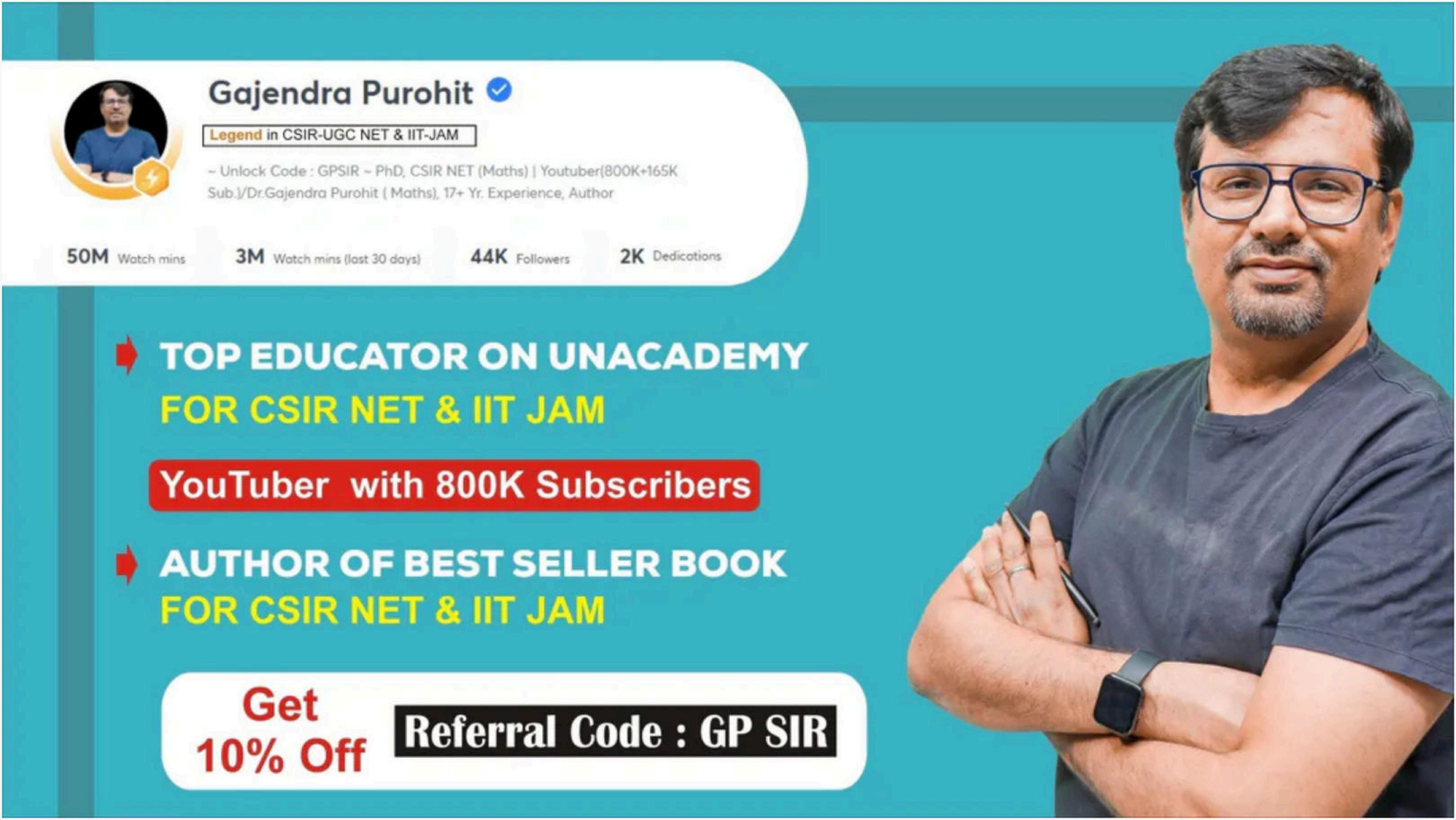
Types of Matrix and Their Functions

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 10 • Sept 20, 2022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Consistent-

(I) $\rho(A) = \rho(A:B) = n$

Unique soln

(II) $\rho(A) = \rho(A:B) < n$

Infinite soln

Inconsistent-

$\rho(A) \neq \rho(A:B)$

No soln

Non-homogeneous system of equation : A system of equation $Ax = b$ is called non-homogeneous iff $b \neq 0$.

Consistency & inconsistency : A non-homogeneous system $Ax = b$ is called consistency if it has a solution otherwise it is called inconsistent.

Augmented matrix : Let $Ax = b$ be a given system of equation then $[A : b]$ is called augmented matrix.

Necessary and sufficient condition for solution :

$Ax = b$ has a solution iff

(i) $\rho(A : b) = \rho(A)$

(ii) b is linear combination of $c_1, c_2, \ldots, c_n$ where c_i are column of A.

Note : If $\rho(A : b) \neq \rho(A)$ then $Ax = b$ has no solution.

Analysis of solution of non-homogeneous system of equation :

(1) **Unique solution :** Let $Ax = b$ has a unique solution iff $\text{Ker}(A) = \{0\}$ i.e. $\eta(A) = 0$ and $\rho(A : b) = \rho(A)$

(2) **Infinite solution :** A non-homogeneous system $Ax = b$ has infinite solution iff $\rho(A : b) = \rho(A)$ and $\eta(A) > 0$.

(3) **No solution :**

If $\rho(A : b) \neq \rho(A)$

Then system $AX = b$ has no solution.

Q1. Consider the system

$$2x + ky = 2 - k$$

$$kx + 2y = k$$

$$ky + kz = k - 1$$

in three unknowns and one real parameter k. For which of the following values of k is the system of linear equation consistent?

(a) 1

(b) 2

(c) -1

(d) -2

$$\left[\begin{array}{ccc:c} 2 & k & 0 & 2-k \\ k & 2 & 0 & k \\ 0 & k & k & k-1 \end{array}\right]$$

$$(0 + 4k + 0) - k^3 = 0$$

$$4k - k^3 = 0$$

$$k(4 - k^2) = 0$$

$$k = 0, 2, -2$$

2020

$$x+y+5z=3$$
$$x+2y+mz=5$$
$$x+2y+4z=k$$

· The system is consistent if

$$\left[\begin{array}{ccc:c} 1 & 1 & 5 & 3 \\ 1 & 2 & m & 5 \\ 1 & 2 & 4 & k \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 5 & 3 \\ 0 & 1 & m-5 & 2 \\ 0 & 1 & -1 & k-3 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 5 & 3 \\ 0 & 1 & m-5 & 2 \\ 0 & 0 & 4-m & k-5 \end{array}\right]$$

$4-m \neq 0$ $\quad$ $k-5 \neq 0$

$m \neq 4$

$k = 5$

$m \neq 4 \quad k \neq 5$

$m = 4, \quad k = 5$

2019

$$x_1 + x_2 + x_3 + x_4 = 4$$
$$x_1 + 2x_2 + 3x_3 + 4x_4 = 5$$
$$x_1 + 3x_2 + 5x_3 + kx_4 = 5$$

(a) 3 (b) 5

(c) 7 (d) 9

$$\left(\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & k & 5 \end{array}\right)$$

$$\left(\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & k-1 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc:c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & \underset{=0}{k-7} & -1 \end{array}\right)$$

Q.2. The system of equation

$$x + 3y + 2z = k$$

$$2x + y - 4z = 4$$

$$5x - 14z = 10$$

(a) has unique solution for k = 2

(b) has infinitely many solution for k = 2

(c) has no solution for k = 2

(d) has unique solution for any k ≠ 2

$R_2 \to 2R_2 - 5R_1$

$R_3 \to 2R_3 - R_1$

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & 4 \\ 5 & 0 & -14 & 10 \\ 1 & 3 & 2 & k \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & 4 \\ 0 & -5 & -8 & 0 \\ 0 & 5 & +8 & 2k-4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 1 & -4 & 4 \\ 0 & -5 & -8 & 0 \\ 0 & 0 & 0 & 2k-4 \end{array}\right]$$

$-28+20$

$20-20$

Q.3. Let $A = \begin{pmatrix} 2 & 0 & 5 \\ 1 & 2 & 3 \\ -1 & 5 & 1 \end{pmatrix}$. The system of linear equations AX = Y has a solution

(a) only for $Y = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, x ∈ R

(b) only for $Y = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}$, y ∈ R

(c) only for $Y = \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}$, y, z ∈ R

(d) for all Y ∈ R^3

$|A| \neq 0$

$AX = Y$

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Q4. Let A be an $m \times m$ matrix with real entries and let x be an $m \times 1$ vector of unknowns. Now consider the two statements given below :

$\rho(A) = 1$ $\quad [\ \ ]_{2\times n}$

I There exists non-zero vector $b_1 \in R^m$ such that the linear system $Ax = b_1$ has no solution.

$m \neq 2$

II There exist non-zero vectors $b_2, b_3 \in R^m$, with $b_2 \neq cb_3$ for any $c \in R$, such that the linear systems $Ax = b_2$ and $Ax = b_3$ have solutions.

$\rho(A) = 2$

$m = 3$

Which of the following statements are true?

(a) II is true whenever A is singular

(b) I is true whenever A is singular

(c) Both I and II can be true simultaneously

(d) If $m = 2$, then at least one of I and II is false.

Q.5. The system of equations :

$$1 . x + 2 . x^2 + 3 . xy + 0 . y = 6$$

$$2 . x + 1 . x^2 + 3 . xy + 1 . y = 5$$

$$1. x - 1 . x^2 + 0 . xy + 1 . y = 7$$

(a) has solution in rational numbers

(b) has solutions in real numbers

(c) has solutions in complex numbers

(d) has no solution

$$\left[\begin{array}{cccc:c} 1 & 2 & 3 & 0 & 6 \\ 2 & 1 & 3 & 1 & 5 \\ 1 & -1 & 0 & 1 & 7 \end{array}\right]$$

$$\left(\begin{array}{cccc:c} 1 & 2 & 3 & 0 & 5 \\ 0 & -3 & -3 & 1 & -7 \\ 0 & -3 & -3 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc:c} 1 & 2 & 3 & 0 & 5 \\ 0 & -3 & -3 & 1 & -7 \\ 0 & 0 & 0 & 0 & 8 \end{array}\right)$$

Q.6 Let m > 1 and n > 1 be integers. Let A be an m × n matrix such that for some m × 1 matrix b_1, the equation $AX = b_1$ has infinitely many solutions. Let b_2 denote an m × 1 different from b_1. Then $AX = b_2$ has

(a) infinitely many solutions for some b_2.

(b) a unique solution for some b_2.

(c) finitely many solutions for some b_2.

(d) no solution for some b_2.

$\rho(A) = \rho(A|B) < n$

$\begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$

$|A| = 0$

$AX = b_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Q.7 Let A be an $n \times n$ real matrix. Let b be an $n \times 1$ vector. Suppose $Ax = b$ has no solution. Which of the following statements are true?

(a) There exists an $n \times 1$ vector c such that $Ax = c$ has a unique solution

(b) There exist infinitely many vectors c such that $Ax = c$ has no solution

(c) If y is the first column of A then $Ax = y$ has a unique solution

(d) $\det A = 0$

Q.8. Number of solution of system of linear equation

$$3x + 4y - z - 6w = 0$$

$$2x + 3y + 2z - 3w = 0$$

$$2x + y - 14z - 9w = 0$$

$$x + 3y + 13z + 3w = 0$$

(a) unique solution (b) two solution

(c) more than 2 but finite solution

(d) infinite solution


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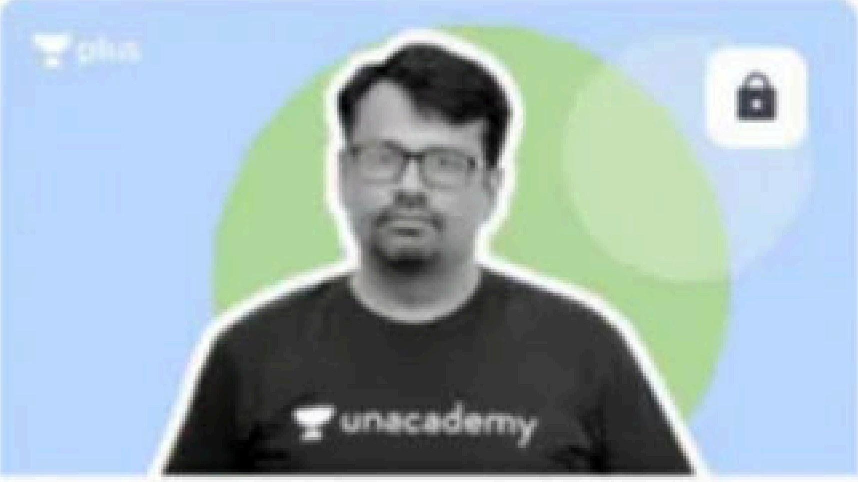

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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