Basis and Dimension

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 11 • Sept 20, 2022

Gajendra Purohit

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Eigen value &eigen vector

Characteristic polynomial : Let A be square matrix of order n then $C_A(x) = \det(xI - A) = \det(A - xI)$ is a polynomial of degree n called the characteristic polynomial of A and the equation $C_A(x) = \det(A - xI) = 0$ is called characteristic equation.

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▲ 2 • Asked by Dr Anurag

Please help me with this doubt

19. Let A be an $n \times n$ matrix over $\mathbb{C}$ such that every nonzero vector of $\mathbb{C}^n$ is an eigenvector of A. Then

(a.) All eigenvalues of A are equal.

(b.) All eigenvalues of A are distinct.

(c.) $A = \lambda I$ for some $\lambda \in \mathbb{C}$, where I is the $n \times n$ identity matrix.

(d.) If χ_A and m_A denote the characteristic polynomial and the minimal polynomial respectively, then $\chi_A = m_A$.

Eigen value and Eigen vector : Let A be any matrix of order n then roots of characteristic equation is called eigen value.

i.e. If A is matrix and $[A - \lambda I]X = 0$ then λ is eigen value and X is eigen vector corresponding to λ

Note : Eigen vector corresponding to distinct eigen value are LI

Result : If λ is eigen value of A then

(1) Eigen value of αA is $\alpha\lambda$

(2) Eigen value of A^n is λ^n.

(3) Sum of all eigen value = Trace (A)

(4) Product of all eigen value = det(A)

(5) Eigen value of A^{-1} is λ^{-1}.

(6) Eigen value of Adj(A) is $\frac{|A|}{\lambda}$

(7) If sum of each row in A is equal to k then k must be eigen value and it is largest eigen value.

Q.1. Let A be a 3×3 matrix with eigen value 1, -1, 0. Then the determinant of $I + A^{100}$ is

(a) 6 (b) 4

(c) 9 (d) 100

Q.2. Let $A=\begin{pmatrix} 2 & -1 & 3 \\ 2 & -1 & 3 \\ 3 & 2 & -1 \end{pmatrix}$. Then the largest eigenvalue of A is

(a) 1 (b) 2

(c) 3 (d) 4

Eigen value for different type of matrix

(1) Eigen values of symmetric matrix and hermitian matrix are real.

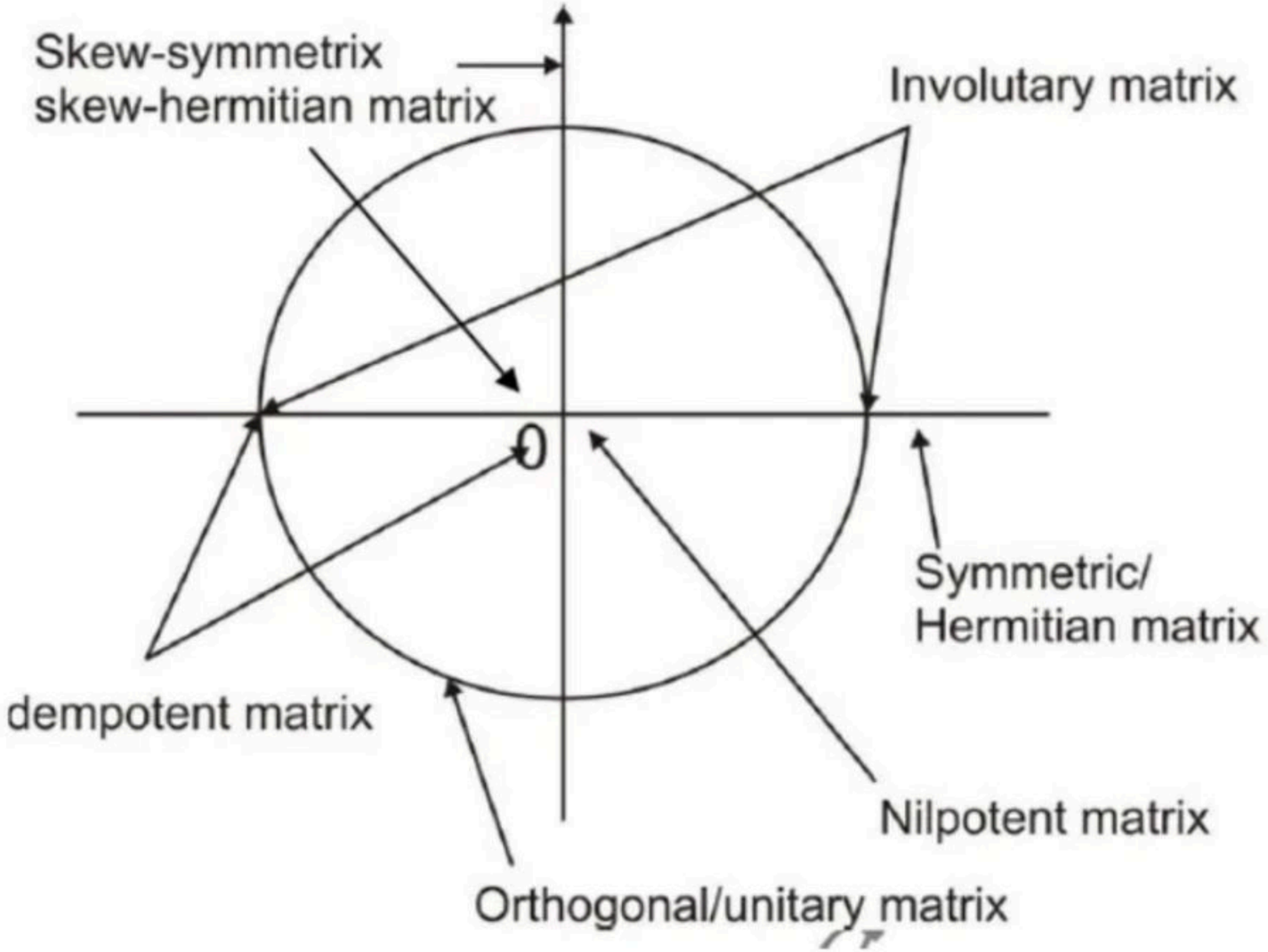
(2) Eigen value of skew-symmetric and skew-hermitian matrix are either zero or purely imaginary.

(3) Eigen values of involutory matrix are either 1 or -1 or both.

(4) Eigen values of idempotent matrix are either 0 or 1 or both.

(5) Eigen values of nilpotent matrix are 0.

(6) Eigen values of orthogonal matrix and unitary matrix are unit modulus.



(7) Eigen value of permutation matrix.

Let $\sigma = c_1.c_2. \ldots . c_k$ product of disjoint cycles such that $l(c_i) = r_i$ where $l(c_i)$ = length of c_i.

Then characteristic of A is $c(x) = \prod_{ri}(x^{r_i} - 1)$

i.e. $\sigma = (12)\,(3) \in S_3$

$c_1 = (12)$ and $l(c_1) = 2 = r_1$

$c_2 = (3)$ and $l(c_2) = 1 = r_2$ then $c(x) = (x^2 - 1)(x - 1)$

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Q.3. Which of the following properties are true?

(a) If λ is an eigen value of A then 2λ is an eigen value of A^{-1}.

(b) If λ is an eigen value of A then $1/\lambda$ is an eigen value of A^{-1}.

(c) If λ is an eigen value of an orthogonal matrix, then $1/\lambda$ is also its eigen value.

(d) All of the above.

Q.4. The square matrix A is said to be an idempotent if $A^2 = A$.

An idempotent matrix is non-singular iff

(a) All E.V. are real

(b) All E.V. are real non-negative

(c) All E.V. are either 0 or 1

(d) All E.V. are 1

Q.5 The trace of the matrix $A = \begin{bmatrix} 3 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{15}$ is

(a) $1 + 3^{15}$

(b) $2 + 3^{15}$

(c) 3^{15}

(d) 0

Q.6. Let A be 3×3 matrix with real entries such that 1, -1, 2 are its eigenvalues if $B = A^3 + 2A^2 + I$, then

(a) det (B) = 50 (b) det (B) = 136

(c) det (B) = 23 (d) det(B) = 17

Q.7.. Let A and B be n × n real matrices and let $C = \begin{pmatrix} A & B \\ B & A \end{pmatrix}$

.Which of the following statements are true?

(a) If λ is an eigenvalue of A + B then λ is an eigen value of C

(b) If λ is an eigenvalue of A – B then λ is an eigen value of C

(c) If λ is an eigen value of A or B then λ is an eigen value of C

(d) All eigen values of C are real

Q.8. Which of the following eigen values of the matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

(a) 1 (b) -1

(c) i (d) –i

Q.1. Let $M_n(R)$ be the set of $n \times n$ matrices with real entries.

Which of the following is true?

(a) Any matrix $A \in M_4(R)$ has a real eigen value.

(b) Any matrix $A \in M_5(R)$ has a real eigen value.

(c) Any matrix $A \in M_2(R)$ has a real eigen value

(d) None of these

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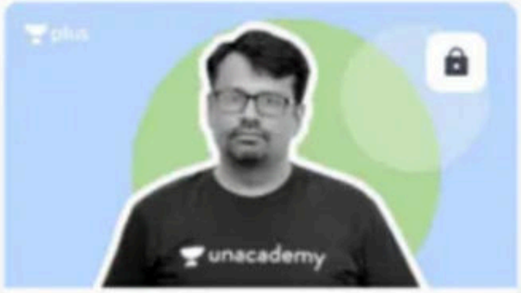

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Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
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