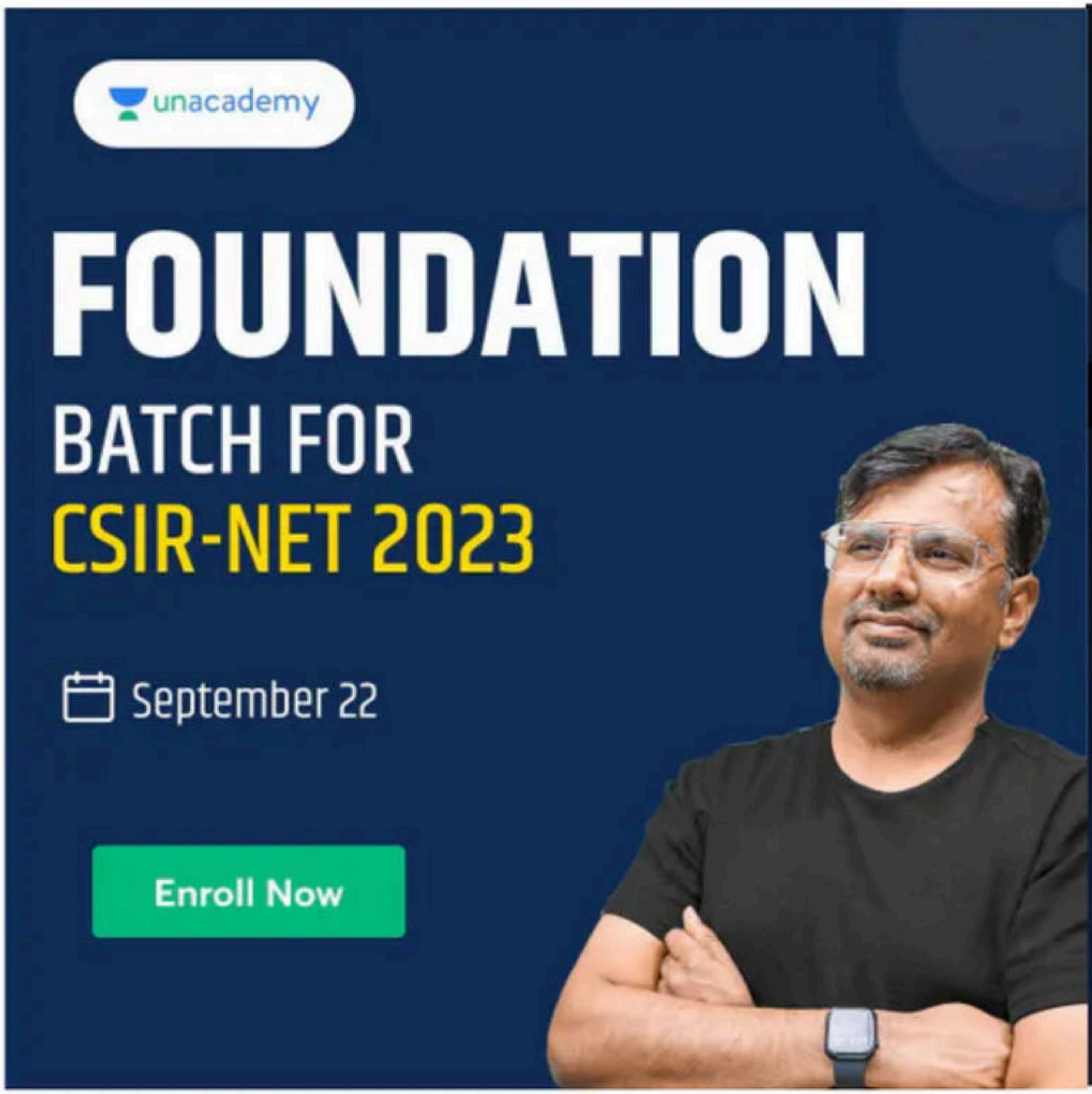
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Sum of two subspace : Let W_1 and W_2 are two subspace then Sum of two subspace define as $W_1 + W_2 = \{ x + y | x \in W_1 , y \in W_2\}$

Example : $W_1 = \{ (x , 0) | x \in R \}$ and $W_2 = \{ (0 ,y) | y \in R\}$

then $W_1 + W_2 = \{ (x ,y) | x ,y \in R\} = R^2$

Note :

(i) Sum of two subspace is also a subspace of vector space

(ii) If $V = W_1 + W_2$ then V is linear sum of W_1 and W_2

(iii) $W_1 + W_2$ is smallest subspace containing both W_1 and W_2
i.e. $W_1 \in W_1 + W_2$ and $W_2 \in W_1 + W_2$

Disjoint subspace : Let W_1 and W_2 are two subspace s.t. $W_1 \cap W_2 = \{0\}$ then Both subspace are called disjoint subspace

Direct Product of subspace :

Let V be a vector space then V is called direct product of W_1 and W_2 if

(i) $V = W_1 + W_2$

(ii) $W_1 \cap W_2 = \{0\}$

It is denoted by $\boxed{V = W_1 \oplus W_2}$

Result :

(i) Intersection of any number of subspaces of a vector space V is always a subspace of V.

(ii) Union of two subspaces is also a subspace iff one is contain in another.

$W_1 + W_2$

$W_1 \cup W_2$

Note : $W_1 + W_2$ and $W_1 \cup W_2$ are different term

$W_1 = \{(x, 0)\}$ $W_2 = \{(0, y)\}$

$W_1 + W_2 = \{(x, y)\} = \{(0,0), (1,0), (0,1), (2,0), (0,2), (2,2) \ldots\}$

$W_1 \cup W_2 = \{(x, 0), (0, y)\} = (0,0), (1,0), (2,0), (3,0) \ldots$

$(0,1), (0,2), (0,3) \ldots$

$(1,0)$ $(0,1)$

Q.1. Let $W_1 = \{(a, 2a, 0) \mid a \in R\}$, $W_2 = \{(a, 0, -a) \mid a \in R\}$. Then

(a) $W_1 + W_2$ is a subspace of R_3 but $W_1 \cup W_2$ is not

(b) $W_1 + W_2$, $W_1 \cup W_2$ are both subspaces of R^3.

(c) Neither $W_1 + W_2$ nor $W_1 \cup W_2$ is a subspace of R^3.

(d) $W_1 \cup W_2$ is a subspace of R^3 but $W_1 + W_2$ is not.

$(1,2,0) + (1,0,-1)$

$(2,2,-1)$

$(a, 2a, 0), (a, 0, -a)$

Q.2. Let $V = \{[a_{ij}]_{m\times n}; a_{ij} \in F\}$ be a vector space

$$W_1 = \{A = [a_{ij}]_{m\times n} / A^k = 0; k \in N; A \text{ is diagonalizable matrix}\}$$

and $W_2 = \{A = [a_{ij}]_{m\times n} / A \text{ is diagonal matrix}\}$

$A = 0$

$W_1 \subset W_2$

then which of the following is true

(a) W_1 is subspace of V

(b) W_2 is subspace of V

(c) ~~$W_1 \cap W_2$ is non - subspace of V~~

(d) $W_1 \cup W_2$ is subspace of V

$H = \{(x_1, x_1)\}$ $\quad$ $H_2 = \{(x_1, -x_1)\}$

Q.3. Let $H_1 = \{(x, y) \mid y = x\}$ and $H_2 = \{(x, y) \mid y = -x\}$ be subspaces of a vector space $R^2(R)$.

Then which of the following statement is correct?

(a) $H_1 + H_2$ is an improper subspace of R^2

(b) $H_1 + H_2$ is a proper subspace of R^2

(c) $H_1 + H_2$ is not a subspace of R^2

(d) $H_1 + H_2$ is a trivial subgroup of R^2.

$H_1 + H_2 = \{(2x_1, 0)\}$

V

$\{0\}$

improper

trivial

proper

non trivial

Linear Combination, LI & LD set of vectors

Linear combination of a set of vectors : Let $v_1, v_2, \ldots, v_n$ are vectors in a vector space V. A linear combination of vectors $v_1, v_2, \ldots, v_n$ in V is a vector of the form

$\alpha_1 v_1 + \alpha_2 v_2 + \ldots + \alpha_n v_n$, where $\alpha_i \in F$ for all i = 1 to m.

Linear Span or Spanning set or Generating Set :

Let S be a non-empty subset of vector space then the set of all possible linear combination of elements of S is called linear span or spanning set of S and denoted by L(S) or <S> or span (S)

$\alpha_1 \in F$ $\quad v_i \in V$

$v_1\alpha_1 + v_2\alpha_2 \ldots v_n\alpha_n = 0$

LI $\quad \alpha_1 = \alpha_2 = \alpha_3 = \ldots = \alpha_n = 0$

LD $\quad \alpha_i \neq 0$ for some i

$S = \{(1,0), (0,1)\}$

$\mathbb{R}^2(\mathbb{R})$

$L(S) = \{\alpha(1,0) + \beta(0,1) \mid \alpha, \beta \in \mathbb{R}\}$

$= \{(\alpha, 0) + (0, \beta) \mid \alpha, \beta \in \mathbb{R}\}$

$= \{(\alpha, \beta) \mid \alpha, \beta \in \mathbb{R}\} = \mathbb{R}^2(\mathbb{R})$

$(x, y) \mid x, y \in \mathbb{R}$

$S = \{ (1,0,0), (0,1,0), (0,0,1) \}$ $\quad \alpha, \beta, \gamma \in \mathbb{R}$

$L(S) = \alpha(1,0,0) + \beta(0,1,0) + \gamma(0,0,1)$

$= \{ (\alpha, \beta, \gamma) \mid \alpha, \beta, \gamma \in \mathbb{R} \} = \mathbb{R}^3(\mathbb{R})$

Result :

(1) Let V(F) be a vector space and $S \subseteq V$ then L(S) is subspace of V called subspace spanned by S.

i.e. L(S) is subspace of V if S is subset of V.

(2) Let S be a subspace of a vector space V then $L(S) = S$

We know that if S is subspace then S is closed i.e. all possible linear combination of elements of S belonging in S. So , Linear span of S is also S

(3) Let $A \subseteq B \subseteq V$ then $L(A) \subseteq L(B)$

(4) $L(\phi) = \{0\}$

$S = \{(x,0)\}$

$L(S) = \{\alpha(x,0) \mid \alpha \in R\}$

$= \{(\alpha x, 0) \mid \alpha \in R\}$

$\subset R^2$

$S = \{\phi\}$

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Linear Dependent : A subset S of a vector space V is said to be dependent if $\exists$ $x_1, x_2, \ldots, x_n$ in S and scalar $\alpha_1, \alpha_2, \ldots\ldots, \alpha_n$ in F, not all zero s.t. $\alpha_1 x_1 + \alpha_2 x_2 + \ldots\ldots \alpha_n x_n = 0$

Linear Independent : Any set containing the vectors $x_1, x_2, \ldots.,$ x_n defined over a field F is said to be LI if $\alpha_1 x_1 + \alpha_2 x_2 + \ldots\ldots \alpha_n x_n = 0 \Rightarrow \alpha_1 = \alpha_2 = \ldots\ldots \alpha_n = 0$

$v_2 = 2v_1$

$S = \{(1,0,1), (2,0,2)\}$

$\alpha(1,0,1) + \beta(2,0,2) = 0$

$(\alpha+2\beta, 0, \alpha+2\beta) = (0,0,0)$

$\alpha + 2\beta = 0 \quad | \quad \alpha + 2\beta = 0$

$\beta = k$

$\alpha = -2k$

$\alpha = -2$

$\beta = 1$

$-2(1,0,1) + 1(2,0,2) = 0$

$$\begin{pmatrix} 1 & 2 \\ 0 & 0 \\ 1 & 2 \end{pmatrix}$$

$S = \{(1,0,0), (0,2,0), (0,1,1)\}$

$\alpha(1,0,0) + \beta(0,2,0) + \gamma(0,1,1) = 0$

$\alpha\beta\gamma$

$(\alpha, 2\beta+\gamma, \gamma) = (0,0,0)$

$\alpha = 0 \quad | \quad 2\beta+\gamma = 0 \quad | \quad \gamma = 0$

$\beta = 0$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$|A| \neq 0$

Result :

(1) Any set containing 0 vector is LD

(2) The empty set is LI

(3) Two vectors are LD, iff they are scalar multiple to each other.

(4) Every subset of LI set is LI

(5) Every superset of LD set is LD

$|A| \neq 0$

Note : **(i)** If a matrix of order n and its rank is n then all columns/rows are LI

(ii) If $|A| \neq 0$ then all columns/rows are LI

Q.4. Which one of the following is correct?

(a) $S = \{(1, 0, 0), (0, -1, 0), (1, 1, 0)\}$ is a linearly independent set of vectors in R^3.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(b) $S = \{(1, 0, 0), (0, 2, 0), (1, 1, 0)\}$ is a linearly independent set of vectors in R^3. LD

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(c) A subset of a linearly dependent set of vectors is linearly independent.

(d) A subset of a linearly independent set of vectors is linearly independent.

$\{(1,0,1),(2,0,2),(0,2,1)\}$

$\{(1,0,1),(2,0,2)\}$

$\{(1,0,1)\}$

Q.5. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \alpha M_1 + \beta M_2 + \gamma M_3$, where $M_1 = I_{2\times 2}, M_2 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \& M_3 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ then

(a) $\alpha = \beta = 1, \gamma = 2$ (b) $\alpha = \beta = -1, \gamma = 2$

(c) $\alpha = 1, \beta = -1, \gamma = 2$ (d) $\alpha = -1, \beta = 1, \gamma = 2$

$$\alpha \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \beta \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} + \gamma \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$\alpha + \beta = 1$

$\beta + \gamma = 1$

$\alpha + \beta + \gamma = 0$

Q.6. If the set $\left\{\begin{bmatrix} x & -x \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ x & x \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\right\}$ is linearly dependent in the vector space of all 2 × 2 matrices with real entries, then x is equal to

(a) 1 (b) -1

(c) 2 (d) -2

$$\begin{bmatrix} x & -x & -1 & 0 \\ 0 & -1 & x & x \\ 1 & -1 & 1 & 0 \end{bmatrix}$$

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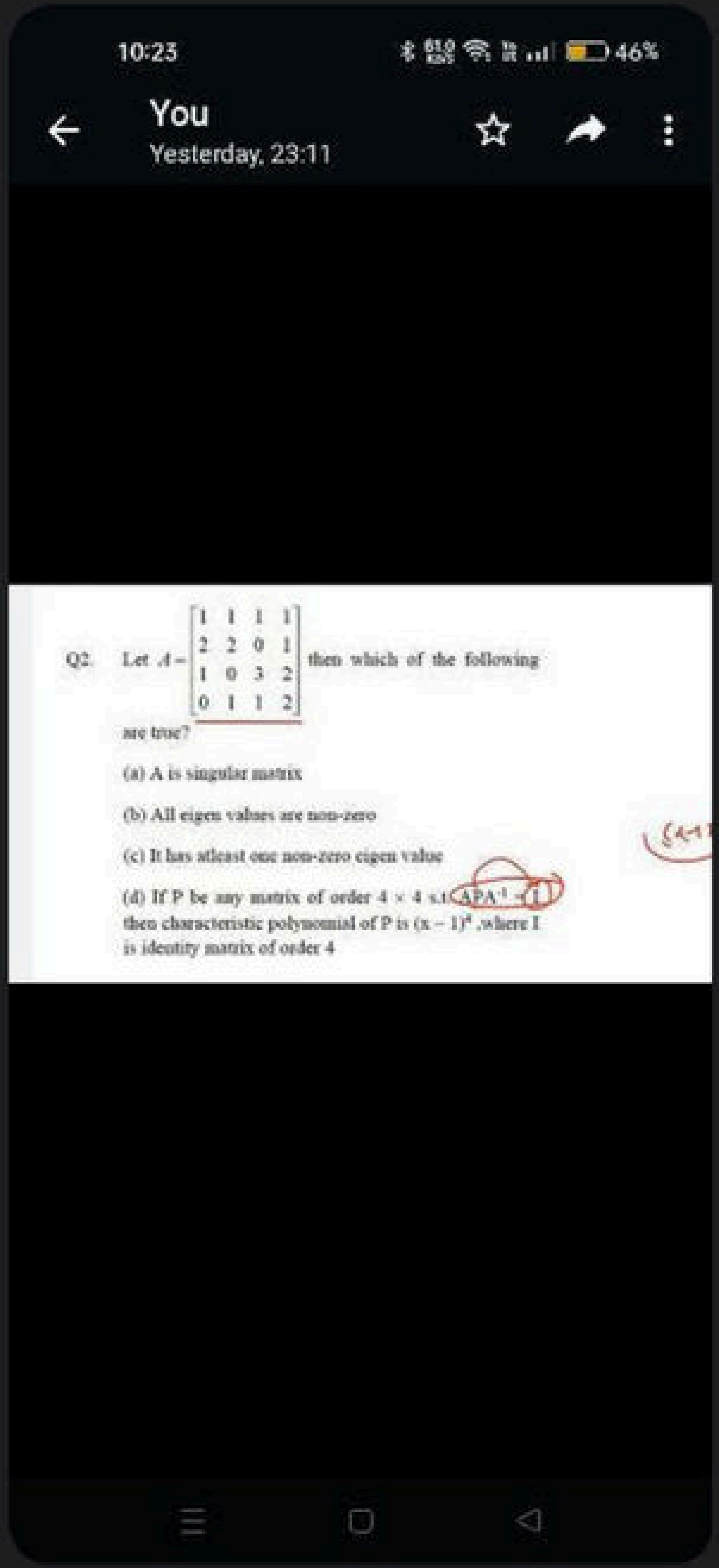
▲ 2 • Asked by Rishabh

Ye wala sir



Q2. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 0 & 1 \\ 1 & 0 & 3 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ then which of the following are true?

(a) A is singular matrix

(b) All eigen values are non-zero

(c) It has atleast one non-zero eigen value

(d) If P be any matrix of order 4×4 s.t. $APA^{-1} = I$ then characteristic polynomial of P is $(x-1)^4$, where I is identity matrix of order 4

Result :

Let $V_1, V_2, \ldots, V_n$ are either column vector or row vector of a matrix A then $V_1, V_2, \ldots, V_n$ are LI or LD if $|A| \neq 0$ or $|A| = 0$.

Q.7. In vector space $R^3(R)$ over the field of real numbers R t
hen the set $S = \{(-1, 2, 1), (3, 0, -1), (-5, 4, 3)\}$ is

(a) LI

(b) LD

(c) Data is insufficient

(d) None of these

$$\begin{bmatrix} -1 & 3 & -5 \\ 2 & 0 & 4 \\ 1 & -1 & 3 \end{bmatrix}$$

$|A| = 0$

Q.8. If α, β, γ are LI vector of V(F) then which of the following is LI.

(a) 2α, β, 2β

(b) α + β, α - β, α - 2β + γ

(c) α - β, β + γ, γ + α

(d) α + β, 2α + γ, α - β + γ

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -2 \\ 0 & 1 & 1 \end{pmatrix} \quad |A| \neq 0$$

$$\begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad |A| \neq 0$$

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} \quad |A| = 0$$

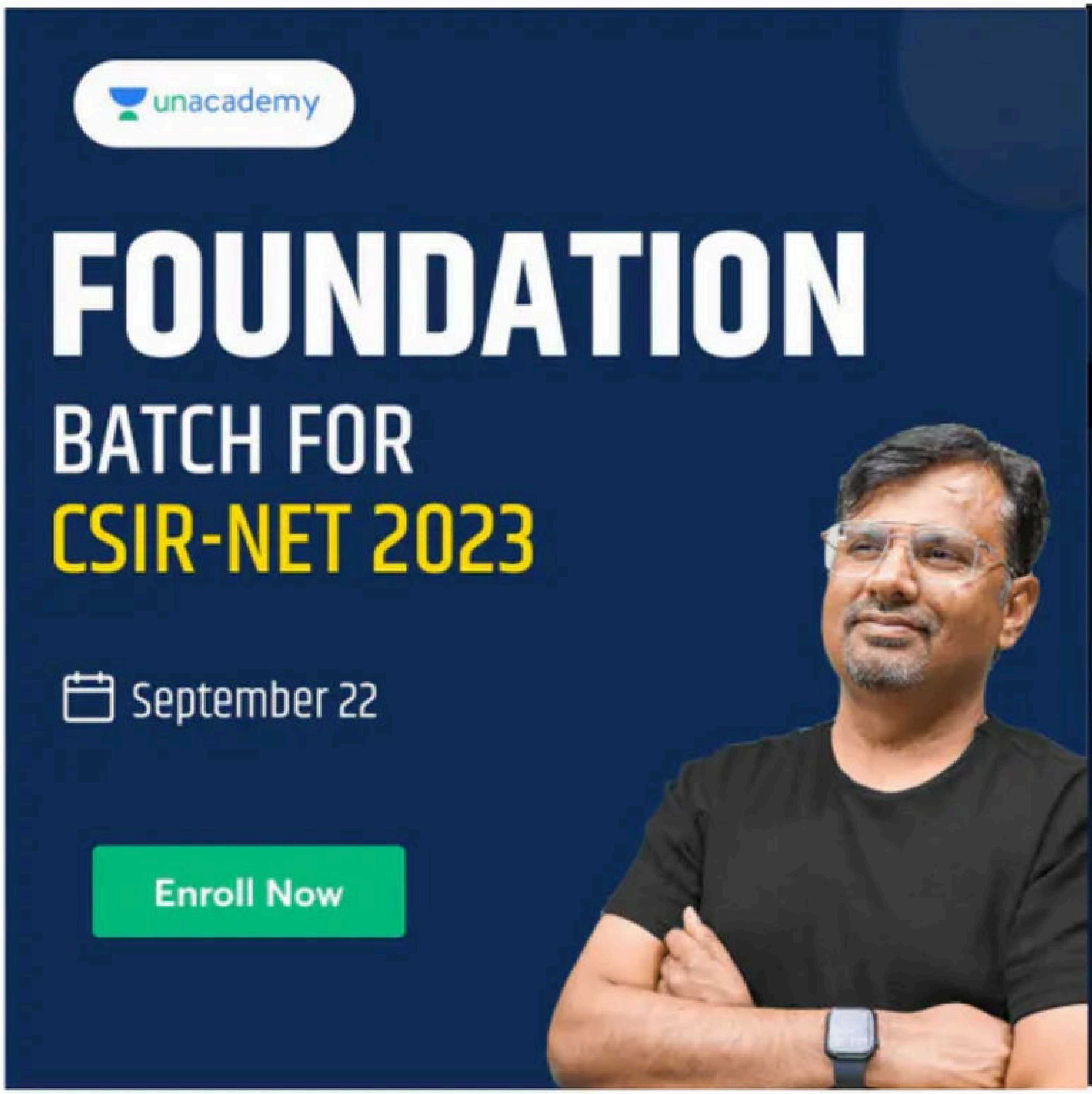
Q.9. Let $p_n(x) = x^n$ for $x \in R$ and let $\wp = \text{span}\{p_0, p_1, p_2, ...\}$. Then

(a) $\wp$ is the vector space of all real valued continuous function on R.

(b) $\wp$ is a subspace of all real valued continuous function on R.

(c) $\{p_0, p_1, p_3, ...\}$ is a linearly independent set in the vector space of all continuous functions on R.

(d) Trignometric functions belong to $\wp$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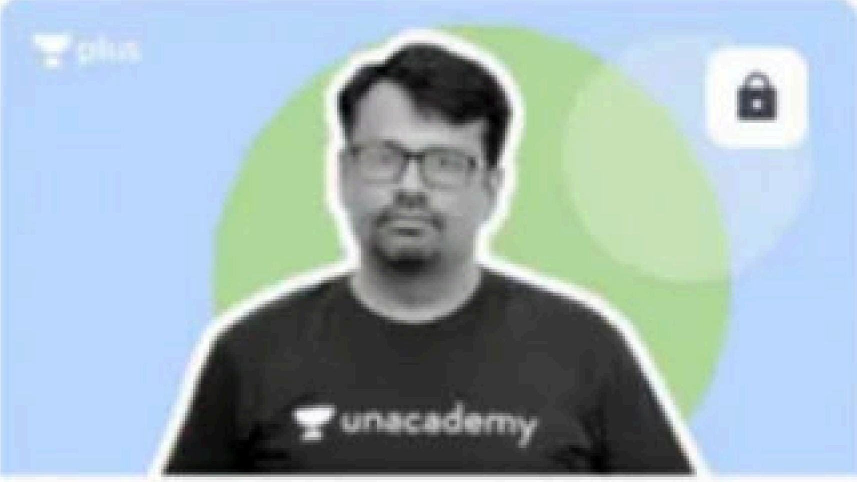

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- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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