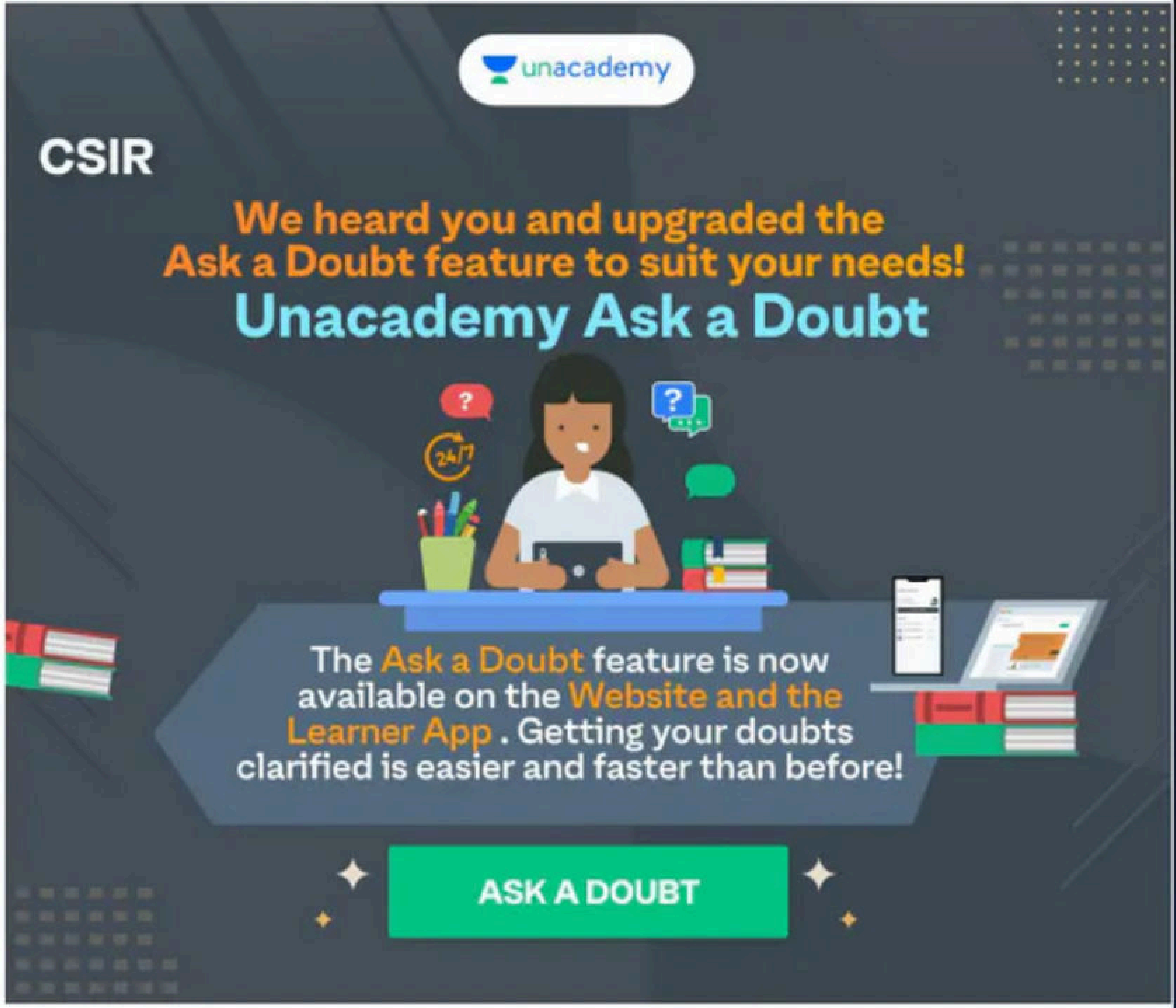
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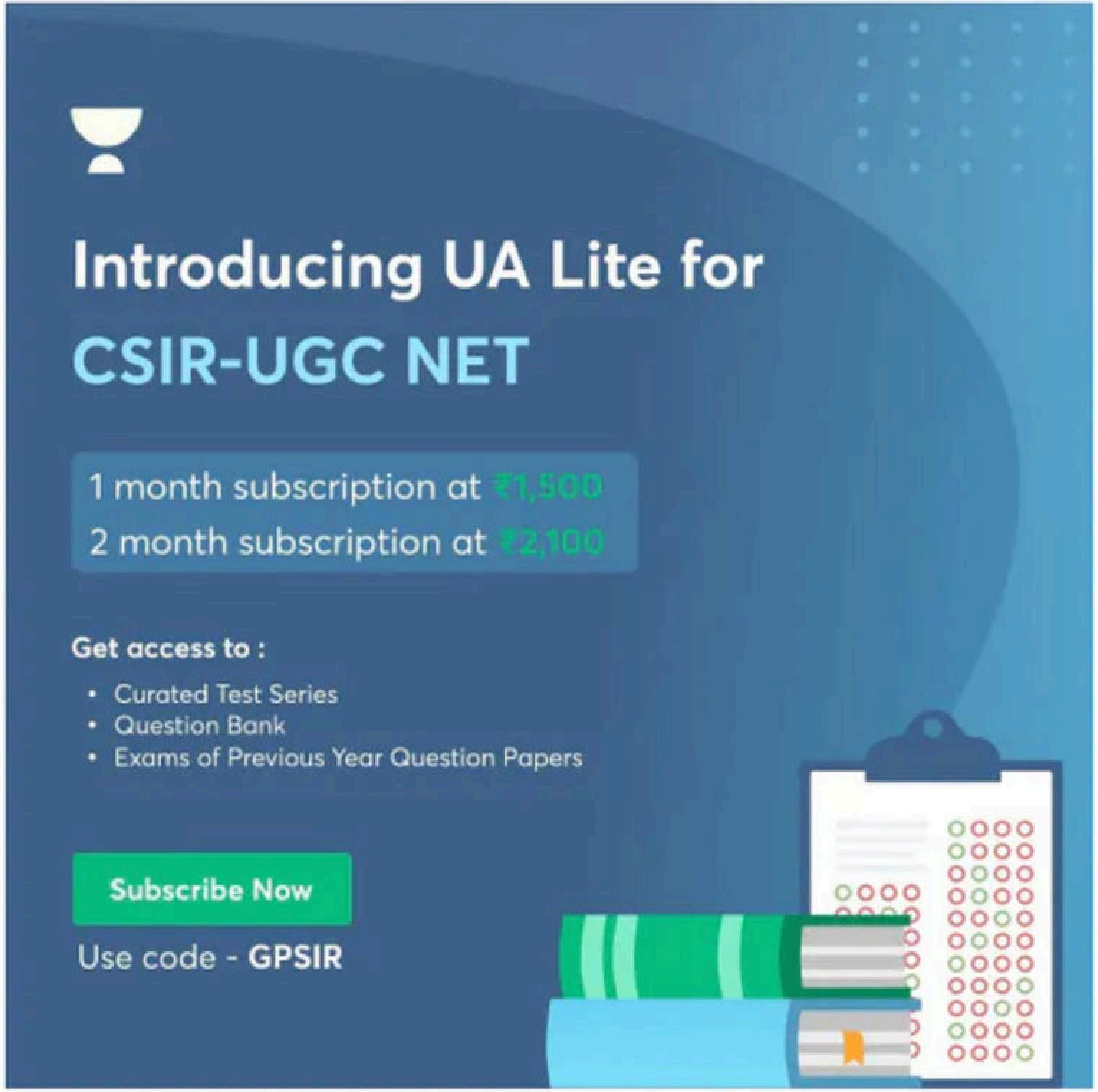
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Triple Integration

We know that double integral is the integral of function w.r.t. x & y variable but if we want to integral of function w.r.t. three variable x, y & z, then we use the concept of triple integral and it is denoted by $\iiint f(x, y, z)dxdydz$.

$$\int_0^1\int_0^1\int_0^1 (x+y)\,dx\,dy\,dz$$

$$\int_0^1\int_0^{1-x}\int_0^{1-x-y} dx\,dy\,dz$$

Procedure of solving triple integral :

Step – 1 : Suppose the triple integral is $\iiint f(x, y, z)\,dx\,dy\,dz$, then first integral w.r.t. x and put the limit.

Step – 2 : Then second integration w.r.t. y and put limit further third and last integral w.r.t. z and put limit.

$$\int_0^1\int_0^1\int_0^1 e^{x+y+z}\,\underline{dx}\,dy\,dz$$

$$\int_0^1 e^x\,dx \int_0^1 e^y\,dy \int_0^1 e^z\,dz$$

$$(e^x)_0^1\,(e^y)_0^1\,(e^z)_0^1$$

$$(e-1)\quad(e-1)\quad(e-1)$$

$$(e-1)^3$$

$$\int_{z=-1}^{1}\int_{y=0}^{2}\int_{y=x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$$

$$\frac{2x^2-2z^2+x^2-2xz+z^2}{2}$$

$$\frac{3x^2-z^2-2xz}{2}$$

$$\int_{z=-1}^{1}\int_{x=0}^{2}\left[(x+z)y+\frac{y^2}{2}\right]_{x-z}^{x+z}dx\,dz$$

$$\int_{-1}^{1}\int_{0}^{2}\left(\left[(x+z)^2+\frac{(x+z)^2}{2}\right]-\left[x^2-z^2+\frac{(x-z)^2}{2}\right]\right)dx\,dz=\int_{-1}^{1}\int_{0}^{2}\left(\frac{3}{2}(x+z)^2-\left(\frac{3x^2-z^2-2xz}{2}\right)\right)$$

$$=0$$

Find the limit of triple integration :

Let $\iiint f \, dxdydz$ be a given integral then $\int\limits_{x=\alpha}^{\beta} \int\limits_{y=g_1(x)}^{g_2(x)} \int\limits_{z=f_1(x,y)}^{f_2(x,y)} f \, dxdydz$.

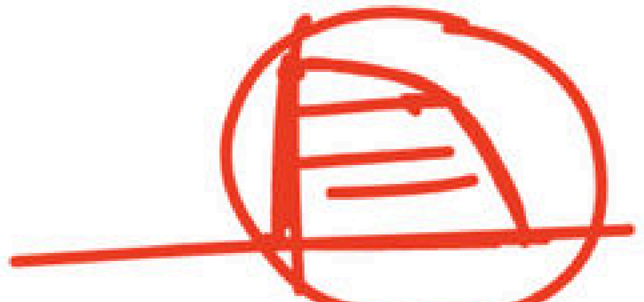
Note : In above integral

(1) In third integral limits are in x & y variable i.e. limit is of z variable. So, we first integrated this integral w.r.t. z.

(2) In middle integral limits are of y. So, we will do second integrated w.r.t. y.

(3) And we will do last integral which is integrated w.r.t. x.

Q.1. Evaluate $\iiint\limits_W z\,dx\,dy\,dz$ where is the bounded by the plane $x = 0$, $y = 0$, $z = 0$, $z = 1$ and the cylinder $x^2 + y^2 = 1$ with $x \geq 0$, $y \geq 0$ **IIT JAM 2006**



(a) π

(b) $\pi/2$

(c) $\pi/4$

(d) $\pi/8$

$$\int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}}\int_{z=0}^{1} z\,dz\,dy\,dx$$

$$\frac{\pi(1)^2}{4}$$

$$\int_0^1\int_0^{\sqrt{1-y^2}}\left(\frac{z^2}{2}\right)_0^1 dy\,dx$$

$$\frac{1}{2}\int_0^1\int_0^{\sqrt{1-x^2}} dy\,dx$$

$$\frac{1}{2}\int_0^1\sqrt{1-x^2}\,dx$$

$$\frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_0^1$$

$$\frac{1}{2}\left(0+\left(\frac{1}{2}\cdot\frac{\pi}{2}\right)\right) = \pi/8$$

$$x^2+y^2+z^2=a^2$$

$$\iint dx\,dy$$

$$V=\iiint dx\,dy\,dz$$

$$\int_{-a}^{a}\int^{\sqrt{a^2-x^2}}\int^{\sqrt{a^2-x^2-y^2}} dz\,dy\,dx$$

$$x^2+y^2=a^2$$

$$x^2=a^2$$

$$z=-\sqrt{a^2-x^2-y^2}$$

$$x=-a \qquad y=-\sqrt{a^2-x^2}$$

$$8\int_{x=0}^{a}\int_{y=0}^{\sqrt{a^2-x^2}}\int_{z=0}^{\sqrt{a^2-x^2-y^2}} dz\,dy\,dx$$

$$=8\int_0^a\int_0^{\sqrt{a^2-x^2}}\sqrt{a^2-x^2-y^2}\,dy\,dx$$

$$=8\int_0^a\left(\frac{y}{2}\sqrt{a^2-x^2-y^2}+\frac{a^2-x^2}{2}\sin^{-1}\frac{y}{\sqrt{a^2-x^2}}\right)_0^{\sqrt{a^2-x^2}}dx$$

$$=8\int_0^a\left[0+\frac{a^2-x^2}{2}\sin^{-1}1\right]dx$$

$$=\frac{8}{4}\pi\int_0^a(a^2-x^2)\,dx$$

$$=2\pi\left(a^2x-\frac{x^3}{3}\right)_0^a=2\pi\left(a^3-\frac{a^3}{3}\right)$$

$$=2\pi\left(\frac{2a^3}{3}\right)=\frac{4\pi a^3}{3}$$

$$\int_1^{n^2} dx$$

$$\int_{n^2}^{N} dx$$

$$2\int_0^{n^2} dx$$

Q.2. The value of $\int_{x=0}^{1}\int_{y=0}^{x^2}\int_{z=0}^{y}(y+2z)\,dz\,dy\,dx$ is **IIT JAM 2014**

(a) 1/53 (b) 2/21

(c) 1/6 (d) 5/3

$$\int_0^1\int_0^{x^2}\left[yz+z^2\right]_0^y dy\,dx$$

$$\int_0^1\int_0^{x^2}(y^2+y^2)\,dy\,dx$$

$$2\int_0^1\left(\frac{y^3}{3}\right)_0^{x^2}dx$$

$$\frac{2}{3}\int_0^1 x^6\,dx$$

$$\frac{2}{3}\left(\frac{x^7}{7}\right)_0^1=\frac{2}{21}$$

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Q.3. The value of $\int_{z=0}^{1}\int_{y=0}^{z}\int_{x=0}^{y} xy^2z^3\,dx\,dy\,dz$ is **IIT JAM – 2012**

(a) 1/90 (b) 1/50

(c) 1/45 (d) 1/10

$$\int_0^1\int_0^z \left(\frac{x^2}{2}\right)_0^y y^2 z^3\,dy\,dz$$

$$\frac{1}{2}\int_0^1\int_0^z y^4 z^3\,dy\,dz$$

$$\frac{1}{2\times 5}\int_0^1 (z^8)\,dz$$

$$\frac{1}{10}\times\left(\frac{z^9}{9}\right)_0^1$$

$$\frac{1}{90}$$

Q.4. If the triple integral over the region bounded by the plane $2x + y + z = 4$, $x = 0$, $y = 0$, $z = 0$ is given by $\int_{x=0}^{2}\int_{y=0}^{\lambda(x)}\int_{z=0}^{\mu(x,y)} dz\,dy\,dx$. Then the function $\lambda(x) - \mu(x, y)$ is

IIT JAM – 2016

$\mu = 4 - 2x - y$

(a) x + y
(b) x – y

$\lambda = 4 - \cancel{2x}$

(c) x
(d) y

$\lambda - \mu = y$

Q.5. Let V be the region bounded by the plane x = 0, x = 2, y = 0, z = 0 and y + z = 1, then the value of the integral $\iiint_V y\,dx\,dy\,dz$ is **IIT JAM – 2011**

(a) 1/2 (b) 4/3

(c) 1 (d) 1/3

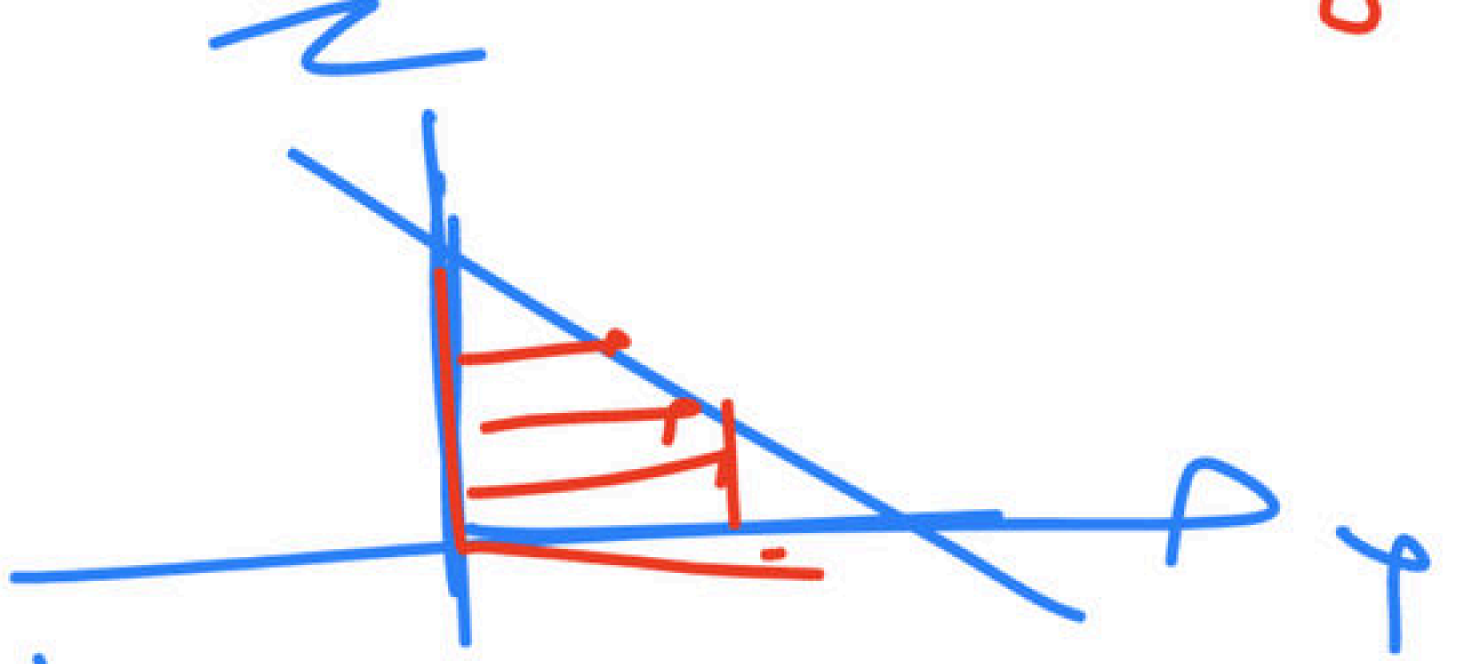
$$\int^1\int^{1-z}\int^2 y\,dx\,dy\,dz$$

$z=0$ $y=0$ $x=0$

$$\int_0^1\int_0^{1-z} y(x)_0^2\,dy\,dz$$

$$2\int_0^1\int_0^{1-z} y\,dy\,dz$$

$$\cancel{2}\int_0^1\left(\frac{y^2}{\cancel{2}}\right)_0^{1-z} dz$$



$$\int_0^1 (1-z)^2\,dz$$

$$-\left[\frac{(1-z)^3}{3}\right]_0^1$$

$$-\left(0-\frac{1}{3}\right)=\frac{1}{3}$$

Q.6. The value of the integral $\iiint\limits_V (x^2y+1)\,dx\,dy\,dz$. where V is region given by $x^2+y^2 \le 1$, $0 \le z \le 2$ is **IIT JAM 2020**

(a) π (b) 2π

(c) 3π (d) 4π

$$\int_0^{2\pi} f(x)\,dx = \int_0^{\pi} \left(f(x) + f(2\pi - x)\right) dx$$

$$2\int_0^{2\pi}\int_0^1 (r^2\cos^2\theta\, r\sin\theta + 1)\, r\,dr\,d\theta$$

$\theta = 0 \quad r = 0$

$$2\int_0^{2\pi} \left(\frac{r^5}{5}\cos^2\theta\sin\theta + \frac{r^2}{2}\right)_0^1 d\theta$$

$$\frac{8}{5}\left(\frac{\frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2}}{2\sqrt{\frac{8}{2}}}\right) + 2\pi$$

$$\frac{2}{5}\int_0^{2\pi} \sin\theta\cos^2\theta\, d\theta = \frac{2}{5}\int_0^{\pi}\left(\sin\theta\cos^2\theta - \sin\theta\cos^2\theta\right) d\theta$$

$= 0$

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$$\int_U C\,\eta\,v\,d\sigma$$

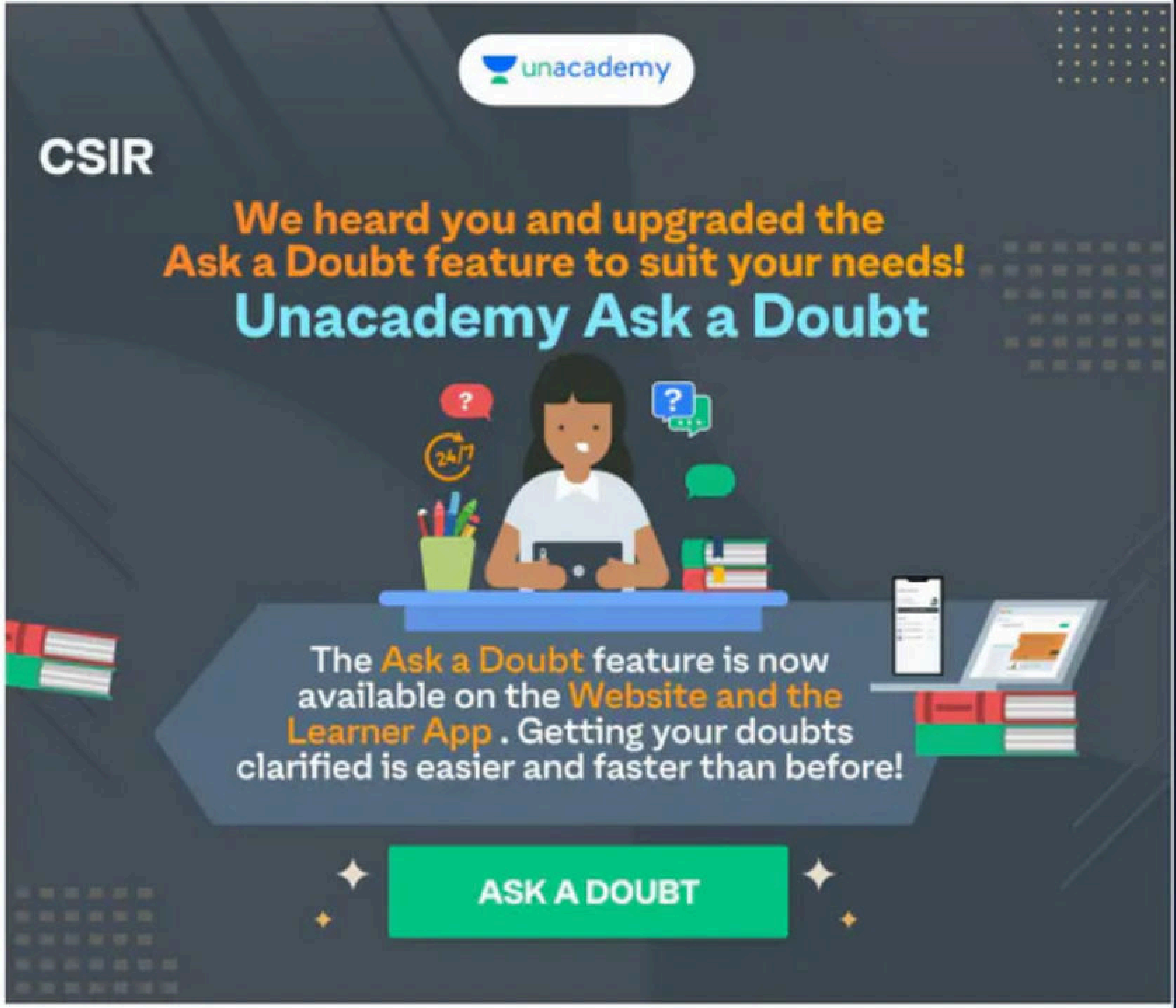
$$f(x) = f(2a-x)$$

$$\int_0^{2a} f(x)\,dx = \begin{cases} 0 \\ 2\int_0^{a} f(x)\,dx & f(x) = -f(2a-x) \end{cases}$$

Q.7. The value of $\iiint_V \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$, where V is volume of $x^2+y^2+z^2=1$.

(a) π (b) π^2

(c) $\pi/2$ (d) 8π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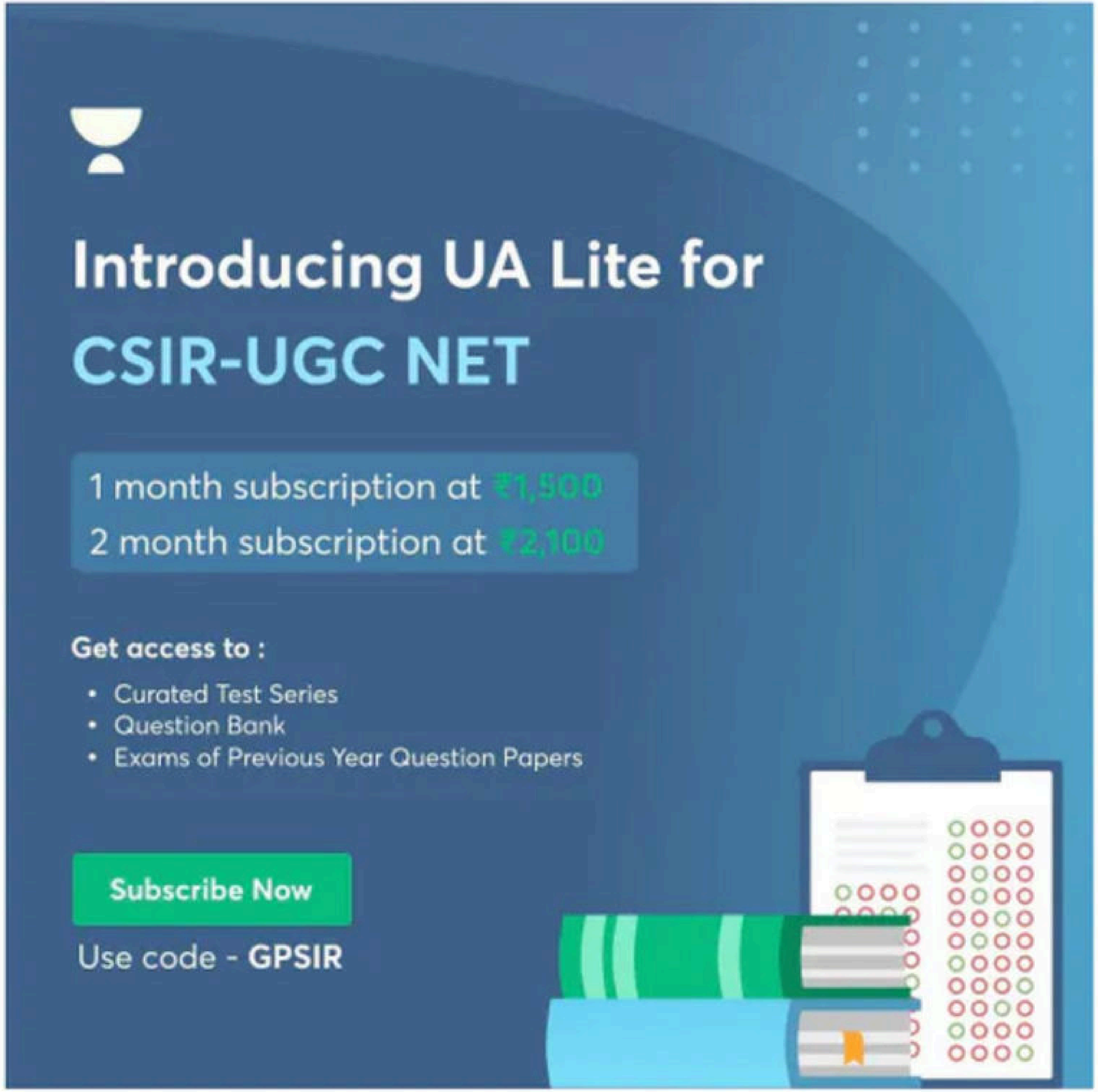
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