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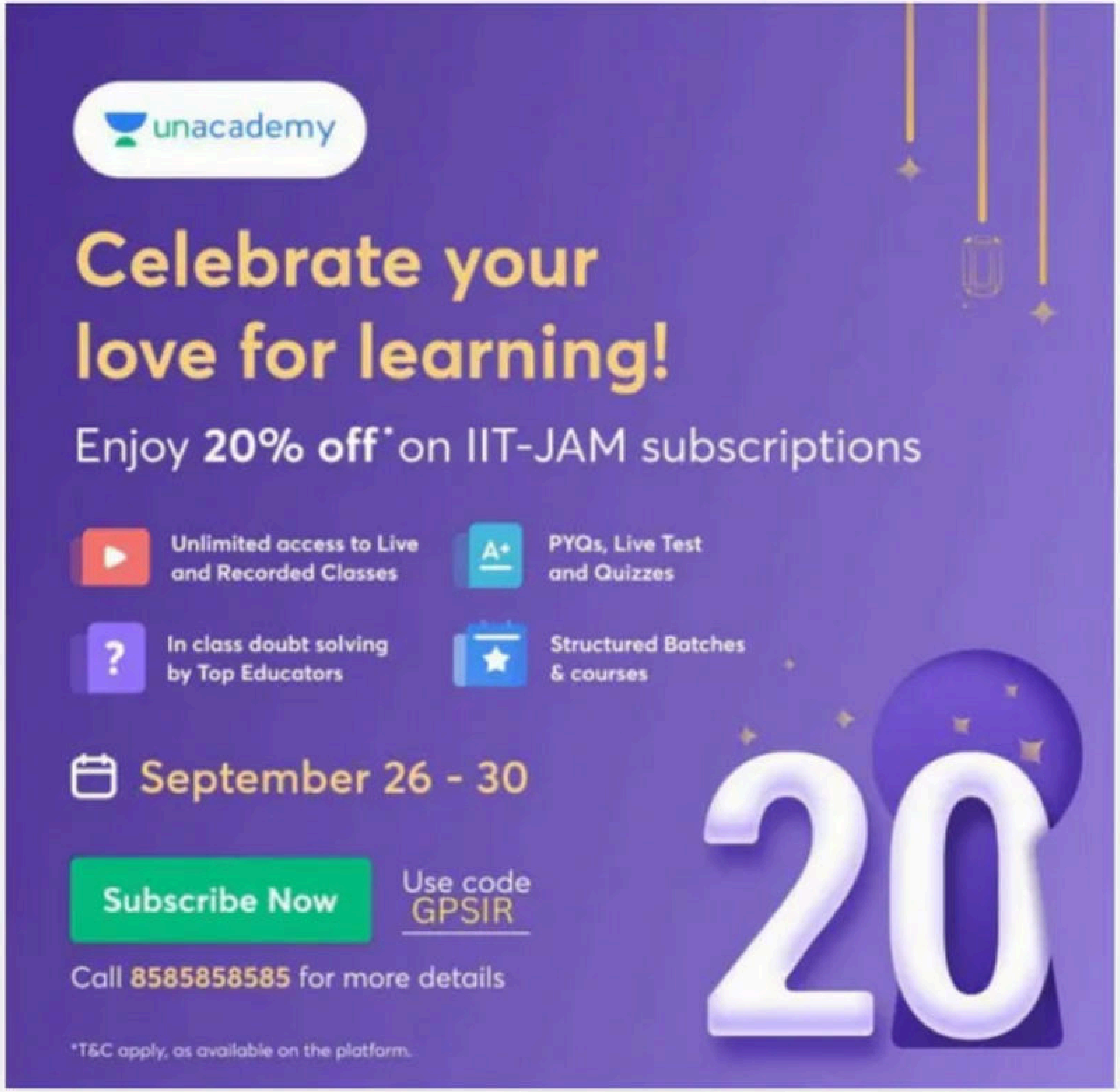
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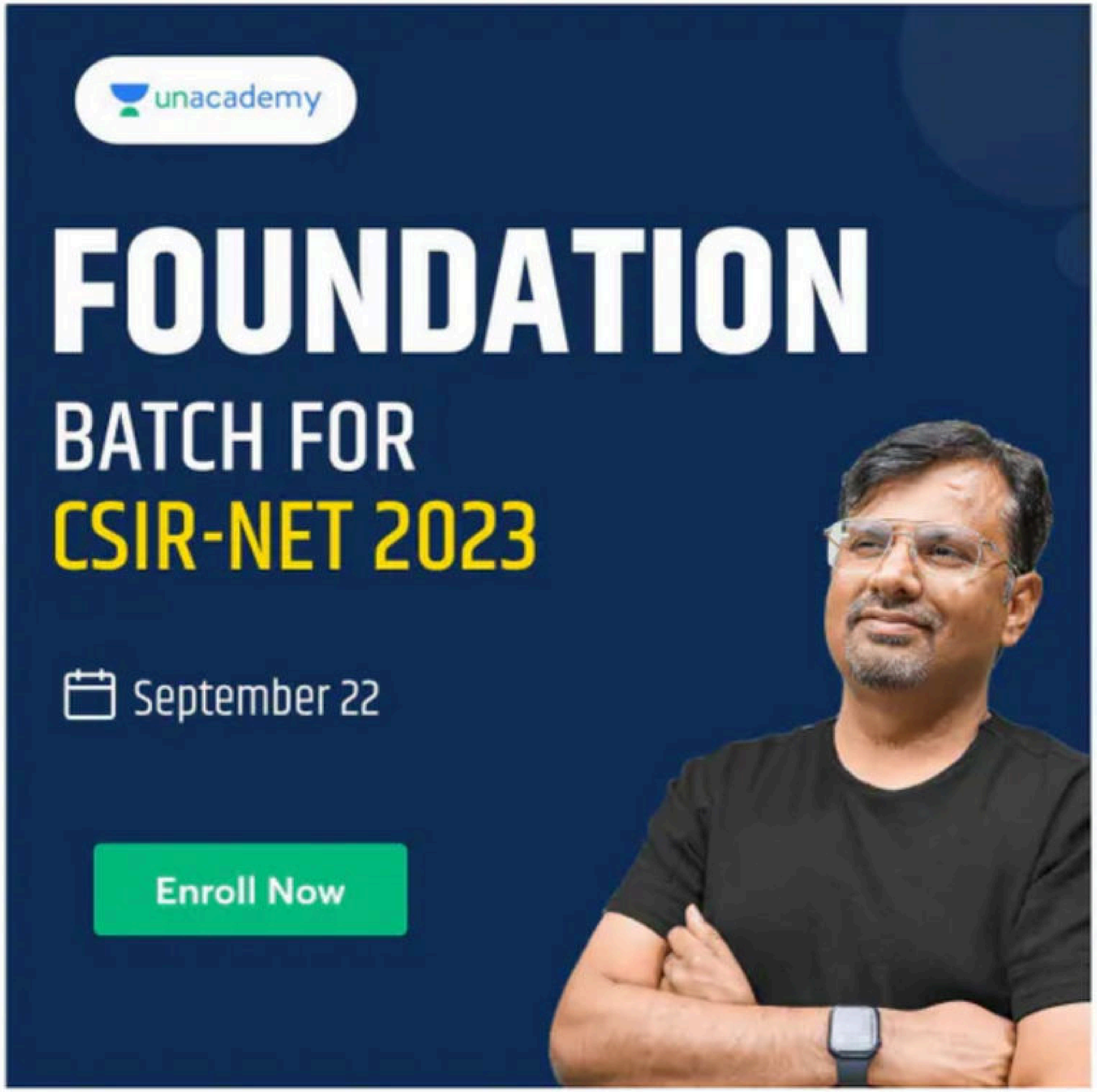
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~~One-One linear transformation~~ : Let $T : V \to V'$ be a linear transformation with $\eta(T) = 0$ then T is called one-one linear transformation.

$\eta(T) = 0$

$\eta(T) = 0$

$\rho(T) = \dim V'$

Onto linear transformation : Let $T : V \to V'$ be a linear transformation with $\rho(T) = \dim V'$ Then T is called onto linear transformation.

$$\begin{bmatrix} 2 & 3 & 1 \\ 5 & 7 & 2 \end{bmatrix}$$

2×3

$\rho(T) \le 2$

$\boxed{\eta(T) \ge 1}$

$\boxed{\eta(T) + \rho(T) = \dim V}$

$\boxed{T : R^3 \to R^2}$

$$\begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 1 & 6 \end{bmatrix}$$

3×2

$\boxed{T : R^2 \to R^3}$

$\rho(T) = 2$

Singular and non-singular linear transformation :

A linear transformation $T : V \to V'$ is called singular linear transformation if $\eta(T) \geq 1$ and if $\eta(T) = 0$ then T will be non-singular.

$T : R^3 \to R^2$

2×3

Matrix representation : Let V(F) be an n-dimensional vector space and V'(F) be an m-dimensional vector space over F.

Let $\beta_1 = \{x_1, x_2, ..., x_n\}$ & $\beta_2 = \{y_1, y_2, ..., y_m\}$ are ordered basis of V(F) & V'(F) respectively and T : V(F) → V'(F) be a linear transformation s.t.

$$T(x_1) = a_{11}y_1 + a_{21}y_2 + \ldots . a_{m1}y_m$$
$$T(x_2) = a_{12}y_1 + a_{22}y_2 + \ldots . a_{m2}y_m$$
$$\vdots$$
$$T(x_n) = a_{1n}y_1 + a_{2n}y_2 + \ldots . a_{mn}y_m$$

Then matrix representation of T relative to the ordered basis β_1 & β_2 is denoted by

$$[T:\beta_1,\beta_2] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & \cdots & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

$T : R^3 \to R^2$

$T(x_1) = a_{11}y_1 + a_{21}y_2$
$T(x_2) = a_{12}y_1 + a_{22}y_2$
$T(x_3) = a_{13}y_1 + a_{23}y_2$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \cancel{3\times 2} \; 2\times 3$$

$T: R^3(R) \to R^3(R)$

$|T| \neq 0$

$T(x, y, z) = (x, z, y)$

$T(1, 0, 0) = (1, 0, 0)$

$T(0, 1, 0) = (0, 0, 1)$

$T(0, 0, 1) = (0, 1, 0)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$n(T) = 0$

$\rho(T) = 3$

$$T: \mathbb{R}^2(\mathbb{R}) \longrightarrow \mathbb{R}^3(\mathbb{R})$$

$$B = \{(1,0), (1,1)\}$$

$$\beta' = \{(1,0,0), (0,1,0), (1,1,1)\}$$

$$T(x,y) = (x+y, x-y, 0)$$

$$T(v_1) = a_{11}w_1 + a_{21}w_2 + a_{31}w_3$$

$$T(v_2) = a_{12}w_1 + a_{22}w_2 + a_{32}w_3$$

$$T(1,0) = a_{11}(1,0,0) + a_{21}(0,1,0) + a_{31}(1,1,1)$$

$$(1,1,0) = (a_{11}+a_{31}, a_{21}+a_{31}, a_{31})$$

$$T(1,1) = a_{12}(1,0,0) + a_{22}(0,1,0) + a_{32}(1,1,1)$$

$$a_{11}+a_{31} = 1 \quad a_{21}+a_{31} = 1$$

$$a_{31} = 0, \; a_{21} = 1$$

$$a_{11} = 1$$

$$(2,0,0) = (a_{12}+a_{32}, a_{22}+a_{32}, a_{32})$$

$$a_{32} = 0$$

$$a_{22} = 0$$

$$a_{12} = 2$$

$$T = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Q.2. Let $T : R^3 \to R^3$ be the linear transformation defined by $T(x, y, z) = (x + y, y + z, z + x)$ for all $(x, y, z) \in R^3$. Then

(a) rank (T) = 0, nullity (T) = 3

(b) rank(T) = 2, nullity (T) = 1

(c) rank(T) = 1, nullity (T) = 2

(d) rank (T) = 3, nullity (T) = 0

$T(1,0,0) = (1,0,1)$

$T(0,1,0) = (1,1,0)$

$T(0,0,1) = (0,1,1)$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$$

$T: R^3 \to R^2$

$T(1,0,0) = (1,1)$
$T(0,0,1) = (0,0)$
$T(0,1,0) = (-1,1)$

$T(x,y,z)$
$T^{-1}(p,q)$

$(x,y,z) = \alpha(1,0,0) + \beta(0,0,1) + \gamma(0,1,0) = (\alpha+0+0,\ 0+0+\gamma,\ 0+\beta+0)$
$= (\alpha, \gamma, \beta)$

$\alpha = x$
$\gamma = y$
$\beta = z$

$T(x,y,z) = xT(1,0,0) + zT(0,0,1) + yT(0,1,0)$
$= x(1,1) + z(0,0) + y(-1,1)$
$= (x+0-y,\ x+0+y)$

$T(x,y,z) = (x-y, x+y)$

$x-y = p$
$x+y = q$

$2x = p+q$
$x = \frac{p+q}{2}$
$y = \frac{q-p}{2}$

$(x,y,z) = T^{-1}(x-y, x+y)$
$T^{-1}(p,q) = \left(\frac{p+q}{2}, \frac{q-p}{2}, 0\right)$

Q.3 Let $T : R^4 \to R^4$ be a linear map defined by $T(x, y, z, w) = (x + z, 2x + y + 3z, 2y + 2z, w)$. The rank of T is equal to

(a) 1

(b) 2

(c) 3

(d) 4

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T: R^2 \to R^2$

$T(1,2) = (2,3)$

$T(0,1) = (1,4)$

$T(5,6) = ?$

(a) $(6,-1)$

(b) $(-6,1)$

(c) $(-1,6)$

(d) $(1,-6)$

$(5,6) = \alpha(1,2) + \beta(0,1)$

$(5,6) = (\alpha, 2\alpha+\beta)$

$\alpha = 5$

$2\alpha + \beta = 6$

$\beta = -4$

$T(5,6) = 5T(1,2) - 4T(0,1)$

$T(5,6) = 5(2,3) - 4(1,4)$

$= (10-4, 15-16)$

$= (6,-1)$

Q.4. Let N be the vector space of all real polynomial of degree atmost 3. Define S : N $\rightarrow$N by (S)p(x) = p(x + 1), p $\in$ N. and the matrix of S in the basis $\{1, x, x^2, x^3\}$ considered as column vector then which of the following is true?

(a) S is upper triangular matrix with determinant 1.

(b) S is singular matrix

(c) S is upper triangular matrix with trace 1.

(d) S is identity matrix.

$$\begin{array}{c|c} 1 & \\ x & \\ x^2 & \\ x^3 & \end{array}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T(x^3) = (x+1)^3$
$= 1 + 3x + 3x^2 + x^3$

$T(1) = 1$

$T(x) = x+1$

$T(x^2) = (x+1)^2$
$= 1 + 2x + x^2$

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$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$\lambda_1, \lambda_2, \lambda_3$

Result :

(1) Let $T : M_2(R) \to M_2(R)$ be a linear transformation such that $T(X) = AX$, where A is given matrix. If A is diagonalizable then T is also diagonalizable.

(2) If $T : R^{m\times n} \to R^{n\times p}$ be a linear transformation such that $T(X) = AX$ then

(a) Rank (T) = n.Rank (A)

(b) Trace(T) = n.Trace(A)

(c) Nullity(T) = n.Nullity(A)

Q.5. Let A be a matrix of order n and let V be the vector space of all real $n \times n$ matrix X such that $AX = 0$. what is dimension of V. **CSIR NET JUNE 2022**

$\rho(A) = r$

(a) nr (b) n^2r

(c) $n^2 - nr$ (d) n

$T : M_{n\times n} \rightarrow M_{n\times n}$

$T(X) = AX$

$\rho(T) = n\,\rho(A) = nr$

$n(T) = ?$

$n(T) + \rho(T) = n^2$

$n(T) = n^2 - \rho(T)$

$= n^2 - nr$

Q.6 Let $M_2(R)$ denote the set of 2×2 real matrices. Let $A \in M_2(R)$ be a trace 2 and determinant -3. Identifying $M_2(R)$ with R^4, consider the linear transformation $T : M_2(R) \rightarrow M_2(R)$ defined by $T(B) = AB$. Then which of the following statements are true?

(a) T is diagonalizable

(b) 2 is an eigenvalues of T

(c) T is invertible

(d) $T(B) = B$ for some $0 \neq B$ is $M_2(R)$

$\lambda_1 + \lambda_2 = 2$

$\lambda_1 \lambda_2 = -3$

$A = \begin{pmatrix} & \end{pmatrix}$

$\rho(A)$

$\rho(T) =$

$n(T) =$

(a) a c

(b) b d

(c) c b

(d) d a

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$(\lambda_1 - \lambda_2)^2 = (\lambda_1 + \lambda_2)^2 - 4\lambda_1\lambda_2$

$= 4 + 12 = 16$

$\lambda_1 - \lambda_2 = 4$

Q.7 Let $T : R^4 \to R^4$ be the linear map satisfying $T(e_1) = e_2$, $T(e_2) = e_3$, $T(e_3) = 0$, $T(e_4) = e_3$ where $\{e_1, e_2, e_3, e_4\}$ is the standard basis of R^4. Then

(a) T is idempotent
(b) T is invertible
(c) Rank(T) = 3
(d) T is nilpotent

$T(1,0,0,0) = (0,1,0,0)$
$T(0,1,0,0) = (0,0,1,0)$
$T(0,0,1,0) = (0,0,0,0)$
$T(0,0,0,1) = (0,0,1,0)$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$|A| = 0$

$A^2 \neq A$

$$A^2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Q.8 Let $R^{2\times 2}$ be the real vector space of all 2×2 real matrices for $Q=\begin{bmatrix}1 & -2\\ -2 & 4\end{bmatrix}$, define a linear transformation T on $R^{2\times 2}$ as $T(P)=QP$. Then the rank of T is

(a) 1 (b) 2

(c) 3 (d) 4

$\rho(Q)=1$

4SU

$T(x)=Ax$

$=$

$\rho(T)=n\rho(Q)$

$=2\times 1$


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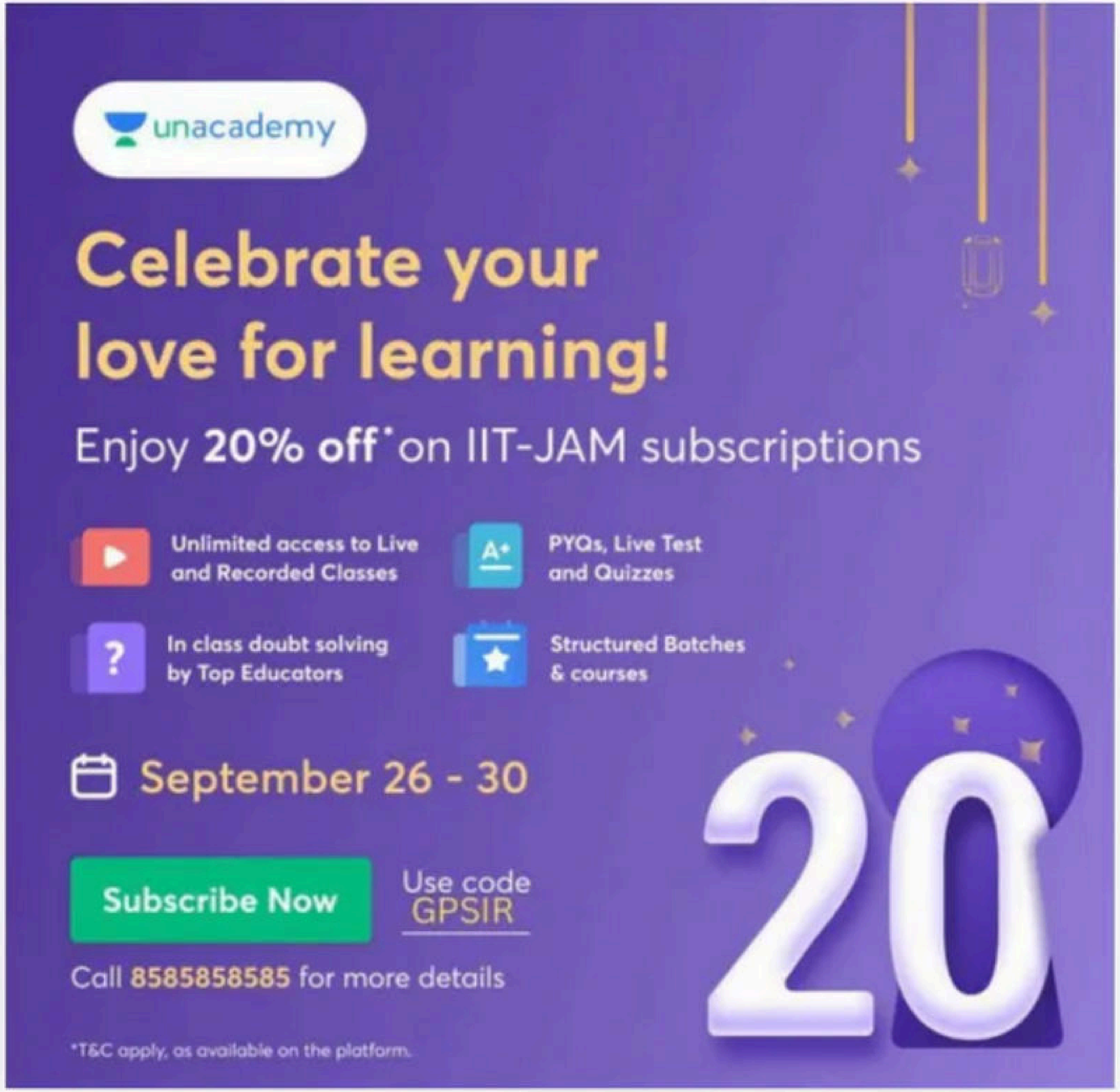
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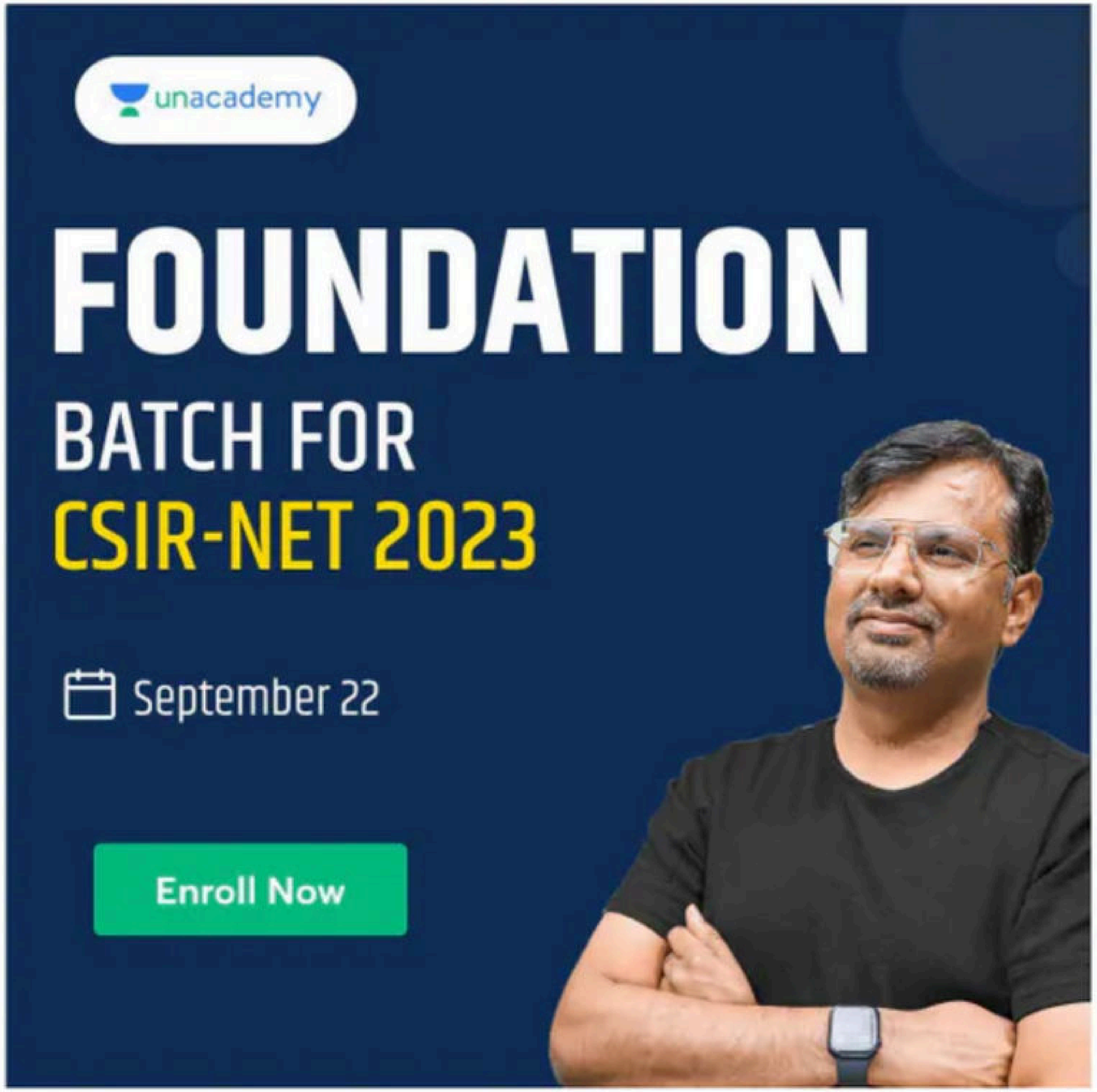
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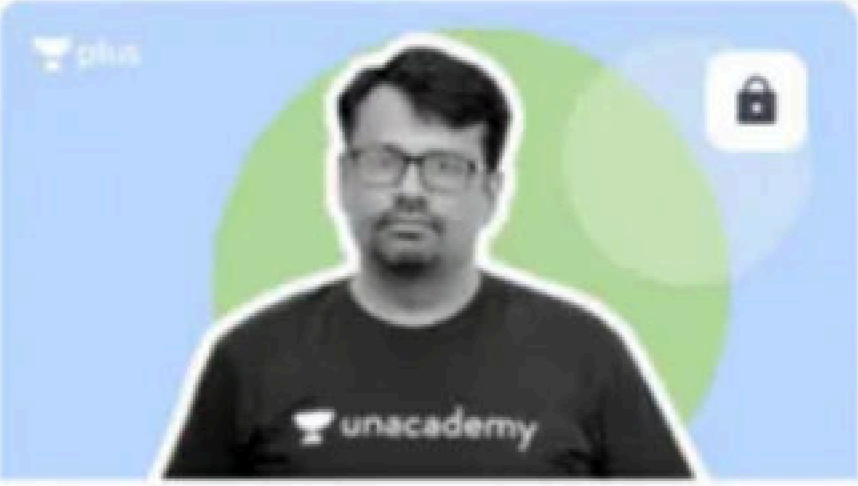

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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