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**Automorphism** : Let  $G$  be a group, then the mapping  $f : G \rightarrow G$  is called automorphism if

- (i)  $f$  is one-one
- (ii)  $f$  is onto
- (iii)  $f$  is homomorphism

i.e. A mapping  $f : G \rightarrow G$  is called automorphism if it is isomorphism

**Note :** Let  $Z$  = group of integer under addition then  $f : Z \rightarrow Z$  s.t.  
 $f(x) = mx; m \neq \{1, -1\}$

Then it will not be onto mapping.

So, it will not be automorphism.

$\Rightarrow$   $Z$  have only two automorphism.

## **Automorphism Group :**

Let  $G$  be a group, then the set of all automorphism of  $G$  form a group under the composition of mapping and this is denoted by  $\text{Aut } G$ .

- Q1.** For any group  $G$ ,  $\text{Aut}(G)$  denote the group of automorphism of  $G$ . Which of the following are true?
- (a) If  $G$  is finite, then  $\text{Aut}(G)$  is finite
  - (b) If  $G$  is cyclic, then  $\text{Aut}(G)$  is cyclic
  - (c) If  $G$  is infinite, then  $\text{Aut}(G)$  is infinite
  - (d) If  $\text{Aut}(G)$  is isomorphic to  $\text{Aut}(H)$  then  $G$  is isomorphic to  $H$ .

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**Q2.** Let  $G = \mathbb{Z}_3 \times \mathbb{Z}_3$  be a group then order of  $\text{Aut}(\mathbb{Z}_3 \times \mathbb{Z}_3)$  is

- (a) 48
- (b) 168
- (c) 50
- (d) 150

**Q3.** The order of  $\text{Aut}(\text{Aut}(\text{Aut}(K_4)))$  is

(a) 4

(b) 5

(c) 6

(d) 8

# **Group Homomorphism**

**Counting of homomorphism :**

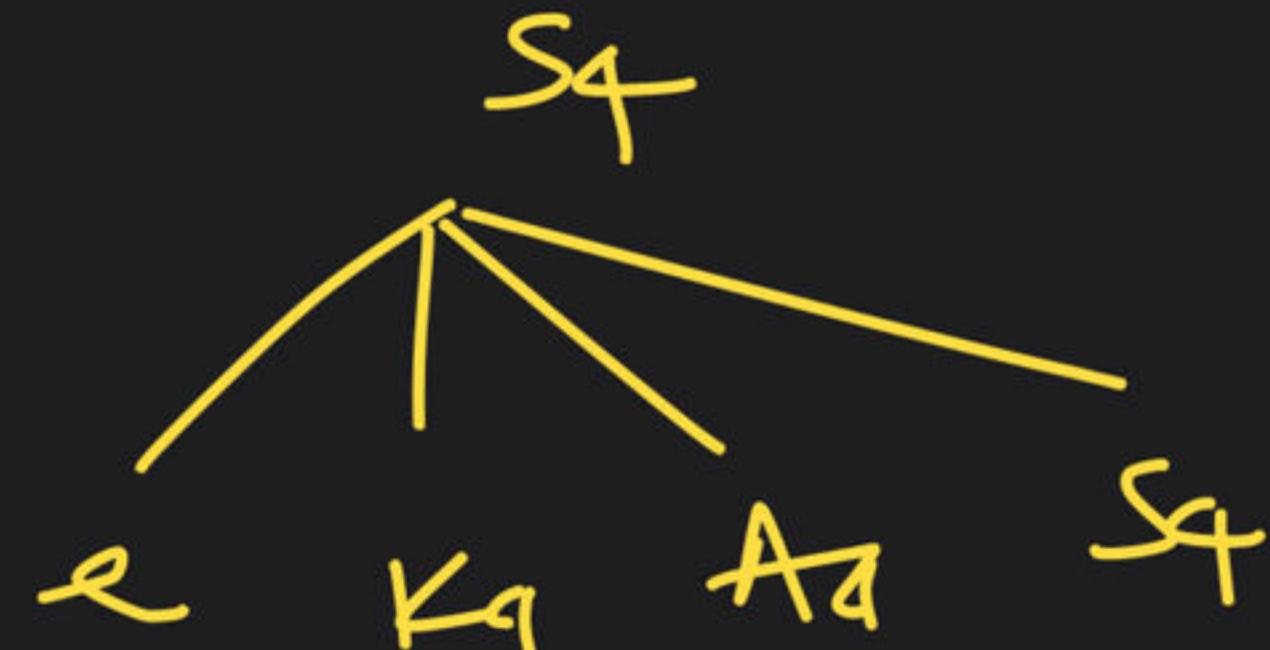
$$f : S_4 \longrightarrow K_4$$

(i)  $\frac{S_4}{K_4} \cong S_4 \times K_4$

(ii)  $\frac{S_4}{K_4} \cong S_3 \times K_4$

(iii)  $\frac{S_4}{A_4} \cong Z_2 \times K_4$

(iv)  $\frac{S_4}{A_4} \cong Z_1 \times K_4$   $n_1 n_2 n_3 = 1 \times 2 \times 1 \Rightarrow$   
 $n_1 = \sigma(\text{Aut } Z_1) = 1, \quad n_2 = 1, \quad n_3 = 1, \quad \epsilon = 1$



$$\text{Hulls} = 4$$

$$\text{one-one} = 0$$

$$\text{onto} = 6$$

$$n_2 = 2$$

$$n_3 = 1$$

$$\epsilon = 1$$

$$f: S_3 \rightarrow \mathbb{C}^{\times}$$

$$\text{(I)} \frac{S_3}{\langle \zeta_1 \rangle} \cong S_3 \neq \mathbb{C}^{\times}$$

$$\text{(II)} \frac{S_3}{A_3} = \mathbb{Z} \subset \mathbb{C}^{\times}$$

$$o(\text{Aut}(\mathbb{Z})) = 1$$

---

$$\text{(III)} \frac{S_3}{\langle \zeta_3 \rangle} \cong \mathbb{Z} \subset \mathbb{C}^{\times}$$

$$o(\text{Aut}(\mathbb{Z})) = \infty$$

$$\begin{array}{c} S_3 \\ \downarrow \\ A_3 \end{array}$$

$$f: S_3 \rightarrow \mathbb{C}^{\times}$$

$$\text{(I)} \frac{S_3}{\langle e \rangle} \cong S_3 \neq \mathbb{C}^{\times}$$

$$\text{(II)} \frac{S_3}{A_3} \cong \mathbb{Z} \neq \mathbb{C}^{\times}$$

$$\text{(III)} \frac{S_3}{\langle \zeta_3 \rangle} \cong \mathbb{Z} \subset \mathbb{C}^{\times}$$

$$o(\text{Aut}(\mathbb{Z})) = \infty$$

Homs from  $f: S_m \rightarrow \mathbb{Z}_m$

$$= \begin{cases} 2 & \\ 1 & \end{cases}$$

$m$  even

$m$  odd

one-one = 0

$$\text{onto } \underline{\text{Homs}} = \begin{cases} 1 & \\ 0 & \end{cases}$$

$m = 1, 2$

$m > 2$

## **Procedure for Counting of homomorphism :**

Let  $f : G \rightarrow G'$  be a homomorphism

- (i) Find Normal subgroup of  $G$  namely  $H_1, H_2$ , etc.
- (ii) Using FTH

$\frac{G}{H_i} = f(G)$  which is subgroup of  $G'$

- (iii) Find  $\text{Aut}(f(G))$
- (iv) During all the above steps, we have collected three numbers,  $n_1$  the number of normal subgroup ( $H_i$ ) in  $G$  ,  $n_2$  number of subgroup( $f(G)$ ) in  $G'$  ,  $n_3$  order of  $\text{Aut}(f(G))$  then product of  $n_1 n_2 n_3$
- (v) Do each step for other  $H_i$  and sum all of them

**(1) Counting of group homomorphism from finite cyclic group to finite cyclic group.**

**Result :** Let  $f : Z_m \rightarrow Z_n$  be a mapping then number of homomorphism are  $\gcd(m, n)$

**2. Number of one-one homomorphism from finite cyclic group to finite cyclic group.**

**Result :** Let  $f : Z_m \rightarrow Z_n$  be a homomorphism then number of one-one

homomorphism are  $\begin{cases} \phi(m) & \text{if } m | n \\ 0 & \text{if } m \nmid n \end{cases}$ .

**(3) Number of onto homomorphism from  $Z_m$  to  $Z_n$ .**

**Result :** Let  $f : Z_m \rightarrow Z_n$  be a homomorphism then number of onto

homomorphism are  $\begin{cases} \phi(n) & \text{if } n | m \\ 0 & \text{if } n \nmid m \end{cases}$ .

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- (4) Counting of group homomorphism from  $S_4$  to  $K_4$ .
- (5) Counting of one – one group homomorphism from  $S_4$  to  $K_4$ .
- (6) Counting of onto group homomorphism from  $S_4$  to  $K_4$ .

## **Homomorphism**

1. Counting of homomorphism from  $S_n \rightarrow Z_m$ , ( $n \geq 3$ )

1. Number of homomorphism  $\begin{cases} 1, & m \text{ is odd} \\ 2, & m \text{ is even} \end{cases}$

2. Number of onto homomorphism  $\begin{cases} 1, & m = 1, 2 \\ 0, & \text{otherwise} \end{cases}$ .

3. Number of one-one homomorphism does not exist.

2. Counting of homomorphism from  $A_4 \rightarrow Z_m$

1. Number of homomorphism are  $\begin{cases} 3 & \text{if } 3 \mid m \\ 1 & \text{if } 3 \nmid m \end{cases}$

$$f: A_4 \longrightarrow Z_3$$

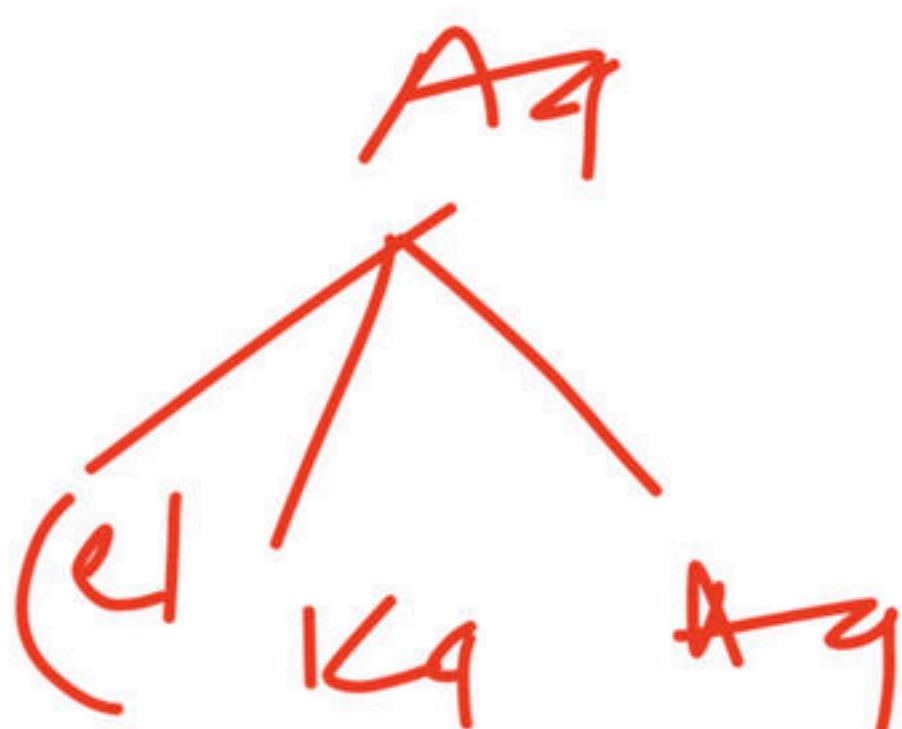
$$\cup \quad \frac{A_4}{E_1} = A_4 / Z_4$$

$$\begin{array}{c} m=1 \\ m=3 \\ \text{otherwise} \end{array} \quad \frac{A_4}{K_4} = Z_3 \subset Z_3$$

~~$\cup (Aut(Z_3)) = \{1\}$~~

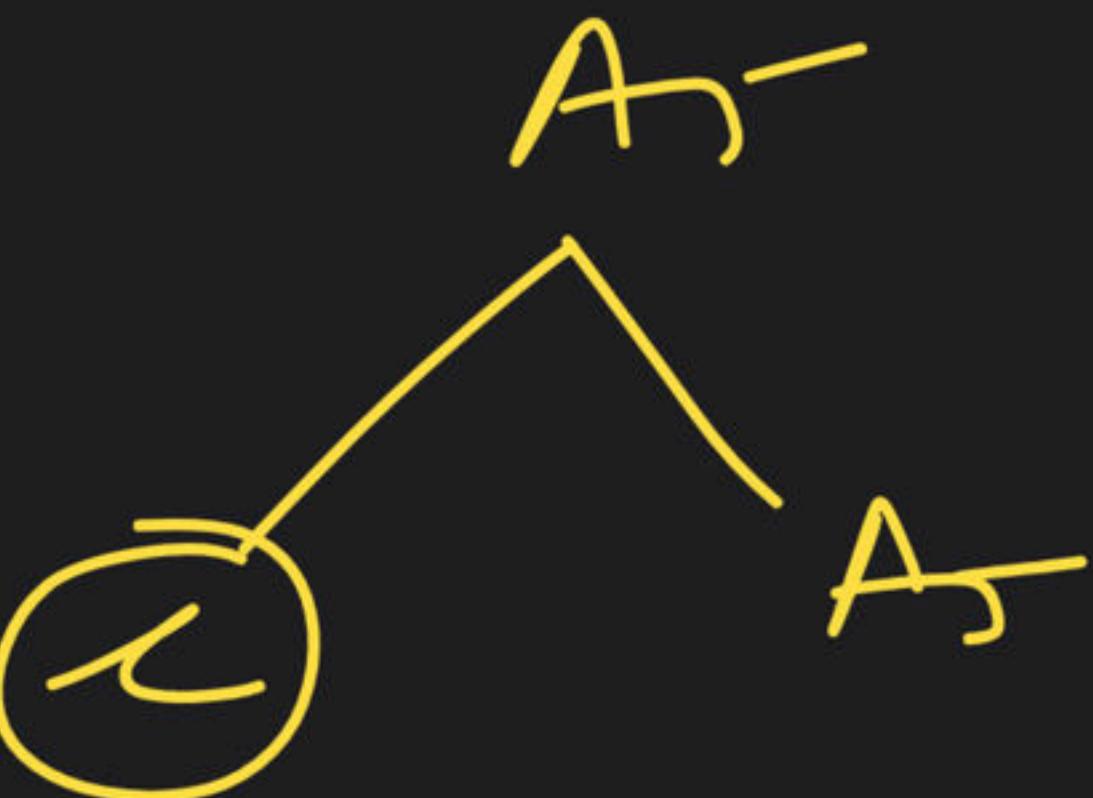
$$(11) \quad \frac{A_4}{A_4} = 3 \subset Z_3$$

~~$\cup (Aut(Z_3)) / E_1$~~



$f: A^{\perp} \rightarrow \mathcal{Z}_S$

(i)  $\frac{A^{\perp}}{\omega} \leq A^{\perp} \times \mathcal{Z}_S$



(ii)  $\frac{A^{\perp}}{A} = G \times \mathcal{Z}_S$

3. Counting of homomorphism from  $A_n \rightarrow Z_m$ , ( $n \geq 5$ )

1. Number of homomorphism only trivial.

2. Number of onto homomorphisms are  $\begin{cases} 1 & \text{if } m = 1 \\ 0 & \text{if } \text{otherwise} \end{cases}$

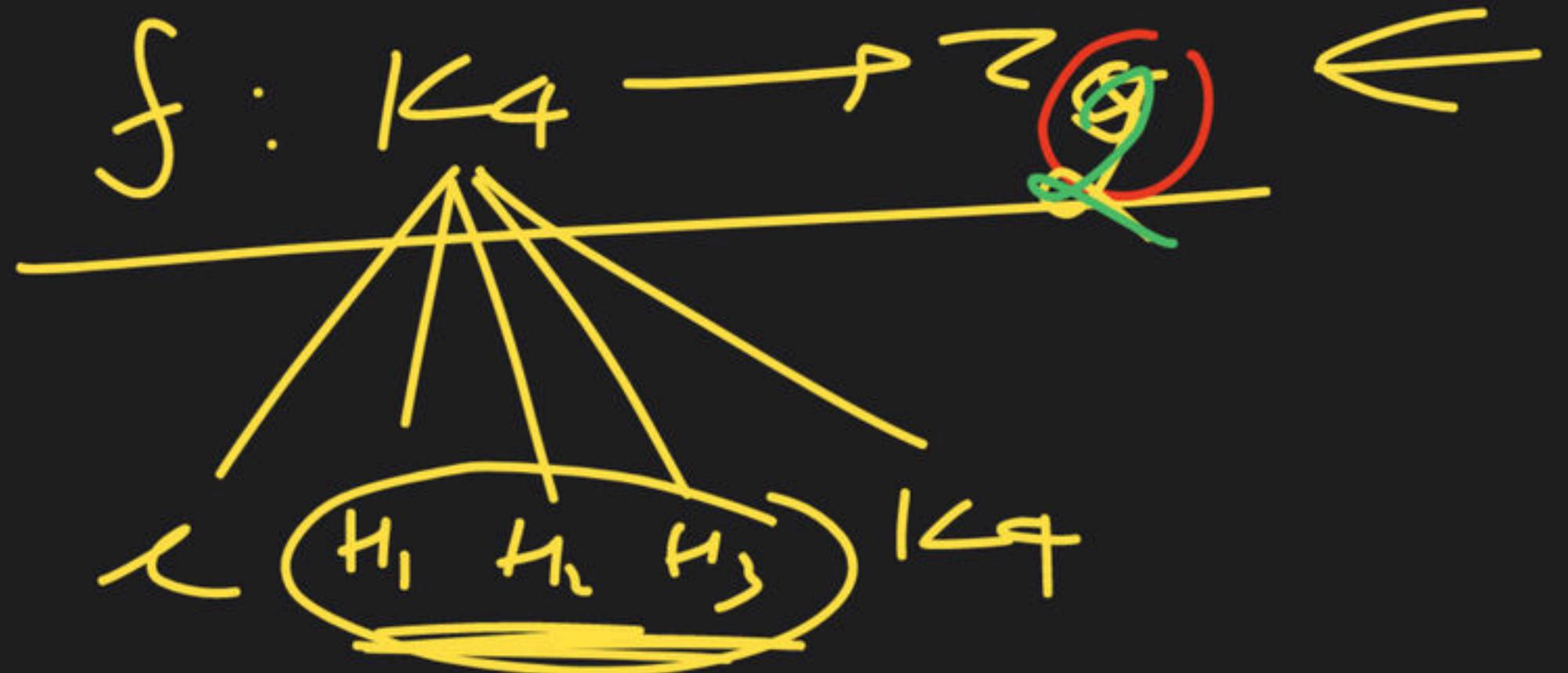
3. Number of one-one homomorphism does not exist.

4. Counting of homomorphism from  $K_4 \rightarrow Z_m$

1. Number of homomorphism are  $\begin{cases} 4, & m \text{ is even} \\ 1, & m \text{ is odd} \end{cases}$

2. Number of onto homomorphisms are  $\begin{cases} 1, & m = 1 \\ 3, & m = 2 \\ 0, & \text{otherwise} \end{cases}$

3. Number of one-one homomorphism does not exists.



$$\text{(I)} \quad \frac{K_4}{(e)} = K_4 \not\subset \mathbb{Z}_2$$

$$\text{(II)} \quad \frac{K_4}{H_1} \equiv \mathbb{Z}_2 \subset \mathbb{Z}_2$$

$$\text{(III)} \quad \frac{K_4}{K_4} \equiv \mathbb{Z}_2 \subset \mathbb{Z}_2$$

$$o(\text{Aut}(K_4)) = 1$$


---

$f: K_4 \rightarrow \mathbb{Z}_2$

$$\frac{K_4}{(e)} = K_4 \not\subset \mathbb{Z}_2$$

$$\frac{K_4}{H_1} = \mathbb{Z}_2 \subset \mathbb{Z}_2$$

$$\frac{K_4}{K_4} = \mathbb{Z}_2 \subset \mathbb{Z}_2$$

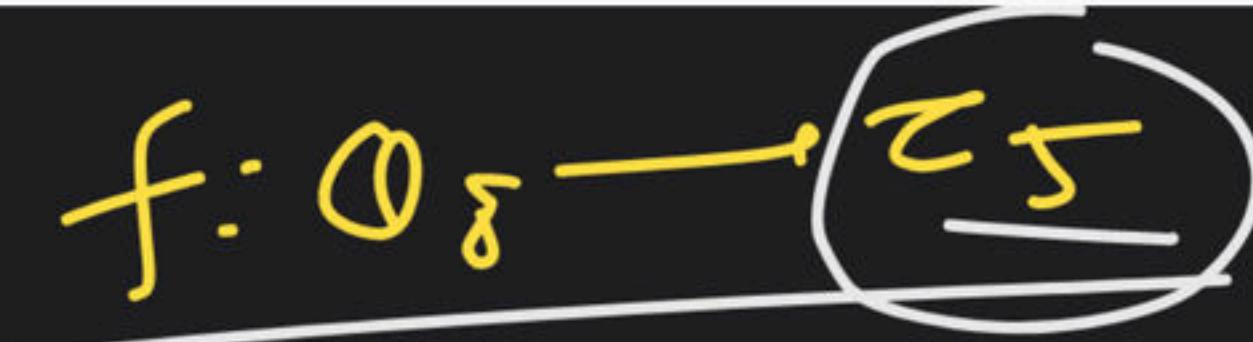
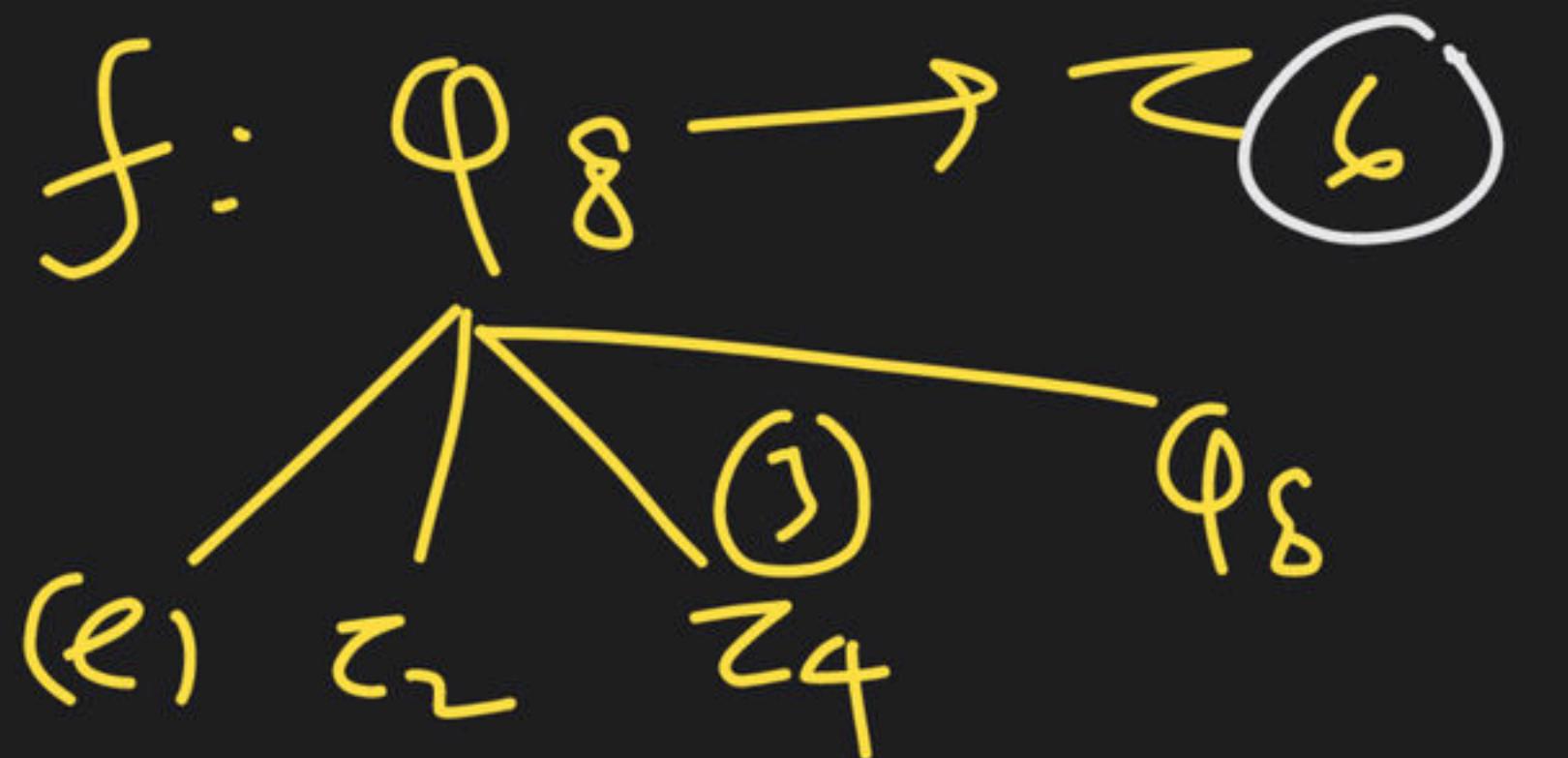
$$n_1=1, n_2=1, n_3=3$$

$$m_1=m_2=3$$

$$o(\text{Aut}(K_4)) = 1$$

$\text{(I)} \quad \frac{K_4}{K_4} = \mathbb{Z}_2 \subset \mathbb{Z}_2$

$$o(\text{Aut}(K_4)) = 1$$



(I)  $\frac{\varphi_8}{z_1} \leq \varphi_8 + \overline{z_6}$

(II)  $\frac{\varphi_8}{z_2} \leq \varphi_8 + \overline{z_6}$

(III)  $\frac{\varphi_8}{z_3} \leq \varphi_8 + \overline{z_6}$

$h_1 h_2 h_3 = 3$

(IV)  $\frac{\varphi_8}{\varphi_8} = \varphi_8 + \overline{z_6} \quad o(\text{Aut}(\varphi_8)) = 1$

$o(\text{Aut}(z_6)) = 1 = h_3$

$h_1 = 1, h_2 = 3$

5. Counting of homomorphism from  $Q_8 \rightarrow Z_m$

1. Number of homomorphisms are

$$\begin{cases} 4, & m \text{ is even} \\ 1, & m \text{ is odd} \end{cases}$$

2. Number of onto homomorphisms are

$$\begin{cases} 1, & m = 1 \\ 3, & m = 2 \\ 0, & \text{otherwise} \end{cases}$$

3. Number of one-one homomorphism does not exists.

6. Counting of homomorphism from  $S_n \rightarrow K_4$ , ( $n \geq 3$ )

1. Number of homomorphism are 4.

2. Number of onto homomorphism does not exist.

3. Number of one-one homomorphism does not exist.

7. Counting of homomorphism from  $S_n \rightarrow Q_8$ ,  $n \geq 3$ .

1. Number of homomorphism are 2.
2. Number of onto homomorphism does not exist.
3. Number of one-one homomorphism does not exist

**Q.1.** The number of group homomorphism from the cyclic group  $Z_4$  to the cyclic group  $Z_7$  is

- (a) homomorphism
- (b) isomorphism
- (c) unique homomorphism
- (d) None of these

**Q.2.** The number of homomorphism from  $Z_4$  to  $Z_{12}$  is

- (a) 4
- (b) 3
- (c) 48
- (d) 12

Q.3. Let  $G$  be the cyclic group of order 8 and  $H = S_3$  be the permutation group of 3 elements.

$\zeta_1, \zeta_2, \zeta_4, \zeta_8$

$$\frac{\zeta_8}{\zeta_4} = \zeta_8 \not\models$$

Which of the following statements are necessarily true?

$$f: \mathbb{Z}_8 \rightarrow S_3$$

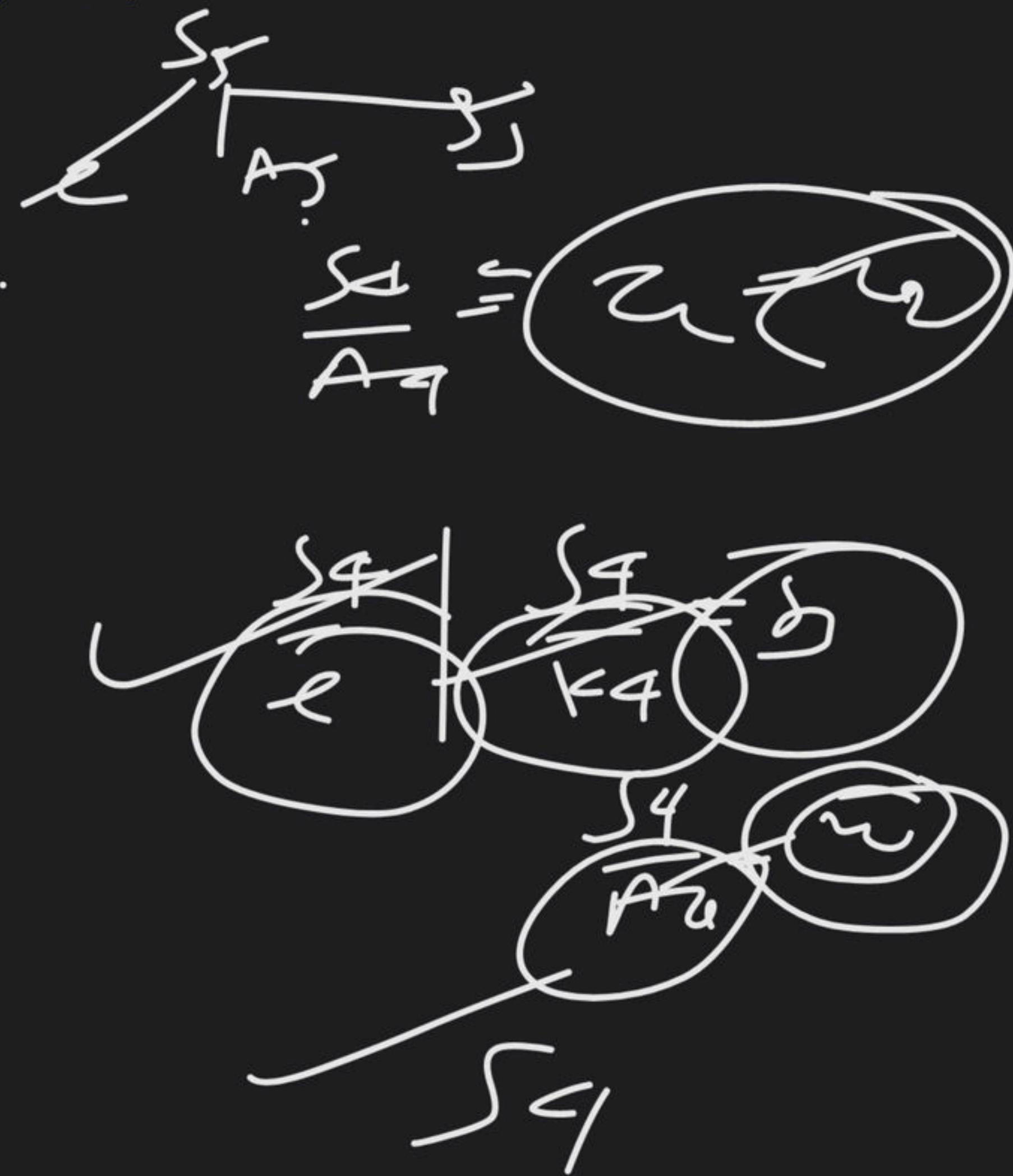
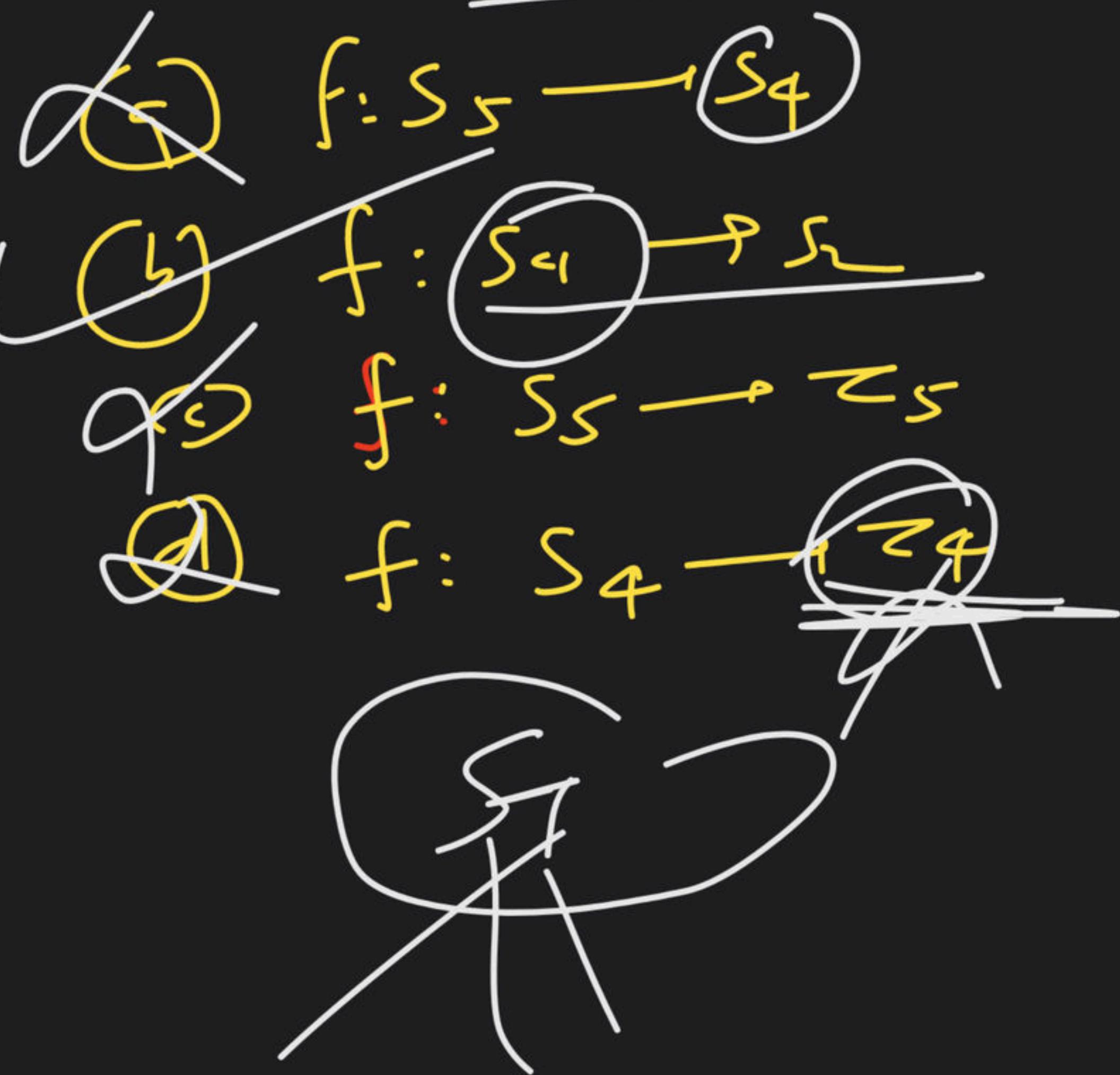
$$\frac{\zeta_8}{\zeta_4} = \zeta_2 = (12)$$

(3)

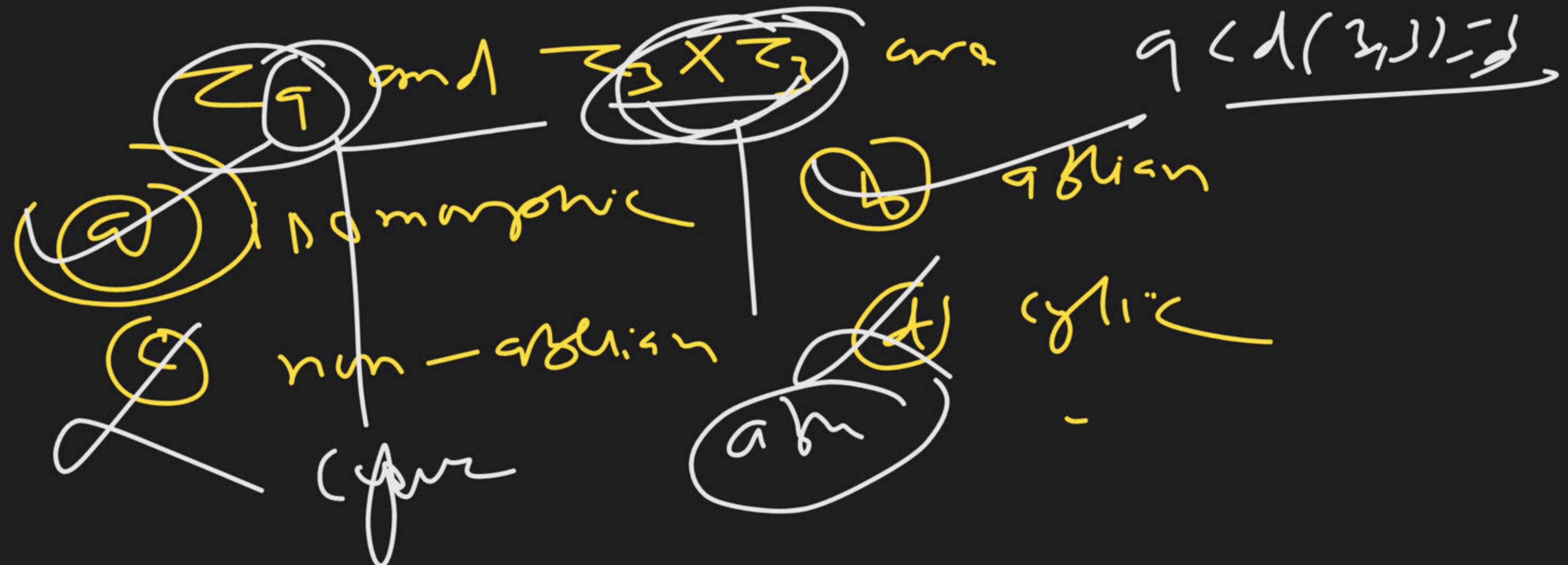
$$\frac{\zeta_8}{\zeta_4} = \zeta_2 = (12)$$

- (a) There exists no non-trivial group homomorphism from  $G$  to  $H$
- (b) There exists no injective group homomorphism from  $G$  to  $H$
- (c) There exists no surjective group homomorphism from  $G$  to  $H$
- (d) There are more than 20 different group homomorphism from  $G$  to  $H$

There exist onto  $\lambda$ -maps.



$\alpha =$



$$H = \langle z_2 \times z_6 \rangle$$

$$K = \langle z_3 \times z_4 \rangle$$

$$L = \langle z_1 \times z_2 \rangle$$

(a)  $H \cong K$  : both are cyclic

(b)  $H \cong K$  b.c. 2 divisors of 8 &  $\gcd(3, 4) = 1$

(c)  $H \not\cong K$  :  $K$  is cyclic but  $H$  is not

(d)  $H \not\cong K$  - : There is no homo from  $H$  to  $K$

$$\langle z_4 \rangle \neq \langle z_4 \rangle$$

$$f: \mathbb{Z}_m \times \mathbb{Z}_n \longrightarrow \mathbb{Z}_d$$

$\gcd(m, d) \times \gcd(n, d)$

$$f: \mathbb{Z}_2 \times \mathbb{Z}_4 \longrightarrow \mathbb{Z}_{10}$$

$\gcd(2, 10) \neq \gcd(4, 10)$

2x2

†

$$f: \mathbb{Z}_m \times \mathbb{Z}_n \longrightarrow \mathbb{Z}_p \times \mathbb{Z}_q$$

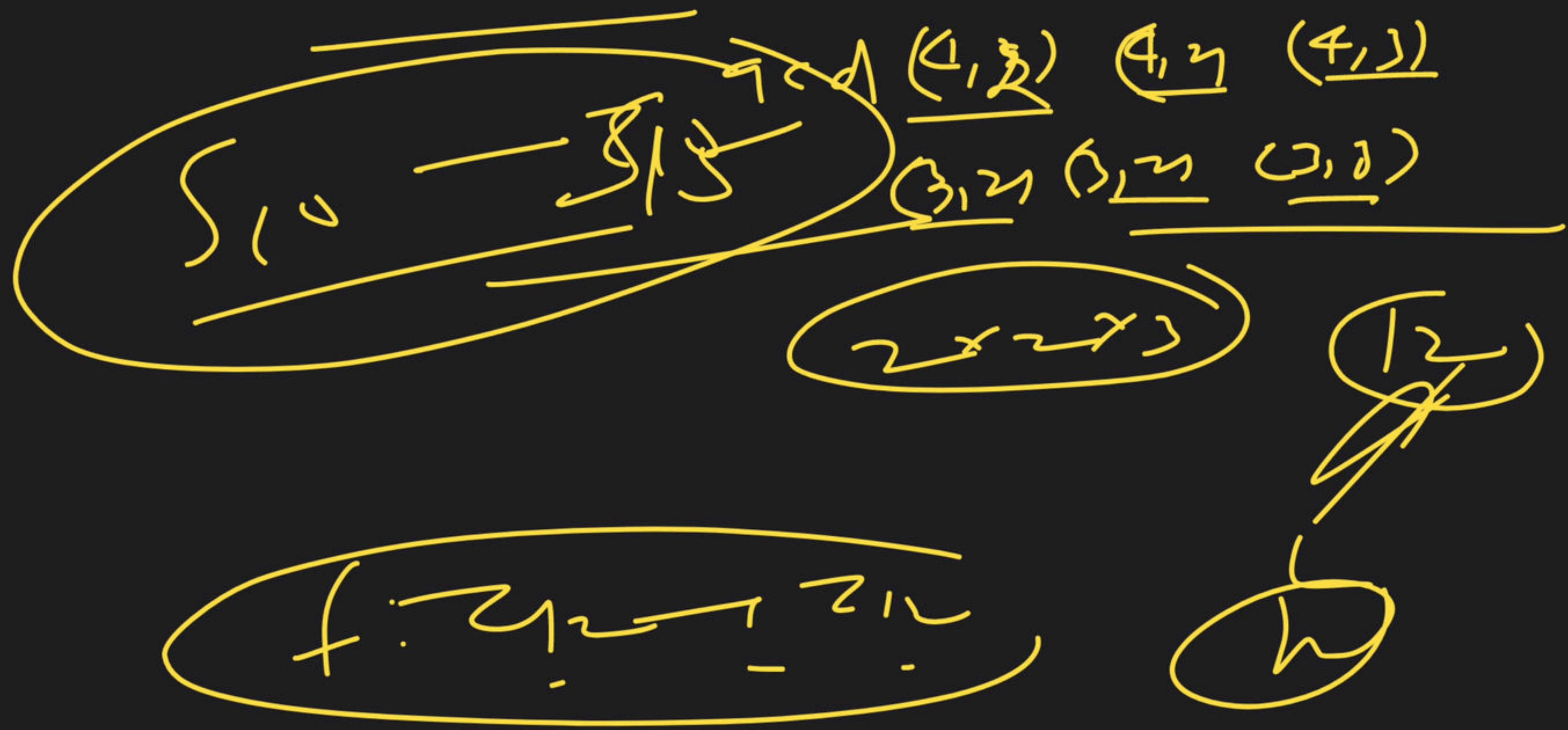
gcd(m, n) must divide  
gcd(n, r) q < q (n, r)

$$f: \mathbb{Z}_2 \times \mathbb{Z}_4 \longrightarrow \mathbb{Z}_3 \times \mathbb{Z}_6$$

gcd(2, 3) | (2, 4) gcd(4, 6) | 4 (4, 6)

1 × 2 × 1 × 2 = 4

$$f: \mathbb{Z}_4 \times \mathbb{Z} \longrightarrow \overline{\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_3}$$



**Q.4. The number of homomorphism from  $S_5$  to  $Z_5$**

- (a) 1
- (b) 2
- (c) 3
- (d) 4

**Q.5.** The number of onto homomorphism from  $Z_8$  to  $Z_4$

- (a) 1
  - (b) 2
  - (c) 3
  - (d) 4

**Q.6.** The number of onto homomorphism from  $Q_8$  to  $Z_2$

- (a) 1
  - (b) 2
  - (c) 3
  - (d) 4

**Q.7.** The number of onto homomorphism from  $Z_{200}$  to  $Z_{100}$

- (a) 46
- (b) 42
- (c) 40
- (d) 38

**Q.8.** The number of group homomorphisms from  $\mathbb{Z}_{10}$  to  $\mathbb{Z}_{20}$  is

- (a) zero
- (b) one
- (c) five
- (d) ten



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### Educator highlights

- 📍 Works at Pacific Science College
- 📍 Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
- 📍 PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
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