Volume of solid generated by revolution of solid

Revolution of solid :

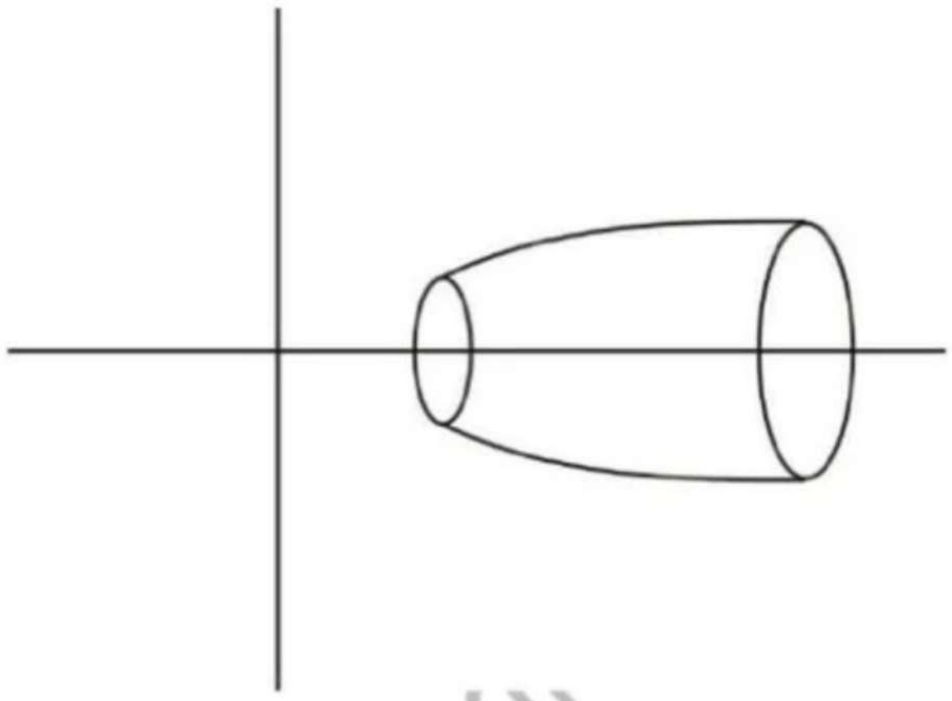
(i) **Revolution of solid about x-axis :** Suppose we have a curve, $y = f(x)$. Imagine that the part of curve between the ordinate $x = a$ and $x = b$ is rotated about the x-axis through 360°. The curve would then map out the surface of solid as it rotated and such solid are called solid of revolution.

(ii) Revolution of solid about y-axis :

Let $x = f(y)$ be a curve and the part of curve between the ordinate $y = c$ and $y = d$ is rotated about y-axis through 360°.

Volume of solid of revolution by x-axis :

Let $y = f(x)$ be a curve and solid of revolution between $x =$ to $x = b$.

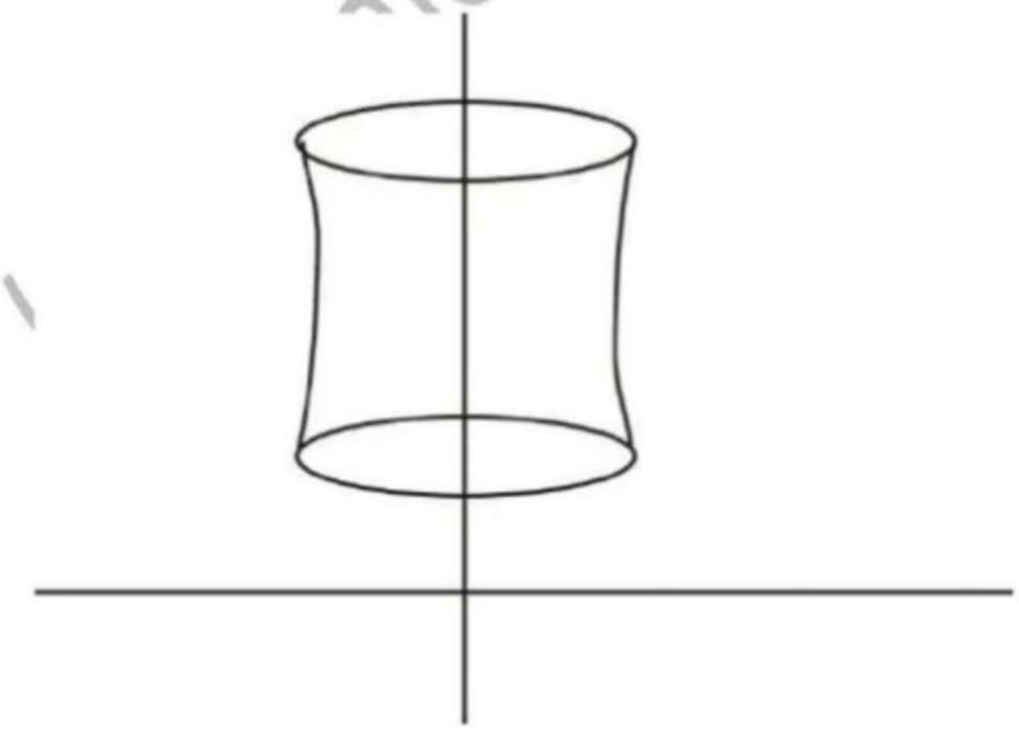


$$\text{The volume} = \int_a^b \pi y^2 dx$$

$$= \int_a^b \pi [f(x)]^2 dx$$

Volume of solid of revolution about y-axis :

Let x = f(y) be a curve and solid of revolution between y = c to y = d.



The volume = $\pi \int_c^d x^2 dy$

Volume of revolution when the equation of generating curve are given in parametric form :

(i) Let $x = \phi(t)$, $y = \psi(t)$

Then volume of solid generated by x-axis is

$$\int_a^b \pi y^2 dx = \pi \int_a^b y^2 \frac{dx}{dt} dt .$$

(ii) Let $x = \phi(t)$ and $y = \psi(t)$

Then volume of solid generated by y-axis is

$$\pi \int_c^d x^2 dy = \pi \int_c^d x^2 \frac{dy}{dt} dt .$$

The axis of rotation being any line :

(i) If this line parallel to x-axis i.e. $y = \alpha$, then volume is

$$\int_a^b \pi(y-\alpha)^2\,dx\,.$$

(ii) If the line is $x = \beta$, then volume is $\int_a^b \pi(x-\beta)^2\,dy\,.$

Q.1. The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x+2)$ about the x- axis (round off to 2 decimal places) is **IIT-JAM 2019**

(a) 6.69 (b) 6.75

(c) 6.80 (d) 6.93

Q.2 Find the volume of the solid formed by revolving the cycloid about its base.

(a) $3\pi^2a^3$ (b) $5\pi^2a^3$

(c) $6\pi^2a^3$ (d) None of these

Q.3 Find the volume formed by the revolutation of the loop of the curve $y^2(a+x) = x^2(a-x)$ about x-axis.

(a) $\pi a^2\left[2\log 2 - \frac{4}{3}\right]$

(b) $\pi a^2\left[2\log 2 + \frac{4}{3}\right]$

(c) $\pi a^2\left[2\log 3 - \frac{4}{3}\right]$

(d) None of these

Q.4. The volume of the solid of revolution of $y = \frac{a}{2}(e^{x/a} + e^{-x/a})$ about x-axis between x = 0 and x = b is **IIT JAM – 2009**

(a) $\frac{\pi a^3}{8}(e^{2b/a} - e^{-2b/a}) - \frac{\pi a^2 b}{2}$

(b) $-\frac{\pi a^3}{8}(e^{2b/a} - e^{-2b/a}) + \frac{\pi a^2 b}{2}$

(c) $-\frac{\pi a^3}{8}(e^{2b/a} - e^{-2b/a}) - \frac{\pi a^2 b}{2}$

(d) $\frac{\pi a^3}{8}(e^{2b/a} - e^{-2b/a}) + \frac{\pi a^2 b}{2}$

Q.5. Volume of the solid generated by revolving the region bounded by the lines $x = 0$, $y = 1$ and the curve $y = \sqrt{x}$ about the line $y = 1$ is equal to **IIT JAM – 2007**

(a) $\pi/6$ (b) $\pi/2$

(c) $5\pi/2$ (d) $3\pi/2$