Triple Integration

Dirichlet's theorem :

The theorem states that $\iiint x^{l-1}y^{m-1}z^{n-1}dxdydz = \dfrac{\gamma(l).\gamma(m).\gamma(n)}{\gamma(l+m+n+1)}$;

Where subject to condition $x + y + z \leq 1$ and all variables are positive.

Note :

In general this theorem states that $\iint .....\int x_1^{m_1-1}x_2^{m_2-1}......x_n^{m_{n-1}}dx_1.....dx_n = \dfrac{\gamma(m_1).\gamma(m_2)....\gamma(m_n)}{\gamma(m_1+m_2+....+1)}$.

Where subject to condition $x_1 + x_2 + ... + x_n \leq 1$

Q.1. Evaluate $\iiint xyz\, dxdydz$ taking throughout the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$ in which variable are positive.

(a) $\frac{abc}{8}$

(b) $\left(\frac{abc}{2}\right)^2$

(c) $\frac{abc}{4}$

(d) $\frac{a^2b^2c^2}{48}$

Volume by Triple Integrals :

The volume by triple integral is $\iiint\limits_D dV$, where D is a solid region and dV = dxdydz.

Q.2. Consider the region $G = \{(x,y,z) \in R^3 : 0 < z < x^2 - y^2 , x^2 + y^2 < 1\}$. Then the volume of G is equal to **IIT JAM 2022**

(a) 1 (b) 0

(c)2 (d) 1.4

Q.3. Volume of the solid $\left\{(x,y,z)\in R^3 \mid 1\le x\le 2, 0\le y\le \frac{2}{x}, 0\le z\le x\right\}$ is expressible as **IIT – JAM 2017**

(a) $\int_1^2\int_0^{2/x}\int_0^x dz\,dy\,dx$ (b) $\int_1^2\int_0^x\int_0^{2/x} dy\,dz\,dx$

(c) $\int_0^2\int_1^z\int_0^{2/x} dy\,dx\,dz$ (d) $\int_0^2\int_{\max(z,1)}^2\int_0^{2/x} dy\,dx\,dz$

Q.4. If the volume of the solid in R^3 bounded by the surface by the surface $x=-1, x=1$, $y = -1$, $y = 1$, $z = 2$, $y^2 + z^2 = 2$ is $\alpha - \pi$, then α equal to **IIT JAM 2018**

(a) 4 (b) 5

(c) 6 (d) 7

Q.5. The volume of the closed region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 5y + 10z = 10$ is **[JAM CA-2008]**

(a) 20/3 (b) 5

(c) 10/3 (d) 5/3

Q.6. The volume of the solid bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$ and $z = 0$ is **[JAM CA 2010]**

(a) $\int_0^1 \int_0^{2-2y} \int_0^{2-x-2y} dz\, dx\, dy$

(b) $\int_0^1 \int_{x/2}^{1-\frac{x}{2}} \int_0^{2-x-2y} dz\, dy\, dx$

(c) $\int_0^1 \int_0^{2y} \int_0^{2-x-2y} dz\, dx\, dy$

(d) $\int_0^1 \int_0^{1/2} \int_0^{2-x-2y} dz\, dx\, dy$

Q.7. The volume of the region in R^3 given by $3|x| + 4|y| + 3|z| \le 12$ is **[JAM CA-2011]**

(a) 64 (b) 48

(c) 32 (d) 24

Q.8. Find the volume of the region bounded by the plane $x = 0$, $y = 0$, $z = 0$ and $6x + 4y + 3z = 12$.

[JAM MS-2008]

(a) 1 (b) 2

(c) 3 (d) 4

Q.9. Find the finite volume enclosed by the paraboloids $z = 2 - x^2 - y^2$ and $z = x^2 + y^2$. **IIT JAM – 2007**

(a) π

(b) $-\pi$

(c) 2π

(d) None