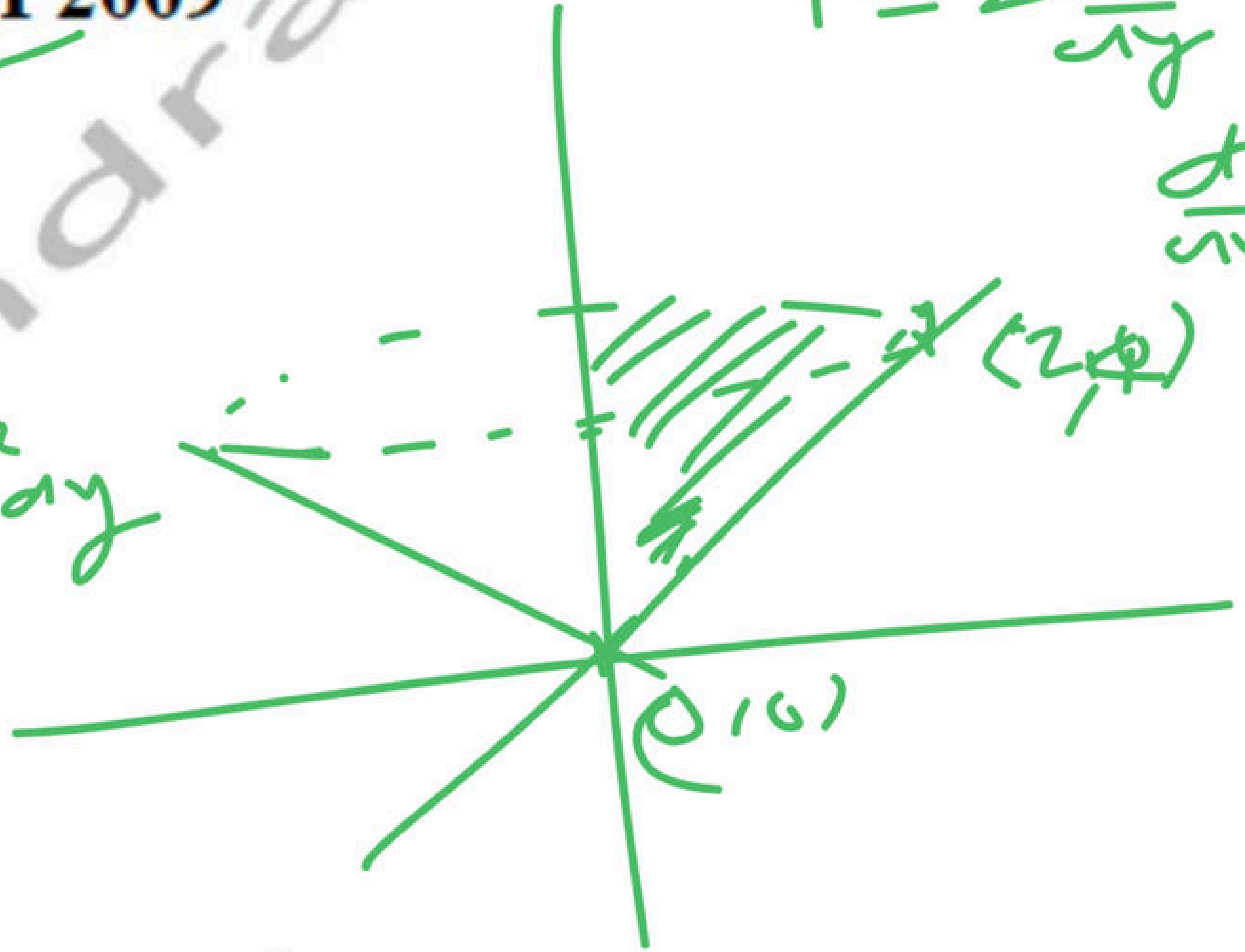
Q.1 The surface area obtained by revolving $y = 2x$, for $x \in [0, 2]$, about y-axis is **IIT JAM 2009**

(a) $2\pi\sqrt{5}$ (b) $4\pi\sqrt{5}$

(c) $2\sqrt{5\pi}$ (d) $4\sqrt{5\pi}$

$1 = 2\frac{dx}{dy}$

$\frac{dx}{dy} = \frac{1}{2}$



$$S = \int 2\pi x \, ds = 2\pi \int_0^4 x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= 2\pi \int_0^4 x\sqrt{1+\frac{1}{4}}\, dy$$

$$= \frac{2\pi\sqrt{5}}{2} \int_0^4 \frac{y}{2}\, dy = \frac{\sqrt{5}\pi}{2}\left(\frac{y^2}{2}\right)_0^4 = \frac{\sqrt{5}\pi}{4} \times 16$$

Area of the surface :

Let the equation of surface is z = f(x, y),

then the area of surface = $S = \iint\limits_R \sqrt{1 + {z_x}^2 + {z_y}^2}\, dx dy$.

Where R is projection of S on xy-plane.

Note :

(1) If the equation of surface is y = f(x, z), then surface area

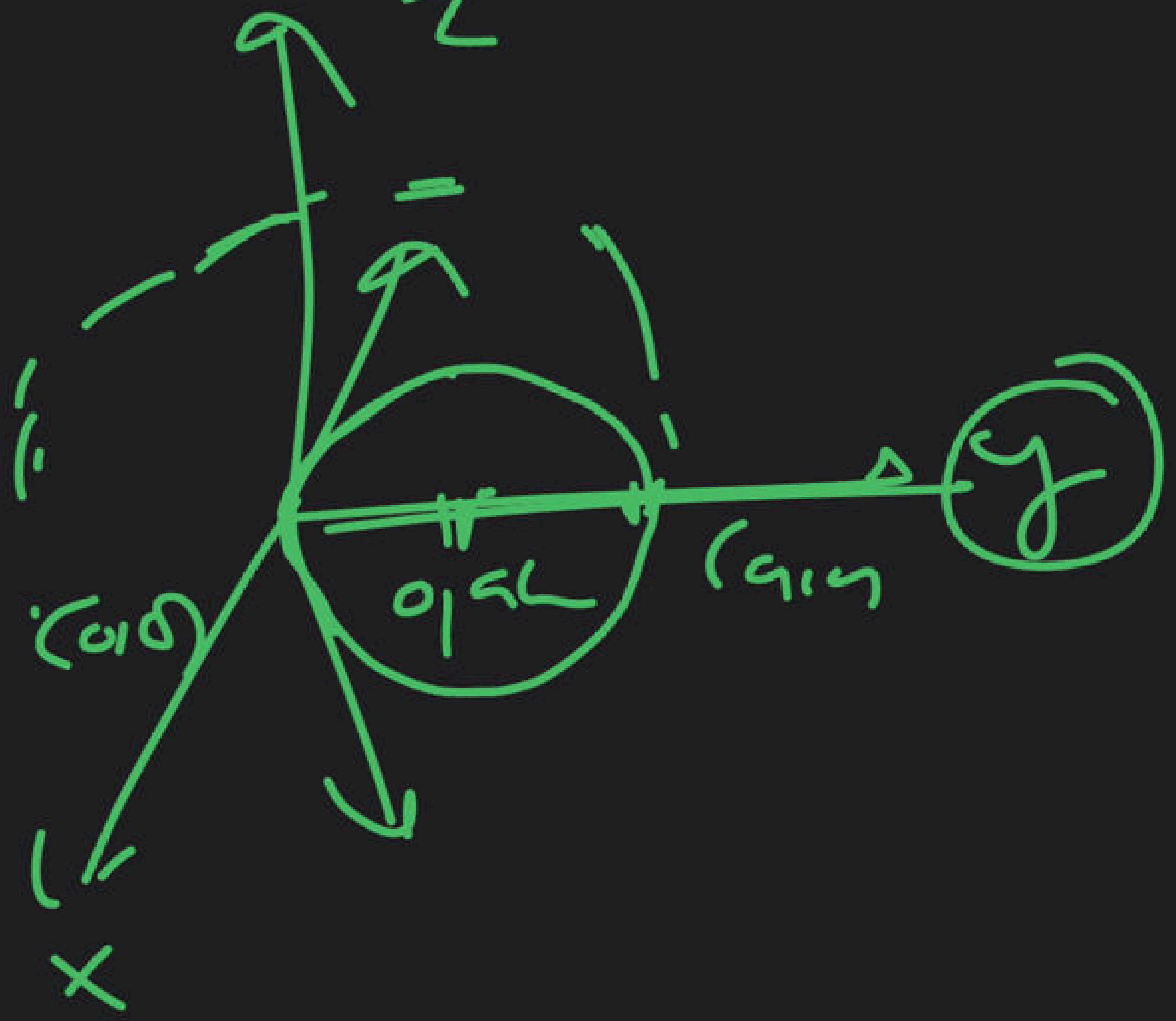
$$S = \iint_R \sqrt{1 + {y_x}^2 + {y_z}^2}\, dxdz.$$

(2) If the equation of surface is x = f(y, z), then surface area

$$S = \iint_R \sqrt{1 + {x_y}^2 + {x_z}^2}\, dydz.$$

$$x^2+y^2+z^2=a^2$$

$x = r\cos\theta, \quad y = r\sin\theta$



$$x^2+y^2-ay=0 \qquad x^2+y^2=ay$$

$$\iint\sqrt{1+z_x^2+z_y^2}\,dx\,dy$$

$$\iint\sqrt{1+\frac{x^2}{z^2}+\frac{y^2}{z^2}}\,dx\,dy$$

$$\iint\sqrt{\frac{z^2+x^2+y^2}{z^2}}\,dx\,dy$$

$$\iint\frac{a}{z}\,dx\,dy$$

$$a\iint\frac{dx\,dy}{\sqrt{a^2-x^2-y^2}}$$

Q.2. The surface area of the portion of the plane $y + 2z = 2$ within the circle $x^2 + y^2 = 3$ is **IIT JAM – 2016**

(a) $\frac{3\sqrt{5}}{2}\pi$ (b) $\frac{5\sqrt{5}}{2}\pi$

(c) $\frac{7\sqrt{5}}{2}\pi$ (d) $\frac{9\sqrt{5}}{2}\pi$

$\pi(\sqrt{3})^2$

$z = \frac{2-y}{2} = 1 - \frac{y}{2}$

$z_x = 0$

$z_y = -\frac{1}{2}$

$\iint \sqrt{1+z_x^2+z_y^2}\,dx\,dy$

$\iint \sqrt{1+0+1/4}\,dx\,dy$

$\iint \frac{\sqrt{5}}{2}\,dx\,dy$

$\frac{\sqrt{5}}{2}\boxed{\iint dx\,dy} = \frac{\sqrt{5}}{2}\pi(\sqrt{3})^2$

$= \frac{3\sqrt{5}\pi}{2}$

Q.3. The area of the surface $z = \frac{xy}{3}$ intercepted by the cylinder $x^2 + y^2 \le 16$ lies in the interval **IIT JAM 2017**

(a) $(20\pi, 22\pi]$ (b) $(22\pi, 24\pi]$

(c) $(24\pi, 26\pi]$ (d) $(26\pi, 28\pi]$

$9+r^2 = p^2$

$2r\,dr = 2p\,dp$

$r\,dr = p\,dp$

$z_x = \frac{y}{3}$

$z_y = \frac{x}{3}$

$x = r\cos\theta$

$y = r\sin\theta$

$$\iint \sqrt{1+\frac{y^2}{9}+\frac{x^2}{9}}\,dx\,dy$$

$$\frac{1}{3}\iint \sqrt{9+x^2+y^2}\,dx\,dy = \frac{1}{3}\int_0^{2\pi}\int_0^4 \sqrt{9+r^2}\,r\,dr\,d\theta$$

$$= \frac{1}{3}\int_0^{2\pi} d\theta \int_3^5 p \cdot p\,dp$$

$$\frac{1}{3}(\theta|_0^{2\pi}\left(\frac{p^3}{3}\right)_3^5$$

$$\frac{1}{9}2\pi(125-27) = \frac{2\pi}{9}\times 98 = 21.78\pi$$

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Q.4. The area of the part of the surface of the paraboloid $x^2 + y^2 + z = 8$ lying inside the cylinder $x^2 + y^2 = 4$ is

IIT JAM – 2019

(a) $\frac{\pi}{2}(17^{3/2} - 1)$ (b) $\pi(17^{3/2} - 1)$

(c) $\frac{\pi}{6}(17^{3/2} - 1)$ (d) $\frac{\pi}{3}(17^{3/2} - 1)$

$z = 8 - x^2 - y^2$

$z_x = -2x$

$z_y = -y$

$\iint \sqrt{1 + z_x^2 + z_y^2}\, dx\, dy$

$\iint \sqrt{1 + 4x^2 + 4y^2}\, dx\, dy$

$\int_0^{2\pi}\int_0^{2} \sqrt{1 + 4r^2}\, r\, dr\, d\theta$

$x = r\cos\theta$

$y = r\sin\theta$

$1 + 4r^2 = p^2$

$8r\,dr = 2p\,dp$

$r\,dr = \frac{p}{4}\,dp$

$\theta = 0 \quad r = 0 \quad \sqrt{17}$

$\frac{1}{4}\int_0^{2\pi} d\theta \int_1^{\sqrt{17}} p \cdot p\, dp$

$\frac{1}{4}(2\pi)\left(\frac{p^3}{3}\right)_1^{\sqrt{17}}$

$\frac{\pi}{6}\left(17^{3/2} - 1\right)$

Q.5. Find the area of the portion of the surface $z = x^2 - y^2$ in R^3 which lies inside the solid cylinder $x^2 + y^2 \leq 1$.

$z_x = 2x,\quad z_y = -2y$

IIT JAM – 2012

$x = r\cos\theta$

$y = r\sin\theta$

$1 + 4r^2 = p^2$

$8r\,dr = 2p\,dp$

$r\,dr = \frac{p}{4}\,dp$

(a) $\frac{\pi}{6}[5^{3/2} - 1]$ (b) $\frac{\pi}{6}[13^{3/2} - 1]$

(c) $\frac{\pi}{3}[5^{3/2} - 1]$ (d) $\frac{\pi}{2}[5^{3/2} - 1]$

$$\iint \sqrt{1 + z_x^2 + z_y^2}\,dx\,dy$$

$$\iint \sqrt{1 + 4x^2 + 4y^2}\,dx\,dy$$

$$\int_0^{2\pi}\int_0^{1} \sqrt{1 + 4r^2}\; r\,dr\,d\theta$$

$0 \text{ to } r = 1$

$$\int_0^{2\pi} d\theta \int_1^{\sqrt{5}} p\,\frac{p}{4}\,dp$$

$$\frac{1}{4}(\theta)_0^{2\pi}\left(\frac{p^3}{3}\right)_1^{\sqrt{5}}$$

$$\frac{1}{12}(2\pi)(5^{3/2} - 1)$$

$$\frac{\pi}{6}(5^{3/2} - 1)$$

Q.6. Find the surface area generated by the revolution of the cardioid $r = a(1 + \cos\theta)$ about the initial line.

(a) $\frac{72}{5}\pi a^2$

(b) $\frac{64}{5}\pi a^2$

(c) $\frac{32}{5}\pi a^2$

(d) None

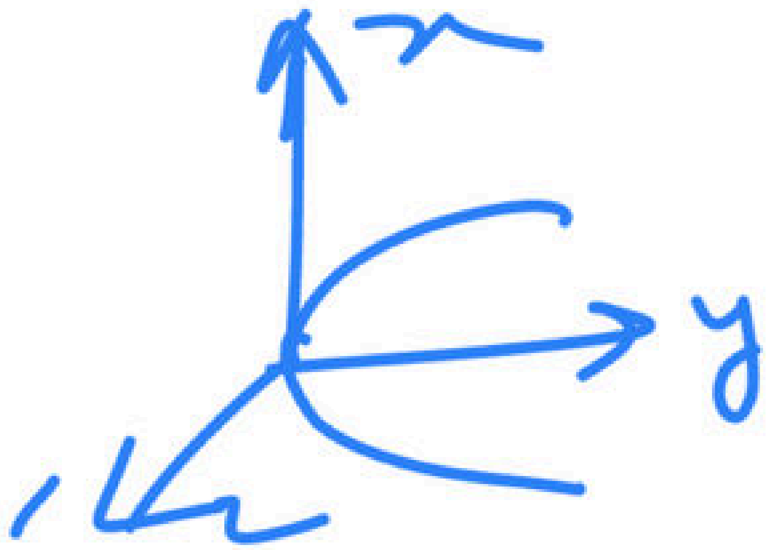
Q.7. Find the surface area of the portion of the cone $z^2 = x^2 + y^2$ that is inside the cylinder $z^2 = 2y$. **IIT JAM – 2008**

$y = r\cos\theta$

$z = r\sin\theta$

(a) $2\sqrt{2}\pi$ (b) $4\sqrt{2}\pi$

(c) $6\sqrt{2}\pi$ (d) $8\sqrt{2}\pi$



$$\iint \sqrt{1 + x_y^2 + x_z^2}\, dy\, dz$$

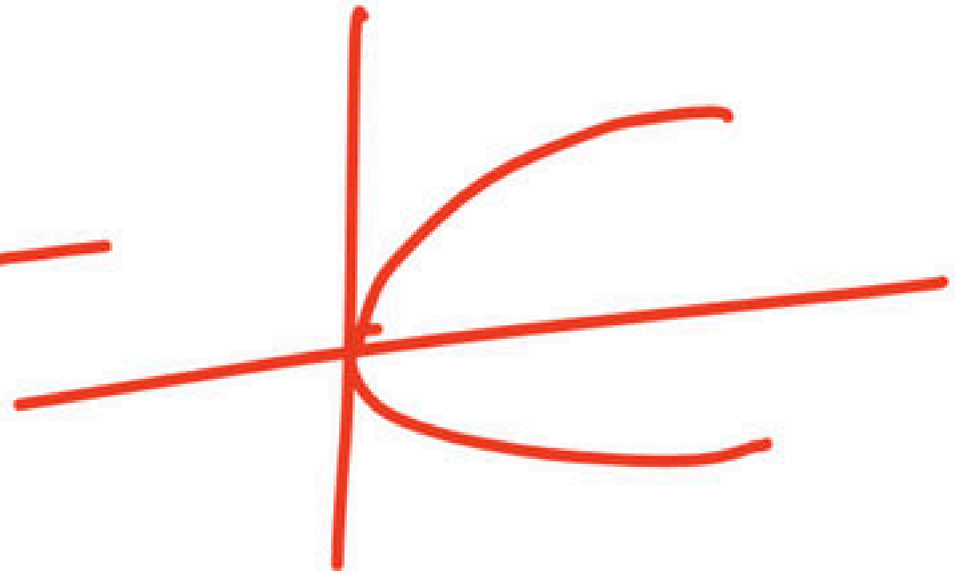
$$\iint \sqrt{1 + \frac{z^2}{x^2} + \frac{y^2}{x^2}}\, dy\, dz$$

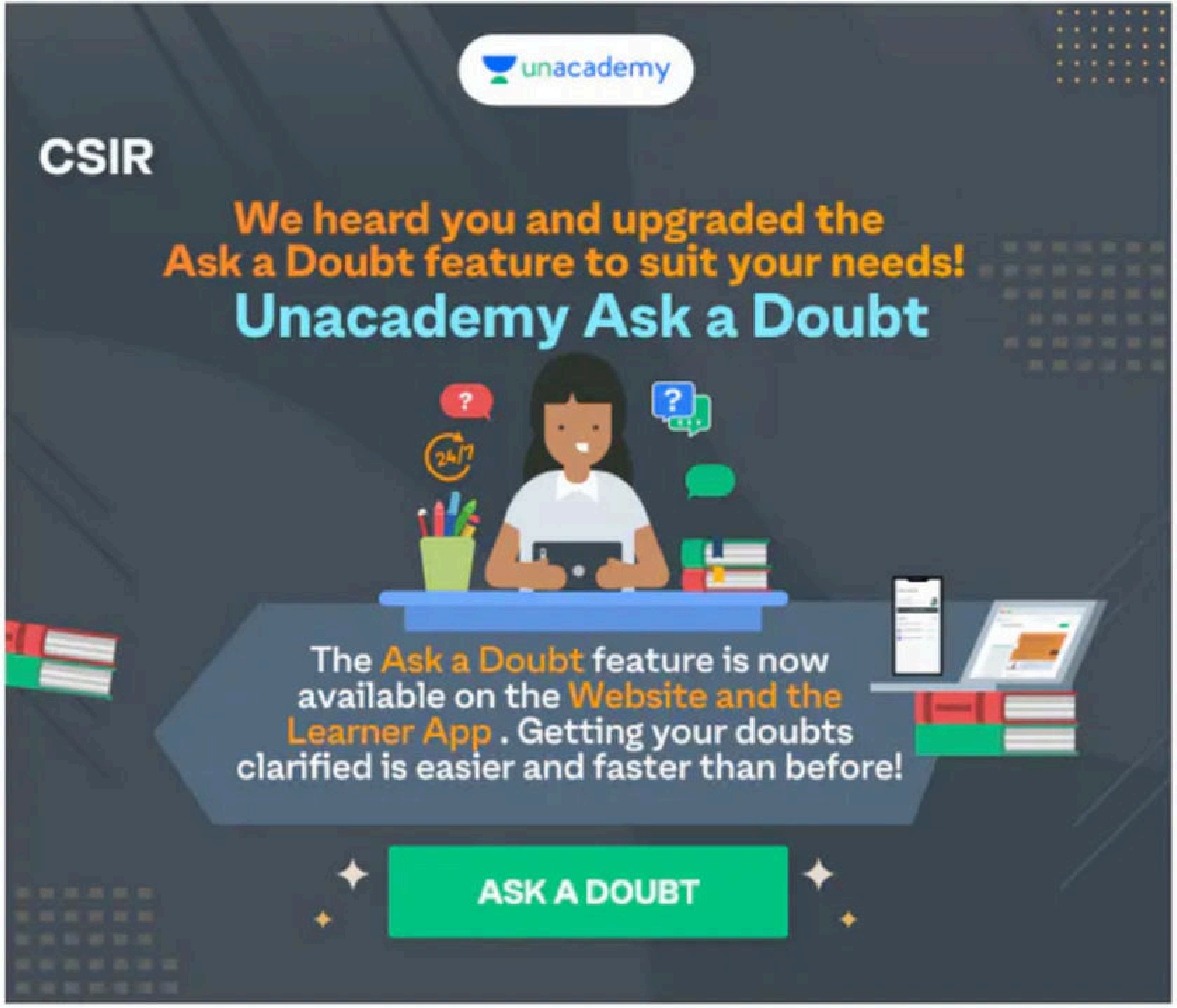
$$\iint \sqrt{\frac{x^2 + z^2 + y^2}{x}}\, dy\, dz$$

$$\iint \frac{z}{x}\, dy\, dz$$

$$\iint \frac{z}{\sqrt{z^2 - y^2}}\, dy\, dz$$

$y =$ $z =$

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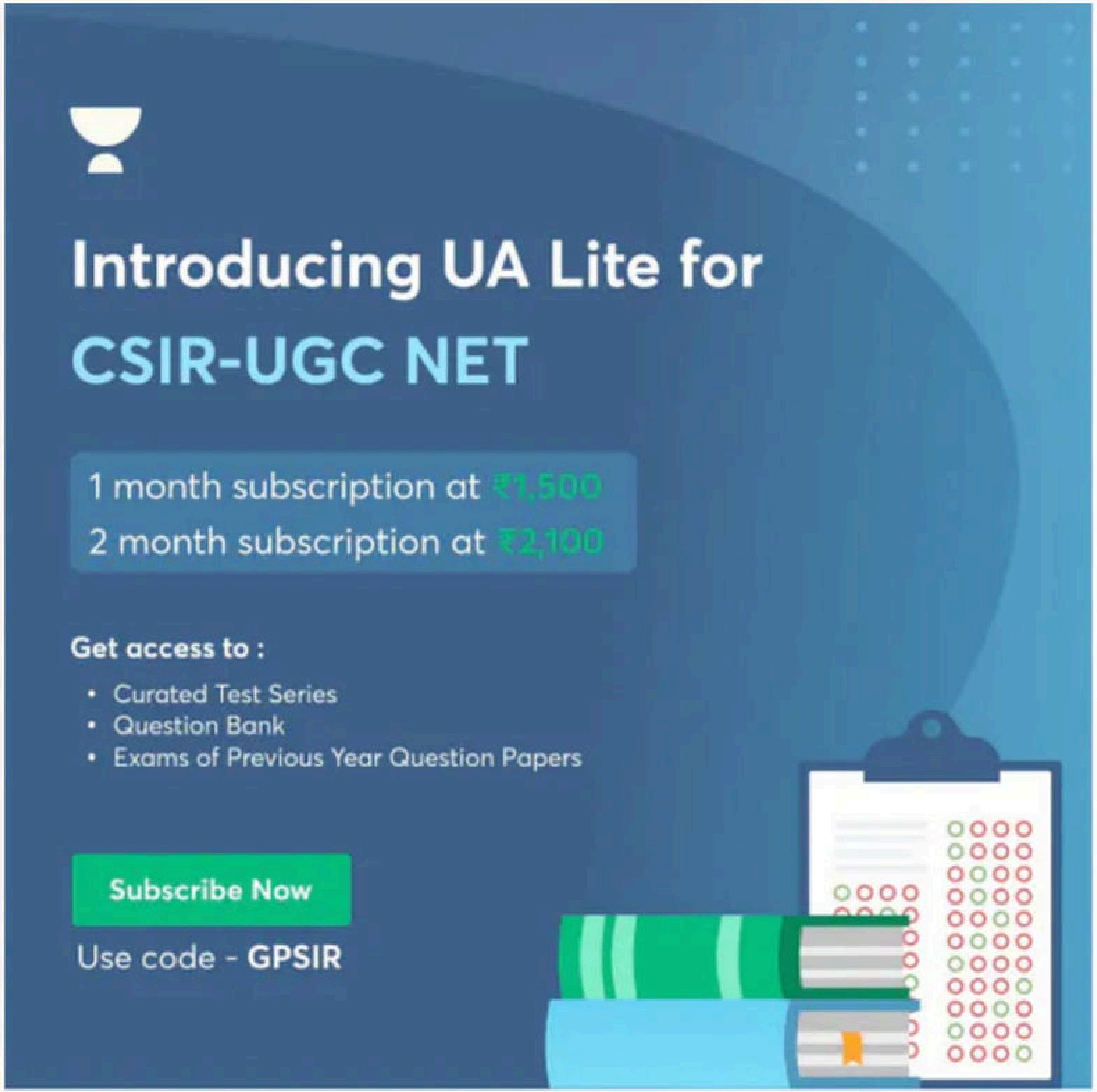
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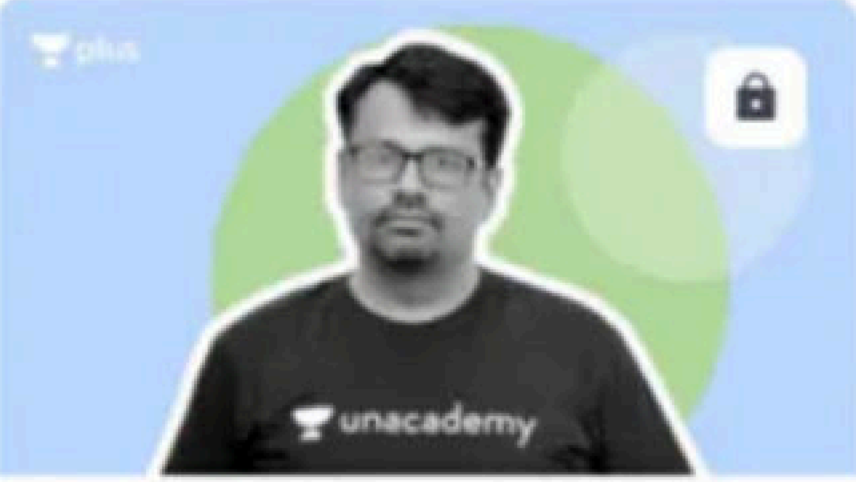

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