Function of Several Variables - Part II

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 10 • Nov 21, 2022

Gajendra Purohit

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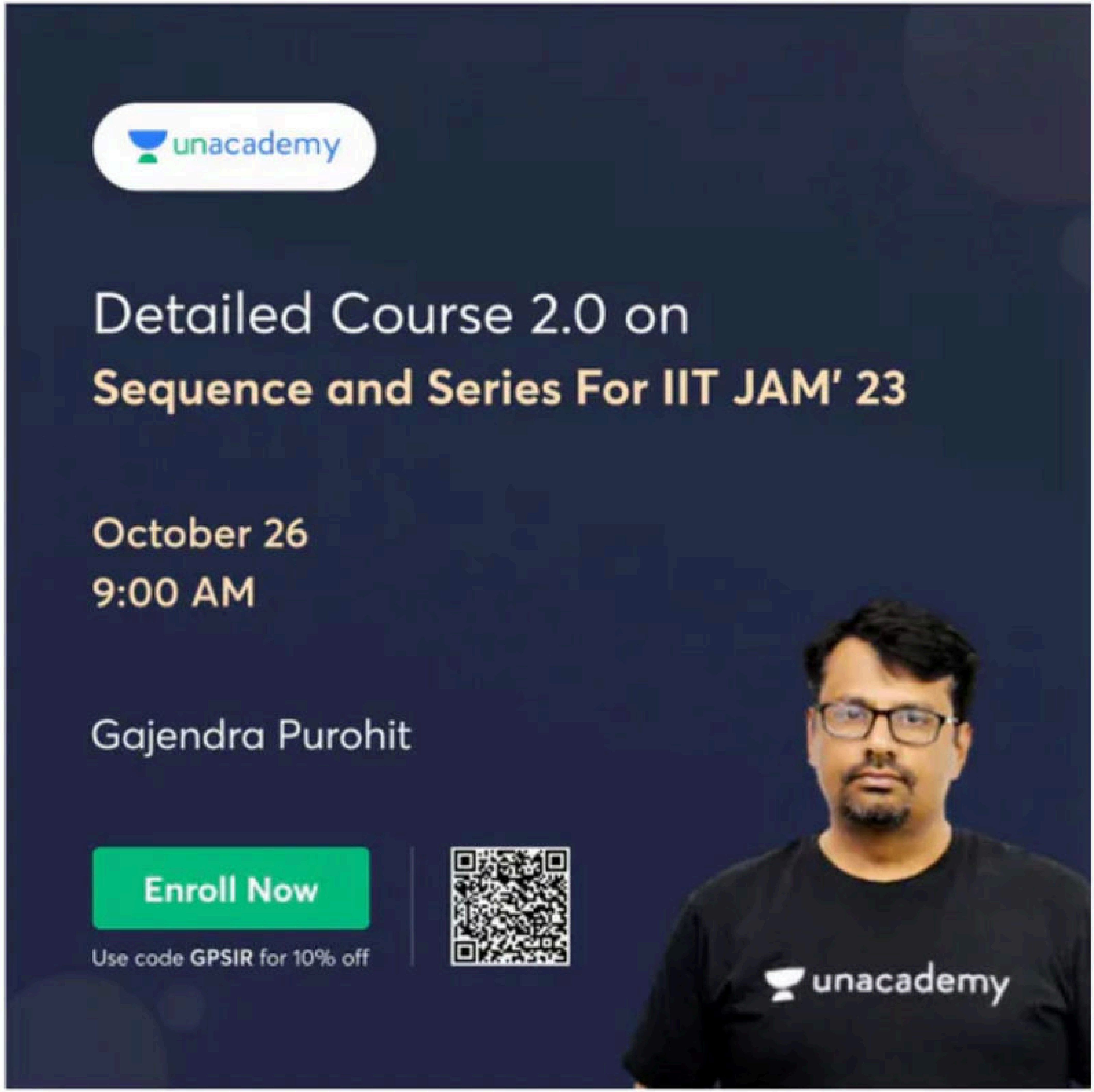
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Riemann Integration

Interval :

(1) **Open Interval :** Let $S \subseteq R$ and $S = \{x \mid a < x < b\}$. It is denoted by (a, b)

(2) **Closed Interval :** Let $S \subseteq R$ and $S = \{x \mid a \le x \le b\}$. It is denoted by $[a, b]$

$\frac{1}{2} - 0 = \frac{1}{2}$

$\frac{1}{3}$

Partition :

Let $[a, b]$ be a closed and bounded interval. A partition of $[a, b]$ is a finite order set $P = \{a = x_0, x_1, \ldots\ldots x_i \ldots., x_n = b \mid x_0 < x_1 < \ldots. < x_n\}$

$\frac{1}{12}$

$\frac{3}{4}$

$S = [0, 1]$

$P_1 = \{0, \frac{1}{2}, 1\}$

$P_2 = \{0, \frac{1}{3}, \frac{2}{3}, \frac{3}{4}, 1\}$

$[0, \frac{1}{3})$ $[\frac{1}{3}, \frac{2}{3})$ $[\frac{2}{3}, \frac{3}{4})$ $[\frac{3}{4}, 1)$

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Subinterval : Let P be a partition of [a, b], then $[x_0, x_1]$, $[x_1, x_2]$ ….., $[x_{n-1}, x_n]$ are called subinterval.

Norm of partition :

Let the greatest length of subinterval $[x_{r-1}, x_r]$ partition P is called norm of partition P and denoted by $||P||$.

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Refinement of partition :

Let P_1 and P_2 are two partition of $[a, b]$ s.t. $P_1 \subset P_2$, then P_2 is called refinement of partition P_1.

Note : Let P_1 and P_2 are two partition of $[a, b]$ then $P_1 \cup P_2$ is called a common refinement of P_1 & P_2.

$[0, 1]$

$P_1 = \{0, \frac{1}{2}, 1\}$

$P_2 = \{0, \frac{1}{4}, 1\}$

$P_1 \cup P_2 = \{0, \frac{1}{4}, \frac{1}{2}, 1\}$

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Supremum and infimum of a function :

Let $f(x)$ be a function defined on $[a, b]$ then supremum of function is subinterval $[x_{r-1}, x_r]$ is maxi mum value of function in $[x_{r-1}, x_r]$

Similarly infimum of this function in $[x_{r-1}, x_r]$ is minimum value of this function in $[x_{r-1}, x_r]$

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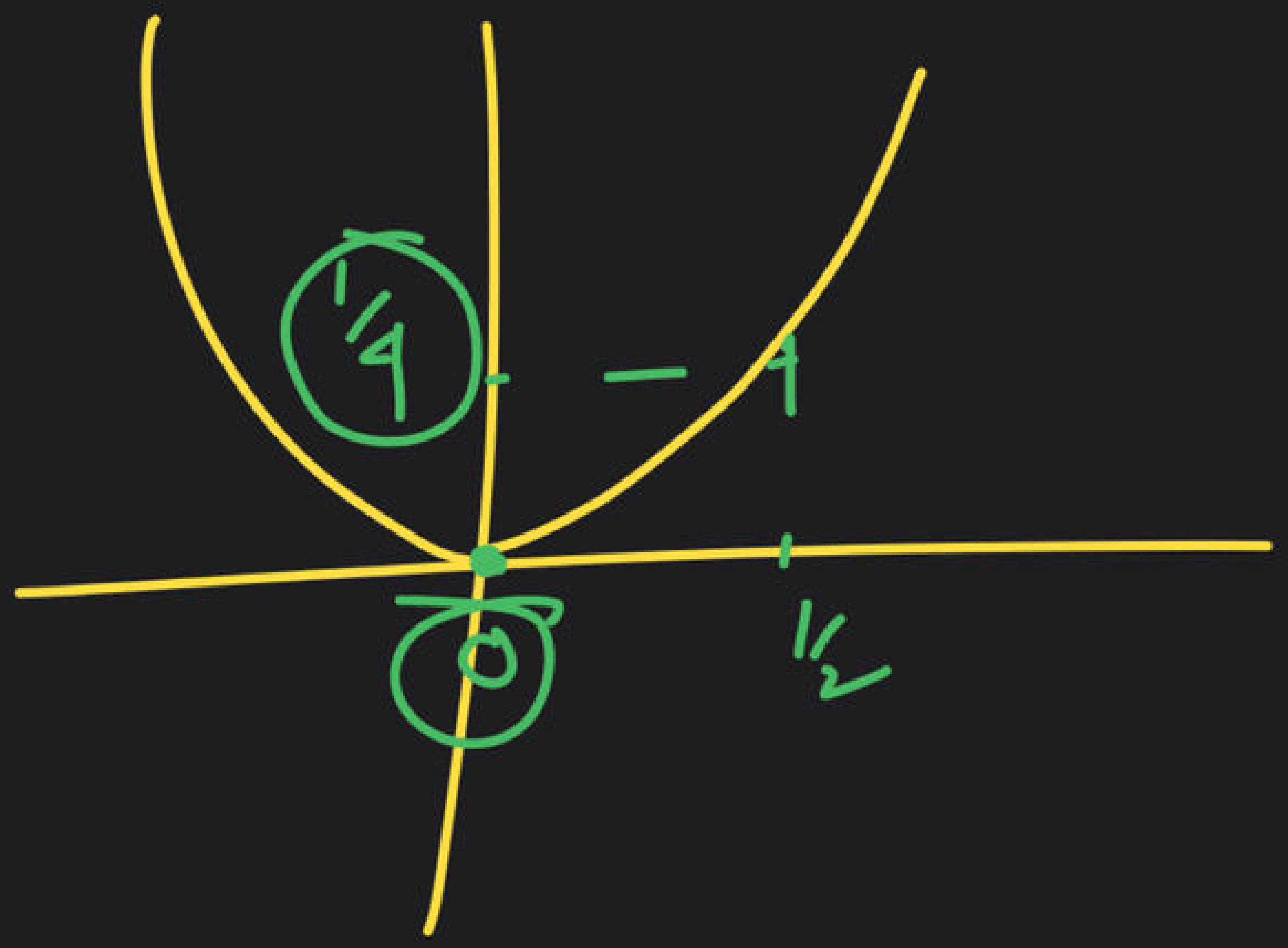
$f(x) = x^2$ $[-1, 1]$

$P = \{-1, 0, \frac{1}{2}, 1\}$

$f(0) = 0 = m$

$f(1/2) = 1/4 = m$ $\underline{[0, 1/2]}$



$[\frac{1}{2}, 1]$

$f(1/2) = 1/4 = m$

$f(1) = 1 = M$

(i) **Upper Riemann sum :**

Let M_r is supremum of function in $[x_{r-1}, x_r]$

And Δx_r is length of interval $[x_{r-1}, x_r]$, then

$U(P,f)=\sum_{r=1}^{n} Mr\Delta xr= M_1\Delta x_1 + M_2\Delta x_2 + \ldots.\ M_n\Delta x_n$

(ii) **Lower Reimann Sum :**

Let mr is infimum of function in $[x_{r-1}, x_r]$, then

$L(p, f) = m_1\Delta x_1 + m_2\Delta x_2 + \ldots.\ m_n\Delta x_n$

$L(P,f)=\sum_{r=1}^{n} mr\Delta xr$

is called lower Riemann sum.

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$f(x) = x$ $\quad [0,1]$ $\quad \int_{\underline{0}}^{1} f(x)\,dx = \int_{0}^{\overline{1}} f(x)\,dx$

$P = \{0, 1/4, 3/4, 1\}$

$M_1 = 1/4 \quad \int_{\underline{0}}^{1} f(x)\,dx = \sup L(P,f)$

$M_2 = 3/4 \quad \int_{0}^{\overline{1}} f(x)\,dx = \inf U(P,f)$

$M_3 = 1$

$\Delta x_1 = 1/4 \qquad m_1 = 0$

$\Delta x_2 = 1/2 \qquad m_2 = 1/4$

$\Delta x_3 = 1/4 \qquad m_3 = 3/4$

$$U(P,f) = \sum_{r=1}^{3} M_r \Delta x_r = M_1 \Delta x_1 + M_2 \Delta x_2 + M_3 \Delta x_3$$

$$= 1/4 \times 1/4 + 3/4 \times 1/2 + 1 \times 1/4$$

$$= 1/16 + 3/8 + 1/4 = 11/16$$

$$L(P,f) = \sum_{r=1}^{3} m_r \Delta x_r = m_1 \Delta x_1 + m_2 \Delta x_2 + m_3 \Delta x_3$$

$$= 0 + 1/8 + 3/16 = 5/16$$

$f(x) = x$ $\quad [0,1]$ $\quad P = \{0, \frac{1}{n}, \frac{2}{n}, \dots \frac{r-1}{n}, \frac{r}{n} \dots \frac{n}{n} = 1\}$

$\left[\frac{r-1}{n}, \frac{r}{n}\right]$, $\Delta x_r = \frac{1}{n}$

$M_r = \frac{r}{n}$

$m_r = \frac{(r-1)}{n}$

$$U(P,f) = \sum_{r=1}^{n} M_r \Delta x_r = \sum_{r=1}^{n} \frac{r}{n}\frac{1}{n}$$

$$= \frac{1}{n^2}\sum_{r=1}^{n} r = \frac{1}{n^2}(1+2+3+\dots n)$$

$$= \frac{1}{n^2}\frac{\cancel{n}(n+1)}{2} = \frac{n+1}{2n} \Leftarrow$$

$$L(P,f) = \sum_{r=1}^{n} m_r \Delta x_r = \sum_{r=1}^{n} \frac{(r-1)}{n}\frac{1}{n} = \frac{1}{n^2}\sum_{r=1}^{n}(r-1) = \frac{1}{n^2}(0+1+2\dots n-1)$$

$$= \frac{1}{n^2}\frac{(n-1)\cancel{n}}{2} = \frac{n-1}{2n}$$

$$\int_0^1 f(x)\,dx = \inf U(P,f) \Rightarrow \int_0^1 x\,dx = \lim_{n\to\infty}\frac{n+1}{2n} = \frac{1}{2}$$

$$\int_0^1 f(x)\,dx = \sup L(P,f) \Rightarrow \int_0^1 x\,dx = \lim_{n\to\infty}\frac{n-1}{2n} = \frac{1}{2}$$

Upper and Lower integral :

(i) **Upper integral :** The infimum of the set of the upper sup U(P, f) is called upper integral of f over [a, b] and is denoted by $\int_a^b f(x)dx$

i.e. $\int_a^b f(x)dx = \inf\{U(P,f): P$ is a partition of [a, b]}.

(ii) **Lower integral :** The supremum of the set of the upper sum U(P, f) is called upper integral of f over [a, b] and is denoted by $\int_a^b f(x)dx$

i.e. $\int_{\underline{a}}^b f(x)dx$ = sup {L(P, f) : P is a partition of [a, b]}.

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Riemann integration :

A function is Riemann integration if

$$\int_{\underline{a}}^{b} f(x)dx = \int_{a}^{\overline{b}} f(x)dx$$

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Q.1. Let $f(x) = x^3$ is defined on $[0, a]$, then

(a) $\int_{\underline{0}}^{a} f(x)dx = \frac{a^4}{3}$

(b) $\int_{\underline{0}}^{a} f(x)dx = \frac{a^4}{4}$

(c) $\int_{0}^{\overline{a}} f(x)dx = \frac{a^3}{4}$

(d) $\int_{0}^{\overline{a}} f(x)dx = a^4$

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Q.2. Function f : [0, 1] → R s.t. $f(x) = \begin{cases} 0 & x \in Q^c \\ 1 & x \in Q \end{cases}$

$\int_0^1 0\,dx = 0$

$\overline{\int_0^1} dx = 1$

Then

(a) f is R.I.

(b) f is not R.I.

(c) $\int_0^1 f(x)dx = 1$

(d) None of these

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Q.3. Define f on [0, 1] by $f(x) = \begin{cases} x^2 & if \quad x \text{ is rational} \\ x^3 & if \quad x \text{ is irrational} \end{cases}$

Then, **CSIR NET JUNE 2016**

(a) f is not Riemann integrable on [0, 1]

(b) f is Riemann integrable and $\int_0^1 f(x)dx = \frac{1}{4}$.

(c) f is Riemann integrable and $\int_0^1 f(x)dx = \frac{1}{3}$.

(d) $\frac{1}{4} = \underline{\int_0^1} f(x)dx < \overline{\int_0^1} f(x)dx = \frac{1}{3}$ where $\underline{\int_0^1} f(x)dx$ and $\overline{\int_0^1} f(x)dx$ are the lower and upper Riemann integrals of f.

$\overline{\int_0^1} x^2 dx = 1/3$

$\underline{\int_0^1} x^3 dx = 1/4$

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Q.4. Consider the identity function $f(x) = x$ on $I := [0, 1]$.

$\left[0, \frac{1}{n}, \frac{2}{n} \cdots \frac{r-1}{n}, \frac{r}{n} \cdots \frac{n}{n} = 1\right]$

Let P_n be the partition that divides I into n equal parts. If $U(f, P_n)$ and $L(f, P_n)$ are the upper and lower Riemann sums, respectively, & $A_n = U(f, P_n) - L(f, P_n)$ then **CSIR NET NOV 2020**

$\left[\frac{r-1}{n}, \frac{r}{n}\right]$

$L(P, f) = \frac{n-1}{2n}$

$U(P, f) = \frac{n+1}{2n}$

$\lim_{n\to\infty} 1 = 1$

$\sum \frac{1}{n}$

(a) $\lim_{n\to\infty} nA_n = 0$

(b) $\sum_{n=1}^{\infty} A_n$ is convergent

$A_n = \frac{n+1}{2n} - \frac{n-1}{2n} = \frac{1}{n}$

(c) A_n is strictly monotonically decreasing.

(d) $\sum_{n=1}^{\infty} A_n A_{n+1} = 1$

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{1}{1!\,1} = 1$

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Q.5. Define a function $[0, \pi/2] \to R$ by $g(x) = \begin{cases} \cos^2 x & \text{if } x \in Q \\ 0 & \text{otherwise} \end{cases}$. Then

(a) The lower integral, $\underline{\int_0^{\pi/2}} g = 1$.

(b) The lower integral, $\underline{\int_0^{\pi/2}} g = 0$.

(c) The lower integral $\underline{\int_0^{\pi/2}} g = \frac{1}{2}$.

(d) None of these

$$\int_0^{\pi/2} \left(\frac{1+\cos 2x}{2}\right) dx$$

$$\frac{1}{2}\left(x + \frac{\sin 2x}{2}\right)_0^{\pi/2}$$

$$\frac{1}{2}\left[\pi/2\right) = \pi/4$$

$$\int_0^{\pi/2} 0\, dx = 0$$

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Q.6. If $f(x) = x^2$ on $I = [0, 1]$ then find the value of lower Reimann Integral?

$\frac{1}{3}$

(a) $(n + 1)(n - 1)$

(b) $(n + 1)(n - 1)^2$

(c) $\frac{(n-1)(2n-1)}{6n^2}$

(d) $\frac{(n-2)(2n-1)}{n^2}$

$\sum n^2 = \frac{n(n+1)(2n+1)}{6}$

$[0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{r-1}{n}, \frac{r}{n}, \dots, \frac{n}{n} = 1]$

$[\frac{r-1}{n}, \frac{r}{n}]$ $\Delta x_r = \frac{1}{n}$

$m_r = \frac{(r-1)^2}{n^2}$

$$L(P, f) = \sum_{r=1}^{n} m_r \Delta x_r = \sum_{r=1}^{n} \frac{(r-1)^2}{n^2} \cdot \frac{1}{n}$$

$$= \frac{1}{n^3} \sum_{r=1}^{n} (r-1)^2$$

$$= \frac{1}{n^3} [0^2 + 1^2 + 2^2 + \dots + (n-1)^2]$$

$$= \frac{1}{n^3} \cdot \frac{(n-1)n(2n-1)}{6}$$

$$\frac{(n-1)(2n-1)}{6n^2}$$

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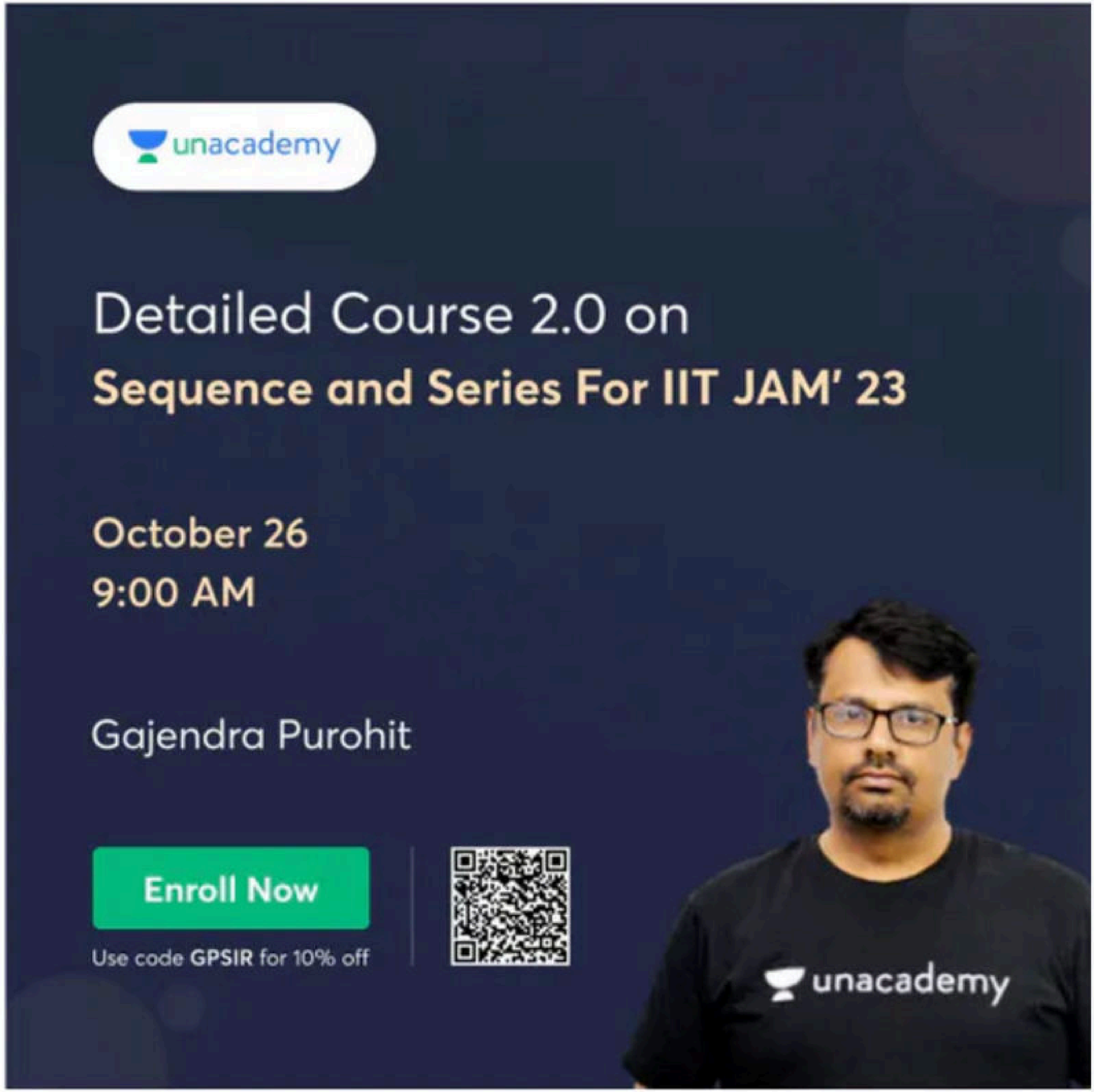
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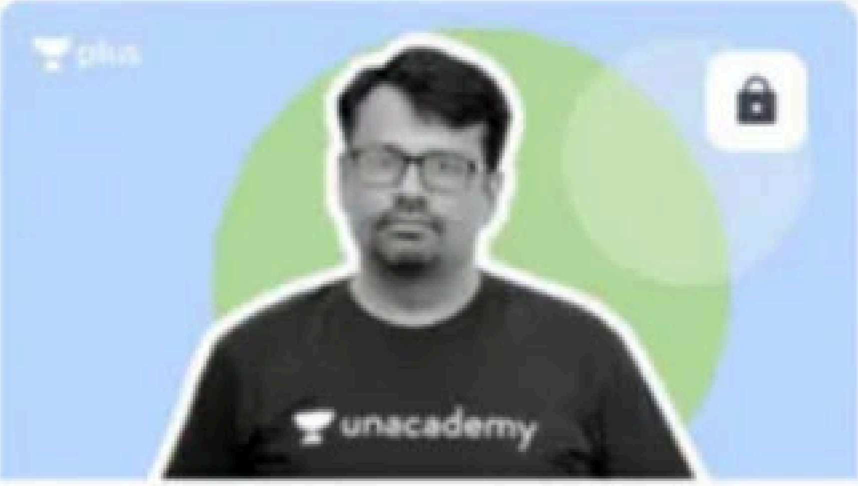

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- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
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