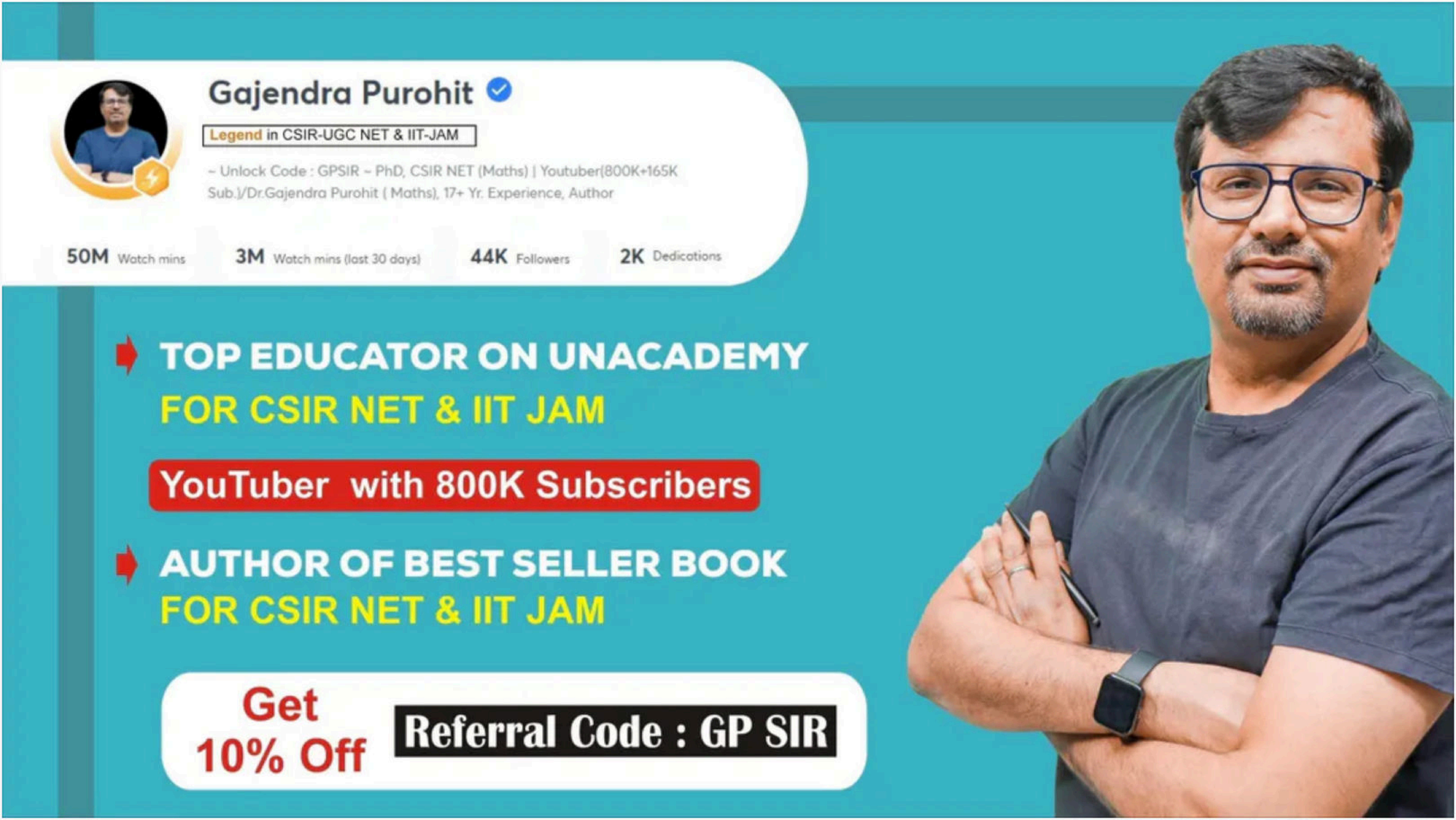
Higher Order Constant Coefficient ODE - II

Detailed Course on Differential Equation for IIT JAM' 23 - II

Gajendra Purohit • Lesson 9 • Aug 18, 2022



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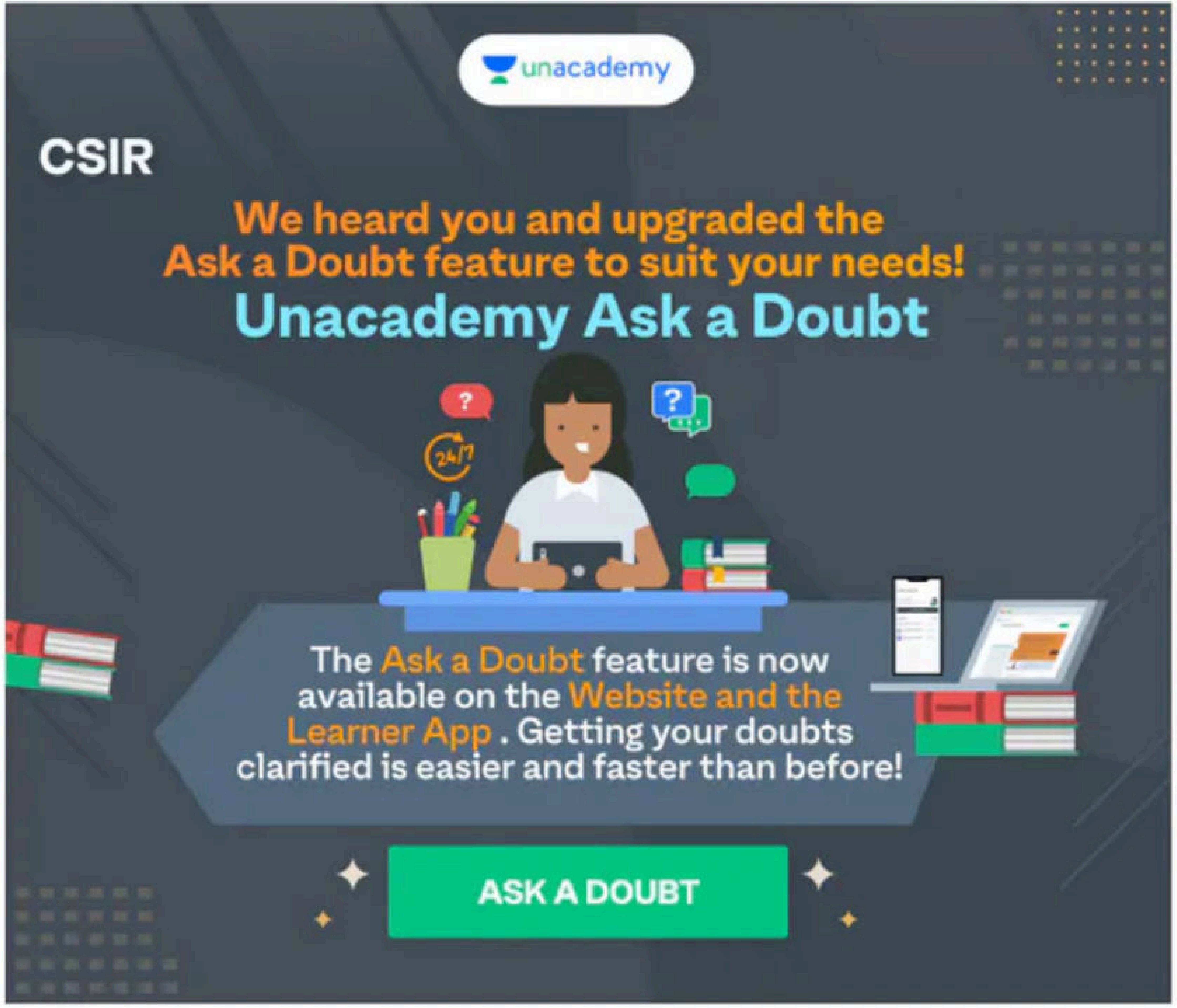
$$y = f(x)$$

$$y(0) =$$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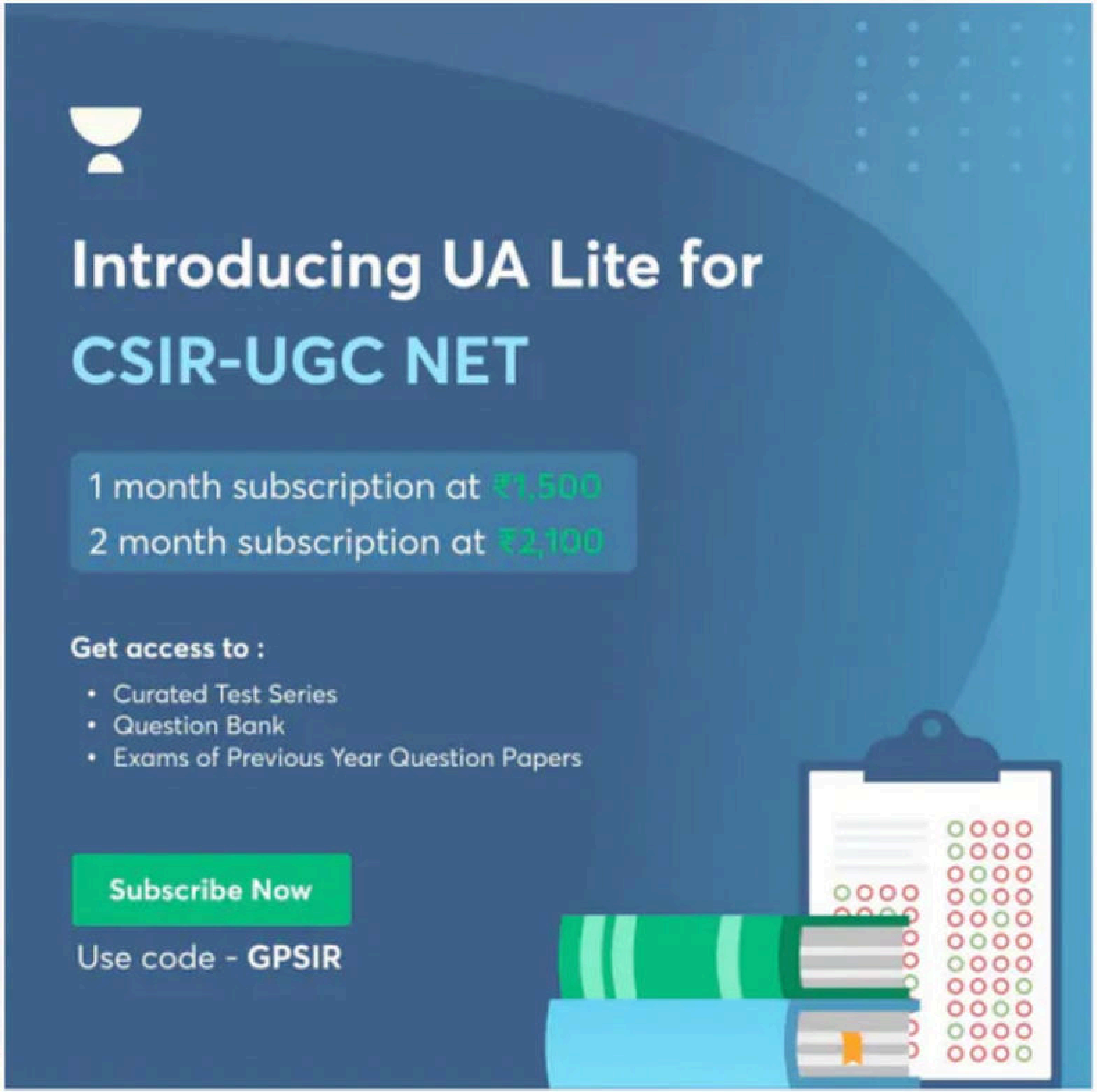
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DETERMINATION OF ORTHOGONAL TRAJECTORIES IN POLAR COORDINATES $f(r, \theta_1 c) = 0$

$\frac{dr}{d\theta}$ $-r^2\frac{d\theta}{dr}$

Working Rule :

1. Differentiate the given equation of family of curves w.r.t. θ (generally take logarithm). Eliminate the parameter.
2. Replace $(dr/d\theta)$ by $-\ r^2(d\theta/dr)$ and obtain the differential equation of orthogonal trajectories.
3. Obtain the general solution of differential equation obtained above.

$$r = \frac{2a}{1+\cos\theta}$$

$$\frac{dr}{d\theta} = -r^2\frac{d\theta}{dr}$$

$$\log r = \log\left(\frac{2a}{1+\cos\theta}\right)$$

$$\log r = \log 2a - \log(1+\cos\theta)$$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{1}{1+\cos\theta}(-\sin\theta)$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{\sin\theta}{1+\cos\theta}$$

$$\cancel{\frac{1}{r}}\left(-r^{\cancel{2}}\frac{d\theta}{dr}\right) = \frac{\cancel{2}\sin\frac{\theta}{2}\cancel{\cos\frac{\theta}{2}}}{\cancel{2}\cos^2\frac{\theta}{2}}$$

$$-r\frac{d\theta}{dr} = \tan\frac{\theta}{2}$$

$$\int \cot\frac{\theta}{2}\,d\theta = -\int\frac{dr}{r} + \log c$$

$$2\log\sin\frac{\theta}{2} = -\log r + \log c$$

$$\log\sin^2\frac{\theta}{2} = \log\frac{c}{r}$$

$$\frac{c}{r} = \sin^2\frac{\theta}{2}$$

$$r = c\operatorname{cosec}^2\frac{\theta}{2}$$

$$r^n \cos n\theta = a^n$$

$$\log(r^n \cos n\theta) = \log a^n$$

$$n \log r + \log \cos n\theta = n \log a$$

$$\frac{n}{r}\frac{dr}{d\theta} + \frac{-n \sin n\theta}{\cos n\theta} = 0$$

$$\frac{1}{r}\frac{dr}{d\theta} = \tan n\theta$$

$$\frac{1}{\cancel{r}}\left(-r^{\cancel{2}}\frac{d\theta}{dr}\right) = \tan n\theta$$

$$\frac{dr}{d\theta} = -r^2 \frac{d\theta}{dr}$$

$$\int \cot n\theta \, d\theta = -\int \frac{dr}{r} + \log c$$

$$\frac{\log \sin n\theta}{n} = -\log r + \log c$$

$$\log(\sin n\theta) = n \log\left(\frac{c}{r}\right)$$

$$\log(\sin n\theta) = \log\left(\frac{c}{r}\right)^n$$

$$\left(\frac{c}{r}\right)^n = \sin n\theta$$

$$c^n = r^n \sin n\theta$$

Q.1. The orthogonal trajectory of the family of curves given by the equation $r = a(1 - \cos\theta)$.

$1 - \cos\theta = 2\sin^2\frac{\theta}{2}$ $\quad 1 + \cos\theta = 2\cos^2\frac{\theta}{2}$

(a) $r = b(1 + \sec\theta)$ (b) $r = b(1 + \cos\theta)$

(c) $r^2 = b(1 + \cos\theta)$ (d) $r = b(1 + \tan\theta)$

$\log r = \log a + \log(1 - \cos\theta)$

$\frac{1}{r}\frac{dr}{d\theta} = 0 + \frac{+\sin\theta}{1 - \cos\theta}$

$\frac{1}{\cancel{r}}\left(-r^2\frac{d\theta}{dr}\right) = \frac{\sin\theta}{1 - \cos\theta}$

$\int \frac{1 - \cos\theta}{\sin\theta}\, d\theta = -\int \frac{dr}{r} + \log c$

$\int \frac{\cancel{2}\sin^2\frac{\theta}{2}}{\cancel{2}\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}\, d\theta = -\log r + \log c$

$\int \tan\frac{\theta}{2}\, d\theta = -\log r + \log c$

$\log \sec^2\frac{\theta}{2} = \log\frac{c}{r}$

$\sec^2\frac{\theta}{2} = \frac{c}{r}$

$r = c\cos^2\frac{\theta}{2}$

$r = \frac{c(1 + \cos\theta)}{2}$

$r = b(1 + \cos\theta)$

GENERAL THEORY OF LINEAR DIFFERENTIAL EQUATION

e^{x} e^{2x} e^{3x}

1 3 5

$1\cdot e^{x} + 3e^{2x} + 5e^{3x}$

1. DEFINITIONS

(i) Linear Combination: If $f_1, f_2, \ldots f_n$ are n functions defined on the interval I and $c_1, c_2, \ldots\ c_n$ are n arbitrary constants, then the function $c_1f_1 + c_2f_2 + \ldots + c_nf_n$ iscalled linear combination of $f_1, f_2, \ldots f_n$ over I.

x, $2x$

2 −1

$2x + (-1)(2x) = 0$

(ii) Linearly Dependent: The functions $f_1, f_2 \ldots, f_n$ of x are said to be linearly dependent over an interval I iff there exist constants $c_1, c_2, \ldots, c_n$ (not all zero) such that $c_1f_1 + c_2f_2 + \ldots + c_nf_n = 0$ for all x in I.

(iii) **Linearly Independent :** The functions $f_1, f_2, \ldots, f_n$ are of x are said to be linearly independent over an interval I iff there exist constants $c_1, c_2, \ldots, c_n$ such that $c_1f_1 + c_2f_2 + \ldots + c_nf_n = 0$ for all x in I, then $c_i = 0 \ \forall\ i = 1, 2, \ldots, n$.

x

(iv) **Convex Combination :** A linear combination $\sum_{i=1}^{n} c_i f_i(x)$ is called a convex combination if $\sum_{i=1}^{n} c_i = 1$ & $c_i \geq 0$ for all i.

x, x^2

$2x - x^2 = 0$

$2 \quad -1$

$2 - 1 = 1$

2. GENERAL LINEAR DIFFERENTIAL EQUATION

Definition: A general linear differential equation of order n is

$$P_0 \frac{d^n y}{dx^n} + P_1 \frac{d^{n-1} y}{dx^{n-1}} + P_2 \frac{d^{n-2} y}{dx^{n-2}} + \ldots + P_n y = Q \quad \ldots\ldots (1)$$

where $P_0 \neq 0$, P_1, P_2, ... P_n and Q are functions of x defined on some interval I.

3. Homogenous Linear Differential Equation

Equation (1) is said to be homogeneous if $Q = 0$.

4. Non-homogenous linear equation

Equation (1) is said to be non-homogeneous if $Q \neq 0$.

Principal of super position :

(i) Let $f_1(x)$ and $f_2(x)$ be two linearly independent solution of $P_0 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = 0$ over an open interval I, where P_0, P_1, P_2 are all continuous functions of x and $P_0(x) \neq 0$ on I. Then $f = \alpha f_1 + \beta f_2$, where α, β are some constants is also its solution.

(ii) Let $f_1(x)$ and $f_2(x)$ be linearly independent solution of

$P_0 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = Q$, $Q \neq 0$ then $f = \alpha f_1 + \beta f_2$ is a solution iff $\alpha + \beta = 1$

$$\frac{d^2y}{dx^2} - y = e^x$$

e^x e^{-x}

$\alpha e^x + \beta e^{-x}$

$\alpha + \beta = 1$

Q.1. Let y_1 & y_2 are two solution of $e^x y'' + \sin x\, y' + e^{\sin x} y = x\cos x$. Then which of the following are TRUE?

(a) $\frac{3}{2}y_1 - \frac{1}{2}y_2$ $\quad \frac{3}{2} - \frac{1}{2} = 1$

(b) $y_1 - 2y_2$

(c) $\frac{1}{2}y_1 + \frac{1}{2}y_2$

(d) $\frac{1}{2}y_1 - \frac{1}{2}y_2$

a	a	c
b	b	d
c	c	b
d	d	a

Q1 $P_0 \frac{d^2y}{dx^2} + P_1 \frac{dy}{dx} + P_2 y = Q$

$y_1 = 1 + x e^{x^2}$, $y_2 = (1+x)e^{x^2} - 1$, $y_3 = 1 + e^{x^2}$

which of the following is general sol'n of DE

(a) $(c_1+1)y_1 + (c_2 - c_1)y_2 - c_2 y_3$

$\cancel{c_1} + 1 + \cancel{c_2} - \cancel{c_1} - \cancel{c_2}$

(b) $c_1(y_1 - y_2) + c_2(y_2 - y_3) = c_1 y_1 + y_2(-c_1 + c_2) - c_2 y_3$

(c) $c_1(y_1 - y_2) + c_2(y_2 - y_3) + c_3(y_3 - y_1)$

$\cancel{c_1} - \cancel{c_1} + \cancel{c_2} - \cancel{c_2}$

(d) $c_1(y_1 - y_3) + c_2(y_3 - y_2) + y_1$

(a) a c
(b) b d
(c) c b
(d) d a

$(c_1+1)y_1 - c_2 y_2 + (-c_1 + c_2)y_3$

$\cancel{c_1} + 1 - \cancel{c_2} - \cancel{c_1} + \cancel{c_2}$

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5 **The Wronskian :**

Definition : The Wronskian of n functions $y_1(x)$, $y_2(x)$ ...... $y_n(x)$ is denoted by w(x) or $w(y_1, y_2, ..., y_n)$

$$= \begin{vmatrix} y_1 & y_2 \ldots\ldots & y_n \\ y'_1 & y'_2 \ldots\ldots & y'_n \\ \vdots & & \\ y_1^{(n-1)} & y_2^{(n-1)} \ldots\ldots & y_n^{(n-1)} \end{vmatrix}$$

$y_1 = e^x \quad y_2 = e^{-x}$

$w(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$= \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$

$= -1-1 = -2$

Wronskian of second order DE

Let $a_0(x)\, y'' + a_1(x)\, y' + a_2(x)\, y = 0$

Where $a_0(x)$, $a_1(x)$, $a_2(x)$ are continuous and $a_0(x) \neq 0 \quad \forall$ x

If $\quad y_1(x)$ and $y_2(x)$ are solution

Then $\quad w(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

Able's Formula:

Let $a_0(x)\, y`` + a_1(x)\, y` + a_2(x)\, y = 0$

$y'' + \frac{a_1(x)}{a_0(x)} y' + \frac{a_2(x)}{a_0(x)} y = 0$

Where $a_0(x)$, $a_1(x)$, $a_2(x)$ are continuous and $a_0(x) \neq 0\ \forall$ x

$y'' + Py' + Qy = 0$

$w(y_1, y_2) = Ae^{-\int \frac{a_1(x)}{a_0(x)} dx}$ is called Able's formula.

$w(x) = Ae^{-\int P dx}$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

$$y = c_1 e^{x} + c_2 e^{2x}$$

$$W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} e^{x} & e^{2x} \\ e^{x} & 2e^{2x} \end{vmatrix}$$

$$= 2e^{3x} - e^{3x}$$

$$W(x) = e^{3x}$$

$$m^2 - 3m + 2 = 0$$

$$m = 1, 2$$

$$W = A e^{-\int P\,dx}$$

$$= A e^{-\int -3\,dx}$$

$$W(x) = A e^{3x}$$

$(\cos x)\, y'' + (\sin x)\, y' - (1 + e^{-x^2})\, y = 0$

y_1 & y_2 are sol^n of D.E

$y_1(0) = \sqrt{2}$, $y_1'(0) = 1$

$y_2(0) = -\sqrt{2}$, $y_2'(0) = 2$

find $W(\pi/4) = ?$

$$W(0) = \begin{vmatrix} y_1(0) & y_2(0) \\ y_1'(0) & y_2'(0) \end{vmatrix} = \begin{vmatrix} \sqrt{2} & -\sqrt{2} \\ 1 & 2 \end{vmatrix}$$

$W(0) = 3\sqrt{2}$

$A = 3\sqrt{2}$

$W(x) = A e^{-\int P\,dx} = A e^{-\int \tan x\,dx}$

$= A e^{\log \cos x}$

$W(x) = A \cos x$

$W(0) = A$

$W(x) = 3\sqrt{2} \cos x$

$W(\pi/4) = 3\sqrt{2} \cos \pi/4$

$= 3\sqrt{2} \cdot \frac{1}{\sqrt{2}}$

$= 3$

(a) $3\sqrt{2}$

(b) 6

(c) 3

(d) $-3\sqrt{2}$

RESULTS:

(1) If w $(y_1, y_2, \ldots y_n) \neq 0$ then $y_1, y_2 \ldots\ldots y_n$ are L.I. solution.

(2) If $w(y_1, y_2, \ldots\ldots y_n) = 0$ then $y_1, y_2, \ldots\ldots y_n$ are LD solution.

(3) Wronskian is either identically zero or non-zero.

(4) If Wronskian is non – zero at least one point then Wronskian is identically non – zero

(5) If Wronskian is zero at least one point then Wronskian is identically zero

(6) Wronskian can never change its sign

Q2. Consider two solution $x(t) = x_1(t)$ and $x(t) = x_2(t)$ of differential equation $\frac{d^2x(t)}{dt^2} + x(t) = 0,\ t > 0$ such that $x_1(0) = 1, \left.\frac{dx_1(t)}{dt}\right|_{t=0} = 0, x_2(0) = 0, \left.\frac{dx_2(t)}{dt}\right|_{t=0} = 1$ the Wronskian $W(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_1(t)}{dt} \end{vmatrix}$ and at $t = \pi/2$ is

(a) 1
(b) – 1
(c) 0
(d) $\pi/2$

$P = 0$

$\frac{d^2x}{dx^2} + 0\frac{dx}{dx} + x$

$W(0) = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$W(x) = Ae^{-\int 0\,dx} = A$

$W(0) = A$

$A = 1$

$W(x) = 1$

Q3. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of the differential equation $x^2 y''(x) - 2xy'(x) - 4y(x) = 0$ for $x \in [1,10]$. Considered the wronskian $W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x)$. If W(1) = 1, then W(3) – W(2) equals

(a) 1
(b) 2
(c) 3
(d) 5

Q4. Let $y_1(x)$ and $y_2(x)$ be the linearly independent solutions of $xy'' + 2y' + xe^x y = 0$ If $W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x)$ with W (1) = 2 find W (5)

(a) $\frac{2}{25}$

(b) $\frac{1}{25}$

(c) $\frac{2}{5}$

(d) None of the above

Q.5. Consider the ODE

$$u''(t)+P(t)u'(t)+Q(t)u(t)=R(t), t\in[0,1]$$

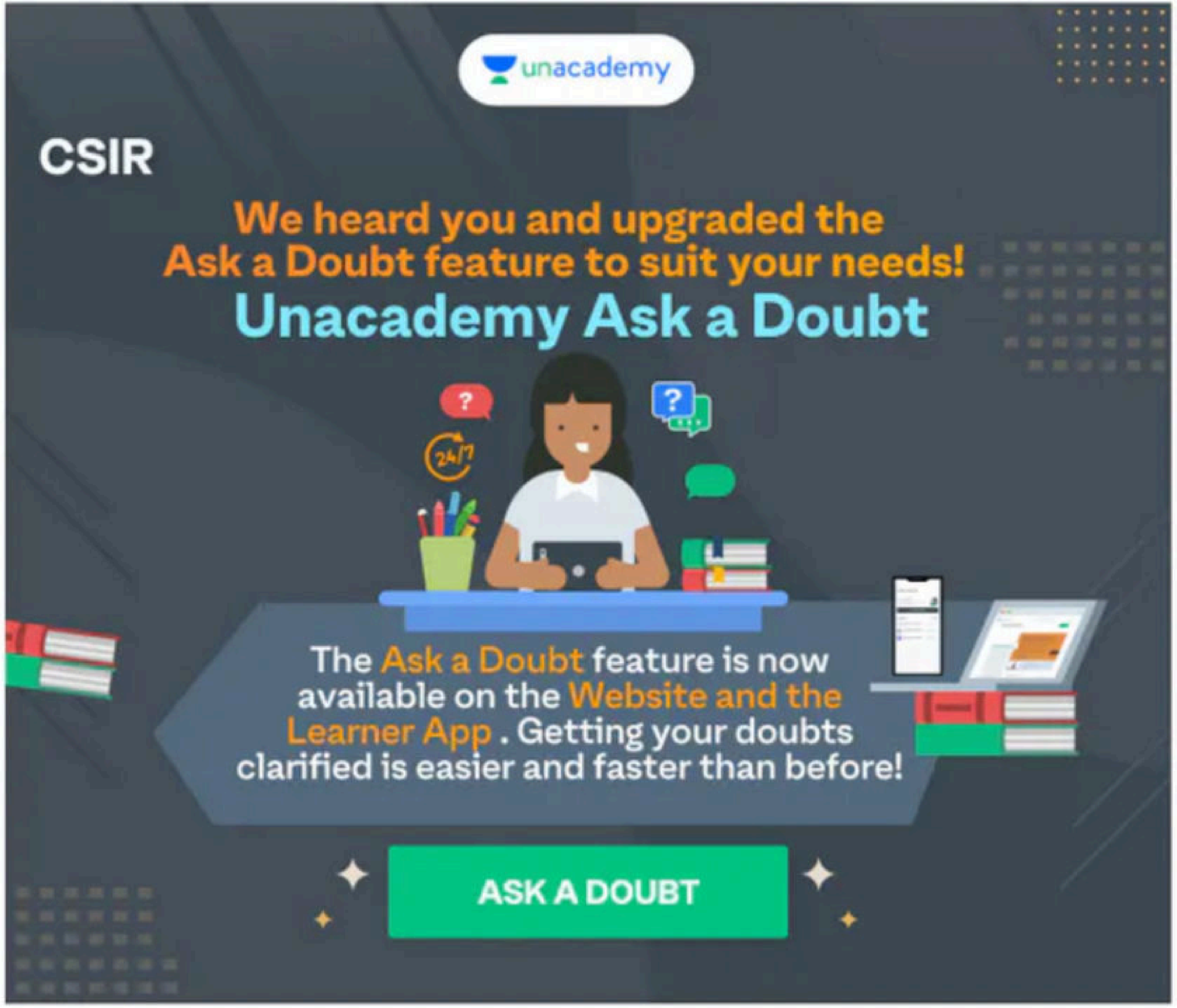
There exist continuous function P, Q and R defined on $[0,1]$ and two solutions u_1 and u_2 of the ODE such that the Wronskian W of u_1 and u_2 is

(a) $W(t)=2t-1, 0\le t\le 1$

(b) $W(t)=\sin 2\pi t, 0\le t\le 1$

(c) $W(t)=\cos 2\pi t, 0\le t\le 1$

(d) $W(t)=1, 0\le t\le 1$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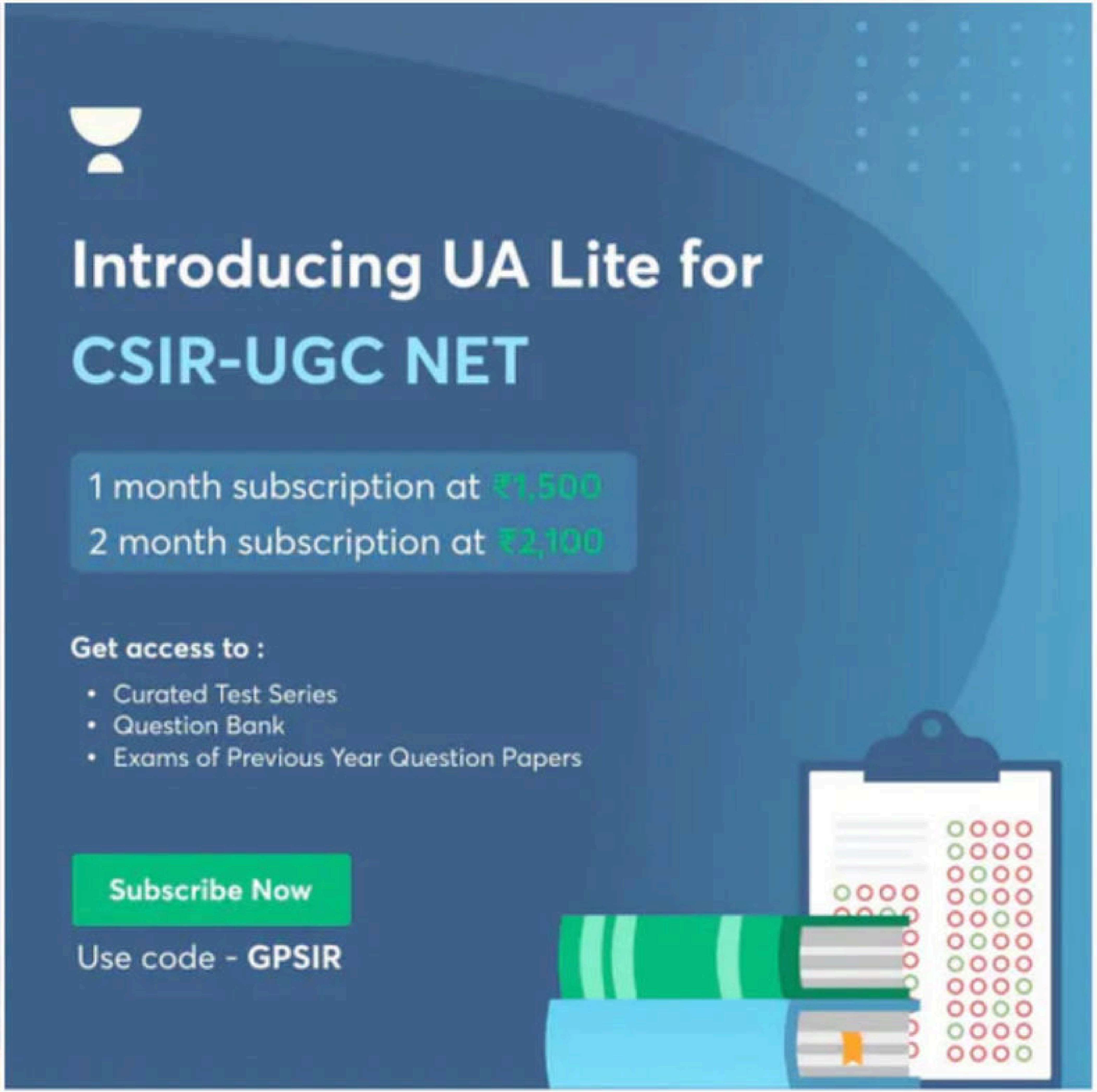

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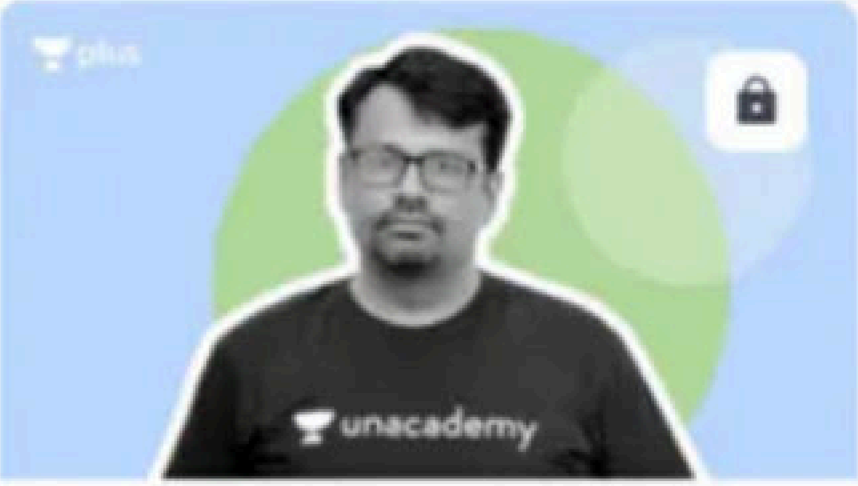

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