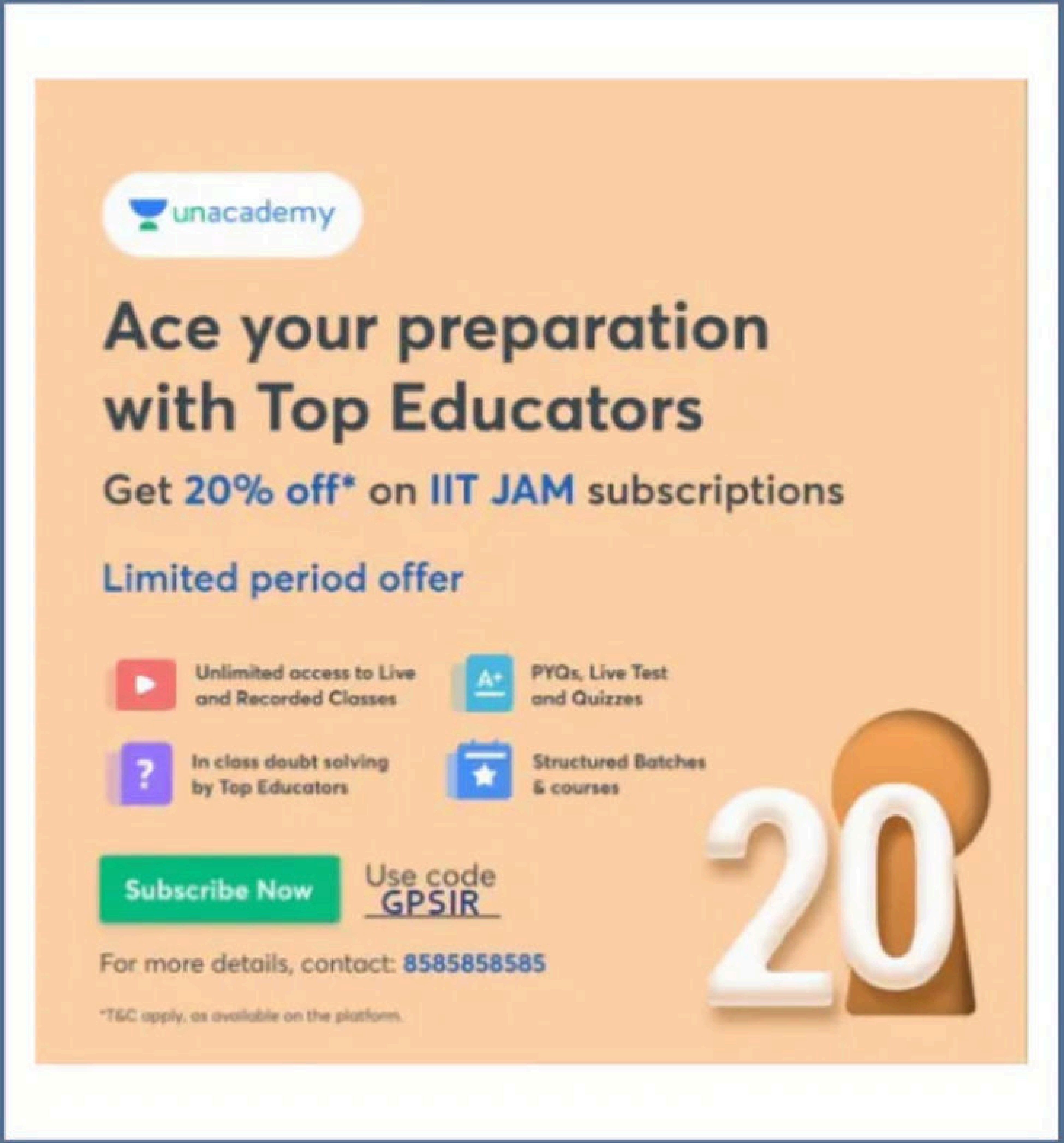
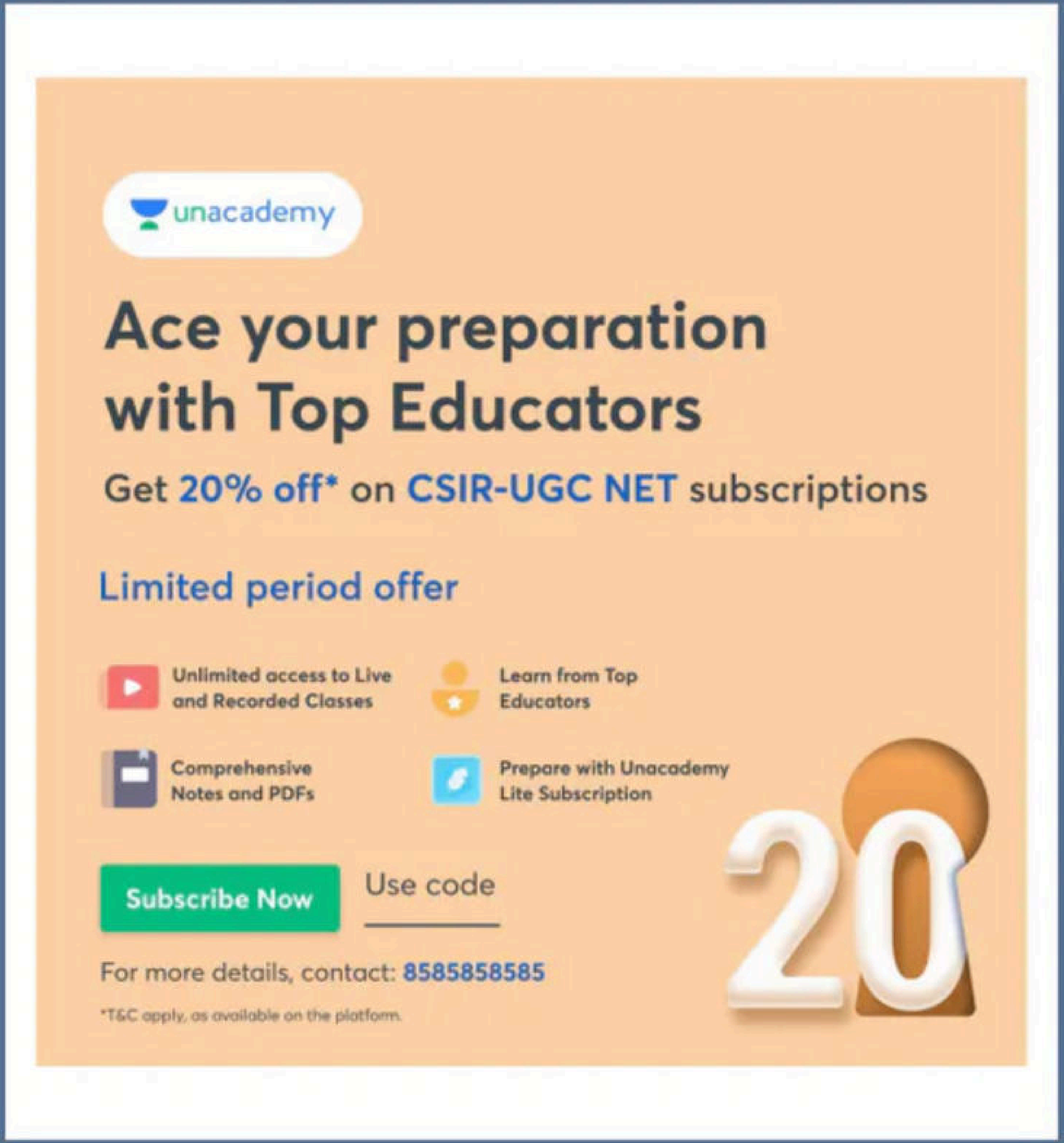
Linear Independence of Vectors

Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 3 • Sept 10, 2022


unacademy
Ace your preparation with Top Educators
Get 20% off* on IIT JAM subscriptions
Limited period offer
Unlimited access to Live and Recorded Classes
PYQs, Live Test and Quizzes
In class doubt solving by Top Educators
Structured Batches & courses
Subscribe Now
Use code GPSIR
For more details, contact: 8585858585
*T&C apply, as available on the platform.
20


unacademy
Ace your preparation with Top Educators
Get 20% off* on CSIR-UGC NET subscriptions
Limited period offer
Unlimited access to Live and Recorded Classes
Learn from Top Educators
Comprehensive Notes and PDFs
Prepare with Unacademy Lite Subscription
Subscribe Now
Use code
For more details, contact: 8585858585
*T&C apply, as available on the platform.
20


unacademy
RANK BOOSTER COURSE UNIT 2
CSIR NET 2022
23rd AUGUST
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy


unacademy
DETAILED COURSE 2.0
LINEAR ALGEBRA FOR IIT JAM 2023
8th SEPTEMBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780	You get 6 months extra for free · Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974	You get 6 months extra for free · Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475	
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252	
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807	
1 month		₹ 2,723 / mo	Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code? | Proceed to pay

Get 10% Off

After Using My Referral Code

GPSIR

Awesome! You get 10% off

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	₹ ~~21,700~~ ₹ 19,602	You get 6 months extra for free · Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo	₹ ~~14,974~~ ₹ 13,477	You get 6 months extra for free · Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo	₹ ~~13,475~~ ₹ 12,128	
6 months	Save 25%	₹ 1,838 / mo	₹ ~~12,252~~ ₹ 11,027	
3 months	Save 17%	₹ 2,042 / mo	₹ ~~6,807~~ ₹ 6,126	

Proceed to pay

Diagonal Matrix : Let $A = (a_{ij})_{n\times n}$ then A is diagonal matrix if

$a_{ij} = 0$ for all $\cancel{i > j, i < j}$ $i \neq j$ or $a_{ij} \neq 0$ for all $i \neq j$.

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scalar Matrix : A diagonal matrix A is said to be scalar matrix

$$\text{if } a_{ij} = \begin{cases} 0 & i \neq j \\ k & i = j \end{cases}$$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & \cancel{4} \end{bmatrix}$$

Note :

1. Number of non-trivial entries of diagonal matrix of order n.

   = n (number of diagonal entries)

2. If D be a diagonal matrix then AD = DA for all $A \in M_{n\times n}$

   3>

   Iff D is a scalar matrix.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix} = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$p = q$

$$AP = DA \qquad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$\begin{pmatrix} \boxed{a} & \boxed{b} & \cancel{2c} \\ \boxed{d} & \boxed{e} & \cancel{2f} \\ \cancel{g} & \cancel{h} & \boxed{2i} \end{pmatrix} = \begin{pmatrix} \boxed{a} & \boxed{b} & \cancel{c} \\ \boxed{d} & \boxed{e} & \cancel{f} \\ \cancel{2g} & \cancel{2h} & \boxed{2i} \end{pmatrix}$$

$$2i = 2i$$

$$i = i$$

Symmetric matrix : A square matrix A is said to be symmetric if $A = A^T$ i.e. $a_{ij} = a_{ji}$; for all i, j.

Properties :

(1) A be any square real matrix then $A + A^T$ is always symmetric.

$(A + A^T)^T = A^T + A = A + A^T$

(2) If A is any square matrix then AA^T is always symmetric.

$(AA^T)^T = (A^T)^T A^T = AA^T$

(3) If A is symmetric matrix then P^TAP is always symmetric. When P is any square matrix of order same is A.

$(P^TAP)^T = P^TA^T(P^T)^T = P^TA^TP$

Given that $A^T = A$

So, $(P^TAP)^T = P^TAP$

(4) If A is symmetric matrix then kA is also symmetric matrix for all $k \in R$

$(kA)^T = k^TA^T = kA$

$$A = \begin{bmatrix} a & d \\ d & b \end{bmatrix}$$

$1+2$

$$A = \begin{bmatrix} a & d & e \\ d & b & f \\ e & f & c \end{bmatrix}$$

$1+2+3$

$$A = \begin{bmatrix} a & e & f & g \\ e & b & h & i \\ f & h & c & j \\ g & i & j & d \end{bmatrix}$$

$1+2+3+4$

$A_{n \times n}$

$\boxed{1+2+3+ \cdots n}$

$\frac{n(n+1)}{2}$

(5) If A & B are symmetric then

(i) $A + B$ is symmetric

(ii) AB is symmetric if $AB = BA$

(iii) $\alpha A + \beta B$ is symmetric for all $\alpha, \beta \in R$

(iv) Ak is symmetric ; for all $k \in R$

(v) $AB + BA$ is symmetric.

$$(AB + BA)^T = (AB)^T + (BA)^T$$

$$= B^T A^T + A^T B^T$$

$$= BA + AB = AB + BA$$

Skew Symmetric Matrix : A square matrix is said to be skew symmetric matrix if $A^T = -A$

i.e. $a_{ij} = -a_{ij}$ for all i, j

for i = j (for diagonal elements)

$a_{ii} = -a_{ii}$

$\Rightarrow 2a_{ii} = 0 \Rightarrow a_{ii}$

$\Rightarrow$ All diagonal entries of a skew-symmetric matrix are zero.

Note : Odd order skew-symmetric matrix are always singular i.e. $|A| = 0$

$A = -A^T$

$A = -A^T$

$$A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & a & b & d \\ -a & 0 & -c & -e \\ -b & c & 0 & f \\ -d & e & -f & 0 \end{bmatrix}$$

$A^T = -A$

$|A^T| = |-A|$

$|A| = (-1)^n |A|$

$\alpha = (-1)^n \alpha$

n - odd

n - even

$\alpha = -\alpha$

$2\alpha = 0$

$\alpha = 0$

$|A^T| = |A|$

$(|kA| = k^n |A|)$

$|A| = \alpha = 0$

Q.1 Let $S = \{A \in M_{n\times n}; A = A^T \ \& \ A^T = -A\}$, then cardinality of S is

(a) 1

(b) ϕ

(c) infinite

(d) 2022

$$A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Property :

(1) A be a square matrix then $A - A^T$ is skew symmetric matrix.

$$(A - A^T)^T = A^T - A = -(A - A^T)$$

(2) If A is skew symmetric matrix then P^TAP is also skew symmetric matrix where P be any square matrix of order same is A.

(3) A & B are skew-symmetric matrix then

(i) A + B is skew-symmetric matrix.

(ii) AB is skew-symmetric if AB = BA

(iii) kA is skew-symmetric.

(iv) $\alpha A + \beta B$ is skew-symmetric.

(v) A^k → k is odd then skew-symmetric.
→ k is even then symmetric.

$(A^k)^T = (A^T)^k = (-A)^k = (-1)^k A^k$

$$= \begin{cases} -A^k & \text{if } k \text{ is odd} \\ A^k & \text{if } k \text{ is even} \end{cases}$$

A → A^{10} Symmetric
A → A^{11} Skew

Q.2. Let A is skew-symmetric matrix then

(a) A^{2020} is symmetric matrix.

(b) A^{2021} is symmetric matrix

(c) A^{2022} is skew symmetric matrix

(d) A^{2021} is skew symmetric matrix

a	a	c
b	b	d
c	c	b
d	d	a

$A^3 = -A^3$

COMPLETE COURSE ON

MATHEMATICS

FOR IIT-JAM 2022

TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINAER ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 908 / mo Total ₹ 21,780
You get 6 months extra for free	Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,248 / mo Total ₹ 14,974
You get 6 months extra for free	Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,497 / mo Total ₹ 13,475
6 months Save 25%	₹ 2,042 / mo Total ₹ 12,252
3 months Save 17%	₹ 2,269 / mo Total ₹ 6,807
1 month	₹ 2,723 / mo Total ₹ 2,723

To be paid as a one-time payment

Have a referral code? | Proceed to pay

Get 10% Off

After Using My Referral Code

GPSIR

Awesome! You get 10% off

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 817 / mo ~~₹ 21,700~~ ₹ 19,602
You get 6 months extra for free	Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,123 / mo ~~₹ 14,974~~ ₹ 13,477
You get 6 months extra for free	Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,348 / mo ~~₹ 13,475~~ ₹ 12,128
6 months Save 25%	₹ 1,838 / mo ~~₹ 12,252~~ ₹ 11,027
3 months Save 17%	₹ 2,042 / mo ~~₹ 6,807~~ ₹ 6,126

Proceed to pay

FOUNDATION COURSE OF
MATHEMATICS
FOR CSIR-NET

$$A = \begin{bmatrix} 0 & a \\ -b & 0 \end{bmatrix} \qquad A^T = \begin{pmatrix} 0 & -b \\ a & 0 \end{pmatrix}$$

$$A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{bmatrix} \qquad 1+2$$

$$A_{n \times n} \qquad 1+2+\cdots(n-1) \qquad \frac{n(n-1)}{2}$$

(3) **Hermitian matrix :** A square complex matrix A is Hermitian if $A^{\theta} = A$

$$\boxed{A^{\theta} = (\bar{A})^{T} = A}$$

i.e. $a_{ij} = \overline{a_{ji}}$; for all i, j

For i = j (for diagonal elements)

$$a_{ii} = \overline{a_{ii}}$$

$\Rightarrow$ a_{ii} is real.

$\Rightarrow$ All diagonal entrie of Hermitian matrix are purely real.

Note : Number of non-trivial entries in Hermitian matrix.

$$A = \begin{bmatrix} a & \alpha + i\beta \\ \alpha - i\beta & b \end{bmatrix}$$

$$A = \begin{bmatrix} a & \alpha + i\beta \\ \alpha - i\beta & b \end{bmatrix}$$

4

$$A = \begin{bmatrix} a & \alpha_1 + i\beta_1 & \alpha_2 + i\beta_2 \\ \alpha_1 - i\beta_1 & b & \alpha_3 + i\beta_3 \\ \alpha_2 - i\beta_2 & \alpha_3 - i\beta_3 & c \end{bmatrix}$$

T

n^2

$A_{n \times m}$

n^2

Property : If A is Hermitian matrix then kA

(a) If $k \in R$, then $(kA)^\theta = kA^\theta = kA$

$\Rightarrow$ kA is Hermitian matrix.

(b) If $k \in C$

(i) k is purely imaginary, then $(kA)^\theta = k^\theta A^\theta = -kA$

$\Rightarrow$ kA is skew-Hermitian matrix.

(ii) $k = \alpha + i\beta$, then $[(\alpha + \beta i)A]^\theta$

$= (\alpha - i\beta)A \neq (\alpha + i\beta)A$

Then kA is not Hermitian matrix.

(c) A & B are Hermitian matrix then A + B is also Hermitian matrix and AB is Hermitian if AB = BA

(d) If A is Hermitian matrix, then A^n is Hermitian matrix.

$(A^n)^\theta = (A^\theta)^n = A^n$

Skew-Hermitian matrix : If $A\theta = -A$ then A is skew-Hermitian matrix i.e. $a_{ij} = -\overline{a_{ji}}$.

$(\overline{A})^T = -A$

$A^\theta = -A$

For i = j (for diagonal element)

$$a_{ii} = -\overline{a_{ii}}$$

$\Rightarrow$ a_{ii} is either zero or purely imaginary.

$A = \begin{bmatrix} 0 & 1+i \\ -1-i & 0 \end{bmatrix}$

Property : If A is skew-Hermitian matrix then kA is

(i) If $k \in R$ then $(kA)^\theta = kA^\theta = -kA$

kA is skew-symmetric matrix.

(ii) If k is purely imaginary.

Q.3. Suppose A is a real square matrix of odd order such that $A + A^T = 0$. Then which of the following is true?

(a) A is singular matrix

(b) A is non-singular matrix

(c) A is symmetric matrix

(d) A is skew-symmetric matrix

(a) a c

(b) b d

(c) c b

(d) d a

$\boxed{\alpha - i\beta}$

$\boxed{\alpha - i\beta}$

$A = -A^T$

$|A| = 0$

Q.6. The diagonal elements of Hermitian matrix are –

(a) Complex number

(b) Real numbers

(c) Natural numbers

(d) None of these

unacademy

▲ 1 • Asked by Millan

Happy teacher's day, sir. Appke liye aye chota sa gift he. Appke photo meri first face drawing he . Plz sir and class mates bhura math mana, drawing achi nehi he to.

Q.7. The diagonal elements of Skew-Hermitian matrix are –

(a) Pure real numbers or zero

(b) Pure imaginary or zero

(c) Complex number

(d) None of these

Q.1 It is known that $X = X_0 \in M_2(Z)$ is a solution of $\boxed{AX - XA = A}$ for some $A \in \left\{ \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \right\}$. Which of the following values are not possible for the determinant of X_0?

CSIR NET FEB 2022

(a) $\det(X_0) = 0$ (b) $\det(X_0) = 2$

(c) $\det(X_0) = 6$ (d) $\det(X_0) = 10$

$$X = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\begin{pmatrix} x & y \\ z & w \end{pmatrix} - \begin{pmatrix} x & y \\ z & w \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\begin{bmatrix} x+z & y+w \\ -x-z & -y-w \end{bmatrix} - \begin{bmatrix} x-y & x-y \\ z-w & z-w \end{bmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\begin{bmatrix} \cancel{x}+z-\cancel{x}+y & y+w-x+y \\ -x-z-z+w & -y-\cancel{w}-z+\cancel{w} \end{bmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\boxed{z+y=1}$$

$$2y-x+w=1$$

$$\boxed{-2z-x+w=-1}$$

$$z = 1-y$$

$$-2+2y-x+w=-1$$

$$w = 1+x-2y$$

$$X_0 = \begin{bmatrix} x & y \\ 1-y & 1+x-2y \end{bmatrix}$$

$$\det(X_0) = (x-y)(x-y+1)$$

$$(3-1)(3-1+1)$$

$x - y = 1$

$x = 2,\ y =$

$x = 3,\ y = 1$

Let $A = \begin{bmatrix} a & 0 & \cdots\cdots & b \\ 0 & a & \cdots\cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ b & 0 & 0 & a \end{bmatrix}_{n\times n}$ then $|A| = (a^2 - b^2)^{\left[\frac{n}{2}\right]}$.

Q2. The determinant of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$ is

(a) 0 (b) -9

(c) -27 (d) 1

Q.4. Let $D_1 = \det\begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}, D_2 = \det\begin{bmatrix} -x & a & -p \\ y & -b & q \\ z & -c & r \end{bmatrix}$. Then

$D_2 = 5$

(a) $D_1 = D_2$ $\qquad D_1 = -5$ $\qquad$ (b) $D_1 = 2D_2$

(c) $D_1 = -D_2$ $\qquad$ (d) $2D_1 = D_2$

$a = b = c = 1$

$x = 4 \quad y = 3, z = 5$

$p = 1$

$q = 2$

$r = 3$

Dr.Gajendra Purohit (PhD,NET)
Referral Code **GPSIR**


unacademy
RANK BOOSTER COURSE UNIT 2
CSIR NET 2022
23rd AUGUST
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy


unacademy
DETAILED COURSE 2.0
LINEAR ALGEBRA FOR IIT JAM 2023
8th SEPTEMBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

unacademy CSIR-UGC NET Search Login

Educator Profile

Gajendra Purohit

#5 Educator in CSIR-UGC NET

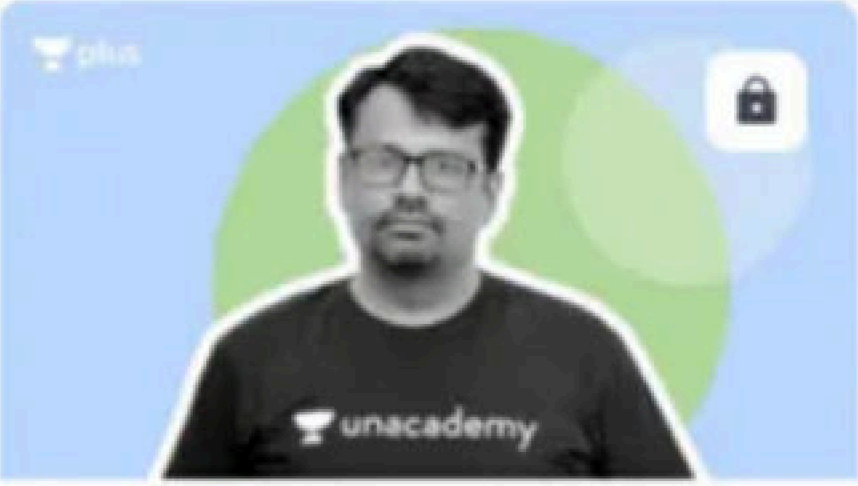
Dr.Gajendra Purohit PhD, CSIR NET (Maths) | Youtuber(330K+30k Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author of Bestseller

Follow

11M Watch mins | 1M Watch mins (last 30 days) | 22k Followers | 1k Dedications

CSIR-UGC NET

SEE ALL



HINDI MATHEMATICAL SCIENCES

Course on Linear Algebra, Partial Diff. Equation & Calculus

Starts on Mar 1, 2021 • 24 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Course on Complex Analysis & Integral Equation

Starts on Jan 14, 2021 • 16 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Foundation Course on Mathematics for CSIR 2021

Starts on Dec 7, 2020 • 20 lessons

Gajendra Purohit

Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo, Total ₹ 21,780
You get 6 months extra for free		Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo, Total ₹ 14,974
You get 6 months extra for free		Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo, Total ₹ 13,475
6 months	Save 25%	₹ 2,042 / mo, Total ₹ 12,252
3 months	Save 17%	₹ 2,269 / mo, Total ₹ 6,807
1 month		₹ 2,723 / mo, Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo, ₹ ~~21,700~~ ₹ 19,602
You get 6 months extra for free		Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo, ₹ ~~14,974~~ ₹ 13,477
You get 6 months extra for free		Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo, ₹ ~~13,475~~ ₹ 12,128
6 months	Save 25%	₹ 1,838 / mo, ₹ ~~12,252~~ ₹ 11,027
3 months	Save 17%	₹ 2,042 / mo, ₹ ~~6,807~~ ₹ 6,126

GPSIR

Awesome! You get 10% off

Proceed to pay