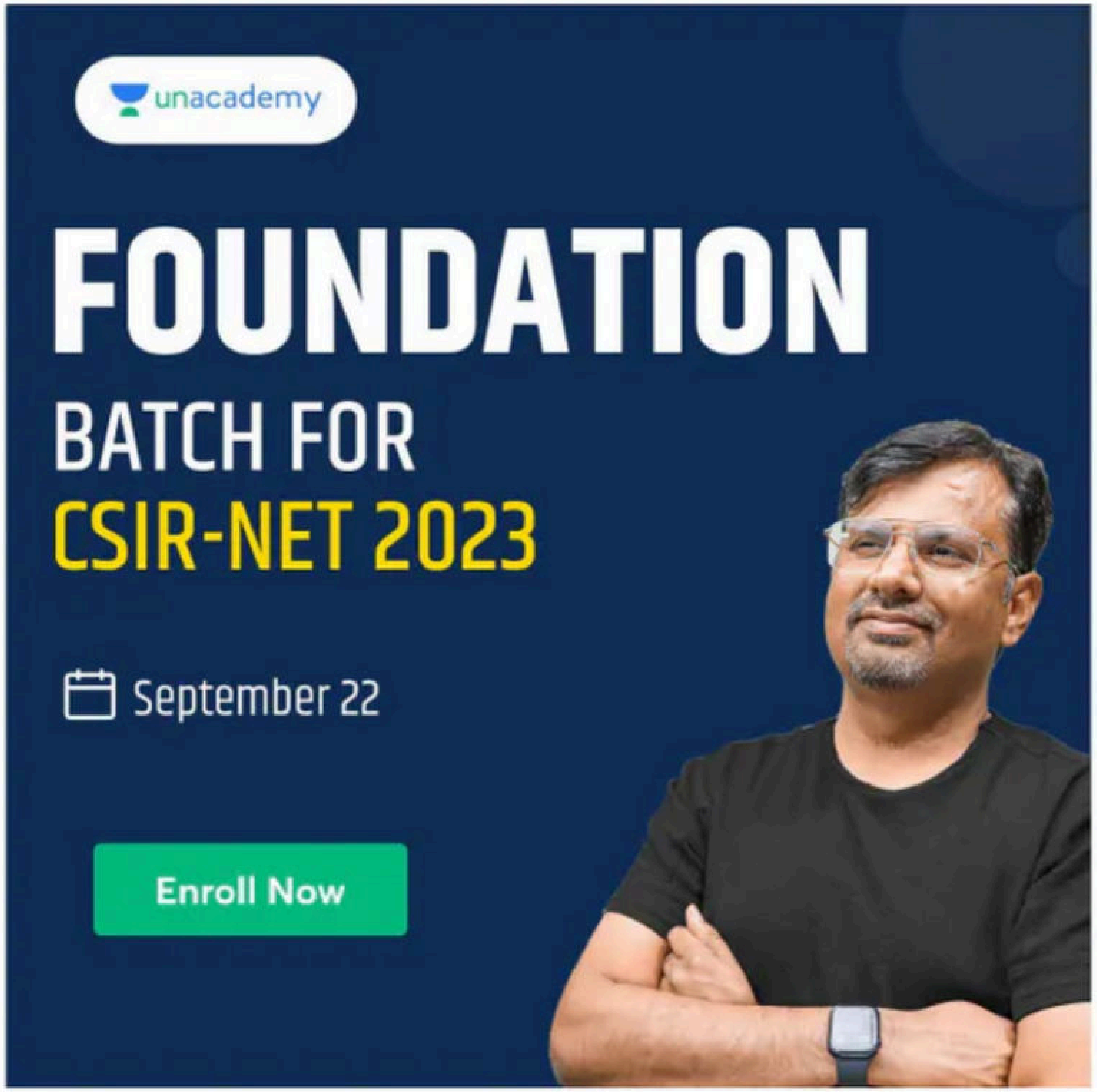
Normal Subgroup

Detail Course 2.0 on Group Theory for IIT JAM '23

Gajendra Purohit • Lesson 7 • Oct 14, 2022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Subgroup

Subgroup : Let (G, *) be a group a non-empty subset H of G is said to be subgroup of G if H itself a group under composition *.

One step test : Let (G, *) be a group. A non-empty subset H of G is called a subgroup of G if

$a * b^{-1} \in H$; for all $a \in H$ & for all $b \in H$ where b^{-1} is inverse of b.

$\times$

$GL(2, R)$

$SL(2, R)$

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$AB^{-1} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \left(\frac{1}{1} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \right) = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{bmatrix} -3 & 8 \\ -2 & 5 \end{bmatrix} = -15 + 16 = 1$$

$AB^{-1} \in SL$

$o(S_3) = 6$

1, 2, 3, 6

$\langle e \rangle$

S_3

No. of cyclic subgroup of order 2 = $\dfrac{\text{No. of elements of order}}{\phi(2)}$

$= \dfrac{3}{\phi(2)} = \dfrac{3}{1} = 3$

No. of cyclic subgroup of order 3

$= \dfrac{\text{No. of elements of order 3}}{\phi(3)} = \dfrac{2}{2} = 1$

$S_3 = \langle e, (12), (13), (23), (123), (132) \rangle$

Trivial

$\langle e \rangle$

$\langle e, (12) \rangle$ $\langle e, (13) \rangle$ $\langle e, (23) \rangle$ $\langle e, (123), (132) \rangle$ S_3

A_3

improper

Proper Subgroup

Non Trivial

(abc)(ef)

$\boxed{(ac)(ab)(ef)}$

S10

~~16~~

Q.1. Consider the alternating group A_4 .Which of the following is FALSE?

(a) A_4 has 12 elements

(b) A_4 has exactly one subgroup of order 4

(c) A_4 has a subgroup of order 6

(d) Number of 3 cycles in A_4 is 8

Result :

(1) Let G be a cyclic group of order n, then number of subgroup are $\tau(n)$ where $\tau(n)$ are number of +ve divisor of n.

$G = (\mathbb{Z}, +)$

(2) Number of cyclic subgroup of order d

$$= \frac{\text{number of elements of order d}}{\phi(d)}$$

$H_1 = (2\mathbb{Z}, +),\ H_2 = (3\mathbb{Z}, +)$

$(H_1 \cup H_2) = \langle 2, 3, 4, 6, \ldots \rangle$

$5 \notin H_1 \cup H_2$

(3) Intersection of two subgroup is always a subgroup.

(4) Union of two subgroup is a subgroup iff one is contained in other.

$H_1 = (2\mathbb{Z}, +) \quad H_2 = (4\mathbb{Z}, +)$

$H_1 \cup H_2$

(5) Let H & K are two subgroup of group G, then $O(H,K) = \dfrac{O(H).O(K)}{O(H \cap K)}$.

$O(G) = n$

$O(H) = p$

$\Rightarrow p|n$

(6) $H \cap K$ is a subgroup of H & K both.

Lagrange's theorem

Order of group is divisible by its every subgroup but converse need not be true

$O(A_4) = 12$

1

2

3

4

6

12

$p|n$

$\cancel{6|12}$

Q.2. Let H & K be subgroups of Z_{144}. If the order of H is 24 and the order of K is 36 then the order of the subgroup $H \cap K$ is

(a) 3 (b) 4

(c) 12 (d) None of these

$H \cap K$

$o(H) = 24$: 1, 2, 3, 4, 6, ~~8~~, 12, 24

$o(K) = 36$: 1, 2, 3, 4, 6, 9, 12, 18, 36

$o(HK) = \dfrac{o(H)\,o(K)}{o(H \cap K)} = \dfrac{24 \times 36}{12} = 72$

$= \dfrac{24 \times 36}{4} = 216$

Q.3. Let S_5 be the symmetric group on five symbols. Then which of the following statements is false? **CSIR NET DEC. 2019**

(a) S_5 contains a cyclic subgroup of order 6

(b) S_5 contains a non-abelian subgroup of order 8

(c) S_5 does not contain a subgroup isomorphic to $Z/2Z \times Z/2Z$

(d) S_5 does not contain a subgroup of order 7

Q.4. Which of the following is true? **JAM 2018**

(a) Z_n is cyclic iff n is prime

(b) Every proper subgroup of Z_n is cyclic.

(c) Every proper subgroup of S_n is cyclic

(d) If every proper subgroup of a group is cyclic then the group is cyclic.

Z_4

Z_3

S_3

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Centre of group : Let G be a group. A collection $Z(G) = \{x \in G \mid xa = ax;\ \text{for all } a \in G\}$ is also a subgroup of G, which is called centre of group.

$Q_8 = \{\pm 1, -1, \pm i, \pm j, \pm k\}$

$Z(Q_8) = \{1, -1\}$

$i \cdot 1 = 1 i$

$i(-1) = (-1) i$

$i(-i) = (-i) i$

$ij = ji$

$k \quad -k$

$K_4 = \{e, a, b, ab\}$

$ab = ba$

$a(ab) = (ab)a$

$\cancel{Z(K_4) = \{e, a, b, ab\}}$

$Z(K_4) = K_4$

$Z(S_3) = \langle e \rangle$

$n > 3$

$S_3 = \langle e, (12), (13), (23), (123), (132) \rangle$

$Z(S_4) = \langle e \rangle$

$(12)(13) = (13)(12)$

$(132) \neq (123)$

$Z(S_2) = S_2$

$Z(S_{14}) = \langle e \rangle$

$Z(S_1) = S_1$

$GL(3, \mathbb{R})$ | $SL(3, \mathbb{R})$

$$Z(G) = \{A \in G \mid AB = BA\}$$

$a^3 = 1$

$a = 1, \cancel{-1 \pm \sqrt{3}}$

$$Z(\ ) = \left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}, a \in \mathbb{R} \right\}$$

$= \text{infinite}$

$o(Z(GL(2, \mathbb{R})))$

$$o(Z(SL(2, \mathbb{R}))) = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\}$$

Q.5. $G=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a,b,c,d \in Z\right\}$ under addition.

(a) $H=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G \mid a+b+c+d=0\right\}$ then H is a subgroup of G.

(b) $H=\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G \mid a+b+c+d=1\right\}$ then H is a subgroup of G.

$\begin{matrix} 0 & b \\ 0 & 0 \end{matrix}$

(c) Both (a) and (b) are correct.

(d) None of above.

Q.6. Pick out the true statements

(a) $H_1 = \{A \in GL(2, R) \mid \det A \text{ is an integer power of } 2\}$ then H_1 is a subgroup.

(b) $H_2 = \{A \in GL(2, R) \mid \det A \text{ is an integer power of } 6\}$ then H_2 is a subgroup.

(c) Both (a) and (b) are correct.

(d) None of the above

Q.7. Given a group G, its center is $\{h \in G : hg = gh \text{ for all } g \in G\}$.

Let G be the group $G = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} : a, b, c \in Z \right\}$. Then the center of G

(a) is G itself

(b) is $\left\{ \begin{pmatrix} 1 & a & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} : x \in Z \right\}$.

(c) consists only of the 3×3 identity matrix.

(d) $\left\{ \begin{pmatrix} 1 & x & x \\ 0 & 1 & x \\ 0 & 0 & 1 \end{pmatrix} : x \in Z \right\}$

Q.8. : If H_1 and H_2 are two subgroups of G, then following is also a subgroup of G

(a) $H_1 \cap H_2$ (b) $H_1 \cup H_2$

(c) H_1H_2 (d) None of these

Q.9 If $G = \{1, \omega, \omega^2\}$, then there are

(a) 2 subgroups (b) 3 subgroups

(c) 4 subgroups (d) No subgroup possible

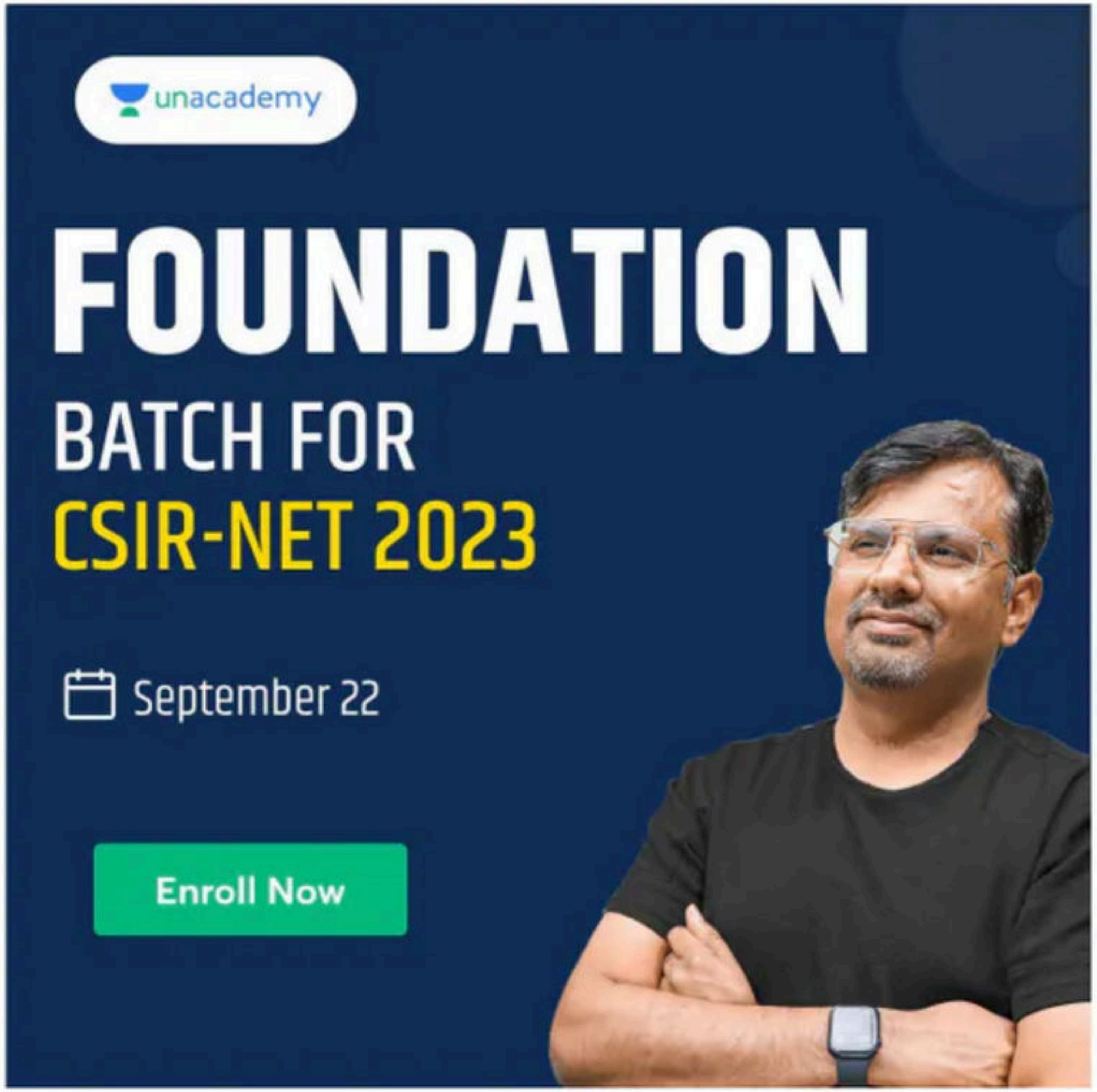
Q.10 Which of the following statement(s) is/are true?

(a) Any group of prime order can have no proper subgroups.

(b) Any group of prime order can have proper subgroup.

(c) The order of a subgroup divides the order of a group.

(d) The order of a subgroup does not divides the order of a group.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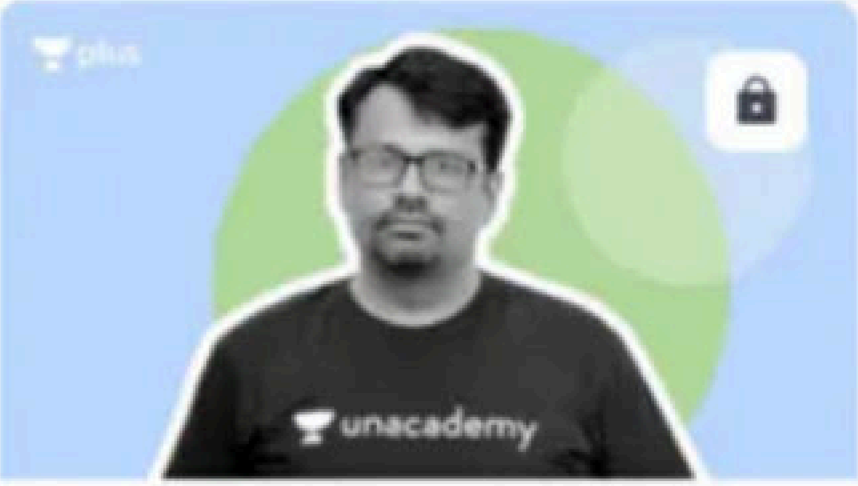

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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