



Gajendra Purohit

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Function of one variable

Function : A mapping $f : A \rightarrow B$ is said to be a function if every element of A assign to unique element of B.

Domain and range of function :

(i) Domain of function : Let $f : D \rightarrow R$ is a real value function then D is called domain of function

Another definition : Let $f : D \rightarrow R$ is a real valued function. Then the set of all points of R at which the function is well define then this set is called as domain of function.

(ii) Range of function : Let $f : A \rightarrow B$ is a function. then the set of all images of distinct elements of A is called range of function.

Bounded and Monotonic function :

(i) Bounded function : A function $f : D \rightarrow R$ is called bounded function if range of this function is bounded.

(ii) Monotonic function :

(1) Monotonic increasing function :

Let $f : D \rightarrow R$ be a function, then f is called monotonic increasing function is for

$$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2); \text{ for all } x_1, x_2 \in D$$

(2) Monotonic decreasing function :

A function $f : D \rightarrow R$ is called monotonic decreasing if for $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in D$

One-One function : A real valued function $f : D \rightarrow R$ is called one-one function if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Conclusion :

- (1) If function is non-monotonic then this function is not one-one function.
- (2) If function is strictly monotonic then this function is one-one function.

Another trick for one-one function :

- (1) If graph of function intersect each line which parallel to x-axis at atmost one point.
- (2) If $f'(x) > 0$ or $f'(x) < 0$ i.e. $f(x)$ is strictly monotonic function then $f(x)$ is one – one

Q.1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ is one-one function, then which of the following is/are true?

- (a) $f(x)$ is strictly increasing function.
- (b) $f(x)$ is strictly decreasing function.
- (c) $f(x)$ is strictly monotonic function.
- (d) $f(x)$ is monotonic function.

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Onto Function : A function $f : A \rightarrow R$ is said to be onto if $f(A) = R$.

Note : A function which is onto or not onto it is totally depend on co-domain.



Check onto function by graph :

If graph of function intersect each line which parallel to x-axis and contain in co-domain, then this function is onto function.

Q.2. Let $f : \mathbb{R} \rightarrow \mathbb{R}^+$ is a non-monotonic function, then which of the following is true?

- (a) A function is always one-one.
- (b) A function may be one-one.
- (c) A function is always onto
- (d) A function may be onto

Composition of Function :

Let $f : A \rightarrow B$ and $g : B \rightarrow C$ are two real valued function
then $\underline{gof(x)}$ is composition of function, where
 $\underline{gof(x) = g[f(x)]}$.

Result : If $\underline{gof(x)}$ is one-one and onto, then $f(x)$ is one-one
and $g(x)$ is onto.

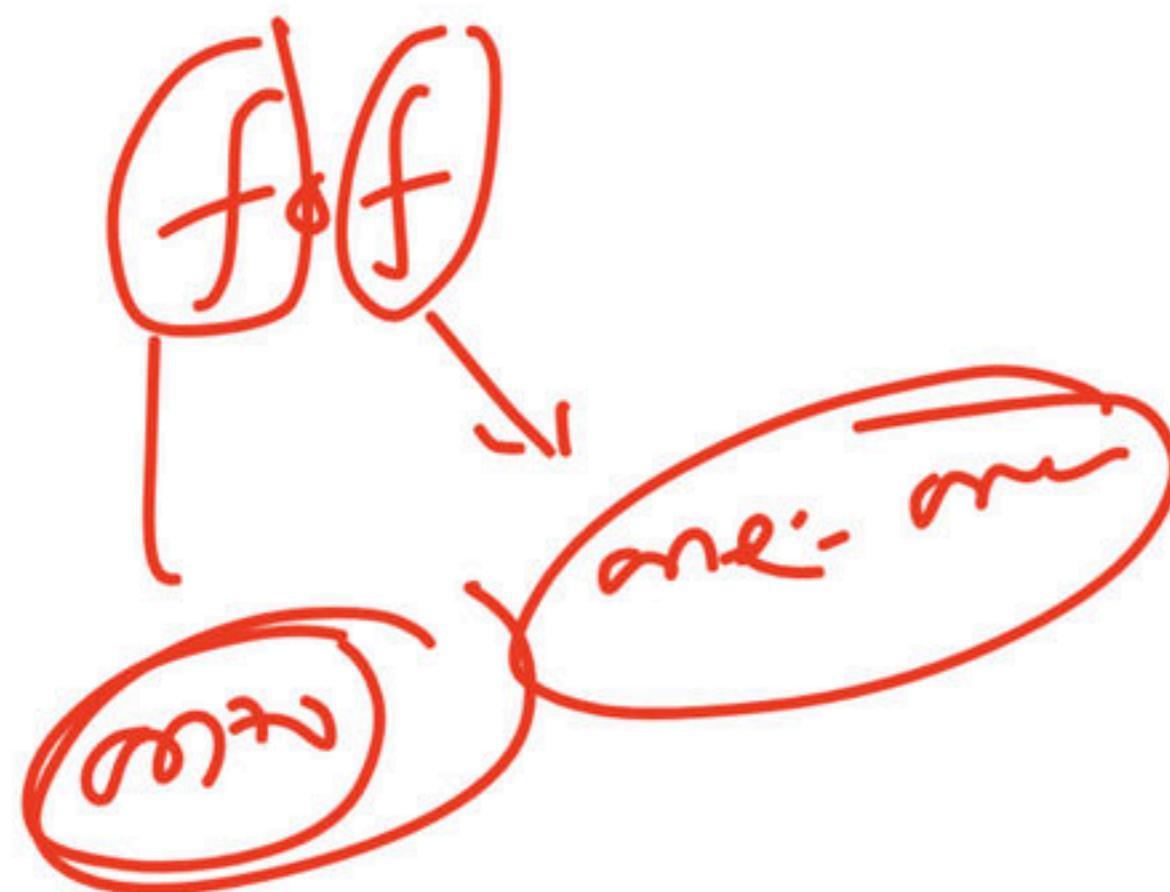
$f(x_1) = y$

$f : x \rightarrow x$

gof - one-one onto

Q.3. Let $f : X \rightarrow X$ such that $f(f(x)) = x$ for all $x \in X$. Then

- (a) f is one-to-one and onto.
- (b) f is one-to-one, but not onto
- (c) f is onto but not one-to-one
- (d) f need not be either one-to-one or onto



Limit of function : Let f be a function defined for all points in some neighbourhood of a point a .

We say that f tends to the limit l as x tends to a .

i.e. $\lim_{x \rightarrow a} f(x) = l$

Then l is called limit of function.

$$\frac{0}{0}$$

$$\lim_{n \rightarrow \infty} \frac{n^2 - 9}{n - 9}$$

$$\lim_{n \rightarrow \infty} (n^2 + n + 1) = \cancel{n^2} + 1 + 1 = 3$$

$$\lim_{n \rightarrow \infty} \frac{(n-\varsigma)(n+\varsigma)}{n-\varsigma} = 2\varsigma$$

$$\lim_{n \rightarrow \infty} \frac{2n}{1} = \underline{\underline{2\varsigma}}$$

Right hand limit : Let $f(x)$ be a function,

$\lim_{x \rightarrow a+0} f(x) = \underline{f(a+0)} = l$ then is called right hand limit.

Left hand limit : Let $f(x)$ be a function, then

$\lim_{x \rightarrow a-0} f(x) = \underline{f(a-0)} = l$ is called left hand limit of $f(x)$ at 'a'.

Note : Let $f(x)$ be a function then

$\lim_{x \rightarrow a} f(x) = \underline{f(a+0)} = \underline{f(a-0)}$.

$$f(x) = \begin{cases} \frac{x^2+1}{2x+1} & x < 0 \\ x^2, 0 & x \geq 0 \end{cases}$$

LHL

$$\lim_{x \rightarrow 0^-} (x^2 + 1) = 1$$

RHL

$$\lim_{x \rightarrow 0^+} (2x+1) = 1$$

$$f(x) = \begin{cases} \frac{x^2+1}{2x+1} & x < 1 \\ x^2, 1 & x \geq 1 \end{cases}$$

LHL

$$\lim_{x \rightarrow 1^-} (x^2 + 1) = 2$$

RHL

$$\lim_{x \rightarrow 1^+} (2x+1) = 3$$

$$\lim_{n \rightarrow \infty}$$

$$\frac{x - \underline{1m}}{n}$$

$$f(x) =$$
$$\begin{cases} \frac{x-n}{n}, & n < 0 \\ \frac{x}{n}, & n \geq 0 \end{cases}$$

Q.4. Let $f : (-1,1) \rightarrow \mathbb{R}$ and $g : (-1,1) \rightarrow \mathbb{R}$ be thrice continuously differentiable function such that $f(x) \neq g(x)$ for every non-zero $x \in (-1,1)$. Suppose $f(0) = \log 2$, $f'(0) = \pi$, $f''(0) = \pi^2$, $f'''(0) = \pi^9$ and $g(0) = \log 2$, $g'(0) = \pi$, $g''(0) = \pi^2$, $g'''(0) = \pi^3$ then the

value of limit $\lim_{x \rightarrow 0} \frac{e^{f(x)} - e^{g(x)}}{f(x) - g(x)}$ is IIT JAM 2022

(a) 1

$$\lim_{x \rightarrow 0} \frac{e^{f(x)} - e^{g(x)}}{f(x) - g(x)}$$

(c) 3

(d) 4

$$\boxed{\begin{aligned} f(0) &= \underline{\log 2} = \underline{g(0)} \\ f'(0) &= \underline{\pi} = \underline{g''(0)} \\ f''(0) &= \underline{\pi^2} = \underline{g'''(0)} \\ f'''(0) &= \underline{\pi^9}, \quad g'''(0) = \underline{\pi^3} \end{aligned}}$$

$$\frac{e^{f(x)} - e^{g(x)}}{f(x) - g(x)} = \frac{e^{f(x)} \cancel{[f'(x)[(f'(x))^2 + f''(x)]} + e^{f(x)} [2f''(x) + f''']}{e^{g(x)} \cancel{[g'(x)[(g'(x))^2 + g''(x)]} + e^{g(x)} [2g''(x) + g''']}$$

$$\lim_{n \rightarrow \infty} \frac{f^{(n)}[\underline{f''}] - e^{g^{(n)}} \underline{g''}}{f''' - g''} = \lim_{n \rightarrow \infty} \frac{e^{\log 2} \underline{\pi^9} - e^{\log 2} \underline{\pi^3}}{\pi^9 - \pi^3}$$

$$= \frac{e^{\log 2}}{2}$$

φ fin

$$\log \lim_{n \rightarrow \infty} y = \frac{1}{\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n}$$

$$\log \lim_{n \rightarrow \infty} y = 1$$

$$\lim_{n \rightarrow \infty} y = e$$

1 ∞

$$\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$$

$$y = (1 + \frac{1}{n})^n$$

$$\log y = \log (1 + \frac{1}{n})^n$$

$$\lim_{n \rightarrow \infty} \log y = \lim_{n \rightarrow \infty} \frac{\log (1 + \frac{1}{n})}{\frac{1}{n}}$$

$$\varphi \lim_{x \rightarrow a} \left[\frac{f(x)}{f(\varsigma)} \right]^{\frac{1}{\log a - \log \varsigma}}$$

(a) $\frac{af'(a)}{f(\varsigma)}$

(c) $\frac{af(\varsigma)}{f'(\varsigma)}$

$$\ln \log y - \ln \frac{\frac{1}{\varsigma} f'(x_1) - 0}{\frac{1}{\varsigma}} = \frac{af'(\varsigma)}{f(\varsigma)}$$

$$y = e^{\frac{af'(\varsigma)}{f(\varsigma)}}$$

(b) $e^{\frac{f''(a)}{f'(\varsigma)}}$

$$\log y = \frac{1}{\log a - \log \varsigma} \left(\frac{f(x)}{f(\varsigma)} \right)$$

$$\lim_{n \rightarrow \infty} \log y_n = \lim_{n \rightarrow \infty} \frac{1}{\log a - \log \varsigma} \cdot \log \left(\frac{f(x_n)}{f(\varsigma)} \right)$$

$$\lim_{n \rightarrow \infty} \log y_n = \lim_{n \rightarrow \infty} \frac{\log f(x_n) - \log f(\varsigma)}{\log a - \log \varsigma}$$

$\phi =$

$$\lim_{n \rightarrow \infty} n \left[\log \left(1 + \frac{x}{n} \right) - \log \frac{x}{n} \right]$$



$$\begin{aligned}
 & \lim_{n \rightarrow \infty} \log \left(\frac{1 + \frac{x}{n}}{\frac{x}{n}} \right) \\
 & \lim_{n \rightarrow \infty} \frac{\log \left(\frac{2}{n} + 1 \right)}{\frac{1}{n}} \\
 & \lim_{n \rightarrow \infty} \frac{\frac{1}{\frac{2}{n} + 1} \left(-\frac{2}{n^2} \right)}{-\frac{1}{n^2}} \\
 & = 2
 \end{aligned}$$

$$\lim_{n \rightarrow 0} \left(\frac{3n}{\eta^3} + \frac{a}{\eta^2} + b \right) = 0 \quad \text{then} \quad \frac{a^2}{2} + b$$

(a) $\frac{q_2}{2}$

(b) $\frac{q_4}{4}$

(c) 0

(d) $\frac{q_8}{8}$

$$\lim_{n \rightarrow 0} \frac{3n + q_n + b\eta^2}{\eta^3}$$

$$\lim_{n \rightarrow 0} \frac{3c_3\eta + q + 3b\eta^2}{3\eta^3}$$

$$\lim_{n \rightarrow 0} \frac{-9c_3\eta + 0 + 6b\eta^2}{5\eta^3}$$

$$\lim_{n \rightarrow 0} \frac{-27c_3\eta + 6b}{\eta^3} = -\frac{27 + cb}{\eta} = 0$$

$$cb = 27 \\ b = \frac{27}{c}$$

$q = -3$

Q.5. Which of the following is false

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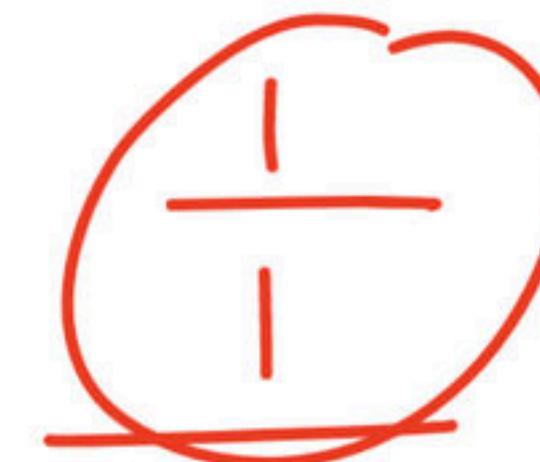
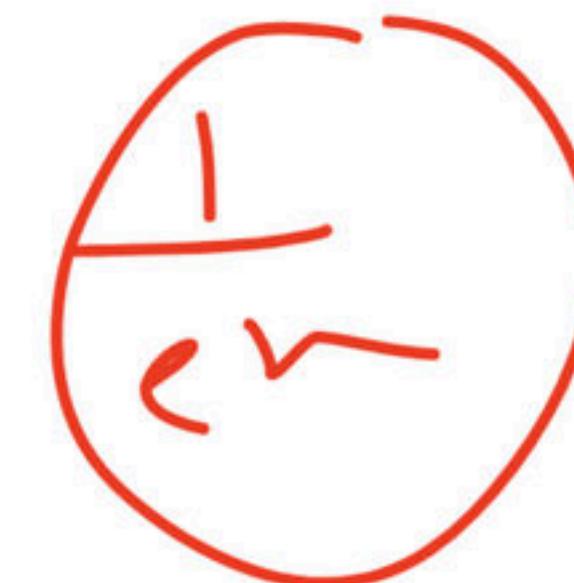
T (a) $\lim_{x \rightarrow \infty} \frac{x}{e^x} = 0$

T (b) $\lim_{x \rightarrow 0^+} \frac{\sin x}{1+2x} = 0$

T (c) $\lim_{x \rightarrow 0^+} \frac{1}{xe^{\frac{1}{x}}} = 0$

(d) $\lim_{x \rightarrow 0^+} \frac{\cos x}{1+2x} = 0$

$$\frac{-\frac{1}{x}}{x}$$



Q.6. The value of $\lim_{x \rightarrow 0} \frac{|x| - x}{2x}$ is

- (a) 0
- (b) 2
- (c) 1
- (d) does not exist

Handwritten notes for the limit problem:

- Top right: $(-1)^n < 0$
- Top left: $\frac{2^n}{2^n}$
- Bottom: $\frac{n-n}{2^n}$
- Right side: $n \geq 0$

A red line connects the note $\frac{n-n}{2^n}$ to the option (d) "does not exist".

Q.7. The value of $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1 - \cos x}}$

(a) $\sqrt{2}$

(b) $-\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) $-\frac{1}{\sqrt{2}}$

$\lim_{x \rightarrow 0}$

$\frac{\sqrt{2}}{\sqrt{1 - \cos x}}$

$\lim_{x \rightarrow 0}$

$\frac{1}{\sqrt{1 - \cos x}}$

$\frac{1}{\sqrt{1 - \frac{1}{2}}} = \sqrt{\frac{1}{\frac{1}{2}}} = \sqrt{2}$

Q.8. Let f be a monotone non-decreasing real-valued function on \mathbb{R} . Then

- (a) $\lim_{x \rightarrow a} f(x)$ exists at each point a .
- (b) If $a < b$, then $\lim_{x \rightarrow a^+} f(x) \leq \lim_{x \rightarrow b^-} f(x)$.
- (c) f is an unbounded function
- (d) The function $g(x) = e^{-f(x)}$ is a bounded function.

Q.9. Let p be a real polynomial of the real variable x of the form $p(x) = x^n + a_{n-1}x^{n-1} + \dots + a_1x + 1$. Suppose that p has no roots in the open unit disc and $p(-1) = 0$. Then

- (a) $p(1) = 0$
- (b) $\lim_{x \rightarrow \infty} p(x) = \infty$
- (c) $p(2) > 0$
- (d) $p(3) = 0$

Continuity

Continuous function at a point ‘a’ :

Let $f : A \rightarrow R$ be a real valued function. This function is continuous at $x = \alpha \in A$

if $\lim_{x \rightarrow \alpha} f(x) = f(\alpha)$.

Continuity of some important function :

(1) Polynomial function : Let $f(x) = a_nx^n + \dots + a_1x + a_0$
then $\lim_{x \rightarrow \alpha} f(x) = f(\alpha) \Rightarrow f(x)$ is continuous at any point.

(2) Constant function : Let $f(x) = \alpha$.

Here $\lim_{x \rightarrow a} f(x) = f(\alpha) = \alpha$.

Constant function is always continuous at a .

(3) Rational function :

Let $p(x)$ and $q(x)$ are polynomial, then $f(x) = \frac{p(x)}{q(x)}$

is called rational function.

Hence $f(x)$ is continuous at c , if $q(c) \neq 0$

(4) Trigonometry function :

(a) $f(x) = \sin x$

(5) Exponential function :

(6) Logarithmic function :

Sequential definition : Let $f : A \rightarrow R$ be a function and f is continuous at ' c ' iff for every sequence $\langle x_n \rangle$ converging to c , then $\langle f(x_n) \rangle$ converging to $f(c)$.

i.e. if \exists two sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ s.t. $\langle x_n \rangle \rightarrow c$ and $\langle y_n \rangle \rightarrow c$.

but if $\langle f(x_n) \rangle$ and $\langle f(y_n) \rangle$ converging to distinct limits then $f(x)$ is not continuous at c .

Result :

Let $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by

$$f(x) = \begin{cases} g(x), & x \in Q \\ h(x), & x \in Q^c \end{cases}$$

Then $f(x)$ is continuous at all zeros of $g(x) - h(x) = 0$.

Q.1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} x^6 - 1 & x \in Q \\ 1 - x^6 & x \in Q^c \end{cases}$.

The number of points at which $f(x)$ is continuous is

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- (a) 1 (b) 2
(c) 3 (d) 4

Result : Let $f : [0, 1] \rightarrow \mathbb{R}$ is a function such that

$$f(x) = \begin{cases} x^\alpha \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Then $f(x)$ is continuous iff $\alpha > 0$

Q.2. Which of the following values of α , the function

$$f(x) = \begin{cases} x^\alpha \sin \frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$
 is continuous

- (a) $1/2$
- (b) 1
- (c) 2
- (d) $-1/3$

Q.3. Let the function $f(x)$ be defined by

$$f(x) = \begin{cases} e^x & x \text{ is rational} \\ e^{1-x} & x \text{ is irrational} \end{cases}$$

for x in $(0,1)$ then

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- (a) f is continuous at every point in $(0,1)$
- (b) f is discontinuous at every point in $(0,1)$
- (c) f is discontinuous at only one point in $(0,1)$
- (d) f is continuous at only one point in $(0,1)$



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- 📍 Works at Pacific Science College
- 📍 Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
- 📍 PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- 📍 Lives in Udaipur, Rajasthan, India
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