Gajendra Purohit

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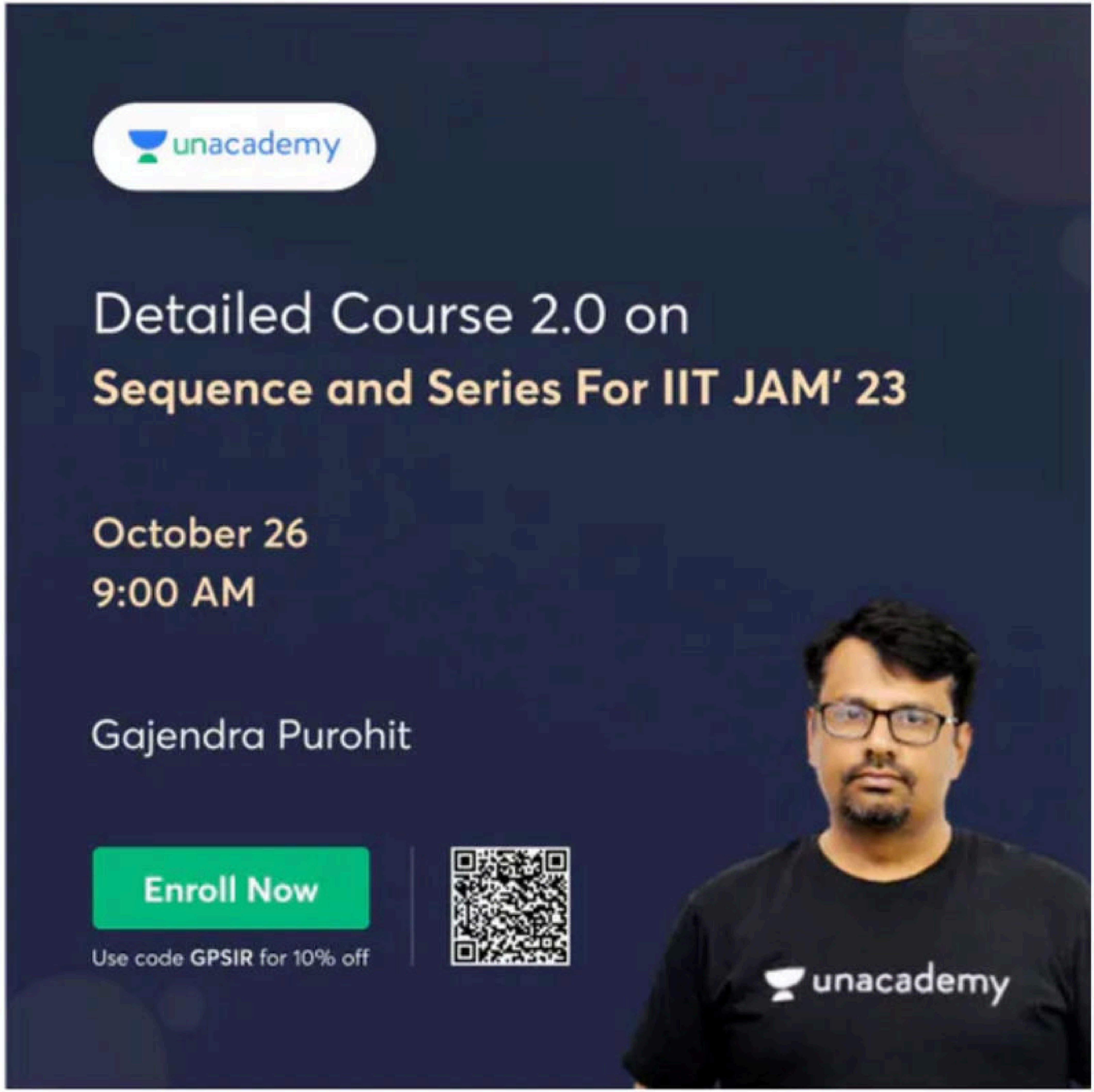
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Differentiability

Let f : [a, b] → R is a real valued function it is said to be a differentiable at x = c.

If $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$ finitely exist.

Right Hand Derivative :

$$Rf`(c)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

Left Hand Derivative :

$$Rf`(c)=\lim_{h\to 0}\frac{f(c-h)-f(c)}{-h}$$

Note : If f(x) is differentiable at x = c

Iff Rf`(c) = Lf`(c)

$$f(x)=\begin{cases} x\sin\frac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0}\frac{\cancel{x}\sin 1/x-0}{\cancel{x}}=\text{DNE}$$

$$f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$$

$\alpha>0$ cont.

$\alpha>1$ diff.

$$f(x) = |x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$f(x) = |x|^2 = \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -2x & x < 0 \\ 2x & x \geq 0 \end{cases}$$

$$f(x) = |x|^{\alpha}$$

$$\alpha > 1$$

$$f(x) = \begin{cases} x^2 & x \in \mathbb{Q} \\ 2x-1 & x \in \mathbb{Q}^c \end{cases}$$

$x^4 = 4x^2$

$x^4 - 4x^2 = 0$

$x^2(x^2 - 4) = 0$

$x = 0, 0, 2, -2$

$$f(x) = \begin{cases} x^4 & x \in \mathbb{Q} \\ 4x^2 & x \in \mathbb{Q}^c \end{cases}$$

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$x^2 = 2x - 1$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1, 1$

$x^2 \sin 1/x$

$\alpha > 0$

Q.1. Let $f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if} \quad x \neq 0 \\ 1 & \text{if} \quad x = 0 \end{cases}$, then f is

$x^{\alpha} \sin \frac{1}{x}$

$\alpha > 1$

(a) Discontinuous

(b) Continuous but not differentiable

(c) Differentiable only once

$f'(x) = 2x \sin \frac{1}{x}$

(d) Differentiable more than once.

Necessary condition for differentiable :

If a function is differentiable at $x = c$, then it is continuous at $x = c$ but converse may not be true.

$|x|$

Conclusion :

$$f(x)=\begin{cases} x^{\alpha}\sin\dfrac{1}{x^{\beta}}; & x\neq 0 \\ 0; & x=0 \end{cases}$$

(i) f(x) is continuous at x = 0 for $\alpha > 0$

(ii) f(x) is differentiable at x = 0 for $\alpha > 1$

Result :

(1) $f(x)=\begin{cases} x^{\alpha}\sin\dfrac{1}{x^{\beta}}; & x\neq 0 \\ 0; & x=0 \end{cases}$

then f(x) is differentiable $\left[\dfrac{p}{q}\right]$ times and

$f^{\left[\frac{p}{q}\right]}=\begin{cases} \cancel{\text{continuous} \quad \text{if } p \text{ is odd}} \\ \cancel{\text{discontinuous} \quad \text{if } p \text{ is even}} \end{cases}$

$f'(x)$

$x^{3}\sin\frac{1}{x^{2}}$

$f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x^{\beta}} & x\neq 0 \\ 0 & x=0 \end{cases}$

$\left[\frac{\alpha}{2}\right]$

$\left[\frac{3}{2}\right]=1$

(2) Let f : R → R be defined by $f(x)=\begin{cases}\phi_1(x) & x\in Q\\ \phi_2(x) & x\in Q^c\end{cases}$

f(x) is differentiable at the double root of $\phi_1(x) - \phi_2(x) = 0$

(3) A function is not differentiable at that point at which graph of function is sharp edge.

Q.2. Consider the function $f(x) = |\cos x| + |\sin(2 - x)|$.

$= 0$ $\quad$ $= 0$

At which of the following points is f not differentiable?

(a) $\left\{(2n+1)\dfrac{\pi}{2} : n \in Z\right\}$ $\qquad$ (b) $\{n\pi : n \in Z\}$

(c) $\{n\pi + 2 : n \in Z\}$ $\qquad$ (d) $\left\{\dfrac{n\pi}{2} : n \in Z\right\}$

$\sin(2-x) = 0$

$\cos x = 0$

$\cos x = \cos (2n+1)\pi/2$

$x = (2n+1)\pi/2$

$x - 2 = n\pi$

$x = n\pi + 2$

Q.3. The function $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$ is differentiable at $x = 0$

(a) for no values of a_0, a_1, a_2, a_3

(b) for any value of a_0, a_1, a_2, a_3

(c) only if $a_1 = 0$

(d) only if both $a_1 = 0$ and $a_3 = 0$

$|x|^2$

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Q.4. Let $f : R \rightarrow R$ be a differentiable function with $f(0) = 0$. If for all $x \in R$, $1 < f'(x) < 2$, then which one of the following statements is true on $(0, \infty)$?

IIT JAM 2015

(a) f is unbounded

(b) f is increasing and bounded

(c) f has at least one zero

(d) f is periodic

$f(x) = \frac{3x}{2}$

$f'(x) = \frac{3}{2}$

Q.5. Let the function $f : R \to R$ as

$$f(x)=\begin{cases} x^2 & x\in(0,2)\cap Q \\ 2x-1 & x\in(0,2)\cap(R/Q) \end{cases},$$

Which one of the following is not true?

(a) f is continuous at x = 1

(b) f is differentiable at x = 1

(c) f is not differentiable at x = 1

(d) f is differentiable only at x = 1

$x^2 = 2x-1$

$x^2-2x+1=0$

$x = 1, 1$

Q.6. Let S be the set of all function $f : R \to R$ satisfying $|f(x) - f(y)|^2 \leq |x - y|^3$ for all x,y in R.Then which of the following is /are true **IIT JAM 2022**

(a) Every function in S is differentiable

(b) $\exists$ a function $f \in S$ s.t. f is twice differentiable but f is not thrice differentiable

(c) $\exists$ a function $f \in S$ s.t. f is differentiable but f is not twice differentiable

(d) f is infinitely differentiable

$$\left|\frac{f(x) - f(y)}{(x-y)}\right|^2 \leq |x-y|$$

$$\lim_{x \to y}\left|\frac{f(x) - f(y)}{x-y}\right|^2 \leq \lim_{x \to y}|x-y|$$

$$|f'(y)|^2 \leq 0$$

$$f'(y) = 0$$

$$f(y) = c$$

Q.7. Let f : R → R be such that f, f′ and f′′ are all continuous functions with f > 0, f′ ≥ 0 and f′′(0), then

$$\lim_{x\to-\infty} \frac{f(x)+f`(x)}{2} \text{ is?}$$

IIT JAM 2020

(a) 0 (b) 1

(c) 2 (d) 3

$f = e^{2x}$

$f' = e^{2x}$

$f'' = e^{2x}$

$$\lim_{x\to-\infty} \frac{e^{x}+e^{x}}{2}$$

$$\lim_{x\to-\infty} \frac{\cancel{2}e^{x}}{\cancel{2}} = e^{-\infty} = 0$$

Q.8. Let f : [-1, 3] → R be a continuous function such that f differentiable on (-1, 3), $|f'(x)| \le \frac{3}{2}$ for all x ∈ (-1, 3), f(-1) = 1 and f(3) = 7 then f(1) = ? **IIT JAM 2020**

(a) 2

(b) 3

(c) 4

(d) 5

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$(-1, 1) \longrightarrow (3, 7)$

$y - 1 = \frac{7-1}{3+1}(x+1)$

$y - 1 = \frac{3}{2}(x+1)$

$y = \frac{3}{2}x + \frac{3}{2} + 1$

$f(x) = y = \frac{3}{2}x + \frac{5}{2}$

$f(1) = \frac{3}{2} + 5/2 =$

Q.9. Let $f : R \to R$ be a differentiate function such that $f'(x) > f(x)$, for all $x \in R$ and $f(0) = 1$, then $f(1)$ lies in the interval.

(a) $(0, e^{-1})$ (b) $(\sqrt{e}, e)$

(c) $(e^{-1}, \sqrt{e})$ (d) (e, ∞)

Q.10. Let f : [0 , 1] $\to$ R be a continouos function such that $f\left(\frac{1}{2}\right) = -\frac{1}{2}$ and $|f(x) - f(y) - (x - y)| \le \sin(|x - y|^2)$, $\forall$ x,y $\in$ [0,1]. then $\int_0^1 f(x)dx$ **IIT JAM 2020**

(a) -1/2 (b) -1/4

(c) 1/4 (d) 1/2


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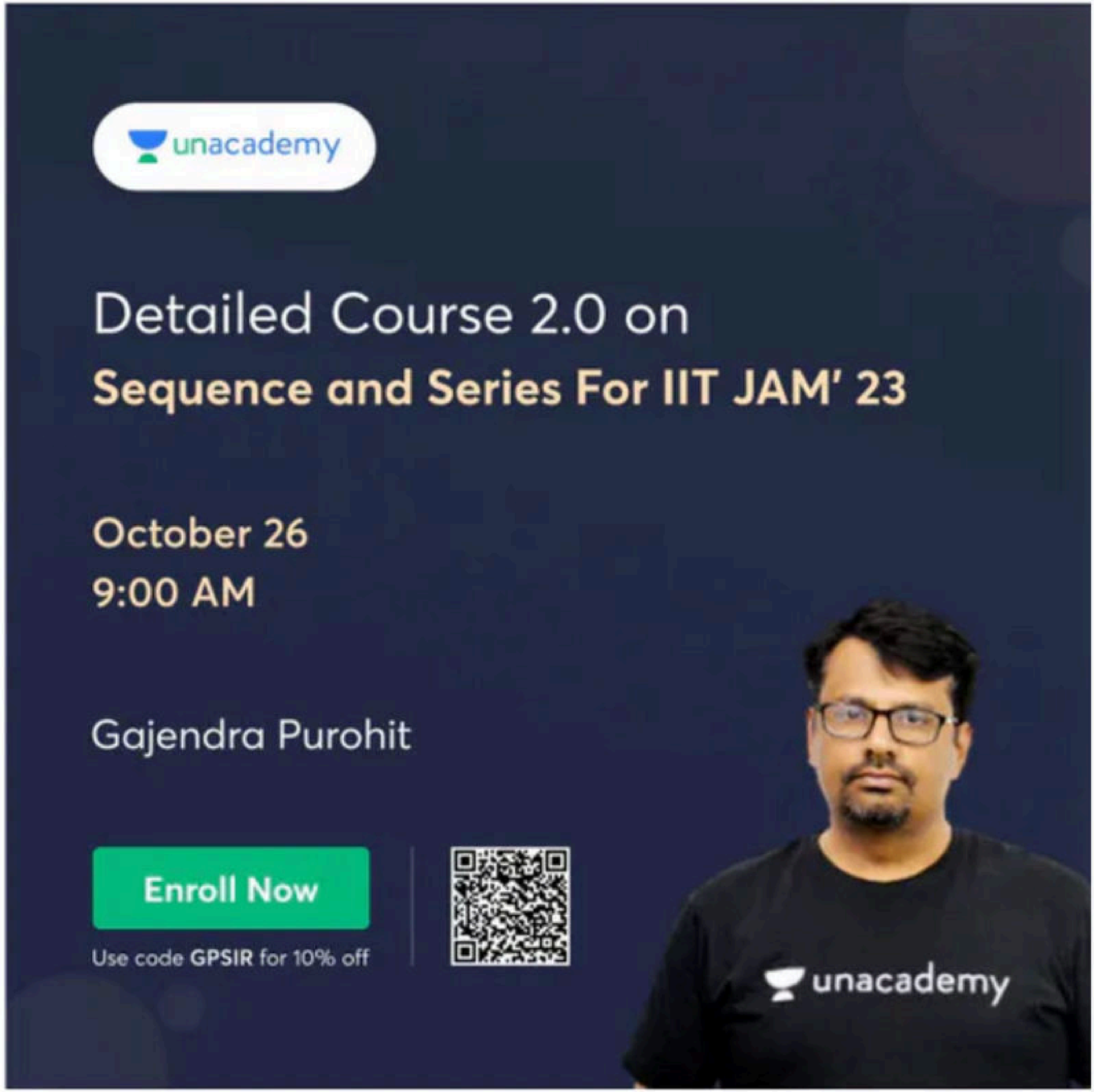
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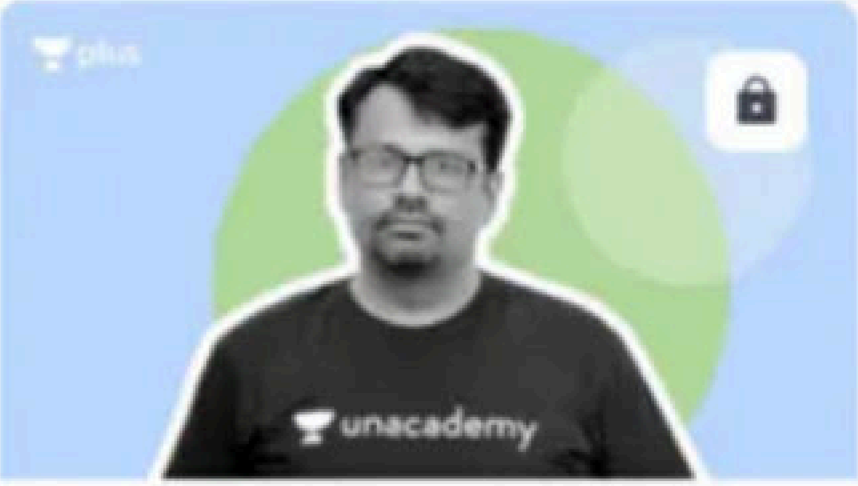

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Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

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