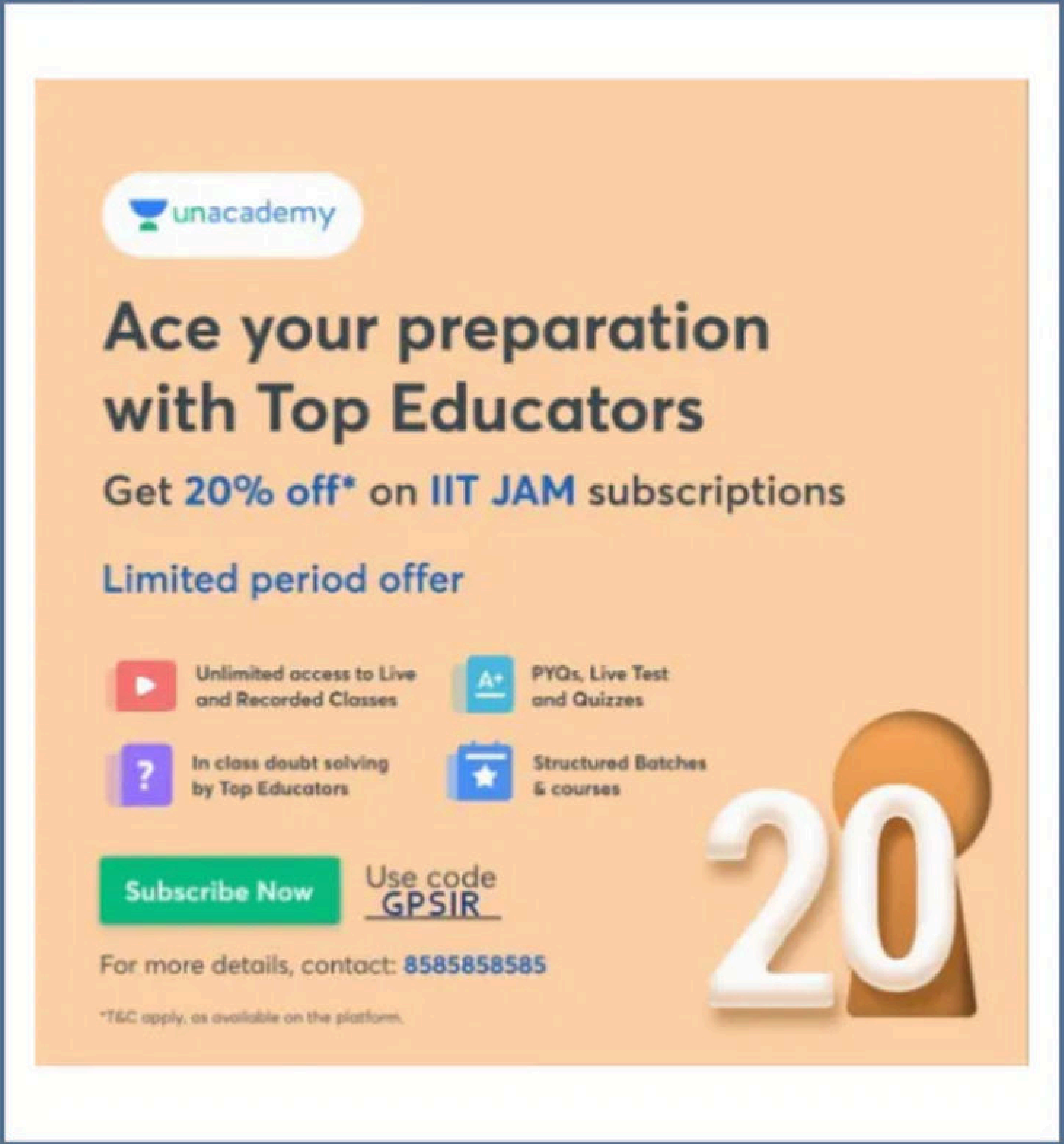
Matrix Representation - Part II

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 8 • Sept 19, 2022



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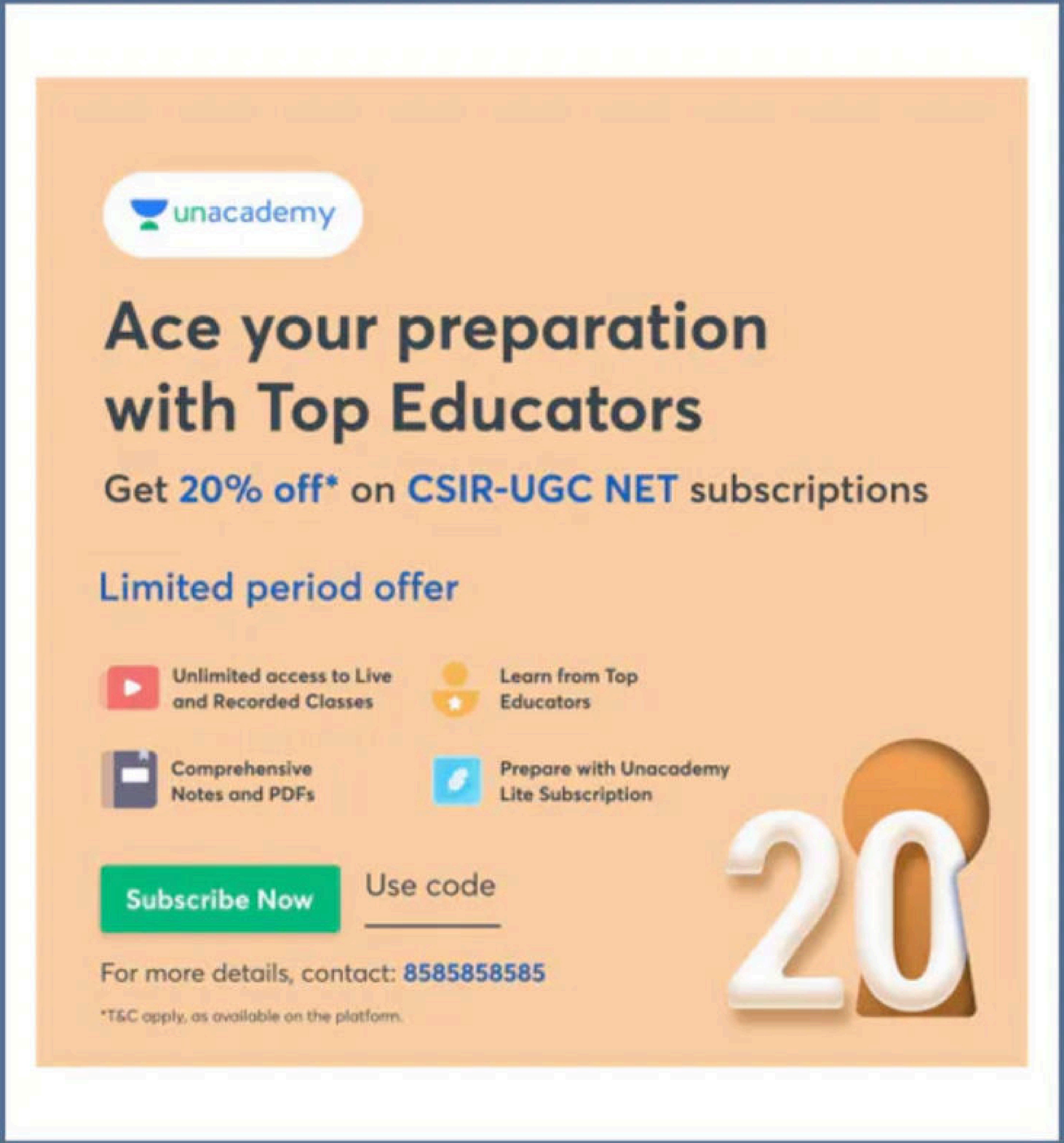

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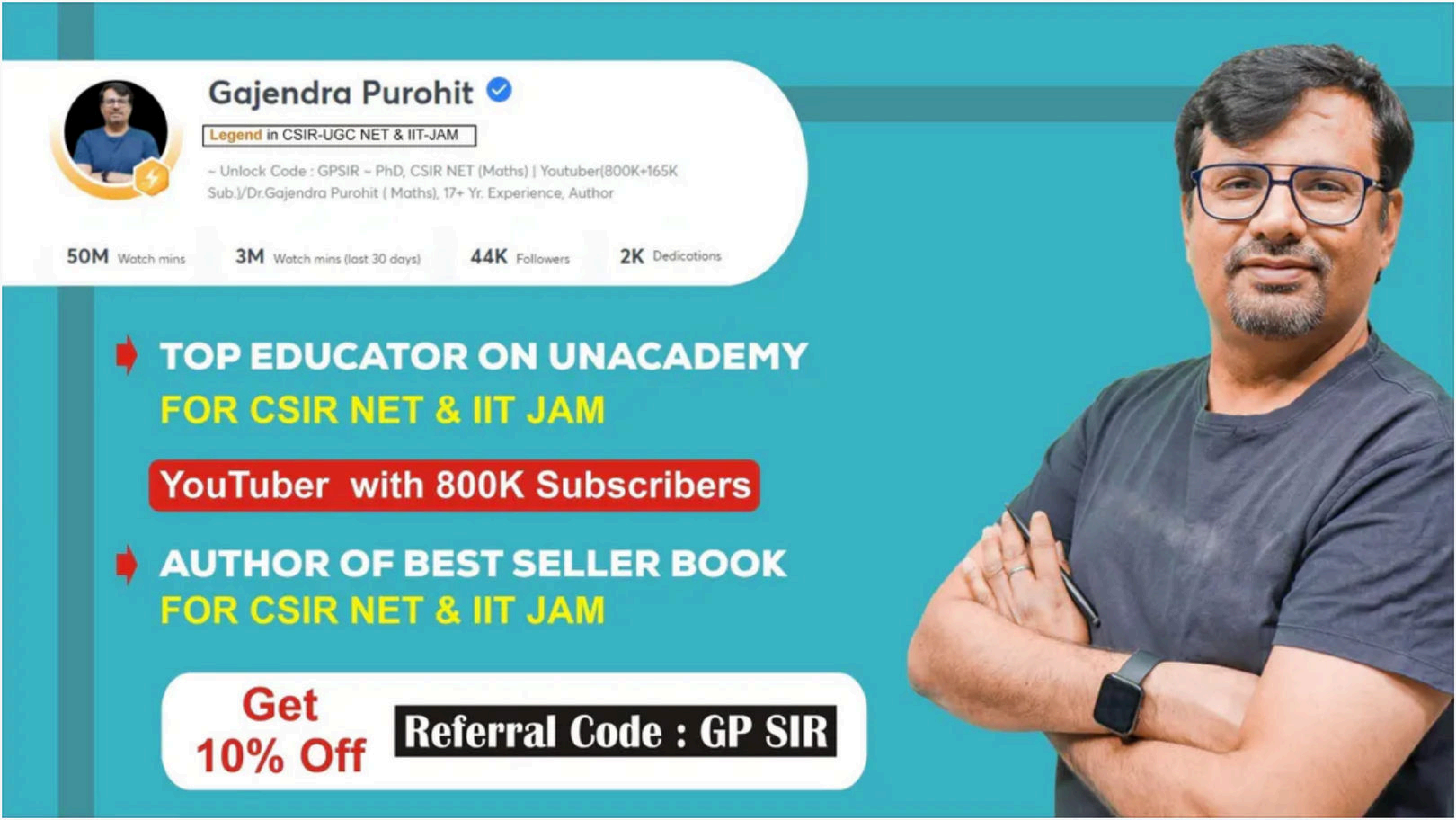
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Rank of matrix : If A is matrix then a non-negative integer is said to be rank of A. if $\exists$ a non-singular submatrix of order r of A and all matrix of order greater than r are singular.

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \quad 2\times 2$$

$$|A| = 1-6 = -5 \neq 0$$

$$\rho(A) = 2$$

Note :

(1) A matrix $A_{n\times n}$ is a non-singular iff rank(A) = n

(2) Rank of matrix is denoted by $\rho(A)$

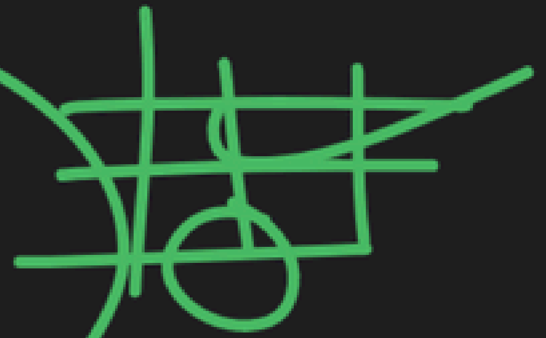
(3) If $|A| \neq 0$ then $\rho(A) = n$ where n is order of A

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 7 & 9 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$R_2 \to R_2 - 2R_1$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



$\rho(A) = 2$

Elementary Transformation : Those transformation that does not effect on rank of matrix are called elementary transformation.

There are 3 elementary row (column) transformation.

Elementary matrix : A matrix obtained by a single elementary operation over identity matrix is known as elementary matrix.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Q.1. Find elementary matrix

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 & 2 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

(a) a, b

(b) b, c

(c) c, d

(d) a, d

Note :

(1) Every elementary matrix is non-singular.

(2) Every permutation matrix is elementary matrix.

Q.2. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 151 & 262 & 373 & 484 \end{bmatrix}$.

Then Rank(A) is

(a) 1 (b) 2

(c) 3 (d) 4

$R_2 \to R_2 - 5R_1$

$R_3 \to R_3 - 9R_1$

$R_4 \to R_4 - 151R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24 \\ 0 & -40 & -80 & -120 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$R_4 \to R_4 - 10R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Q.3. Let $A = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 2 & \cdots & 2 \\ 1 & 2 & 3 & \cdots & 3 \\ 1 & 2 & 3 & \cdots & 4 \\ \vdots & & & & \\ 1 & 2 & 3 & \cdots & n \end{bmatrix}$, then $\rho(A)$ is

(a) 0 (b) 1

(c) 2 (d) n

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Property :

1. A matrix is of Rank zero iff A is null matrix i.e. if A is non-null matrix then $\rho(A) \geq 1$

2. Rank of idempotent matrix of order n are always less then n except identity matrix.

3. Rank of involutory matrix of order are always n because determinant of involutory matrix are non-zero.

4. Rank of orthogonal matrix of order n is n.

5. Rank of nilpotent matrix of order n are always less than n.

$A^2 = I$

$|A| = \pm 1$

$\rho(A) = n$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$1 \leq \rho(A) < n$$

$$\rho(A) = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Q.4. Let $S = \{A = [a_{ij}]_{n\times n} \mid A^k = 0 \;\&\; \rho(A) = n, \text{ for some } k\}$ then cardinality of S is

(a) ϕ (b) 1

(c) 2 (d) n

$A = [0]$

$A = (1)$

$A^k \neq 0$

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1 • Asked by Nitesh

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Test 5

Syllabus

Mathematics

- Range & Null Space
- Linear Independence of Vectors
- Basis & Dimension
- Linear Transformations
- Solution of Linear Eqn.
- Rank & Inverse of a Matrix
- Determinant
- Finite Dimension Vector Space
- Connected & Compact Sets
- Taylor's Series
- Power Series Differentiation
- Interior & Limit Points
- Integration of Power Series
- Eigenvalues & Eigenvectors
- Matrix Representation
- Radius & Interval Convergence
- Cayley-Hamilton Theorem
- Open, Closed & Bounded Sets
- Group Homomorphisms
- Completeness of R.Power Series
- Consistency Conditions
- Lagrange's Theory-Finite Group

Instructions

Time Duration = 30 minutes
Total number of questions = 20

Test Series

Result will be available after live test is over

(6) Let A be a matrix of order $m \times n$ then

$\rho(A) \leq \min\{m, n\}$

$\rho(A) = \rho(A^T) = \rho(AA^T)$

AB $m \times p$

(7) Let A & B are matrix of order $m \times n$ & $n \times p$, then $\rho(A) + \rho(B) - n \leq \rho(AB) \leq \min\{\rho(A), \rho(B)\}$

$\rho(A) + \rho(B) - n \leq \rho(AB) \leq \min(\rho(A), \rho(B))$

(8) Let A and A^T are matrix then $\rho(A) = \rho(A^T)$

(9) $\rho(A + B) \leq \rho(A) + \rho(B)$

$A_{5\times 3}$ $B_{3\times 5}$

$\rho(A) + \rho(B) - 3 \leq \rho(AB) \leq \min(3, 3)$

$\rho(A) + \rho(B) - 3 \leq \rho(AB) \leq 3$

$$\rho(A+B) \leq \rho(A)+\rho(B)$$

$$0 \leq 2+2$$

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\rho(A) = 2 \qquad \rho(B) = 2$$

$$A+B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\rho(A+B) = 0$$

Q A & B are matrix of same order $m \times m$ of rank 2 & 3 then which of following true

$$\rho(A) + \rho(B) - n \le \rho(AB) \le \min(\rho(A), \rho(B))$$

(a) $5 - m \le \rho(AB) \le 1$

$$2 + 3 - m \le \rho(AB) \le 2$$

(b) $5 - m \le \rho(AB) \le 2$

$$5 - n \le \rho(AB) \le 2$$

(c) $2 \le \rho(AB) \le 5 - m$

(d) $\rho(A+B) \le 3$

Q Let A be matrix of 3x3 of rank 2 and A^T be matrix, then we conclude that -

$\rho(A) = 2 = \rho(A^T)$

(a) $\rho(AA^T) \neq \rho(A)$

(b) ~~$\rho(AA^T) = 3$~~

(c) $1 \leq \rho(AA^T) \leq 2$

(d) $3 \leq \rho(AA^T) \leq 4$

$\rho(A) + \rho(A^T) - 3 \leq \rho(AA^T) \leq$

$2 + 2 - 3 \leq \rho(AA^T) \leq 2$

$1 \leq \rho(AA^T) \leq 2$

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Q.5. Let $A=[a_1\ a_2\ \ldots\ldots\ a_n]^T_{1\times n}$ and $B=[b_1\ b_2\ \ldots\ldots\ b_n]_{1\times n}$ both are non-zero matrix then $\rho(AB)$ is

(a) 1 (b) n

(c) n + 1 (d) n – 1

$A = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_{n\times 1}$ $B = [\ \ ]_{1\times n}$

$$\rho(A)+\rho(B)-1 \le \rho(AB) \le \min(\rho(A),\rho(B))$$

$m\times n$

$$1+1-1 \le \rho(AB) \le \min(1,1)$$

$$1 \le \rho(AB) \le 1$$

Q.6 Let A be a 3×4 matrix and B be a 4×3 matrix with real entries such that AB is non- singular. Consider the following statements :

P : Nullity of A is 0.

Q : BA is a non-singular matrix.

Then

(a) both P and Q are true

(b) P is true and Q is false

(c) P is false and Q is true

(d) Both P and Q are false

Q.7. Let A be an $n \times n$ matrix such that the first 3 rows of A are linearly independent and the first 5 columns of A are linearly independent. Which of the following statements are true?

(a) A has at least 5 linearly independent rows

(b) $3 \le \text{rank } A \le 5$

(c) Rank $A \ge 5$ (d) Rank $A^2 \ge 5$

Q.8. What is the rank of the matrix $\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$.

(a) 2 (b) 3

(c) 4 (d) 5

Q.9 Let J denote the matrix of order n × n with all entries 1

and let B be a (3n)×(3n) matric given by $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$

$n \times n$ $n \times n$ $(n \times n)$ $3n \times 3n$

. Then the rank of B is

(a) 2n (b) 3n – 1

(c) 2 (d) 3

$n=1$

$J = (1)_{1\times 1}$

$\rho(A) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} = (13)$

$= (-1)^{1} = -1 \neq 0$

$\rho(A) = 3$

Q.10. Let A be an $n \times m$ matrix with each entry equal to +1, -1 or 0 such that every column has exactly one +1 and exactly one -1. We can conclude that

(a) Rank $A \le n - 1$ (b) Rank $A = m$

(c) $n \le m$ (d) $n - 1 \le m$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & +1 & -1 \end{bmatrix} \quad 2 \times 3$$

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1 • Asked by Srinibas

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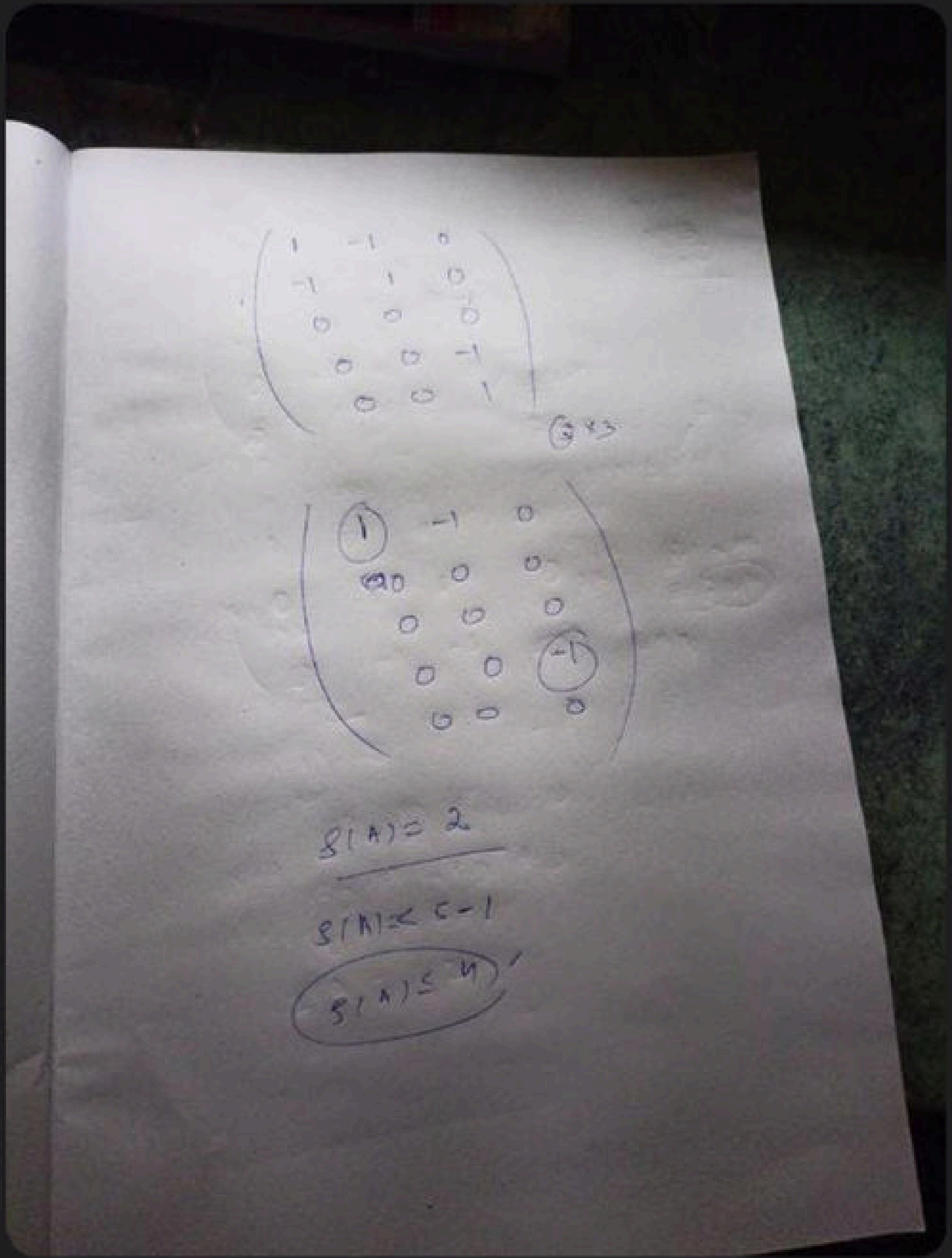

Q.11 Let A and B be $n \times n$ real matrices such that $AB = BA = 0$ and $A + B$ is invertible.

Which of the following are always true?

(a) Rank (A) = rank (B)

(b) Rank (A) + rank (B) = n

(c) Nullity (A) + nullity (B) = n

(d) A – B is invertible


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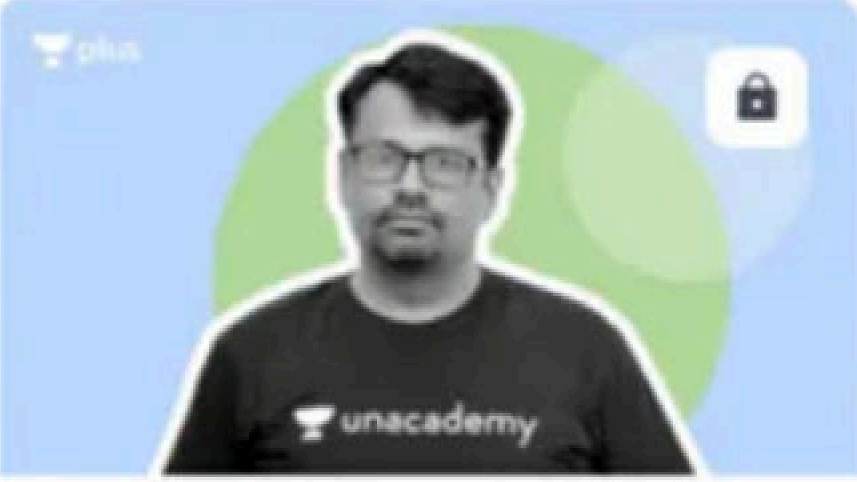

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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