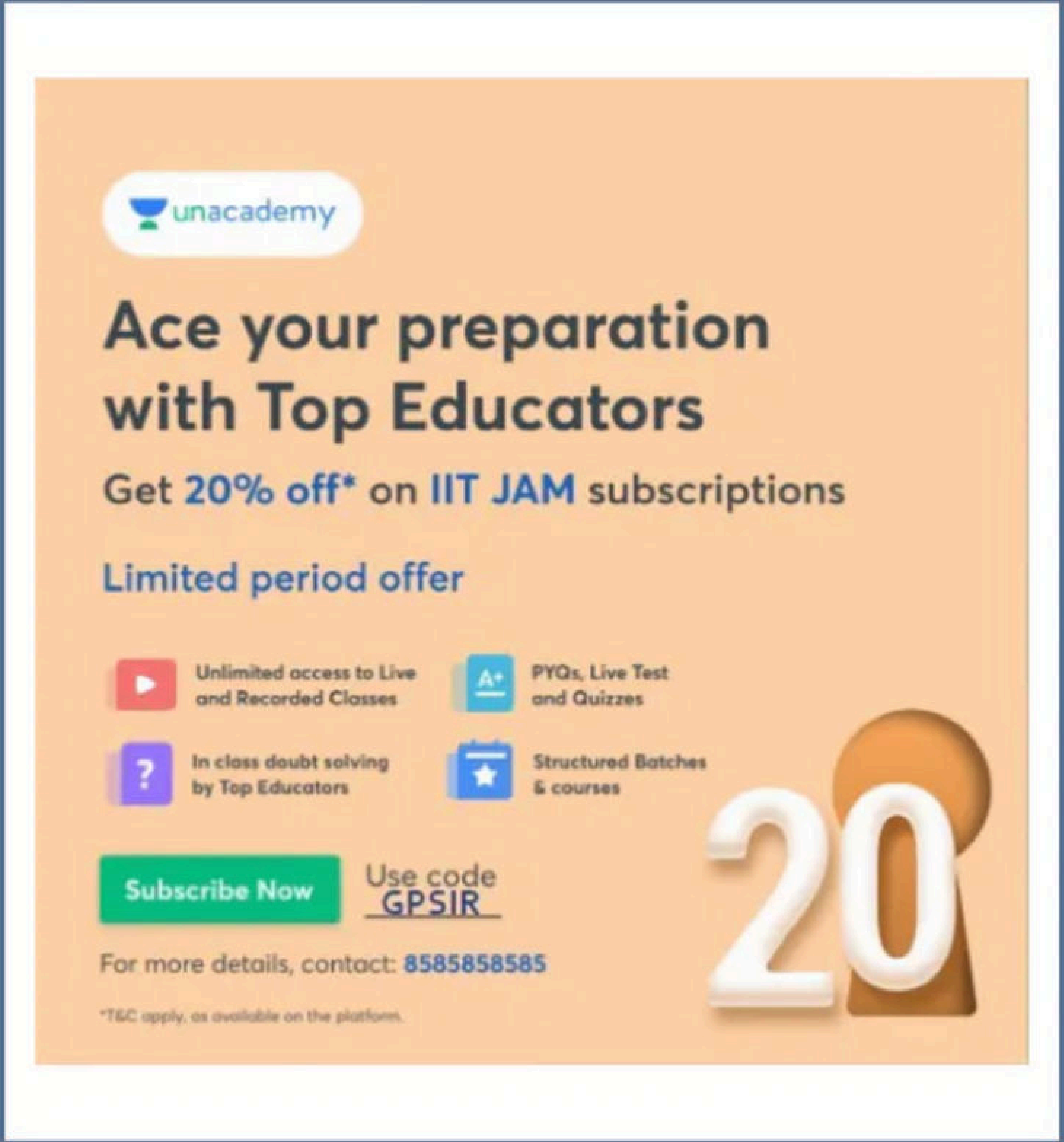
Linear Transformations - Part II

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 5 • Sept 15, 2022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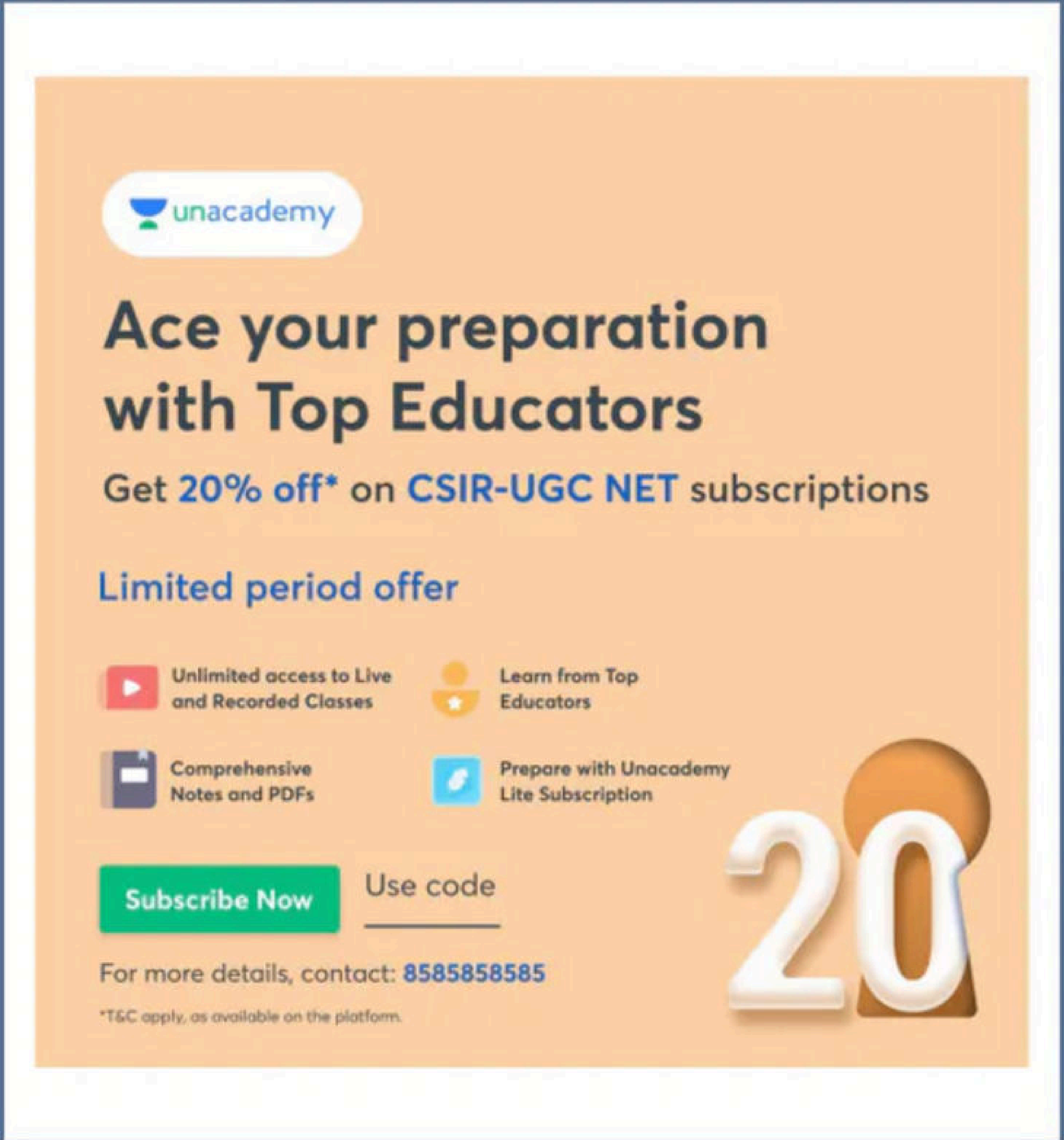
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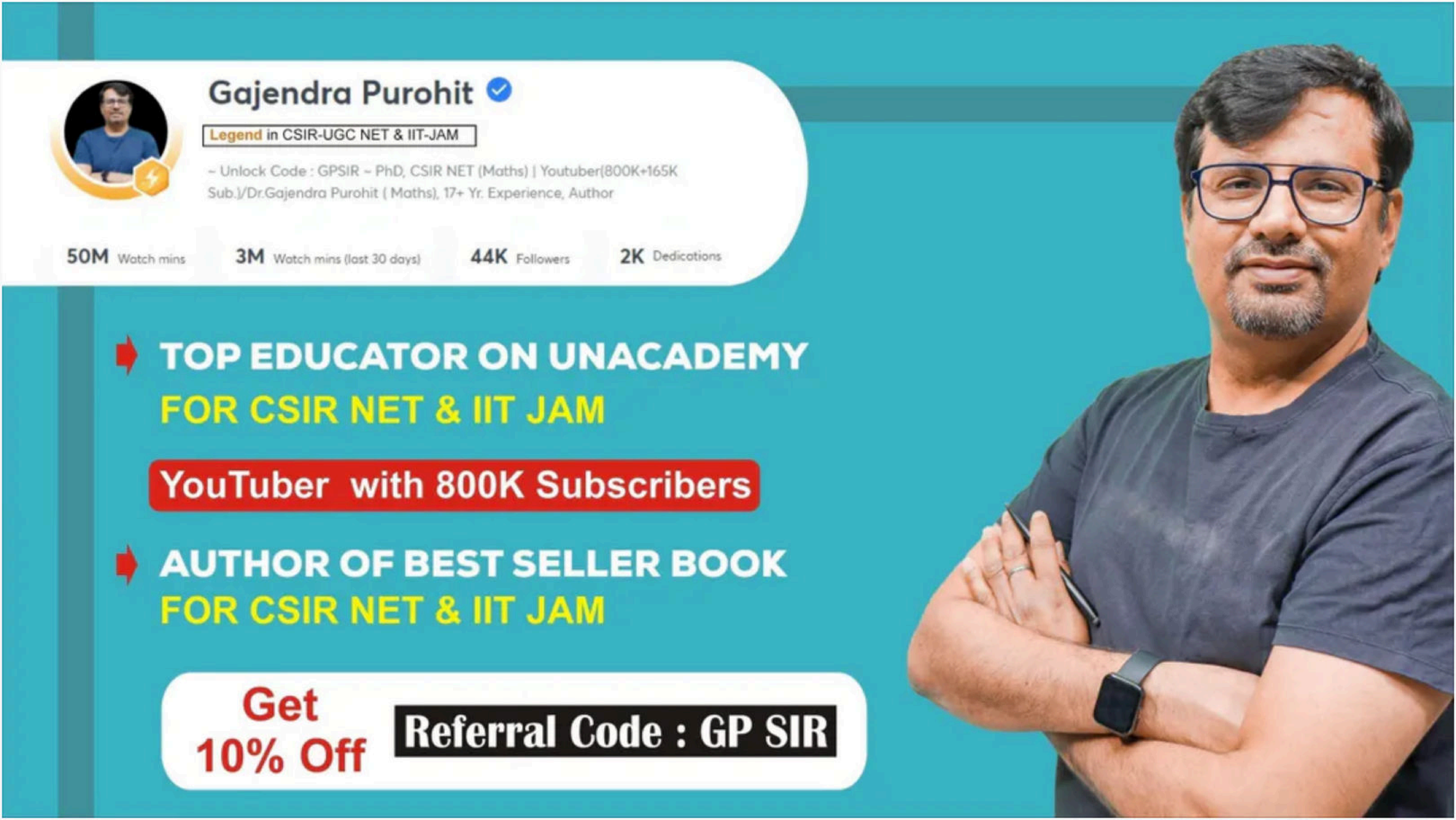
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Orthogonal matrix : A be a matrix A is called orthogonal matrix if $AA^T = A^TA = I$

$$A = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$\sin\theta\cos\theta - \cos\theta\sin\theta$

Note :

(1) $AA^T = I$

$|AA^T| = 1 \Rightarrow |A|\,|A^T| = 1$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

Determinant of an othrogonal matrix is ± 1 and $A^T = A^{-1}$.

(2) Sum of square of elements of each row or column are 1 and sum of the product of element of any row or column with corresponding elements of any other (column) is always zero.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad 2 \qquad (123) \quad (132)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad 3!$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Q.1. Number of orthogonal matrix of order n whose entries are 0 & 1 only

(a) n

(b) n!

(c) n – 1

(d) None of these

$n \times n$

$n!$

$3!$

Property :

(1) If A is orthogonal then kA is orthogonal if $k = \pm 1$

Example : If A is orthogonal then 3A is not orthogonal.

(2) If A & B are orthogonal then A + B cannot orthogonal but AB is always orthogonal.

(3) If A is orthogonal then A^n is orthogonal.

A

Q.2. If A is orthogonal matrix then which of the following are true?

(a) 2A is orthogonal

(b) A^2 is orthogonal

(c) –A is orthogonal

(d) None of these

(a) a, b

(b) b, c

(c) a, c

(d) d

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$-A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Q.3. The number of orthogonal matrix of order 5 whose entries are 0 & 1 only

(a) 5^2 (b) $5!$

(c) 120 (d) 0

Unitary matrix : A matrix A is said to be unitory if $AA^{\theta} = A^{\theta}A = I$

$$A = \frac{1}{2}\begin{pmatrix} 1-i & 1+i \\ 1+i & 1-i \end{pmatrix}$$

$$\bar{A} = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$$

$$(\bar{A})^T = \frac{1}{2}\begin{pmatrix} 1+i & 1-i \\ 1-i & 1+i \end{pmatrix}$$

$$A(\bar{A})^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Q4. The matrix $M = \begin{bmatrix} \cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha \end{bmatrix}$ is a unitary matrix when α is

(a) $(2n+1)\dfrac{\pi}{2}, n \in Z$ (b) $(3n+1)\dfrac{\pi}{3}, n \in Z$

(c) $(4n+1)\dfrac{\pi}{4}, n \in Z$ (d) $(5n+1)\dfrac{\pi}{5}, n \in Z$

$\bar{M} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -i\sin\alpha & -i\cos\alpha \end{bmatrix}$

$M^\dagger = (\bar{M})^T = \begin{pmatrix} \cos\alpha & -i\sin\alpha \\ \sin\alpha & -i\cos\alpha \end{pmatrix}$

$MM^\dagger = \begin{pmatrix} \cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha \end{pmatrix} \begin{pmatrix} \cos\alpha & -i\sin\alpha \\ \sin\alpha & -i\cos\alpha \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$n = 0$

$2\sin\alpha\cos\alpha = 0$

$\sin\alpha = 0$ $\cos\alpha = 0$

π

Q.5. If A and B are orthogonal matrix then which of the following is true?

(a) $A + B$ is orthogonal (b) AB is orthogonal

(c) $2A$ is orthogonal (d) B^2 is orthogonal

(a) a, b
(b) b, d
(c) c a
(d) d c

Q.6. Let $\underline{u}$ be a real $n \times 1$ vector satisfying $\underline{u}'\underline{u} = 1$ where $\underline{u}'$ is the transpose of $\underline{u}$. Define $A = I - 2\underline{u}\underline{u}'$ where I is the nth order identity matrix. Which of the following statements are true?

(a) A is singular (b) $A^2 = A$

(c) Trace (A) = n – 2 (d) $A^2 = I$

$n=1$

$Tr(UU^T) = Tr(U^TU)$

$Tr(A) = Tr(I - 2UU^T)$

$= Tr\, I - 2\, Tr(UU^T)$

$= Tr\, I - 2\, Tr(U^TU)$

$= n - 2 \times 1$

$= n - 2$

$A^2 = (I - 2UU^T)(I - 2UU^T)$

$= I - 2UU^T - 2UU^T + 4U\boxed{U^TU}U^T$

$= I - \cancel{4UU^T + 4UU^T}$

$= I$

$U^T_{1\times n} U_{n\times 1} = 1\times 1 = 1$

$(UU^T)_{n\times n} =$

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Permutation Matrix :

A matrix whose entries are either 0 or 1 and each row sum and each column sum is 1 then this matrix is called permutation matrix.

Properties :

(1) Number of permutation matrix of order n are n!.

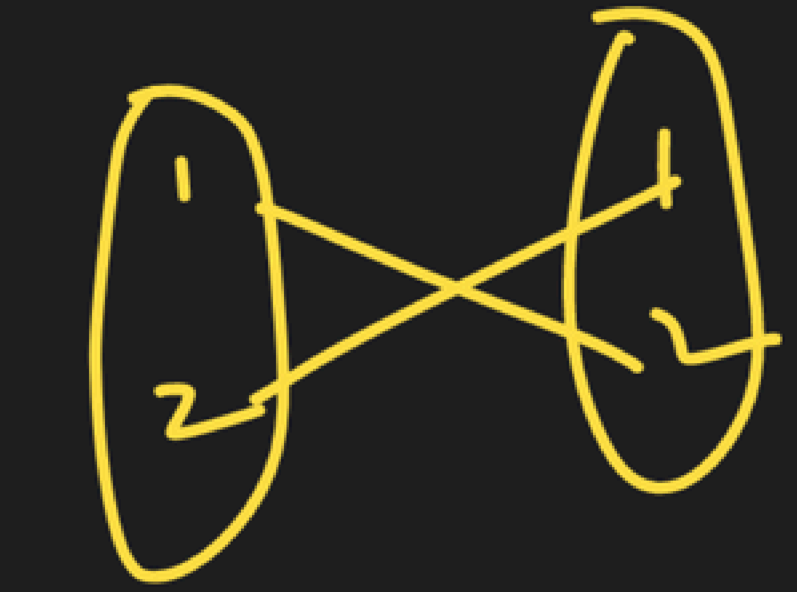
(2) Permutation $\sigma \in S_n$ corresponding to permutation matrix.

Let $I = [c_1\ c_2\ ....\ c_n]$ is a identity matrix where c_i are column and $A = [c_{\sigma(1)}, c_{\sigma(2)}\ ....\ c_{\sigma(n)}]$ is permutation matrix then permutation

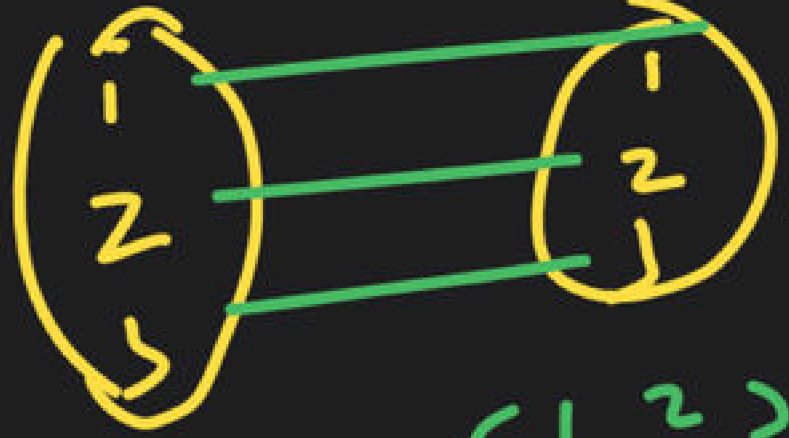
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & .... & \sigma(n) \end{pmatrix}.$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

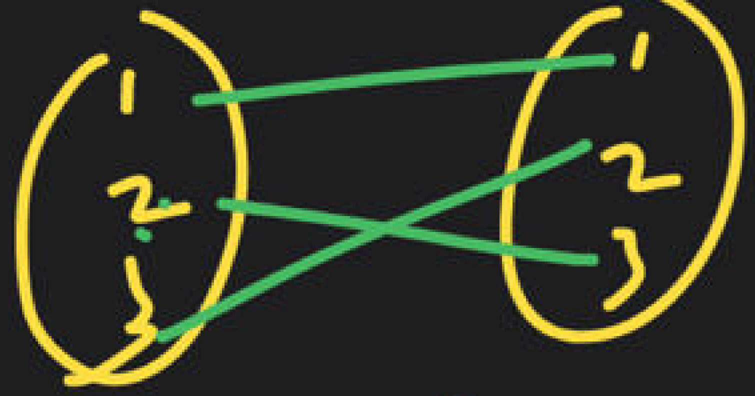


$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



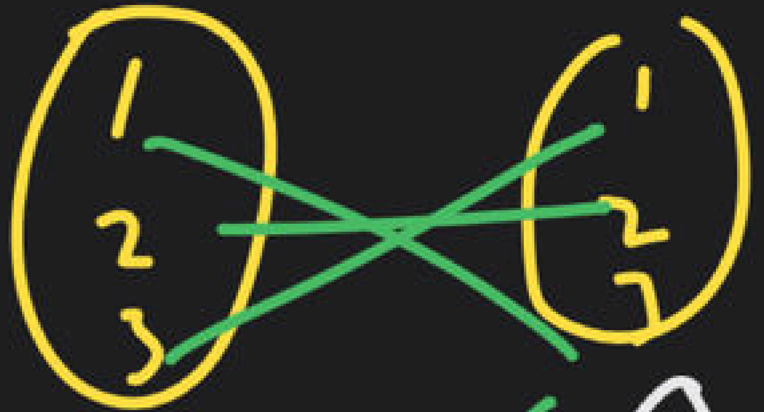
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} = \underline{e}$ | $A^1 = 1$



$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix} = \underline{(23)}$ | $A^2 = I$



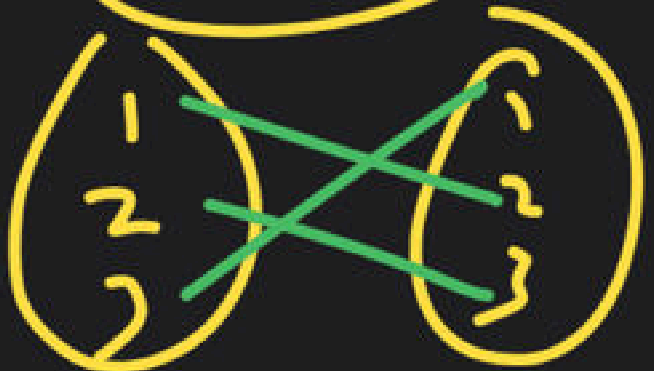
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = \underline{(13)}$ $A^2 = I$

$A^2 = I$

$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix} = \underline{(12)}$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

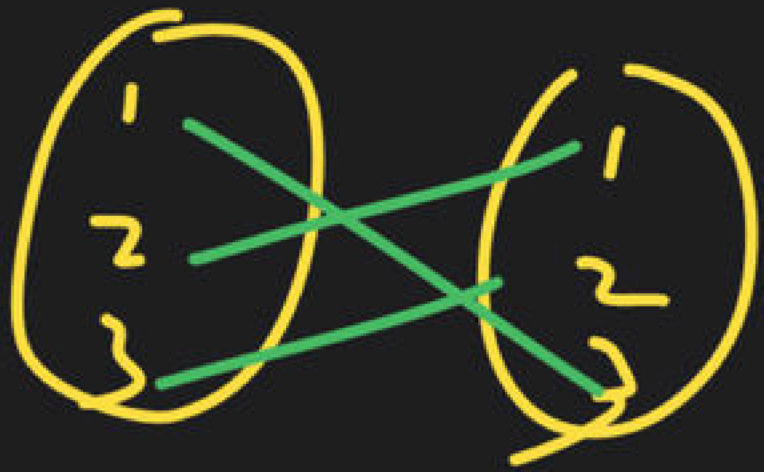


$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$\underline{(123)}$

$A^3 = I$



$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$\underline{(132)}$

$A^3 = I$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$$

$$= (12)(34)$$

$$\text{lcm}(2,2) = 2$$

$$A^2 = I$$

$$(1234)$$

$$A^4 = I$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$$

$$(1\,5)(2\,4)$$

$$\operatorname{lcm}(2,2) = 2$$

$$A^2 = I$$

(3) Permutation matrix corresponding to permutation $\sigma \in S_n$

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ \sigma(1) & \sigma(2) & \sigma(3) & \ldots & \sigma(n) \end{pmatrix}$$

i

permutation then $A = [c_{\sigma(1)} \quad c_{\sigma(2)} \quad \ldots\ldots \quad c_{\sigma(n)}]$ is a permutation matrix.

(4) Trace of permutation matrix is number of self inverse element in permutation .

(5) **Determinant of matrix** : Let A be a matrix corresponding to permutation $\sigma \in S_n$.

Then $\det(A) = |A| = (-1)^d$

Where d is number of transposition.

(6) Every permutation matrix is orthogonal/unitary matrix.

(7) If $\sigma \in S_n$ be a permutation and $O(\sigma) = k$, then $A^k = I$

Let $\sigma = (1\ 2\ 3\ 4) \in S_4$

Q.7. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$ and matrix A is denoted to the one whose i^{th} column is the $\sigma(i)^{th}$ column of the identity matrix I. Which of the following is true?

(a) $A^2 = A$

(b) $A^{-4} = A$

(c) $A^{-5} = A$

(d) $A = A^{-1}$

$(1\ 3\ 2)(4\ 5)$

$\text{lcm}(3, 2) = 6$

$A^6 = I$

$A = A^{-5}$

Q.8. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix} \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix}$ and matrix A is denoted to the one whose i^{th} column is the $\sigma(i)^{th}$ column of the identity matrix I. Which of the following is true?

(a) A is involutory matrix (b) $|A| = 1$

(c) $Tr(A) = 1$ (d) $A^2 = A^{-1}$

Q.9. Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{6\times 6}$. Then which of the following is true?

(a) A is involutory matrix (b) $|A| = 1$

(c) A is idempotent matrix (d) $|A| = 0$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 2 & 5 & 4 & 6 \end{pmatrix}$

$(2\ 3)(4\ 5)$

$\text{Lcm}(2, 2) = 2$

$A^2 = I$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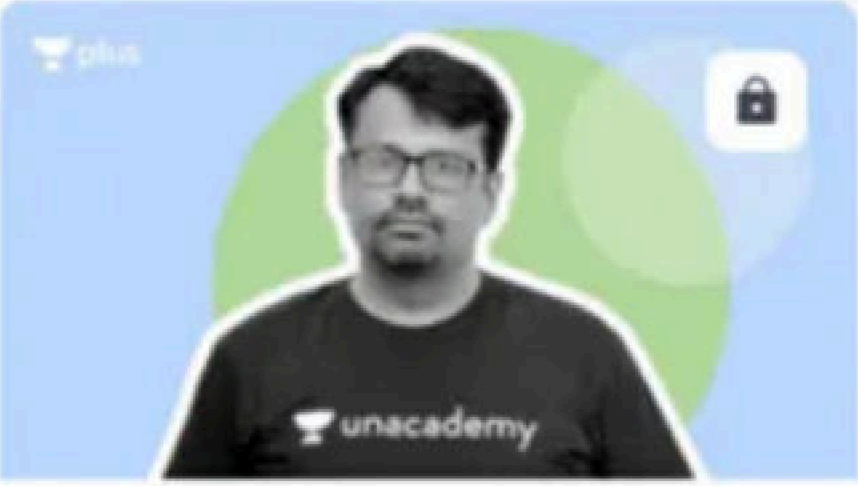

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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