Doubt Clearing Session

Detail Course 2.0 on Group Theory for IIT JAM '23

Gajendra Purohit • Lesson 9 • Oct 17, 2022


unacademy
Detailed Course 2.0 on
Sequence and Series For IIT JAM' 23
October 26
9:00 AM
Gajendra Purohit
Enroll Now
Use code GPSIR for 10% off
unacademy


unacademy
DETAILED COURSE 2.0
GROUP THEORY FOR IIT JAM 2023
6th OCTOBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 908 / mo Total ₹ 21,780
You get 6 months extra for free	Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,248 / mo Total ₹ 14,974
You get 6 months extra for free	Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,497 / mo Total ₹ 13,475
6 months Save 25%	₹ 2,042 / mo Total ₹ 12,252
3 months Save 17%	₹ 2,269 / mo Total ₹ 6,807
1 month	₹ 2,723 / mo Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months Save 67%	₹ 817 / mo ~~₹ 21,780~~ ₹ 19,602
You get 6 months extra for free	Offer expires 15 Jun 2022
12 months Save 54%	₹ 1,123 / mo ~~₹ 14,974~~ ₹ 13,477
You get 6 months extra for free	Offer expires 15 Jun 2022
9 months Save 45%	₹ 1,348 / mo ~~₹ 13,475~~ ₹ 12,128
6 months Save 25%	₹ 1,838 / mo ~~₹ 12,252~~ ₹ 11,027
3 months Save 17%	₹ 2,042 / mo ~~₹ 6,807~~ ₹ 6,126

GPSIR

Awesome! You get 10% off

Proceed to pay

Quotient Group

Let N be a normal subgroup of G. If $a \in G$, then Na is a right coset of N in G. Since N is normal in G, left coset aN will be equal to the right coset Na.

Let $\frac{G}{N}$ be the collection of all distinct coset of N in G i.e. $\frac{G}{N} = \{Na; a \in G\}$, then $\frac{G}{N}$ is a group w.r.t. multiplication of coset i.e. $(Na).(Nb) = Nab$

It is called quotient group or factor group of G by N as the composition in $\frac{G}{N}$.

$$(N+a) + (N+b) = N+a+b$$

$$(Na)(Nb) = Nab$$

$G = \mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$

$H = \{0, 3\}$

$H + 0 = H$

$H + 1 = \{1, 4\}$

$H + 2 = \{2, 5\}$

$H + 3 = \{3, 0\} = H$

$H + 4 = \{4, 1\} = H + 1$

$\frac{G}{N} = \{H, H+1, H+2\}$

+	H	H+1	H+2
H	H	H+1	H+2
H+1	H+1	H+2	H
H+2	H+2	H	H+1

$o\left(\frac{G}{H}\right) = G : H = \frac{o(G)}{o(H)} = \frac{6}{2} = 3$

Note : We know that index of N in G are number of distinct coset of N in G.

Conclusion : Let N be a normal subgroup of a finite group G.

Then index of N in G = $\dfrac{O(G)}{O(N)}$. If G is finite

Note : Order of Quotient group $O\left(\dfrac{G}{N}\right)=\dfrac{O(G)}{O(N)}$. If G is finite

Centre of Group : Let G be a group then Z(G) = { x ∈ G | xa = ax | ∀ a ∈ G} is called centre of group

$Q_8 = \langle \pm 1, \pm i, \pm j, \pm k \rangle$, $H = \langle 1, -1 \rangle$

$\dfrac{Q_8}{H} = \langle H, Hi, Hj, Hk \rangle$

$\times$	H	Hi	Hj	Hk
$\tau = 4$	H	Hi	Hj	Hk
Hi	Hi	$\boxed{H}$	Hk	Hj
Hj	Hj	Hk	$\boxed{H}$	Hi
Hk	Hk	Hj	Hi	H

$H1 = H$

$H(-1) = H$

$Hi = \{i, -i\} = H(-i)$

$Hj = H(-j)$

$Hk = H(-k)$

$(Ha)(Hb) = Hab$

$\dfrac{Q_8}{Z(Q)} = \boxed{K_4}$

Result :

(1) The quotient group of abelian group is abelian but converse is not true.

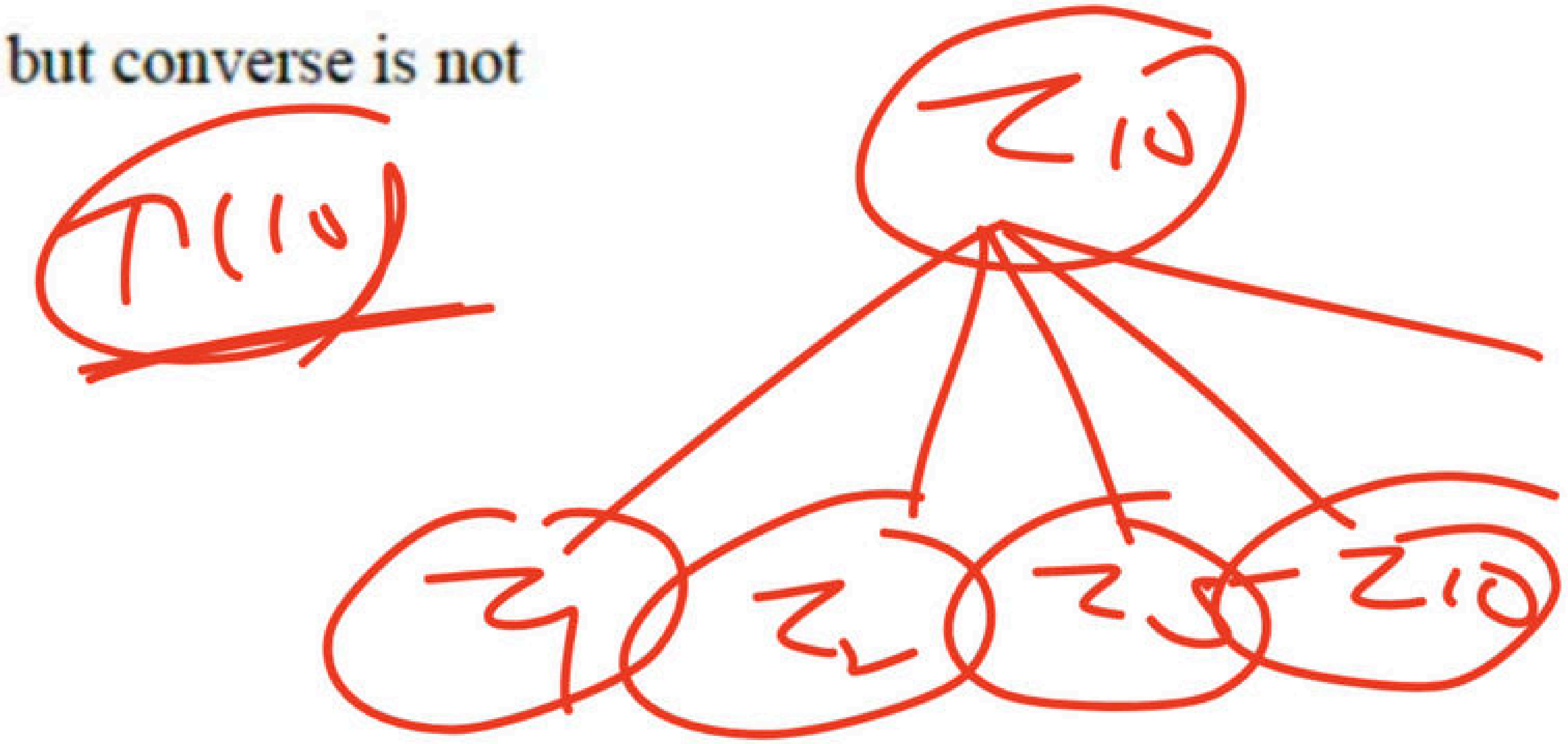
Example : Let $G = S_4$ & $N = A_4$, then $\frac{G}{N} = Z_2$.

Here $\frac{G}{N}$ is abelian but G is not abelian.

(2) The quotient group of cyclic group is cyclic but converse is not true.

(3) Let Z(G) be a centre of a group G, then G is abelian if $\frac{G}{Z(G)}$ is cyclic.

(4) Let G be a cyclic group of order n, then number of factor group of G are $\tau(n)$ because number of normal subgroups are $\tau(n)$.



$\frac{Z_{10}}{Z_1} \mid \frac{Z_{10}}{Z_2} \mid \frac{Z_{10}}{Z_5} \mid \frac{Z_{10}}{Z_{10}}$

(5) $\dfrac{K}{N}$ is a subgroup of $\dfrac{G}{N}$, if K is a subgroup of G.

$$\frac{K}{N} \triangleleft \frac{G}{N} \quad if \quad K \triangleleft G$$

$G = Q_8$

$K = \langle \pm 1, \pm i \rangle$ $\quad H = \langle \pm 1 \rangle$

Result : If H be a subgroup of G and O(G) / O(H) = 2, then H is normal in G.

$\dfrac{K}{H} \cong Z_2$ $\qquad$ $\dfrac{Q_8}{H} \cong \dfrac{K_4}{\,}$

Group $\left(\frac{Q}{Z},+\right)$.

$$\frac{Q}{Z}=\{Z+a \mid a\in Q\}=\left\{Z,\left(Z+\frac{1}{2}\right),\left(Z+\frac{3}{2}\right)\left(Z+\frac{1}{3}\right)\ldots\ldots\right\}$$

(1) $\frac{Q}{Z}$ is abelian group but not cyclic group.

(2) Number of elements of order n are $\phi(n)$

(3) $\left(\frac{1}{p}+Z\right)\in\frac{Q}{Z}$ *then* $O\left(\frac{1}{p}+Z\right)=p$

$$And\ O\left(\frac{k}{p}+Z\right)=p\ \ if\ \gcd(k,p)=1$$

(3) We know that $\frac{1}{4}+Z\in\frac{Q}{Z}$ for order of $\frac{1}{4}+Z$.

(5) There exist a unique cyclic subgroup of each order i.e. For every positive integer n , there is a cyclic subgroup of order n which is unique which is $H = \left\langle \frac{1}{n} + Z \right\rangle$

Q.1. Consider the quotient group of $\frac{Q}{Z}$ of the additive group of rational number, the order of element $\frac{2}{3} + Z$ in $\frac{Q}{Z}$ is

(a) 2 (b) 3 (c) 5
(d) 6

Q.3. Consider the following subsets of the group of 2×2 non – singular matrices over R

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in R, ad = 1 \right\}$$

$$H = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} : b \in R \right\}$$

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$$

Which of the following statements are correct

(a) G form a group under matrix multiplication

(b) H is normal subgroup of G

(c) G/H is well define and is abelian

(d) None of these

COMPLETE COURSE ON
MATHEMATICS
FOR IIT-JAM 2022

TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINAER ALGEBRA
- MODERN ALGEBRA

TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780	You get 6 months extra for free — Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974	You get 6 months extra for free — Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475	
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252	
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807	
1 month		₹ 2,723 / mo	Total ₹ 2,723	

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	₹ ~~21,780~~ ₹ 19,602	You get 6 months extra for free — Offer expires 15 Jun 2022
12 months	Save 54%	₹ 1,123 / mo	₹ ~~14,974~~ ₹ 13,477	You get 6 months extra for free — Offer expires 15 Jun 2022
9 months	Save 45%	₹ 1,348 / mo	₹ ~~13,475~~ ₹ 12,128	
6 months	Save 25%	₹ 1,838 / mo	₹ ~~12,252~~ ₹ 11,027	
3 months	Save 17%	₹ 2,042 / mo	₹ ~~6,807~~ ₹ 6,126	

GPSIR

Awesome! You get 10% off

Proceed to pay

FOUNDATION COURSE OF

MATHEMATICS

FOR CSIR-NET

Q.4. Let G be a non-abelian group and Z(G) is its centre, then which of the following is cannot be possible of $O\left(\frac{G}{Z(G)}\right)$.

(a) 7 (b) 8

(c) 4 (d) 6

$$\frac{Q_8}{H} = K_4$$

Q.5. Number of factor group of Klein's 4 group is

(a) 5 (b) 4

(c) 6 (d) 7

K_4

$e \quad (e,a) \quad (e,b) \quad (e,c) \quad K_4$

$\frac{K_4}{e} \,\Big|\, \frac{K_4}{H_1} \,\Big|\, \frac{K_4}{H_2}$

$\frac{K_4}{H_3}, \frac{K_4}{K_4}$

Q.6 If G be a group such that $G/Z(G)$ is cyclic, then G is

(a) cyclic

(b) commutative

(c) non-commutative

(d) None of these

Q.7. If H be a subgroup of a cyclic group G, the G/H is

(a) cyclic (b) non-cyclic

(c) normal (d) none of these

$\frac{S_4}{A_4} = Z_2$

$\frac{S_4}{\{e\}} = S_4$

G — cyclic

$\Downarrow$

$\frac{G}{H}$ cyclic

G abelian

$\Downarrow$

$\frac{G}{H}$ abelian

Q.8. If H be a subgroup of a commutative group G, then G/H is

(a) cyclic (b) normal

(c) commutative (d) none of these

3,2

Q.9. If G be a non-commutative group G with centre Z, then G/Z is

(a) cyclic (b) non-cyclic

(c) commutative (d) none of these

$\frac{S_3}{Z(e)} \cong S_3$

$\frac{Q_8}{H} \cong K_4$

Q.10 If H be a subgroup of a group G and [G : H] = 2, then

(a) H is cyclic (b) H is commutative

(c) H is normal

(d) none of these

Q.11 If H be a normal subgroup of a finite group G, then

(a) ~~O(G/H) = O(G).O(H)~~

(b) ~~$O(G/H) = O(G).O\left(\frac{H}{G}\right)$~~

(c) ~~$O(G/H) = \frac{O(G)}{\gcd\{O(H),O(G)\}}$~~

(d) $O(G/H) = \frac{O(G)}{O(H)}$

Q.12. If $H \subset K$ are two normal subgroups of a group G and if $[G : H] = 10$ and $[G : K] = 5$, then $[K : H]$ is

(a) 5 (b) 2

(c) 10 (d) 50

$$\frac{\dfrac{\cancel{o(G)}}{o(H)} = 10}{\dfrac{\cancel{o(G)}}{o(K)} = 5}$$

$$\frac{o(K)}{o(H)} = \frac{10}{5} = 2$$

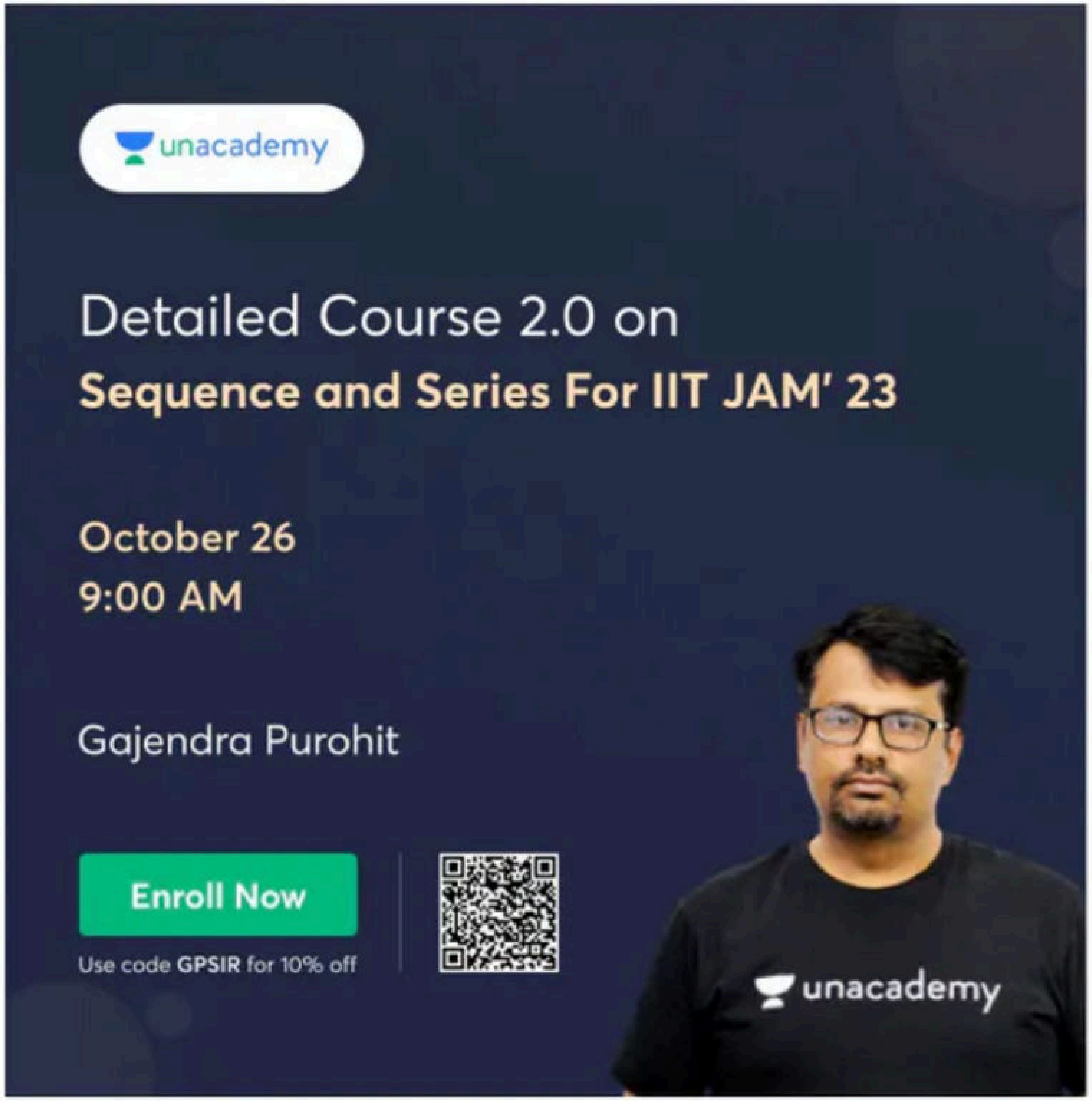
Q.13. Suppose N is a normal subgroup of a group G. Which one of the following is true?

(a) If G is an infinite group, then G/N is an infinite group.

(b) If G is a non-abelian group, then G/N is a non-abelian group.

(c) If G is a cyclic group, then G/N is an abelian group.

(d) If G is an abelian group, then G/N is a cyclic group.


unacademy
Detailed Course 2.0 on
Sequence and Series For IIT JAM' 23
October 26
9:00 AM
Gajendra Purohit
Enroll Now
Use code GPSIR for 10% off
unacademy


unacademy
DETAILED COURSE 2.0
GROUP THEORY FOR IIT JAM 2023
6th OCTOBER
Gajendra Purohit
Enroll Now
USE CODE
GPSIR
FOR 10% OFF
unacademy

unacademy CSIR-UGC NET Search Login

Educator Profile

Gajendra Purohit

#5 Educator in CSIR-UGC NET

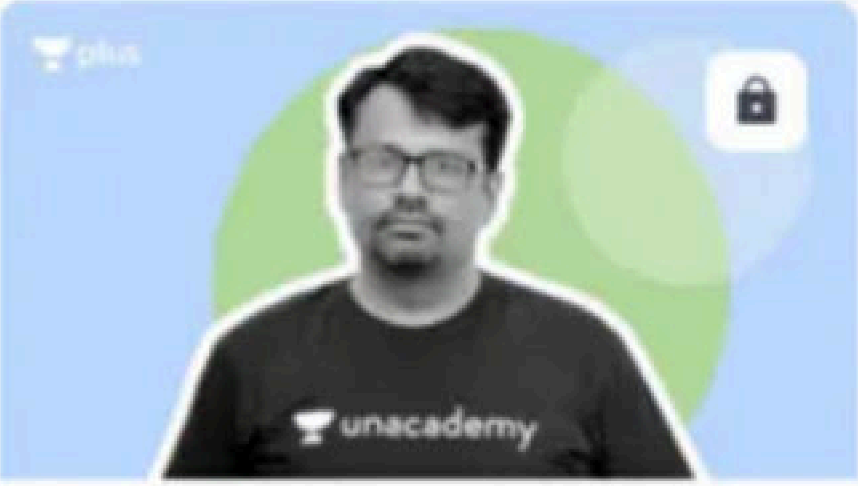
Dr.Gajendra Purohit PhD, CSIR NET (Maths) | Youtuber(330K+30k Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author of Bestseller

Follow

11M Watch mins **1M** Watch mins (last 30 days) **22k** Followers **1k** Dedications

CSIR-UGC NET

SEE ALL



HINDI MATHEMATICAL SCIENCES

Course on Linear Algebra, Partial Diff. Equation & Calculus

Starts on Mar 1, 2021 • 24 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Course on Complex Analysis & Integral Equation

Starts on Jan 14, 2021 • 16 lessons

Gajendra Purohit

HINDI MATHEMATICAL SCIENCES

Foundation Course on Mathematics for CSIR 2021

Starts on Dec 7, 2020 • 20 lessons

Gajendra Purohit

Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 908 / mo	Total ₹ 21,780
12 months	Save 54%	₹ 1,248 / mo	Total ₹ 14,974
9 months	Save 45%	₹ 1,497 / mo	Total ₹ 13,475
6 months	Save 25%	₹ 2,042 / mo	Total ₹ 12,252
3 months	Save 17%	₹ 2,269 / mo	Total ₹ 6,807
1 month		₹ 2,723 / mo	Total ₹ 2,723

24 months: You get 6 months extra for free. Offer expires 15 Jun 2022

12 months: You get 6 months extra for free. Offer expires 15 Jun 2022

To be paid as a one-time payment

Have a referral code?

Proceed to pay

Get 10% Off

After Using My Referral Code

GPSIR

Awesome! You get 10% off

No cost EMI available on 6 months & above subscription plans

24 months	Save 67%	₹ 817 / mo	₹ ~~21,780~~ ₹ 19,602
12 months	Save 54%	₹ 1,123 / mo	₹ ~~14,974~~ ₹ 13,477
9 months	Save 45%	₹ 1,348 / mo	₹ ~~13,475~~ ₹ 12,128
6 months	Save 25%	₹ 1,838 / mo	₹ ~~12,252~~ ₹ 11,027
3 months	Save 17%	₹ 2,042 / mo	₹ ~~6,807~~ ₹ 6,126

24 months: You get 6 months extra for free. Offer expires 15 Jun 2022

12 months: You get 6 months extra for free. Offer expires 15 Jun 2022

Proceed to pay