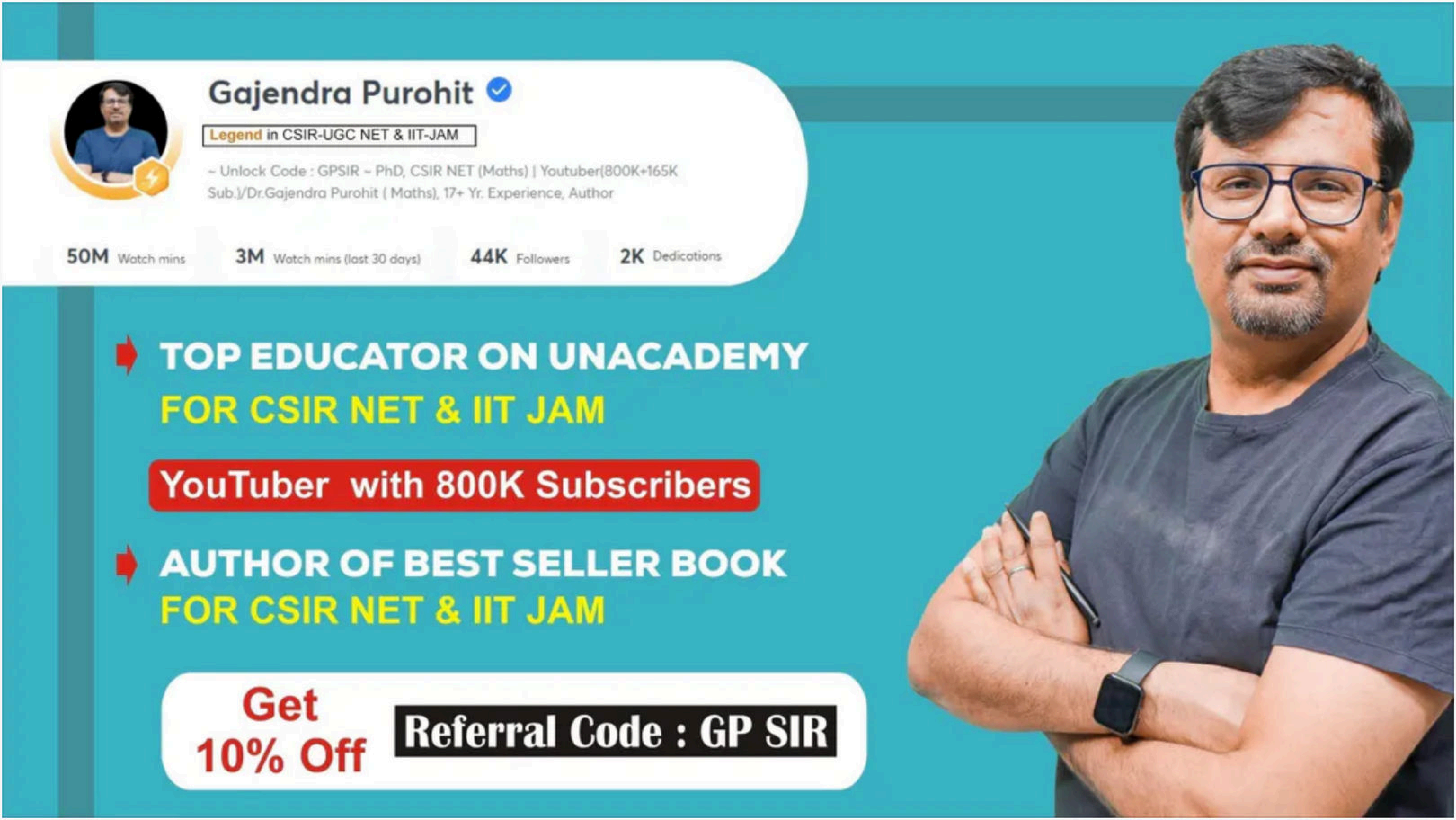
Continuity - Part II

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 3 • Nov 12, 2022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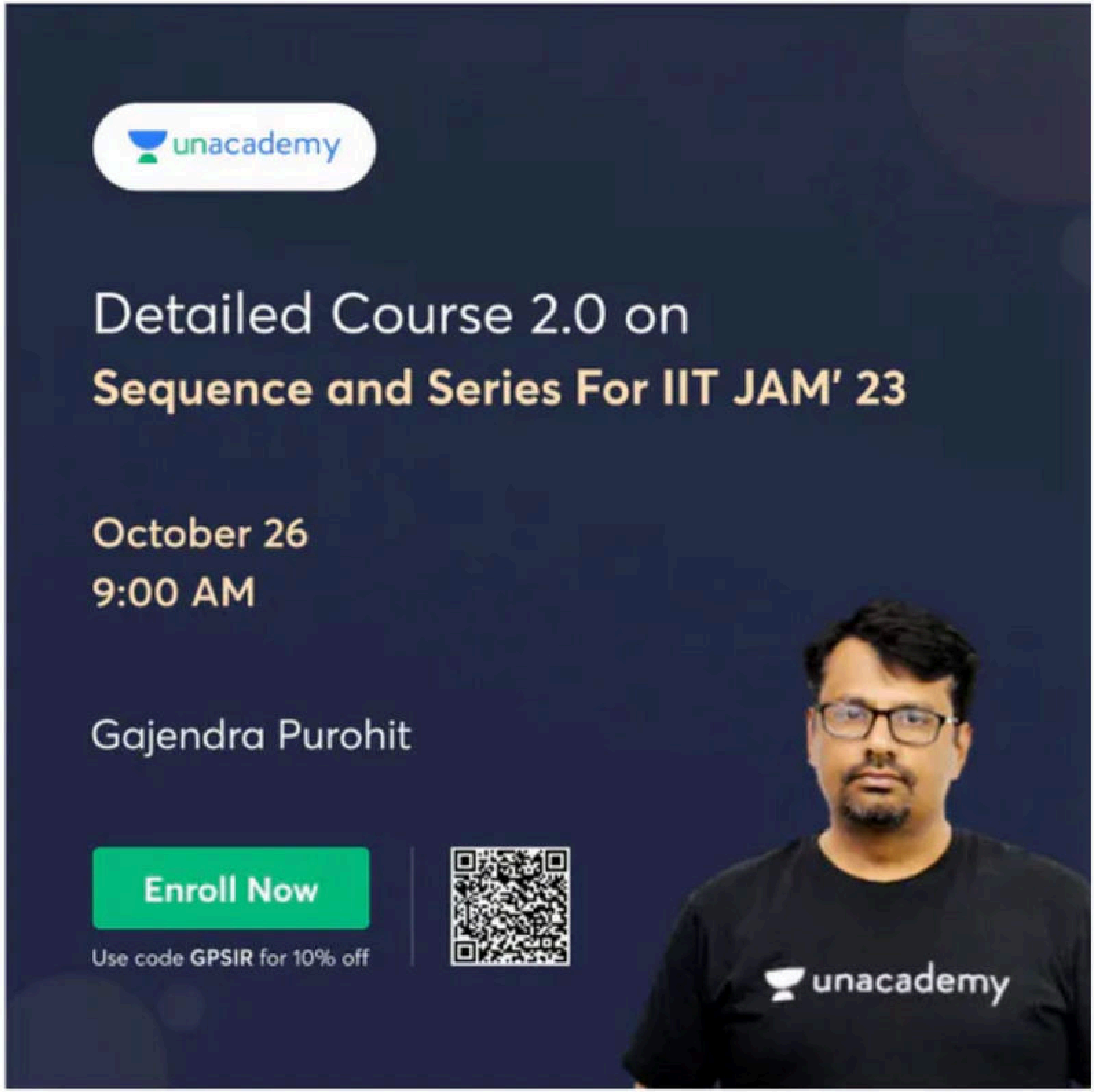
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Continuity

Continuous function at a point 'a' :

Let f : A $\rightarrow$ R be a real valued function. This function is continuous at x = $\alpha \in$ A

if $\lim_{x\to\alpha} f(x) = f(\alpha)$.

Continuity of some important function :

(1) **Polynomial function :** Let $f(x) = a_n x^n + \ldots\ a_1 x + a_0$ then $\lim_{x \to \alpha} f(x) = f(\alpha) . \Rightarrow$ f(x) is continuous at any point.

(2) **Constant function :** Let f(x) = α.

Here $\lim_{x \to a} f(x) = f(\alpha) = \alpha$.

Constant function is always continuous at a.

(3) **Rational function :**

Let p(x) and q(x) are polynomial, then $f(x) = \frac{p(x)}{q(x)}$ is called rational function.

Hence f(x) is continuous at c, if $q(c) \neq 0$

(4) Trigonometry function :

(a) $f(x) = \sin x$

(5) Exponential function :

(6) Logarithmic function :

Sequential definition : Let $f : A \to R$ be a function and f is continuous at 'c' iff for every sequence $\langle x_n \rangle$ converging to c, then $\langle f(x_n) \rangle$ converging to f(c).

i.e. if $\exists$ two sequence $\langle x_n \rangle$ and $\langle y_n \rangle$ s.t. $\langle x_n \rangle \to c$ and $\langle y_n \rangle \to c$.

but if $\langle f(x_n) \rangle$ and $\langle f(y_n) \rangle$ converging to distinct limit then f(x) is not continuous at c.

Result :

Let f : R $\rightarrow$ R is a function defined by

$$f(x)=\begin{cases} g(x), & x \in Q \\ h(x), & x \in Q^C \end{cases}$$

Then f(x) is continuous at all zeros of g(x) – h(x) = 0.

Q.1. Let $f : R \to R$ be defined by $f(x) = \begin{cases} x^6 - 1 & x \in Q \\ 1 - x^6 & x \in Q^C \end{cases}$.

The number of points at which f(x) is continuous is

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(a) 1 (b) 2

(c) 3 (d) 4

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Result : Let f : [0 ,1] → R is a function such that

$$f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$$

Then f(x) is continuous iff $\alpha > 0$

Q.2. Which of the follwing values of α , the function

$$f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$$ is continuous

(a) 1/2 (b) 1

(c) 2 (d) -1/3

Q.3. Let the function f(x) be defined by

$$f(x)=\begin{cases} e^{x} & x \text{ is rational} \\ e^{1-x} & x \text{ is irrational} \end{cases}$$ for x in (0,1) then

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(a) f is continuous at every point in (0,1)

(b) f is discontinuous at every point in (0,1)

(c) f is discontinuous at only one point in (0,1)

(d) f is continuous at only one point in (0,1)

DISCONTINUITIES : If a function is not continuous then it is called discontinuous

Type of Discontinuity :

(i) Removable Discontinuity

A function f(x) is said to have a discontinuity of removable kind at x = a if $\lim_{x \to a} f(x)$ exist but not equal to the value of function at x = a i.e., f(a + 0) = f(a – 0) ≠ f(a). Or $\lim_{x \to a} f(x) \neq f(a)$

(ii) Discontinuity of First kind / Jump Discontinuity

A function $f(x)$ is said to have a discontinuity of first kind at $x = a$ if both $f(a - 0)$ and

$f(a + 0)$ exist but are unequal. The point $x = a$ is said the point of discontinuity of first kind

i.e. $\quad f(a - 0) \neq f(a + 0)$

It is also known as ***ordinary discontinuity***.

(iii) Discontinuity of Second kind

A function f(x) is said to have a discontinuity of second kind at $x = a$ if none of the limit $f(a - 0)$ and $f(a + 0)$ exist at $x = a$. The point $x = a$ is the point of discontinuity of second kind.

(iv) Mixed Discontinuity

A function $f(x)$ is said to have a discontinuity of mixed kind at $x = a$ if $f(x)$ has a discontinuity of second kind on one side of a and on the other side it has discontinuity of first kind or may be continuous.

(v) Infinite Discontinuity

A function $f(x)$ at $x = a$ is said to have discontinuity of infinite kind if $f(a + 0)$ or $f(a - 0)$ is ∞ or $-\infty$.

Q.4. Let $F : R \to R$ be a monotone function. Then

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(a) F has no discontinuities

(b) F has only finitely many discontinuities

(c) F can have at most countably many discontinuities

(d) F can have uncountably many discontinuities.

Q.5. Let $f : R \to R$ be a strictly increasing continuous function. If $\{a_n\}$ is a sequence in $[0, 1]$ then the sequence $\{f(a_n)\}$ is

(a) Increasing (b) Bounded

(c) Convergent

(d) Not necessarily bounded

Q.6. Consider the function f : R → R defined by

$$f(x)=\begin{cases}2x & if\ x\in Q\\ 3-x & if\ x\in Q^{C}\end{cases}.$$

Then f is continuous at

(a) x = 1 (b) x = 2

(c) x = 0 (d) not continuous

Q.8. Let f be a monotone non-decreasing real-valued function on R. Then

(a) $\lim_{x\to a} f(x)$ exists at each point a.

(b) If a < b, then $\lim_{x\to a^+} f(x) \le \lim_{x\to b^-} f(x)$.

(c) f is an unbounded function

(d) The function $g(x) = e^{-f(x)}$ is a bounded function.

Q.9. Let p be a real polynomial of the real variable x of the form $p(x) = x^n + a_{n-1}x^{n-1} + \ldots.. + a_1x + 1$. Suppose that p has no roots in the open unit disc and $p(-1) = 0$. Then

(a) $p(1) = 0$

(b) $\lim_{x \to \infty} p(x) = \infty$

(c) $p(2) > 0$

(d) $p(3) = 0$


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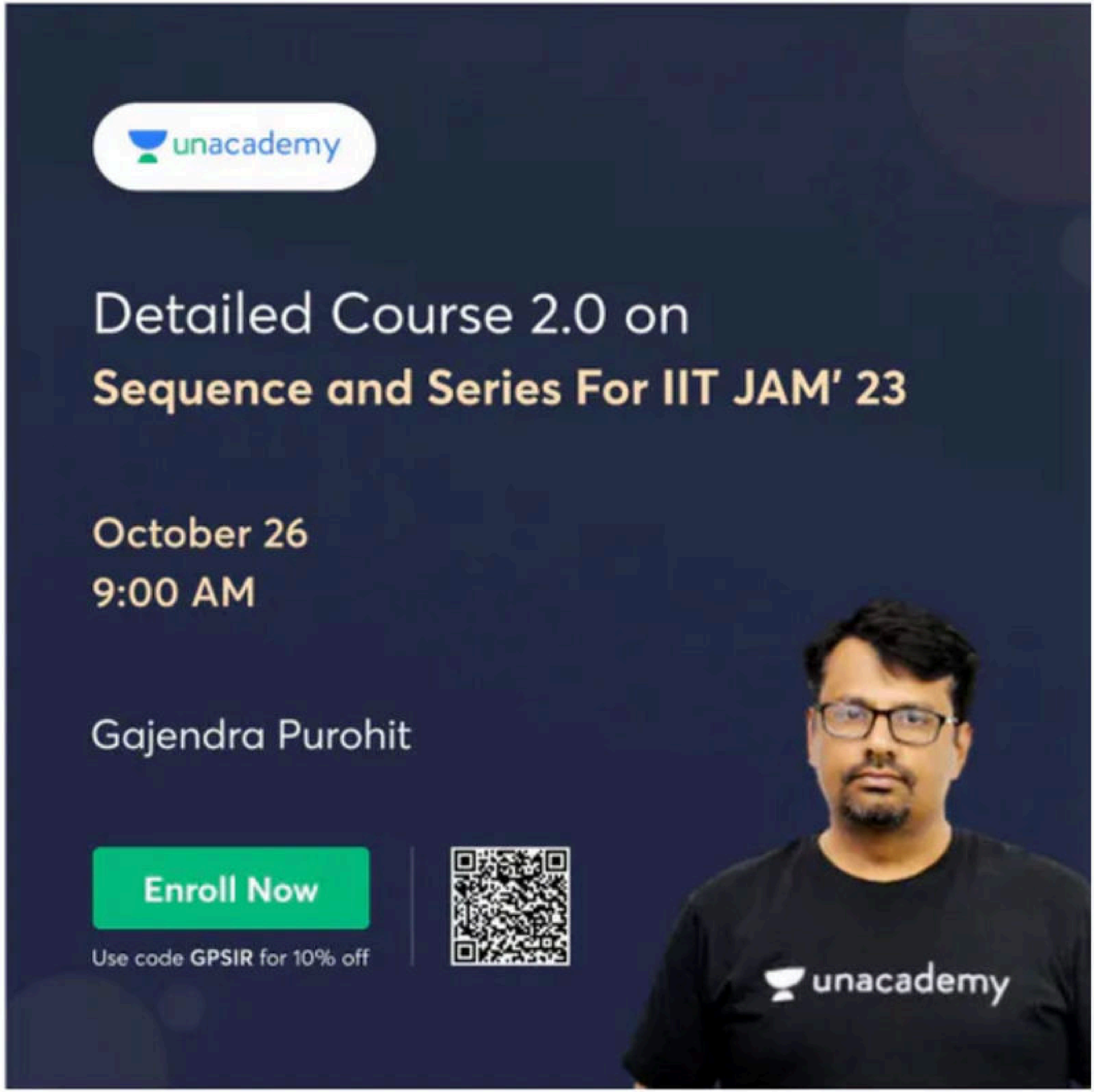
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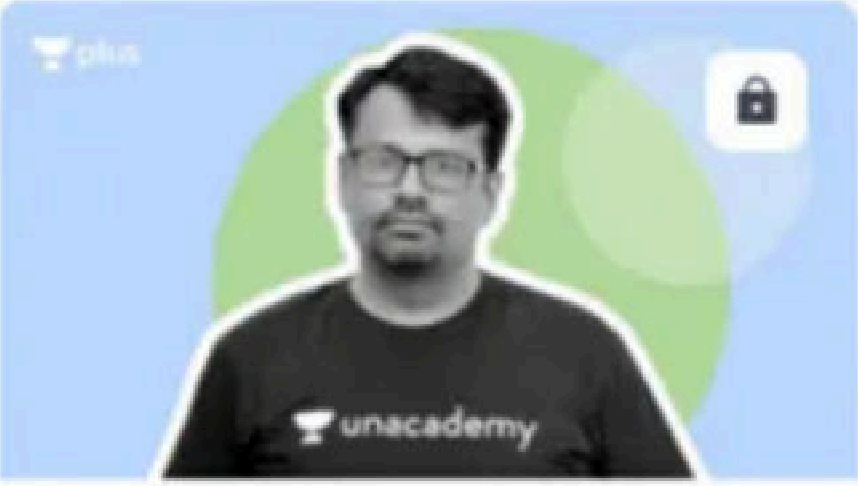

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