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Legendre's linear differential equation :

A linear differential equation of the form

$$[a_0(a+bx)^2\frac{d^2y}{dx^2}+a_1(a+bx)\frac{dy}{dx}+a_2y=X. \quad \ldots(1)$$

For solution

We put $a + bx = e^z$ & $z = \log(a + bx)$

$$\frac{dz}{dx}=\frac{b}{a+bx}$$

$$\frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}$$

$$\Rightarrow \frac{b}{a+bx}\frac{dy}{dz}$$

$$\Rightarrow (a+bx)\frac{dy}{dx}=bDy \text{ where } D=\frac{d}{dz}$$

$$(1+2x)^2\frac{d^2y}{dx^2}+(1+2x)\frac{dy}{dx}+y=4\cos\log(1+2x)$$

Put $1+2x=t$

$dx=dt$

$dx^2=dt^2$

$dx=dz$

$$t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+y=4\cos\log t$$

Put $t=e^z$ or $z=\log t$

$$t^2\frac{dy}{dx^2}=D(D-1)y,\quad t\frac{dy}{dx}=Dy$$

$$(D^2-D+D+1)y=4\cos z$$

$$(D^2+1)y=4\cos z$$

$$m^2+1=0$$

$$m=\pm i$$

$y=$ cf + PI

$$y=C_1\cos z+C_2\sin z+\frac{4z\sin z}{2D}$$

$$y=C_1\cos(\log t)+C_2\sin(\log t)+2\log t\sin(\log t)$$

$$y=C_1\cos(\log(1+2x))+C_2\sin(\log(1+2x))+2\log(1+2x)\sin(\log(1+2x))$$

Q.1. Solution $(5+2x)^2 \frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 0.$

(a) $y(x) = c_1(5+2x) + c_2(5-2x)$

(b) $y(x) = c_1(5+2x)^{2+\sqrt{2}} + c_2(5+2x)^{2-\sqrt{2}}$

(c) $y(x) = c_1(2+\sqrt{2})x + c_2(2-\sqrt{2}x)$

(d) none of these

$5+2x = t$

$2dx = dt$

$dx = dt/2$

$t^2 \frac{d^2y}{dt^2/4} - 6t\frac{dy}{dt/2} + 8y = 0$

$4t^2\frac{d^2y}{dt^2} - 12t\frac{dy}{dt} + 8y = 0$

$t = e^z, \quad z = \log t$

$4D(D-1)y - 12Dy + 8y = 0$

$(4D^2 - 4D - 12D + 8)y = 0$

$(4D^2 - 16D + 8)y = 0$

$4m^2 - 16m + 8 = 0$

$m^2 - 4m + 2 = 0$

$m = \frac{4 \pm \sqrt{16-8}}{2} = 2 \pm \sqrt{2}$

$y = c_1 e^{(2+\sqrt{2})z} + c_2 e^{(2-\sqrt{2})z}$

$y = c_1 t^{2+\sqrt{2}} + c_2 t^{(2-\sqrt{2})}$

$y = c_1(5+2x)^{2+\sqrt{2}} + c_2(5+2x)^{2-\sqrt{2}}$

Q.2. The general solution

$(1 + 2x)^2 y'' - 6(1 + 2x)y' + 16y = 8(1 + 2x)^2.$

(a) $c_1 \cos\{\log(1 + x)\} + c_2 \sin\{\log(1 + x)\} + 2\log(1 + x).\sin\{\log(1 + x)\}$

(b) $\frac{c_1}{x} + \frac{c_2}{x^2} - \frac{\sin x}{x^2}$

(c) $\{c_1 + c_2 \log(1 + 2x)\}(1 + 2x)^2 + (11 + 2x)^2 \{\log(1 + 2x)\}$

(d) None of these

$1+2x = e^t$
$2dx = dt$ | $dx = \frac{dt}{2}$

$4e^{2t}\frac{d^2y}{dt^2} - 6e^t\frac{dy}{dt} + 16y = 8e^{2t}$

$4e^{2t}\frac{d^2y}{dt^2} - 12e^t\frac{dy}{dt} + 16y = 8e^{2t}$

$e^t = e^z, \; z = \log t$

$e^{2z}\frac{d^2y}{dt^2} - 3e^z\frac{dy}{dz} + 4y = 2e^{2z}$

$(D(D-1) - 3D + 4)y = 2e^{2z}$

$(D^2 - 4D + 4)y = 2e^{2z}$ | $m^2 - 4m + 4 = 0$, $m = 2, 2$

$y = (c_1 + c_2 z)e^{2z} + z^2 e^{2z}$ $\quad y = (c_1 + c_2 z)e^{2z} + \frac{2z^2 e^{2z}}{2}$

$y = (c_1 + c_2 \log t)t^2 + (\log t)^2 t^2$

$y = (c_1 + c_2 \log(1+2x))(1+2x)^2 + (\log(1+2x))^2 (1+2x)^2$

Q.3. General solution of

$$(1+x^2)y'' + (1+x)y' + y = 4\cos\{\log(1+x)\}$$

(a) $\frac{c_1}{x} + \frac{c_2}{x^2} - \frac{\sin x}{x^2}$

(b) $\cos\{\log(1+x)\} + c_2\sin\{\log(1+x)\} + 2\log(1-$ 1x).sin{log(1 + x)}

(c) $\{c_1 + c_2\log(1+2x)\}(1+2x)^2 + (1+2x)^2\{\log(1+$ 2x)}

(d) None of these

Second Order Variable Coefficient Differential Equation

$$\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$$

When One Part of CF is known

The given differential equation is of the forms

$$\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R \qquad \ldots (1)$$

Where P, Q, R are as function of x alone

Suppose y = u be a known integral of CF

Now let CS is $y = u\,v$

Then v can be found by solution of equation

$$\frac{d^2v}{dx^2}+\left(P+\frac{2}{u}\frac{du}{dx}\right)\frac{dv}{dx}=\frac{R}{u}$$

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Rules for finding the one part of C.F

	Condition	An integral of CF
1	$P+Qx=0$	$y=x$
2	$2+2Px+Qx^2=0$	$y=x^2$
3	$m(m-1)+Pmx+Qx^2=0$	$y=x^m$
4	$1+P+Q=0$	$y=e^x$
5	$1-P+Q=0$	$y=e^{-x}$
6	$m^2+mP+Q=0$	$y=e^{mx}$

$x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$

$P = \frac{-2}{x}(1+x)$, $Q = \frac{2(1+x)}{x^2}$

$P + Qx = 0$

$k = x$

$\frac{d^2y}{dx^2} - \frac{2(1+x)}{x}\frac{dy}{dx} + \frac{2(1+x)}{x^2}y = x$

$y = uv = xv$

$(D^2 - 2D)v = 1$

$m^2 - m = 0$

$m = 0, 2$

$\frac{d^2v}{dx^2} + \left(P + \frac{2}{u}\frac{du}{dx}\right)\frac{dv}{dx} = \frac{R}{u}$

If $v = c_1 + c_2 e^{2x}$

$v = \frac{e^{0x}}{D^2 - 2D} = \frac{xe^{0x}}{2D - 2} = \frac{xe^{0x}}{-2}$

$\frac{d^2v}{dx^2} + \left[-\frac{2(1+x)}{x} + \frac{2}{x}\right]\frac{dv}{dx} = \frac{x}{x}$

$v = -x/2$

$\frac{d^2v}{dx^2} + \left(-\frac{2}{x} - 2 + \frac{2}{x}\right)\frac{dv}{dx} = 1$

$v = c_1 + (c_2 e^{2x} - x/2)$

$\frac{d^2v}{dx^2} - 2\frac{dv}{dx} = 1$

$y = x(c_1 + c_2 e^{2x} - x/2)$

$y = xc_1 + xc_2 e^{2x} - x^2/2$

y_c $\quad$ y_p

Q4. Given that $y(x)=x$ is a solution of differential equation

$$(1+x^2)y''-2xy'+2y=0,\ x>0$$

Find second linearly independent solution

(a) (x^2-1) (b) $\frac{1}{x}$

(c) e^x (d) e^{-x}

$y = v^2 - 1$

$y' = 2v$

$y'' = 2$

$2(1+v^2) - 2v(2v) + 2(v^2-1)$

$2 + 2v^2 - 4v^2 + 2v^2 - 2 = 0$

Q5. Let $y = e^x$ be a solution of $x\frac{d^2y}{dx^2} - \frac{dy}{dx} + (1-x)y = 0$.

Then the second linearly independent solution of this ordinary differential equation is

(a) $xe^{-2x} + \frac{1}{2}$ (b) $\frac{1}{2}\left(x - \frac{1}{2}\right)e^{-x}$

(c) $\frac{1}{2}\left(x + \frac{1}{2}\right)e^{-2x}$ (d) $xe^{-2x} - \frac{1}{2}$

Q.6. If $y = x^2$ is a solution of the differential equation $y`` - \left(\frac{2}{x^2} + \frac{2}{x}\right)(xy` - y) = 0$, $0 < x < \infty$, then its general solution is

(a) $\alpha x^2 \int x^{-2} e^x dx + \beta$

(b) $\alpha x^{-2} \int x^2 e^x dx + \beta$

(c) $\alpha x^2 \int x^2 e^x dx + \beta$

(d) None of these

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

$$I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4} = \text{const}$$

$$\text{Reduced } u = e^{-\frac{1}{2}\int P\,dx}$$

$$y = uv$$

$$\frac{d^2v}{dx^2} + Iv = Rv$$

$$\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} - y = 0$$

Part d- $u = e^{-\frac{1}{2}\int P\,dx}$

$$u = e^{-\frac{1}{2}\int -2\tan x\,dx}$$

$$= e^{\int \tan x\,dx}$$

$$= e^{\log \sec x} = \sec x$$

$y = \sec x\, v$ = $\sec x(c_1 x + c_2)$

$\frac{d^2v}{dx^2} + I v = R$ ⇒ $\frac{d^2v}{dx^2} = 0$

$\frac{dv}{dx} = c_1$

$v = c_1 x + c_2$

$P = -2\tan x,\quad Q = -1,\quad R = 0$

$I = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4}$

$= -1 - \frac{1}{2}(-2\sec^2 x) - \frac{4\tan^2 x}{4}$

$= -1 + \sec^2 x - \tan^2 x$

$= -1 + 1 = 0$


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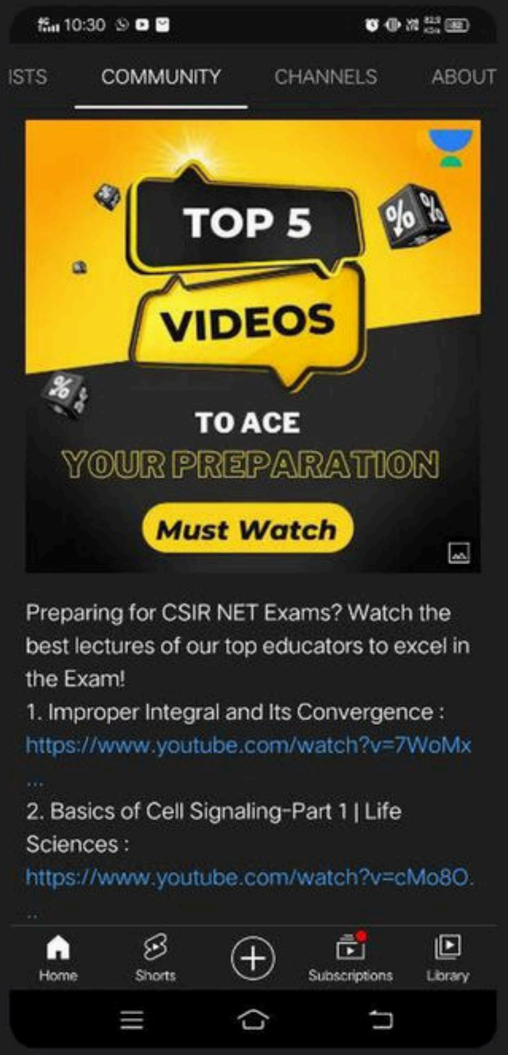

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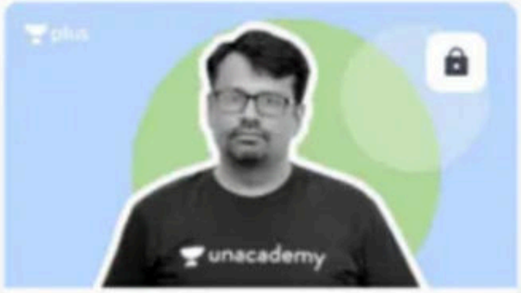

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