First Order & First Degree ODE - I

Detailed Course on Differential Equation for IIT JAM' 23 - II

Gajendra Purohit • Lesson 1 • Aug 4, 2022

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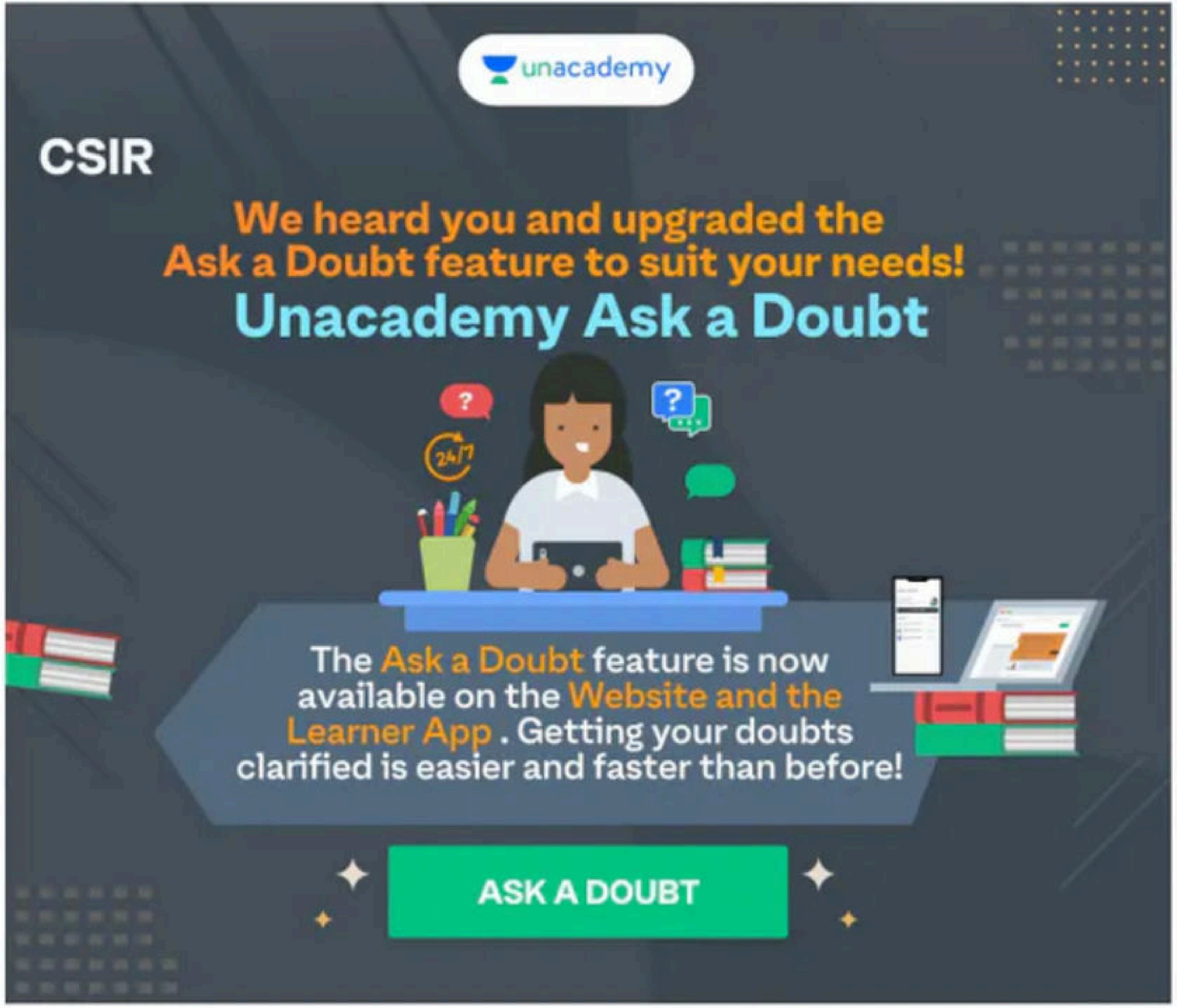
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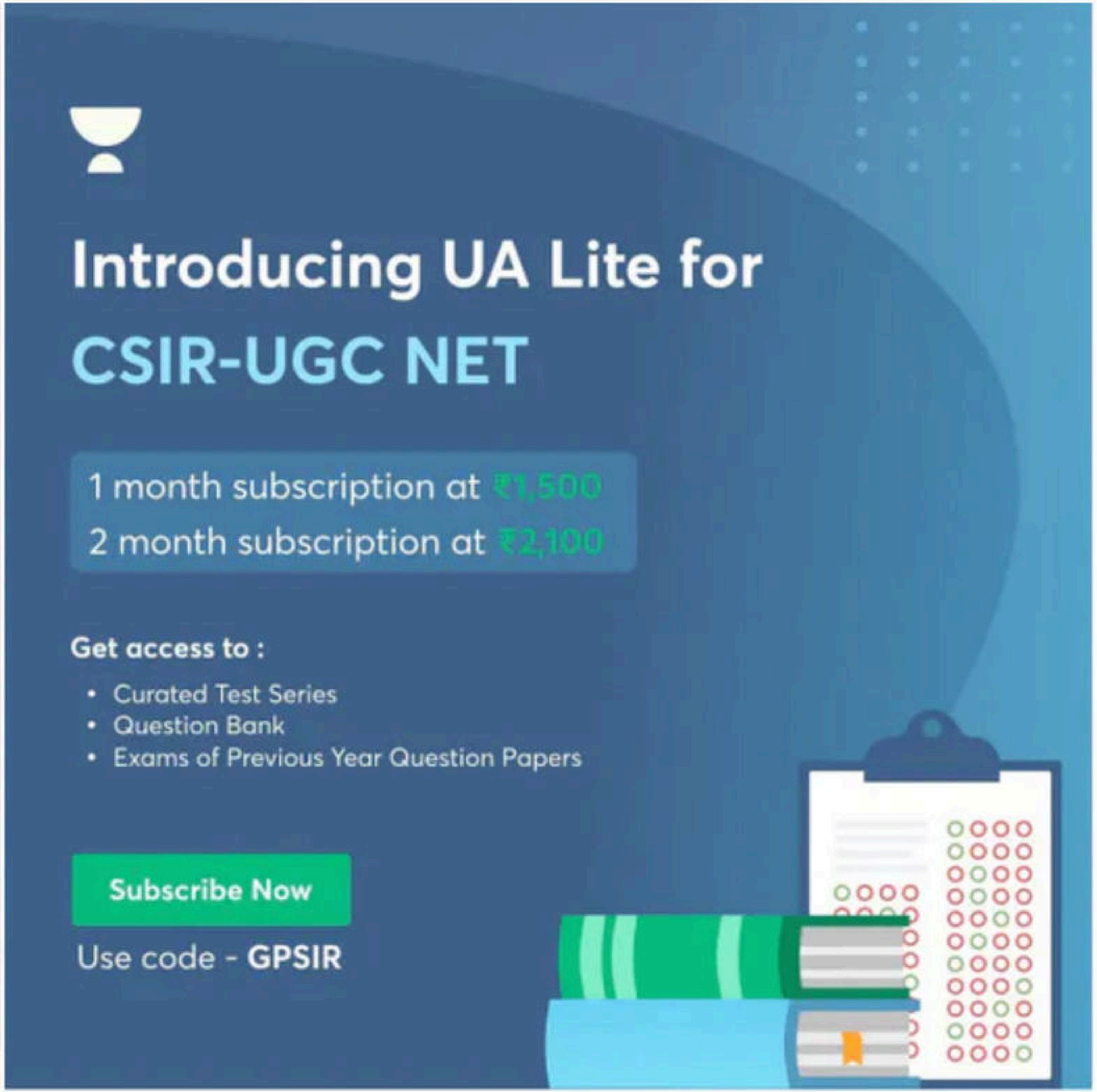
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ORDER & DEGREE

1. Independent variables & dependent variables : The variable whose value is assign is called independent variable and the variable whose value is obtained corresponding to independent variable is called dependent variable.

Suppose f : A → B be a function s.t. y = f(x)

Where x is independent variable and y is dependent variable which is depend on x.

$\frac{dy}{dx}$

$y = x+1$

$x = 0$
$x = 1$

$y = 1$
$y = 2$

Derivative of dependent variable :

Suppose y = x + 1 be a function then derivative of dependent variable is dy/dx.

$y = x+1$

$\frac{dy}{dx} = 1$

Where y is dependent variable, x is independent variable.

Notation :

(1) For dy/dx

y is dependent variable.

x is independent variable.

$y = x+1$

$x = y-1$

$\frac{dx}{dy} = 1$

(2) For dx/dy

x is dependent variable.

y is independent variable

Differential equation : An equation involving derivative of one or more dependent variable with respect to one or more independent variable is called a differential equation.

Ordinary differential equation : A differential equation involving derivative w.r.t. single independent variable is called an ordinary differential equation.

$$\frac{d^2y}{dx^2}+y=\sin x$$

Partial differential equation : A differential equation involving partial derivatives with respect to more than one independent variable is called partial differential equation.

$$\frac{\partial^2 y}{\partial x^2}+\frac{\partial^2 y}{\partial y^2}=0$$

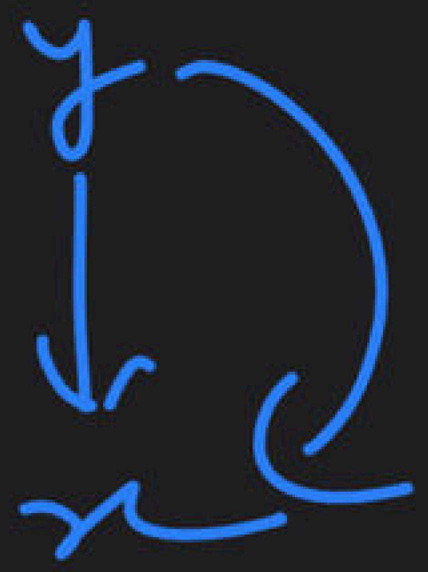
$$\frac{\partial^2 y}{\partial x^2}+4\frac{\partial^2 y}{\partial x\,\partial y}+\frac{\partial^2 y}{\partial x}=0$$

$y = x^2 + t$

variable — constant

$$\frac{dy}{dx} = 2x$$

$$\frac{dy}{dt} = 0$$



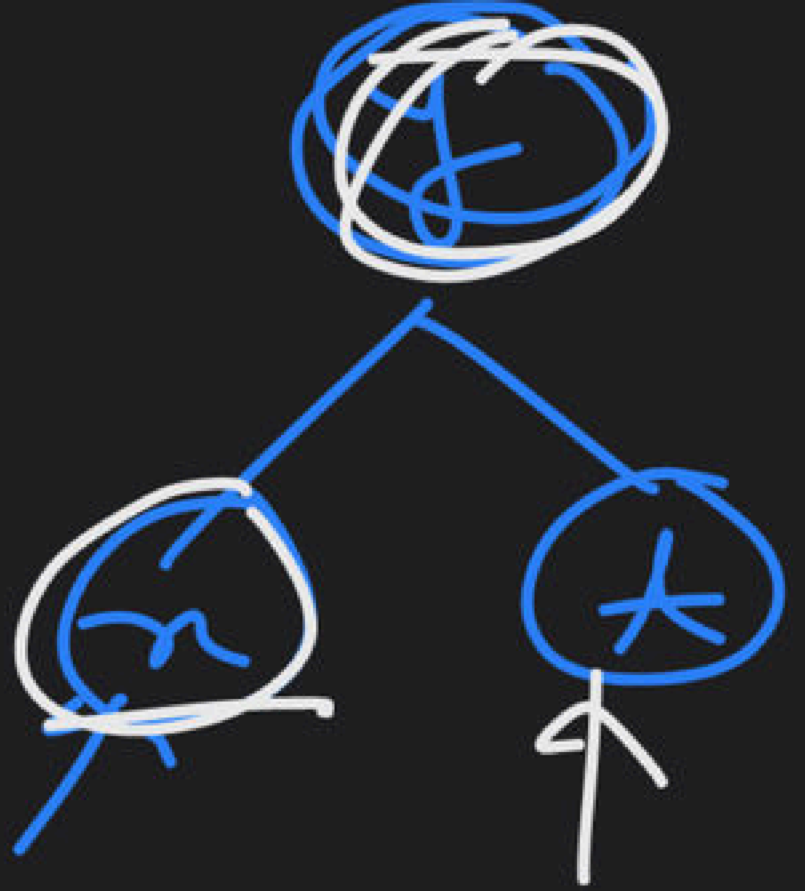
$y = x^2 + t^2$

variable

$$\frac{\partial y}{\partial x} = 2x$$

$$\frac{\partial y}{\partial t} = 2t$$

$$\frac{\partial y}{\partial x}$$



Formation of ODE :

Let $f(x, y, c_1, c_2, \ldots, c_n) = 0$ be the relation between x, y, c_1, c_2, ..., c_n.

Where x is independent variable.

y is dependent variable.

c is arbitrary constant.

Now, $f`(x, y, y`, c_1, c_2, \ldots, c_n) = 0$

$$f``(x, y, y`, y``, c_1, c_2, \ldots\ldots, c_n) = 0$$

$$\vdots$$

$$f^{(n)}(x, y, \ldots\ldots y^{(n)}, c_1, \ldots\ldots c_n) = 0$$

We eliminate $c_1, c_2, \ldots, c_n$ from (n + 1) equation

We get $\Psi(x, y`, y``, \ldots\ldots y^{(n)}) = 0$

$x^2 + y^2 = a^2$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} + \frac{x}{y} = 0$

$$y = \underline{c_1} e^{x} + \underline{c_2} e^{2x}$$

$$\boxed{\frac{dy}{dx} - \frac{y}{2x} = 0}$$

$$2\int \frac{dy}{y} - \int \frac{dx}{2x} = 0$$

$$2\log y - \log x = \log c$$

$$2\log y = \log c + \log x$$

$$y^2 = xc$$

$$\boxed{y^2 = \underline{4a}x}$$

$$\frac{y^2}{4x}$$

$$2y\frac{dy}{dx} = 4a$$

$$y\frac{dy}{dx} = 2a$$

$$y\frac{dy}{dx} = 2\frac{y^2}{4x}$$

$$\cancel{y}\frac{dy}{dx} = \frac{y^{\cancel{2}}}{2x}$$

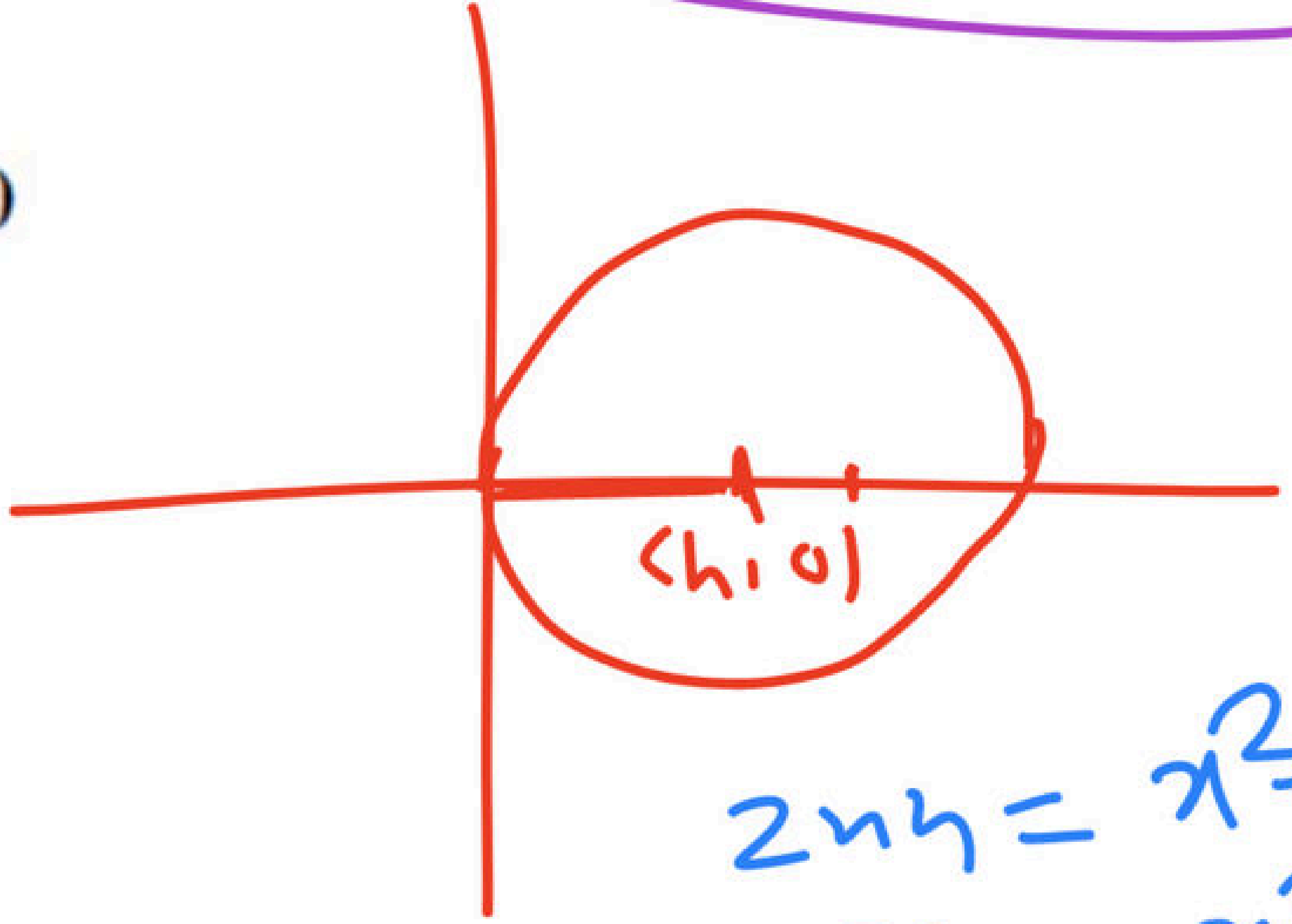
$$\frac{dy}{dx} - \frac{y}{2x}$$

Q.1 The differential equation of all circles which pass through the origin and whose centers are on the x-axis.

(a) $\frac{dy}{dx} = x^2 + y^2$ (b) $2xy\frac{dy}{dx} + x^2 - y^2 = 0$

(c) $\frac{dy}{dx} + e^x y = 0$ (d) None of these

$x^2 - y^2 + 2xy\frac{dy}{dx} = 0$



$(x-h)^2 + y^2 = h^2$

$x^2 - 2xh + \cancel{h^2} + y^2 = \cancel{h^2}$

$\boxed{x^2 - 2xh + y^2 = 0}$

$2x - \underline{2h} + 2y\frac{dy}{dx} = 0$

$2xh = x^2 + y^2$

$2h = \frac{x^2+y^2}{x}$

$2x - \frac{x^2+y^2}{x} + 2y\frac{dy}{dx}$

$\frac{2x^2 - x^2 - y^2}{x} + 2y\frac{dy}{dx} = 0$

$\frac{x^2-y^2}{x} + 2y\frac{dy}{dx} = 0$

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Order of DE :

The highest order derivative occur in differential equation is called order of the DE.

$$\frac{d^3y}{dx^3} + 2\frac{dy}{dx} + y = e^y$$

$$y = Ae^{2x} + Be^{-2x}$$

$$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$$

$$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x}$$

$$\frac{d^2y}{dx^2} = 4\left(Ae^{2x} + Be^{-2x}\right)$$

$$\frac{d^2y}{dx^2} = 4y$$

$$\frac{d^2y}{dx^2} - 4y = 0$$

Q.2. $y = Ae^{Bx+c}$ then order of the DE.

(a) 1 (b) ~~2~~

(c) 3 (d) 4

$y = Ae^{Bx}e^{c}$

$y = De^{Bx}$ — (1)

$y' = \frac{dy}{dx} = BDe^{Bx}$ — (2)

$y'' = B^2De^{Bx}$ — (3)

$\frac{(2)}{(1)}$ $\boxed{\frac{y'}{y} = B}$

$\frac{(3)}{(2)}$ $\boxed{\frac{y''}{y'} = B}$

$\frac{y'}{y} = \frac{y''}{y'}$

$yy'' = (y')^2$

$\underline{yy'' - (y')^2 = 0}$

Degree of DE : The highest power of the highest derivative of DE is called degree of DE.

We should notice that the derivative should be free from radicals function or transcendental function or other than natural number.

$$\left(\frac{d^2y}{dx^2}\right)^2 + \frac{dy}{dx} = 0$$

$$y''' + \sqrt{y''} = 0$$

$$y''' = -\sqrt{y''}$$

$$(y''')^2 = y''$$

$$y = x\frac{dy}{dx} + \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{1/2}$$

$$y - x\frac{dy}{dx} = \left(1+\left(\frac{dy}{dx}\right)^2\right)^{1/2}$$

$$\left(y - x\frac{dy}{dx}\right)^{2} = \left(1+\left(\frac{dy}{dx}\right)^2\right)$$

Q.3. The Order and degree of differential equation

$$\frac{d^3y}{dx^3}+4\sqrt{\left(\frac{dy}{dx}\right)^3+y^2}=0 \text{ are}$$

(a) 3 and 2

(b) 2 and 3

(c) 3 and 3

(d) 3 and 1

$$\frac{d^3y}{dx^3} = -4\sqrt{\left(\frac{dy}{dx}\right)^2+y^2}$$

$$\left(\frac{d^3y}{dx^3}\right)^{2} = 16\left(\left(\frac{dy}{dx}\right)^2+y^2\right)$$

$$\tan\left(\frac{dy}{dx}\right) = x + y$$

$$\frac{dy}{dx} + \sin(y) = c$$

$$\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = x$$

$$a_0 \frac{d^ny}{dx^n} + a_1 \frac{d^{n-1}y}{dx^{n-1}} + a_2 \frac{d^{n-2}y}{dx^{n-2}} + \cdots \quad a_n y = 0$$

Q What are order & degree of D.E

$$\frac{d^2}{dx^2}\left[\left(\frac{d^2y}{dx^2}\right)^{-1/2}\right] = 0$$

$$\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)^{-1/2}\right) = 0$$

$$\frac{d}{dx}\left(\boxed{\left(\frac{d^2y}{dx^2}\right)^{-3/2}} \frac{d^3y}{dx^3}\right) = 0$$

$$\frac{3}{2}\left(\frac{d^2y}{dx^2}\right)^{-5/2}\left(\frac{d^3y}{dx^3}\right)^2 = \left(\frac{d^2y}{dx^2}\right)^{-3/2}\frac{d^4y}{dx^4} = 0$$

$$\frac{3}{2}\left(\frac{d^3y}{dx^3}\right)^2 = \left(\frac{d^2y}{dx^2}\right)\left(\frac{d^4y}{dx^4}\right)^1$$

Q.4. If $y = \ln(\sin(x + a)) + b$, where a and b are constants, is the primitive, then the corresponding lowest order differential equation is

(a) $y`` = -(1 + (y`)^2)$

(b) $y`` = y^2 - (y`)^2$

(c) $y`` = 1 + (y`)^2$

(d) $y`` = y` + y^2$

Q.5. Let y(x) = x sin x be one of the solution of an nth order linear differential equation with constant coefficients then the minimum value of n is

(a) 1 (b) 2

(c) 3 (d) 4

$y = x\sin x$

$y' = \sin x + x\cos x$

$y'' = \cos x + \cos x - x\sin x$

$\boxed{y'' + y = 2\cos x}$

$y''' + y' = -2\sin x$

$\boxed{y^{iv} + y'' = -2\cos x}$

$y^{iv} + y'' + y'' + y = 0$

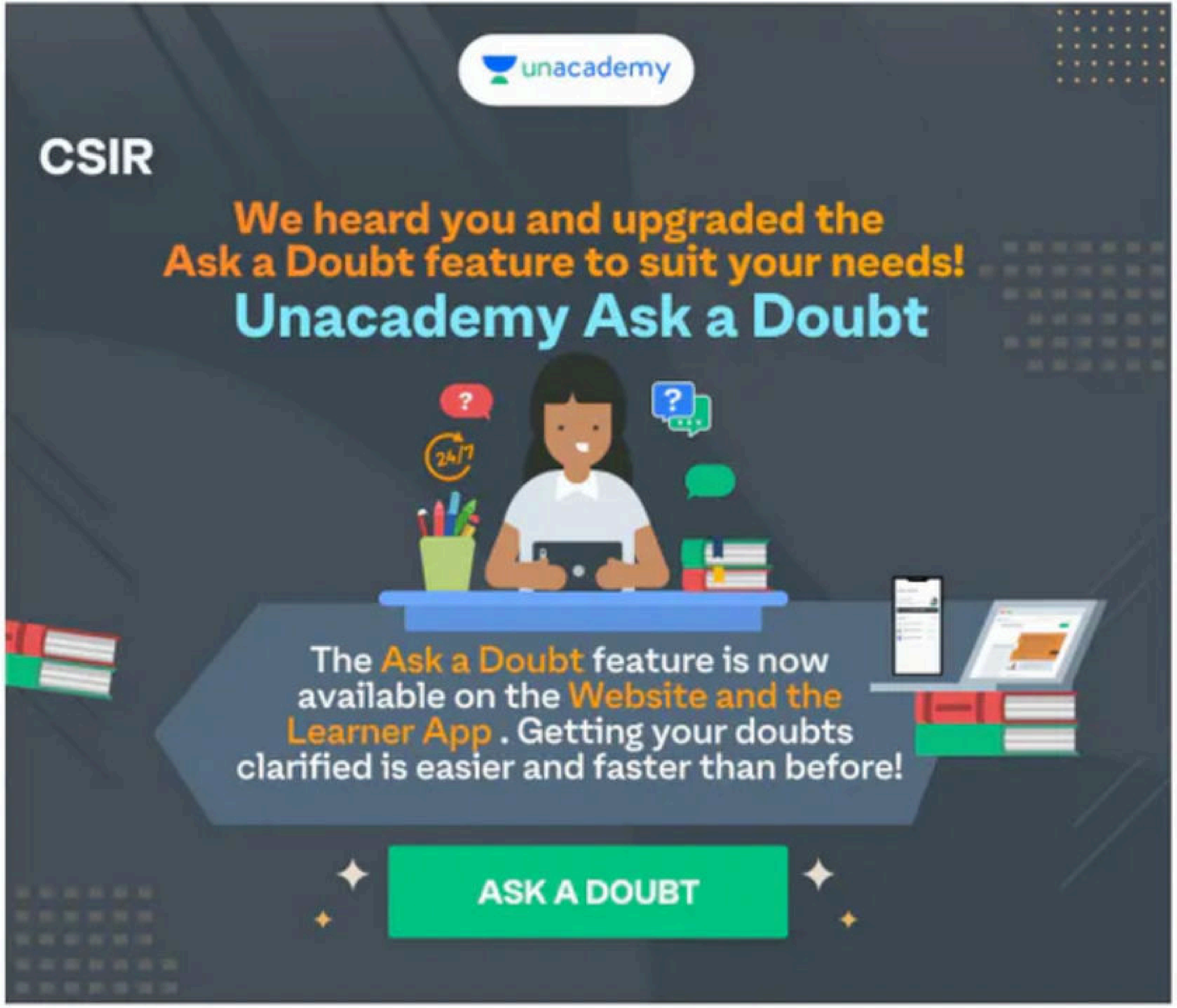
Q.6 Which one of the following differential equations represent all circles with radius a?

(a) $1+\left(\frac{dy}{dx}\right)^2+\sqrt{a^2-x^2}\,\frac{d^2y}{dx^2}=0$

(b) $1+\left(\frac{dy}{dx}\right)^2+\sqrt{a^2-y^2}\,\frac{d^2y}{dx^2}=0$

(c) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3+a^2\left(\frac{d^2y}{dx^2}\right)^2=0$

(d) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=a^2\left(\frac{d^2y}{dx^2}\right)^2$



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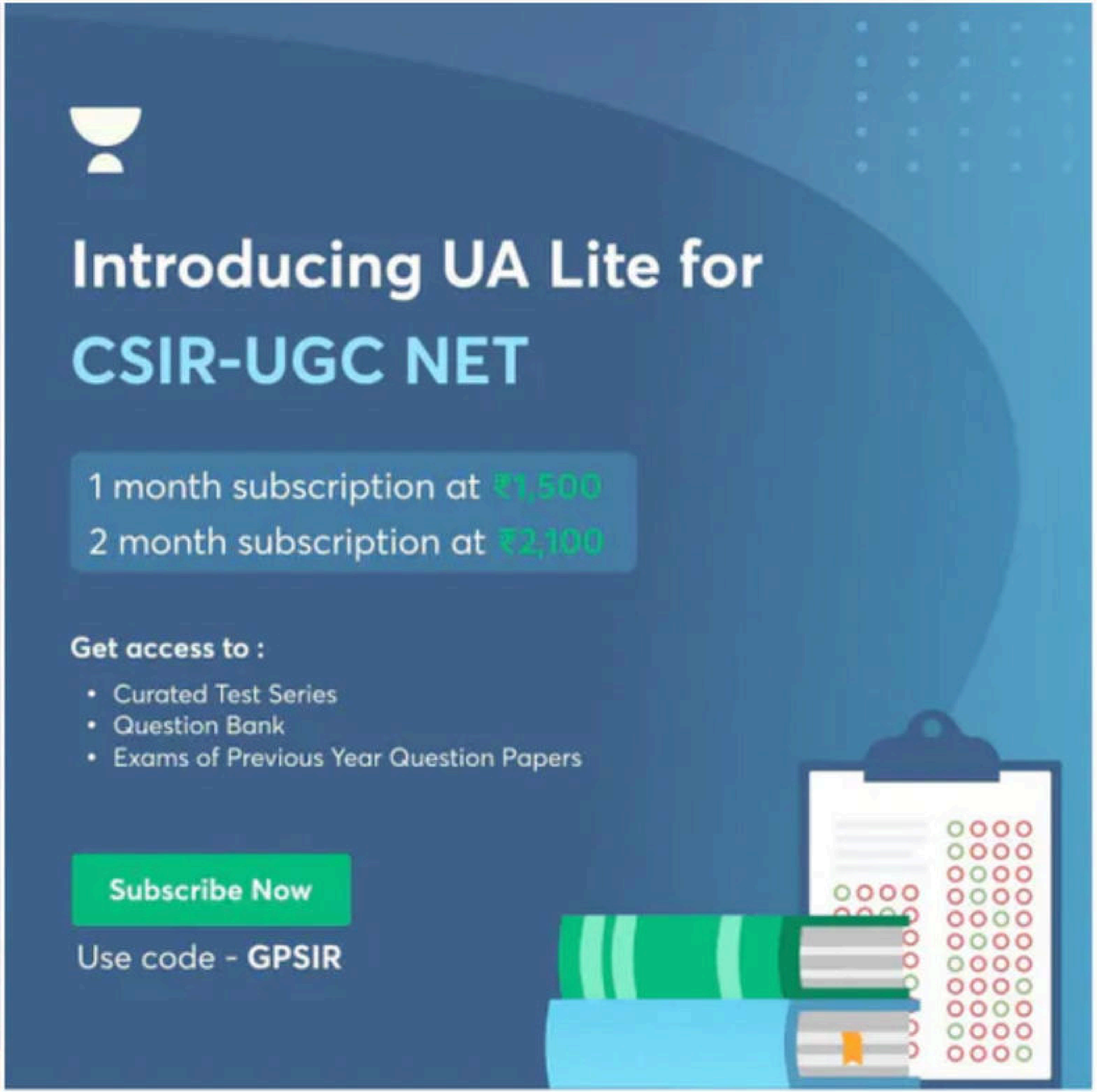
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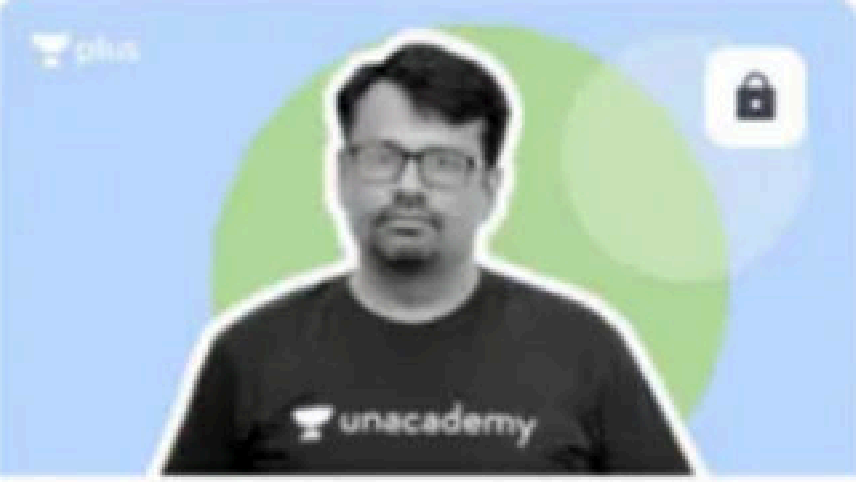

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