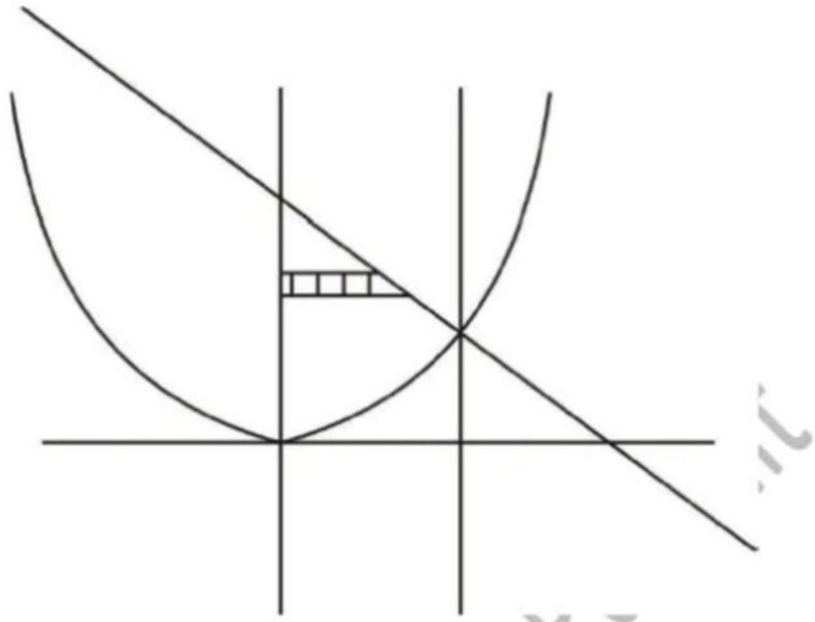
Change of order in mixed region

We know that if strip move on more than two curve then tl
region is called mixed region.

Example :



Then for double integration, we divide into simple region.

Q1. Change the order of integration $\int_0^1 \int_{x-1}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$.

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(a) $\int_0^1 \int_0^{\sqrt{1-y^2}} f(x,y)\,dx\,dy + \int_{-1}^0 \int_0^{1+y} f(x,y)\,dx\,dy$

(b) $\int_0^1 \int_0^{\sqrt{1-y^2}} f(x,y)\,dx\,dy - \int_{-1}^0 \int_0^{1+y} f(x,y)\,dx\,dy$

(c) $\int_0^1 \int_0^{1+y} f(x,y)\,dx\,dy - \int_{-1}^0 \int_0^{1+y} f(x,y)\,dx\,dy$

(d) None of these

Q2. The value of $\int_0^{\pi/2}\int_{\pi/2}^{\pi}\frac{\sin x}{x}dxdy+\int_{\pi/2}^{\pi}\int_y^{\pi}\frac{\sin x}{x}dxdy$

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(a) 0 (b) 1

(c) 2 (d) 3

Q3. Change the order of integration in $\int_{-1}^{2}\int_{-x}^{2-x^2} f(x,y)\,dy\,dx$.

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(a) $\int_{-1}^{2}\int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x,y)\,dx\,dy - \int_{-2}^{1}\int_{-y}^{\sqrt{2-y}} f(x,y)\,dx\,dy$

(b) $\int_{1}^{2}\int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x,y)\,dx\,dy + \int_{0}^{1}\int_{-y}^{\sqrt{2-y}} f(x,y)\,dx\,dy$

(c) $\int_{-1}^{2}\int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x,y)\,dx\,dy + \int_{-2}^{1}\int_{-y}^{\sqrt{2-y}} f(x,y)\,dx\,dy$

(d) None of these

Q4. Change the order of $\int_0^1 \int_0^{1/x} \frac{y}{(1+xy)^2(1+y^2)} dydx$

(a) $\int_0^1 \int_0^{\infty} \frac{y}{(1+xy)^2(1+y^2)} dxdy + \int_1^{\infty} \int_0^{1/y} \frac{-y}{(1+xy)^2(1+y^2)} dxdy$

(b) $\int_0^1 \int_0^1 \frac{y}{(1+xy)^2(1+y^2)} dxdy + \int_1^{\infty} \int_0^{1/y} \frac{y}{(1+xy)^2(1+y^2)} dxdy$

(c) $\int_0^1 \int_0^1 \frac{y}{(1+xy)^2(1+y^2)} dxdy - \int_1^{\infty} \int_0^{1/y} \frac{y}{(1+xy)^2(1+y^2)} dxdy$

(d) None of these

Q5. Change the order of integration in $\int_0^{2a}\int_{x^2/4a}^{3a-x} f(x,y)\,dy\,dx$.

(a) $I=\int_0^{a}\int_0^{\sqrt{4ay}} f(x,y)\,dx\,dy+\int_a^{3a}\int_0^{3a-y} f(x,y)\,dx\,dy$

(b) $I=\int_0^{a}\int_0^{\sqrt{4ay}} f(x,y)\,dx\,dy-\int_a^{3a}\int_0^{3a-y} f(x,y)\,dx\,dy$

(c) $I=\int_0^{a}\int_0^{a-y} f(x,y)\,dx\,dy+\int_a^{3a}\int_0^{3a-y} f(x,y)\,dx\,dy$

(d) None of these

Q6 Change the order of double integration $I = \int_0^1 \int_{\sqrt{y}}^{2-y} f dx dy$

(a) $I = \int_0^1 \int_0^{x^2} f dy dx - \int_1^2 \int_0^{2-x} f dy dx$

(b) $I = \int_0^1 \int_0^{x^2} f dy dx + \int_1^2 \int_0^{2-x} f dy dx$

(c) $I = \int_0^1 \int_0^{x^2} f dy dx + \int_0^{x^2} \int_0^{2-x} f dy dx$

(d) None of these

Q7. Change the order of integration in the double integral

$$\int_{-1}^{2}\left(\int_{-x}^{2-x^2} f(x,y)dy\right)dx$$ **IIT-JAM – 2011**

(a) $I=\int_{-2}^{1}\left(\int_{-y}^{\sqrt{2-y}} f(x,y)dx\right)dy-\int_{1}^{2}\left(\int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x,y)dx\right)dy$

(b) $I=\int_{-2}^{1}\left(\int_{-y}^{\sqrt{2-y}} f(x,y)dx\right)dy+\int_{1}^{2}\left(\int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x,y)dx\right)dy$

(c) $I=\int_{0}^{1}\left(\int_{-y}^{\sqrt{2+y}} f(x,y)dx\right)dy+\int_{0}^{2}\left(\int_{-\sqrt{2-y}}^{\sqrt{2-y}} f(x,y)dx\right)dy$

(d) None of these