



Gajendra Purohit

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1. Order of Group :- Number of elements in G Group is called Order of group. It is denoted by $o(G) = |G|$

OR

Let $(G, *)$ be a group. The cardinality of G (finite or Infinite) is defined as the order of the group.

Order of some special group.

- i. $o(\mathbb{R}) = o(\mathbb{C}) = o(\mathbb{Z}) = o(\mathbb{Q}) = \infty$
- ii. $o(\mathbb{R}^*) = o(\mathbb{Q}^*) = \infty$
- iii. $o(U_n) = \phi(n)$, where ϕ is a euler's function.
- iv. $o(D_n) = 2n$

- v. $o(Q_8) = 8$
- vi. $o(K_4) = 4$
- vii. $o[Gl_n(\mathbb{R})] = o[Sl_n(\mathbb{R})] = \infty$
- viii. $o[Gl_n(\mathbb{Z}_p)] = (p^n - p^{n-1})(p^n - p^{n-2})(p^n - p^{n-3})\dots$
 $(p^n - 1)$
- ix. $o[Sl_n(\mathbb{Z}_p)]$
 $= \frac{(p^n - p^{n-1})(p^n - p^{n-2})(p^n - p^{n-3})\dots(p^n - 1)}{(p-1)}$
- x. $o(S_n) = n!$, (S_n will be covered in the next lecture.)
- xi. $o(Z_n) = n$

2. **Order of Elements-:** Let G be a group, the order of an elements $a \in G$ is the least positive integer n such that $a^n = e$ if such n does not exist then order of element a is infinite.

The order of element $a \in G$ is denoted by $o(a)$.

Basic example of order of elements of some special Group-

(i) Possible Order of Each Element of $(\mathbb{Z}, +)$

$$\mathbb{Z} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$$

$0 \in \mathbb{Z}$ is identity of \mathbb{Z}

$$o(0) = 1$$

$n \in \mathbb{Z}; |n| \geq 1$. It is not possible; order of n can't belongs to any natural number.

$$\text{Then } o(n) = \infty, n \in \mathbb{Z} - \{0\}$$

Similarly, order of elements of $(\mathbb{Q}, +)$, $(\mathbb{R}, +)$ & $(\mathbb{C}, +)$ are 1 &

∞ .

(ii) Possible Order of elements of (\mathbb{Z}^*, \cdot)

Order of element of identity, $o(1) = 1$

Order of element, -1 is 2 i.e. $o(-1) = 2$

All other elements are of order ∞

(iii) Possible Order of elements of (\mathbb{Q}^*, \cdot)

$1 \in \mathbb{Q}^*$ is identity of \mathbb{Q}^* , then $o(1) = 1$

$o(-1) = 2$; $o(a) = \infty$, $a \in \mathbb{Q}^* - \{1, -1\}$

Similarly, order of elements of (\mathbb{R}^*, \cdot)
are $1, 2$ and ∞ .

(iv) Possible Order of each element in \mathbb{Z}_n .

NOTE-:

- a) In \mathbb{Z}_n , possible order of elements are $d \mid n$ [n is divisible by d].
- b) If $d \mid n$ then numbers of elements of order d in \mathbb{Z}_n
 $= \phi(d)$.

Properties of order in Product

- i. If G_1 is group with identity e_1 and G_2 is group with identity e_2 then $G_1 \times G_2$ is group with identity (e_1, e_2) and with inverse $(a_1^{-1}$ in G_1, a_2^{-1} in $G_2)$.
- ii. If $G_1, G_2, G_3, \dots, G_n$ are finite groups. Then
$$o(G_1 \times G_2 \times G_3 \times \dots \times G_n) = o(G_1) \times o(G_2) \\ \times o(G_3) \times \dots \times o(G_n).$$
- iii. $(a_1, a_2, a_3, \dots, a_n) \in (G_1 \times G_2 \times G_3 \times \dots \times G_n)$ then
$$o(a_1, a_2, a_3, \dots, a_n) = \text{L.C.M.} [o(a_1) \text{ in } G_1, \\ o(a_2) \text{ in } G_2, o(a_3) \text{ in } G_3, \dots, o(a_n) \text{ in } G_n.]$$
- iv. No. of elements of order d in $\mathbb{Z}_n \times \mathbb{Z}_m =$
$$\sum \phi(d_1) \cdot \phi(d_2) \text{ where } d_1 | n, d_2 | m \text{ &} \\ \text{L.C.M.}(d_1, d_2) = d$$

- v. $o(a) = o(xax^{-1}) = o(x^{-1}ax)$, where $a, x \in G$
and G is a group.
- vi. $o(ba) = o(ab)$, where $a, b \in G$
- vii. $o(a) = o(a^{-1})$, where $a \in G$
- viii. $(ab)^{-1} = b^{-1}a^{-1}$, where $a, b \in G$
- ix. $(a_1, a_2, \dots, a_{n-1}, a_n)^{-1} = (a_n^{-1}, a_{n-1}^{-1}, \dots,$
 $a_2^{-1}, a_1^{-1})$, where $a_i \in G$

Result : Let $G = Z_k \times Z_k \dots \times Z_k$

(p-times)

Then G has an element of order either 1 or k .

- (1) G has exactly one element of order 1.
- (2) Other all elements of G are of order k

i.e. number of elements of order k in G are $k^p - 1$

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Q1. Number of elements of order 7 in $\mathbb{Z}_7 \times \mathbb{Z}_7 \dots \mathbb{Z}_7$ (50-times)

- (a) 7^{50}
- (b) $7^{50} - 1$
- (c) $7^{50} - 2$
- (d) $7^{49} - 1$

Abelian Group :- A group G is said to be abelian if $xy = yx$ such that, $x, y \in G$.

Properties of Abelian group:-

- (i) If every element of G has self-inverse then G is abelian but converse need not be true.

Example : K_4 is abelian because every elements of K_4 has self-inverse. \mathbb{Z}_6 is abelian but every elements of \mathbb{Z}_6 is not self-inverse.

- (ii) Let G_1 and G_2 are two groups $G_1 \times G_2$ is abelian iff G_1 and G_2 are abelian.

- (iii) $G_1 \times G_2 \times G_3 \times \dots \times G_{n-1} \times G_n$ is abelian iff each G_i , $1 \leq i \leq n$ is abelian

- (iv) Let G be a group and $(ab)^{-1} = a^{-1}b^{-1}$ $\forall a, b \in G$ iff G is abelian.

- (v) Let G be a group and $(ab)^2 = a^2b^2$ $\forall a, b \in G$ iff G is abelian.

- (vi) But G is a group and $(ab)^n = a^n b^n$ ($n \geq 3$) $\forall a, b \in G$ iff G is abelian.

Cyclic Group : A group (G, o) is called a cyclic group if every elements of G is of the form a^n , for some $a \in G$ where $n \in \mathbb{Z}$.

Symbolically, $G = \{a^n \mid n \in \mathbb{Z}\}$

$$\Rightarrow G = \{\dots, a^{-2}, a^{-1}, e, a, a^2, \dots\}$$

The element a is called generator of G .

Note : If G is a cyclic group generated by a , then we write $G = \langle a \rangle$

The element of G will be of the form

$\langle a \rangle = \{\dots, -2a, -a, 0, a, 2a, \dots\}$ for additive composition.

$\langle a \rangle = \{\dots, a^{-2}, a^{-1}, e, a, a^2, \dots\}$ for multiplication composition.

Result :

- (1) Let G be a group of order n and if G have an element of order n , then G must be cyclic group.

Example : Let $G = \{1, -1, i, -i\}$ is a group of order 4 under multiplicative.

Now, $O(1) = 1$, $O(-1) = 2$, $O(i) = 4 = O(-i)$

Here, order of i & $-i$ are equal to order of group.

So, G is cyclic and i & $-i$ are generator of G .

- (2) Every cyclic group is abelian group.
- (3) Let G be a cyclic group of order n , then number of generator of group are $\phi(n)$.

Non-Cyclic group: K_4 , Q_8 , S_n , (\mathbb{Q}^*, \cdot) , (\mathbb{R}^*, \cdot) , (\mathbb{C}^*, \cdot) , $(\mathbb{Q}, +)$, $(\mathbb{R}, +)$, $(\mathbb{C}, +)$, $Gl_n(\mathbb{Z}_p)$, $Sl_n(\mathbb{Z}_p)$
 $(n > 1)$, D_n

Some important points of cyclic group:-

- (i) If G is group under addition then $G = \langle a \rangle = \{na \mid n \in \mathbb{Z}\}$.
- (ii) If G is cyclic group and ' a ' is generator of G then $o(G) = o(a)$. But Converse need not be true.

Example : $G = (\mathbb{Z}, +)$

All elements other than 1 and -1 are of order infinite which is equal to order of G but they are not generator

- (iii) G is a finite cyclic group and ' a ' is generator of G iff $o(G) = o(a)$.
- (iv) If $a^1, a^2, a^3, \dots, a^n = e$ are distinct element then G is generated by a .
- (v) If G is non abelian then G is non-cyclic group

- (vi) If $o(G) = p$ (prime number) then G is cyclic group.
- (vii) $\mathbb{Z}_m \times \mathbb{Z}_n$ is cyclic group iff $\gcd(m, n) = 1$ then $\mathbb{Z}_m \times \mathbb{Z}_n \approx \mathbb{Z}_{mn}$
- (ix) If 'a' is generator of G then a^{-1} is also generator of G .
- (x) If G be a finite cyclic group of order n then $G \approx \mathbb{Z}_n$.
- (xi) If G be an infinite cyclic group then $G \approx \mathbb{Z}$.
- (xii) Let $G = \langle a \rangle$ be a cyclic group of order n then $G = \langle a^k \rangle$ iff g.c.d.(k, n) = 1
- (xiii) If G_1 and G_2 are cyclic group then $G_1 \times G_2$ need not be cyclic group.
- (xiv) If G be infinite cyclic group of order n then G has exactly two generators. Namely {1, -1}
- (xv) If $O(G) = p^2$ then G is abelian group but converse need not be true

(xvi) Let G be a finite group of order pq i.e. $O(G) = pq$ (where $p < q$ & p, q are prime)

Case – 1 : If $p \nmid q - 1$, then G is cyclic.

Case – 2 : If $p \mid q - 1$, then G need not be cyclic.

Q.1. Consider the set of matrices $G = \left\{ \begin{bmatrix} s & b \\ 0 & 1 \end{bmatrix} : b \in \mathbf{Z}, s \in \{-1, 1\} \right\}$. Then which of the following is true

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- (a) G forms a group under addition
- (b) G forms a abelian group under multiplication
- (c) Every elements in G is diagonalizable over C
- (d) G is finitely generated group under multiplication

Q.2. Let G be finite abelian group and $a, b \in G$ with order $(a) = m$, order $(b) = n$. which of the following are necessarily true

CSIR NET DEC. 2017

- (a) Order $(ab) = mn$
- (b) Order $(ab) = \text{lcm}\{m,n\}$
- (c) There is an elements of G whose order is $\text{lcm}\{a, b\}$
- (d) Order $(ab) = \text{gcd}(m, n)$

Q.3. Let G be a non – abelian group. then its order can be

Q.4. The number of generator of the additive group Z_{36} is equal to

IIT - JAM 2017

- (a) 6
- (b) 12
- (c) 18
- (d) 36

Q.5. The value of α for which $G = \{\alpha, 1, 3, 9, 19, 27\}$ is a cyclic group under multiplication modulo 56 is

- a) 5
- b) 15
- c) 25
- d) 35

Q.6. Which of the following statements is/are TRUE?

- a) If G is abelian then $(ab)^n = a^n b^n \forall n \in \mathbb{N} \text{ & } \forall a, b \in G$
- b) If G is abelian then $\forall n \in \mathbb{N} \text{ & } \forall a, b \in G, (aba^{-1})^n = ab^n a^{-1}$.
- c) If G is a group & $\forall n \in \mathbb{N} \text{ & } \forall a, b \in G, (aba^{-1})^n = ab^n a^{-1}$ then G is abelian.
- d) None of these

Q.7 Let $G = \mathbb{Z}_{15} \times \mathbb{Z}_{10}$ be a group. Then which of the following is true

- (a) G has exactly one element of order 2
- (b) G has exactly 5 elements of order 3
- (c) G has exactly 24 elements of order 5
- (d) G has exactly 24 elements of order 10

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Q.8. What is the order of $GL_3(\mathbb{Z}_4)$?

(a) $54 \times 72 \times 78 \times 80$

(c) $\frac{54 \times 72 \times 78 \times 80}{3}$

(b) $48 \times 60 \times 63$

(d) $\frac{48 \times 60 \times 63}{3}$



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Educator highlights

- 📍 Works at Pacific Science College
- 📍 Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
- 📍 PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- 📍 Lives in Udaipur, Rajasthan, India
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