Doubt Clearing Session

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 4 • Nov 14, 2022

Gajendra Purohit

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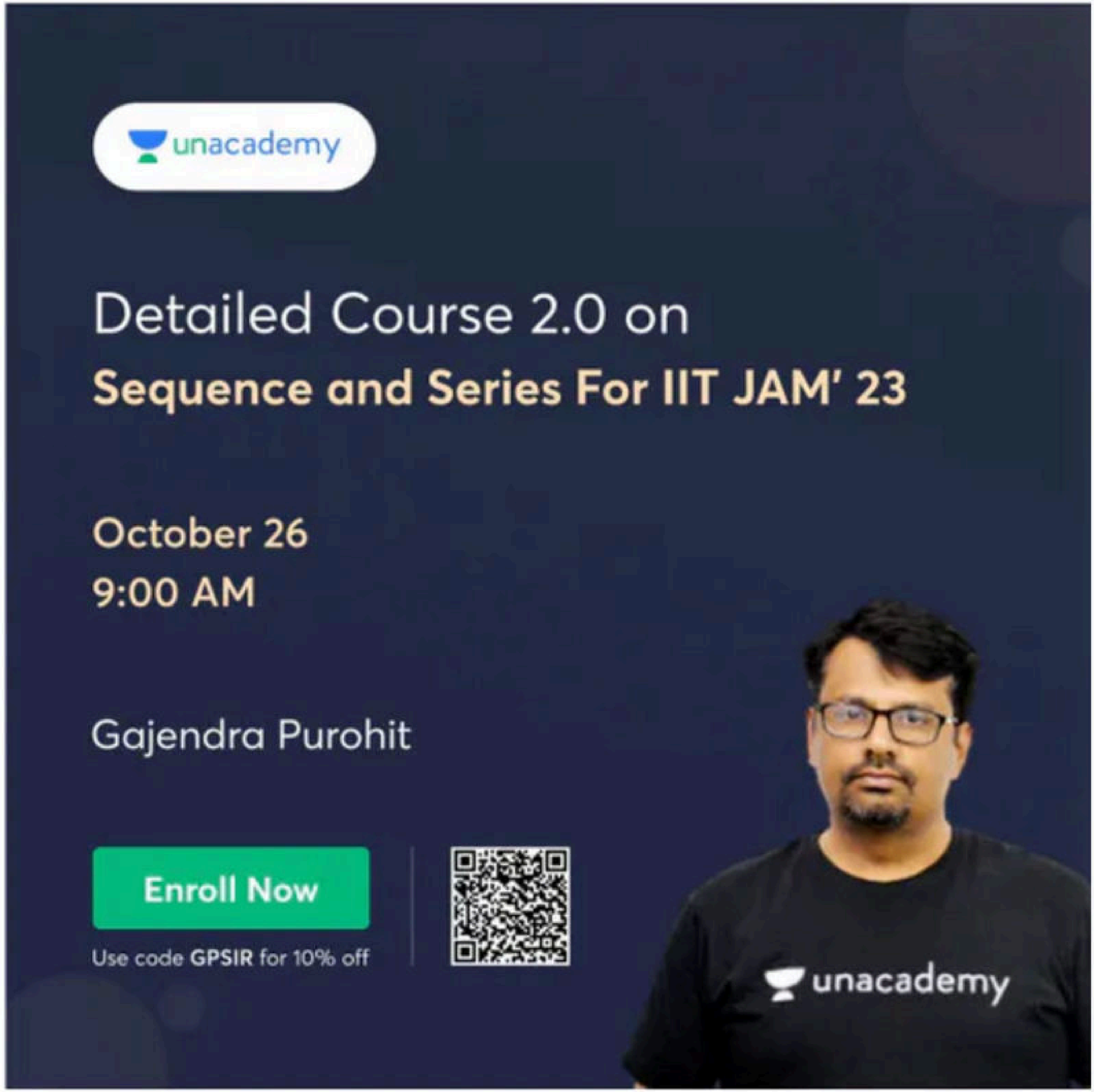
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Results on continuity :

(1) **Boundedness :** If f is continuous on closed interval on [a, b], then f is bounded on [a, b]

i.e.f(x) lies in [m, M] ; where m and M are infimum and supremum of f(x) in [a , b]

(2) **Attainment property :**

Let f : [a, b] $\to$ R is a continuous function and

$$M = \sup_{x \in [a,b]} f(x), \qquad m = \inf_{x \in [a,b]} f(x)$$

Then f(x) attains its infimum and supremum in [a , b]

Then f(x) $\in$ [m, M]

(3) Opposite sign theorem :

If $f(x)$ is continuous on $[a, b]$ and $f(a)$, $f(b)$ have opposite sign,

then $\exists\, c \in (a, b)$, s.t. $f(c) = 0$.

(4) Intermediate value property :

If f is continuous on $[a, b]$, then f assumes every value between $f(a)$ & $f(b)$.

(5) Image of compact set under continuous function is compact

(6) Inverse image of open set under continuous function is open

Q.1. Let $f : R \to R$ be defined by $f(x) = x^2$ and let U be any non-empty open subset of R. Then, **IIT JAM 2005**

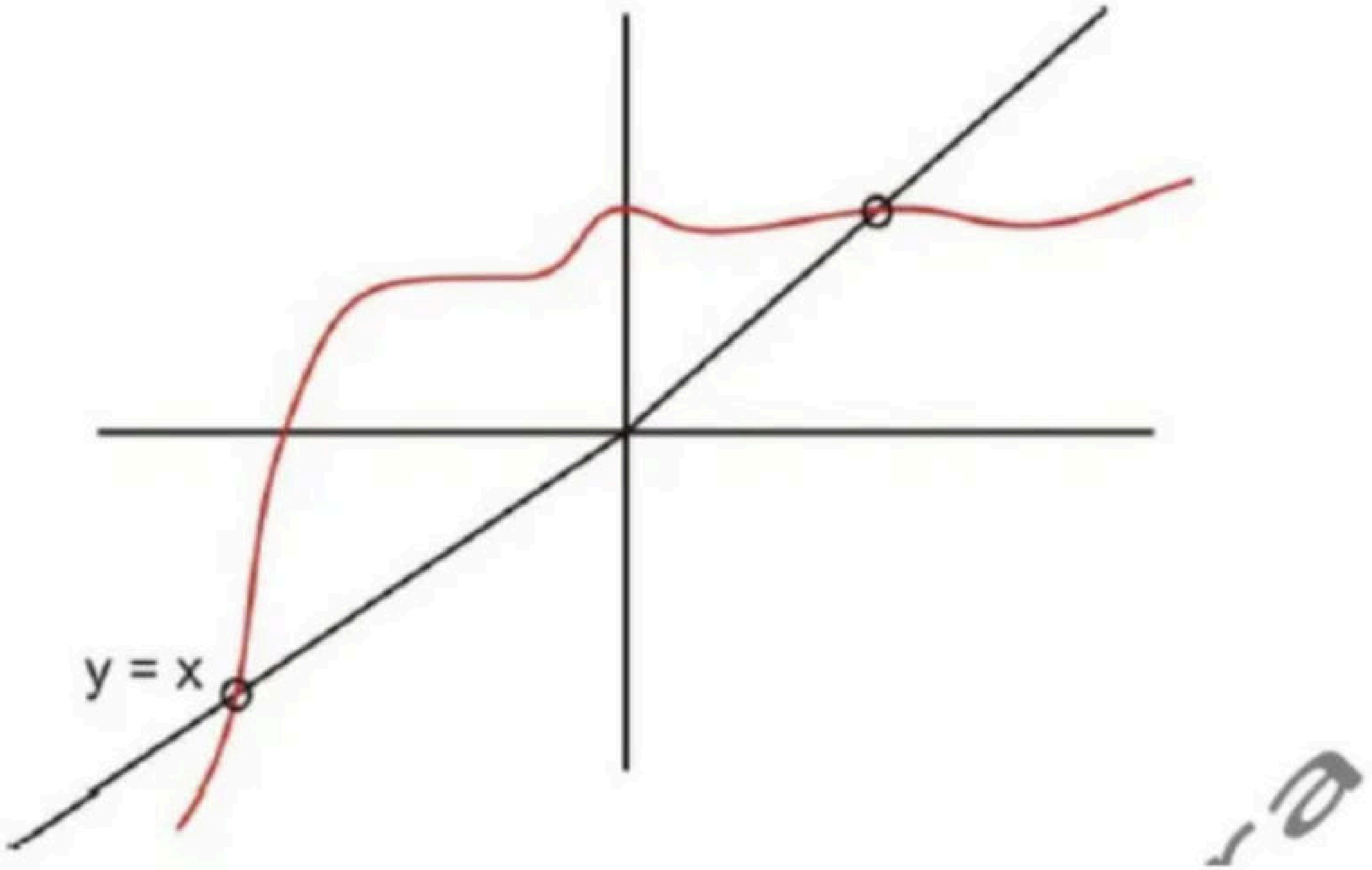
(a) $f(U)$ is open (b) $f^{-1}(U)$ is open

(c) $f(U)$ is closed (d) $f^{-1}(U)$ is closed

Fixed Point :

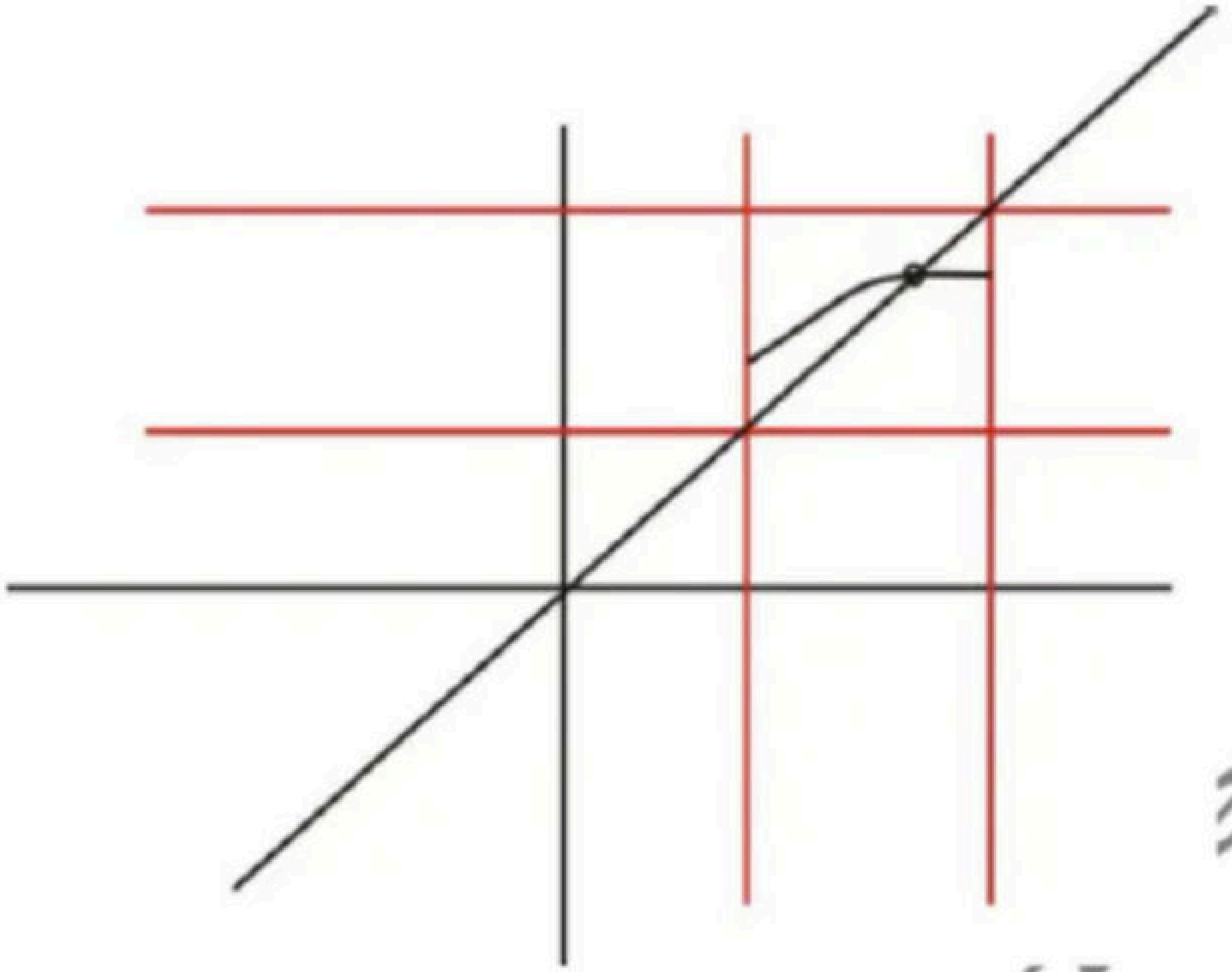
If $f(x)$ is continuous on closed interval $[a, b]$. then a point $c \in [a, b]$ is called fixed point if $f(c) = c$.

Note : Fixed point of f mean those points where graph of f intersects the graph of y = x.



Note :

(a) Let $f : [a, b] \to [a, b]$ is a continuous function.



Then $\exists$ atleast one fixed point.

(b) Let $f : (a, b) \to (a, b)$ is continuous function, then $f(x)$ may not have fixed point.

Example : $f : (0, 1) \to (0, 1)$

s.t. $f(x) = 2 - x$

$f(x)$ does not have any fixed point.

(c) $f : [a, b] \to B$ is continuous, where B is any subset of $[a, b]$, then there must exist $x \in B$ such that $f(x) = x$

Some observation :

(1) A function $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$ for all $x, y \in R$ and f is continuous,

Then $f(x) = ax; a \in R$

So, $f(x)$ is continuous at every point.

(2) A function $f : R \to R$ satisfies $f(x + y) = f(x).f(y)$ for all $x, y \in R$ and f is continuous.

Then $f(x) = a^x ; a > 0$

So, $f(x)$ is continuous.

(3) If a function satisfying $f(x.y) = f(x) + f(y)$

Then $f(x) = \log_a x ; a > 0$

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Q.2. Let f be a strictly monotonic continuous real values function defined on $[a, b]$ such that $f(a) < a$ and $f(b) > b$. Then which one of the following is TRUE?

IIT JAM 2016

(a) There exists atleast one $c \in (a, b)$ s.t. $f(c) = c$

(b) There exists exactly two points $c_1, c_2 \in (a, b)$ such that $f(c_i) = c_i$, $i = 1, 2$

(c) There exists no $c \in (a, b)$ such that $f(c) = c$

(d) There exist infinitely many points $c \in (a, b)$ such that $f(c) = c$

Q.3. Let $f : R \to R$ be a function satisfying $f(x + y) = f(x) f(y), \forall x, y \in R$ and $\lim_{x \to 0} f(x) = 1$. Which of the following are necessarily true? **CSIR NET DEC. 2017**

(a) f is strictly increasing

(b) f is either constant or bounded

(c) $f(rx) = f(x)^r$ for every rational $r \in Q$

(d) $f(x) \geq 0, \forall x \in R$

Q.4. If a, b, c $\in$ R (a < b < c). Then which of the following is/are true for any continuous function satisfying $f(a) = b, f(b) = c, f(c) = a$

(a) There exist $\alpha \in (a, c)$ such that $f(\alpha) = \alpha$

(b) There exist $\beta \in (a, b)$ such that $f(\beta) = \beta$

(c) There exist $\gamma \in (a, b)$ such that $fof(\gamma) = \gamma$

(d) There exist $\delta \in (a, c)$ such that $fofof(\delta) = \delta$

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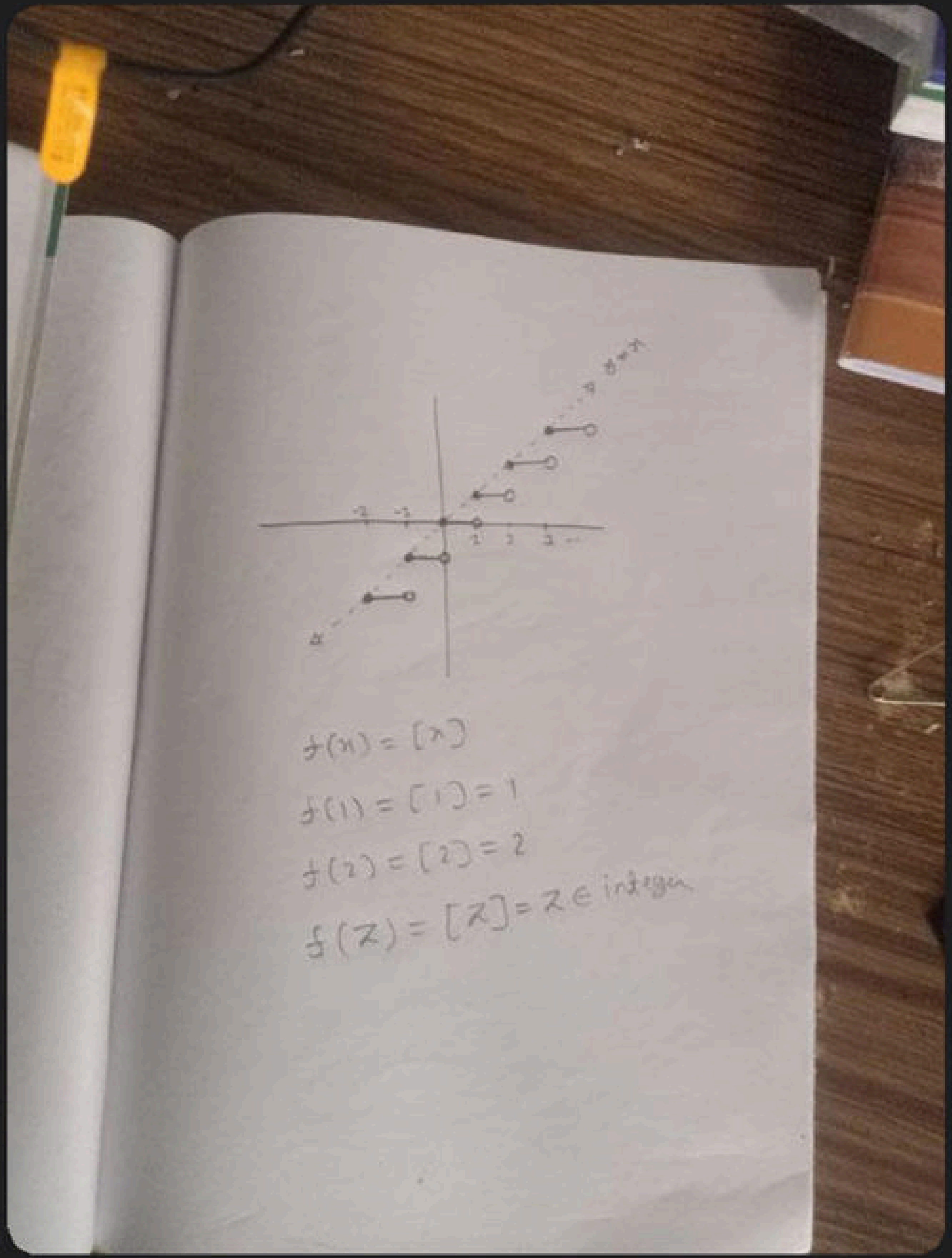
▲ 1 • Asked by Aryan

Sir ye doubt tha.. please clear this..



Difference between continuity and uniformly continuity :

Continuity of a function is at point and uniformly continuity is in interval.

Example : $f(x) = x$ is continuous at 0.

But it is uniformly continuous in $[0, 1]$

Lipschitz function :

A function $f : I \to R$ is said to be satisfy a Lipschitz condition

on I, if $\exists$ a positive integer M such that

$|f(x_1) - f(x_2)| \leq M|x_1 - x_2|$, for any two $x_1, x_2 \in I$

Some direct result for uniformly continuity:

(1) If a function f(x) is Lipschitz function then f is uniformly continuous.

(2) If f(x) is continuous on closed interval [a, b] then it is uniformly continuous on [a, b].

(3) If a function is UC then it is continuous

i.e. If a function is not continuous then it is not UC

Sequential definition :

Let $f : D \to R$ be a function. If $< x_n>$ & $<y_n>$ are two convergent sequence which converge to same limit and $f(<x_n>)$ and $f(<y_n>)$ are also converges to same limit then this function is uniformly continuous on D.

Conclusion : If f(x) is bounded and continuous on I, then f(x) may not be uniformly continuous on I.

(4) Let a function f be continuous on an open bounded interval (a, b), then f is uniformly continuous on (a, b) if $\lim_{x \to a^+} f(x)$ & $\lim_{x \to b^-} f(x)$ both exist finitely. it is necessary condition.

(5) If derivative of f(x) is bounded on I, then f(x) is uniformly continous on I.

(6) If f(x) is uniformly continuous on [a, c] and [c, b] both & f(x) is continuous at c, then f(x) is uniformly continuous on [a, b].

Q5. If a function $f : R \to R$ is continuous and $f(x+y) = f(x) + f(y)$, $\forall\, x, y$ then $f(x)$ is **CUCET 2016**

(a) $x^3 f(1)$ (b) $x^2 f(1)$

(c) $x f(1)$ (d) $x^4 f(1)$

Q.6. If a function $f : R \to R$ is continuous then choose correct statements **B.H.U.2018**

(a) $f(A)$ is bounded for all bounded subset A of R

(b) f is bounded

(c) The image of f is an open subset of R

(d) $f^{-1}(A)$ is compact for all compact subset A of R

Q.7. Which of the following functions is uniformly continuous on the domain as stated? **IIT JAM**

(a) $f(x) = x^2, x \in R$

(b) $f(x) = \frac{1}{x}, x \in [1, \infty)$

(c) $f(x) = \tan x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(d) $f(x) = [x], x \in [0, 1]$

([x] is the greatest integer less than or equal to x]

Q.8. Let f : (0 , ∞) → R be defined by $f(x)=\dfrac{\sin(x^3)}{x}$.

then f(x) is **TIFR 2019**

(a) bounded and uniformly continuous

(b) bounded but not uniformly continuous

(c) Not bounded but uniformly continuous

(d) Neither bounded nor uniformly continuous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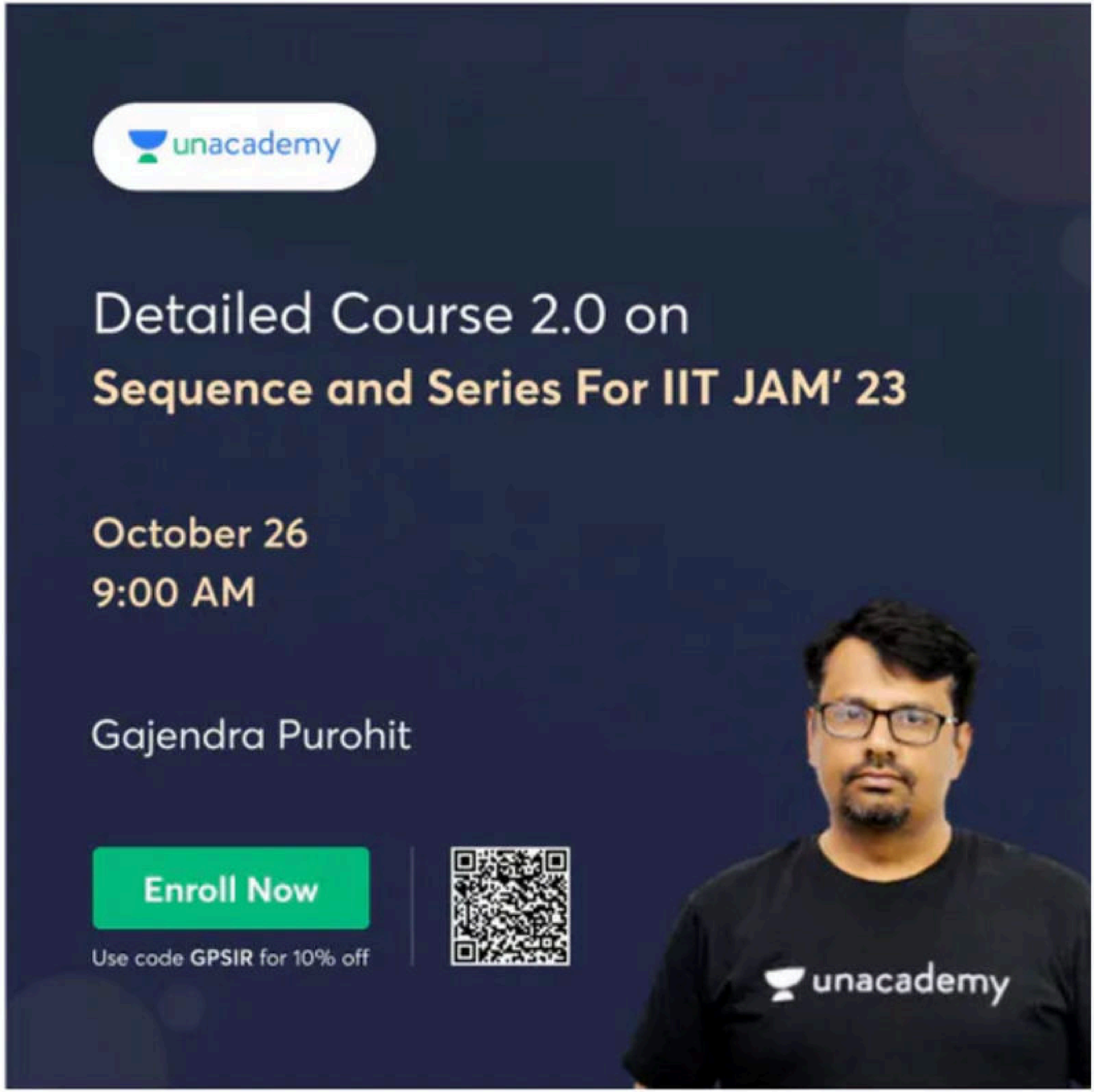
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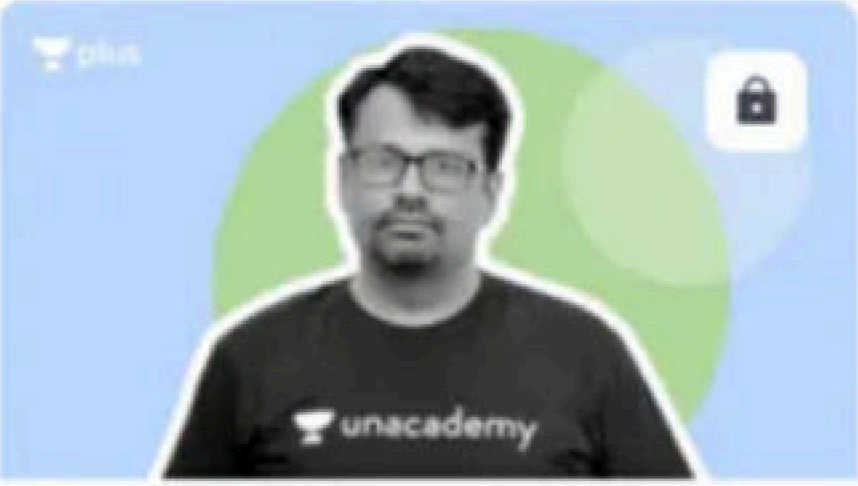

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Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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