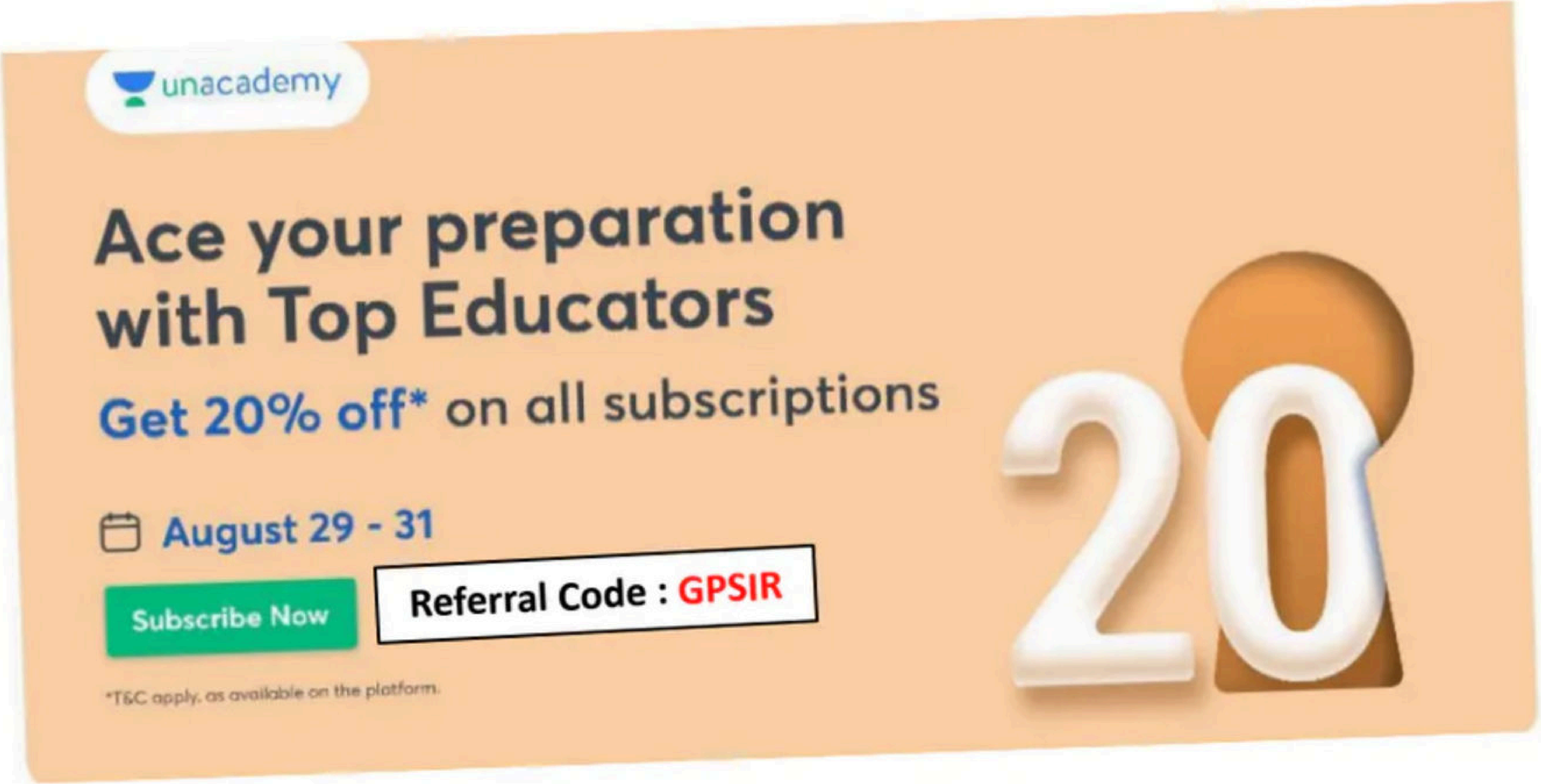
Doubt Clearing Session

Detailed Course on Differential Equation for IIT JAM' 23 - II

Gajendra Purohit • Lesson 17 • Aug 30, 2022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COMPLETE COURSE ON

MATHEMATICS

FOR IIT-JAM 2022

VARIATION OF PARAMETERS

Consider a second order differential equation as follows

$$y'' + Py' + Qy = R$$

$y = c_1u + c_2v$

Let u and v are parts of CF then, Complete solution of given differential

$y = c_1u + c_2v + Au + Bv$

equation is $y = c_1u + c_2v + Au + Bv$

$A = -\int \frac{Rv}{W} dx$

Where c_1, c_2 are arbitrary constants & A and B are to be determined.

$B = \int \frac{Ru}{W} dx$

$$A = -\int \frac{Rv}{W} dx, B = \int \frac{Ru}{W} dx \text{ where } W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix}$$

Example1. Using the method of variation of parameters, solve $y'' + y = \sec x$

$$\frac{d^2y}{dx^2} + y = \tan x$$

$$(D^2+1)y = \tan x$$

$$m^2+1=0$$

$$m = \pm i$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos^2 x + \sin^2 x = 1$$

$$y = c_1 \underbrace{\cos x}_{y_1} + c_2 \underbrace{\sin x}_{y_2}$$

$$y = c_1 \cos x + c_2 \sin x + \boxed{A} \cos x + \boxed{B} \sin x$$

$$A = -\int \frac{y_2 R}{W}\, dx$$

$$A = -\int \frac{\sin x \cdot \tan x}{1}\, dx$$

$$A = -\int \frac{\sin^2 x}{\cos x}\, dx$$

$$A = -\int \left(\frac{1-\cos^2 x}{\cos x}\right) dx$$

$$A = -\int \sec x\, dx + \int \cos x\, dx$$

$$= -\log(\sec x + \tan x) + \sin x$$

$$B = \int \frac{y_1 R}{W}\, dx = \int \frac{\cos x \tan x}{1}\, dx$$

$$= \int \cancel{\cos x} \frac{\sin x}{\cancel{\cos x}}\, dx = -\cos x$$

$$y = c_1 \cos x + c_2 \sin x + \left(-\log(\sec x + \tan x) + \cancel{\sin x}\right)\cos x$$

$$+ (-\cos x)\cancel{\sin x}$$

$$y = c_1 \cos x + c_2 \sin x - \cos x \log(\sec x + \tan x)$$

$$y'' - 3y' + 2y = \frac{e^x}{1+e^x}$$

$$(D^2 - 3D + 2)y = \frac{e^x}{1+e^x}$$

$$m^2 - 3m + 2 = 0$$

$$m = 1, 2$$

CF $y = c_1 \underbrace{e^x}_{y_1} + c_2 \underbrace{e^{2x}}_{y_2}$

$$W = \begin{vmatrix} e^x & e^{2x} \\ e^x & 2e^{2x} \end{vmatrix} = 2e^{3x} - e^{3x} = e^{3x}$$

$$y = c_1 e^x + c_2 e^{2x} + A e^x + B e^{2x}$$

$$A = -\int \frac{y_2 R}{W}\,dx$$

$$A = -\int \frac{e^{2x}\,e^x}{e^{3x}(1+e^x)}\,dx$$

$$\boxed{A = -\int \frac{1}{1+e^x}\,dx}$$

$$A = -\int \frac{e^{-x}}{e^{-x}+1}\,dx$$

$$A = +\log(1+e^{-x})$$

$$B = \int \frac{y_1 R}{W}\,dx$$

$$B = \int \frac{e^x}{e^{3x}} \cdot \frac{e^x}{1+e^x}\,dx$$

$$B = \int \frac{1+e^x-e^x}{e^x(1+e^x)}\,dx$$

$$B = \int \frac{\cancel{1+e^x}}{e^x\cancel{(1+e^x)}}\,dx - \int \frac{e^x}{e^x(1+e^x)}\,dx$$

$$B = \int e^{-x}\,dx - \int \frac{1}{1+e^x}\,dx$$

$$B = -e^{-x} + \log(1+e^{-x})$$

$$y = c_1 e^x + c_2 e^{2x} + \log(1+e^{-x})\cdot e^x + \left(-e^{-x} + \log(1+e^{-x})\right)e^{2x}$$

$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^5$

Let $x = e^z$ or $z = \log x$

$D(D-1)y - 3Dy + 3y = e^{5z}$

$(D^2 - 4D + 3)y = 0$ $\quad \boxed{R = x^3}$

$m^2 - 4m + 3 = 0$

$m = 1, 3$

$y = c_1 e^z + c_2 e^{3z}$

$y = c_1 x + c_2 x^3$

$W = \begin{vmatrix} x & x^3 \\ 1 & 3x^2 \end{vmatrix}$

$= 3x^3 - x^3 = 2x^3$

$y = c_1 x + c_2 x^3 + Ax + Bx^3$

$A = -\int \frac{y_2 R}{W} dx$

$A = -\int \frac{x^3 \cdot \cancel{x^3}}{2\cancel{x^3}} dx$

$\boxed{A = -\frac{x^4}{8}}$

$B = \int \frac{y_1 R}{W} dx$

$B = \int \frac{x \cdot \cancel{x^3}}{2\cancel{x^3}} dx$

$\boxed{B = \frac{x^2}{4}}$

$y = c_1 x + c_2 x^3 + \left(-\frac{x^4}{8}\right)x + \left(\frac{x^2}{4}\right)x^3$

$y = c_1 x + c_2 x^3 - \frac{x^5}{8} + \frac{x^5}{4}$

$\boxed{y = c_1 x + c_2 x^3 + \frac{x^5}{8}}$

1. $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

Example2. Using the method of variation of parameters, solve

$$y'' - 3y' + 2y = \frac{e^x}{1+e^x}$$

Q1 Let $y(x)=u(x)\sin x+v(x)\cos x$ be a solution of differential equation $y''+y=\sec x$ then $u(x)$ is **[IIT: JAM-2015]**

(a) $\ln|\cos x|+C$ (b) $-x+C$

(c) $x+C$ (d) $\ln|\sec x|+C$

y_1, y_2

$$u(x) = -\int \frac{y_2 R}{W}\,dx$$

$$= -\int \frac{\cos x \sec x}{-1}\,dx$$

$$= x$$

$$W = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}$$

$$= -1$$

$$y = c_1 \sin x + c_2 \cos x + u \sin x + v \cos x$$

$$= (c_1 + x)\sin x$$

Q2. Assume that $y_1(x) = x$ and $y_2(x) = x^3$ are two linearly independent solutions of the homogeneous differential equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$ using the method of variation of parameters find a particular solution of the differential equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^5$

[IIT: JAM-2012]

(a) $\frac{x^5}{8}$ (b) $-\frac{x^5}{8}$

(c) $\frac{x^5}{4}$ (d) None of these

Q3. A particular integral of the differential equation $y'' + 3y' + 2y = e^{e^x}$ is

[IIT-JAM: 2018]

(a) $e^{e^x} e^{-x}$

(b) $e^{e^x} e^{-2x}$

(c) $e^{e^x} e^{2x}$

(d) $e^{e^x} e^{x}$

$e^x = t$

$e^x dx = dt$

$W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix}$

$= -2e^{-3x} + e^{-3x}$

$= -e^{-3x}$

$m^2 + 3m + 2 = 0$

$m = -1, -2$

$y = c_1 e^{-x} + c_2 e^{-2x}$

y_1, y_2

$y = c_1 e^{-x} + c_2 e^{-2x} + \boxed{Ae^{-x} + Be^{-2x}}$

$A = -\int \frac{y_2 R}{W} dx = -\int \frac{e^{-2x} e^{e^x}}{-e^{-3x}} dx$

$= \int e^x e^{e^x} dx$

$= \int e^t dt = e^t$

$\boxed{A = e^{e^x}}$

$B = \int \frac{y_1 R}{W} dx = \int \frac{e^{-x} \cdot e^{e^x}}{-e^{-3x}} dx$

$= -\int e^{2x} e^{e^x} dx = -\int t e^t dt$

$= -[te^t - e^t]$

$= -e^x e^{e^x} + e^{e^x}$

$y_p = Ae^{-x} + Be^{-2x}$

$y_p = e^{e^x} e^{-x} + (-e^x e^{e^x} + e^{e^x}) e^{-2x}$

$y_p = \cancel{e^{e^x} e^{-x}} - \cancel{e^{-x} e^{e^x}} + e^{e^x} e^{-2x}$

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- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINAER ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

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Q4. Using the method of variation of parameters solve the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2$ given that $x\ \&\ \frac{1}{x}$ are two solutions of the corresponding homogeneous equation **[IIT: JAM-2007]**

$R = 1$

(a) $c_1x+c_2\frac{1}{x}+\frac{x}{2}$ (b) $c_1x+c_2\frac{1}{x}+\frac{x^2}{3}$

(c) $c_1x+c_2\frac{1}{x}-\frac{x^3}{6}$ (d) None of these

$$y = c_1 \underset{y_1}{x} + c_2 \underset{y_2}{\frac{1}{x}} + A x + B\frac{1}{x}$$

$$w = \begin{vmatrix} x & \frac{1}{x} \\ 1 & -\frac{1}{x^2} \end{vmatrix} = -\frac{1}{x} - \frac{1}{x} = -\frac{2}{x}$$

$$A = -\int \frac{y_2 R}{w}\,dx = -\int \frac{\frac{1}{x}\cdot 1}{-2/x}\,dx = +\frac{x}{2}$$

$$B = \int \frac{y_1 R}{w}\,dx = \int \frac{x(1)}{-2/x}\,dx = -\frac{x^3}{6}$$

$$y = c_1x + c_2\frac{1}{x} + \left(\frac{x}{2}\right)x - \frac{x^3}{6}\left(\frac{1}{x}\right)$$

$$= c_1x + c_2\frac{1}{x} + \frac{x^2}{2} - \frac{x^2}{6}$$

$$= c_1x + c_2\frac{1}{x} + \frac{x^2}{3}$$

Q5. PI Of $\frac{d^2y}{dx^2}+y=\frac{1}{1+\sin x}$

(a) $-1+\sin x+x\cos x-\sin x.\log(1-\sin x)$

(b) $-1+\sin x+x\cos x+\sin x.\log(1-\sin x)$

(c) $-1+\sin x-x\cos x+\sin x.\log(1+\sin x)$

(d) $-1+\sin x+x\cos x+\sin x.\log(1+\sin x)$

$y = c_1\cos x + c_2\sin x + \boxed{A\cos x + B\sin x}$

$A = -\int \frac{y_2 R}{W} dx = -\int \frac{\sin x}{1+\sin x} dx$

$= -\int \frac{\sin x(1-\sin x)}{1-\sin^2 x} dx = -\int \frac{\sin x - \sin^2 x}{\cos^2 x} dx$

$= -\int \tan x\sec x + \int \tan^2 x\, dx$

$= -\sec x + \tan x - x$

$B = \int \frac{y_1 R}{W} dx = \int \frac{\cos x}{1+\sin x} dx$

$= \log(1+\sin x)$

$y_P = A\cos x + B\sin x$

$y = (-\sec x + \tan x - x)\cos x + \sin x\log(1+\sin x)$

$= -1 + \sin x - x\cos x$

Q6. Solving by variation of parameters $y'' - 2y' + y = e^x \log x$, the value of wronskian w is

(a) e^{2x} (b) 2

(c) e^{-2x} (d) None of these

Q7. For $\frac{d^2y}{dx^2}+4y=\tan 2x$, solving by variation of parameters. The value of wronskian w is

(a) 1 (b) 2

(c) 3 (d) 4

Q8. Using the method of variation of parameters for the particular solution of the differential equation $y'' + 4y = \dfrac{3}{\sin 2x}$; $0 < x < \dfrac{\pi}{2}$

(a) $\dfrac{3}{4}\sin 2x \log \cos 2x - \dfrac{3}{4}\cos 2x$

(b) $\dfrac{3}{2}\sin 2x \log \cos 2x - \dfrac{3}{4}\cos 2x$

(c) $\dfrac{3}{2}\sin 2x \log \sin 2x - \dfrac{3}{2}x\cos 2x$

(d) $\dfrac{3}{4}\sin 2x \log \sin 2x - \dfrac{3}{2}x\cos 2x$

$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2$

$y = c_1 \overset{y_1}{\cos 2x} + c_2 \overset{y_2}{\sin 2x}$

$y_p = A\cos 2x + B\sin 2x$

$A = -\int \dfrac{y_2 R}{W}\,dx$

$= -\int \dfrac{\sin 2x}{2}\,(\;) -$

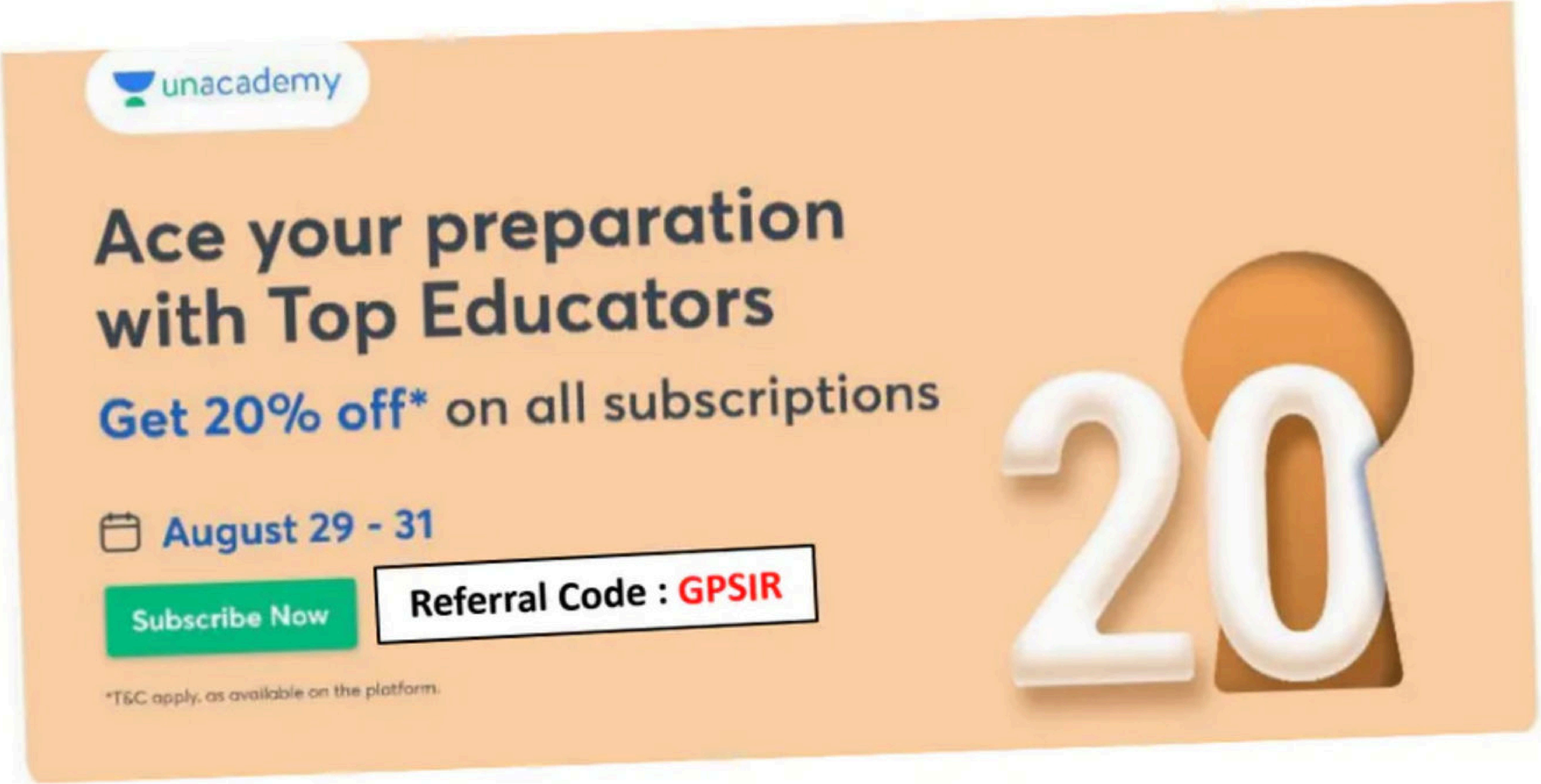
$A = -\int \dfrac{\sin 2x}{2}\,\dfrac{3}{\sin 2x}\,dx = -\dfrac{3}{2}x$

$B = \int \dfrac{y_1}{W} R\,dx = \int \dfrac{\cos 2x}{2}\,\dfrac{3}{\sin 2x}\,dx$

$= \dfrac{3}{2}\int \cot 2x\,dx$

$= \dfrac{3}{2}\,\dfrac{\log \sin 2x}{2}$

$\dfrac{1}{1+\sin x}\cdot\dfrac{1-\sin x}{(1-\sin x)}$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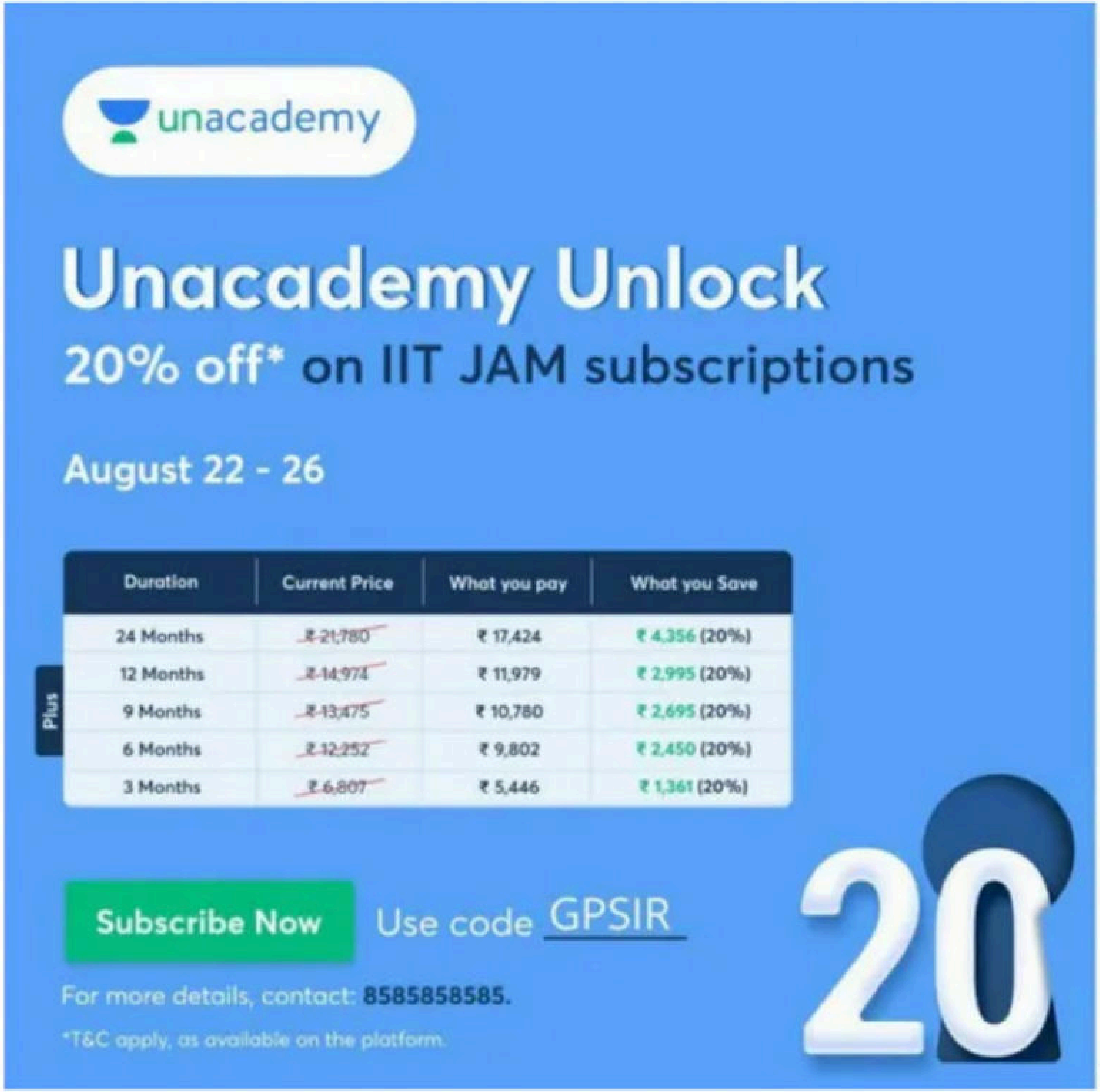
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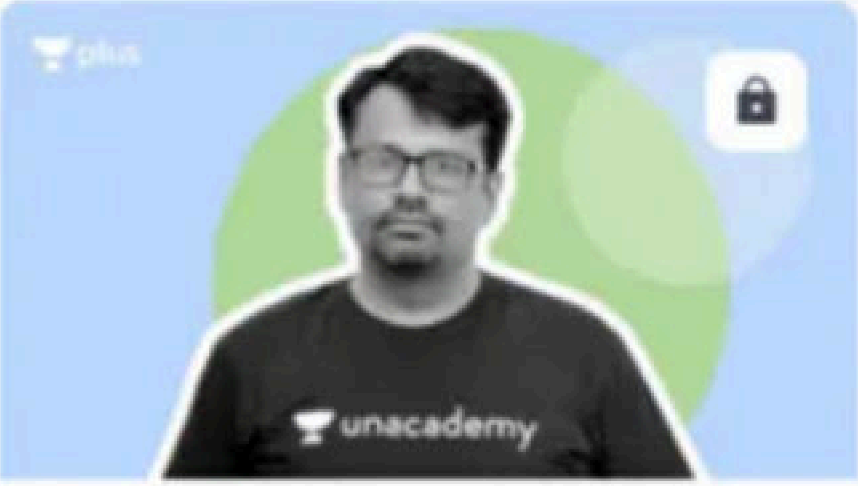

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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