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Automorphism : Let G be a group, then the mapping $f : G \to G$ is called automorphism if

(i) f is one-one

(ii) f is onto

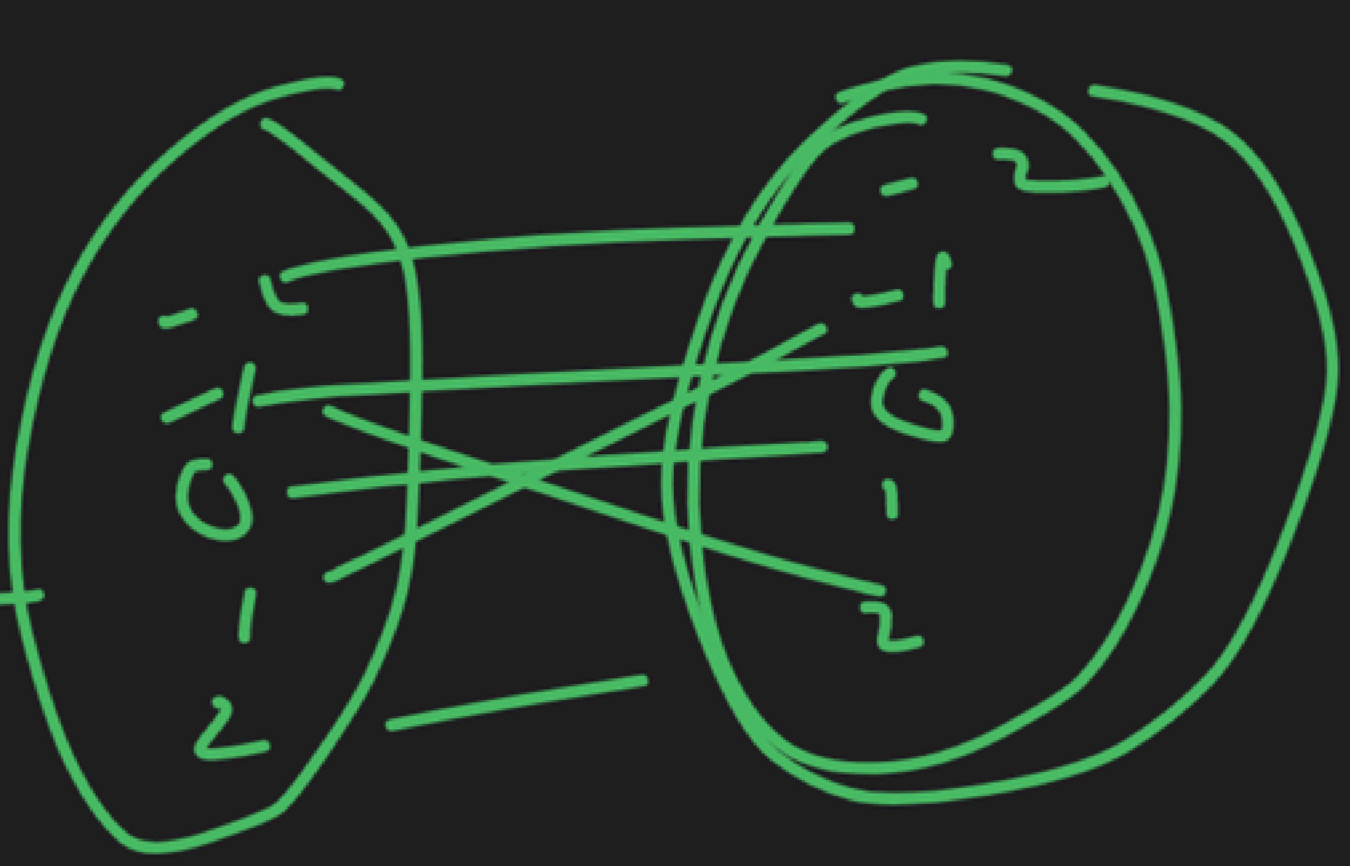
(iii) f is homomorphism

i.e. A mapping $f : G \to G$ is called automorphism if it is isomorphism

$f : G \to G$ $\quad$ $f(x) = x$ trivial homo.

$f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = x$

$f(x) = -x$



$((1, -1), x)$

$f(x) = mx$

$m = -1, 1$

$f: \mathbb{Z}_4 \longrightarrow \mathbb{Z}_4$

$f(x) = ax$

$f(x) = 0 \cdot x$

$f(x) = 1 \cdot x$

$f(x) = 2 \cdot x$

$f(x) = 3 \cdot x$

$f(x) = 1 \cdot x$

$f(0) = 0$

$f(1) = 1$

$f(2) = 2$

$f(3) = 3$

$f(x) = 2x$

$f(0) = 0$

$f(1) = 2$

$f(2) = 0$

$f(3) = 2$

$f(x) = 3x$

$f(0) = 0$

$f(1) = 3$

$f(2) = 2$

$f(3) = 1$

$\langle 1, 3 \rangle, \times$

$\mathbb{Z}_2$

$\mathrm{Aut}(\mathbb{Z}_4) \cong \mathbb{Z}_2$

$\mathrm{Aut}(\mathbb{Z}_n) \cong U(n)$

$\mathrm{Aut}(\mathbb{Z}_4) \cong U(4) = \mathbb{Z}_2$

$f : \mathbb{Z}_{10} \to \mathbb{Z}_{10}$

$\mathrm{Aut}(\mathbb{Z}_{10}) \cong U(10)$

$\cong \mathbb{Z}_4$

Note : Let Z = group of integer under addition then f : Z → Z s.t. $f(x) = mx$; $m \neq \{1, -1\}$

Then it will not be onto mapping.

So, it will not be automorphism.

⇒ Z have only two automorphism.

Aut (Z) ≅ Z_2

$f : Z_n \to Z_n$

Automorphism Group :

Let G be a group, then the set of all automorphism of G form a group under the composition of mapping and this is denoted by Aut G.

I $f: \mathbb{Z} \to \mathbb{Z}$ $\quad Aut(\mathbb{Z}) \cong \mathbb{Z}_2$

II $f: \mathbb{Z}_n \to \mathbb{Z}_n$ $\quad Aut(\mathbb{Z}_n) \cong U(n)$

III $f: K_4 \to K_4$ $\quad Aut(K_4) \cong S_3$

IV $f: S_n \to S_n$ $\quad n \geq 3 \quad Aut(S_n) \cong S_n$

V $Aut(\underbrace{\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \times \cdots \times \mathbb{Z}_p}_{n}) \cong GL(n, \mathbb{Z}_p)$

$$\underline{\mathrm{Aut}(\mathbb{Z}_3 \times \mathbb{Z}_3)} \cong GL(2, \mathbb{Z}_3)$$

$$\mathrm{Aut}(\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_5) \cong GL(3, \mathbb{Z}_5)$$

$$\underline{O(\mathrm{Aut}(\mathbb{Z}_3 \times \mathbb{Z}_3))} = O(GL(2, \mathbb{Z}_3))$$

$$= \underline{(3^2 - 1)(3^2 - 3)}$$

$$= 8 \times 6$$

$$= \underline{48}$$

Q1. For any group G, Aut(G) denote the group of automorphism of G. Which of the following are true?

(a) If G is finite, then Aut(G) is finite

(b) If G is cyclic, then Aut(G) is cyclic

(c) If G is infinite, then Aut(G) is infinite

(d) If Aut (G) is isomorphic to Aut(H) then G is isomorphic to H.

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$Aut(Z_n) \cong U(n)$

$Aut(K_4) \cong S_3$

$Aut(S_n) \cong S_n$ $(n \geq 3)$

$Aut(Z_p \times Z_p) \cong GL(2, Z_p)$

S_3 S_3

K_4 S_3

$Aut(Z_8) \cong U(8) \cong Z_2 \times Z_2$

Q2. Let $G = Z_3 \times Z_3$ be a group then order of $\text{Aut}(Z_3 \times Z_3)$ is

(a) 48 0

(b) 168 + 12

(c) 50 0

(d) 150 + 20

$(5^2-1)(5^2-5)$

$(25-1)(25-5)$

24×20

Q3. The order of Aut(Aut(Aut(K_4))) is

(a) 4 (b) 5

(c) 6 (d) 8

Group Homomorphism

Counting of homomorphism :

Homo — 2 $\gcd(4, 10) = 2$

one-one Homo — 0

onto Homo. $\longrightarrow 0$

$$\frac{G}{\ker f} \cong f(G) \subset G$$

$f : Z_4 \rightarrow Z_{10}$

$\frac{Z_4}{Z_4} \cong Z_1 \subset Z_{10}$

$\frac{Z_4}{Z_2} \cong Z_2 \subset Z_{10}$

$O(\mathrm{Aut}(Z_2)) = 1$

$\frac{Z_4}{Z_4} \cong Z_1 \subset Z_{10}$

$O(\mathrm{Aut}(Z_1)) = 1$

$\mathbb{Z}_3$ Homo $= 3$

One-one $- 2$

onto $= 0$

$\mathbb{Z}_1$ $\quad$ $\mathbb{Z}_3$

$f: \mathbb{Z}_3 \to \mathbb{Z}_6$

(I) $\dfrac{\mathbb{Z}_3}{\mathbb{Z}_1} = \mathbb{Z}_3 \subset \mathbb{Z}_6$

$Aut(\mathbb{Z}_3) = U(3) \equiv \mathbb{Z}_2$

2

(II) $\dfrac{\mathbb{Z}_3}{\mathbb{Z}_3} = \mathbb{Z}_1 \subset \mathbb{Z}_6$

$Aut(\mathbb{Z}_1) = U(1) = \mathbb{Z}_1$

1

$f: \mathbb{Z}_m \to \mathbb{Z}_n$

Homo — $\gcd(m, n)$

one-one $\begin{cases} \phi(m) & m \mid n \\ 0 & m \nmid n \end{cases}$

onto $\begin{cases} \phi(n) & n \mid m \\ 0 & n \nmid m \end{cases}$

$f : Z_6 \to Z_6$

Homo – 3

onto – 2

$\phi(3)$

(i) $\frac{Z_6}{Z_1} = Z_6 \not\subset Z_3$

(ii) $\frac{Z_6}{Z_2} = Z_3 = Z_3$

$Aut(Z_3) = U(3) = Z_2$

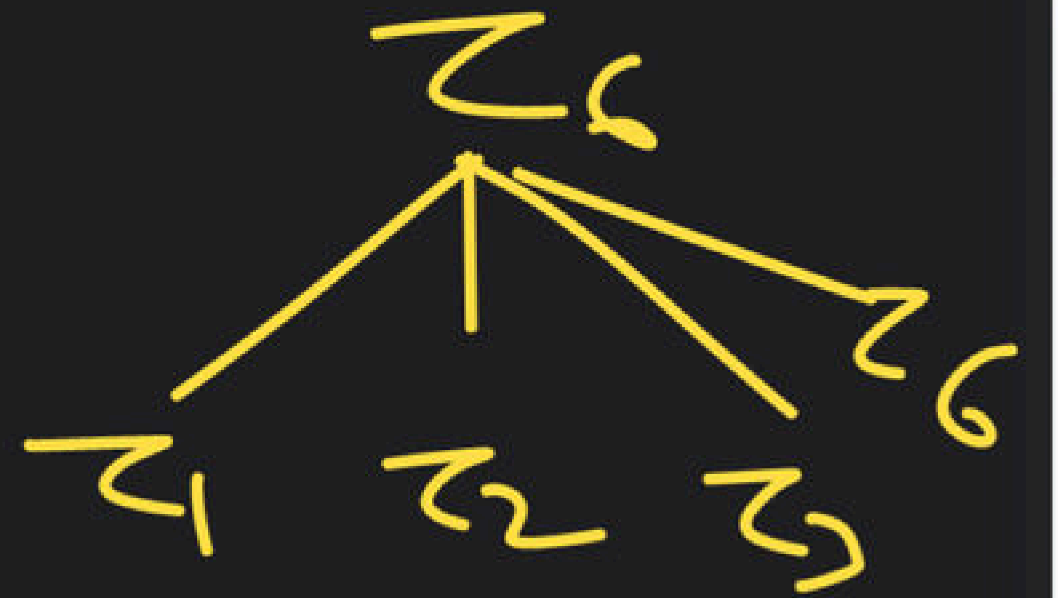
2

(iii) $\frac{Z_6}{Z_3} = Z_2 \not\subset Z_3$

(iv) $\frac{Z_6}{Z_6} = Z_1 \subset Z_3$

$Aut(Z_1) = 1$

1



Procedure for Counting of homomorphism :

Let f : G → G' be a homomorphism

(i) Find Normal subgroup of G namely H_1 , H_2 , etc.

(ii) Using FTH

$\frac{G}{H_i} = f(G)$ which is subgroup of G'

(iii) Find Aut(f(G))

(iv) During all the above steps, we have collected three numbers, n_1 the number of normal subgroup (H_i) in G , n_2 number of subgroup(f(G)) in G' , n_3 order of Aut(f(G)) then product of $n_1n_2n_3$

(v) Do each step for other H_i and sum all of them

(1) Counting of group homomorphism from finite cyclic group to finite cyclic group.

Result : Let $f : Z_m \to Z_n$ be a mapping then number of homomorphism are gcd(m, n)

2. Number of one-one homomorphism from finite cyclic group to finite cyclic group.

Result : Let f : $Z_m \to Z_n$ be a homomorphism then number of one-one homomorphism are $\begin{cases} \phi(m) & if \quad m \mid n \\ 0 & if \quad m \nmid n \end{cases}$.

(3) Number of onto homomorphism from Z_m to Z_n.

Result : Let f : $Z_m \to Z_n$ be a homomorphism then number of onto homomorphism are $\begin{cases} \phi(n) & \text{if } n \mid m \\ 0 & \text{if } n \nmid m \end{cases}$.

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(4) Counting of group homomorphism from S_4 to K_4.

(5) Counting of one – one group homomorphism from S_4 to K_4.

(6) Counting of onto group homomorphism from S_4 to K_4.

Homomorphism

1. Counting of homomorphism from $S_n \to Z_m$, $(n \geq 3)$

   1. Number of homomorphism $\begin{cases} 1, & m \text{ is odd} \\ 2, & m \text{ is even} \end{cases}$

   2. Number of onto homomorphism $\begin{cases} 1, & m = 1,2 \\ 0, & \text{otherwise} \end{cases}$.

   3. Number of one-one homomorphism does not exist.

2. Counting of homomorphism from $A_4 \to Z_m$

1. Number of homomorphism are $\begin{cases} 3 & if \quad 3 \mid m \\ 1 & if \quad 3 \nmid m \end{cases}$

2. Number of onto homomorphisms are $\begin{cases} 1 & if \quad m = 1 \\ 2 & if \quad m = 3 \\ 0 & otherwise \end{cases}$

3. Number of one-one homomorphism does not exist.

3. Counting of homomorphism from $A_n \to Z_m$, $(n \geq 5)$

1. Number of homomorphism only trivial.

2. Number of onto homomorphisms are $\begin{cases} 1 & if \quad m = 1 \\ 0 & if \quad otherwise \end{cases}$

3. Number of one-one homomorphism does not exist.

4. Counting of homomorphism from $K_4 \to Z_n$.

1. Number of homomorphism are $\begin{cases} 4, & m \text{ is even} \\ 1, & m \text{ is odd} \end{cases}$.

2. Number of onto homomorphisms are $\begin{cases} 1, & m = 1 \\ 3, & m = 2 \\ 0, & otherwise \end{cases}$.

3. Number of one-one homomorphism does not exists.

5. Counting of homomorphism from $Q_8 \to Z_m$

1. Number of homomorphisms are $\begin{cases} 4, & m \text{ is even} \\ 1, & m \text{ is odd} \end{cases}$.

2. Number of onto homomorphisms are $\begin{cases} 1, & m=1 \\ 3, & m=2 \\ 0, & \text{otherwise} \end{cases}$.

3. Number of one-one homomorphism does not exists.

6. Counting of homomorphism from $S_n \to K_4$, $(n \geq 3)$

1. Number of homomorphism are 4.

2. Number of onto homomorphism does not exist.

3. Number of one-one homomorphism does not exist.

7. Counting of homomorphism from $S_n \to Q_8$, $n \geq 3$.

1. Number of homomorphism are 2.
2. Number of onto homomorphism does not exist.
3. Number of one-one homomorphism does not exist

Q.1. The number of group homomorphism from the cyclic group Z_4 to the cyclic group Z_7 is

$\gcd(4, 7) = 1$

(a) homomorphism

(b) isomorphism

(c) unique homomorphism

(d) None of these

Q.2. The number of homomorphism from Z_4 to Z_{12} is

(a) 4 (b) 3

(c) 48 (d) 12

Q.3. Let G be the cyclic group of order 8 and H = S_3 be the permutation group of 3 elements.

Which of the following statements are necessarily true?

(a) There exists no non-trivial group homomorphism from G to H

(b) There exists no injective group homomorphism from G to H

(c) There exists no surjective group homomorphism from G to H

(d) There are more than 20 different group homomorphism from G to H

Q.4. The number of homomorphism from S_5 to Z_5

(a) 1 (b) 2

(c) 3 (d) 4

Q.5. The number of onto homomorphism from Z_8 to Z_4

(a) 1 (b) 2

(c) 3 (d) 4

$\phi(4) = 2$

Q.6. The number of onto homomorphism from Q_8 to Z_2

(a) 1 (b) 2

(c) 3 (d) 4

Q.7. The number of onto homomorphism from Z_{200} to Z_{100}

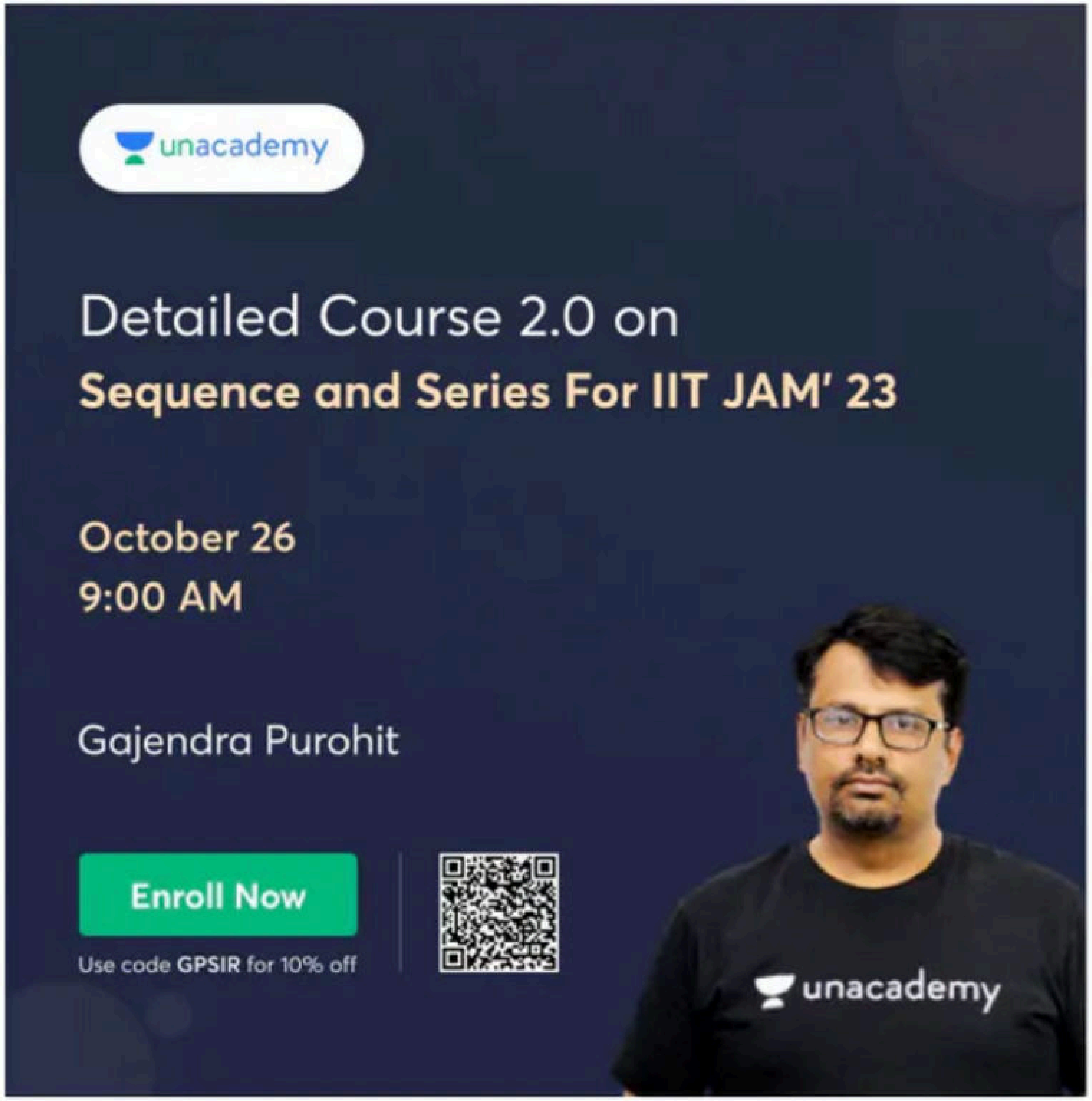
(a) 46 (b) 42

(c) 40 (d) 38

Q.8. The number of group homomorphisms from to Z_{10} is Z_{20}

(a) zero (b) one

(c) five (d) ten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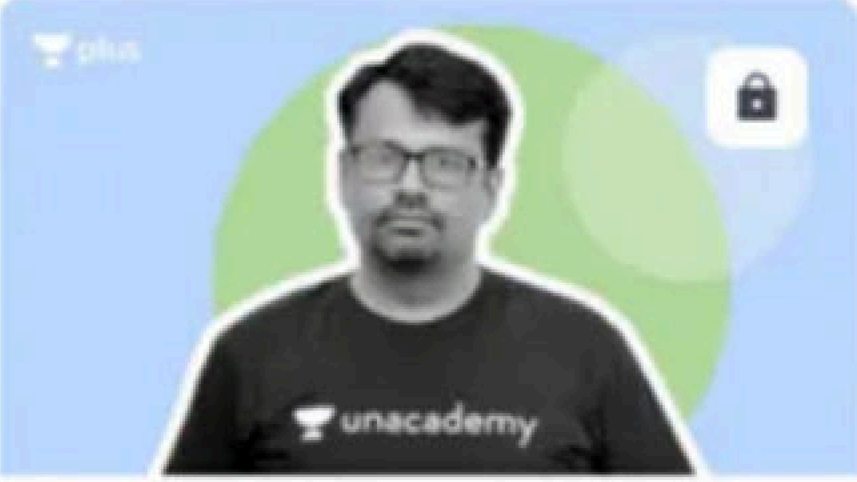

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Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
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