Definite integral

Definition : If $\frac{d}{dx}[f(x)] = \phi(x)$ and a & b are constant, then

$$\int_a^b \phi(x)dx = [f(x)]_a^b = f(b) - f(a)$$

is called definite integration of $\phi(x)$ within limit a & b.

Note : This is also called fundamental theorem of calculus.

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Basic properties of definite integrals.

(1) $$\int_a^b f(t)dt = \int_a^b f(x)dx$$

(2) $$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

(3) $$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

For any c $\in$ (a, b)

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(4) $$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

(5) $$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & if\ f(x)\ is\ even \\ 0 & if\ f(x)\ is\ odd \end{cases}$$

(6) $$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx; & \text{if } f(2a-x) = f(x) \\ 0; & \text{if } f(2a-x) = -f(x) \end{cases}$$

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Definite integral as the limit of a sum :

$$\int_0^1 f(x)dx = \lim_{n\to\infty} \frac{1}{n}\sum_{r=0}^{n} f\left(\frac{r}{n}\right)$$

Where f(x) is continuous function on closed interval [0, 1]

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Leibnitz's Rule :

If g is continuous on [a, b] and $f_1(x)$ & $f_2(x)$ are differentiable function whose value lies in [a, b] then

$$\frac{d}{dx}\int_{f_1(x)}^{f_2(x)} g(t)dt = g[f_2(x)]f_2`(x) - g(f_1(x))f_1`(x)$$

General form : If g is continuous on [a, b] and $f_1(x)$ & $f_2(x)$ are differentiable function whose value lies in [a, b] then

$$\frac{d}{dx}\int_{f_1(x)}^{f_2(x)} g(x,t)dt = \int_{f_1(x)}^{f_2(x)} \frac{\partial}{\partial x}g(x,t)dt + g[x, f_2(x)]f_2`(x) - g(x, f_1(x))f_1`(x)$$

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Gamma Function:

If m and n are non-negative integers, then

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$$

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where $\Gamma(n)$ is called gamma function which satisfied the following properties

$\Gamma(n+1) = n\Gamma(n) = n!$ $\quad$ *i.e.* $\Gamma(1) = 1$ and $\Gamma(1/2) = \sqrt{\pi}$

In place of gamma function, we can also use the following formula :

$$\int_0^{\pi/2} \sin^m x \cos^n x dx =$$

$$\frac{(m-1)(m-3).....(2 \text{ or } 1)(n-1)(n-3).....(2 \text{ or } 1)}{(m+n)(m+n-2)....(2 \text{ or } 1)}$$

It is important to note that we multiply by $(\pi/2)$; when both m and n are even.

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Q1. The value of $\int_0^{\pi/2} \sin^4 x \cos^6 x dx$

(a) $3\pi/312$

(b) $5\pi/512$

(c) $3\pi/512$

(d) $5\pi/312$

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Reduction formulae Definite Integration

(1) $\int_0^{\infty} e^{-ax} \sin bx dx = \frac{b}{a^2 + b^2}$

(2) $\int_0^{\infty} e^{-ax} \cos bx dx = \frac{a}{a^2 + b^2}$

(3) $\int_0^{\infty} e^{-ax} x^n dx = \frac{n!}{a^n + 1}$

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Q2. If $I_n = \int_0^\infty e^{-x} x^{n-1} dx$, then $\int_0^\infty e^{-\lambda x} x^{n-1} dx$ is equal to

(a) λI_n

(b) $\frac{1}{\lambda} I_n$

(c) $\frac{I_n}{\lambda^n}$

(d) $\lambda^n I_n$

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Q3. $\int_0^{\pi/2} \sin^7 x dx$ has value

(a) $\frac{37}{184}$ (b) $\frac{17}{45}$

(c) $\frac{16}{35}$ (d) $\frac{16}{45}$

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Q.4. Let a,b be positive real numbers such that a < b Given that $\lim_{n\to\infty}\int_0^n e^{-t^2}\,dt = \frac{\sqrt{\pi}}{2}$ Then value of $\lim_{n\to\infty}\int_0^n \frac{1}{t^2}\left(e^{-at^2} - e^{-bt^2}\right)dt$ is

IIT JAM 2022

(a) $\sqrt{\pi}\left(\sqrt{b}-\sqrt{a}\right)$

(b) $\sqrt{\pi}\left(\sqrt{b}+\sqrt{a}\right)$

(c) $-\sqrt{\pi}\left(\sqrt{b}-\sqrt{a}\right)$

(d) $\sqrt{\pi}\left(-\sqrt{b}+\sqrt{a}\right)$

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Q.6. If $g(x) = \int_{x(x-2)}^{4x-5} f(t)dt$, where $f(x) = \sqrt{1+3x^4}$ for $x \in R$, then g`(1) is **JAM – 2019**

(a) 6 (b) 7

(c) 8 (d) 10

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Q.7. Let f : [0 ,1] $\to$ [0, ∞) be continuous function such that $(f(t))^2 < 1 + 2\int_0^t f(s)ds$, $\forall t \in [0,1]$

IIT JAM 2021

(a) $f(t) < 1 + t$; $\forall t \in [0,1]$

(b) $f(t) > 1 + t$; $\forall t \in [0,1]$

(c) $f(t) = 1 + t$; $\forall t \in [0,1]$

(d) $f(t) < 1 + t/2$; $\forall t \in [0,1]$

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Q.8. The value of the integral $\int_{-\pi}^{\pi} |x| \cos nx\,dx, n \geq 1$ is

JAM - 2016

(a) 0, when n is even

(b) 0, when n is odd

(c) $-\frac{4}{n^2}$, when n is even

(d) $-\frac{4}{n^2}$, when n is odd

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Q.9. Let $f(x) = \int_{\sin x}^{\cos x} e^{-t^2} dt$, then $f`\left(\frac{\pi}{4}\right)$ equals

IIT JAM 2006

(a) $\sqrt{\frac{1}{e}}$ (b) $-\sqrt{\frac{2}{e}}$

(c) $\sqrt{\frac{2}{e}}$ (d) $-\sqrt{\frac{1}{e}}$

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Q.10. Let f : R →R be continuous function if $\int_0^x f(2t)dt = \frac{x}{\pi}\sin(\pi x)$ for all x ∈ R, then f(2) is equal to **JAM 2007**

(a) -1 (b) 0

(c) 1 (d) 2

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Q.11. Let $f(x)=\int_0^x (x^2+t^2)g(t)dt,$ where g is a real valued continuous function on R, then f`(x) is equal to

JAM – 2008

(a) 0

(b) $x^3 g(x)$

(c) $\int_0^x g(t)dt$

(d) $2x\int_0^x g(t)dt$

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Q.12. Let a be a non-zero real number, then

$$\lim_{x \to a} \frac{1}{x^2 - a^2} \int_a^x \sin(t^2)\,dt$$ equals **JAM – 2009**

(a) $\frac{\sin(a^2)}{2a}$ (b) $\frac{\cos(a^2)}{2a}$

(c) $-\frac{\sin(a^2)}{2a}$ (d) $-\frac{\cos(a^2)}{2a}$

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