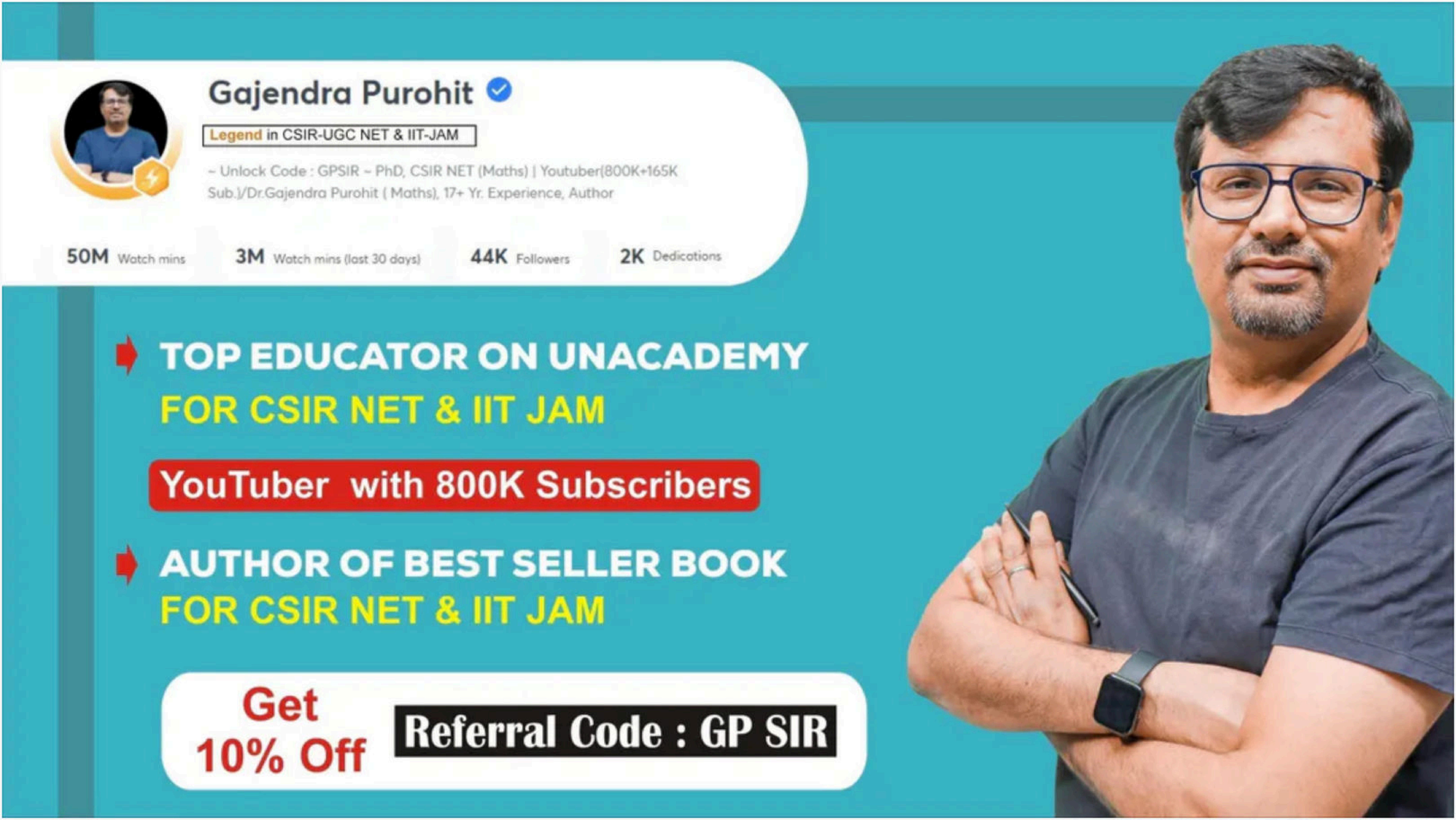
Higher Order Constant Coefficient ODE - I

Detailed Course on Differential Equation for IIT JAM' 23 - II

Gajendra Purohit • Lesson 8 • Aug 17, 2022



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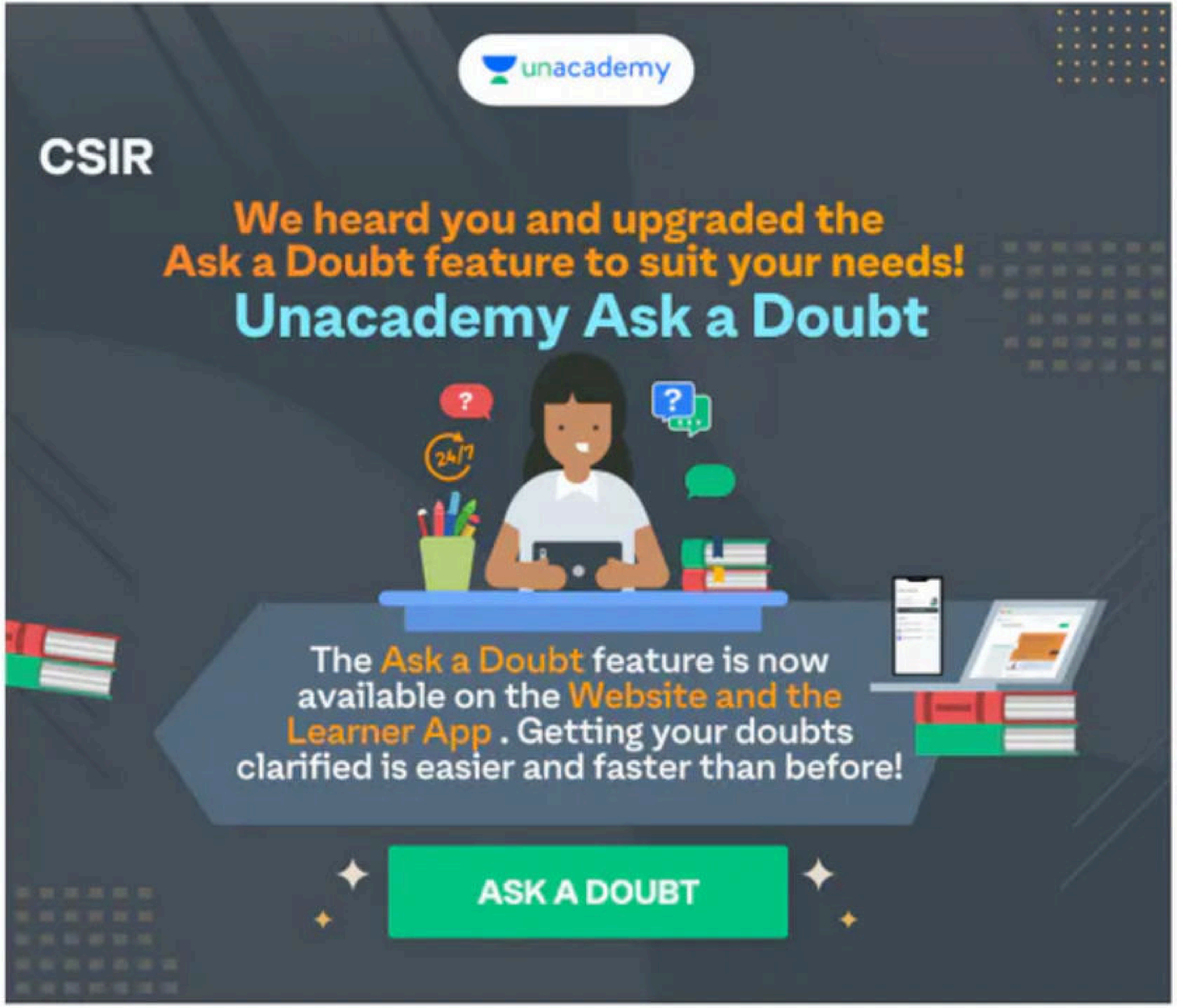
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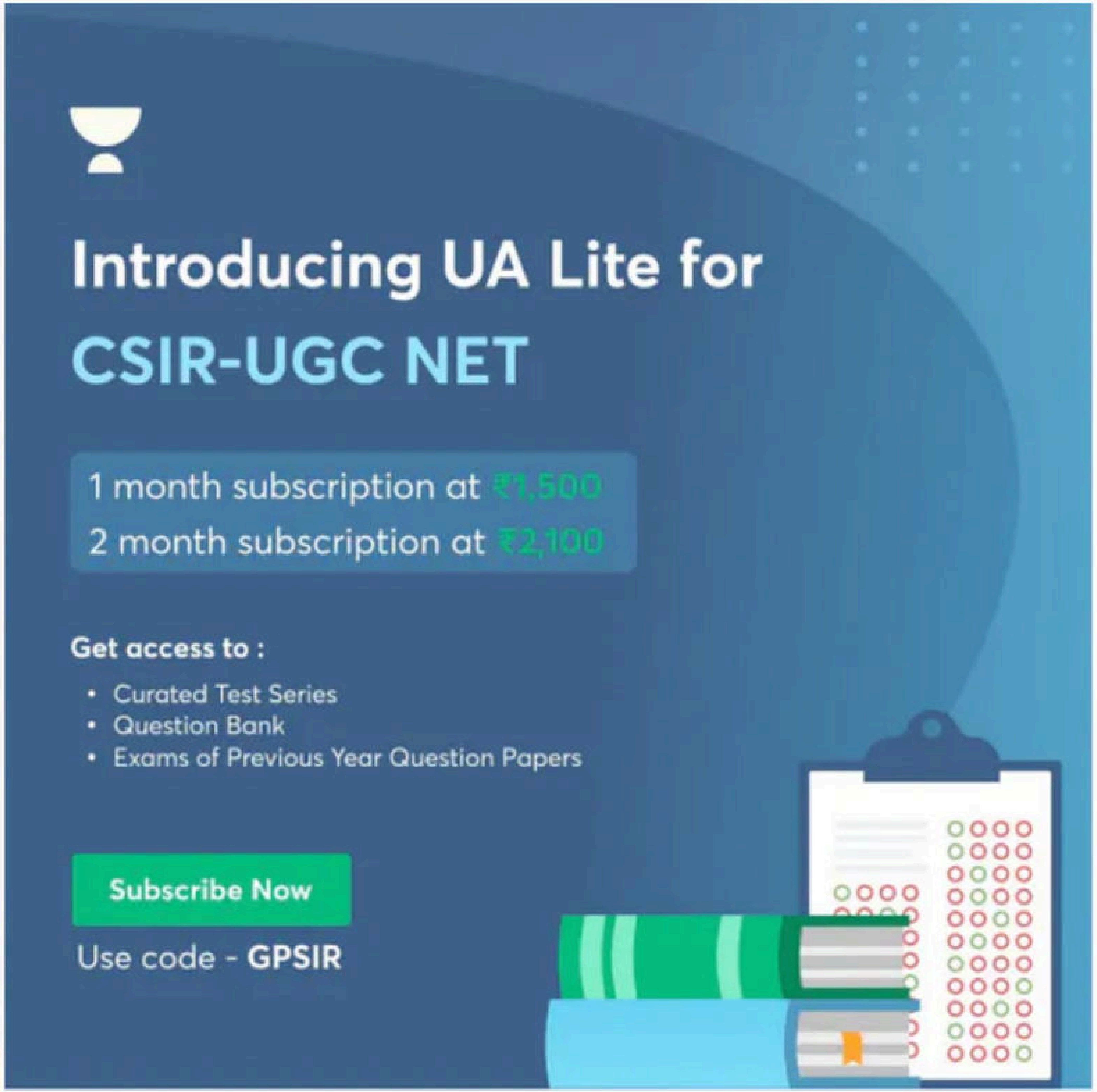

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TRAJECTORIES

A curve which cuts every members of a given family of curve in accordance with same given law is called trajectory of the given family of curve.

Types of Trajectory

(1) **Orthogonal trajectory :** If a curve cuts every member of given family of curves at right angle, it is called an orthogonal trajectory.

(2) **Oblique trajectory :** If a curve cuts every member of a given family of curves at an angle $\alpha(\neq 90^\circ)$, it is called an oblique trajectory.

$x^2+y^2=a^2$

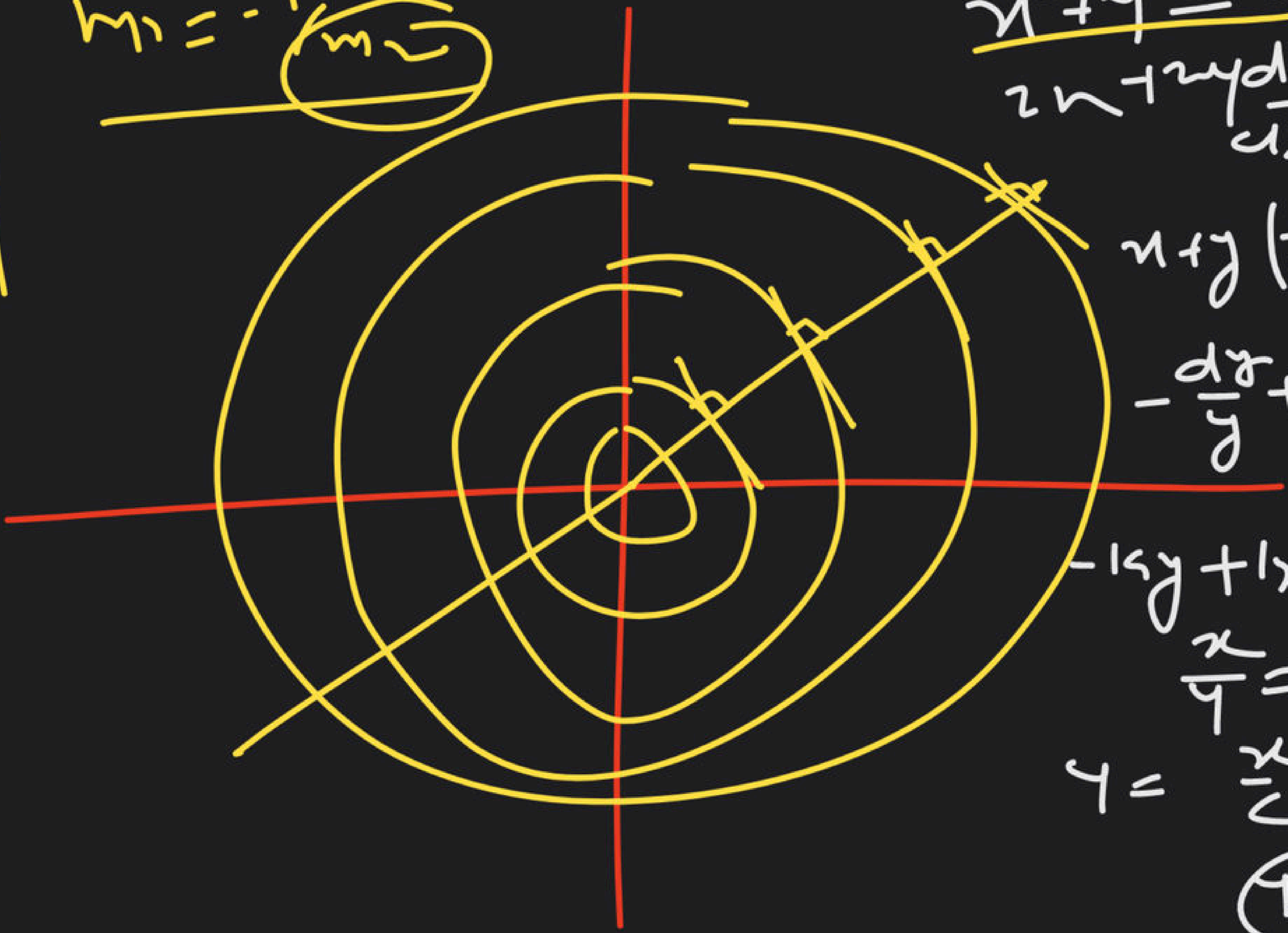
$y=mx$

$m_1 m_2 = -1$

$\left(\frac{dy}{dx}\right) = \frac{-1}{\frac{dy}{dx}}$

$\frac{dy}{dx} = -\frac{dx}{dy}$

$m_1 = -\frac{1}{m_2}$



$x^2+y^2=a^2$

$2x+2y\frac{dy}{dx}=0$

$x+y\left(\frac{-dx}{dy}\right)=0$

$-\frac{dy}{y}+\frac{dx}{x}=0$

$-\ln y+\ln x=\ln c$

$\frac{x}{y}=c$

$y=\frac{x}{c}$

$y=mx$

Determination of orthogonal trajectories in Cartisian coordinates :

Step – 1 : Differentiate the given equation of the family of curves, eliminate the parameter.

Step – 2 : Replace dy/dx by $-dx/dy$.

Step – 3 : Solve this new DE and we obtain orthogonal trajectory.

$$y = ax^2$$

$$a = \frac{y}{x^2}$$

$$\frac{dy}{dx} = 2\boxed{a}x$$

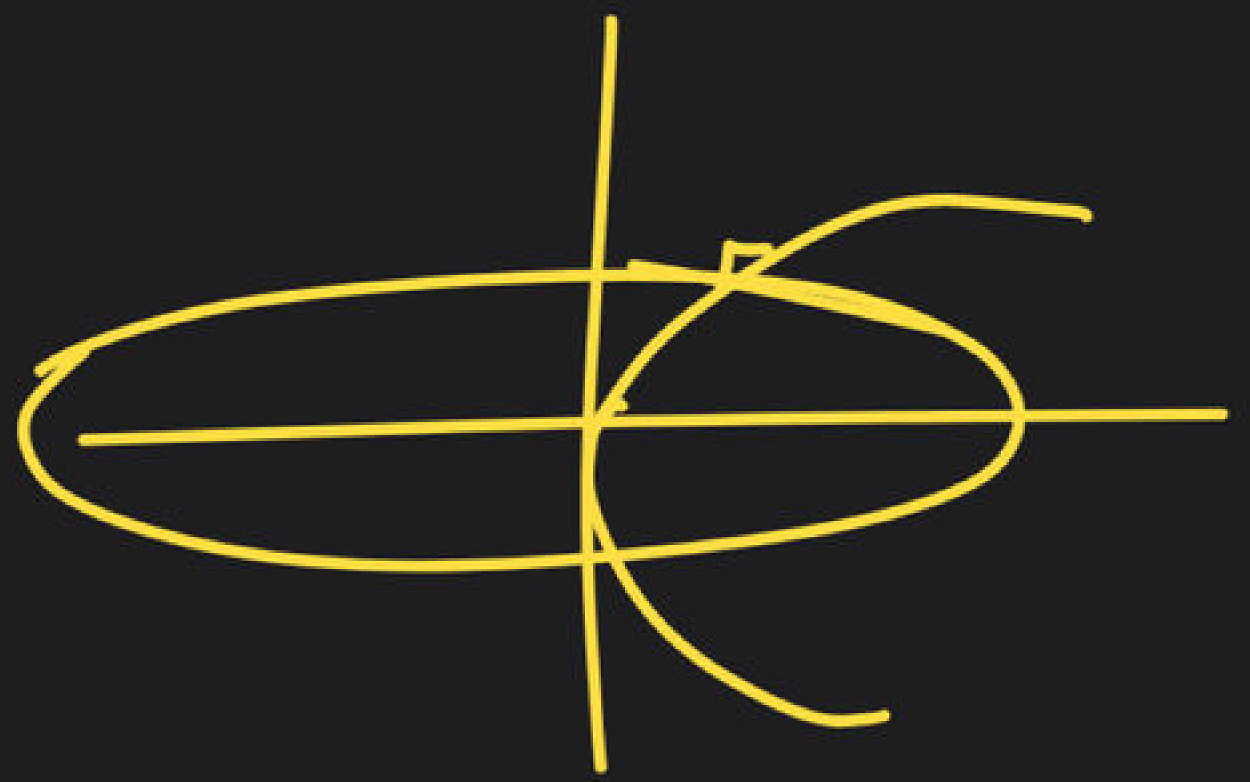
$$\frac{dy}{dx} = \frac{2y}{\cancel{x^2}}\cancel{x}$$

$$-\frac{dx}{dy} = \frac{2y}{x}$$

$$-x\,dx = 2y\,dy$$

$$x\,dx + 2y\,dy = 0$$

$$\frac{x^2}{2} + \frac{y^2}{1} = c$$



$$x^2 - y^2 + 2\lambda xy = 1$$

$$x - y\frac{dy}{dx} + \lambda\left(y + x\frac{dy}{dx}\right) = 0$$

$$x - y\frac{dy}{dx} + \left(\frac{1 - x^2 + y^2}{2xy}\right)\left(y + x\frac{dy}{dx}\right) = 0$$

$$x + y\frac{dx}{dy} + \left(\frac{1 - x^2 + y^2}{2x}\right) - \left(\frac{1 - x^2 + y^2}{2y}\right)\frac{dx}{dy}$$

$$\left(y - \frac{1 - x^2 + y^2}{2y}\right)\frac{dx}{dy} = -\left[x + \frac{1 - x^2 + y^2}{2x}\right]$$

$$\left(\frac{2y^2 - 1 + x^2 - y^2}{2y}\right)\frac{dx}{dy} = -\left[\frac{2x^2 + 1 - x^2 + y^2}{2x}\right]$$

$$\left(\frac{x^2 + y^2 - 1}{2y}\right)\frac{dx}{dy} = -\frac{(x^2 + y^2 + 1)}{2x}$$

$$(x^2 + y^2 - 1)\,x\,dx + (x^2 + y^2 + 1)\,y\,dy = 0$$

$$(x^3 + xy^2 - x)\,dx + (x^2 y + y^3 + y)\,dy$$

$$\int (x^3 + xy^2 - x)\,dx + \int (y^3 + y)\,dy = c$$

$$\frac{x^4}{4} + \frac{x^2y^2}{2} - \frac{x^2}{2} + \frac{y^4}{4} + \frac{y^2}{2} = c$$

Self orthogonal family of curves :

If each member of a given family of curves intersects all other members orthogonally then given family of curves is said to be self orthogonal.

Note : The differential equation of the family of curve is identical with differential equation of its orthogonal trajectories.

$$y^2 = 4a(x+a)$$

$$2y\frac{dy}{dx} = 4a$$

$$a = \frac{y}{2}\frac{dy}{dx}$$

$$y^2 = 4\frac{y}{2}x\frac{dy}{dx} + \cancel{4}\frac{y^2}{\cancel{4}}\left(\frac{dy}{dx}\right)^2$$

$$\cancel{y^2} = \cancel{y}\left[2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2\right]$$

$$\boxed{y = 2x\left(\frac{dy}{dx}\right) + y\left(\frac{dy}{dx}\right)^2}$$

$$y = 2x\left(-\frac{dx}{dy}\right) + y\left(-\frac{dx}{dy}\right)^2$$

$$y = \frac{2x}{-\frac{dy}{dx}} + \frac{y}{\left(\frac{dy}{dx}\right)^2}$$

$$y\left(\frac{dy}{dx}\right)^2 = -2x\left(\frac{dy}{dx}\right) + y$$

$$\boxed{y = 2x\left(\frac{dy}{dx}\right) + y\left(\frac{dy}{dx}\right)^2}$$

Q.1 Which one of the following curves is the orthogonal trajectory of straight line passing through fixed point (5, 6) ?

A) $(x-5)^2 = c(y-6)$

B) $(x-5) = c(y-6)$

C) $(x-5)^2 + (y-6)^2 = c$

D) None of these

$y - y_1 = m(x - x_1)$

$y - 6 = m(x - 5)$

$\frac{dy}{dx} = m$

$y - 6 = \frac{dy}{dx}(x - 5)$

$(y - 6) = -\frac{dx}{dy}(x - 5)$

$(y-6)dy + (x-5)dx = 0$

$\frac{(y-6)^2}{2} + \frac{(x-5)^2}{2} = c$

$(x-5)^2 + (y-6)^2 = c'$

Q find OT of $y^2 = 3x^3 + x + c$

(a) $2\tan^{-1}3x + 3\log y = K$

(b) $3\tan^{-1}3x + 2\log y = K$

(c) $3\tan^{-1}3x - 2\log y = K$

(d) $3\log x - 2\tan^{-1}3y = K$

$$2y\frac{dy}{dx} = 9x^2+1$$

$$2y\left(-\frac{dx}{dy}\right) = 9x^2+1$$

$$\frac{dx}{9x^2+1} + \frac{dy}{2y} = 0$$

$$\int\frac{dx}{1+(3x)^2} + \frac{1}{2}\int\frac{dy}{y} = C$$

$$\frac{\tan^{-1}3x}{3} + \frac{1}{2}\log y = C$$

$$2\tan^{-1}3x + 3\log y = C'$$

Q OT of $(x-1)^2 + y^2 + 2ax = 0$ are soln of

differential equation $2a = -\left(\frac{(x-1)^2+y^2}{x}\right)$

$$2(x-1) + 2y\frac{dy}{dx} + 2a = 0$$

$$2(x-1) + 2y\frac{dy}{dx} - \left[\frac{(x-1)^2+y^2}{x}\right] = 0$$

$$2x(x-1) + 2xy\frac{dy}{dx} - (x-1)^2 - y^2 = 0$$

$$2x^2 - \cancel{2x} + 2xy\frac{dy}{dx} - x^2 + \cancel{2x} - 1 - y^2$$

$$x^2 - y^2 - 1 + 2xy\frac{dy}{dx} = 0$$

$$x^2 - y^2 - 1 - 2xy\frac{dx}{dy} = 0$$

Q.2. The value of $a \in R$ for which the curves $x^2 + ay^2 = 1$ and $y = x^2$ intersect orthogonally is

$2x + 2ay\frac{dy}{dx} = 0$

$m_2 = \frac{dy}{dx} = 2x$

$m_1 = \frac{dy}{dx} = -\frac{x}{ay}$

(a) -2

(b) $\frac{-1}{2}$

(c) $\frac{1}{2}$

(d) 2

$m_1 m_2 = -1$

$\left(-\frac{x}{ay}\right) 2x = -1$

$\frac{+2x^2}{ay} = +1$

$\frac{2\cancel{y}}{a\cancel{y}} = 1$

$a = 2$

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Q.3. The integral curves of the first order linear differential equation $x\frac{dy}{dx}+ay=0$ will be orthogonal to the family of hyperbolas $x^2-by^2=c$ if and only if

A) $a+b=0$ B) $a\ b=1$

C) $a-\ b=0$ D) $a\ b=-1$

$m_1=\frac{dy}{dx}=-\frac{ay}{x}$

$2x-2by\frac{dy}{dx}=0$

$m_2=\frac{dy}{dx}=\frac{x}{by}$

$m_1 m_2=-1$

$\left(-\frac{ay}{x}\right)\left(\frac{x}{by}\right)=-1$ $-\frac{a}{b}=-1$

$+a=b$

$a-b=0$

Q.4. The orthogonal trajectories of family of curves $3xy = x^3 - a^3$, a being paramter of family, is of the form $x^2 = y + f(y)$ and satisfies $y(0) = 0$. then $f(\log 2)$ is

(a) 0.37 $f(y) = -\frac{1}{2} + \frac{e^{-2y}}{2}$

~~(b) -0.37~~

(c) 0.40 $f(\ln 2) = -\frac{1}{2} + \frac{1}{2}e^{-2\ln 2} = -\frac{1}{2} + \frac{1}{8} = -\frac{3}{8}$

(d) 0.5

$x^2 = t$

$2x\frac{dx}{dy} = \frac{dt}{dy}$

$3y + 3x\frac{dy}{dx} = 3x^2$

$y + x\left(-\frac{dx}{dy}\right) = x^2$

$-x\frac{dx}{dy} + y = x^2$

$x\frac{dx}{dy} + x^2 = y$

$\frac{dt}{dy} + 2t = 2y$

$IF = e^{\int 2dy} = e^{2y}$

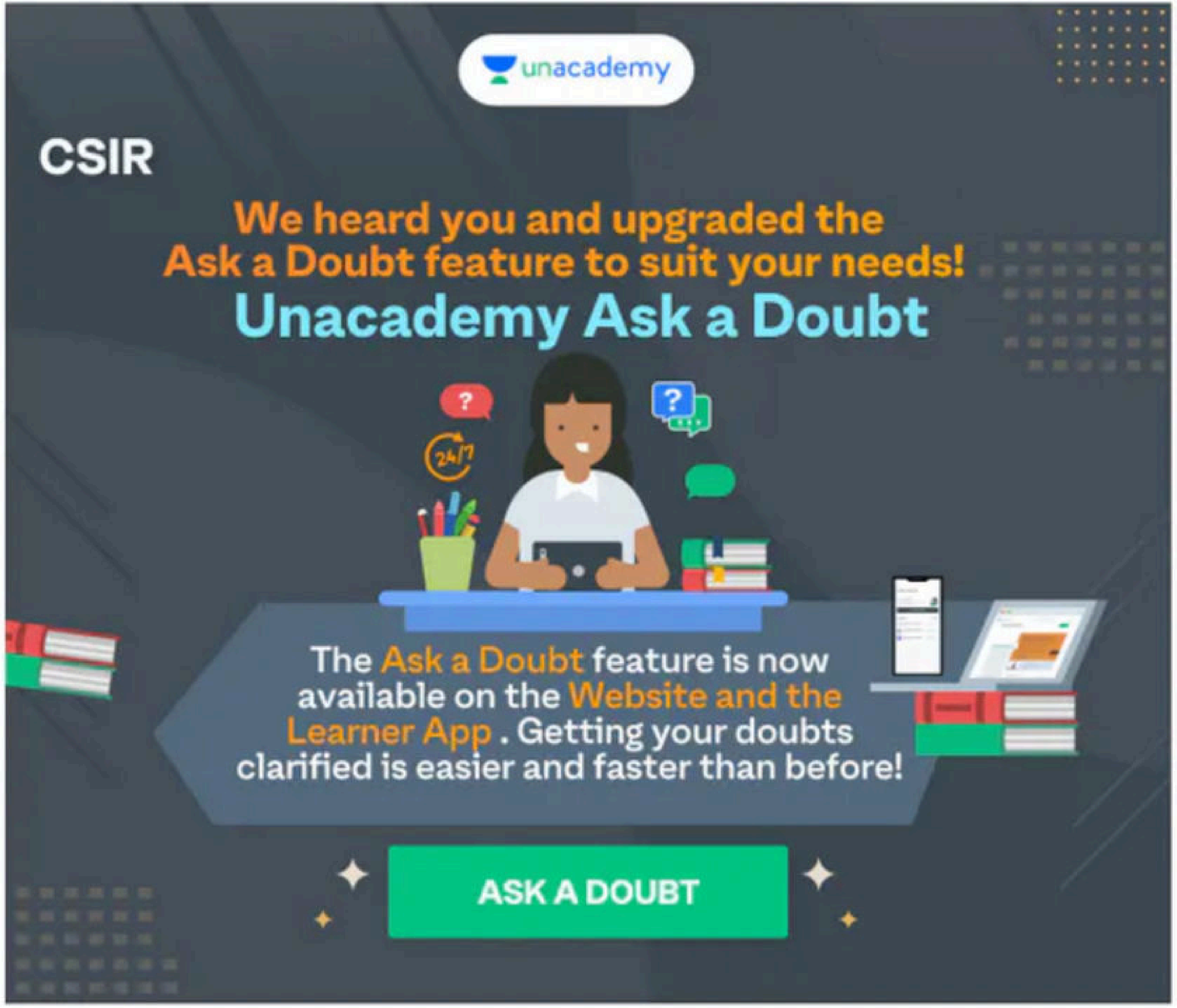
$t \cdot e^{2y} = 2\int e^{2y}\, y\, dy + c$

$x^2 e^{2y} = 2\left(\frac{ye^{2y}}{2} - \frac{e^{2y}}{4}\right) + c$

$x^2 = y - \frac{1}{2} + ce^{-2y}$

$c = \frac{1}{2}$

$x^2 = y - \frac{1}{2} + \frac{1}{2}e^{-2y}$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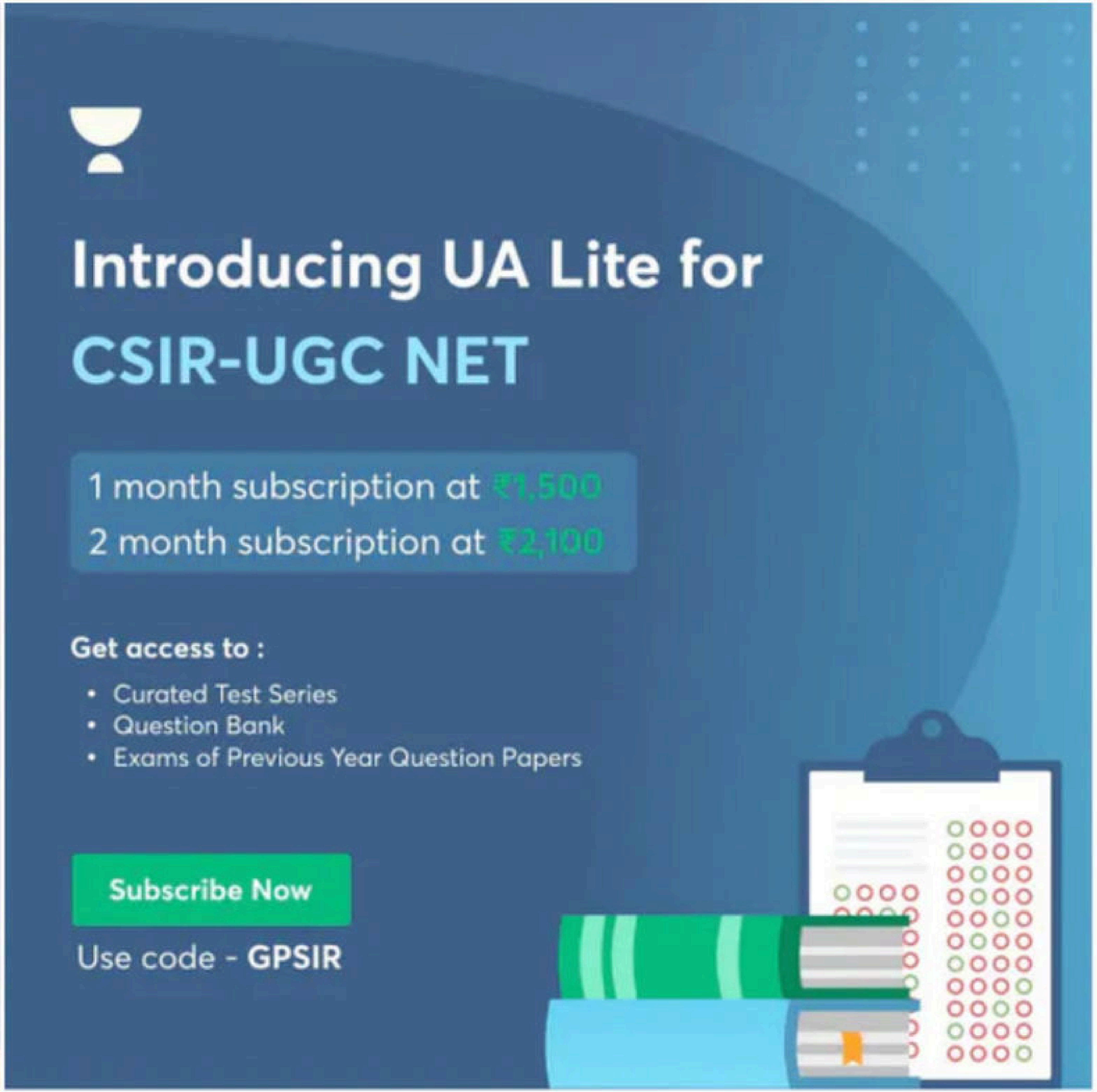
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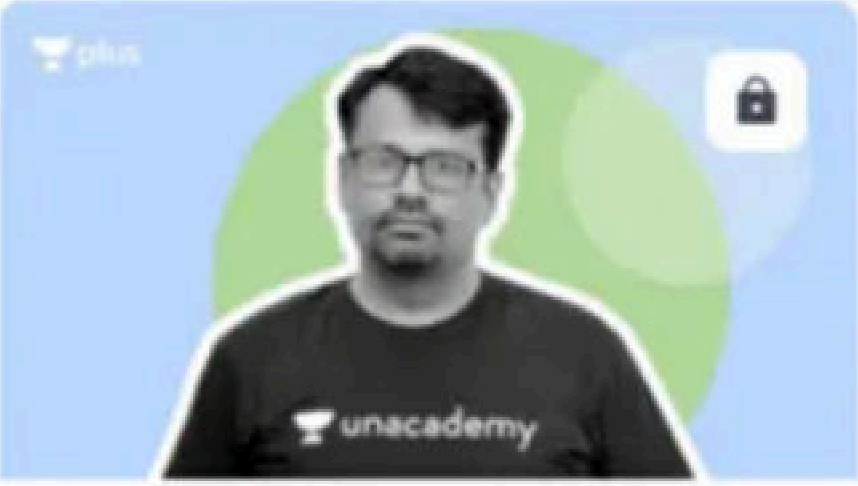

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