



Gajendra Purohit ✓

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✓ Limit inferior and limit superior of sequence :

Let $\langle a_n \rangle$ be a sequence of real number then limit superior and limit inferior are denoted by $\overline{\lim}_{n \rightarrow \infty} a_n$ and $\liminf_{n \rightarrow \infty} a_n$.

Case – 1 : If sequence is convergent Then $\overline{\lim}_{n \rightarrow \infty} a_n = \liminf_{n \rightarrow \infty} a_n = l = \lim_{n \rightarrow \infty} a_n$

$$a_n = \left(1 + \frac{(-1)^n}{n}\right)$$

$$\lim_{n \rightarrow \infty} a_n = 1 = \lim_{n \rightarrow \infty} a_n$$

$$\langle (-1)^n \rangle$$

$$\langle 1, -1, 1, -1, \dots \rangle$$

$$\lim_{n \rightarrow \infty} a_n = 1$$

$$\lim_{n \rightarrow \infty} a_n = 1$$



Case – 2 : If sequence is divergent. There arise two cases.

(a) If sequence is unbounded above,

Then $\overline{\lim}_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_n = \infty$

(b) If sequence is unbounded below, Then,

$\overline{\lim}_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} a_n = -\infty.$

$\langle n \rangle$

$a_n = \langle e^n \rangle$

$\langle -n \rangle$

$\langle -2^{-n} \rangle$

Case - 3: If sequence is oscillatory. There arise again two cases.

(a) If sequence is finitely oscillate.

Let $\langle a_n \rangle$ be a finitely oscillate sequence and P be the set of all limit points.

Then $\overline{\lim}_{n \rightarrow \infty} a_n = \sup P$ and $\underline{\lim}_{n \rightarrow \infty} a_n = \inf P$.

$$\langle (-1)^n \rangle$$

$$\langle 1 + (-1)^n \rangle$$

$$\langle 1, 0, 1, 0, \dots \rangle$$

$$\langle 0, 1, 0, 1 \rangle$$

$$\langle 0, 2, 0, 2, \dots \rangle$$
$$\langle 0, 2 \rangle$$

(b) If sequence is infinitely oscillate:

(i) Let $\langle a_n \rangle$ be a infinitely oscillate sequence. which is unbounded above and unbounded below then $\overline{\lim}_{n \rightarrow \infty} a_n = \infty$ and $\underline{\lim}_{n \rightarrow \infty} a_n = -\infty$.

(ii) Let $\langle a_n \rangle$ is infinitely oscillate sequence which is unbounded above but bounded below.

$\overline{\lim}_{n \rightarrow \infty} a_n = \infty$ & $\underline{\lim}_{n \rightarrow \infty} a_n = \text{smallest limit point}$

$\langle (-1)^n n \rangle$

$-1, 2, -3, 4, -5, 6$

if n is p.m.
if n is n.m.p

$\sqrt{-n}$

$\lim_{n \rightarrow \infty} a_n = \infty$
 $\lim_{n \rightarrow \infty} a_n = -\infty$

$1, 2, 3, 4, 5, 6, \dots$

$-1, -2, -3, -4, -5, -6, \dots$

(iii) Let $\langle a_n \rangle$ is infinitely oscillate sequence which is unbounded below but bounded above.

$$\overline{\lim}_{n \rightarrow \infty} a_n = \text{biggest limit point} \ \& \ \underline{\lim}_{n \rightarrow \infty} a_n = -\infty$$

Q1. Let $x_n = n^{\frac{1}{n}}$ and $y_n = e^{1-x_n}$, $n \in \mathbb{N}$. then which of the following is true

(a) $\limsup y_n = \liminf y_n = 0$

(b) $\limsup y_n \neq \liminf y_n$

✓ (c) $\limsup y_n = \liminf y_n = 1$

✓ (d) $\limsup x_n = \liminf y_n$

$$\lim_{n \rightarrow \infty} x_n = \lim_{n \rightarrow \infty} n^{1/n} = 1$$

$$\begin{aligned} \lim_{n \rightarrow \infty} y_n &= \lim_{n \rightarrow \infty} e^{1-x_n} \\ &= e^{\lim_{n \rightarrow \infty} (1-x_n)} \\ &= e^{1-\lim_{n \rightarrow \infty} x_n} \\ &= e^{1-1} \\ &= e^0 \\ &= 1 \end{aligned}$$

Q.2. Consider the sequence $\langle a_n \rangle$,

where $a_n = 3 + 5\left(-\frac{1}{2}\right)^n + \underline{(-1)^n} \left(\frac{1}{4} + (-1)^n \frac{2}{n}\right)$

then the interval $\left(\liminf_{n \rightarrow \infty} a_n, \limsup_{n \rightarrow \infty} a_n\right)$

(a) $(-2, 8)$

~~(b) $\left(\frac{11}{4}, \frac{13}{4}\right)$~~

(c) $(3, 5)$

(d) $\left(\frac{1}{4}, \frac{7}{4}\right)$
 $3 + \frac{1}{4}$
 $3 - \frac{1}{4}$

$$\left\{ \begin{array}{l} 3 + 5\left(-\frac{1}{2}\right)^n + \left(\frac{1}{4} + \frac{2}{n}\right) \quad \underline{n\text{-even}} \\ 3 - 5\left(\frac{1}{2}\right)^n - \left(\frac{1}{4} - \frac{2}{n}\right) \quad \underline{n\text{-odd}} \end{array} \right.$$

9

$$x_n = 1 + 4^n + \frac{1}{3}n$$

$$\lim_{n \rightarrow \infty} a_n + \lim_{n \rightarrow \infty} a_n =$$

(a) 0

(b) 1

(c) -1

(d) 2

$1 + 1 + \left(\frac{1}{3}\right)^n$ n even
 $0 + \left(\frac{1}{3}\right)^n$ n odd

Q $\Rightarrow \langle a_n \rangle = \langle 1, 1, \frac{1}{2}, 1, \frac{1}{2}, \frac{1}{3}, 1, \frac{1}{4}, \frac{1}{5}, \frac{1}{4}, 1, \dots \rangle$

~~(a) $\langle a_n \rangle$ has infinite number of limit points.~~

~~(b) $\limsup a_n = 1$~~

~~(c) $\liminf a_n = 0$~~

~~(d) $\langle a_n \rangle$ has infinite convergent~~

$\langle 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots \rangle$

$\langle \frac{1}{n} \rangle$

subseq.

(a) a, b

(b) b, c

(c) a, b, c

(d) a, b, c, d

$\langle 1, 1, 1, \dots \rangle$
 $\langle \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \dots \rangle$
 $\langle \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \dots \rangle$

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Q3. Consider the sequence $a_n = \left[1 + (-1)^n \frac{1}{n} \right]^n$, then

(a) $\limsup a_n = \liminf a_n = 1$

(b) $\limsup a_n = \liminf a_n = e$

(c) $\limsup a_n = e, \liminf a_n = 1/e$

(d) $\limsup a_n = \liminf a_n = 1/e$

Handwritten notes in green ink:

- e (circled)
- $(1 + 1/n)^n$ (circled)
- $(1 - 1/n)^n$ (circled)
- e^{-1} (circled)

$n - \text{even}$
 $n - \text{odd}$

Q4. Let $a_n = \begin{cases} 2 + \frac{(-1)^{\frac{n-1}{2}}}{n} & \text{if } n \text{ is odd} \\ 1 + \frac{1}{2^n} & \text{if } n \text{ is even} \end{cases}$ then which of the following are true?

Handwritten notes: $n=1$, $(2) 2^{-1/3}, 2^{-1/5}, \dots$, $1 + \frac{1}{4}, 1 + \frac{1}{8}, \dots$, $1 + \frac{1}{2^n}$, $1 + \frac{1}{2}$

- (a) $\sup \{a_n \mid n \in \mathbb{N}\} = 3$ & $\inf \{a_n \mid n \in \mathbb{N}\} = 1$
- (b) $\liminf a_n = \limsup a_n = 3/2$
- (c) $\sup \{a_n \mid n \in \mathbb{N}\} = 2$ and $\inf \{a_n \mid n \in \mathbb{N}\} = 1$
- (d) $\liminf a_n = 1$ & $\limsup a_n = 3$

$$2 + \frac{(-1)^0}{1}$$



Q5. Let $\langle a_n \rangle = 1 + \frac{(-1)^n}{n}$, $n \in N$, then which of the following is/are true.

(a) $\langle a_n \rangle$ is finitely oscillatory sequence.

(b) $\langle a_n \rangle$ is convergent sequence

(c) $\overline{\lim}_{n \rightarrow \infty} a_n \neq \lim_{n \rightarrow \infty} a_n$

(d) $\langle a_n \rangle$ is divergent sequence.

(e) lvl

Q7. The limit inferior of the sequence $\langle a_n \rangle$, where

$$\langle a_n \rangle = 1 + (-1)^n + \frac{1}{3^n}.$$

(a) 1

(b) 2

(c) 3

(d) 0

Q8. If $\langle a_n \rangle = 1 + (-1)^n + \sin \frac{1}{n}$ then

- (a) $\langle a_n \rangle$ is convergent sequence
- (b) $\limsup a_n \neq \liminf a_n$
- (c) $\limsup a_n = \liminf a_n$
- (d) $\limsup a_n = 2$

Handwritten notes and diagrams:

- A red bracket groups the terms $2 + \sin \frac{1}{n}$ (for n -even) and $\sin \frac{1}{n}$ (for n -odd).
- A green circle contains the number 2.
- A green circle contains the expression $\sin \frac{1}{n}$.
- A green circle contains the number 0.
- Red text: n -even, n -odd.

Some important theorem on Limit :

(1) If $\lim_{n \rightarrow \infty} a_n = l$ then $\lim_{n \rightarrow \infty} |a_n| = |l|$ But converse may not true

Example : Let $\langle a_n \rangle = \langle (-1)^n \rangle$ then $\langle |a_n| \rangle = \langle 1 \rangle$

Here $\lim |a_n| = 1$ but limit a_n does not exist

(2) **Cauchy's First Theorem :** Let $\langle a_n \rangle$ be a sequence of real numbers

and $\lim_{n \rightarrow \infty} a_n = l$ then $\lim_{n \rightarrow \infty} \frac{a_1 + a_2 + \dots + a_n}{n} = l$

Q.9. Find the Limit of $\frac{1 + \sqrt{2} + \sqrt[3]{3} + \dots + \sqrt[n]{n}}{n}$

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(a) 1

(b) 2

(c) 3

(d) 4

(2) Cauchy's Second theorem : Let $\langle a_n \rangle$ be a sequence of real number

and $\lim_{n \rightarrow \infty} a_n = l$ Then $\lim_{n \rightarrow \infty} (a_1 \cdot a_2 \cdot \dots \cdot a_n)^{\frac{1}{n}} = l$



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Educator highlights

- Works at Pacific Science College
- Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in Udaipur, Rajasthan, India
- Unacademy Educator since



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