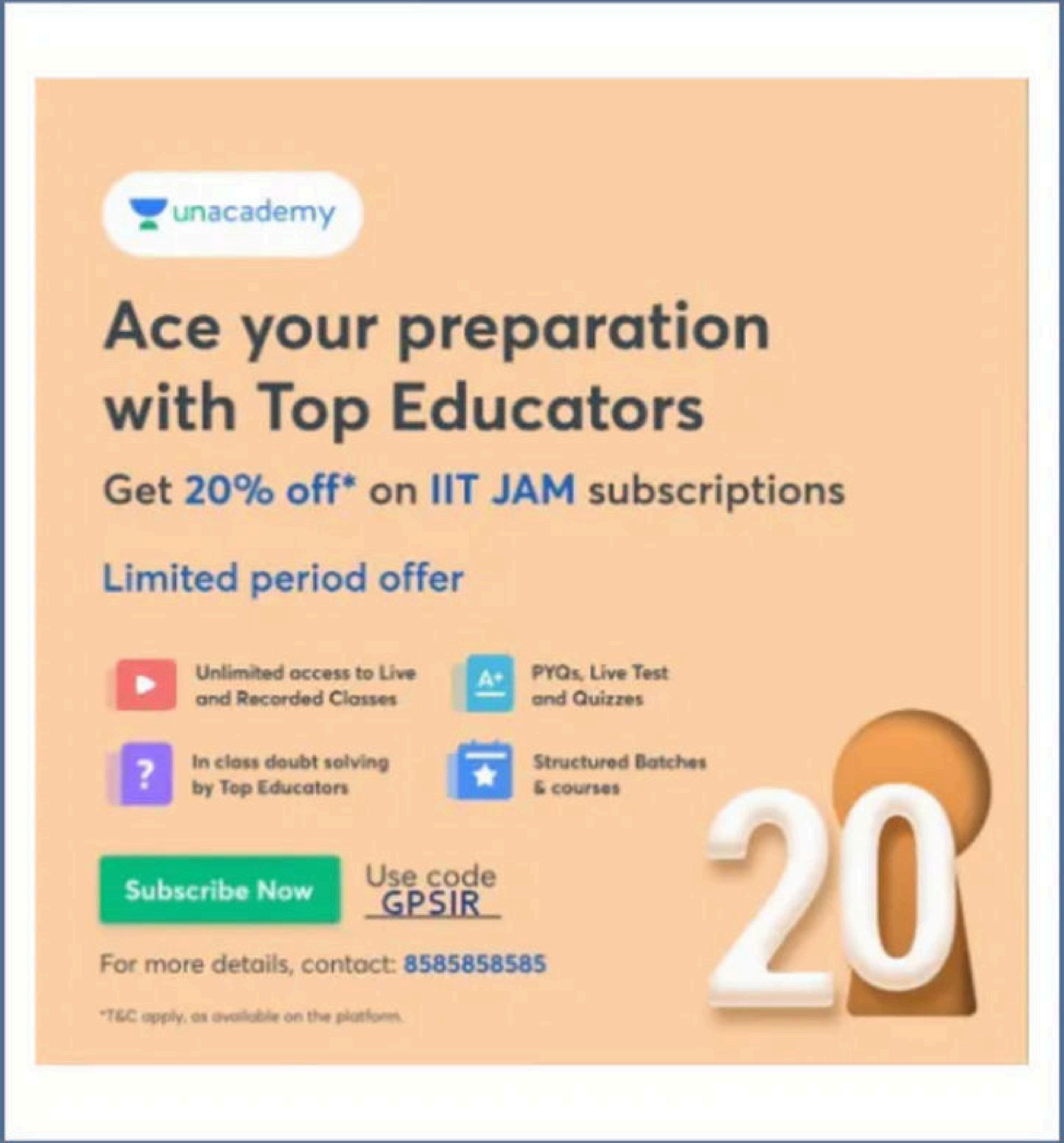
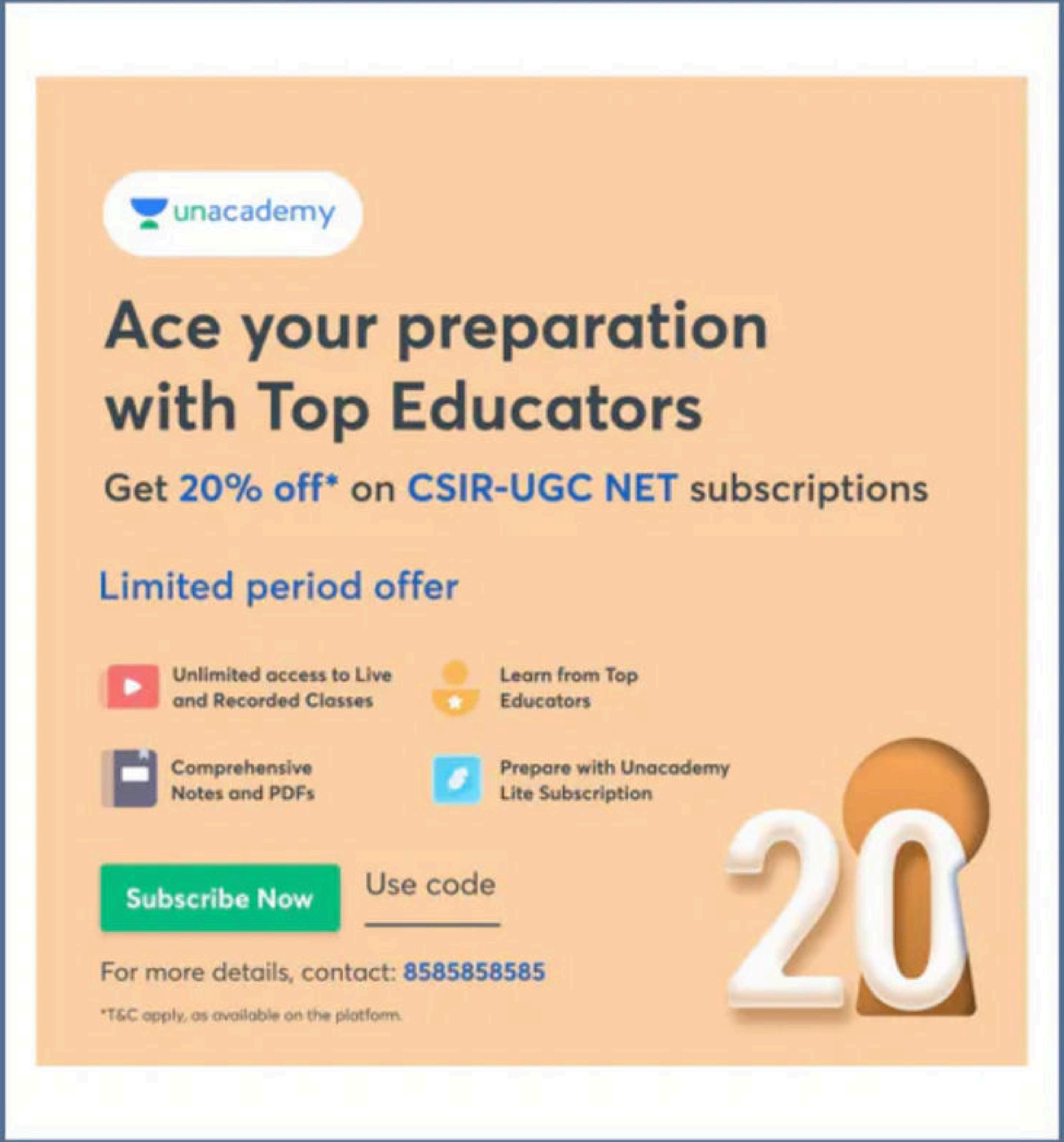
Vector Space - Part I

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 1 • Sept 8, 2022


unacademy
Ace your preparation with Top Educators
Get 20% off* on IIT JAM subscriptions
Limited period offer
Unlimited access to Live and Recorded Classes
PYQs, Live Test and Quizzes
In class doubt solving by Top Educators
Structured Batches & courses
Subscribe Now
Use code GPSIR
20
For more details, contact: 8585858585
*T&C apply, as available on the platform.


unacademy
Ace your preparation with Top Educators
Get 20% off* on CSIR-UGC NET subscriptions
Limited period offer
Unlimited access to Live and Recorded Classes
Learn from Top Educators
Comprehensive Notes and PDFs
Prepare with Unacademy Lite Subscription
Subscribe Now
Use code
20
For more details, contact: 8585858585
*T&C apply, as available on the platform.


unacademy
RANK BOOSTER COURSE UNIT 2
CSIR NET 2022
23rd AUGUST
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy


unacademy
DETAILED COURSE 2.0
LINEAR ALGEBRA FOR IIT JAM 2023
8th SEPTEMBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780	You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974	You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475		
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252		
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807		
1 month		₹ 2,723 / mo	Total ₹ 2,723		

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

GPSIR

Awesome! You get 10% off

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	~~₹ 21,780~~ ₹ 19,602	You get 6 months extra for free	Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo	~~₹ 14,974~~ ₹ 13,477	You get 6 months extra for free	Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo	~~₹ 13,475~~ ₹ 12,128		
6 months	Save 25%	₹ 1,838 / mo	~~₹ 12,252~~ ₹ 11,027		
3 months	Save 17%	₹ 2,042 / mo	~~₹ 6,807~~ ₹ 6,126		

Proceed to pay

~~I~~

~~II~~

~~III~~

~~IV~~

I

Matrix and their properties

Definition : A set of mn numbers arranged in the form of rectangular array consisting of m-rows and n-columns is called an m × n matrix or matrix of order m × n and denoted by $A = [a_{ij}]_{m\times n}$.

Matrix over any field : $A = [a_{ij}]_{m\times n}$ is matrix over any field F if $a_{ij} \in F$, for all i,j and matrix A is known as F – matrix.

RG

$$(a_{ij}) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \quad 4\times 4$$

~~(z, +, x)~~

~~(+ x)~~

~~(r, +, x)~~

~~(zp, *, r × p)~~

(q, +, x)

$$\begin{bmatrix} 2 & 1 & 7 \\ 5 & 3 & 1 \\ 7 & 2 & 5 \\ 2 & 8 & 7 \\ 5 & 1 & 7 \end{bmatrix}$$

5×3

3×3

Algebra of matrix :

RC

1. **Equality of two matrix :** If $A = [a_{ij}]_{m\times n}$ & $B = [b_{ij}]_{m\times n}$ are said to be equal matrix if $a_{ij} = b_{ij}$; for all i,j

2. **Multiplication of two matrices :** Two matrices A & B are conformable for multiplication if number of columns in A is equal to number of rows in B i.e. AB exist if $A = [a_{ij}]_{m\times n}$ & $B = [b_{ij}]_{n\times p}$. Then $AB = C = [C_{ij}]_{m\times p}$.

$$A = \begin{bmatrix} 2 & 7 \\ 5 & 1 \end{bmatrix}_{2\times 2} \qquad B = \begin{bmatrix} 3 & 1 \\ 2 & 9 \end{bmatrix}_{2\times 2}$$

$$AB = \begin{bmatrix} 2 & 7 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 9 \end{bmatrix} = \begin{bmatrix} 6+14 & 2+63 \\ 15+2 & 5+9 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 5 \\ 7 & 7 \\ 2 & 9 \end{bmatrix}_{3\times 2}$$

$$B = \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 7 & 9 \end{bmatrix}_{3\times 2}$$

$$AB = \begin{bmatrix} 2 & 5 \\ 7 & 1 \\ 2 & 9 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 1 \\ 7 & 9 \end{bmatrix}$$

3×3 $\quad$ 3×2

KC

$A_{\underline{m}\times \cancel{n}}$ $B_{\cancel{p}\times \underline{q}}$

$B_{\underline{p\times q}}$ $A_{m\times n}$

$AB_{m\times q}$ $\underline{n = p}$

$(BA)_{p\times n}$ $q = m$

$A_{5\times 2} \quad B_{2\times 7} = (AB)_{5\times 7}$

$2\times 7 \quad 5\times 2$

(BA)

Q.1: Let $A=\begin{bmatrix}0 & 1\\0 & 2\\1 & 0\end{bmatrix}$ & $B=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$, then which of the following is true?

3×2 $\quad$ 1×3

(a) AB exist

(b) BA exist

(c) Given data is insufficient

(d) None of these

$(AB) = (3\times 2)(1\times 3)$

$(BA) = \cancel{1\times 3}\ \cancel{3\times 2}$

1×2

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 7 \end{bmatrix}_{3\times 2}$$

$$A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 7 \end{bmatrix}_{2\times 3}$$

$$\underbrace{(AA^T)}_{3\times 3} =$$

$$\underbrace{(A^TA)}_{2\times 2} = 2\times\cancel{3} \quad \cancel{3}\times 2$$

$A_{5\times 2}$ $\qquad$ $B_{7\times 2}$

$AB^T = \qquad A_{5\times \cancel{2}}\, B^T_{\cancel{2}\times 7} = (AB^T)_{5\times 7}$

$A_{n\times n}$ $\qquad$ $A^T_{n\times n}$

Q $X_{4\times 3}$, $Y_{4\times 3}$, $P_{2\times 3}$

order of $\left[P(X^TY)^{-1}P^T\right]^T$

(a) 2×2

(b) 3×3

(c) 4×3

(d) 3×4

$\left(P_{2\times 3}\,(X^TY)^{-1}_{3\times 3}\,P^T_{3\times 2}\right)^T$

$(PP^T)^T_{2\times 2}$

$$\left(P_{2\times 3}\left[X^T_{3\times \cancel{4}}\,Y_{\cancel{4}\times 3}\right]^{-1}P^T_{3\times 2}\right)^T$$

Q.2. Let $A_i = \begin{bmatrix} \cos^2\theta_i & \cos\theta_i \sin\theta_i \\ \cos\theta_i \sin\theta_i & \sin^2\theta_i \end{bmatrix}$, i = 1, 2. Then

$A_1A_2 = 0$ if

(a) $\theta_1 = \theta_2 + (2k+1)\pi/2$, k = 0, 1, 2....

(b) $\theta_1 = \theta_2 + k\pi$,, k = 0, 1, 2 ....

(c) $\theta_1 = \theta_2 + 2k\pi$, k = 0, 1, 2, ...

(d) $\theta_1 = \theta_2 + k\pi/2$, k = 0, 1, 2, ...

$$A_1A_2 = \begin{bmatrix} \cos^2\theta_1 & \cos\theta_1\sin\theta_1 \\ \cos\theta_1\sin\theta_1 & \sin^2\theta_1 \end{bmatrix}\begin{bmatrix} \cos^2\theta_2 & \cos\theta_2\sin\theta_2 \\ \cos\theta_2\sin\theta_2 & \sin^2\theta_2 \end{bmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta_1\cos^2\theta_2 + \cos\theta_1\cos\theta_2\sin\theta_1\sin\theta_2 & \cos^2\theta_1\cos\theta_2\sin\theta_2 + \sin^2\theta_2\cos\theta_1\sin\theta_1 \\ & \end{pmatrix}$$

$$\begin{bmatrix} \cos\theta_1\cos\theta_2\cos(\theta_1-\theta_2) & \cos\theta_1\sin\theta_2\cos(\theta_1-\theta_2) \\ \cos\theta_2\sin\theta_1\cos(\theta_1-\theta_2) & \sin\theta_1\sin\theta_2\cos(\theta_1-\theta_2) \end{bmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$

$\cos(\theta_1-\theta_2) = 0 = \cos(2n+1)\pi/2$

$\theta_1 - \theta_2 = (2n+1)\pi/2$

$\theta_1 =$

$\cos\theta = 0$

$\theta = (2n+1)\frac{\pi}{2}$

$\sin\theta = 0$

$\theta = n\pi$

Q.3 How many elements do the set $S=\left\{A=\begin{bmatrix}2 & 3x\\ \frac{3}{x} & 2\end{bmatrix}: x\in R\setminus\{0\}\right\}$ Have, such that each element of the set satisfies the equation $A^2-4A-5I=0$

(a) Infinitely many (b) 1

(c) 2 (d) 3

$$A^2=\begin{pmatrix}2 & 3x\\ \frac{3}{x} & 2\end{pmatrix}\begin{pmatrix}2 & 3x\\ \frac{3}{x} & 2\end{pmatrix}=\begin{pmatrix}13 & 12x\\ \frac{12}{x} & 13\end{pmatrix}$$

$$\begin{pmatrix}13 & 12x\\ \frac{12}{x} & 13\end{pmatrix}-4\begin{pmatrix}2 & 3x\\ \frac{3}{x} & 2\end{pmatrix}-5\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\begin{pmatrix}13-8-5 & 12x-12x-0\\ \frac{12}{x}-\frac{12}{x}-0 & 13-8-5\end{pmatrix}=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

COMPLETE COURSE ON
MATHEMATICS
FOR IIT-JAM 2022

TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINAER ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 908 / mo	Total ₹ 21,780
You get 6 months extra for free		Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,248 / mo	Total ₹ 14,974
You get 6 months extra for free		Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,497 / mo	Total ₹ 13,475
6 months Save 25%	₹ 2,042 / mo	Total ₹ 12,252
3 months Save 17%	₹ 2,269 / mo	Total ₹ 6,807
1 month	₹ 2,723 / mo	Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 817 / mo	₹ ~~21,780~~ ₹ 19,602
You get 6 months extra for free		Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,123 / mo	₹ ~~14,974~~ ₹ 13,477
You get 6 months extra for free		Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,348 / mo	₹ ~~13,475~~ ₹ 12,128
6 months Save 25%	₹ 1,838 / mo	₹ ~~12,252~~ ₹ 11,027
3 months Save 17%	₹ 2,042 / mo	₹ ~~6,807~~ ₹ 6,126

GPSIR

Awesome! You get 10% off

Proceed to pay

FOUNDATION COURSE OF
MATHEMATICS
FOR CSIR-NET

3. Positive integral power of a square matrix :

If $A = [a_{ij}]_{m\times n}$ then $A^2 = A.A$ &$A^k = A.A. \ldots\ldots A$ (k-times)

Note :

(1) If sum of element in each row or each column of a square matrix A is 'a' then sum of element in each row or each column of a square matrix A^n is a^n.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix} \quad \begin{matrix} 4 \\ 4 \\ 4 \end{matrix}$$

$$A^2 = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 2 & 0 & 2 \end{pmatrix}$$

$$= \begin{bmatrix} 5 & 2 & 9 \\ 6 & 1 & 9 \\ 6 & 2 & 8 \end{bmatrix} \quad \begin{matrix} 16 \\ 16 \\ 16 \end{matrix}$$

$$A^{10} = \begin{pmatrix} \text{———} \\ \text{———} \\ \text{———} \end{pmatrix} \quad \begin{matrix} 4^{10} \\ 4^{10} \\ 4^{10} \end{matrix}$$

$$4^{10} + 4^{10} + 4^{10}$$

$$3 \cdot 4^{10}$$

$$A = \begin{bmatrix} 1 & 2 \\ 7 & 6 \end{bmatrix}$$

$$\begin{matrix} 8 & 8 \end{matrix}$$

$$A^2 = \begin{pmatrix} 1 & 2 \\ 7 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 7 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & 14 \\ 49 & 50 \end{pmatrix}$$

$$\begin{matrix} 64 & 64 \end{matrix}$$

$$\begin{matrix} 8 & 8 \end{matrix}$$

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix} \quad \begin{matrix} 6 \\ 6 \end{matrix}$$

$$B = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix} \quad \begin{matrix} 7 \\ 7 \end{matrix}$$

$$AB = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 29 \\ 14 & 28 \end{bmatrix} \quad \begin{matrix} 42 \\ 42 \end{matrix}$$

$$42$$

Note :

(2) If each row or column sum in A is a & each row or column sum in B is b then each row or column sum in AB is ab.

Q.5 If $A = \begin{bmatrix} 3 & -2 \\ 2 & -1 \end{bmatrix}$, then A^{20} equals

(a) $\begin{bmatrix} 41 & 40 \\ -40 & -39 \end{bmatrix}$ (b) $\begin{bmatrix} 41 & -40 \\ 40 & -39 \end{bmatrix}$

(c) $\begin{bmatrix} 41 & -40 \\ -40 & -39 \end{bmatrix}$ (d) $\begin{bmatrix} 41 & 40 \\ 40 & -39 \end{bmatrix}$

Q.6. If $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then P^{50} eqals

(a) $\begin{bmatrix} 1 & 100 & 500 \\ 0 & 1 & 100 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 50 & 100 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 50 & 100 & 150 \\ 0 & 50 & 100 \\ 0 & 0 & 50 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 50 & 1275 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}$

$P^2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, $P^3 = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$, $P^4 = \begin{bmatrix} 1 & 4 & 10 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$

$T_1 \quad T_2 \quad T_3 \quad T_4 \qquad T_{50}$

$\boxed{1, 3, 6, 10 \ldots}$

$\dfrac{n(n+1)}{2}$

$\cancel{T_2} - T_1 = 2$

$\cancel{T_3} - \cancel{T_2} = 3$

$\cancel{T_4} - \cancel{T_3} = 4$

$\vdots$

$T_{50} - T_{49} = 50$

$T_{50} - T_1 = 2 + 3 + 4 + \cdots 50$

$T_{50} = 1 + 2 + 3 + \cdots 50 = \dfrac{50 \times 51}{2}$

Q.7. The least positive integer n, such that $\begin{Bmatrix} \cos\pi/4 & \sin\pi/4 \\ -\sin\pi/4 & \cos\pi/4 \end{Bmatrix}^n$ is the identity matrix of order 2, is

(a) 4 (b) 8

(c) 12 (d) 16

Q.8 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then A^{50} is

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 48 & 1 & 0 \\ 48 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$ (d) $\cancel{\begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix}}$

$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$A^8 = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix}$

$A^4 = A^2 \cdot A^2$

$A^8 = \underline{A^4 \cdot A^4}$


unacademy
RANK BOOSTER COURSE UNIT 2
CSIR NET 2022
23rd AUGUST
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy


unacademy
DETAILED COURSE 2.0
LINEAR ALGEBRA FOR IIT JAM 2023
8th SEPTEMBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

unacademy CSIR-UGC NET Search Login

Educator Profile

Gajendra Purohit

#5 Educator in CSIR-UGC NET

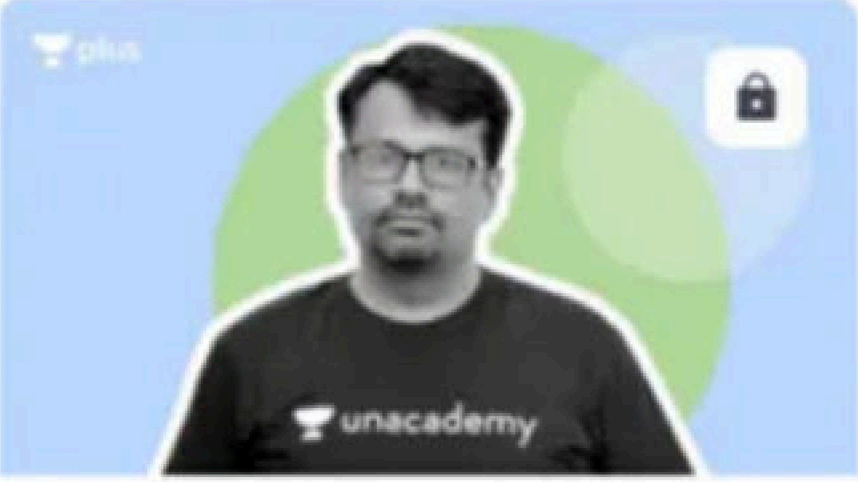
Dr.Gajendra Purohit PhD, CSIR NET (Maths) | Youtuber(330K+30k Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author of Bestseller

Follow

11M Watch mins | 1M Watch mins (last 30 days) | 22k Followers | 1k Dedications

CSIR-UGC NET

SEE ALL



HINDI MATHEMATICAL SCIENCES

Course on Linear Algebra, Partial Diff. Equation & Calculus

Starts on Mar 1, 2021 • 24 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Course on Complex Analysis & Integral Equation

Starts on Jan 14, 2021 • 16 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Foundation Course on Mathematics for CSIR 2021

Starts on Dec 7, 2020 • 20 lessons

Gajendra Purohit

Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 908 / mo Total ₹ 21,780
You get 6 months extra for free	Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,248 / mo Total ₹ 14,974
You get 6 months extra for free	Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,497 / mo Total ₹ 13,475
6 months Save 25%	₹ 2,042 / mo Total ₹ 12,252
3 months Save 17%	₹ 2,269 / mo Total ₹ 6,807
1 month	₹ 2,723 / mo Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 817 / mo ~~₹ 21,780~~ ₹ 19,602
You get 6 months extra for free	Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,123 / mo ~~₹ 14,974~~ ₹ 13,477
You get 6 months extra for free	Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,348 / mo ~~₹ 13,475~~ ₹ 12,128
6 months Save 25%	₹ 1,838 / mo ~~₹ 12,252~~ ₹ 11,027
3 months Save 17%	₹ 2,042 / mo ~~₹ 6,807~~ ₹ 6,126

GPSIR

Awesome! You get 10% off

Proceed to pay