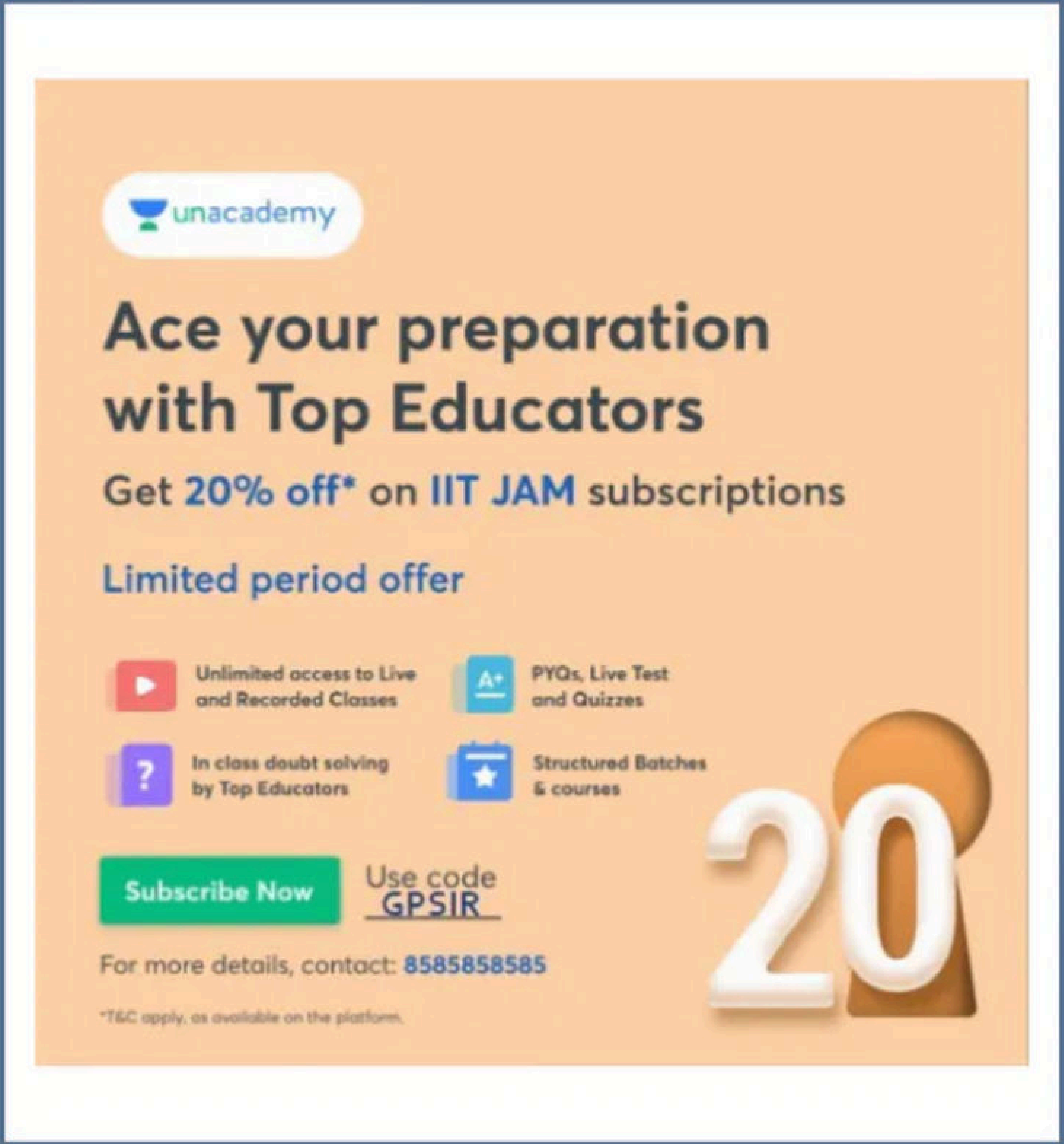
Matrix Representation - Part I

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 7 • Sept 17, 2022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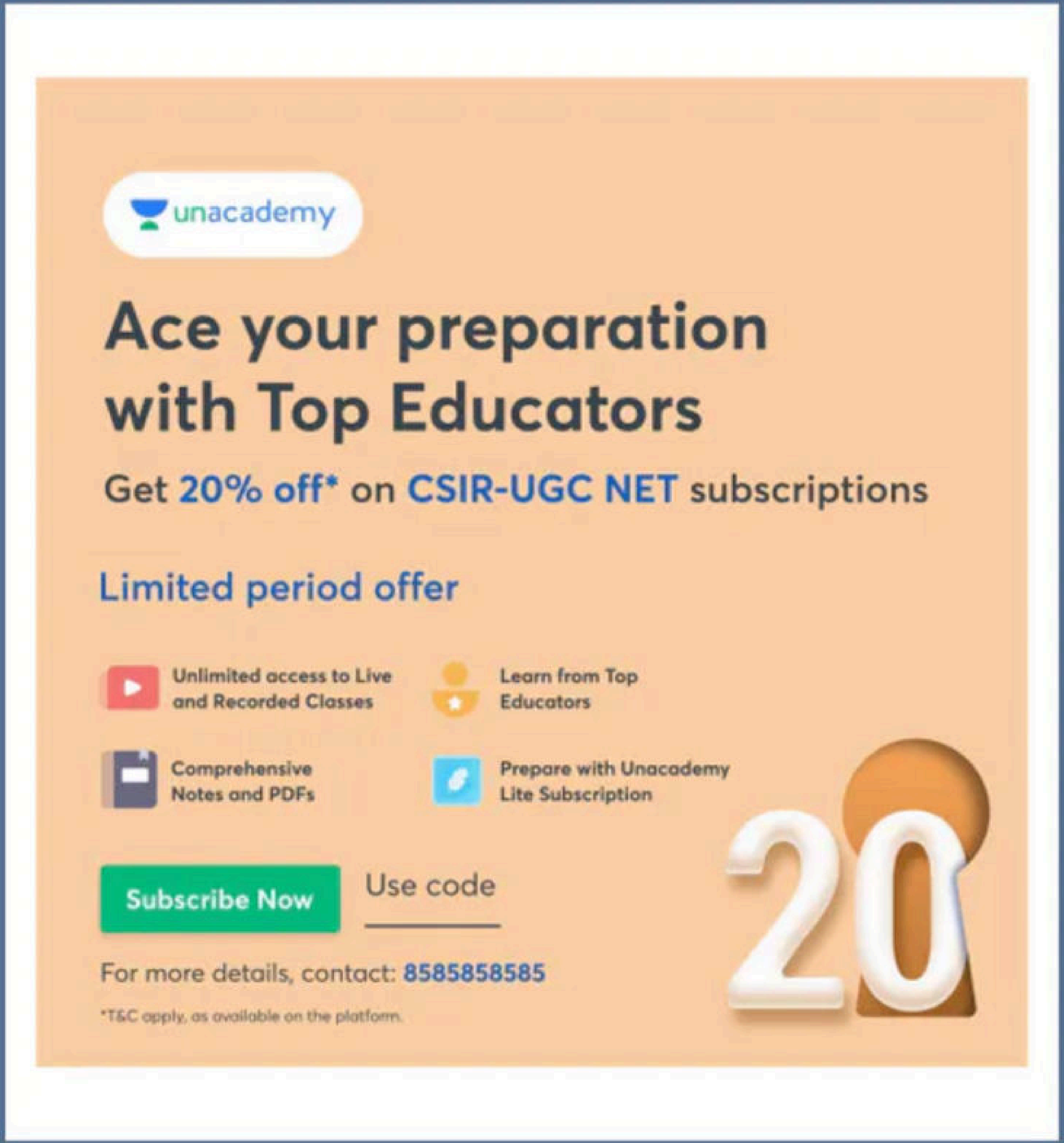
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Rank of matrix : If A is matrix then a non-negative integer is said to be rank of A. if ∃ a non-singular submatrix of order r of A and all matrix of order greater than r are singular.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 5 \\ 3 & 4 & 2 \end{bmatrix}$$

Note :

(1) A matrix $A_{n\times n}$ is a non-singular iff rank(A) = n

(2) Rank of matrix is denoted by $\rho(A)$

(3) If $|A| \neq 0$ then $\rho(A) = n$ where n is order of A

$\rho(A) = 3$ — $|A| \neq 0$

$\rho(A) = ?$ — $|A| = 0$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$R_3 \to R_3 - 4R_1$

$4 - 4 \qquad 3 - 3$

$6 - 8 \qquad 4 - 6$

$8 - 12 \qquad 5 - 9$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & -2 & -4 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

$$A = \begin{bmatrix} 1 & 4 & 8 & 7 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & -14 & -29 & -27 \\ 0 & 0 & 0 & -19 \\ 0 & 0 & 3 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1$

$R_3 \to R_3 - 3R_1$

(a) 3 (b) 1

(c) 2 (d) 4

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 \rightleftharpoons R_3$

$R4 \rightarrow R4 + R_2$

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R4 \rightarrow R_4 + R_3$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_5 \rightarrow R_5 + R_4$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & 5 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 5 & 2 & 0 \\ 3 & -1 & 1 & 3 \\ -1 & 2 & 1 & -1 \\ 0 & 5 & 2 & 0 \end{bmatrix}$$

$$|A| = -15 + 10 - 5 = -10$$

$$\operatorname{adj} A = \begin{bmatrix} -15 & 7 & 6 \\ 5 & -3 & -4 \\ -5 & -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{-10}\begin{bmatrix} -15 & 7 & 6 \\ 5 & -3 & -4 \\ -5 & -1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}, \quad \operatorname{adj} A = \begin{bmatrix} -11 & a & 1 \\ b & -2 & -3 \\ 10 & c & 7 \end{bmatrix}$$

$a+b+c =$

(a) 5 (b) -5

(c) 10 (d) -10

$$-9+4-5 = -10$$

$$\begin{bmatrix} 3 & 5 & -2 & 3 \\ 1 & -2 & -1 & 1 \\ 2 & 0 & 1 & 2 \\ 3 & 5 & -2 & 3 \end{bmatrix}$$

$$\begin{pmatrix} -11 & -9 & 1 \\ 4 & -2 & -3 \\ 10 & -5 & 7 \end{pmatrix}$$

Q7

$$A = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} a & \cdot & \cdot \\ \cdot & \cdot & b \\ c & \cdot & \cdot \end{bmatrix}$$

$(5+0-12=)$

$\underline{a+b+c=}$

(a) 1

(b) -2

(c) -1

(d) 2

$$\begin{bmatrix} 3 & 0 & 0 & \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 5 & 0 \\ 3 & 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 3 & 0 & -6 \\ 0 & 1 & 0 \\ -6 & 0 & 15 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -2 \\ 0 & \frac{1}{3} & 0 \\ -2 & 0 & 5 \end{pmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \Bigg| \qquad \operatorname{adj} A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix} \qquad A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{12-2}\begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix}$$

$$A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix}$$

$a + b =$

~~(a) $\frac{7}{20}$~~

(b) $\frac{3}{20}$

(c) $\frac{19}{60}$

(d) $\frac{11}{20}$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 0.1 \\ 0 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{0.1}{6} \\ 0 & \frac{1}{3} \end{bmatrix}$$

$$\frac{0.1}{6} + \frac{1}{3} = \frac{0.1 + 2}{6} = \frac{21}{60}$$

$$M = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & 3/5 \end{bmatrix}$$

$$M^T = M^{-1}$$

$$MM^T = I$$

$$x = ?$$

(a) $-\frac{4}{5}$ (b) $-3/5$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

$$\frac{3}{5}x + \frac{12}{25} = 0$$

$$\frac{3}{5}x = -\frac{12}{25}$$

$$x = -4/5$$

$$\varphi = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix} \qquad \varphi^{-1} = \begin{bmatrix} a & b & \cdot \\ \cdot & \cdot & c \\ \cdot & \cdot & d \end{bmatrix}$$

$a+b+c+d=$

(a) $\frac{13}{49}$ (b) $\frac{26}{49}$

(c) $\frac{9}{49}$ (d) 0

$$\varphi\varphi^{T} = \underline{I}$$

$$\varphi^{T} = \underline{\varphi^{-1}}$$

$$\varphi^{T} = \begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$$

$$-\frac{18}{49} + \frac{6}{49} + \frac{12}{49}$$

Elementary Transformation : Those transformation that does not effect on rank of matrix are called elementary transformation.

There are 3 elementary row (column) transformation.

Elementary matrix : A matrix obtained by a single elementary operation over identity matrix is known as elementary matrix.

Q.1. Find elementary matrix

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 2 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

Note :

(1) Every elementary matrix is non-singular.

(2) Every permutation matrix is elementary matrix.

Q.2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 151 & 262 & 373 & 484 \end{bmatrix}$.

Then Rank(A) is

(a) 1 (b) 2

(c) 3 (d) 4

Q.3. Let $A=\begin{bmatrix}1 & 1 & 1 & \cdots & 1\\ 1 & 2 & 2 & \cdots & 2\\ 1 & 2 & 3 & \cdots & 3\\ 1 & 2 & 3 & \cdots & 4\\ \vdots & & & & \\ 1 & 2 & 3 & \cdots & n\end{bmatrix}$, then $\rho(A)$ is

(a) 0 (b) 1

(c) 2 (d) n

Property :

1. A matrix is of Rank zero iff A is null matrix i.e. if A is non-null matrix then $\rho(A) \geq 1$

2. Rank of idempotent matrix of order n are always less then n except identity matrix.

3. Rank of involutory matrix of order are always n because determinant of involutory matrix are non-zero.

4. Rank of orthogonal matrix of order n is n.

5. Rank of nilpotent matrix of order n are always less than n.

Q.4. Let $S = \{A = [a_{ij}]_{n\times n} \mid A^k = 0 \ \& \ \rho(A) = n, \text{ for some } k\}$ then cardinality of S is

(a) ϕ (b) 1

(c) 2 (d) n

(6) Let A be a matrix of order $m \times n$ then

$\rho(A) \leq \min\{m, n\}$

(7) Let A & B are matrix of order $m \times n$ & $n \times p$, then $\rho(A) + \rho(B) - n \leq \rho(AB) \leq \min\{\rho(A), \rho(B)\}$

(8) Let A and A^T are matrix then $\rho(A) = \rho(A^T)$

(9) $\rho(A + B) \leq \rho(A) + \rho(B)$

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Q.5. Let $A = [a_1\ a_2\ \ldots..\ a_n]^T_{1\times n}$ and $B = [b_1\ b_2\ \ldots..\ b_n]_{1\times n}$ both are non-zero matrix then $\rho(AB)$ is

(a) 1 (b) n

(c) n + 1 (d) n – 1

Q.6 Let A be a 3×4 matrix and B be a 4×3 matrix with real entries such that AB is non- singular. Consider the following statements :

P : Nullity of A is 0.

Q : BA is a non-singular matrix.

Then

(a) both P and Q are true

(b) P is true and Q is false

(c) P is false and Q is true

(d) Both P and Q are false

Q.7. Let A be an $n \times n$ matrix such that the first 3 rows of A are linearly independent and the first 5 columns of A are linearly independent. Which of the following statements are true?

(a) A has at least 5 linearly independent rows

(b) $3 \leq \text{rank } A \leq 5$

(c) Rank $A \geq 5$ (d) Rank $A^2 \geq 5$

Q.8. What is the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$.

(a) 2 (b) 3

(c) 4 (d) 5

Q.9 Let J denote the matrix of order $n \times n$ with all entries 1

and let B be a $(3n)\times(3n)$ matric given by $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$

. Then the rank of B is

(a) $2n$ (b) $3n - 1$

(c) 2 (d) 3

Q.10. Let A be an $n \times m$ matrix with each entry equal to +1, -1 or 0 such that every column has exactly one +1 and exactly one -1. We can conclude that

(a) $\text{Rank } A \le n-1$ (b) $\text{Rank } A = m$

(c) $n \le m$ (d) $n-1 \le m$

Q.11 Let A and B be $n \times n$ real matrices such that $AB = BA = 0$ and $A + B$ is invertible.

Which of the following are always true?

(a) Rank (A) = rank (B)

(b) Rank (A) + rank $(B) = n$

(c) Nullity (A) + nullity $(B) = n$

(d) $A - B$ is invertible


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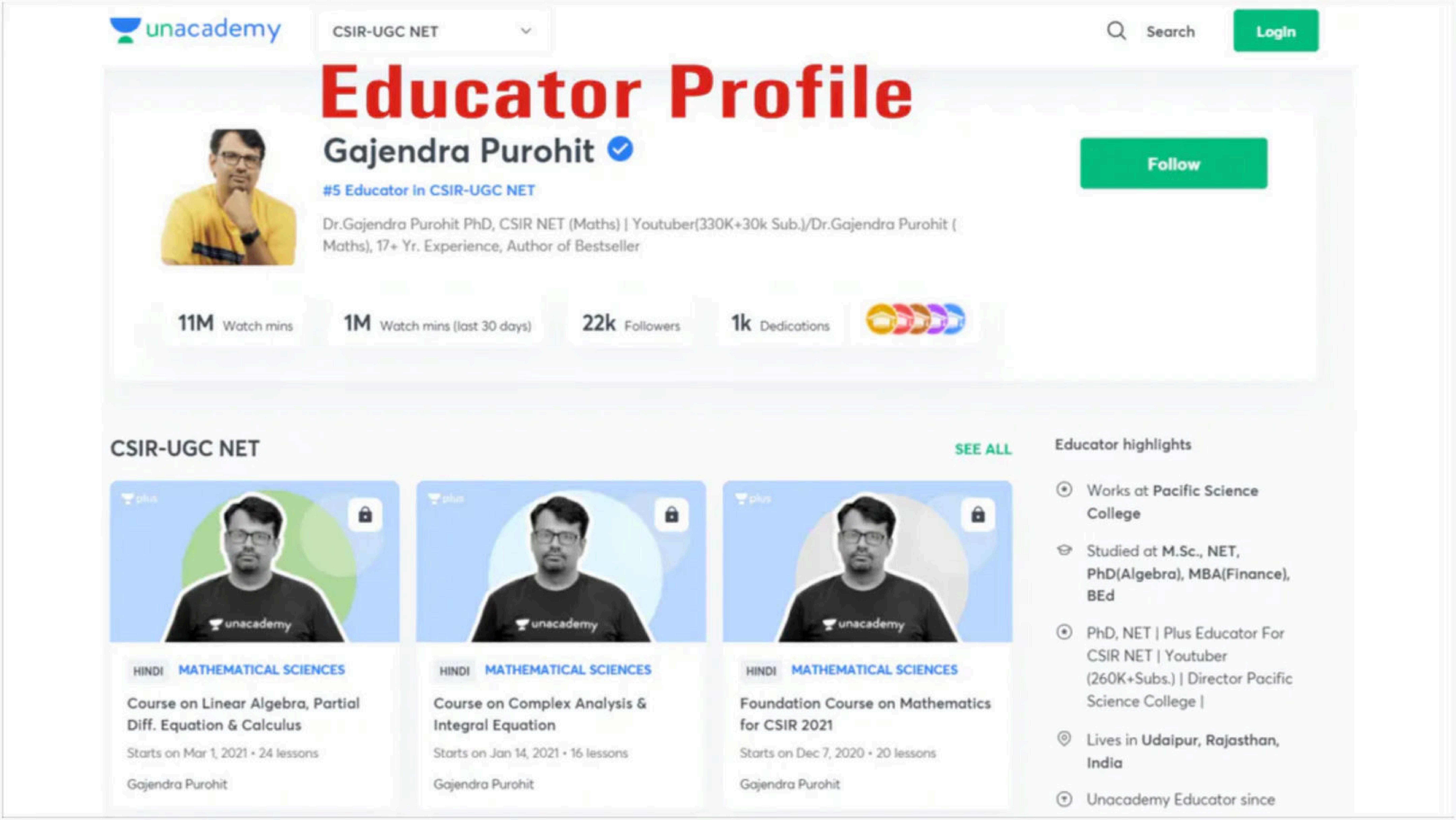
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