Tracing of curve

(1) Tracing of cartisiancurve :

(a) Symmetric :

(i) If f(x, y) be a given curve and the power of x is even then it is symmetric about y- axis.

(ii) If f(x, y) be a given curve and the power of y is even power then it is symmetric about x-axis.

(iii) If power of x & y both even then this curve is symmetric about x & y axis.

(b) Curve passing through origin :

If $f(x, y)$ be a given curve and $f(0, 0) = 0$ then this curve passes through origin.

(c) Intersection with coordinate axis :

(i) If we put $y = 0$ in given curve then we get intersection point of curve and x-axis.

(ii) If we put $x = 0$ in given curve we get intersection point of curve and y-axis

(d) **Asymtote :**

If f(x, y) is a curve

(i) At $x = a$, if we get $y = \infty$, then Asymtote is parallel to y-axis.

(ii) At $y = a$, if we get $x = \infty$, then Asymtote is parallel to x-axis.

Curve tracing of polar form :

(1) Symmetry :

(i) If $f(r, \theta) = f(r, -\theta)$

Then this curve is symmetric about initially line (i.e. $\theta = 0$ line)

(ii) If $f(r, \theta) = f(r, \pi - \theta)$

Then this curve is symmetric about $\theta = \frac{\pi}{2}$ line.

(iii) **Pole :** Put $r = 0$, then find value of θ.

Hence (r, θ) is a pole.

(iv) **Tangent at pole :** Put $r = 0$, then value of θ is tangent at pole.

(v) **Table :**

r									
θ	0	30	45	60	90	120	135	150	180

(vi) **Asymtotes :**

For any value of θ if r become ∞, then a curve has asymtotes.

Q.1. The curve $ay^2 = x^2 (a - x)$ is passing through

(a) (0, 1) (b) (0, 0)

(c) (1, 0) (d) (1, 2)

Q.2. The cardiod $r = a(1 + \cos\theta)$ is symmetric about

(a) $\theta = 0$ line (b) $\theta = \pi/4$ line

(c) $\theta = \pi/2$ line (d) none of these

Q.3. The tangent at origin of the curve $2y^2 = x^2(2 - x)$ is

(a) $x = +2y$ and $x=-2y$

(b) $y = 2x$ and $y=-2x$

(c) $x = y$ and $x=-y$

(d) none of these

Double Integrals

Let f(x, y) is a function of two variable then double integral of f(x, y) is denoted by $\iint f(x,y)dxdy$ or $\iint f(x,y)dydx$.

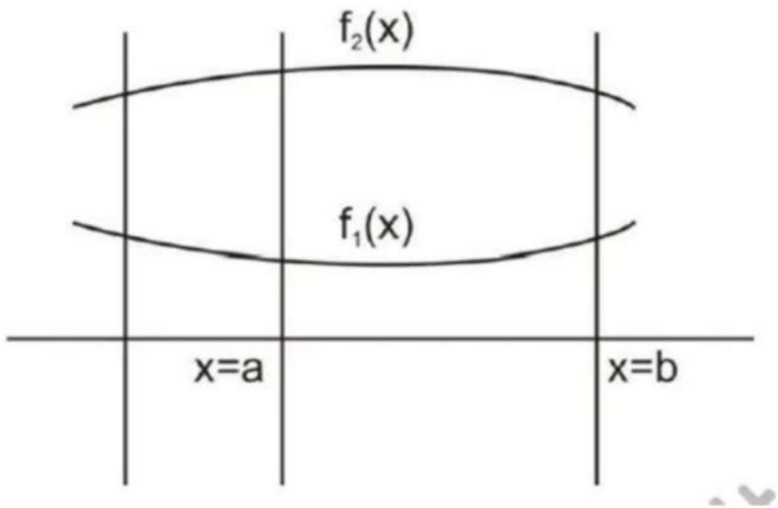
Note :

(1) For $\iint f(x, y)dxdy$, f(x,y) is first integrated w.r.t. x and then it is integrated w.r.t. y.

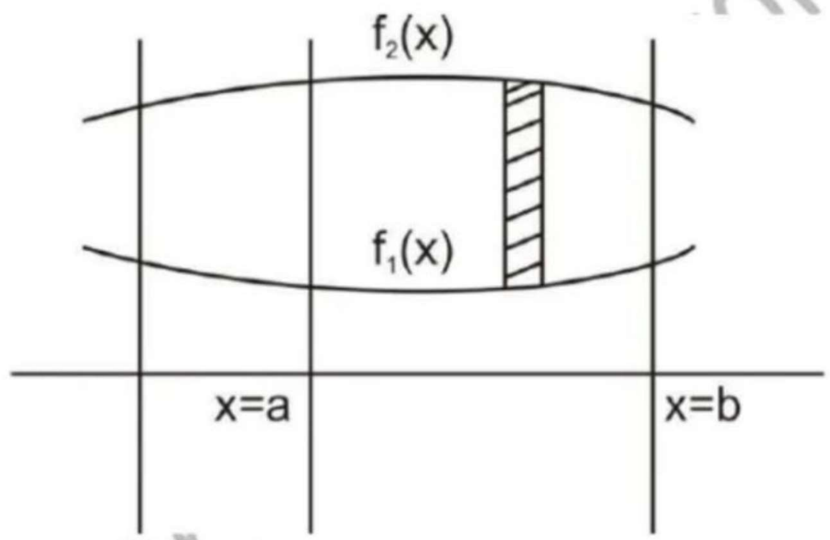
(2) For $\iint f(x, y)dydx$, f(x, y) is first integrated w.r.t. y and then it is integrated w.r.t. x.

Find limit by a given curve :

(a) If the region A is bounded by the curves $y = f_1(x)$ & $= f_2(x)$ and the coordinate $x = a$ and $x = b$.

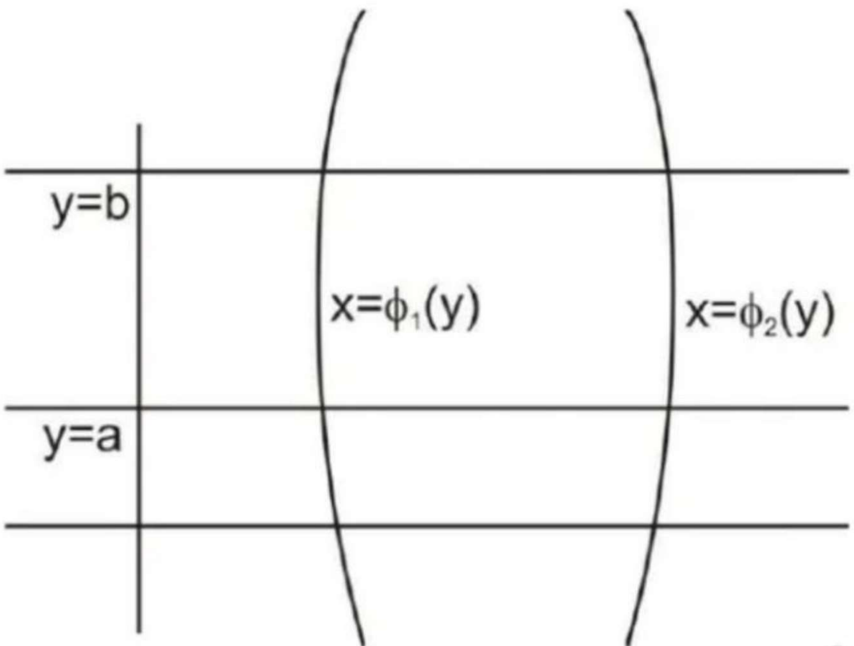


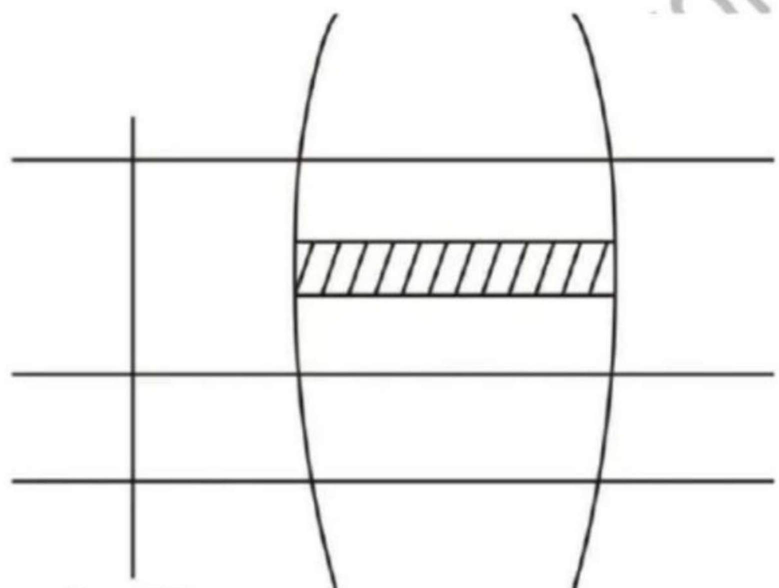
Then strip is parallel to y-axis



Then $\iint_A f(x,y)dA = \int_{x=a}^{b} \int_{y=f_1(x)}^{y=f_2(x)} f(x,y)dydx$

(b) If the region A is the area bounded by the curve $x = \phi_1(y)$ & $x = \phi_2(y)$ and coodinate $y = a$ and $y = b$.





Then strip is parallel to x-axis

So, $$\iint_A f(x,y)\,dA = \int_{y=a}^{y=b} \int_{x=\phi_1(y)}^{x=\phi_2(y)} f(x,y)\,dx\,dy$$

Q.1. The value of $\iint x e^{y^2} dx dy$, where R is the region bounded by the line x = 0, y = 1 and the parabola y = x^2. **IIT JAM-2006**

(a) $-\frac{1}{4}[e-1]$ (b) $\frac{1}{4}[e-1]$

(c) $\frac{1}{4}[e+1]$ (d) None

Q.2. The value of $\iint xy(x+y)\,dx\,dy$ over the area between $y^2 = x$ and $y = x$

(a) $\frac{1}{56}$ (b) $\frac{3}{56}$

(c) $\frac{5}{56}$ (d) $\frac{3}{55}$

Q.3. The value of $\iint_R xy\,dx\,dy$, where R is the quadrant of the circle $x^2 + y^2 = a^2$

(a) $a^4/8$ (b) $a^2/8$

(c) $a/8$ (d) $3a/2$