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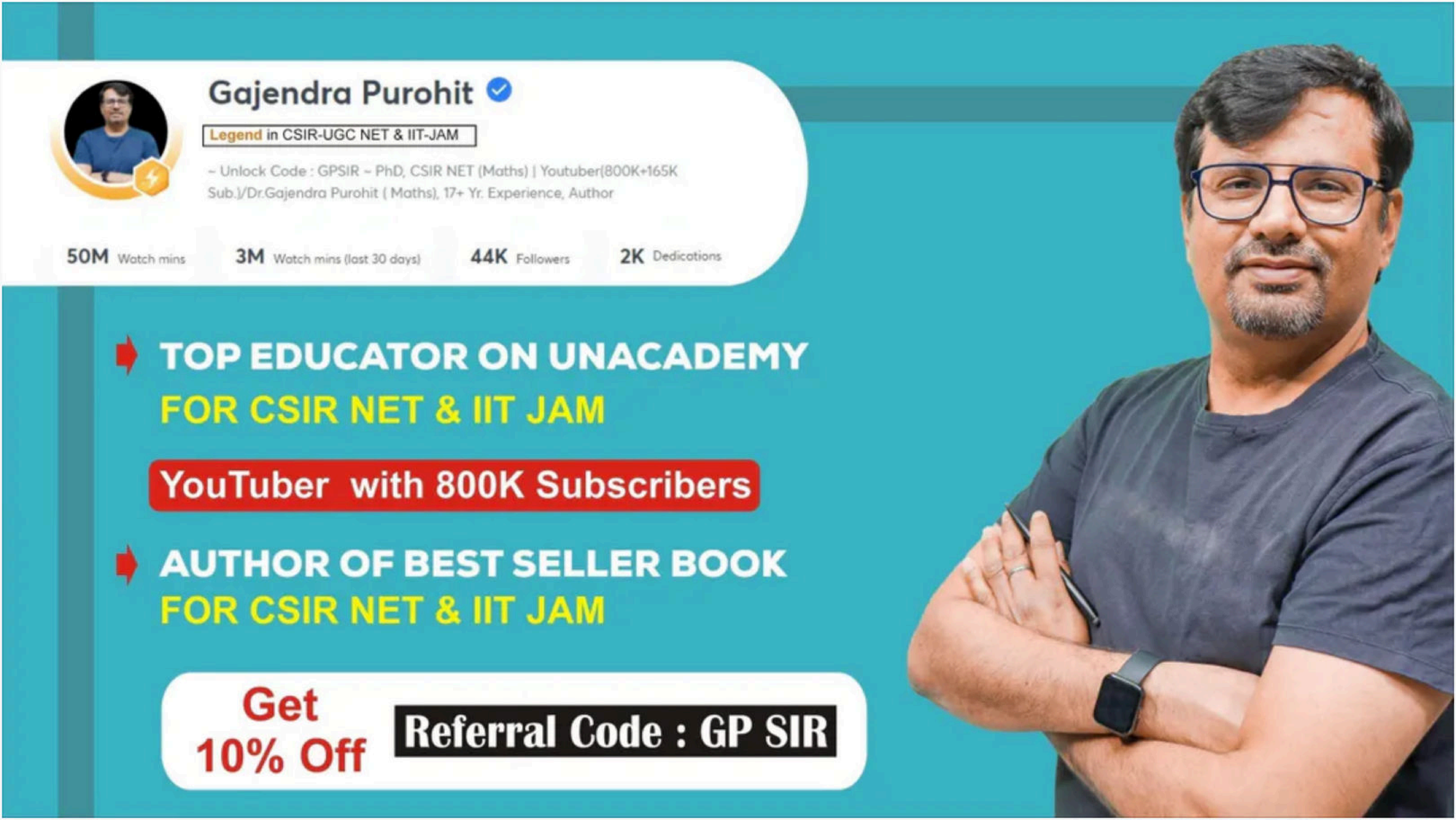
Detail Course 2.0 on Group Theory for IIT JAM '23

Gajendra Purohit • Lesson 9 • Oct 17, 2022



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Quotient Group

Let N be a normal subgroup of G. If a $\in$ G, then Na is a right coset of N in G. Since N is normal in G, left coset aN will be equal to the right coset Na.

Let $\frac{G}{N}$ be the collection of all distinct coset of N in G i.e. $\frac{G}{N} = \{Na; a \in G\}$, then $\frac{G}{N}$ is a group w.r.t. multiplication of coset i.e. (Na).(Nb) = Nab

It is called quotient group or factor group of G by N as the composition in $\frac{G}{N}$.

Note : We know that index of N in G are number of distinct coset of N in G.

Conclusion : Let N be a normal subgroup of a finite group G.

Then index of N in G = $\frac{O(G)}{O(N)}$. If G is finite

Note : Order of Quotient group $O\left(\frac{G}{N}\right)=\frac{O(G)}{O(N)}$. If G is finite

Centre of Group : Let G be a group then Z(G) = { x ∈ G | xa = ax | ∀ a ∈ G}is called centre of group

Result :

(1) The quotient group of abelian group is abelian but converse is not true.

Example : Let G = S_4 & N = A_4, then $\frac{G}{N} = Z_2$.

Here $\frac{G}{N}$ is abelian but G is not abelian.

(2) The quotient group of cyclic group is cyclic but converse is not true.

(3) Let Z(G) be a centre of a group G, then G is abelian if $\frac{G}{Z(G)}$ is cyclic.

(4) Let G be a cyclic group of order n, then number of factor group of G are $\tau(n)$ because number of normal subgroups are $\tau(n)$.

(5) $\frac{K}{N}$ is a subgroup of $\frac{G}{N}$, if K is a subgroup of G.

$$\frac{K}{N} \triangleleft \frac{G}{N} \quad if \quad K \triangleleft G$$

Result : If H be a subgroup of G and O(G) / O(H) = 2, then H is normal in G.

Group $\left(\frac{Q}{Z},+\right)$.

$$\frac{Q}{Z}=\{Z+a \mid a\in Q\}=\left\{Z,\left(Z+\frac{1}{2}\right),\left(Z+\frac{3}{2}\right)\left(Z+\frac{1}{3}\right)\ldots\ldots\right\}$$

(1) $\frac{Q}{Z}$ is abelian group but not cyclic group.

(2) Number of elements of order n are $\phi(n)$

(3) $\left(\frac{1}{p}+Z\right)\in\frac{Q}{Z}$ *then* $O\left(\frac{1}{p}+Z\right)=p$

$$\textit{And } O\left(\frac{k}{p}+Z\right)=p \quad \textit{if } \gcd(k,p)=1$$

(3) We know that $\frac{1}{4}+Z\in\frac{Q}{Z}$ for order of $\frac{1}{4}+Z$.

(5) There exist a unique cyclic subgroup of each order i.e. For every positive integer n , there is a cyclic subgroup of order n which is unique which is $H = \left\langle \frac{1}{n} + Z \right\rangle$

Q.1. Consider the quotient group of $\frac{Q}{Z}$ of the additive group of rational number, the order of element $\frac{2}{3} + Z$ in $\frac{Q}{Z}$ is

(a) 2 (b) 3 (c) 5
(d) 6

Q.3. Consider the following subsets of the group of 2 × 2 non – singular matrices over R

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in R, ad = 1 \right\}$$

$$H = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in R \right\}$$

Which of the following statements are correct

(a) G form a group under matrix multiplication

(b) H is normal subgroup of G

(c) G/H is well define and is abelian

(d) None of these

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Q.4. Let G be a non-abelian group and Z(G) is its centre, then which of the following is cannot be possible of $O\left(\frac{G}{Z(G)}\right)$.

(a) 7 (b) 8

(c) 4 (d) 6

Q.5. Number of factor group of Klein's 4 group is

(a) 5 (b) 4

(c) 6 (d) 7

Q.6 If G be a group such that $G/Z(G)$ is cyclic, then G is

(a) cyclic (b) commutative

(c) non-commutative

(d) None of these

Q.7. If H be a subgroup of a cyclic group G, the G/H is

(a) cyclic (b) non-cyclic

(c) normal (d) none of these

Q.8. If H be a subgroup of a commutative group G, then G/H is

(a) cyclic (b) normal

(c) commutative (d) none of these

Q.9. If G be a non-commutative group G with centre Z, then G/Z is

(a) cyclic (b) non-cyclic

(c) commutative (d) none of these

Q.10 If H be a subgroup of a group G and [G : H] = 2, then

(a) H is cyclic

(b) H is commutative

(c) H is normal

(d) none of these

Q.11 If H be a normal subgroup of a finite group G, then

(a) O(G/H) = O(G).O(H) (b) $O(G/H) = O(G).O\left(\frac{H}{G}\right)$

(c) $O(G/H) = \frac{O(G)}{\gcd\{O(H), O(G)\}}$

(d) $O(G/H) = \frac{O(G)}{O(H)}$

Q.12. If $H \subset K$ are two normal subgroups of a group G and if $[G : H] = 10$ and $[G : K] = 5$, then $[K : H]$ is

(a) 5 (b) 2

(c) 10 (d) 50

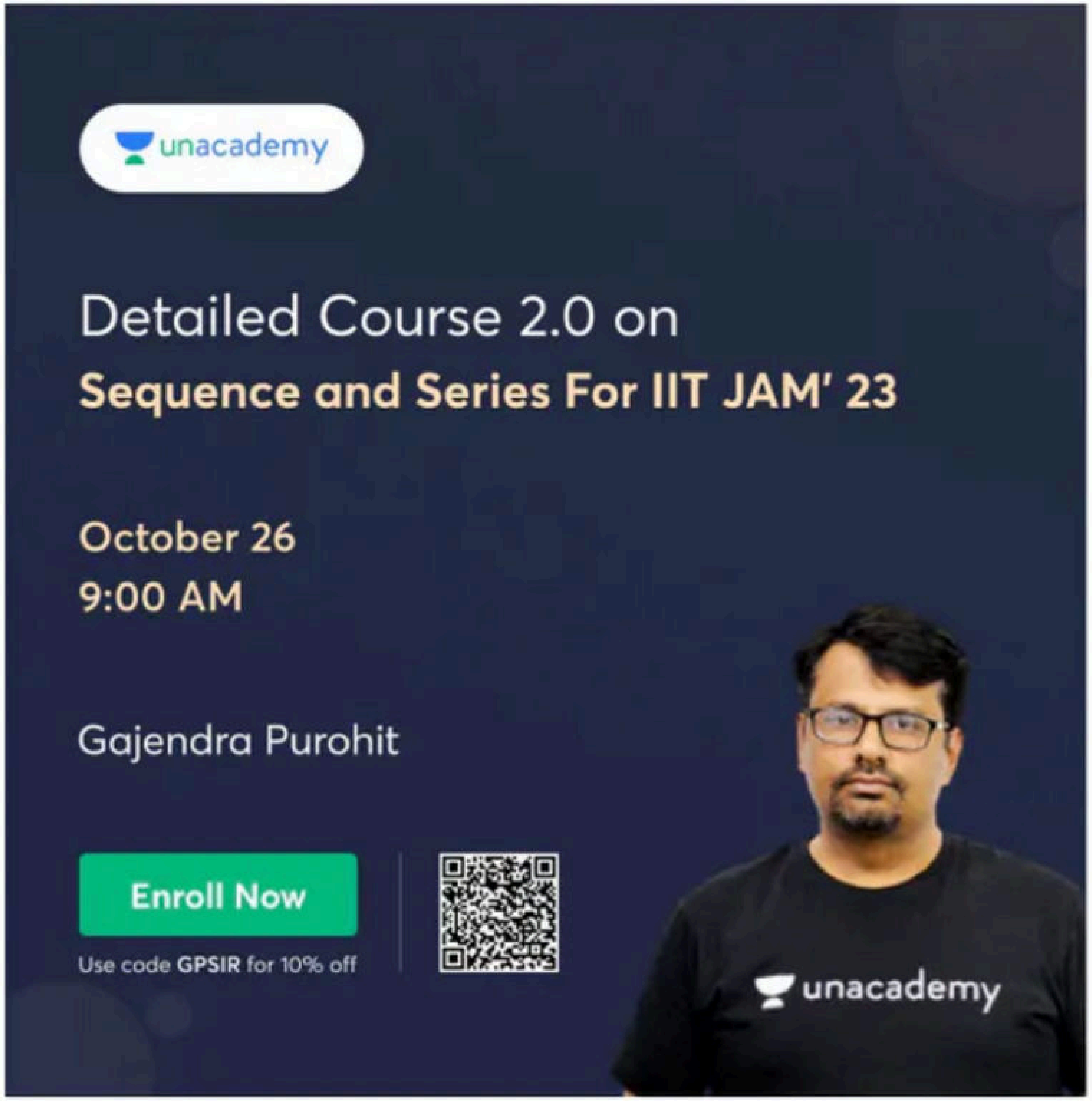
Q.13. Suppose N is a normal subgroup of a group G. Which one of the following is true?

(a) If G is an infinite group, then G/N is an infinite group.

(b) If G is a non-abelian group, then G/N is a non-abelian group.

(c) If G is a cyclic group, then G/N is an abelian group.

(d) If G is an abelian group, then G/N is a cyclic group.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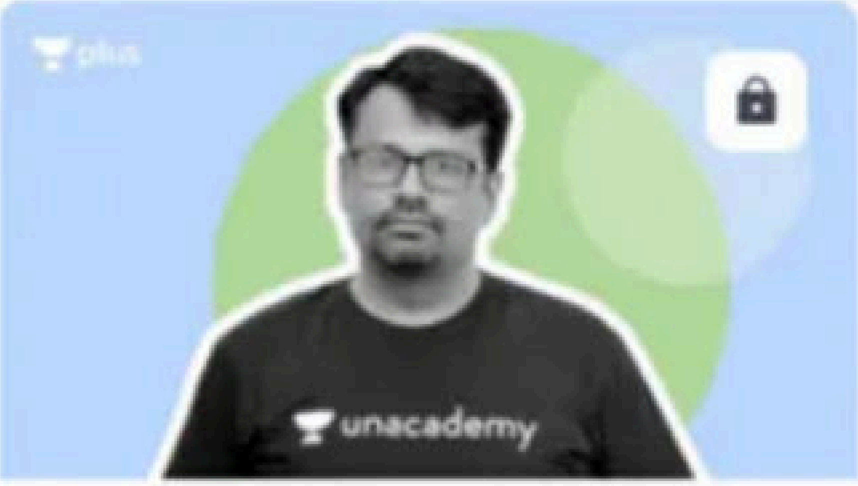

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