Group Homomorphism

Detail Course 2.0 on Group Theory for IIT JAM '23

Gajendra Purohit • Lesson 10 • Oct 18, 2022

Gajendra Purohit

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Conjugate Class and Class Equation

Conjugate elements in a group :

Let G be a group, a $\in$G and b $\in$ G. Then b is said to be conjugate to a.

If $b = xax^{-1}$ for some $x \in G$.

Conclusion : Two elements of S_n are said to be conjugate to each other if they have same cycle decomposition

Note : In abelian group, every element is conjugate to itself.

Self Conjugate element :

Let G be a group of an element $a \in G$ is called self conjugate element

of G if a is conjugate to itself only.

Conjugacy classes : The set of all conjugate elements of 'a' is called conjugacy classes of a in G and it is denoted by Cl(a) i.e. $Cl(a) = \{xax^{-1} ; x \in G\}$

Result : Number of distinct conjugate classes in S_n are partion of S_n.

Result :

(1) If a is self conjugate element then $Cl(a) = \{a\}$

(2) $Cl(e) = \{e\}$

(3) $Z(G)$, the centre of the group is the collection of all self conjugate elements.

i.e. if $a \in Z(G)$ then $Cl(a) = \{a\}$

(4) If G be a group a and b are conjugate element then $O(a) = O(b)$

i.e. if $O(a) \neq O(b)$ then a and b not conjugate to each other.

Example : In S_3, (1 2) and (1 2 3) can not be conjugate because order are different.

(5) If $O(a) = O(b)$ then we cannot say that a and b are conjugate.

Example : Let $G = Z_4$

Here 1 & 3 have same order but both are not conjugate to each other, they are self conjugate element.

Q.1. Let G be a group of order p^2 then $|Cl(a)|$ is, $a \in G$.

(a) 0 (b) 1

(c) p (d) p^2

Q.1. Total number of distinct conjugate classes S_4 are

(a) 2 (b) 3

(c) 5 (d) 7

Class equation : Let G be a finite group of order n and $c_1, c_2, \ldots c_k$ be k-distinct conjugate classes of cardinality $n_1, n_2, \ldots, n_k$ then the expression is called class equation of G.

Result :

(1) Let G be a finite group then $O(G) = O[Z(G)] + \sum_{a \notin Z(G)} O[Cl(a)]$

Here $O[Cl(a)] = \frac{O(G)}{O[N(a)]}$

$\Rightarrow$ $O[Z(G)] + \sum_{a \notin Z(G)} \frac{O(G)}{O[N(a)]}$

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Normalizer or Centralizer of an elements :

Let G be a group and $a \in G$ then $N(a) = \{ x \in G \mid ax = xa \text{ , for all } x \in G$

Note :

(i) N(a) is a subgroup of G

(ii) $a \in Z(G) \Leftrightarrow N(a) = G \Leftrightarrow cl(a) = \{a\}$

(iii) Let G be a finite group and $a \in G$ then $O(cl(a)) = \frac{O(G)}{O(N(a))}$

(iv) Number of elements of G which commute to a is $O(N(a)) = \frac{O(G)}{O(cl(a))}$

Q.2. Determine which of the following cannot be the class equation of a group. **CSIR NET DEC 2013**

(a) $10 = 1 + 1 + 1 + 2 + 5$

(b) $4 = 1 + 1 + 2$

(c) $8 = 1 + 1 + 3 + 3$

(d) $6 = 1 + 2 + 3$

Q.3. Which of the following cannot be class equation of group of order 10. **CSIR NET JUNE 2015**

(a) $10 = 1 + 1 + 1 + 2 + 5$ (b) $10 = 1 + 2 + 3 + 4$

(c) $10 = 1 + 2 + 2 + 5$ (d) $10 = 1 + 1 + 2 + 2 + 2 + 2$

Q.4. Let G be a group of order 9, then class equation is

(a) $9 = 3 + 2 + 3 + 1$

(b) $9 = 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1$

(c) $9 = 1 + 1 + 2 + 2 + 3$

(d) $9 = 2 + 2 + 2 + 2 + 1$

Q.5. Which of the following class equation for group?

(a) $3 = 1 + 2$

(b) $6 = 1 + 2 + 3$

(c) $14 = 1 + 7 + 2 + 2 + 2$

(d) $7 = 1 + 1 + 3 + 2$

Q.6. Let G be a group of all non – singular matrices of order n under multiplication i.e. $G = GL(n, Z_8)$, then number of self conjugate elements of G are

(a) Infinite

(b) Only one

(c) Finite but more than 10

(d) Finite but less than 10

Q.7. Number of elements of S_4 which commute to $\sigma = (1\ 2\ 3)$ are

(a) 1 (b) 2

(c) 3 (d) 5

Q.8. Number of elements of S_5 which commute to $\sigma = (1\ 2\ 3)(4\ 5)$ are

(a) 1 (b) 10

(c) 15 (d) 6

Q.9. which of the following are possible class equation of given order

(A) $39 = 1 + 3 + 3 + 3 + 3 + 13 + 13$

(b) $14 = 1 + 1 + 2 + 2 + 2 + 2 + 2 + 2$

(c) $21 = 1 + 3 + 3 + 7 + 7$

(d) $15 = 1 + 1 + 5 + 5 + 3$

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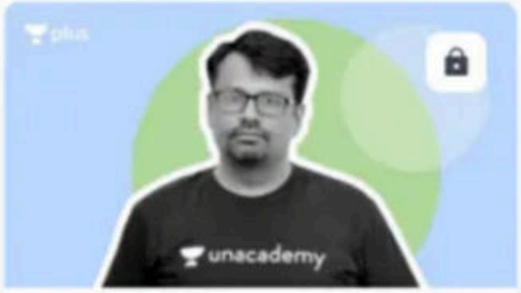

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- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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