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Definition : A function whose domain is the set of natural numbers N and range a subset of real numbers R is called a real sequence.

i.e. A function $f : N \rightarrow R$ is called a sequence and we denoted by $f(n) = a_n$.

Notation : $<a_n>$ or $\{a_n\}$ or $<a_1, a_2, \ldots.. a_n \ldots.>$

Terms of Sequence : Let $<a_n>$ be a sequence then $a_1, a_2, \ldots.., a_n, \ldots.$ are called terms of sequence.

$a_n = \langle \frac{1}{n} \rangle = \langle 1, \frac{1}{2}, \frac{1}{3}, \cdots \rangle$

$a_n = \langle (-1)^n \rangle = \langle -1, 1, -1, 1 \cdots \rangle$

$a_n = \langle \frac{(-1)^n}{n} \rangle = \langle -1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5} \rangle$

$a_n = \langle 1 + (-1)^n \rangle = \langle 0, 2, 0, 2 \cdots \rangle$

$a_n = \langle \frac{n}{1+n} \rangle = \langle \frac{1}{2}, \frac{2}{3}, \frac{3}{4} \cdots \rangle$

$a_n = \langle n \rangle = \langle 1, 2, 3 \cdots \rangle$

$a_n = \langle -n \rangle = \langle -1, -2, -3 \cdots \rangle$

$a_n = \langle (-1)^n n \rangle = \langle -1, 2, -3, 4 \cdots \rangle$

$a_n = \begin{cases} 2 & \text{if } n \text{ is prime} \\ n & \text{if } n \text{ is not prime} \end{cases}$

$= \langle 1, 2, 2, 4, 2, 6, 2, 8 \cdots \rangle$

$R = \{1, \frac{1}{2}, \frac{1}{3}, \cdots\}$ — Sup = 1, Inf = 0

$R = \{-1, 1\}$ — Sup = 1, Inf = -1

$R = \{\cdots -\frac{1}{3}, -1, \frac{1}{2}, \frac{1}{4} \cdots\}$ — Sup = $\frac{1}{2}$, Inf = -1

$R = \{0, 2\}$ — Sup = 2, Inf = 0

$R = \{\frac{1}{2}, \frac{2}{3}, \frac{3}{4} \cdots 1\}$ — Sup = 1, Inf = $\frac{1}{2}$

$R = \{1, 2, 3 \cdots \infty\}$ — Sup = ∞, Inf = 1

$R = \{-\infty \cdots -3, -2, -1\}$

$R = \{-\infty \cdots -3, -1, 2, 4 \cdots\}$

$R = \{1, 2, 4, 6, 8 \cdots\}$

$$\frac{(-1)^n}{n} =$$

$a_1 = -1$

$a_2 = \frac{1}{2}$

$a_3 = -\frac{1}{3}$

$a_4 = \frac{1}{4}$

$a_5 = -\frac{1}{5}$

$a_6 = \frac{1}{6}$

0

0

$\sup = \frac{1}{2}$

$\inf = -1$

$$\langle n \rangle = \langle 1, 2, 3 \ldots \infty \rangle$$

$$\langle -n \rangle = \langle -1, -2, -3 \ldots -\infty \rangle$$

a

r

Range of a sequence : The set of all distinct terms of sequence is called its range.

Bounded and unbounded sequence :

1. **Bounded Above Sequence :** A sequence $\langle a_n \rangle$ is said to be bounded above if there exist a real number k s.t. $a_n \leq k$, for all $n \in N$.

   i.e. if the range set of the sequence is bounded above.

$\langle \frac{1}{n} \rangle$

$\langle 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} - - 0 \rangle$

2. **Bounded Below Sequence :** A sequence $\langle a_n \rangle$ is said to be bounded below if there exist a real number k s.t. $a_n \geq k$; for all $n \in N$ i.e. if the range set of the sequence is bounded below.

$\langle n \rangle$

$\langle 1, 2, 3 - - \quad \infty \rangle$

Bounded Sequence : A sequence is said to be bounded iff its range set is bounded. i.e. if a sequence is bounded above and bounded below both, the this sequence is called a bounded sequence.

Unbounded above sequence : A sequence $<a_n>$ is said to be unbounded above if the Range set of this sequence is not bounded above.

Unbounded below sequence : A sequence $<a_n>$ is said to unbounded below if the range set of this sequence is not bounded below.

Unbounded Sequence : A sequence is said to be unbounded if it is not bounded.

i.e. if a sequence is neither bounded above nor bounded below then it is called a unbounded sequence.

Supremum (least upper bound) of sequence :

The supremum of the range set of the sequence is called supremum of sequence.

Infimum or greatest lower bound (glb) :

The infimum of the range set of the sequence is called infimum of sequence.

Note :

(1) If sequence is unbounded above then supremum of sequence is ∞.

$\langle n \rangle$

$\langle 1, 2, 3 \ldots \infty \rangle$

(2) If sequence is unbounded below then infimum of sequence is $-\infty$.

$\langle -n \rangle$

(3) If supremum and infimum of the sequence are finite then sequence is bounded.

$\langle -\infty, \; -2, -1 \rangle$

Monotonicity of Sequence :

(1) **Monotonic increasing sequence :** Let $\langle a_n \rangle$ be a sequence of real number, this sequence is called monotonic increasing sequence if $a_{n+1} \geq a_n$ for all $n \in N$.

$\langle n \rangle$

$\langle 1, 2, 3 - - - \rangle$ | $\langle 1, 1, 1, 1, 1 \cdots \rangle$

(2) **Strictly increasing sequence :** Let $\langle a_n \rangle$ be a sequence and $a_{n+1} > a_n$; for all $n \in N$, then $\langle a_n \rangle$ is called strictly increasing sequence.

(3) **Monotonic decreasing sequence :** Let $<a_n>$ be a sequence and $a_{n+1} \le a_n$ for all $n \in N$. Then this sequence is called monotonic decreasing sequence.

$< 1, 1, 1, 1 - - - >$

$<-n> = < -1, -2, -3 - - - >$

$< \frac{1}{n} >$

$< 1, \frac{1}{2}, \frac{1}{3} - - >$

(4) **Strictly decreasing sequence :** Let $<a_n>$ be a sequence and $a_{n+1} < a_n$; for all $n \in N$.

$< 1, 1, 1, 1 - - - >$

$< 1, \frac{1}{2}, \frac{1}{3}$

Q.1. Which of the following is bounded sequence.

$\langle -1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}$

(a) a, b

(b) b, c

(c) c, d

(d) a, d

(a) $<n(-1)^n>$

(b) $a_n = \left\langle \frac{(-1)^n}{n} \right\rangle$

(c) $<1 + (-1)^n>$

(d) $a_n = \begin{cases} 2 & if \quad n \text{ is prime} \\ n & if \quad n \text{ is not prime} \end{cases}$

$<0, 2, 0, 2, \dots>$

$<-1, 2, -3, +4, -5 \dots$

$<1, 2, 2, 4, 2, 6, 2, 8 \dots>$

$-\infty \; -5, \; -3, -1, 2, 4 \; \dots \; \infty$

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Q.2. Let $\langle a_n \rangle$ be a sequence of real number given by
$$a_n = \begin{cases} 2 & n \text{ is prime} \\ n & \text{n is not prime} \end{cases}$$, then which of the following is true

$1, 2, 2, 4, 2, 6.$

(a) a_n is monotonic

(b) It has supremum

(c) It has infimum

(d) None of these

Q.3. The greatest lower bound of the sequence $\{(e^n + 2^n)^{1/n};\ n \in N\}$ is

(a) e (b) e/2

(c) 1 (d) 0

$0 < 2 < e$

$0^n < 2^n < e^n$

$e^n + 0 < 2^n + e^n < 2e^n$

$(e^n)^{1/n} < (2^n + e^n)^{1/n} < 2^{1/n} e$

$e < (2^n + e^n)^{1/n} < 2^{1/n} e$

$e < \lim_{n \to \infty} (2^n + e^n)^{1/n} < e$

Q.4. Let <a_n> be a sequence of real number where $a_n = \sin\frac{n\pi}{2}, n \in N$.

Then supremum and infimum of the sequence is

(a) 1 & -1 (b) -1 & -2

(c) 1 & 0 (d) -1 & 0

< 1, 0, -1, ...

Q.5. Which of the following is true?

(a) Every bounded sequence is monotonic sequence

(b) Every monotonic sequence is bounded sequence

(c) Every unbounded sequence is monotonic sequence

(d) A monotonic sequence need not be bounded.

$1+(-1)^n$

$\langle (-1)^n \rangle$

$\langle -1, 1, -1, 1 \ldots \rangle$

$\langle n \rangle$

$\langle 1, 2, 3 \ldots \rangle$

$\langle (-1)^n n \rangle$

$-1, 2, -3, 4, -5 \ldots$

$(-5, -3, -1, 2, 4 \ldots)$

Q.6. Let $\langle a_n \rangle$ be a sequence of real number given by $a_n = 2^{(-1)^n}\left(1-\frac{1}{n}\right)\sin\frac{n\pi}{2}; n \in N$, then the least upper bound of sequence is

(a) 1/6 (b) ~~1/2~~

(c) 1/3 (d) 1/4

$$a_n = \begin{cases} 0 & n\text{-even} \\ \frac{1}{2}\left(1-\frac{1}{n}\right)\sin\frac{(2n+1)\pi}{2} & n\text{-odd} \end{cases}$$

$$\frac{1}{2} \times 1 \times 1$$

$$2^{-1}\left(1-\frac{1}{n}\right)\sin\frac{(2n+1)\pi}{2}$$

Q.7. The sequence $\frac{20^n}{n!}$ is

(a) Monotonic increasing sequence

(b) Monotonic decreasing sequence

(c) Eventually monotonic increasing

(d) Eventually monotonic decreasing

$\frac{2^n \times (10)^n}{n!}$

$n = 20$

Q.8. Consider the sequence $\{a_n\}$ defined by $a_1 = 2,\ a_{n+1} = \frac{1}{2}(a_n + 6)$.

Then

(a) $\{a_n\}$ is bounded but not monotonic

(b) $\{a_n\}$ is monotonic but not bounded

(c) $\{a_n\}$ is neither bounded or monotonic

(d) $\{a_n\}$ is bounded and monotonic

$l = \frac{1}{2}(l+6)$

$2l = l + 6$

$l = 6$

$a_1 = 2$

$n = 1$

$a_2 = \frac{1}{2}(a_1 + 6)$

$a_2 = 4$

$n = 2$

$a_3 = \frac{1}{2}(a_2 + 6) = 5$

$a_4 = \frac{1}{2}(a_3 + 6) = 5.5$

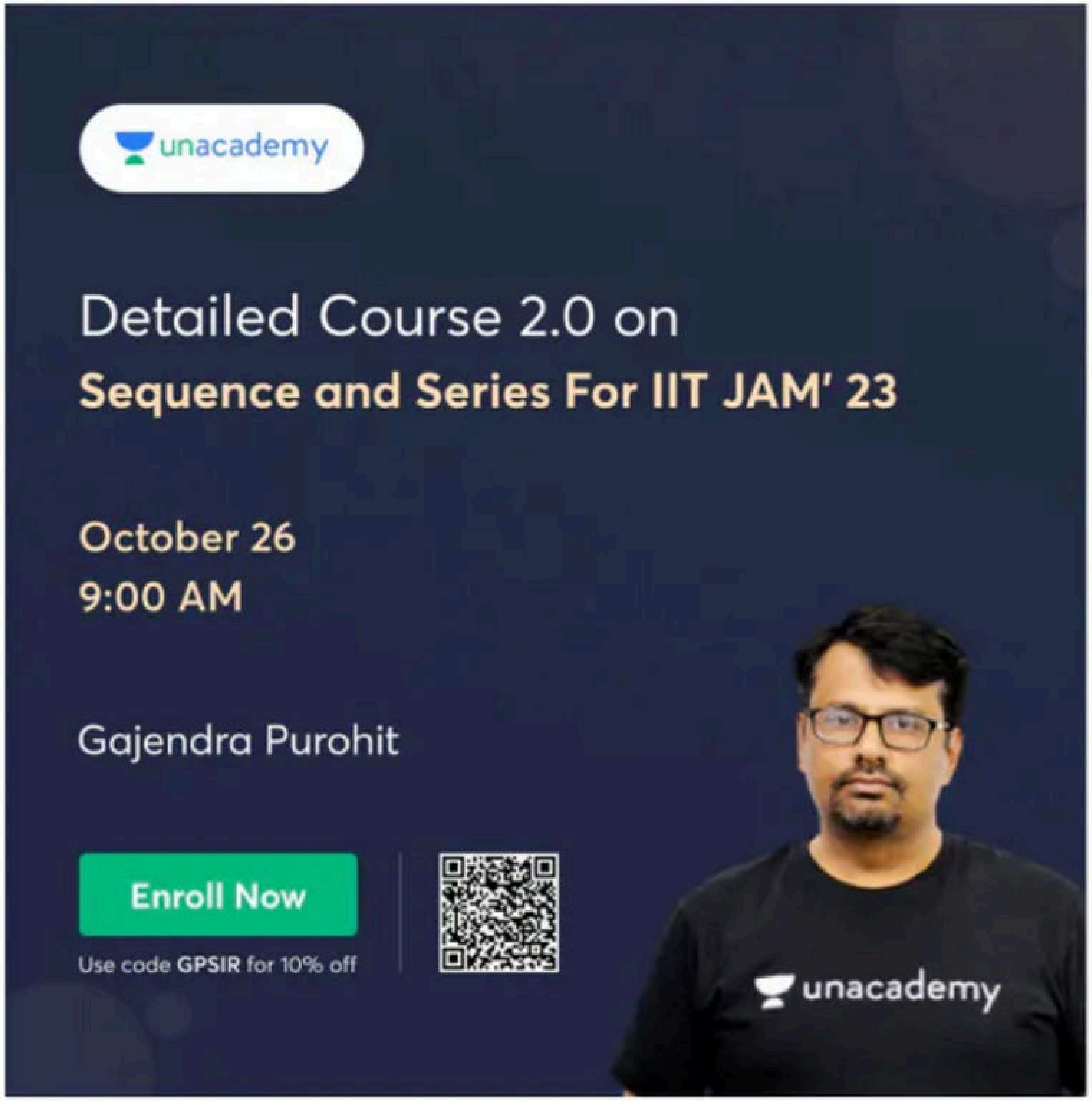
Q.9. Consider the sequence $\{a_n\}$ of real number where $a_1 > 1$ and $a_{n+1} = 2 - \frac{1}{a_n}$, $n \geq 1$, then the sequence $\{a_n\}$ is

(a) Bounded but not monotone

(b) Not bounded but monotone

(c) Both bounded and monotone

(d) Neither bounded nor monotone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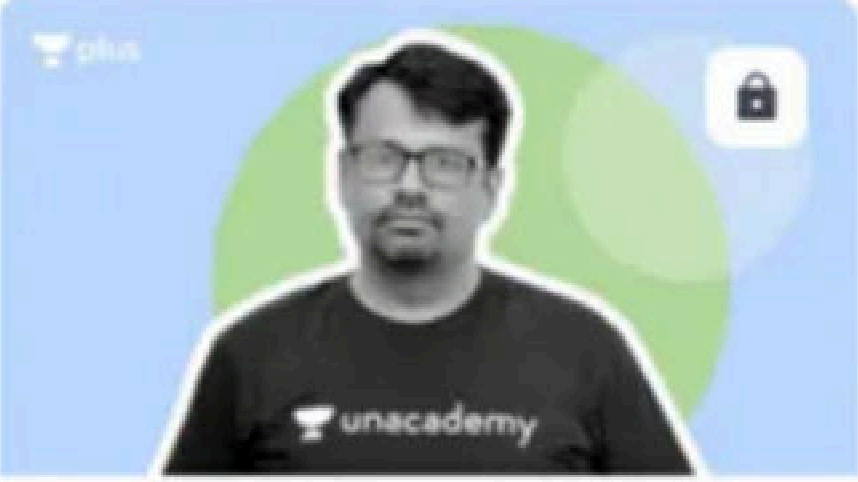

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- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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