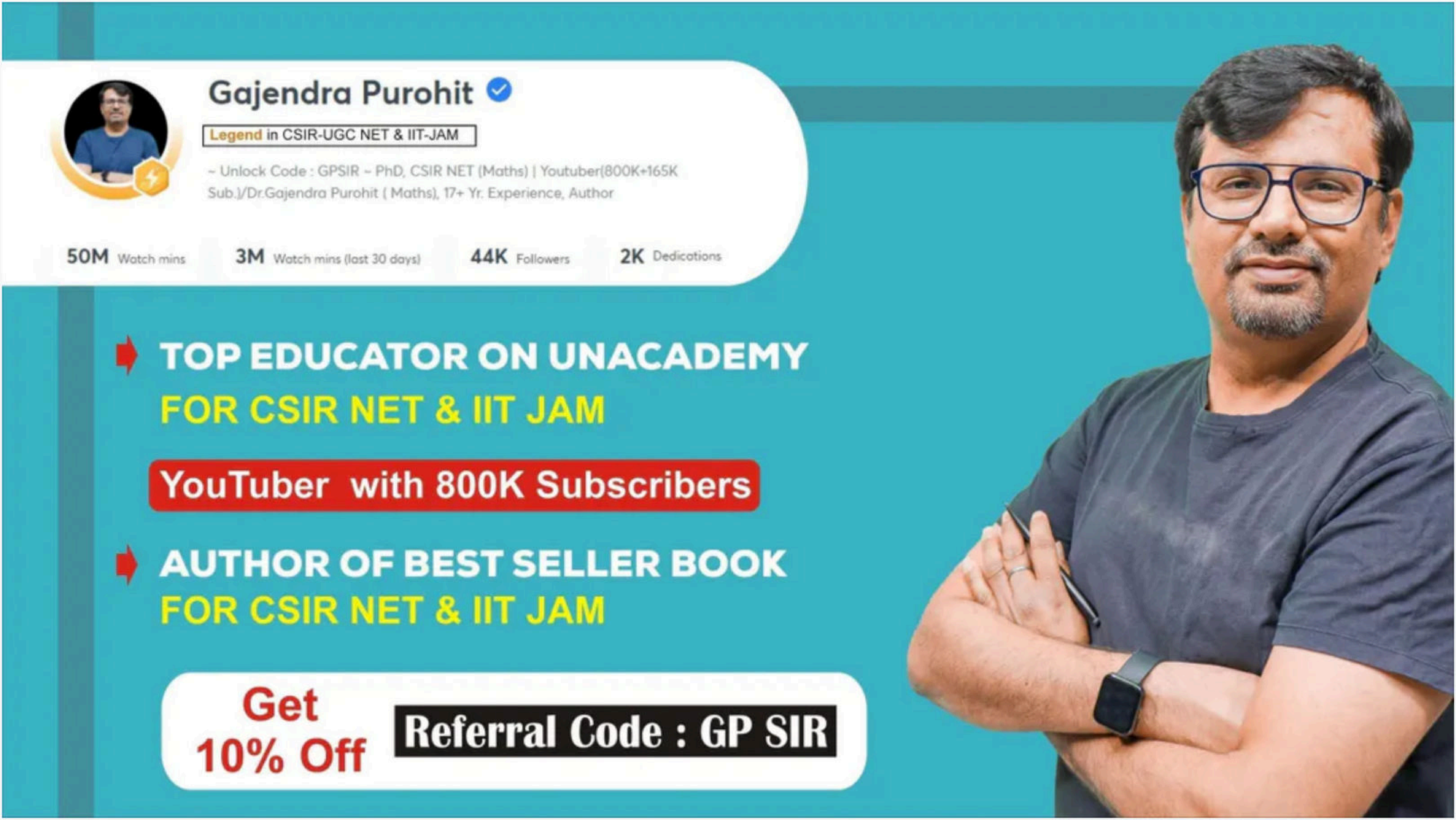


Gajendra Purohit
Legend in CSIR-UGC NET & IIT-JAM
~ Unlock Code : GPSIR ~ PhD, CSIR NET (Maths) | Youtuber(800K+165K Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author
50M Watch mins
3M Watch mins (last 30 days)
44K Followers
2K Dedications
TOP EDUCATOR ON UNACADEMY
FOR CSIR NET & IIT JAM
YouTuber with 800K Subscribers
AUTHOR OF BEST SELLER BOOK
FOR CSIR NET & IIT JAM
Get 10% Off
Referral Code : GP SIR

I

II

III

IV

45

45

Nov

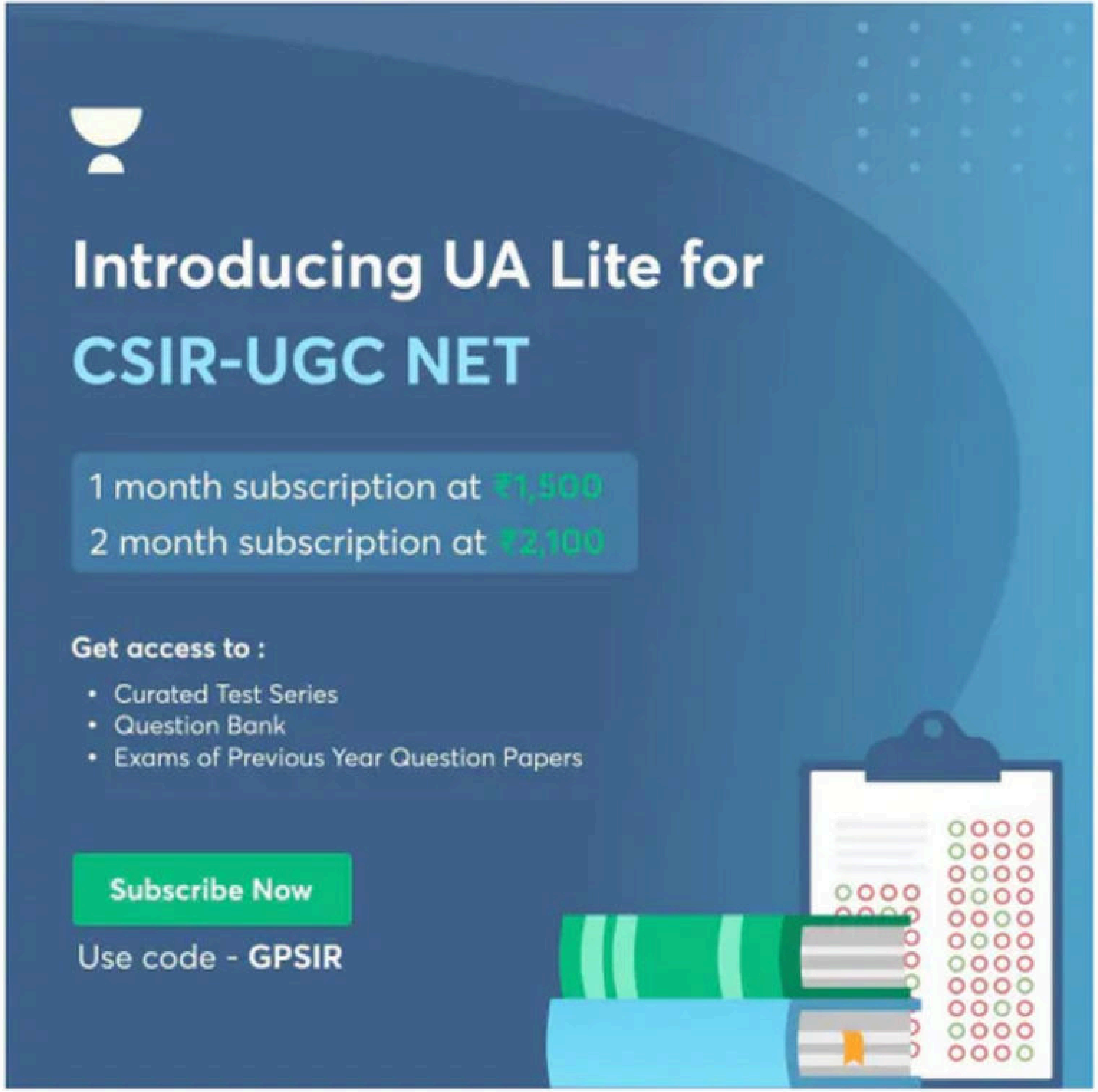
Dec

Jan

12m

15


unacademy
RANK BOOSTER COURSE FOR
CSIR NET 2022
13th JULY
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy


Introducing UA Lite for
CSIR-UGC NET
1 month subscription at ₹1,500
2 month subscription at ₹2,100
Get access to :
• Curated Test Series
• Question Bank
• Exams of Previous Year Question Papers
Subscribe Now
Use code - GPSIR

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780	You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974	You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475		
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252		
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807		
1 month		₹ 2,723 / mo	Total ₹ 2,723		

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	~~₹ 21,700~~ ₹ 19,602	You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo	~~₹ 14,974~~ ₹ 13,477	You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo	~~₹ 13,475~~ ₹ 12,128		
6 months	Save 25%	₹ 1,838 / mo	~~₹ 12,252~~ ₹ 11,027		
3 months	Save 17%	₹ 2,042 / mo	~~₹ 6,807~~ ₹ 6,126		

GPSIR

Awesome! You get 10% off

Proceed to pay

Definite integral

Definition : If $\dfrac{d}{dx}[f(x)] = \phi(x)$ and a & b are constant, then

$$\int_a^b \frac{d}{dx} f(x) = \int_a^b \phi(x)\,dx$$

$$\int_a^b \phi(x)\,dx = [f(x)]_a^b = f(b) - f(a)$$

is called definite integration of $\phi(x)$ within limit a & b.

Note : This is also called fundamental theorem of calculus.

$$\int_0^1 e^x\,dx = \left(e^x\right)_0^1 = \left(e^1 - e^0\right) = \left(e^1 - 1\right)$$

$$\int_1^2 \frac{dx}{x} = \left(\ln x\right)_1^2 = \ln 2 - \ln 1 = \underline{\ln 2}$$

Basic properties of definite integrals.

(1) $$\int_a^b f(t)dt = \int_a^b f(x)dx$$

(2) $$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

(3) $$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

For any c $\in$ (a, b)

$$\int_1^3 x^2 dx = \int_1^2 x^2 dx + \int_2^3 x^2 dx$$

(4) $$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

(5) $$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd} \end{cases}$$

(6) $$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx; & \text{if } f(2a-x)=f(x) \\ 0; & \text{if } f(2a-x)=-f(x) \end{cases}$$

$\int_0^{\pi} \sin x\, dx = 2\int_0^{\pi/2} \sin x\, dx$

$\sin(\pi - x) = \sin x$

$-(\cos x)_0^{\pi}$

$-(\cos\pi - \cos 0)$

$-(-1-1)$

$= 2$

$= -2(\cos x)_0^{\pi/2}$

$= -2(\cos\pi/2 - \cos 0)$

$= -2(0-1) = 2$

$\cos(\pi - x) = -\cos x$

$\int_0^{\pi} \cos x\, dx = 0$

Definite integral as the limit of a sum :

$$\int_0^1 f(x)dx = \lim_{n\to\infty} \frac{1}{n}\sum_{r=0}^{n} f\left(\frac{r}{n}\right)$$

Where f(x) is continuous function on closed interval [0, 1]

$$\sum_{k=1}^{2n} \frac{1}{k+n} = \frac{1}{n}\sum_{k=1}^{2n} \frac{1}{\frac{k}{n}+1}$$

$$\frac{1}{n}\int_0^2 \frac{1}{x+1}dx$$

Leibnitz's Rule :

If g is continuous on [a, b] and $f_1(x)$ & $f_2(x)$ are differentiable function whose value lies in [a, b] then

$$\frac{d}{dx}\int_{f_1(x)}^{f_2(x)} g(t)dt = g[f_2(x)]f_2`(x) - g(f_1(x))f_1`(x)$$

General form : If g is continuous on [a, b] and $f_1(x)$ & $f_2(x)$ are differentiable function whose value lies in [a, b] then

$$\frac{d}{dx}\int_{f_1(x)}^{f_2(x)} g(x,t)dt = \int_{f_1(x)}^{f_2(x)} \frac{\partial}{\partial x}g(x,t)dt + g[x, f_2(x)]f_2`(x) - g(x, f_1(x))f_1`(x)$$

$$f(x) = \int_{x}^{x^2} e^{-t}\,dt$$

$$f'(x)$$

$$f'(0)$$

$$f'(x) = \frac{d}{dx}\int_{x}^{x^2} e^{-t}\,dt$$

$$= \int_{x}^{x^2} \frac{d}{dx}\left(e^{-t}\right)dt + e^{-x^2}\frac{d}{dx}(x^2) - e^{-x}\frac{d}{dx}(x)$$

$$f'(x) = 0 + e^{-x^2}(2x) - e^{-x}\cdot 1$$

$$f'(0) = 0 - e^{0} = -1$$

$$f(x,\pi) = \int_0^{\pi} \sin(x+t)\,dt \qquad \frac{\partial f}{\partial x}$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\int_0^{\pi} \sin(x+t)\,dt$$

$$= \int_0^{\pi} \frac{\partial}{\partial x}\sin(x+t)\,dt + \sin(x+\pi)\frac{\partial}{\partial x}(\pi) - \sin(0+x)\frac{\partial}{\partial x}(0)$$

$$= +\int_0^{\pi} \cos(x+t)\,dt \qquad \frac{\partial^2 f}{\partial x^2} + f = 0$$

$$\frac{\partial^2 f}{\partial x^2} = -\int_0^{\pi} \sin(x+t)\,dt = -f$$

Gamma Function:

If m and n are non-negative integers, then

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$

COMPLETE COURSE ON

MATHEMATICS

FOR IIT-JAM 2022

TARGETED AUDIENCE

- IIT-JAM
- M.Sc. Entrance Exam

TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINAER ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months Save 67% You get 6 months extra for free	₹ 908 / mo Total ₹ 21,780 Offer expires 15 Jun 2022
12 months Save 54% You get 6 months extra for free	₹ 1,248 / mo Total ₹ 14,974 Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,497 / mo Total ₹ 13,475
6 months Save 25%	₹ 2,042 / mo Total ₹ 12,252
3 months Save 17%	₹ 2,269 / mo Total ₹ 6,807
1 month	₹ 2,723 / mo Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months Save 67% You get 6 months extra for free	₹ 817 / mo ~~₹ 21,700~~ ₹ 19,602 Offer expires 15 Jun 2022
12 months Save 54% You get 6 months extra for free	₹ 1,123 / mo ~~₹ 14,974~~ ₹ 13,477 Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,348 / mo ~~₹ 13,475~~ ₹ 12,128
6 months Save 25%	₹ 1,838 / mo ~~₹ 12,252~~ ₹ 11,027
3 months Save 17%	₹ 2,042 / mo ~~₹ 6,807~~ ₹ 6,126

GPSIR

Awesome! You get 10% off

Proceed to pay

where $\Gamma(n)$ is called gamma function which satisfied the following properties

$$\Gamma(n+1) = n\Gamma(n) = n! \quad i.e.\ \Gamma(1) = 1 \text{ and } \Gamma(1/2) = \sqrt{\pi}$$

In place of gamma function, we can also use the following formula :

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{(m-1)(m-3).....(2 \text{ or } 1)(n-1)(n-3).....(2 \text{ or } 1)}{(m+n)(m+n-2)....(2 \text{ or } 1)}$$

It is important to note that we multiply by $(\pi/2)$; when both m and n are even.

Q1. The value of $\int_0^{\pi/2} \sin^4 x \cos^6 x dx$

(a) $3\pi/312$

(b) $5\pi/512$

(c) $3\pi/512$

(d) $5\pi/312$

Reduction formulae Definite Integration

(1) $\int_0^{\infty} e^{-ax} \sin bx dx = \dfrac{b}{a^2+b^2}$

(2) $\int_0^{\infty} e^{-ax} \cos bx dx = \dfrac{a}{a^2+b^2}$

(3) $\int_0^{\infty} e^{-ax} x^n dx = \dfrac{n!}{a^n+1}$

Q2. If $I_n = \int_0^\infty e^{-x} x^{n-1} dx$, then $\int_0^\infty e^{-\lambda x} x^{n-1} dx$ is equal to

(a) λI_n

(b) $\frac{1}{\lambda} I_n$

(c) $\frac{I_n}{\lambda^n}$

(d) $\lambda^n I_n$

Q3. $\int_0^{\pi/2} \sin^7 x dx$ has value

(a) $\frac{37}{184}$

(b) $\frac{17}{45}$

(c) $\frac{16}{35}$

(d) $\frac{16}{45}$

Q.4. Let a,b be positive real numbers such that a < b Given that $\lim_{n\to\infty}\int_0^n e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$ Then value of $\lim_{n\to\infty}\int_0^n \frac{1}{t^2}\left(e^{-at^2} - e^{-bt^2}\right)dt$ is

IIT JAM 2022

(a) $\sqrt{\pi}\left(\sqrt{b}-\sqrt{a}\right)$

(b) $\sqrt{\pi}\left(\sqrt{b}+\sqrt{a}\right)$

(c) $-\sqrt{\pi}\left(\sqrt{b}-\sqrt{a}\right)$

(d) $\sqrt{\pi}\left(-\sqrt{b}+\sqrt{a}\right)$

Q.6. If $g(x)=\int_{x(x-2)}^{4x-5} f(t)\,dt$, where $f(x)=\sqrt{1+3x^4}$ for $x \in R$, then $g'(1)$ is **JAM – 2019**

(a) 6 (b) 7

(c) 8 (d) 10

$f(-1)=\sqrt{1+3(-1)^4}=2$

$g'(x)=\frac{d}{dx}\int_{x(x-2)}^{4x-5} f(t)\,dt$

$= 0 + f(4x-5)\frac{d}{dx}(4x-5) - f(x^2-2x)\frac{d}{dx}(x^2-2x)$

$g'(x)= 4f(4x-5)-(2x-2)f(x^2-2x)$

$g'(1)= 4f(-1)-0$

$= 4\times 2 = 8$

Q.7. Let f : [0 ,1] $\to$ [0, ∞) be continuous function such that $(f(t))^2 < 1 + 2\int_0^t f(s)ds \quad, \forall t \in [0,1]$

IIT JAM 2021

(a) $f(t) < 1 + t \quad ;\forall t \in [0,1]$

(b) $f(t) > 1 + t \quad ;\forall t \in [0,1]$

(c) $f(t) = 1 + t \quad ;\forall t \in [0,1]$

(d) $f(t) < 1 + t/2 \quad ;\forall t \in [0,1]$

Q.8. The value of the integral $\int_{-\pi}^{\pi} |x| \cos nx\,dx, n \geq 1$ is

JAM - 2016

(a) 0, when n is even (b) 0, when n is odd

(c) $-\frac{4}{n^2}$, when n is even (d) $-\frac{4}{n^2}$, when n is odd

Q.9. Let $f(x)=\int_{\sin x}^{\cos x} e^{-t^2}\,dt$, then $f'\left(\frac{\pi}{4}\right)$ equals

IIT JAM 2006

(a) $\sqrt{\frac{1}{e}}$ (b) $-\sqrt{\frac{2}{e}}$

(c) $\sqrt{\frac{2}{e}}$ (d) $-\sqrt{\frac{1}{e}}$

$f'(x) = \frac{d}{dx}\int_{\sin x}^{\cos x} e^{-t^2}\,dt$

$= 0 + e^{-\cos^2 x}\frac{d}{dx}\cos x - e^{\sin^2 x}\frac{d}{dx}(\sin x)$

$f'(x) = -e^{-\cos^2 x}\sin x - e^{-\sin^2 x}\cos x$

$f'(\pi/4) = -e^{-\cos^2 \pi/4}\sin \pi/4 - e^{-\sin^2 \pi/4}\cos \pi/4$

$= -e^{-1/2}\frac{1}{\sqrt{2}} - e^{1/2}\frac{1}{\sqrt{2}} = \frac{-2e^{-1/2}}{\sqrt{2}}$

$-\sqrt{2}\,e^{-1/2}$

$\frac{-\sqrt{2}}{\sqrt{e}}$

$-\sqrt{\frac{2}{e}}$

Q.10. Let f : R $\to$R be continuous function if $\frac{d}{dx}\int_0^x f(2t)\,dt = \frac{d}{dx}\left(\frac{x}{\pi}\sin(\pi x)\right)$ for all x $\in$ R, then f(2) is equal to **JAM 2007**

(a) -1 (b) 0

(c) 1 (d) 2

$$0 + f(2x)\frac{d}{dx}(x) - f(0)\frac{d}{dx}(0) = \frac{1}{\pi}\left(\sin\pi x\frac{d}{dx}(x) + x\frac{d}{dx}(\sin\pi x)\right)$$

$x=1$

$$f(2x) = \frac{1}{\pi}\left(\sin\pi x + x\pi\cos\pi x\right)$$

$$f(2) = \frac{1}{\pi}\left(\sin\pi + 1\cdot\pi\underbrace{\cos\pi}_{=-1}\right) = \frac{1}{\pi}(\pi(-1))$$

Q.11. Let $f(x)=\int_0^x (x^2 - t^2)g(t)\,dt$, where g is a real valued continuous function on R, then f`(x) is equal to

JAM – 2008

(a) 0

(b) $x^3g(x)$

(c) $\int_0^x g(t)\,dt$

(d) $2x\int_0^x g(t)\,dt$

$f'(x) = \frac{\partial}{\partial x}\int_0^x (x^2 - t^2)g(t)\,dt$

$f'(x) = \int_0^x 2x\,g(t)\,dt$

$+ (x^2 - x^2)\,g(x)\frac{\partial}{\partial x}(x)$

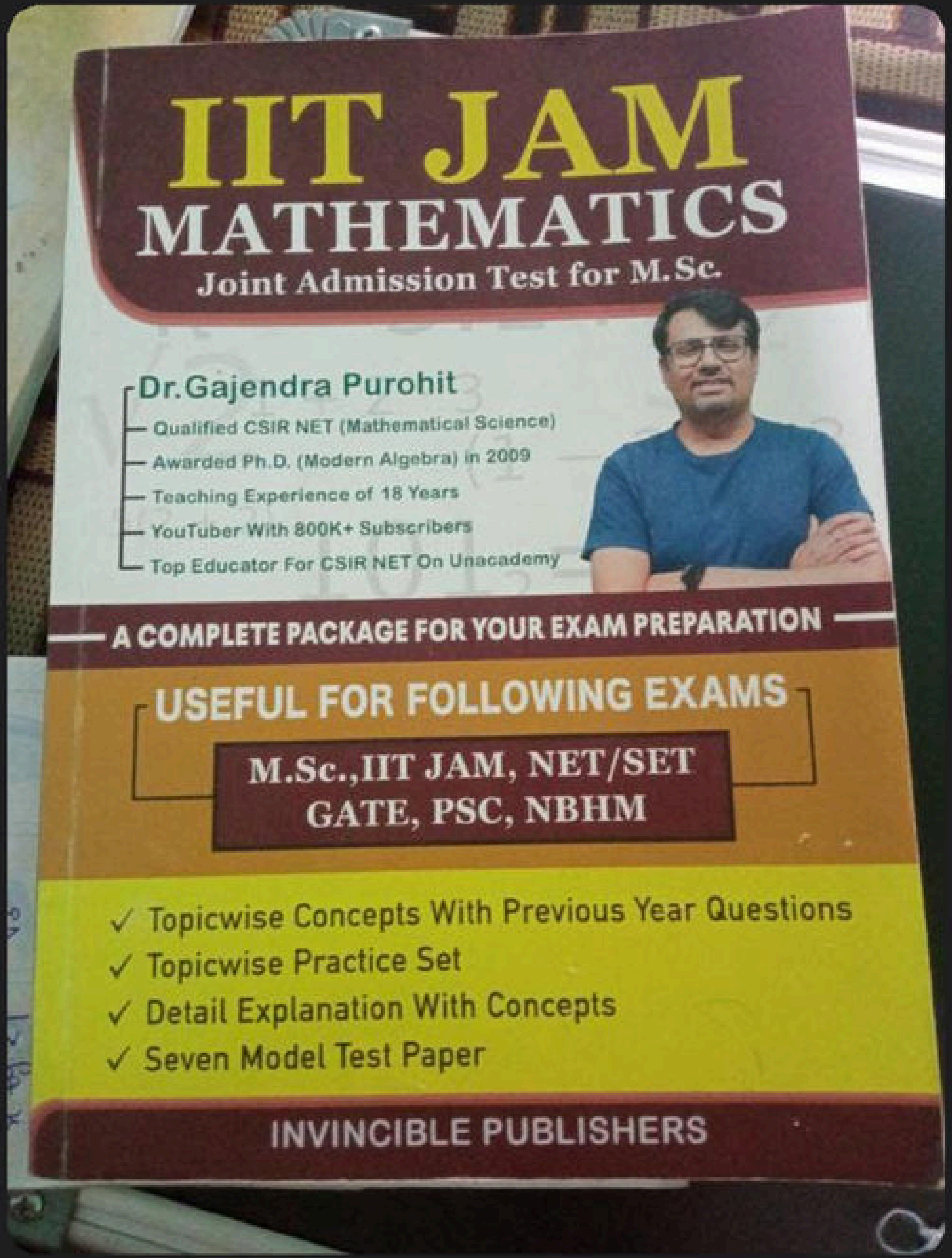
$- (x^2 - 0)\,g(0)\frac{\partial}{\partial x}(0)$

$2x\int_0^x g(t)\,dt$

unacademy

▲ 2 • Asked by Mr Aayush

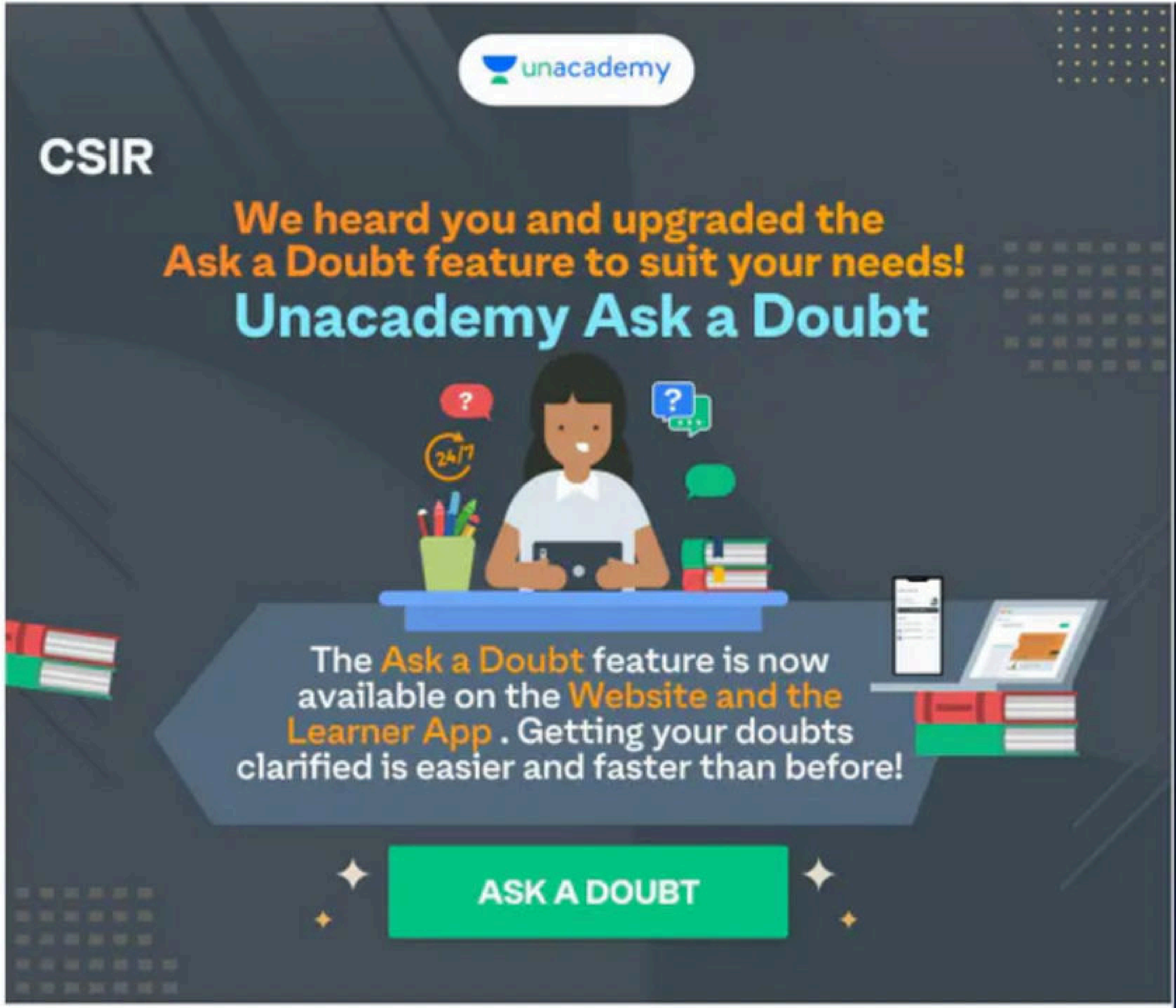
Please help me with this doubt

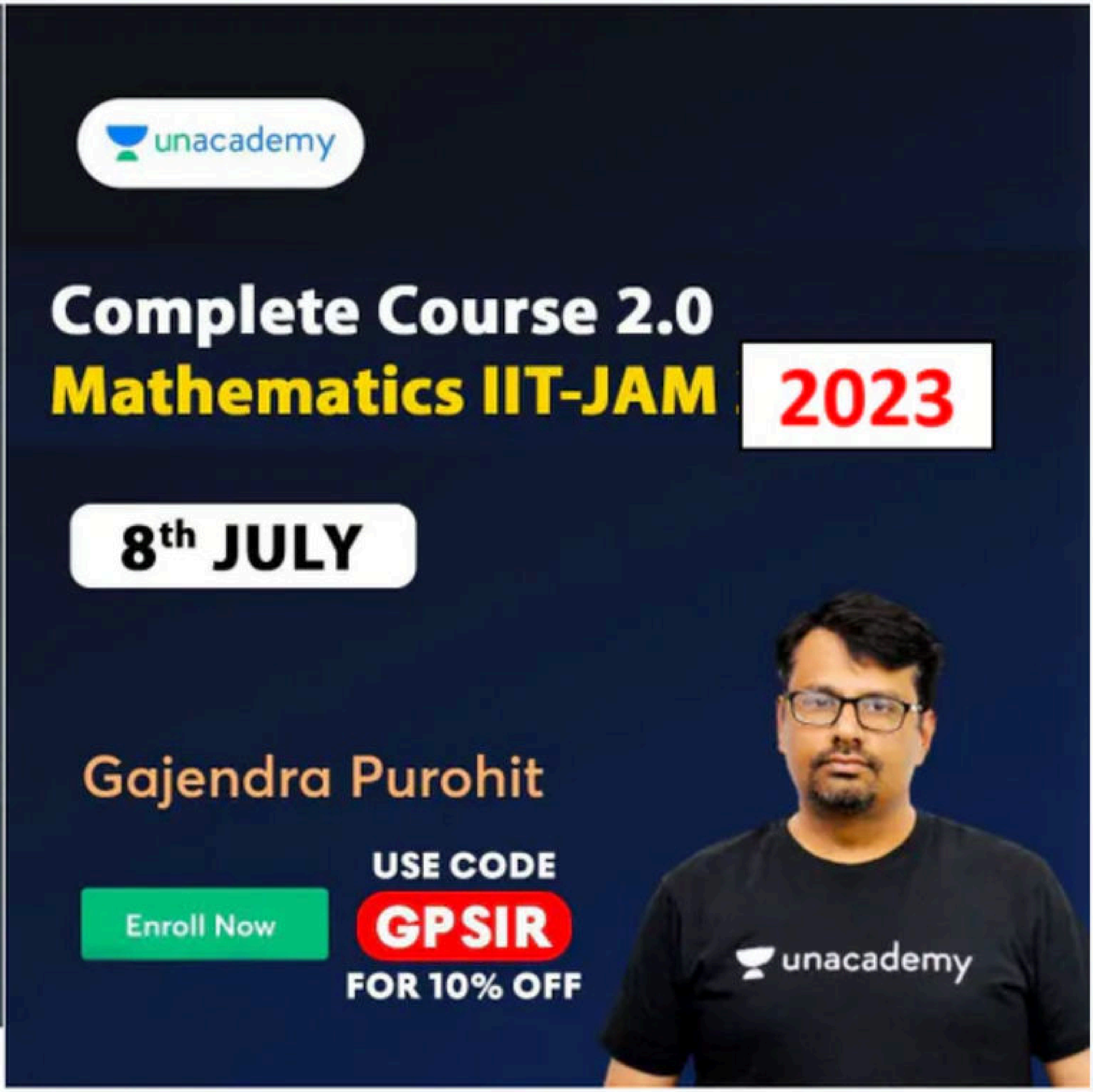


Q.12. Let a be a non-zero real number, then $\lim_{x \to a} \frac{1}{x^2 - a^2} \int_a^x \sin(t^2)\,dt$ equals **JAM – 2009**

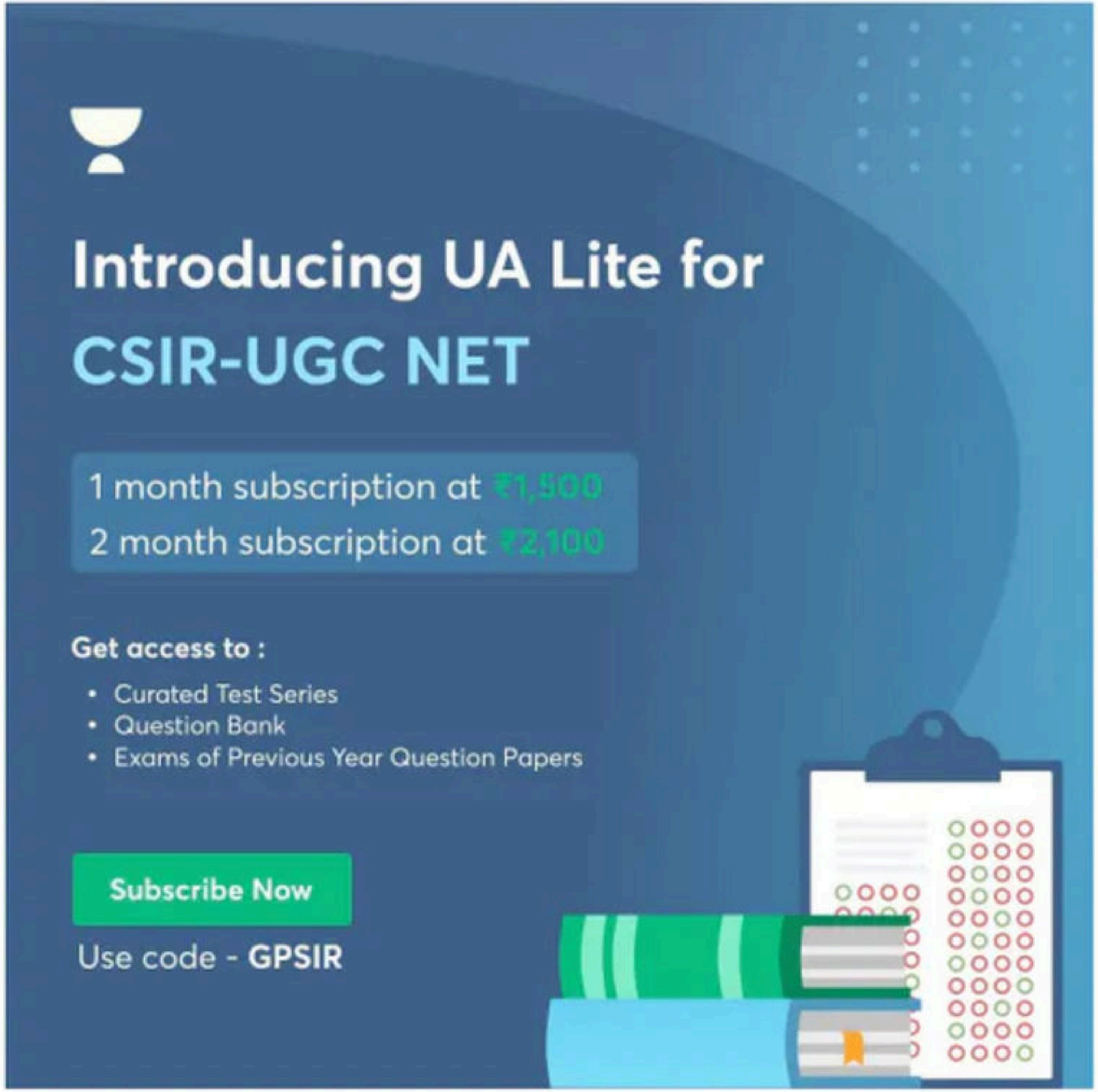
(a) $\frac{\sin(a^2)}{2a}$ (b) $\frac{\cos(a^2)}{2a}$

(c) $-\frac{\sin(a^2)}{2a}$ (d) $-\frac{\cos(a^2)}{2a}$


unacademy
CSIR
We heard you and upgraded the
Ask a Doubt feature to suit your needs!
Unacademy Ask a Doubt
24/7
The Ask a Doubt feature is now available on the Website and the Learner App . Getting your doubts clarified is easier and faster than before!
ASK A DOUBT


unacademy
Complete Course 2.0
Mathematics IIT-JAM
2023
8th JULY
Gajendra Purohit
USE CODE
GPSIR
FOR 10% OFF
Enroll Now
unacademy


unacademy
RANK BOOSTER COURSE FOR
CSIR NET 2022
13th JULY
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy


Introducing UA Lite for
CSIR-UGC NET
1 month subscription at ₹1,500
2 month subscription at ₹2,100
Get access to :
• Curated Test Series
• Question Bank
• Exams of Previous Year Question Papers
Subscribe Now
Use code - GPSIR

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	₹ 908 / mo	
Save 67%	Total ₹ 21,780	
You get 6 months extra for free	Offer expires 15 Jun 2022	
12 months	₹ 1,248 / mo	
Save 54%	Total ₹ 14,974	
You get 6 months extra for free	Offer expires 15 Jun 2022	
9 months	₹ 1,497 / mo	
Save 45%	Total ₹ 13,475	
6 months	₹ 2,042 / mo	
Save 25%	Total ₹ 12,252	
3 months	₹ 2,269 / mo	
Save 17%	Total ₹ 6,807	
1 month	₹ 2,723 / mo	
	Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months	₹ 817 / mo
Save 67%	~~₹ 21,780~~ ₹ 19,602
You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	₹ 1,123 / mo
Save 54%	~~₹ 14,974~~ ₹ 13,477
You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	₹ 1,348 / mo
Save 45%	~~₹ 13,475~~ ₹ 12,128
6 months	₹ 1,838 / mo
Save 25%	~~₹ 12,252~~ ₹ 11,027
3 months	₹ 2,042 / mo
Save 17%	~~₹ 6,807~~ ₹ 6,126

GPSIR

Awesome! You get 10% off

Proceed to pay