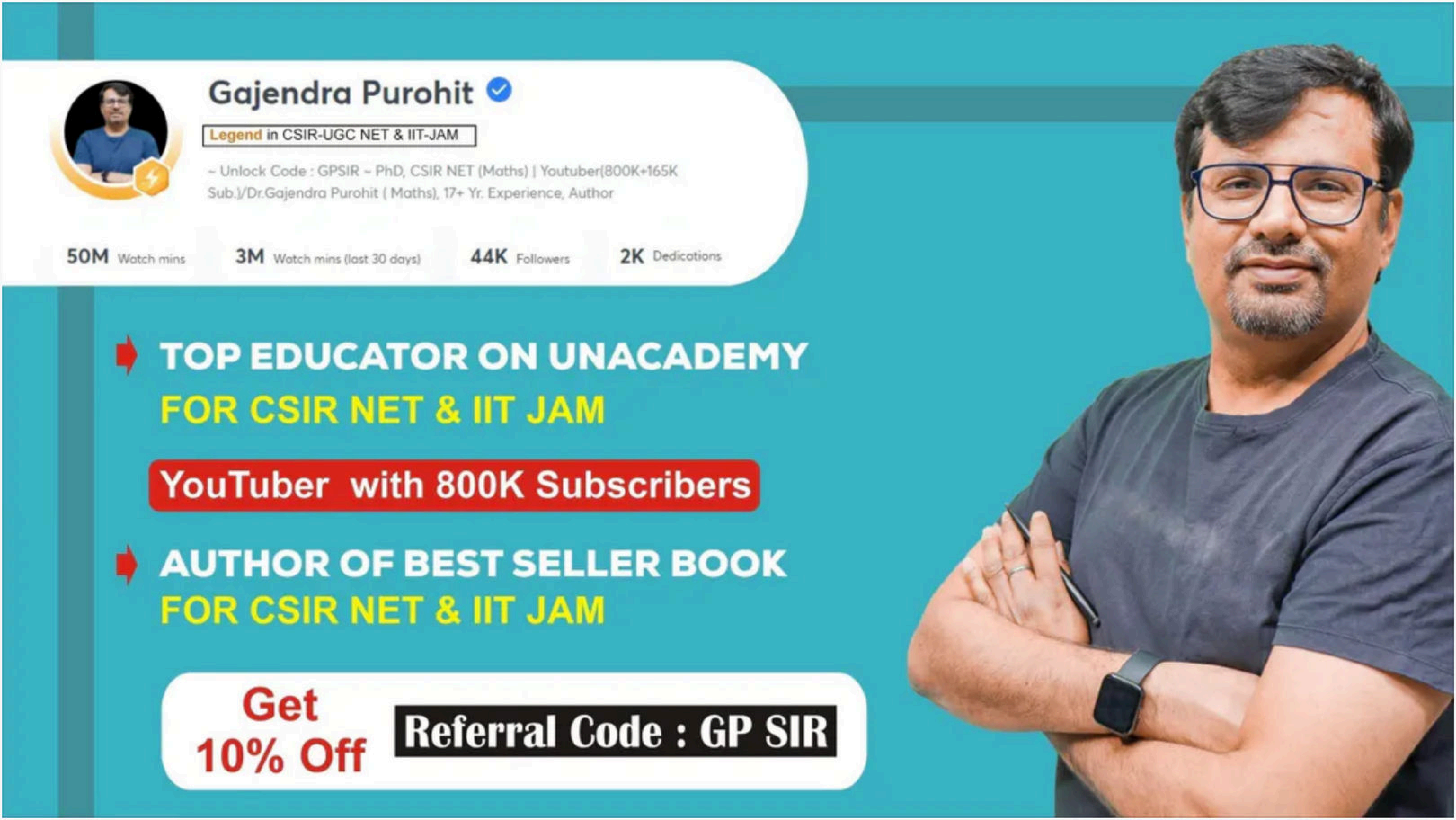
Continuity - Part II

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 3 • Nov 12, 2022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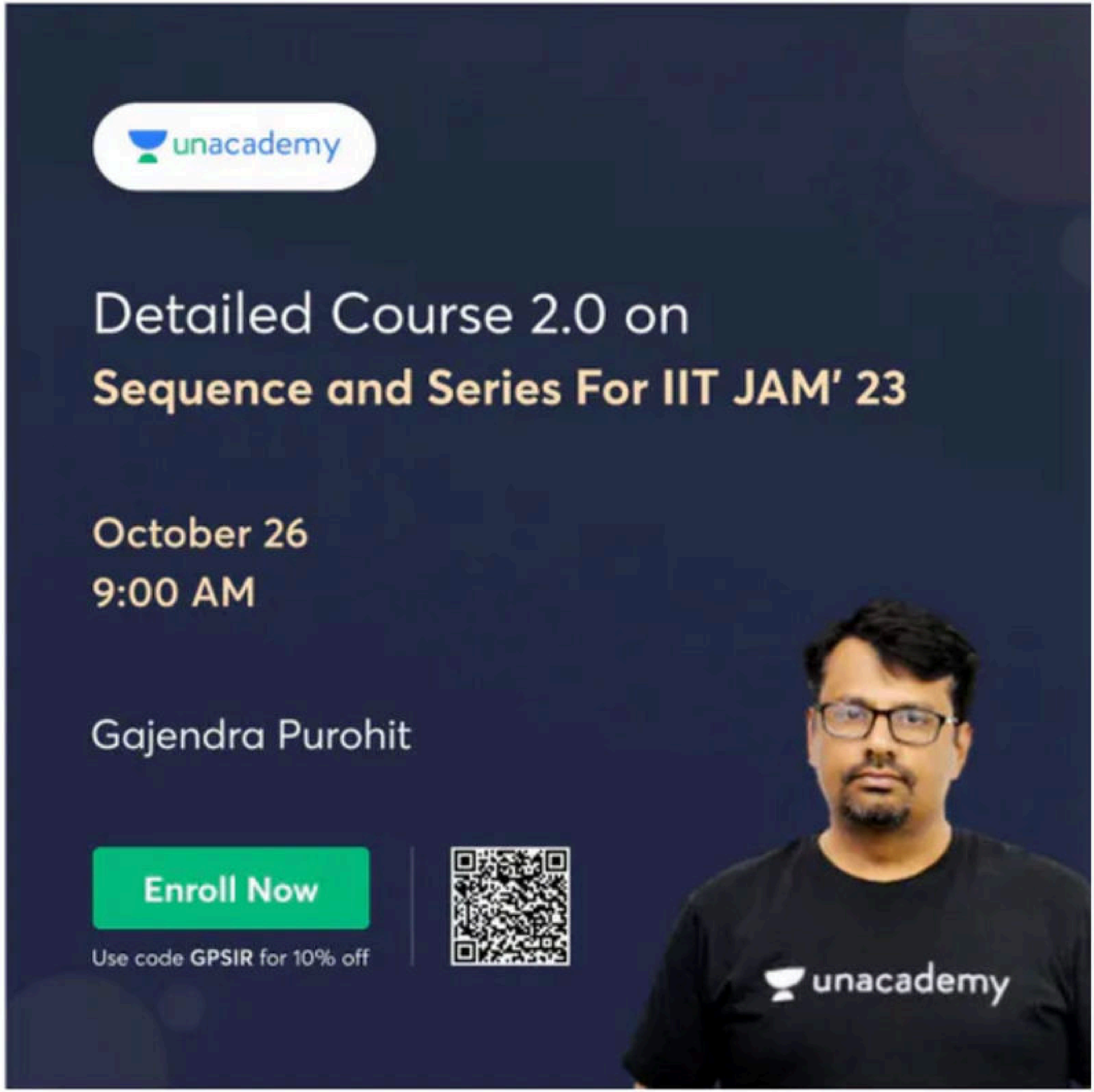
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Continuity

Continuous function at a point 'a' :

Let $f : A \to R$ be a real valued function. This function is continuous at $x = \alpha \in A$

if $\lim_{x \to \alpha} f(x) = f(\alpha)$.

Continuity of some important function :

(1) **Polynomial function :** Let $f(x) = a_n x^n + ... a_1 x + a_0$ then $\lim_{x\to\alpha} f(x) = f(\alpha) . \Rightarrow$ f(x) is continuous at any point.

(2) **Constant function :** Let f(x) = α.

Here $\lim_{x\to a} f(x) = f(\alpha) = \alpha$.

Constant function is always continuous at a.

(3) **Rational function :**

Let p(x) and q(x) are polynomial, then $f(x) = \frac{p(x)}{q(x)}$ is called rational function.

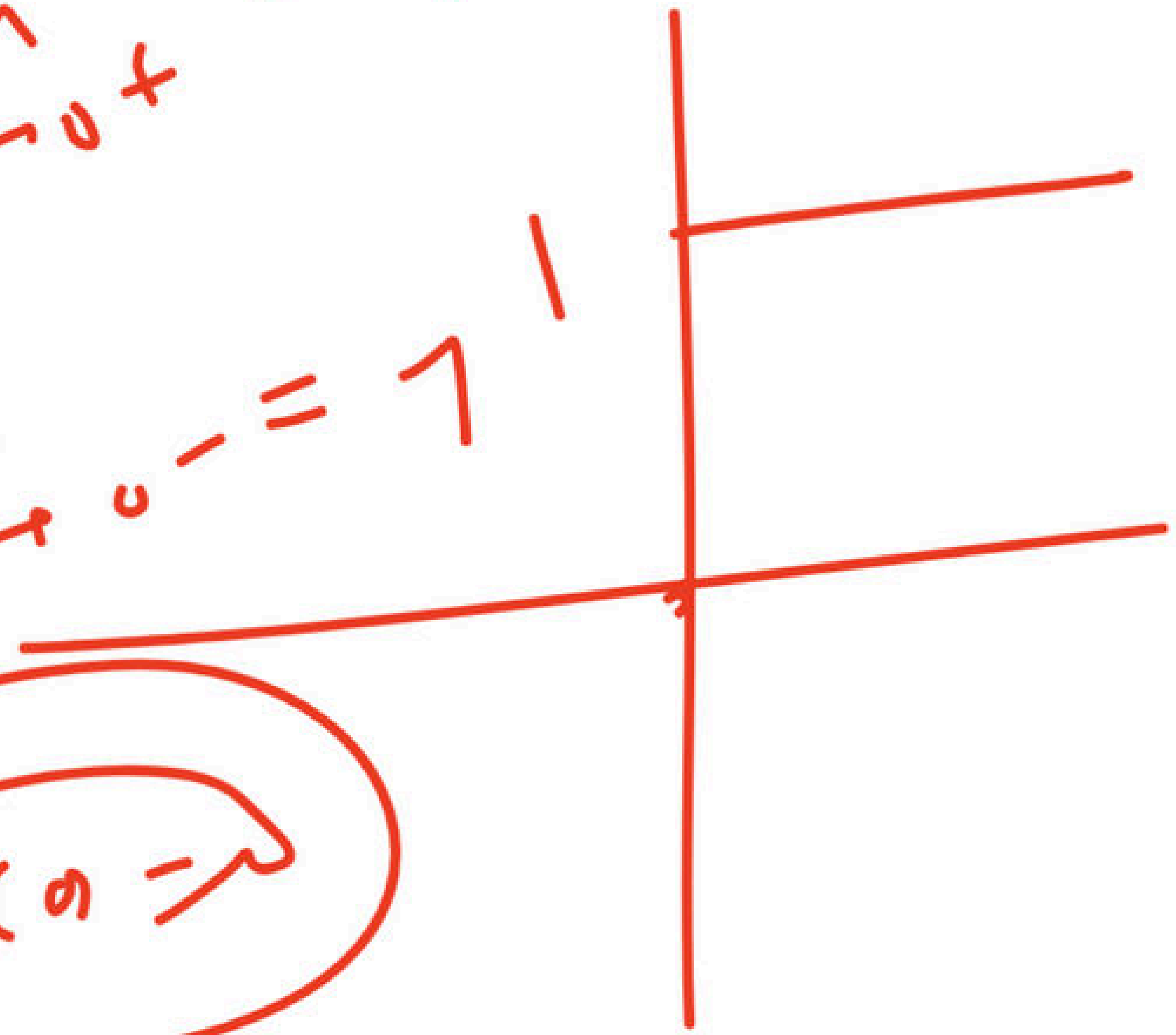
Hence f(x) is continuous at c, if $q(c) \neq 0$

(4) Trigonometry function :

(a) $f(x) = \sin x$

$\lim_{x \to 0^+} = 1$

$\lim_{x \to 0^-} = -1$



$f(x) =$

(5) Exponential function :

(6) Logarithmic function :

Sequential definition : Let f : A → R be a function and f is continuous at 'c' iff for every sequence $<x_n>$ converging to c, then $<f(x_n)>$ converging to f(c).

i.e. if ∃ two sequence $<x_n>$ and $<y_n>$ s.t. $<x_n> \to c$ and $<y_n> \to c$.

but if $<f(x_n)>$ and $<f(y_n)>$ converging to distinct limit then f(x) is not continuous at c.

$x = 0$

$f(x) = \sin \frac{1}{x}$

$<x_n> = <\frac{1}{n\pi}> \longrightarrow 0$

$<y_n> = <\frac{1}{(4n+1)\frac{\pi}{2}}> \longrightarrow 0$

$<f(x_n)> = <\sin n\pi> = 0$

$<f(y_n)> = <\sin(4n+1)\frac{\pi}{2}> = 1$

Result :

Let f : R → R is a function defined by

$$f(x)=\begin{cases} g(x), & x\in Q \\ h(x), & x\in Q^C \end{cases}$$

$g(x) = h(x)$

Then f(x) is continuous at all zeros of g(x) – h(x) = 0.

$$f(x) = \begin{cases} x^2-1 & x \in Q \\ 2x-1 & x \in Q^c \end{cases}$$

$x^2 - \cancel{1} = 2x - \cancel{1}$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

Q.1. Let $f : R \to R$ be defined by $f(x) = \begin{cases} x^6 - 1 & x \in Q \\ 1 - x^6 & x \in Q^C \end{cases}$.

The number of points at which f(x) is continuous is

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(a) 1 (b) 2

(c) 3 (d) 4

$x^6 - 1 = 1 - x^6$

$2x^6 - 2 = 0$

$2(x^6 - 1) = 0$

$2(x^3 - 1)(x^3 + 1) = 0$

$x = 1, -1$

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Result : Let f : [0 ,1] → R is a function such that

$$f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$$

Then f(x) is continuous iff α > 0

$\alpha > 0$

$x^{\alpha} \sin\frac{1}{x}$

$\lim_{x\to 0}$ $0 \times (-1,1) = 0$

$x^{0.000001} \sin\frac{1}{x}$

Q For what value of α, the function

$$f(x) = \begin{cases} x^{\alpha} \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$ is cont.

(a) $1/2$

(b) 0

(c) -1

(d) None

Q.2. Which of the follwing values of α , the function

0

$$f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x}, & x\neq 0 \\ 0 & x=0 \end{cases}$$ Not is continuous

(a) 1/2 (b) 1

(c) 2 (d) -1/3

Q.3. Let the function f(x) be defined by

$$f(x)=\begin{cases} e^{x} & x \text{ is rational} \\ e^{1-x} & x \text{ is irrational} \end{cases} \text{ for x in } (0,1) \text{ then}$$

$e^x = e^{1-x}$

$x = 1-x$

$2x = 1$

$x = 1/2$

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(a) f is continuous at every point in (0,1)

(b) f is discontinuous at every point in (0,1)

(c) f is discontinuous at only one point in (0,1)

(d) f is continuous at only one point in (0,1)

DISCONTINUITIES : If a function is not continuous then it is called discontinuous

Type of Discontinuity :

(i) Removable Discontinuity

A function f(x) is said to have a discontinuity of removable kind at x = a if $\lim_{x \to a} f(x)$ exist but not equal to the value of function at x = a i.e., f(a + 0) = f(a – 0) ≠ f(a). Or $\lim_{x \to a} f(x) \neq f(a)$

(ii) Discontinuity of First kind / Jump Discontinuity

A function f(x) is said to have a discontinuity of first kind at x = a if both $f(a-0)$ and

$f(a+0)$ exist but are unequal. The point x = a is said the point of discontinuity of first kind

i.e. $f(a-0) \neq f(a+0)$

It is also known as ***ordinary discontinuity***.

1

$$\begin{cases} x^2+1 & x<0 \\ 2x+5 & x\geq 0 \end{cases}$$

5

(iii) Discontinuity of Second kind

A function f(x) is said to have a discontinuity of second kind at x = a if none of the limit $f(a - 0)$ and $f(a + 0)$ exist at x = a. The point x = a is the point of discontinuity of second kind.

LHL $\neq$ RHL

$\lim \frac{1}{x}$ $x \neq 0$

0 $x = \infty$

(iv) Mixed Discontinuity

A function f(x) is said to have a discontinuity of mixed kind at x = a if f(x) has a discontinuity of second kind on one side of a and on the other side it has discontinuity of first kind or may be continuous.

$\sin$

L.H.L

$\lim_{x \to 0^-} e^{\frac{1}{x}} \sin\frac{1}{x} = e^{-\infty}(-1,1) = 0$

$e^{\frac{1}{x}} \sin\frac{1}{x}$

$x \neq 0$

$x = 0$

$\lim_{x \to 0^+} e^{\frac{1}{x}} \sin\frac{1}{x} = e^{\infty}(-1,1) = \infty$

(v) Infinite Discontinuity

A function f(x) at x = a is said to have discontinuity of infinite kind if f(a + 0) or f(a – 0) is ∞ or $-\infty$.

$f(x) = \frac{1}{x}$ $\quad x = 0$

Q.4. Let F : R → R be a monotone function. Then

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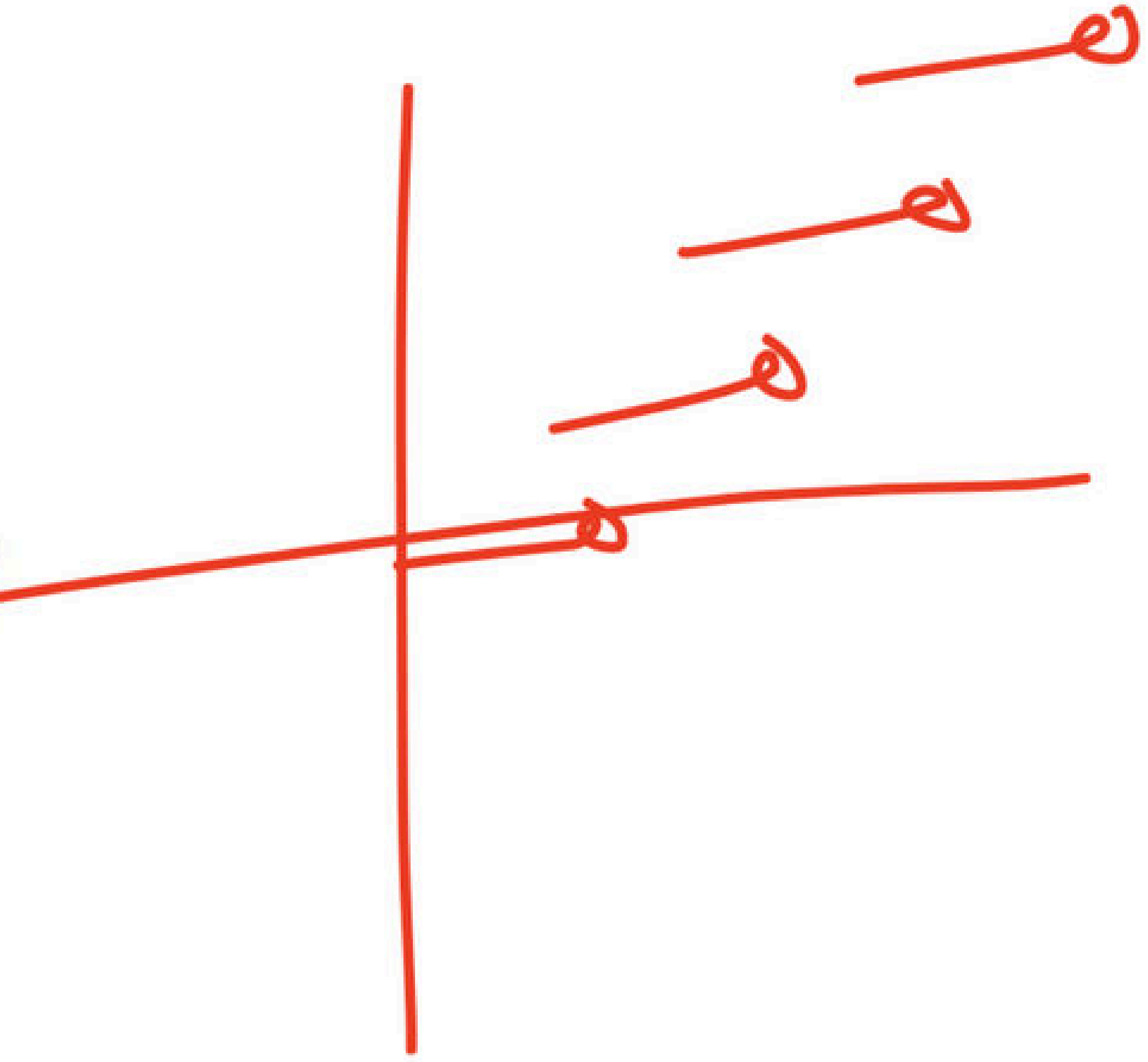
(a) F has no discontinuities

(b) F has only finitely many discontinuities

(c) F can have at most countably many discontinuities

(d) F can have uncountably many discontinuities.



Q.5. Let $f : R \to R$ be a strictly increasing continuous function. If $\{a_n\}$ is a sequence in $[0, 1]$ then the sequence $\{f(a_n)\}$ is

(a) Increasing

(b) Bounded

(c) Convergent

(d) Not necessarily bounded

$$\langle a_n \rangle = \begin{cases} 0 & n\text{-even} \\ \frac{1}{n} & n = \text{odd} \end{cases}$$

$f(x) = x$

$$f(a_n) = \begin{cases} 0 & n\text{-odd} \\ \frac{1}{n} & n\text{-even} \end{cases}$$

$f(x) = x$

$f(x_n) = \langle \frac{1}{n} \rangle$

Q.6. Consider the function f : R → R defined by

$$f(x)=\begin{cases} 2x & if\ x\in Q \\ 3-x & if\ x\in Q^C \end{cases}.$$

2x = 3 - x

3x = 3

x = 1

Then f is continuous at

(a) x = 1 (b) x = 2

(c) x = 0 (d) not continuous

Q.8. Let f be a monotone non-decreasing real-valued function on R. Then

(a) $\lim\limits_{x \to a} f(x)$ exists at each point a.

(b) If a < b, then $\lim\limits_{x \to a^+} f(x) \leq \lim\limits_{x \to b^-} f(x)$.

(c) f is an unbounded function

(d) The function $g(x) = e^{-f(x)}$ is a bounded function.

Q.9. Let p be a real polynomial of the real variable x of the form $p(x) = x^n + a_{n-1}x^{n-1} + \ldots.. + a_1x + 1$. Suppose that p has no roots in the open unit disc and $p(-1) = 0$. Then

(a) $p(1) = 0$

(b) $\lim_{x \to \infty} p(x) = \infty$

(c) $p(2) > 0$

(d) $p(3) = 0$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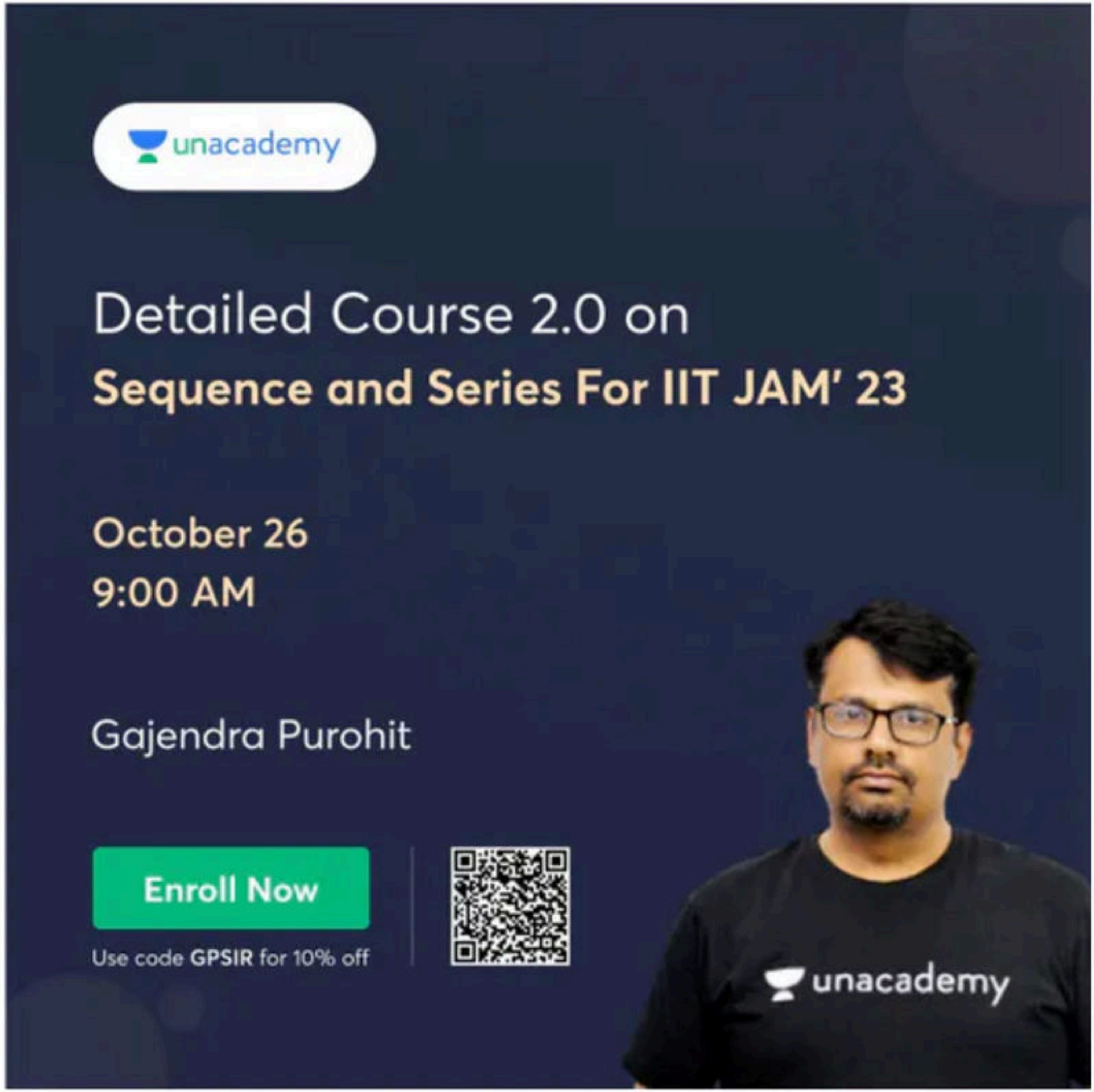
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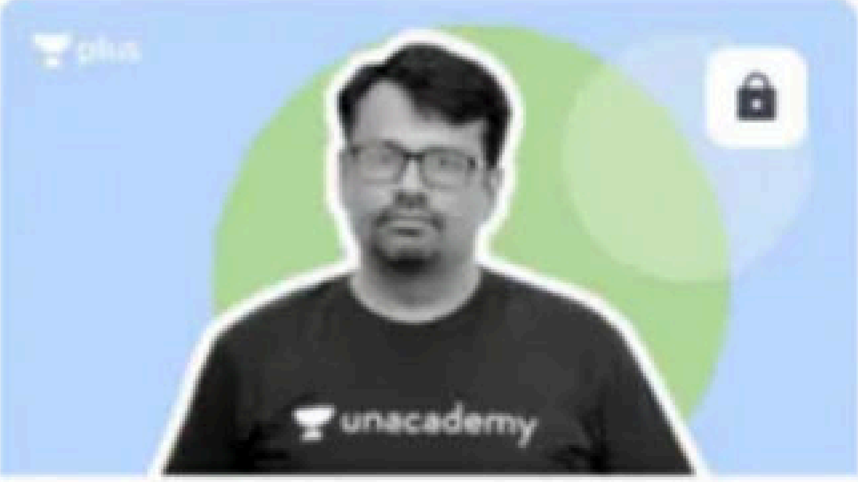

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