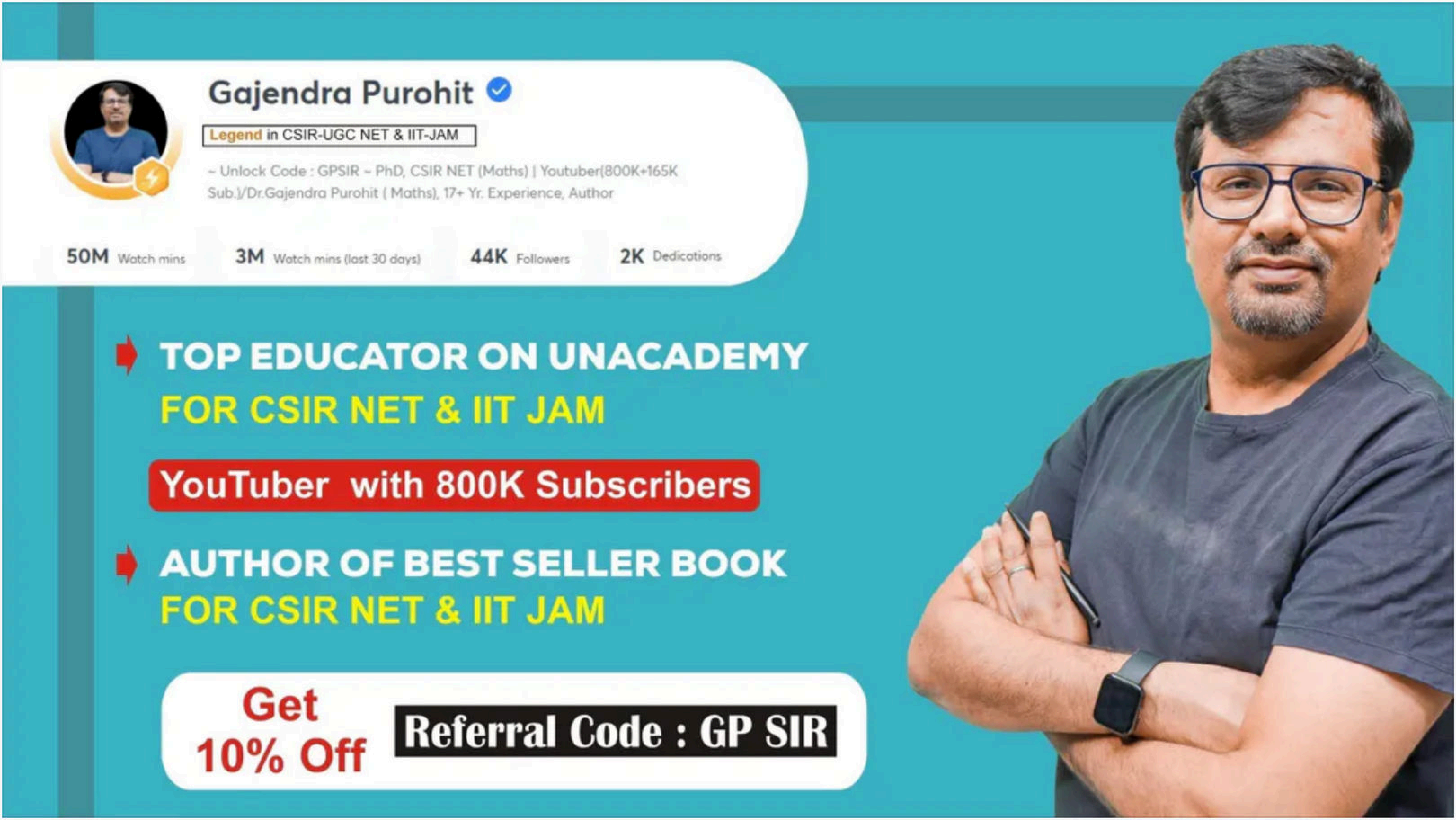
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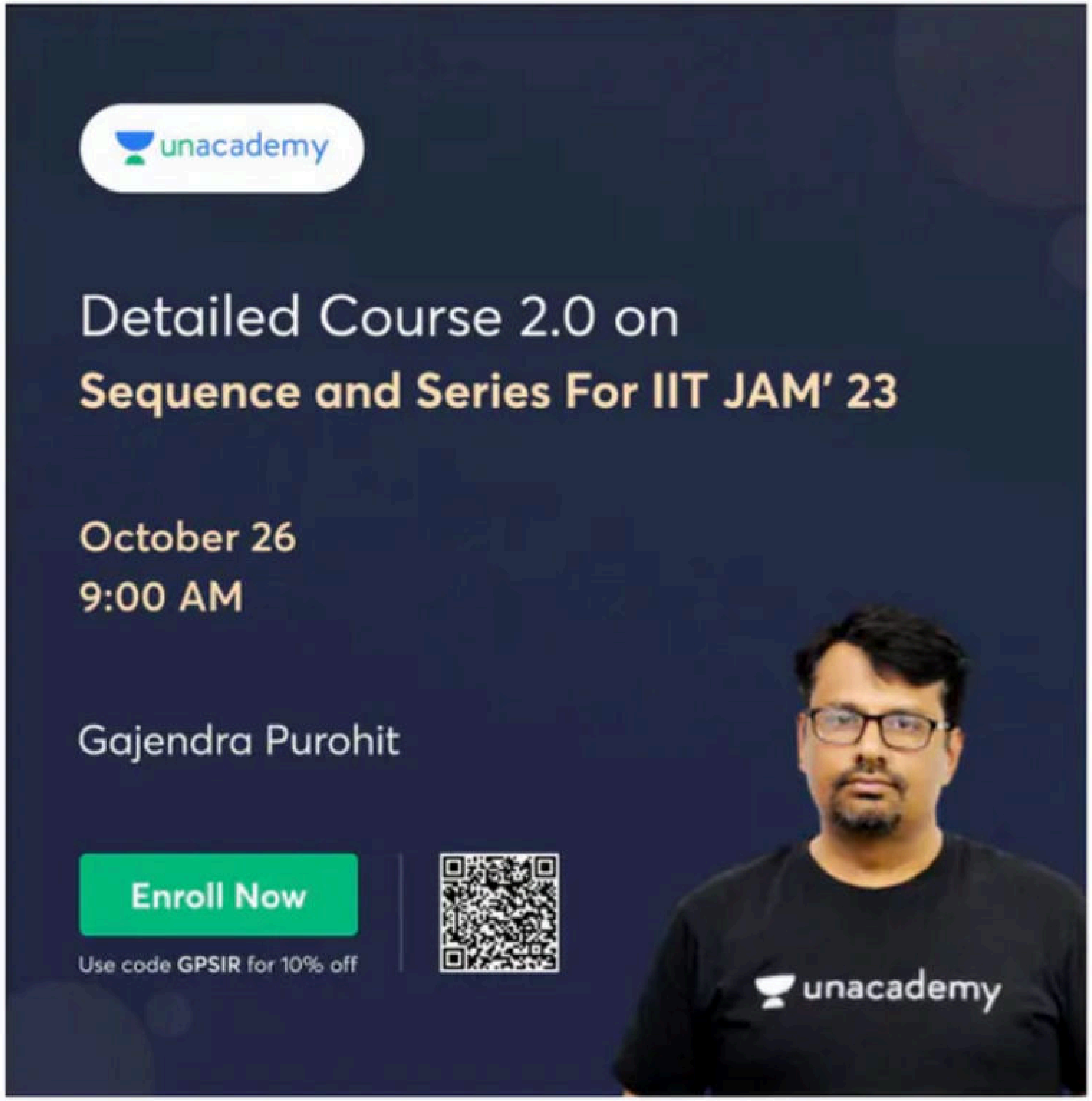
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Series of real numbers

D'Alembert Ratio test – Let $\sum u_n$ be a positive terms series such that

(A) $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l$ Then the series is

(i) Convergent if $l < 1$

(ii) Divergent if $l > 1$

(iii) Test fails for $l = 1$

(B) $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \infty$ then $\sum u_n$ is divergent.

$$\frac{2!}{3} + \frac{3!}{3^2} + \frac{4!}{3^3} + \cdots \frac{(n+1)!}{3^n} + \cdots \qquad \sum_{n=1}^{\infty} \frac{(n+1)!}{3^n}$$

$$a_n = \frac{(n+1)!}{3^n} \qquad \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+2)!}{3^{n+1}} \cdot \frac{3^n}{(n+1)!}$$

$$= \lim_{n\to\infty} \frac{(n+2)\cancel{(n+1)!}\cdot\cancel{3^n}}{\cancel{3^n}\cdot 3 \quad \cancel{(n+1)!}} = \infty$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n \qquad u_n = \left(\tfrac{1}{2}\right)^n$$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{\left(\tfrac{1}{2}\right)^{n+1}}{\left(\tfrac{1}{2}\right)^n} = \lim_{n\to\infty} \frac{\left(\tfrac{1}{2}\right)\cancel{\left(\tfrac{1}{2}\right)^n}}{\cancel{\left(\tfrac{1}{2}\right)^n}} = \underline{\tfrac{1}{2} < 1}$$

Q.1. Suppose

$a_n = \dfrac{3^n + 3}{5^n - 5}$ and $b_n = \dfrac{1}{(1+n^2)^{1/4}}$ for $n = 2, 3, 4....$ Then

which of the following is true

(a) Both $\sum_{n=2}^{\infty} a_n$ and $\sum_{n=2}^{\infty} b_n$ are convergent

(b) Both $\sum_{n=2}^{\infty} a_n$ and $\sum_{n=2}^{\infty} b_n$ are divergent

(c) $\sum_{n=2}^{\infty} a_n$ is convergent and $\sum_{n=2}^{\infty} b_n$ are divergent

(d) $\sum_{n=2}^{\infty} a_n$ is divergent and $\sum_{n=2}^{\infty} b_n$ are convergent

$$v_n = \frac{1}{\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3^{n+1}+3}{5^{n+1}-5} \cdot \frac{5^n-5}{3^n+3}$$

$$= \lim_{n\to\infty} \frac{3^{n+1}\left(1+\frac{1}{3^n}\right)5^n\left(1-\frac{1}{5^n}\right)}{5^{n+1}\left(1-\frac{1}{5^n}\right)3^n\left(1+\frac{1}{3^n}\right)}$$

$$= \frac{3}{5} < 1$$

Q2. Let $<a_n>$ be a sequence of positive real numbers, the series $\sum_{n=1}^{\infty} a_n$ converges if the series

(a) $\sum_{n=1}^{\infty} a_n^2$ converges

(b) $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ converges

(c) $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges

(d) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

$\lim_{n\to\infty} \frac{a_n}{a_{n+1}} = 0$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \infty > 1$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0 < 1$

Q6. The series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\ldots\ldots$ is convergent if x belong to the interval

$l<1$

(a) $\left(0,\frac{1}{e}\right)$

(b) $\left(\frac{1}{e},\infty\right)$

(c) $\left(\frac{2}{e},\frac{3}{e}\right)$

(d) $\left(\frac{3}{e},\frac{4}{e}\right)$

$l<1$

$x<1/e$

$\left(0,\frac{1}{e}\right)$

$l<1$

$a_n=\frac{n^n x^n}{n!}$

$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(n+1)^{n+1}x^{n+1}}{(n+1)!}\frac{n!}{n^n x^n}$

$=\lim_{n\to\infty}\frac{\cancel{(n+1)}(n+1)^n\cancel{x^n}.x\,\cancel{n!}}{\cancel{(n+1)n!}\,n^n\,\cancel{x^n}}$

$=\lim_{n\to\infty}\left(1+1/n\right)^n x$

$=ex<1$

Q3. What value of x, the series $\sum_{n=1}^{\infty} \frac{x^n}{n^2+1}$ is divergent?

$u_n = \frac{x^n}{n^2+1}$

(a) (0, 1)

(b) $(2, \infty)$

(c) $(1, \infty)$

(d) (0, 2)

$\lim_{n\to\infty} \frac{u_{n+1}}{u_n}$

$\lim_{n\to\infty} \frac{x^{n+1}}{(n+1)^2+1} \cdot \frac{n^2+1}{x^n}$

$\lim_{n\to\infty} \frac{x \cancel{n^2}(1+1/n^2)}{\cancel{n^2}((1+1/n)^2+1/n^2)} = x > 1$

Cauchy's condensation test :

If f(n) is a positive monotonically decreasing function of n, then the two infinite series $\sum f(n)$ and $\sum a^n f(a^n)$ converge or diverge together, where $a > 1$, $a \in Z$.

$$\frac{(\log 2)^2}{2^2} + \frac{(\log 3)^2}{3^2} + \dots + \frac{(\log n)^2}{n^2}$$

$\log 2^4$

$4 \log 2$

$$f_n = \frac{(\log n)^2}{n^2}$$

$$u_n = \frac{n^2}{2^n}$$

$$\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = \lim_{n\to\infty} \frac{(n+1)^2}{2^{n+1}} \frac{2^n}{n^2}$$

$$= \lim_{n\to\infty} \frac{(1+\frac{1}{n})^2}{2} = \frac{1}{2} < 1$$

$$\sum_{n=1}^{\infty} 2^n f(2^n)$$

$$\sum_{n=1}^{\infty} \frac{\cancel{2^n}(\log 2^n)^2}{(2^n)^{\cancel{2}}} = \sum_{n=1}^{\infty} \frac{(n\log 2)^2}{2^n}$$

$$(\log 2)^2 \boxed{\sum_{n=1}^{\infty} \frac{n^2}{2^n}}$$

$$\frac{\log 2}{2} + \frac{\log 3}{3} + \frac{\log 4}{4} + \cdots \frac{\log n}{n}$$

$$\sum 2^n f(2^n)$$

$$f(n) = \frac{\log n}{n}$$

$$\sum_{n=2}^{\infty} f(n)$$

$$\sum_{n=2}^{\infty} \cancel{2^n} \frac{\log 2^n}{\cancel{2^n}}$$

$$\sum_{n=2}^{\infty} n \log 2$$

$$\log 2 \sum_{n=1}^{\infty} n$$

Q4. For which value of 'p' the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ is convergent?

(a) 1

(b) 2

(c) 1/2

(d) 1/3

$$\sum_{n=2}^{\infty} \cancel{2^4} \frac{1}{\cancel{2^4}(\log 2^4)^p}$$

$$\sum_{n=2}^{\infty} \frac{1}{(n\log 2)^p}$$

$$\frac{1}{(\log 2)^p} \sum_{n=2}^{\infty} \frac{1}{n^p}$$

Q5. Which of the following series is Divergent

(a) $\sum_{n=1}^{\infty}\frac{1}{n}\sin^2\frac{1}{n}$

(b) $\sum_{n=2}^{\infty}\frac{1}{n}\log n$

(c) $\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{1}{n}$

(d) $\sum_{n=1}^{\infty}\frac{1}{n}\tan\frac{1}{n}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \cdots$

$\tan x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5$

$\frac{1}{n^2}\left[\frac{1}{n} - \frac{1}{3!}\frac{1}{n^3} + \cdots\right] = \frac{1}{n^3}\left(1 - \frac{1}{3!}\frac{1}{n^2} + \cdots\right)$

$v_n = \cancel{\frac{1}{n^3}}$

$\frac{1}{n}\left(\frac{1}{n} - \frac{1}{3}\left(\frac{1}{n}\right)^3 + \cdots\right.$

$\frac{1}{n^2}$

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Cauchy's n^th^ root test : Let $\sum u_n$ be a positive terms series and let $\lim\limits_{n\to\infty}(u_n)^{1/n} = l$.

Then the series is

(a) Convergent if $l < 1$
(b) Divergent if $l > 1$
(c) Test fails if $l = 1$

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2}$$

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{-n^2}\right)^{1/n}$$

$$\lim_{n\to\infty}\frac{1}{(1+1/n)^n} = 1/e < 1$$

Note : If n^th^ term of series is in the power of n then we can use Cauchy's n^th^ root test.

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\cdots+\left(\frac{(n+1)^{n+1}}{n^{n+1}}-\frac{n+1}{n}\right)^{-n}$$

$$\sum a_n=\sum\left[\left(\frac{n+1}{n}\right)^{n+1}-\frac{n+1}{n}\right]^{-n}$$

$$\lim_{n\to\infty}(a_n)^{1/n}=\lim_{n\to\infty}\frac{1}{\left(\frac{n+1}{n}\right)^{n+1}-\frac{n+1}{n}}$$

$$=\lim_{n\to\infty}\frac{1}{\underline{\left(1+\frac{1}{n}\right)^{n}}\left(1+\frac{1}{n}\right)-\left(1+\frac{1}{n}\right)}=\frac{1}{e-1}<1$$

Q1. Which of the following series is/are convergent?

(a) $\sum_{n=1}^{\infty}\left(\frac{5n+1}{4n+1}\right)^n$

(b) $\sum_{n=1}^{\infty}\frac{\sin\frac{1}{n}}{n^{1/n}}$

(c) $\sum_{n=1}^{\infty}\left(1-\frac{1}{n}\right)^n$

(d) $\sum_{n=1}^{\infty}\sqrt{n}\left(1-\cos\left(\frac{1}{n}\right)\right)$

$u_n = \frac{\sin 1/n}{n^{1/n}}$

$v_n = \sin 1/n$

$\sqrt{n}\left[1-\left(1-\frac{1}{2!}\left(\frac{1}{n}\right)^2+\cdots\right)\right]$

$\sqrt{n}\left(\frac{1}{2!}\frac{1}{n^2}-\frac{1}{4!}\frac{1}{n^4}\cdots\right)$

$\frac{1}{n^{3/2}}\left(\frac{1}{2}-\frac{1}{4!}\frac{1}{n^2}-\cdots\right)$

Q2. For $n \geq 1$, let $a_n = \begin{cases} n2^{-n} & \text{if } n \text{ is odd} \\ 3^{-n} & \text{if } n \text{ is even} \end{cases}$. Which of the following statements is/are convergent?

(a) The sequence $\langle a_n \rangle$ is convergent

(b) The sequence $\langle a_n \rangle$ is divergent

(c) The series $\sum_{n=1}^{\infty} a_n$ is convergent

(d) The series $\sum_{n=1}^{\infty} a_n$ is divergent

Cauchy's integral test :

If u(x) is non-negative decreasing integrable function such that $u(n) = u_n$ then $\sum_{n=1}^{\infty} u_n$ is convergent iff the value of $\int_1^{\infty} u(x)dx$ is finite.

Q3 . The convergence for series $\sum_{n=1}^{\infty} \frac{1}{n(\log n)}$. is

(a) Convergent (b) Divergent

(b) Oscillatory (d) None of these

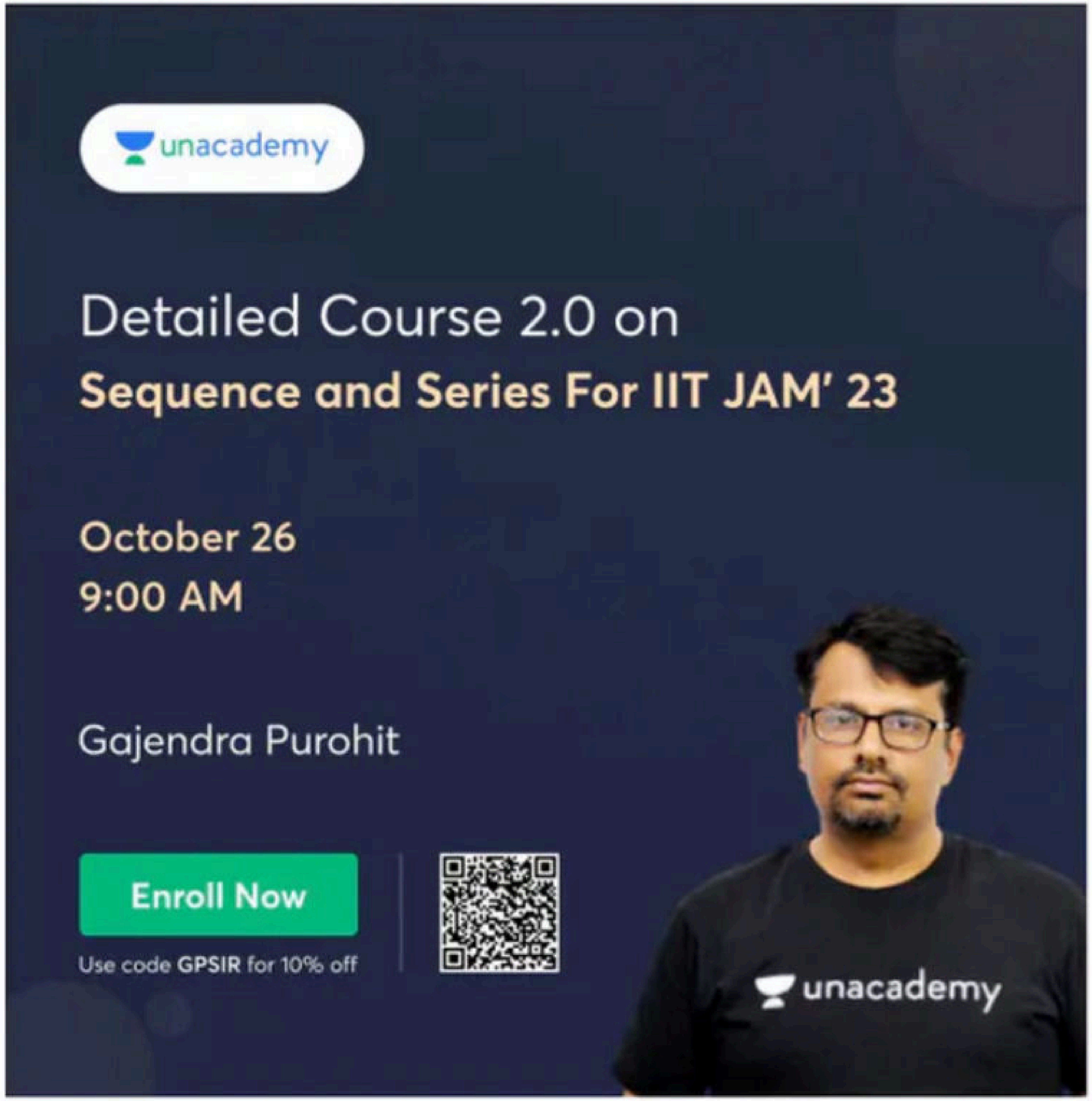
Q4 . Which of the following series is divergent?

(a) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n} \log n$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$

(d) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$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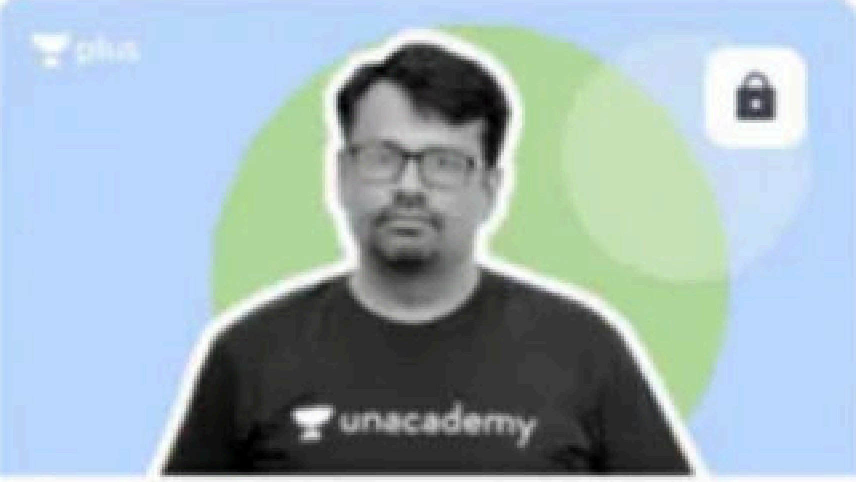

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- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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