Q.1 The differential equation of all circles which pass through the origin and whose centers are on the x-axis.

(a) $\frac{dy}{dx} = x^2 + y^2$ (b) $2xy\frac{dy}{dx} + x^2 - y^2 = 0$

(c) $\frac{dy}{dx} + e^x y = 0$ (d) None of these

Q.2. $y = Ae^{Bx+c}$ then order of the DE.

(a) 1 (b) 2

(c) 3 (d) 4

Q.3. The Order and degree of differential equation

$$\frac{d^3y}{dx^3}+4\sqrt{\left(\frac{dy}{dx}\right)^3+y^2}=0 \text{ are}$$

(a) 3 and 2 (b) 2 and 3

(c) 3 and 3 (d) 3 and 1

Q.4. If $y = \ln(\sin(x + a)) + b$, where a and b are constants, is the primitive, then the corresponding lowest order differential equation is

(a) $y'' = -(1 + (y')^2)$

(b) $y'' = y^2 - (y')^2$

(c) $y'' = 1 + (y')^2$

(d) $y'' = y' + y^2$

Q.5. Let $y(x) = x \sin x$ be one of the solution of an nth order linear differential equation with constant coefficients then the minimum value of n is

(a) 1 (b) 2

(c) 3 (d) 4

Q.6 Which one of the following differential equations represent all circles with radius a?

(a) $1+\left(\frac{dy}{dx}\right)^2+\sqrt{a^2-x^2}\,\frac{d^2y}{dx^2}=0$

(b) $1+\left(\frac{dy}{dx}\right)^2+\sqrt{a^2-y^2}\,\frac{d^2y}{dx^2}=0$

(c) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3+a^2\left(\frac{d^2y}{dx^2}\right)^2=0$

(d) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=a^2\left(\frac{d^2y}{dx^2}\right)^2$

Q1. The differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\sin y=0$ is

(a)linear (b) nonlinear

(c) Order 3 (d) of degree 2

Q2. The differential equation $\frac{d^2y}{dx^2}+(x^2+4x)\frac{dy}{dx}+y=x^8-8$ is

(a) Partial differential equation

(b) nonlinear

(c) second order linear

(d) linear

Q3. The differential equation representing the family of circles touching y-axis at the origin is **[IIT: JAM-2006]**

(a)Linear and of first order

(b) Linear and of second order

(c) Nonlinear and of first order

(d) Nonlinear and of second order

Q.4. Let y(x) be the solution of DE $y` = y^2 + t$, then

(a) y is thrice differential but 3^{rd} order does not exist.

(b) y is thrice differential but higher order derivative does not exist.

(c) y is infinite time differentiable.

(d) None of these

Singular Solution : A solution of DE which is neither is general solution nor is particular solution is called singular solution.

Q.6. The number of arbitrary constant in the complete primitive of the differential equation

$\phi(x, y, dy/dx, d^5y/dx^5)$

(a) 1 (b) 2

(c) 5 (d) 4

Q.7. If f(x) and g(x) are two solutions of the differential equation $\left(a\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+y=e^x\right)$, then f(x) – g(x) is the solution of :

(a) $a^2\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+y=e^x$

(b) $a^2\frac{d^2y}{dx^2}+y=0$

(c) $a^2\frac{d^2y}{dx^2}+x^2\frac{dy}{dx}+y=0$

(d) $a\frac{d^2y}{dx^2}+y=0$

Q.8. The solution of $\frac{dy}{dx}-y^2=1$ satisfying the condition

y(0) =1 is

(a) $y=e^{x^2}$

(b) $y=\sqrt{x}$

(c) $y=\cot\left(x+\frac{\pi}{4}\right)$

(d) $y=\tan\left(x+\frac{\pi}{4}\right)$

Q9. If y is the solution of the differential equation

$y^3 \dfrac{dy}{dx} + x^3 = 0, y(0) = 1,$ the value of y(– 1) is

(a)– 2 (b)– 1

(c) 0 (d) 1

Q. 9. Match each differential equation in Group I to its family solution curves from Group II

Group I	**Group II**
A. $\frac{dy}{dx}=\frac{y}{x}$	1. Circles
B. $\frac{dy}{dx}=-\frac{y}{x}$	2. Straight lines
C. $\frac{dy}{dx}=\frac{x}{y}$	3. Hyperbola
D. $\frac{dy}{dx}=-\frac{x}{y}$	

(a) A-2 ,B-3 ,C-3 ,D-1

(b) A-1 ,B-3 ,C-2 ,D-1

(c) A-2 ,B-1 ,C-3 ,D-3

(d) A-3 ,B-2 ,C-1 ,D-2

Q10. Let y(x) be the solution of the differential equation

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right)=x;\ y(1)=0,\ \left.\frac{dy}{dx}\right|_{x=1}=0.$$ Then y(2) is

[IIT-JAM: 2016]

(a) $\frac{3}{4}+\frac{1}{2}\ln 2$ (b) $\frac{3}{4}-\frac{1}{2}\ln 2$

(c) $\frac{3}{4}+\ln 2$ (d) $\frac{3}{4}-\ln 2$

Q.1. The DE $2x\frac{dy}{dx}-3(2y-1), y(0)=\frac{1}{2}$ has

(a) No solution

(b) Infinite many solution

(c) A unique solution

(d) More than one but only finitely many solutions

Q.2. The differential equation $y - x\frac{dy}{dx} = 3\left(1 + x^2\frac{dy}{dx}\right)$ s.t $y(0) = 3$ has

(a) Unique solution

(b) Infinite solution

(c) Two solution

(d) More than two but finite solution

Q.3. Consider the equation $y` = \frac{-t}{y}$ then which of the following is correct.

(below c_1 is a constant and $y` = \frac{dy}{dt}$)

(a) There exist a solution for $|t| \le |c_1|$

(b) Solution is not defined for $|t| \ge |c_1|$

(c) Both (a) and (b) are true

(d) Neither (a) nor (b) is true.

Q.4. y(t) be the solution of ODE $y`(t) = 1 - y^2(t)$,

$t \in R$. $y : R \to R$, $y(0) = 0$

(a) $y(t) = 1$ for some $t_1 \in R$

(b) y(t) is strictly increasing in R

(c) $y(t) > -1$ for all $t \in R$

(d) y(t) is increasing in (0, 1) and decreasing in $(1, \infty)$

Q.5. Let $(x+y+1)\dfrac{dy}{dx}=1$ s.t. y(1) = 1, then

(a) $x + y + 2 = 4\, e^{y-1}$

(b) $x + y - 3 = -\, e^{y-1}$

(c) $x + y + 1 = 3\, e^{y-1}$

(d) $x + y + 3 = 5\, e^{y-1}$

Q.6. The general solution of the differential equation $(x + y - 3)dx - (2x + 2y + 1)\,dy = 0$ is

(a) $\ln|3x + 3y - 2| + 3x + 6y = k$

(b) $\ln|3x + 3y - 2| - 3x - 6y = k$

(c) $7\ln|3x + 3y - 2| + 3x + 6y = k$

(d) $7\ln|3x + 3y - 2| - 3x + 6y = k$

Q. 9. Match each differential equation in Group I to its family solution curves from Group II

Group I	**Group II**
A. $\frac{dy}{dx}=\frac{y}{x}$	1. Circles
B. $\frac{dy}{dx}=-\frac{y}{x}$	2. Straight lines
C. $\frac{dy}{dx}=\frac{x}{y}$	3. Hyperbola
D. $\frac{dy}{dx}=-\frac{x}{y}$	

(a) A-2 ,B-3 ,C-3 ,D-1

(b) A-1 ,B-3 ,C-2 ,D-1

(c) A-2 ,B-1 ,C-3 ,D-3

(d) A-3 ,B-2 ,C-1 ,D-2

Q10. Let y(x) be the solution of the differential equation

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right)=x;\ y(1)=0,\ \left.\frac{dy}{dx}\right|_{x=1}=0.$$ Then y(2) is

[IIT-JAM: 2016]

(a) $\frac{3}{4}+\frac{1}{2}\ln 2$ (b) $\frac{3}{4}-\frac{1}{2}\ln 2$

(c) $\frac{3}{4}+\ln 2$ (d) $\frac{3}{4}-\ln 2$

Q.1. The differential equation $(x^2+y^2)\frac{dy}{dx}=xy$ s.t. y(0) = 1 has

(a) Unique solution (b) Infinite solution

(c) Two solution (d) None of these

Q.2. The general solution of the differential equation $(x^2 - y^2)dx + 2xydy = 0$ is

(a) $x^2 - y^2 = c$

(b) $x^2 + y^2 = c$

(c) $x^2 - y^2 = cx$

(d) None

Q.3. Consider the following difference equation

$$x(ydx+xdy)\cos\frac{y}{x}=y(xdy-ydx)\sin\frac{y}{x}$$

Which of the following is solution of the above equation?

(a) $\frac{x}{y}\cos\frac{y}{x}=c$

(b) $\frac{x}{y}\sin\frac{y}{x}=c$

(c) $xy\cos\frac{y}{x}=c$

(d) $xy\sin\frac{y}{x}=c$

Q.6. Solution of $\frac{dy}{dx}=\frac{(xy^2-x^2y)}{x^3}$ s.t. $y(1)=2$

(a) Unique solution

(b) No solution

(c) Infinite solution

(d) None of these

Q.1. Let $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$; x > 0, then the solution y(x) of DE is

(a) y(x) → 0 as x → ∞ (b) y(x) → ∞ as x → ∞

(c) y(x) → -∞ as x → ∞ (d) None of these

Q.2. $\frac{dy}{dx}+2xy=e^{-x^2}$, then the general solution of the DE is

(a) y(x) is bounded on R

(b) y(x) is bounded on R^+

(c) $y(x) \to 0$ as $x \to \infty$

(d) $y(x) \to 0$ as $x \to \infty$

Q.3. Solve $\frac{dy}{dx}+y=f(x)$ where $f(x)=\begin{cases}1 & 0\le x\le 1\\ 0 & x>1\end{cases}$ s.t.

y(0) = 0.

Q.4. $\frac{dy}{dx} - ay = e^{ax}$; y(0) = 0; a ∈ R then

(a) If a > 0, y(x) → ∞ as x → ∞

(b) If a < 0; y(x) is bounded on R^+.

(c) If a < 0; y(x) → -∞ as x → -∞

(d) None of these

Q.5 The equation of the curve passing through the point $\left(\frac{\pi}{2},1\right)$ and having slope $\frac{\sin(x)}{x^2}-\frac{2y}{x}$ at each point (x, y) with x $\neq$ 0 is

(a) $-x^2y+\cos(x)=\frac{-\pi^2}{4}$

(b) $x^2y+\cos(x)=\frac{\pi^2}{4}$

(c) $x^2y-\sin(x)-\frac{\pi^2}{4}-1$

(d) $x^2y+\sin(x)=\frac{\pi^2}{4}+1$

Q.6. Consider the ODE $ty' - 3y = t^2y^{1/2}$, $y(1) = 1$. Find the value of $y(2)$

(a) 14 (b) 16

(c) 0 (d) 8

Q.7. Solution of the differential equation

$xy' + \sin 2y = x^3 \sin^2 y$ is

(a) $\cot y = -x^3 + c\,x^2$ (b) $2\cot y = x^3 + 2c\,x^2$

(b) $\tan y = -x^3 + c\,x^2$ (d) $2\tan y = x^3 + 2c\,x^2$

Q.8. The general solution of differential equation $\frac{dy}{dx}=(1+y^2)\left(e^{-x^2}-2x\tan^{-1}y\right)$ is

(a) $e^{x^2}\tan^{-1}y=x+c$

(b) $e^{-x^2}\tan y=x+c$

(c) $e^{x}\tan y=x^2+c$

(d) $e^{-x}\tan^{-1}y=x^3+c$

Q.1. Consider the ODE $ty' - 3y = t^2y^{1/2}$, $y(1) = 1$. Find the value of $y(2)$

(a) 14 (b) 16

(c) 0 (d) 8

Q.2. Solution of the differential equation

$$xy' + \sin 2y = x^3 \sin^2 y \text{ is}$$

(a) $\cot y = -x^3 + c x^2$ (b) $2\cot y = x^3 + 2c x^2$

(b) $\tan y = -x^3 + c x^2$ (d) $2\tan y = x^3 + 2c x^2$

Q4. Consider the differential equation $\frac{dy}{dx} = ay - by^2$, where $a, b > 0$ and $y(0) = y_0$. As $x \to +\infty$ the solution $y(x)$ tends to

(a) 0

(b) a/b

(c) b/a

(d) y_0

Q6. Let y(x) be the solution of the differential equation $(xy + y + e^{-x})dx + (x + e^{-x})dy = 0$ satisfying $y(0) = 1$. Then y(-1) is equal to **IIT JAM- 2017**

(a) $\dfrac{e}{e-1}$ (b) 0

(c) $\dfrac{e}{1-e}$ (d) e

Q7 . Let y(x) is a integrating factor of the differential equation

$$\left(y+\frac{1}{3}y^3+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)dy=0$$

then y(x) is **IIT JAM -2018**

(a) Even function (b) Odd function

(c) Periodic function (d) Trignometric function

Q8. If $x^h y^k$ is an integrating factor of the differential equation $y(1+xy)dx + x(1-xy)dy = 0$, then the value of $h + k$ is

IIT JAM 2019

(a) Divisible by 8 (b) Divisible by 2

(c) Divisible by 5 (d) None of these

Q9. The non-zero value of n for which the differential equation $(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)dy = 0$, $x \neq 0$, becomes exact is **IIT JAM 2016**

(a) -3 (b) -2

(c) 2 (d) 3

Q.1 An integrating factor of the differential equation $\left(y+\frac{1}{3}y^2+\frac{1}{2}x^2\right)dx+\frac{1}{4}(x+xy^2)dy=0$ is

(a) x^2

(b) $3\log_e x$

(c) x^3

(d) $2\log_e x$

Q.2 If $x^h y^k$ is an integrating factor of the differential equation $y(1 + xy)dx + x(1 - xy)dy = 0$, then the ordered pair (h, k) is equal to

(a) (-2, -2) (b) (-2, -1)

(c) (-1, -2) (d) (-1, -1)

Q.3. The solution of differential equation

$(1 + y^2)\,dx + \left(x - e^{\tan^{-1} y}\right) dy = 0$ is

(a) $(x-2) = ke^{-\tan^{-1} y} + k$

(b) $xe^{\tan^{-1} y} - 2e^{2\tan^{-1} y} = k$

(c) $xe^{-\tan x} = \tan^{-1} y + k$

(d) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

Q.4 Let y(x) be the solution of the differential equation $(xy + y + e^{-x})dx + (x + e^{-x})\,dy = 0$ satisfying $y(0) = 1$. Then $y(-1)$ is equal to

(a) $\dfrac{e}{e-1}$ (b) $\dfrac{2e}{e-1}$

(c) $\dfrac{e}{1-e}$ (d) 0

Q5. Consider the differential equation $\left(x-\frac{1}{y}\right)dy + y^2dx = 0$; $y(1) = 1$ then as $y \to \infty$, x equals

(a) 0

(b) $\frac{1}{e}$

(c) $1+\frac{1}{e}$

(d) $1-\frac{1}{e}$

Q.1 Which one of the following curves is the orthogonal trajectory of straight line passing through fixed point (5, 6) ?

A) $(x-5)^2 = c(y-6)$

B) $(x-5) = c(y-6)$

C) $(x-5)^2 + (y-6)^2 = c$

D) None of these

Q.2. The value of $a \in R$ for which the curves $x^2 + ay^2 = 1$ and $y = x^2$ intersect orthogonally is

(a) -2

(b) $\frac{-1}{2}$

(c) $\frac{1}{2}$

(d) 2

Q.3. The integral curves of the first order linear differential equation $x\frac{dy}{dx}+ay=0$ will be orthogonal to the family of hyperbolas $x^2-by^2=c$ if and only if

A) $a+b=0$

B) $a\ b=1$

C) $a-\ b=0$

D) $a\ b=-1$

Q.4. The orthogonal trajectories of family of curves $3xy = x^3 - a^3$, a being paramter of family, is of the form $x^2 = y + f(y)$and satisfies $y(0) = 0$. then $f(\log 2)$is

(a) 0.37 (b) -0.37

(c) 0.40 (d) 0.5

Q.1. The orthogonal trajectory of the family of curves given by the equation $r = a(1 - \cos\theta)$.

(a) $r = b(1 + \sec\theta)$ (b) $r = b(1 + \cos\theta)$

(c) $r^2 = b(1 + \cos\theta)$ (d) $r = b(1 + \tan\theta)$

Q.1. Let y_1 & y_2 are two solution of $e^x y'' + \sin x\, y' + e^{\sin x} y = x\cos x$. Then which of the following are TRUE?

(a) $\frac{3}{2}y_1 - \frac{1}{2}y_2$

(b) $y_1 - 2y_2$

(c) $\frac{1}{2}y_1 + \frac{1}{2}y_2$

(d) $\frac{1}{2}y_1 - \frac{1}{2}y_2$

Q2. Consider two solution $x(t)=x_1(t)$ and $x(t)=x_2(t)$ of differential equation $\frac{d^2x(t)}{dt^2}+x(t)=0,\ t>0$ such that $x_1(0)=1, \left.\frac{dx_1(t)}{dt}\right|_{t=0}=0, x_2(0)=0, \left.\frac{dx_2(t)}{dt}\right|_{t=0}=1$ the Wronskian $W(t)=\begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_1(t)}{dt} \end{vmatrix}$ and at $t=\pi/2$ is

(a) 1 (b) – 1

(c) 0 (d) $\pi/2$

Q3. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of the differential equation $x^2 y''(x) - 2xy'(x) - 4y(x) = 0$ for $x \in [1,10]$. Considered the wronskian $W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x)$. If W(1) = 1, then W(3) – W(2) equals

(a) 1 (b) 2

(c) 3 (d) 5

Q4. Let $y_1(x)$ and $y_2(x)$ be the linearly independent solutions of $xy''+2y'+xe^x y=0$ If $W(x)=y_1(x)y_2'(x)-y_2(x)y_1'(x)$ with W (1) = 2 find W (5)

(a) $\frac{2}{25}$

(b) $\frac{1}{25}$

(c) $\frac{2}{5}$

(d) None of the above

Q.5. Consider the ODE

$$u''(t)+P(t)u'(t)+Q(t)u(t)=R(t), t\in[0,1]$$

There exist continuous function P, Q and R defined on $[0,1]$ and two solutions u_1 and u_2 of the ODE such that the Wronskian W of u_1 and u_2 is

(a) $W(t)=2t-1,\ 0\le t\le 1$

(b) $W(t)=\sin 2\pi t,\ 0\le t\le 1$

(c) $W(t)=\cos 2\pi t,\ 0\le t\le 1$

(d) $W(t)=1,\ 0\le t\le 1$

Q2. Consider two solution $x(t)=x_1(t)$ and $x(t)=x_2(t)$ of differential equation $\frac{d^2x(t)}{dt^2}+x(t)=0,\ t>0$ such that

$x_1(0)=1, \left.\frac{dx_1(t)}{dt}\right|_{t=0}=0, x_2(0)=0, \left.\frac{dx_2(t)}{dt}\right|_{t=0}=1$ the

Wronskian $W(t)=\begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_1(t)}{dt} \end{vmatrix}$ and at $t=\pi/2$ is

(a) 1 (b) – 1
(c) 0 (d) $\pi/2$

Q3. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of the differential equation $x^2y''(x) - 2xy'(x) - 4y(x) = 0$ for $x \in [1,10]$. Considered the wronskian $W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x)$. If $W(1) = 1$, then $W(3) - W(2)$ equals

(a) 1 (b) 2

(c) 3 (d) 5

Q4. Let $y_1(x)$ and $y_2(x)$ be the linearly independent solutions of $xy'' + 2y' + xe^x y = 0$ If $W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x)$ with W (1) = 2 find W (5)

(a) $\frac{2}{25}$

(b) $\frac{1}{25}$

(c) $\frac{2}{5}$

(d) None of the above

Q.5. Consider the ODE

$$u''(t)+P(t)u'(t)+Q(t)u(t)=R(t), t\in[0,1]$$

There exist continuous function P, Q and R defined on $[0,1]$ and two solutions u_1 and u_2 of the ODE such that the Wronskian W of u_1 and u_2 is

(a) $W(t)=2t-1,\ 0\le t\le 1$

(b) $W(t)=\sin 2\pi t,\ 0\le t\le 1$

(c) $W(t)=\cos 2\pi t,\ 0\le t\le 1$

(d) $W(t)=1,\ 0\le t\le 1$

Q.6. Consider the ordinary DE $y'' + P(x)\, y' + Q(x)\, y = 0$, Where P and Q are smooth functions. Let y_1 and y_2 be any two solution of the ODE. Let w(x) be the Wronskian. Then which of the following is always true.

(a) If y_1 & y_2 are LD then $\exists\, x_1, x_2$ s.t. $w(x_1) = 0$ and $w(x_2) \neq 0$

(b) If y_1 & y_2 are LI then $w(x) = 0\ \forall\ x$

(c) If y_1 & y_2 are LD then $w(x) \neq 0\ \forall\ x$

(d) If y_1 & y_2 are LI then $w(x) \neq 0\ \forall\ x$

Q.7. Let P,Q be continuous real valued functions defined on $[-1,1]$ and $u_i:[-1,1]\to R, i=1,2$ be solutions of the ODE: $\frac{d^2u}{dx^2}+P(x)\frac{du}{dx}+Q(x)d=0, x\in[-1,1]$ satisfying $u_1\geq 0, u_2\leq 0$ and $u_1(0)=u_2(0)=0$. Let w denote the Wronskian of u_1 and $u_{2,}$ then

(a) u_1 and $u_{2,}$ are linearly independent

(b) u_1 and $u_{2,}$ are linearly dependent

(c) $w(x)=0$ for all $x\in[-1,1]$

(d) $w(x)\neq 0$ for some $x\in[-1,1]$

Q.8. Let $Y_1(x)$ and $Y_2(x)$ defined on $[0,1]$ be twice continuously differentiable functions satisfying $Y''(x)+Y'(x)+Y(x)=0$. Let $W(x)$ be the Wronskian of Y_1 and Y_2 and satisfy $W\left(\frac{1}{2}\right)=0$. Then

(a) $W(x)=0$ for $x\in[0,1]$

(b) $W(x)\neq 0$ for $x\in[0,1/2)\cup(1/2,1]$

(c) $W(x)>0$ for $x\in(1/2,1]$

(d) $W(x)<0$ for $x\in[0,1/2)$

Q.9. Let $y_1(x)$ and $y_2(x)$ be two solutions of $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\sec x.y=0$ with Wronskian W(x).If $y_1(0)=1,\left(\frac{dy_1}{dx}\right)_{x=0}=0$ and $W\left(\frac{1}{2}\right)=\frac{1}{3}$, then $\left(\frac{dy_2}{dx}\right)_{x=0}$ equals

(a) 1/4 (b) 1

(c) 3/4 (d) 4/3

Q.10. Let $y=\phi(x)$ and $y=\psi(x)$ be solutions of $y''-2xy'+(\sin x^2)y=0$ such that $\phi(0)=1, \phi'(0)=1$ and $\psi(0)=1, \psi'(0)=2$. Then the value of the Wronskian $W(\phi,\psi)$ at $x=1$ is

(a) 0 (b) 1

(c) e (d) e^2

Q.11. Consider the ordinary DE $y'' + P(x)\, y' + Q(x)\, y = 0$

Where P and Q are smooth function. Let y1 and y2 be any two solution of the ODE. Let w(x) be the Wronskian. Then which of the following is always true.

(a) If y_1& y_2 are LD then $\exists\ x_1, x_2$ s.t. $w(x_1)=0$ & $w(x_2) \neq 0$

(b) If y_1& y_2 are LI then $w(x) = 0\ \forall\ x$

(c) If y_1& y_2 are LD then $w(x) \neq 0\ \forall\ x$

(d) If y_1& y_2 are LI then $w(x) \neq 0\ \forall\ x$

Q.12 The wronskian of two solutions of the differential equation $t^2y'' - t(t+2)y' + (t+2)y = 0$ satisfies $W(1) = 1$ is

(a) t^2e^t (b) t^2e^{t-1}

(c) $t\,e^t$ (d) $t\,e^{t-1}$

Q.1. Let y(x) be a solution of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0$ .s.t. $\lim_{x\to\infty} e^{-x} y(x)$ is finitely exist. Then y(log 2) is

(a) Constant (b) in term of x

(c) in term of e^x (d) None of these

Q.2. If y(x) is the solution of the initial value problem $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0, y(0)=1, \frac{dy}{dx}(0)=-2$, then y(ln 2) is

(a) ln2 (b) $(1-\ln 2)\frac{1}{2}$

(c) integer number (d) 0

Q.3. If $y(x) = \lambda e^{2x} + e^{\beta x}$, $\beta \neq 2$, is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ satisfying $\frac{dy}{dx}(0) = 5$, then $y(0)$ is equal to

(a) 3 (b) 4

(c) 5 (d) 6

Q.4. The differential equation whose linearly independent solutions are cos 2x, sin 2x and e^x, is

(a) $(D^3 + D^2 + 4D)y = 0$ (b) $(D^3 - D^2 + 4D - 4)y = 0$

(c) $(D^3 + D^2 - 4D - 4)y = 0$ (d) $(D^3 - D^2 - 4D + 4)y = 0$

Q.5. The number of arbitrary constants in the complete primitive of differential equation $\frac{d^5y}{dx^5}+2\frac{d^4y}{dx^4}=0$ is/are not

(a) 5 (b) 4

(c) 1 (d) 6

Q.6 Let $P : R \to R$ be a continuous function such that $P(x) > 0$ for all $x \in R$. Let y be a twice differentiable function on R satisfying $y``(x) + P(x)y`(x) - y(x) = 0$ for all $x \in R$. Suppose that there exist two real numbers a, b $(a < b)$ such that $y(a) = y(b) = 0$. Then

(a) $y(x) > 0$ for all $x \in (a, b)$

(b) $y(x) < 0$ for all $x \in (a, b)$

(c) $y(x)$ changes sign on (a, b)

(d) $y(x) = 0$ for all $x \in [a, b]$

Q.1. The solution of Differential following differential equation $y''+4y'+4y=x^2, y(0)=1, y(1)=1$ is

(a) y(x) = 1

(b) y(x) = 0

(c) $y(x) = \left(\frac{5}{8}+\frac{7}{8}e^2x-\frac{5}{8}x\right)e^{-2x}+\frac{1}{4}\left(x^2-2x+\frac{3}{2}\right)$

(d) $y(x) = 2\cos 4x+5\sin 4x$

Q2. The particular integral of the differential equation $y'' + y' + 3y = 5\cos(2x+3)$ is

(a) $2\cos(2x+3) - \sin(2x+3)$

(b) $2\sin(2x+3) + \cos(2x+3)$

(c) $\sin(2x+3) - 2\cos(2x+3)$

(d) $2\sin(2x+3) - \cos(2x+3)$

Q.3. The solution of differential equation $y'' - y' - 2y = 3e^{2x}$

where $y(0) = 0$ and $y'(0) = -2$ is

(a) $y = e^{-x} - e^{2x} + xe^{2x}$

(b) $y = e^{x} - e^{-2x} - xe^{2x}$

(c) $y = e^{-x} + e^{2x} + xe^{2x}$

(d) $y = e^{x} - e^{-2x} + xe^{2x}$

Q4. The solution of the differential equation

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 3e^{2x}$ wherey(0) = 0 and

$y'(0)$ = -2 is

(a) $y = e^{-x} - e^{2x} + xe^{2x}$

(b) $y = e^{-x} - e^{-2x} - xe^{2x}$

(c) $y = e^{-x} + e^{2x} + xe^{2x}$

(d) $y = e^{-x} - e^{-2x} + xe^{2x}$

Q5. Consider the following second order differential equation

$$y'' - 4y' + 3y = 2t - 3t^2$$

The particular solution of the differential equation is

(a) $-2 - 2t - t^2$ (b) $-2t - t^2$

(c) $2t - t^2$ (d) $-2 - 2t - 3t^2$

Q6. The solution of the differential equation for $y(t): \frac{d^2y}{dt^2} - y = 2cosh(t)$, subject to the initial conditions: $y(0) = 0\ and\ \left.\frac{dy}{dt}\right|_{t=0} = 0$ is:

(a) $\frac{1}{2}cosh(t) + t\,sinh(t)$

(b) $-sinh(t) + t\,cosh(t)$

(c) $t\,cosh(t)$ (d) $t\,sinh(t)$

Q.7. The homogeneous part of the differential equation

$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = r$ has real distinct real roots if

(a) $p^2 - 4q > 0$

(b) $p^2 - 4q < 0$

(c) $p^2 - 4q = 0$

(d) $p^2 - 4q = r$

Q1 A particular solution of the differential equation

$y''' - 3y'' + 3y' - y = e^x \cos 2x$ is

(a) $-\frac{1}{8} e^x \sin 2x$

(b) $\frac{1}{8} e^x \sin 2x$

(c) $\frac{1}{8} e^x \cos 2x$

(d) $e^x \sin 2x$

Q2. A particular integral of the differential equation

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^{2x}\sin x \text{ is}$$

(a) $\frac{e^{2x}}{10}(3\cos x-2\sin x)$ (b) $-\frac{e^{2x}}{10}(3\cos x-2\sin x)$

(c) $-\frac{e^{2x}}{5}(2\cos x+\sin x)$ (d) $\frac{e^{2x}}{5}(2\cos x-\sin x)$

Q3. The general solution of the differential equation

$$y''(x)-4y'(x)+8y(x)=10e^{x}\cos x$$

(a) $e^{2x}(k_1\cos 2x+k_2\sin 2x)+e^{x}(2\cos x+\sin x)$

(b) $e^{2x}(k_1\cos 2x+k_2\sin 2x)+e^{x}(2\cos x-\sin x)$

(c) $e^{-2x}(k_1\cos 2x+k_2\sin 2x)-e^{x}(2\cos x-\sin x)$

(d) $e^{-2x}(k_1\cos 2x+k_2\sin 2x)+e^{x}(2\cos x+\sin x)$

Q.4. Solve $(D^2 - 2D + 1)y = \sin x + x^2e^x$.

(a) $(c_1 + c_2x)e^x + \left(\frac{1}{2}\right)\cos x + \left(\frac{1}{12}\right)x^4e^x$

(b) $(c_1 + c_2x)e^{-x} + \left(\frac{1}{2}\right)\cos x - \left(\frac{1}{12}\right)x^4e^x$

(c) $(c_1 + c_2x)e^{-x} + \left(\frac{1}{2}\right)\cos x + \left(\frac{1}{12}\right)x^4e^x$

(d) None of these

Q.5. Let p(x) be the particular integral of

$(D^2 + 1)y = e^x \cos x + \sin 3x$ then p(0)

(a) $-\dfrac{119}{730}$

(b) $\dfrac{119}{730}$

(c) $\dfrac{19}{730}$

(d) None of these

Q.6. Let p(x) be the particular solution integral of the equation

$(D^2 + 4)y = x\sin x$, then $p\left(\frac{\pi}{2}\right)$.

(a) $\pi/3$ (b) $-\pi/6$

(c) $\pi/2$ (d) $\pi/6$

Q7. If y(t) is a solution of the differential equation $y'' + 4y = e^{2t}$ then $\lim_{t\to\infty} e^{-t} y(t)$ is equal to

(a) 2/3
(b) 2/5
(c) 2/7
(d) 2/9

Q1. A particular integral of the differential equation $y'' + 3y' + 2y = e^{e^x}$ is

(a) $e^{e^x} e^{-x}$

(b) $e^{e^x} e^{-2x}$

(c) $e^{e^x} e^{2x}$

(d) $e^{e^x} e^{x}$

Q2. Solve $\left(D^2-3D+2\right)y=\sin\left(e^{-x}\right)$

(a) $y=c_1e^x+c_2e^{2x}-e^x\sin\left(e^{-x}\right)$

(b) $y=c_1e^x+c_2e^{2x}-e^{-2x}\sin\left(e^{-x}\right)$

(c) $y=c_1e^x+c_2e^{2x}+e^{-2x}\sec\left(e^{-x}\right)$

(d) $y=c_1e^x+c_2e^{2x}-e^{-2x}\sin\left(e^{2x}\right)$

Q3. The solution of differential equation $\frac{d^2y}{dx^2} - y = e^x$ satisfying $y(0) = 0 \ \& \ \frac{dy}{dx}(0) = \frac{3}{2}$ is

(a) $y(x) = \sinh x + \frac{x}{2}e^x$ (b) $y(x) = x\cosh x + \frac{x}{2}e^x$

(c) $y(x) = \sinh x - \frac{x}{2}e^x$ (d) $y(x) = 2x\cosh x - \frac{x}{2}e^x$

Q.4. A particular integral of $\frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby = Q(x)$ is

(a) $e^{ax}\{\int e^{(a-b)x}\int Qe^{bx}dx\}dx$

(b) $e^{ax}\{\int e^{(b-a)x}\int Qe^{-bx}dx\}dx$

(c) $e^{-ax}\{\int e^{(b-a)x}\int Qe^{bx}dx\}dx$

(d) None of these

Q.5. Consider the differential equation

$$y'' + ay' + y = \sin x \text{ for } x \in R \qquad (**).$$

Then which one of the following is true ?**IIT JAM 2022**

(a) If $a = 0$, then all the solutions of (**) are unbounded over R.

(b) If $a = 1$, then all the solutions of (**) are unbounded over $(0, \infty)$.

(c) If $a = 1$, then all the solutions of (**) tend to zero as $x \to \infty$

(d) If $a = 2$, then all the solutions of (**) are bounded over $(-\infty, 0)$

Q.6. The real valued function $y(x)$ defined on R is said to be periodic if there exists a real number T > 0 such that $y(x+T) = y(x)$ for all $x \in$ R. Consider the differential equation $\frac{d^2y}{dx^2} + 4y = \sin ax,\ x \in (R,)$ (*)**IIT JAM 2022**
where $a \in R$ is a constant. Then Which of the following is are true ?

(a) All solutions of (*) are periodic for every choice of a.

(b) All solutions of (*) are periodic for every choice of $a \in R - \{-2, 2\}$

(c) All solutions of (*) are periodic for every choice of $a \in Q - \{-2, 2\}$

(d) $a \in R - Q$ Then there is a unique periodic solution of (*)

Q1. Consider the differential equation $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y = 0$ with the boundary condition of y(0) = 0 and y(1) = 1. The complete solution of differential equation is

(a) x^2

(b) $\sin\left(\frac{\pi x}{2}\right)$

(c) $e^x \sin\left(\frac{\pi x}{2}\right)$

(d) $e^{-x} \sin\left(\frac{\pi x}{2}\right)$

Q2. The general solution of $x^2\frac{d^2y}{dx^2}-5x\frac{dy}{dx}+9y=0$ is

(a) $(c_1 + c_2x)e^{3x}$ (b) $(c_1 + c_2 \ln x)x^3$

(c) $(c_1 + c_2x)x^3$ (d) $(c_1 + c_2 \ln x)\ e^{x^3}$

Q3 The general solution of differential equation $4x^2y'' - 8xy' + 9y = 0$ is

(a) $c_1e^{5x/2} + c_2e^{-3x/2}$ (b) $c_1e^{3x/2} + c_2e^{-3x/2}$

(c) $(c_1 + c_2 \log x)x^{3/2}$ (d) $c_1x^{3/2} + c_2x^{-3/2}$

Q4. If $\dfrac{(c_1+c_2\ln x)}{x}$ is the general solution of the differential equation $x^2\dfrac{d^2y}{dx^2}+kx\dfrac{dy}{dx}+y=0$, x > 0 the k equals

(a) 3 (b) -3

(c) 2 (d) -1

Q5. A solution of the differential equation

$$2x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-y=0, x>0$$ that passes through the point $(1,1)$ is

(a) $y=\frac{1}{x}$

(b) $y=\frac{1}{x^2}$

(c) $y=\frac{1}{\sqrt{x}}$

(d) $y=\frac{1}{x^{3/2}}$

Q6. Let $y(x); x > 0$ be the solution of differential equation $x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = 0$ satisfying the conditions $y(1) = 1 \,\&\, y'(1) = 0$ Then the value of $e^2 y(e)$ is

(a) 3 (b) 1

(c) 2 (d) -1

Q7. Let y(x) be the solution of $x^2 y''(x) - 2y(x) = 0;\ y(1) = 1, y(2) = 1$
Then the value of y(3) is

(a) $\frac{11}{21}$ (b) 1

(c) $\frac{17}{21}$ (d) $\frac{11}{7}$

Q8 A particular solution of $x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+\frac{y}{4}=\frac{1}{\sqrt{x}}$ is

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\log x}{2\sqrt{x}}$

(c) $\frac{(\log x)^2}{2\sqrt{x}}$ (d) $\frac{(\log x)\sqrt{x}}{2}$

Q.1. Solution $(5+2x)^2 \frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 0$.

(a) $y(x) = c_1(5 + 2x) + c_2(5 - 2x)$

(b) $y(x) = c_1(5+2x)^{2+\sqrt{2}} + c_2(5+2x)^{2-\sqrt{2}}$

(c) $y(x) = c_1(2+\sqrt{2})x + c_2(2-\sqrt{2}x)$

(d) none of these

Q.2. The general solution

$(1 + 2x)^2 y`` - 6(1 + 2x)y` + 16y = 8(1 + 2x)^2.$

(a) $c_1 \cos \{\log(1 + x)\} + c_2 \sin \{\log(1 + x)\} + 2 \log (1 - 1x).\sin\{\log(1 + x)\}$

(b) $\frac{c_1}{x} + \frac{c_2}{x^2} - \frac{\sin x}{x^2}$

(c) $\{c_1 + c_2 \log(1 + 2x)\}(1 + 2x)^2 + (11 + 2x)^2 \{\log(1 + 2x)\}$

(d) None of these

Q.3. General solution of

$(1 + x^2)y`` + (1 + x)y` + y = 4\cos\{\log(1 + x)\}$

(a) $\dfrac{c_1}{x}+\dfrac{c_2}{x^2}-\dfrac{\sin x}{x^2}$

(b) $\cos\{\log(1 + x)\} + c_2\sin\{\log(1 + x)\} + 2\log(1 - 1x).\sin\{\log(1 + x)\}$

(c) $\{c_1 + c_2\log(1 + 2x)\}(1 + 2x)^2 + (11 + 2x)^2\{\log(1 + 2x)\}$ (d) None of these

Q4. Given that $y(x)=x$ is a solution of differential equation $(1+x^2)y''-2xy'+2y=0,\ x>0$

Find second linearly independent solution

(a) (x^2-1) (b) $\frac{1}{x}$

(c) e^x (d) e^{-x}

Q5. Let $y = e^x$ be a solution of $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + (1-x)y = 0$.

Then the second linearly independent solution of this ordinary differential equation is

(a) $xe^{-2x} + \frac{1}{2}$

(b) $\frac{1}{2}\left(x - \frac{1}{2}\right)e^{-x}$

(c) $\frac{1}{2}\left(x + \frac{1}{2}\right)e^{-2x}$

(d) $xe^{-2x} - \frac{1}{2}$

Q.6. If $y = x^2$ is a solution of the differential equation $y``-\left(\frac{2}{x^2}+\frac{2}{x}\right)(xy`-y)=0$, $0 < x < \infty$, then its general solution is

(a) $\alpha x^2\int x^{-2}e^x dx+\beta$

(b) $\alpha x^{-2}\int x^2 e^x dx+\beta$

(c) $\alpha x^2\int x^2 e^x dx+\beta$

(d) None of these

Q1. Let $y(x)=u(x)\sin x+v(x)\cos x$ be a solution of differential equation $y''+y=\sec x$ then $u(x)$ is **[IIT: JAM-2015]**

(a) $\ln|\cos x|+C$ (b) $-x+C$

(c) $x+C$ (d) $\ln|\sec x|+C$

Q2. Assume that $y_1(x) = x$ and $y_2(x) = x^3$ are two linearly independent solutions of the homogeneous differential equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$ using the method of variation of parameters find a particular solution of the differential equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^5$

[IIT: JAM-2012]

(a) $\frac{x^5}{8}$ (b) $-\frac{x^5}{8}$

(c) $\frac{x^5}{4}$ (d) None of these

Q3. A particular integral of the differential equation $y''+3y'+2y=e^{e^x}$ is

[IIT-JAM: 2018]

(a) $e^{e^x}e^{-x}$ (b) $e^{e^x}e^{-2x}$

(c) $e^{e^x}e^{2x}$ (d) $e^{e^x}e^{x}$

Q4. Using the method of variation of parameters solve the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2$ given that $x\ \&\ \frac{1}{x}$ are two solutions of the corresponding homogeneous equation **[IIT: JAM-2007]**

(a) $c_1x+c_2\frac{1}{x}+\frac{x}{2}$ (b) $c_1x+c_2\frac{1}{x}+\frac{x^2}{3}$

(c) $c_1x+c_2\frac{1}{x}-\frac{x^3}{6}$ (d) None of these

Q5. PI Of $\dfrac{d^2y}{dx^2}+y=\dfrac{1}{1+\sin x}$

(a) $-1+\sin x+x\cos x-\sin x.\log(1-\sin x)$

(b) $-1+\sin x+x\cos x+\sin x.\log(1-\sin x)$

(c) $-1+\sin x-x\cos x+\sin x.\log(1+\sin x)$

(d) $-1+\sin x+x\cos x+\sin x.\log(1+\sin x)$

Q6. Solving by variation of parameters $y'' - 2y' + y = e^x \log x$, the value of wronskian w is

(a) e^{2x} (b) 2

(c) e^{-2x} (d) None of these

Q7. For $\frac{d^2y}{dx^2} + 4y = \tan 2x$, solving by variation of parameters. The value of wronskian w is

(a) 1 (b) 2

(c) 3 (d) 4

Q8. Using the method of variation of parameters for the particular solution of the differential equation $y'' + 4y = \frac{3}{\sin 2x}; 0 < x < \frac{\pi}{2}$

(a) $\frac{3}{4}\sin 2x \log \cos 2x - \frac{3}{4}\cos 2x$

(b) $\frac{3}{2}\sin 2x \log \cos 2x - \frac{3}{4}\cos 2x$

(c) $\frac{3}{2}\sin 2x \log \sin 2x - \frac{3}{2}x \cos 2x$

(d) $\frac{3}{4}\sin 2x \log \sin 2x - \frac{3}{2}x \cos 2x$