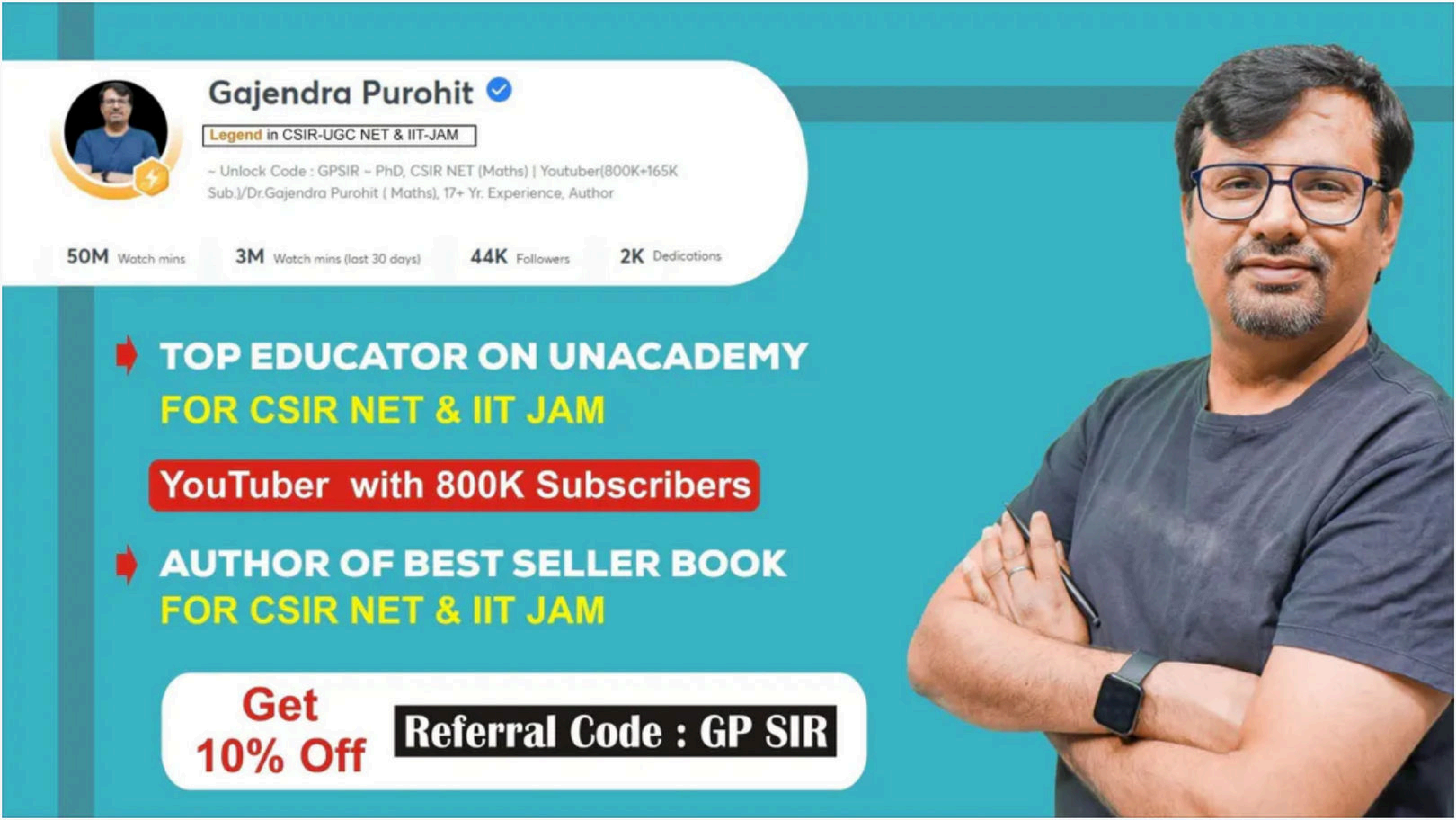
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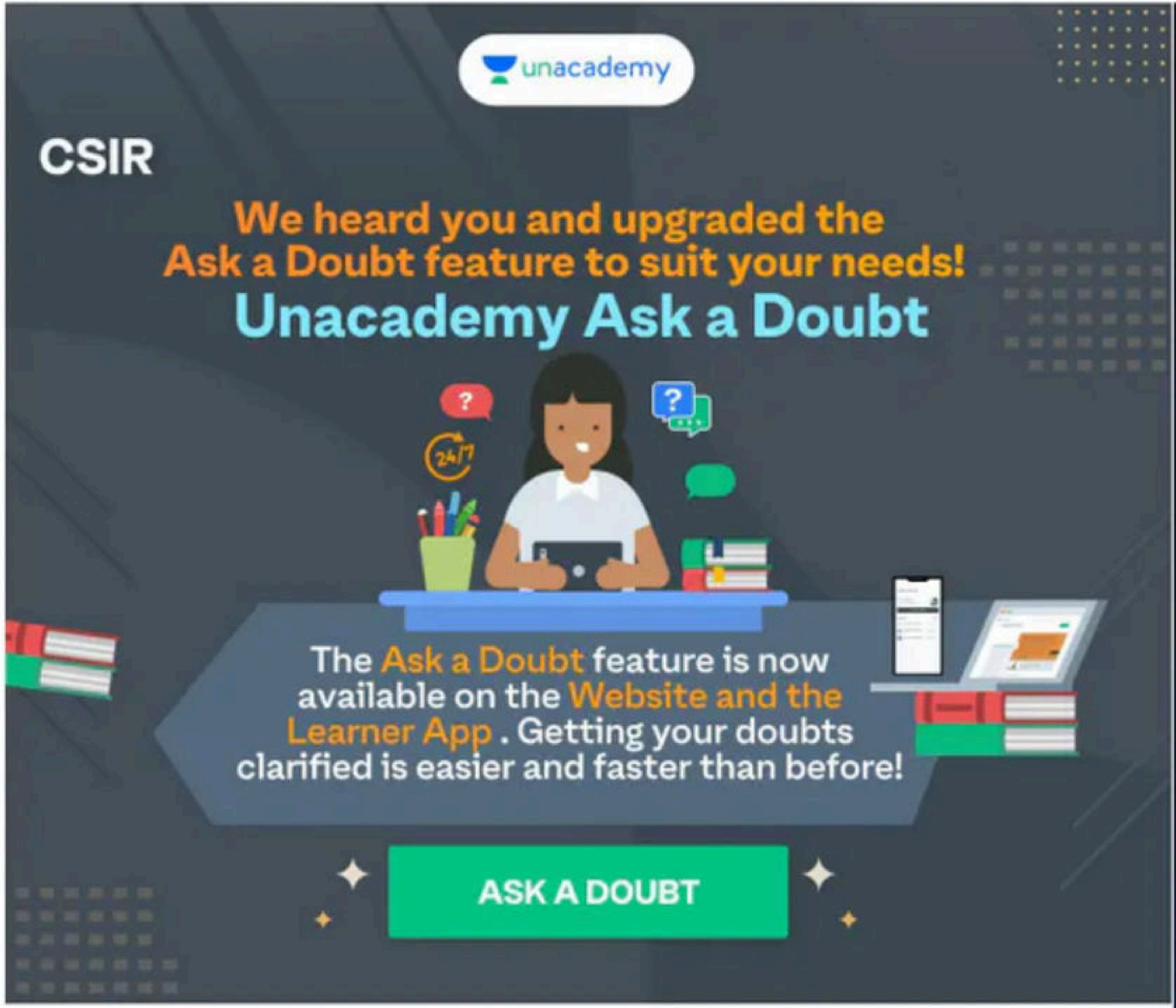

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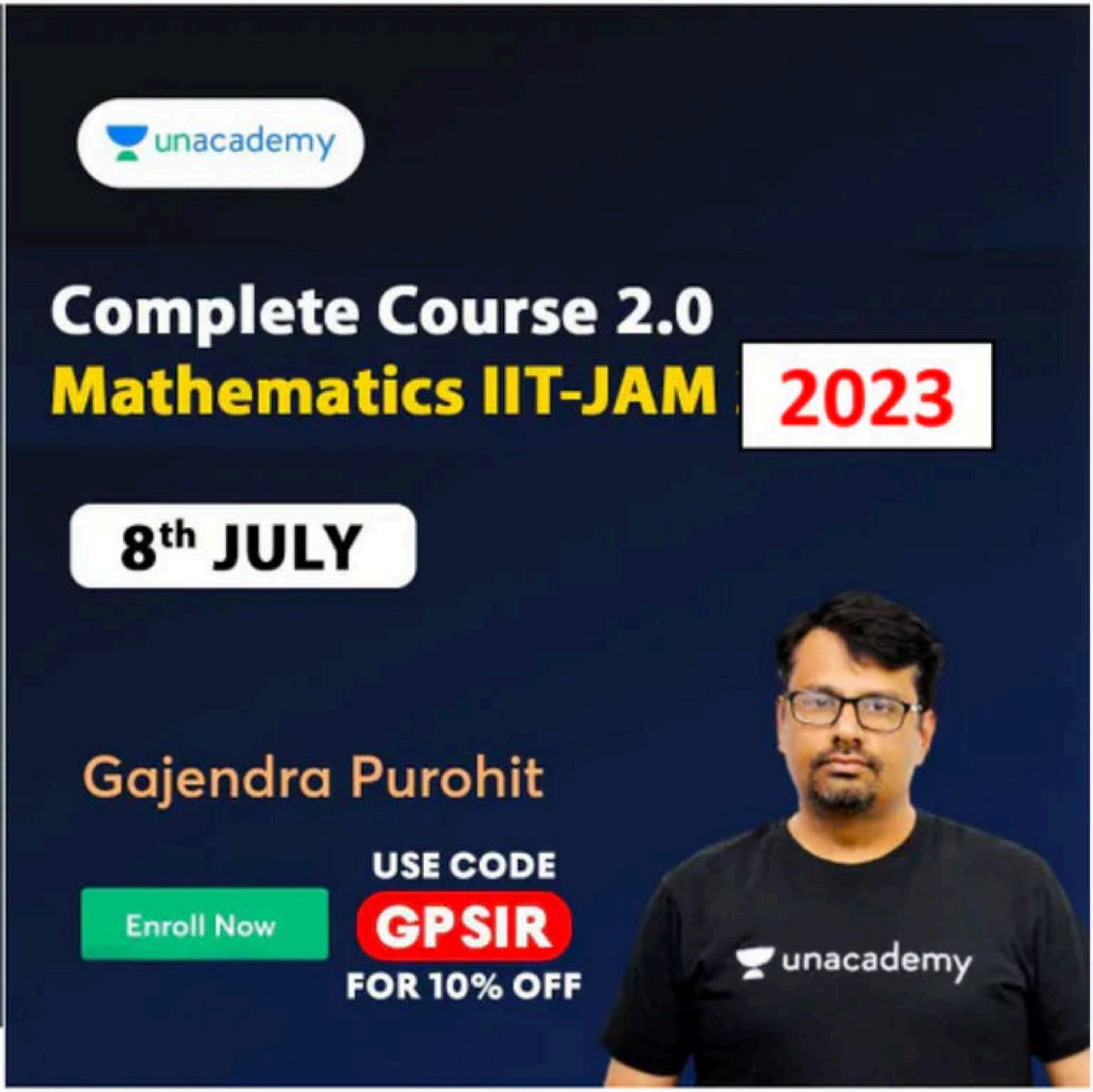

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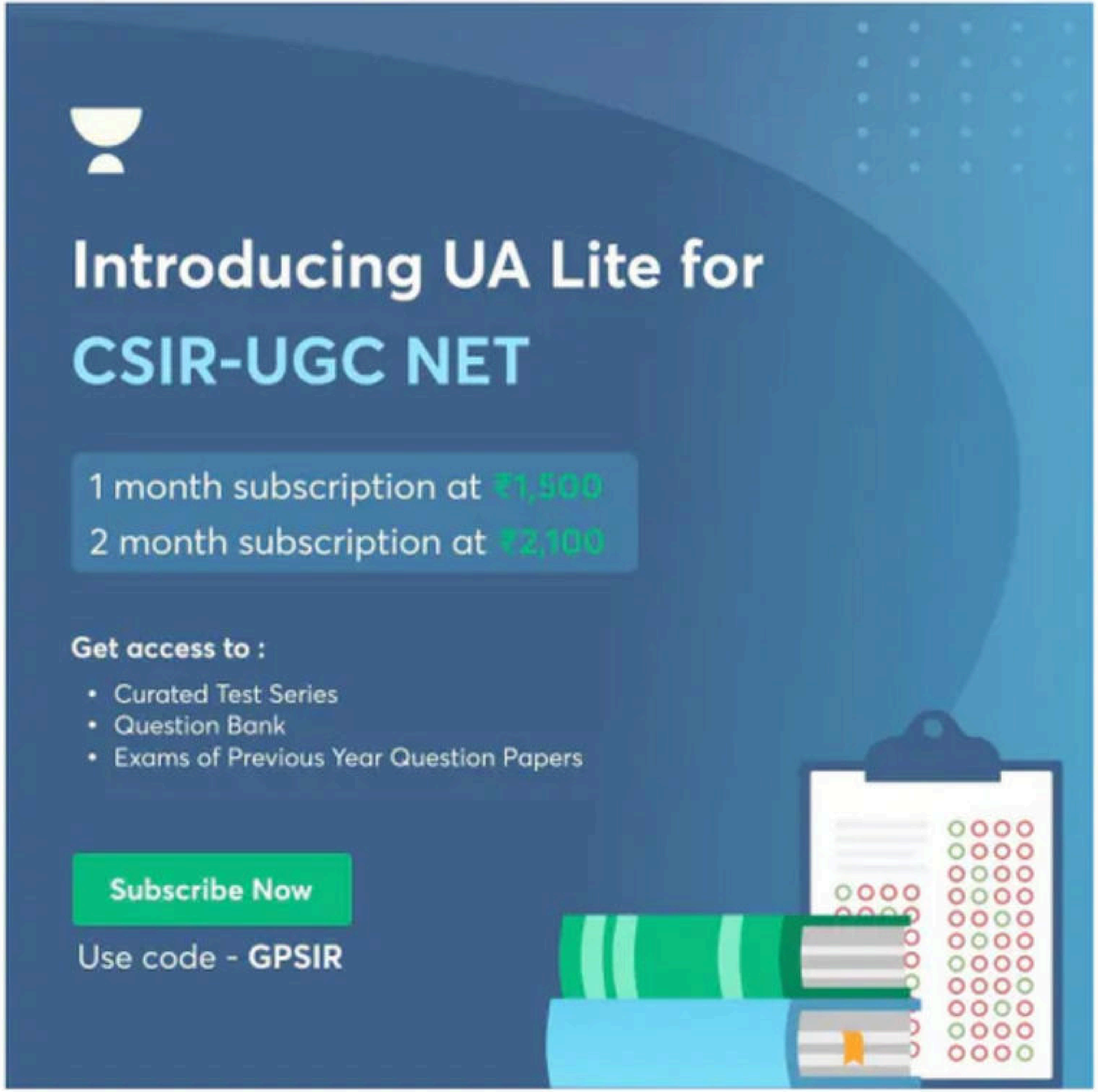
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Curve tracing of polar form :

(1) Symmetry :

(i) If $f(r, \theta) = f(r, -\theta)$

Then this curve is symmetric about initially line (i.e. $\theta = 0$ line)

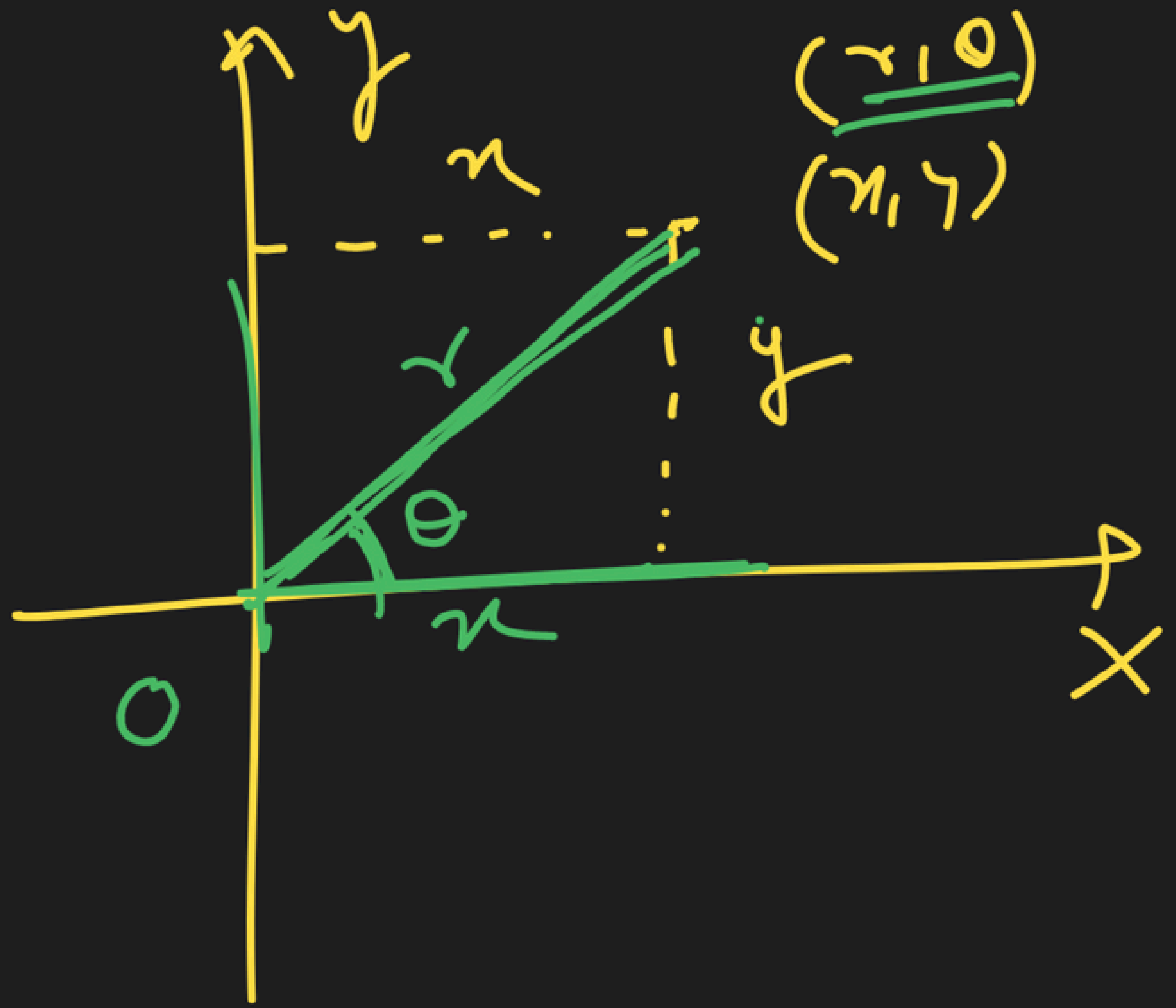
(ii) If $f(r, \theta) = f(r, \pi - \theta)$

Then this curve is symmetric about $\theta = \frac{\pi}{2}$ line.

(iii) **Pole :** Put $r = 0$, then find value of θ.

Hence (r, θ) is a pole.

(iv) **Tangent at pole :** Put $r = 0$, then value of θ is tangent at pole.



$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

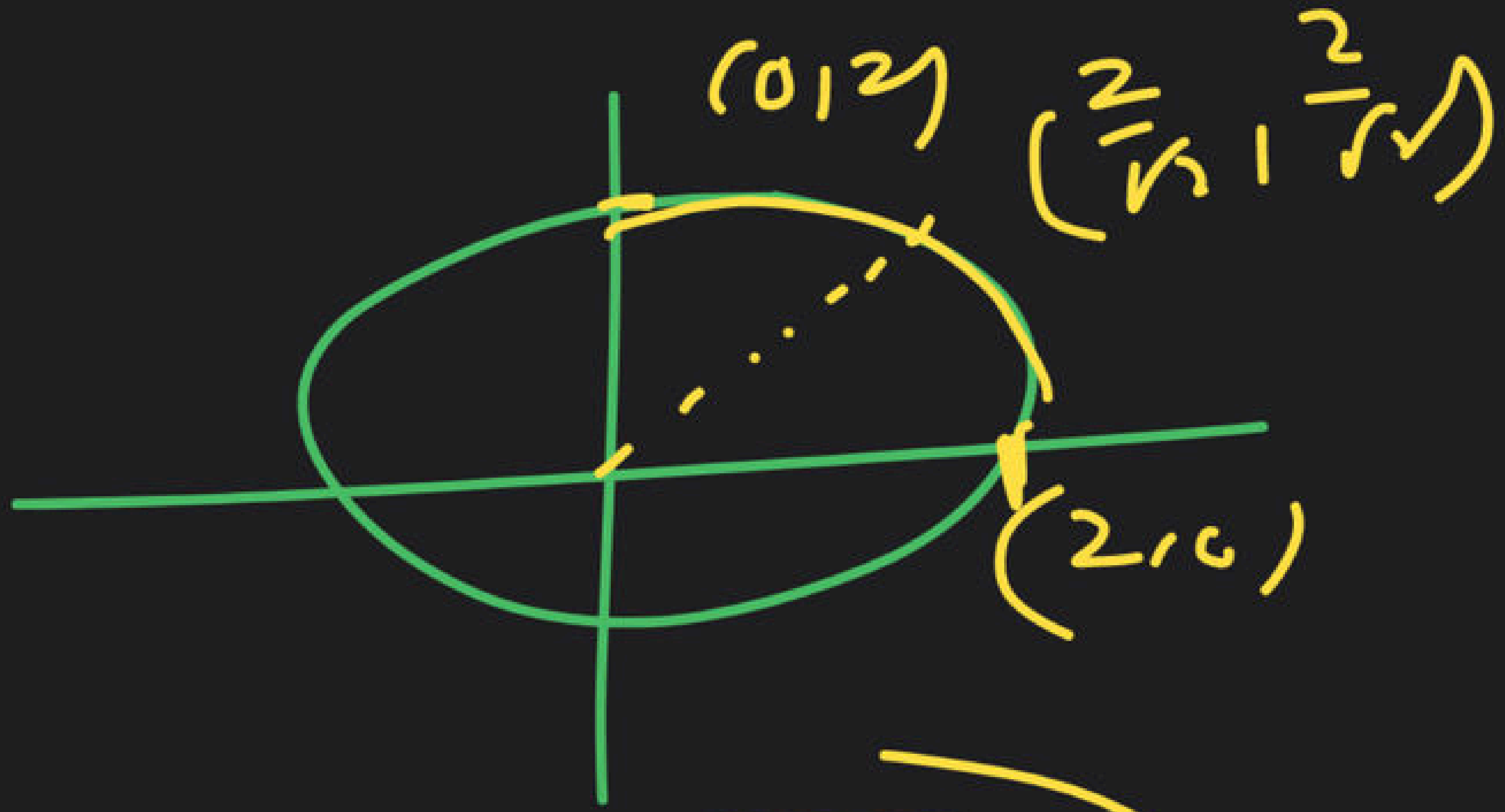
$x = 2\cos\theta$

$y = 2\sin\theta$

$\theta = 45^\circ$

$r = 2$

$\theta =$

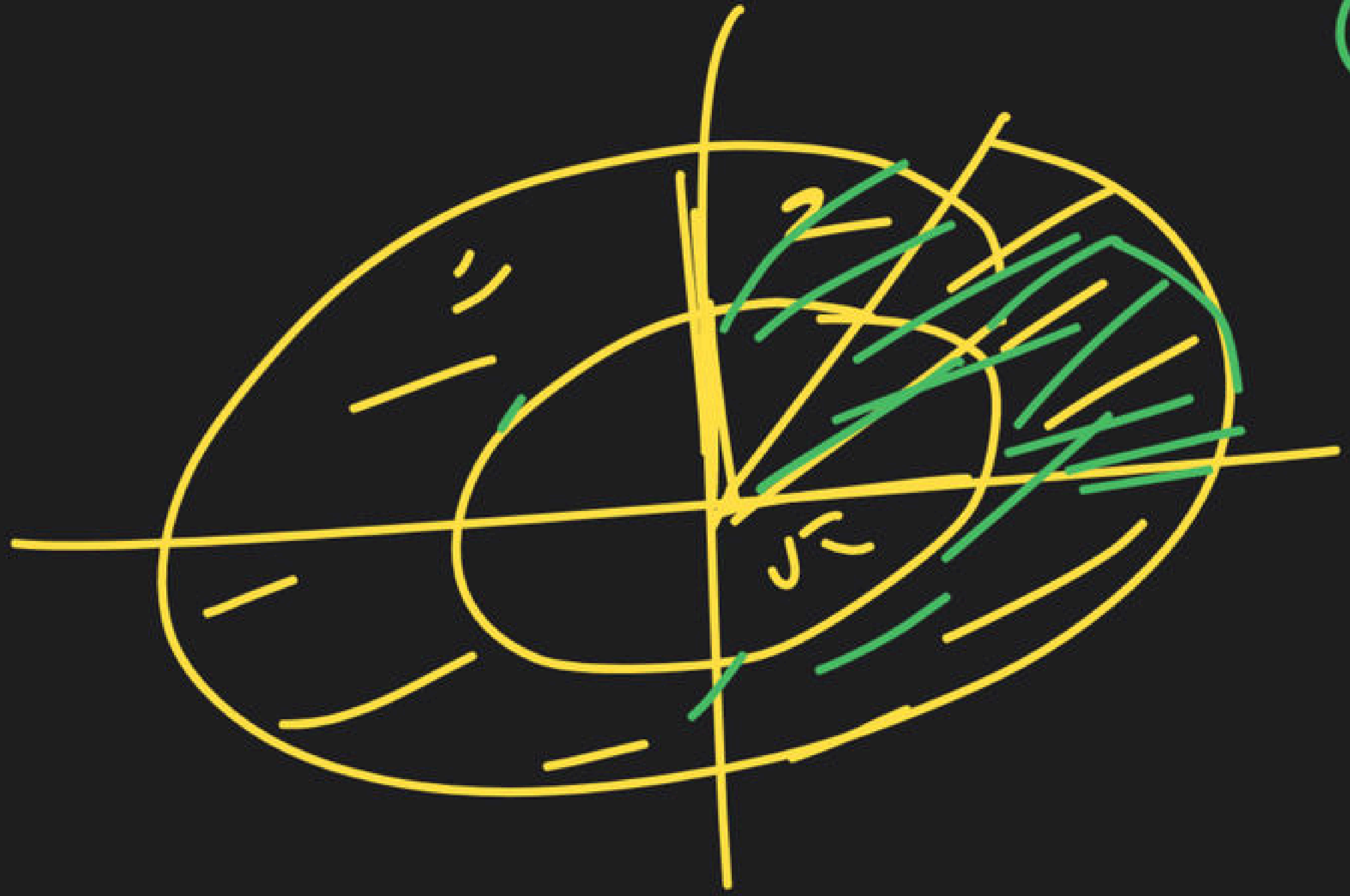


$x^2 + y^2 = 4$

$x = r\cos\theta$

$y = r\sin\theta$

$2 \le x^2 + y^2 \le 4$



$2 \le r^2 \le 4$

$\sqrt{2} \le r \le 2$

$r = \sqrt{2} \rightarrow 2$

$\theta = 0$ to $\pi/2$

$\theta = 0$ to 2π

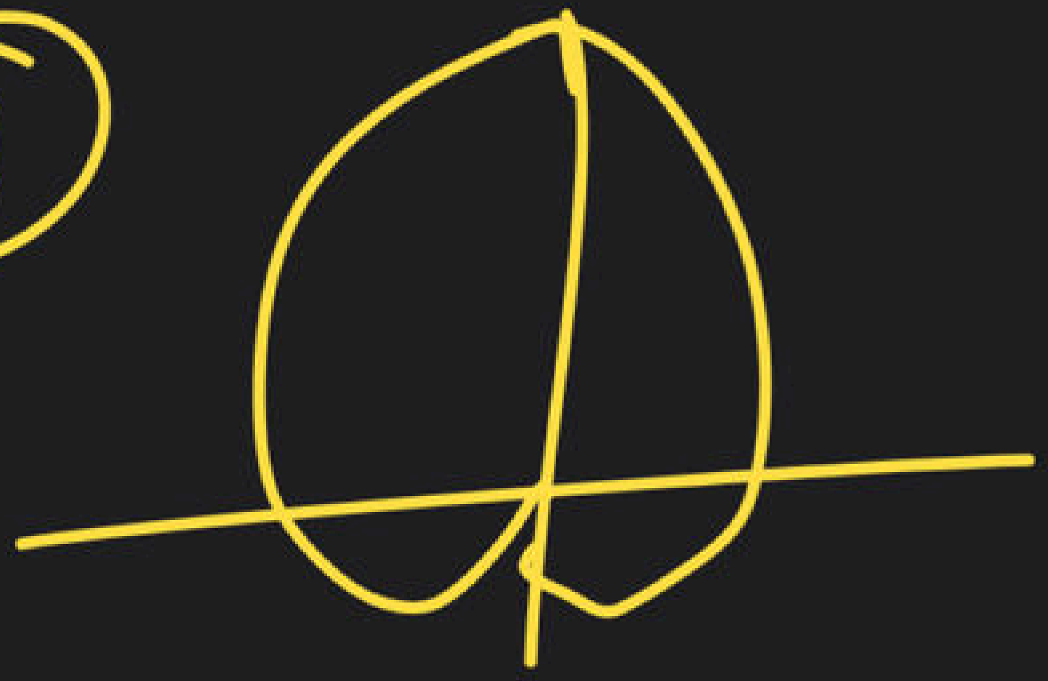
1 Symmetry

$$f(r, \theta) = f(r, -\theta)$$

$$r = a(1 + \cos\theta)$$

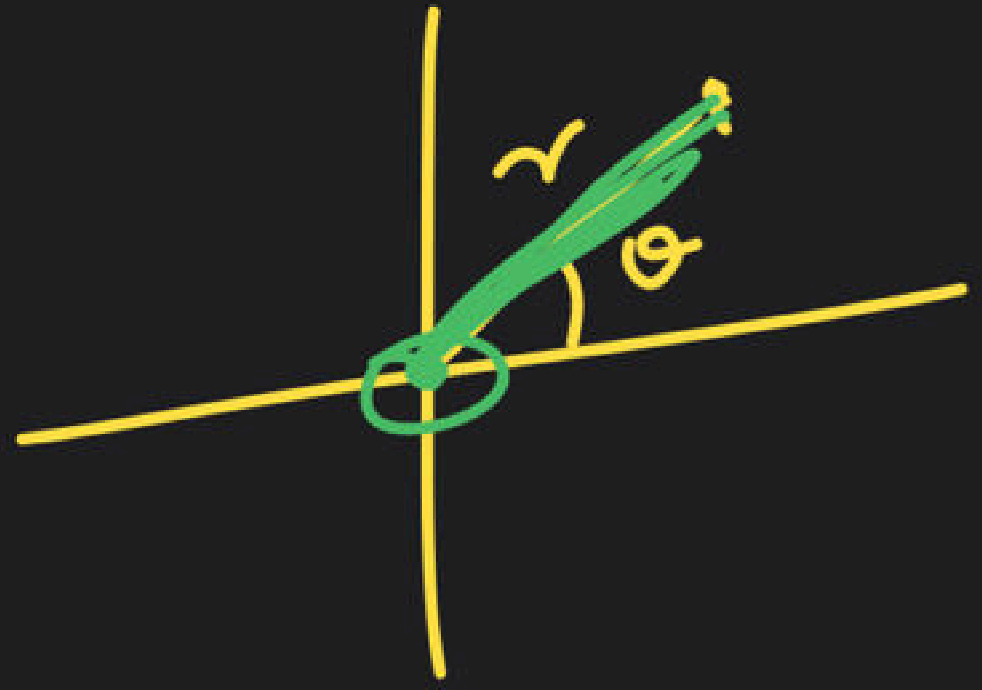
$$f(r, \theta) = f(r, \pi - \theta)$$

$$r = a(1 + \sin\theta)$$



Pole

$r = a(1+\cos\theta) = 0$



for $r=0$

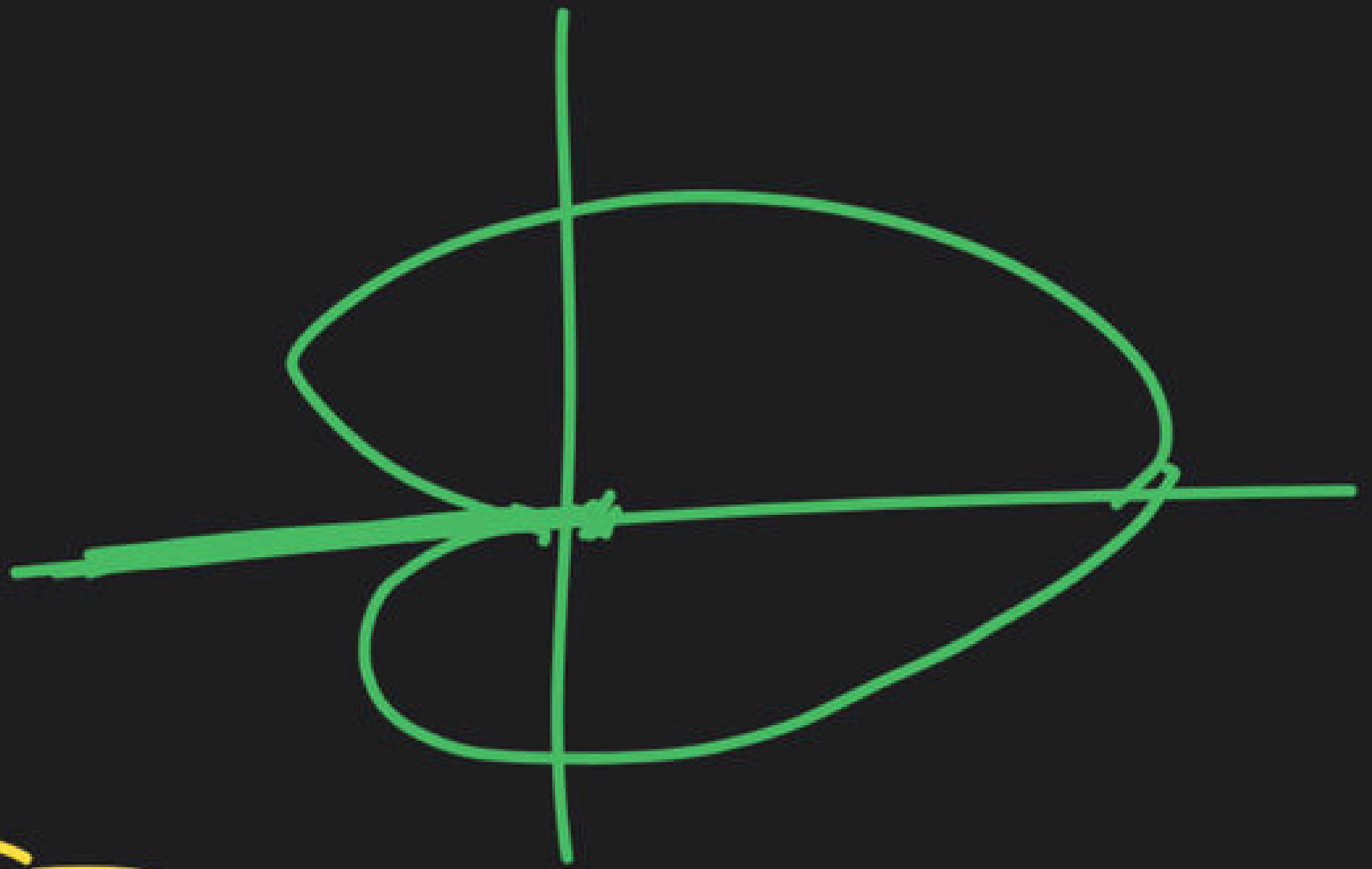
$a(1+\cos\theta)=0$

$1+\cos\theta=0$

$\cos\theta=-1$

$a<b$

$\theta=\pi$



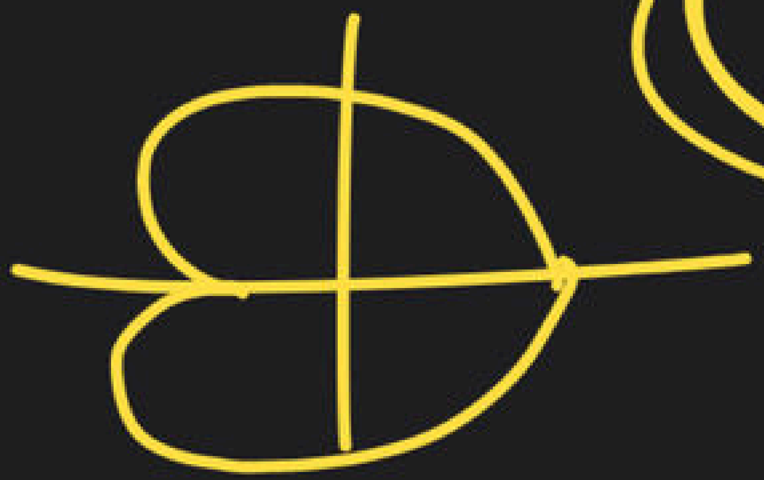
$r = a+b\cos\theta$, $a>b$

$\frac{a}{b} \geq 1$

2)

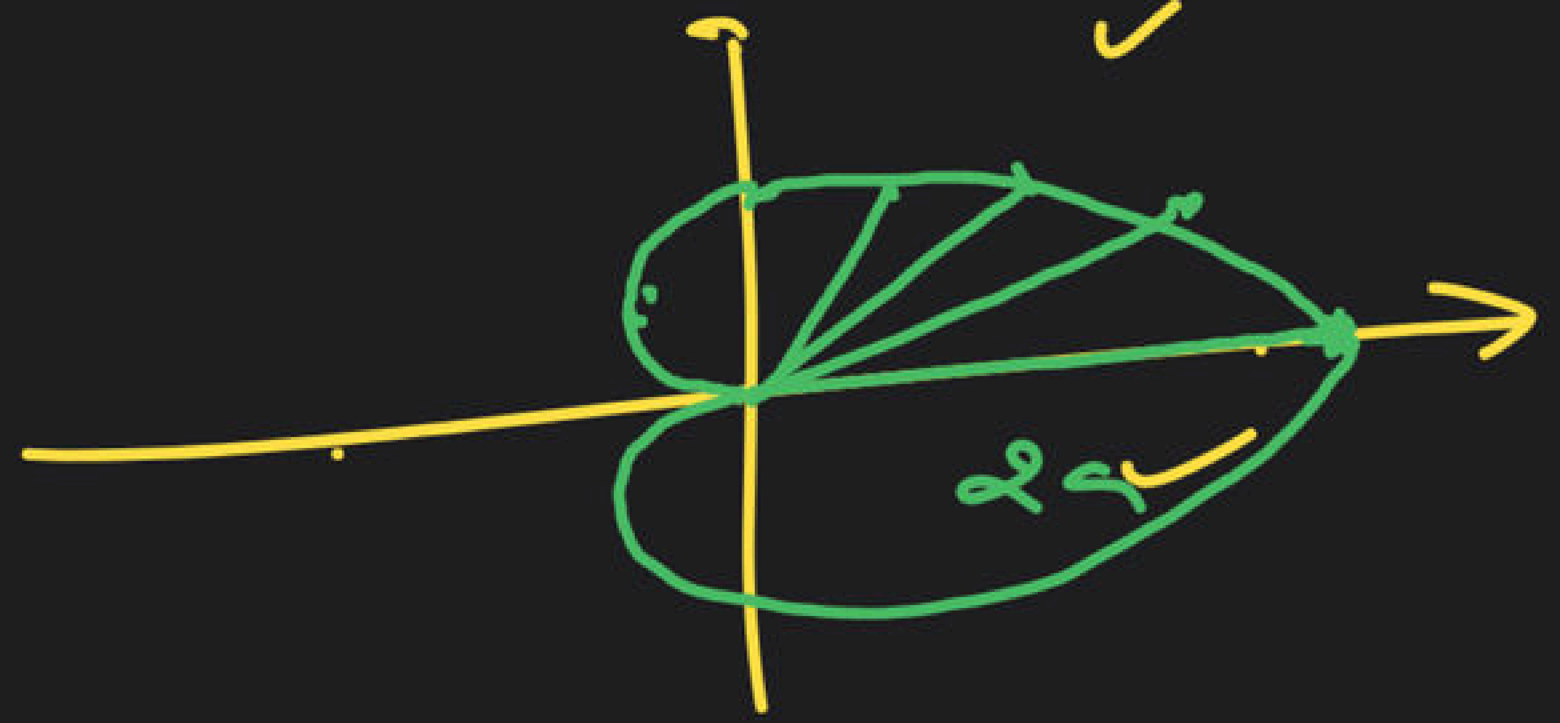
$a+b\cos\theta=0$

$\cos\theta=-\frac{a}{b}$



θ	0	30	45	60	90	120	135	150	180
r	$2a$	$a(1+\frac{\sqrt{3}}{2})$	$a(1+\frac{1}{\sqrt{2}})$	$a(1+\frac{1}{2})$	a	$a(1-\frac{1}{2})$	$a(1-\frac{1}{\sqrt{2}})$	$a(1-\frac{\sqrt{3}}{2})$	0

$$r = a(1+\cos\theta)$$



(v) **Table :**

r									
θ	0	30	45	60	90	120	135	150	180

(vi) **Asymtotes :**

For any value of θ if r become ∞, then a curve has asymtotes.

Transformation of Variables :

Sometime, it is convenient to solve the double integral by transforming the variables.

$\iint f(r,\theta)\, r\, dr\, d\theta$

$r =$ $\theta -$

(A) Transformation in polar form :

1. Let $\iint f(x,y)dxdy$ is a integration in cartisian form, then put x = r cosθ, y = r sin θ in givenintegration.

2. $$dxdy = \frac{\partial(x,y)}{\partial(r,\theta)} dr d\theta$$

$$dxdy = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} d\theta dr$$

dxdy = r dθdr

Putting this value, then we get $\iint f(r,\theta) r d\theta dr$.

$\iint \frac{\sqrt{1-x^2-y^2}}{\sqrt{1+x^2+y^2}}\,dx\,dy$ region I Quadrant of $x^2+y^2 \le 1$

$x = r\cos\theta$

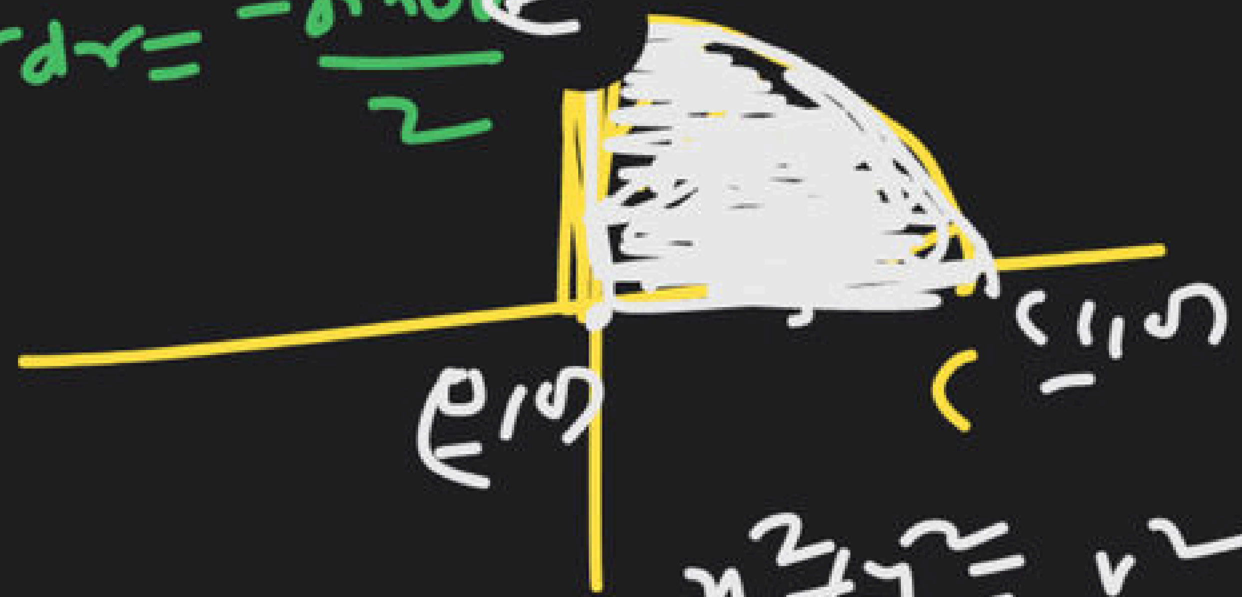
$y = r\sin\theta$

$r^2 = \cos t$

$2r\,dr = -\sin t\,dt$

$r\,dr = -\frac{\sin t\,dt}{2}$

$$\iint \frac{\sqrt{1-(x^2+y^2)}}{\sqrt{1+(x^2+y^2)}}\,dx\,dy = \int_{\theta=0}^{\pi/2}\int_{r=0}^{1} \frac{\sqrt{1-r^2}}{\sqrt{1+r^2}}\,r\,dr\,d\theta$$



$x^2+y^2 = r^2$

$dx\,dy = r\,dr\,d\theta$

$1-\cos\theta = 2\sin^2\theta/2$

$1+\cos\theta = 2\cos^2\theta/2$

$$\int_{\theta=0}^{\pi/2}\int_{\pi/2}^{0} \sqrt{\frac{1-\cos t}{1+\cos t}}\left(-\frac{\sin t\,dt}{2}\right)d\theta$$

$$\frac{1}{2}\int_0^{\pi/2}\int_0^{\pi/2} \sqrt{\frac{\cancel{2}\sin^2 t/2}{\cancel{2}\cos^2 t/2}}\;\cancel{2}\sin\theta/2\,\cos\theta/2\,dt\,d\theta = \int_0^{\pi/2}\int_0^{\pi/2} \sin^2 t/2\,dt\,d\theta$$

$$\int_{\theta=0}^{\pi/2}\int_{t=0}^{\pi/2} \sin^2 t/2 \, dt \, d\theta$$

$$\int_{\theta=0}^{\pi/2}\int_{t=0}^{\pi/2} \left(\frac{1-\cos t}{2}\right) dt \, d\theta$$

$$\frac{1}{2}\int_{\theta=0}^{\pi/2} (t - \sin t)\Big|_0^{\pi/2} d\theta = \frac{1}{2}\int_0^{\pi/2}(\pi/2 - 1)\, d\theta$$

$$\frac{1}{2}(\pi/2 - 1)\cancel{(d\theta)}\Big|_0^{\pi/2} = \underline{\frac{1}{2}(\pi/2 - 1)\,\pi/2}$$

$$\pi/4(\pi/2 - 1)$$

$$\iint_R \log(1+x^2+y^2)\,dx\,dy$$

1 quadrant of $x^2+y^2=a^2$

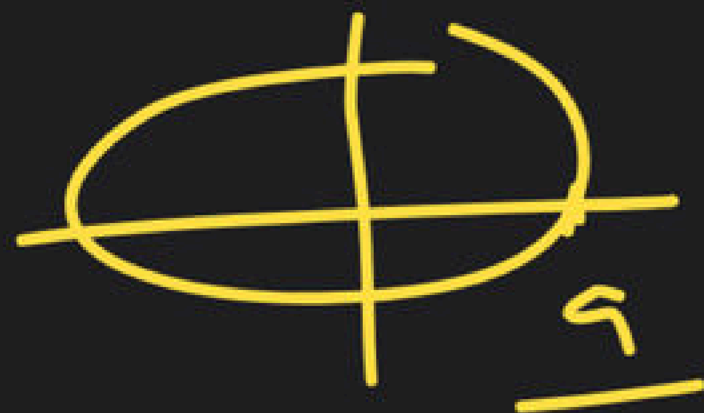
$x^2+y^2=r^2$

$dx\,dy = r\,dr\,d\theta$

$1+r^2 = t$

$2r\,dr = dt$

$r\,dr = dt/2$



$$\int_0^{\pi/2}\int_0^{a} \log(1+r^2)\,r\,dr\,d\theta$$

$\theta=0 \quad r=0$

$$\int_{\theta=0}^{\pi/2}\int_1^{1+a^2} \log t\,\frac{dt}{2}\,d\theta = \frac{1}{2}\int_0^{\pi/2}\int_1^{1+a^2} \log t\,dt\,d\theta$$

$4a^2$

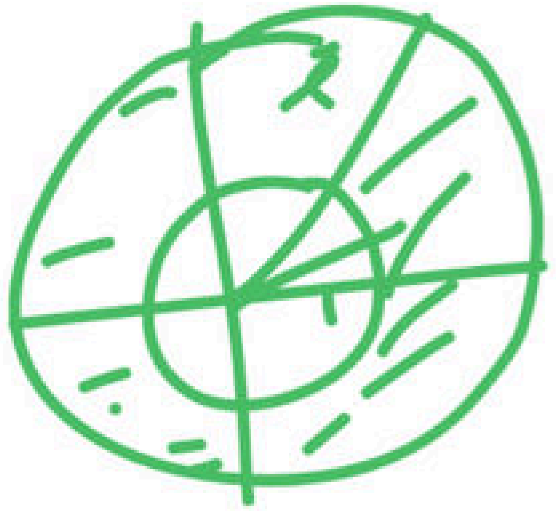
$$\frac{1}{2}\int_0^{\pi/2}\left(\log t\int dt - \int\left(\frac{d(\log t)}{dt}\int dt\right)dt\right)_1^{1+a^2} d\theta = \frac{1}{2}\int_0^{\pi/2}\left(t\log t - t\right)_1^{1+a^2} d\theta$$

$$\frac{1}{2}\int_0^{\pi/2}\left((1+a^2)\log(1+a^2) - (1+a^2)+1\right)d\theta$$

Q.1. The value of $\iint_G \frac{\log(x^2+y^2)}{x^2+y^2}\,dxdy$ where $G = \{(x, y) \in R^2;\ 1 \le x^2 + y^2 \le e^2\}$ is **IIT JAM 2010**

(a) π (b) 2π

(c) 3π (d) 4π



$x^2+y^2=r^2$

$dxdy = r\,dr\,d\theta$

$$\int_{\theta=0}^{2\pi}\int_{r=1}^{e} \frac{\log r^2}{r^2}\, r\,dr\,d\theta = \int_0^{2\pi} d\theta \int_{r=1}^{e} \frac{2\log r}{r}\,dr = 2(\theta)_0^{2\pi}\int_1^e \frac{\log r}{r}\,dr$$

$\log r = t$

$\frac{1}{r}dr = dt$

$$= 2(2\pi-0)\int_{t=0}^{1} t\,dt = 4\pi\left(\frac{t^2}{2}\right)_0^1$$

$$= 4\pi\left(\frac{1}{2}-0\right) = 2\pi$$

Q.2. The value of $\iint \sqrt{x^2+y^2}\,dx\,dy$ over the region lying in xy-plane and bounded by $x^2+y^2=4$, $x^2+y^2=9$

(a) $\frac{37\pi}{3}$ (b) $\frac{38}{3}\pi$

(c) $\frac{19}{2}\pi$ (d) $\frac{19}{3}\pi$

$$\int_{\theta=0}^{2\pi}\int_{r=2}^{3} r \cdot r\,dr\,d\theta$$

$$\int_0^{2\pi} d\theta \int_2^3 r^2\,dr$$

$$(\theta)_0^{2\pi}\left(\frac{r^3}{3}\right)_2^3$$

$$\frac{2\pi}{3}(27-8)$$

$$38\pi/3$$

Q.3. Let D be the region in the first quadrant lying between $x^2 + y^2 = 1$ & $x^2 + y^2 = 4$, value of integral $\iint_D \sin(x^2 + y^2)\,dx\,dy$. **IIT JAM – 2007**

(a) $\frac{\pi}{4}(\cos 1 - \cos 2)$ (b) $\frac{\pi}{4}(\cos 1 - \cos 4)$

(c) $\frac{\pi}{2}(\cos 1 - \cos 2)$ (d) $\frac{\pi}{2}(\cos 1 - \cos 4)$

$r^2 = t$

$2r\,dr = dt$

$$\int_{\theta=0}^{\pi/2}\int_{r=1}^{2} \sin r^2 \cdot r\,dr\,d\theta$$

$$\frac{1}{2}\int_{\theta=0}^{\pi/2} d\theta \int_{t=1}^{4} \sin t\,dt$$

$$-\frac{1}{2}(\theta)_0^{\pi/2}(\cos t)_1^4$$

$$-\pi/4(\cos 4 - \cos 1)$$

$$\pi/4(\cos 1 - \cos 4)$$

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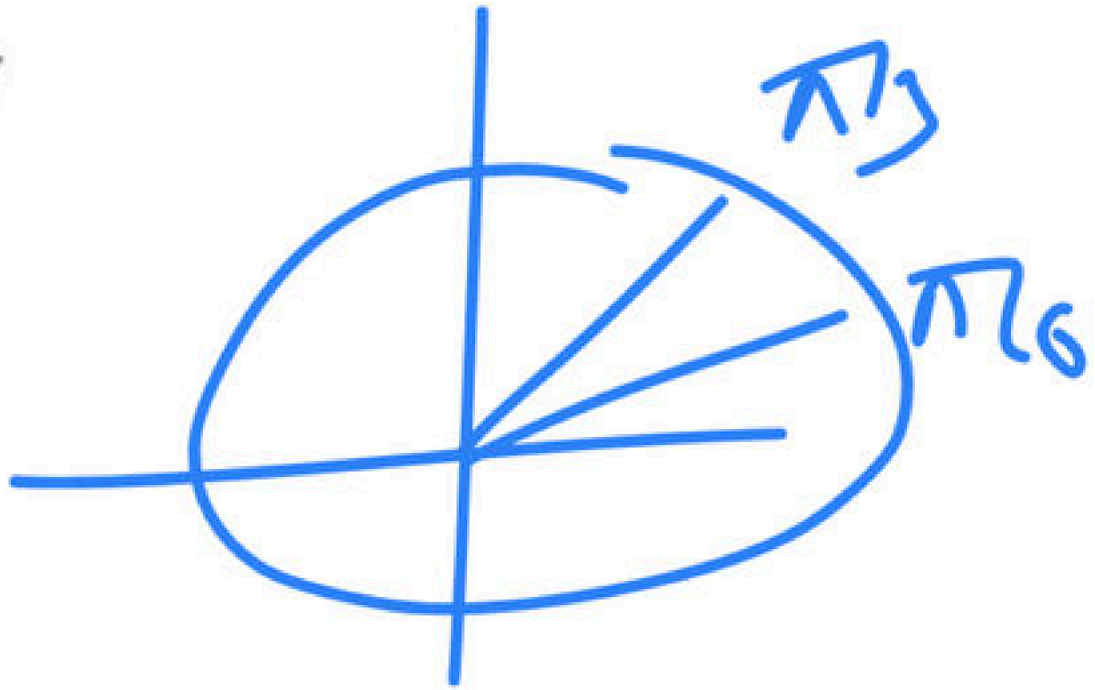
Q.4 If G is the region in R^2 given by

$$G=\left\{(x,y)\in R^2 : x^2+y^2<1, \frac{x}{\sqrt{3}}<y<\sqrt{3}x, x>0, y>0\right\}$$



$$\frac{25}{\pi}\left(\frac{\pi}{6}\right)=\frac{25}{6}=4.1$$

then the value of $\frac{200}{\pi}\iint_G x^2\,dxdy$ is **IIT JAM 2022**

$$\frac{200}{\pi}\int_{\pi/6}^{\pi/3}\int_{r=0}^{1} r^2\cos^2\theta\; r\,dr\,d\theta$$

(a) 2.5 (b) 4.16

(c) 3 (d) 5.5

$$\frac{200}{\pi}\int_{\pi/6}^{\pi/3}\cos^2\theta\,d\theta\int_0^1 r^3\,dr=\frac{200}{\pi}\int_{\pi/6}^{\pi/3}\left(\frac{1+\cos 2\theta}{2}\right)d\theta\left(\frac{r^4}{4}\right)_0^1$$

$$\frac{200}{\pi}\times\frac{1}{4\times 2}\left[\theta+\frac{\sin 2\theta}{2}\right]_{\pi/6}^{\pi/3}=\frac{25}{\pi}\left(\left(\frac{\pi}{3}-\frac{\pi}{6}\right)+\frac{1}{2}\left(\sin\frac{2\pi}{3}-\sin\frac{2\pi}{6}\right)\right)$$

Q.5. Let p and t be positive real numbers. Let D_t be the closed disc of radius t center (0,0) i.e. $D_t = \{(x,y) : x^2 + y^2 \leq t^2$.

Define $I(p,t) = \iint_{D_t} \frac{dxdy}{\left(p^2 + x^2 + y^2\right)^p}$

Then $\lim_{t\to\infty} I(p,t)$ is finite **IIT JAM 2021**

(a) only if p > 1

(b) only if p < 1

(c) only if p = 1

(d) for no value of p

Q.6. The value of the real number m in the following equation

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \left(x^2+y^2\right) dy\,dx = \int_{m\pi}^{\pi/2} \int_0^{\sqrt{2}} r^3\,dr\,d\theta \text{ is } \textbf{IIT JAM 2016}$$

(a) 0 (b) 1

(c) 2 (d) 1/4

Q.7. Let $I = \int_0^2 \int_{\sqrt{4-y^2}}^{\sqrt{9-y^2}} 2xy\,dx\,dy + \int_2^3 \int_2^{\sqrt{9-y^2}} 2xy\,dx\,dy$. **IIT-JAM 2010**

Then using the transformation x = r cosθ, y = r sin θ, integral I is equal to

(a) $\int_0^{\pi/2} \int_0^3 r^2 \sin 2\theta\,dr\,d\theta$

(b) $\int_0^{\pi/2} \int_0^2 r^3 \sin 2\theta\,dr\,d\theta$

(c) $\int_0^{\pi/2} \int_0^3 r^3 \sin 2\theta\,dr\,d\theta$

(d) $\int_0^{\pi/2} \int_0^{-3} r^2 \sin 2\theta\,dr\,d\theta$

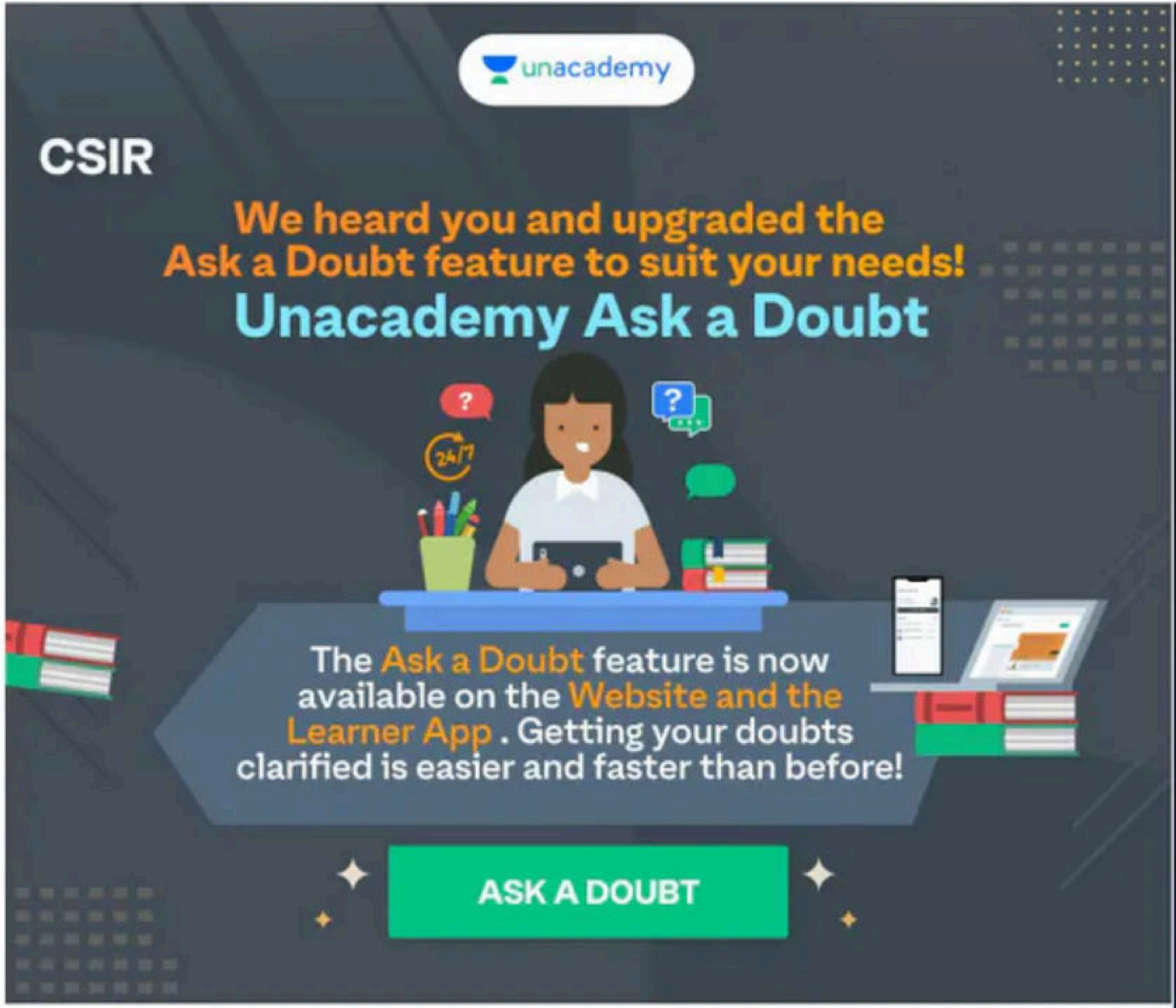
Q.8. The integral $\iint_R e^{x^2+y^2}\,dydx$, where R is the semicircle region bounded by the x – axis and the curve $y=\sqrt{1-x^2}$ equals **SAU 2017**

(a) $\frac{\pi}{2}(e+1)$

(b) $\frac{\pi}{2}(e-1)$

(c) $\frac{\pi}{2}(e^2)$

(d) $\frac{\pi}{2}e$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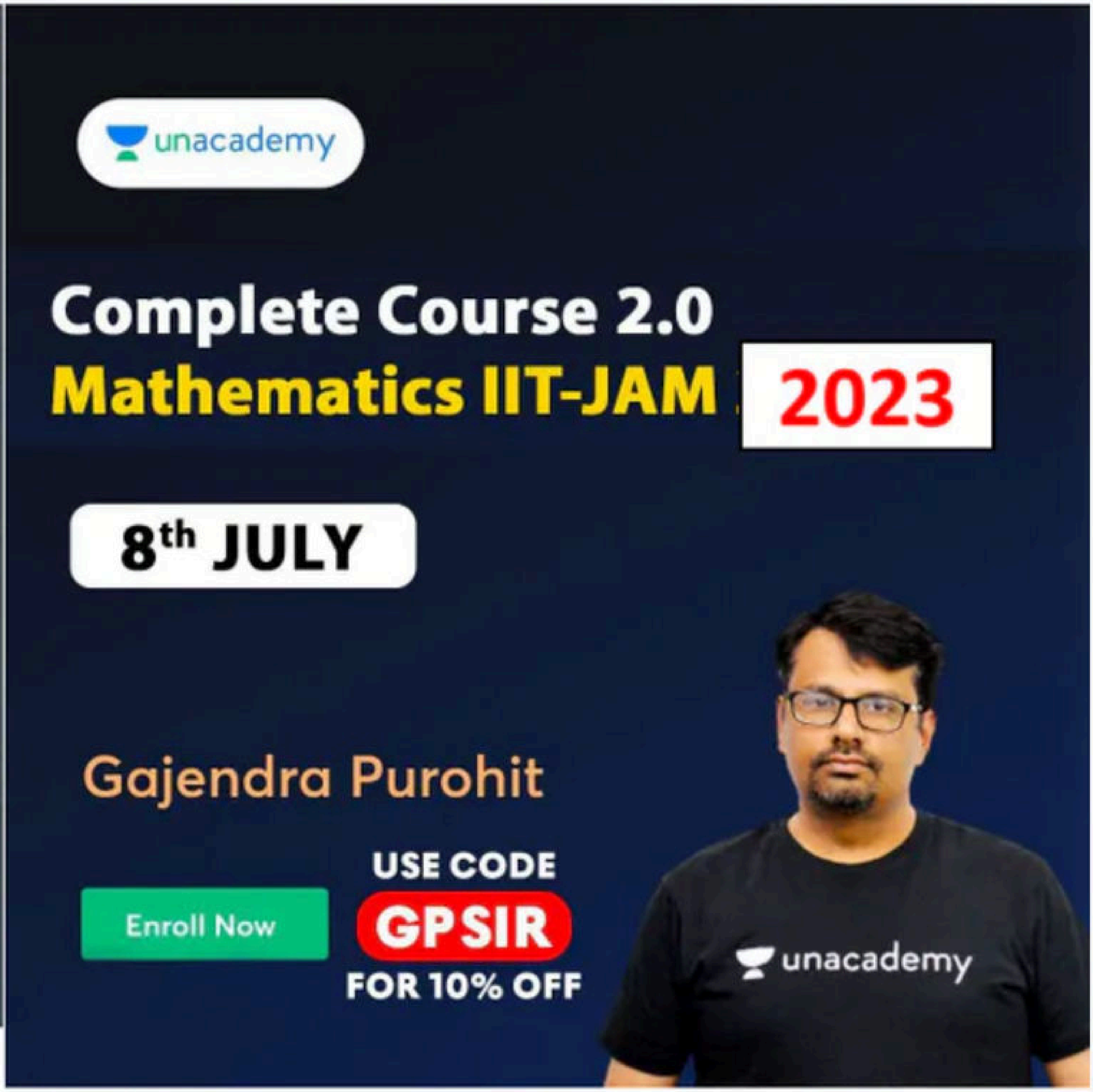
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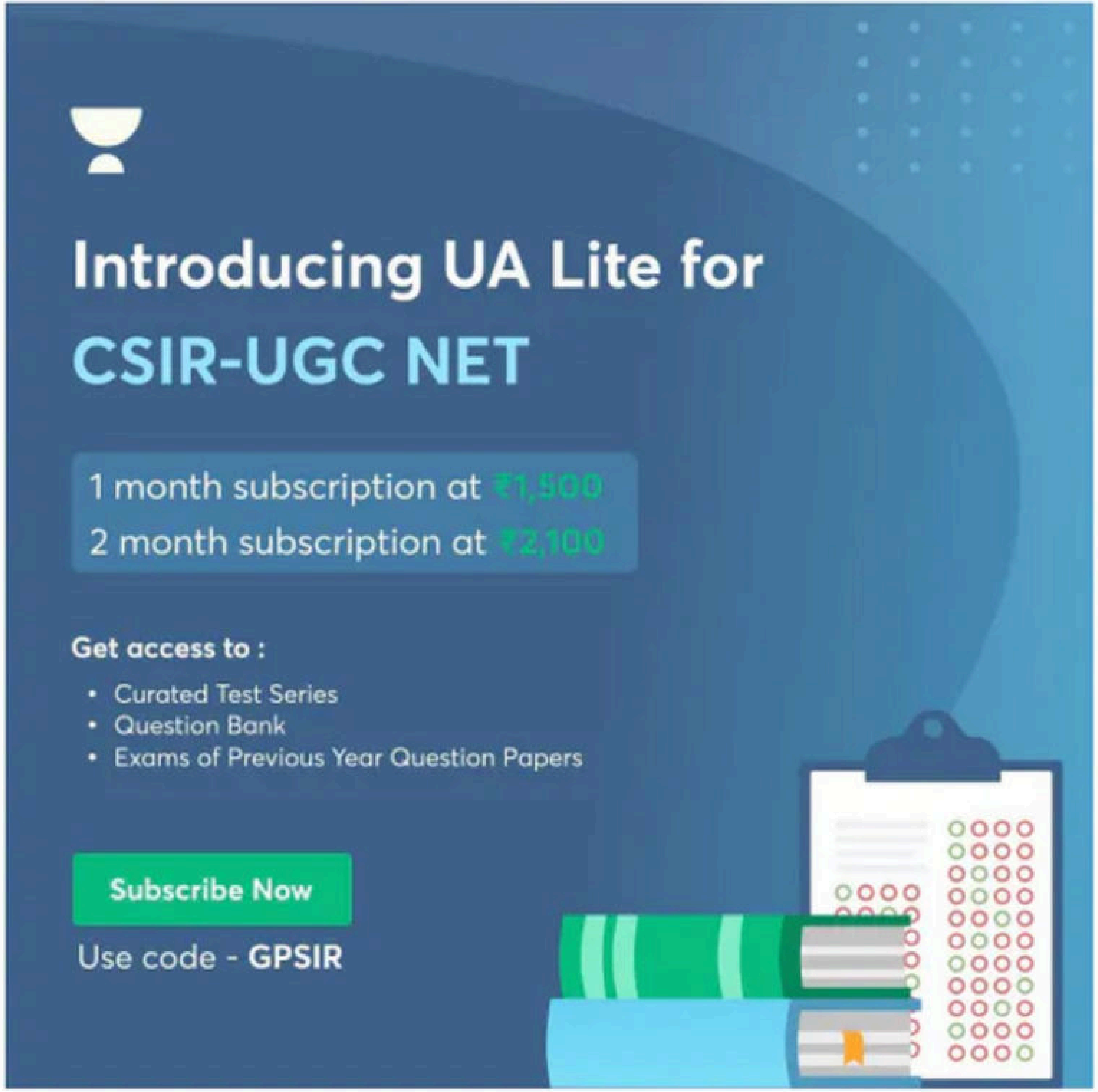
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