Basis and Dimension

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 11 • Sept 20, 2022

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Eigen value &eigen vector

Characteristic polynomial : Let A be square matrix of order n then $C_A(x) = \det(xI - A) = \det(A - xI)$ is a polynomial of degree n called the characteristic polynomial of A and the equation $C_A(x) = \det(A - xI) = 0$ is called characteristic equation.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$(A - \lambda I) = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$

$$= \lambda^2 - (a+d)\lambda + (ad - bc)$$

$$A = \begin{pmatrix} 1 & 4 \\ 7 & 3 \end{pmatrix}$$

$$(A - \lambda I) = \lambda^2 - 4\lambda + (3 - 28)$$

$$\lambda^2 - 4\lambda - 25$$

$$= 0$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \qquad \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix}$$

$+ \quad - \quad + \quad -$

$$= \lambda^3 - 5\lambda^2 + [3+3-5]\lambda - (-19)$$

$$= \lambda^3 - 5\lambda^2 + \lambda + 19$$

$$|A| = \begin{array}{c} 0 \\ \\ 3 \end{array} \begin{vmatrix} 1 & 2 & 0 \\ 3 & 1 & -1 \\ 2 & 0 & 3 \end{vmatrix} \begin{array}{c} 1 \\ \\ 2 \end{array}$$

$$= (0 + 3 - 4) - (18)$$

$$= -19$$

$$(A - \lambda I) = \begin{bmatrix} 1-\lambda & 2 & 0 \\ 3 & 1-\lambda & -1 \\ 2 & 0 & 3-\lambda \end{bmatrix}$$

$$= (1-\lambda)\left[(1-\lambda)(3-\lambda) - 0\right] - 2\left[3(3-\lambda) + 2\right]$$

$$= (1-\lambda)(\lambda^2 - 4\lambda + 3) - 2(11 - 3\lambda)$$

$$= \lambda^2 - 4\lambda + 3 - \lambda^3 + 4\lambda^2 - 3\lambda - 22 + 6\lambda$$

$$= -\lambda^3 + 5\lambda^2 - \lambda - 19$$

$$= -(\lambda^3 - 5\lambda^2 + \lambda + 19)$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$(0+0+0) - 2$

(a) 2 (b) 3

(c) 5 (d) 1

$\lambda = 1, 1, 2$ 4

$\lambda^3 - 2\lambda^2 + [1-2]\lambda - (-2)$

$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$

$\lambda^2(\lambda-2) - 1(\lambda-2) = 0$

$(\lambda-2)(\lambda^2-1)$

$(\lambda-2)(\lambda-1)(\lambda+1)$

$\lambda = 1, -1, 2$

$Tr(A) = \lambda_1 + \lambda_2 + \lambda_3$

$\det(A) = \lambda_1 \lambda_2 \lambda_3 = -2$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 2 & 3 \\ 1 & -\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$\lambda = 1, -1, 2$

$\lambda = 1$

$$(A - I)X = \begin{bmatrix} 0 & 2 & 3 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_3 = 2k$

$x_2 = -3k$

$x_1 = -5k$

$2x_2 + 3x_3 = 0$

$x_1 - x_2 + x_3 = 0$

$x_1 + 3k + 2k = 0$

$x_1 = -5k$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = k \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix}$$

$$(A + I)X = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$2x_1 + 2x_2 + 3x_3 = 0$

$x_1 + x_2 + x_3 = 0$

$2x_3 = 0$

$x_3 = 0$

$x_1 + x_2 = 0$

$x_2 = k$

$x_1 = -k$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = k \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Which of the following is

(a) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

(c) $\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 6 - 10 \\ -10 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ -10 \\ 0 \end{pmatrix}$$

$$= -2 \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$$

Q

$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 \\ a \end{bmatrix}$ & $\begin{bmatrix} 1 \\ b \end{bmatrix}$ $a+b = ?$ $\frac{1}{2}$

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

$a = \frac{1}{2}$

$b = 0$

$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}\begin{pmatrix} 1 \\ a \end{pmatrix} = 1\begin{pmatrix} 1 \\ a \end{pmatrix}$

$1 + 2a = 1$

$2a = 0$

$a = 0$

$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}\begin{pmatrix} 1 \\ b \end{pmatrix} = 2\begin{pmatrix} 1 \\ b \end{pmatrix}$

$1 + 2b = 2$

$2b = 1$

$b = \frac{1}{2}$

$$\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 2 & 7 & 5 \\ 0 & 0 & 7 & 8 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix} \qquad Ax = \lambda x$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -5-6+6 \\ -5+0+2 \\ 0+0+2 \end{pmatrix} = 1 \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix}1\\2\\3\end{pmatrix}$$

$$A = \begin{bmatrix}4 & 1 & 2\\ p & 2 & 1\\ 14 & -4 & 10\end{bmatrix}\begin{pmatrix}1\\2\\3\end{pmatrix} = \lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$$

(a) 12 (b) 13

(c) 17 (d) 19

$$\begin{pmatrix}4+2+6\\ p+4+3\\ 14-8+30\end{pmatrix} = \begin{pmatrix}12\\ p+7\\ 36\end{pmatrix} = 12\begin{pmatrix}1\\ \frac{p+7}{12}\\ 3\end{pmatrix}$$

$$\frac{p+7}{12} = 2$$

$$p+7 = 24$$

$$p = 24-7$$

$$= 17$$

$$\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix} \quad \begin{matrix} -3 \\ -3 \end{matrix}$$

(a) $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$

(c) $\begin{pmatrix} 2 \\ 7 \end{pmatrix}$ (d) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = \begin{pmatrix} 1+2+3+4+5 \\ 5+4+1+2+3 \\ \\ \end{pmatrix} = 15 \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

a -25 b 0

~~c 15~~   d ≤ 5

$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

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2 • Asked by Dr Anurag

Please help me with this doubt

19. Let A be an $n \times n$ matrix over $\mathbb{C}$ such that every nonzero vector of $\mathbb{C}^n$ is an eigenvector of A. Then

(a.) All eigenvalues of A are equal.

(b.) All eigenvalues of A are distinct.

(c.) $A = \lambda I$ for some $\lambda \in \mathbb{C}$, where I is the $n \times n$ identity matrix.

(d.) If χ_A and m_A denote the characteristic polynomial and the minimal polynomial respectively, then $\chi_A = m_A$.

λI

Eigen value and Eigen vector : Let A be any matrix of order n then roots of characteristic equation is called eigen value.

i.e. If A is matrix and $[A - \lambda I]X = 0$ then λ is eigen value and X is eigen vector corresponding to λ

Note : Eigen vector corresponding to distinct eigen value are LI

Result : If λ is eigen value of A then

(1) Eigen value of αA is $\alpha\lambda$

(2) Eigen value of A^n is λ^n.

(3) Sum of all eigen value = Trace (A)

(4) Product of all eigen value = det(A)

(5) Eigen value of A^{-1} is λ^{-1}.

(6) Eigen value of Adj(A) is $\frac{|A|}{\lambda}$

(7) If sum of each row in A is equal to k then k must be eigen value and it is largest eigen value.

Q.1. Let A be a 3×3 matrix with eigen value 1, -1, 0. Then the determinant of $I + A^{100}$ is

(a) 6 (b) 4

(c) 9 (d) 100

Q.2. Let $A=\begin{pmatrix}2 & -1 & 3\\ 2 & -1 & 3\\ 3 & 2 & -1\end{pmatrix}$. Then the largest eigenvalue of A is

(a) 1 (b) 2

(c) 3 (d) 4

Eigen value for different type of matrix

(1) Eigen values of symmetric matrix and hermitian matrix are real.

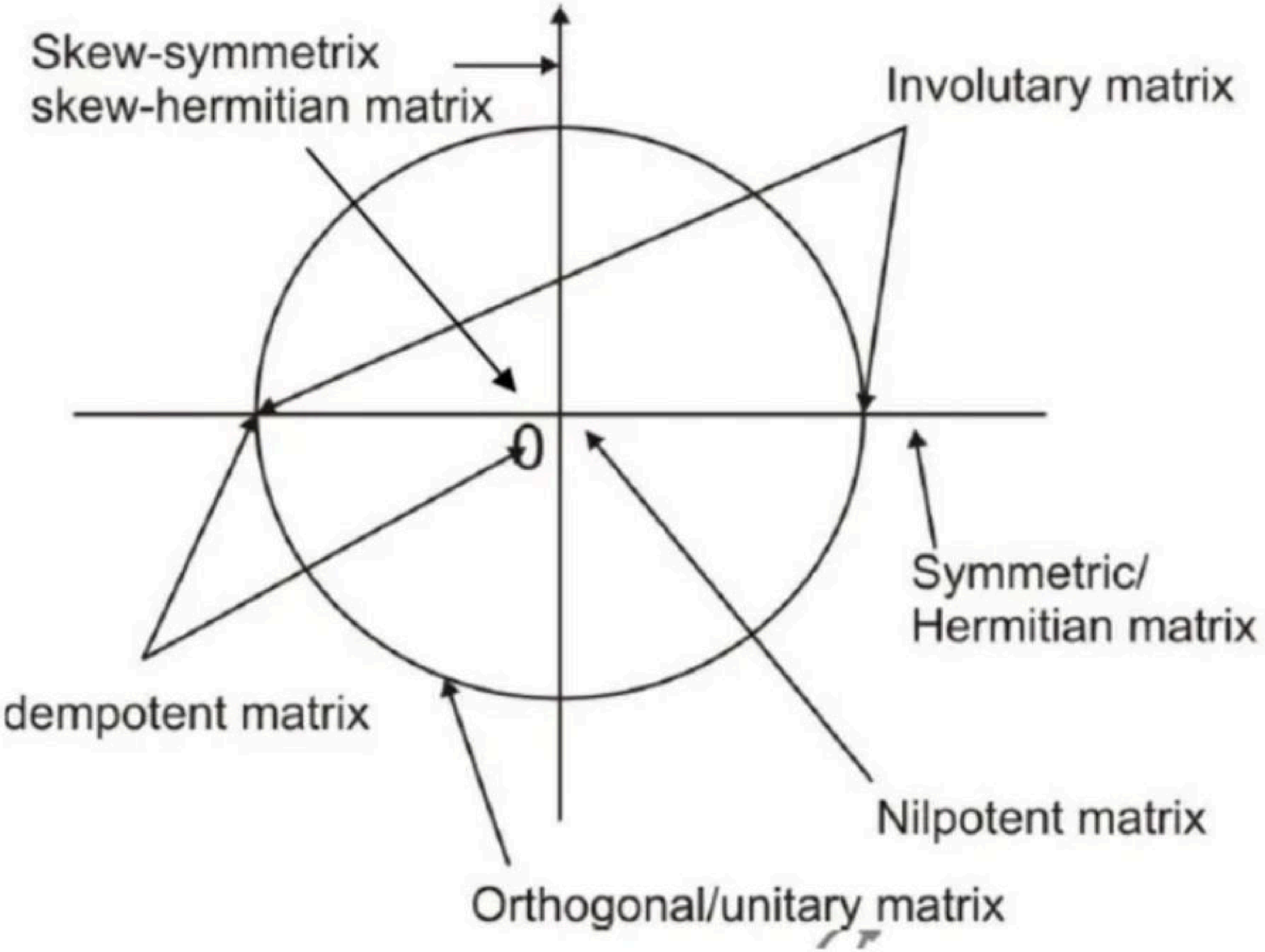
(2) Eigen value of skew-symmetric and skew-hermitian matrix are either zero or purely imaginary.

(3) Eigen values of involutory matrix are either 1 or -1 or both.

(4) Eigen values of idempotent matrix are either 0 or 1 or both.

(5) Eigen values of nilpotent matrix are 0.

(6) Eigen values of orthogonal matrix and unitary matrix are unit modulus.


Skew-symmetrix
skew-hermitian matrix
Involutary matrix
0
Symmetric/
Hermitian matrix
dempotent matrix
Nilpotent matrix
Orthogonal/unitary matrix

(7) Eigen value of permutation matrix.

Let $\sigma = c_1.c_2. \ldots .. c_k$ product of disjoint cycles such that $l(c_i) = r_i$ where $l(c_i)$ = length of c_i.

Then characteristic of A is $c(x) = \prod_{ri} (x^{r_i} - 1)$

i.e. $\sigma = (12)\,(3) \in S_3$

$c_1 = (12)$ and $l(c_1) = 2 = r_1$

$c_2 = (3)$ and $l(c_2) = 1 = r_2$ then $c(x) = (x^2 - 1)(x - 1)$

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Q.3. Which of the following properties are true?

(a) If λ is an eigen value of A then 2λ is an eigen value of A^{-1}.

(b) If λ is an eigen value of A then $1/\lambda$ is an eigen value of A^{-1}.

(c) If λ is an eigen value of an orthogonal matrix, then $1/\lambda$ is also its eigen value.

(d) All of the above.

Q.4. The square matrix A is said to be an idempotent if $A^2 = A$.

An idempotent matrix is non-singular iff

(a) All E.V. are real

(b) All E.V. are real non-negative

(c) All E.V. are either 0 or 1

(d) All E.V. are 1

Q.5 The trace of the matrix $A=\begin{bmatrix}3 & 2 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}^{15}$ is

(a) $1+3^{15}$ (b) $2+3^{15}$

(c) 3^{15} (d) 0

Q.6. Let A be 3×3 matrix with real entries such that 1, -1, 2 are its eigenvalues if $B = A^3 + 2A^2 + I$, then

(a) det (B) = 50 (b) det (B) = 136

(c) det (B) = 23 (d) det(B) = 17

Q.7.. Let A and B be n × n real matrices and let $C = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$

.Which of the following statements are true?

(a) If λ is an eigenvalue of A + B then λ is an eigen value of C

(b) If λ is an eigenvalue of A – B then λ is an eigen value of C

(c) If λ is an eigen value of A or B then λ is an eigen value of C

(d) All eigen values of C are real

Q.8. Which of the following eigen values of the matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(a) 1 (b) -1

(c) i (d) –i

Q.1. Let $M_n(R)$ be the set of $n \times n$ matrices with real entries.

Which of the following is true?

(a) Any matrix $A \in M_4(R)$ has a real eigen value.

(b) Any matrix $A \in M_5(R)$ has a real eigen value.

(c) Any matrix $A \in M_2(R)$ has a real eigen value

(d) None of these


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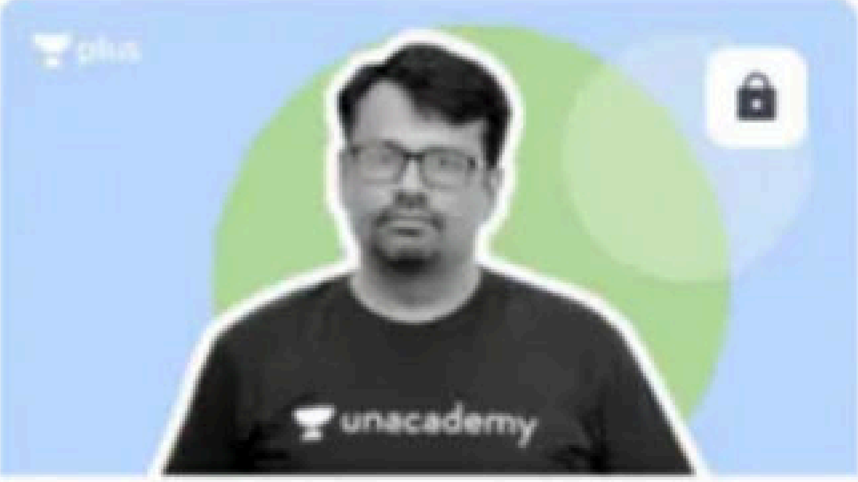

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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