



Function and their Properties

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23



Gajendra Purohit

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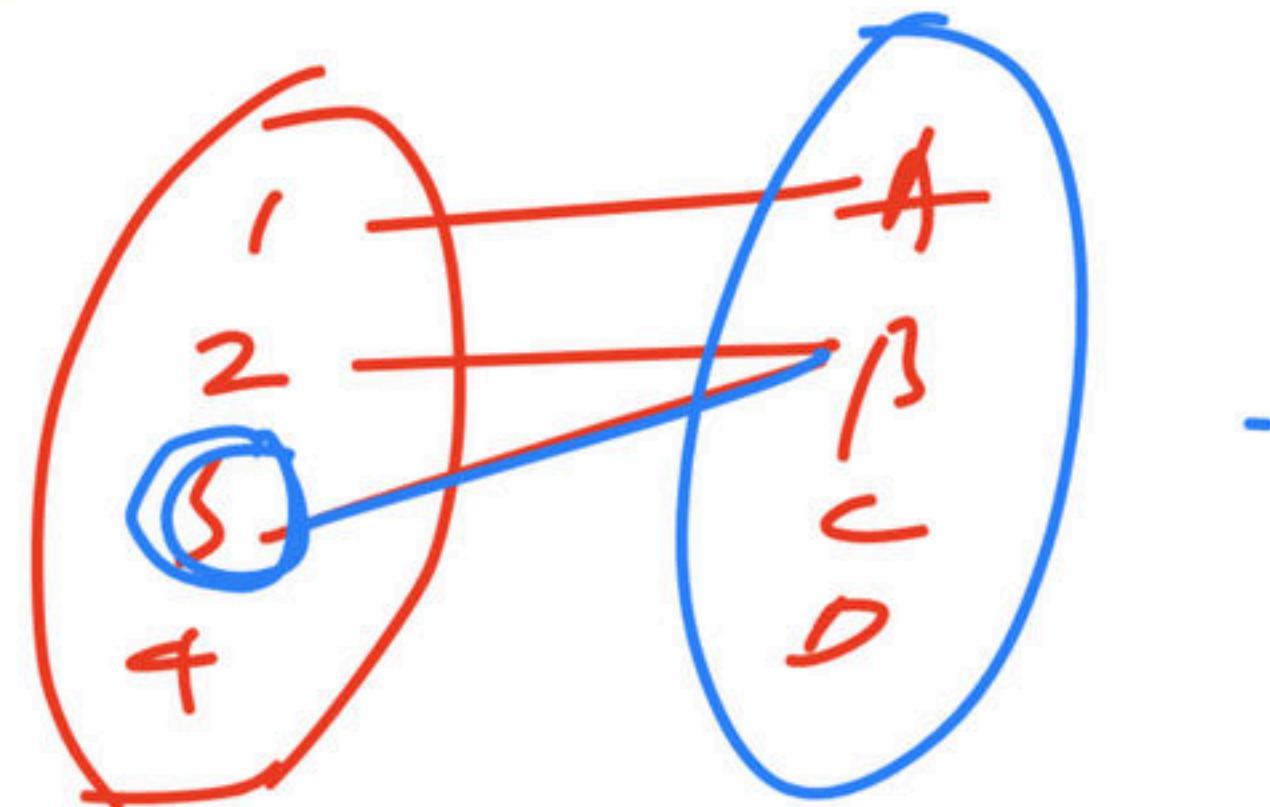
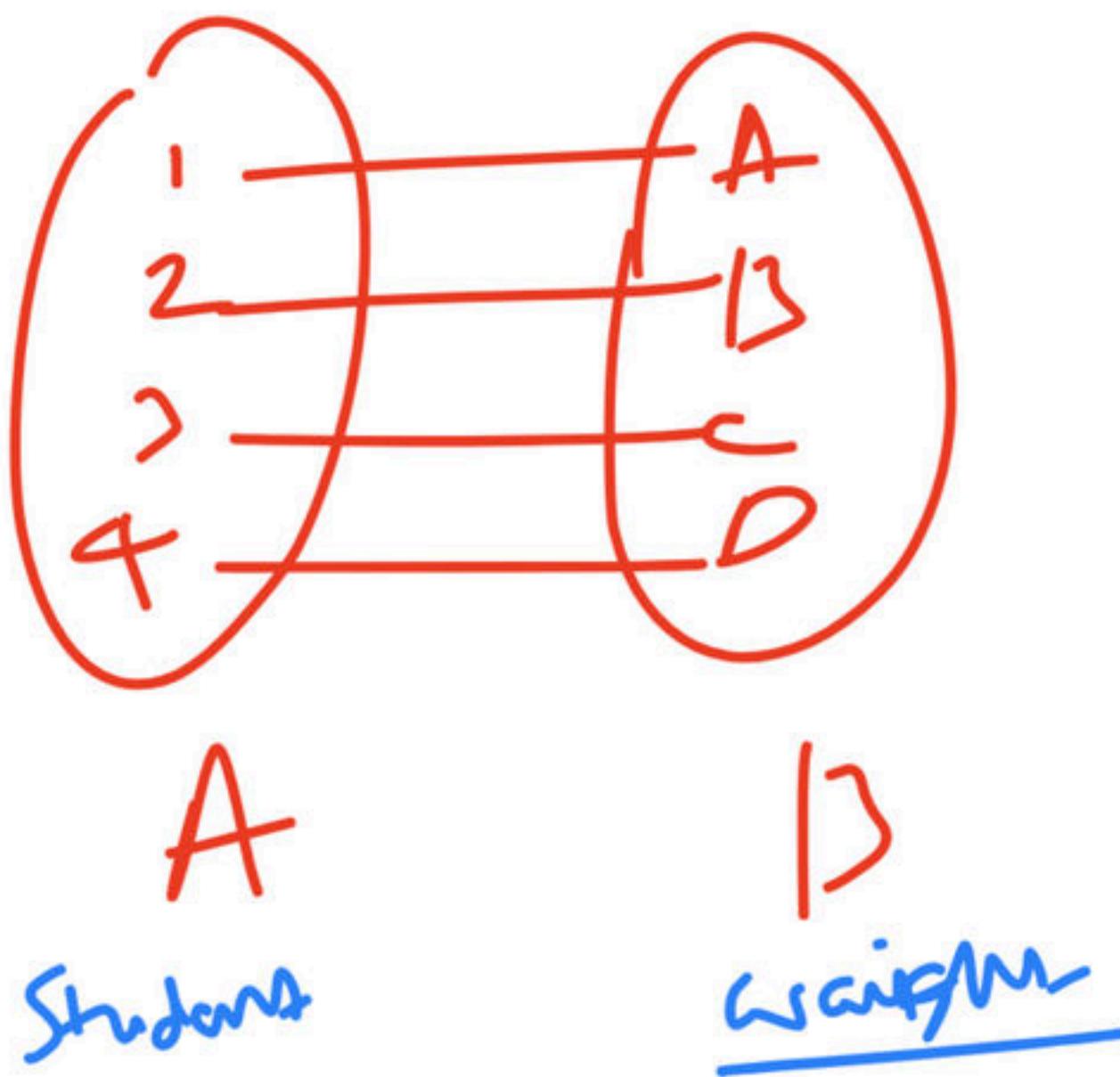
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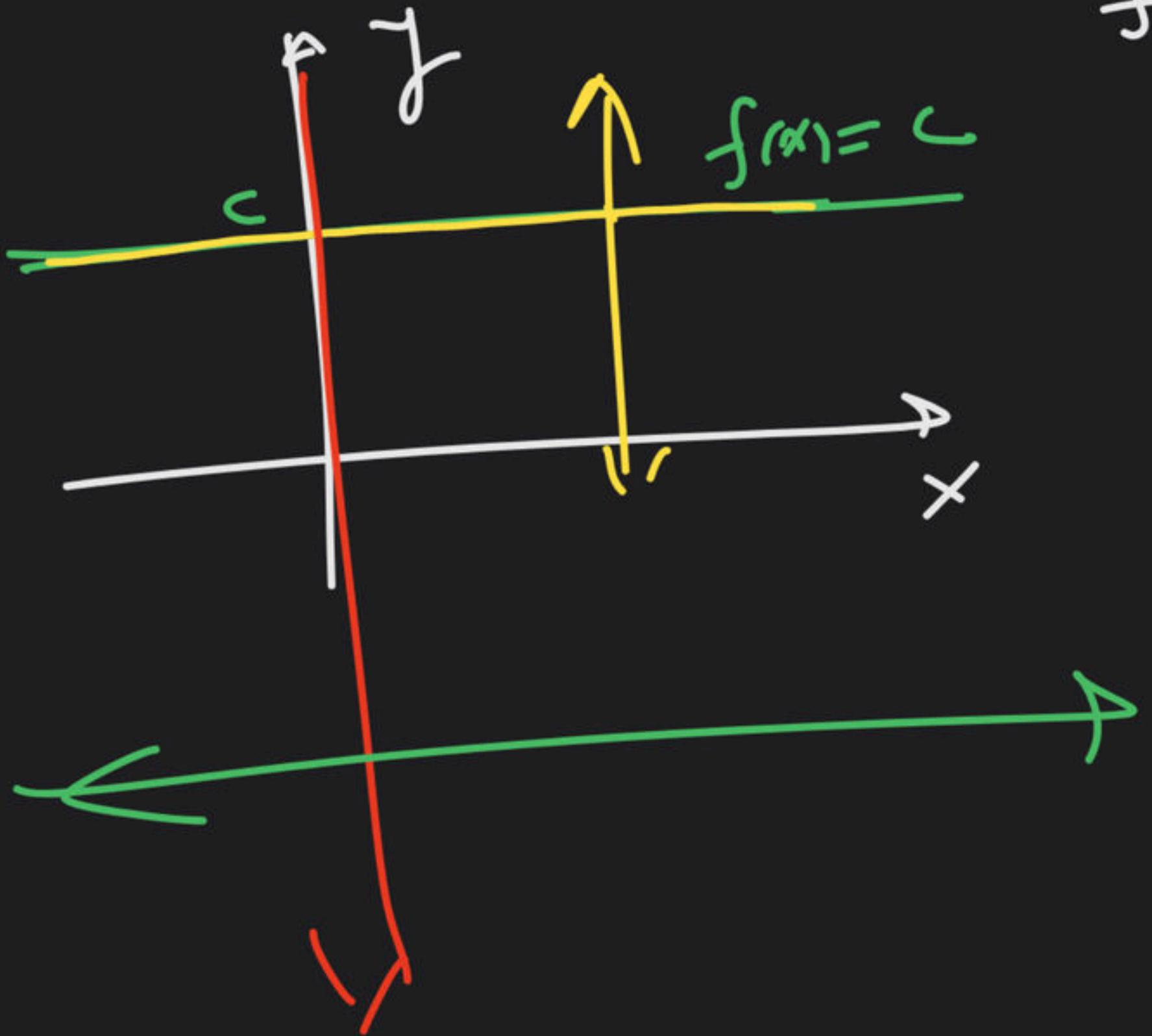
Function of one variable

Function : A mapping $f : A \rightarrow B$ is said to be a function if every element of A assign to unique element of B.



Example

Constant-functions

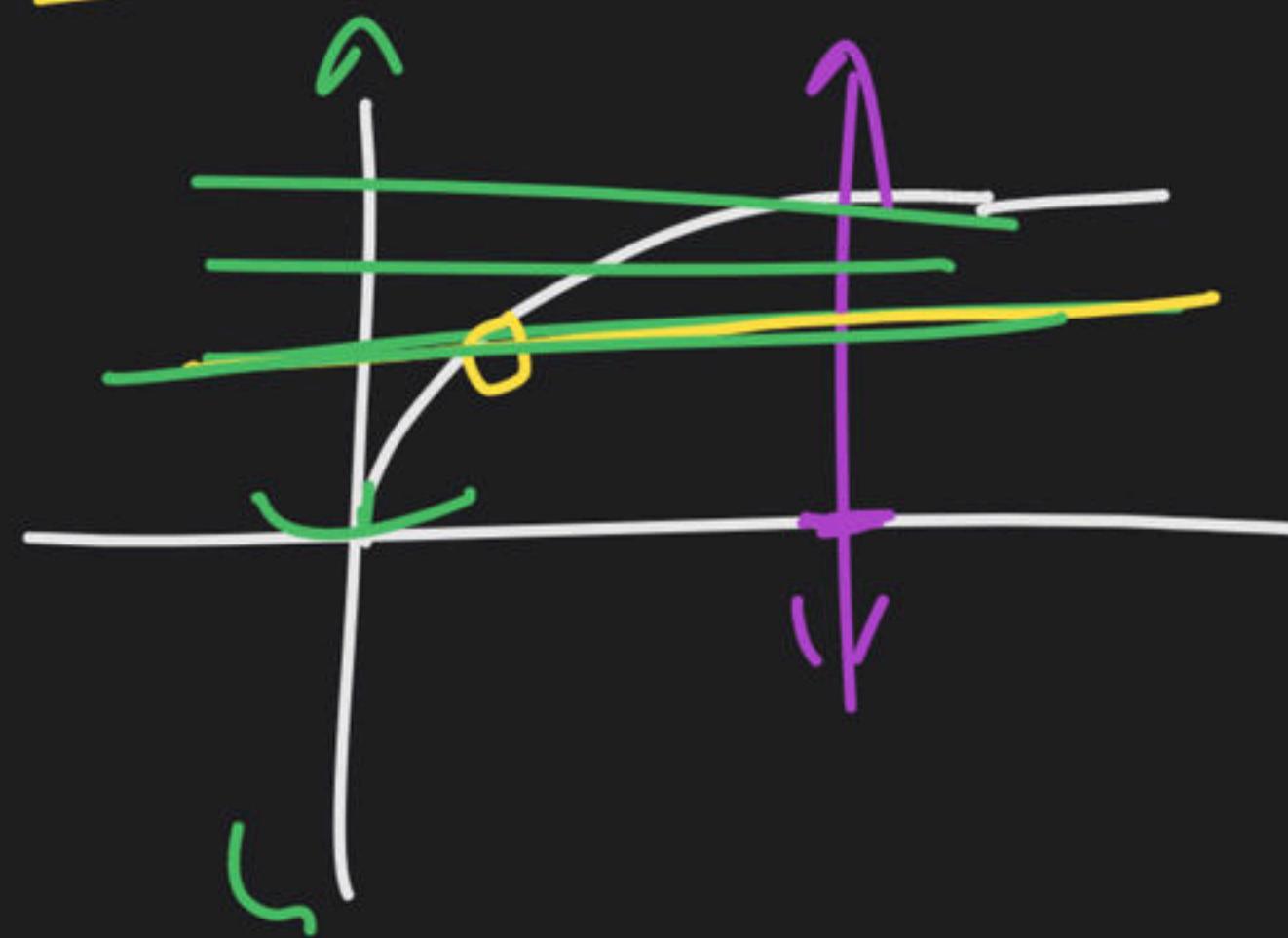


$$f: \mathbb{R} \rightarrow \mathbb{R}$$
$$f(x_1) = c$$

Domain \mathbb{R}
Range $= \{c\}$

Example

Square



Range of function

$$f: [0, \infty) \rightarrow [0, \infty)$$

Domain : \mathbb{R}^+

Range = \mathbb{R}^+

$$f(x) = \sqrt{x}$$

$$y = \sqrt{x}$$

$$y^2 = x$$

Absolute Value function

$$f(x) = \begin{cases} -x & x \leq 0 \\ x & x > 0 \end{cases}$$



$$f: \mathbb{R} \rightarrow \mathbb{R}^+$$

$$f(x) = |x|$$

Domain = \mathbb{R}

Range = $\mathbb{R}^+ = [0, \infty)$

Signum function

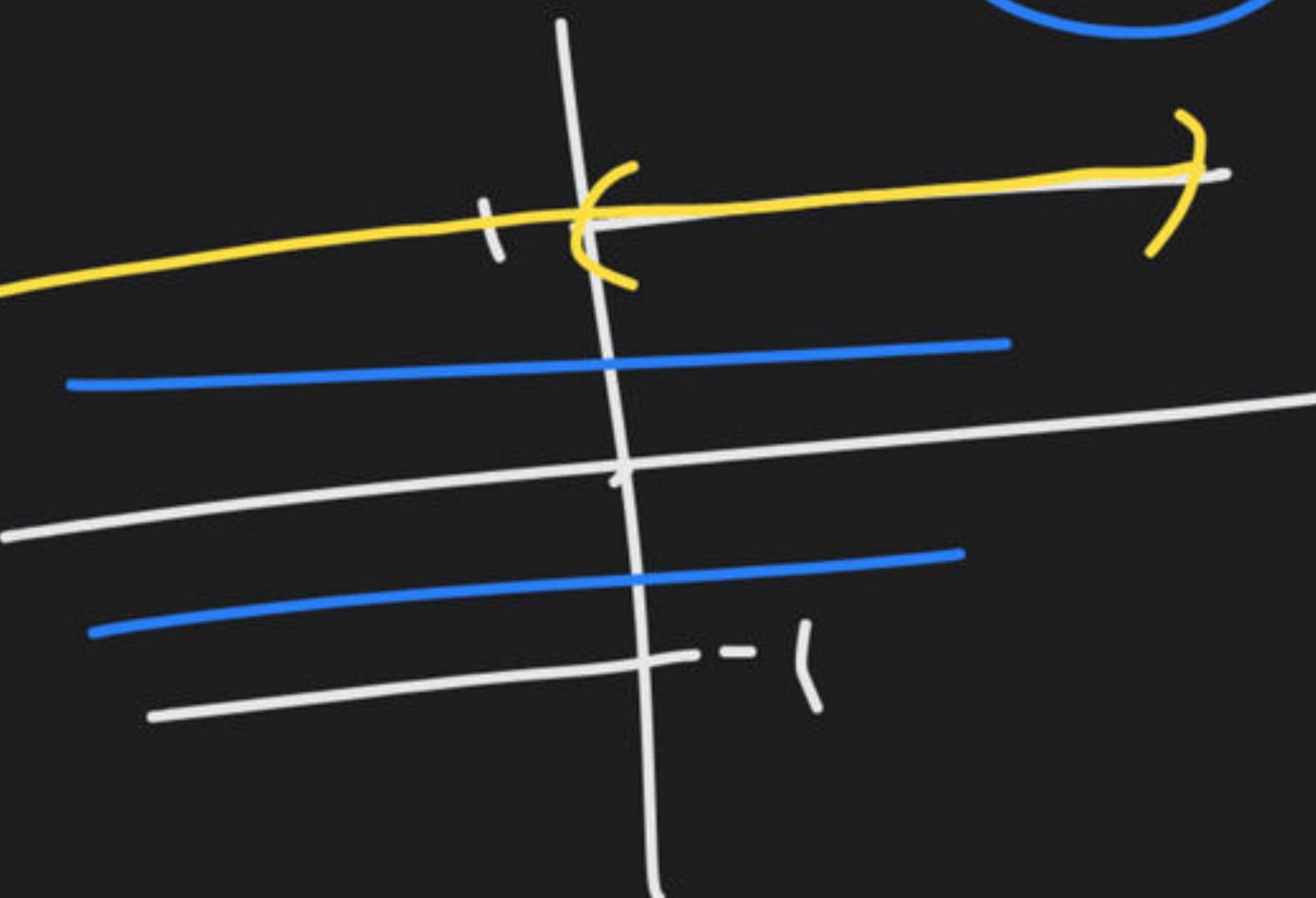
$f: \mathbb{R} \rightarrow \boxed{\{-1, 0, 1\}}$

$\textcircled{(-1, 1)}$

$$f(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

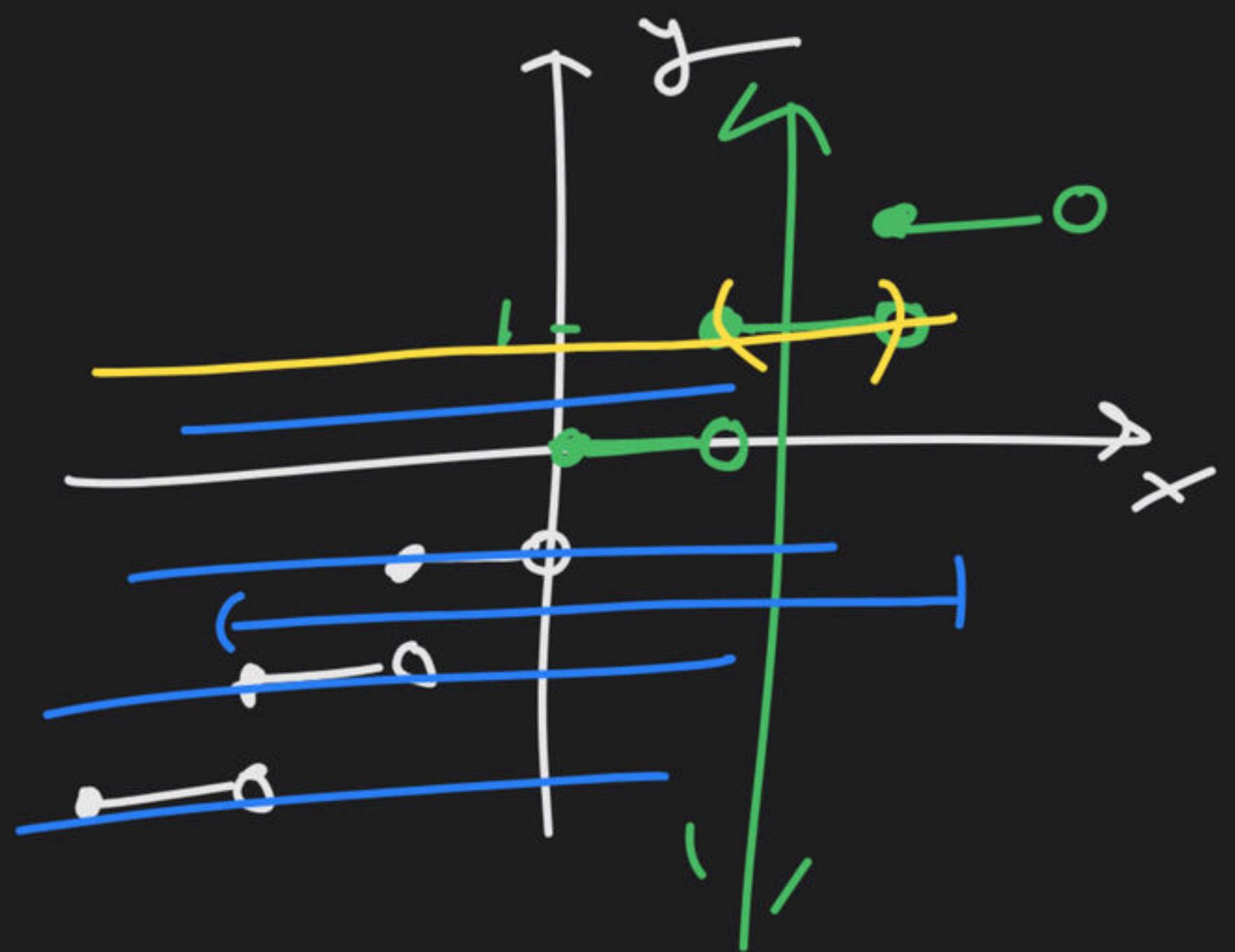
Domain = \mathbb{R}

Range = $\boxed{\mathbb{R} \setminus \{-1, 0, 1\}}$

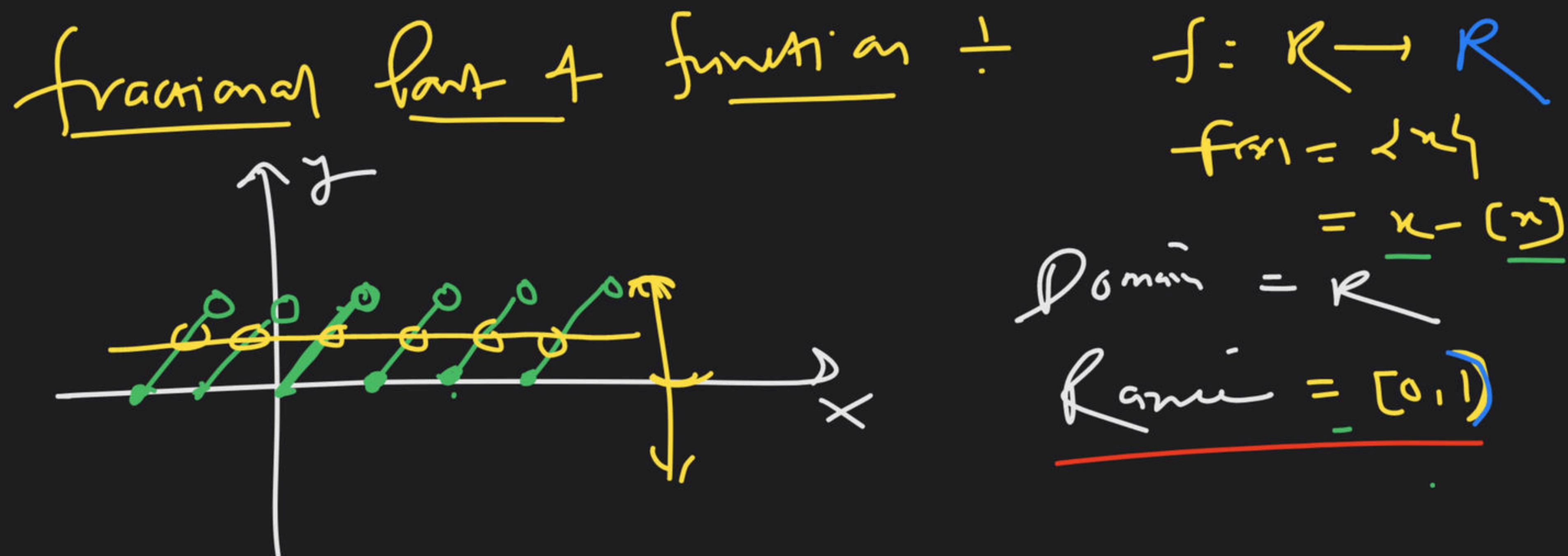


Greatesh integer function for $[n]$

$f: \mathbb{R} \rightarrow \mathbb{Z}$



Domain = \mathbb{R}
Range = $\{-2, -1, 0, 1, 2, \dots\}$
 $= \mathbb{Z}$



Logarithmic functions

(Case I) $a > 1$ \log_e^y

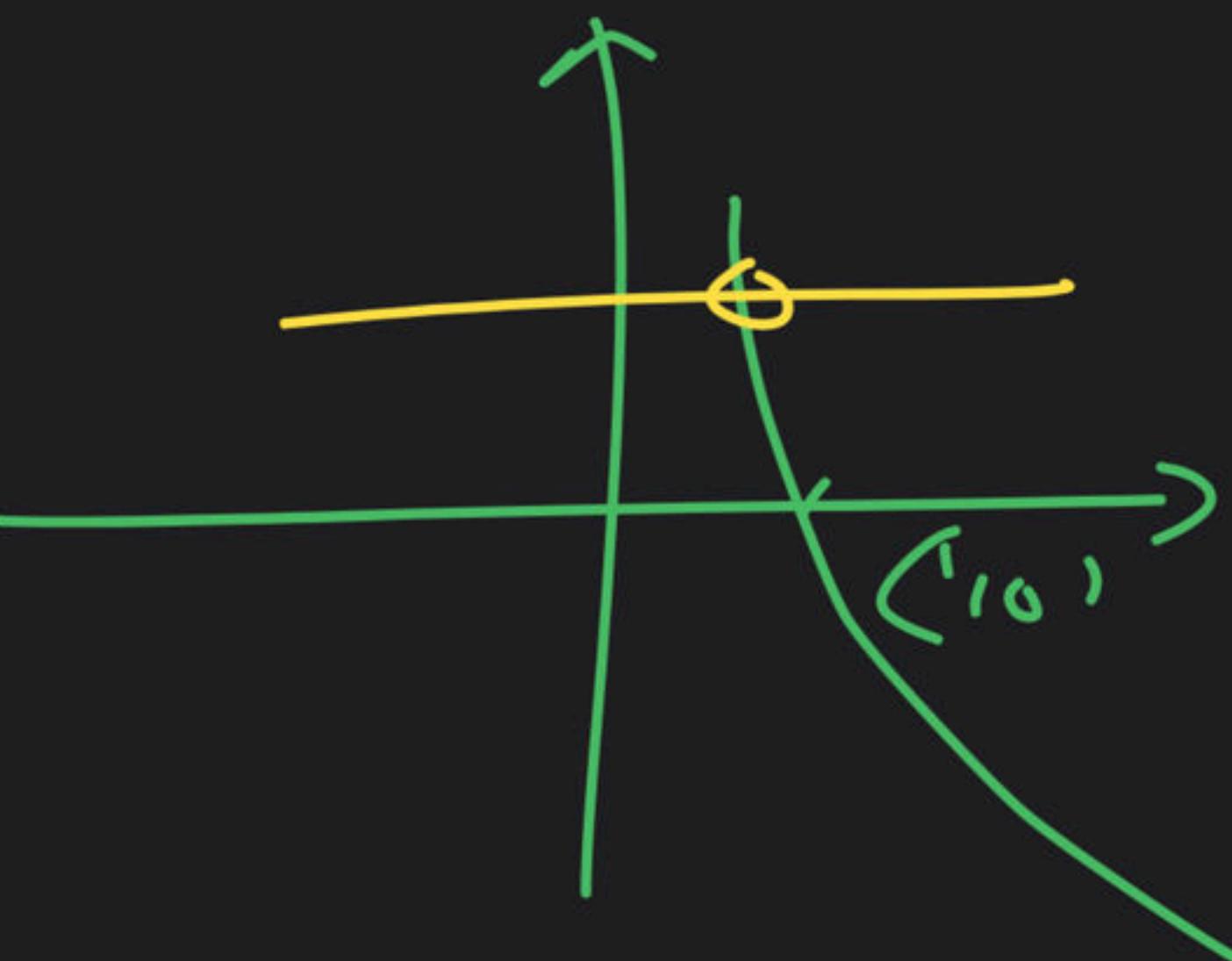


$$\text{Domain} = (0, \infty)$$

$$\text{Range} = \mathbb{R}$$

$f: \mathbb{R}^+ \rightarrow \mathbb{R}$, $f(x) = \log_a x$

(Case II) $a < 1$

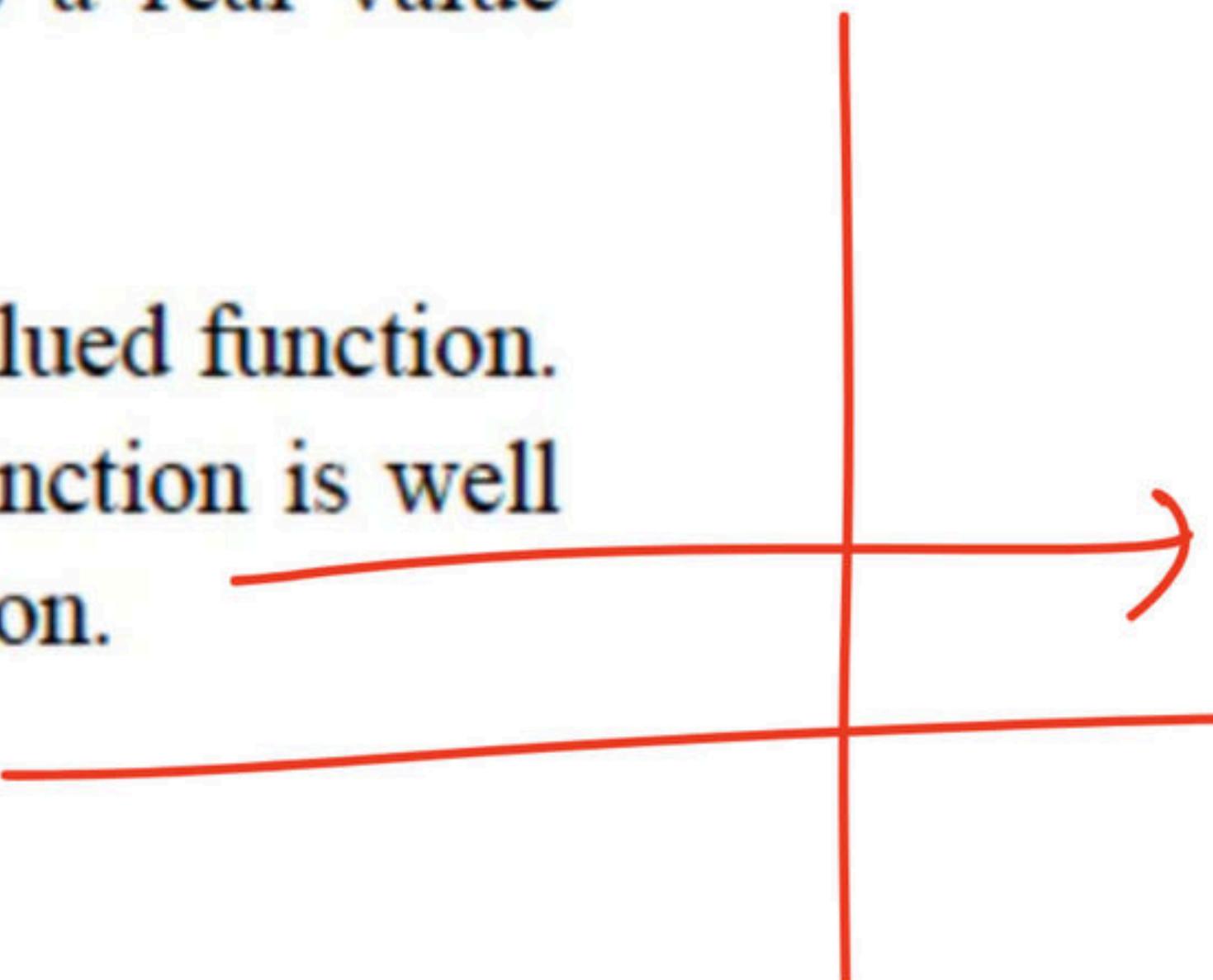




Domain and range of function :

(i) Domain of function : Let $f : D \rightarrow R$ is a real value function then D is called domain of function

Another definition : Let $f : D \rightarrow R$ is a real valued function. Then the set of all points of R at which the function is well define then this set is called as domain of function.



(ii) Range of function : Let $f : A \rightarrow B$ is a function. then the set of all images of distinct elements of A is called range of function.

Bounded and Monotonic function :

(i) Bounded function : A function $f : D \rightarrow R$ is called bounded function if range of this function is bounded.

(ii) Monotonic function :

(1) Monotonic increasing function :

Let $f : D \rightarrow R$ be a function, then f is called monotonic increasing function is for

$$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2); \text{ for all } x_1, x_2 \in D$$

(2) Monotonic decreasing function :

A function $f : D \rightarrow R$ is called monotonic decreasing if for $x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$ for all $x_1, x_2 \in D$

One-One function : A real valued function $f : D \rightarrow R$ is called one-one function if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Conclusion :

- (1) If function is non-monotonic then this function is not one-one function.
- (2) If function is strictly monotonic then this function is one-one function.

Another trick for one-one function :

- (1) If graph of function intersect each line which parallel to x-axis at atmost one point.
- (2) If $f'(x) > 0$ or $f'(x) < 0$ i.e. $f(x)$ is strictly monotonic function then $f(x)$ is one – one

Q.1. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ is one-one function, then which of the following is/are true?

- (a) $f(x)$ is strictly increasing function.
- (b) $f(x)$ is strictly decreasing function.
- (c) $f(x)$ is strictly monotonic function.
- (d) $f(x)$ is monotonic function.

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Onto Function : A function $f : A \rightarrow R$ is said to be onto if $f(A) = R$.

Note : A function which is onto or not onto it is totally depend on co-domain.



Check onto function by graph :

If graph of function intersect each line which parallel to x-axis and contain in co-domain, then this function is onto function.

Q.2. Let $f : \mathbb{R} \rightarrow \mathbb{R}^+$ is a non-monotonic function, then which of the following is true?

- (a) A function is always one-one.
- (b) A function may be one-one.
- (c) A function is always onto
- (d) A function may be onto

Composition of Function :

Let $f : A \rightarrow B$ and $g : B \rightarrow C$ are two real valued function then $gof(x)$ is composition of function, where $gof(x) = g[f(x)]$.

Result : If $gof(x)$ is one-one and onto, then $f(x)$ is one-one and $g(x)$ is onto.

Q.3. Let $f : X \rightarrow X$ such that $f(f(x)) = x$ for all $x \in X$. Then

- (a) f is one-to-one and onto.
- (b) f is one-to-one, but not onto
- (c) f is onto but not one-to-one
- (d) f need not be either one-to-one or onto

Limit of function : Let f be a function defined for all points in some neighbourhood of a point a .

We say that f tends to the limit l as x tends to a .

i.e. $\lim_{x \rightarrow a} f(x) = l$

Then l is called limit of function.

Right hand limit : Let $f(x)$ be a function,

$\lim_{x \rightarrow a+0} f(x) = f(a+0) = l$ then is called right hand limit.

Left hand limit : Let $f(x)$ be a function, then

$\lim_{x \rightarrow a-0} f(x) = f(a-0) = l$ is called left hand limit of $f(x)$ at 'a'.

Note : Let $f(x)$ be a function then

$\lim_{x \rightarrow a} f(x) = f(a+0) = f(a-0).$

Q.4. Let $f : (-1,1) \rightarrow \mathbb{R}$ and $g : (-1,1) \rightarrow \mathbb{R}$ be thrice continuously differentiable function such that $f(x) \neq g(x)$ for every non-zero $x \in (-1,1)$. Suppose $f(0) = \log 2$, $f'(0) = \pi$, $f''(0) = \pi^2$, $f'''(0) = \pi^9$ and $g(0) = \log 2$, $g'(0) = \pi$, $g''(0) = \pi^2$, $g'''(0) = \pi^3$ then the

value of limit $\lim_{x \rightarrow 0} \frac{e^{f(x)} - e^{g(x)}}{f(x) - g(x)}$ is **IIIT JAM 2022**

- (a) 1
- (b) 2
- (c) 3
- (d) 4

Q.5. Which of the following is false

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(a) $\lim_{x \rightarrow \infty} \frac{x}{e^x} = 0$

(b) $\lim_{x \rightarrow 0^+} \frac{\sin x}{1 + 2x} = 0$

(c) $\lim_{x \rightarrow 0^+} \frac{1}{\frac{1}{xe^x}} = 0$

(d) $\lim_{x \rightarrow 0^+} \frac{\cos x}{1 + 2x} = 0$

Q.6. The value of $\lim_{x \rightarrow 0} \frac{|x| - x}{2x}$ is

- (a) 0
 - (b) 2
 - (c) 1
 - (d) does not exist

Q.7. The value of $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1 - \cos x}}$

(a) $\sqrt{2}$

(b) $-\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) $-\frac{1}{\sqrt{2}}$

Q.8. Let f be a monotone non-decreasing real-valued function on \mathbb{R} . Then

- (a) $\lim_{x \rightarrow a} f(x)$ exists at each point a .
- (b) If $a < b$, then $\lim_{x \rightarrow a^+} f(x) \leq \lim_{x \rightarrow b^-} f(x)$.
- (c) f is an unbounded function
- (d) The function $g(x) = e^{-f(x)}$ is a bounded function.

Q.9. Let p be a real polynomial of the real variable x of the form $p(x) = x^n + a_{n-1}x^{n-1} + \dots + a_1x + 1$. Suppose that p has no roots in the open unit disc and $p(-1) = 0$. Then

- (a) $p(1) = 0$
- (b) $\lim_{x \rightarrow \infty} p(x) = \infty$
- (c) $p(2) > 0$
- (d) $p(3) = 0$

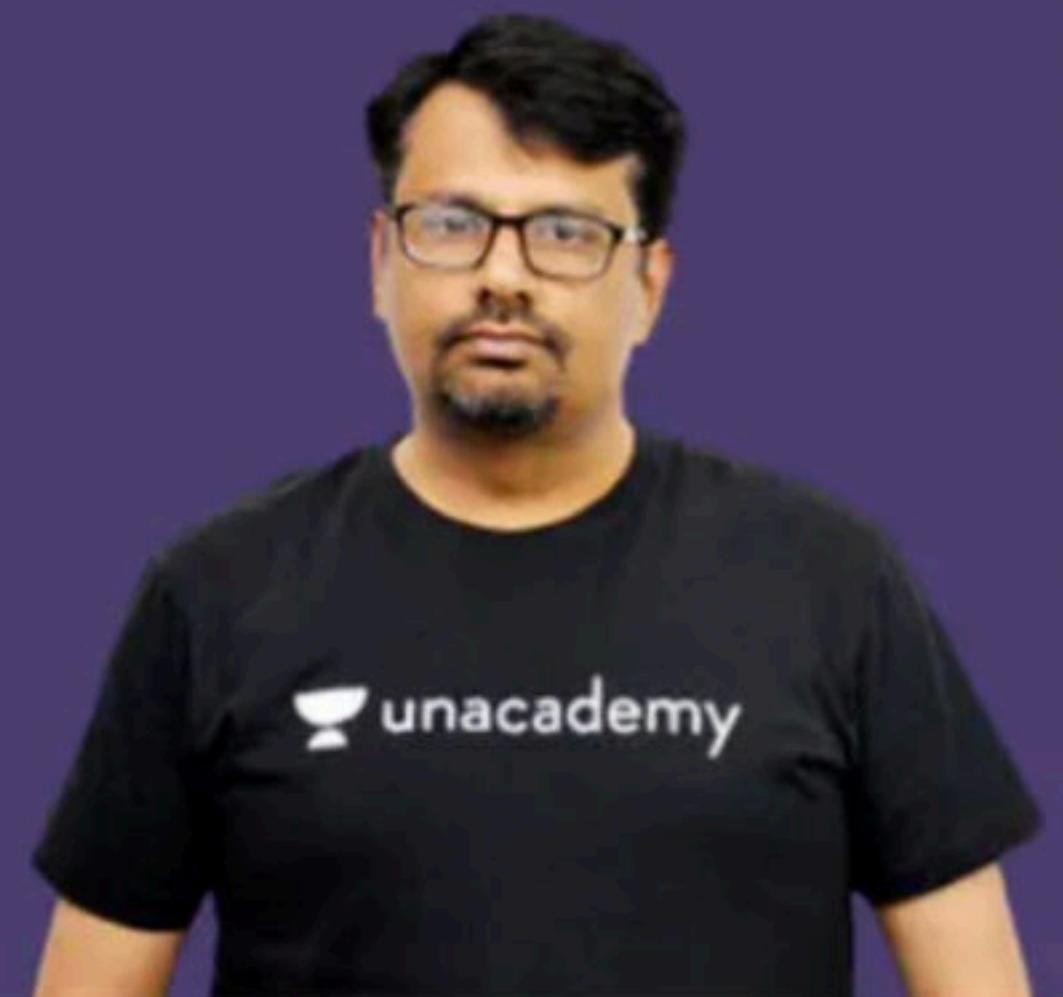


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Educator highlights

- 📍 Works at Pacific Science College
- 📍 Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
- 📍 PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
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