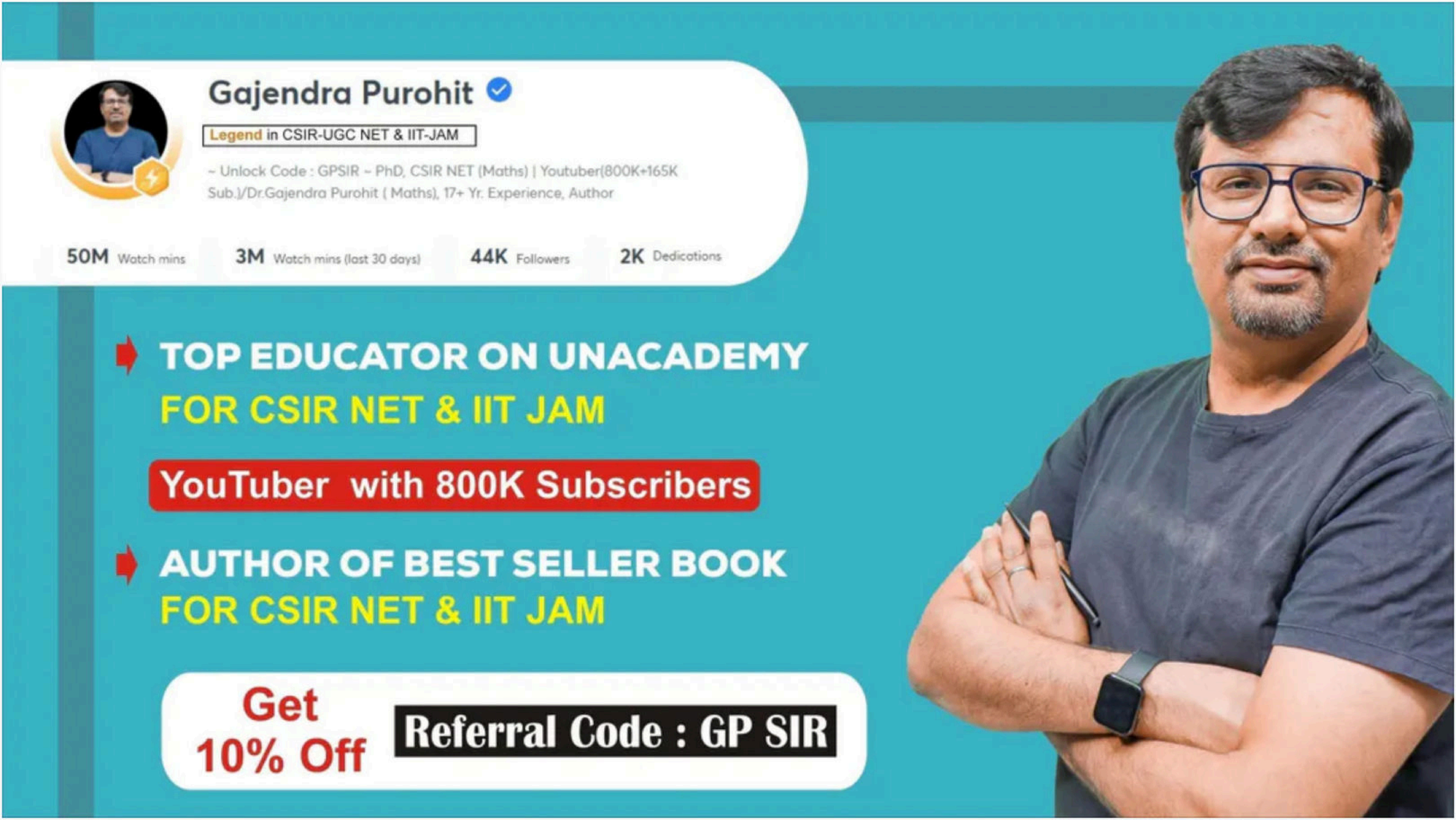
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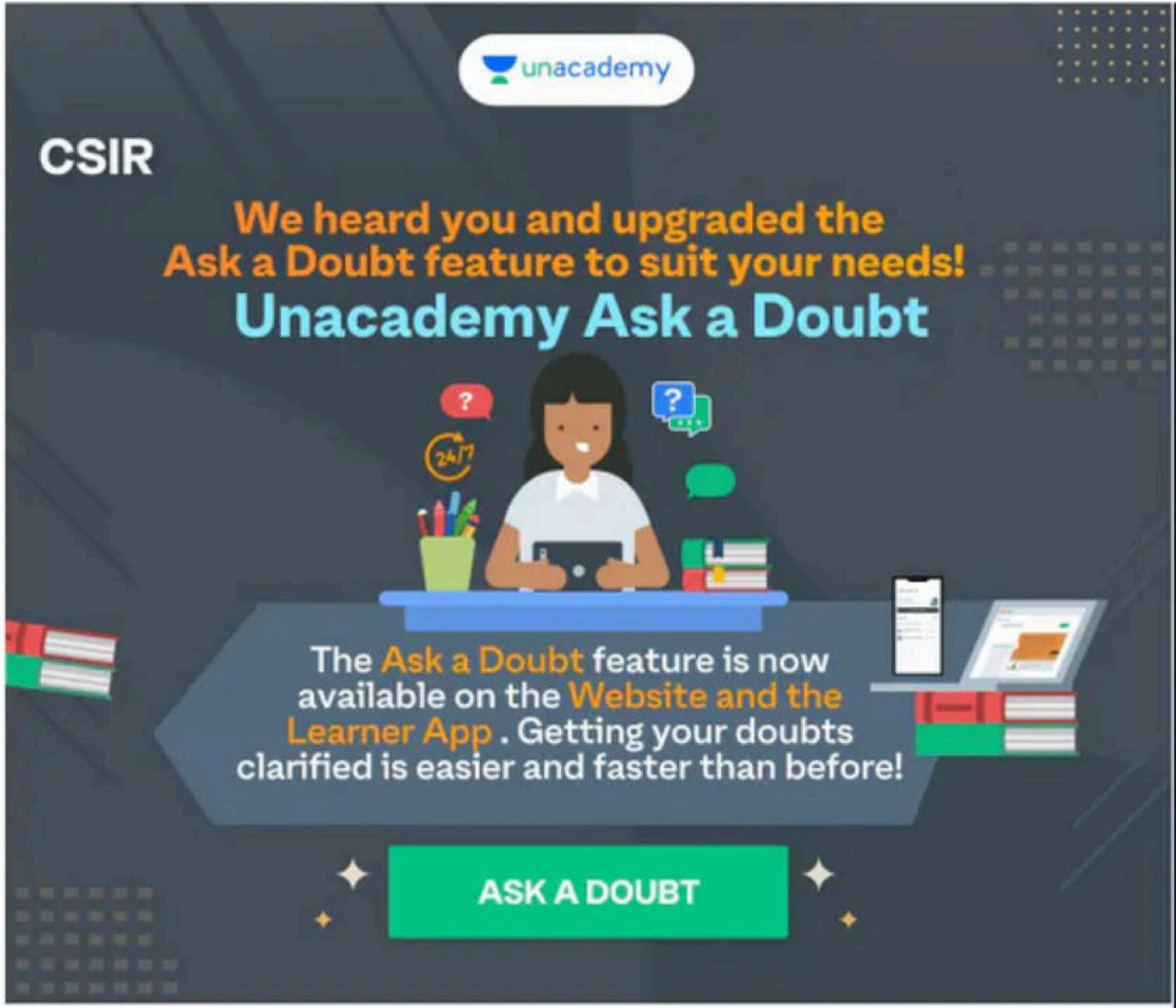
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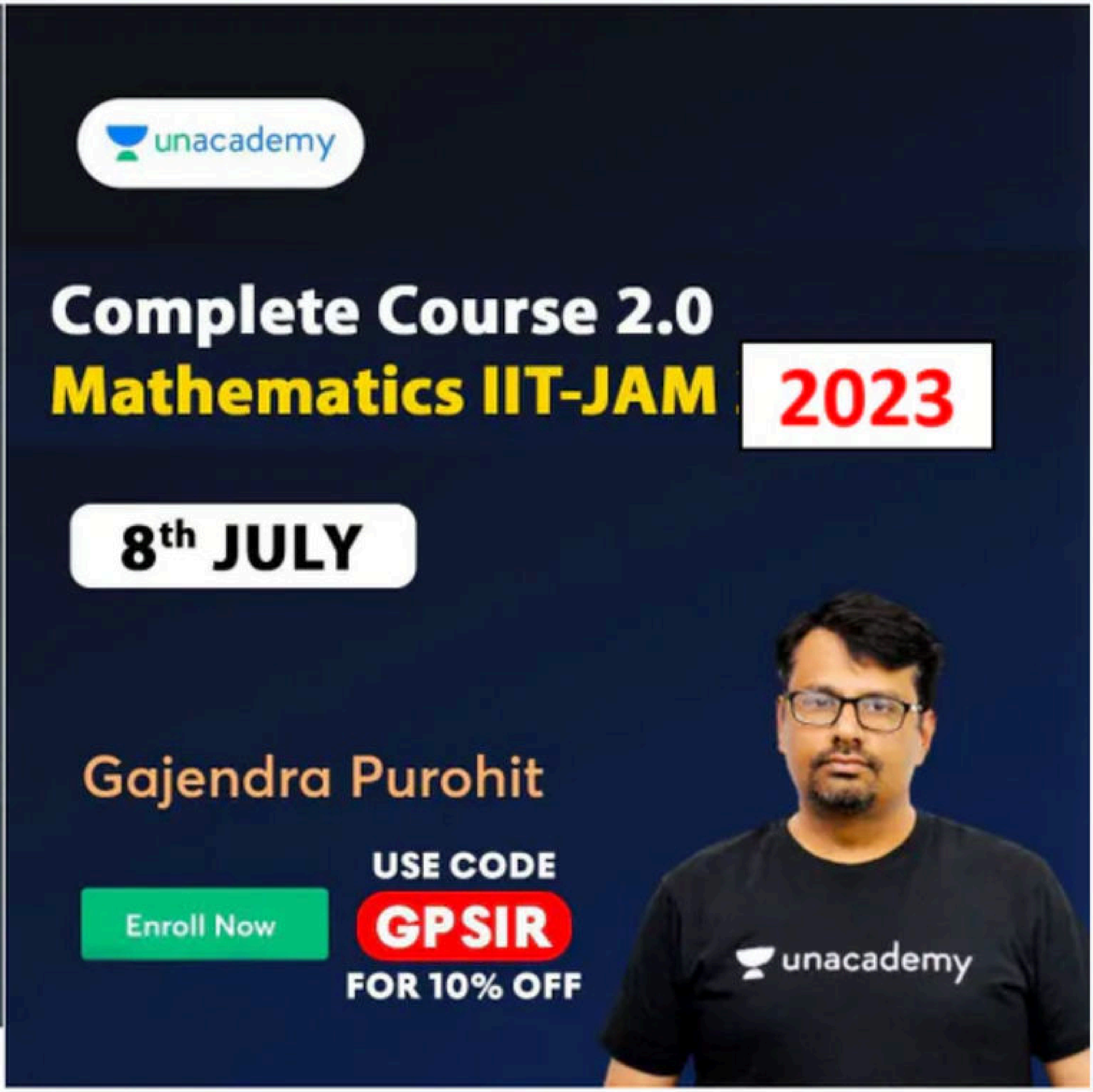
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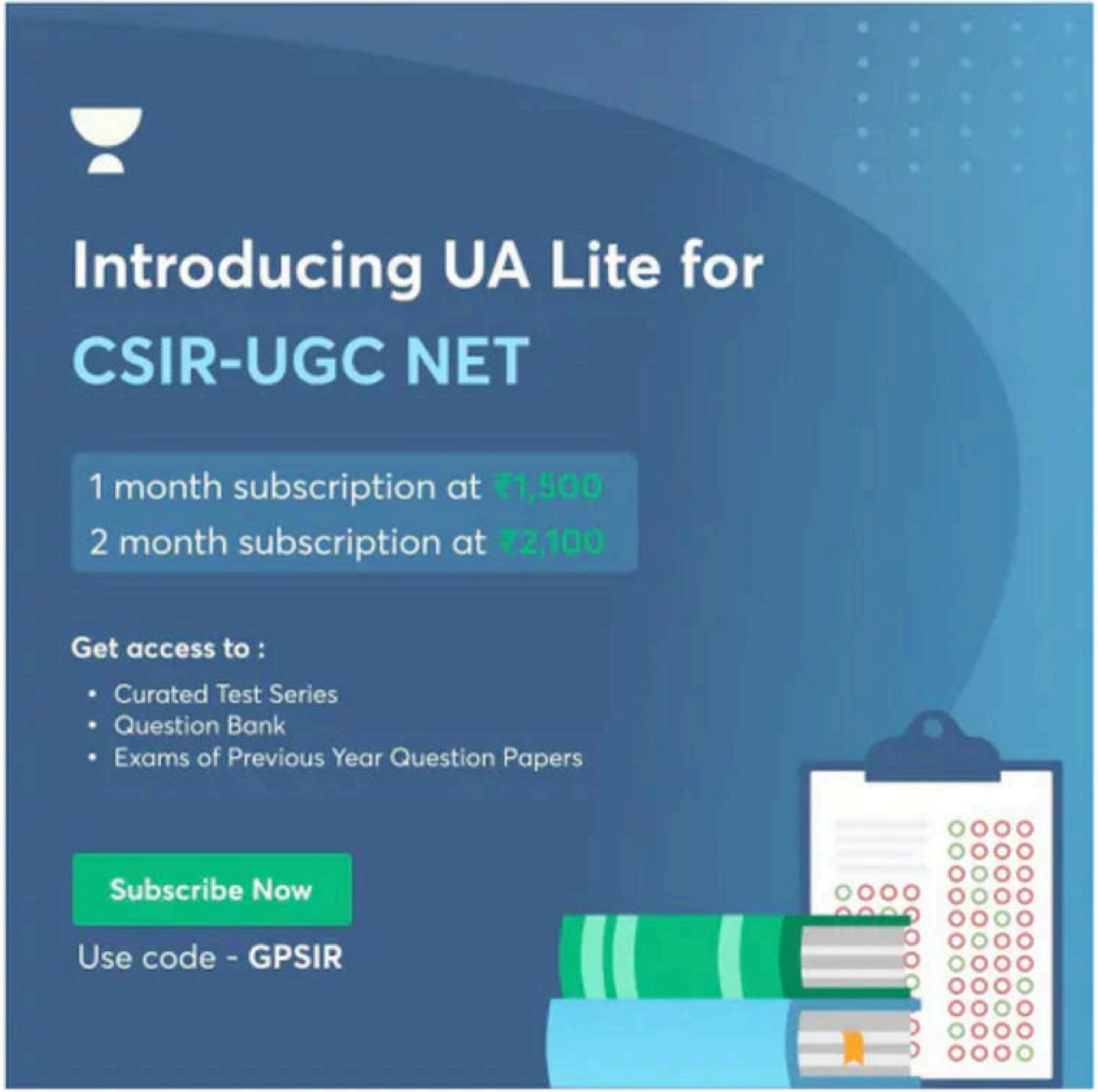

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Area and volume by double integral

Area of the region D by double integral :

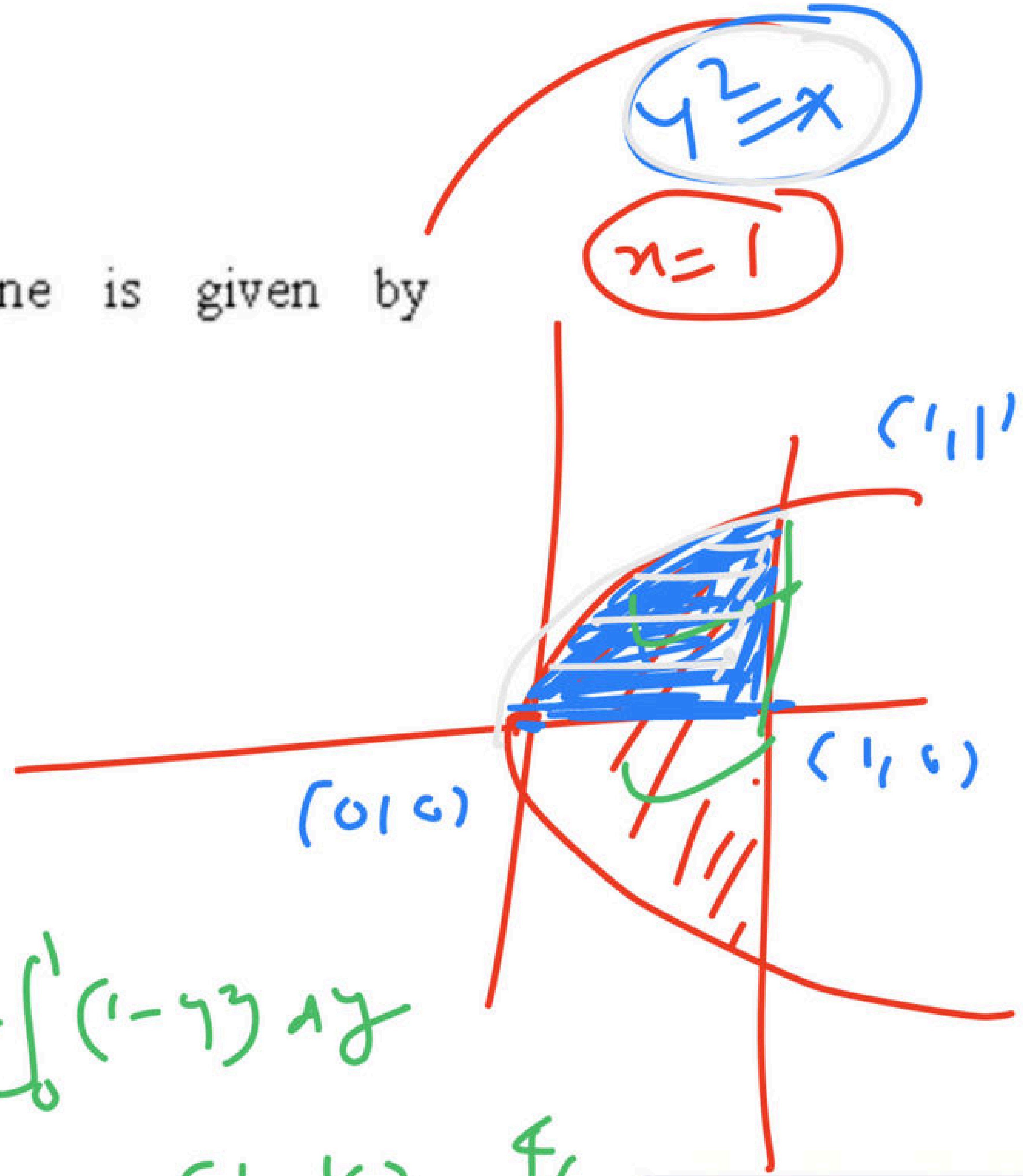
The area of the region D in the xy-plane is given by

$$A = \iint_D dx\,dy = \iint_D dA\,.$$

$$A = 2\int_{y=0}^{1}\int_{x=y^2}^{1} dx\,dy$$

$$= 2\int_0^1 (x)_{y^2}^{1}\,dy = 2\int_0^1 (1-y^2)\,dy$$

$$= 2\left(y - \frac{y^3}{3}\right)_0^1 = 2\left(1-\frac{1}{3}\right) = \frac{4}{3}$$



Q1. The area enclosed between the curves $|x| + |y| \geq 2$ and $y^2 = 4\left(1 - \frac{x^2}{9}\right)$ is

$6\left[\frac{4}{2}\sin^{-1}(1)\right] - 4(4-2)$

$6\left(2\cdot\frac{\pi}{2}\right) - 8 = 6\pi - 8$

$\pi 4$

πab

$\pi(2)(3) - (2\sqrt{2})^2$

(a) $(6\pi - 4)$ sq. units

(b) $(6\pi - 8)$ sq. units

(c) $(3\pi - 4)$ sq. units

(d) $(3\pi - 2)$ sq. units

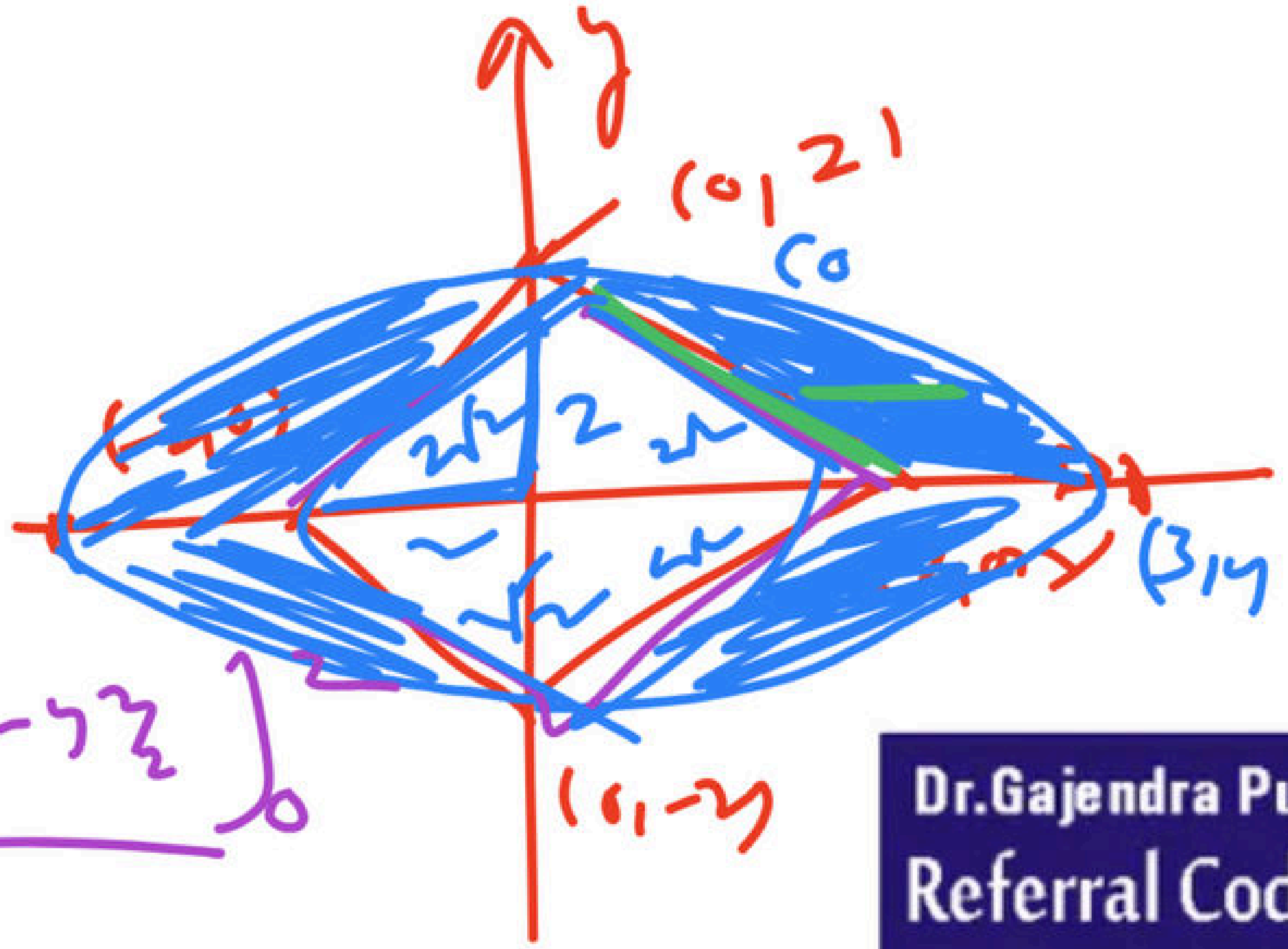
$6\pi - 8$

$A = 4\int^{2}\int^{3\sqrt{1-\frac{y^2}{4}}} dx\,dy$

$y = 0 \quad x = 2 - y$

$A = 4\int_0^2 \left(\frac{3}{2}\sqrt{4-y^2} - (2-y)\right) dy$

$4\left(\frac{3}{2}\right)\left[\frac{y}{2}\sqrt{4-y^2} + \frac{4}{2}\sin^{-1}\frac{y}{2}\right]_0^2 - 4\left[2y - \frac{y^2}{2}\right]_0^2$



Q2. Let the straight line x = b divides the area enclosed by $y = (1-x)^2$, $y = 0$ and $x = 0$ into two parts $R_1 (0 \le x \le b)$ and $R_2 (b \le x \le 1)$ such $R_1 - R_2 = \frac{1}{4}$. Then b equals

(a) 3/4 (b) 1/2

(c) 1/3 (d) 1/4

$$\frac{1}{3} - \frac{(1-b)^3}{3} - \frac{(1-b)^3}{3} = \frac{1}{4}$$

$$\frac{2}{3}(1-b)^3 = \frac{1}{3} - \frac{1}{4}$$

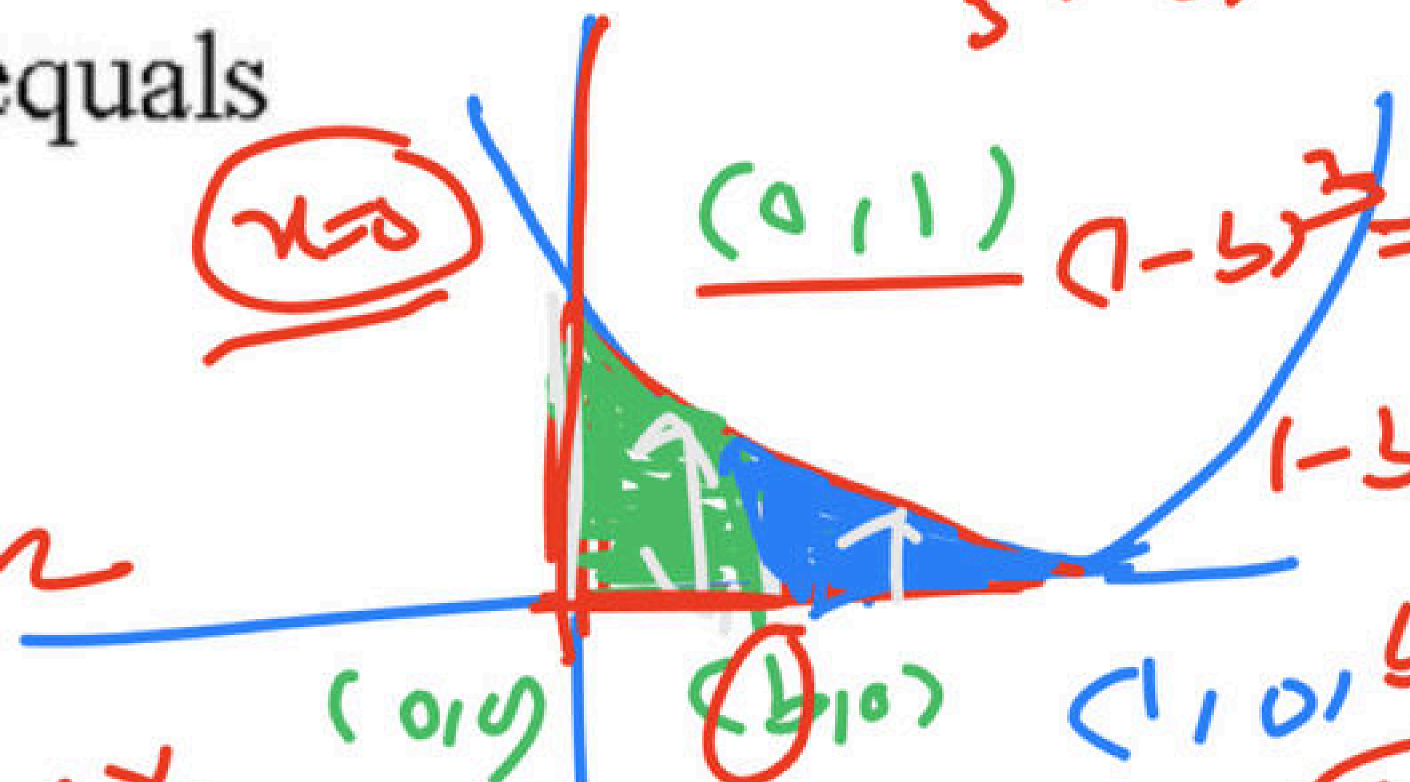
$$\frac{2}{3}(1-b)^3 = \frac{1}{12}$$

$$(1-b)^3 = \frac{1}{8}$$

$$1-b = \frac{1}{2}$$

$$b = 1 - \frac{1}{2}$$

$$b = \frac{1}{2}$$



$$R_1 = \int_{x=0}^{b} \int_{y=0}^{(1-x)^2} dy\, dx$$

$$= \int_0^b (1-x)^2 dx$$

$$= -\left(\frac{(1-x)^3}{3}\right)_0^b = \frac{1}{3} - \frac{(1-b)^3}{3}$$

$$R_2 = \int_b^1 \int_{y=0}^{(1-x)^2} dy\, dx$$

$$\int_b^1 (1-x)^2 dx = -\left(\frac{(1-x)^3}{3}\right)_b^1$$

$$= -\left(0 - \frac{(1-b)^3}{3}\right)$$

$$= \frac{(1-b)^3}{3}$$

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Volume by double integration :

The volume of solids by double integration is $\iint z\,dx\,dy$, where z = f(x, y) is given surface in x & y variable.

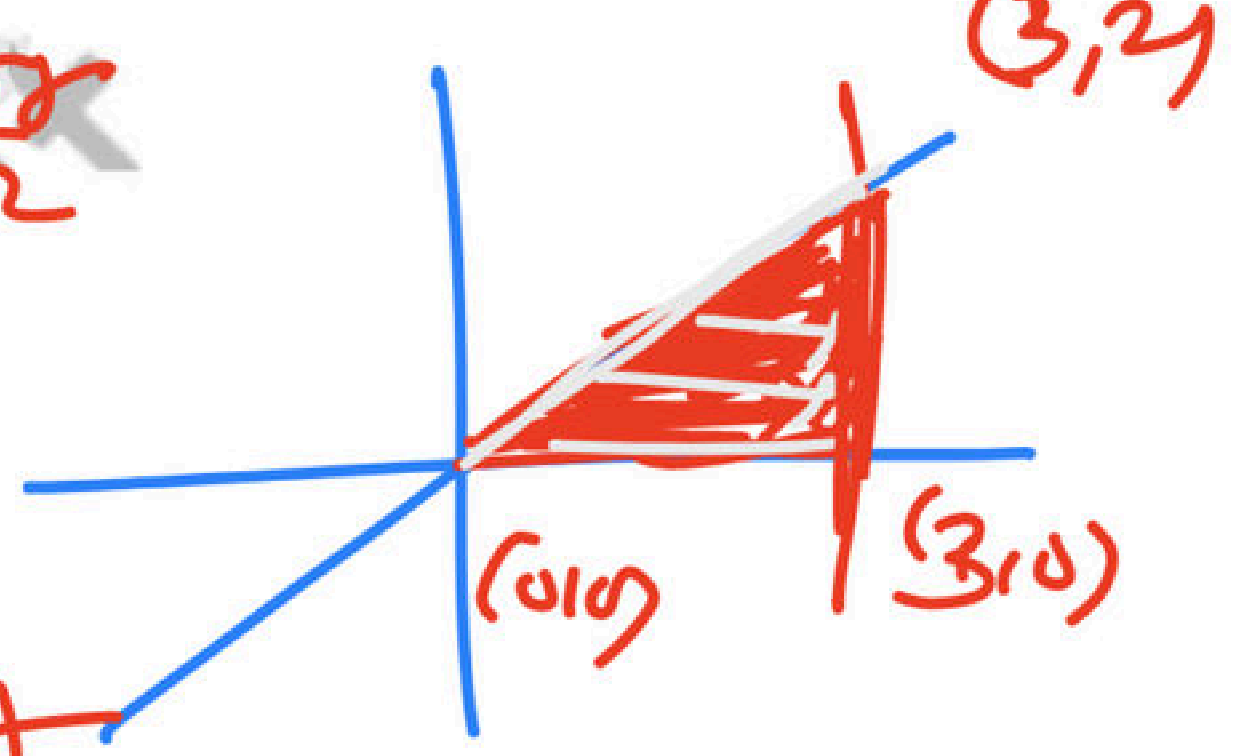
Q.3. A triangle in xy-plane is bounded by straight line $2x = 3y$, $y = 0$ and $x = 3$, then volume above the triangle and under the plane $x + y + z = 6$ **GATE-2016**

$\iint z\,dx\,dy$

(a) 5 $\quad 27 - 24 + 7 \quad 34 - 24$ (b) 10

$\int_{y=0}^{2}\int_{x=\frac{3y}{2}}^{3} (6 - x - y)\,dx\,dy$

(c) 15 (d) 20



$$\int_0^2 \left(6x - \frac{x^2}{2} - xy\right)_{3y/2}^{3} dy$$

$$\int_0^2 \left[\left(18 - \frac{9}{2} - 3y\right) - \left(\frac{18y}{2} - \frac{9y^2}{8} - \frac{3y^2}{2}\right)\right] dy$$

$$\int_0^2 \left(\frac{27}{2} - 12y + \frac{21y^2}{8}\right) dy = \left(\frac{27}{2}y - 6y^2 + \frac{7y^3}{8}\right)_0^2$$

Q.4. The volume of the solid cut off by the surface $z = (x + y)^2$ from the right prism whose base in the plane $z = 0$ is the triangle by the lines $x = 0, y = 0, x + y = 1$.

(a) 0 (b) 1/2

(c) 1/3 (d) 1/4

$$\iint z \, dx \, dy$$

$$\int_0^1 \int_0^{1-y} (x+y)^2 \, dx \, dy$$

$y = 0 \quad x = 0$

$$\int_0^1 \left(\frac{(x+y)^3}{3} \right)_0^{1-y} dy$$

$$\int_0^1 \left(\frac{1}{3} - \frac{y^3}{3} \right) dy$$

$$\frac{1}{3}\left(y - \frac{y^4}{4} \right)_0^1$$

$$\frac{1}{3}\left(1 - \frac{1}{4} \right)$$

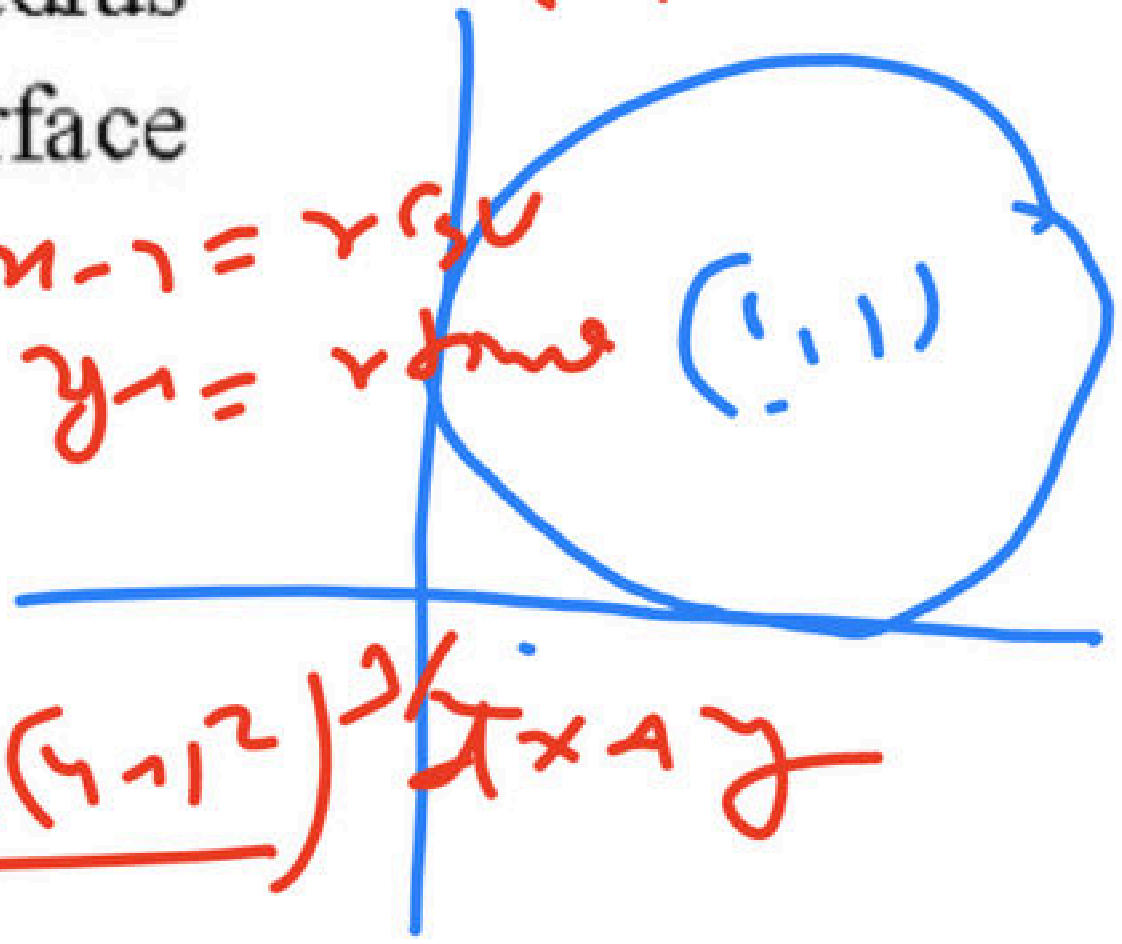
$$\frac{1}{3}\left(\frac{3}{4} \right) = \frac{1}{4}$$

Q.5. The volume of the cylinder with base as the disc of unit radius in the xy-plane centred at (1, 1) and top being the surface $z = [(x-1)^2 + (y-1)^2]^{3/2}$. **IIT JAM – 2005**

(a) π (b) 2π

(c) $\frac{2\pi}{3}$ (d) $\frac{2\pi}{5}$

$(x-1)^2+(y-1)^2=1$



$x-1 = r\cos\theta$

$y-1 = r\sin\theta$

$V = \iint z\,dx\,dy$

$V = \iint \left((x-1)^2+(y-1)^2\right)^{3/2} dx\,dy$

$\int_0^{2\pi}\int_0^{1} (r^2)^{3/2}\, r\,dr\,d\theta$

$\theta = 0 \quad r = 0$

$\int_0^{2\pi} d\theta \int_0^{1} r^4\,dr$

$(2\pi)\left(\frac{r^5}{5}\right)_0^1 = 2\pi\left(\frac{1}{5}\right)$

Q.5. Consider the open rectangle $G = \{(s,t) \in R^2 : 0 < s < 1$ and $0 < t < 1\}$ and the map $T : G \to R^2$ given by $T(s,t) = \left(\frac{\pi s(1-t)}{2}, \frac{\pi(1-s)}{2}\right)$ for $(s,t) \in G$ Then the area of the image $T(G)$ of the map T is equal to **IIT JAM 2022**

(a) $\pi/4$ (b) $\pi^2/4$

(c) $\pi^2/8$ (d) 1

$\frac{\pi s(1-t)}{2} = x$

$\frac{\pi(1-s)}{2} = y$

$1-s = \frac{2y}{\pi}$

$\boxed{s = 1 - \frac{2y}{\pi}}$

$0 < s < 1$

$0 < 1 - \frac{2y}{\pi} < 1$

$0 < \pi - 2y < \pi$

$-\pi < -2y < 0$

$0 < 2y < \pi$

$0 < y < \pi/2$

$\pi\left(1 - \frac{2y}{\pi}\right)(1-t) = 2x$

$(\pi - 2y)(1-t) = 2x$

$1 - t = \frac{2x}{\pi - 2y}$

$t = 1 - \frac{2x}{\pi - 2y}$

$0 < t < 1$

$0 < 1 - \frac{2x}{\pi - 2y} < 1$

$-1 < -\frac{2x}{\pi - 2y} < 0$

$0 < \frac{2x}{\pi - 2y} < 1$

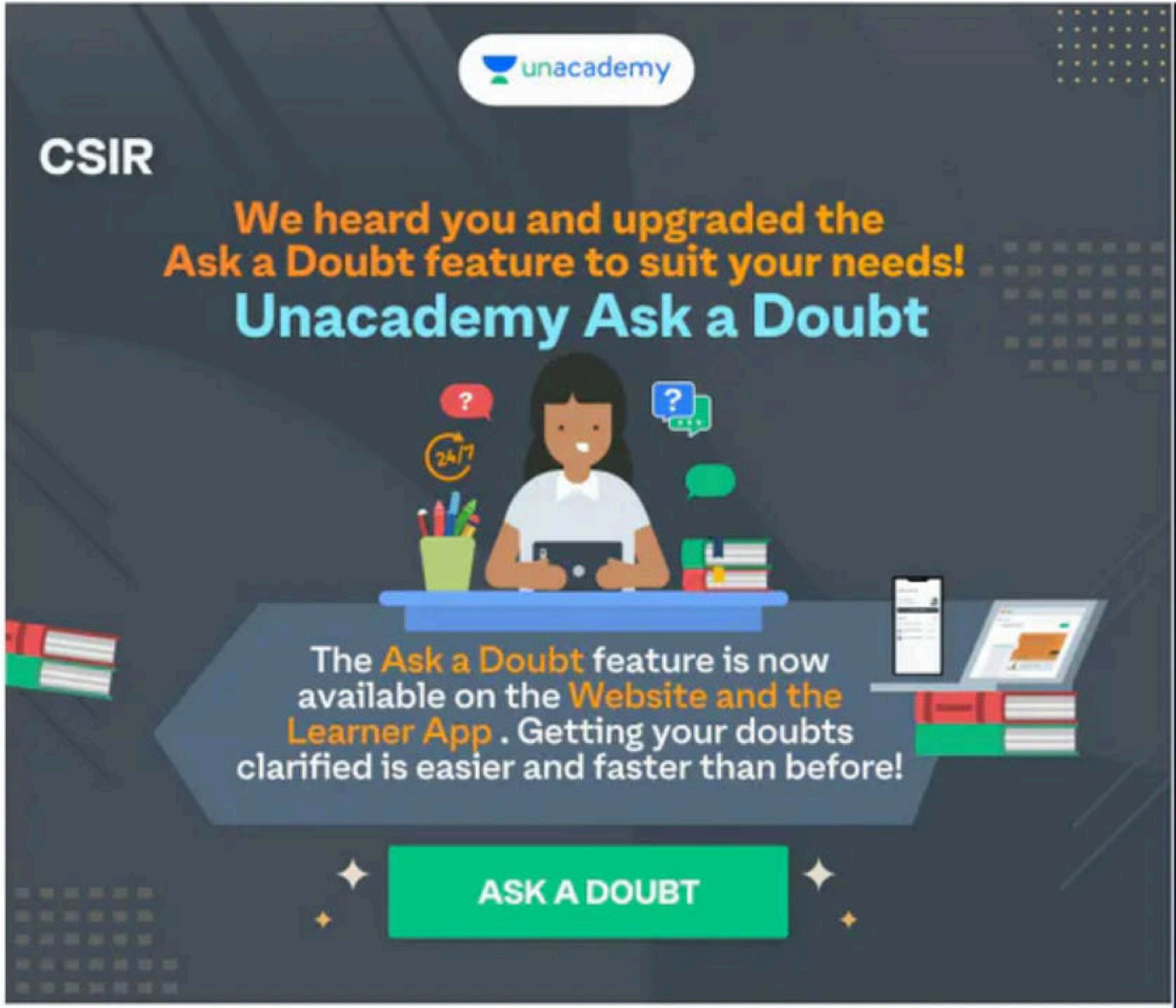
$0 < 2x < \pi - 2y$

$0 < x < \frac{\pi - 2y}{2}$

$$\int_{y=0}^{\pi/2} \int_{x=0}^{\frac{\pi - 2x}{2}} dx\,dy$$

$$\int_0^{\pi/2} \left(\frac{\pi - 2y}{2}\right) dy = \frac{1}{2}\int_0^{\pi/2} (\pi y - y^2)\,dy$$

$$= \frac{1}{2}\left(\frac{\pi^2}{2} - \frac{\pi^2}{4}\right) = \frac{1}{2}\left(\frac{\pi^2}{4}\right) = \frac{\pi^2}{8}$$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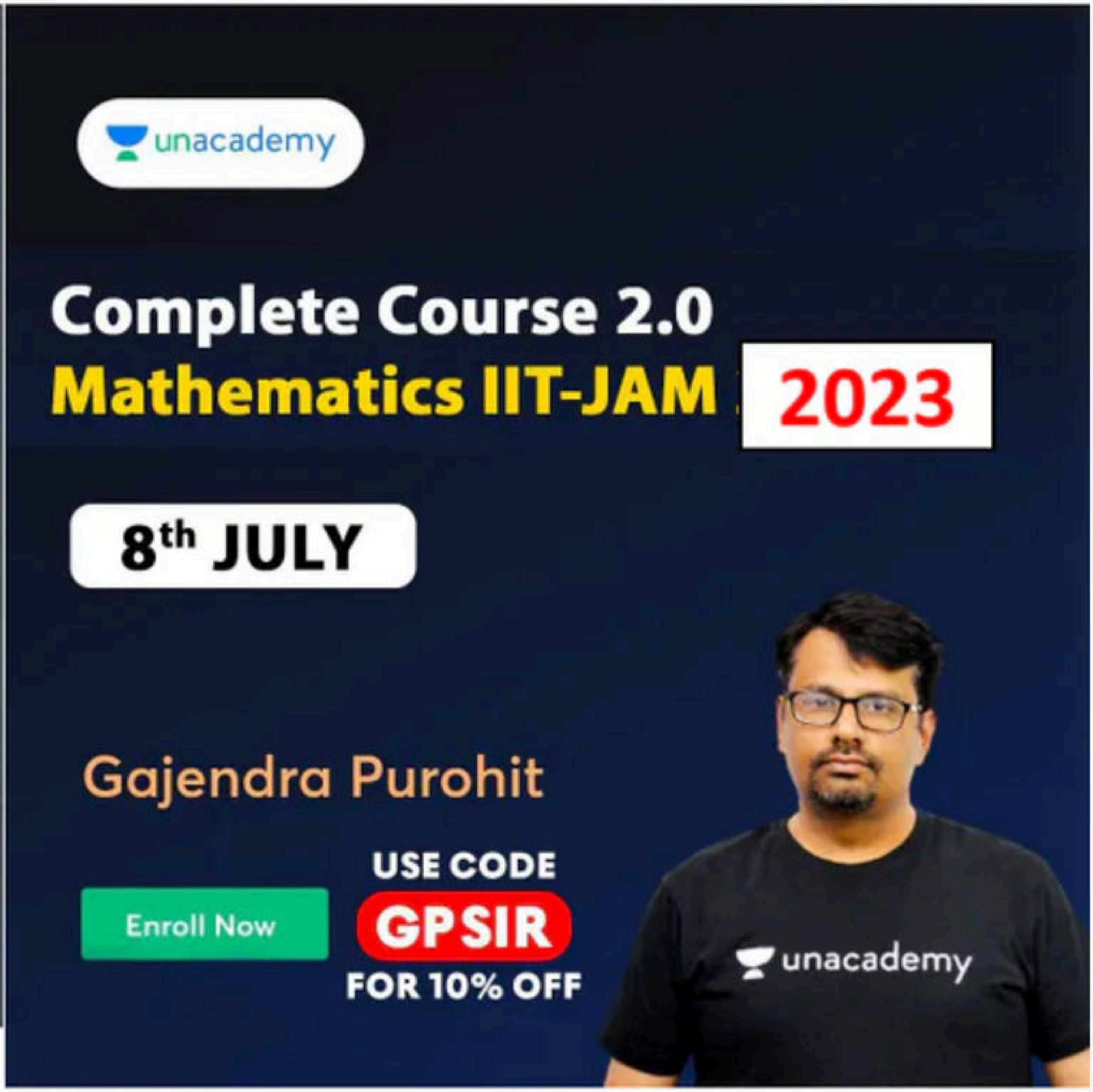
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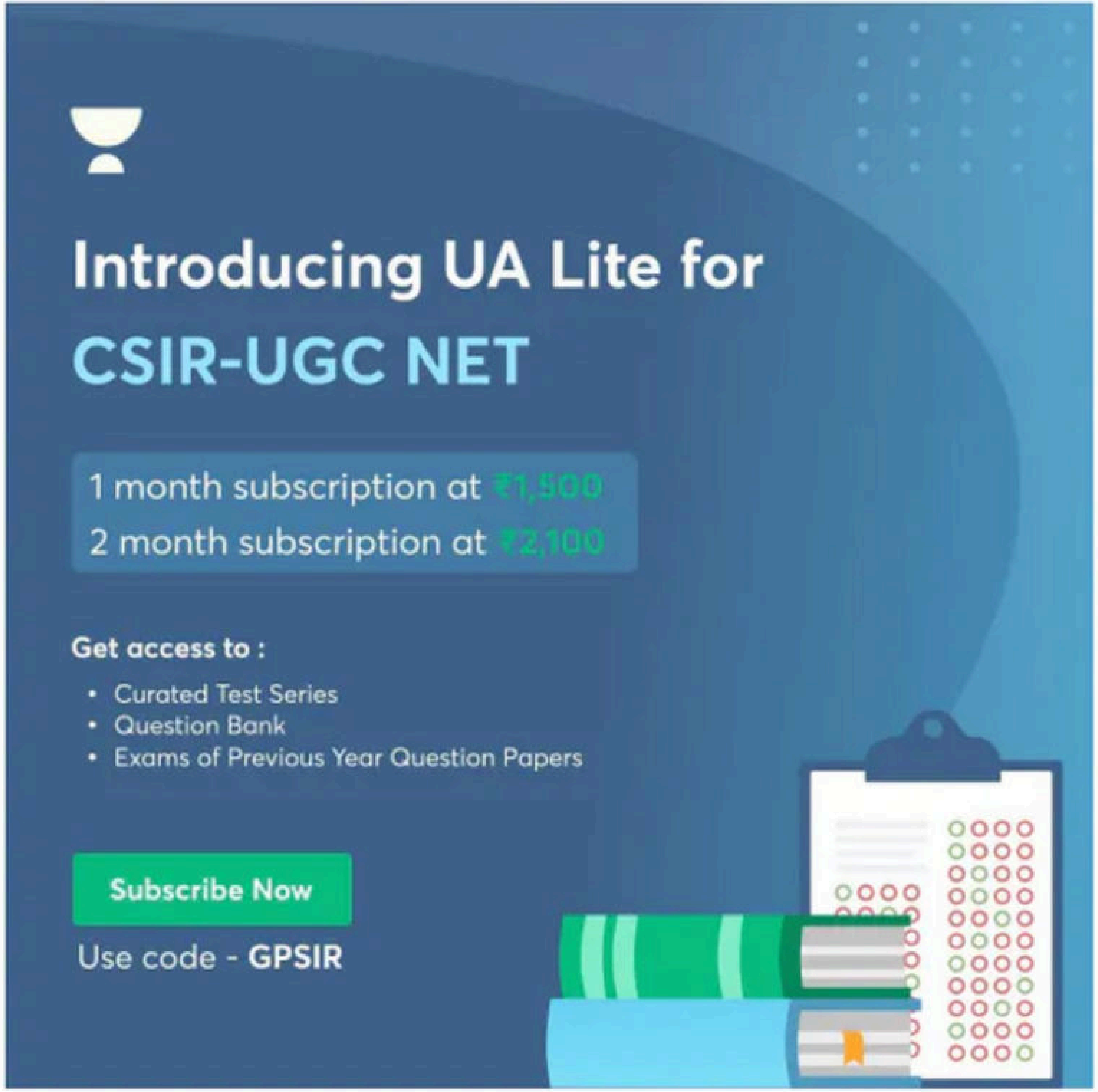
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