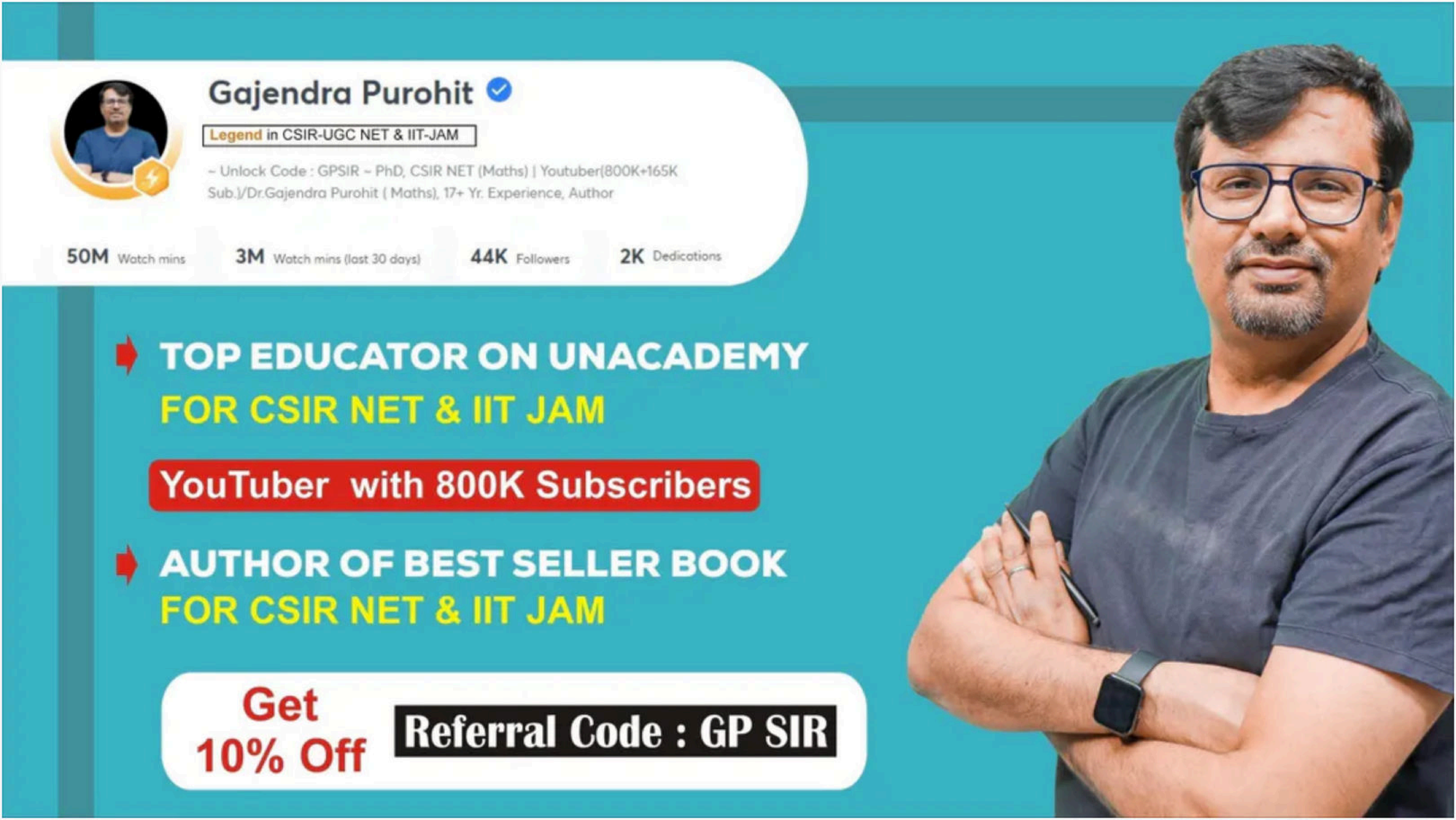

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1 • Asked by Tanu

Please help me with this doubt

Q.9 If $\sum a_n$ is a convergent series of positive real number then $\sum \frac{a_n}{n}$

(a) may not convergent

(b) divergent

(c) is convergent

(d) may or may not be convergent

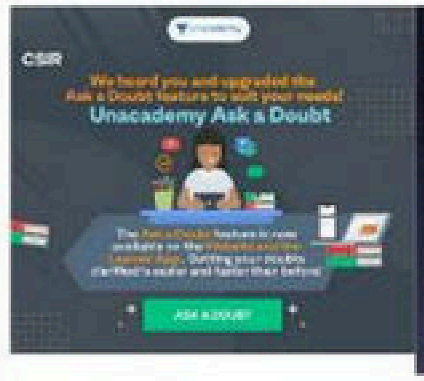
Q.10 Let $a_n = \begin{cases} \frac{1}{\sqrt{n}} & \text{When n is odd} \\ \frac{1}{n^2} & \text{When n is even} \end{cases}$ Then $\sum_{n=1}^{\infty} a_n$ is

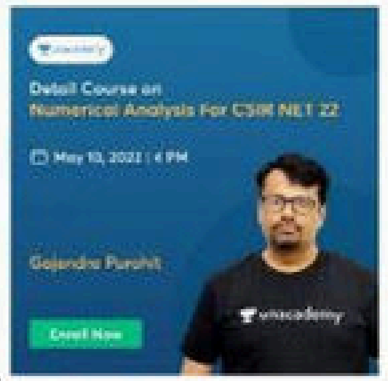
(a) divergent

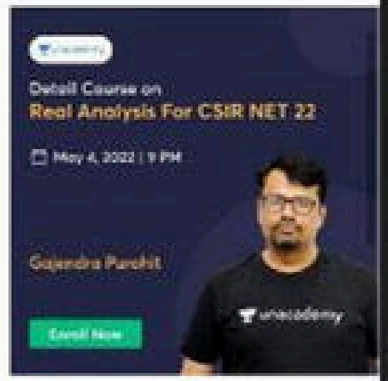
(b) convergent

(c) oscillatory

(d) NOT







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▲ 2 • Asked by Tanu

Please help me with this doubt

$= 1$

Q.5. What is the sum of series $\left(\frac{1}{2\cdot 3}+\frac{1}{2^2\cdot 3}\right)+\left(\frac{1}{2^2 3^2}+\frac{1}{2^3 3^2}\right)+\cdots\left(\frac{1}{2^n 3^n}+\frac{1}{2^{n+1} 3^n}\right)+\cdots$

CSIR NET JUNE 2019

(a) $\frac{3}{8}$ (b) $\frac{3}{10}$

(c) $\frac{3}{14}$ (d) $\frac{3}{16}$

Q.6. The sum of the series $\frac{1}{1!}+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\ldots$ equals :

(a) e (b) e/2

(c) 3e/2 (d) $1+\frac{e}{2}$

Q.7. $\lim_{n\to\infty}\frac{1}{n^3}\sum_{j=0}^{2n-1} j^2$ equals : **CSIR NET DEC 2016**

(a) 4 (b) 16 (c) 1
(d) 8


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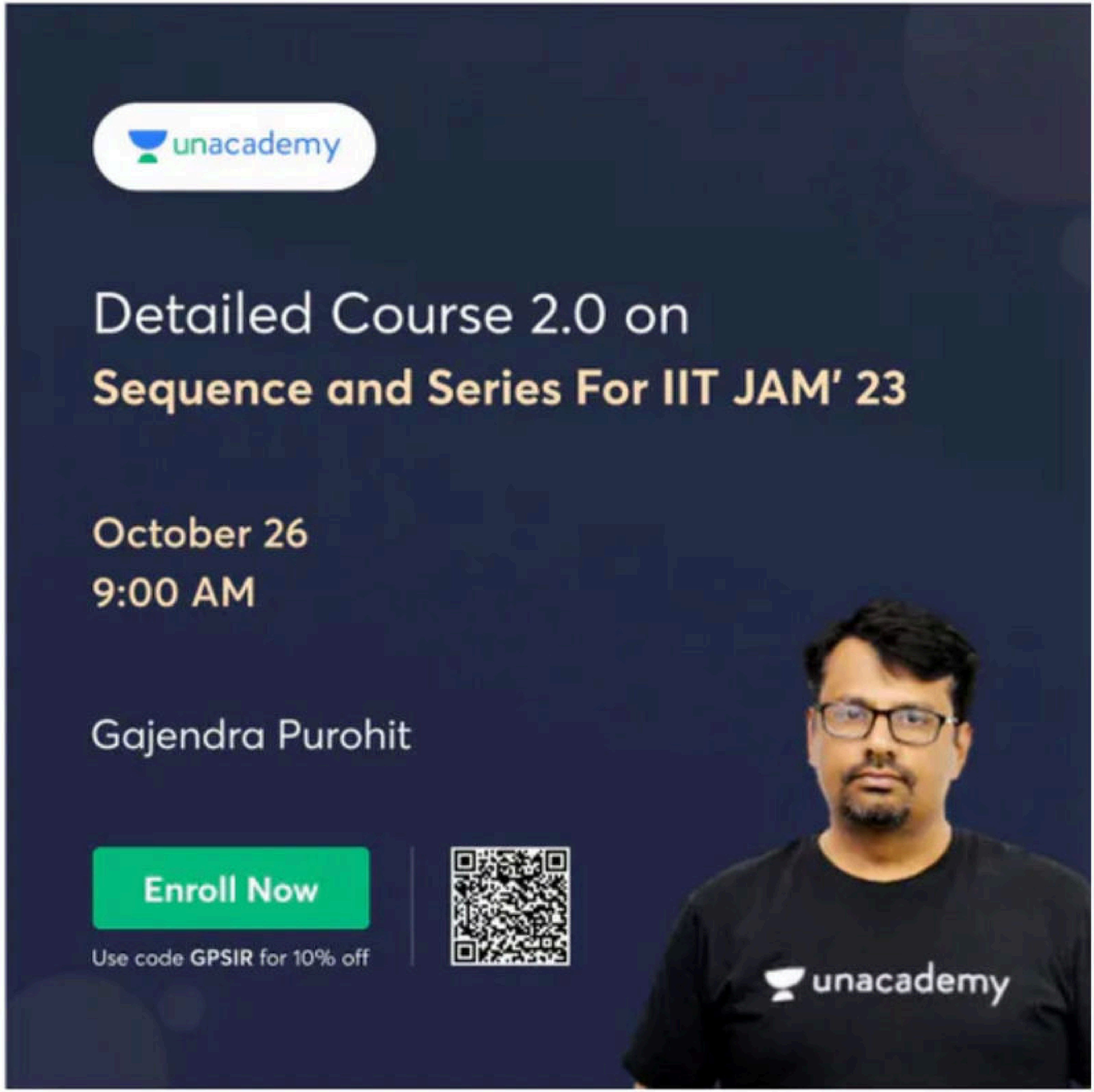
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Sum of the telescoping series : $\sum(t_n - t_{n+1})$ is telescoping series, then

$t_1 - t_2 + t_2 - t_3 + t_3 - t_4 + \ldots\ldots + t_n - t_{n+1} = t_1 - t_{n+1}.$

$\Rightarrow$ Sum of given series is $t_1 - \lim_{n\to\infty} t_{n+1}$.

Note : If telescoping series is convergent then this series is converge to sum of this series.

$$\sum \frac{1}{n(n+1)} = \sum\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$t_n = 1/n,\ t_{n+1} = \frac{1}{n+1}$

$S = t_1 - \lim_{n\to\infty} t_{n+1}$

$= 1$

Q1. Let $a_1 = 1$ and $a_n = 2 - \frac{1}{n}$ for $n \geq 2$, then

$$\sum_{n=1}^{\infty}\left(\frac{1}{a_n^{\,2}} - \frac{1}{a_{n+1}^{2}}\right) \text{ converges to}$$

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{3}{4}$

$$a_1 - \lim_{n\to\infty} a_{n+1}$$

$$1 - \lim_{n\to\infty} \frac{1}{\left(2 - \frac{1}{n+1}\right)^2}$$

$$1 - \frac{1}{4} = \frac{3}{4}$$

Q2. Let $<a_n>$ be a sequence of real numbers such that $a_1 = 1$ and $\lim_{n\to\infty} a_n = 3$, then the value of $\sum_{n=1}^{\infty}\left(a_{n+1}^2 - a_n^{\ 2}\right)$ is

(a) 7

(b) 8

(c) 9

(d) 10

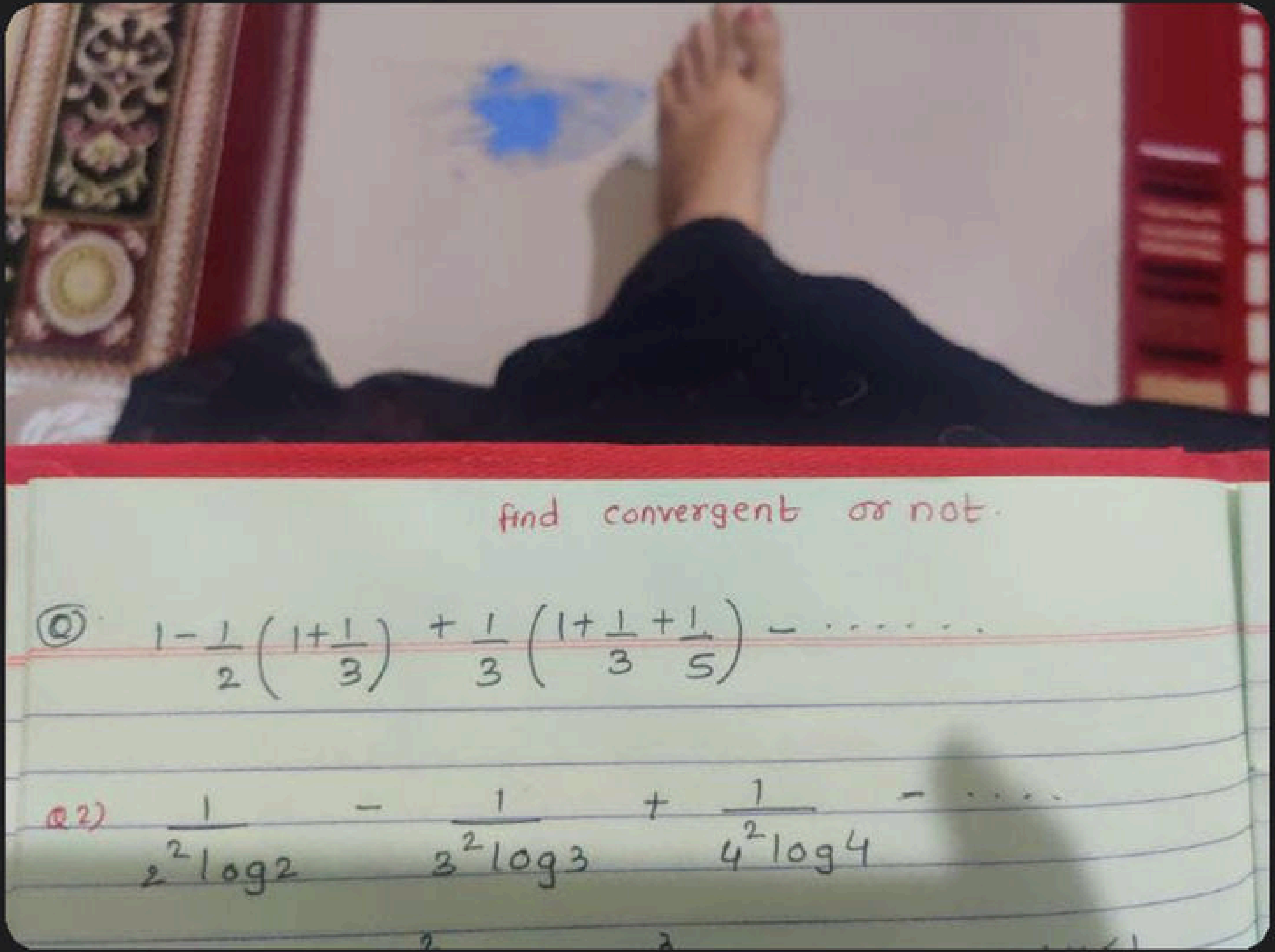
$\lim_{n\to\infty} a_{n+1}^2 - a_1$

$\lim_{n\to\infty} (3)^2 - 1$

$9-1$

▲ 2 • Asked by Harsha

Sir last doubt plz explain kar dijiye



$(-1)^n \frac{1}{n}$

$1+\frac{1}{3}$

$\frac{4}{3}$

$\frac{15+5+3}{15}=\frac{23}{15}$

Result :

(1) Sum of series $\sum\frac{1}{n(n+1)}$ is $1=\frac{1}{1}\cdot\frac{1}{1!}$.

(2) Sum of series $\sum\frac{1}{n(n+1)(n+2)}$ is $\frac{1}{4}=\frac{1}{2}\cdot\frac{1}{2!}$

(3) Sum of series $\sum\frac{1}{n(n+1)(n+2)(n+3)}$ is $\frac{1}{3}\cdot\frac{1}{3!}=\frac{1}{3\times 6}=\frac{1}{18}$.

(4) Sum of series $\sum\frac{1}{n(n+1).....(n+m)}$ is $\frac{1}{m}\cdot\frac{1}{m!}$.

Q3. The sum of the series $\sum \frac{1}{n(n+1)(n+2)}$. $= \frac{1}{2 \cdot 2!} = 1/4$

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Sum of Series of type $\sum_{k=1}^{n} f(n,k) = \frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)$

Step – 1 : Re – write the series in the form $\frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)$

Step – 2 : Put $\frac{1}{n} = dx$ & $\frac{k}{n} = x$ then Change summition into integration with limit 0 to 1

Step – 3 : Integrate and get final solution

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+k^2}}$$

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \frac{1}{\sqrt{1+\left(\frac{k}{n}\right)^2}}$$

$$\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$$

$$\int_0^{\pi/4} \frac{\sec^2\theta\, d\theta}{\cancel{\sec\theta}}$$

Let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$$\left(\log(\sec\theta + \tan\theta)\right]_0^{\pi/4}$$

$\log(1+\sqrt{2})$

Q4. The value of $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{\sqrt{n^2+kn}}$ is [IIT-JAM 2011]

(A) $2\left(\sqrt{2}-1\right)$

(B) $2\sqrt{2}-1$

(C) $2-\sqrt{2}$

(D) $\frac{1}{2}\left(\sqrt{2}-1\right)$

$$\frac{(1+x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\sqrt{1+\frac{k}{n}}}$$

$$\int_0^1\frac{1}{\sqrt{1+x}}\,dx$$

$$2\left(\sqrt{1+x}\right)_0^1$$

$$2\left(\sqrt{2}-1\right)$$

Dr.Gajendra Purohit (PhD,NET)
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Sum of series by expansion :

We know that (i) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots.$

(ii) $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots.$

(iii) $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots$

(iv) $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} \ldots$

(v) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots\ldots$

(vi) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \ldots\ldots$

(vii) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots\ldots$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} = \log 2$$

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}(-1)^n$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n+1}}{(2n+1)!} = \sin 2$$

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Q5. Let $a_n = n + \frac{1}{n},\ n \in N$. Then the sum of the series

$\sum_{n=1}^{\infty} (-1)^n \frac{a_{n+1}}{\lfloor n}$ is **[IIT-JAM 2018]**

(A) $e^{-1} - 1$

(B) e^{-1}

(C) $1 - e^{-1}$

(D) $1 + e^{-1}$

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{(n+1) + \frac{1}{(n+1)}}{n!} \right)$$

$$\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n!} + \frac{1}{n!} + \frac{1}{(n+1)\,n!} \right)$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{(n-1)!} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} + \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)!}$$

$$\left[-1 + \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \cdots\right) + \left(-1 + \frac{1}{2!} - \frac{1}{3!} + \cdots - 1\right)$$

$$+ \left(-\frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \cdots\right)$$

$$-e^{-1} + (e^{-1} - 1) + (1 - 1 - e^{-1})$$

$$-1 - e^{-1}$$

$$\sum \frac{(-1)^n}{n!} = e^{-1}$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \cdots$$

$$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!}$$

$$1 - \frac{1}{1!} - e^{-1}$$

Q6. The value of the series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is

(A) 1 (B) 3

(C) 2 (D) 4

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3}$$

$$1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$$

$$1 + 2x + 3x^2 + \dots = \frac{1}{(1-x)^2}$$

$$x + 2x^2 + 3x^3 + \dots = \frac{x}{(1-x)^2}$$

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots = \frac{1/2}{(1-1/2)^2}$$

$$= \frac{1/2}{1/4}$$

$$= 2$$

Q7. The sum of the series $\sum_{n=2}^{\infty}\frac{(-1)^n}{n^2+n-2}$ is

[IIT-JAM 2016]

(A) $\frac{1}{3}\ln 2-\frac{5}{18}$

(B) $\frac{1}{3}\ln 2-\frac{5}{6}$

(C) $\frac{2}{3}\ln 2-\frac{5}{18}$

(D) $\frac{2}{3}\ln 2-\frac{5}{6}$

$$= \sum_{n=2}^{\infty}\frac{(-1)^n}{(n-1)(n+2)}$$

$$\frac{1}{3}\sum_{n=2}^{\infty}\frac{(-1)^n}{n-1}-\frac{(-1)^n}{n+2}$$

$$\frac{1}{3}\left(\left(1-\frac{1}{2}+\frac{1}{3}\cdots\right)-\left(\frac{1}{4}-\frac{1}{5}+\cdots\right)\right)$$

$$\frac{1}{3}\left(\log 2-(1-\tfrac{1}{2}+\tfrac{1}{3}-\log 2)\right)$$

$$\tfrac{1}{3}\left(2\log 2-(1-\tfrac{1}{2}+\tfrac{1}{3})\right)$$

$$\tfrac{1}{3}\left(2\log 2-\tfrac{5}{6}\right)$$

$$\frac{6-3+2}{6}$$

$$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots$$

$$\log 2 = 1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\cdots$$

$$1-\tfrac{1}{2}+\tfrac{1}{3}-\log 2$$

Q8. $\sum_{n=1}^{\infty} \tan^{-1}\frac{2}{n^2} =$ is **[IIT-JAM 2016]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$

(C) $\frac{3\pi}{4}$ (D) π

$\sum \tan^{-1}\left(\frac{2}{1+n^2-1}\right)$

$\sum \tan^{-1}\left(\frac{2}{1+(n^2-1)}\right)$

$\sum \tan^{-1}\left(\frac{n+1-(n-1)}{1+(n-1)(n+1)}\right)$

$\sum_{n=1}^{\infty} \left[\tan^{-1}(n+1) - \tan^{-1}(n-1)\right]$

$\lim_{n\to\infty} \left[(\tan^{-1}2 - \tan^{-1}0) + (\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}4 - \tan^{-1}2) + \cdots + (\tan^{-1}(n+1) - \tan^{-1}(n-1))\right]$

$= -\frac{\pi}{4} + \frac{\pi}{2} + \frac{\pi}{2}$

$-\frac{\pi}{4} + \pi$

Q9. The sum of the finite series $S = \frac{1}{2} - \frac{1}{3\times 1!} + \frac{1}{4\times 2!} - \frac{1}{5\times 3!} + ...$ is equal to

[CSIR-NET Nov. 2020]

(A) $2 - \frac{1}{e}$ (B) $1 - \frac{2}{e}$

(C) $\frac{2}{e} - 1$ (D) $\frac{1}{e} - 2$

Q10. Let $S_1 = \frac{1}{3} - \frac{1}{2} \times \frac{1}{3^2} + \frac{1}{3} \times \frac{1}{3^3} - \frac{1}{4} \times \frac{1}{3^4} + ...$ and $S_2 = \frac{1}{4} + \frac{1}{2} \times \frac{1}{4^2} + \frac{1}{3} \times \frac{1}{4^3} + \frac{1}{4} \times \frac{1}{4^4} + ...$ Which of the following identities is true?

[CSIR-NET Feb. 2022]

(A) $3S_1 = 4S_2$ (B) $4S_1 = 3S_2$

(C) $S_1 + S_2 = 0$ (D) $S_1 = S_2$


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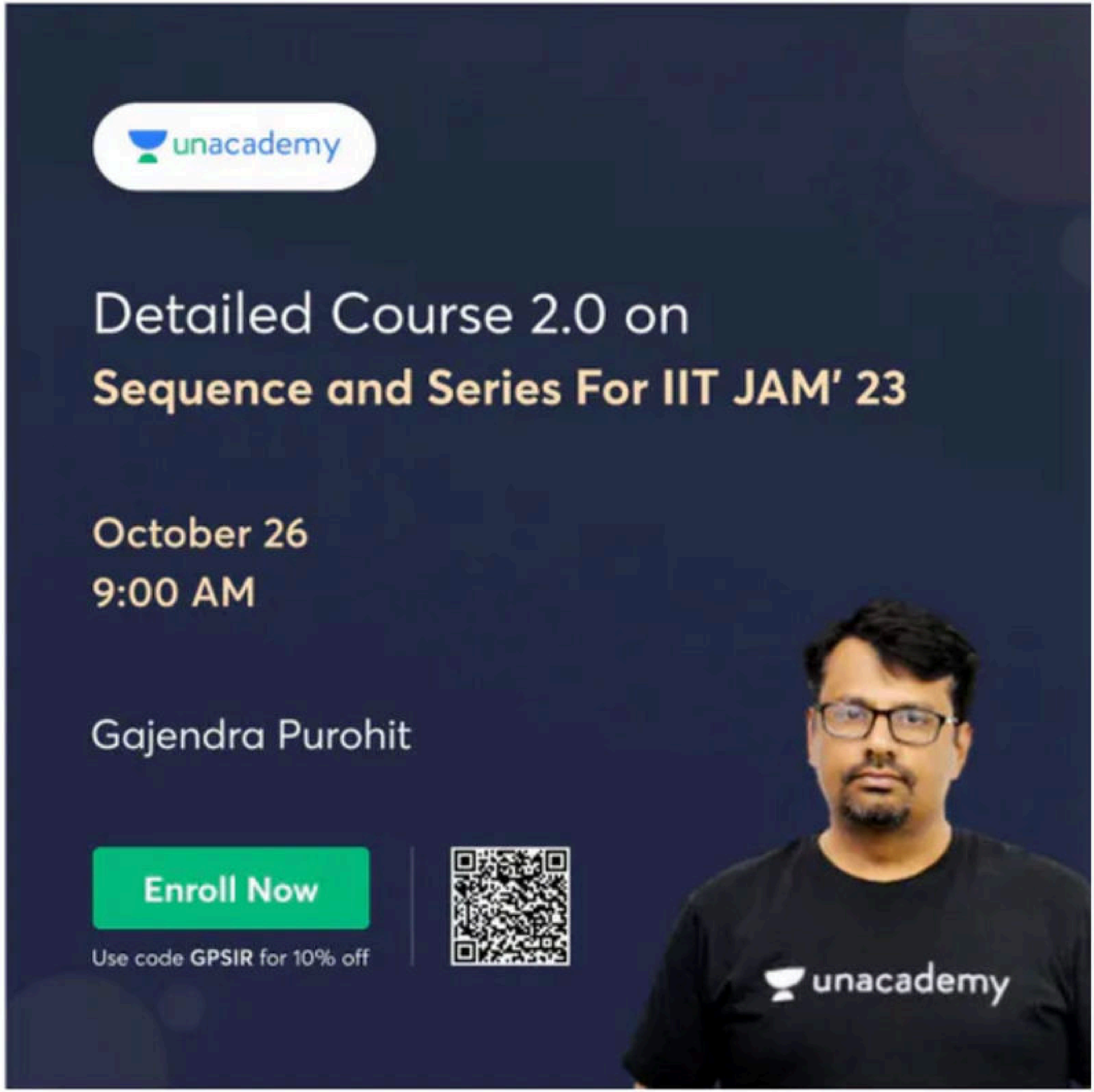
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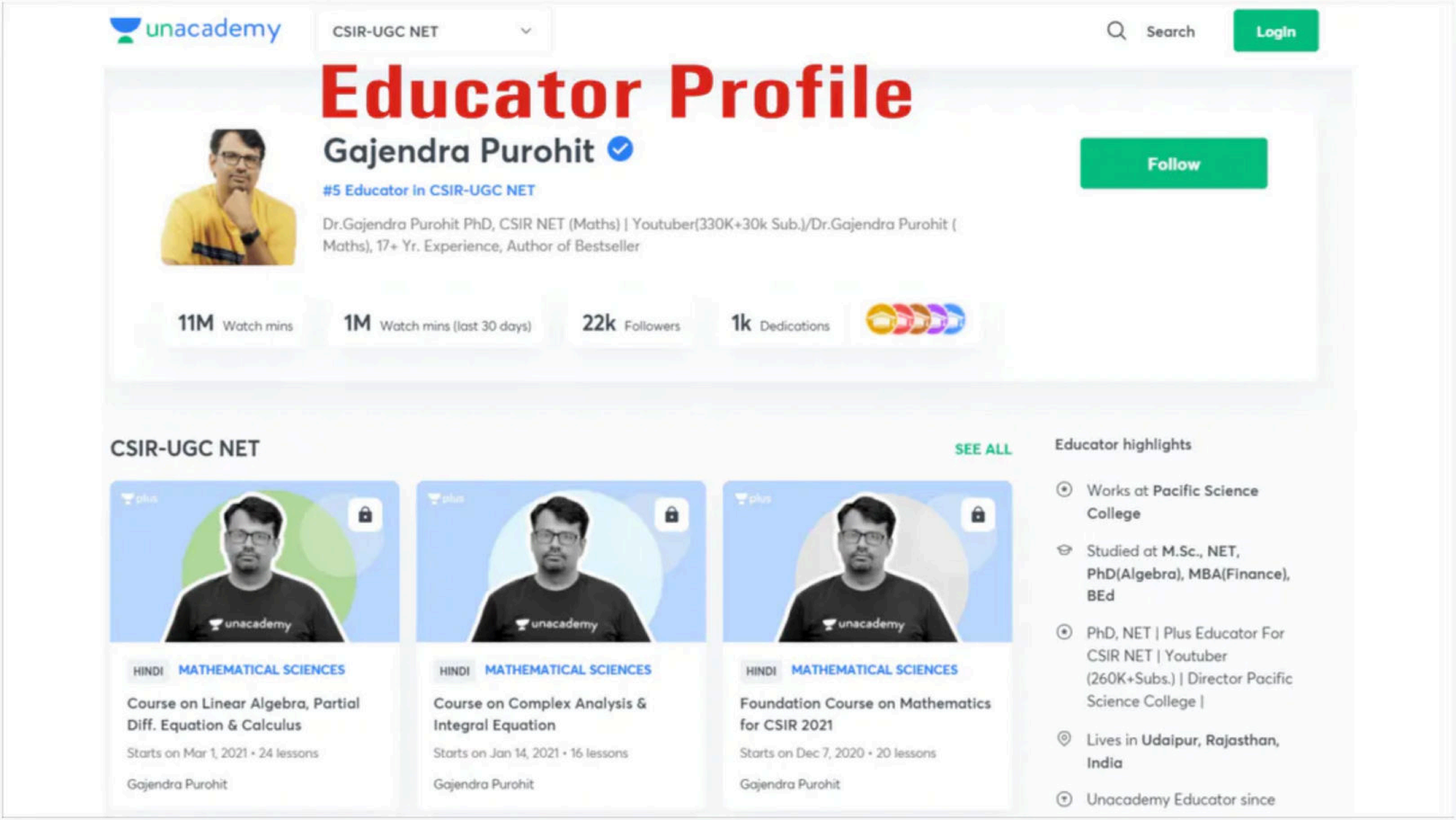
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