## Gajendra Purohit

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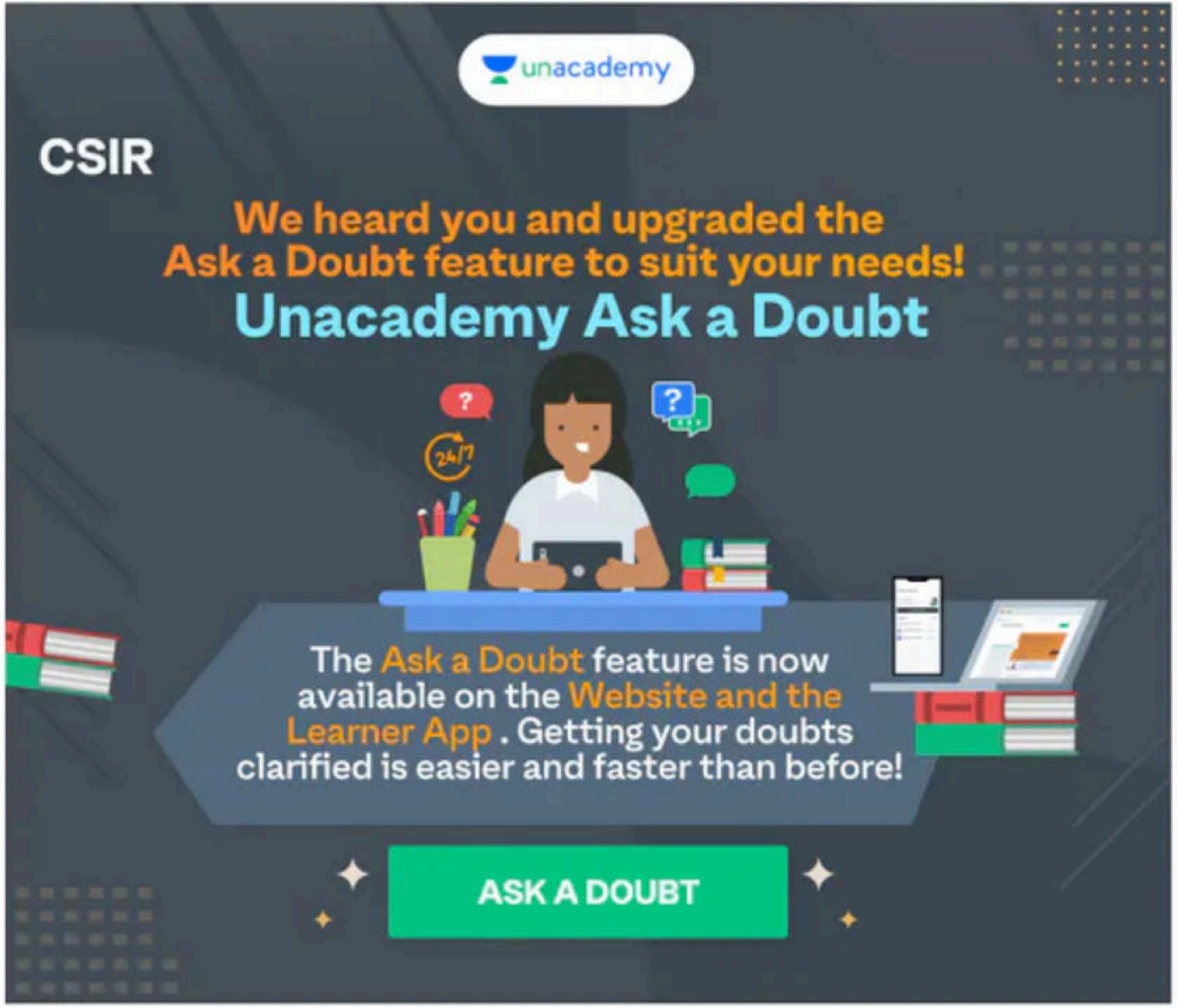
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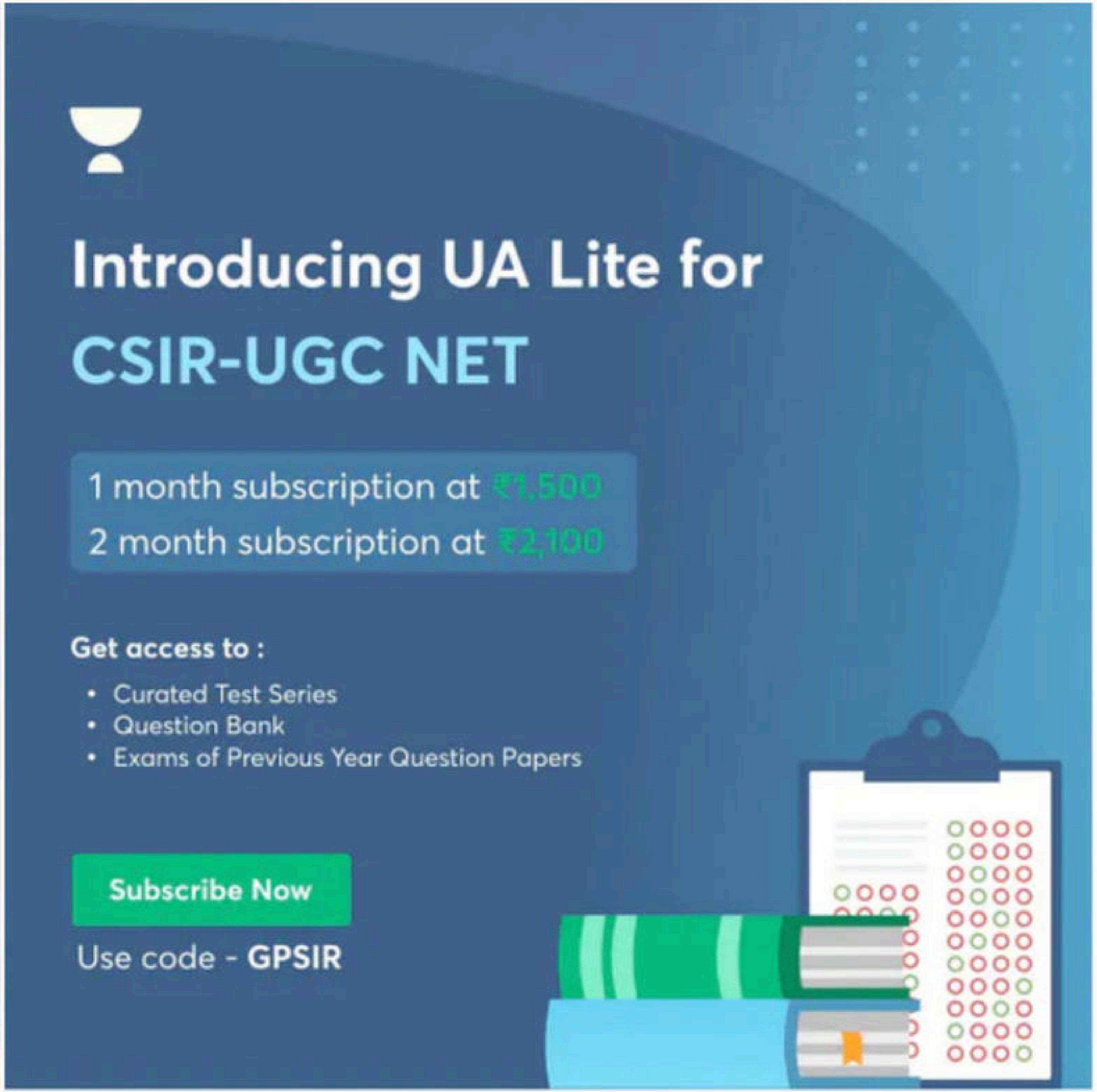
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## Linear differential equation with constant coefficients

A linear differential equation with constant coefficients is that in which dependent variables and its differentials occur only in first degree, and not multiplied together, and coefficients are all constants

$$\frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \ldots\ldots + a_{n-1} \frac{dy}{dx} + a_n y = X \quad ..(1)$$

Where X is a function of x only and $a_1, a_2, \ldots, a_n$ are constants is called linear differential equation with constant coefficients of $n^{th}$ order.

We can write (1) as

$$D^n y + a_1 D^{n-1} y + a_2 D^{n-2} y + \ldots.. + a_n y = X$$

$$[D_n + a_n D^{n-1} + a_2 D^{n-2} + \ldots.. + a_n]\, y = X$$

$D = \frac{d}{dx}$

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^x$$

$$(D^3 + 3D^2 + 2D + 1)y = e^x$$

CF

PI

$y = CF + PI$

$f(D)y = X$ .... (2)

Where $f(D) = D^n + a_1 D^{n-1} + a_2 D^{n-2} + \ldots.. + a_n$.

Now consider the differential equation $f(D)y = 0$.... (3)

The general solution of $n^{th}$ order differential equation involved arbitrary constants. So, the general solution of (3) is of the form

$y = c_1 y_1 + c_2 y_2 + \ldots\ldots + c_n y_n$ .. (4)

Which is also called complementary function (CF) of (2).

Let V be the particular solution of (2) (due to X called PI)

Hence, $f(D)y = X$, has the complete solution as

$y = CF + PI$.

CF involves n arbitrary constants and PI does not involve any constant.

## 1 Complementary Function

For the sake of convenience, we consider a second order linear equation

$$\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2 y = 0 \qquad \ldots\ldots (1)$$

$(D^2 + a_1 D + a_2)y = 0$

$m^2 + a_1 m + a_2 = 0$

$m_1$ & $m_2$

$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

Then auxiliary equation is $m^2 + a_1 m + a_2 = 0$ .....(2)

**Case I : The roots of (2) are real and distinct :**

Let $m_1$, $m_2$ be the two real and distinct roots of (2).

Then $e^{m_1 x}, e^{m_2 x}$ are the solutions of (1)

Hence, the complementary function of (1) is

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x} \qquad \ldots. (3)$$

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

$$(D^2 - 3D + 2)y = 0$$

$$m^2 - 3m + 2 = 0$$

$$(m-1)(m-2) = 0$$

$$m = 1, 2$$

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$y = c_1 e^{x} + c_2 e^{2x}$$

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$$

$$(D^2 + 4D + 4)y = 0$$

$$m^2 + 4m + 4 = 0$$

$$(m+2)^2 = 0$$

$$m = -2, -2$$

$$y = c_1 e^{-2x} + c_2 e^{-2x}$$

$$= (c_1 + c_2 x) e^{-2x}$$

$$y = (c_1 + c_2 x) e^{-2x}$$

$$m = 2, 2, 2$$

$$y = (c_1 + c_2 x + c_3 x^2) e^{2x}$$

$$m = 1, 2, 2, -3$$

$$y = c_1 e^{x} + (c_2 + c_3 x) e^{2x} + c_4 e^{-3x}$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4y = 0$$

$$m^2 + m + 4 = 0$$

$$m = \frac{-1 \pm \sqrt{1-16}}{2} = \frac{-1 \pm \sqrt{15}\,i}{2} = -\frac{1}{2} \pm \frac{\sqrt{15}}{2} i$$

$$y = e^{-\frac{1}{2}x}\left(c_1 \cos \frac{\sqrt{15}}{2}x + c_2 \sin \frac{\sqrt{15}}{2}x\right)$$

$\alpha \pm i\beta$

$$y = c_1 e^{\alpha + i\beta} + c_2 e^{\alpha - i\beta}$$

$$y = c_1 e^{\alpha} e^{i\beta} + c_2 e^{\alpha} e^{-i\beta}$$

$$y = e^{\alpha}\left(c_1' \cos\beta x + c_2' \sin\beta x\right)$$

$2 \pm \sqrt{3}$

$e^{ix} = \cos x + i\sin x$

$e^{x} = \cosh x + i\sinh x$

$$y = c_1 e^{(2+\sqrt{3})x} + c_2 e^{(2-\sqrt{3})x}$$

$$y = e^{2x}\left(c_1 e^{\sqrt{3}x} + c_2 e^{-\sqrt{3}x}\right)$$

$$= e^{2x}\left(c_1\left(\cosh\sqrt{3}x + i\sinh\sqrt{3}x\right) + c_2\left(\cosh\sqrt{3}x - i\sinh\sqrt{3}x\right)\right)$$

$$= e^{2x}\left((c_1+c_2)\cosh\sqrt{3}x + i(c_1-c_2)\sinh\sqrt{3}x\right)$$

**Case II : The roots of (2) are real and equal :**

Let roots $m_1 = m_2 = m$

then $y = (c_1 + c_2x)e^{mx}$

is a complementary function.

**Case – III : The roots of (ii) are complex**

Let a + ib and a – ib are the roots of (2)

Then the general solution of (1)

$$y = e^{ax}[A\cos bx + B\sin bx]$$

**Q.1.** Let y(x) be a solution of the differential equation $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+4y=0$ s.t. $\lim_{x\to\infty} e^{-x}y(x)$ is finitely exist. Then y(log 2) is

(a) Constant (b) in term of x

(c) in term of $e^x$ (d) None of these

$e^{-\infty}=0$

$e^{\infty}$ DNE

$m^2-5m+4=0$

$m=1,4$

$y=c_1e^x+c_2e^{4x}$

$\lim_{x\to\infty} e^{-x}y = c_1+c_2e^{3x}$

$c_2=0$

$y=c_1e^x$

$y(\log 2)=c_1e^{\log 2}$

$=2c_1$

**Q.2.** If y(x) is the solution of the initial value problem $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=0, y(0)=1, \frac{dy}{dx}(0)=-2$, then y(ln 2) is

(a) ln2

(b) $(1-\ln 2)\frac{1}{2}$

(c) integer number

(d) 0

$m^2+2m+1=0$

$m=-1,-1$

$y=(c_1+c_2x)e^{-x}$

$1=c_1$

$y=e^{-x}+c_2xe^{-x}$

$y'=-e^{-x}+c_2e^{-x}-c_2xe^{-x}$

$-2=-1+c_2$

$c_2=-1$

$y=e^{-x}-xe^{-x}$

$y=e^{-\ln 2}-\ln 2\, e^{-\ln 2}$

$y=\frac{1}{2}-\frac{\ln 2}{2}$

Soln a-

$$\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$ approaches zero as $x \to \infty$ if

(a) $b < 0, \; c > 0$

(b) $b > 0, \; c < 0$

(c) $b > 0, \; c > 0$

(d) $b < 0, \; c < 0$

$\bar{e}$

$m^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$

$b > 0, \; c > 0$

$$\frac{d^2y}{dx^2} + 4\omega\frac{dy}{dx} + y = 0$$

$$m = \frac{-4\omega \pm \sqrt{16 - 4}}{2} = \frac{-4\omega \pm 2\sqrt{3}}{2}$$

$= -2\omega \pm \sqrt{3}$

$y = e^{-2\omega x}(c_1 \cos h\sqrt{3}x + c_2 \sin h\sqrt{3}x)$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 4y = 0$$

$m^2 + m + 4 = 0$

$$m = \frac{-1 \pm \sqrt{1 - 16}}{2} = -\frac{1}{2} \pm \frac{\sqrt{15}}{2} i$$

$y = e^{-\frac{1}{2}x}\left(c_1 \cos\frac{\sqrt{15}}{2}x + c_2 \sin\frac{\sqrt{15}}{2}x\right)$

$k, l \in R$ s.t.

sqn $\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + ly = 0$

$m^2 + 2km + l = 0$

$\lim_{x \to \infty} y(x) = 0$ then

$m = \frac{-2k \pm \sqrt{4k^2 - 4l}}{2}$

(a) $3k^2 + l < 0$, $k > 0$

$m = -k \pm \sqrt{k^2 - l}$

(b) $k^2 + l > 0$, $k < 0$

(c) $k^2 - l \leq 0$, $k > 0$

$k^2 - l = 4$

(d) $k^2 - l > 0$, $k > 0$, $l > 0$

$k = -4$

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**Q.3.** If $y(x) = \lambda e^{2x} + e^{\beta x}$, $\beta \neq 2$, is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ satisfying $\frac{dy}{dx}(0) = 5$, then $y(0)$ is equal to

(a) 3 (b) 4

(c) ~~5~~ (d) 6

$\beta = -3$

$c_2 = 1$

$m^2 + m - 6 = 0$

$m = 2, -3$

$y = c_1 e^{2x} + c_2 e^{-3x}$

$y = \lambda e^{2x} + e^{-3x}$

$y' = 2\lambda e^{2x} - 3e^{-3x}$

$5 = 2\lambda - 3$

$2\lambda = 8$

$\lambda = 4$

$y = 4e^{2x} + e^{-3x}$

$y(0) = 4 + 1 = 5$

**Q.4.** The differential equation whose linearly independent solutions are cos 2x, sin 2x and $e^x$, is

$\pm 2i$, $1$

(a) $(D^3+D^2+4D)y=0$ (b) $(D^3-D^2+4D-4)y=0$

$(m^2+4)(m-1)$

$m^3-m^2+4m-4$

(c) $(D^3+D^2-4D-4)y=0$ (d) $(D^3-D^2-4D+4)y=0$

$(D^3-D^2+4D-4)$

**Q.5.** The number of arbitrary constants in the complete primitive of differential equation $\frac{d^5y}{dx^5}+2\frac{d^4y}{dx^4}=0$ is/are not

(a) 5 (b) 4

(c) 1 (d) 6

**Q.6** Let $P : R \to R$ be a continuous function such that $P(x) > 0$ for all $x \in R$. Let y be a twice differentiable function on R satisfying $y``(x) + P(x)y`(x) - y(x) = 0$ for all $x \in R$. Suppose that there exist two real numbers a, b $(a < b)$ such that $y(a) = y(b) = 0$. Then

(a) $y(x) > 0$ for all $x \in (a, b)$

(b) $y(x) < 0$ for all $x \in (a, b)$

(c) $y(x)$ changes sign on $(a, b)$

(d) $y(x) = 0$ for all $x \in [a, b]$

**Q.7.** The homogeneous part of the differential equation

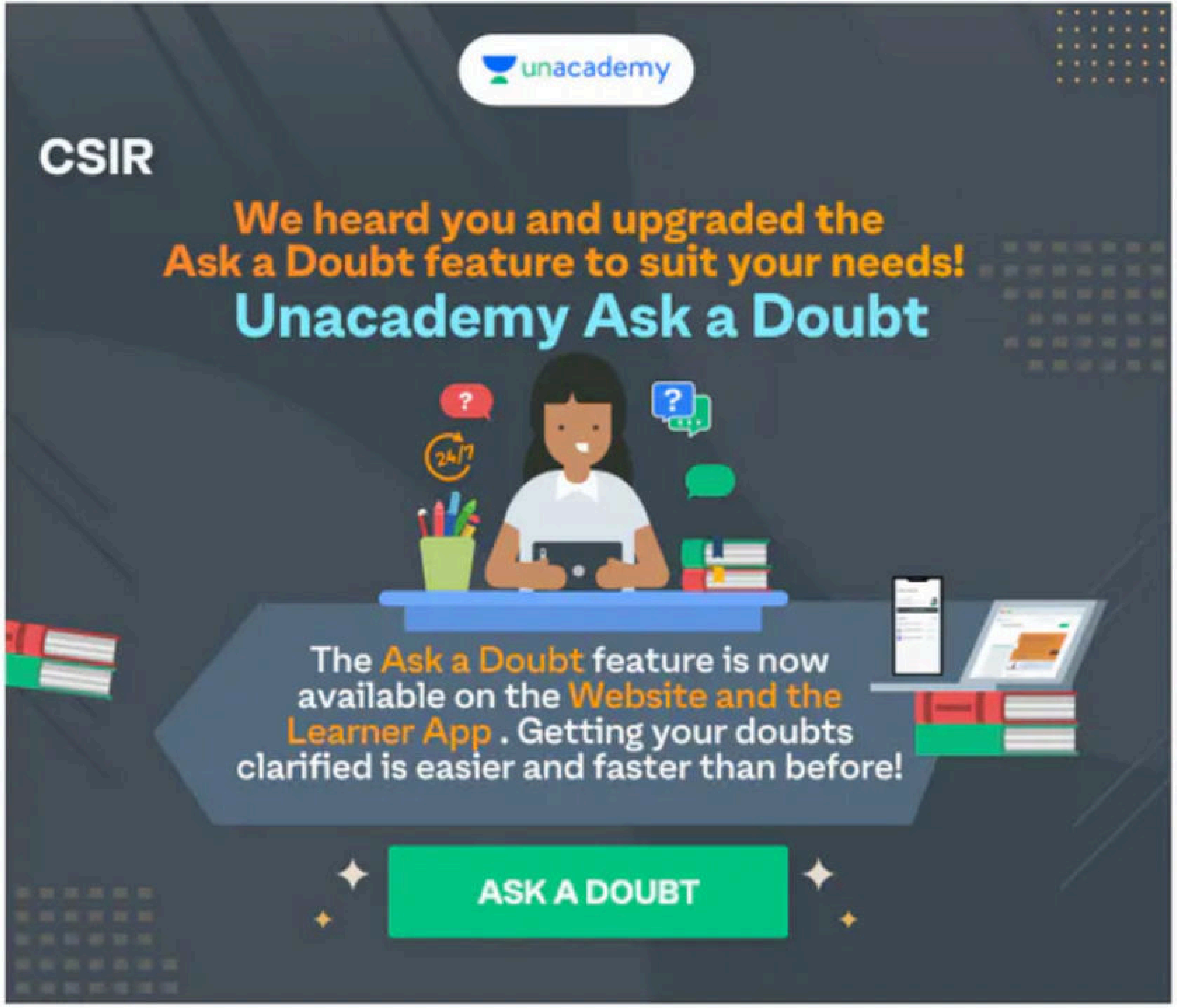
$$\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = r$$ has real distinct real roots if

(a) $p^2 - 4q > 0$

(b) $p^2 - 4q < 0$

(c) $p^2 - 4q = 0$

(d) $p^2 - 4q = r$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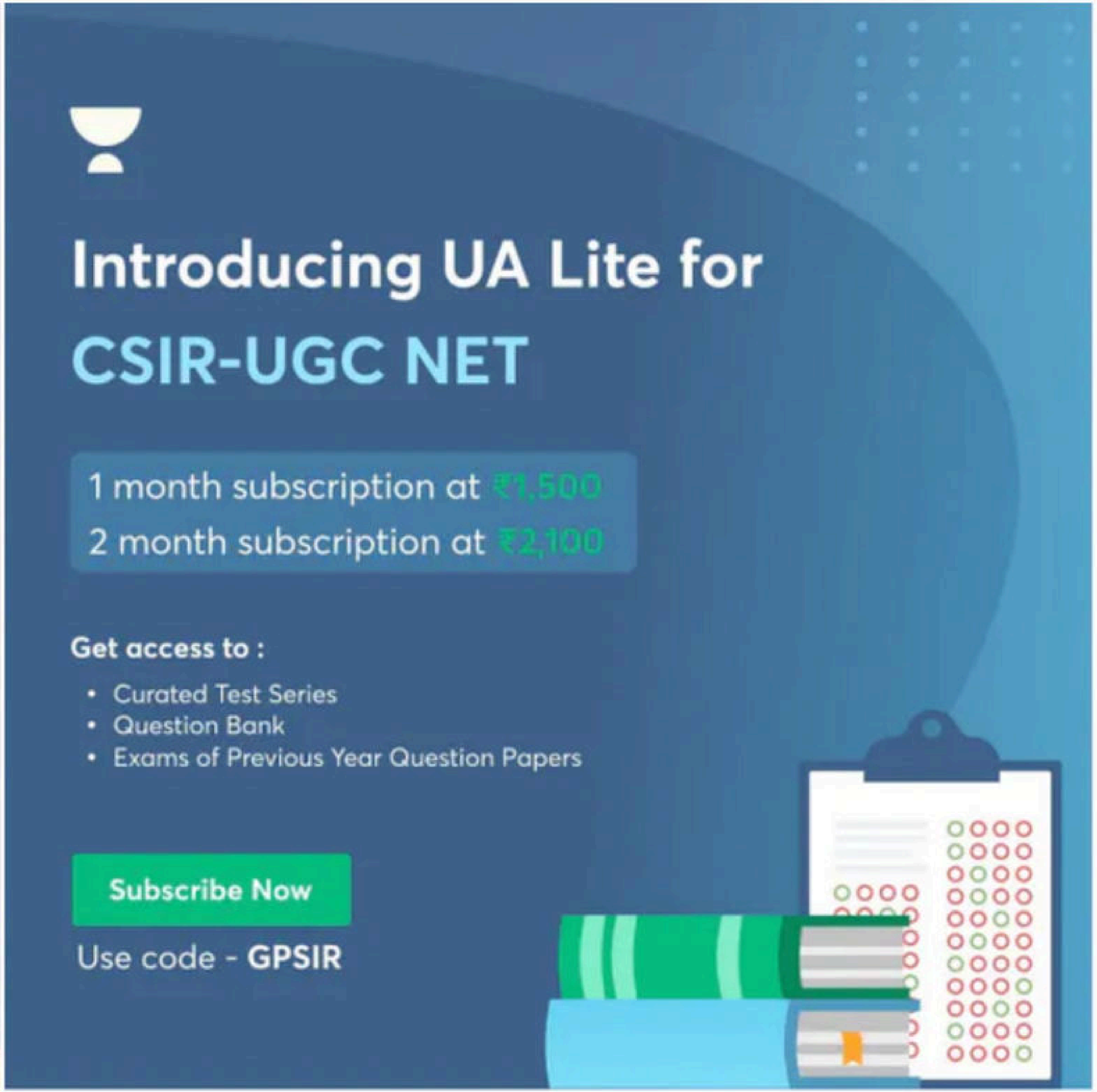
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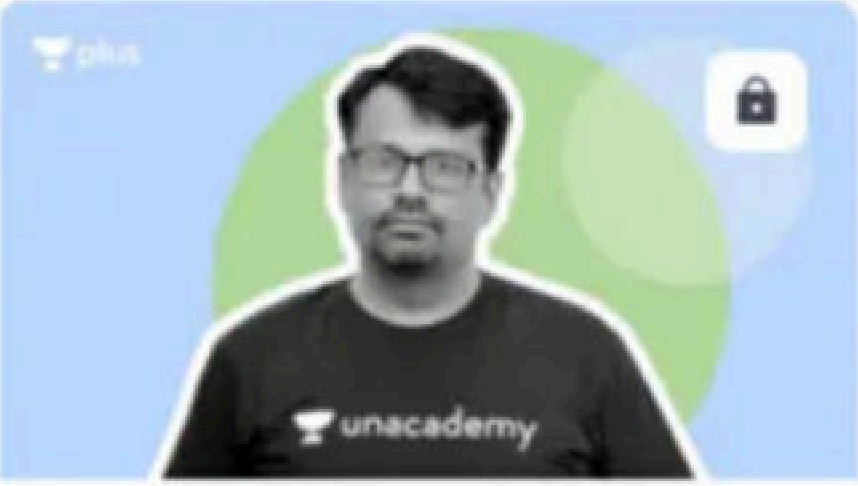


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### Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

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