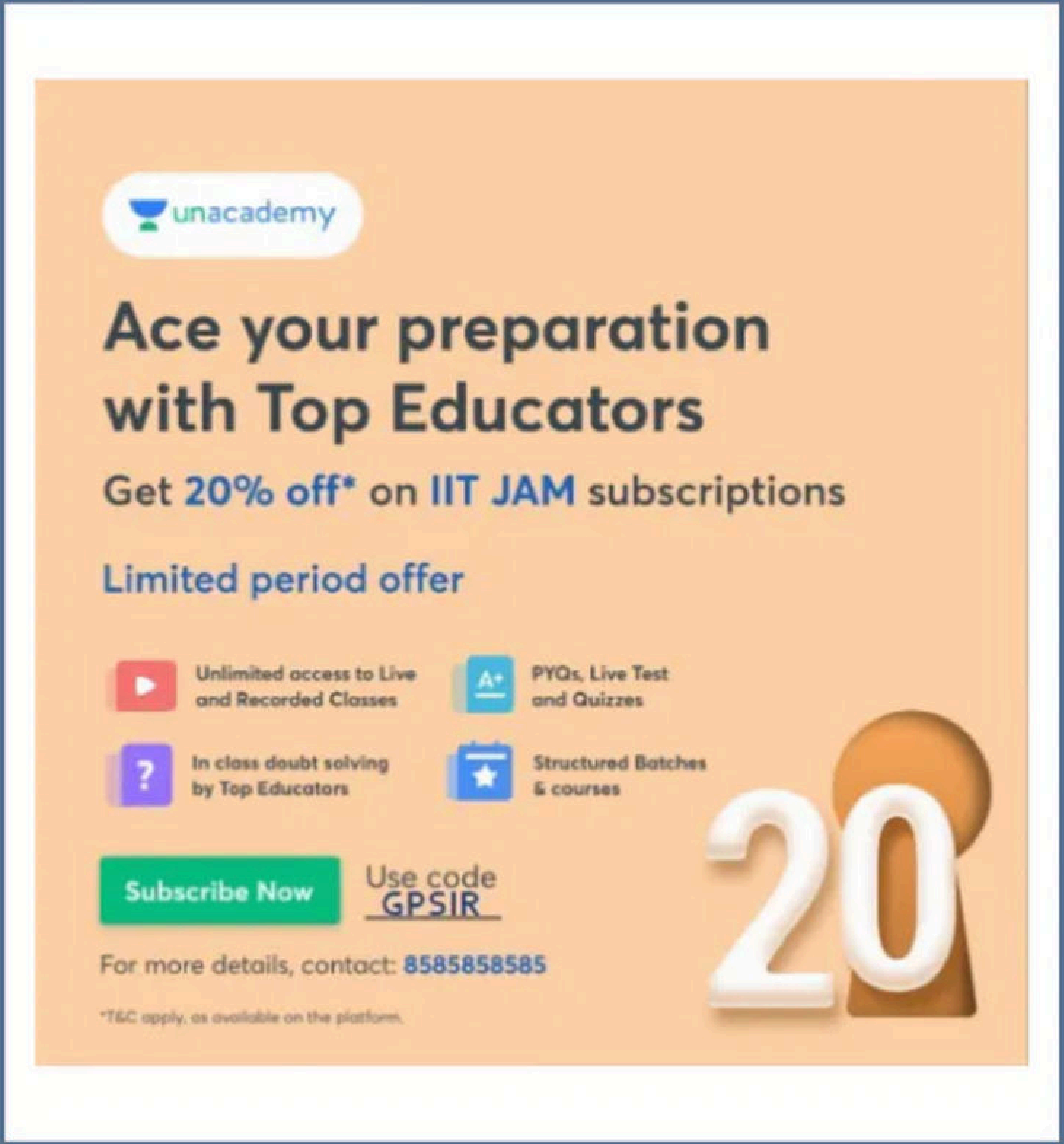
Linear Transformations - Part I

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 4 • Sept 14, 2022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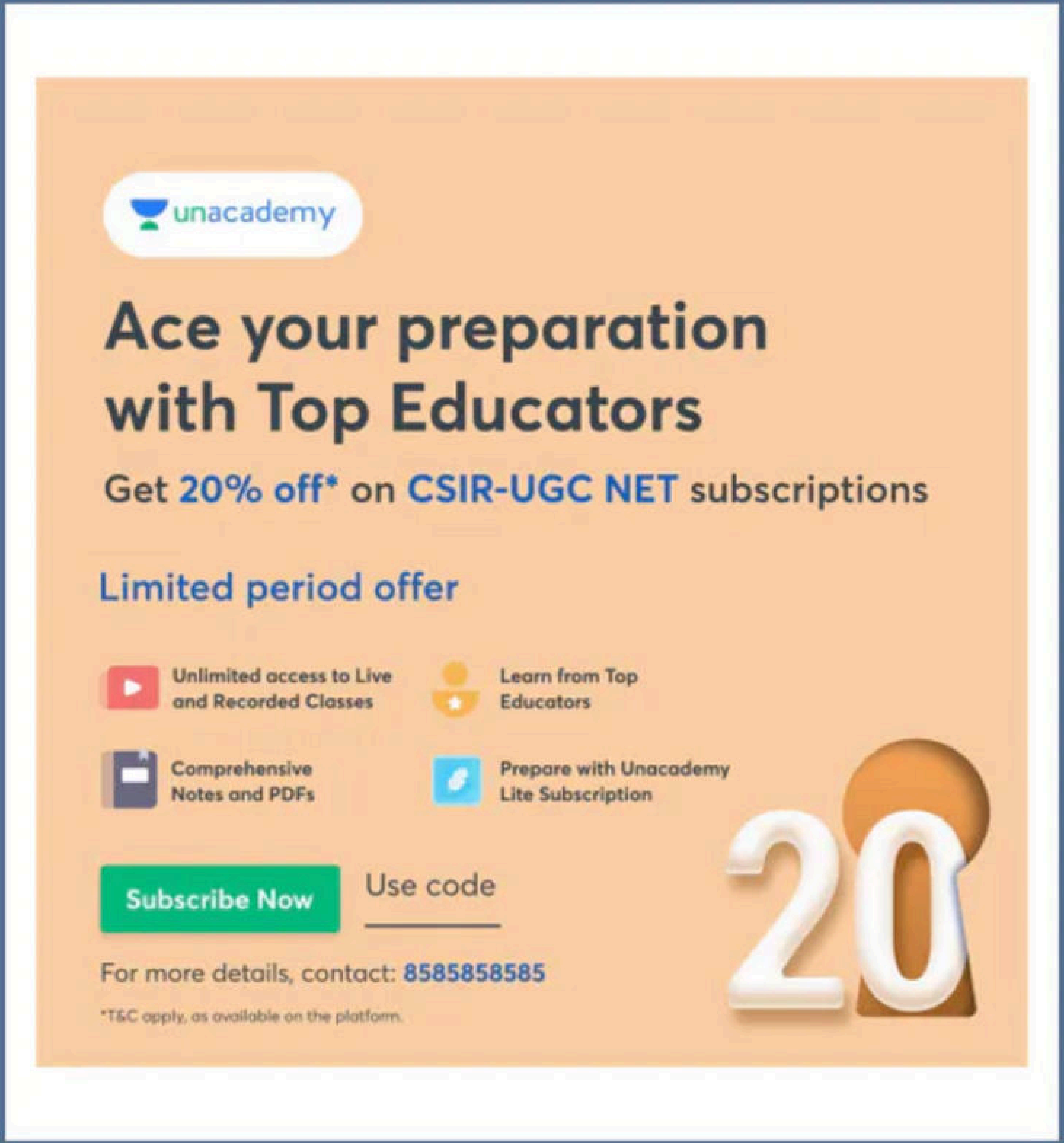
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Idempotent matrix : A square matrix A is said to be idempotent if $A^2 = A$.

Note :

(1) $A^2 - A = 0 \Rightarrow A(A - I) = 0$

$\Rightarrow A = 0$ (Null) or $A = I$

Identity and null matrix are trivial example.

(2) If A is idempotent and $|A| = \alpha$

Then $|A|^2 = |A| = \alpha^2 = \alpha$

$\Rightarrow \alpha = 0$ or $\alpha = 1$

Determinant of A is either 0 or 1.

(3) Identity is the only idempotent matrix where determinant is 1 and all other idempotent matrix have determinant zero.

$A^2 = A$

$|A|^2 = |A|$

$\alpha^2 = \alpha$

$\alpha^2 - \alpha = 0$

$\alpha(\alpha - 1) = 0$

$\alpha = 0$ | $\alpha = 1$

$$A^2 = A$$

$$|A| = 0 \text{ or } 1$$

$$A = 0$$

$$A = \begin{pmatrix} \frac{1}{n} & \cdots & \frac{1}{n} \\ \vdots & & \vdots \\ \frac{1}{n} & \cdots & \frac{1}{n} \end{pmatrix}_{n \times n}$$

$$\boxed{A = I}$$

$$A_{2\times 2} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Property :

(1) A is idempotent matrix then I – A is also idempotent.

$(I-A)^2 = I + A^2 - 2A = I + A - 2A$

$= I - A$

(2) If A & B are idempotent matrix then AB is idempotent if AB = BA and A + B is idempotent if $\boxed{AB = -BA}$

$A^2 = A, B^2 = B$

$(A+B)^2 = (A+B)(A+B)$

$= A^2 + AB + BA + B^2$

$= (A+B) + AB + BA$

$(A+B)^2 = A+B$

Q.1. A & B are square matrix such that AB = A & BA = B, then

(a) $A^2 = A, B^2 = B$ **(b)** $A^2 = A, B^2 \neq B$

(c) $A^2 \neq A, B^2 = B$ (d) $A^2 \neq A, B^2 \neq B$

$BA = B$

$B(AB) = B^2$

$BA = B^2$

$B = B^2$

$AB = A$

$A(BA) = A^2$

$AB = A^2$

$A = A^2$

(3) If A is idempotent then kA is also idempotent iff k = 0 or k = 1

$(kA)^2 = k^2A^2 = k^2A = kA$

$\Rightarrow k^2 = k$

$\Rightarrow k = 0$ or $k = 1$

(4) If A is idempotent then A^k is idempotent for positive integer k.

A =

3A

A

A

A^2

A^3

A^2

A^3

Involutory matrix : A square matrix A is involutory if $A^2 = I$.

Note :

(1) If A is involutary and $|A| = \alpha$ then $|A|^2 = |I| = 1$

$\Rightarrow \alpha^2 = 1$

$\Rightarrow \alpha = \pm 1$

Determinant of an involutory matrix is ± 1.

(2) If A is involutory matrix then kA is involutory iff $k = \pm 1$

(3) If A is involutory matrix then A^k is involutory.

$(A^k)^2 = (A^2)^k = I$

For every positive integers.

(4) If A & B are two involutory matrix then AB is involutory

if $AB = BA$ and $A + B$ is involutory if $AB + BA = -I$

$$A = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$a \in \mathbb{R}$

$$A^2 = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Nilpotent matrix : A square matrix A is said to be nilpotent if $\exists$ $m \in N$ such that $A^m = 0$

Index of nilpotent matrix : The smallest positive integer K s.t. $A^k = 0$ is known as index of nilpotent matrix where $A^{k-1} \neq 0$ and $A^{k+1} = A^{k+2} = \ldots\ldots = 0$

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

A^1

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 3 \times 3$$

$A^2 = 0$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$A^3 = 0$

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Property :

(1) If A & B are nilpotent then AB is nilpotent if AB = BA.

(2) If A is nilpotent matrix of index P then A^k is also nilpotent of index [P/k], where [.] is ceilling function.

$|A| = 0$

$A + B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Note :

If A & B are nilpotent matrix then A + B may or may not be nilpotent.

$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$A = \begin{bmatrix} \quad \end{bmatrix}_{3\times 3}$$

$A^5 = 0$

$A^3 = 0$

$$A = \left(\quad \right)_{5\times 5}$$

$$B = A^2$$

$$A^4 = 0$$

$$\frac{4}{2}$$

$$\boxed{P = \text{Index} = 4}$$

A^{20}

$A = \begin{pmatrix} & \end{pmatrix}_{100\times 100}$

$\binom{50}{20} = ?$

$A^{50} = 0$

$p = 50$

Q.2. Let A & B are two nilpotent matrix of index 15 & 13 then which of the following are true?

(a) Index of A^2 is 9.

(b) Index of A^4 is 4

(c) Index of B^3 is 6

(d) Index of B^5 is 3

(a) a c

(b) b d

(c) c b

(d) d a

$\left[\frac{15}{2}\right] = 8$

$\left[\frac{15}{4}\right] = 4$

$\left[\frac{13}{3}\right] = 5$

$\left[\frac{13}{5}\right] = 3$

Note :

(1) $AA^T = I$

$|AA^T| = 1 \Rightarrow |A|\,|A^T| = 1$

$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$

Determinant of an othrogonal matrix is ± 1 and $A^T = A^{-1}$.

othogonal matrix

$AA^T = I$

(2) Sum of square of elements of each row or column are 1 and sum of the product of element of any row or column with corresponding elements of any other (column) is always zero.

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Q.3. Number of orthogonal matrix of order n whose entries are 0 & 1 only

(a) n (b) $n!$

(c) $n - 1$ (d) None of these

Property :

(1) If A is orthogonal then kA is orthogonal if $k = \pm 1$

Example : If A is orthogonal then 3A is not orthogonal.

(2) If A & B are orthogonal then A + B cannot orthogonal but AB is always orthogonal.

(3) If A is orthogonal then A^n is orthogonal.

Unitary matrix : A matrix A is said to be unitory if

$AA^{\theta} = A^{\theta}A = I$

$$A = \frac{1}{2}\begin{bmatrix} 1-i & 1+i \\ 1+i & 1-i \end{bmatrix}$$

Now $\quad A^{\theta} = (\overline{A})^{T} = \frac{1}{2}\begin{bmatrix} 1+i & 1-i \\ 1-i & 1+i \end{bmatrix}$

$$AA^{\theta} = \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

and $A^{\theta}A = I$

So, A is unitory matrix.

Q.4. If A is orthogonal matrix then which of the following are true?

(a) 2A is orthogonal

(b) A^2 is orthogonal

(c) $-A$ is orthogonal

(d) None of these

Q.5. The number of orthogonal matrix of order 5 whose entries are 0 & 1 only

(a) 5^2 (b) $5!$

(c) 120 (d) 0

Q.6. The matrix $M = \begin{bmatrix} \cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha \end{bmatrix}$ is a unitary matrix when α is

(a) $(2n+1)\dfrac{\pi}{2}, n \in Z$ (b) $(3n+1)\dfrac{\pi}{3}, n \in Z$

(c) $(4n+1)\dfrac{\pi}{4}, n \in Z$ (d) $(5n+1)\dfrac{\pi}{5}, n \in Z$

Q.7 Suppose A is idempotent matrix of order n , then which of the following is true?

(a) Tr(A) > n (b) |A| > n

(c) Tr(A) ∈ N (d) Tr(A) ∈ Z

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 3$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 3\times3$$

$$\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix} \quad 1$$

Q.8. Suppose A is involutory matrix of order n , then which of the following is true?

(a) $I - A$ is involutory (b) $3A$ is involutory

(c) $\text{Tr}(A)$ may be $n/2$ (d) $\text{Tr}(A) \in Z$

Q.9. If A is nilpotent matrix of index 2022, then matrix A^{2011} is

(a) nilpotent matrix of index 1

(b) nilpotent matrix of index 2

(c) nilpotent matrix of index 2022

(d) nilpotent matrix of index 2011

$P = 2022$

$\left(\frac{2022}{2011}\right) =$

(A^3) $= A^4 = 0$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left(\frac{4}{3}\right) = 1.33$$

$$\boxed{A^4 = 0}$$

$k = 4$

$$A^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$A^4 = 0$

Q.10. If A and B are orthogonal matrix then which of the following is true?

(a) $A + B$ is orthogonal (b) AB is orthogonal

(c) $2A$ is orthogonal (d) B^2 is orthogonal


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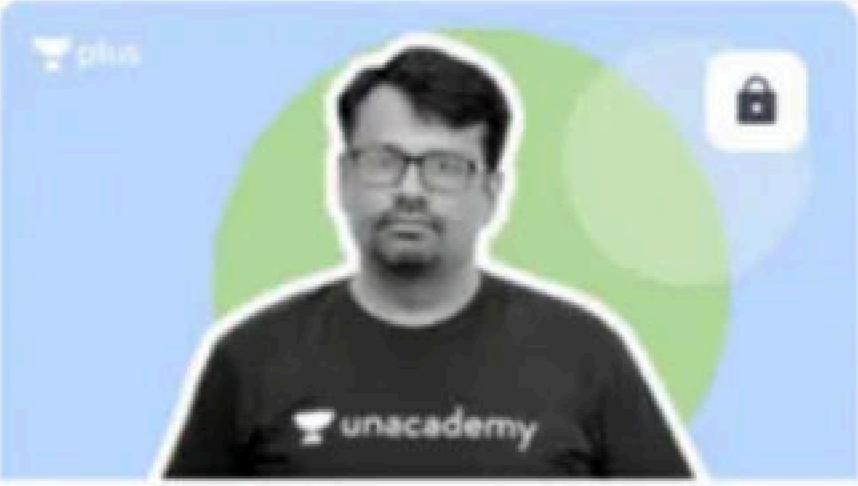

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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