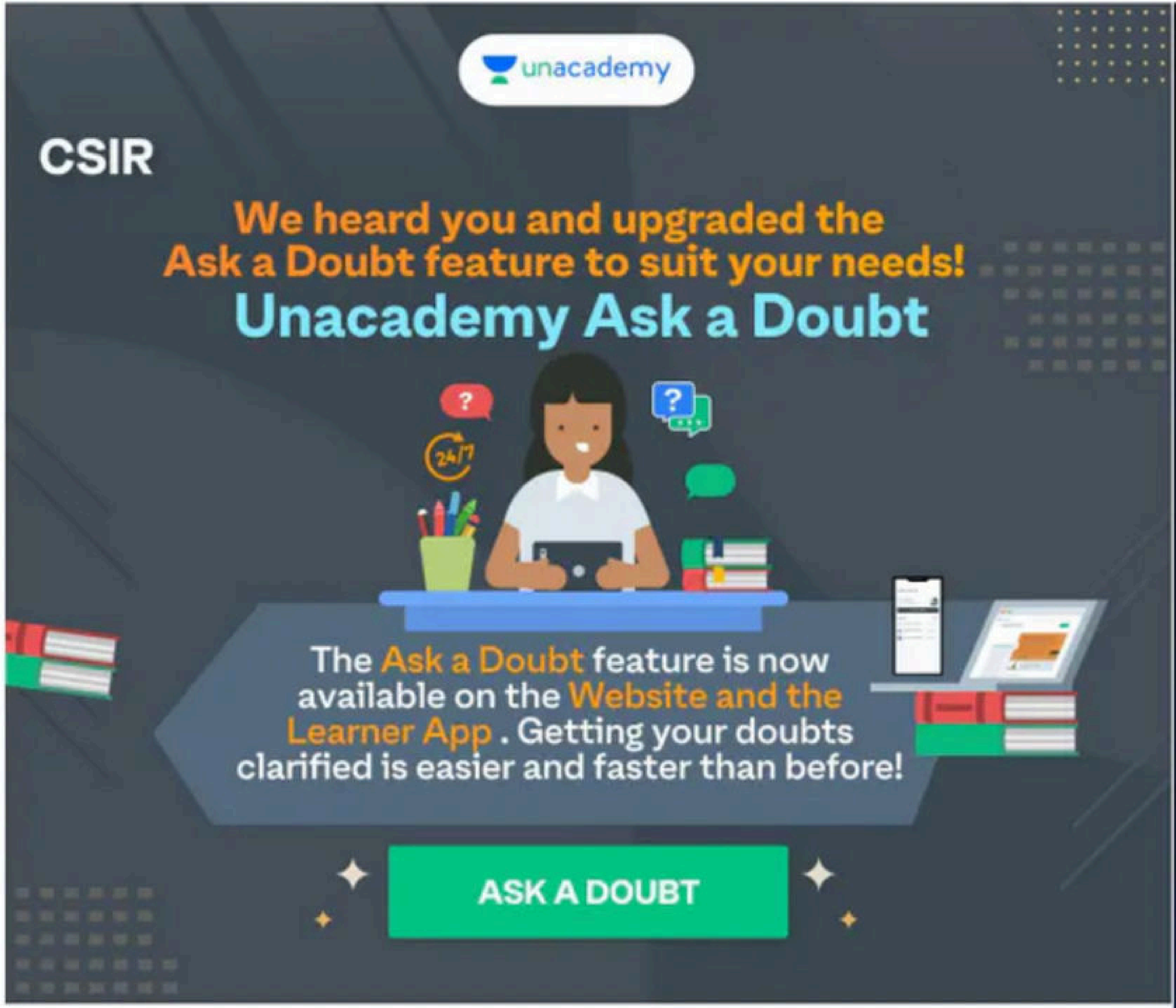
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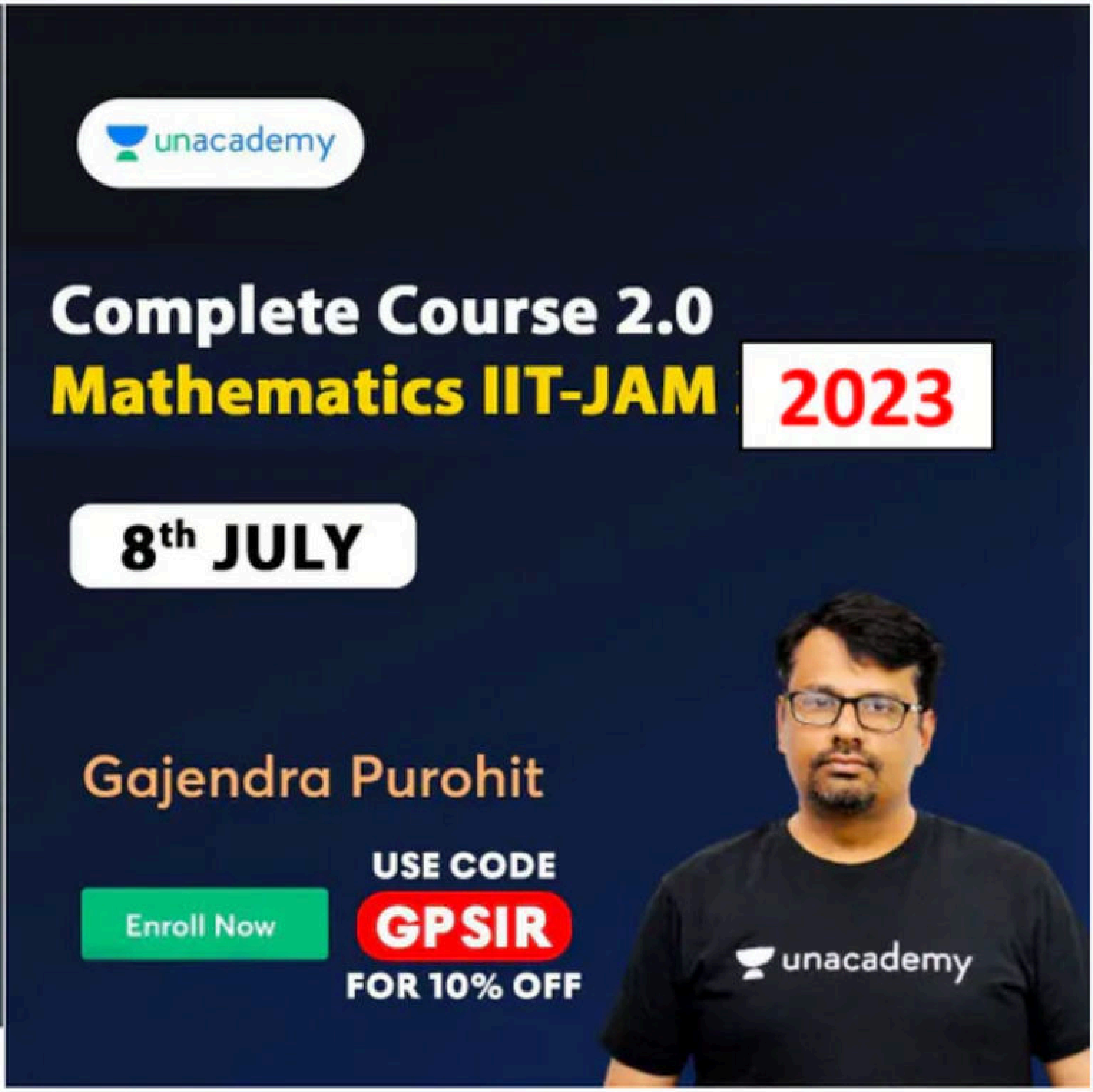
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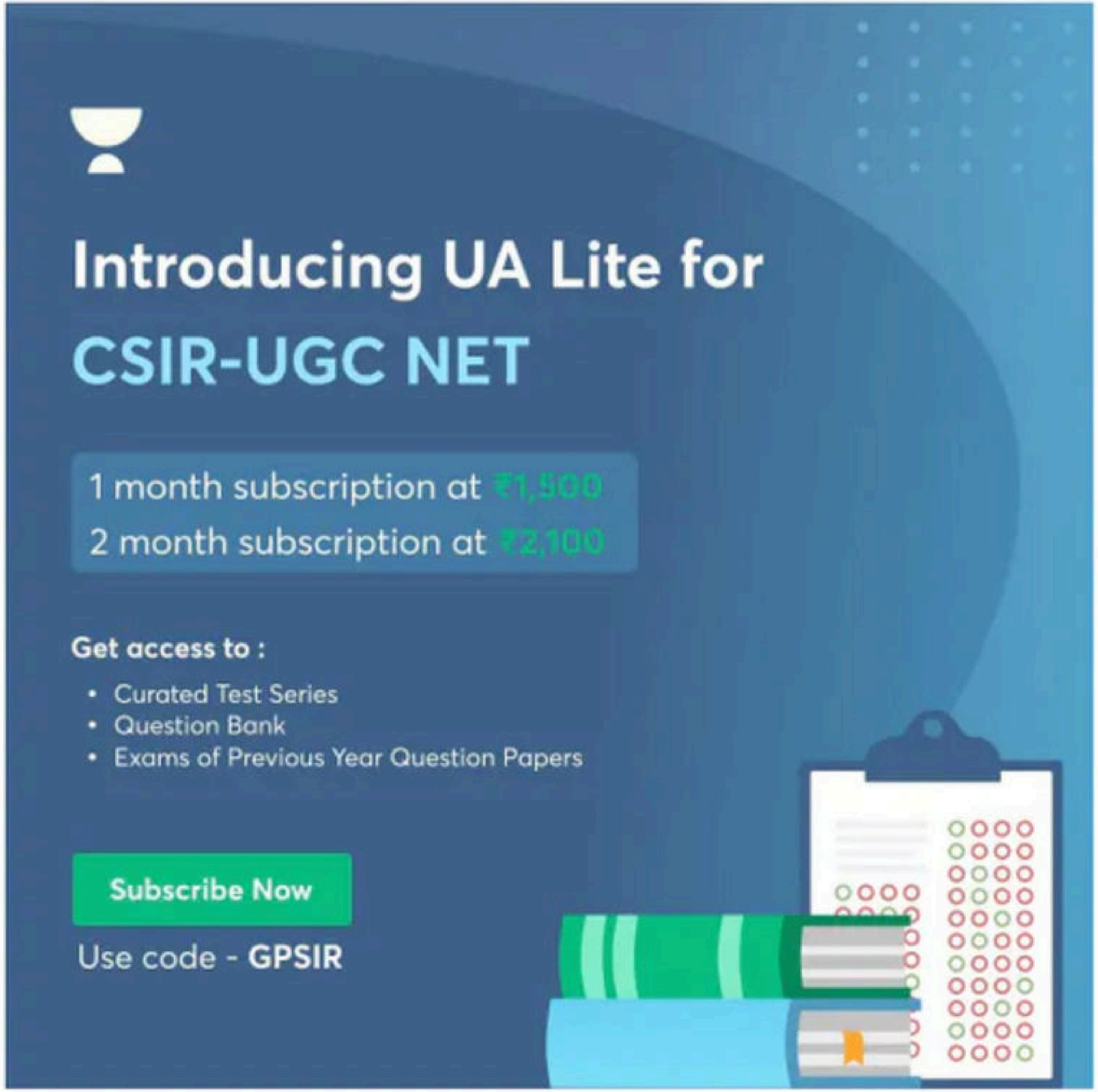
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Transformation of Variables :

Sometime, it is convenient to solve the double integral by transforming the variables.

$\iint f(x,y)\,dx\,dy = \iint f(r,\theta)$

$= \iint r\,dr\,d\theta$

(A) Transformation in polar form :

1. Let $\iint f(x,y)dxdy$ is a integration in cartisian form, then put $x = r\cos\theta$, $y = r\sin\theta$ in givenintegration.

2. $$dxdy = \frac{\partial(x,y)}{\partial(r,\theta)} dr d\theta$$

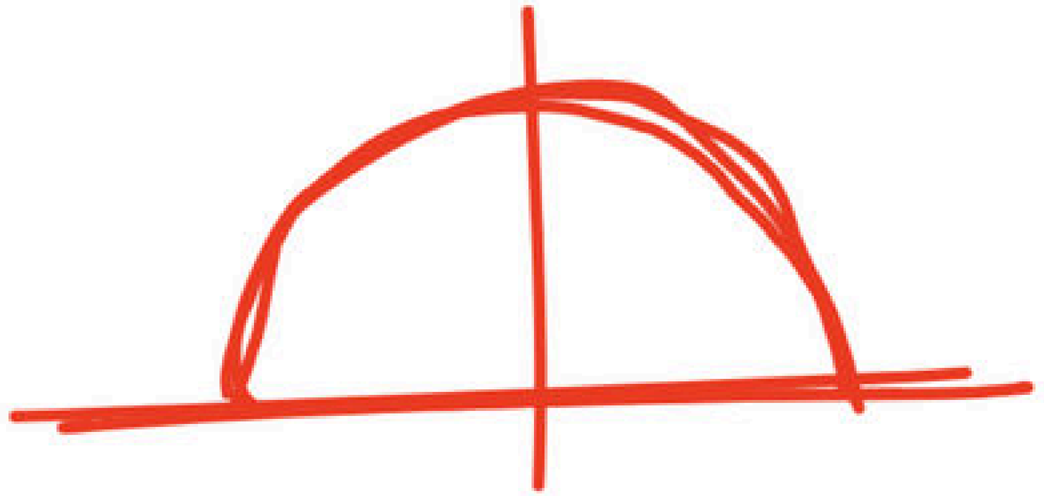
$$dxdy = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} d\theta dr$$

$$dxdy = r\, d\theta dr$$

Putting this value, then we get $\iint f(r,\theta) r d\theta dr$.

Q.1. The integral $\iint_R e^{x^2+y^2}\,dy\,dx$, where R is the semicircle region bounded by the x – axis and the curve $y=\sqrt{1-x^2}$ equals

SAU 2017



(a) $\frac{\pi}{2}(e+1)$

(b) $\frac{\pi}{2}(e-1)$

(c) $\frac{\pi}{2}(e^2)$

(d) $\frac{\pi}{2}e$

$r^2 = t$

$2r\,dr = dt$

$r\,dr = \frac{dt}{2}$

$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 = r^2$

$dx\,dy = r\,dr\,d\theta$

$$\int_{\theta=0}^{\pi}\int_{r=0}^{1} e^{r^2}\, r\,dr\,d\theta$$

$$\int_0^{\pi} d\theta \int_0^1 e^t\,\frac{dt}{2} = \frac{1}{2}(\theta)_0^{\pi}(e^t)_0^1$$

$$= \frac{\pi}{2}(e-1)$$

Q The value of integral $\iint \sqrt{x^2+y^2}\, dx\, dy$

$D = \{(x,y) \in R^2 \mid x \le \boxed{x^2+y^2 \le 2x}\}$ is

(a) 0 (b) 7/9

(c) $\frac{14}{9}$ (d) $\frac{28}{9}$

$$\int_{-\pi/2}^{\pi/2}\int_{\cos\theta}^{2\cos\theta} r \cdot r\, dr\, d\theta$$

$\theta = \pi/2$ $r = \cos\theta$

$$\int_{-\pi/2}^{\pi/2} \left(\frac{r^3}{3}\right)_{\cos\theta}^{2\cos\theta} d\theta = \frac{1}{3}\int_{-\pi/2}^{\pi/2} (8\cos^3\theta - \cos^3\theta)\, d\theta$$

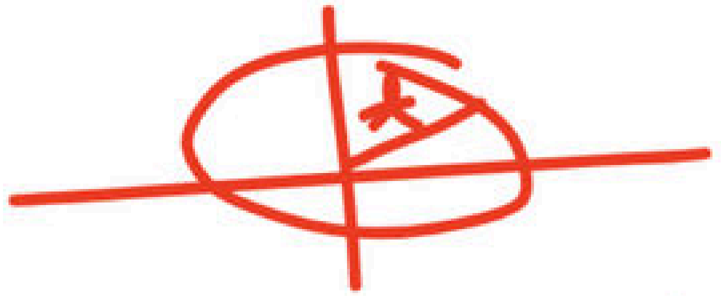
$x^2+y^2-2x \le 0$

$\theta = -\pi/2$

$$= \frac{7}{3}\int_{-\pi/2}^{\pi/2} \cos^3\theta\, d\theta = \frac{14}{3}\int_0^{\pi/2} \sin^0\theta \cos^3\theta\, d\theta$$

$$= \frac{14}{3}\, \frac{\Gamma\frac{1}{2}\,\Gamma\frac{4}{2}}{2\,\Gamma\frac{5}{2}} = \frac{14}{3}\left(\frac{\cancel{\sqrt{\pi}} \cdot 1}{\cancel{2} \cdot \frac{3}{2} \cdot \frac{1}{2}\,(\cancel{\sqrt{\pi}})}\right) \frac{28}{9}$$

Q.2. Let p and t be positive real numbers. Let D_t be the closed disc of radius t center (0,0) i.e. $D_t = \{(x,y) : x^2 + y^2 \le t^2\}$.



Define $I(p,t) = \iint_{D_t} \frac{dx\,dy}{(p^2+x^2+y^2)^p} = \int_{\theta=0}^{2\pi}\int_{r=0}^{t} \frac{r\,dr\,d\theta}{(p^2+r^2)^p}$

$r^2 = z$

$2r\,dr = dz$

$r\,dr = \frac{dz}{2}$

Then $\lim_{t\to\infty} I(p,t)$ is finite **IIT JAM 2021**

(a) only if p > 1

(b) ~~only if p < 1~~

(c) ~~only if p = 1~~

(d) ~~for no value of p~~

$\int_0^{2\pi} d\theta \int_0^{t^2} \frac{dz/2}{(p^2+z)^p}$

$\frac{2\pi}{2}\int_0^{t^2}\frac{dz}{(p^2+z)^p}$

$\pi\int_0^{t^2}\frac{dz}{(p^2+z)^p}$

P=1 $\quad \pi\int_0^{t^2}\frac{dz}{(1+z)} = \pi\left(\log(1+z)\right)_0^{t^2} = \pi\log(1+t^2) = DNE$

P=2 $\quad \pi\int_0^{t^2}\frac{dz}{(4+z)^2} = \pi\left[-\frac{1}{(4+z)}\right]_0^{t^2} = -\pi\left(\frac{1}{4+t^2}-\frac{1}{4}\right) = \pi/4$

Q.3. The value of the real number m in the following equation

$$\int_{x=0}^{1}\int_{y=x}^{\sqrt{2-x^2}}\left(x^2+y^2\right)dy\,dx=\int_{m\pi}^{\pi/2}\int_{0}^{\sqrt{2}} r^3\,dr\,d\theta$$ is IIT JAM 2016

(a) 0 (b) 1

(c) 2 (d) 1/4

$x=0$, $x=1$

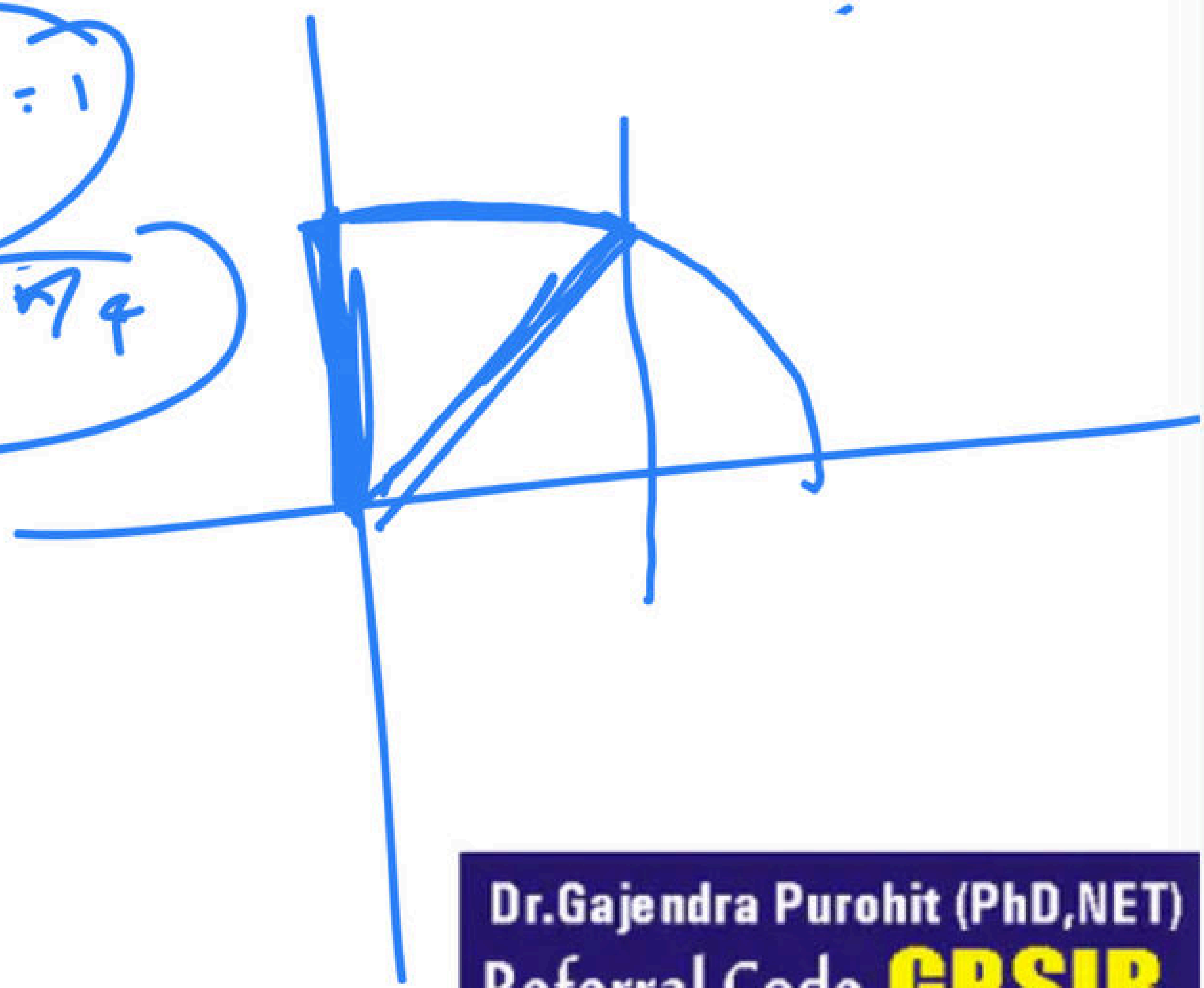
$y=x$, $y=\sqrt{2-x^2}$

$x^2+y^2=2$

$r\cos\theta = r\sin\theta$

$\tan\theta = 1$

$\theta = \pi/4$



$$\int_{\theta=\pi/4}^{\pi/2}\int_{r=0}^{\sqrt{2}}$$

$\pi/4 = m\pi$

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Q.4. Let $I=\int_{0}^{2}\int_{\sqrt{4-y^2}}^{\sqrt{9-y^2}} 2xy\,dx\,dy+\int_{2}^{3}\int_{2}^{\sqrt{9-y^2}} 2xy\,dx\,dy$. IIT-JAM 2010

$y=$ $x=$ $y=$ $x=2$

$y=0$, $y=2$ | $x^2+y^2=4$, $x^2+y^2=9$

$y=2$, $y=3$ | $x=2$, $x^2+y^2=9$

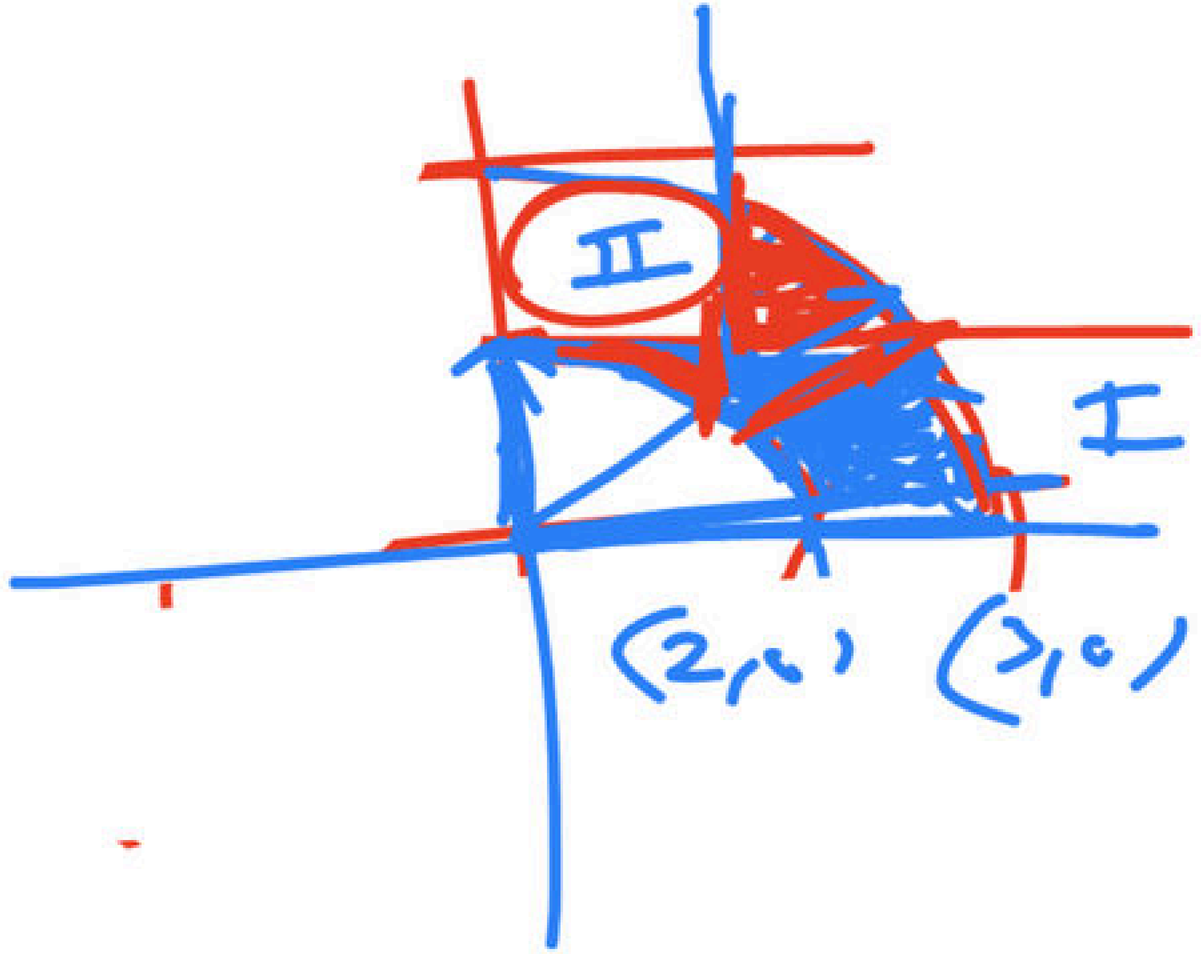
Then using the transformation x = r cosθ, y = r sin θ, integral I is equal to

(a) $\int_{0}^{\pi/2}\int_{0}^{3} r^2\sin 2\theta\,dr\,d\theta$

(b) $\int_{0}^{\pi/2}\int_{0}^{2} r^3\sin 2\theta\,dr\,d\theta$

(c) $\int_{0}^{\pi/2}\int_{0}^{3} r^3\sin 2\theta\,dr\,d\theta$

(d) $\int_{0}^{\pi/2}\int_{0}^{-3} r^2\sin 2\theta\,dr\,d\theta$

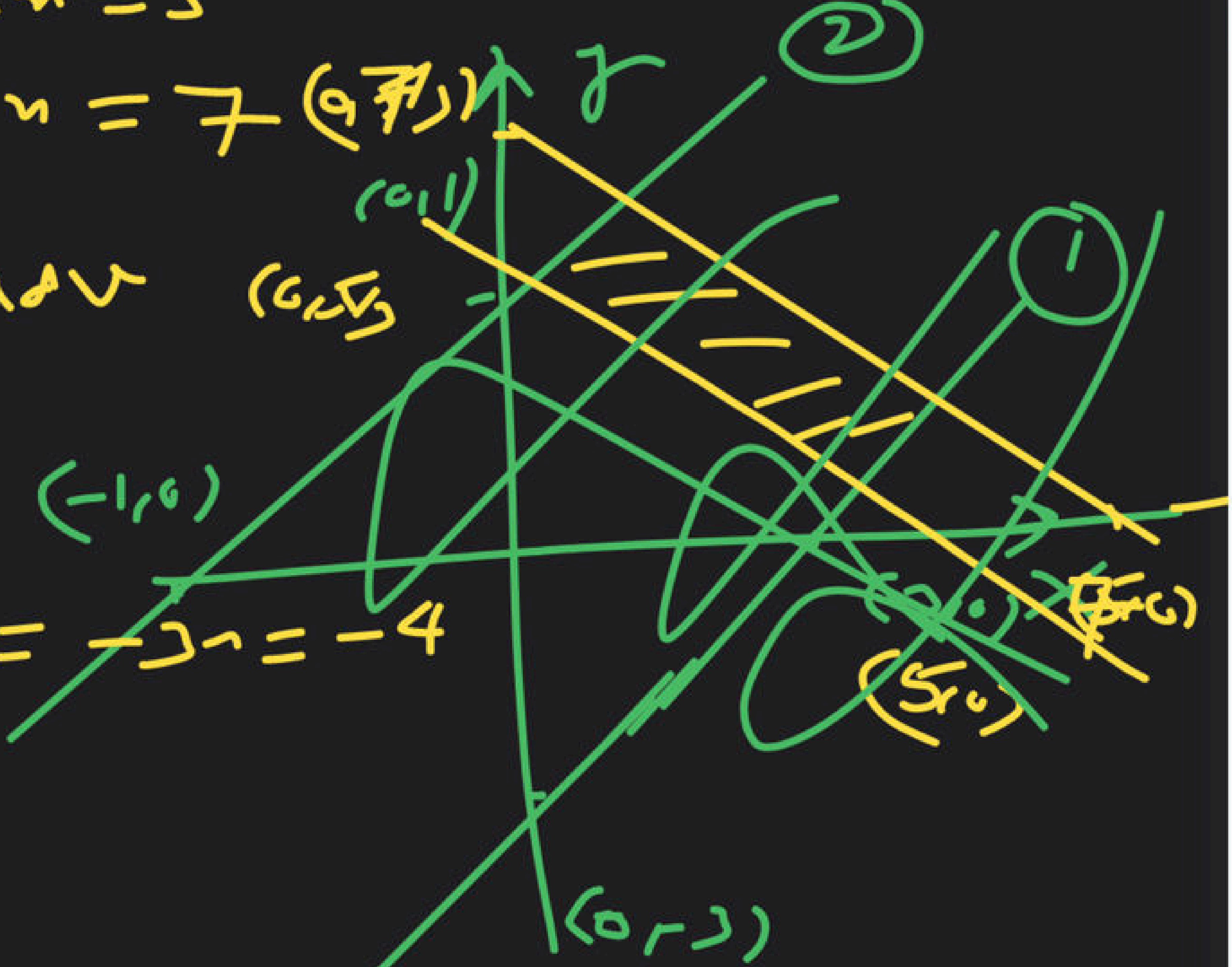


$\int_{\theta=0} \int_{r=0}^{3}$

$\iint (y-x)\,dx\,dy$, $y = x-3$ (1), $y = x+1$ (2) | $3y+x=5$, $3y+x=7$

$\int_{u=-3}^{1}\int_{v=5}^{7} u\,\frac{1}{4}\,du\,dv$

$dx\,dy = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$



$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ 1 & 3 \end{vmatrix} = -3-1 = -4$$

$$\frac{\partial(x,y)}{\partial(u,v)} = -1/4$$

$u=-3$: $y-x=-3$	$x+3y=5$
$u=1$: $y-x=1$	$x+3y=7$
$y-x=u$	
$x+3y=v$	

$$\frac{1}{4}\int_{u=-3}^{1} u\,du \int_{5}^{7} dv = \frac{1}{4}\left(\frac{u^2}{2}\right)_{-3}^{1}(v)_{5}^{7}$$

$$= \frac{1}{8}(1-9)(7-5) = -2$$

Q.5. The value of $\iint_D (x+2y)\,dx\,dy$, where D is region in the xy-plane bounded by the straight line y = x + 3, y = x – 3, y = -2x + 4 and y = -2x + 2. **IIT JAM – 2007**

(a) 10 (b) 11

(c) 12 (d) 13

$2x+y=4 \qquad y-x=3$

$2x+y=2 \qquad y-x=-3$

$u = 2x+y \qquad v = y-x$

$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 3$

$\int_{v=-3}^{3}\int_{u=2}^{4} (u+v)\frac{1}{3}\,du\,dv = 12$

$\frac{1}{3}\int_{v=-3}^{3}\left(\frac{u^2}{2}+uv\right)_2^4 dv$

$\frac{1}{3}\int_{v=-3}^{3}\left(\frac{16-4}{2}\right)+(4-2v)\,dv$

$\frac{1}{3}\int_{-3}^{3}(6+2v)\,dv$

$\frac{1}{3}\left(6\int_{-3}^{3}dv + 2\int_{-3}^{3}v\,dv\right)$

$\frac{2}{3}\times 6\int_0^3 dv$

$4\,(v)_0^3$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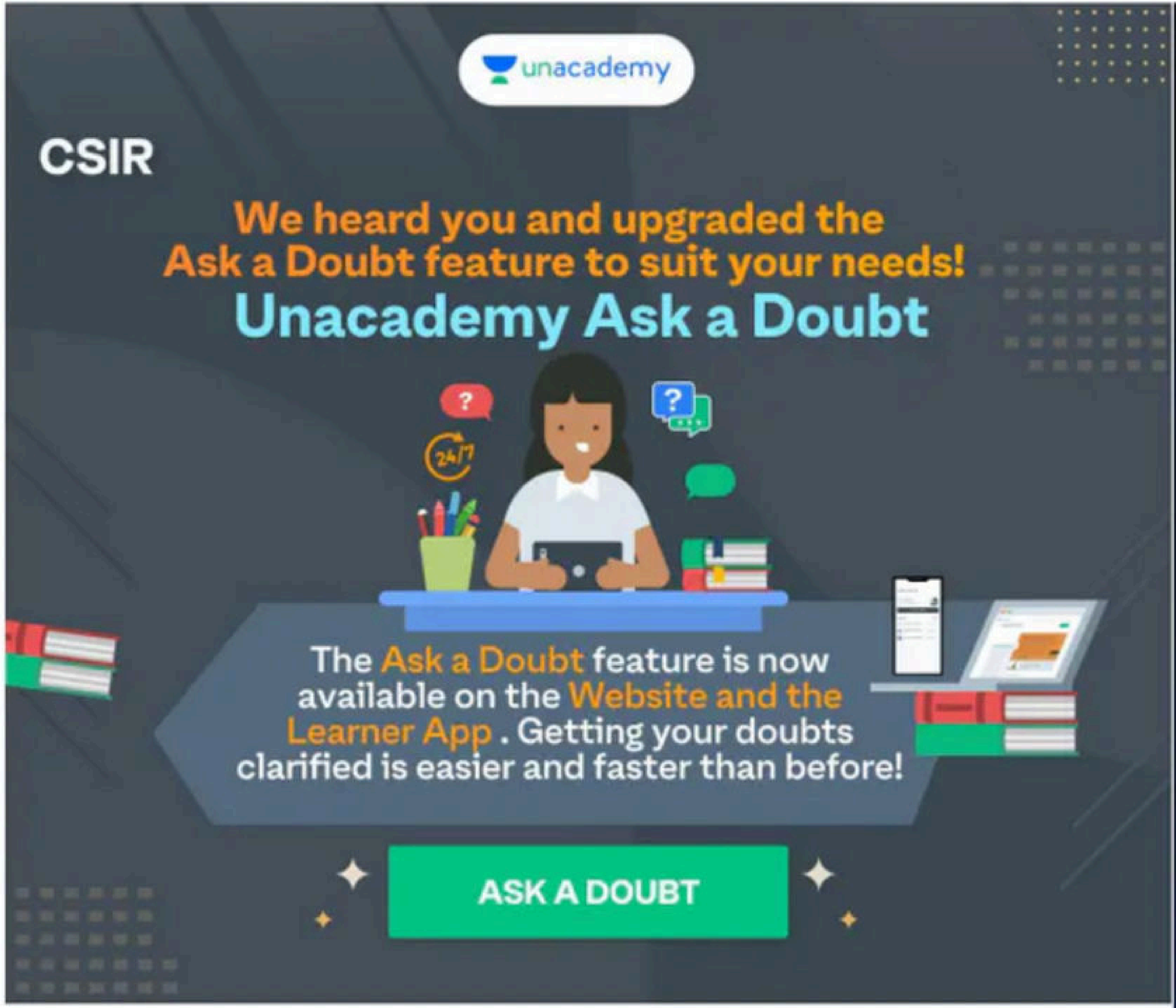
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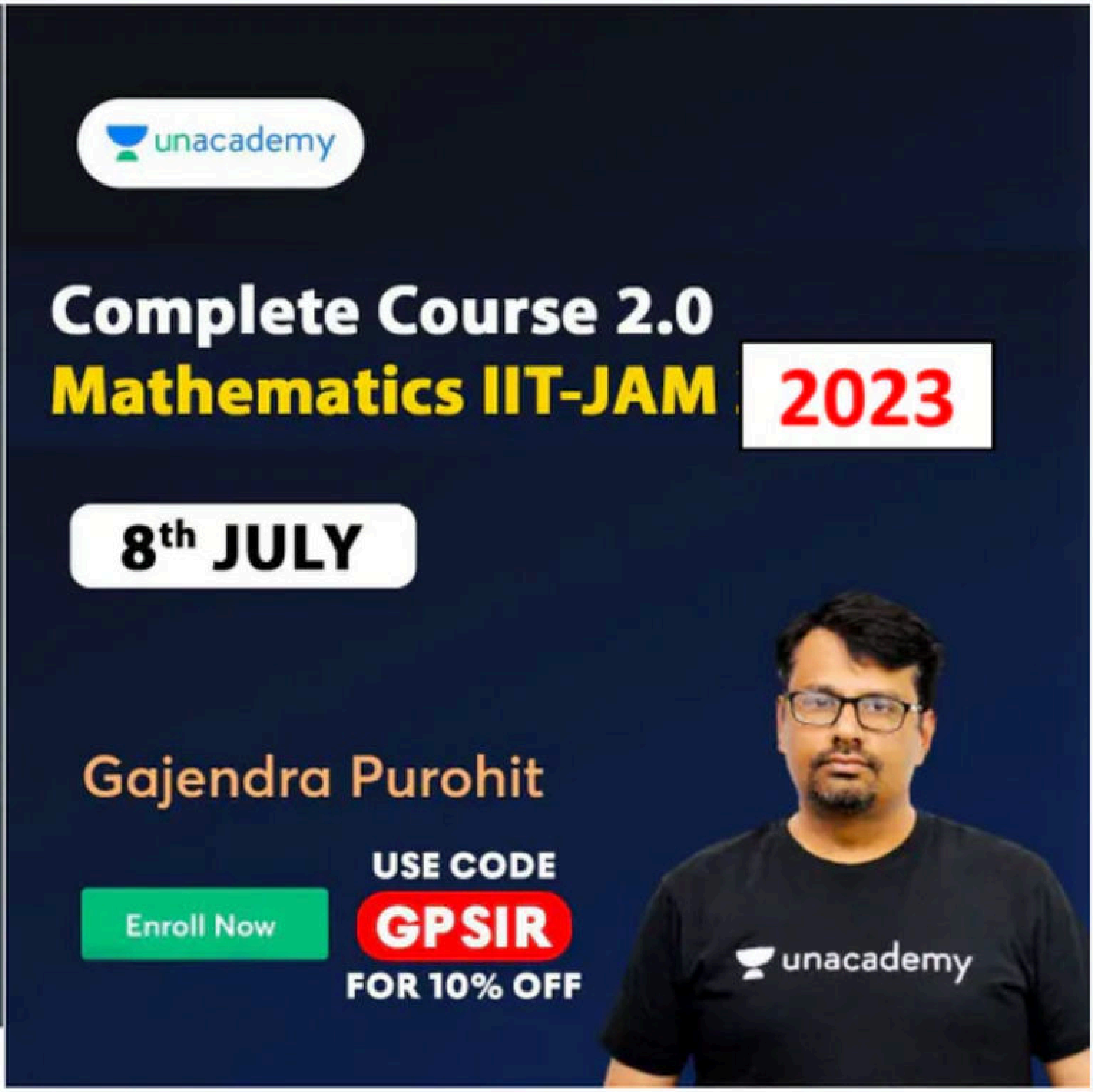
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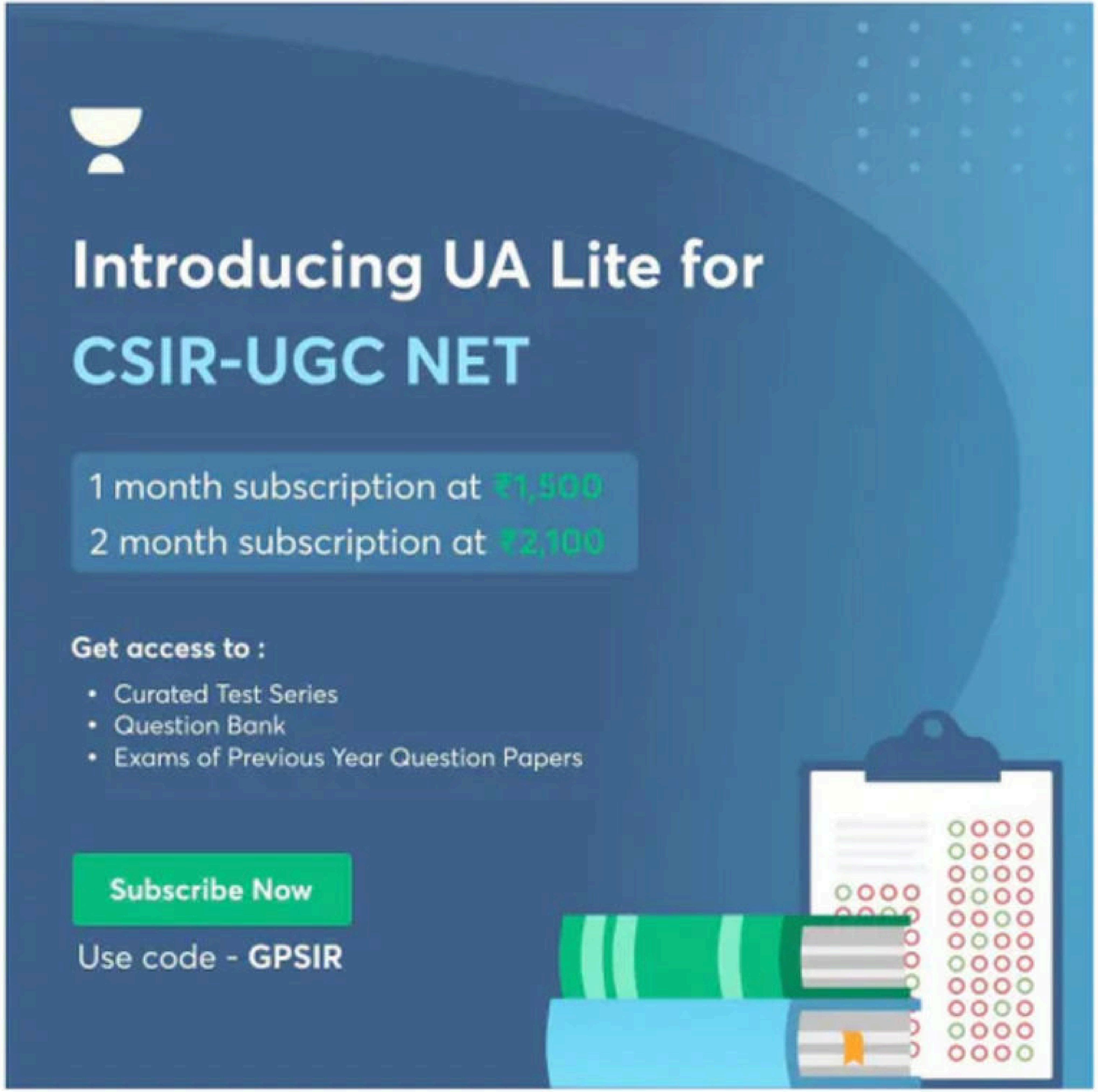
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