Gajendra Purohit

Legend in CSIR-UGC NET & IIT-JAM

~ Unlock Code : GPSIR ~ PhD, CSIR NET (Maths) | Youtuber(800K+165K Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author

50M Watch mins | **3M** Watch mins (last 30 days) | **44K** Followers | **2K** Dedications

- **TOP EDUCATOR ON UNACADEMY**
  FOR CSIR NET & IIT JAM

YouTuber with 800K Subscribers

- **AUTHOR OF BEST SELLER BOOK**
  FOR CSIR NET & IIT JAM

Get 10% Off — **Referral Code : GP SIR**


unacademy
Detailed Course 2.0 on
Sequence and Series For IIT JAM' 23
October 26
9:00 AM
Gajendra Purohit
Enroll Now
Use code GPSIR for 10% off
unacademy


unacademy
DETAILED COURSE 2.0
GROUP THEORY FOR IIT JAM 2023
6th OCTOBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 908 / mo Total ₹ 21,780	You get 6 months extra for free · Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,248 / mo Total ₹ 14,974	You get 6 months extra for free · Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,497 / mo Total ₹ 13,475	
6 months Save 25%	₹ 2,042 / mo Total ₹ 12,252	
3 months Save 17%	₹ 2,269 / mo Total ₹ 6,807	
1 month	₹ 2,723 / mo Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

GPSIR

Awesome! You get 10% off

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 817 / mo ₹ ~~21,780~~ ₹ 19,602	You get 6 months extra for free · Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,123 / mo ₹ ~~14,974~~ ₹ 13,477	You get 6 months extra for free · Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,348 / mo ₹ ~~13,475~~ ₹ 12,128	
6 months Save 25%	₹ 1,838 / mo ₹ ~~12,252~~ ₹ 11,027	
3 months Save 17%	₹ 2,042 / mo ₹ ~~6,807~~ ₹ 6,126	

Proceed to pay

Basic Properties of Homomorphism

1. $f(e) = e`$, where e and e` are identity elements of G and G` respectively.

2. $f(x^{-1}) = (f(x))^{-1}$, for all $x \in G$

3. $f(x^n) = (f(x))^n$ for all $x \in G, n \in Z$

4. Let $f : G \to G`$ be a homomorphism, then ker f is a normal subgroup of G.

Fundamental Theorem of Homomorphism :

Let $f : G \to G`$ is a onto homomorphsim from G to G` if ker(f) is a kernel of f, then

$$\frac{G}{\ker(f)} \approx f(G) \approx G`$$

$$\Rightarrow \quad \frac{G}{\ker(f)} \approx G`$$

Note : Let f : G → G\` is a group homomorphism then $\frac{G}{\ker(f)}$ is a subgroup of G\` because $\frac{G}{\ker(f)} \approx f(G)$ & f(G) is a subgroup of G\`, then

$$O\left[\frac{G}{\ker(f)}\right] \Big| O(G`)$$.

Q.3. Let $f : Z_{14} \rightarrow Z_{10}$ be a homomorphism with $O(\ker f) = 7$ then order of range set of f is

(a) 1 (b) 2

(c) 5 (d) 10

Q.4. Let $f : Z_{10} \to Z_8$ be a homomorphism then which of the following possible order for Kernal of f

(a) 2 (b) 5

(c) 8 (d) 1

Group Homomorphism

Group isomorphism : A mapping $f : (G, o) \to (G`, o`)$ is said to be a group isomorphism if

(a) f is group homomorphism.

(b) f is one-one

(c) f is onto

Note : If a mapping $f : G \to G`$ is a group isomorphism then G & G` are called isomorphic group.

General method :

To show that G & G\` are isomorphism group.

Given that $G = (\{1, w, w^2\}, .\}$ & $G` = (0, 1, 2\}, +_3)$

	1	w	w^2
1	1	w	w^2
w	w	w^2	1
w^2	w^2	1	w

&

$+_3$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

(1) $O(G) = O(G`) = 3$

(2) Number of elements of order 3 are 2 in both group.

So, both are isomorphic.

We can write $G \cong G`$.

Note : If G & G\` are two isomorphic group, then we can write $G \cong G`$.

Conclusion : The additive group of integer $G = (Z, +)$ is isomorphic to the additive group $G` = (mZ, +)$; $m \neq 0$.

Important results :

1. Any two cyclic group of equal order are isomorphic.
2. Any cyclic group of order n is isomorphic to $(Z_n, +_n)$
3. An infinite cyclic group is isomorphic to the addditive group of integer i.e. isomorphic to $(Z, +)$
4. Every group of prime order is cyclic.
5. Let G be a finite group of order pq i.e. $O(G) = pq$ (where $p < q$ & p, q are prime)

   Case – 1 : If $p \nmid q - 1$, then G is cyclic.

   Case – 2 : If $p \mid q - 1$, then G need not be cyclic.

Q.1. Let G be a group of order 15. Then total number of non-isomorphic subgroup of G are

(a) 1 (b) 2

(c) 3 (d) 4

Q.2. Let G be a cyclic group of order 12, then the number of non-isomorphic subgroup of G is

(a) 2 (b) 4

(c) 6 (d) 12

Result : Let G be a abelian group of order n, then A mapping $f : G \to G$ s.t. $f(x) = x^m$ is isomorphism iff $\gcd(m, n) = 1$.

Example : Let G be a abelian group of orde 6, then mapping $\phi(x) = x^5$ is isomorphism.

Q.3. Let G be a group of order 7 and $\phi(x) = x^4$; $x \in G$. Then ϕ is

(a) Not one-one (b) Not a homomorphism

(c) Not onto (d) Isomorphism

COMPLETE COURSE ON

MATHEMATICS

FOR IIT-JAM 2022

TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINAER ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 908 / mo Total ₹ 21,780	You get 6 months extra for free — Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,248 / mo Total ₹ 14,974	You get 6 months extra for free — Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,497 / mo Total ₹ 13,475	
6 months Save 25%	₹ 2,042 / mo Total ₹ 12,252	
3 months Save 17%	₹ 2,269 / mo Total ₹ 6,807	
1 month	₹ 2,723 / mo Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code? Proceed to pay

Get 10% Off

After Using My Referral Code

GPSIR

Awesome! You get 10% off

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 817 / mo ₹ ~~21,700~~ ₹ 19,602	You get 6 months extra for free — Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,123 / mo ₹ ~~14,974~~ ₹ 13,477	You get 6 months extra for free — Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,348 / mo ₹ ~~13,475~~ ₹ 12,128	
6 months Save 25%	₹ 1,838 / mo ₹ ~~12,252~~ ₹ 11,027	
3 months Save 17%	₹ 2,042 / mo ₹ ~~6,807~~ ₹ 6,126	

Proceed to pay

FOUNDATION COURSE OF

MATHEMATICS

FOR CSIR-NET

Q.4. G is a group of order 51. Then which of the following statement is false?

(a) All proper subgroup of G are cyclic.

(b) If G has only one subgroup of order 3 and only one subgroup of order 17 then G is cyclic.

(c) G must have an element of order 17.

(d) None of these

Cauchy Theorem : Let G be a finite group and p | O(G); where p is prime, then G has atleast one element of order p.

Result

(1) Let G be a group of order $n = p_1^{n_1} p_2^{n_2} \ldots\ldots p_m^{n_m}$, then number of non-isomorphic abelian group are $X(n_1) . X(n_2) \ldots\ldots X(n_m)$, where X(n) is partition of n.

Example : Number of non-isomorphic abelian group of order $8 = 2^3$ are $X(3) = 3$.

Q.5. Number of abelian group of order 72, which are non-isomorphic

(a) 5 (b) 6 (c) 7
(d) 8

Automorphism : Let G be a group, then the mapping $f : G \to G$ is called automorphism if

(i) f is one-one

(ii) f is onto

(iii) f is homomorphism

i.e. A mapping $f : G \to G$ is called automorphism if it is isomorphism

Note : Let Z = group of integer under addition then $f : Z \to Z$ s.t. $f(x) = mx$; $m \neq \{1, -1\}$

Then it will not be onto mapping.

So, it will not be automorphism.

$\Rightarrow$ Z have only two automorphism.

Automorphism Group :

Let G be a group, then the set of all automorphism of G form a group under the composition of mapping and this is denoted by Aut G.

Example

(1) Let $G = Z$, then $\text{Aut } Z = \{f(x) = x \ \& \ f(x) = -x\}$

$\Rightarrow \text{Aut } Z \approx Z_2$.

(2) Let $G = Z_m$, then $\text{Aut } Z = (Z_m) \approx U(m)$

(i) Suppose $G = Z_4$, then $\text{Aut}(Z_4) \approx U(4) \approx Z_2$

(ii) Suppose $G = Z_5$ then $\text{Aut}(Z_5) \approx U(5) \approx Z_4$.

(3) $\text{Aut}(K_4) \approx S_3$

(4) $\text{Aut }(S_n) \approx S_n$; for all $n \geq 3$

(5) $\text{Aut}(Z_p \times Z_p \ldots\ldots \times Z_p) \approx GL(n, Z_p)$

'n' times

Q6. For any group G, Aut(G) denote the group of automorphism of G. Which of the following are true?

(a) If G is finite, then Aut(G) is finite

(b) If G is cyclic, then Aut(G) is cyclic

(c) If G is infinite, then Aut(G) is infinite

(d) If Aut (G) is isomorphic to Aut(H) then G is isomorphic to H.

CSIR NET DEC-2018

Q7. Let $G = Z_3 \times Z_3$ be a group then order of $Aut(Z_3 \times Z_3)$ is

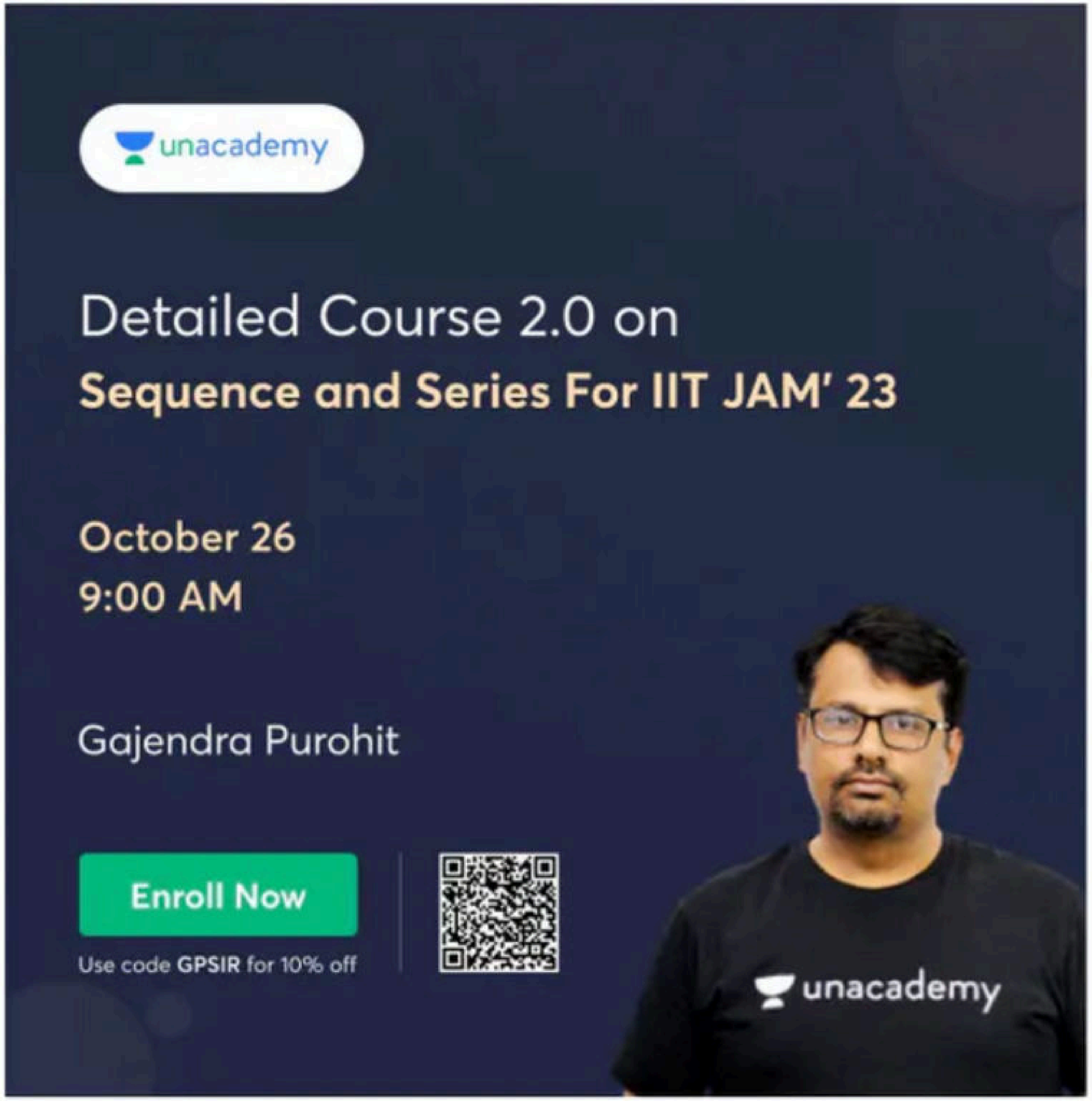
(a) 48 (b) 168

(c) 50 (d) 150

Q8. The order of $\mathrm{Aut}(\mathrm{Aut}(\mathrm{Aut}(K_4)))$ is

(a) 4 (b) 5

(c) 6 (d) 8


unacademy
Detailed Course 2.0 on
Sequence and Series For IIT JAM' 23
October 26
9:00 AM
Gajendra Purohit
Enroll Now
Use code GPSIR for 10% off
unacademy


unacademy
DETAILED COURSE 2.0
GROUP THEORY FOR IIT JAM 2023
6th OCTOBER
Gajendra Purohit
USE CODE
GPSIR
FOR 10% OFF
Enroll Now
unacademy

unacademy CSIR-UGC NET Search Login

Educator Profile

Gajendra Purohit

#5 Educator in CSIR-UGC NET

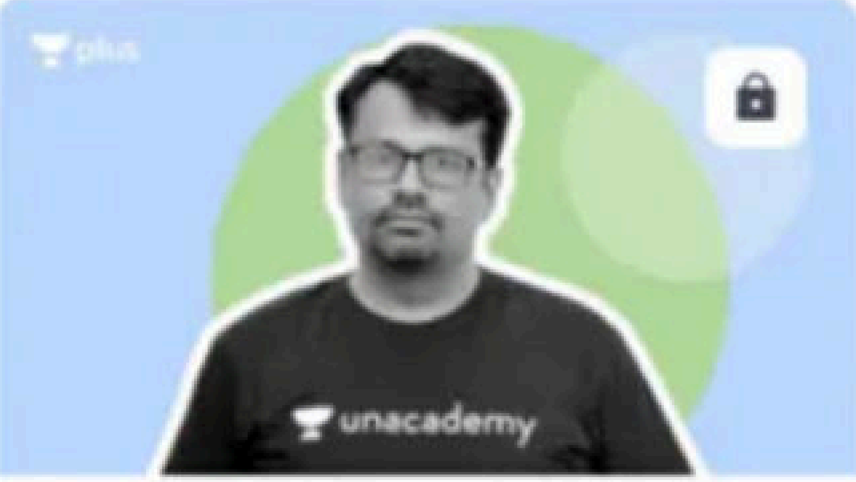
Dr.Gajendra Purohit PhD, CSIR NET (Maths) | Youtuber(330K+30k Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author of Bestseller

Follow

11M Watch mins | 1M Watch mins (last 30 days) | 22k Followers | 1k Dedications

CSIR-UGC NET

SEE ALL



HINDI MATHEMATICAL SCIENCES

Course on Linear Algebra, Partial Diff. Equation & Calculus

Starts on Mar 1, 2021 • 24 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Course on Complex Analysis & Integral Equation

Starts on Jan 14, 2021 • 16 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Foundation Course on Mathematics for CSIR 2021

Starts on Dec 7, 2020 • 20 lessons

Gajendra Purohit

Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 908 / mo Total ₹ 21,780	You get 6 months extra for free Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,248 / mo Total ₹ 14,974	You get 6 months extra for free Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,497 / mo Total ₹ 13,475	
6 months Save 25%	₹ 2,042 / mo Total ₹ 12,252	
3 months Save 17%	₹ 2,269 / mo Total ₹ 6,807	
1 month	₹ 2,723 / mo Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 817 / mo ~~₹ 21,780~~ ₹ 19,602	You get 6 months extra for free Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,123 / mo ~~₹ 14,974~~ ₹ 13,477	You get 6 months extra for free Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,348 / mo ~~₹ 13,475~~ ₹ 12,128	
6 months Save 25%	₹ 1,838 / mo ~~₹ 12,252~~ ₹ 11,027	
3 months Save 17%	₹ 2,042 / mo ~~₹ 6,807~~ ₹ 6,126	

GPSIR

Awesome! You get 10% off

Proceed to pay