Gajendra Purohit

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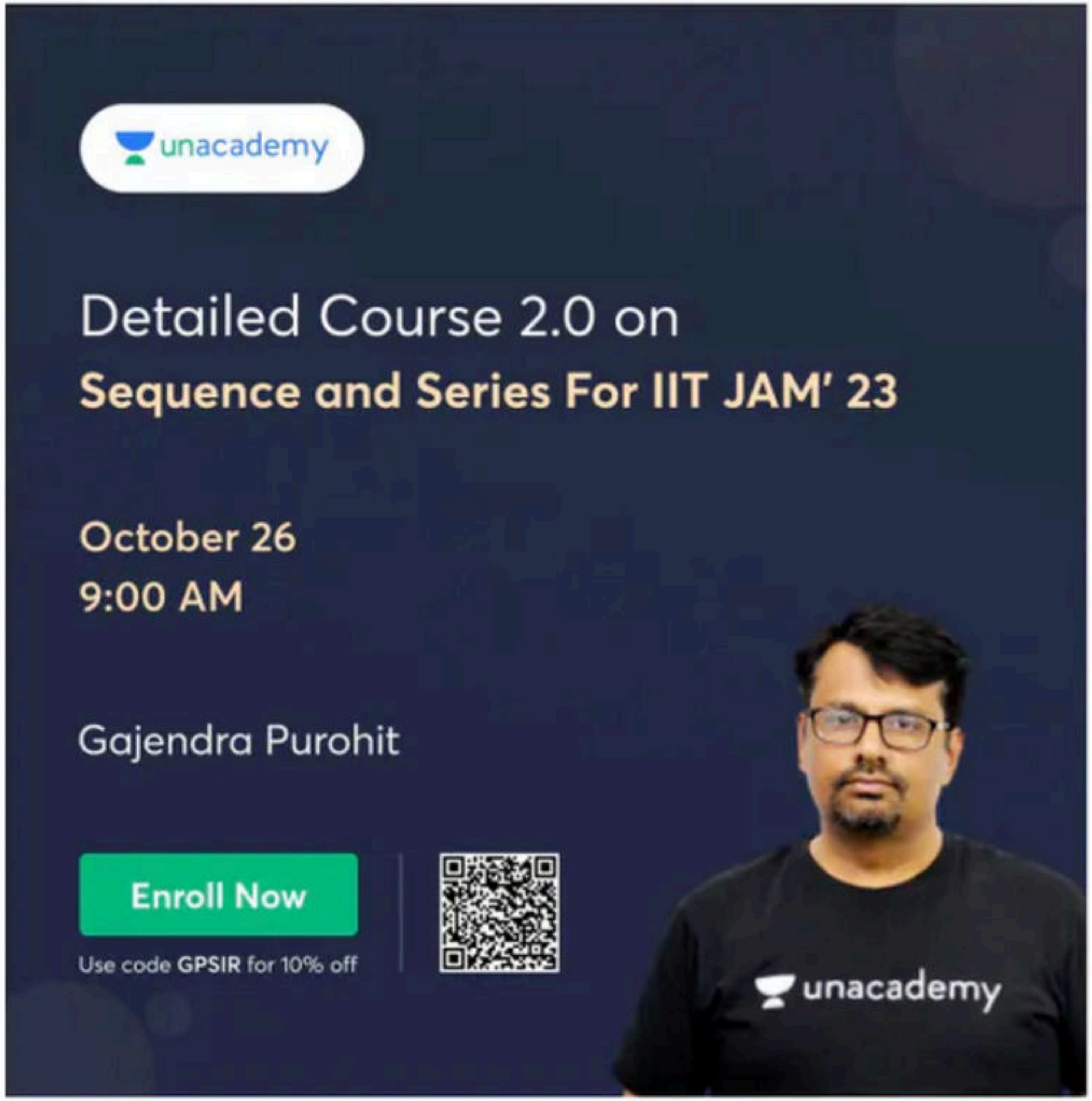
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SERIES OF REAL NUMBERS

Infinite Series : Let $<u_n>$ be a given sequence then a symbol of the form $u_1 + u_2 + \dots u_n + \dots$. is called infinite series. it is denoted by $\sum_{n=1}^{\infty} u_n$.

$$\sum_{n=1}^{\infty} \cos\frac{2}{n}$$

$$\sum_{n=1}^{\infty} \sin\frac{1}{n}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Sequence of partial sums of the series : Let $<u_n>$ be a sequence and $\sum u_n$ be a series. Then

the sequene $<S_n>$ defined by

$S_1 = u_1,$

$S_2 = u_1 + u_2,$

.....

$S_n = u_1 + u_2 + ... + u_n$

The elements of the sequence $<S_n>$ are called the partial sums of the series and the sequence $<S_n>$ is called sequence of partial sums (SOPS).

$<S_1, S_2, S_3 \ldots S_n>$

$<S_n>$

$$\sum u_n = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \cdots$$

$$S_1 = \frac{1}{1.2} = 1 - \frac{1}{2}$$

$$S_2 = \frac{1}{1.2} + \frac{1}{2.3} = \left(1 - \cancel{\tfrac{1}{2}}\right) + \left(\cancel{\tfrac{1}{2}} - \tfrac{1}{3}\right) = 1 - \frac{1}{3}$$

$$S_3 = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} = 1 - \frac{1}{4}$$

$$\cdot \quad - \quad - \quad \cdot$$

$$S_n = 1 - \frac{1}{n+1}$$

$$= \langle S_1, S_2, S_3 \ldots S_n \rangle$$

$$= \langle 1 - \tfrac{1}{2},\ 1 - \tfrac{1}{3},\ 1 - \tfrac{1}{4} \ldots\ 1 - \tfrac{1}{n+1} \rangle$$

$$\lim_{n \to \infty} S_n = \lim_{n \to \infty} 1 - \frac{1}{n+1}$$

$$= 1$$

$$\sum_{n=1}^{\infty} n = 1+2+3+4+\cdots$$

$S_1 = 1$

$S_2 = 1+2$

$S_3 = 1+2+3$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n(n+1)}{2} = \infty$$

$$S_n = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$\left\langle 1, 3, 6, \ldots \frac{n(n+1)}{2} \right\rangle$$

$$\sum_{n=1}^{\infty} (-1)^n = -1+1-1+1-\dots$$

$\langle -1, 0, -1, 0 \dots \rangle$

$S_1 = -1$

$S_2 = 0$

$S_3 = -1$

$\lim_{n\to\infty} S_n =$

$$S_n = \begin{cases} -1 & n - \text{odd} \\ 0 & n - \text{even} \end{cases}$$

$$\sum u_n = \sum_{n=1}^{\infty} \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$$

$$\frac{1}{1-r}$$

$1 + r + r^2 + r^3 + \cdots$

$$\frac{1-r^n}{1-r}$$

$\langle S_1, S_2 - S_n \rangle$

$S_1 = 1$

$S_2 = 1 + 1/2$

$S_3 = 1 + 1/2 + 1/2^2 -$

$$S_n = 1 + 1/2 + 1/2^2 + \cdots \frac{1}{2^{n-1}}$$

$$\frac{1-r^n}{1-r}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1-(1/2)^{n}}{1-1/2}$$

$$= \frac{1}{1-1/2} = 2$$

Sum of Series : Let $\sum u_n$ be a series and S_n is sequence of partial sum of the series then sum of the series = $\lim_{n\to\infty} S_n$

Geometric Series :

$1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + - - -$

Let us consider the series $1 + a + a^2 + ... + a^n + ...$

Case – 1 If $|a| < 1$.

Then SOPS of this series is $S_n = 1 + a + a^2 + ... + a^{n-1}$

Then $S_n = \dfrac{1-a^n}{1-a}$ given that $|a| < 1$

So, $\lim_{n\to\infty} S_n = \dfrac{1}{1-a}$ = Sum of the Series

So, the series is convergent because SOPS of the series is convergent.

Case – 2 If |a| > 1

$$S_n = \frac{a^n - 1}{a - 1} \qquad \lim_{n\to\infty} S_n = \infty$$

The series is divergent because SOPS of the series is divergent.

Case – 3 If a = 1

$$\sum u_n = \sum_{n=1}^{\infty} (1)^{n-1}$$

$S_1 = 1$, $S_2 = 1 + 1$, $S_3 = 1 + 1 + 1$, ...

$$\lim_{n\to\infty} S_n = \infty$$

So, the given series is divergent.

Result : Every constant series is divergent.

Case – 4 If a = -1

$$\sum u_n = \sum_{n=1}^{\infty} (-1)^{n-1}$$

$S_1 = 1, S_2 = 0, S_3 = 1, \ldots$

SOPS oscillate so the series is oscillating.

Telescopic Series : Let $<a_n>$ be a seqence of real numbers, then the series $\sum_{n=1}^{\infty}(a_n - a_{n+1})$ and $\sum_{n=1}^{\infty}(a_{n+1} - a_n)$ are convergent iff the sequence $<a_n>$ is convergent.

$$\sum_{n=1}^{\infty}(a_n - a_{n+1}) = a_1 - \lim_{n\to\infty} a_n \text{ and } \sum_{n=1}^{\infty}(a_{n+1} - a_n) = \lim_{n\to\infty} a_n - a_1 =$$

= Sum of the Series

$$\sum_{n=1}^{\infty}(a_n - a_{n+1})$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= a_1 - \lim_{n\to\infty} a_n$$

$$= 1 - \lim_{n\to\infty} \frac{1}{n}$$

$$= 1 - 0$$

$$= 1$$

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$= \frac{1}{n} - \frac{1}{n+1}$$

$$= \left(1 - \cancel{\tfrac{1}{2}}\right) + \left(\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}}\right) + \left(\cancel{\tfrac{1}{3}} - \tfrac{1}{4}\right)$$

$$+ \cdots \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S_1 = 1 - \tfrac{1}{2}$$

$$S_2 = 1 - \tfrac{1}{3}$$

$$S_3 = 1 - \tfrac{1}{4}$$

$$S_n = 1 - \tfrac{1}{n+1}$$

Converges of positive term series :

The convergence of a positive terms series $\sum_{n=1}^{\infty} u_n$ depend on the sequence $<S_n>$ of the partial sum of the series.

i.e. A series $\sum u_n$ is convergent $\Leftrightarrow$ SOPS is convergent.

A series $\sum u_n$ is divergent $\Leftrightarrow$ SOPS is divergent.

A series $\sum u_n$ oscillate $\Leftrightarrow$ SOPS oscillate.

Q.1. The sum of the series $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^{n-1}$ is $= 1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \cdots + \left(\frac{3}{4}\right)^{n-1}$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1-\left(\frac{3}{4}\right)^n}{1-\frac{3}{4}}$$

(a) 1 (b) 4

(c) 0 (d) does not exist

$$= \frac{1}{1-\frac{3}{4}} = 4$$

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Q.2. What is the sum of series

$$\left(\frac{1}{2.3}+\frac{1}{2^2.3}\right)+\left(\frac{1}{2^2.3^2}+\frac{1}{2^3.3^2}\right)+\ldots\left(\frac{1}{2^a.3^a}+\frac{1}{2^{a+1}.3^a}\right)+\ldots$$

CSIR NET JUNE 2019

(a) $\frac{3}{8}$

(b) $\frac{3}{10}$

(c) $\frac{3}{14}$

(d) $\frac{3}{16}$

$\frac{1-(1/6)^a}{1-1/6}$

$\left(\frac{1}{6}+\frac{1}{6^2}+\cdot\ +\frac{1}{6^a}\right)$

$+\left(\frac{1}{2^2.3}+\frac{1}{2^3.3^2}+\ldots+\frac{1}{2^{a+1}3^a}\right)$

$\left(\frac{1}{6}+\frac{1}{6^2}+\cdots\right)+\frac{1}{2}\left(\frac{1}{6}+\frac{1}{6^2}\cdots\right)$

$\left(\frac{1}{6}+\frac{1}{6^2}+\cdots\right)(1+1/2)$

$(3/2)\left(\frac{1}{6}\right)\left(1+1/6+1/6^2+\ldots\right)$

$\frac{1}{4}\left(\frac{1}{1-1/6}\right)$

$1/4\left(\frac{6}{5}\right)=\frac{3}{10}$

Results :

(1) $$\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)\ldots.(n+m)}=\frac{1}{m}\cdot\frac{1}{\lfloor\!\underline{m}}$$

(2) $$\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}=\sum_{n=2}^{\infty}\frac{1}{(n-1)n(n+1)}=\frac{1}{2}\cdot\frac{1}{\lfloor\!\underline{2}}$$

$\sum \frac{1}{n(n+1)} = \frac{1}{1.1!} = 1$

$\sum \frac{1}{n(n+1)(n+2)} = \frac{1}{2.2!} = 1/4$

$\sum \frac{1}{(n-1)n(n+1)}$

$\sum \frac{1}{n(n+1)(n+2)(n+3)(n+4)} = \frac{1}{4.4!}$

$\sum \frac{1}{(n-2)(n-1)n(n+1)(n+2)}$

Q.3. The sum of the series

$$\frac{1}{2(2^2-1)}+\frac{1}{3(3^2-1)}+\frac{1}{4(4^2-1)}+\ldots\ldots?$$

$= \sum_{n=2}^{\infty} \frac{1}{n(n^2-1)}$

$\sum_{n=2}^{\infty} \frac{1}{(n-1)\,n\,(n+1)} = \frac{1}{2\cdot 2!}$

$= 1/4$

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(a) 1

(b) 0.25

(c) 0.5

(d)2

Q.4. The sum of the series $\sum_{n=1}^{\infty} \frac{1}{(4n-3)(4n+1)}$ is equal to

IIT JAM 2022

(a) 1 (b) 0.5

(c) 0.25 (d) 2

$$\sum_{n=1}^{\infty} \frac{1}{(7n-2)(7n+5)}$$

Q.5. The sum of the series $\frac{1}{1!}+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\ldots$ equals :

(a) e (b) e/2

(c) 3e/2 (d) $1+\frac{e}{2}$

$\frac{\Sigma n}{n!}$

$\Sigma \frac{n(n+1)}{2n!}$

$\frac{1}{2}\Sigma \frac{\cancel{n}(n+1)}{\cancel{n}(n-1)!}$

$\frac{1}{2}\Sigma \frac{n-1+2}{(n-1)!}$

$\frac{1}{2}\left[\Sigma \frac{n-1}{(n-1)!} + 2\Sigma \frac{1}{(n-1)!}\right]$

$\frac{1}{2}\left(\Sigma \frac{1}{(n-2)!} + 2\Sigma \frac{1}{(n-1)!}\right)$

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots = \Sigma \frac{x^n}{n!}$

$e^1 = \Sigma \frac{1}{n!} = \Sigma \frac{1}{(n-1)!} = \Sigma \frac{1}{(n-2)!}$

$\frac{1}{2}(e+2e) = \frac{3e}{2}$

Q.6. $\lim\limits_{n\to\infty} \frac{1}{n^4} \sum\limits_{j=0}^{2n-1} j^3$ equals :

CSIR NET DEC 2016

(a) 4 (b) 16

(c) 1 (d) 8

$$\left(\frac{n(n+1)}{2}\right)^2$$

$$\lim_{n\to\infty} \frac{1}{n^4}\left(\frac{(2n-1)(2n+1)}{2}\right)^2$$

$$\lim_{n\to\infty} \frac{1}{n^4}\left(\frac{4n^2\left(1-\frac{1}{2n}\right)\left(1+\frac{1}{2n}\right)}{2}\right)^2$$

$$\lim_{n\to\infty} \frac{1}{\cancel{n^4}} 4\cancel{n^4}\left(\left(1-\frac{1}{2n}\right)\left(1+\frac{1}{2n}\right)\right)^2$$

$$= 4$$

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1 • Asked by Manvi

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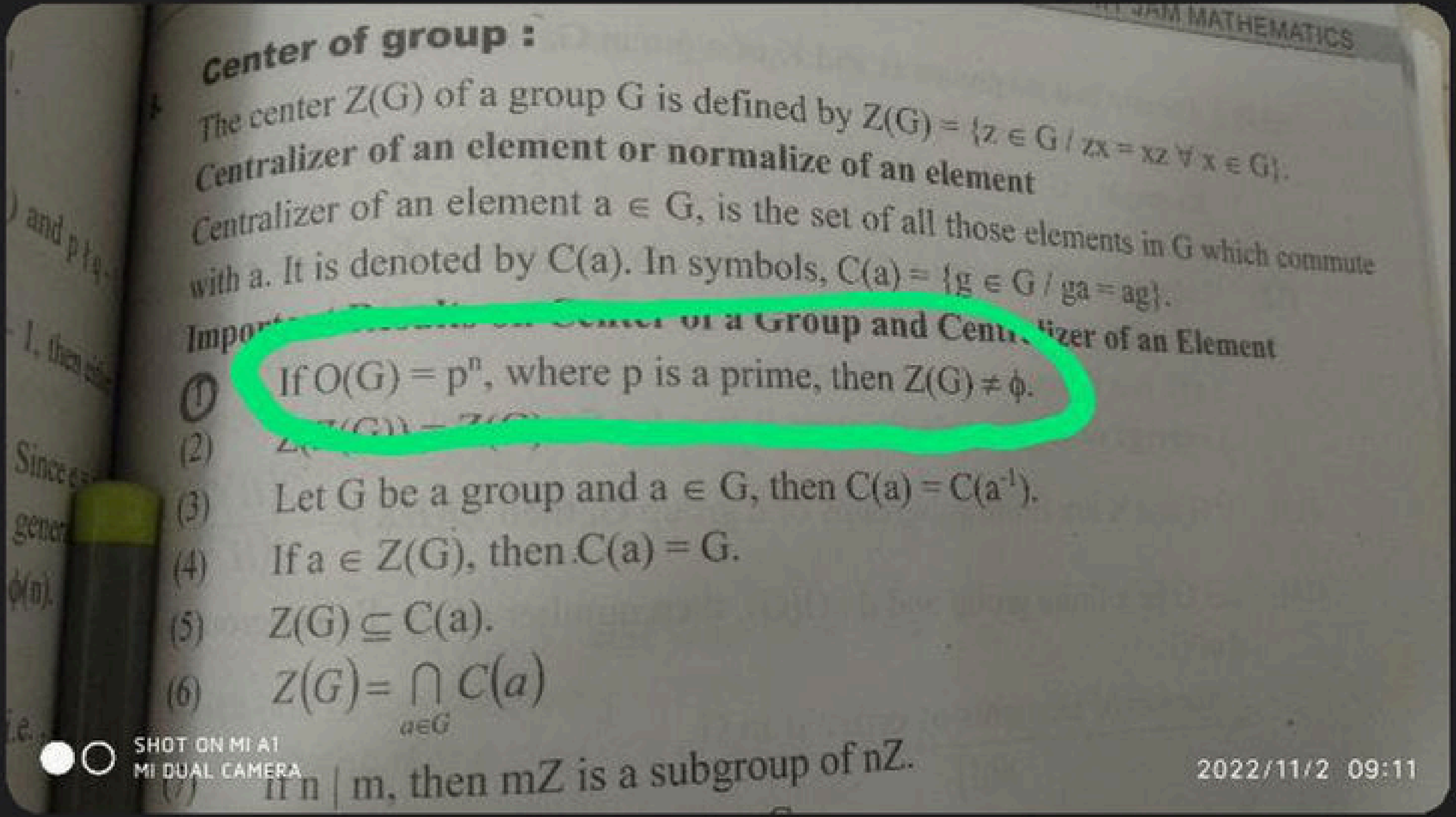


Center of group :

The center Z(G) of a group G is defined by $Z(G) = \{z \in G \,/\, zx = xz \ \forall\, x \in G\}$.

Centralizer of an element or normalize of an element

Centralizer of an element $a \in G$, is the set of all those elements in G which commute with a. It is denoted by C(a). In symbols, $C(a) = \{g \in G \,/\, ga = ag\}$.

Impor[illegible] of a Group and Cent[illegible]lizer of an Element

(1) If $O(G) = p^n$, where p is a prime, then $Z(G) \neq \phi$.

(2) [illegible]

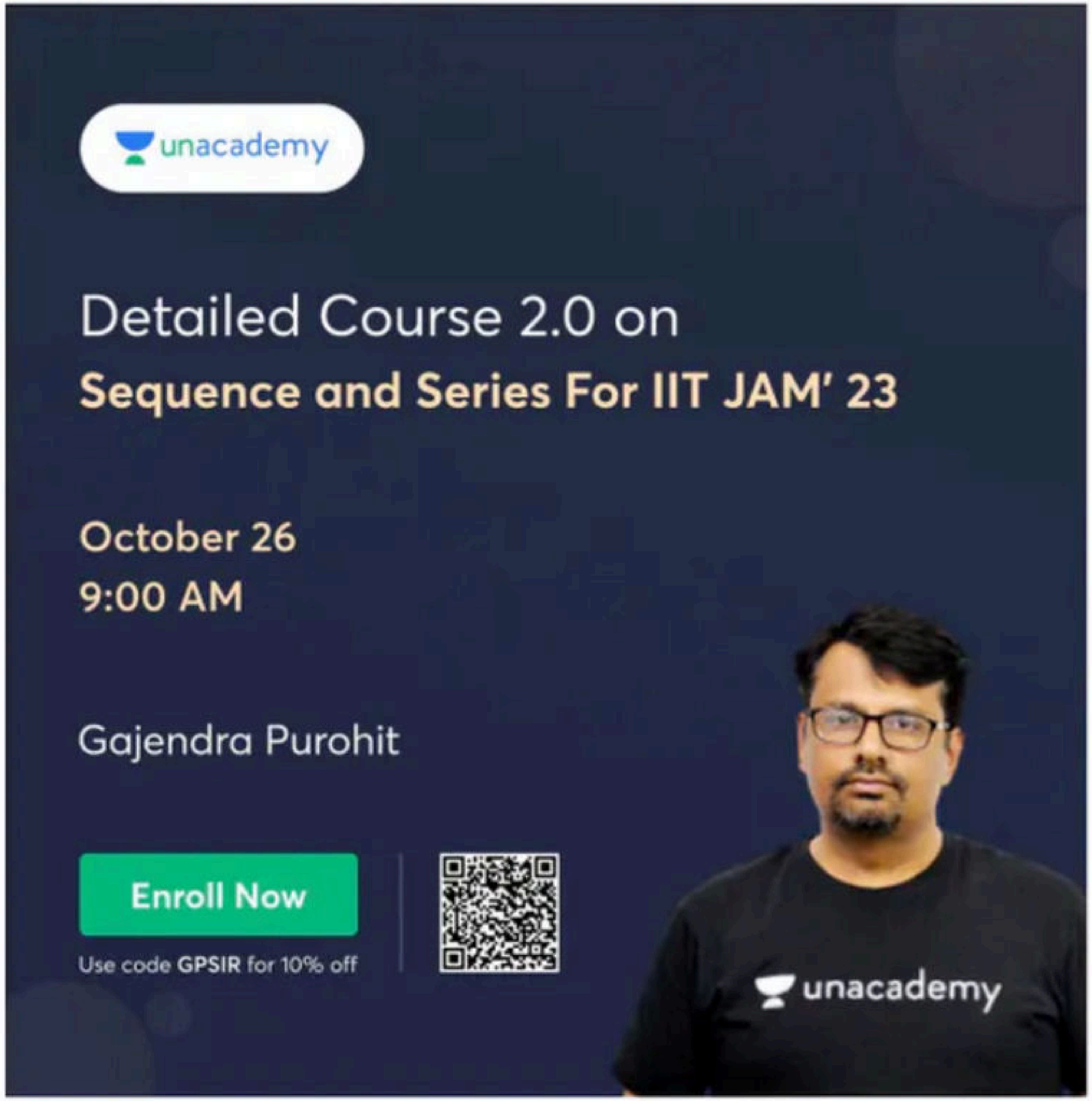
(3) Let G be a group and $a \in G$, then $C(a) = C(a^{-1})$.

(4) If $a \in Z(G)$, then $C(a) = G$.

(5) $Z(G) \subseteq C(a)$.

(6) $Z(G) = \bigcap_{a \in G} C(a)$

(7) If $n \mid m$, then mZ is a subgroup of nZ.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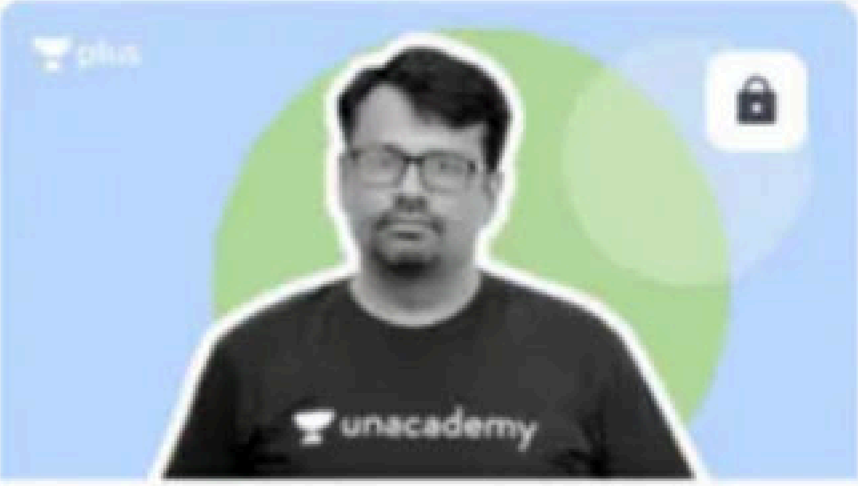

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Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

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