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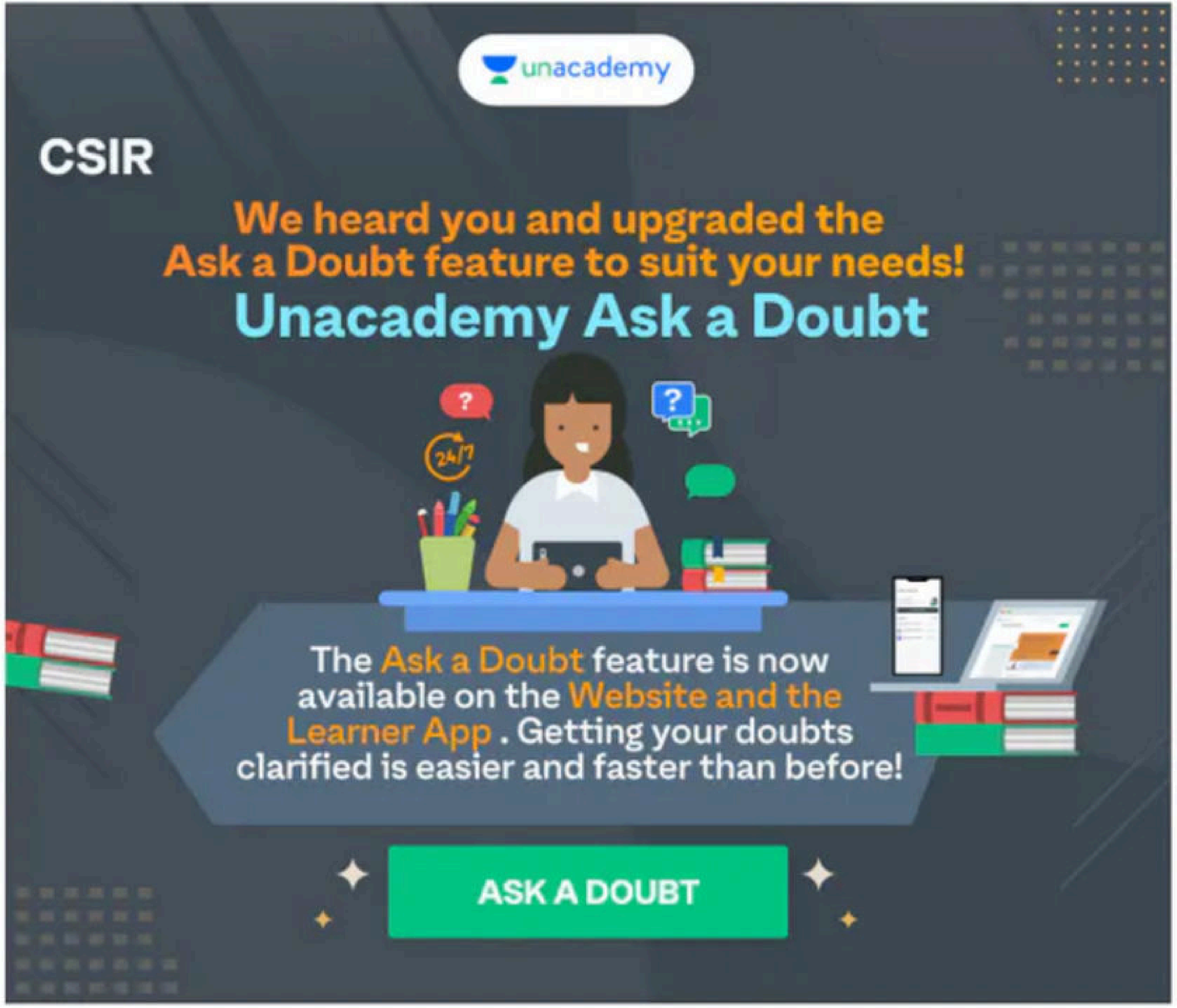
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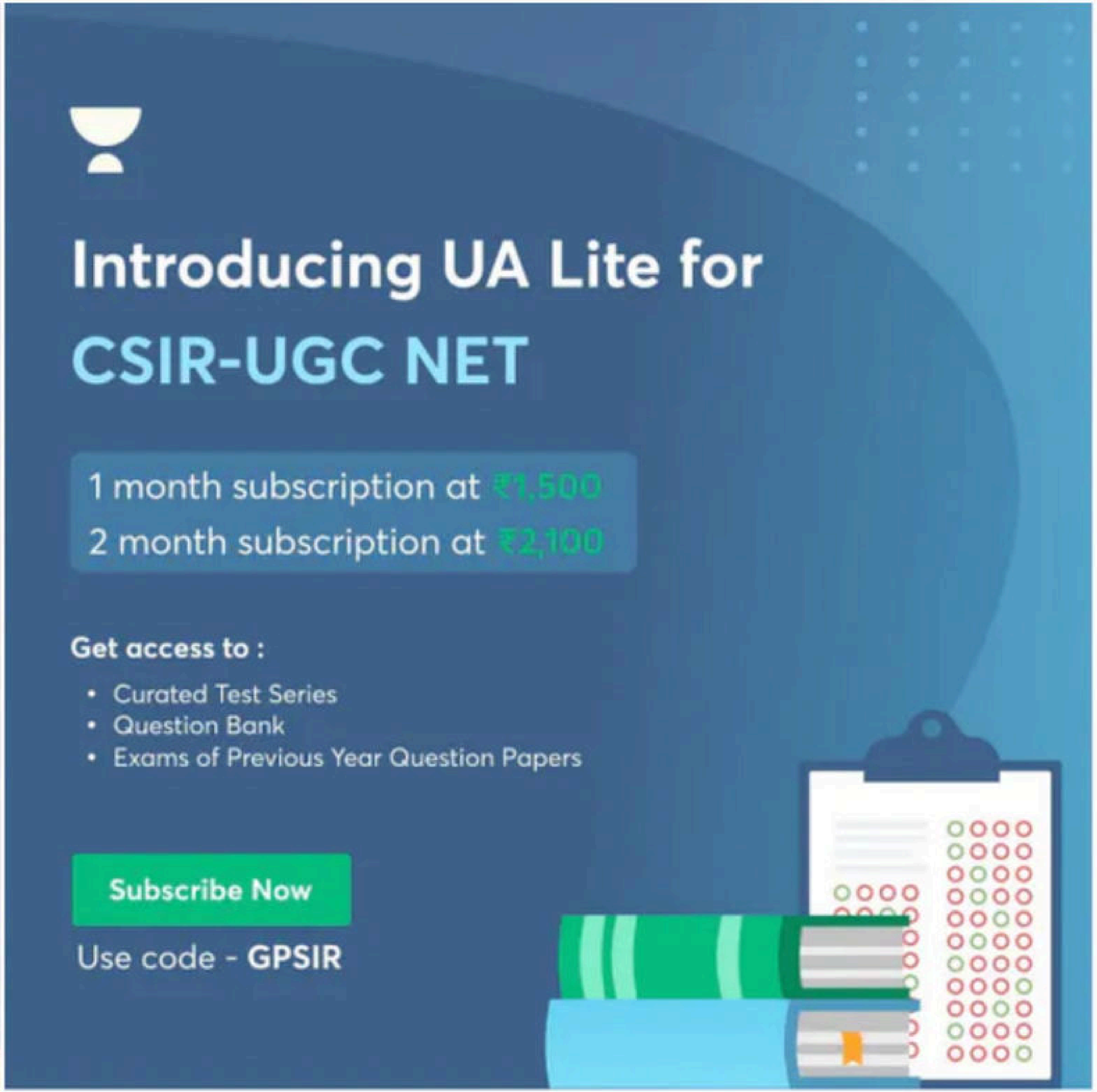
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Volume of solid generated by revolution of solid

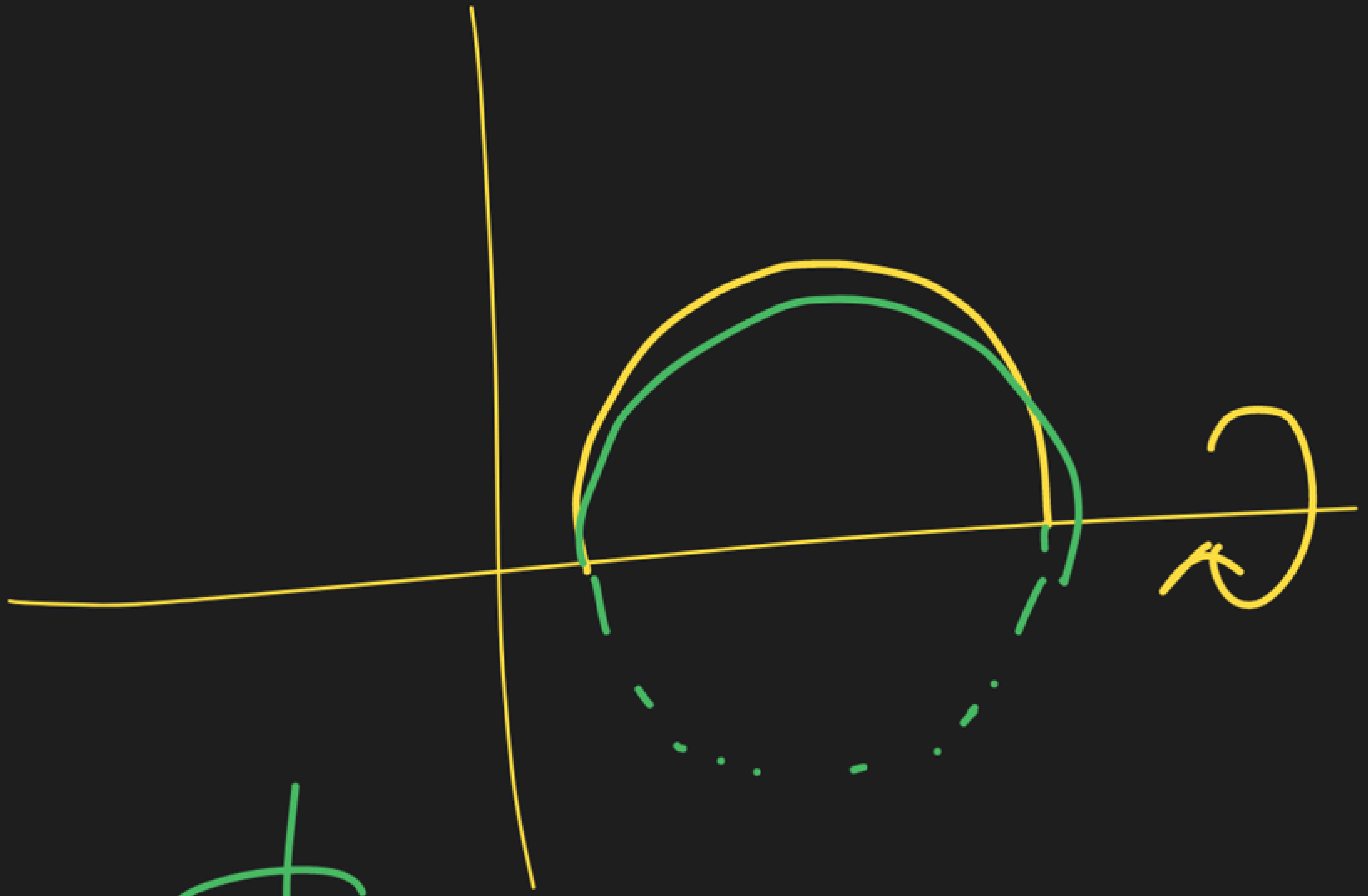
Revolution of solid :

(i) **Revolution of solid about x-axis :** Suppose we have a curve, $y = f(x)$. Imagine that the part of curve between the ordinate $x = a$ and $x = b$ is rotated about the x-axis through 360°. The curve would then map out the surface of solid as it rotated and such solid are

called solid of revolution.

$$V = \int_{x_1}^{x_2} \pi y^2 \, dx$$

$$V = \int_{y_1}^{y_2} \pi x^2 \, dy$$



$$V = \int_{x_1}^{x_2} \pi y^2 \, dx$$

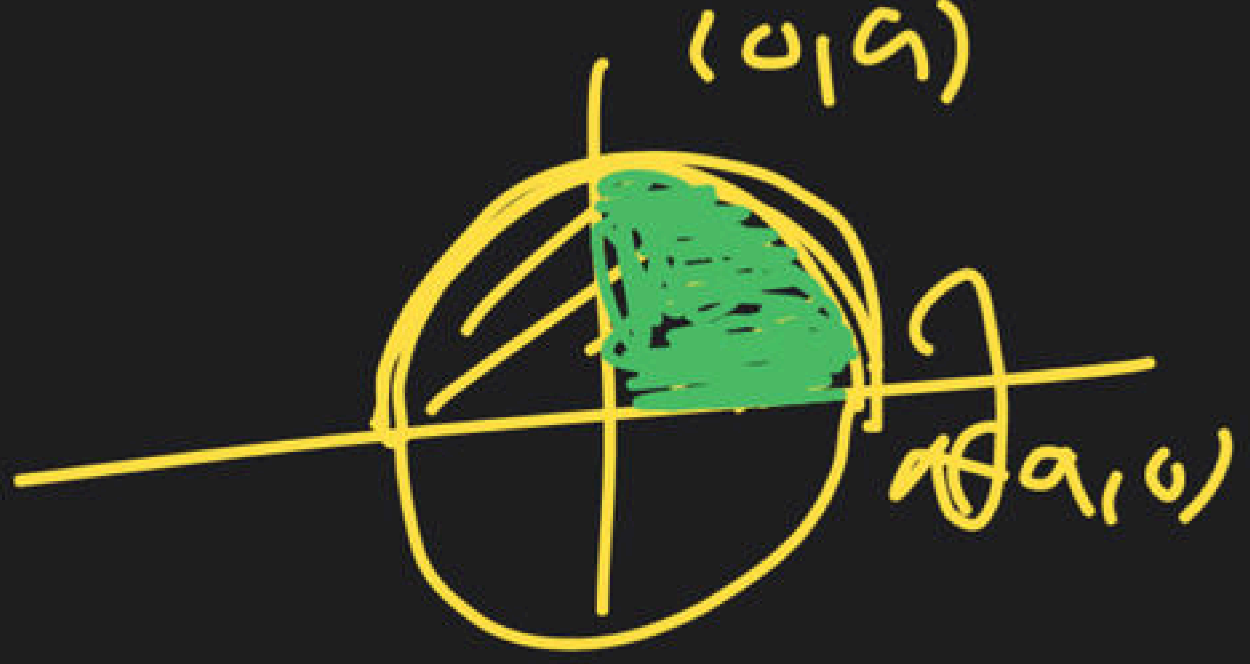
$$V = 2\pi \int_0^a (a^2 - x^2)\, dx$$

$$V = 2\pi \left(a^2 x - \frac{x^3}{3} \right)_0^a$$

$$= 2\pi \left(a^3 - a^3/3 \right)$$

$$= 2\pi \left(\frac{2a^3}{3} \right) = \frac{4\pi a^3}{3}$$

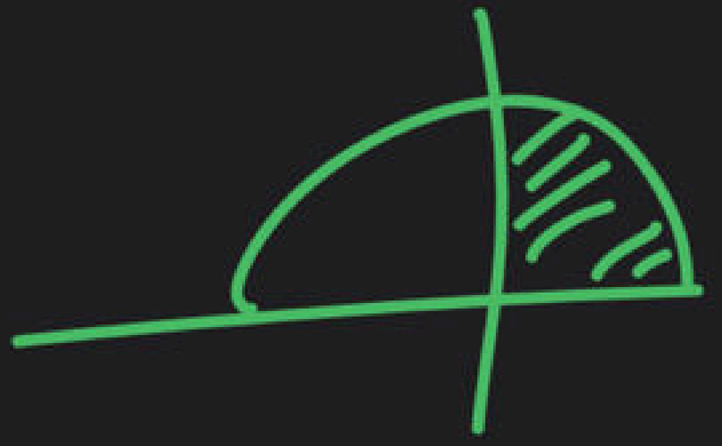
$x^2 + y^2 = a^2$



$\frac{4}{3}\pi a^3$

$$\boxed{\begin{aligned} x &= a\cos\theta \\ y &= a\sin\theta \end{aligned}}$$

$$V = \int \pi y^2 \frac{dx}{d\theta}\, d\theta$$



$$V = 2\pi \int_0^{\pi/2} a^2 \sin^2\theta \left(-a\sin\theta\, d\theta\right)$$

$$V = -2\pi a^3 \int_0^{\pi/2} \sin^3\theta \cos^0\theta\, d\theta = -\cancel{2}\pi a^3 \frac{\Gamma\frac{4}{2}\,\Gamma\frac{1}{2}}{\cancel{2}\,\Gamma\frac{5}{2}}$$

$$= -\pi a^3 \frac{1 \cdot \cancel{\Gamma\frac{1}{2}}}{\frac{3}{2}\cdot\frac{1}{2}\,\cancel{\Gamma\frac{1}{2}}} = -\frac{4\pi a^3}{3}$$

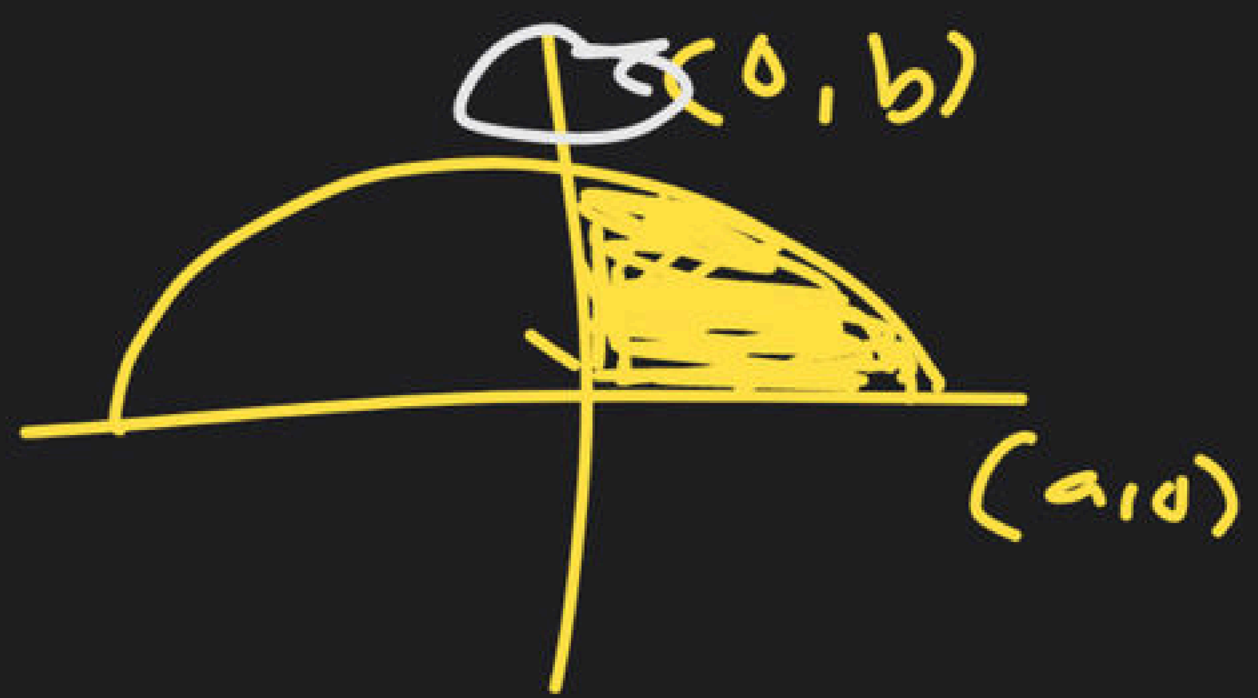
$y = x^2 + 1$, $x = 1$, $x = 3$

$$V = \int_1^3 \pi y^2 \, dx$$

$$V = \pi \int_1^3 (x^2+1)^2 \, dx$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$2\int \pi x^2\,dy$$



$$V = 2\int \pi y^2\,dx$$

$$V = 2\pi\int_0^a b^2\left(1-x^2/a^2\right)dx = 2\pi b^2\left(x-\frac{x^3}{3a^2}\right)_0^a$$

$$= 2\pi b^2\left(a-\frac{a^{\cancel{3}}}{3\cancel{a^2}}\right)$$

$$= 2\pi b^2\left(a-a/3\right)$$

$$= 2\pi b^2\left(\frac{2a}{3}\right) = \frac{4\pi a b^2}{3}$$

find volume of $x = a\cos^3 t$ $y = a\sin^3 t$ (x-axis)

$$V = 2\int_0^{\pi/2} \pi y^2 \frac{dx}{dt}\, dt$$

$$V = 2\pi \int_0^{\pi/2} a^2 \sin^6 t \; 3a\cos^2 t\,(-\sin t\, dt)$$

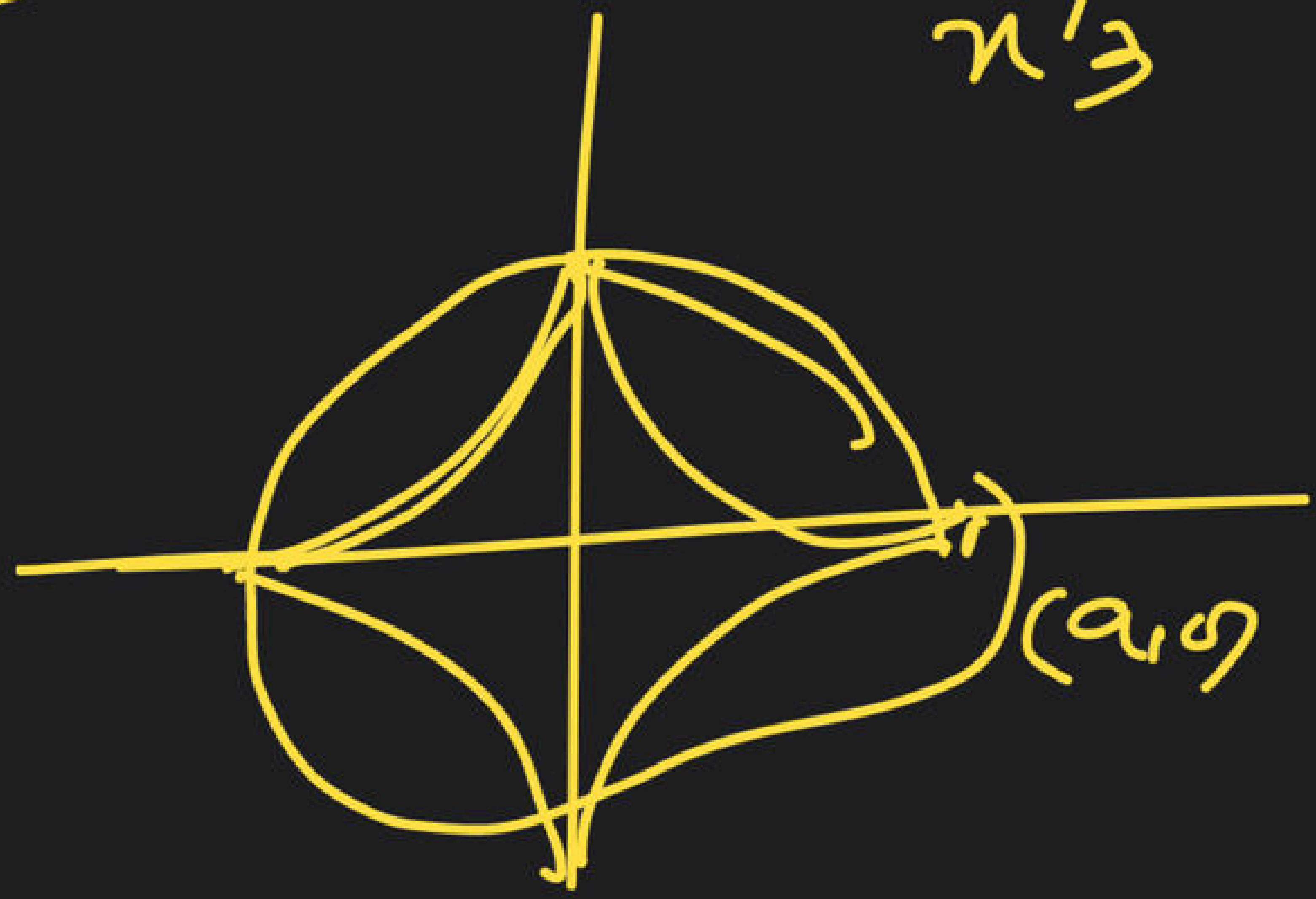
$$V = -6\pi a^3 \int_0^{\pi/2} \sin^7 A \cos^2 t\, dt = (\pi/2)\left(\frac{\Gamma\frac{8}{2}\,\Gamma\frac{3}{2}}{2\,\Gamma\frac{11}{2}}\right)$$

$$= 6\pi a^3 \frac{6}{2} \frac{\frac{1}{2}\Gamma\frac{1}{2}}{\frac{9}{2}\frac{7}{2}\frac{5}{2}\frac{3}{2}\,\frac{1}{2}\Gamma\frac{1}{2}} = \frac{6\pi a^3 \times 2 \times 16}{9 \times 7 \times 5 \times 3} = \frac{32\pi a^3}{105}$$

$x = a\cos^3 t$ $\quad$ $y = a\sin^3 t$

θ	0	30	45	60	90
x					
y					

$x^{2/3}$

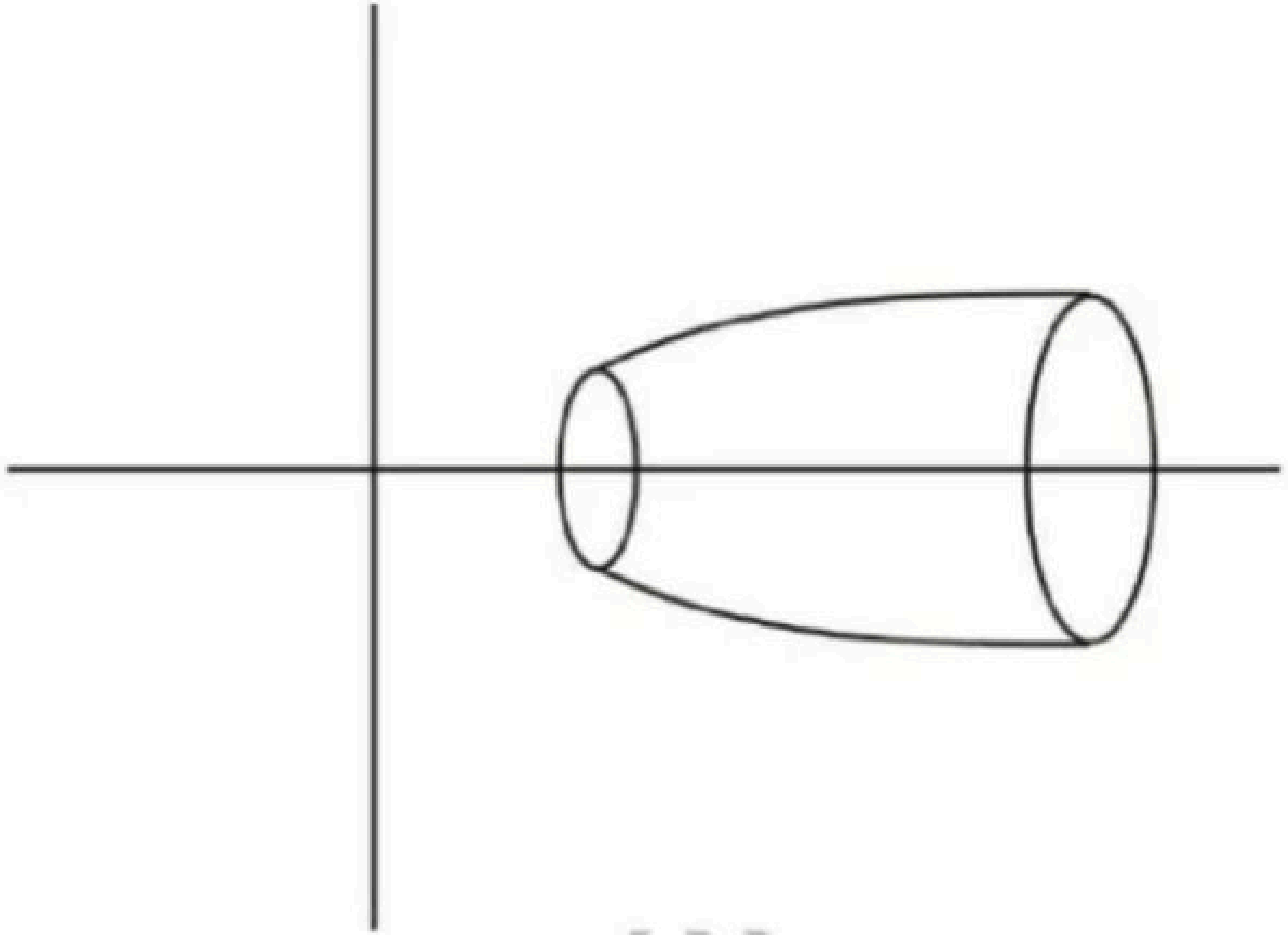


(ii) Revolution of solid about y-axis :

Let $x = f(y)$ be a curve and the part of curve between the ordinate $y = c$ and $y = d$ is rotated about y-axis through 360°.

Volume of solid of revolution by x-axis :

Let y = f(x) be a curve and solid of revolution between x =
to x = b.

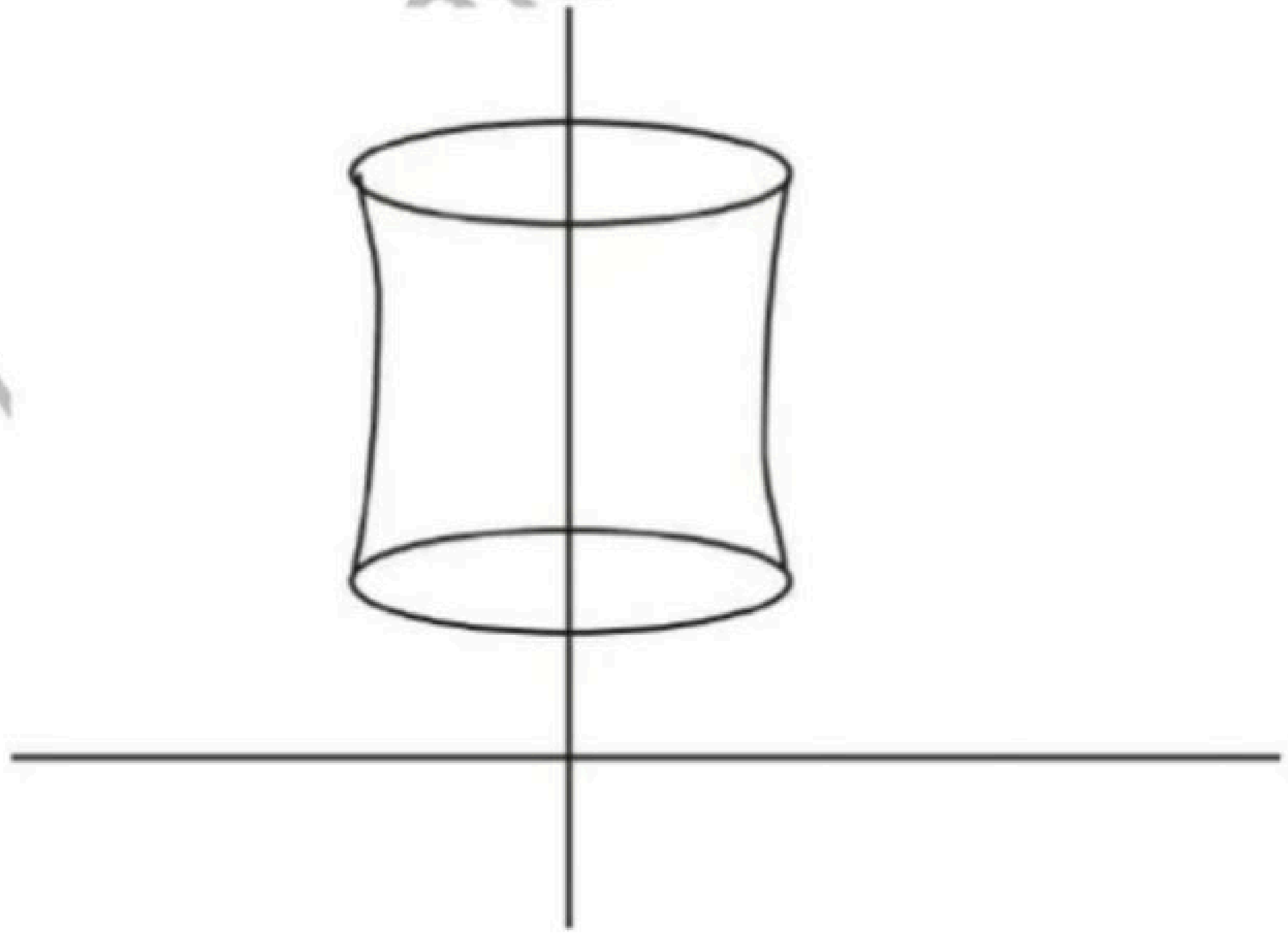


The volume = $\int_a^b \pi y^2 dx$

$= \int_a^b \pi [f(x)]^2 dx$

Volume of solid of revolution about y-axis :

Let x = f(y) be a curve and solid of revolution between y = c to y = d.



The volume = $\pi \int_{c}^{d} x^2 dy$

Volume of revolution when the equation of generating curve are given in parametric form :

(i) Let x = $\phi(t)$, y = $\psi(t)$

Then volume of solid generated by x-axis is $\int_a^b \pi y^2 dx = \pi \int_a^b y^2 \frac{dx}{dt} dt$.

(ii) Let x = $\phi(t)$ and y = $\psi(t)$

Then volume of solid generated by y-axis is $\pi \int_c^d x^2 dy = \pi \int_c^d x^2 \frac{dy}{dt} dt$.

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The axis of rotation being any line :

(i) If this line parallel to x-axis i.e. y = α, then volume is

$$\int_a^b \pi(y-\alpha)^2\,dx.$$

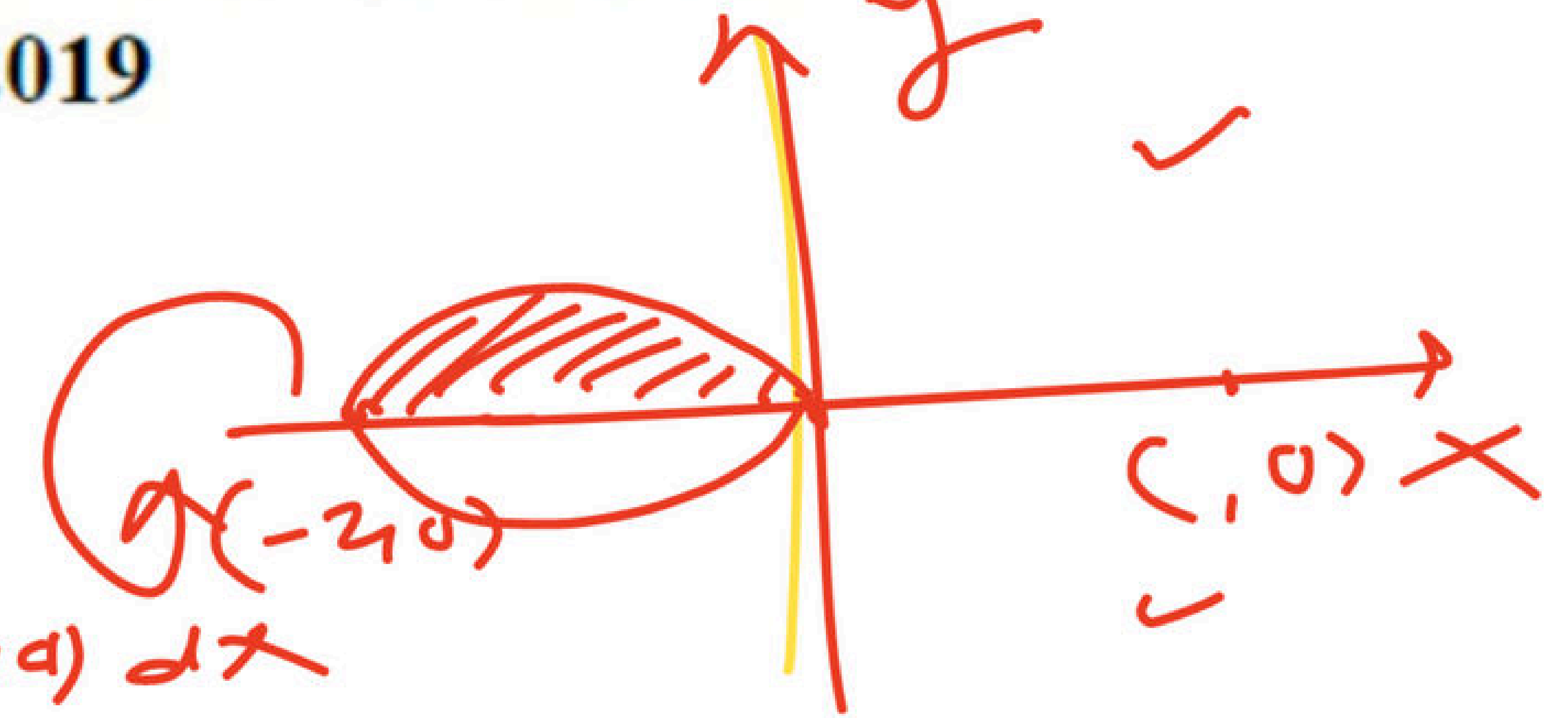
(ii) If the line is x = β, then volume is $\int_a^b \pi(x-\beta)^2\,dy$.

Q.1. The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x + 2)$ about the x- axis (round off to 2 decimal places) is **IIT-JAM 2019**

$\cancel{\text{(a) } 6.69}$ $x = x_1, -2$ (b) 6.75

(c) 6.80 (d) 6.93

$\frac{22}{7} \times \frac{\cancel{64}\ 32}{\cancel{30}\ 15}$



$$V = \int_{x_1}^{x} \pi y^2\, dx = \pi \int_{-2}^{0} (x^5 + 2x^4)\, dx$$

$$= \pi \left[\frac{x^6}{6} + \frac{2x^5}{5}\right]_{-2}^{0} = \pi\left(0 - \frac{2^6}{6} + \frac{2^6}{5}\right)$$

$$= \pi\, 2^6 \left(\frac{1}{5} - \frac{1}{6}\right)$$

$$= \pi \times \frac{64}{30}$$

Q.2 Find the volume of the solid formed by revolving the cycloid about its base.

(a) $3\pi^2a^3$

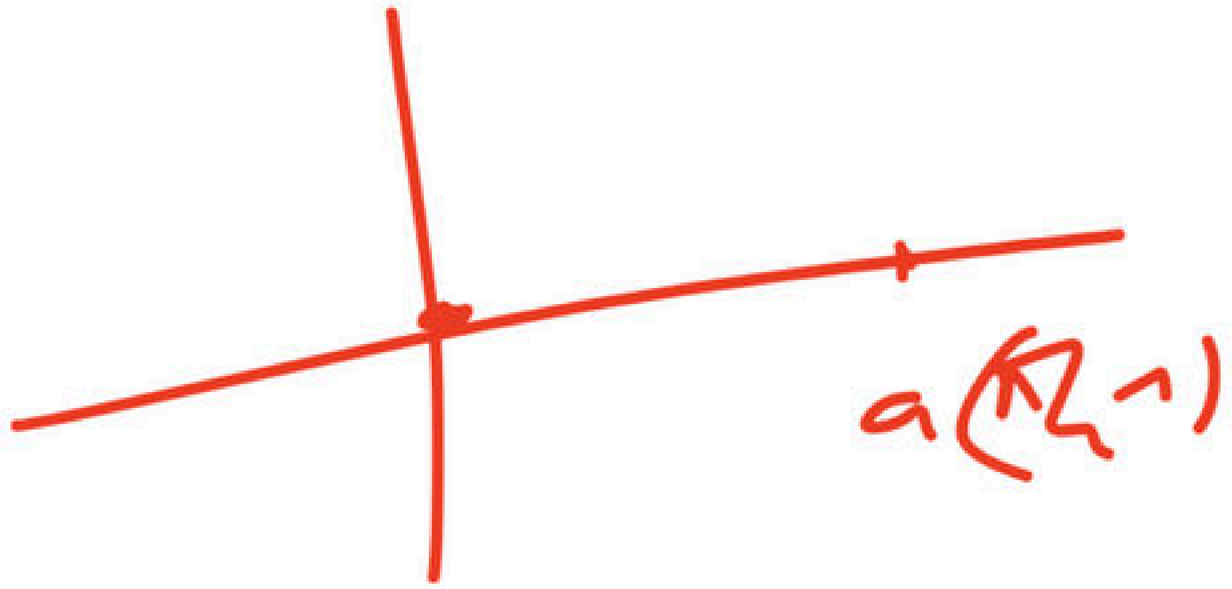
(b) $5\pi^2a^3$

(c) $6\pi^2a^3$

(d) None of these

$\underline{x = a(\theta - \sin\theta)}$

$\underline{y = a(1-\cos\theta)}$



$\pi/2$

$x = a(\pi/2 - 1)$

$\pi\left[-\frac{a^3}{3}+a^3-2a^3+2a^3\log 2a-2a^3\log a\right]$

Q.3 Find the volume formed by the revolutation of the

$a^3\pi\left(-\frac{1}{3}-1+2\log 2\right)$

loop of the curve $y^2(a+x) = x^2(a-x)$ about x-axis.

(a) $\pi a^2\left[2\log 2-\frac{4}{3}\right]$

(b) $\pi a^2\left[2\log 2+\frac{4}{3}\right]$

(c) $\pi a^2\left[2\log 3-\frac{4}{3}\right]$

(d) None of these

$$-x^2+2xa-2a^2$$

$$a+x\,\big)\,-x^3+x^2a$$

$$-x^3-x^2a$$

$$2x^2a$$

$$2x^2a+2xa^2$$

$$-2xa^2$$

$$-2xa^2-2a^3$$

$$2a^3$$

$$V=\pi\int_0^a y^2\,dx=\pi\int_0^a \frac{x^2(a-x)}{a+x}\,dx$$

$$=\pi\int_0^a\left(-x^2+2xa-2a^2+\frac{2a^3}{a+x}\right)dx$$

$$=\pi\left(-\frac{x^3}{3}+x^2a-2a^2x+2a^3\log(a+x)\right)_0^a$$

Q.4. The volume of the solid of revolution of $y = \frac{a}{2}(e^{x/a} + e^{-x/a})$ about x-axis between x = 0 and x = b is **IIT JAM – 2009**

(a) $\frac{\pi a^3}{8}(e^{2b/a} - e^{-2b/a}) - \frac{\pi a^2 b}{2}$

(b) $-\frac{\pi a^3}{8}(e^{2b/a} - e^{-2b/a}) + \frac{\pi a^2 b}{2}$

(c) $-\frac{\pi a^3}{8}(e^{2b/a} - e^{-2b/a}) - \frac{\pi a^2 b}{2}$

(d) $\frac{\pi a^3}{8}(e^{2b/a} - e^{-2b/a}) + \frac{\pi a^2 b}{2}$

$$V = \pi \int_0^b \frac{a^2}{4} (e^{x/a} + e^{-x/a})^2 dx$$

$$V = \frac{\pi a^2}{4} \int_0^b (e^{2x/a} + e^{-2x/a} + 2) dx$$

$$V = \frac{\pi a^2}{4} \left(\frac{e^{2x/a}}{2/a} + \frac{e^{-2x/a}}{-2/a} + 2x \right)_0^b$$

$$V = \frac{\pi a^2}{4} \left(\frac{a}{2} e^{2b/a} - \frac{a}{2} e^{-2b/a} + 2b \right)$$

$$V = \frac{\pi a^2}{4} \left(\frac{a}{2}\right) (e^{2b/a} - e^{-2b/a}) + \frac{\pi a^2 b}{2}$$

$V = \int \pi x^2 \, dy$

$x = 0$

$V = \int \pi y^2 \, dx$

$y = 0$

$V = \int \pi (x-1)^2 \, dy$

$y = 1$

$y - 1 = 0$

$x = -1$

$x = 1$

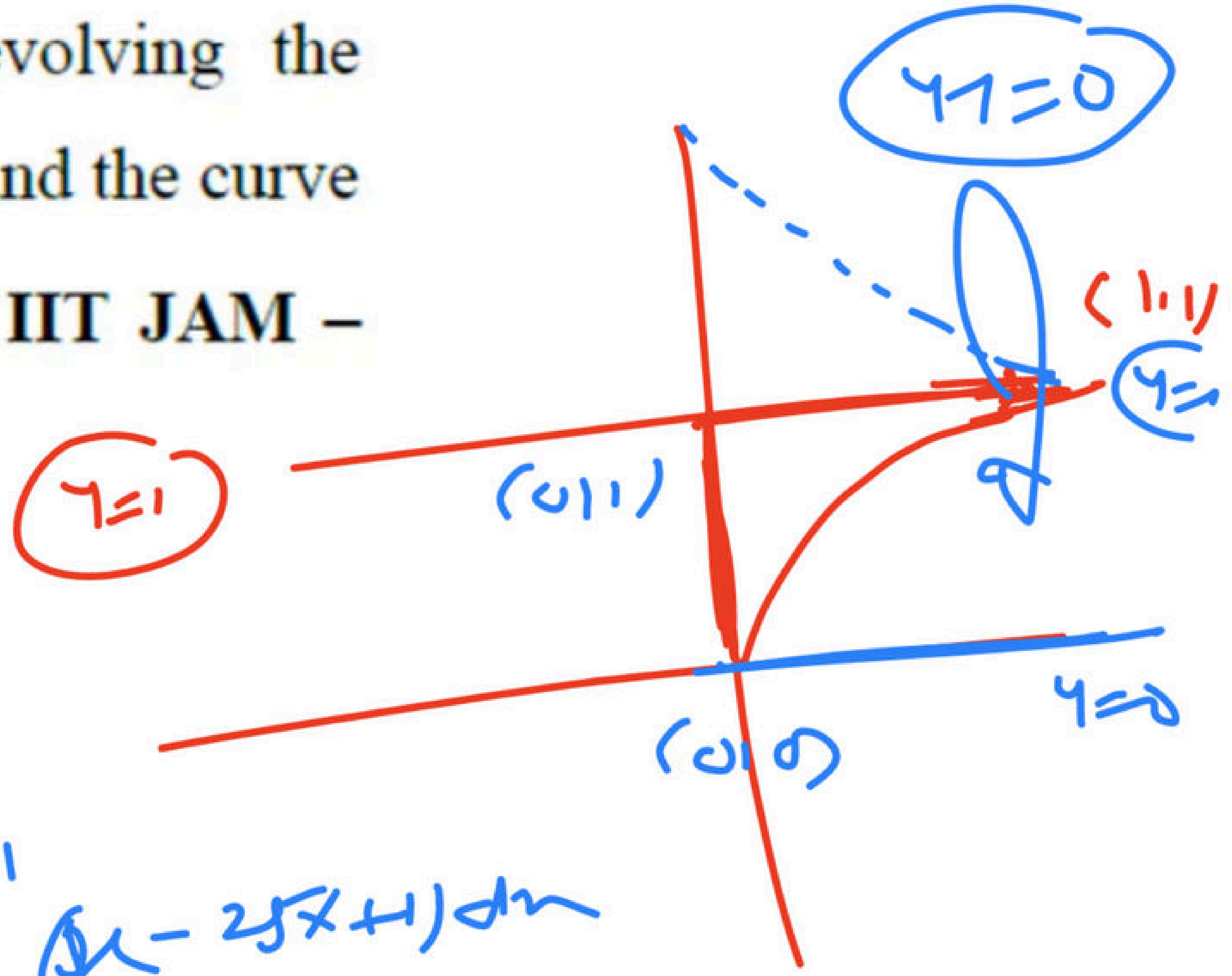
$x - 1 = 0$

$V = \pi \int (y-1)^2 \, dx$

Q.5. Volume of the solid generated by revolving the region bounded by the lines x = 0, y = 1 and the curve $y = \sqrt{x}$ about the line y = 1 is equal to **IIT JAM – 2007**

(a) $\pi/6$ (b) $\pi/2$

(c) $5\pi/2$ (d) $3\pi/2$



$$V = \pi \int_0^1 (y_1)^2 \, dx$$

$$V = \pi \int_0^1 (\sqrt{x} - 1)^2 \, dx = \pi \int_0^1 (x - 2\sqrt{x} + 1) \, dx$$

$$= \pi \left(\frac{x^2}{2} - 2\frac{x^{3/2}}{3/2} + x \right)\Bigg|_0^1 = \pi \left(\frac{1}{2} - \frac{4}{3} + 1 \right) =$$

$$= \pi \left(\frac{3 - 8 + 6}{6} \right) = \pi/6$$

Q. $\frac{d}{dx}\int_0^{2x^2} f(t)\,dt = \frac{d}{dx} e^{\cos x^2}$ $\qquad f'(\pi) = ?$

$$f(2x^2)\,4x = e^{\cos x^2}(-\sin x^2)\,2x$$

$$\frac{d}{dx} f(2x^2) = \frac{d}{dx}\left(-\frac{1}{2}\sin x^2\, e^{\cos x^2}\right)$$

$$4x\, f'(2x^2) = -\frac{1}{2}\left(2x\cos x^2\, e^{\cos x^2} - (\sin x^2)^2 e^{\cos x^2}\, 2x\right)$$

$$f'(2x^2) = -\frac{1}{4}\left(\cos x^2\, e^{\cos x^2} - (\sin x^2)^2 e^{\cos x^2}\right)$$

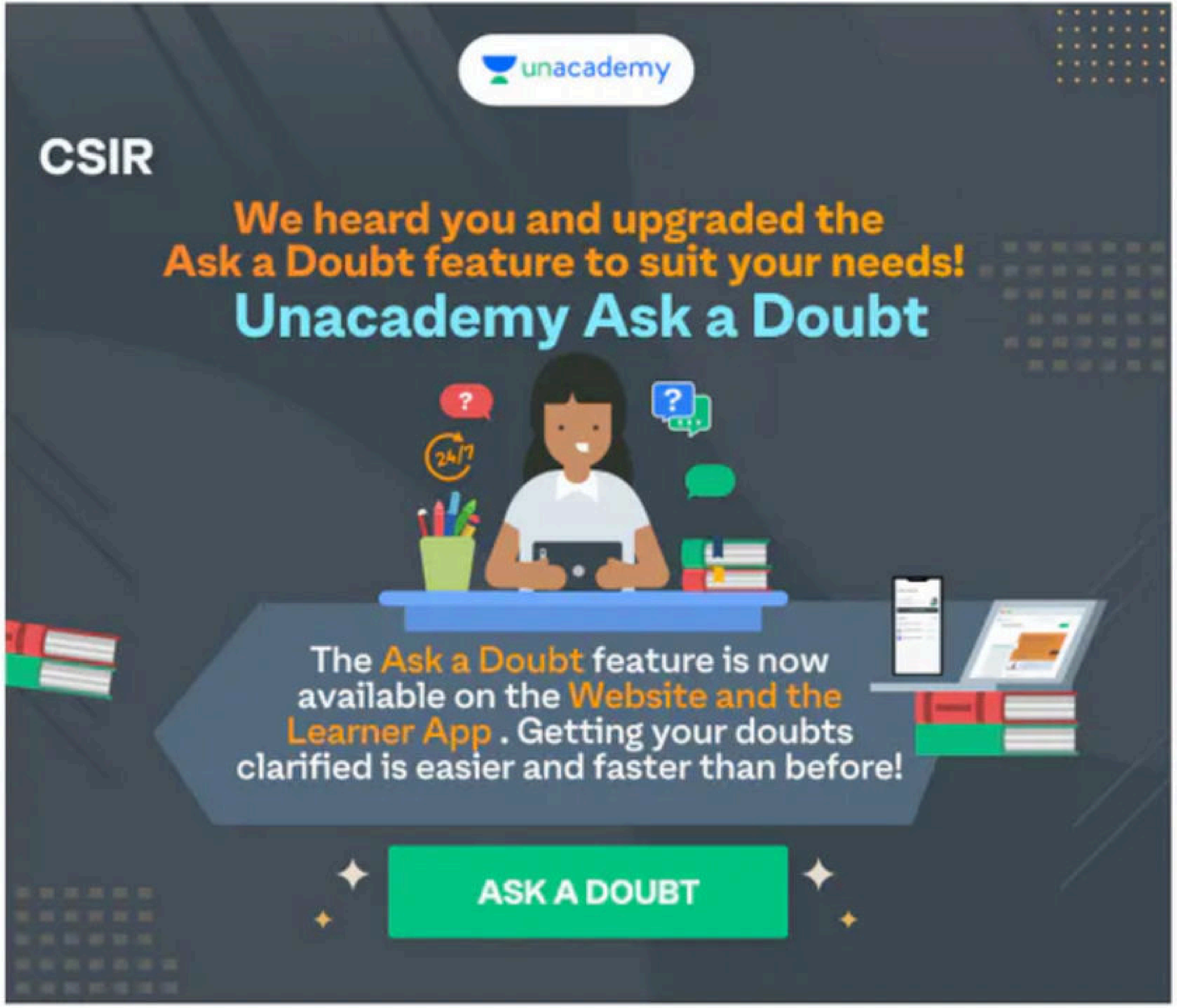
$$f'(\pi) = -\tfrac{1}{4}(0 - 1) = \tfrac{1}{4}$$

1

$f'(\pi)$

$-\frac{2x}{2} \times \frac{1}{4x}$

$2x^2 = \pi$ $\qquad x^2 = \pi/2$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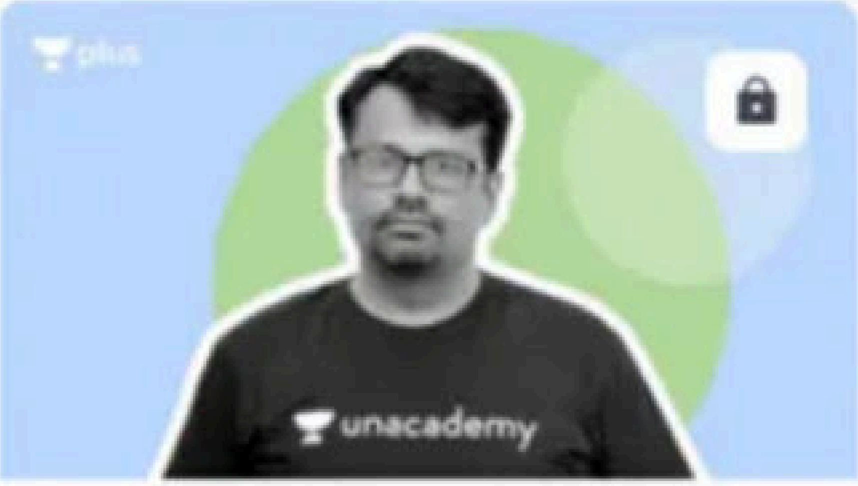

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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