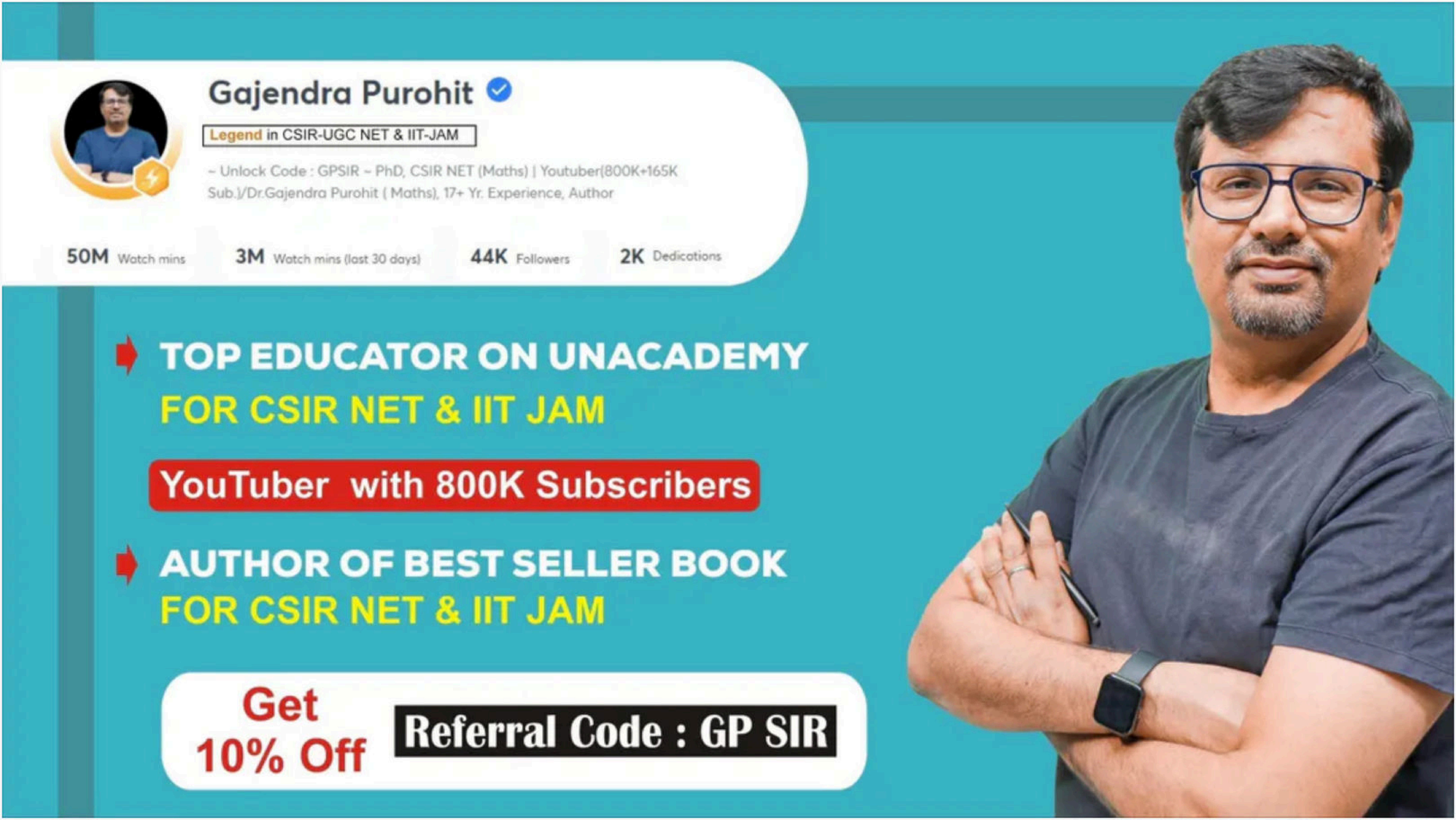
Differentiability - Part I

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 5 • Nov 15, 2022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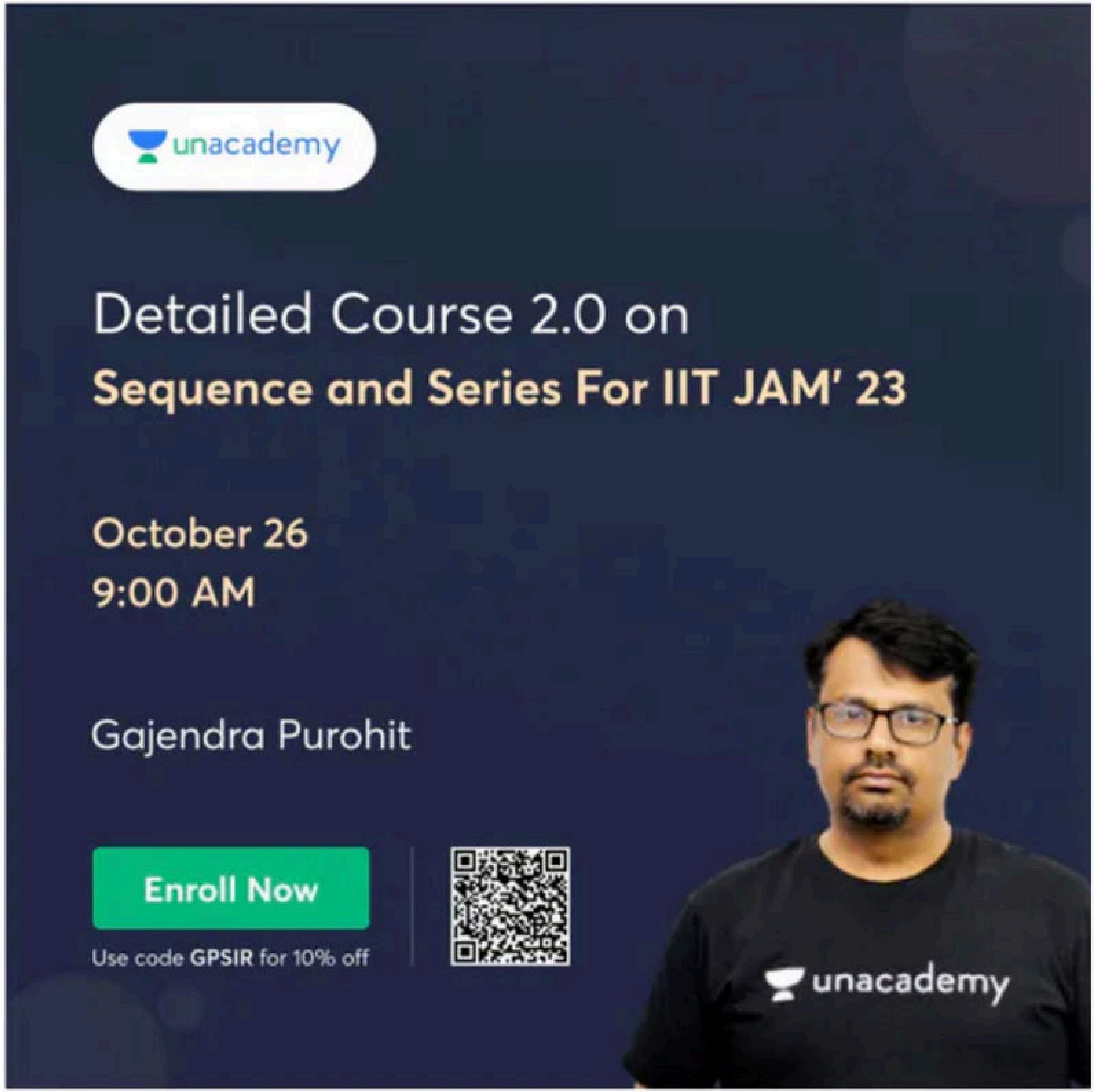
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Difference between continuity and uniformly continuity :

Continuity of a function is at point and uniformly continuity is in interval.

Example : $f(x) = x$ is continuous at 0.

But it is uniformly continuous in $[0, 1]$

Lipschitz function :

A function $f : I \to R$ is said to be satisfy a Lipschitz condition

on I, if $\exists$ a positive integer M such that

$|f(x_1) - f(x_2)| \leq M|x_1 - x_2|$, for any two $x_1, x_2 \in I$

Some direct result for uniformly continuity:

(1) If a function f(x) is Lipschitz function then f is uniformly continuous.

(2) If f(x) is continuous on closed interval [a, b] then it is uniformly continuous on [a, b].

(3) If a function is UC then it is continuous

i.e. If a function is not continuous then it is not UC

Sequential definition :

Let $f : D \to R$ be a function. If $< x_n>$ & $<y_n>$ are two convergent sequence which converge to same limit and $f(<x_n>)$ and $f(<y_n>)$ are also converges to same limit then this function is uniformly continuous on D.

Conclusion : If f(x) is bounded and continuous on I, then f(x) may not be uniformly continuous on I.

(4) Let a function f be continuous on an open bounded interval (a, b), then f is uniformly continuous on (a, b) if $\lim_{x\to a^+} f(x)$ & $\lim_{x\to b^-} f(x)$ both exist finitely. it is necessary condition.

(5) If derivative of f(x) is bounded on I, then f(x) is uniformly continous on I.

(6) If f(x) is uniformly continuous on [a, c] and [c, b] both & f(x) is continuous at c, then f(x) is uniformly continuous on [a, b].

Q.1. Let $f,g : (0,1) \to R$. Let $f(x) = x\sin(1/x^2)$ and $g(x) = x^2$ then

(a) Both are uniformly continuous

(b) f is uniformly continuous but g is not

(c) g is uniformly continuous but f is not

(d) None of the above

Q.2. Which of the following functions is uniformly continuous on the domain as stated? **IIT JAM**

(a) $f(x) = x^2, x \in R$

(b) $f(x) = \frac{1}{x}, x \in [1, \infty)$

(c) $f(x) = \tan x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(d) $f(x) = [x], x \in [0, 1]$

([x] is the greatest integer less than or equal to x]

Q.3. Let f : $(0, \infty) \to R$ be defined by $f(x) = \frac{\sin(x^3)}{x}$.

then f(x) is **TIFR 2019**

(a) bounded and uniformly continuous

(b) bounded but not uniformly continuous

(c) Not bounded but uniformly continuous

(d) Neither bounded nor uniformly continuous

Q4. Let $f(x) = e^{-x}$ and $g(x) = e^{-x^2}$.

Which of the following statements are true?

(a) Both f and g are uniformly continuous on R

(b) f is uniformly continuous on every interval of the form $[a, +\infty)$, $a \in R$

(c) g is uniformly continuous on R

(d) f(x) g(x) is uniformly continuous on R

Q5. Which of the following functions are uniformly continuous on $(0, 1)$? **CSIR NET NOV 2020**

(a) $\frac{1}{x}$

(b) $\sin\frac{1}{x}$

(c) $x\sin\frac{1}{x}$

(d) $\frac{\sin x}{x}$

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Differentiability

Let f : [a, b] $\rightarrow$ R is a real valued function it is said to be a differentiable at x = c.

If $\lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$ finitely exist.

Right Hand Derivative :

$$Rf'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

Left Hand Derivative :

$$Rf'(c) = \lim_{h \to 0} \frac{f(c-h) - f(c)}{-h}$$

Note : If f(x) is differentiable at x = c

Iff $\quad Rf'(c) = Lf'(c)$

Q.1. Let $f(x)=\begin{cases}\dfrac{\sin x}{x} & if \quad x\neq 0\\ 1 & if \quad x=0\end{cases}$, then f is

(a) Discontinuous

(b) Continuous but not differentiable

(c) Differentiable only once

(d) Differentiable more than once.

Necessary condition for differentiable :

If a function is differentiable at $x = c$, then it is continuous at $x = c$ but converse may not be true.

Conclusion :

$$f(x)=\begin{cases} x^{\alpha}\sin\dfrac{1}{x^{\beta}}; & x\neq 0 \\ 0; & x=0 \end{cases}$$

(i) f(x) is continuous at x = 0 for $\alpha > 0$

(ii) f(x) is differentiable at x = 0 for $\alpha > 1$

Result :

(1) $$f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x^{\beta}}; & x\neq 0 \\ 0; & x=0 \end{cases}$$

then f(x) is differentiable $\left[\frac{p}{q}\right]$ times and

$$f^{\left[\frac{p}{q}\right]}=\begin{cases} \text{continuous} & \text{if } p \text{ is odd} \\ \text{discontinuous} & \text{if } p \text{ is even} \end{cases}$$

(2) Let f : R → R be defined by $f(x) = \begin{cases} \phi_1(x) & x \in Q \\ \phi_2(x) & x \in Q^c \end{cases}$

f(x) is differentiable at the double root of $\phi_1(x) - \phi_2(x) = 0$

(3) A function is not differentiable at that point at which graph of function is sharp edge.

Q.2. Consider the function $f(x) = |\cos x| + |\sin(2 - x)|$.

At which of the following points is f not differentiable?

(a) $\left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$ (b) $\{n\pi : n \in Z\}$

(c) $\{n\pi + 2 : n \in Z\}$ (d) $\left\{\frac{n\pi}{2} : n \in Z\right\}$

Q.3. The function $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$ is differentiable at $x = 0$

(a) for no values of a_0, a_1, a_2, a_3

(b) for any value of a_0, a_1, a_2, a_3

(c) only if $a_1 = 0$

(d) only if both $a_1 = 0$ and $a_3 = 0$


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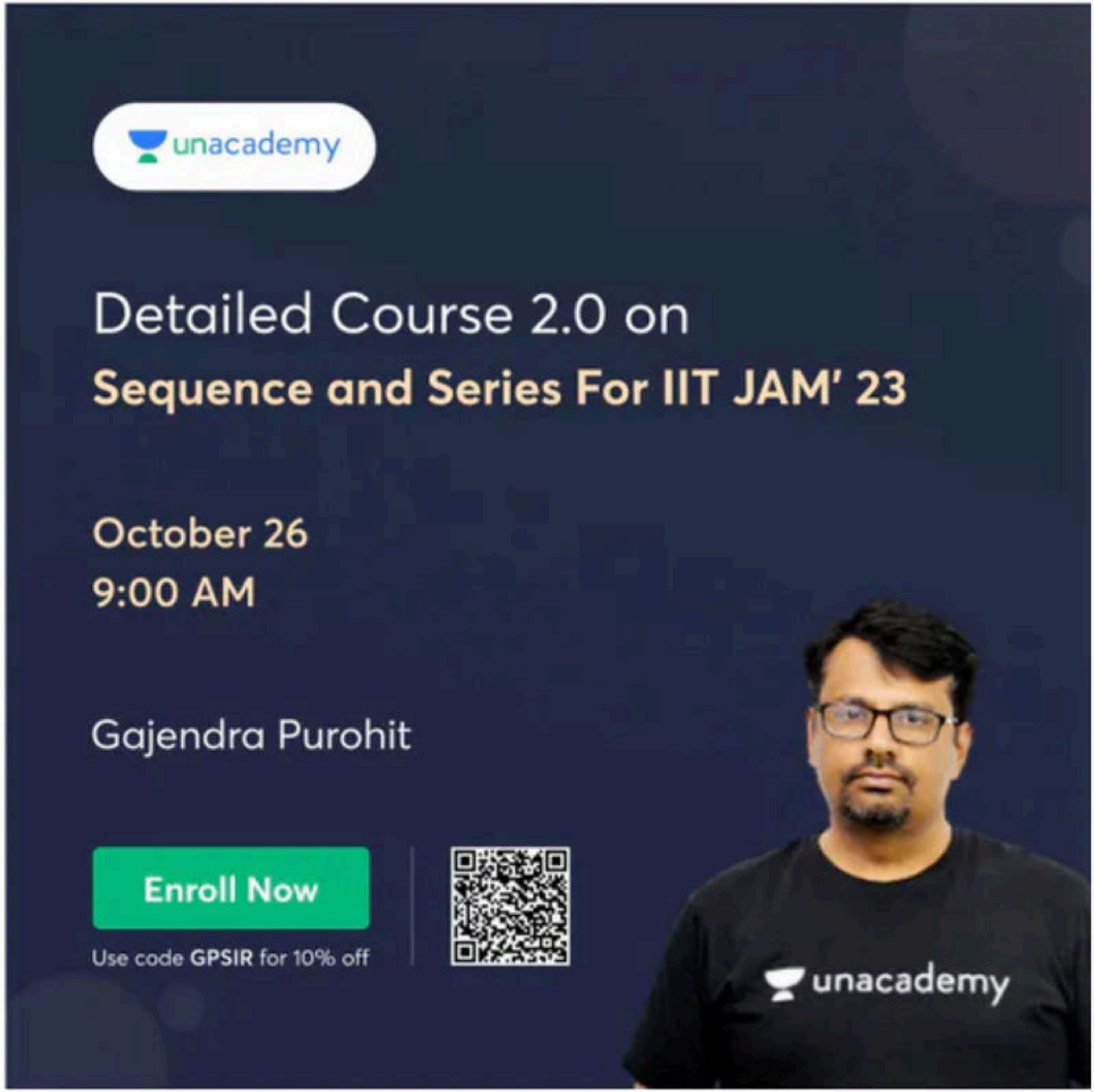
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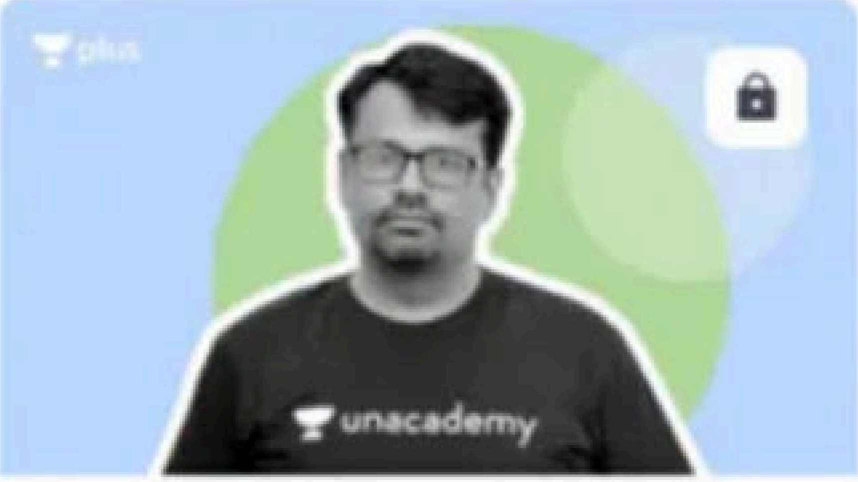

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