1

Let $f: [-1, 1] \to \mathbb{R}$ be a continuous function. Then the integral $\int_0^{\pi} x\, f(\sin x)\, dx$ is equivalent to

(a) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$

(b) $\frac{\pi}{2}\int_0^{\pi} f(\cos x)\, dx$

(c) $\pi\int_0^{\pi} f(\cos x)\, dx$

(d) $\pi\int_0^{\pi} f(\sin x) dx$

1) Let $f : [-1, 1] \longrightarrow R$ be a continuous function.

Let $I = \int_0^{\pi} x\, f(\sin x)\, dx.$

$$= \int_0^{\pi} (\pi - x)\, f(\sin(\pi - x))\, dx.$$

$$I = \int_0^{\pi} (\pi - x)\, f(\sin x)\, dx.$$

$$\Rightarrow \quad I = \int_0^{\pi} \pi\, f(\sin x)\, dx - \int_0^{\pi} x\, f(\sin x)\, dx$$

$$\Rightarrow \quad I = \frac{\pi}{2} \int_0^{\pi} f(\sin x)\, dx.$$

2

If the triple integral over the region bounded by the planes $2x + y + z = 4$, $x = 0$, $y = 0$, $z = 0$ is given by $\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz\, dy\, dx$, then the function $\lambda(x) - \mu(x, y)$ is

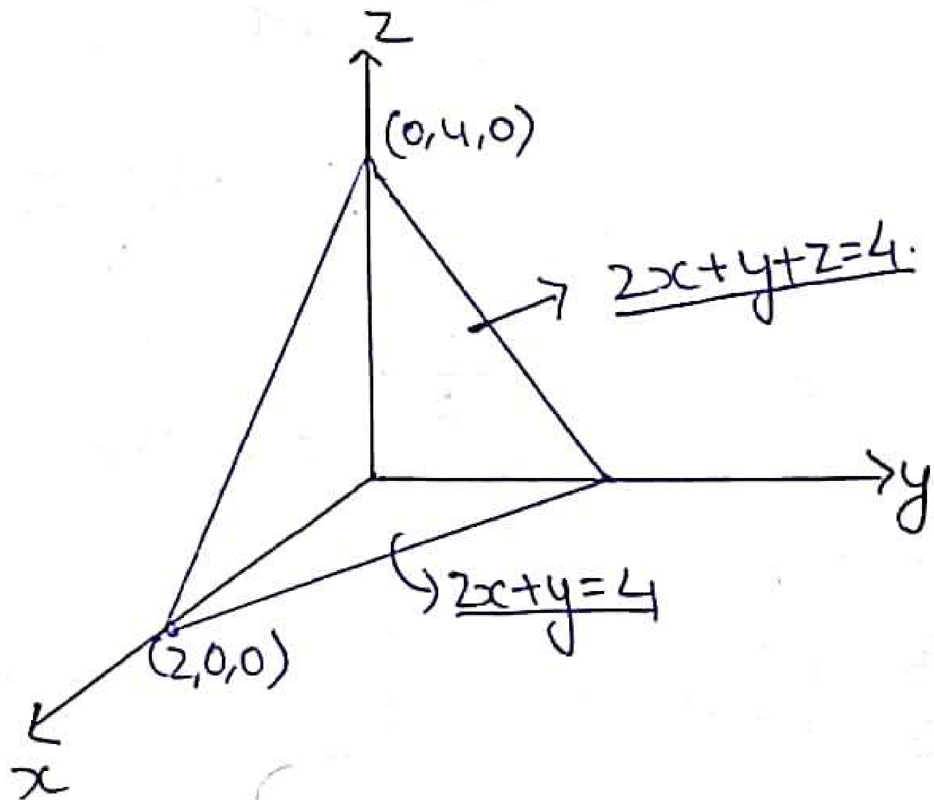
(a) $x + y$ (b) $x - y$

(c) x (d) y

[D]

2) Let $\bar{V} = \iiint_V dz\,dy\,dx$, where V is region bounded by the plane

- $2x+y+z=4$
- $x=0$
- $y=0$
- $z=0$



So, $$\bar{V} = \int_{x=0}^{2} \int_{y=0}^{4-2x} \int_{z=0}^{4-2x-y} dz\, dy\, dx \quad \text{-----} \ (1)$$

But $$I = \int_{0}^{2a} \int_{0}^{\lambda(x)} \int_{0}^{\mu(x,y)} dz\, dy\, dz \quad \text{-------} \ (2)$$

From (1) & (2), we have.

$$\Rightarrow \lambda(x) = 4-2x.$$

$$\Rightarrow \mu(x,y) = 4-2x-y$$

$$\Rightarrow \lambda - \mu = 4-2x-4+2x+y = \underline{y}.$$

3

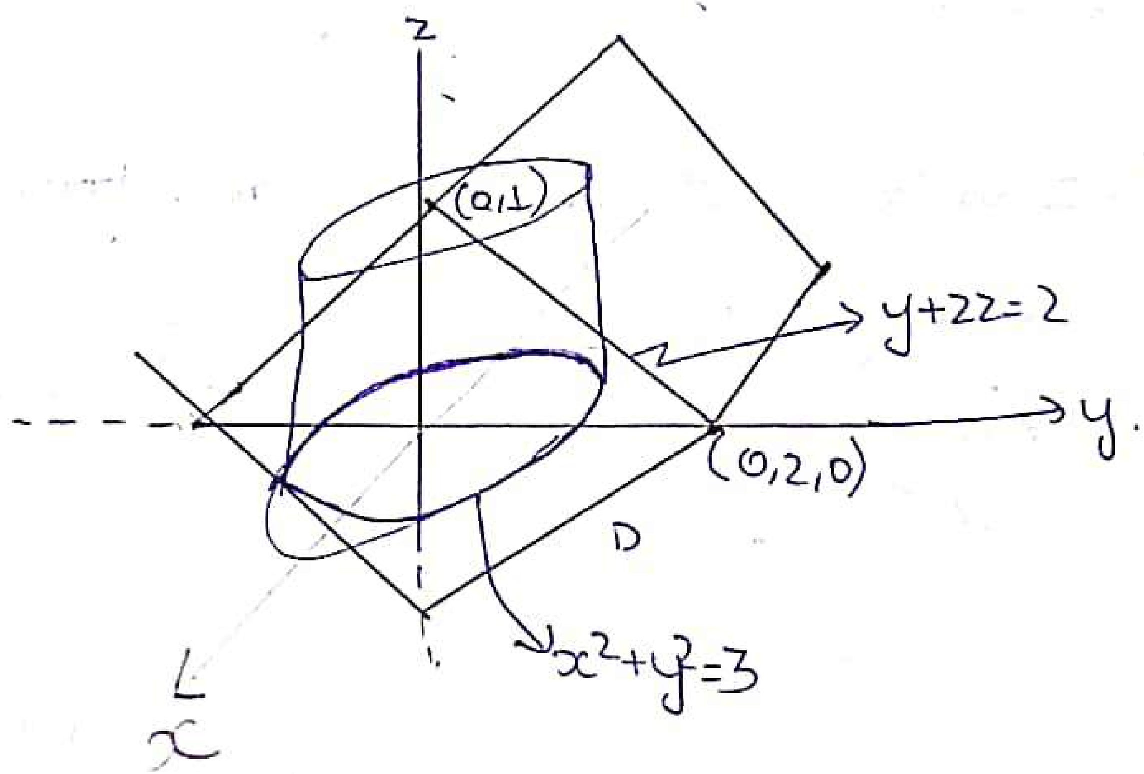
The surfaces area of the portion of the plane $y + 2z = 2$ within the cylinder $x^2 + y^2 =$ is

(a) $\frac{3\sqrt{5}}{2}\pi$ (b) $\frac{5\sqrt{5}}{2}\pi$

(c) $\frac{7\sqrt{5}}{2}\pi$ (d) $\frac{9\sqrt{5}}{2}\pi$

[A]

3) Surface area $= \iint \sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}\, dx\, dy.$



$= \iint_D \sqrt{1+0+\left(\frac{-1}{2}\right)^2}\, dx\, dy.$

$= \frac{\sqrt{5}}{2} \iint_D dx\, dy = \left[\frac{\sqrt{5}}{2} \times \pi (\sqrt{3})^2 - \frac{3\sqrt{5}}{2}\pi\right]$