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Detailed Course on Differential Equation for IIT JAM' 23 - II

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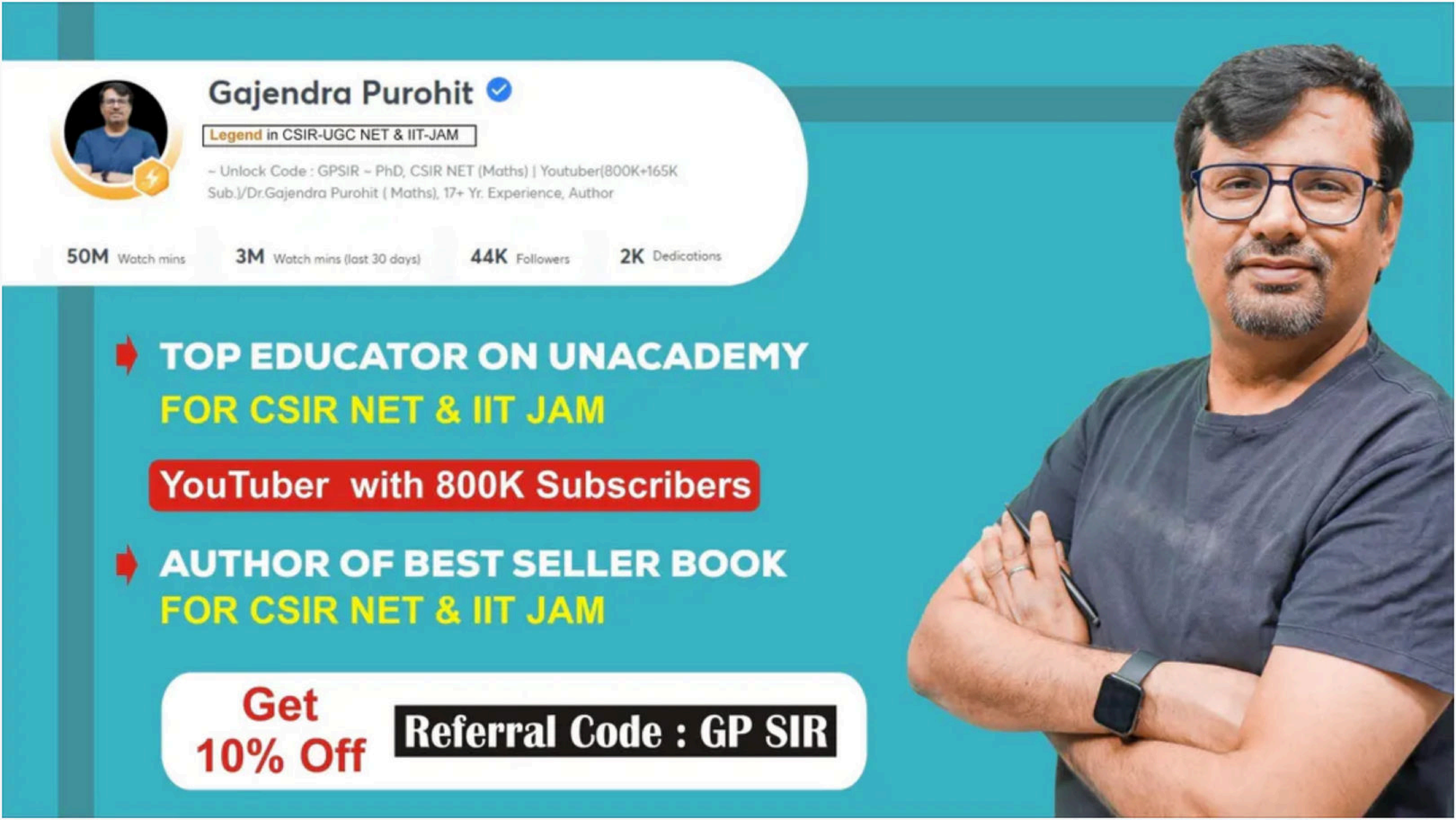
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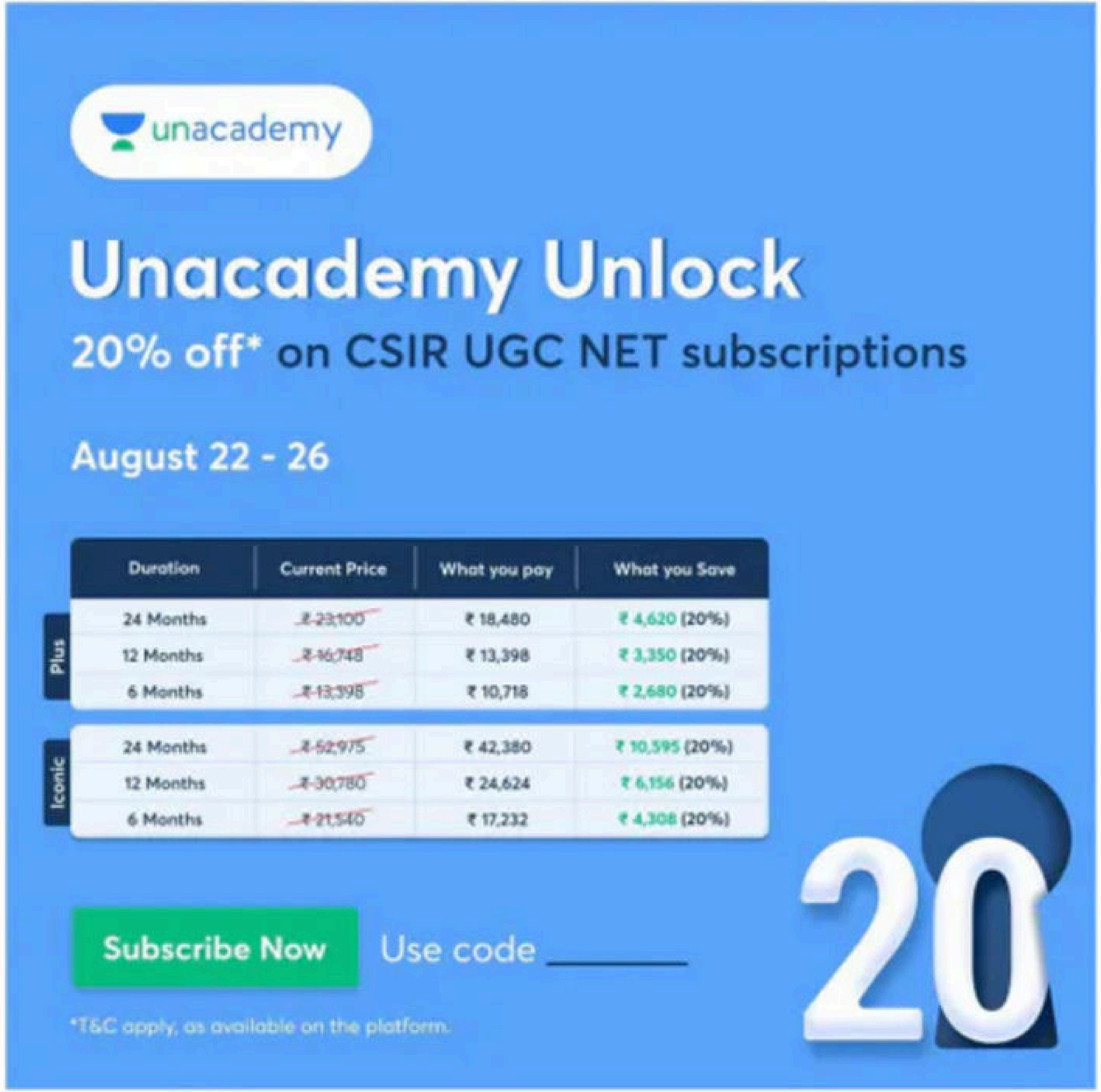

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Homogeneous Differential Equation

Cauchy's Homogeneous Differential Linear Equations

$$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+y=0$$

$$x^3\frac{d^3y}{dx^3}+x\frac{dy}{dx}+y=$$

A linear differential equation of the form

$$x^n\frac{d^n y}{dx^n}+a_1x^{n-1}\frac{dy^{n-1}}{dx^{n-1}}+\ldots\ldots\ldots\ldots+a_{n-1}x\left(\frac{dy}{dx}\right)+a_ny=Q$$

Or $\quad \left(x^nD^n+a_1x^{n-1}D^{n-1}+\ldots..+a_{n-1}xD+a_n\right)y=Q$

Where $a_1,a_2,\ldots..,a_{n-1},a_n$ are constants, and Q is either a constant or function of x only is called homogeneous linear differential equation.

Put $x=e^z \Rightarrow z=\log x$, Let $D_1=\dfrac{d}{dz}$ Then we have,

$$x\frac{dy}{dx}=D_1y,\; x^2\frac{d^2y}{dx^2}=D_1(D_1-1)y,\; x^3\frac{d^3y}{dx^3}=D_1(D_1-1)(D_1-2)y$$

$$\ldots x^n\frac{d^ny}{dx^n}=D_1(D_1-1)(D_1-2)\ldots(D_1-n+1)y$$

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$$

$$D(D-1)y+Dy-y=0$$

$$(D^2-\cancel{D}+\cancel{D}-1)y=0$$

$$(D^2-1)y=0$$

$$m^2-1=0 \Rightarrow \underline{m=\pm 1}$$

$$y=c_1e^{z}+c_2e^{-z}$$

$$\boxed{y=c_1x+c_2x^{-1}}$$

Let $\underline{x=e^{z}}$ | $e^{2z}=x^2$

$$\underline{z=\log x}$$

$$D=\frac{d}{dz}$$

$$\underline{x^2\frac{d^2y}{dx^2}=D(D-1)y}$$

$$\underline{x\frac{dy}{dx}}=\underline{Dy}$$

$$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2x^2$$

Let $x = e^z$, $z = \log x$

$$x^2\frac{d^2y}{dx^2} = D(D-1)y$$

$$x\frac{dy}{dx} = Dy$$

$$D(D-1)y - 3Dy + 4y = 2e^{2z}$$

$$(D^2 - D - 3D + 4)y = 2e^{2z}$$

$$(D^2 - 4D + 4)y = 2e^{2z}$$

$$m^2 - 4m + 4 = 0 \Rightarrow m = 2, 2$$

CF $y = (c_1 + c_2 z)e^{2z}$

PI $y = \dfrac{2e^{2z}}{D^2 - 4D + 4} = \dfrac{2z e^{2z}}{2D - 4}$

$y = \dfrac{\cancel{2} z^2 e^{2z}}{\cancel{2}}$

$y = CF + PI$

$$y = (c_1 + c_2 z)e^{2z} + z^2 e^{2z}$$

$$y = (c_1 + c_2 \log x)x^2 + (\log x)^2 x^2$$

$$x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$$

Let $x=e^z$, $z=\log x$

$$x^3\frac{d^3y}{dx^3}=D(D-1)(D-2)y$$

$$x^2\frac{d^2y}{dx^2}=\underline{D(D-1)y}$$

$$D(D-1)(D-2)y+2\underline{D(D-1)}y+2y=10(e^z+e^{-z})$$

$$\left[D^3-3D^2+\cancel{2D}+2D^2-\cancel{2D}+2\right]y=10(e^z+e^{-z})$$

$$m^2(m+1)-2m(m+1)+2(m+1)$$

$$(D^3-D^2+2)y=10(e^z+e^{-z})$$

$$m^3-m^2+2=0 \qquad m=-1,\ \frac{2\pm\sqrt{4-8}}{2}$$

$$(m+1)(\underline{m^2-2m+2})=0 \qquad m=-1,\ 1\pm i$$

$$PI=\frac{10e^z}{D^3-D^2+2}+\frac{10e^{-z}}{D^3-D^2+2}$$

$$=5e^z+\frac{10ze^{-z}}{3D^2-2D}$$

$$y=c_1e^{-z}+e^z(c_2\cos z+c_3\sin z)+5e^z+2ze^{-z}$$

$$\underline{y=c_1x^{-1}+x\left[c_2\cos(\log x)+c_3\sin(\log x)\right]+5x+2\log x\, x^{-1}}$$

Substituting the values of $x\frac{dy}{dx}, x^2\frac{d^2y}{dx^2}...$ in above equation and changing independent variable from x to z. So

$$\left[D_1(D_1-1)(D_1-2).....(D_1-n+1)+......+a_{n-2}D_1(D_1-1)+a_{n-1}D_1+a_n\right]y=f(z)$$

Which is of the form linear differential equation with constant coefficients, and hence solve using the methods defined earlier, then replacing z with log x, we got the required solution.

Q1. Consider the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$ with the boundary condition of y(0) = 0 and y(1) = 1. The complete solution of differential equation is

$D(D-1)y + Dy - 4y = 0$

(a) x^2

(b) $\sin\left(\frac{\pi x}{2}\right)$

(c) $e^x\sin\left(\frac{\pi x}{2}\right)$

(d) $e^{-x}\sin\left(\frac{\pi x}{2}\right)$

Let $x = e^z$, $z = \log x$

$x^2\frac{d^2y}{dx^2} = D(D-1)y$

$x\frac{dy}{dx} = Dy$

$(D^2-4)y = 0$

$m = \pm 2$

$y = c_1e^{2z} + c_2e^{-2z}$

$y = c_1x^2 + c_2x^{-2}$

$y = x^2c_1 + \frac{c_2}{x^2}$

$x^2y = x^4c_1 + c_2$

$c_2 = 0$

$y = c_1x^2$

$1 = c_1$

$y = x^2$

Q2. The general solution of $x^2 \frac{d^2y}{dx^2} - 5x\frac{dy}{dx} + 9y = 0$ is

Let $x = e^z$

$\underline{z = \log x}$

(a) $(c_1 + c_2 x)e^{3x}$

(b) $(c_1 + c_2 \ln x)x^3$

(c) $(c_1 + c_2 x)x^3$

(d) $(c_1 + c_2 \ln x)\ e^{x^3}$

$D(D-1)y - 5Dy + 9y = 0$

$(D^2 - 6D + 9)y = 0$

$m^2 - 6m + 9 = 0$

$m = 3, 3$

$y = (c_1 + c_2 z)e^{3z}$

$\underline{y = (c_1 + c_2 \log x)x^3}$

Q3 The general solution of differential equation $4x^2y'' - 8xy' + 9y = 0$ is

(a) $c_1e^{5x/2} + c_2e^{-3x/2}$ (b) $c_1e^{3x/2} + c_2e^{-3x/2}$

(c) $(c_1 + c_2 \log x)x^{3/2}$ (d) $c_1x^{3/2} + c_2x^{-3/2}$

Let $x = e^z$, $z = \log x$

$4D(D-1)y - 8Dy + 9y = 0$

$(4D^2 - 12D + 9)y = 0$

$4m^2 - 12m + 9 = 0$

$(2m-3)^2 = 0$

$m = 3/2, 3/2$

$y = (c_1 + c_2 z)e^{3/2 z}$

$y = (c_1 + c_2 \log x)x^{3/2}$

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Q4. If $\frac{(c_1 + c_2 \ln x)}{x}$ is the general solution of the differential equation $x^2 \frac{d^2 y}{dx^2} + kx \frac{dy}{dx} + y = 0$, x > 0 the k equals

(a) 3 (b) -3

(c) 2 (d) -1

$(c_1 + c_2 \ln x) x^{-1}$

$(c_1 + c_2 z) e^{-z}$

$-1, -1$

$(D(D-1) + kD + 1) y = 0$

$(D^2 - D + kD + 1) y = 0$

$m^2 + (k-1) m + 1 = 0$

$1 - (k-1) + 1 = 0$

$2 - k + 1 = 0$

$\boxed{k = 3}$

Q5. A solution of the differential equation

$$2x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-y=0, x>0$$ that passes through the point (1,1) is

(a) $y=\dfrac{1}{x}$

(b) $y=\dfrac{1}{x^2}$

(c) $y=\dfrac{1}{\sqrt{x}}$

(d) $y=\dfrac{1}{x^{3/2}}$

Q6. Let $y(x); x>0$ be the solution of differential equation $x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+4y=0$ satisfying the conditions $y(1)=1$ & $y'(1)=0$ Then the value of $e^2\boxed{y(e)}$ is

(a) 3 (b) 1

(c) 2 (d) -1

$(D(D-1)+5D+4)y=0$

$(D^2+4D+4)y=0$

$m=-2,-2$

$\text{cf } y=(c_1+c_2 z)e^{-2z}$

$y=(c_1+c_2\log x)\frac{1}{x^2}$

$1=c_1$

$y=\frac{1+c_2\log x}{x^2}$

$y'=\frac{x^2\frac{c_2}{x}-(1+c_2\log x)2x}{x^4}$

$0=c_2-2 \Rightarrow c_2=2$

$y=\frac{1+2\log x}{x^2}$

$y(e)=\frac{1+2\log e}{e^2}$

$y(e)=\frac{3}{e^2}$

$e^2 y(e)=3$

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

s.t. $y(1) + \alpha y'(1) = 1$

$y(2) + \beta y'(2) = 2$

Has unique soln if

(a) $\alpha = -1,\ \beta = 2$

(b) $\alpha = -1,\ \beta = -2$

(c) $\alpha = -2,\ \beta = 2$

(d) $\alpha = -3,\ \beta = 2/3$

$(D(D-1)y - 2D + 2)y = 0$

$(D^2 - 3D + 2)y = 0$

$m^2 - 3m + 2 = 0$

$m = 1, 2$

$y = c_1 e^z + c_2 e^{2z}$

$y = c_1 x + c_2 x^2 \quad | \quad y' = c_1 + 2c_2 x$

$c_1 + c_2 + \alpha(c_1 + 2c_2) = 1$

$(1+\alpha)c_1 + (1+2\alpha)c_2 = 1$

$(2+\beta)c_1 + (4+4\beta)c_2 = 2$

$2c_1 + 4c_2 + \beta(c_1 + 4c_2) = 2$

$$\begin{bmatrix} 1+\alpha & 1+2\alpha \\ 2+\beta & 4+4\beta \end{bmatrix}$$

$(1+\alpha)(4+4\beta) - (1+2\alpha)(2+\beta) \neq 0$

Q7. Let y(x) be the solution of $x^2 y''(x) - 2y(x) = 0;\ y(1) = 1, y(2) = 1$
Then the value of y(3) is

(a) $\frac{11}{21}$ (b) 1

(c) $\frac{17}{21}$ (d) $\frac{11}{7}$

Q8 A particular solution of $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+\frac{y}{4}=\frac{1}{\sqrt{x}}$ is

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\log x}{2\sqrt{x}}$

(c) $\frac{(\log x)^2}{2\sqrt{x}}$ (d) $\frac{(\log x)\sqrt{x}}{2}$


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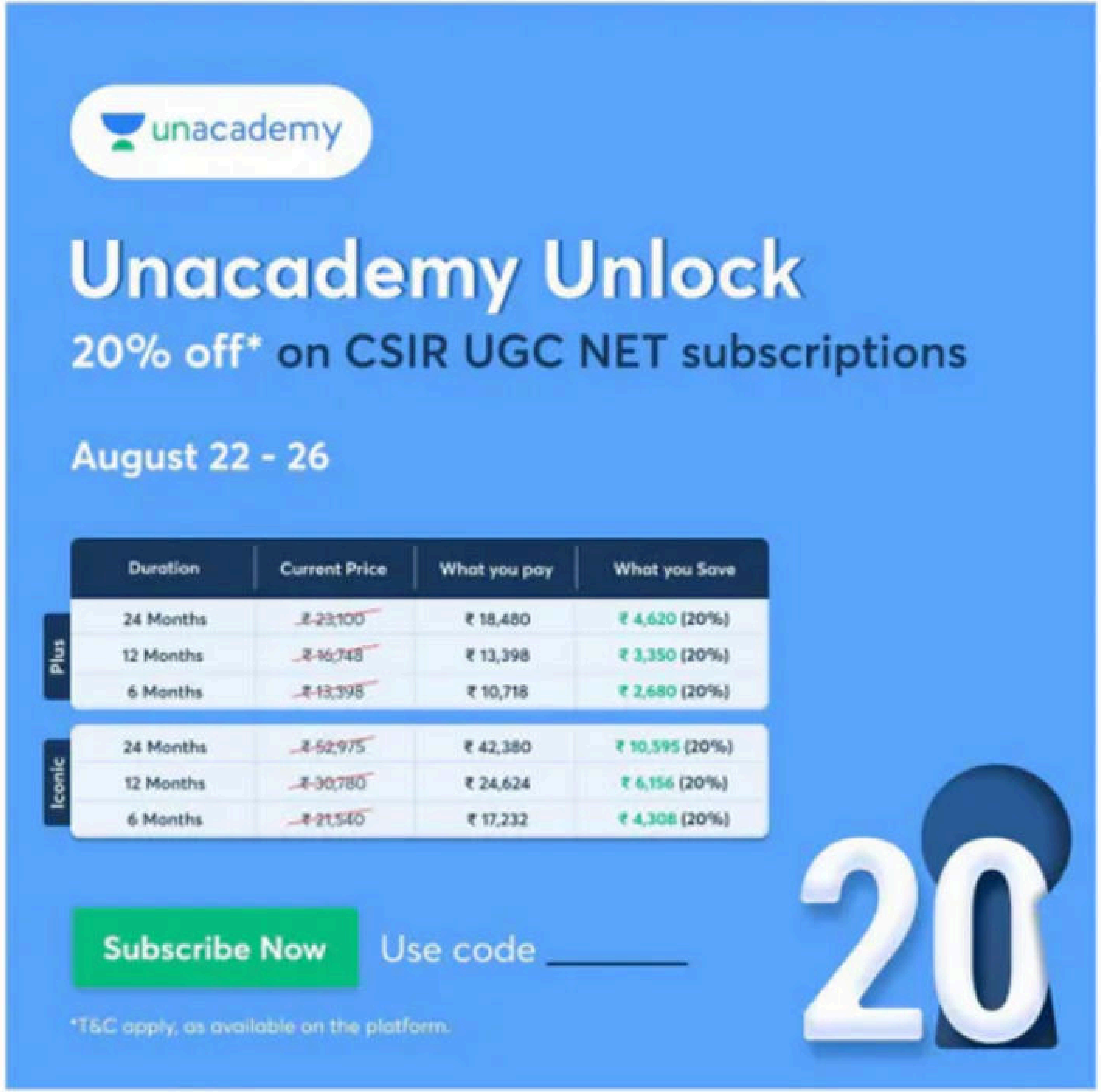
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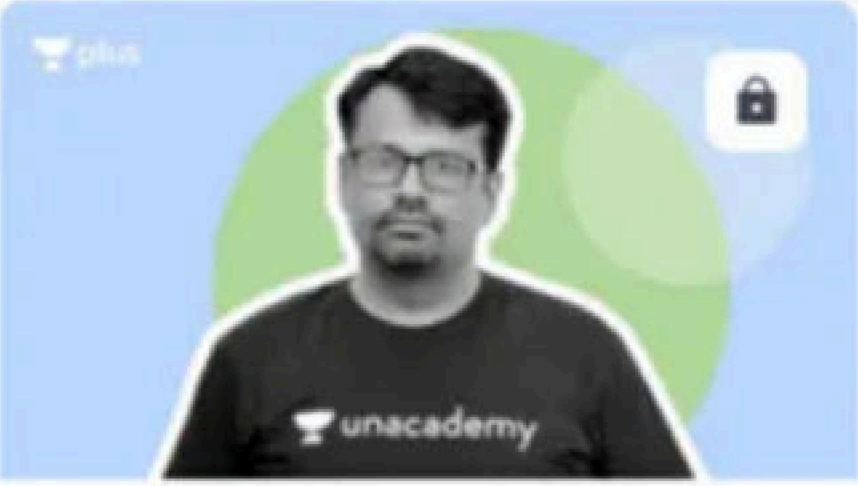

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