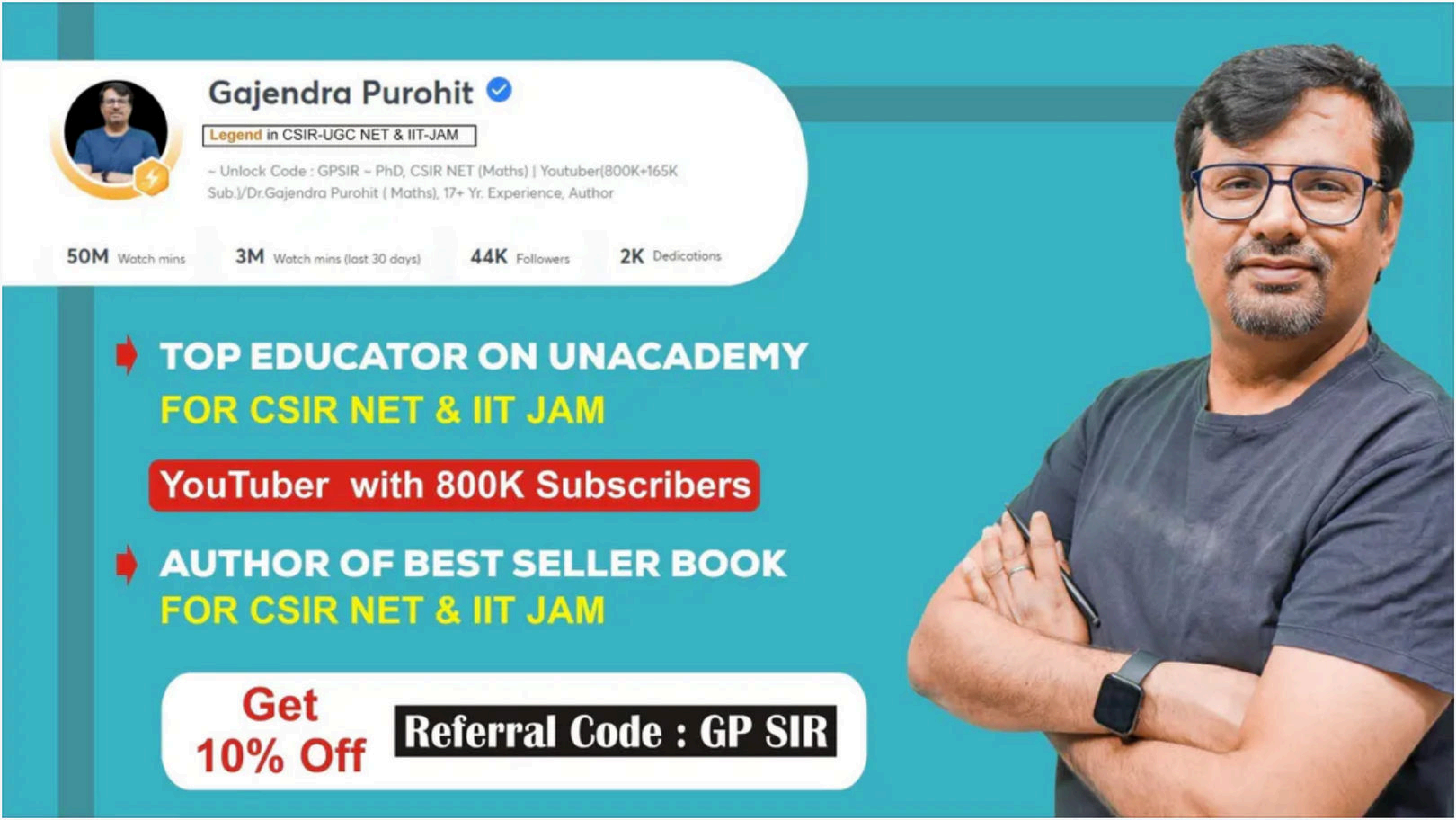
Doubt Clearing Session

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 8 • Nov 18, 2022



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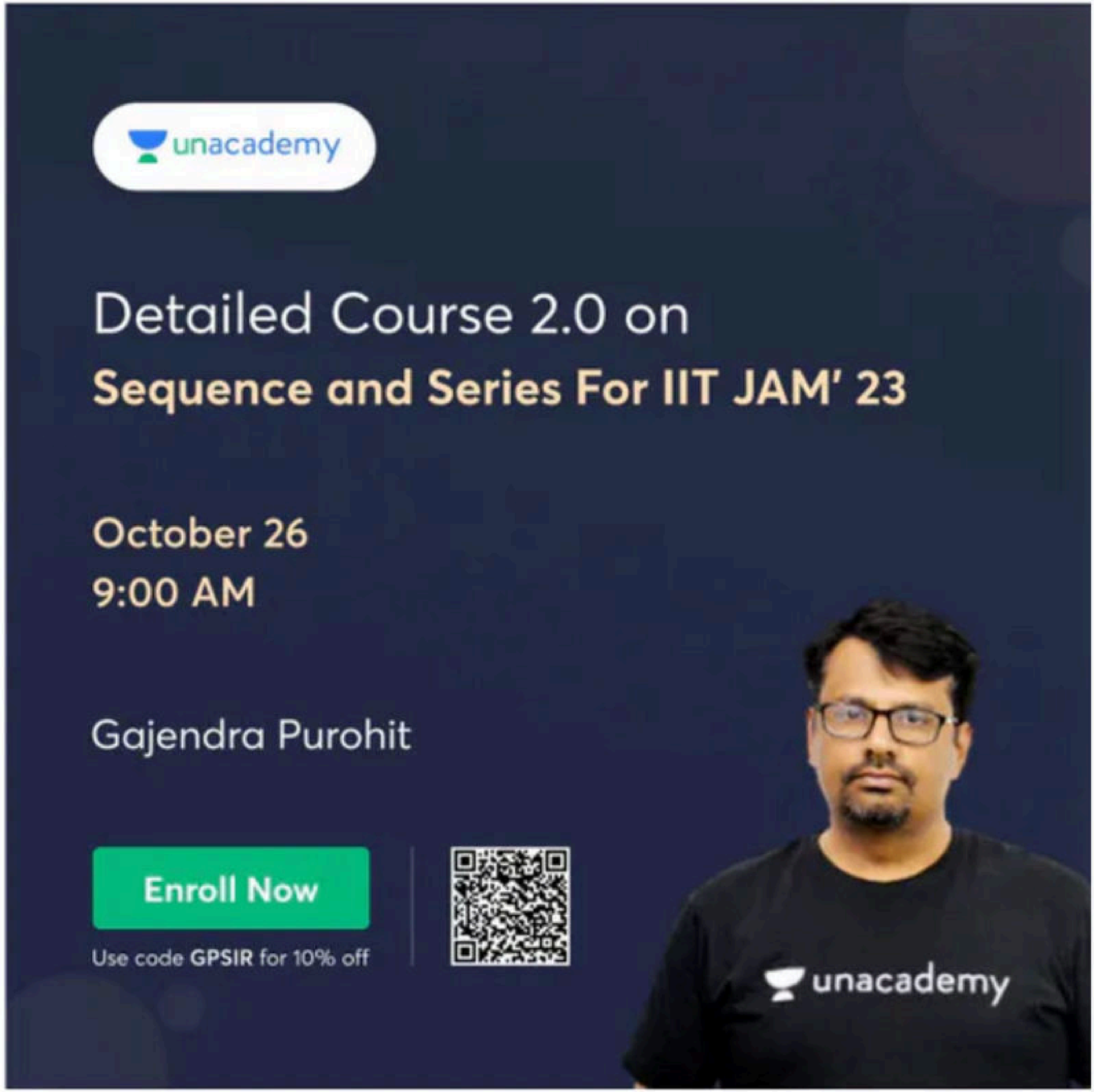
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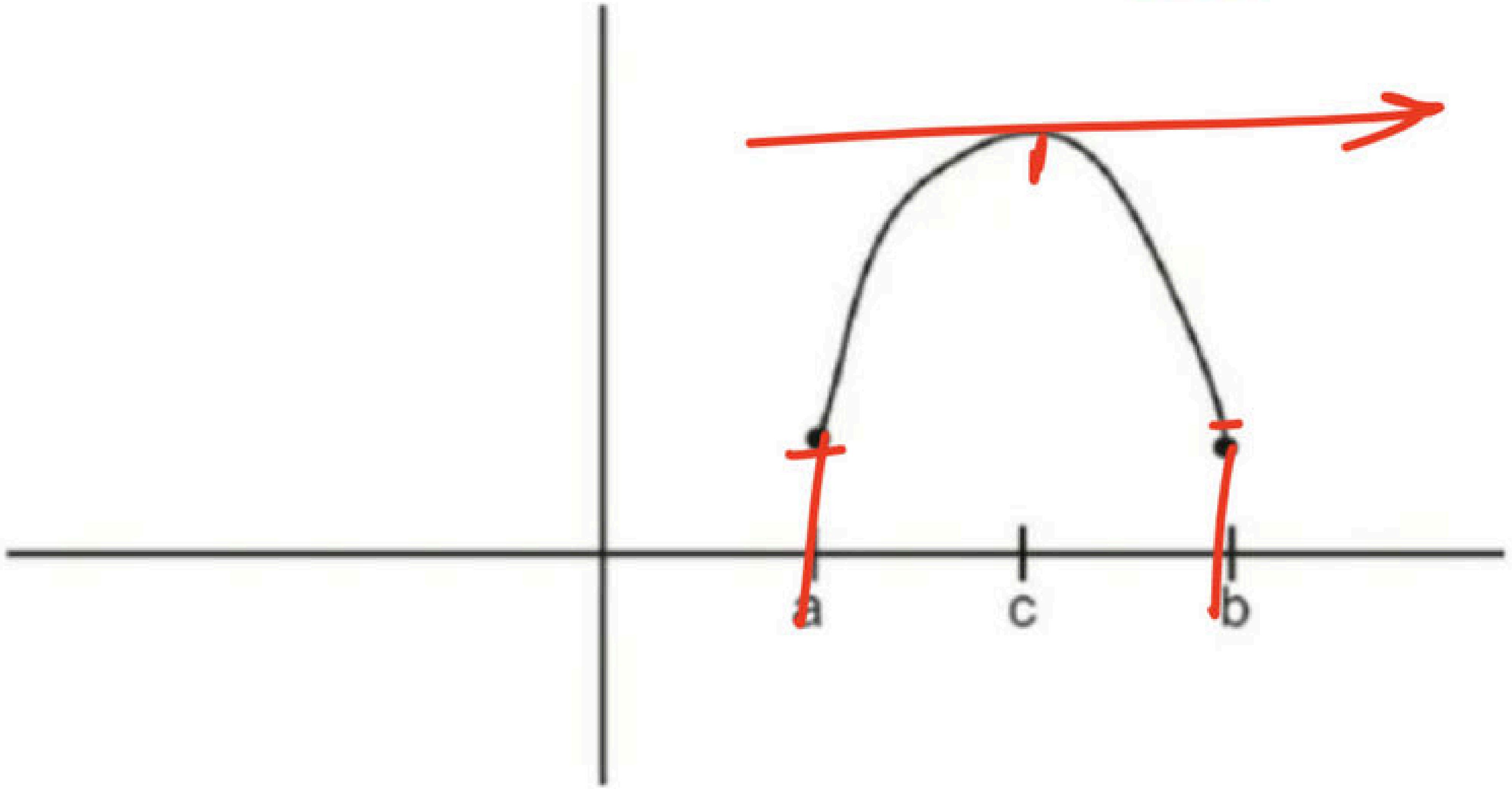
Mean Value Theorem :

(1) **Rolle's Theorem :** Let f be a function defined on [a, b] s.t.

(a) f is continuous on [a, b]

(b) f is differentiable on (a, b)

(c) f(a) = f(b) then $\exists\, c \in (a, b)$ s.t. $f'(c) = 0$

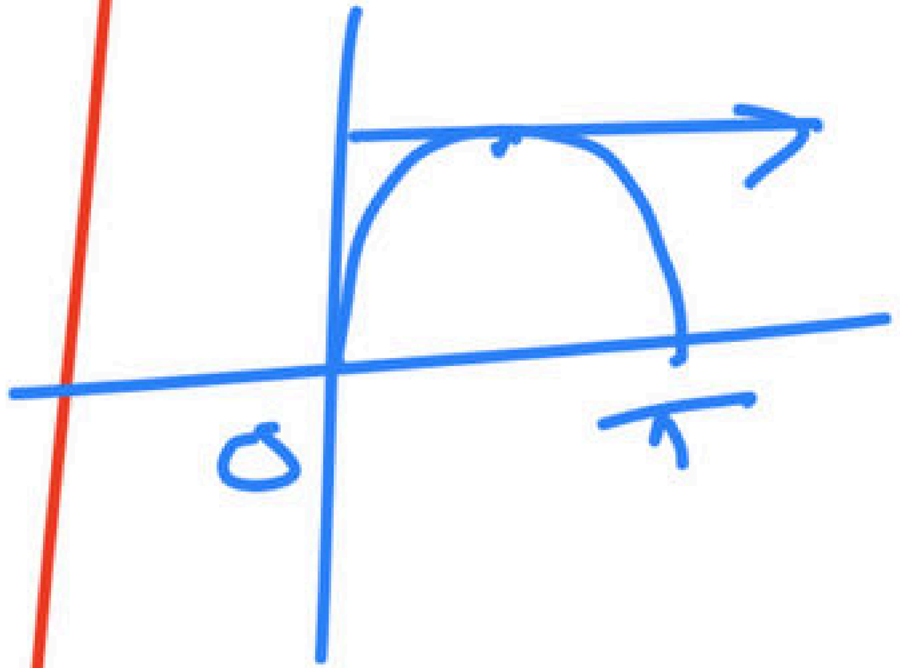


$f(x) = \sin x \quad [0, \pi]$

$f(0) = 0 = f(\pi)$

$f'(c) = \cos c = 0$

$c = \pi/2$



$f(x) = x^2 \quad x \in [-1, 1]$

Q.1. If $f(x)=\begin{cases} 1+x & \text{if } x<0 \\ (1-x)(px+q) & \text{if } x\ge 0 \end{cases}$ satisfies the assumption of Rolle's theorem in the interval [-1, 1] then the order pair (p, q) is **IIT JAM 2017**

~~(a) (2, -1)~~ ~~(b) (-2, -1)~~

~~(c) (-2, 1)~~ (d) (2, 1)

1

$(1-x)p+(-1)(px+q)$

1

$p-q$

$p-q=1$

$p=1+q=2$

Q.3. Using Rolle's theorem ,the equation $a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_n = 0$ has atleast one root between 0 and 1 , If

$f(0) = f(1)$

(a) $\frac{a_0}{n} + \frac{a_1}{n-1} + \ldots\ldots a_{n-1} = 0$

(b) $\frac{a_0}{n-1} + \frac{a_1}{n-2} + \ldots\ldots a_{n-2} = 0$

(c) $\frac{a_0}{n+1} + \frac{a_1}{n} + \ldots\ldots a_n = 0$

(d) $a_0 n + a_1(n-1) + \ldots\ldots a_{n-1} = 0$

$f(x) = \frac{a_0 x^{n+1}}{n+1} + \frac{a_1 x^n}{n} + \cdots a_n x$

$f(0) = 0$

$f(1) = \frac{a_0}{n+1} + \frac{a_1}{n} + \cdots a_n = 0$

$f'(x) = a_0 x^n + a_1 x^{n-1} + \cdots a_n = 0$

Lagrange's mean value theorem (LMVT) :

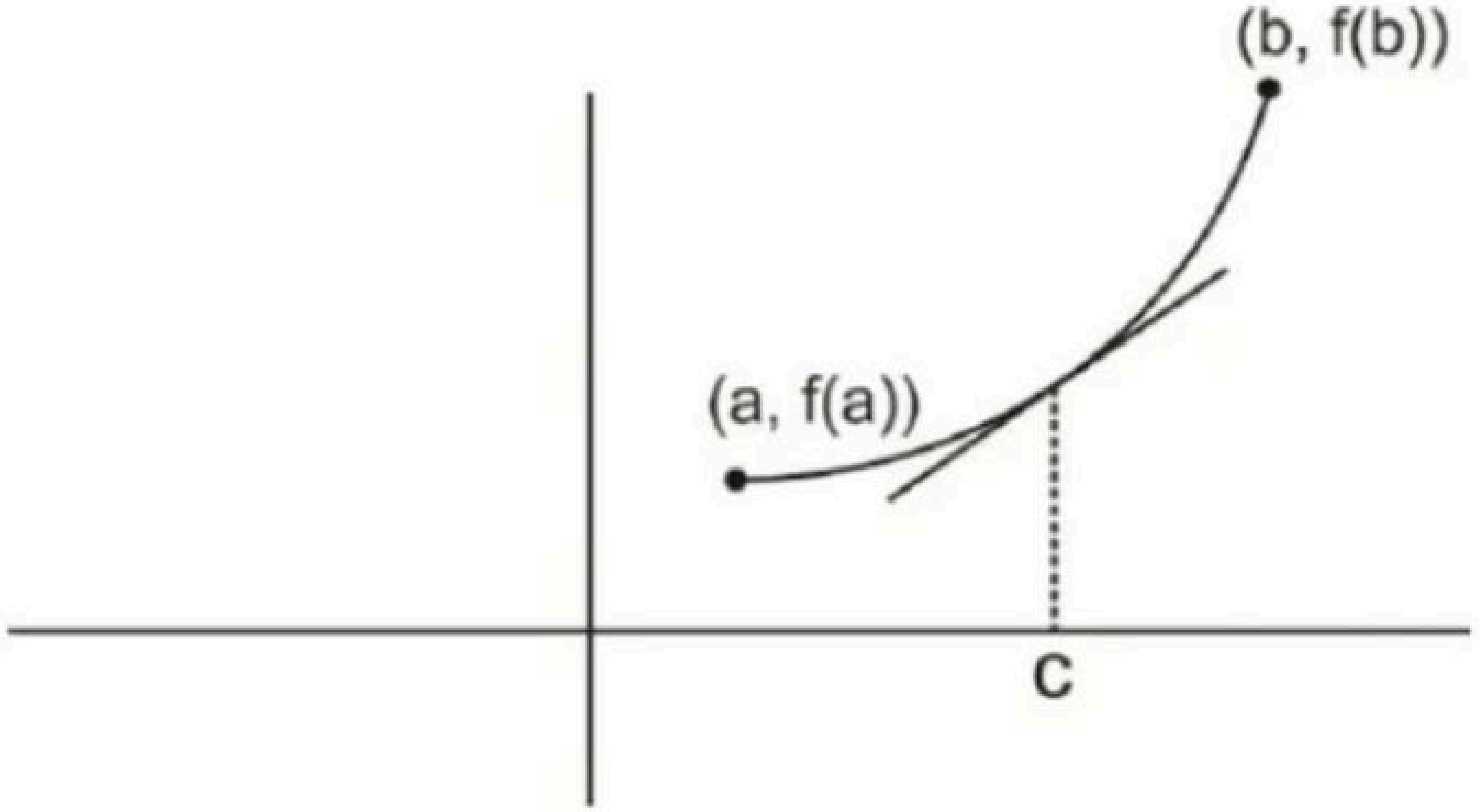
Let f be a function defined on [a, b] s.t.

(i) f is continuous on [a, b]

(ii) f is differentiable on (a, b),

then $\exists$ c $\in$ (a, b)

s.t. $$\frac{f(b)-f(a)}{b-a} = f`(c)$$



$f(x) = 2x^2 + 3x + 4$ $\quad x \in [1, 2]$

$f'(x) = 4x + 3$

$f(1) = 9$ $\quad f(1) \neq f(2)$

$f(2) = 18$

$f'(c) = \frac{f(2) - f(1)}{2 - 1} = 9$

$4c + 3 = \frac{18 - 9}{2 - 1}$

$4c + 3 = 9$

$4c = 6$

$c = 6/4 = 3/2$

$f(x) = |x|$ $\quad [-1, 1]$

Q.3. Let $f : [a, b] \to R$ be a differentiable function. there exist point $c_1, c_2 \in (a, b)$ then which of the following is true **IIT JAM 2005**

(a) $3f(c_1)\, f'(c_1) = f'(c_2)\, [f(a) - f(b)]$

(b) $4f(c_1)\, f'(c_1) = f''(c_2)\, [f(a) + f(b)]$

(c) $5f(c_1)\, f''(c_1) = f'(c_2)\, [f(a) + f(b)]$

(d) $2f(c_1)\, f'(c_1) = f'(c_2)\, [f(a) + f(b)]$

$[f(x)]^2$

$2f(x)f'(x) = \frac{f(b)^2 - f(a)^2}{b-a}$

$2f(c_1)f'(c_1) = \frac{(f(b)-f(a))}{(b-a)}(f(b)+f(a))$

$2f(c_1)f'(c_1) = f'(c_2)(f(a)+f(b))$

Q.4. For a, b $\in$ R with a < b, let f : [a, b] $\to$ R be continuous on [a, b] and twice differentiable on (a, b). Further, assume that the graph of f intersects the straight line segment joining the points (a, f(a)) and (b, f(b)) at point (c, f(c)) for a < c < b. Then which of the following is always true **IIT JAM 2012**

(a) There exists a real number $\xi \in (a, b)$ such that $f``(\xi) = 0$

(b) For all real number $\xi \in (a, b)$ such that $f``(\xi) \neq 0$

(c) we can't say

(d) None of these

$$\rho = \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\frac{d^2y}{dx^2} = 0}$$

Cauchy's Mean Value Theorem :

Let f & g be two function defined on [a, b] s.t.

(i) f and g are continuous in [a, b]

(ii) f & g are differentiable in (a, b)

(iii) $g`(x) \neq 0$ for each $x \in (a, b)$ and $g(a) \neq g(b)$.

Then $\exists$ at least one point $c \in (a, b)$ s.t.

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$$

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$g'(c)=\frac{g(b)-g(a)}{(b-a)}$$

$$\frac{f'(c)}{g'(c)}=\frac{f(b)-f(a)}{g(b)-g(a)}$$

$f(x) = x^2 \qquad g(x) = x^3 \qquad x \in [1, 2]$

$$\frac{f(2) - f(1)}{g(2) - g(1)} = \frac{f'(c)}{g'(c)}$$

$$\frac{4 - 1}{8 - 1} = \frac{2\cancel{c}}{3c^{\cancel{2}}}$$

$$\frac{3}{7} = \frac{2}{3c}$$

$$c = \frac{14}{9}$$

Q.5. The value of ξ in the mean value theorem of $f(b) - f(a) = (b - a)f'(\xi)$ for $f(x) = Ax^2 + Bx + C$ in (a, b) is

(a) $b + a$ (b) $b - a$

(c) $\frac{(b+a)}{2}$ (d) $\frac{(b-a)}{2}$

$f(a) = Aa^2 + Ba + C$

$f(b) = Ab^2 + Bb + C$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$f'(c) = \frac{(Aa^2 + Ba + \cancel{C}) - (Ab^2 + Bb + \cancel{C})}{(b - a)}$

$f'(c) = \frac{-[A(a-b)(a+b) + B(a-b)]}{b - a}$

$f'(c) = A(a+b) + B$

$2Ac + \cancel{B} = A(a+b) + \cancel{B}$

$c = \frac{a+b}{2}$

Q.6 A function $f(x) = 1 - x^2 + x^3$ is defined in the closed interval [-1, 1]. The value of x, in the open interval (-1, 1) for which the mean value theorem is satisfied is

(a) -1/2 (b) -1/3

(c) 1/3 (d) 1/2

$-2x+3x^2$

$f(-1) = -1$

$f(1) = 1$

$f'(c) = \frac{f(1)-f(-1)}{1-(-1)}$

$-2c+3c^2 = \frac{1-(-1)}{1-(-1)}$

$-2c+3c^2 = 1$

$3c^2-2c-1=0$

$(3c+1)(c-1)=0$

$c=-1/3$ $\cancel{1}$

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Taylor's infinite series :

Let $n \in N$, $I = [a,b]$ and $f : I \to R$ be a function f , f' , f''….$f^{(n)}$ are continouous on I and that $f^{(n+1)}$ exist on (a , b). then

$$f(x) = f(a) + (x-a)f`(a) + \ldots\ldots \frac{(x-a)^{n-1}}{(n-1)!} f^{(n-1)}(a) + \frac{(x-a)^{n}}{(n)!} f^{(n)}(a) + ..$$

This is called Taylor's infinite series about x = a.

Maclaurin's infinite series :

In Taylor's series put a = 0.

So, $f(x) = f(0) + xf`(0) + \ldots\ldots \frac{x^n}{n!} f^{(n)}(0)$

which is called Maclaurin's infinite series

$x^5 \frac{f^{V}(0)}{5!} =$

$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \cdots$

$+ \frac{x^3}{3!} f'''(0)$

$\frac{4!}{5!} = \frac{1}{5}$

$\log(1+x) = f(x)$

$f'(x) = \frac{1}{1+x}$

$f''(x) = -\frac{1}{(1+x)^2}$

$f'''(x) = \frac{+2}{(1+x)^3}$

$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$

$\log(1-x)$

Tricks : If f(x) is continuous function and it is vanishes at countably infinte numbers then it will

be identically zero

Q.7. Let S be the set of all continuous function $f : [-1,1] \to R$ satisfying the following three conditions

(i) f is infinitely differentiable on the open interval (-1,1)

(ii) The Taylor's series

$f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\ldots\ldots$ of f at 0

converges to f(x) for each $x \in (-1,1)$

(iii) $f\left(\frac{1}{n}\right)=0$ for all $n \in N$

Which of the following is true**IIT JAM 2022**

(a) $f(0) = 0$ for all $f \in S$

(b) $f'\left(\frac{1}{2}\right)=0$ for all $f \in S$

(c) $\exists\, f \in S$ such that $f'\left(\frac{1}{2}\right)\neq 0$

(d) $\exists\, f \in S$ such that $f(x) \neq 0$ for some $x \in [-1,1]$

Q.8. Let α be the real number such that the coefficient of x^{125} in Maclaurin's series of $(x + \alpha^3)e^x$ is $\frac{28}{(124)!}$, then

α **IIT JAM 2020**

(a) 15 (b) 20

(c) 25 (d) 30

Q.9. Let $f(x) = \sqrt{x} + \alpha x$, x > 0 and g(x) = $a_0 + a_1(x - 1) + a_2(x - 1)^2$ be the sum of the first three terms of the taylor series of f(x) around x = 1. If g(3) = 3, then α is? **IIT JAM 2019**

(a) 1 (b) 1/2

(c) 1/4 (d) 3/4

Q.10. The coefficient of x^2 in the Maclaurin's series expansion of the function $f(x) = xe^x$.

(a) 0 (b) 1

(c) 2 (d) 3


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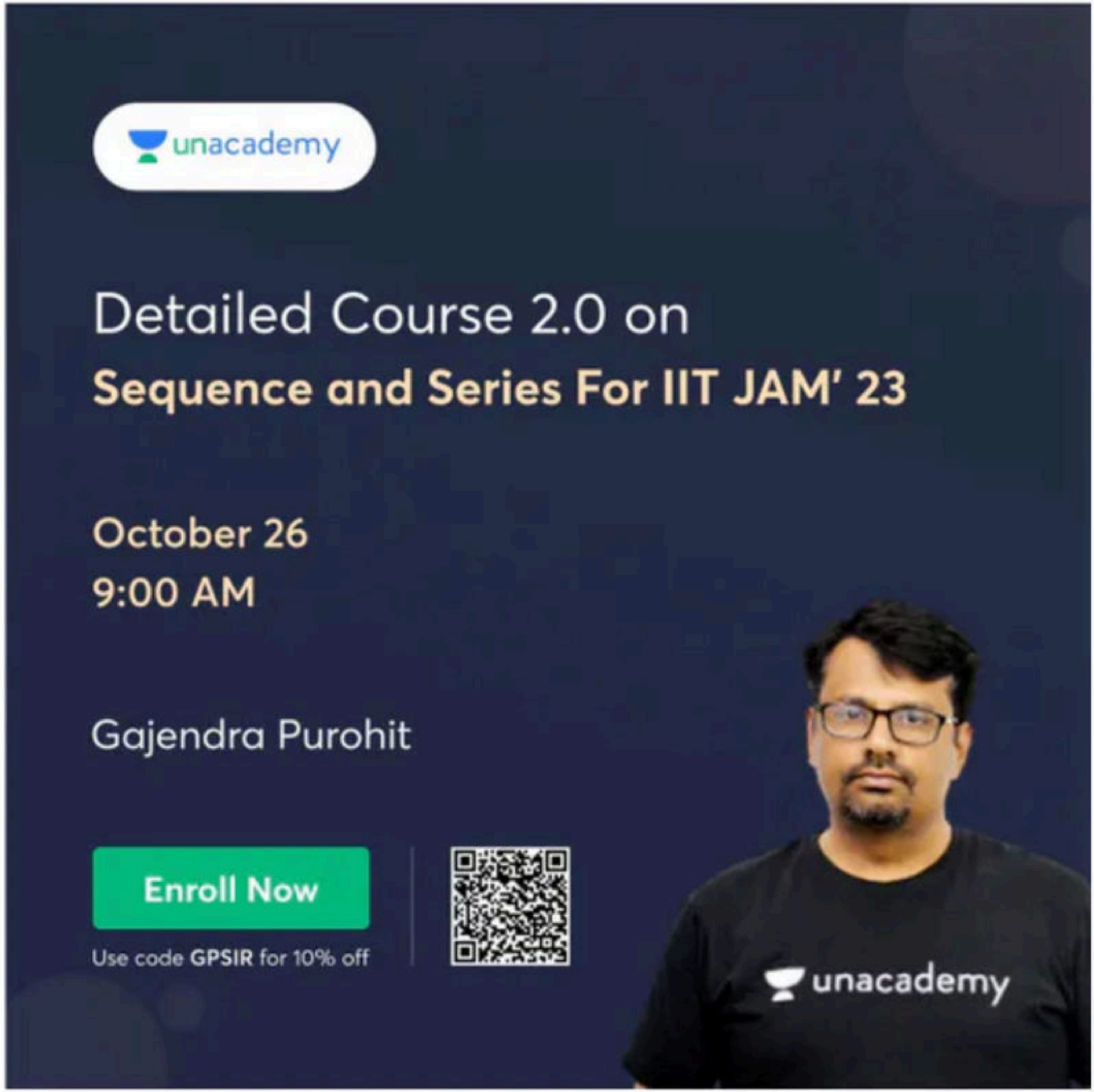
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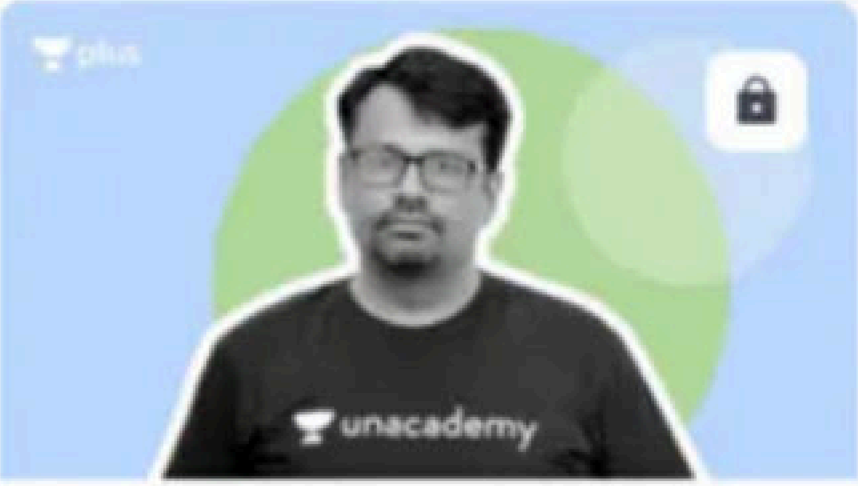

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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