Maxima and Minima - Part I

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 13 • Nov 25, 2022

Gajendra Purohit

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Continuity

A function $f : R^2 \to R$ is said to be continuous at (a, b) iff

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b).$$

Conclusion :

(1) If limit of function exist at (a, b) then this function need not be continuous at (a, b).

(2) If limit of function not exist then this is not continuous at (a, b).

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2}$$

$y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{x^2(mx)}{x^2+m^2x} = 0$$

$f(0,0) \neq 0$

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 2 & (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \neq 0$$

$$f(0,0) = 0$$

$$f(x,y) = \begin{cases} 1 & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = 1$$

$$f(0,0) = 0$$

Q.1. $f(x,y)=\begin{cases}\dfrac{1-\cos(x^2+y^2)}{x^2+y^2}; & (x,y)\neq(0,0)\\ 1/2 & otherwise\end{cases}$, then which of the following is/are true?

(a) f is not continuous at (0, 0)

(b) f is continuous at (0, 0)

(c) limit does not exist at (0, 0)

(d) limit exist at (0, 0)

$y=mx$

$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2}$

$\lim_{x\to 0} \frac{1-\cos(x^2+m^2x^2)}{x^2+m^2x^2}$

$\lim_{x\to 0} \frac{1-\cos x^2(1+m^2)}{x^2(1+m^2)}$

$\lim_{x\to 0} \frac{2\sin x^2(1+m^2)(1+m^2)}{2x(1+m^2)}$

$\lim_{x\to 0} \frac{(1+m^2)^2\cos(1+m^2)x^2}{2(1+m^2)}$

$\frac{(1+m^2)^2}{1+m^2}$

Q.2. Let $D \subseteq R^2$ be defined by $D = R^2 \setminus \{(x,0) : x \in R\}$. consider the function $f: D \to R$ defined by

$f(x,y) = x \sin \frac{1}{y}$ then **IIT JAM 2021**

(a) f is a discontinuous function on D

(b) f is continuous on D and cannot be extended continuously to any point outside D

(c) f is continuous on D and can be extended continuously to on $D \cup (0,0)$

(d) f is a continuous function on whole R^2

Q.3. Let $f(x,y)=\begin{cases}\dfrac{xy}{(x^2+y^2)^{\alpha}} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$. Consider a set $A=\{\alpha\in R \mid f(x,y) \text{ is continuous at } (0,0)\}$. Then

(a) A is a connected set

(b) A is a closed set

(c) A is a compact set

(d) None of the above

$\alpha<1$

$(-\infty,1)$

$2-2\alpha>0$

$-2\alpha>-2$

$2\alpha<2$

$\alpha<1$

$\frac{mn^2}{(n^2+n^2m)^{\alpha}}$

$\frac{mn}{n^{2\alpha}(1+m^2)^{\alpha}}$

$n^{2-2\alpha}>0$

Partial Derivatives :

If f(x, y) is a function of two variable then its partial derivative are the function f_x & f_y defined by

$$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}; h > 0$$

&

$$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}; h > 0$$

$$f = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$f_x = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{0 - 0}{h} = 0$$

$$f_y = \lim_{k \to 0} \frac{f(0,0+k) - f(0,0)}{k}$$

$$= 0$$

$$f = \begin{cases} \frac{x^2 y}{x^4 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$$f_x = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{h^2/h - 0}{h} = 1$$

$$f_y = \lim_{k \to 0} \frac{f(0,0+k) - f(0,0)}{k}$$

$$= \lim_{k \to 0} \frac{(0+k)/0 - 0}{k}$$

$$= 1$$

Q.4. Let $f(x,y)=\begin{cases}\dfrac{|x|\sqrt{x^4+y^2}}{|x|+|y|} & (x,y)\neq(0,0)\\ 0; & (x,y)=(0,0)\end{cases}$.

Then at (0,0) **IIT JAM 2019**

(a) f is continuous

(b) $f_x = 0$ and f_y doesnot exist

(c) f_x doesnot exist and $f_y = 0$

(d) $f_x = 0$ and $f_y = 0$

$f_y = \lim_{h\to 0} \frac{f(h) - f(0)}{h}$

$= \lim_{h\to 0} \frac{|h|h}{h|h|} = 0$

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Q.5. If $f : R^2 \to R$ is defined by

$$f(x,y)=\begin{cases}\dfrac{x^3}{x^2+y^4}; & (x,y)\neq(0,0)\\ 0; & (x,y)=(0,0)\end{cases}$$, then

IIT-JAM – 2012

(a) $f_x(0, 0) = 0$ & $f_y(0, 0) = 0$

(b) $f_x(0, 0) = 0$ & $f_y(0, 0) = 1$

(c) $f_x(0, 0) = 1$ & $f_y(0, 0) = 0$

(d) $f_x(0, 0) = 1$ & $f_y(0, 0) = 1$

$f_x = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h-0}$

$= \lim_{h\to 0} \frac{\frac{h^3}{h^2+0}-0}{h} = 1$

$f_y = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k}$

$= 0$

Q.6. For all $(x, y) \in R^2$, let

$$f(x,y)=\begin{cases}\dfrac{x}{|x|}\sqrt{x^2+y^2} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$$ Then

$\dfrac{\partial f}{\partial x}(0,0)+\dfrac{\partial f}{\partial y}(0,0)$ equals **IIT-JAM – 2014**

(a) -1 (b) 0

(c) 1 (d) 2

$$f_x=\lim_{h\to 0}\frac{f(h,0)-f(0,0)}{h}$$

$$=\lim_{h\to 0}\frac{h\sqrt{h^2+0}-0}{|h|\,h}$$

$$=1$$

$$f_y=\lim_{k\to 0}\frac{f(0,k)-f(0,0)}{k}$$

$$=0$$

Important Result :

We know that $f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

& $f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$

(1) $f_{xx}(a, b) = (f_x)_x (a, b)$

$$= \lim_{h\to 0} \frac{f_x(a+h,b) - f_x(a,b)}{h}$$

(2) $f_{yy}(a, b) = (f_y)_y (a, b)$

$$= \lim_{h\to 0} \frac{f_y(a,b+h) - f_y(a,b)}{h}$$

$$f = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

$f_x(0,0) = 0$

$f_y(0,0) = 0$

$$f_{xx} = (f_x)_x = \lim_{h \to 0} \frac{f_x(h,0) - f_x(0,0)}{h} = \lim_{h \to 0} \frac{f_x(h,0)}{h} = 0$$

$$f_{xy} = (f_x)_y = \lim_{k \to 0} \frac{f_x(0,k) - f_x(0,0)}{k} = \lim_{k \to 0} \frac{f_x(0,k)}{k}$$

$$= \lim_{k \to 0} \frac{1}{k} \left[\lim_{h \to 0} \frac{f(h,k) - f(0,k)}{h} \right]$$

$$= \lim_{h \to 0, k \to 0} \left(\frac{\frac{hk}{h^2+k^2}}{hk} \right) = \infty$$

(3) $f_{xy}(a, b) = (f_x)_y(a, b)$

$$= \lim_{h \to 0} \frac{f_x(a, b+h) - f_x(a,b)}{h}$$

(4) $f_{yx}(a, b) = (f_y)_x(a, b)$

$$= \lim_{h \to 0} \frac{f_y(a+h, b) - f_y(a,b)}{h}$$

Q.7. Let $f : R^2 \to R$ s.t.

$$f(x,y) = \begin{cases} \dfrac{x^2 y^2 (x-y)}{x^2+y^2}; & (x,y) \neq (0,0) \\ 0; & (x,y) = (0,0) \end{cases}.$$

Then $\dfrac{\partial}{\partial x}\left(\dfrac{\partial f}{\partial y}\right) - \dfrac{\partial}{\partial y}\left(\dfrac{\partial f}{\partial x}\right)$ at (0, 0) is

IIT JAM – 2018

(a) 0 (b) 1

(c) 2 (d) -1

$f_{x0} = 0$, $f_{y0} = 0$

$$(f_{yx})_{0,0} = \lim_{h\to 0} \frac{f_y(h,0) - f_y(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h} \lim_{k\to 0} \frac{f(h,k) - f(h,0)}{k}$$

$$= \lim_{h\to 0} \left(\frac{1}{h} \lim_{k\to 0} \frac{h^2 k (h-k)}{(h^2+k^2)k} \right)$$

$$= 1$$

$$(f_{xy})_{0,0} = \lim_{k\to 0} \frac{f_x(0,k) - f_x(0,0)}{k}$$

$$= \lim_{k\to 0} \frac{1}{k} \lim_{h\to 0} \frac{f(h,k) - f(0,k)}{h}$$

$$= \lim_{k\to 0} \frac{1}{k} \lim_{h\to 0} \left(\frac{h^2 k (h-k)}{h^2+k^2} \right)$$

$$= 0$$

$1 - 0$

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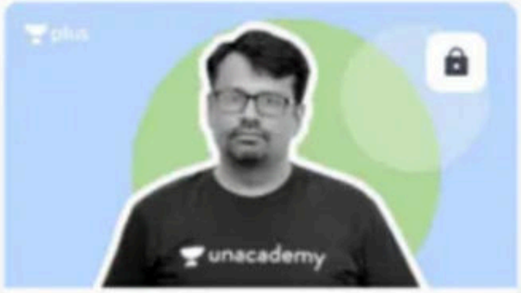

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- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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