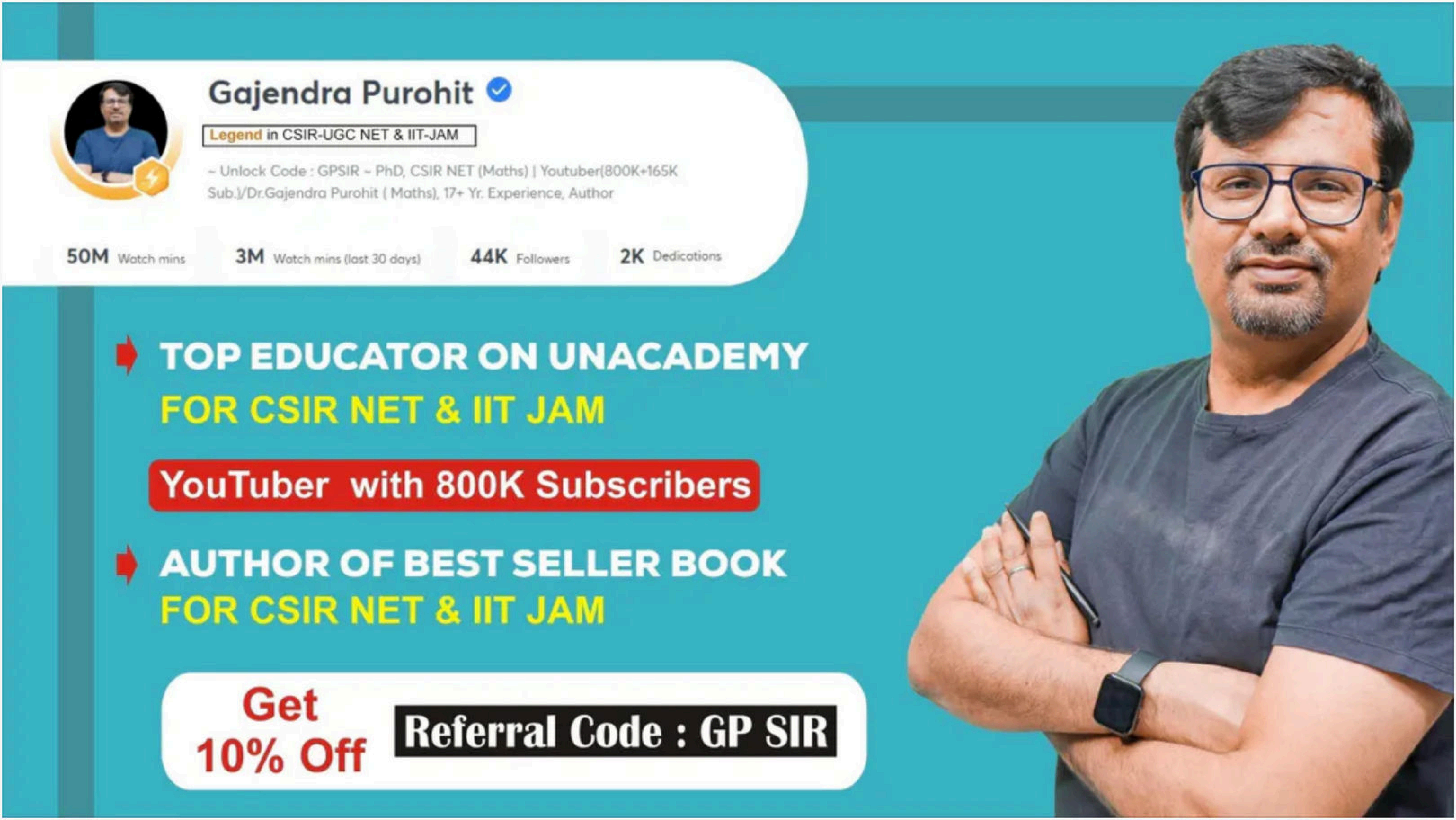
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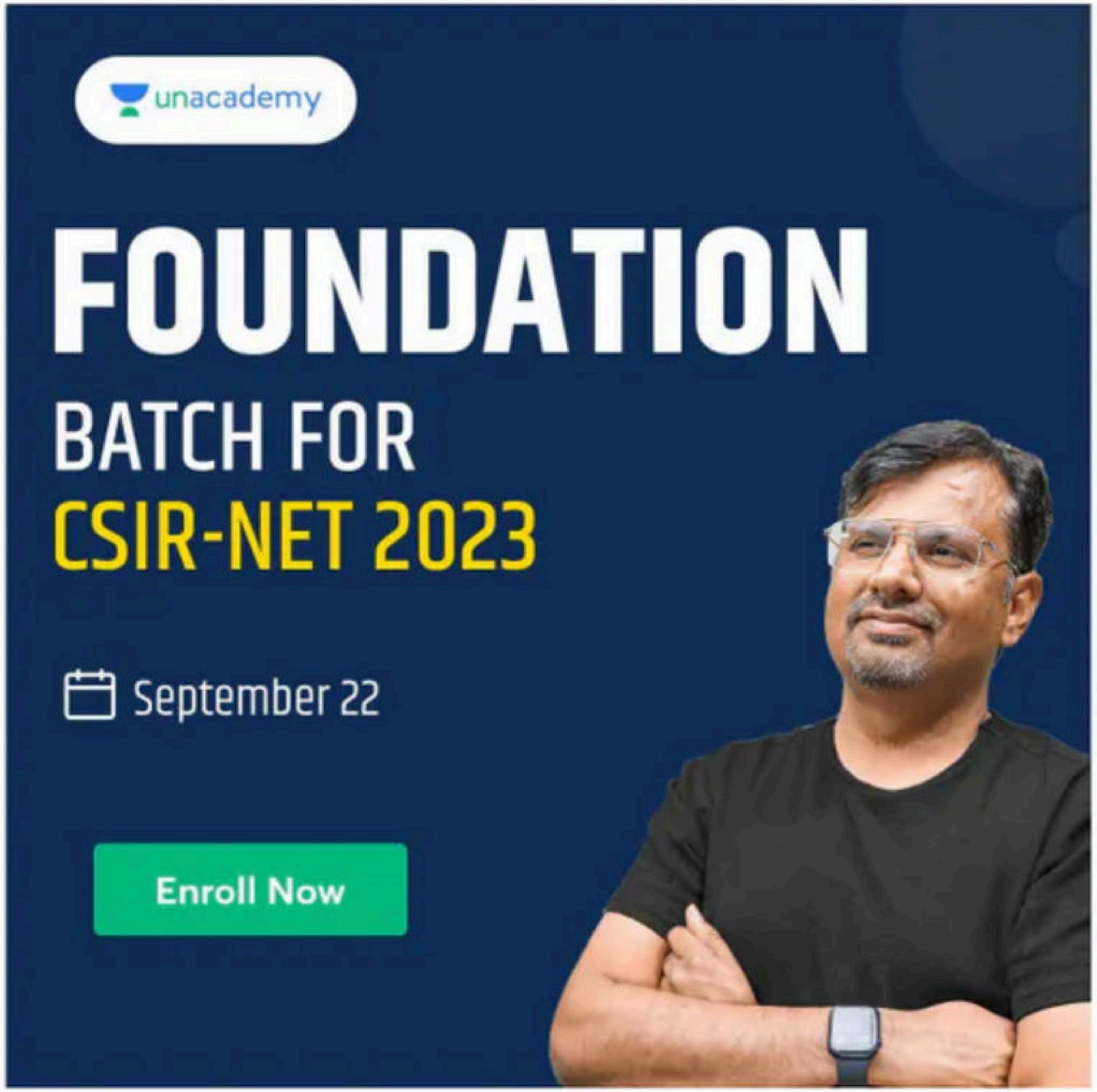
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Bases & Dimension

$R^2(R)$

$R^2(R)$

Basis of vector space : A subset 'S' of a vector space V(F) is said to be basis of V(F), if

(i) S consist of LI vectors.

(ii) L(S) = V i.e. each vector in V is a linear combination of a finite number of element of S.

$$\boxed{S = \{(1,0), (0,1)\}}$$

$$L(S) = \{\alpha(1,0) + \beta(0,1) \mid \alpha, \beta \in R\}$$

$$= \{(\alpha, \beta) \mid \alpha, \beta \in R\} = R^2(R)$$

$R(x)$

$S = \{1, x, x^2, x^3, x^4 \ldots\}$

$L(S) = \alpha_0 \cdot 1 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \ldots = R(x)$

$R_n(x)$

$\boxed{S = \{1, x, x^2, \ldots x^n\}}$

$L(S) = (\alpha_0 \cdot 1 + \alpha_1 x + \cdot \ \alpha_n x^n) = R_n(x)$

$\dim(R_n(x)) = n+1$

$M_{2\times 2}(\mathbb{R})$

$$S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right\}$$

$M_{m\times n}$

$$L(S) = \left\{ \alpha\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \beta\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \gamma\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \delta\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \;\middle|\; \alpha, \beta, \gamma, \delta \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} \alpha & \beta \\ \delta & \gamma \end{pmatrix} \;\middle|\; \alpha, \beta, \gamma, \delta \in \mathbb{R} \right\}$$

$\dim(M_{m\times n}) = \underline{mn}$

Dimension : Let V(F) be a vector space over F then number of elements in basis of V(F) is called dimension of V(F)

Finitely generated & infinitely generated vector space :

If dimension of any vector space is finite then this type of vector space is called finitely generated vector space. Otherwise it is infinitely generated vector space.

Result : Number of LI arbitrary in V constant is dimension of V

$C(C) = 1$

$C(R) = 2$

$C(Q) = \infty$

$C^2(C)$

$R^2(R)$

$Q^2(Q)$

$x + iy$

$B = \langle (1, i) \rangle$

$\alpha \cdot 1 + \beta \cdot i$

Q.1. Let us define a sequence $(a_n)_{n \in N}$ of real number to be a Fibonacci like sequence if $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. What is the dimension of the R-vector space of Fibonacci like sequence?

a_{n-1}

a_{n-2}

(a) 1

(b) 2

$a_2 = a_1 + a_2$

(c) Infinnite & countable

(d) Infinite & uncountable

1, 1, 2, 3, 5

Some Important Example :

(1) Let V = ~~$C^n(C)$~~/ $R^n(R)$

Results :

$$W = \{(a_1, a_2, \ldots, a_n) \mid 'k'\ LI\ condition\} \text{ then } \mathrm{Dim}W = n - k$$

(b) $$W_2 = \left\{(a_1, a_2, \ldots, a_n) \mid \sum_{i=1}^{n} i a_i = 0\right\} \text{ then } \mathrm{Dim}W_2 = n - 1$$

(a) $W_1 = \left\{(a_1, a_2, \ldots, a_n) \mid \sum_{i=1}^{n} a_i = 0\right\}$ then $\text{Dim}W_1 = (n-1) \times 2$

$(<k)$

$2^{①} \times 3^{①}$

$(1+1)\times(1+1) = 4$

(b) $W_2 = \left\{(a_1, a_2, \ldots, a_n) \mid \sum_{i=1}^{n} i a_i = 0\right\}$ then $\text{Dim}W_2 = (n-1) \times$

$w = \{(a_1, a_2, a_3, a_4, a_5, a_6) \mid a_i = 0$

$4 \; i \mid 6)$

(c) $W_3 = \{(a_1, a_2, \ldots, a_n) \mid a_{2i} = 0\}, i = 1, 2, \ldots \left(\left\lceil \frac{n}{2} \right\rceil\right) \times$

$= (0, 0, 0, a_4, a_5, 0)$

then $\text{Dim}W_3 = \left(\left\lfloor \frac{n+1}{2} \right\rfloor\right)$

(d) $W_4 = \{(a_1, a_2, \ldots, a_n) \mid a_i = 0 \text{ if } i|n\}, i = 1, 2, \ldots, n$

then $\text{Dim } W_4 = (n - \tau(n)) = 6 - 4 = 2$

(2) Let V = $C^n(R)$

$i+1$

(a) $W_1 = \left\{(a_1, a_2, \ldots, a_n) \mid \sum_{i=1}^{n} a_i = 0\right\}$ then $\text{Dim}W_1 = 2(n-1)$

(b) $W_2 = \left\{(a_1, a_2, \ldots, a_n) \mid \sum_{i=1}^{n} i a_i = 0\right\}$ then $\text{Dim}W_2 = 2(n-1)$

$C(R) = 2$

$C^2(R) = 2\times 2$

$C^3(R) = \underline{3\times 2}$

(c) $W_3 = \{(a_1, a_2, \ldots, a_n) \mid a_{2i} = 0\}, i = 1, 2, \ldots \left\lfloor \frac{n+1}{2} \right\rfloor$

then $\text{Dim}W_3 = 2\left\lfloor \frac{n+1}{2} \right\rfloor$

(d) $W_4 = \{(a_1, a_2, \ldots, a_n) \mid a_i = 0 \text{ if } i/n\}, i = 1, 2, \ldots, n$

then $\text{Dim } W_4 = 2(n - \tau(n))$

(3) Let $V = C^{m\times n}(C)/\ R^{m\times n}(R)$

(a) $W_1 = \left\{[a_{ij}] \mid \sum_{j=1}^{n} a_{ij} = 0, i = 1,2,\ldots,m\right\}$ then Dim $W_1 = mn - m$

(b) $W_2 = \left\{[a_{ij}] \mid \sum_{i=1}^{m} a_{ij} = 0, j = 1,2,\ldots,n\right\}$ then Dim $W_1 = mn - n$

(c) $W_3 = \{[a_{ij}] \mid a_{ij} = 0 \text{ if } i \mid n\}$ then Dim $W_3 = mn - n.\tau(m)$

(d) $W_4 = \{A = [a_{ij}]_{n\times n} / Tr(A) = 0\}$ then Dim $W_3 = n^2 - 1$

(e) $W_5 = \{A = [a_{ij}]_{m\times n} / A^T = A\}$ then Dim $W_3 = \dfrac{n(n+1)}{2}$

$$\begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix}$$

$1+2+3$

$1+2+3+ \cdots n$

$\dfrac{n(n+1)}{2}$

(f) $W_6 = \{A = [a_{ij}]_{m \times n} / A^T = -A\}$ then Dim $W_3 = \dfrac{n(n-1)}{2}$

(g) $W_7 = \{A = [a_{ij}]_{n \times n} /$ A is upper triangle$\}$

then Dim $W_3 = \dfrac{n(n+1)}{2}$

$$\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

(h) $W_8 = \{A = [a_{ij}]_{n \times n} /$ A is diagonal$\}$ then Dim $W_3 = n$

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

4×4

$(1 + 2 +)$

$n \times m$

$$\begin{pmatrix} 0 & a & b \\ a & 0 & c \\ -b & -c & 0 \end{pmatrix}$$

$(1 + 2 +) + \dots n$

$(1 + 2$ ___

$\frac{n(n)}{2}$

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(4) Let V = R[x]

(a) $W_1 = \{p(x) / \deg(p(x)) \le n\}$ then Dim W_3 = n + 1

(b) $W_5 = \{p(x) \in F_n[x] / p(\alpha) = 0, \alpha \in F\}$ then Dim W_3 = n

(c) $W_7 = \{p(x) \in F_n[x] / p(x) = p(1-x)\}$

then Dim $W_3 = \left[\frac{n}{2}\right] + 1$

$W = \langle a + bx \mid p(x) = p(1-x) \rangle$

$a + bx = a + b(1-x)$

$\cancel{a} + bx = \cancel{a} + b - bx$

$2bx - b = 0$

$b(2x-1) = 0$

$b = 0$

$\left[\frac{2}{2}\right] + 1$

=

$a + bx + cx^2 = a + b(1-x) + c(1-x)^2$

(d) $W_8 = \{p(x) \in F_n[x] \mid p(\alpha) = p(\beta)$ then Dim = n

(e) $W_{11} = \{p(x) \in F_n[x] \mid p(x) = p(-x)\}$

$a + bx + cx^2$

$\cancel{a} + bx + \cancel{cx^2} = \cancel{a} - bx + \cancel{cx^2}$

$b = 0$

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Q.1 Let V be the vector space of all 6×6 real matrices then dimension of subspace of V consisting of all symmetric matrices is

(a) 15 (b) 18

(c) 21 (d) 35

$\frac{n(n+1)}{2}$

$\frac{6(6+1)}{2} = \frac{6 \times 7}{2}$

Q.2. Let V denote the vector space of real valued continuous functions on the closed interval [0, 1]. Let W be the subspace of V spanned by {sin(x), cos(x), tan(x)}. Then the dimension of W over R is

(a) 1 (b) 2

(c) 3 (d) infinite

$\sin x + \cos x \neq \tan x$

$\tan x = \dfrac{\sin x}{\cos x}$

Q.3. Let $M_2(R)$ be the vector space of 2×2 real matrices. Let V be a subspace of $M_2(R)$ defined by

$$V = \left\{ A \in M_2(R); A\begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} A \right\}$$

Then the dimension of V is

(a) 1 (b) 2

(c) 3 (d) 4

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\begin{pmatrix} 3b & 2a+b \\ 3d & 2c+d \end{pmatrix} = \begin{pmatrix} 2c & 2d \\ 3a+c & 3b+d \end{pmatrix}$$

$3b = 2c$

$3d = 3a + c$

$d = \frac{3a + 3b/2}{3}$

$d = a + b/2$

$2a + b = 2d$

$2c + \cancel{d} = 3b + \cancel{d}$

$3b = 2c$

$$\begin{pmatrix} a & b \\ \frac{3}{2}b & a + b/2 \end{pmatrix}$$

Q.4. Consider the following subspace of R^3

$$W = \{(x, y, z) \in R^3 \mid 2x + 2y + z = 0, 3x + 3y - 2z = 0, x + y - 3z = 0\}$$

Then dimension of W is

(a) 0 (b) 1

(c) 2 (d) 3

2

$$\begin{pmatrix} 1 & 1 & -3 \\ 2 & 2 & 1 \\ 3 & 3 & -2 \end{pmatrix}$$

$3 - 2$

$\rho(A) = 2$


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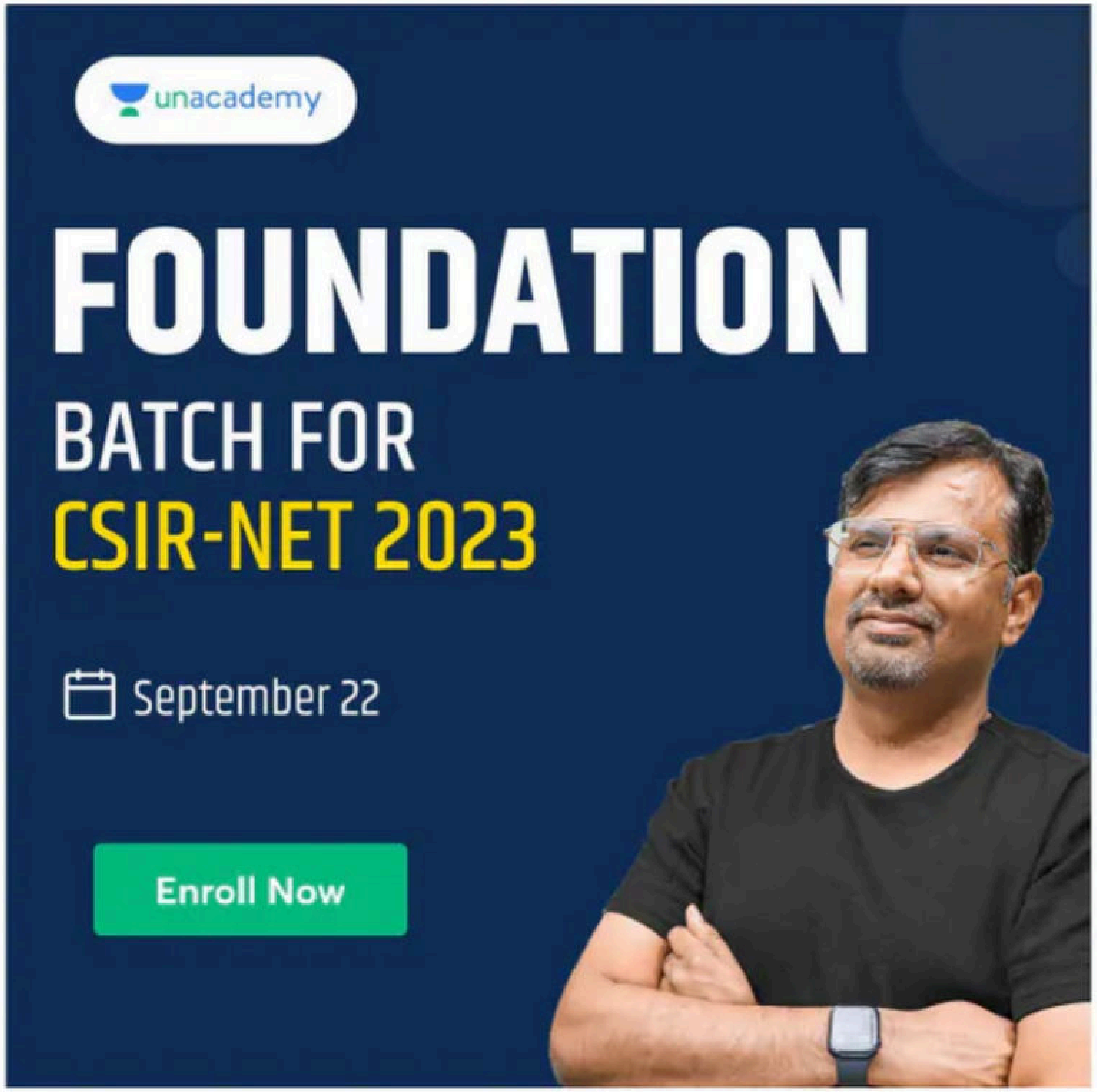
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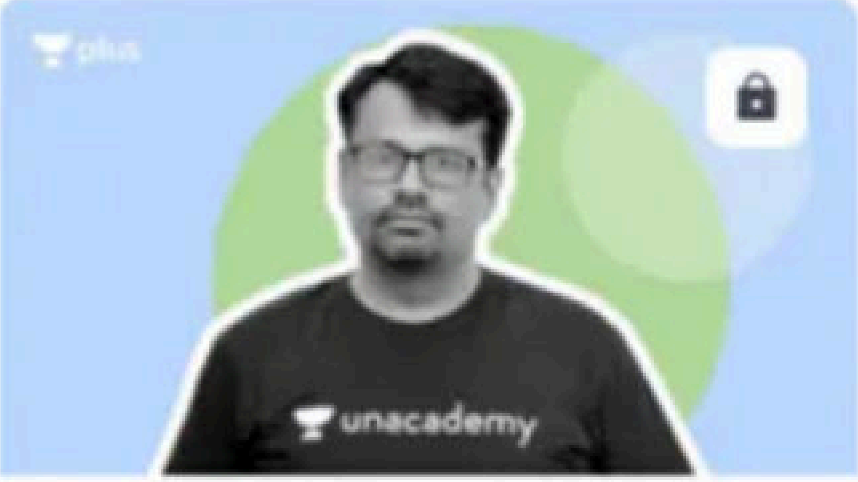

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- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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