

First Order & First Degree ODE - V

Detailed Course on Differential Equation for IIT JAM' 23 - II



Gajendra Purohit

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HOMOGENEOUS DIFFERENTIAL EQUATION

Homogeneous Function : A function $f(x, y)$ is called homogeneous function of degree ' n ' if $f(\lambda x, \lambda y) = \lambda^n f(x, y)$; for all $x, y; \lambda \geq 0$

Example : $f(x, y) = 2x^3 - 3xy^2 + 4y^3$

Solution : $f(\lambda x + \lambda y) = 2\lambda^3 x^3 - 3\lambda^3 xy^2 + 4\lambda^3 y^3$

$$= \underline{\lambda^3} \underline{f(x, y)}$$

This function is homogeneous of degree 3.

Note : A differential equation of first order and first degree is said to be homogeneous if it can be put in the form

$$\frac{dy}{dx} = \phi\left(\frac{y}{x}\right) = \phi\left(\frac{x}{y}\right).$$

$$\begin{aligned}
 f(x, y) &= \frac{y^2 + y^2}{wy + y} \\
 f(x, y) &= \frac{y^2(1 + (\frac{y}{x})^2)}{y^2(\frac{y}{x}) + 1} \\
 &= \underline{y^2} \underline{f\left(\frac{y}{x}\right)}
 \end{aligned}$$

$$f(x,y) = \frac{x^2 + y^2}{x^2 + y^2}$$

$$f(x,y) = \frac{x^2(1+\frac{y^2}{x^2})}{x^2(1+\frac{y^2}{x^2})} = \textcircled{Pf(\frac{y}{x})}$$

$$f'(x,y) = e^{\frac{2x+y}{3x-y}} = e^{\frac{2x+y/x}{3x-y/x}} = \textcircled{f'(\frac{y}{x})}$$

Note :

1. Every homogeneous first order DE is reducible into separable variable.
2. A function $f(x, y)$ is homogeneous of degree n then

$$\underline{x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = nf}$$

$$f = \frac{\sqrt{x^2 + y^2}}{x^2 + y^2}$$

$$\underline{\underline{x^2 + y^2 - nf = 0}}$$

$$f = \frac{z^2 + j}{z + j}$$

$$\frac{z \frac{d}{dz} + j \frac{d}{dj}}{z + j} = 1 + f$$

$$(y^3 + 3xy^2)dx + (x^3 + 3x^2y)dy = 0$$

$$\frac{dy}{dx} = - \frac{(y^3 + 3xy^2)}{(x^3 + 3x^2y)} \quad |$$

$$\frac{dy}{dx} = \frac{\cancel{x}(1 + 2\sqrt{\frac{y}{x}})^2}{\cancel{x}^2((\frac{y}{x})^3 + 2\sqrt{\frac{y}{x}})} = y^2f(\frac{y}{x})$$

Working Rule for solving homogeneous equation :

Let $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ is a homogeneous equation, then

Step - 1 : Put $y/x = v \Rightarrow y = vx$

Step - 2 : $\frac{dy}{dx} = v + x \frac{dv}{dx}$

Step - 3 : Put both value in DE

Step - 4 : Using separation of variable we get required
solution.

$$(y^3 + 3xy^2)dx + (x^2 + 2y^2)dy = 0 \quad \text{div } y - \cancel{vx}$$

$$\frac{dy}{dx} = -\frac{(x^2 + 2y^2)}{y^3 + 3xy^2} \quad 1 + 6v^2 + v^4 = k \quad \frac{dv}{dx} = \frac{v + \cancel{vdv}}{\cancel{dx}}$$

$$y \frac{dv}{dx} = -\frac{(y^3 + 3y^2v^2)}{v^3 + 3v^2v} - v$$

$$y \frac{dv}{dx} = -\left[\frac{(1+3v^2)}{v^3 + 3v} + v \right]$$

$$y \frac{dv}{dx} = -\left[\frac{1+3v^2+v^4+3v^3}{v^3 + 3v} \right]$$

$$y \frac{dv}{dx} = -\frac{(1+6v^2+v^4)}{v^3 + 3v}$$

$$(12v + 4v^3)dv = dt \quad \int \frac{v^2 + 3v}{1 + 6v^2 + v^4} dv = - \int \frac{dt}{v}$$

$$\int \frac{dt}{t} = - \int \frac{dt}{v}$$

$$\log t = -4 \ln v + C$$

$$\log t = \log \frac{C}{v^4}$$

$$t = \frac{C}{v^4}$$

$$\frac{v^4(1+6v^2+v^4)}{v^3 + 3v} = <$$

Q.1. The differential equation $(x^2 + y^2) \frac{dy}{dx} = xy$ s.t. $y(0) = 1$

~~has~~ $\Rightarrow c$

$$\frac{dy}{dx} = \frac{y^2}{x^2}$$

$$\frac{dy}{dx} = v + n \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{ny}{x^2 + y^2}$$

~~(a) Unique solution~~

~~(b) Infinite solution~~

~~(c) Two solution~~

~~(d) None of these~~

$$n \frac{dv}{dx} = \frac{v - v^3}{1 + v^2}$$

$$\int \frac{1+v^2}{v^3} dv = - \int \frac{dv}{v} + 19C$$

$$\int \frac{1}{v^3} dv + \int \frac{1}{v} dv = - 19n + 19C$$

$$-\frac{1}{2v^2} + 19v = 19 \frac{C}{v}$$

$$19v - 19 \frac{C}{v} = \frac{1}{2v^2}$$

$$19 \frac{v^2}{2} = \frac{1}{2v^2}$$

$$v + nv \frac{dv}{dx} = \frac{v}{1+v^2}$$

$$\frac{ndv}{dx} = \frac{v}{1+v^2}$$

$$\frac{v^2}{C} = e^{-\frac{1}{2}v^2}$$

$$x = C e^{\frac{v^2}{2}}$$

$$y = C e^{-\frac{v^2}{2}}$$

Q.2. The general solution of the differential equation

$$(x^2 - y^2)dx + 2xydy = 0 \text{ is}$$

(a) $x^2 + y^2 = c$

(c) $x^2 + y^2 = cx$

$$\frac{dy}{dx} = \frac{y+x}{x-y}$$

$$\frac{dy}{dx} = \frac{x^2 - y^2}{2xy}$$

(b) $x^2 - y^2 = c$

(d) None

$$\frac{v+x}{v-x} = \frac{v^2 - x^2}{2x^2 v}$$

$$\int \frac{2v}{1+v^2} dv = - \int \frac{dx}{x} + C$$

$$v \frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$\log(1+v^2) = -\ln x + C$$

$$v \frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$1/(1+v^2) = 1/x$$

$$(1+v^2)x = C$$

$$(1+y^2)x^2 = C$$

$$\frac{v^2+1}{v} x = C$$

$$v^2 + 1 = C$$

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Q.3. Consider the following difference equation

$$x(ydx + xdy) \cos \frac{y}{x} = y(xdy - ydx) \sin \frac{y}{x}$$

$$\frac{(ydx + xdy)}{x} = \frac{(xdy - ydx)}{x} \Rightarrow \frac{\frac{dy}{dx} + \frac{y}{x}}{\frac{dy}{dx} - \frac{y}{x}} = \frac{y}{x}$$

$$\frac{dy}{dx} = y + \frac{y^2}{x^2}$$

Which of the following is solution of the above equation?

$$y \ln \frac{y}{x} = c$$

$$(a) \frac{x}{y} \cos \frac{y}{x} = c$$

$$(b) \frac{x}{y} \sin \frac{y}{x} = c$$

$$\frac{vndv + nvdm + n^2dv}{nvdx + n^2dv - vxdm} = v \tan v$$

~~$$(c) xy \cos \frac{y}{x} = c$$~~

$$(d) xy \sin \frac{y}{x} = c$$

$$\frac{2vndv + n^2dv}{n^2dv} = v \tan v$$

$$2 \frac{v}{n} \frac{dv}{dv} + 1 = v \tan v$$

$$2 \ln n = -\log(v) - \ln v + k$$

$$\ln n^2 + \ln v + k = \ln C$$

$$n^2 v \ln v = C$$

$$n^2 \ln v = C$$

$$2 \frac{dm}{n} = \int \left(\frac{v \tan v - 1}{v} \right) dv + 1, n$$

$$\left(\eta \frac{\partial u}{\partial x} - y G \frac{\partial v}{\partial x} \right) dx + n \cos \frac{\partial v}{\partial x} dy = 0 \quad | \quad y(0) = 0$$

(a) $n \frac{\partial u}{\partial x} = 1$

(b) $y = \pm n \pi x$

(c) $y = n \tan \frac{y}{x}$

(d) $n = y$

$$n \tan \frac{y}{x} = c$$

$$n \frac{\partial y}{\partial x} = 0$$

$$\frac{\partial y}{\partial x} = 0 = \frac{\partial y}{\partial x} \quad | \quad y = \pm n \pi$$

$$(n \frac{\partial v}{\partial x} - v n (\cos v)) dx$$

$$+ n \cos v (v dx + n dv) = 0$$

$$\text{or } y = v <$$

$$\frac{dy}{dx} = v + n \frac{dv}{dx}$$

$$dy = v dx + n dv$$

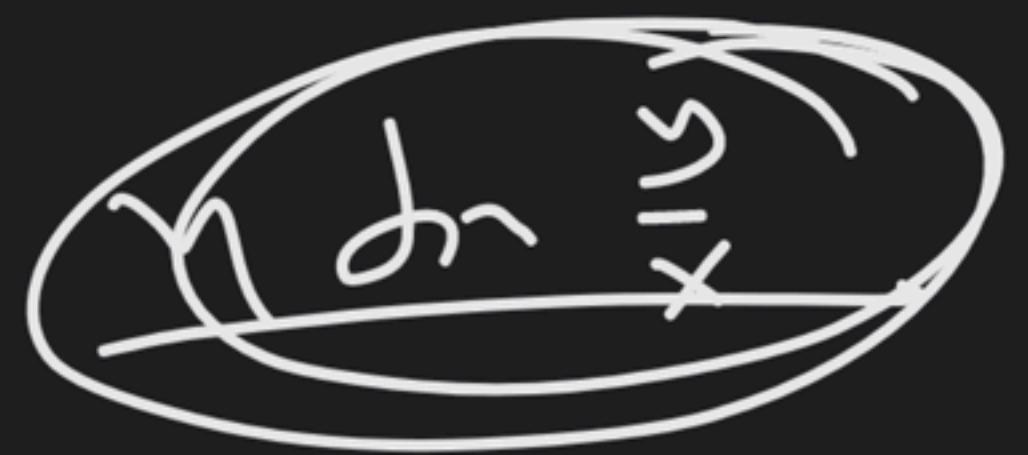
$$dn v - v ds v = - \cos v \frac{(v dx + n dv)}{dx}$$

$$\tan v - v = -v - n \frac{dv}{dx}$$

$$\int \tan v \, dv = - \int \frac{dn}{v} + 1 \leq c$$

$$\log \tan v = 1 \leq c$$

$$n \tan v = c$$



Equation reducible to homogeneous form

Equation of the form $\frac{dy}{dx} = \frac{ax+by+c}{cx+dy+c}$, where $\frac{a}{a'} \neq \frac{b}{b'}$ can be reduced to homogeneous form.

$$\frac{a}{a'} \neq \frac{b}{b'}$$

Working rule :

(1) Take $x = X + h$ & $y = Y + k$, then $\frac{dy}{dx} = \frac{dY}{dX}$.

$$\frac{dy}{dx} = \frac{m+n+5}{2m+n+2}$$

(2) Put all values in above DE

$$\frac{dY}{dX} = \frac{aX+bY+ah+bk+c}{a'x+b'y+a'h+b'k+c'}$$

$$h, k -$$

(3) Find value of h & k for which $ah + bk + c = 0$ & $a'h + b'k + c' = 0$,

Then $\frac{dY}{dX} = \frac{aX+bY}{a'X+b'Y}$ which homogeneous.

(4) Solve this DE and put $X = x - h$ & $Y = y - k$

$$\frac{dy}{dx} = \frac{(n+2y-3)}{2x+y} \rightarrow \left| \begin{array}{l} x = x+h \\ y = y+k \end{array} \right. \quad \left| \begin{array}{l} h+2k-3=0 \\ 2h+k-3=0 \end{array} \right. \quad \left| \begin{array}{l} y+x\frac{dv}{dx} = \frac{1+2v}{1+v} \\ x\frac{dv}{dx} = \frac{1+2v}{2-v}-v \end{array} \right.$$

$$\frac{dy}{dx} = \frac{(x+h)+2(y+k)-3}{2(x+h)+(y+k)} \rightarrow \left| \begin{array}{l} h=k=1 \\ y=y_0 \end{array} \right.$$

$$\frac{dy}{dx} = \frac{y+2y+h+2k-3}{2x+y+2h+k-3}$$

$$\frac{dy}{dx} = \frac{y+2y}{2x+y}$$

$$\text{let } y = vx, \quad \frac{dy}{dx} = v + y\frac{dv}{dx}$$

$$v+x\frac{dv}{dx} = \frac{x+2vx}{2x-vx}$$

$$x\frac{dv}{dx} = \frac{1+2x-2v}{1+v} v -$$

$$\int \frac{1+v}{1-v^2} dv = \int \frac{dx}{x} + \log c$$

$$\int \frac{1}{1-v^2} dv + \int \frac{v}{1-v^2} dv = 19x c$$

$$2\log \frac{1+v}{1-v} - \frac{1}{2} \log(1-v^2) = 19x c$$

$$19 \frac{(1+v)^2}{(1-v)^2} \sqrt{1-v^2} = xc$$

~~Q.6.~~ Solution of $\frac{dy}{dx} = \frac{(xy^2 - x^2y)}{x^3}$ s.t. $y(1) = 2$

$$v + n \frac{dv}{dx} = \frac{v^2 - v}{1}$$

$$n \frac{dv}{dx} = v^2 - nv$$

$$\int \frac{dv}{v(v-n)} = \int \frac{dx}{x} + C_1$$

(a) Unique solution

(b) No solution

(c) Infinite solution

(d) None of these

$$\frac{1}{\sqrt{v-n}} = \frac{1}{x} + \frac{C_1}{\sqrt{v-n}}$$

$$= -\frac{1}{n} + \frac{1}{n}(\sqrt{v-n})$$

$$-\int \frac{1}{\sqrt{v}} dv + \int \frac{1}{\sqrt{v-n}} dv =$$

$$= 2 \ln x + C$$

$$-19v + 18(v-n) = 1, v^2 < 0$$

$$\frac{y^2 - n}{y/x} = n^2 <$$

$$y - 2n = 4n^2 <$$

$$2 - 2 = 2 \cdot 1 <$$

$$C = 0$$

$$\frac{v-n}{v} = v^2 <$$

$$1 - 2n = 0$$



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Educator highlights

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- 📍 Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
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