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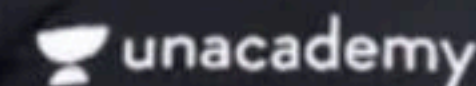
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Double Integrals

Let $f(x, y)$ is a function of two variable then double integral of $f(x, y)$ is denoted by $\iint f(x, y) dx dy$ or $\iint f(x, y) dy dx$.

$$\int_{y=1}^2 \int_{x=1}^2 (\underline{x} + \underline{y}) \underline{dx} dy = \int_{y=1}^2 \left(\frac{x^2}{2} + yx \right) dy$$
$$= \int_{y=1}^2 \left(\left(\frac{4}{2} + 2y \right) - \left(\frac{1}{2} + y \right) \right) dy$$
$$= \int_{y=1}^2 \left(\frac{3}{2} + y \right) dy = \left(\frac{3}{2}y + \frac{y^2}{2} \right)$$

$$\int \int (x^2 + y^3) \, dx \, dy = \int (x^3/3 + y^3 y) \, dy$$

$$= \frac{x^3 y}{3} + \frac{y^4 y}{4} + C$$

$$\int_0^2 \int_0^{\sqrt{2x-y^2}} dy \, dx = \int_0^2 (y) \Big|_0^{\sqrt{2x-y^2}} dx$$

$$\int_0^2 \sqrt{2x-y^2} \, dx = \int_0^2 \sqrt{-(x^2-2x+1)+1} \, dx = \int_0^2 \sqrt{1-(x-1)^2} \, dx$$

$$= \left(\frac{x-1}{2} \sqrt{2x-y^2} + \frac{1}{2} \sin^{-1} \left(\frac{x-1}{1} \right) \right) \Big|_0^2 = \left(0 + \frac{1}{2} \sin^{-1}(1) \right) = \frac{\pi}{4}$$

$$\int_{y=0}^1 \int_{x=0}^y e^{-xy} dx dy = \int_{y=0}^1 \left(\frac{e^{-xy}}{-y} \right)_0^y dy = \int_{y=0}^1 \left(y e^{-xy} \right)_0^y dy$$

$$= \int_{y=0}^1 (y e^{-y} - y) dy = (e-1) \int_0^1 y dy = (e-1) \left(\frac{y^2}{2} \right)_0^1$$

$$= \underline{\underline{\frac{(e-1)}{2}}}$$

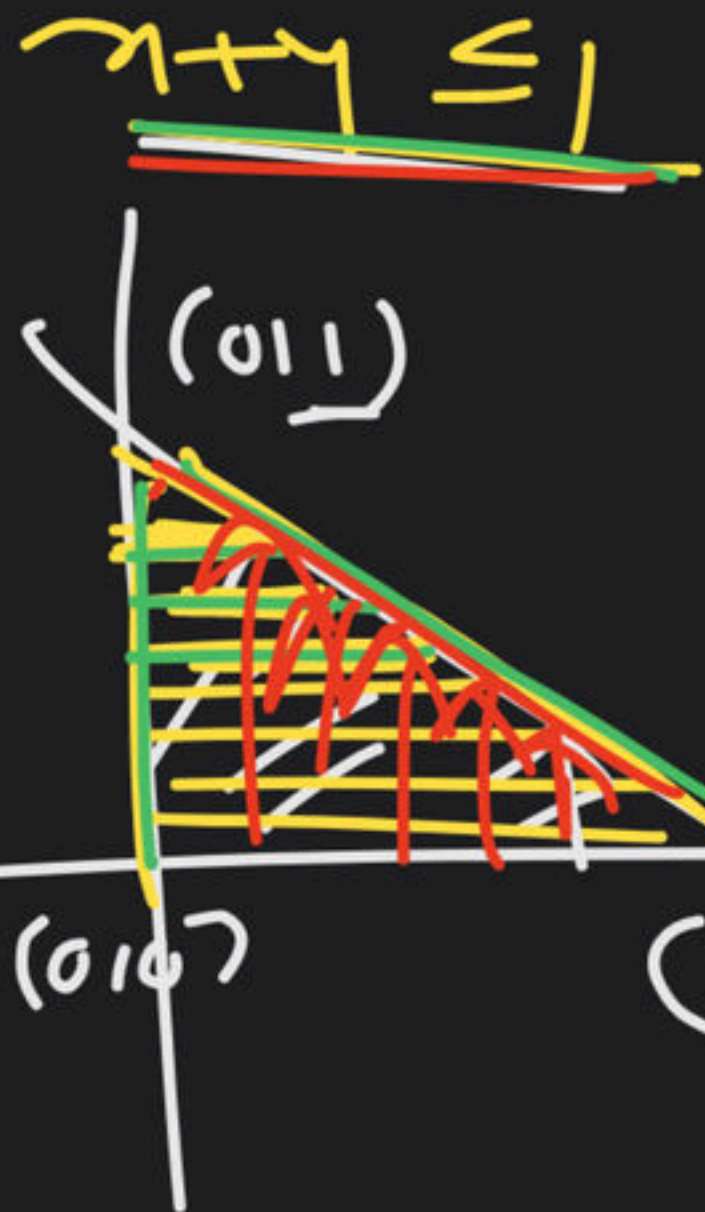
$$\iint (x+y) dx dy$$

first quadrant

x	1	0
y	0	1

$$\frac{1}{2} \int_0^1 (1-y) dy = \frac{1}{2} (y - y^2/2)$$

$$\frac{1}{2} [1 - \frac{1}{3}] = \frac{1}{3}$$



$$\int_0^1 \int_0^{1-y} (x+y) dx dy$$

$y=0$ $x=0$

$$\int_0^1 \left(\frac{x^2}{2} + yx \right) \Big|_0^{1-y} dy$$

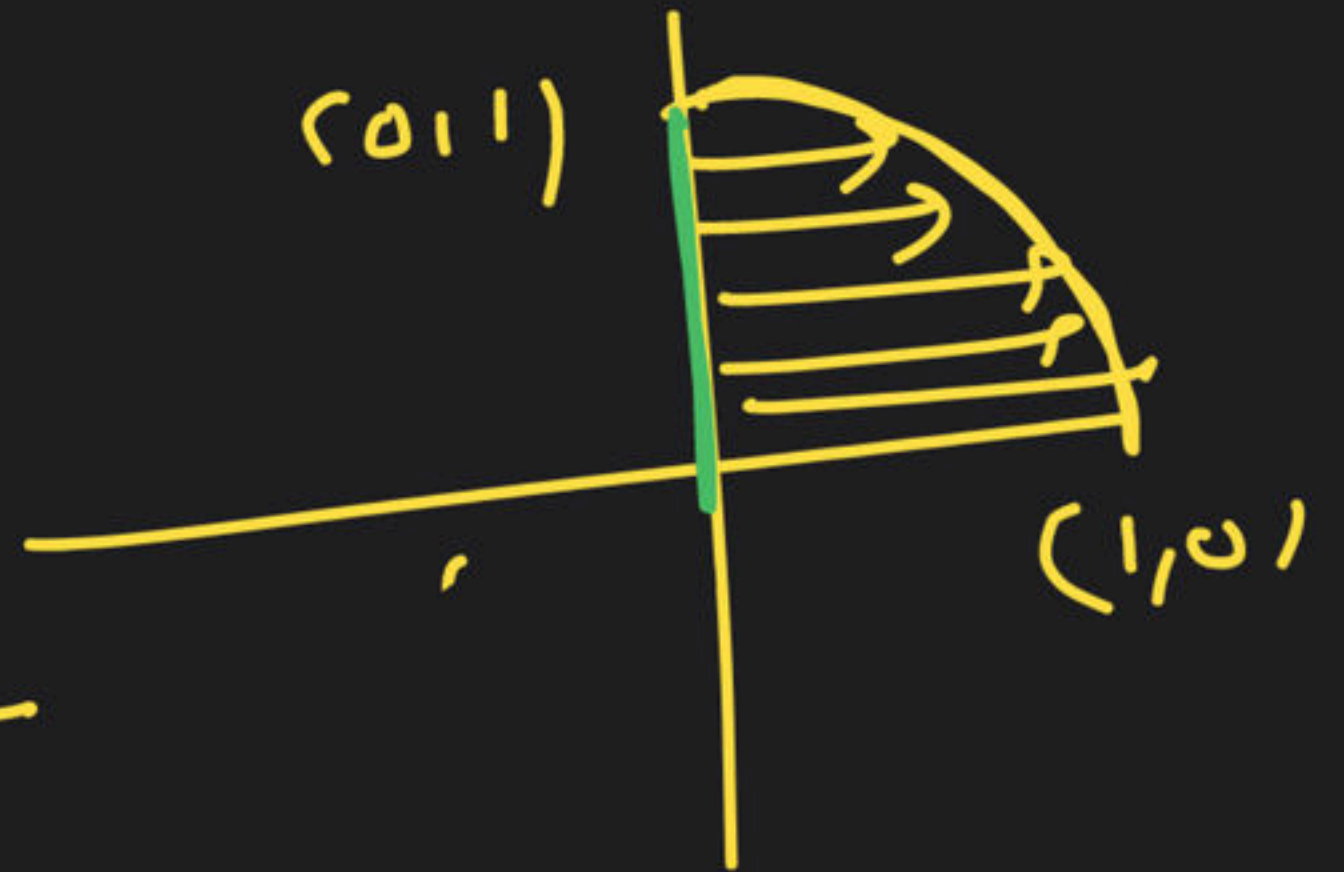
$y=0$

$$\int_0^1 \left(\frac{(1-y)^2}{2} + y(1-y) \right) dy$$

$$\int_0^1 \left(\frac{1-2y+y^2}{2} + y - y^2 \right) dy$$

$$\int_0^1 \int_0^{1-y} (x+y) dx dy = \frac{1}{3}$$

$$x^2 + y^2 \leq 1 \quad \text{in } I \text{ draw}$$



$$\int_0^1 \int_0^{\sqrt{1-y^2}} dx dy$$

$$\int_0^1 \left(x \right)_0^{\sqrt{1-y^2}} dy = \int_0^1 \sqrt{1-y^2} dy$$

$$= \left(\frac{y}{2} \sqrt{1-y^2} + \frac{1}{2} \sin^{-1} y \right)_0^1$$

$$= 0 + \frac{1}{2} \sin^{-1} 1 = \frac{1}{2} \pi = \underline{\underline{\frac{\pi}{2}}}$$

✓

$$\int \int \int u \, dx \, dy \, dz$$

$y =$
 $x =$
 $z = \sqrt{2}$

$$\int_0^1 \int_0^1 \int_0^{\sqrt{2}} u \, dz \, dy \, dx$$

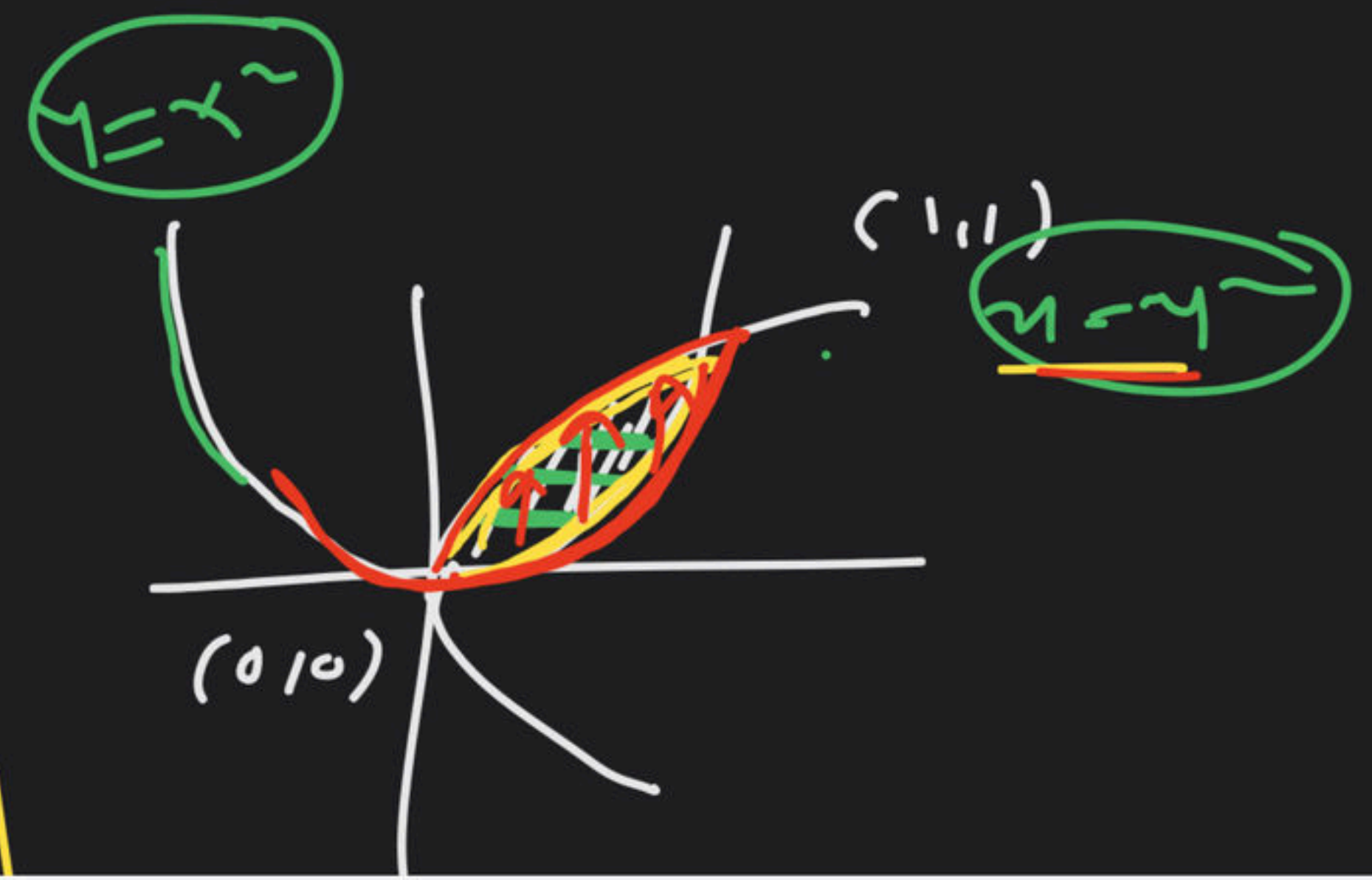
$y=0$
 $u=y^2$

✓

$$y = x^2 \quad z = y^2$$

$$\int_0^1 \int_0^{\sqrt{y}} x \, dx \, dy$$

$z=0$
 $y=y^2$



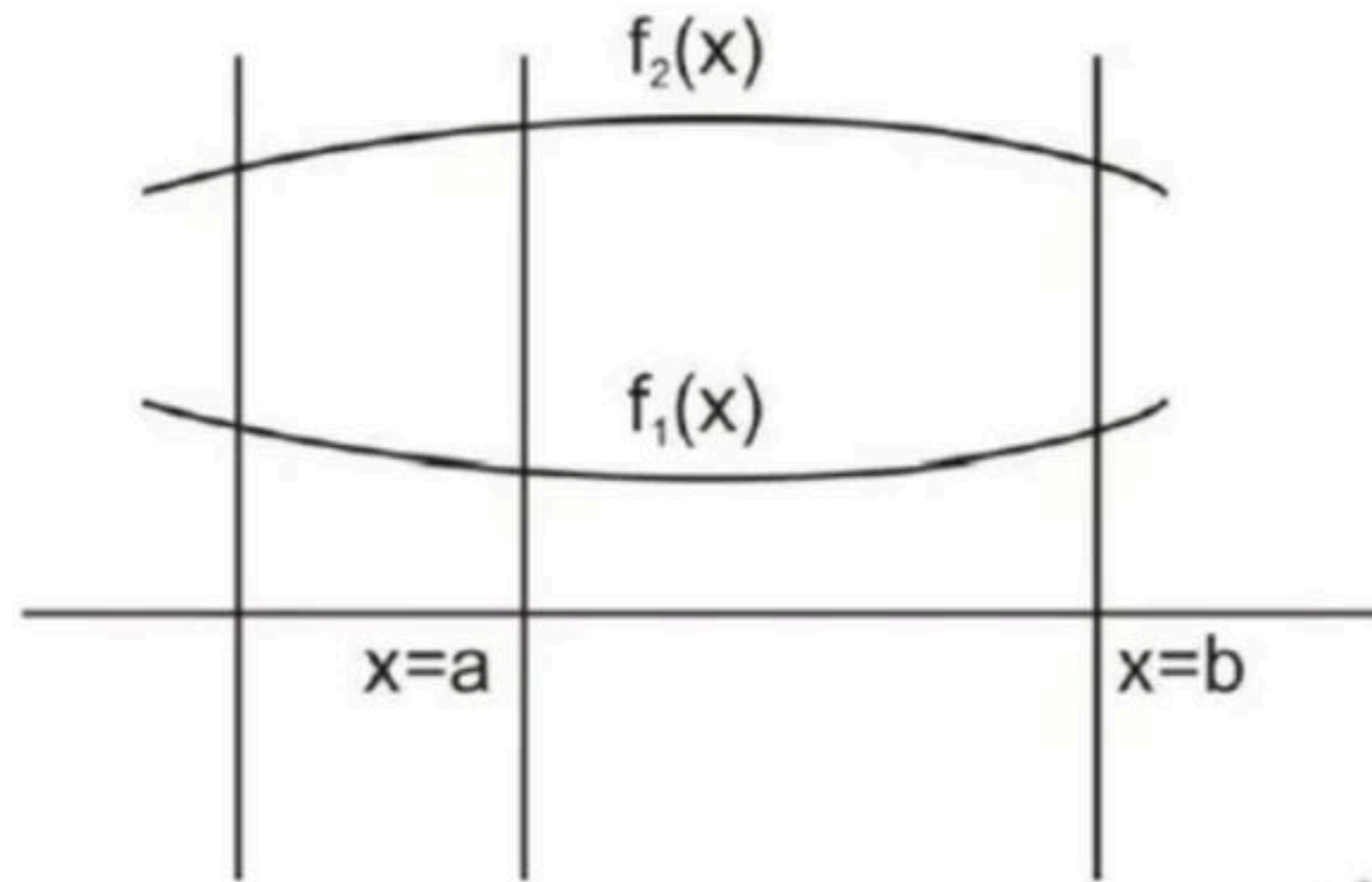
Note :

(1) For $\iint f(x, y) dx dy$, $f(x, y)$ is first integrated w.r.t. x and then it is integrated w.r.t. y .

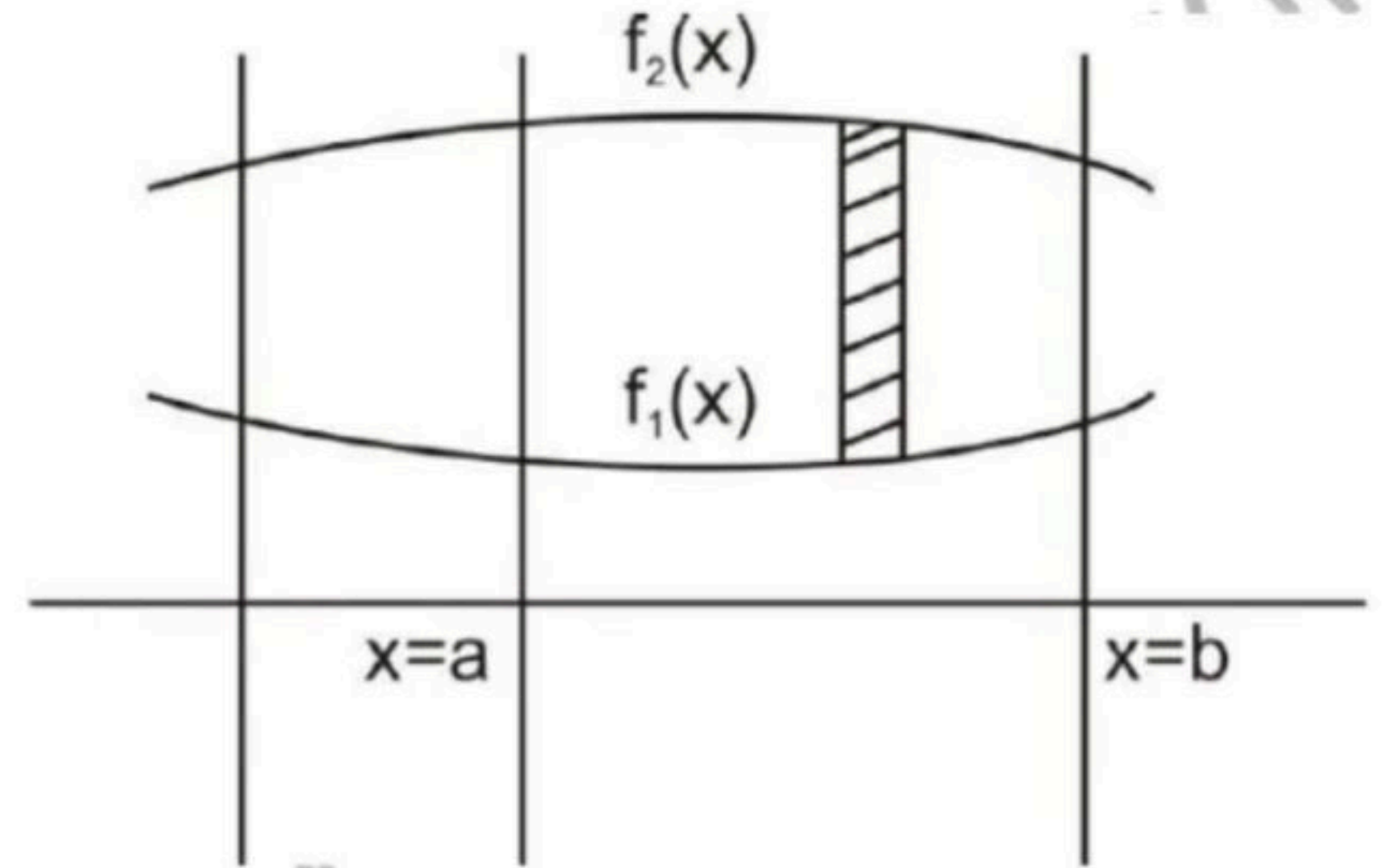
(2) For $\iint f(x, y) dy dx$, $f(x, y)$ is first integrated w.r.t. y and then it is integrated w.r.t. x .

Find limit by a given curve :

- (a) If the region A is bounded by the curves $y = f_1(x)$ & $y = f_2(x)$ and the coordinate $x = a$ and $x = b$.

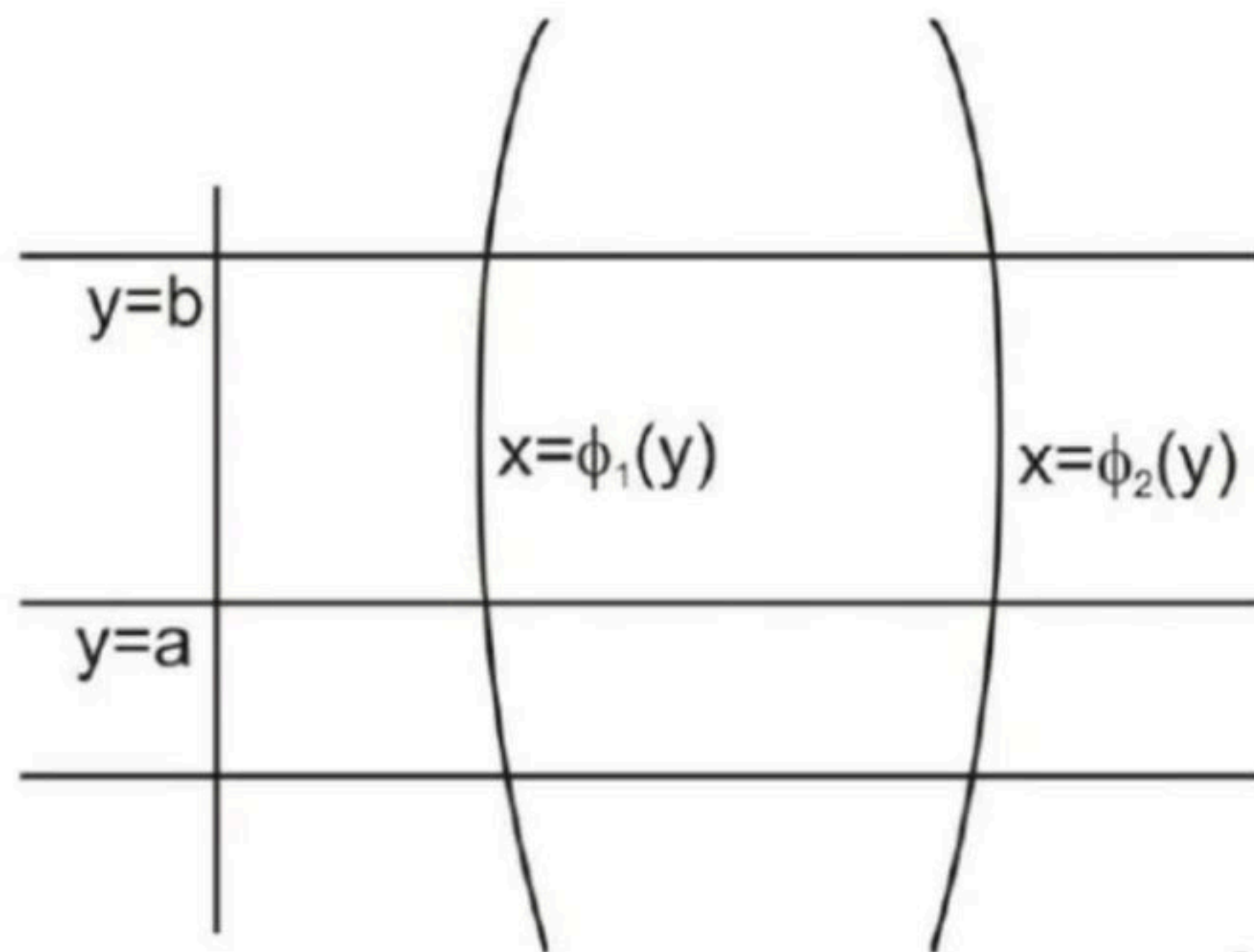


Then strip is parallel to y-axis

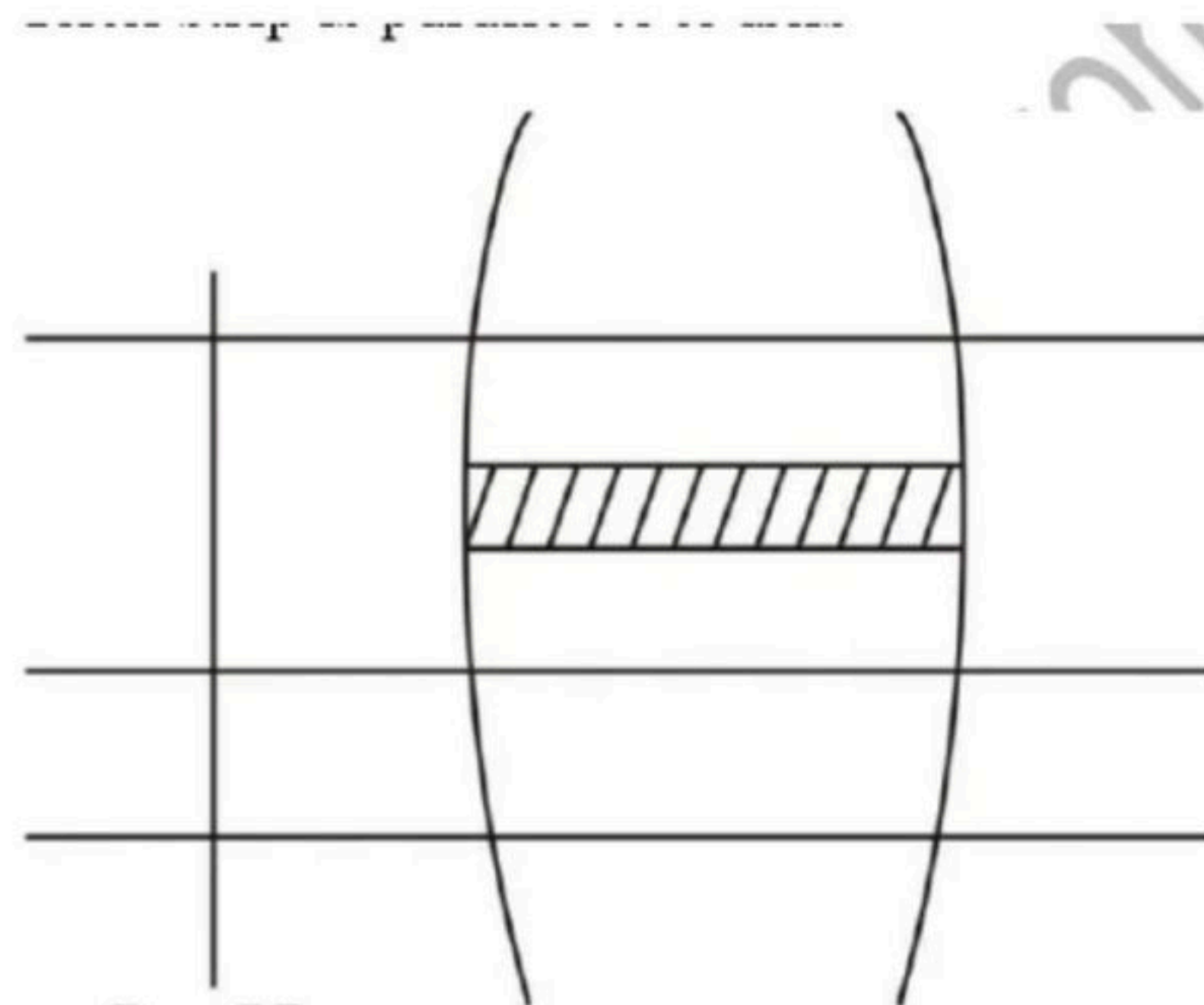


$$\text{Then } \iint_A f(x, y) dA = \int_{x=a}^b \int_{y=f_1(x)}^{y=f_2(x)} f(x, y) dy dx$$

- (b) If the region A is the area bounded by the curve $x = \phi_1(y)$ & $x = \phi_2(y)$ and coordinate $y = a$ and $y = b$.



Then strip is parallel to x-axis



$$\text{So, } \iint_A f(x, y) dA = \int_{y=a}^{y=b} \int_{x=\phi_1(y)}^{x=\phi_2(y)} f(x, y) dx dy$$

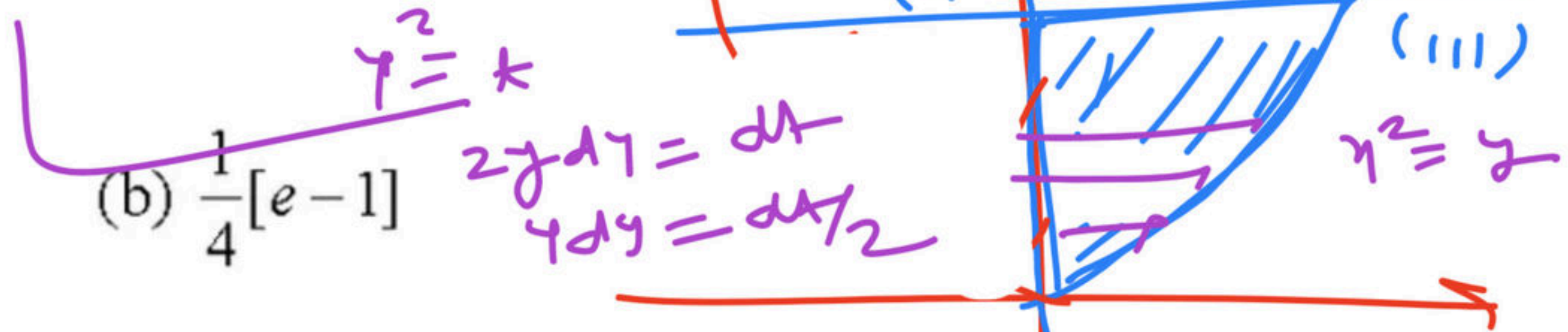
Q.1. The value of $\iint_R x e^{y^2} dx dy$, where R is the region bounded by the line $x = 0$, $y = 1$ and the parabola $y = x^2$. **IIT JAM-2006**

(a) $-\frac{1}{4}[e-1]$

(b) $\frac{1}{4}[e-1]$

(c) $\frac{1}{4}[e+1]$

(d) None



$$\int_{y=0}^1 \int_{x=0}^{\sqrt{y}} x e^{y^2} dx dy = \int_0^1 \left(\frac{x^2}{2} \right)_0^{\sqrt{y}} e^{y^2} dy = \frac{1}{2} \int_0^1 y e^{y^2} dy$$

$$= \frac{1}{2} \int_{t=0}^1 e^t \frac{dt}{2}$$

$$\frac{1}{4} (e^t)_0^1 = \frac{1}{4} (e-1)$$

Q.2. The value of $\iint xy(x+y) dx dy$ over the area between

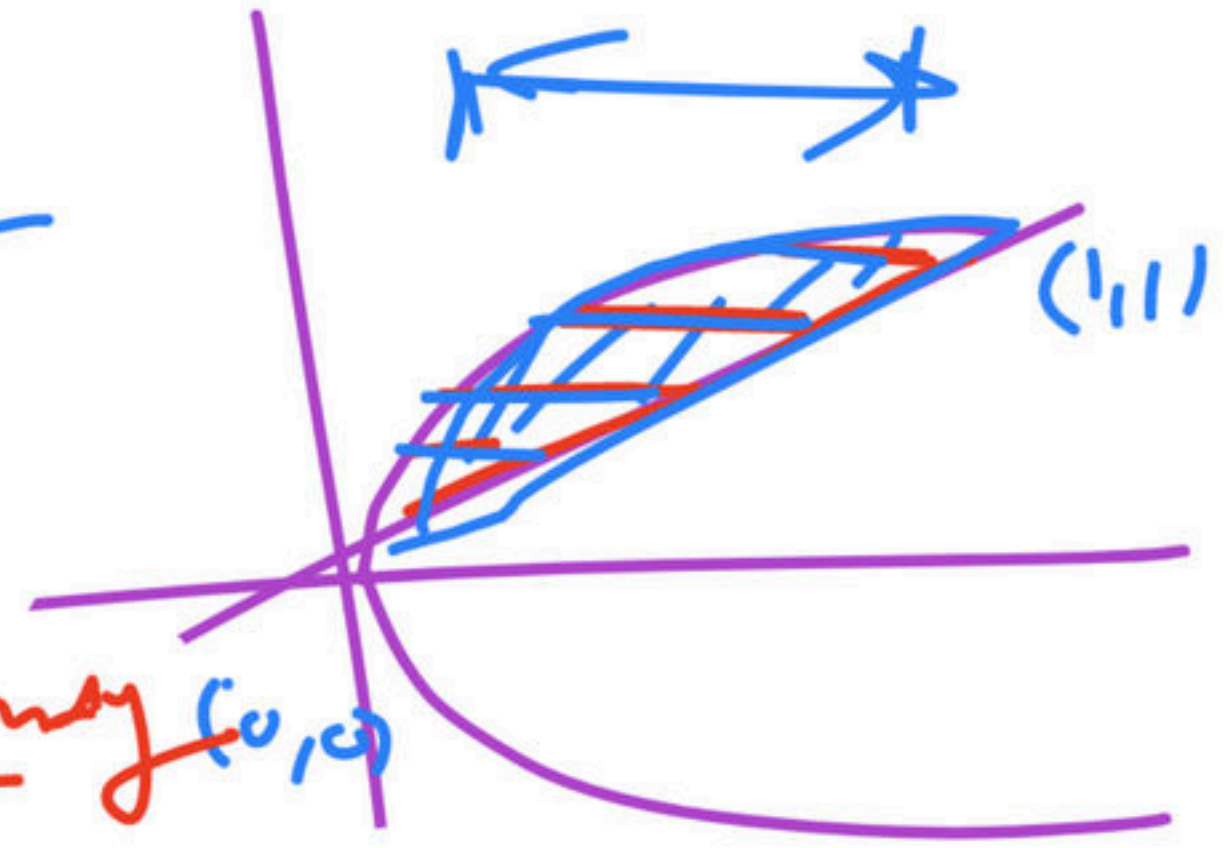
$y^2 = x$ and $y = x$

(a) $\frac{1}{56}$

(b) $\frac{3}{56}$

(c) $\frac{5}{56}$

(d) $\frac{3}{55}$



$$\int_0^1 \int_{y^2}^y xy(x+y) dx dy$$

$$\int_0^1 \int_{y^2}^y (x^2 y + xy^2) dx dy$$

$$\int_0^1 \left(\frac{x^3}{3} + \frac{x^2}{2} y^2 \right)_{y^2}^y dy$$

$$\int_0^1 \left(\left(\frac{y^4}{3} + \frac{y^4}{2} \right) - \left(\frac{y^7}{3} + \frac{y^6}{2} \right) \right) dy$$

$$\int_0^1 \left(\frac{5}{6} y^4 - \frac{y^7}{3} - \frac{y^6}{2} \right) dy$$

$$\left(\frac{y^5}{6} - \frac{y^8}{24} - \frac{y^7}{14} \right)_0^1$$

$$\frac{1}{6} - \frac{1}{24} - \frac{1}{14}$$

$$\frac{3}{24} - \frac{1}{14} = \frac{1}{8} - \frac{1}{14}$$

$$\frac{7-4}{56} = \frac{3}{56}$$

Q.3. The value of $\iint_R xy dx dy$, where R is the quadrant of the circle $x^2 + y^2 = a^2$

~~(a) $a^4/8$~~

(b) $a^2/8$

(c) $a/8$

(d) $3a/2$

$$\frac{1}{2} \left(\frac{a^4}{4} \right)$$

$$a^4/8$$

$$\int_0^a \int_0^{\sqrt{a^2-y^2}} xy dx dy$$

$y=0 \quad x=0$

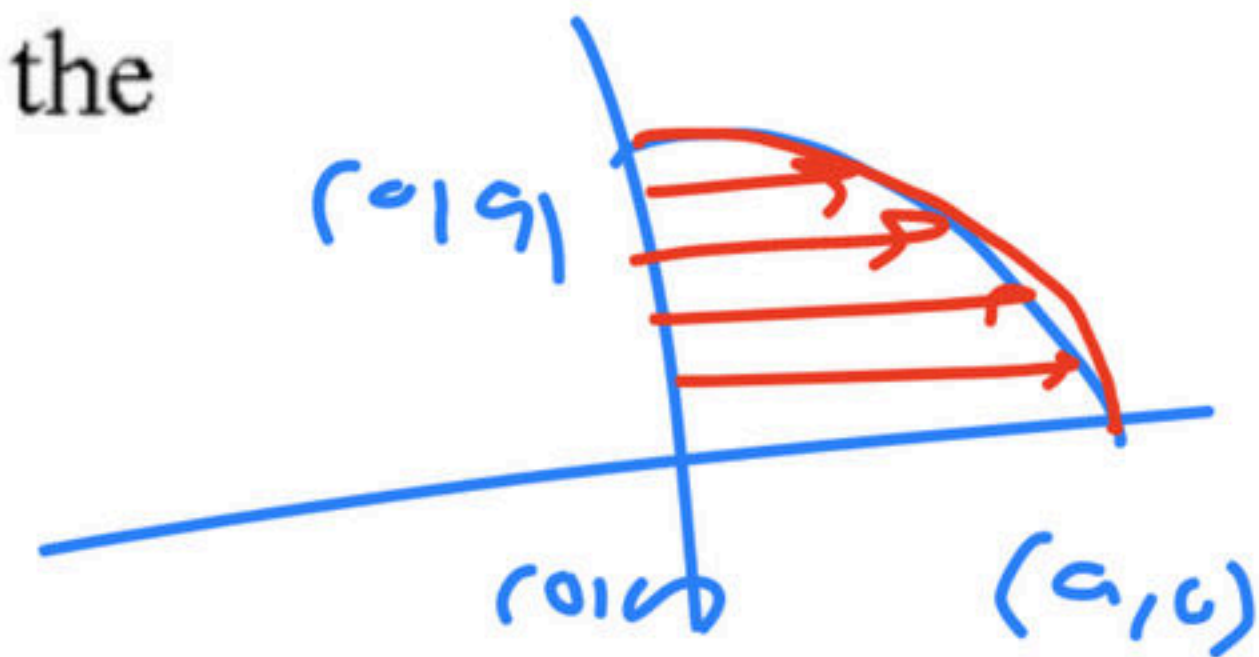
$$\int_0^a \left(\frac{x^2}{2} \right)_0^{\sqrt{a^2-y^2}} dy$$

$$\frac{1}{2} \int_0^a (a^2 - y^2) dy$$

$$\frac{1}{2} \int_0^a (a^2 y - y^3) dy$$

$$\frac{1}{2} \left(a^2 \frac{y^2}{2} - \frac{y^4}{4} \right)_0^a$$

$$\frac{1}{2} \left(\frac{a^4}{2} - \frac{a^4}{4} \right)$$



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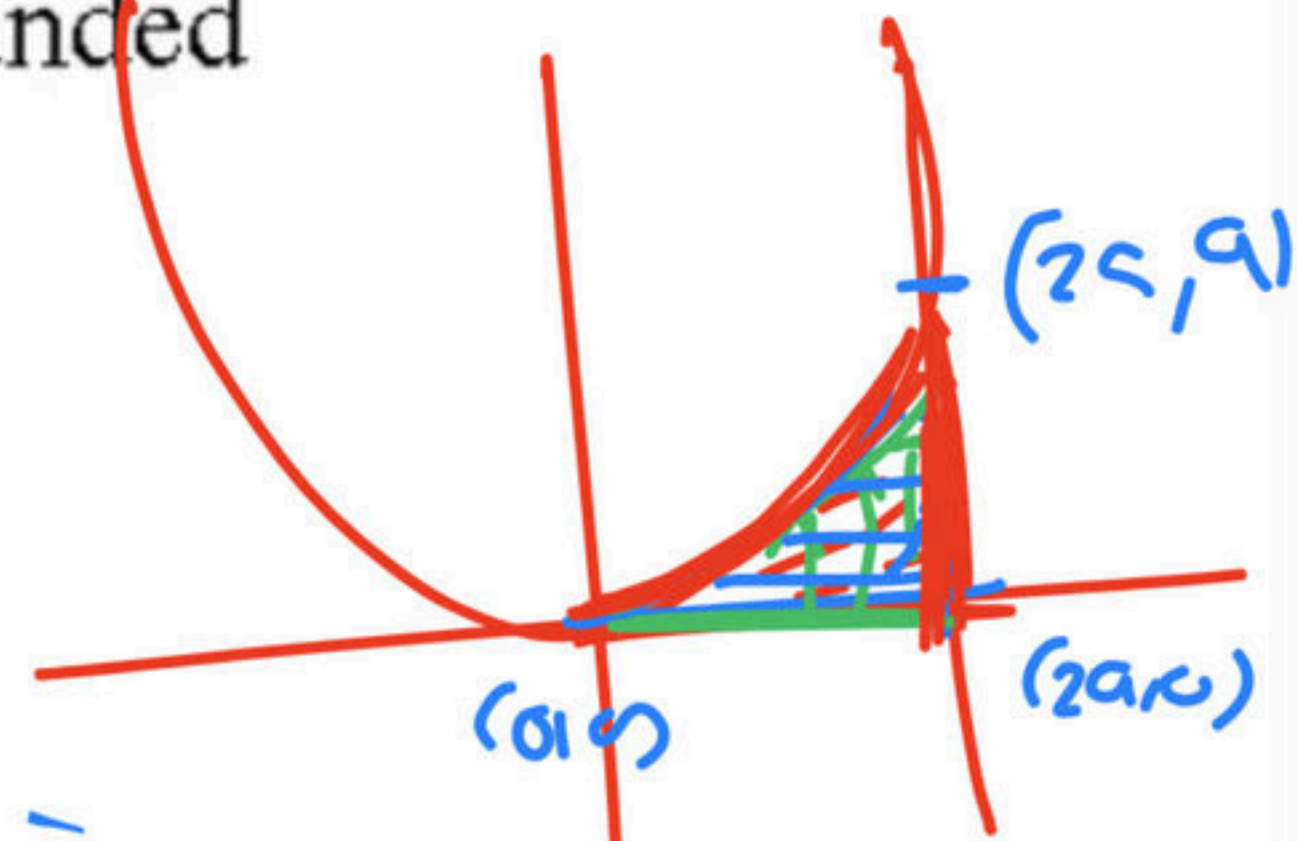
Q.4. The value of $\iint_A xy dy dx$ where A is domain bounded by x-axis ordinate $x = 2a$ and the curve $x^2 = 4ay$.

(a) $a/7$

(b) $a^4/3$

(c) $a^4/5$

(d) $a/5$



$$\begin{aligned} \int_0^a \int_{y=0}^{2\sqrt{ay}} xy \, dx \, dy &= \int_0^a y \left(\frac{x^2}{2} \right)_{x=0}^{x=2\sqrt{ay}} dy \\ &= \frac{1}{2} \int_0^a y (4ay - 0) dy = \frac{1}{2} \int_0^a (4a^2y - 4ay^2) dy \\ &= \frac{1}{2} \left(4a^2 \frac{y^2}{2} - \frac{4ay^3}{3} \right)_0^a = \frac{1}{2} \left(2a^4 - \frac{4a^4}{3} \right) \\ &= \frac{1}{2} \left(\frac{2a^4}{3} \right) = \frac{a^4}{3} \end{aligned}$$

Q.5 The value of integral $\int_0^3 \int_0^{\sqrt{3}x} \frac{dydx}{\sqrt{x^2 + y^2}}$.**JAM-2008**

- (a) $3\log(\sqrt{3} - 2)$ (b) $\log(\sqrt{3} + 2)$
(c) $3\log(\sqrt{3} + 2)$ (d) $-3\log(\sqrt{3} + 2)$

Q.6. The value of the integral $\iint_D \frac{\sin(2x)}{x} dx dy$ where D denotes the region bounded by the x - axis and the lines $y = x$ and $x = 1$. **IIT JAM 2007**

(a) $-\frac{\cos 2}{2} + \frac{1}{2}$

(b) $\frac{\cos 2}{2}$

(c) $\cos 2$

(d) $\sin 2$



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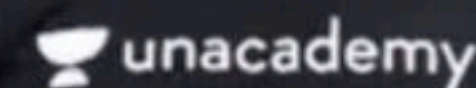
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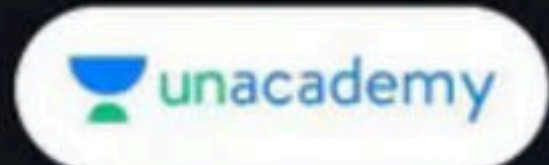
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