Differentiability - Part I

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 5 • Nov 15, 2022

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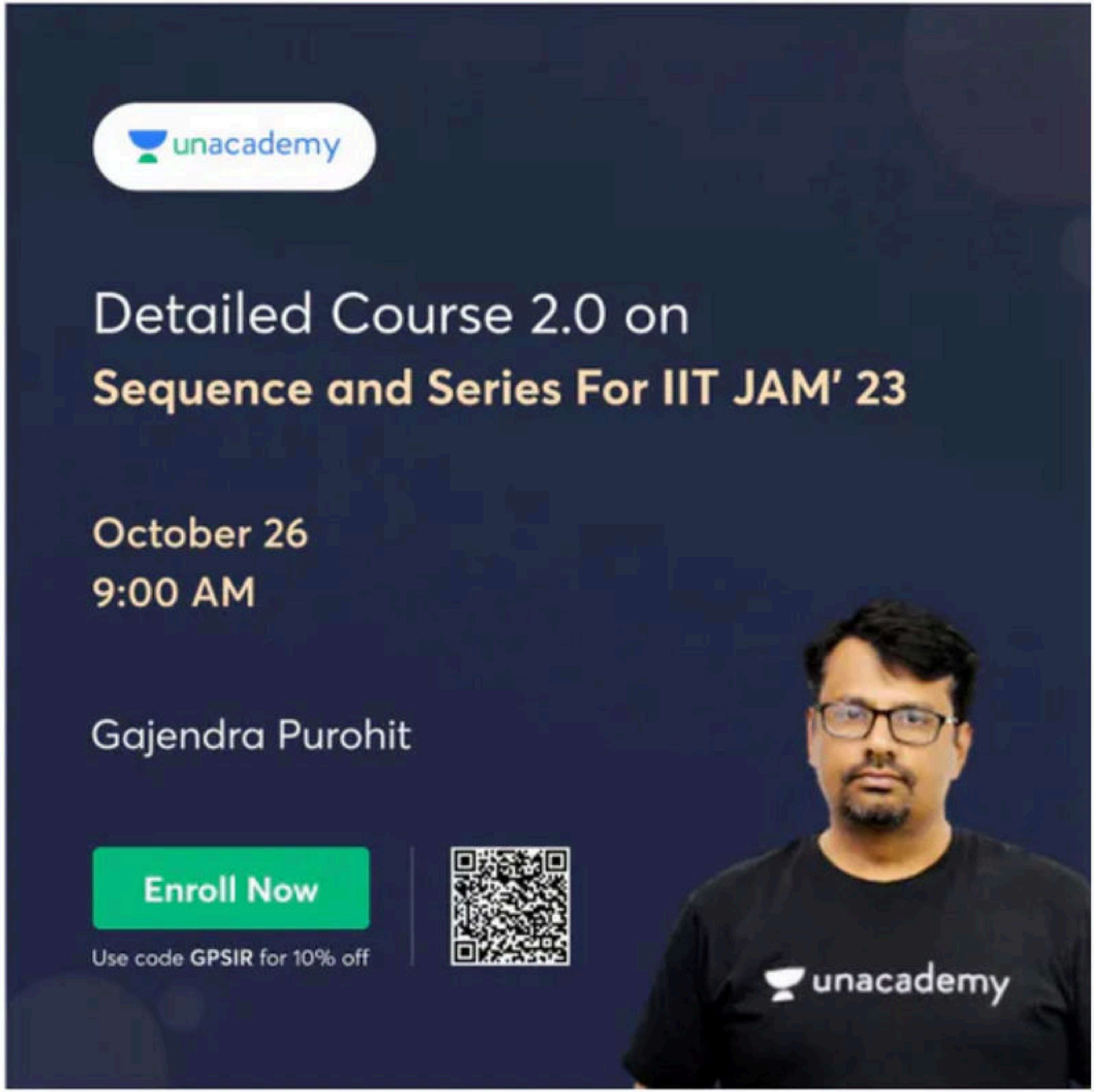
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Difference between continuity and uniformly continuity :

Continuity of a function is at point and uniformly continuity is in interval.

Example : $f(x) = x$ is continuous at 0.

But it is uniformly continuous in $[0, 1]$

Lipschitz function :

A function $f : I \to R$ is said to be satisfy a Lipschitz condition

on I, if $\exists$ a positive integer M such that

$|f(x_1) - f(x_2)| \le M|x_1 - x_2|$, for any two $x_1, x_2 \in I$

$f(x) = x^2 \quad x \in [0, 2]$

$|f(x_1) - f(x_2)| = |x_1^2 - x_2^2|$

$= |(x_1 - x_2)(x_1 + x_2)|$

$\le |x_1 + x_2| |x_1 - x_2|$

$\le (2 + 2) |x_1 - x_2|$

$\le 4 |x_1 - x_2|$

$f(x) = \sin x, \; x \in \mathbb{R}$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$|f(x_1) - f(x_2)| = |\sin x_1 - \sin x_2|$$

$$= \left|2\cos\left(\frac{x_1+x_2}{2}\right)\sin\left(\frac{x_1-x_2}{2}\right)\right|$$

$$\leq 2\left|\cos\left(\frac{x_1+x_2}{2}\right)\right|\left|\sin\left(\frac{x_1-x_2}{2}\right)\right|$$

$$\leq 2 \cdot 1 \cdot \left|\frac{x_1-x_2}{2}\right|$$

$$\leq 1 \cdot |x_1 - x_2|$$

$|\cos x| \leq 1$

$|\sin x| \leq |x|$

Some direct result for uniformly continuity:

(1) If a function f(x) is Lipschitz function then f is uniformly continuous.

(2) If f(x) is continuous on closed interval [a, b] then it is uniformly continuous on [a, b].

(3) If a function is UC then it is continuous

i.e. If a function is not continuous then it is not UC

Sequential definition :

Let f : D → R be a function. If < x_n> & <y_n> are two convergent sequence which converge to same limit and f(<x_n>) and f(<y_n>) are also converges to same limit then this function is uniformly continuous on D.

$$f(x) = \sin\frac{1}{x}$$

$$x \in (0,1) = I$$

$$x_n = \frac{1}{n\pi}$$

$$y_n = \frac{1}{(2n+1)\pi/2}$$

Conclusion : If f(x) is bounded and continuous on I, then f(x) may not be uniformly continuous on I.

(4) Let a function f be continuous on an open bounded interval (a, b), then f is uniformly continuous on (a, b) if $\lim\limits_{x \to a^+} f(x)$ & $\lim\limits_{x \to b^-} f(x)$ both exist finitely. it is necessary condition.

(5) If derivative of f(x) is bounded on I, then f(x) is uniformly continous on I.

(6) If f(x) is uniformly continuous on [a, c] and [c, b] both & f(x) is continuous at c, then f(x) is uniformly continuous on [a, b].

$f(x) = \sqrt{x}$

$x \in [0, \infty)$

$f(x) = \sqrt{x}$ $[0, 1]$

$f(x) = \sqrt{x}$ $(1, \infty)$

$\frac{1}{2\sqrt{x}}$

$f + g$

fg

Q.1. Let $f,g : (0,1) \to R$. Let $f(x) = x\sin(1/x^2)$ and $g(x) = x^2$ then

(a) Both are uniformly continuous

(b) f is uniformly continuous but g is not

(c) g is uniformly continuous but f is not

(d) None of the above

Q.2. Which of the following functions is uniformly continuous on the domain as stated? **IIT JAM**

$[0,4]$

(a) $f(x) = x^2, x \in R$ (b) $f(x) = \frac{1}{x}, x \in [1, \infty)$

(c) $f(x) = \tan x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(d) $f(x) = [x], x \in [0, 1]$

([x] is the greatest integer less than or equal to x] let $f(x) = e^{\frac{1}{x}}$ in which of following Domain

it is not UC

(a) $(0,1)$ (b) $(1,\infty)$ (c) $(1,2)$

(d) $(-\infty,-1)$

which of following is not uc

(a) $f(x) = \sin x \cos\left(\frac{1}{x}\right) \quad \forall x \in (0,1)$

(b) $f(x) = \sin x \cos\left(\frac{1}{x}\right) \quad \forall x \in (0,1)$

(c) $f(x) = \sin^2 x \quad \forall x \in (0,\infty)$

(d) None

$0 \times (-)$

$2 \sin x \cos x$

$0 \times [-1,1]$

$\sin 2x$

$$\lim_{x \to 0} x \sin \frac{1}{x} \qquad = 0 \times (-1, 1)$$

Q.3. Let f : (0 , ∞) → R be defined by $f(x) = \frac{\sin(x^3)}{x}$.

then f(x) is **TIFR 2019**

(a) ~~bounded~~ and uniformly continuous

(b) bounded but not uniformly continuous

(c) Not bounded but uniformly continuous

(d) Neither bounded nor uniformly continuous

$\lim_{x \to 0} \frac{\sin x^3}{x}$

$\lim_{x \to 0} \frac{3x^2 \cos x^3}{1}$

$\frac{1}{x} \sin x^3$

$0 \times [-1, 1]$

$\frac{1}{x} \sin x^3$

$0 \times [-1, 1]$

Q4. Let $f(x) = e^{-x}$ and $g(x) = e^{-x^2}$.

Which of the following statements are true?

(a) Both f and g are uniformly continuous on R

(b) f is uniformly continuous on every interval of the form $[a, +\infty)$, $a \in R$

(c) g is uniformly continuous on R

(d) f(x) g(x) is uniformly continuous on R

$e^{-\infty}$ R $(-\infty, \infty)$

$f(x) = e^{-x}$ e^{∞} $e^{-\infty} = 0$

$g(x) = e^{-x^2}$ $e^{-\infty} = 0$ $e^{-\infty} = 0$

$e^{-x} \cdot e^{-x^2} = e^{-x-x^2} = e^{-x^2\left[\frac{1}{x}+1\right]}$

Q5. Which of the following functions are uniformly continuous on (0, 1)? **CSIR NET NOV 2020**

(a) $\frac{1}{x}$ (b) $\sin \frac{1}{x}$

(c) $x \sin \frac{1}{x}$ (d) $\frac{\sin x}{x}$

(a) a, b

(b) b, c

(c) c, d

(d) a, d

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Differentiability

Let f : [a, b] → R is a real valued function it is said to be a differentiable at x = c.

If $\lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$ finitely exist.

Right Hand Derivative :

$$Rf`(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

Left Hand Derivative :

$$Rf`(c) = \lim_{h \to 0} \frac{f(c-h) - f(c)}{-h}$$

Note : If f(x) is differentiable at x = c

Iff $Rf`(c) = Lf`(c)$

Q.1. Let $f(x)=\begin{cases}\dfrac{\sin x}{x} & if \quad x\neq 0\\ 1 & if \quad x=0\end{cases}$, then f is

(a) Discontinuous

(b) Continuous but not differentiable

(c) Differentiable only once

(d) Differentiable more than once.

Necessary condition for differentiable :

If a function is differentiable at $x = c$, then it is continuous at $x = c$ but converse may not be true.

Conclusion :

$$f(x)=\begin{cases} x^{\alpha}\sin\dfrac{1}{x^{\beta}}; & x\neq 0 \\ 0; & x=0 \end{cases}$$

(i) f(x) is continuous at x = 0 for $\alpha > 0$

(ii) f(x) is differentiable at x = 0 for $\alpha > 1$

Result :

(1) $$f(x)=\begin{cases} x^{\alpha}\sin\dfrac{1}{x^{\beta}}; & x\neq 0 \\ 0; & x=0 \end{cases}$$

then f(x) is differentiable $\left[\dfrac{p}{q}\right]$ times and

$$f^{\left[\frac{p}{q}\right]}=\begin{cases} \text{continuous} & \text{if } p \text{ is odd} \\ \text{discontinuous} & \text{if } p \text{ is even} \end{cases}$$

(2) Let f : R → R be defined by $f(x)=\begin{cases}\phi_1(x) & x\in Q\\ \phi_2(x) & x\in Q^c\end{cases}$

f(x) is differentiable at the double root of $\phi_1(x) - \phi_2(x) = 0$

(3) A function is not differentiable at that point at which graph of function is sharp edge.

Q.2. Consider the function $f(x) = |\cos x| + |\sin(2 - x)|$.

At which of the following points is f not differentiable?

(a) $\left\{(2n+1)\frac{\pi}{2} : n \in Z\right\}$

(b) $\{n\pi : n \in Z\}$

(c) $\{n\pi + 2 : n \in Z\}$

(d) $\left\{\frac{n\pi}{2} : n \in Z\right\}$

Q.3. The function $f(x) = a_0 + a_1|x| + a_2|x|^2 + a_3|x|^3$ is differentiable at $x = 0$

(a) for no values of a_0, a_1, a_2, a_3

(b) for any value of a_0, a_1, a_2, a_3

(c) only if $a_1 = 0$

(d) only if both $a_1 = 0$ and $a_3 = 0$


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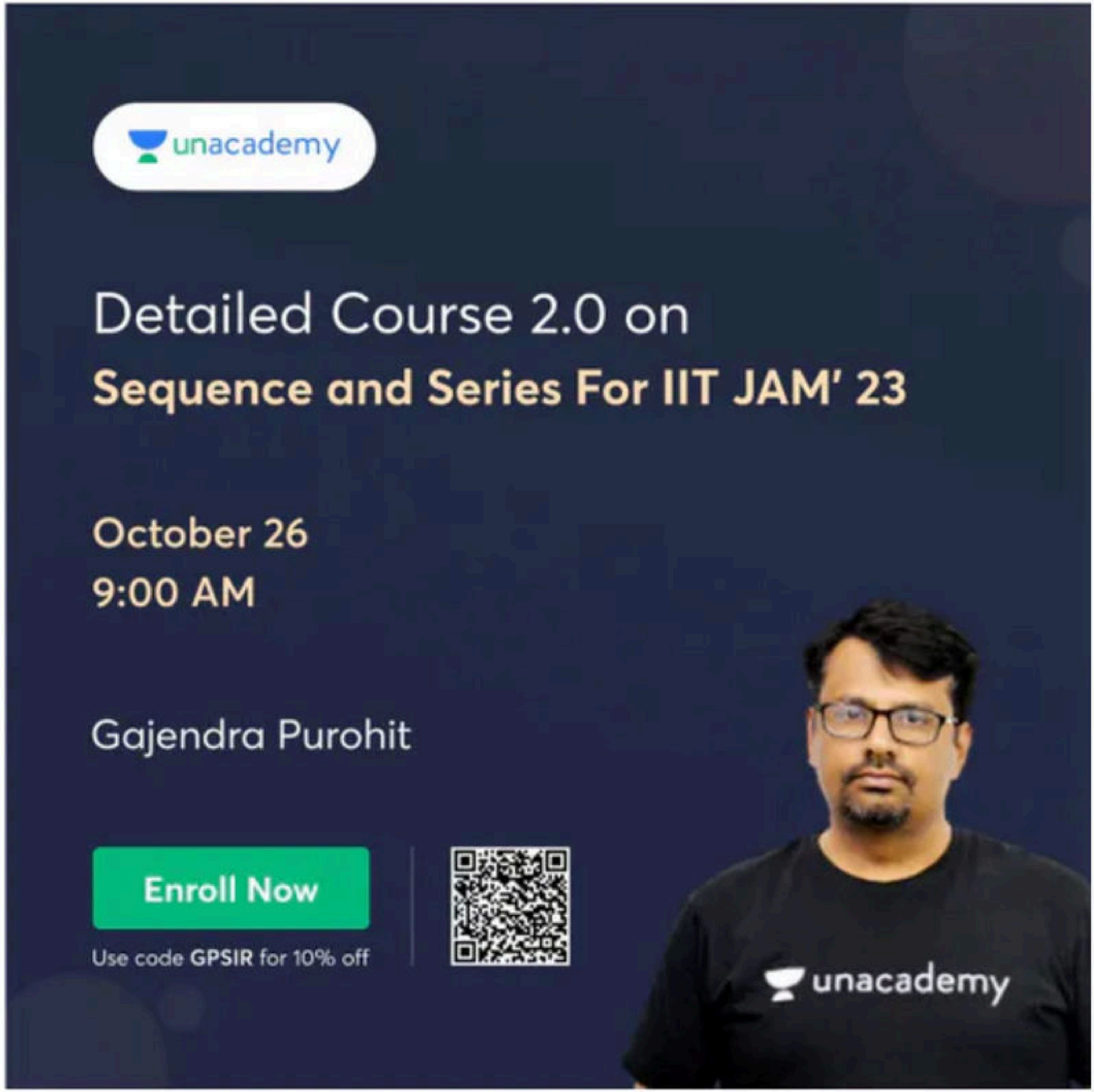
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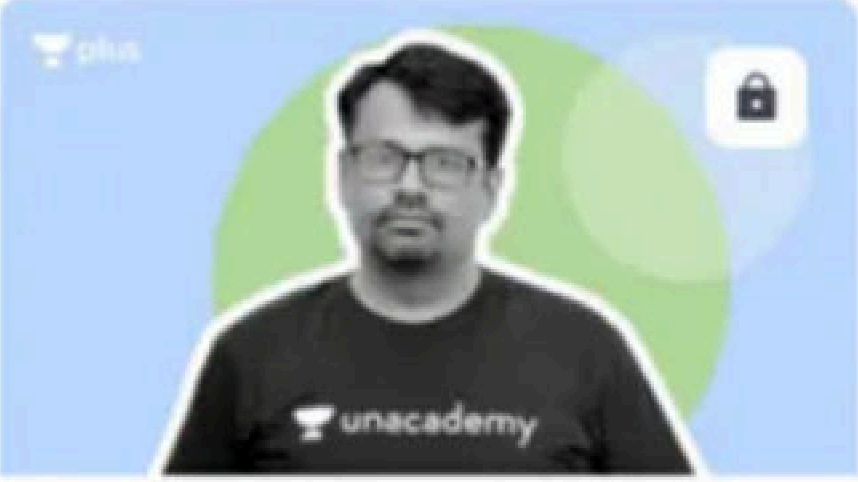

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- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
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- Lives in **Udaipur, Rajasthan, India**
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