Higher Order Constant Coefficient ODE - II

Detailed Course on Differential Equation for IIT JAM' 23 - II

Gajendra Purohit • Lesson 9 • Aug 18, 2022

Gajendra Purohit

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DETERMINATION OF ORTHOGONAL TRAJECTORIES IN POLAR COORDINATES $f(r, \theta_1 c) = 0$

Working Rule :

1. Differentiate the given equation of family of curves w.r.t. θ (generally take logarithm). Eliminate the parameter.
2. Replace $(dr/d\theta)$ by $- r^2(d\theta/dr)$ and obtain the differential equation of orthogonal trajectories.
3. Obtain the general solution of differential equation obtained above.

Q.1. The orthogonal trajectory of the family of curves given by the equation $r = a(1 - \cos\theta)$.

(a) $r = b(1 + \sec\theta)$ (b) $r = b(1 + \cos\theta)$

(c) $r^2 = b(1 + \cos\theta)$ (d) $r = b(1 + \tan\theta)$

GENERAL THEORY OF LINEAR DIFFERENTIAL EQUATION

1. DEFINITIONS

(i) Linear Combination: If $f_1, f_2, \ldots f_n$ are n functions defined on the interval I and $c_1, c_2, \ldots c_n$ are n arbitrary constants, then the function $c_1f_1 + c_2f_2 + \ldots + c_nf_n$ is called linear combination of $f_1, f_2, \ldots f_n$ over I.

(ii) Linearly Dependent: The functions $f_1, f_2 \ldots\ldots, f_n$ of x are said to be linearly dependent over an interval I iff there exist constants $c_1, c_2, \ldots\ldots, c_n$ (not all zero) such that $c_1f_1 + c_2f_2 + \ldots\ldots + c_nf_n = 0$ for all x in I.

(iii) Linearly Independent : The functions $f_1, f_2, \ldots, f_n$ are of x are said to be linearly independent over an interval I iff there exist constants $c_1, c_2, \ldots, c_n$ such that $c_1f_1 + c_2f_2 + \ldots + c_nf_n = 0$ for all x in I, then $c_i = 0 \; \forall \; i = 1, 2, \ldots, n$.

(iv) Convex Combination : A linear combination $\sum_{i=1}^{n} c_i f_i(x)$

is called a convex combination if $\sum_{i=1}^{n} c_i = 1$ & $c_i \geq 0$ for all

i.

2. GENERAL LINEAR DIFFERENTIAL EQUATION

Definition: A general linear differential equation of order n is

$$P_0\frac{d^n y}{dx^n}+P_1\frac{d^{n-1}y}{dx^{n-1}}+P_2\frac{d^{n-2}y}{dx^{n-2}}+\ldots+P_n y=Q \quad \ldots\ (1)$$

where $P_0 \neq 0$, P_1, P_2, ... P_n and Q are functions of x defined on some interval I.

3. Homogenous Linear Differential Equation

Equation (1) is said to be homogeneous if $Q = 0$.

4. Non-homogenous linear equation

Equation (1) is said to be non-homogeneous if $Q \neq 0$.

Principal of super position :

(i) Let $f_1(x)$ and $f_2(x)$ be two linearly independent solution of $P_0\frac{d^2y}{dx^2}+P_1\frac{dy}{dx}+P_2y=0$ over an open interval I, where P_0, P_1, P_2 are all continuous functions of x and $P_0(x) \neq 0$ on I. Then $f = \alpha f_1 + \beta f_2$, where α,β are some constants is also its solution.

(ii) Let $f_1(x)$ and $f_2(x)$ be linearly independent solution of

$$P_0\frac{d^2y}{dx^2}+P_1\frac{dy}{dx}+P_2y=Q \quad ;Q\neq 0$$ then f = $\alpha f_1+\beta f_2$ is a solution iff $\alpha+\beta=1$

Q.1. Let y_1& y_2 are two solution of $e^x y'' + \sin x\, y' + e^{\sin x}\, y = x\cos x$. Then which of the following are TRUE?

(a) $\frac{3}{2}y_1 - \frac{1}{2}y_2$

(b) $y_1 - 2y_2$

(c) $\frac{1}{2}y_1 + \frac{1}{2}y_2$

(d) $\frac{1}{2}y_1 - \frac{1}{2}y_2$

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5. The Wronskian :

Definition :The Wronskian of n functions $y_1(x)$, $y_2(x)$ ..... $y_n(x)$ is denoted by $w(x)$ or $w(y_1, y_2, ..., y_n)$

$$= \begin{vmatrix} y_1 & y_2 \ldots\ldots y_n \\ y'_1 & y_2' \ldots\ldots y'_n \\ \vdots & \\ y_1^{(n-1)} & y_2^{(n-1)} \ldots\ldots y_n^{(n-1)} \end{vmatrix}$$

Wronskian of second order DE

Let $a_0(x)\, y'' + a_1(x)\, y' + a_2(x)\, y = 0$

Where $a_0(x)$, $a_1(x)$, $a_2(x)$ are continuous and $a_0(x) \neq 0 \ \forall$ x

If $y_1(x)$ and $y_2(x)$ are solution

Then $w(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

Able's Formula:

Let $a_0(x)\, y'' + a_1(x)\, y' + a_2(x)\, y = 0$

Where $a_0(x)$, $a_1(x)$, $a_2(x)$ are continuous and $a_0(x) \neq 0\ \forall$ x

$w(y_1, y_2) = Ae^{-\int \frac{a_1(x)}{a_0(x)} dx}$ is called Able's formula.

RESULTS:

(1) If $w(y_1, y_2, \ldots y_n) \neq 0$ then $y_1, y_2 \ldots\ldots y_n$ are L.I. solution.

(2) If $w(y_1, y_2, \ldots\ldots y_n) = 0$ then $y_1, y_2, \ldots\ldots y_n$ are LD solution.

(3) Wronskian is either identically zero or non-zero.

(4) If Wronskian is non – zero at least one point then Wronskian is identically non – zero

(5) If Wronskian is zero at least one point then Wronskian is identically zero

(6) Wronskian can never change its sign

Q2. Consider two solution $x(t)=x_1(t)$ and $x(t)=x_2(t)$ of differential equation $\frac{d^2x(t)}{dt^2}+x(t)=0,\ t>0$ such that

$x_1(0)=1, \left.\frac{dx_1(t)}{dt}\right|_{t=0}=0, x_2(0)=0, \left.\frac{dx_2(t)}{dt}\right|_{t=0}=1$ the

Wronskian $W(t)=\begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_1(t)}{dt} \end{vmatrix}$ and at $t=\pi/2$ is

(a) 1
(b) -1
(c) 0
(d) $\pi/2$

Q3. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of the differential equation $x^2y''(x)-2xy'(x)-4y(x)=0$ for $x\in[1,10]$. Considered the wronskian $W(x)=y_1(x)y_2'(x)-y_2(x)y_1'(x)$. If W(1) = 1, then W(3) – W(2) equals

(a) 1
(b) 2
(c) 3
(d) 5

Q4. Let $y_1(x)$ and $y_2(x)$ be the linearly independent solutions of $xy''+2y'+xe^x y=0$ If $W(x)=y_1(x)y_2'(x)-y_2(x)y_1'(x)$ with W (1) = 2 find W (5)

(a) $\frac{2}{25}$

(b) $\frac{1}{25}$

(c) $\frac{2}{5}$

(d) None of the above

Q.5. Consider the ODE

$$u''(t)+P(t)u'(t)+Q(t)u(t)=R(t), t\in[0,1]$$

There exist continuous function P, Q and R defined on $[0,1]$ and two solutions u_1 and u_2 of the ODE such that the Wronskian W of u_1 and u_2 is

(a) $W(t)=2t-1, 0\le t\le 1$

(b) $W(t)=\sin 2\pi t, 0\le t\le 1$

(c) $W(t)=\cos 2\pi t, 0\le t\le 1$

(d) $W(t)=1, 0\le t\le 1$

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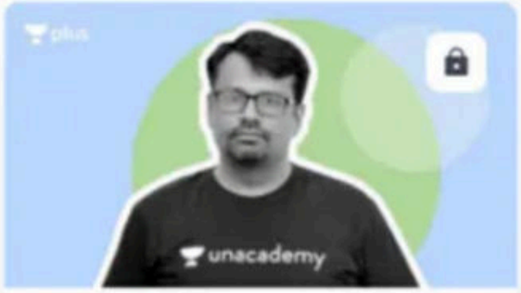

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
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