Integral Calculus - Part V

Detailed Course on Integral Calculus - IIT JAM' 23

Gajendra Purohit • Lesson 6 • July 14, 2022

Gajendra Purohit

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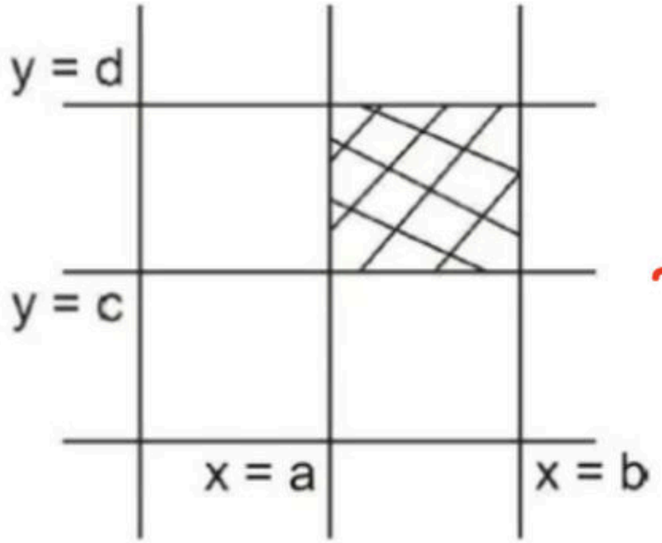
Double Integrals over a rectangular region :

If f(x, y) is continuous over the rectangle R : $a \le x \le b$, $c \le y \le d$, then the double integral.



$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$

Also, $$\iint_R f(x,y)\,dA = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

$\iint xy\,dx\,dy$ $\quad 0 \le y \le 1, \quad 0 \le x \le 2$

$$\int_{y=0}^{1}\int_{x=0}^{2} xy\,dx\,dy$$

$$\int_{y=0}^{1}\left(\frac{x^2}{2}\right)_0^2 y\,dy = \frac{1}{2}\int_0^1 (4-0)\,y\,dy = 2\left(\frac{y^2}{2}\right)_0^1 = 1$$

Q.1. The value of $\iint_A (x^2 + y^2)dA$, where A is rectangle

$2 \le x \le 4$ & $0 \le y \le 1$

(a) 57/6 (b) 58/3

(c) 58/7 (d) None of these

$$\int_0^1 \int_2^4 (x^2+y^2)\,dx\,dy$$

$$\int_0^1 \left(\frac{y^3}{3} + xy^2\right)_2^4 dy$$

$$= \int_0^1 \left(\frac{1}{3}(64-8) + (4-2)y^2\right) dy$$

$$= \int_0^1 \left(\frac{56}{3} + 2y^2\right) dy$$

$$= \left(\frac{56}{3}y + \frac{2y^3}{3}\right)_0^1$$

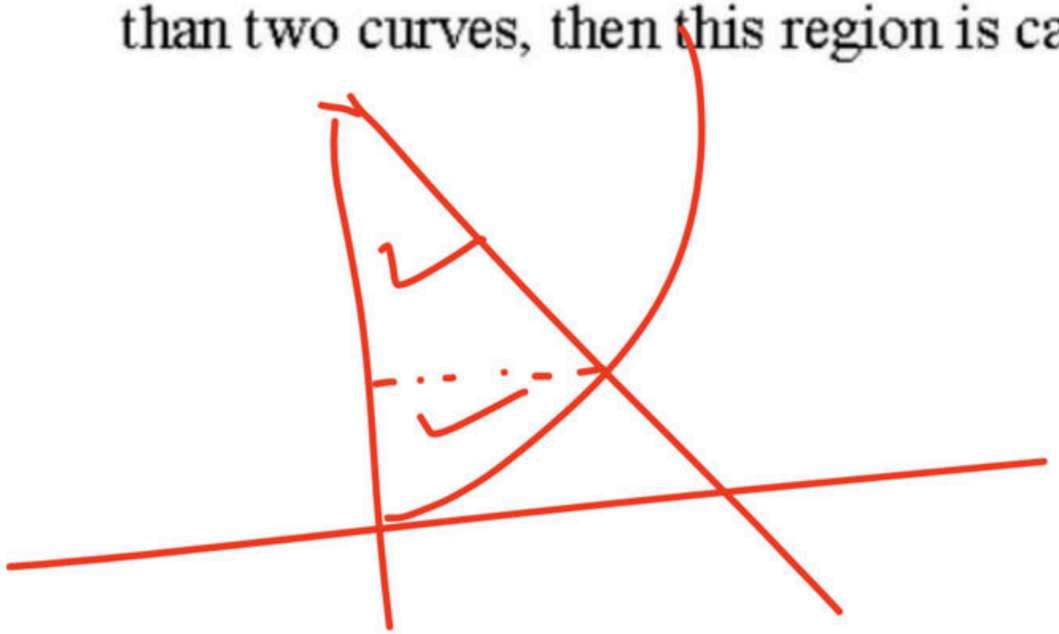
$$\frac{56}{3} + \frac{2}{3} = \frac{58}{3}$$

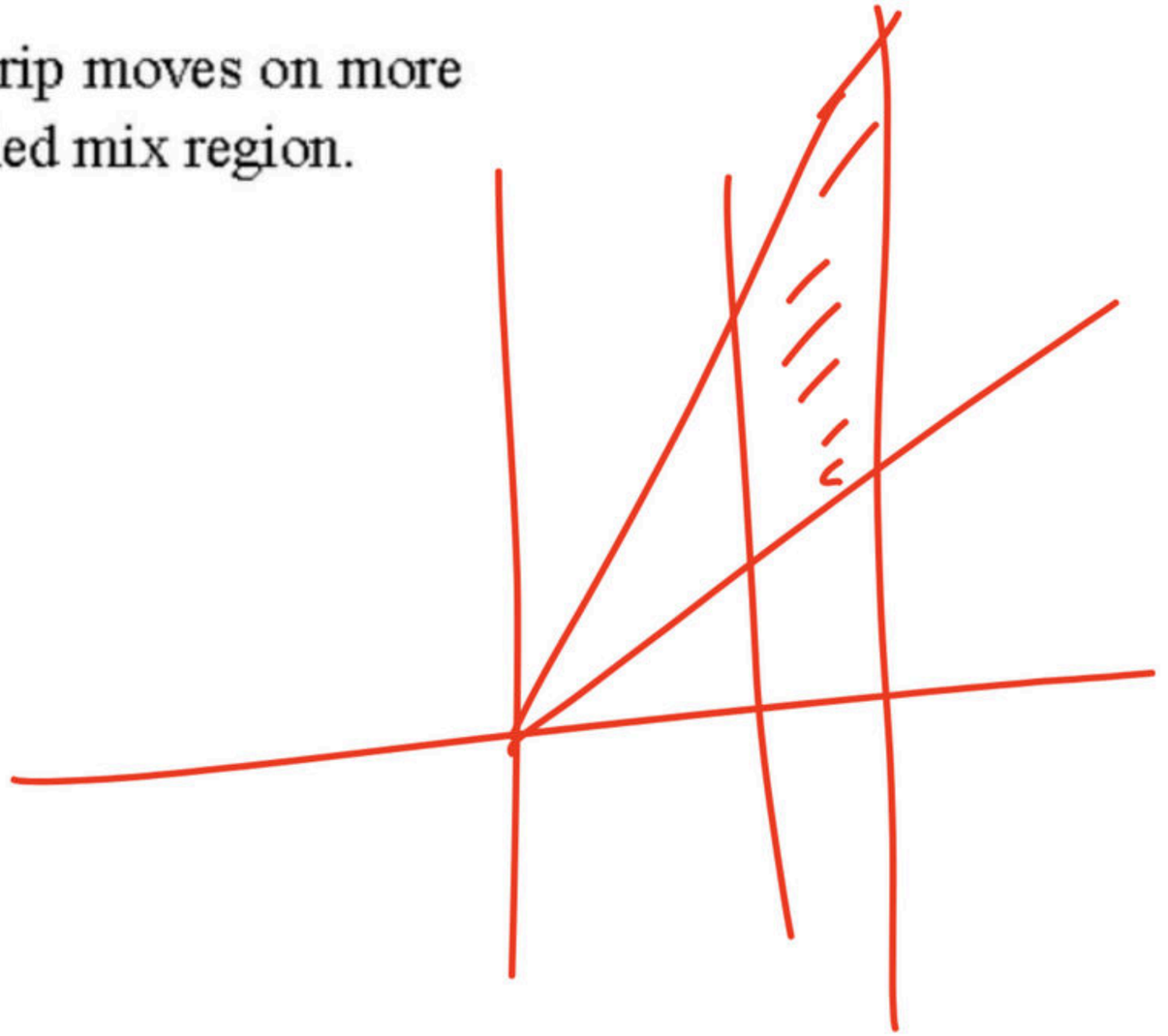
Double integration over mix region :

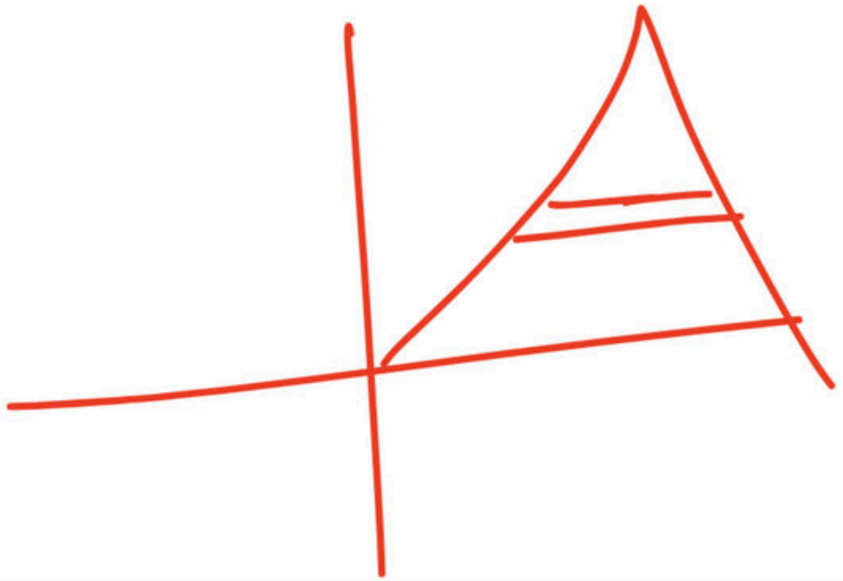
(a) **Simple region :** A region in which strip moves only on two curve, then this region is called simple region.

$0 \le x \le 1$ $\quad 2 \le y \le 4$

(b) **Mix region :** A region in which the strip moves on more than two curves, then this region is called mix region.





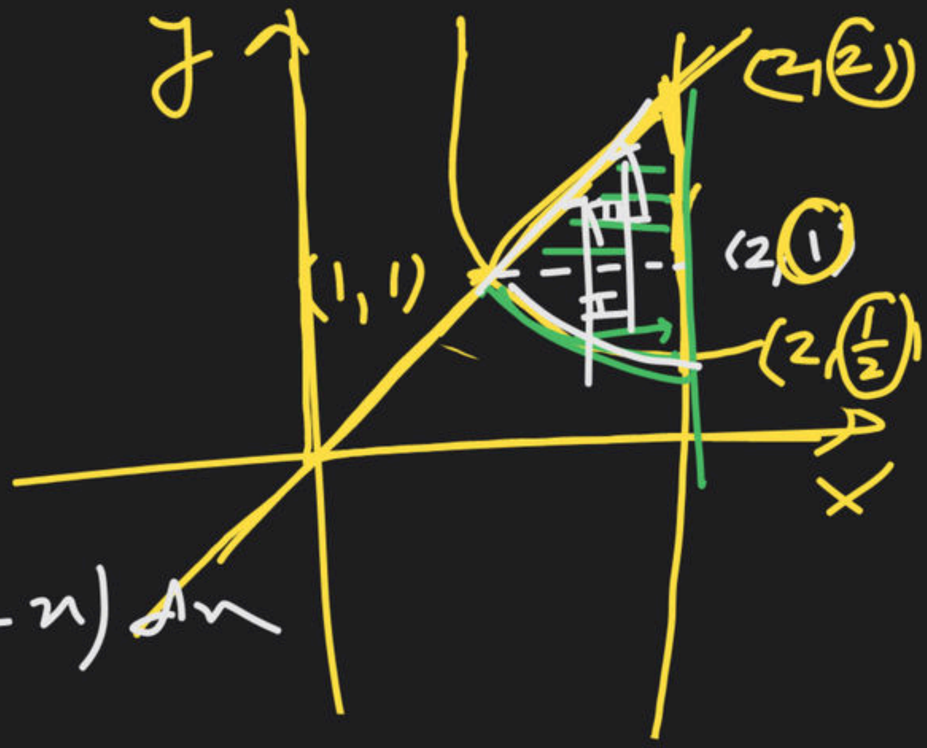


$$\iint \frac{x^2}{y^2}\,dx\,dy$$

$x = 2$

$y = x$

$xy = 1$



$$\int_{x=1}^{2}\int_{y=1/x}^{x} \frac{x^2}{y^2}\,dy\,dx$$

$$\int_{x=1}^{2} x^2\left(-\frac{1}{y}\right)_{1/x}^{x} dx = -\int_1^2 x^2\left(\frac{1}{x} - x\right) dx$$

$$-\int_1^2 (x - x^3)\,dx = -\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_1^2$$

$$= -\left(\frac{4-1}{2} - \frac{16-1}{4}\right)$$

$$= -\left(\frac{3}{2} - \frac{15}{4}\right) = 9/4$$

$$\int_{y=0}^{1}\int_{x=y}^{y+1} x\, e^{x^2-y^2}\,dx\,dy$$

$$\frac{1}{2}\int_{y=0}^{1}\int_{x=y}^{y+1} 2x\, e^{x^2} e^{-y^2}\,dx\,dy$$

$$\frac{1}{2}\int_{y=0}^{1} \left(e^{x^2}\right)_{y}^{y+1} e^{-y^2}\,dy$$

$$\frac{1}{2}\int_{0}^{1}\left(e^{(y+1)^2} - e^{y^2}\right)e^{-y^2}\,dy$$

$$\frac{1}{2}\int_{0}^{1}\left(e^{(y+1)^2-y^2} - 1\right)dy = \frac{1}{2}\int_{0}^{1}\left(e^{y^2+2y+1-y^2} - 1\right)dy$$

$$= -\frac{1}{2}\int_{0}^{1}\left(e^{2y} - 1\right)dy$$

$$= \frac{1}{2}\left(e\,\frac{e^{2y}}{2} - y\right)_0^1$$

$$= \frac{1}{2}\left(\left(e\,\frac{e^2}{2} - 1\right) - \left(\frac{e}{2}\right)\right)$$

$$= \frac{1}{2}\left(\frac{e^3}{2} - 1 - \frac{e}{2}\right)$$

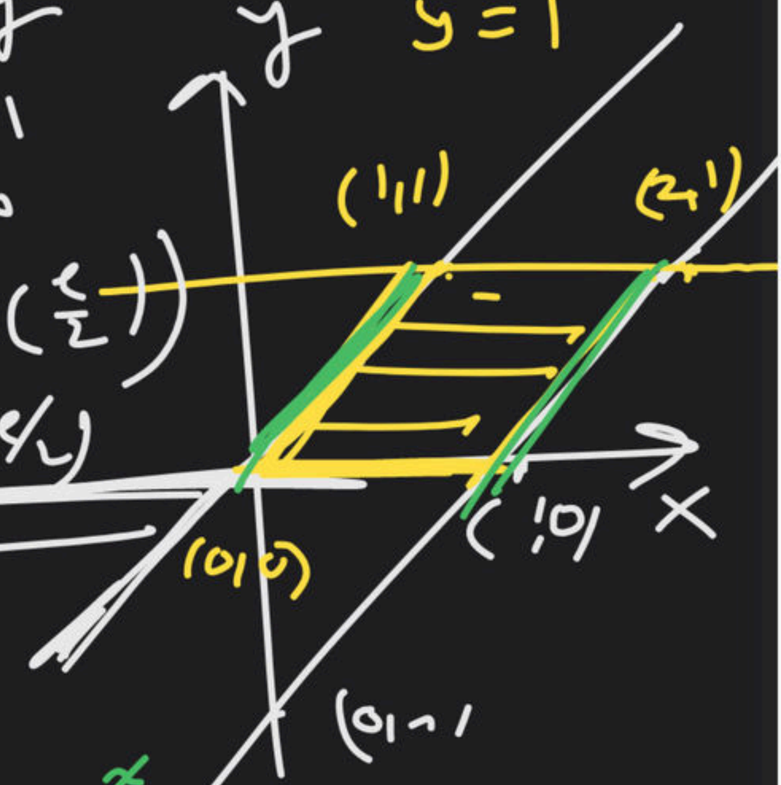
y	1	0
x	0	−1

$y = x$

$y = x-1$

$y = 0$

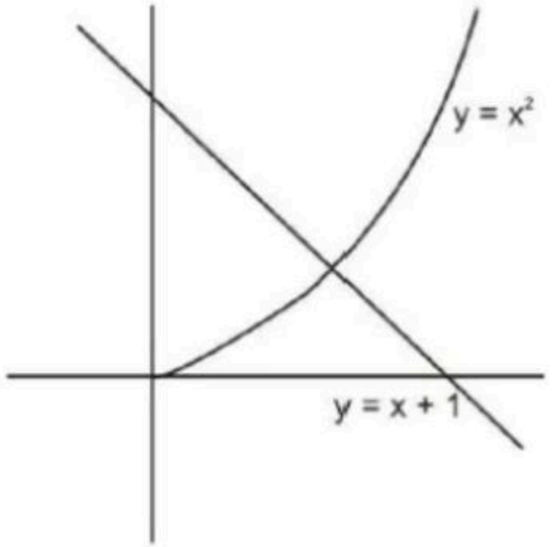
$y = 1$



Procedure of double integration over mix region :

Step – 1 : Trace the region

Example :



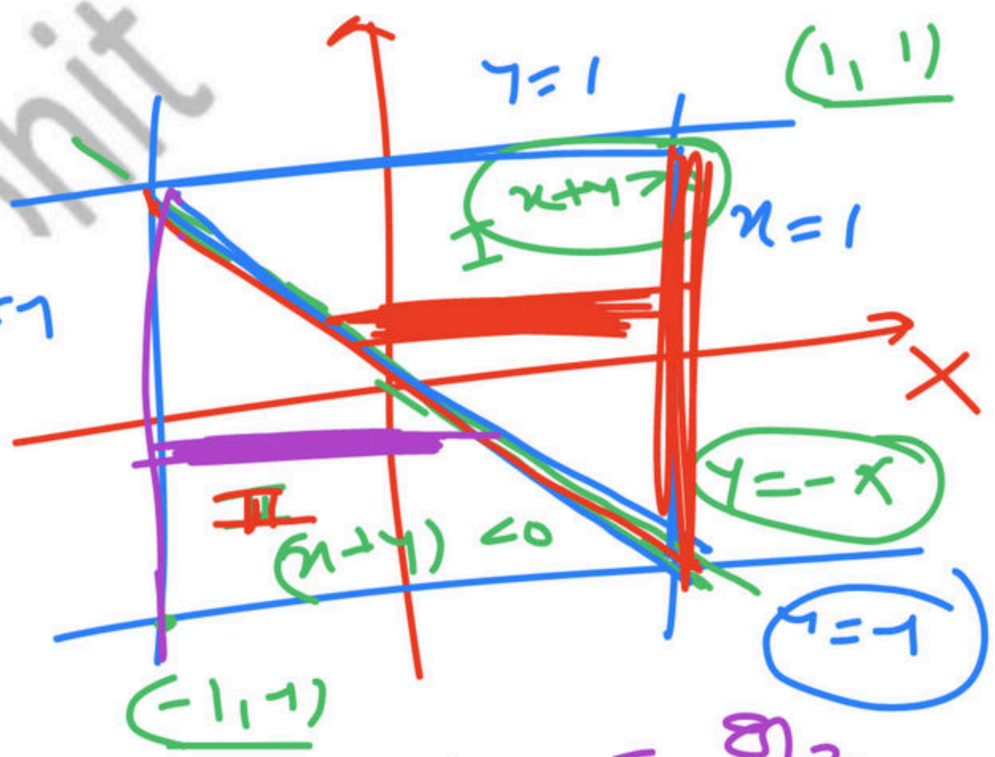
Step – 2 : Divide into parts and all parts are simple region.

Step – 3 : Integrate over all simple region and sum of all quantities.

Q.2. The value of integral $\int_{-1}^{1}\int_{-1}^{1}|x+y|\,dxdy$ is **IIT JAM – 2019**

(a) 1.79 (b) 1

(c) 4.39 (d) 2.66



$$\int_{y=-1}^{1}\int_{x=-y}^{1}(x+y)\,dxdy + \int_{y=-1}^{1}\int_{x=-1}^{-y}-(x+y)\,dxdy$$

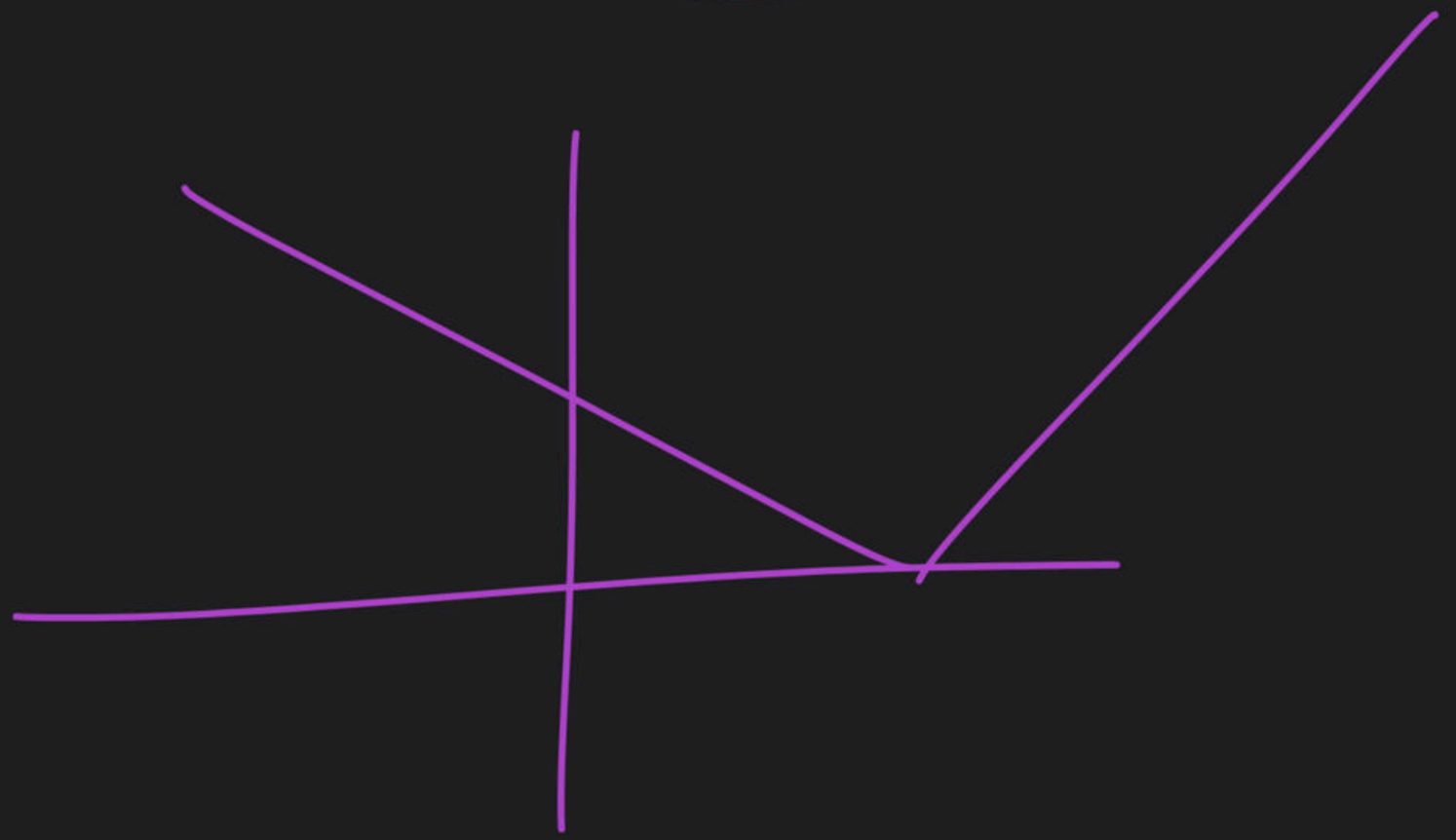
I II

$$\int_{y=-1}^{1}\left(\frac{x^2}{2}+xy\right)_{-y}^{1}dy - \int_{-1}^{1}\left(\frac{x^2}{2}+xy\right)_{-1}^{-y}dy$$

$$\int_{y=-1}^{1}\left(\left(\frac{1}{2}+y\right)-\left(\frac{y^2}{2}-y^2\right)\right)dy - \int_{-1}^{1}\left(\frac{y^2}{2}-y^2-\left(\frac{1}{2}-y\right)\right)dy = \frac{8}{3}$$

$|x+11|$

$m=7$



$$\int_{-1}^{1} \int_{-1}^{1} (x - y)^2 \, dx \, dy$$

Q.3. The value of $\iint_R \cos(\max\{x^3, y^{3/2}\})\,dx\,dy$, where

$R = [0, 1] \times [0, 1]$. **JAM – 2009**

(a) sin 2 (b) sin 3

(c) sin 1 (d) None

$\int\int \cos(\max(x^2, y))\,dx\,dy$

$\int_{x=0}^{1}\int_{y=x^2}^{1} \cos y\,dy\,dx + \int_{x=1}^{1}\int_{y=0}^{x} \cos x^2\,dy\,dx$

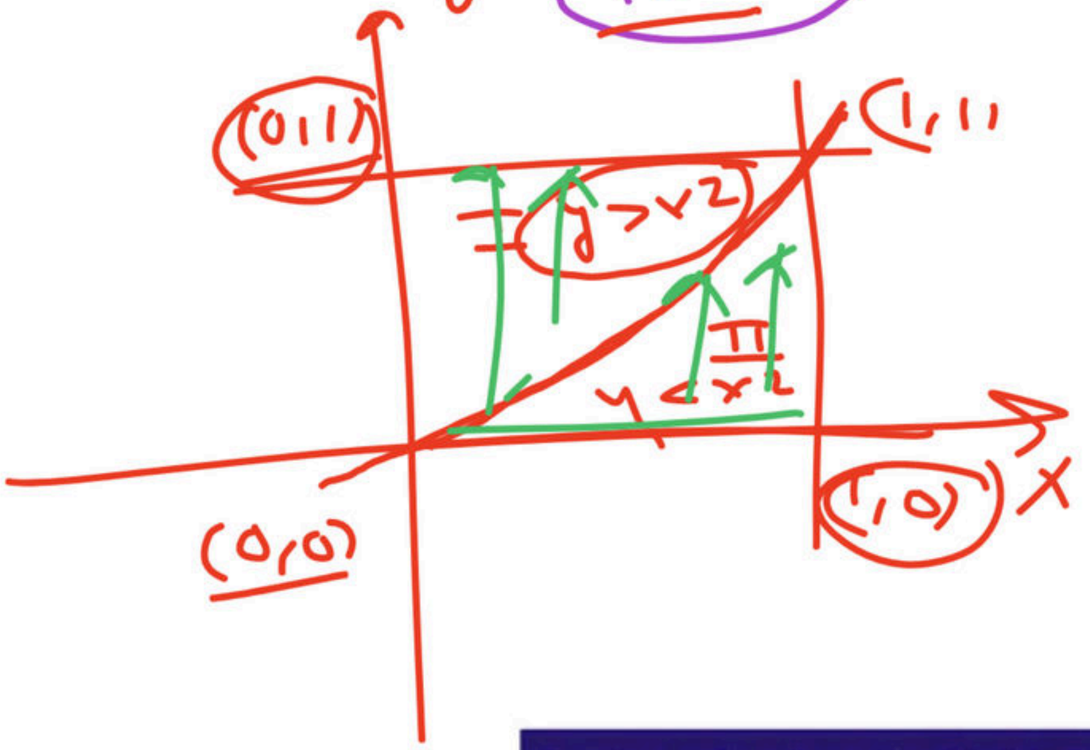
$\int_0^1 (\sin y)_{x^2}^1\,dx + \int_{x=0}^{1} (y)_0^x \cos x^2\,dy$

$\int_0^1 (\sin 1 - \sin x^2)\,dx + \int_0^1 x\cos x^2\,dx$

$x^3 = y^{3/2}$

$x^6 = y^3$

$y = x^2$



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Q.4. Evaluate $\int_{\pi/2}^{\pi}\int_{0}^{x}\frac{\sin x}{x}\,dy\,dx$

(a) 0 (b) 1

(c) 2 (d) -1

Q.5. The value of the integral $\int_0^1 \int_0^1 (x^2 + y^2)\,dx\,dy$ **BHU 2012**

(a) 1 (b) 0

(c) ½ (d) 2/3

$$\int_0^1 \left(\frac{x^3}{3} + y^2 x\right)_0^1 dy$$

$$\int_0^1 \left(\frac{1}{3} + y^2\right) dy$$

$$\left(\frac{1}{3}y + \frac{y^3}{3}\right)_0^1 = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Q.6. Evaluate $\int_0^1 \int_0^1 \left(2x^3 e^{x^2 y}\right) dy dx$ **JNU 2021**

(a) $e^2 - e - 2$

(b) $e^2 - 2$

(c) $e - 2$

(d) 0

Q.7 The value of integral $\int_0^3 \int_0^{\sqrt{3}x} \frac{dydx}{\sqrt{x^2+y^2}}$ **.JAM-2008**

(a) $3\log(\sqrt{3}-2)$ (b) $\log(\sqrt{3}+2)$

(c) $3\log(\sqrt{3}+2)$ (d) $-3\log(\sqrt{3}+2)$

Q.8. The value of the integral $\iint_D \frac{\sin(2x)}{x}\,dxdy$ .where D denotes the region bounded by the x – axis and the lines y = x and x = 1.**IIT JAM 2007**

(a) $-\frac{\cos 2}{2}+\frac{1}{2}$

(b) $\frac{\cos 2}{2}$

(c) cos2

(d) sin 2

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