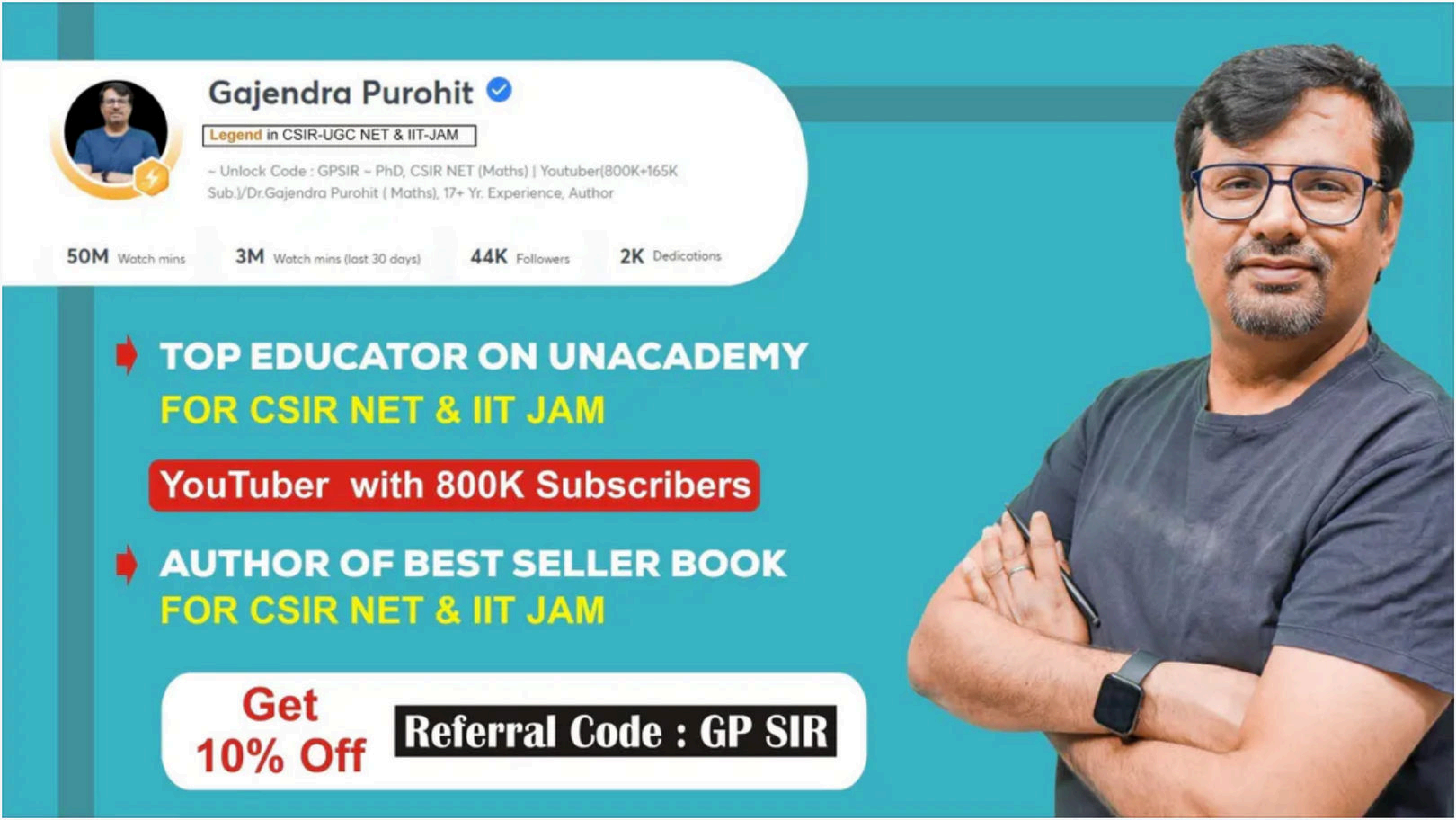
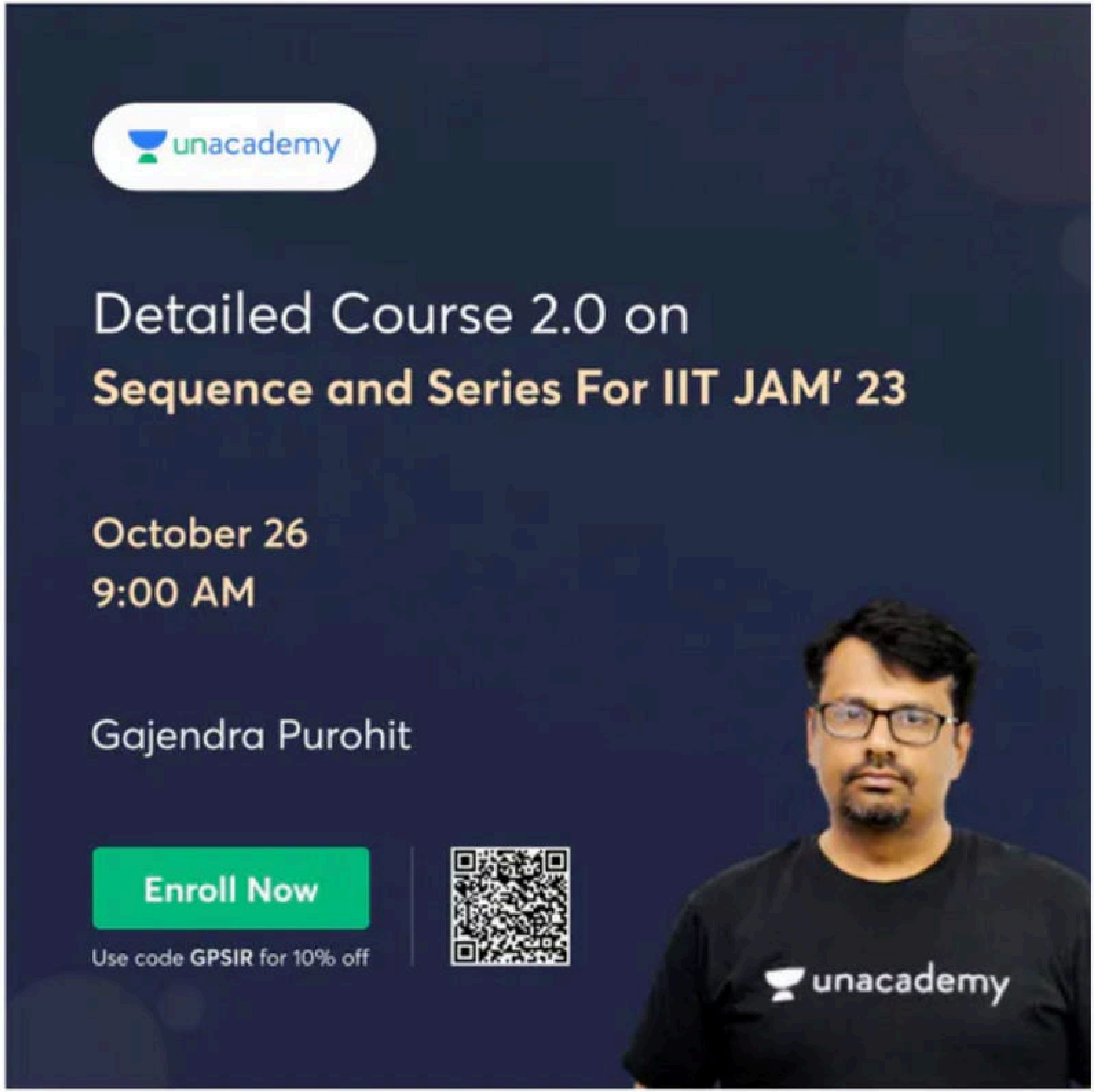
Function of Several Variables - Part II

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 10 • Nov 21, 2022


Gajendra Purohit
Legend in CSIR-UGC NET & IIT-JAM
~ Unlock Code : GPSIR ~ PhD, CSIR NET (Maths) | Youtuber(800K+165K Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author
50M Watch mins
3M Watch mins (last 30 days)
44K Followers
2K Dedications
TOP EDUCATOR ON UNACADEMY
FOR CSIR NET & IIT JAM
YouTuber with 800K Subscribers
AUTHOR OF BEST SELLER BOOK
FOR CSIR NET & IIT JAM
Get 10% Off
Referral Code : GP SIR


unacademy
Detailed Course on Group Theory
For CSIR NET 2023
Gajendra Purohit
November 3
Enroll Now
unacademy


unacademy
Detailed Course 2.0 on
Sequence and Series For IIT JAM' 23
October 26
9:00 AM
Gajendra Purohit
Enroll Now
Use code GPSIR for 10% off
unacademy

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780	You get 6 months extra for free — Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974	You get 6 months extra for free — Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475	
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252	
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807	
1 month		₹ 2,723 / mo	Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code? Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	~~₹ 21,780~~ ₹ 19,602	You get 6 months extra for free — Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo	~~₹ 14,974~~ ₹ 13,477	You get 6 months extra for free — Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo	~~₹ 13,475~~ ₹ 12,128	
6 months	Save 25%	₹ 1,838 / mo	~~₹ 12,252~~ ₹ 11,027	
3 months	Save 17%	₹ 2,042 / mo	~~₹ 6,807~~ ₹ 6,126	

GPSIR

Awesome! You get 10% off

Proceed to pay

Riemann Integration

Interval :

(1) **Open Interval :** Let $S \subseteq R$ and $S = \{x \mid a < x < b\}$. It is denoted by (a, b)

(2) **Closed Interval :** Let $S \subseteq R$ and $S = \{x \mid a \le x \le b\}$. It is denoted by $[a, b]$

Partition :

Let $[a, b]$ be a closed and bounded interval. A partition of $[a, b]$ is a finite order set $P = \{a = x_0, x_1, \ldots\ldots, x_r \ldots., x_n = b \mid x_0 < x_1 < \ldots. < x_n\}$

Subinterval : Let P be a partition of [a, b], then $[x_0, x_1]$, $[x_1, x_2]$ ....., $[x_{n-1}, x_n]$ are called subinterval.

Norm of partition :

Let the greatest length of subinterval $[x_{r-1}, x_r]$ partition P is called norm of partition P and denoted by $||P||$.

Refinement of partition :

Let P_1 and P_2 are two partition of $[a, b]$ s.t. $P_1 \subset P_2$, then P_2 is called refinement of partition P_1.

Note : Let P_1 and P_2 are two partition of $[a, b]$ then $P_1 \cup P_2$ is called a common refinement of P_1 & P_2.

Supremum and infimum of a function :

Let f(x) be a function defined on [a, b] then supremum of function is subinterval $[x_{r-1}, x_r]$ is maxi mum value of function in $[x_{r-1}, x_r]$

Similarly infimum of this function in $[x_{r-1}, x_r]$ is minimum value of this function in $[x_{r-1}, x_r]$

(i) Upper Riemann sum :

Let M_r is supremum of function in $[x_{r-1}, x_r]$

And Δx_r is length of interval $[x_{r-1}, x_r]$, then

$U(P,f)=\sum_{r=1}^{n} Mr\Delta xr= M_1\Delta x_1 + M_2\Delta x_2 + \ldots.\ M_n\Delta x_n$

(ii) Lower Reimann Sum :

Let mr is infimum of function in $[x_{r-1}, x_r]$, then

$L(p, f) = m_1\Delta x_1 + m_2\Delta x_2 + \ldots.\ m_n\Delta x_n$

$L(P,f)=\sum_{r=1}^{n} mr\Delta xr$

is called lower Riemann sum.

Upper and Lower integral :

(i) **Upper integral :** The infimum of the set of the upper sup U(P, f) is called upper integral of f over [a, b] and is denoted by $\int_a^b f(x)dx$

i.e. $\int_a^b f(x)dx = \inf\{U(P,f) : P$ is a partition of [a, b]}.

(ii) **Lower integral :** The supremum of the set of the upper sum U(P, f) is called upper integral of f over [a, b] and is denoted by $\int_a^b f(x)dx$

i.e. $\int_{\underline{a}}^b f(x)dx$ = sup {L(P, f) : P is a partition of [a, b]}.

Riemann integration :

A function is Riemann integration if

$$\int_{\underline{a}}^{b} f(x)dx = \int_{a}^{\overline{b}} f(x)dx$$

Q.1. Let f(x) = x^3 is defined on [0, a], then

(a) $\int_{\underline{0}}^{a} f(x)dx = \frac{a^4}{3}$ (b) $\int_{\underline{0}}^{a} f(x)dx = \frac{a^4}{4}$

(c) $\int_{0}^{\overline{a}} f(x)dx = \frac{a^3}{4}$ (d) $\int_{0}^{\overline{a}} f(x)dx = a^4$

Q.2. Function f : [0, 1] → R s.t. $f(x) = \begin{cases} 0 & x \in Q^C \\ 1 & x \in Q \end{cases}$

Then

(a) f is R.I. (b) f is not R.I.

(c) $\int_0^1 f(x)dx = 1$ (d) None of these

COMPLETE COURSE ON

MATHEMATICS

FOR IIT-JAM 2022

TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINAER ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780	You get 6 months extra for free · Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974	You get 6 months extra for free · Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475	
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252	
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807	
1 month		₹ 2,723 / mo	Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code? | Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	₹ ~~21,780~~ ₹ 19,602	You get 6 months extra for free · Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo	₹ ~~14,974~~ ₹ 13,477	You get 6 months extra for free · Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo	₹ ~~13,475~~ ₹ 12,128	
6 months	Save 25%	₹ 1,838 / mo	₹ ~~12,252~~ ₹ 11,027	
3 months	Save 17%	₹ 2,042 / mo	₹ ~~6,807~~ ₹ 6,126	

GPSIR

Awesome! You get 10% off

Proceed to pay

FOUNDATION COURSE OF

MATHEMATICS

FOR CSIR-NET

Q.3. Define f on [0, 1] by $f(x) = \begin{cases} x^2 & if \quad x \text{ is rational} \\ x^3 & if \quad x \text{ is irrational} \end{cases}$

Then, **CSIR NET JUNE 2016**

(a) f is not Riemann integrable on [0, 1]

(b) f is Riemann integrable and $\int_0^1 f(x)dx = \frac{1}{4}$.

(c) f is Riemann integrable and $\int_0^1 f(x)dx = \frac{1}{3}$.

(d) $\frac{1}{4} = \underline{\int_0^1} f(x)dx < \overline{\int_0^1} f(x)dx = \frac{1}{3}$ where $\underline{\int_0^1} f(x)dx$ and $\overline{\int_0^1} f(x)dx$ are the lower and upper Riemann integrals of f.

Q.4. Consider the identity function $f(x) = x$ on $I := [0, 1]$.

Let P_n be the partition that divides I into n equal parts. If $U(f, P_n)$ and $L(f, P_n)$ are the upper and lower Riemann sums, respectively,& $A_n = U(f, P_n) - L(f, P_n)$ then **CSIR NET NOV 2020**

(a) $\lim\limits_{n\to\infty} nA_n = 0$

(b) $\sum\limits_{n=1}^{\infty} A_n$ is convergent

(c) A_n is strictly monotonically decreasing.

(d) $\sum\limits_{n=1}^{\infty} A_n A_{n+1} = 1$

Q.5. Define a function $[0, \pi/2] \to R$ by $g(x) = \begin{cases} \cos^2 x & if \quad x \in Q \\ 0 & otherwise \end{cases}$. Then

(a) The lower integral, $\int_{-0}^{\pi/2} g = 1$.

(b) The lower integral, $\int_{-0}^{\pi/2} g = 0$.

(c) The lower integral $\int_{-0}^{\pi/2} g = \frac{1}{2}$.

(d) None of these

Q.6. If $f(x) = x^2$ on $I = [0, 1]$ then find the value of lower Reimann Integral?

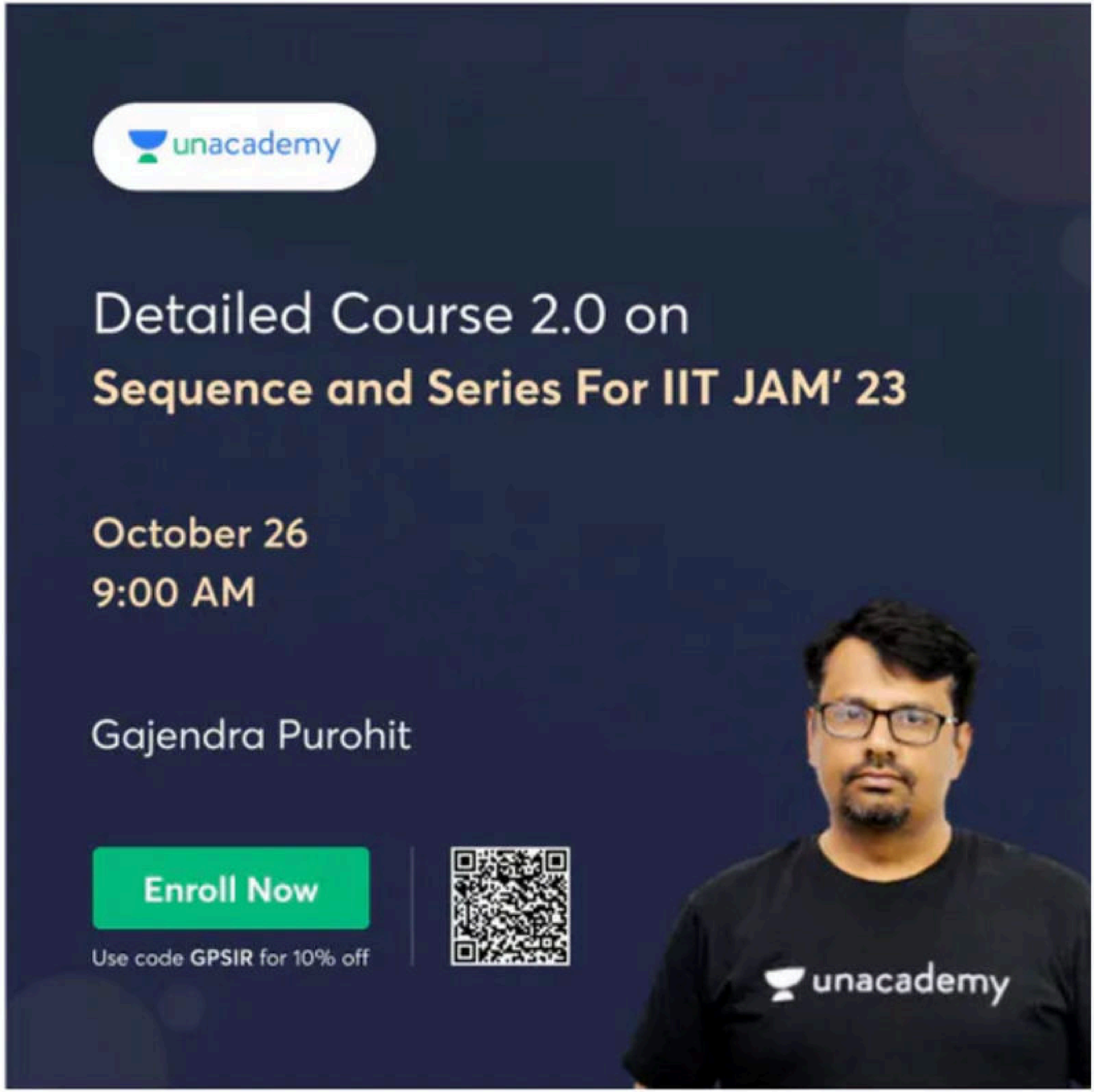
(a) $(n+1)(n-1)$

(b) $(n+1)(n-1)^2$

(c) $\dfrac{(n-1)(2n-1)}{6n^2}$

(d) $\dfrac{(n-2)(2n-1)}{n^2}$


unacademy
Detailed Course on Group Theory
For CSIR NET 2023
Gajendra Purohit
November 3
Enroll Now
unacademy


unacademy
Detailed Course 2.0 on
Sequence and Series For IIT JAM' 23
October 26
9:00 AM
Gajendra Purohit
Enroll Now
Use code GPSIR for 10% off
unacademy

Educator Profile

unacademy   CSIR-UGC NET   Search   Login

Gajendra Purohit

#5 Educator in CSIR-UGC NET

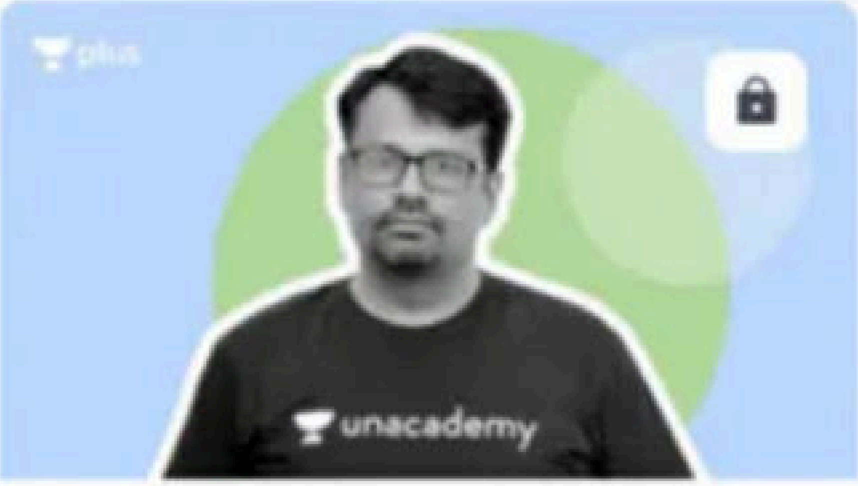
Dr.Gajendra Purohit PhD, CSIR NET (Maths) | Youtuber(330K+30k Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author of Bestseller

Follow

11M Watch mins | **1M** Watch mins (last 30 days) | **22k** Followers | **1k** Dedications

CSIR-UGC NET

SEE ALL



HINDI MATHEMATICAL SCIENCES

Course on Linear Algebra, Partial Diff. Equation & Calculus

Starts on Mar 1, 2021 • 24 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Course on Complex Analysis & Integral Equation

Starts on Jan 14, 2021 • 16 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Foundation Course on Mathematics for CSIR 2021

Starts on Dec 7, 2020 • 20 lessons

Gajendra Purohit

Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since