

Quotient Group and their Properties

Detail Course 2.0 on Group Theory for IIT JAM '23



Gajendra Purohit

Legend in CSIR-UGC NET & IIT-JAM

- Unlock Code : GPSIR ~ PhD, CSIR NET (Maths) | Youtuber(800K+165K Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author

50M Watch mins

3M Watch mins (last 30 days)

44K Followers

2K Dedications

→ **TOP EDUCATOR ON UNACADEMY
FOR CSIR NET & IIT JAM**

YouTuber with 800K Subscribers

→ **AUTHOR OF BEST SELLER BOOK
FOR CSIR NET & IIT JAM**

**Get
10% Off**

Referral Code : GP SIR





Detailed Course 2.0 on Sequence and Series For IIT JAM' 23

October 26
9:00 AM

Gajendra Purohit

[Enroll Now](#)

Use code GPSIR for 10% off



DETAILED COURSE 2.0 GROUP THEORY FOR IIT JAM 2023

6th OCTOBER

Gajendra Purohit

[Enroll Now](#)

USE CODE
GPSIR
FOR 10% OFF



FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months ₹ 908 / mo
Save 67%
Total ₹ 21,780

You get 6 months extra for free Offer expires 15 Jun 2022

✓ 12 months ₹ 1,248 / mo
Save 54%
Total ₹ 14,974

You get 6 months extra for free Offer expires 15 Jun 2022

9 months ₹ 1,497 / mo
Save 45%
Total ₹ 13,475

6 months ₹ 2,042 / mo
Save 25%
Total ₹ 12,252

3 months ₹ 2,269 / mo
Save 17%
Total ₹ 6,807

1 month ₹ 2,723 / mo
Save 0%
Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

No cost EMI available on 6 months & above subscription plans

24 months ₹ 817 / mo
Save 67%
Total ₹ 21,700 ₹ 19,602

You get 6 months extra for free Offer expires 15 Jun 2022

✓ 12 months ₹ 1,123 / mo
Save 54%
Total ₹ 13,477

You get 6 months extra for free Offer expires 15 Jun 2022

9 months ₹ 1,348 / mo
Save 45%
Total ₹ 12,128

6 months ₹ 1,838 / mo
Save 25%
Total ₹ 11,027

3 months ₹ 2,042 / mo
Save 17%
Total ₹ 6,126



After Using
My Referral
Code



GPSIR

Awesome! You get 10% off

Proceed to pay

Coset and normal subgroup

Coset : Let G be a group on which the group operation is multiplication and H be a subgroup of it. Let a be any element of G then the set $Ha = \{ha : h \in H\}$ is called right coset of H in G generated by a .

Similarly, the set $aH = \{ah : h \in H\}$ is called a left coset of H in G generated by a .

Note : If the group operation is addition, then right and left coset of H in G generated by a , as

$$H + a = \{h + a \mid h \in H\}$$

$$a + H = \{a + h \mid h \in H\}$$

Conclusion from both example :

Let G be a group and H be its subgroup then right coset and left coset of

H in G generated by a are need not be equal

In example (1), Right and left coset are equal.

In example (2), Right and left coset are not equal.

Result :

- (1) If H is any subgroup of G and $h \in H$, then $Hh = H = hH$

- (2) If a and b are any two elements of a group G & H is a subgroup of G then $Ha = Hb \Leftrightarrow ab^{-1} \in H$ & $aH = bH \Leftrightarrow b^{-1}a \in H$.
- (3) Any two right (left) cosets of a subgroup are either disjoint or identical.
- (4) Lagrange's theorem : The order of each subgroup of a finite group is a divisor of the order of the group.

Index of a subgroup in a group : The number of distinct coset of a subgroup in a group is called index of subgroup.

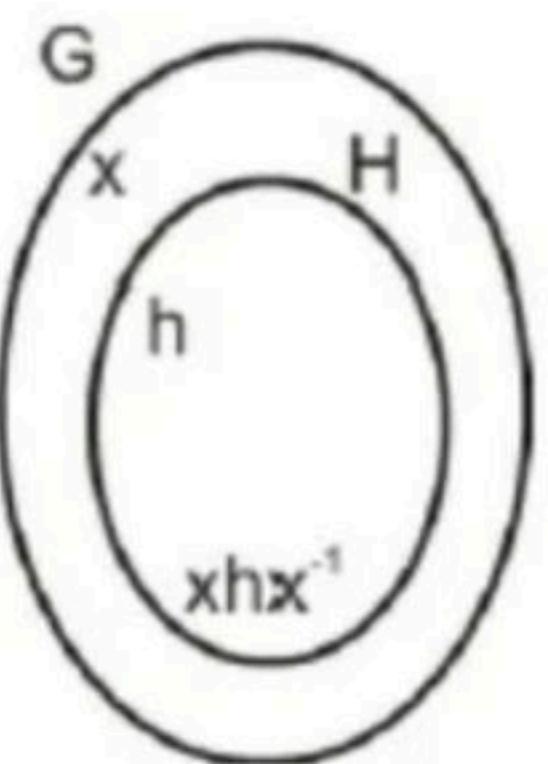
Note : Let G be a finite group then index of H in G .

$$= \frac{O(G)}{O(H)}$$

Normal subgroup : Let G be a group. A subgroup H of G is called a normal subgroup of G iff right coset and left coset are equal i.e. $xH = Hx$; for all $x \in G$.

Another definition : A subgroup H of a group G is said to be normal subgroup of G if for every $x \in G$ & for every $h \in H$

$$xhx^{-1} \in H$$



Q.1. Let G denote the group of all 2×2 invertible matrices with entries from \mathbb{R} . Let $H_1 = \{A \in G; \det(A) = 1\}$ and $H_2 = \{A \in G,$
 $A \text{ is upper triangular}\}$ Consider the following statements :

P : H_1 is a normal subgroup of G

Q : H_2 is a normal subgroup of G

Then

(a) Both P & Q are true

(b) P is true & Q is false

(c) P is false & Q is true

(d) Both P & Q are false.

COMPLETE COURSE ON

MATHEMATICS

FOR IIT-JAM 2022

TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINEAR ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months ₹ 908 / mo
Save 67%
Total ₹ 21,780

You get 6 months extra for free Offer expires 15 Jun 2022

✓ 12 months ₹ 1,248 / mo
Save 54%
Total ₹ 14,974

You get 6 months extra for free Offer expires 15 Jun 2022

9 months ₹ 1,497 / mo
Save 45%
Total ₹ 13,475

6 months ₹ 2,042 / mo
Save 25%
Total ₹ 12,252

3 months ₹ 2,269 / mo
Save 17%
Total ₹ 6,807

1 month ₹ 2,723 / mo
Save 0%
Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

No cost EMI available on 6 months & above subscription plans

24 months ₹ 817 / mo
Save 67%
Total ₹ 21,700 ₹ 19,602

You get 6 months extra for free Offer expires 15 Jun 2022

✓ 12 months ₹ 1,123 / mo
Save 54%
Total ₹ 13,477

You get 6 months extra for free Offer expires 15 Jun 2022

9 months ₹ 1,348 / mo
Save 45%
Total ₹ 12,128

6 months ₹ 1,838 / mo
Save 25%
Total ₹ 11,027

3 months ₹ 2,042 / mo
Save 17%
Total ₹ 6,126



After Using
My Referral
Code



GPSIR

Awesome! You get 10% off

Proceed to pay

FOUNDATION COURSE OF

MATHEMATICS

FOR CSIR-NET

Result :

- (1) A subgroup of index 2 in G is normal subgroup of G.
- (2) $SL(n, F)$ is normal in $GL(n, F)$
- (3) A_n is normal in S_n .
- (4) Every subgroup of abelian group is normal.
- (5) Normal Subgroup of S_n = $\begin{cases} I, K_4, A_n, S_n & \text{If } n = 4 \\ I, A_n, S_n & \text{If } n \neq 4 \end{cases}$
- (6) Normal Subgroup of Q_8 are $\{1\}$, $\{-1, 1\}$, $\{1, -1, i, -i\}$,
 $\{1, -1, j, -j\}$, $\{1, -1, k, -k\}$ and Q_8

(7) Normal Subgroup of $A_n = \begin{cases} I, K_4, A_n & \text{If } n = 4 \\ I, A_n & \text{If } n \neq 4 \end{cases}$

(8) $\{e\}$ and G are always normal subgroup

Q.2. Let $H = \{ e, (1\ 2)(3\ 4) \}$ and $K = \{ e, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3) \}$ be a subgroup of S_4 where e denotes the identify elements of S_4 then

- (a) H and K are normal subgroup of S_4
- (b) H is normal in K and K is normal in A_4
- (c) H is normal in A_4 and K is not normal in S_4
- (d) K is normal in S_4 and H is not

Quotient Group:

Let N be a normal subgroup of G . If $a \in G$, then Na is a right coset of N in G .

Since N is normal in G , left coset aN will be equal to the right coset Na .

Let $\frac{G}{N}$ be the collection of all distinct coset of N in G i.e.

$\frac{G}{N} = \{Na; a \in G\}$, then $\frac{G}{N}$ is a group w.r.t. multiplication of coset i.e.

$$(Na).(Nb) = Nab$$

It is called quotient group or factor group of G by N as the composition

in $\frac{G}{N}$.

Conclusion : Let N be a normal subgroup of a finite group G.

Then index of N in G = $\frac{O(G)}{O(N)}$. If G is finite

Note : Order of Quotient group

$$o\left(\frac{G}{N}\right) = \frac{O(G)}{O(N)}. \text{ If } G \text{ is finite}$$

Centre of Group : Let G be a group then $Z(G) = \{ x \in G \mid xa = ax \mid \forall a \in G \}$ is called centre of group

Result :

- (1) The quotient group of abelian group is abelian but converse is not true.
- (2) The quotient group of cyclic group is cyclic but converse is not true.

(3) Let $Z(G)$ be a centre of a group G , then G is abelian if $\frac{G}{Z(G)}$ is cyclic.

(4) Let G be a cyclic group of order n , then number of factor group of G are $\tau(n)$ because number of normal subgroups are $\tau(n)$.

(5) $\frac{K}{N}$ is a subgroup of $\frac{G}{N}$, if K is a subgroup of G .

$$\frac{K}{N} \triangleleft \frac{G}{N} \text{ if } K \triangleleft G$$

- (1) $\frac{\mathbb{Q}}{\mathbb{Z}}$ is abelian group but not cyclic group.
- (2) Number of elements of order n are $\phi(n)$
- (3) $\left(\frac{1}{p} + \mathbb{Z}\right) \in \frac{\mathbb{Q}}{\mathbb{Z}}$ then $O\left(\frac{1}{p} + \mathbb{Z}\right) = p$
And $O\left(\frac{k}{p} + \mathbb{Z}\right) = p$ if $\gcd(k, p) = 1$
- (5) There exist a unique cyclic subgroup of each order i.e. For every positive integer n , there is a cyclic subgroup of order n which is unique which is $H = \left\langle \frac{1}{n} + \mathbb{Z} \right\rangle$

Q.3. Consider the quotient group of $\frac{\mathbb{Q}}{\mathbb{Z}}$ of the additive group of rational number, the order of element $\frac{2}{3} + \mathbb{Z}$ in $\frac{\mathbb{Q}}{\mathbb{Z}}$ is

- (a) 2
- (b) 3
- (c) 5
- (d) 6

Q.4. Consider the group $G = \mathbb{Q}/\mathbb{Z}$. Let n be a positive integer. Then there is a cyclic subgroup

of order n

(a) Not necessarily

(b) Yes , a unique

(c) Yes , but not necessarily a unique

(d) Never

Q.5. Consider the following subsets of the group of 2×2 non singular matrices over \mathbb{R}

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R}, ad = 1 \right\}$$

$$H = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in \mathbb{R} \right\}$$

Which of the following statements are correct

- (a) G form a group under matrix multiplication
- (b) H is normal subgroup of G
- (c) G/H is well define and is abelian
- (d) None of these

Q.6. Let G be a non-abelian group and $Z(G)$ is its centre, then which

of the following is cannot be possible of $O\left(\frac{G}{Z(G)}\right)$.

- (a) 7
- (b) 8
- (c) 4
- (d) 6

Q.7. If $H \subset K$ are two normal subgroups of a group G and if $[G : H] = 10$ and $[G : K] = 5$, then $[K : H]$ is

- (a) 5
- (b) 2
- (c) 10
- (d) 50

Q.8. Suppose N is a normal subgroup of a group G . Which one of the following is true?

- (a) If G is an infinite group, then G/N is an infinite group.
- (b) If G is a non-abelian group, then G/N is a non-abelian group.
- (c) If G is a cyclic group, then G/N is an abelian group.
- (d) If G is an abelian group, then G/N is a cyclic group.



Detailed Course 2.0 on Sequence and Series For IIT JAM' 23

October 26
9:00 AM

Gajendra Purohit

[Enroll Now](#)

Use code GPSIR for 10% off



DETAILED COURSE 2.0 **GROUP THEORY FOR IIT JAM 2023**

6th OCTOBER

Gajendra Purohit

[Enroll Now](#)

**USE CODE
GPSIR
FOR 10% OFF**



Educator Profile



Gajendra Purohit

#5 Educator in CSIR-UGC NET

[Follow](#)

Dr.Gajendra Purohit PhD, CSIR NET (Maths) | Youtuber(330K+30k Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author of Bestseller

11M Watch mins

1M Watch mins (last 30 days)

22k Followers

1k Dedications



CSIR-UGC NET

[SEE ALL](#)

HINDI MATHEMATICAL SCIENCES

Course on Linear Algebra, Partial Diff. Equation & Calculus

Starts on Mar 1, 2021 • 24 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Course on Complex Analysis & Integral Equation

Starts on Jan 14, 2021 • 16 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Foundation Course on Mathematics for CSIR 2021

Starts on Dec 7, 2020 • 20 lessons

Gajendra Purohit

Educator highlights

- 📍 Works at Pacific Science College
- 📍 Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
- 📍 PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- 📍 Lives in Udaipur, Rajasthan, India
- 📍 Unacademy Educator since

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months ₹ 908 / mo
Save 67%
Total ₹ 21,780

You get 6 months extra for free Offer expires 15 Jun 2022

✓ 12 months ₹ 1,248 / mo
Save 54%
Total ₹ 14,974

You get 6 months extra for free Offer expires 15 Jun 2022

9 months ₹ 1,497 / mo
Save 45%
Total ₹ 13,475

6 months ₹ 2,042 / mo
Save 25%
Total ₹ 12,252

3 months ₹ 2,269 / mo
Save 17%
Total ₹ 6,807

1 month ₹ 2,723 / mo
Save 0%
Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

No cost EMI available on 6 months & above subscription plans

24 months ₹ 817 / mo
Save 67%
Total ₹ 21,700 ₹ 19,602

You get 6 months extra for free Offer expires 15 Jun 2022

✓ 12 months ₹ 1,123 / mo
Save 54%
Total ₹ 13,477

You get 6 months extra for free Offer expires 15 Jun 2022

9 months ₹ 1,348 / mo
Save 45%
Total ₹ 12,128

6 months ₹ 1,838 / mo
Save 25%
Total ₹ 11,027

3 months ₹ 2,042 / mo
Save 17%
Total ₹ 6,126



After Using
My Referral
Code



GPSIR

Awesome! You get 10% off

Proceed to pay

THANK YOU VERY MUCH EVERYONE

GET THE UNACADEMY PLUS SUBSCRIPTION SOON.

TO GET 10% DISCOUNT IN TOTAL SUBSCRIPTION AMOUNT

USE REFERRAL CODE: GPSIR