Integral Calculus - Part IX

Detailed Course on Integral Calculus - IIT JAM' 23

Gajendra Purohit • Lesson 11 • July 22, 2022


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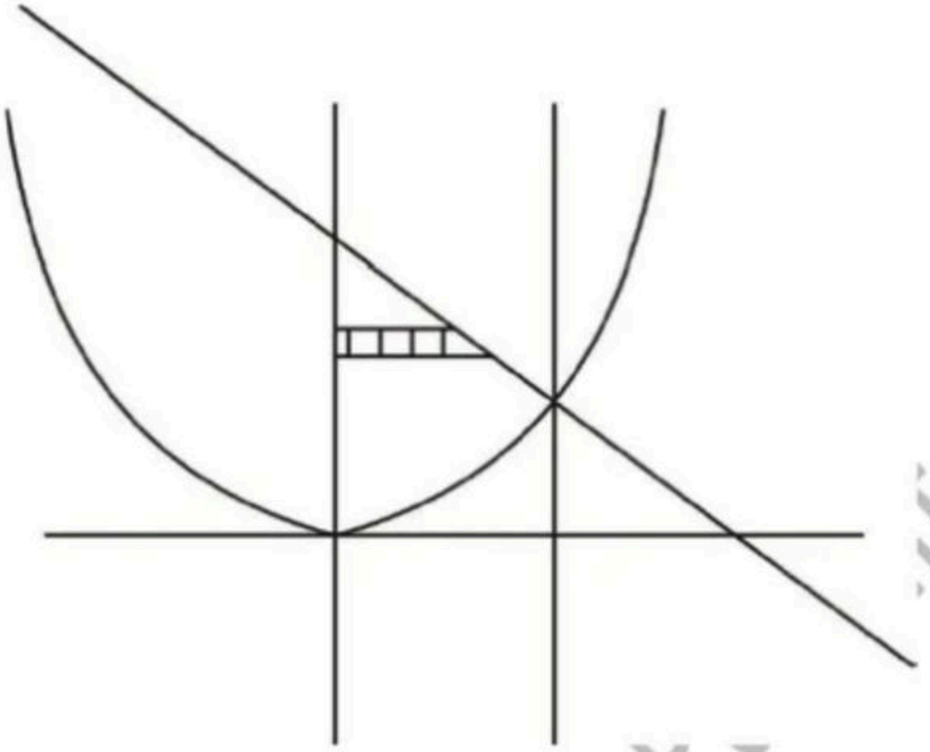
Change of order in mixed region

We know that if strip move on more than two curve then t
region is called mixed region.

Example :

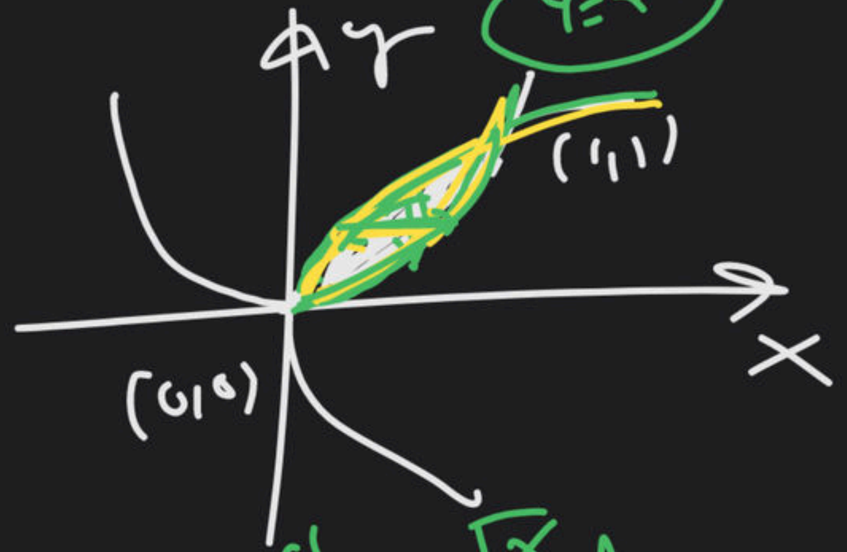


Then for double integration, we divide into simple region.

$$\iint_R dx\,dy$$

$y = x^2$ & $x = y^2$

$$A = \iint dx\,dy$$



$$\int_{x=0}^{1}\int_{y=x^2}^{\sqrt{x}} dy\,dx = \int_0^1 (y)_{x^2}^{\sqrt{x}}\,dx$$

$$\int_0^1 (\sqrt{x} - x^2)\,dx = \left(\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right)_0^1$$

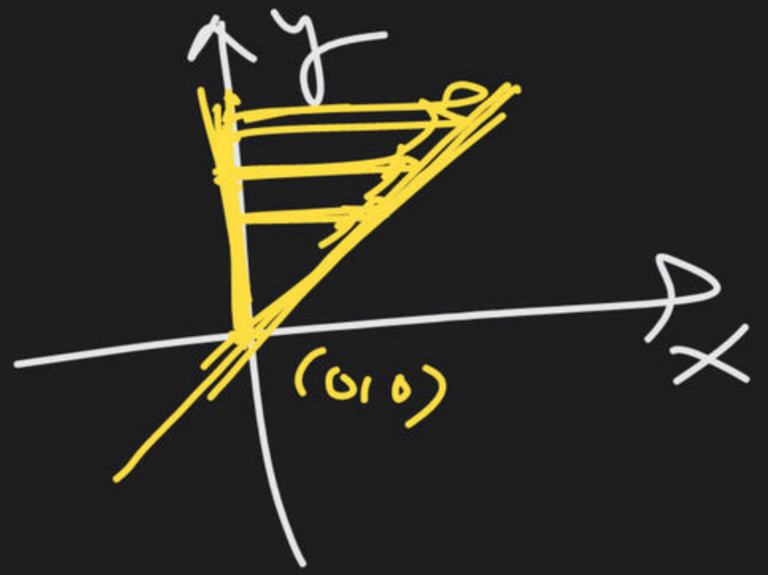
$$= \left(\frac{2}{3} - \frac{1}{3}\right) = \frac{1}{3}$$

$$\int_{y=0}^{1}\int_{x=y^2}^{\sqrt{y}} dx\,dy = \left(\frac{x^2}{2}\right)_{y^2}^{\sqrt{y}}$$

$$\int_{x=0}^{\infty}\int_{y=x}^{\infty}\frac{1}{y}e^{-y/2}\,dy\,dx$$

$x=0$ $\quad$ $y=x$

$x=\infty$ $\quad$ $y=\infty$



$$\int_{y=0}^{\infty}\int_{x=0}^{y}\frac{1}{y}e^{-y/2}\,dx\,dy$$

$$\int_{y=0}^{\infty}\frac{1}{y}e^{-y/2}(x)_0^y\,dy$$

$$\int_0^{\infty}\frac{1}{y}e^{-y/2}(y-0)\,dy = \left(\frac{e^{-y/2}}{-1/2}\right)_0^{\infty} = -2\left[e^{-\infty}-e^{0}\right]$$

$$= 2$$

Q $\int_{x=0}^{1}\int_{y=1}^{e^x} f(x,y)\,dy\,dx$ can be expressed as $\int_{x=0}^{1}\int_{y=1}^{e^x} f(x,y)\,dy\,dx$

(a) $\int_{0}^{1}\int_{1}^{\log y} f(x,y)\,dx\,dy$

(b) $\int_{1}^{e}\int_{0}^{\log y} f(x,y)\,dx\,dy$

(c) $\int_{1}^{e}\int_{1}^{e^y} f(x,y)\,dx\,dy$

(d) $\int_{1}^{e}\int_{\log y}^{1} f(x,y)\,dx\,dy$

$\int_{y=1}^{e}\int_{x=\log y}^{1} f(x,y)\,dx\,dy$

Q.1. After the change of order of integration ,the double integral $\int_0^8 \int_{x^{1/3}}^2 dy\,dx$ becomes **CUCET 2021**

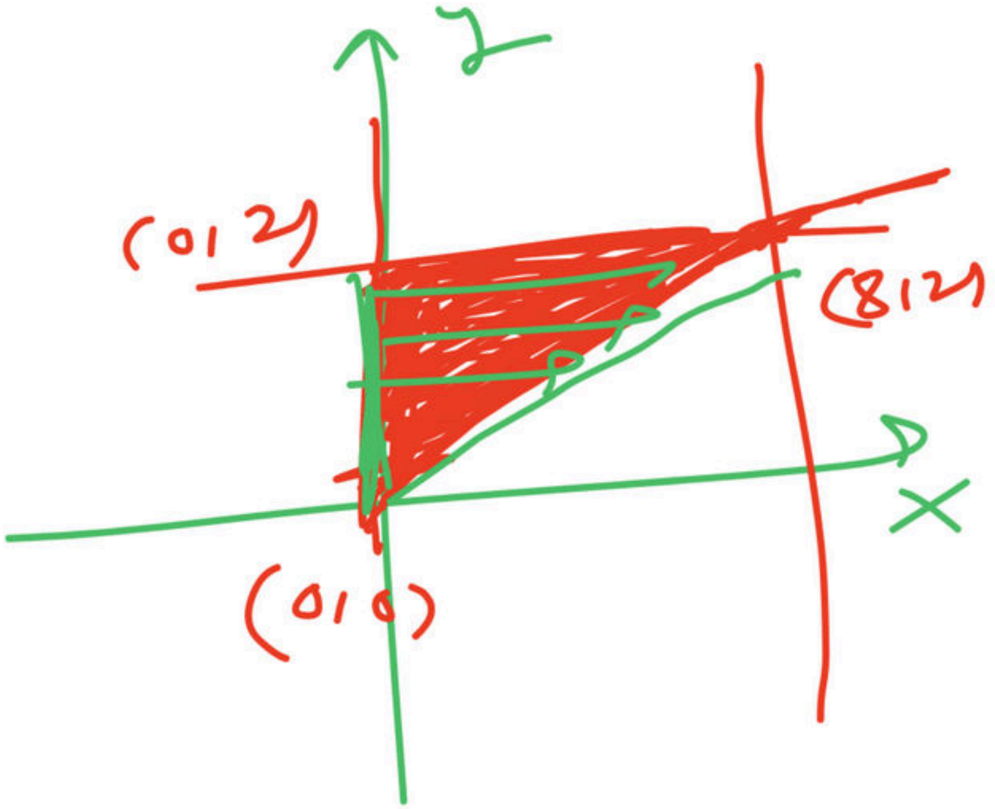
$x = 0$ $y = x^{1/3}$ $x = 8$ $y = 2$ $y = y^3$

(a) $\int_{x^{1/3}}^2 \int_0^8 dx\,dy$

(b) $\int_0^2 \int_0^{y^3} dx\,dy$

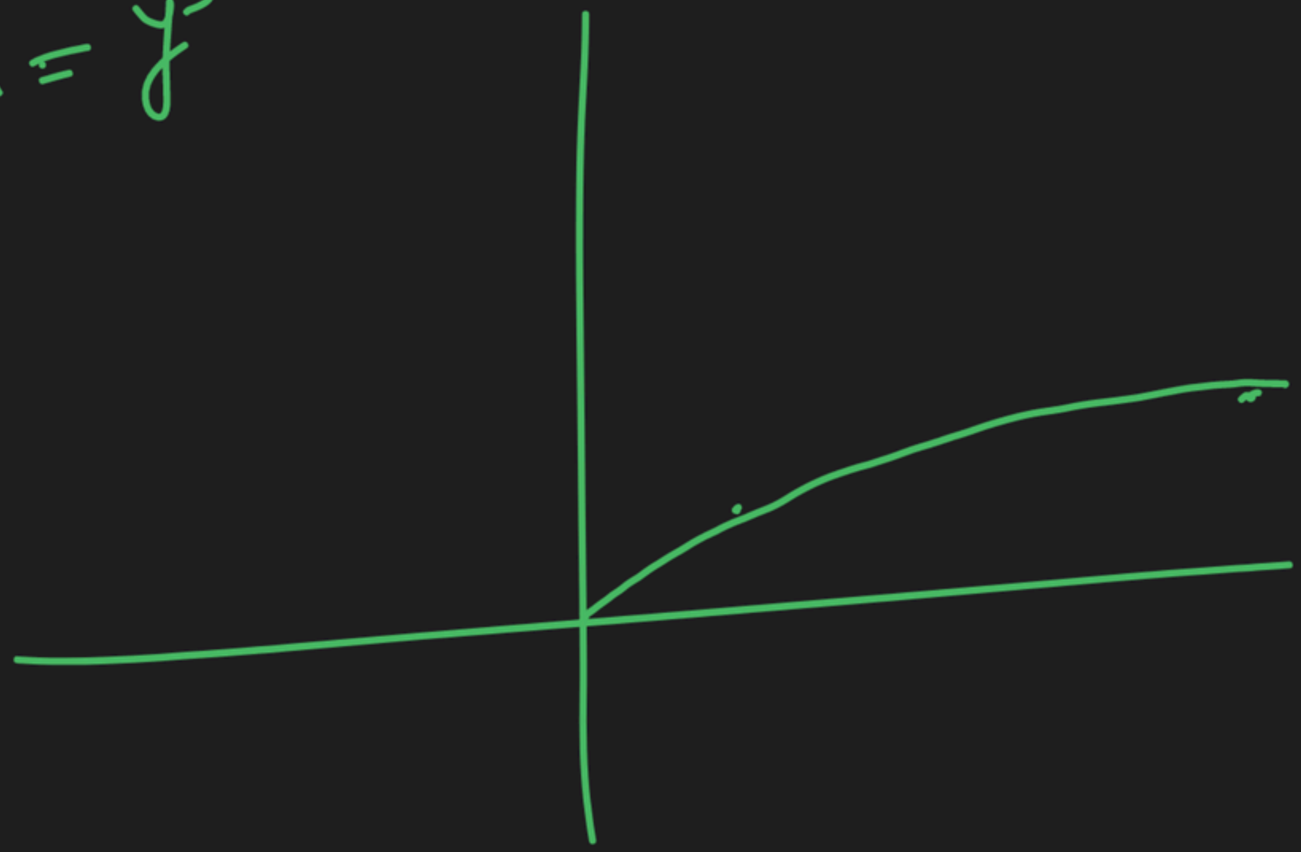
(c) $\int_8^0 \int_2^{x^{1/3}} dx\,dy$

(d) $\int_0^2 \int_{y^3}^0 dx\,dy$



$\int_{y=0}^{2} \int_{x=0}^{y^3} dx\,dy$

$x = y^3$



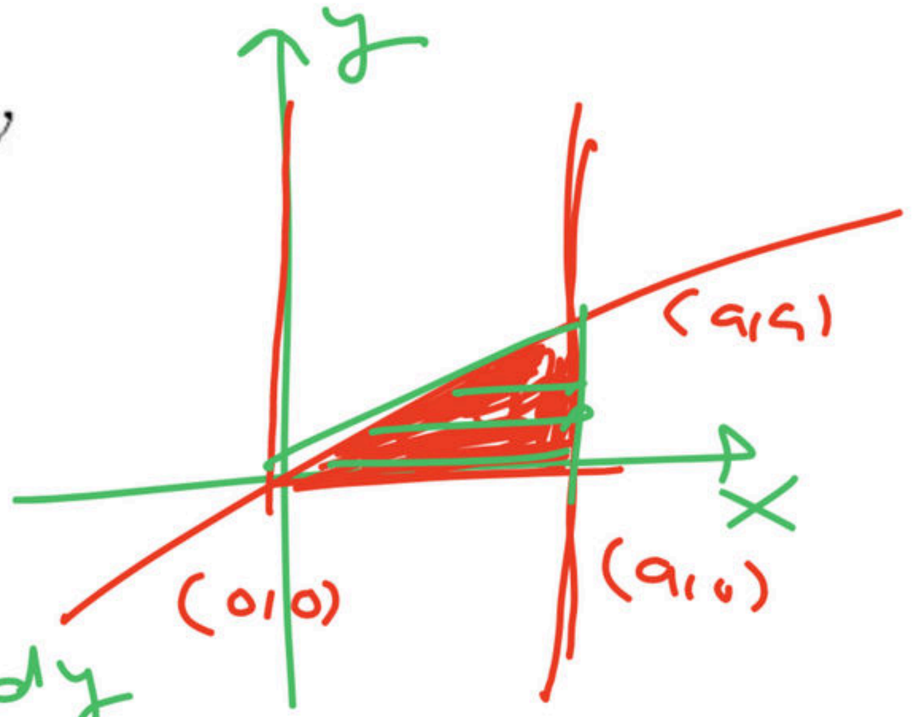
Q.2. Let f : R $\to$R be continuous function and a > 0 then the integral $\int_0^a \int_0^x f(y)\,dy\,dx$ equals **JAM – 2009**

(a) $\int_0^a yf(y)\,dy$

(b) $\int_0^a (a-y)f(y)\,dy$

(c) $\int_0^a (y-a)f(y)\,dy$

(d) $\int_a^0 yf(y)\,dy$



Q3. The value of $I = \int_0^1 \int_0^x x^2 e^{xy}\, dx\, dy$ is

(a) $\frac{e+2}{2}$

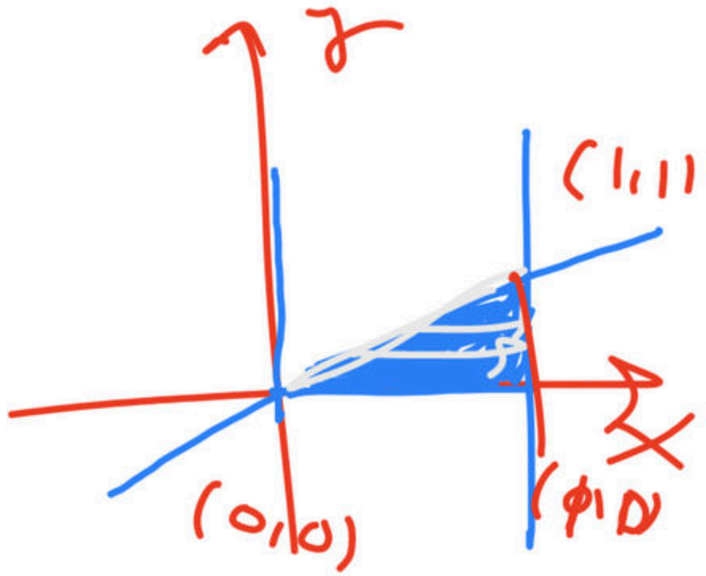
(b) $\frac{e-2}{2}$

(c) $\frac{e-1}{2}$

(d) $\frac{e+1}{2}$

$x=0$ $y=0$

$x=1$ $y=x$



$\int_0^1 x^2 \left(\frac{e^{xy}}{x}\right)_0^x dx$

$\int_0^1 x(e^{x^2}-1)\, dx$

$x^2 = t$

$2x\, dx = dt$

$x\, dx = \frac{dt}{2}$

$\frac{1}{2}\int_0^1 (e^t - 1)\, dt = \frac{1}{2}\left(e^t - t\right)_0^1$

$= \frac{1}{2}((e-1)-(1))$

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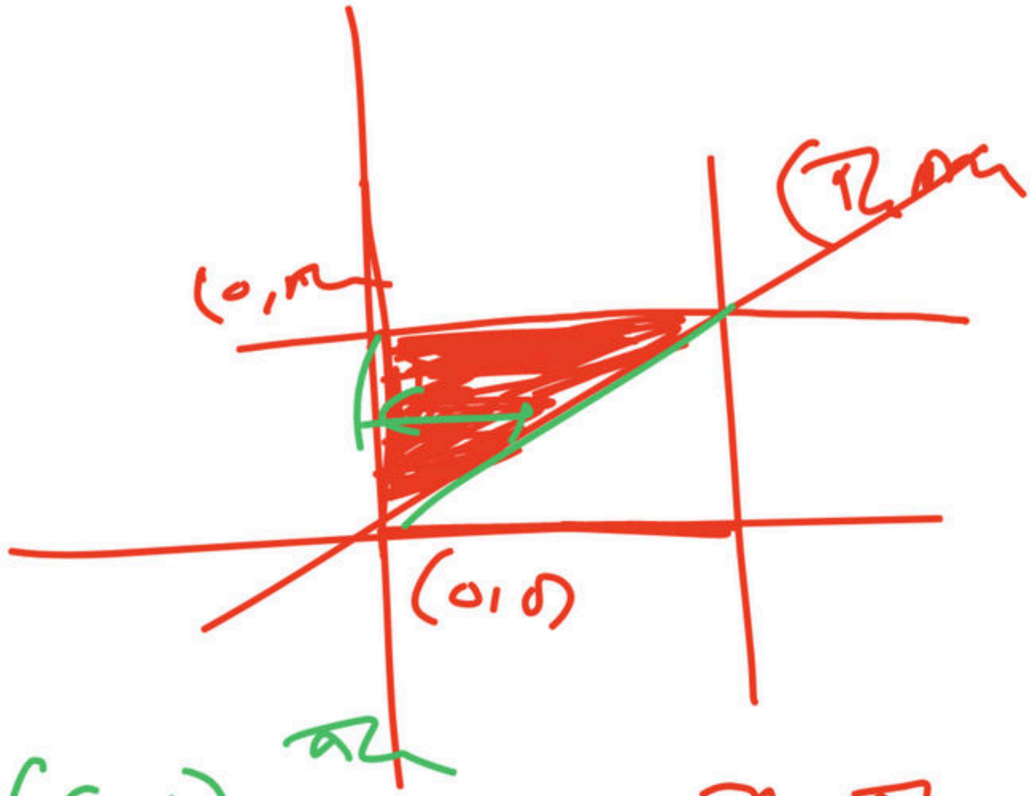
Q4. $\int_{x=0}^{\pi/2}\int_{y=x}^{\pi/2} \frac{\sin y}{y}\,dy\,dx$ is equal to

(a) 1 (b) 2

(c) 3 (d) 4



$$\int_{y=0}^{\pi/2}\int_{x=0}^{y} \frac{\sin y}{y}\,dx\,dy$$

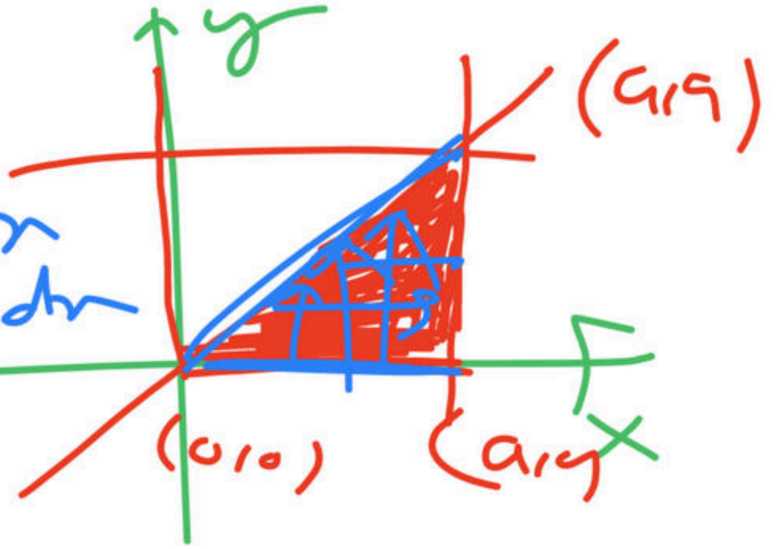
$$\int_{y=0}^{\pi/2} \frac{\sin y}{y}(x)_0^y\,dy = -(\cos y)_0^{\pi/2}$$

$$= -(0-1) = 1$$

Q5. The value of the double integral $\int_{0}^{a}\int_{y}^{a} \frac{x}{x^2+y^2}\,dx\,dy$ is

(a) $\frac{\pi a}{4}$ (b) $\frac{3\pi a}{4}$

(c) $-\frac{\pi a}{3}$ (d) $\frac{2\pi a}{3}$



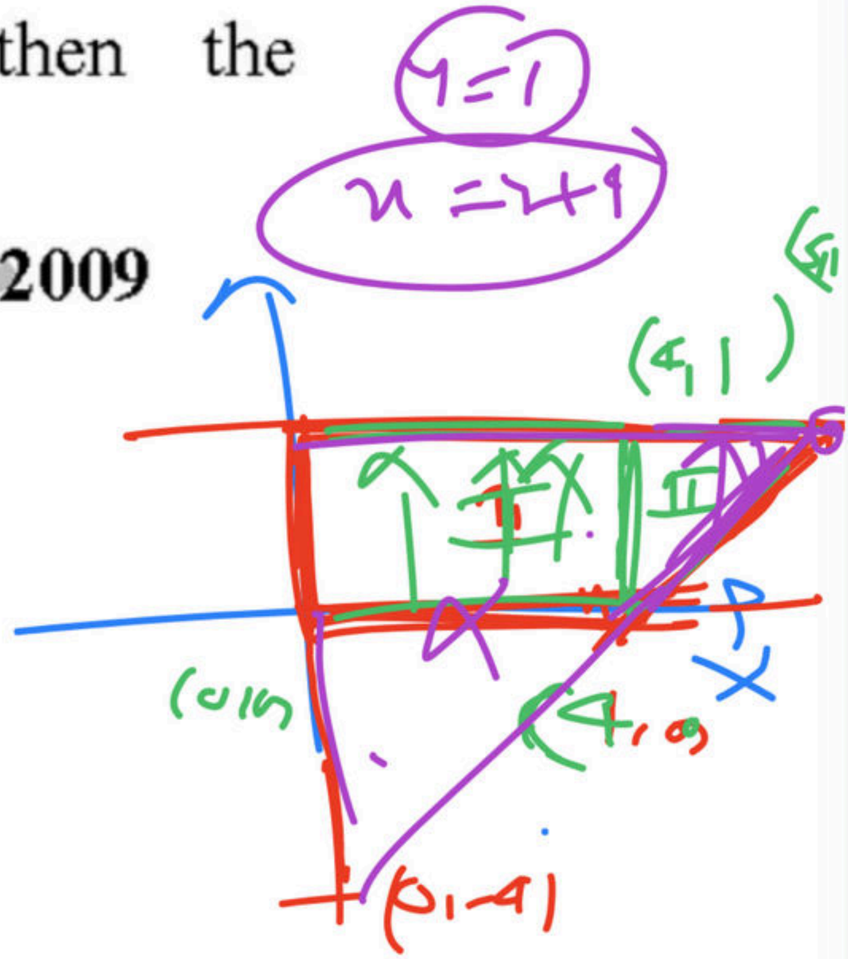
Q6, If $\int_{x=0}^{1}\int_{y=0}^{y+4} dx\,dy = \int_{x=0}^{4}\int_{y=0}^{1} dy\,dx + \int_{x=4}^{5}\int_{y=g(x)}^{h(x)} dy\,dx$, then the function g(x) and h(x) are, respectively **JAM – 2009**

(a) (x – 4) and 1

(b) (x + 4) and 1

(c) 1 and (x – 4)

(d) 1 and (x + 4)



Q7. Evaluate $\iint \sqrt{4x^2 - y^2}\, dx.dy$ over region bounded by

$y = 0, y = x, x = 1$ is

(a) $\frac{\sqrt{3}}{6} + \frac{\pi}{9}$

(b) $\frac{\sqrt{3}}{5} + \frac{\pi}{9}$

(c) $\frac{\sqrt{2}}{3}$

(d) $\frac{\sqrt{7}}{9}$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