Gamma Function:

If m and n are non-negative integers, then

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$

where $\Gamma(n)$ is called gamma function which satisfied the following properties

$\Gamma(n+1) = n\Gamma(n) = n!$ $\quad$ *i.e.* $\Gamma(1) = 1$ and $\Gamma(1/2) = \sqrt{\pi}$

Q.1. Evaluate $\int_0^\infty x^{1/4} e^{-\sqrt{x}} dx$

(a) $2\sqrt{\pi}$

(b) $\frac{3}{2}\pi$

(c) $\sqrt{\pi}$

(d) $\frac{3}{2}\sqrt{\pi}$

Q2. If $I_n = \int_0^{\infty} e^{-x} x^{n-1} dx$, then $\int_0^{\infty} e^{-\lambda x} x^{n-1} dx$ is equal to

(a) λI_n

(b) $\frac{1}{\lambda} I_n$

(c) $\frac{I_n}{\lambda^n}$

(d) $\lambda^n I_n$

Q.3. Let a,b be positive real numbers such that a < b Given that $\lim_{n\to\infty}\int_0^n e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$ Then value of $\lim_{n\to\infty}\int_0^n \frac{1}{t^2}\left(e^{-at^2} - e^{-bt^2}\right)dt$ is

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(a) $\sqrt{\pi}\left(\sqrt{b}-\sqrt{a}\right)$

(b) $\sqrt{\pi}\left(\sqrt{b}+\sqrt{a}\right)$

(c) $-\sqrt{\pi}\left(\sqrt{b}-\sqrt{a}\right)$

(d) $\sqrt{\pi}\left(-\sqrt{b}+\sqrt{a}\right)$

Q4. The value of $\int_0^{\pi/2} \sin^4 x \cos^6 x dx$

(a) $3\pi/312$ (b) $5\pi/512$

(c) $3\pi/512$ (d) $5\pi/312$

Q5. $\int_0^{\pi/2} \sin^7 x dx$ has value

(a) $\frac{37}{184}$ (b) $\frac{17}{45}$

(c) $\frac{16}{35}$ (d) $\frac{16}{45}$

Tracing of curve

(1) Tracing of cartisiancurve :

(a) Symmetric :

(i) If f(x, y) be a given curve and the power of x is even then it is symmetric about y- axis.

(ii) If f(x, y) be a given curve and the power of y is even power then it is symmetric about x-axis.

(iii) If power of x & y both even then this curve is symmetric about x & y axis.

(b) Curve passing through origin :

If $f(x, y)$ be a given curve and $f(0, 0) = 0$ then this curve passes through origin.

(c) Intersection with coordinate axis :

(i) If we put $y = 0$ in given curve then we get intersection point of curve and x-axis.

(ii) If we put $x = 0$ in given curve we get intersection point of curve and y-axis

(d) **Asymtote :**

If f(x, y) is a curve

(i) At $x = a$, if we get $y = \infty$, then Asymtote is parallel to y-axis.

(ii) At $y = a$, if we get $x = \infty$, then Asymtote is parallel to x-axis.

Curve tracing of polar form :

(1) Symmetry :

(i) If $f(r, \theta) = f(r, -\theta)$

Then this curve is symmetric about initially line (i.e. $\theta = 0$ line)

(ii) If $f(r, \theta) = f(r, \pi - \theta)$

Then this curve is symmetric about $\theta = \dfrac{\pi}{2}$ line.

(iii) **Pole :** Put $r = 0$, then find value of θ.

Hence (r, θ) is a pole.

(iv) **Tangent at pole :** Put $r = 0$, then value of θ is tangent at pole.

(v) **Table :**

r									
θ	0	30	45	60	90	120	135	150	180

(vi) **Asymtotes :**

For any value of θ if r become ∞, then a curve has asymtotes.

Q.1. The curve $ay^2 = x^2 (a - x)$ is passing through

(a) (0, 1) (b) (0, 0)

(c) (1, 0) (d) (1, 2)

Q.2. The cardiod $r = a(1 + \cos\theta)$ is symmetric about

(a) $\theta = 0$ line (b) $\theta = \pi/4$ line

(c) $\theta = \pi/2$ line (d) none of these

Q.3. The tangent at origin of the curve $2y^2 = x^2(2 - x)$ is

(a) $x = +2y$ and $x=-2y$ (b) $y = 2x$ and $y=-2x$

(c) $x = y$ and $x=-y$ (d) none of these