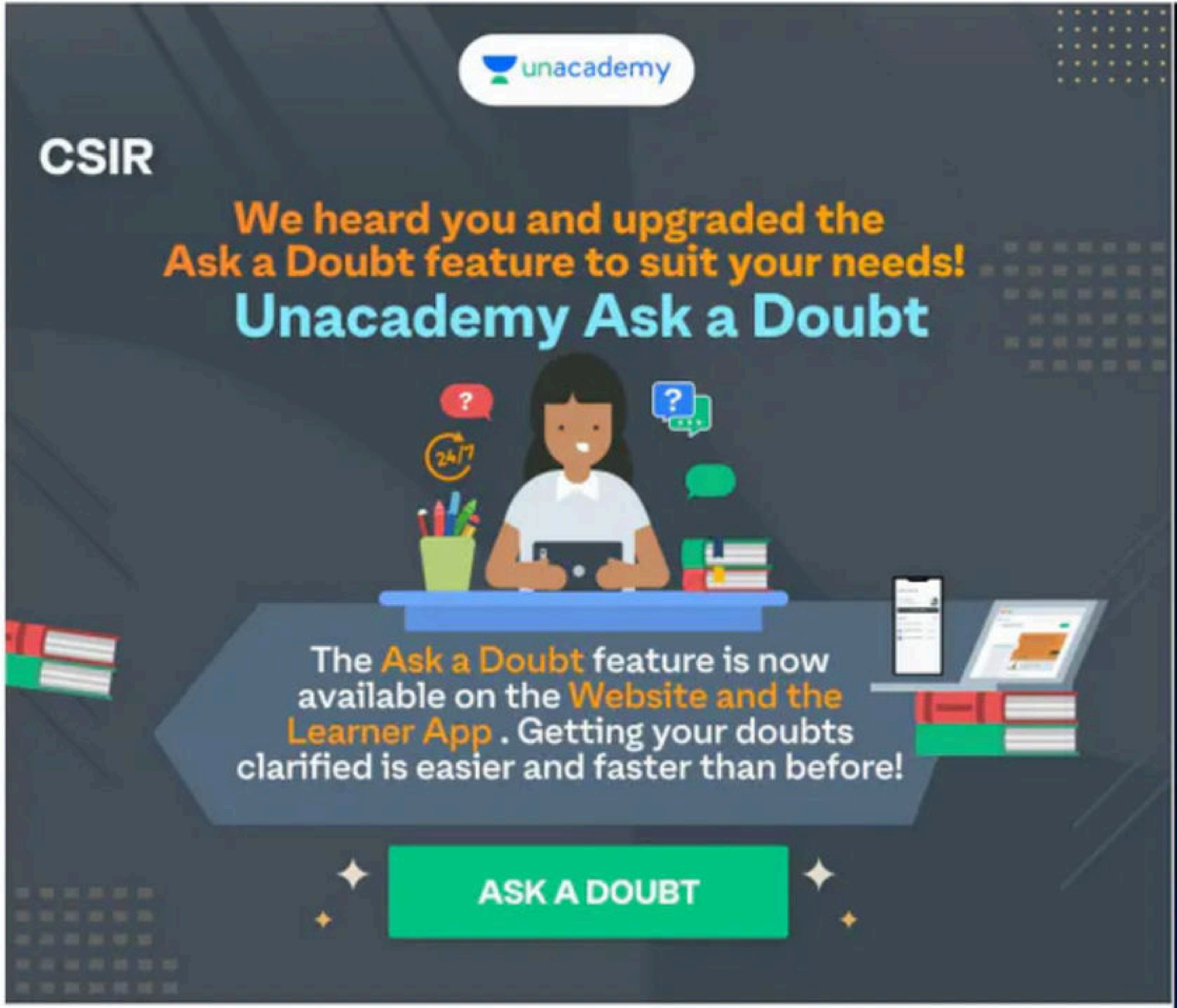
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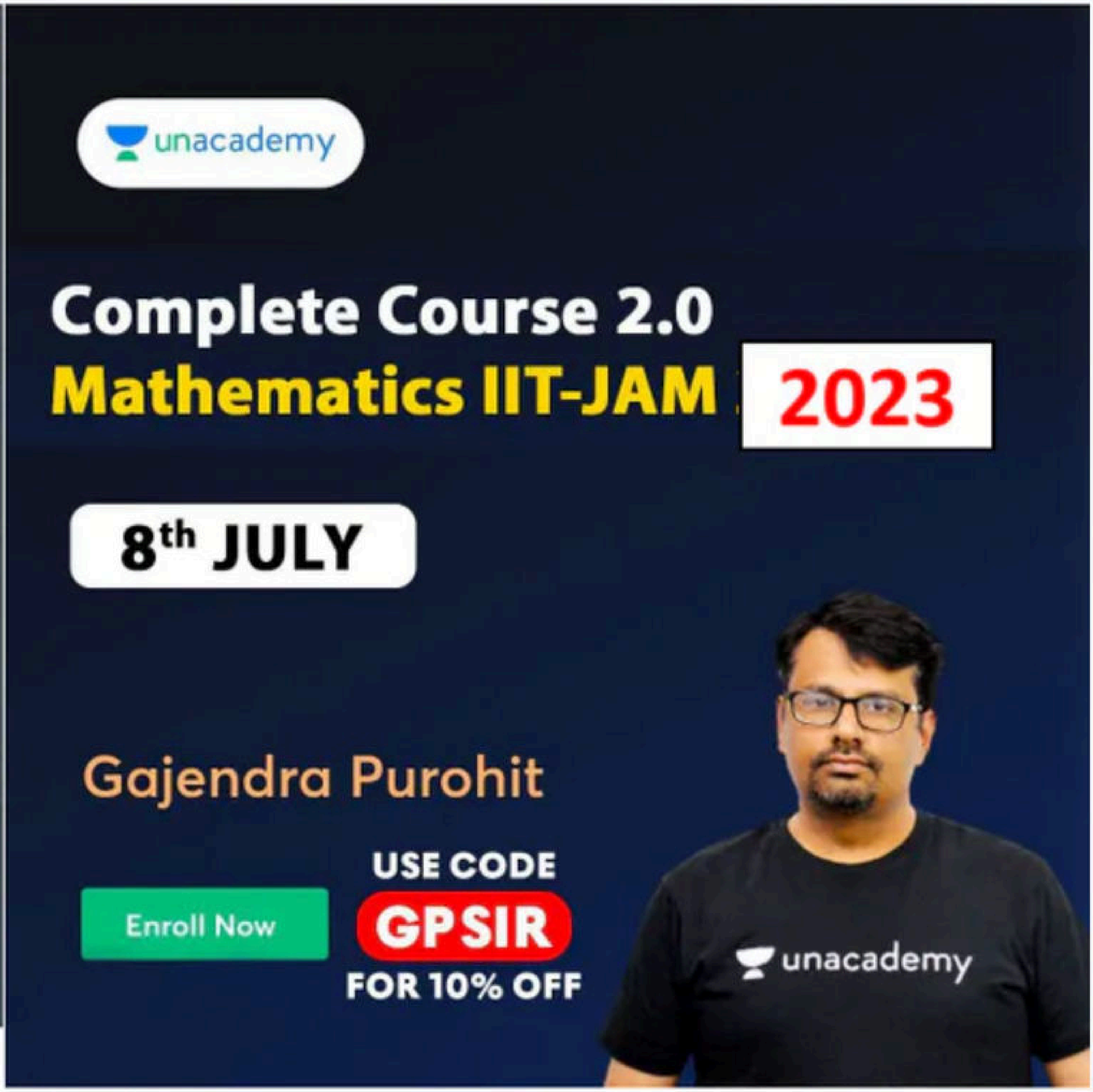
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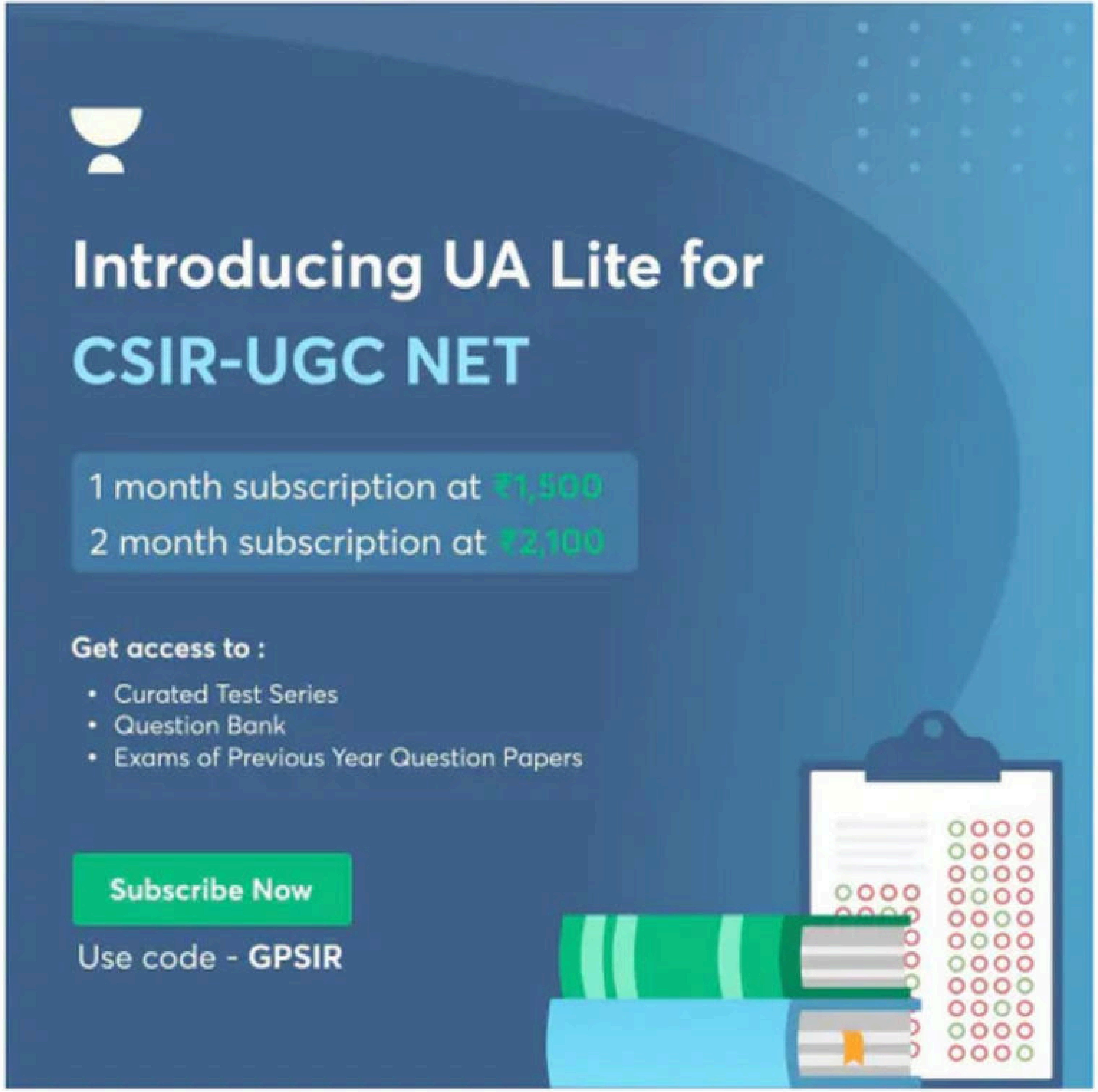
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Definite integral

Definition : If $\frac{d}{dx}[f(x)] = \phi(x)$ and a & b are constant, then

$$\int_a^b \phi(x)dx = [f(x)]_a^b = f(b) - f(a)$$

is called definite integration of $\phi(x)$ within limit a & b.

Note : This is also called fundamental theorem of calculus.

Basic properties of definite integrals.

(1) $$\int_a^b f(t)dt = \int_a^b f(x)dx$$

(2) $$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

(3) $$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

For any $c \in (a, b)$

(4) $$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$I = \int_0^{\pi/2} \frac{x \sin x}{\sin x + \cos x} dx$$

(5) $$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if } f(x) \text{ is even} \\ 0 & \text{if } f(x) \text{ is odd} \end{cases}$$

$$I = \int \frac{(0 + \pi/2 - x)\sin(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)} dx$$

(6) $$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx; & \text{if } f(2a-x) = f(x) \\ 0; & \text{if } f(2a-x) = -f(x) \end{cases}$$

$$I = \int \frac{(\pi/2 - x)\cos x}{\cos x + \sin x} dx$$

$\sin n\pi = 0$

$\cos n\pi = (-1)^n$

Q1. The value of the integral $\int_{-\pi}^{\pi} |x| \cos nx\,dx, n \geq 1$ is

$$= 2\int_0^{\pi} x \cos nx\,dx$$

$$= 2\left[x\int \cos nx\,dx - \int\left(\frac{d(x)}{dx}\int \cos nx\,dx\right)dx\right]_0^{\pi}$$

$$= 2\left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{\pi}$$

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(a) 0, when n is even

(b) 0, when n is odd

$$= 2\left[\left(0 + \frac{(-1)^n}{n^2}\right) - \left(0 + \frac{1}{n^2}\right)\right]$$

(c) $-\frac{4}{n^2}$, when n is even

(d) $-\frac{4}{n^2}$, when n is odd

$$= 2\left(\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right)$$

$$= 2\left(\frac{(-1)^n - 1}{n^2}\right)$$

n - even $\rightarrow 0$

n - odd $\quad -\frac{4}{n^2}$

Q.2. The value of $I = \int_{0}^{\pi/4} \log(1+\tan\theta)\,d\theta$ is

(a) $\frac{\pi}{8}$

(b) $\log 2$

(c) $\frac{\pi}{8}\log 2$

(d) $\frac{\pi}{8}\log 3$

$$I = \int_0^{\pi/4} \log\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$$

$$= \int_0^{\pi/4} \log\left(1+\frac{\tan\pi/4 - \tan\theta}{\tan\pi/4 + \tan\theta}\right)d\theta$$

$$I = \int_0^{\pi/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta$$

$$I = \int_0^{\pi/4} \log\left(\frac{1+\cancel{\tan\theta}+1-\cancel{\tan\theta}}{1+\tan\theta}\right)d\theta$$

$$I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right)d\theta$$

$$2I = \int_0^{\pi/4}\left(\log(1+\tan\theta) + \log\left(\frac{2}{1+\tan\theta}\right)\right)d\theta$$

$$2I = \int_0^{\pi/4} \log 2\, d\theta$$

$$I = \frac{\log 2}{2}(\theta)_0^{\pi/4} = \frac{\pi \log 2}{8}$$

Q.3. Evaluate : $\displaystyle\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$

(a) $\dfrac{\pi^2}{2a^2b^2}$ (b) $\dfrac{\pi^2}{2ab}$

(c) $\dfrac{2\pi^2}{a^2b}$ (d) π^2

$$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}$$

$$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$2I = \int_0^{\pi} \frac{(x + \pi - \cancel{x})\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$I = \cancel{2}\frac{\pi}{\cancel{2}} \int_0^{\pi/2} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x} \Rightarrow I = \frac{\pi}{\cancel{2}} \int_0^{\pi/2} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$$

$b\tan x = t$

$\sec^2 x\,dx = \dfrac{dt}{b}$

$$I = \pi \int_0^{\infty} \frac{dt/b}{a^2 + t^2} = \frac{\pi}{b}\left(\frac{1}{a}\tan^{-1}\frac{t}{b}\right)_0^{\infty}$$

$$= \frac{\pi}{ab}\left[\tan^{-1}\infty - \tan^{-1}0\right] = \frac{\pi^2}{2ab}$$

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Definite integral as the limit of a sum :

$$\int_0^1 f(x)dx = \lim_{n\to\infty} \frac{1}{n}\sum_{r=0}^{n} f\left(\frac{r}{n}\right)$$

Where f(x) is continuous function on closed interval [0, 1]

$$\sum_{k=1}^{2n} \frac{1}{k+n}$$

$$\frac{1}{n}\sum_{k=1}^{n} \frac{1}{\frac{k}{n}+1}$$

$$\int_0^2 \frac{1}{x+1}dx$$

Q.4. The limit of sum $\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{3n}$, when $n\to\infty$ is

(a) log 3 (b) log2

(c) 0 (d) 1

$$\frac{1}{n}+\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{1}{k+n}$$

$$\int_0^2 \frac{1}{1+x}\,dx$$

$$\left(\log(1+x)\right)_0^2$$

$$\log 3-\log 1$$

$$\log 3$$

Q5. The limit when $n \to \infty$ of the product

$$A = \left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\ldots\ldots\left(1+\frac{n}{n}\right)\right\}^{1/n}$$

(a) 2/e (b) 4/e

(c) 5/e (d) e

$\log A = \frac{1}{n}\left(\log\left(1+\frac{1}{n}\right) + \log\left(1+\frac{2}{n}\right) + \log\left(1+\frac{3}{n}\right) + \cdots \log\left(1+\frac{n}{n}\right)\right)$

$\log A = \frac{1}{n}\sum_{r=1}^{n} \log\left(1+\frac{r}{n}\right)$

$\log A = \int_0^1 1.\log(1+x)\,dx$

$\log A = \left(\log(1+x)\int 1\,dx - \int\left(\frac{d}{dx}\log(1+x)\int 1\,dx\right)dx\right)_0^1$

$\log A = \left(x\log(1+x) - \int\frac{x}{1+x}\,dx\right)_0^1$

$\log A = \left[x\log(1+x) - \int\frac{1+x-1}{1+x}\,dx\right]_0^1$

$\log A = \left(x\log(1+x) - (x - \log(1+x))\right)_0^1$

$\log A = (\log 2 - 1 + \log 2)$

$\log A = (2\log 2 - 1) = \log 2^2 - \log_e e$

$\log A = \log 4 - \log_e e \Rightarrow \log A = \log\frac{4}{e}$ | $A = 4/e$

Q6. The value : $\lim_{n\to\infty}\left[\frac{n!}{n^n}\right]^{1/n}$ is

(a) 1/e (b) 2/e

(c) 3/e (d) 4/e

$$A = \left(\frac{n(n-1)(n-2)\cdots 3\cdot 2\cdot 1}{n\cdot n}\right)^{1/n}$$

$$A = \left(\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{(n-1)}{n}\cdot\frac{n}{n}\right)^{\frac{1}{n}}$$

$$\log A = \frac{1}{n}\log(\_\_\_ + \_\_\_)$$

$$\log A = \frac{1}{n}\left[\log\frac{1}{n} + \log\frac{2}{n} + \cdots + \log\frac{n}{n}\right]$$

$$\log A = \frac{1}{n}\sum_{r=1}^{n}\log\frac{r}{n}$$

$$\log A = \int_0^1 1\cdot\log x\,dx$$

$$\log A = \left(\log x\int 1\cdot dx - \int\left(\frac{d}{dx}(\log x)\int dx\right)dx\right)_0^1$$

$$\log A = \left[x\log x - \int\frac{1}{x}\cdot x\,dx\right)_0^1$$

$$\log A = (x\log x - x)_0^1$$

$$\log A = (0-1) - (0-0) = -1$$

$$A = e^{-1}$$

Gamma Function:

If m and n are non-negative integers, then

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$

where $\Gamma(n)$ is called gamma function which satisfied the following properties

$$\Gamma(n+1) = n\Gamma(n) = n! \quad i.e.\ \Gamma(1) = 1 \text{ and } \Gamma(1/2) = \sqrt{\pi}$$

In place of gamma function, we can also use the following formula :

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{(m-1)(m-3).....(2 \text{ or } 1)(n-1)(n-3).....(2 \text{ or } 1)}{(m+n)(m+n-2)....(2 \text{ or } 1)}$$

It is important to note that we multiply by $(\pi/2)$; when both m and n are even.

Q.7. Evaluate $\int_0^\infty x^{1/4} e^{-\sqrt{x}}\, dx$

(a) $2\sqrt{\pi}$

(b) $\frac{3}{2}\pi$

(c) $\sqrt{\pi}$

(d) $\frac{3}{2}\sqrt{\pi}$

Q.8. Let a,b be positive real numbers such that a < b Given that $\lim\limits_{n\to\infty}\int_0^n e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}$ Then value of $\lim\limits_{n\to\infty}\int_0^n \frac{1}{t^2}\left(e^{-at^2} - e^{-bt^2}\right)dt$ is

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(a) $\sqrt{\pi}\left(\sqrt{b} - \sqrt{a}\right)$

(b) $\sqrt{\pi}\left(\sqrt{b} + \sqrt{a}\right)$

(c) $-\sqrt{\pi}\left(\sqrt{b} - \sqrt{a}\right)$

(d) $\sqrt{\pi}\left(-\sqrt{b} + \sqrt{a}\right)$

Q9. The value of $\int_0^{\pi/2} \sin^4 x \cos^6 x dx$

(a) $3\pi/312$

(b) $5\pi/512$

(c) $3\pi/512$

(d) $5\pi/312$

Q10. If $I_n = \int_0^\infty e^{-x} x^{n-1} dx$, then $\int_0^\infty e^{-\lambda x} x^{n-1} dx$ is equal to

(a) λI_n

(b) $\frac{1}{\lambda} I_n$

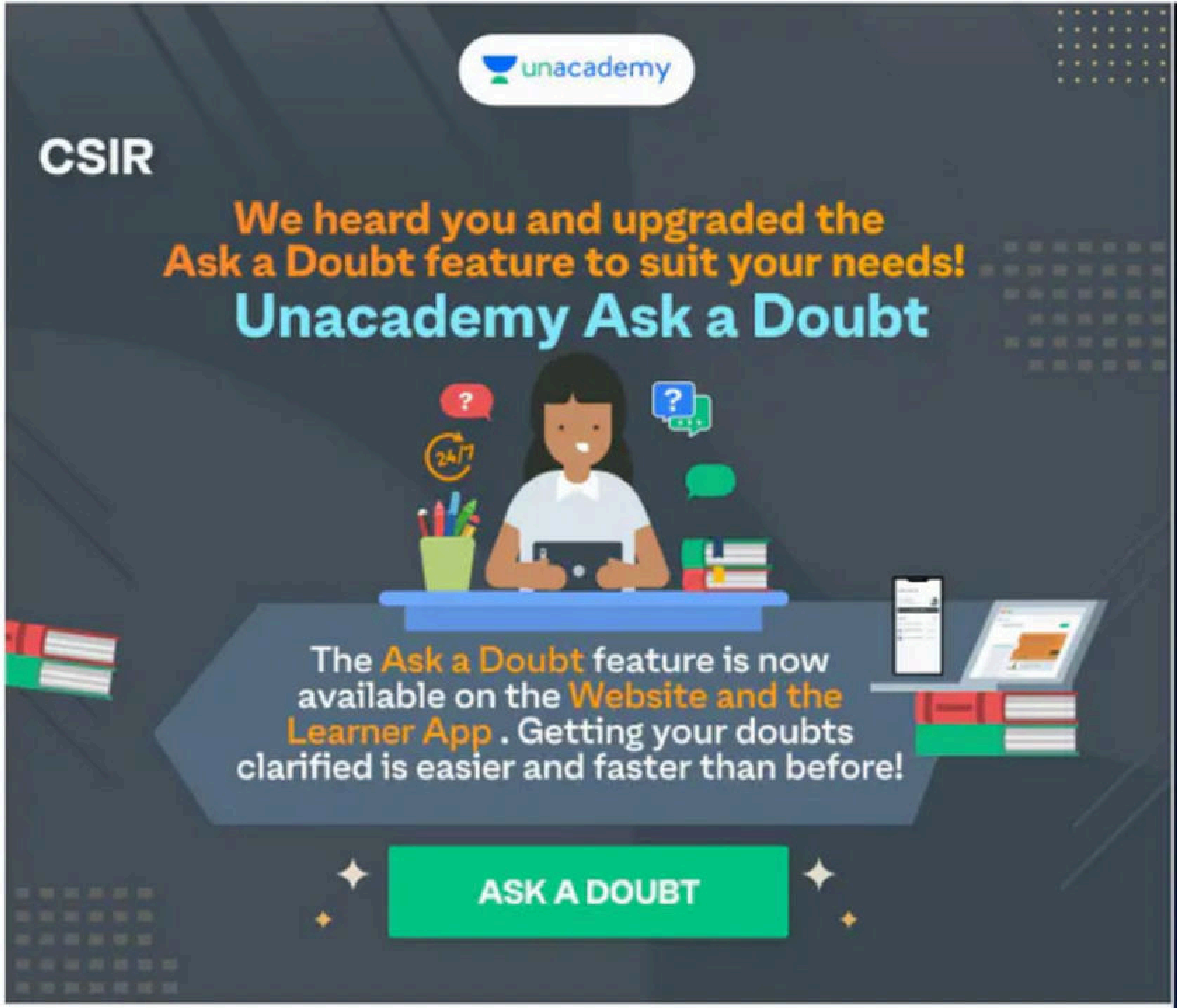
(c) $\frac{I_n}{\lambda^n}$

(d) $\lambda^n I_n$

Q11. $\int_0^{\pi/2} \sin^7 x dx$ has value

(a) $\frac{37}{184}$ (b) $\frac{17}{45}$

(c) $\frac{16}{35}$ (d) $\frac{16}{45}$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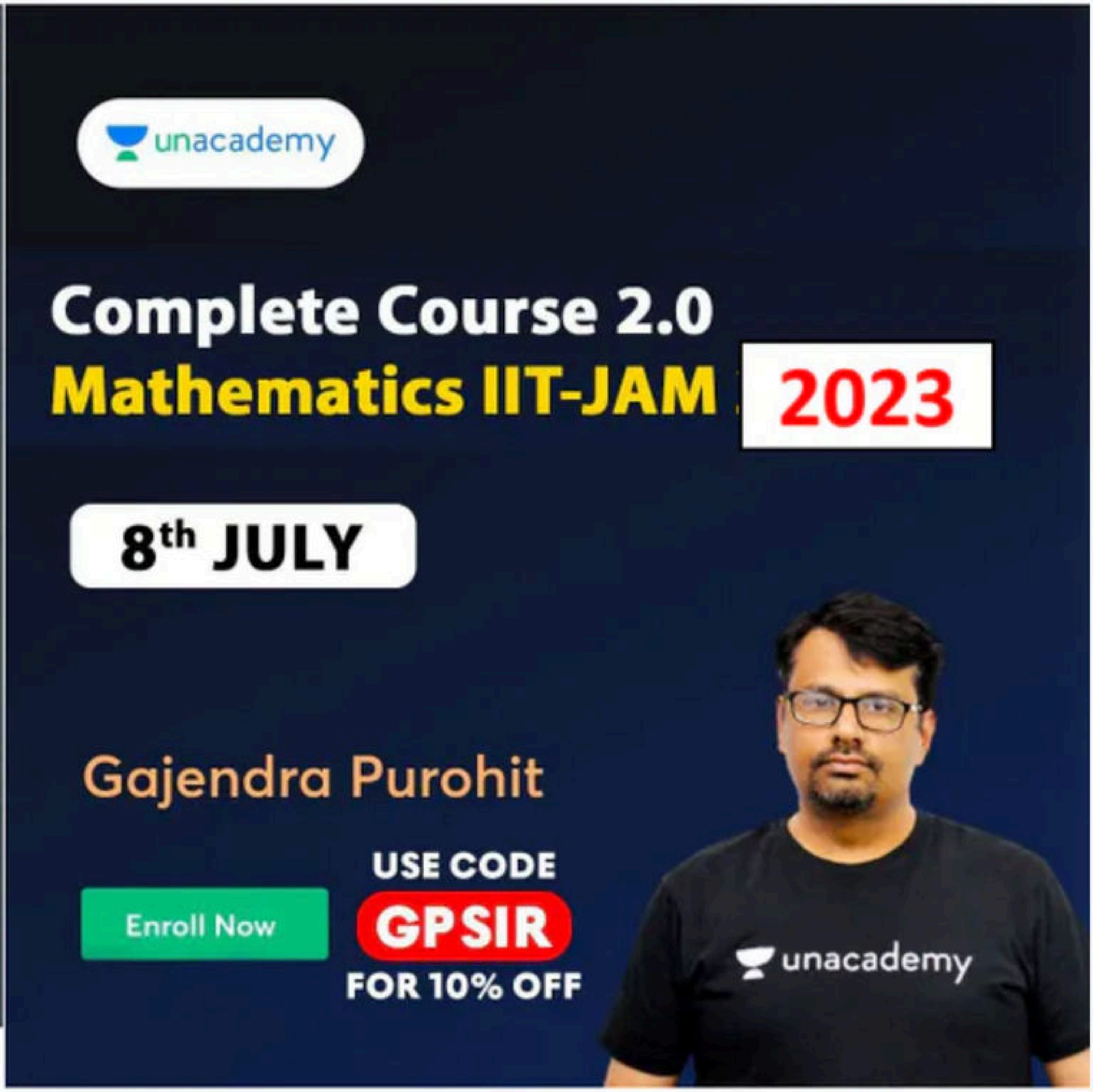
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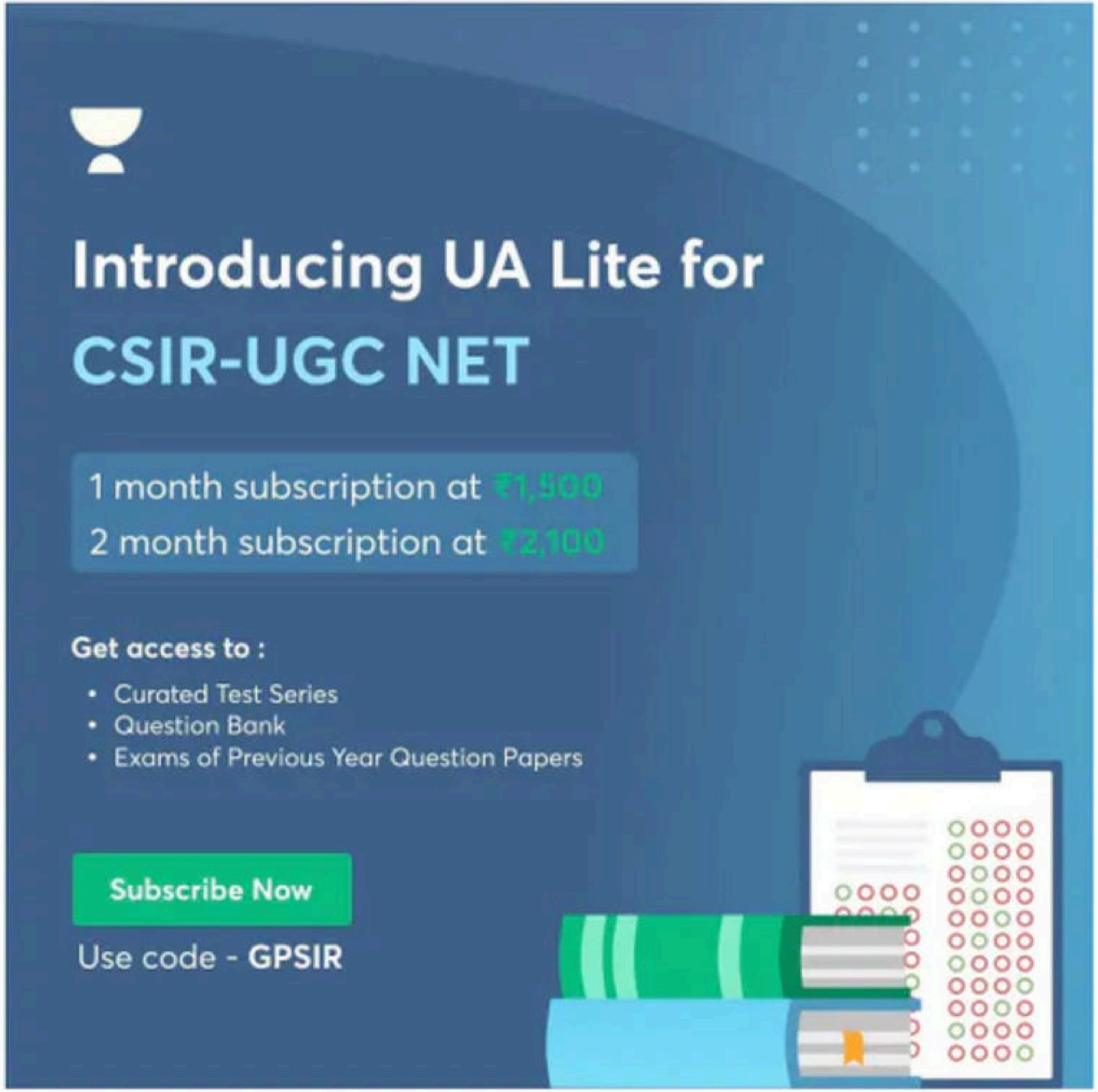

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