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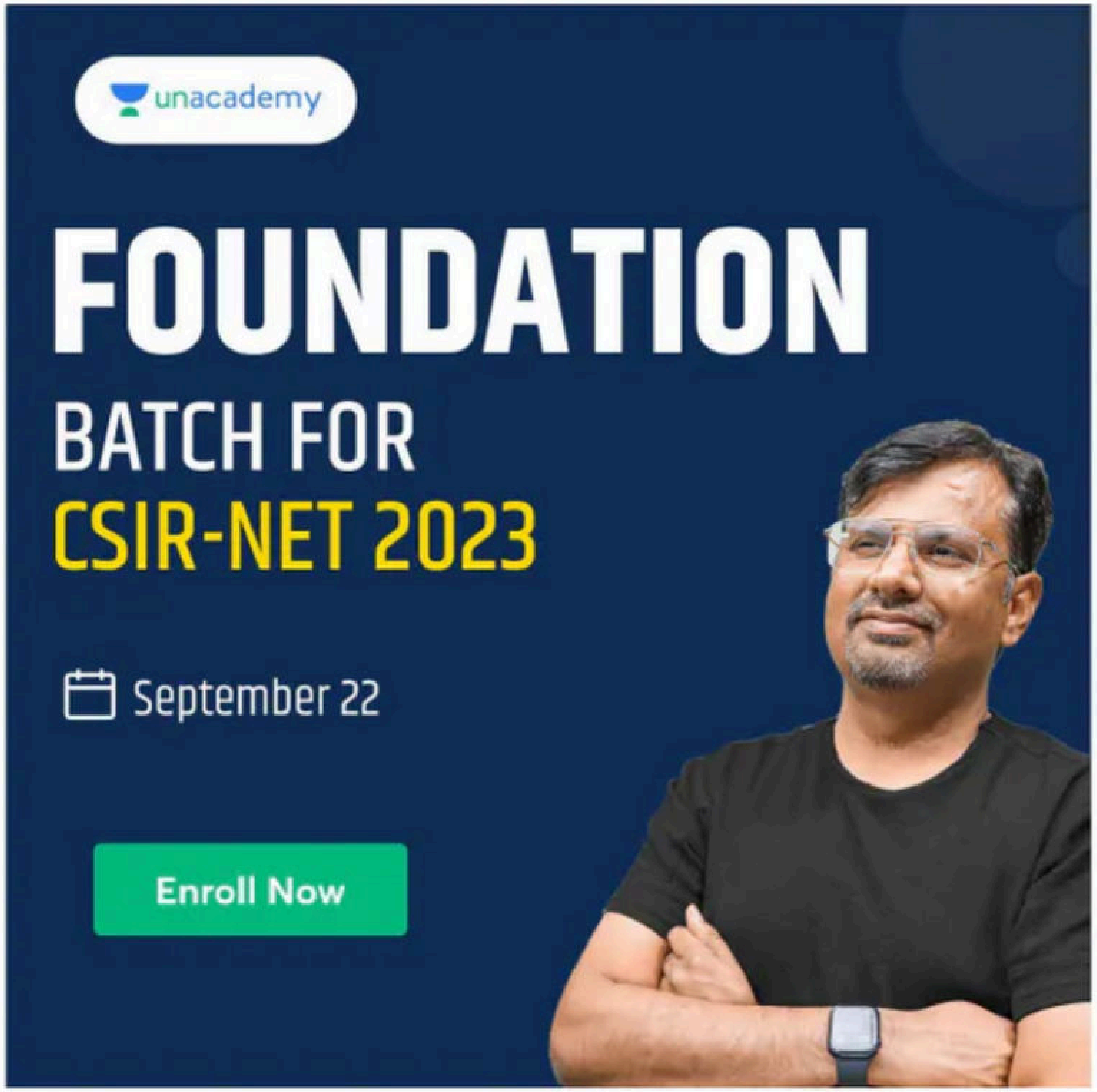
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Result :Let W_1& W_2 are two subspace of V then $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

Result :Let V be a vector space of dimension n and W be a subspace of V with m Linearly independent condition then $\dim V = n - m$

$$W_1 = \{(x,y,z) \in \mathbb{R}^3 \mid 3x+y=0\}$$

$$W_2 = \{(x,y,z) \in \mathbb{R}^3 \mid z=0\}$$

$$W_1 \cap W_2 = \{(x,y,z) \in \mathbb{R}^3 \mid 3x+y=0,\ z=0\}$$

$\dim(W_1 \cap W_2)$

$\dim(W_1 + W_2)$

$$\dim W_1 = 3-1 = 2$$

$$\dim W_2 = 3-1 = 2$$

$$\dim(W_1 \cap W_2) = 3-2 = 1$$

$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$$

$$= 2+2-1$$

$$= 3$$

Q.1 Let $W_1 = \{(u, v, w, x) \in R^4 \mid u + v + w = 0,\ 2v + x = 0,\ 2u + 2w - x = 0\}$ & $W_2 = \{(u, v, w, x) \in R^4 \mid u + w + x = 0,\ u + w - 2x = 0,\ v - x = 0\}$. Then which among the following is true.

(a) $\dim(W_1) = 1$ (b) $\dim(W_2) = 2$

(c) $\dim(W_1 \cap W_2) = 1$ (d) $\dim(W_1 + W_2) = 3$

$\dim W_1 = 4 - 2 = 2$

$\dim W_2 = 4 - 3 = 1$

$\dim(W_1 \cap W_2) = 4 - 3 = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & -2 & 0 & -1 \\ 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

$R_4 \to 2R_4 + R_2$

$R_5 \to 2R_5 + R_2$

$R_6 \to 2R_6 - R_2$

$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_2)$

$= 2 + 1 - 1 = 2$

Q.2. Consider the subspace $W = \{(x_1, x_2, \ldots, x_{10}) \in R^{10};$ $\boxed{x_n = x_{n-1} + x_{n-2}}$ for $3 \le n \le 10\}$ of the vector space R^{10}.

The dimension of W is

(a) 2 (b) 3

(c) 9 (d) 10

$(10 - 8) = 2$

$x_3 = x_2 + x_1$

$x_4 = x_3 + x_2$

$x_5 = x_4 + x_3$

$x_6 = x_5 + x_4$

$x_7 = x_6 + x_5$

$x_8 = x_7 + x_6$

$x_9 = x_8 + x_7$

$x_{10} = x_9 + x_8$

Q.3. Let $W_1 = \{(x, y, z) \in R^3; 3x + y = 0\}$ & $W_2 = \{(x, y, z) \in R^3; z = 0\}$. Then $\dim (W_1 \cap W_2)$ is

(a) 0 (b) 1

(c) 2 (d) 3

Q4. Let V be the vector space of all 2×2 matrices over R.

Consider the subspace $W_1 = \left\{ \begin{bmatrix} a & -a \\ c & d \end{bmatrix}; a, c, d \in R \right\}$ &

$W_2 = \left\{ \begin{bmatrix} a & b \\ -a & d \end{bmatrix}; a, b, d \in R \right\}$.

2

If m = dim($W_1 \cap W_2$) & n = dim($W_1 + W_2$) then m + n is

(a) 5 (b) 6

(c) 7 (d) 8

$\dim W_1 = 3$

$\dim W_2 = 3$

$\underbrace{\dim(W_1+W_2)}_{n} = \dim W_1 + \dim W_2 - \underbrace{\dim(W_1 \cap W_2)}_{m}$

$n + m = 3 + 3$

Q.5. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{bmatrix}$ and let $V = \{(x, y, z) \in R^3; \det A = 0\}$.

$(2y + \cancel{2z} + 3x) - (\cancel{2z} + 2x + 3y) = 0$

$-y + x = 0$

$x - y = 0$

Then dimension of V equals to

(a) 0 (b) 1

(c) 2 (d) 3

$V = \{(x, y, z) \in R^3 : x - y = 0\}$

$3 - 1 = 2$

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Linear Transformation

Let V and V\` be vector space over F. A mapping $T : V \rightarrow V'$ is a linear transformation if for all $u, v \in V$ & $\alpha, \beta \in F$

$$T(\alpha u + \beta v) = \alpha T(u) + \beta T(v) \text{ \& } T(\alpha u) = \alpha T(u); \text{ for all } \alpha \in F$$

$V = \mathbb{R}^2(\mathbb{R})$ $\quad V' = \mathbb{R}^3(\mathbb{R})$

$T : V \to V'$

$T(-x,-y) = -T(x,y)$

$T(x,y) = (x+y,\ x-y,\ 0)$

$T(0,0) = 0$

$T(\alpha u + \beta v)$ $\quad u = (x_1, y_1)$, $v = (x_2, y_2)$

$T(\alpha(x_1,y_1) + \beta(x_2,y_2)) = T(\alpha x_1 + \beta x_2,\ \alpha y_1 + \beta y_2)$

$= (\alpha x_1 + \beta x_2 + \alpha y_1 + \beta y_2,\ \alpha x_1 + \beta x_2 - \alpha y_1 - \beta y_2,\ 0)$

$= (\alpha(x_1+y_1) + \beta(x_2+y_2),\ \alpha(x_1-y_1) + \beta(x_2-y_2),\ 0)$

$= (\alpha(x_1+y_1),\ \alpha(x_1-y_1),\ 0) + (\beta(x_2+y_2),\ \beta(x_2-y_2),\ 0)$

$= \alpha(x_1+y_1,\ x_1-y_1,\ 0) + \beta(x_2+y_2,\ x_2-y_2,\ 0)$

$= \alpha T(u) + \beta T(v)$

$T: R^2(R) \to R^3(R)$

$T(x,y) = (x+y,\ x-y+1,\ 5)$

$T(x,y) = (xy,\ x^2-y^2,\ x+y)$

$T(0) \neq 0$

$\underline{T(0)=0}$

$\underline{T(-v)=-T(v)}$

$T: R^n \to R^n$

$T(x_1, x_2 \ldots x_n) = (x_n, x_{n-1} \ldots x_1) = 0$

$T(x_1, x_2 \ldots x_n) = (x_1+x_2, x_2+x_3, \ldots x_n+x_{n+1})$

$T(x_1, x_2 \ldots x_n) = (x_1, x_2 \ldots 0)$

$T(x_1, x_2 \ldots x_n) = (0, x_1, x_2 \ldots x_{n-1})$

$T: R^2(R) \to R^3(R)$

$T(x, y) = (x+y, x-y, 0) = (0, 0, 0)$

$x+y=0, \quad x-y=0,$

$x=0$

$y=0$

$\ker T = (0, 0)$

$$\mathbb{R}^{n\times n} = \{A = [a_{ij}] \mid a_{ij} \in \mathbb{R}\}$$

$$T: \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$$

$$T: \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$$

$$T: \mathbb{R}^{n\times n} \to \mathbb{R}^{n\times n}$$

$$T(A) = A^T = 0$$

$$A = -A^T$$

$$T(A) = \frac{A + A^T}{2} = 0$$

$$T(A) = \operatorname{tr}(A) = 0$$

$R(x) = \{ p(x) \mid p(x) \text{ is polynomial over } K \}$

~~$T: R(x) \longrightarrow K(x)$~~

$T(p(x)) = p'(x)$

$T(p(x)) = p''(x)$

$T(p(x)) = \int_0^x p(x)\,dx$

$T(p(x)) = p'(0)$

$T: R_n(x) \longrightarrow R_n(x)$

$T(p_n(x)) = \int_0^x p_n(x)\,dx$

$\int x^3 dx$

x^4

Note : Let V & V' be vector space over F & $T : V \rightarrow V'$ be a linear transformation. Then

(a) $T(0) = 0; 0 \in V$

(b) $T(-v) = -T(v)$; for all $u \in V$

Q.1. Which of the following is a linear transformation from R^3 to R^2?

$f(0) \neq 0$

$f(-v) = -f(v)$

(A) $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ x+y \end{pmatrix}$

(B) $g\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} xy \\ x+y \end{pmatrix}$

(C) $h\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z-x \\ x+y \end{pmatrix}$

(a) Only f

(b) Only g

(c) Only h

(d) All of the above

$T(x, y) = ((x, 0)) = (0, 0)$

$x \to$ y

$(0, y)$

$T: R^3(R) \to R^3(R)$

$x =$ $y =$ $z = x$

$T(x, y, z) = \langle x+y+z,\ 2x+2y+3z,\ 3x+3y+4z \rangle = 0$

$x+y+z=0$

$2x+2y+3z=0$

$3x+3y+4z=0$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 3 & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Null space of Linear transformation : Let $T : V(F) \to V'(F)$ be a linear transformation then null space of T is the set of all vectors of V(F) s.t. $T(u) = 0$; (zero vector of V') and it is denoted by ker (T) i.e. $\ker(T) = \{u \in V(F) : T(u) = 0\}$

Note : If V(F) & V'(F) be two vector space & $T : V(F) \to V'(F)$ be linear transformation then ker(T) is subspace of V(F).

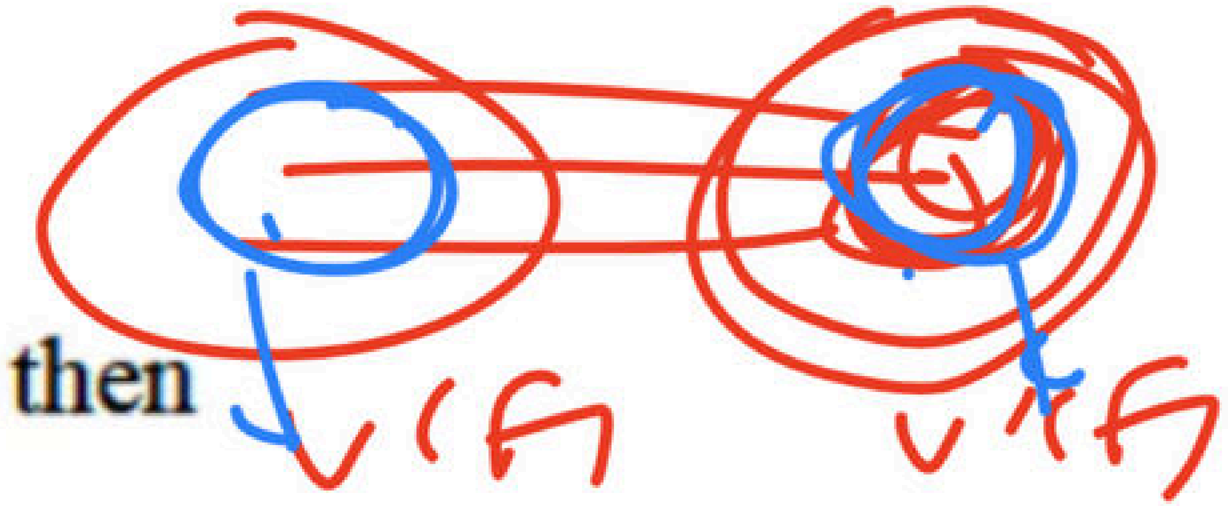
dim(Ker T) = nullity n(T)

dim(null space) = n(T)

Range of linear transformation : Let V(F) & V'(F) be two vector space and $T : V \to V'$ be a linear transformation. Then the Range of T written as R(T) is the set of all vectors β in V' such that $\beta = T(\alpha)$ for some α in V.

i.e. Range $(T) = \{T(\alpha) \in V' \mid \alpha \in V\}$

$T : V(F) \to V'(F)$



Note : Let $T : V(F) \to V'(F)$ be a linear transformation then Range(T) is subspace of V'(F).

dim(Range space) = Rank T

Rank & Nullity of Linear Transformation :

Let $T : V(F) \rightarrow V'(F)$ be a linear transformation & Range (T) & ker(T) are range space of T & null space of T then dim

$\{\text{Range}(T)\} = \rho(T) =$ rank of T

& dim $\{\ker(T)\} = \eta(T) =$ nullity of T

Sylvester's Law : Let $T : \boxed{V(F)} \rightarrow V'(F)$ be linear transformation then $\rho(T) + \eta(T) = \dim V(F)$

$n(T) = 0$ one - one

$\rho(T) = \dim V'(F)$

onto

$T : B \to 6J$

$Ax = 0$

$(1, x, x^2, x^3)$

$T(p(x)) = p''(x) + p(x)$

$T(1) = 0 + 1$

$T(x) = 0 + x$

$T(x^2) = 2 + x^2$

$T(x^3) = 6x + x^3$

$$T = \begin{array}{c|cccc} & T(1) & T(x) & T(x^2) & T(x^3) \\ 1 & 1 & 0 & 2 & 0 \\ x & 0 & 1 & 0 & 6 \\ x^2 & 0 & 0 & 1 & 0 \\ x^3 & 0 & 0 & 0 & 1 \end{array}$$

$\boxed{P(T) = 4}$

$N(T) = 0$

One-One linear transformation : Let $T : V \to V'$ be a linear transformation with $\eta(T) = 0$ then T is called one-one linear transformation.

Onto linear transformation : Let $T : V \rightarrow V'$ be a linear transformation with $\rho(T) = \dim V'$ Then T is called onto linear transformation.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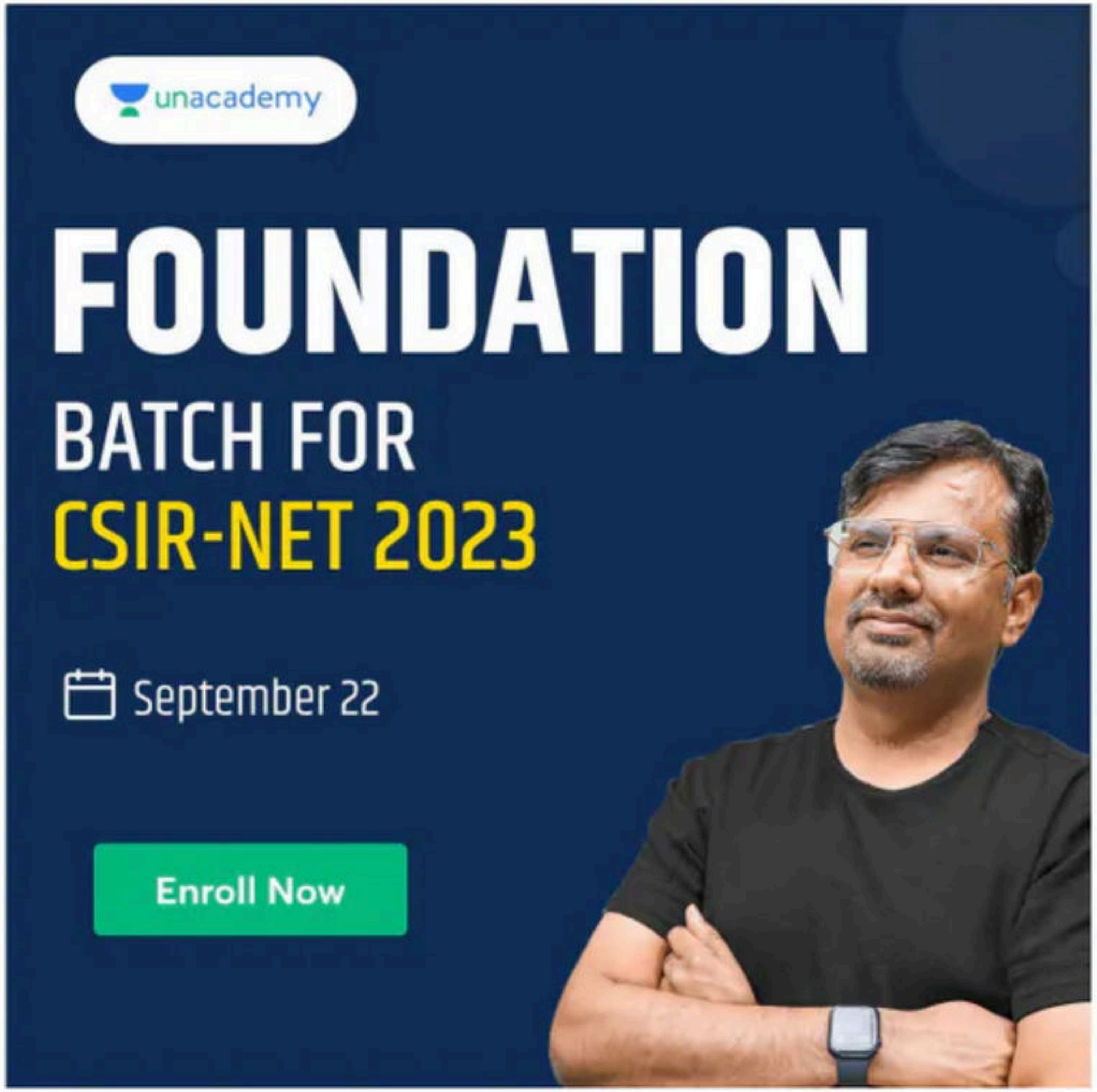
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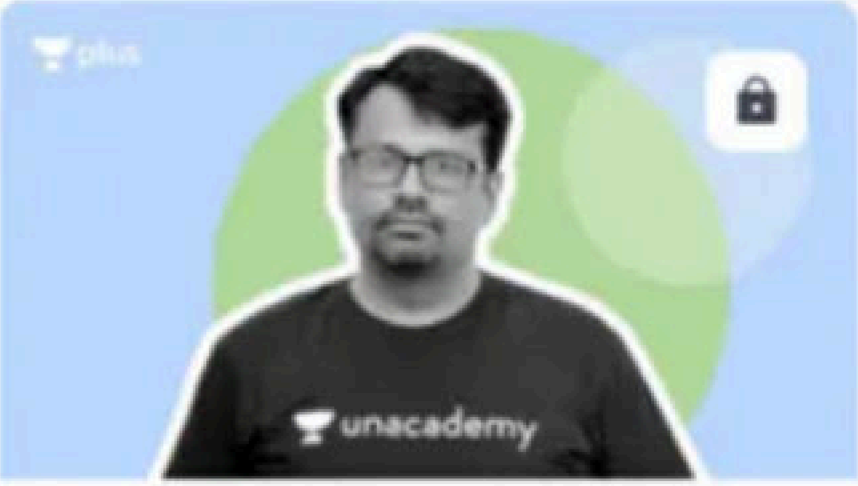

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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