Integral Calculus - Part VII

Detailed Course on Integral Calculus - IIT JAM' 23

Gajendra Purohit • Lesson 9 • July 20, 2022


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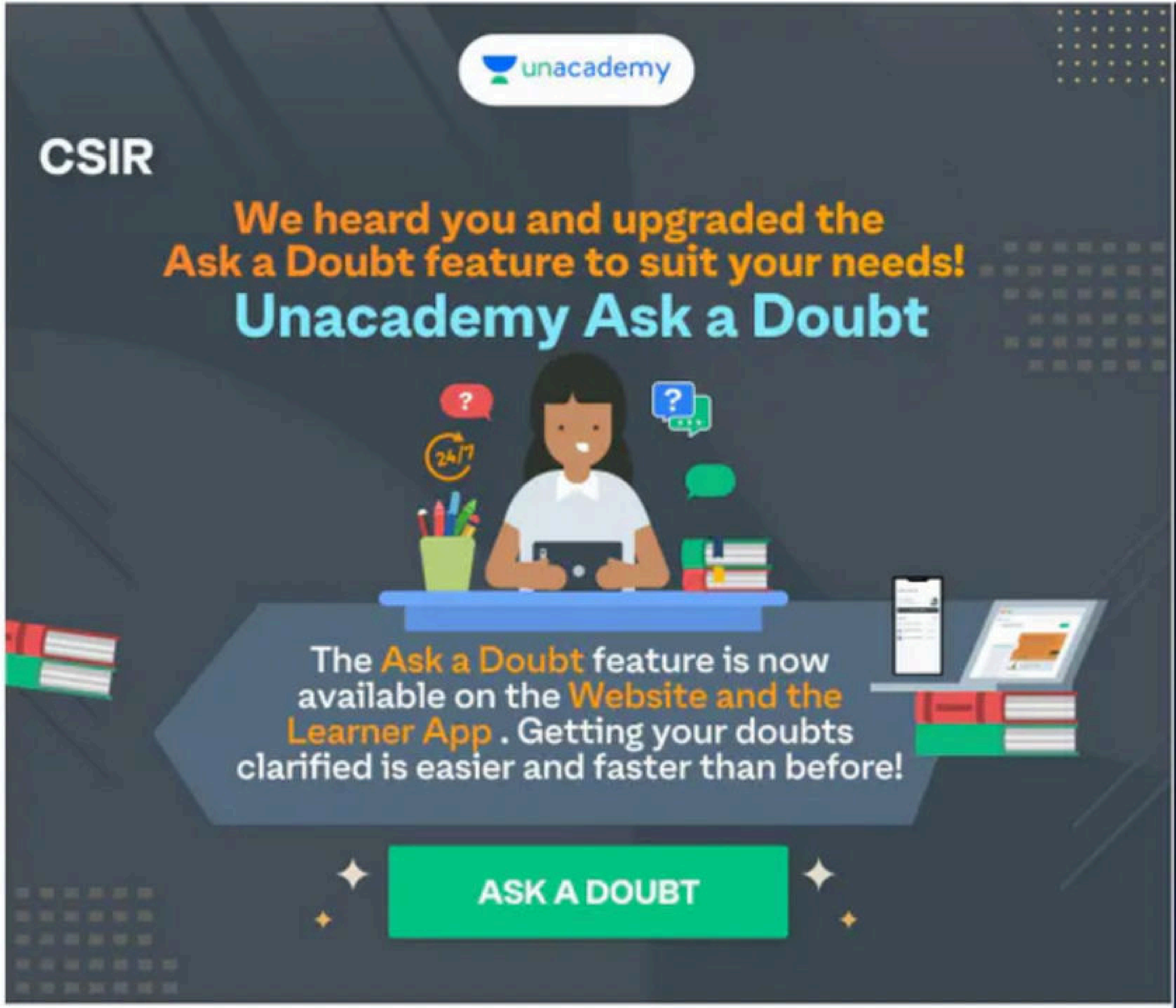
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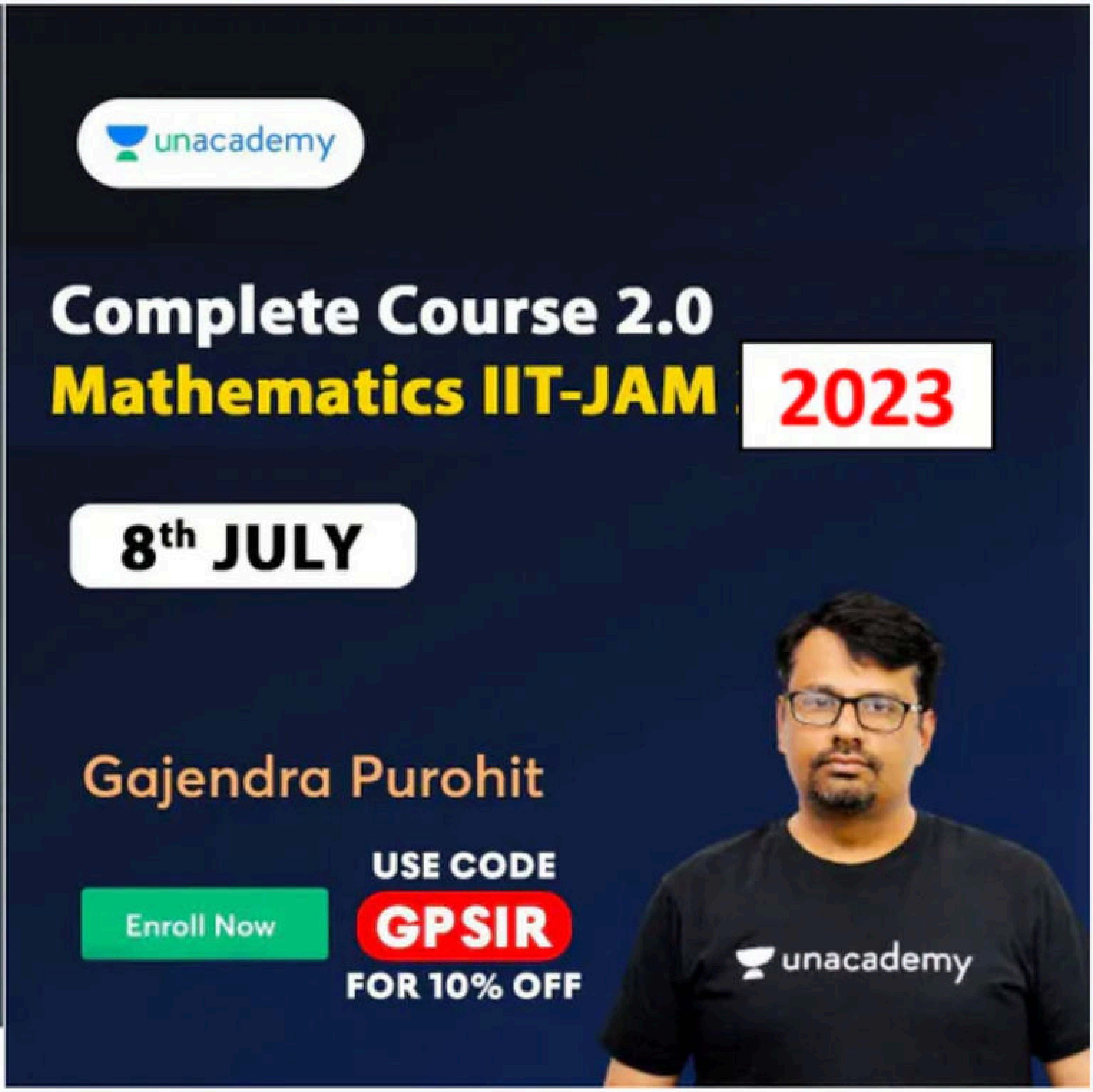
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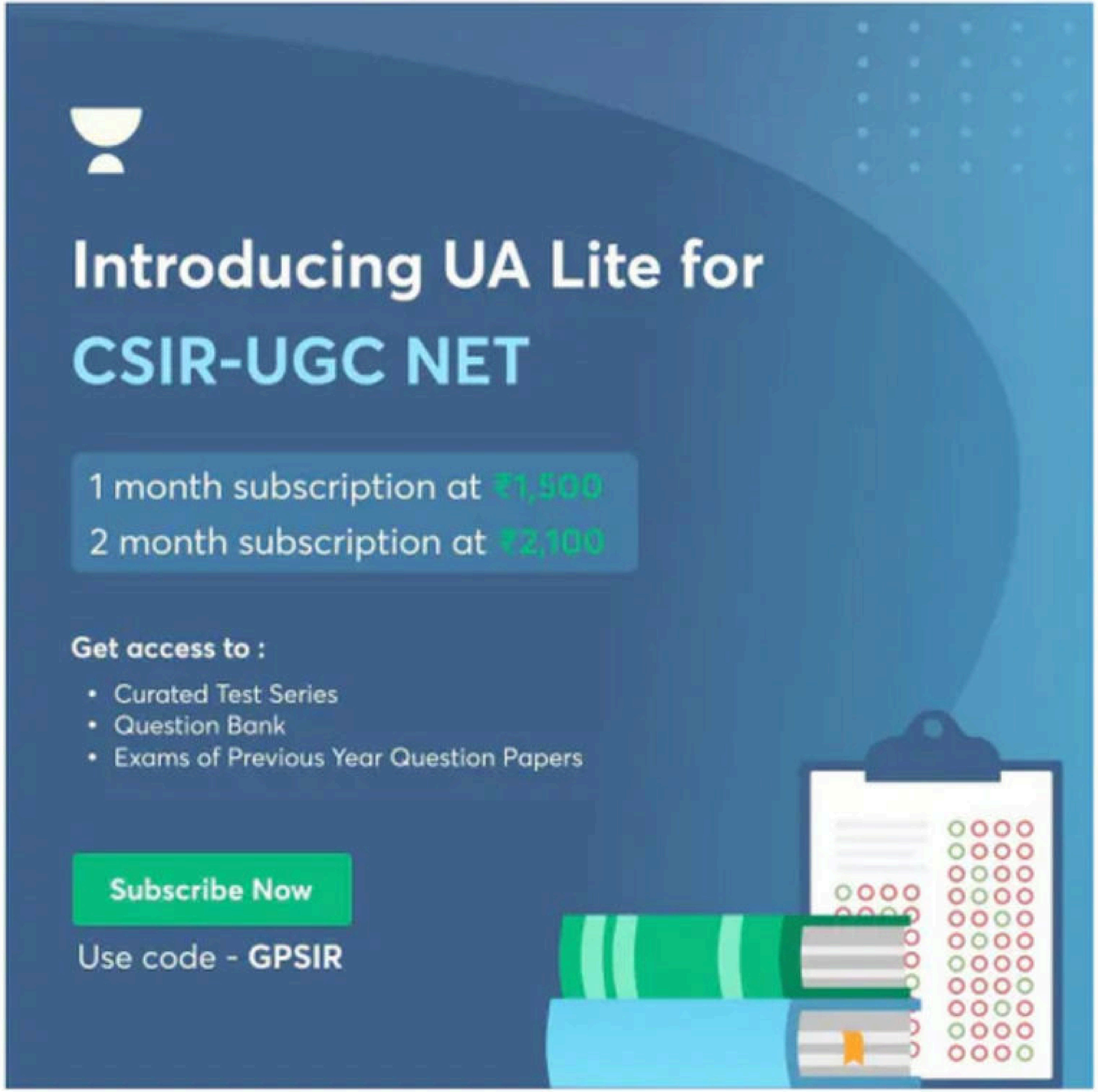
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Change of order of integration

The integral $\iint f(x,y)dydx$ is first integrated w.r.t. the variable 'y', then limit of y are substituted but if we want first integrate w.r.t. 'x' instead of y i.e. we want to change $\iint f(x,y)dydx$ to $\iint f(x,y)dxdy$, then we have to find new limit of x as function of y. This method is called change of order.

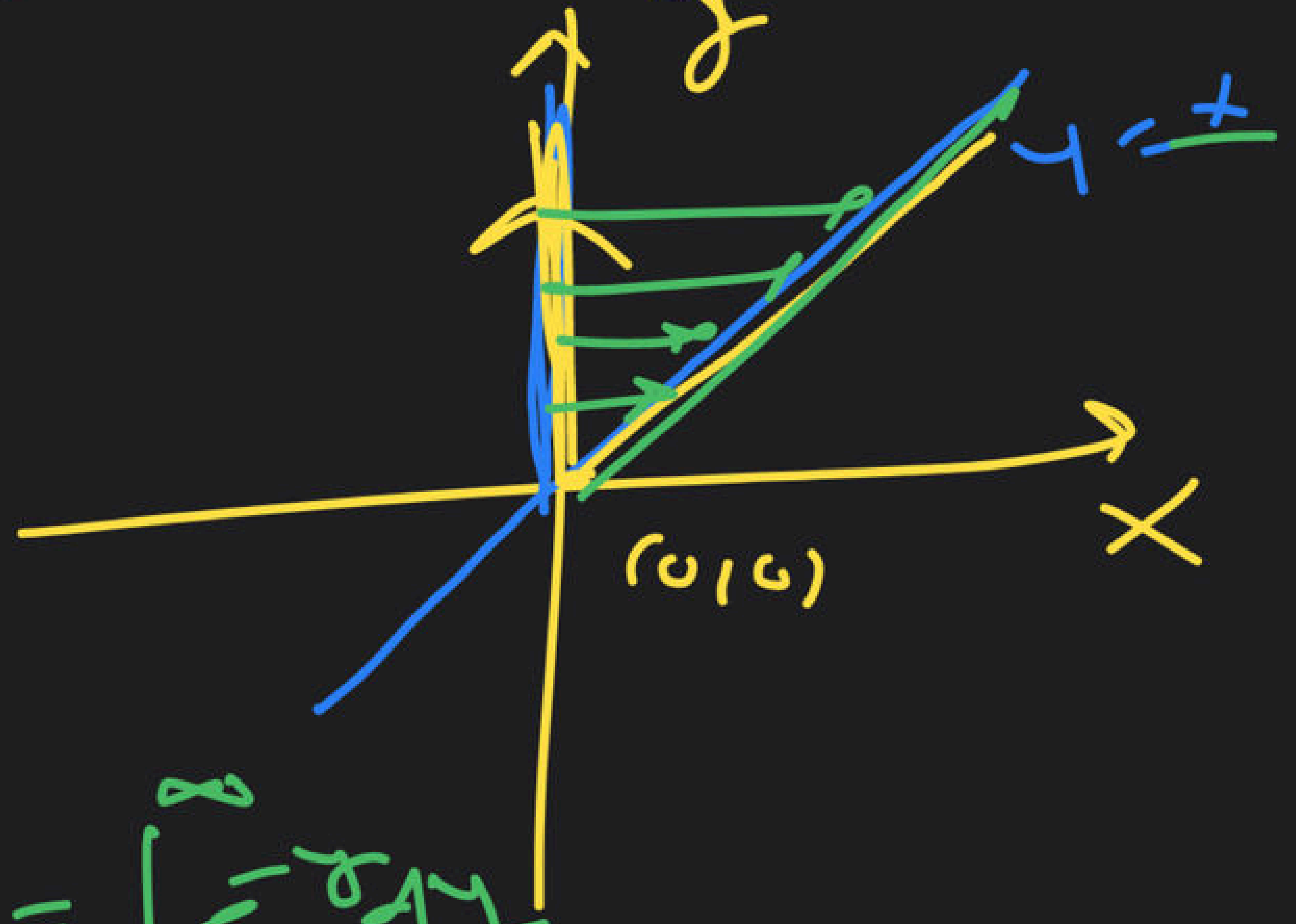
We can do easily by graph

$$\int_{x=0}^{\infty}\int_{y=x}^{\infty}\frac{e^{-y}}{y}\,dy\,dx$$

$$\frac{x=0}{x=\infty}\;\Big|\;\frac{y=x}{y=\infty}$$

$$e^{-\infty}=0$$

$$\int_{y=0}^{\infty}\int_{x=0}^{y}\frac{e^{-y}}{y}\,dx\,dy$$



$$\int_{y=0}^{\infty}\frac{e^{-y}}{y}(x)_0^{y}\,dy$$

$$\int_{y=0}^{\infty}\frac{e^{-y}}{\cancel{y}}(\cancel{y}-0)\,dy=\int_0^{\infty}e^{-y}\,dy$$

$$=-\left(e^{-y}\right)_0^{\infty}=-\left(e^{-\infty}-e^{-0}\right)=1$$

$$\int_{x=0}^{a/2}\int_{y=x^2/a}^{x-x^2/a} f(x,y)\,dy\,dx$$

$x=0$

$x=a/2$

$y=x^2/a$

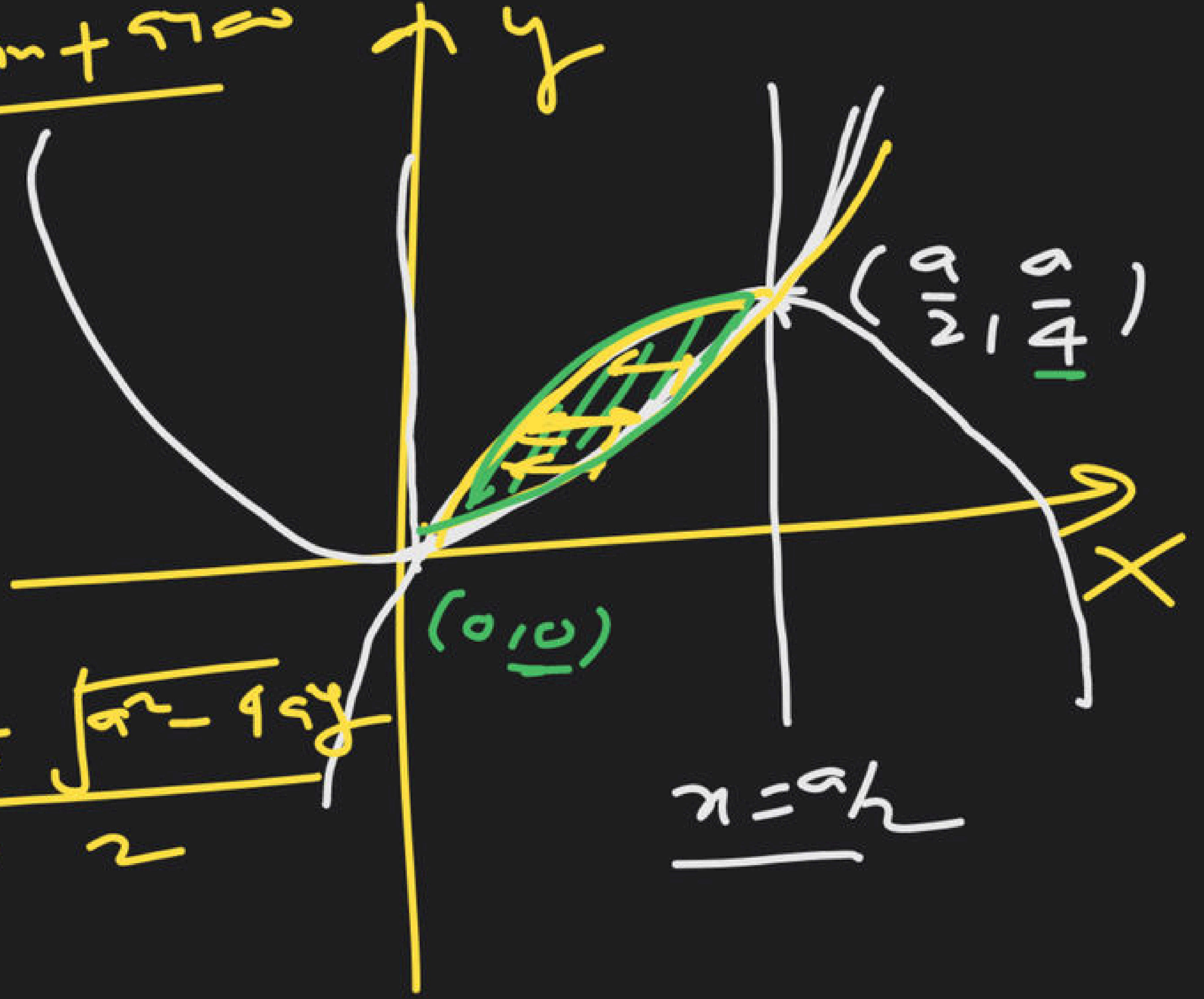
$y=x-x^2/a$

$\frac{x^2}{a}-x+y=0$

$x^2-ax+ay=0$

$$x=\frac{a\pm\sqrt{a^2-4ay}}{2}$$

$$\int_{y=0}^{a/4}\int_{x=f(y)}^{\sqrt{ay}} f(x,y)\,dx\,dy$$

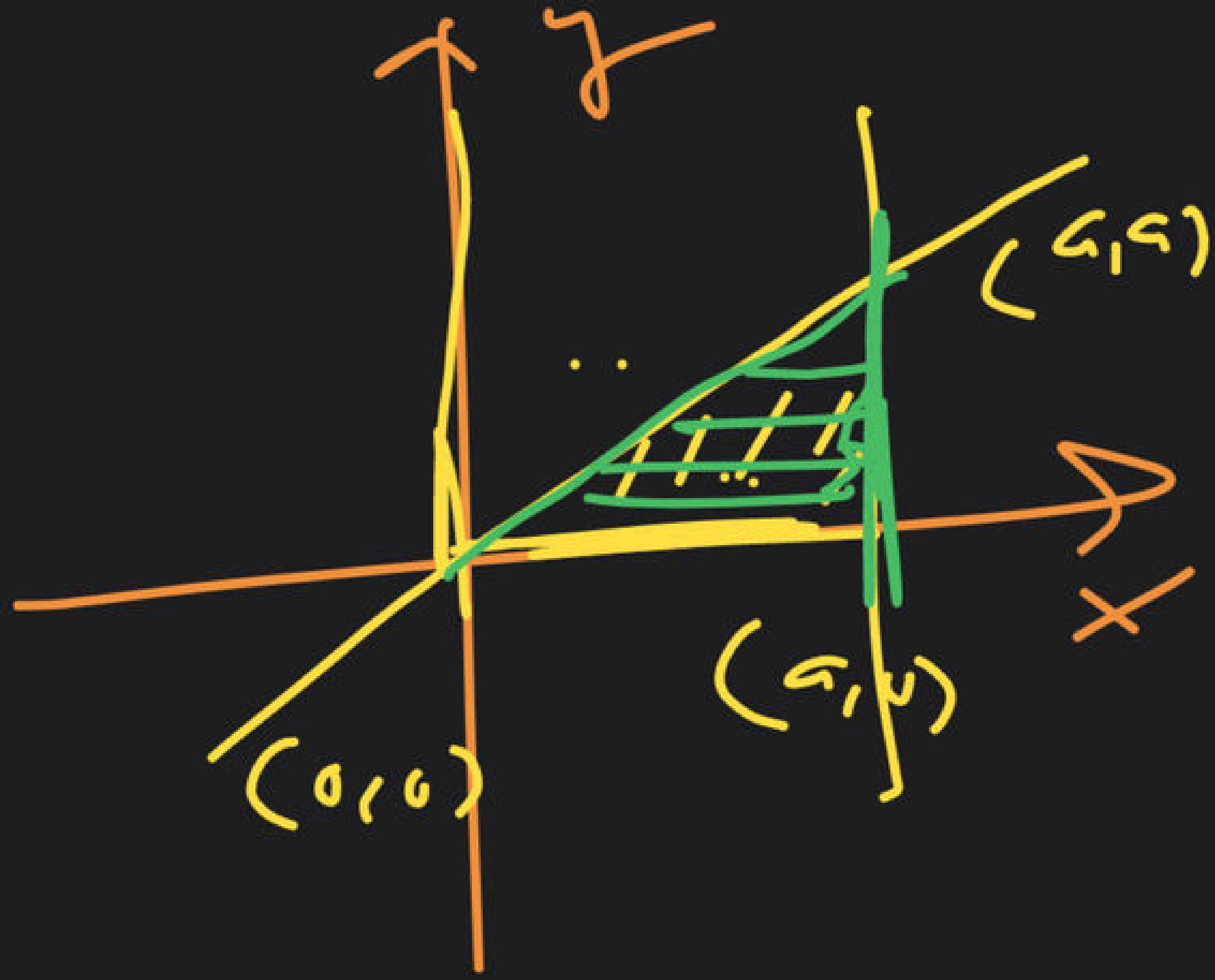


$$\int_{x=0}^{a}\int_{y=0}^{x} f(x,y)\,dy\,dx$$

$$\int_{y=0}^{a}\int_{x=y}^{a} f(x,y)\,dx\,dy$$

$x=0$ | $y=0$

$x=a$ | $y=x$



Q.1. The value of $\int_0^1\int_x^1 \sin(y^2)\,dy\,dx$. **JAM – 2017**

$x=0,\ x=1 \quad \big|\ y=x,\ y=1$

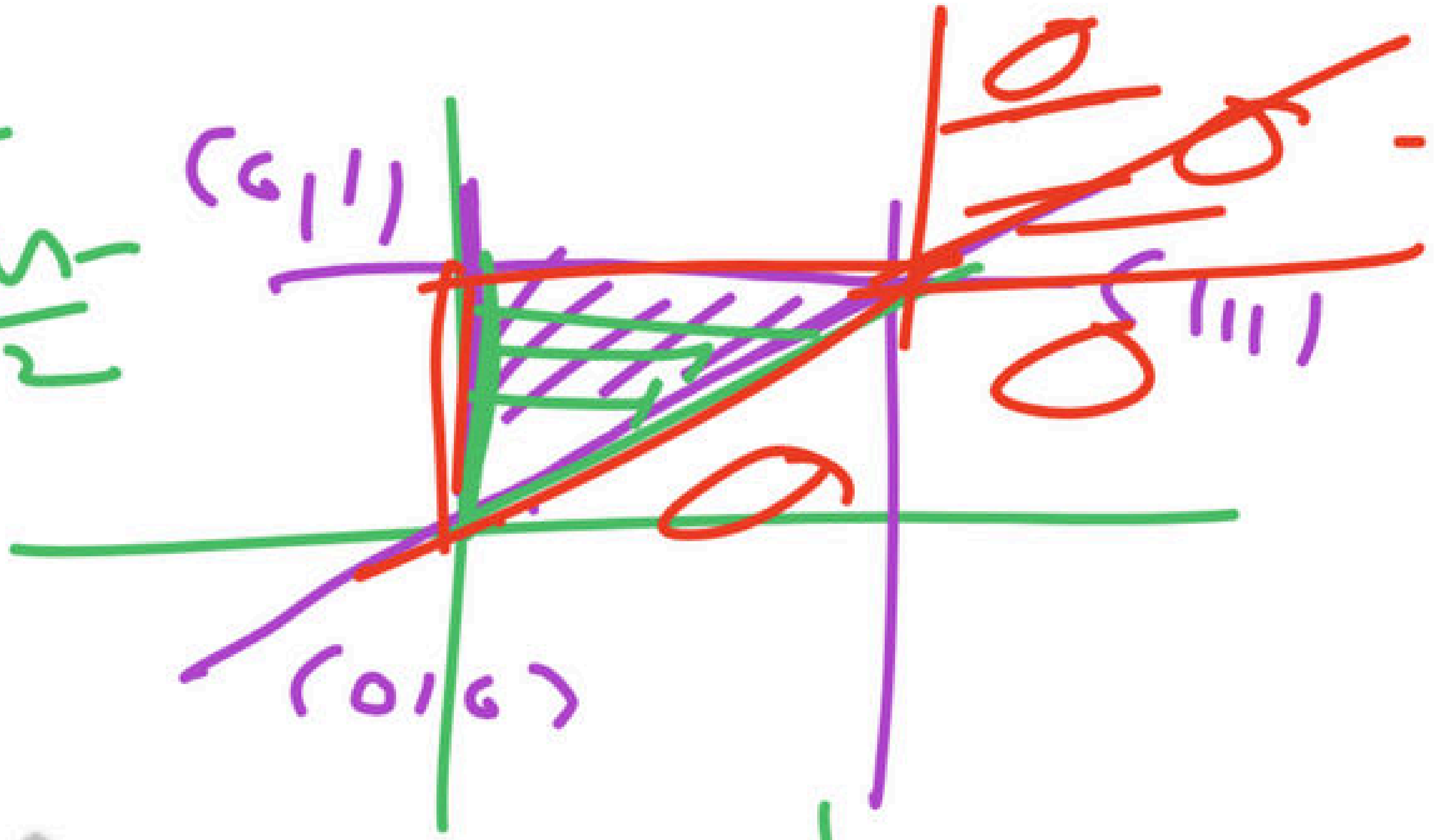
(a) $\frac{1+\cos 1}{2}$ (b) $1-\cos 1$

(c) $1+\cos 1$ (d) $\frac{1-\cos 1}{2}$

$y^2 = t$

$2y\,dy = dt$

$y\,dy = \frac{dt}{2}$



$$\int_{y=0}^{1}\int_{x=0}^{y} \sin y^2\,dx\,dy = \int_{y=0}^{1} \sin y^2\,(x)_0^y\,dy = \int_0^1 y\sin y^2\,dy$$

$$\int_0^1 \sin t\,\frac{dt}{2} = -\frac{1}{2}(\cos t)_0^1 = -\frac{1}{2}(\cos 1 - 1)$$

$$= \frac{1}{2}(1-\cos 1)$$

Q2. The value of the integral $\int_{y=0}^{1}\int_{x=0}^{1-y^2} y\sin(\pi(1-x)^2)\,dx\,dy$ is

$y=0,\ y=1 \quad \Big|\quad x=0,\ x=1-y^2$

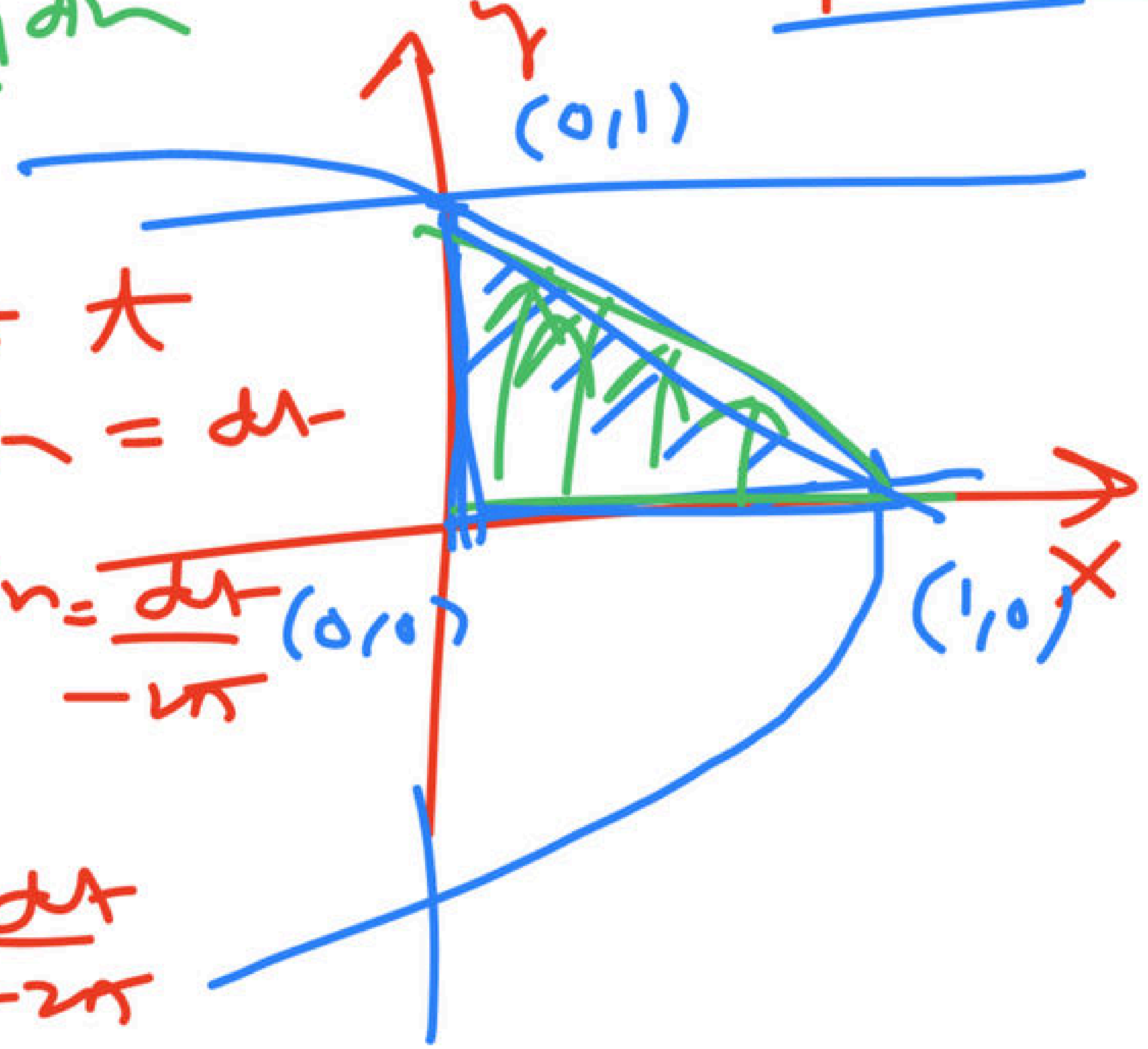
$y^2=-(x-1)$

JAM-2019

$$\int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x}} y\sin[\pi(1-x)^2]\,dy\,dx$$

(a) $\frac{1}{2\pi}$ (b) 2π

(c) $\pi/2$ (d) $2/\pi$



$\pi(1-x)^2=t$

$-2\pi(1-x)\,dx=dt$

$(1-x)\,dx=\frac{dt}{-2\pi}$

$$\int_{x=0}^{1}\left(\frac{y^2}{2}\right)_0^{\sqrt{1-x}}\sin[\pi(1-x)^2]\,dx$$

$$\frac{1}{2}\int_0^1 (1-x)\sin[\pi(1-x)^2]\,dx=\frac{1}{2}\int_{\pi}^{0}\sin t\,\frac{dt}{-2\pi}$$

$$=\frac{1}{4\pi}\int_0^{\pi}\sin t\,dt=\frac{-1}{4\pi}(\cos t)_0^{\pi}=\frac{-1}{4\pi}(\cos\pi-\cos 0)$$

$$=-\frac{1}{4\pi}(-1-1)=2/4\pi=1/2\pi$$

Q.3. The value of integral $\int_0^1\int_x^1 y^4 e^{xy^2}\,dy\,dx$ is **JAM – 2018**

$x=0$, $x=1$ | $y=x$, $y=1$

$x=0$, $y=$

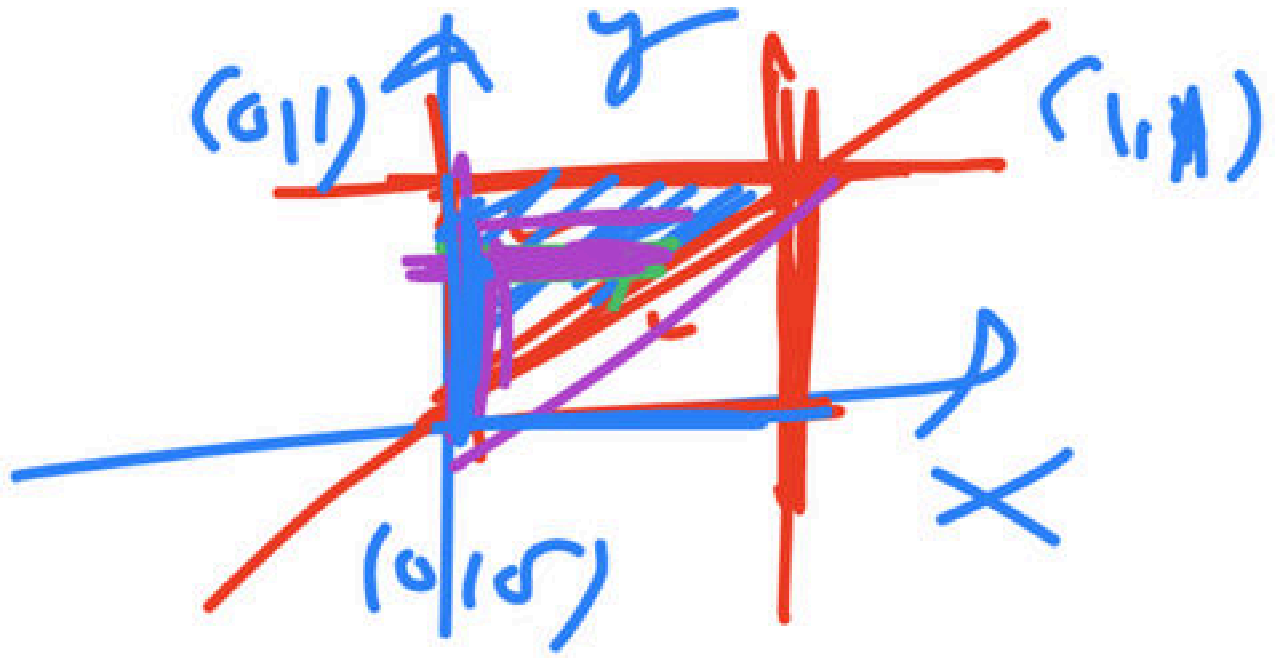
(a) $\frac{e+1}{2}$ (b) $\frac{e-1}{2}$

(c) $\frac{e-2}{3}$ (d) $\frac{e+2}{3}$

$y^3=t$

$\frac{1}{3}(e-1-1)$

$\frac{e-2}{2}$



$$\int_{y=0}^{1}\int_{x=0}^{y} y^4 e^{xy^2}\,dx\,dy = \int_{y=0}^{1} y^4\left(\frac{e^{xy^2}}{y^2}\right)_0^y dy = \int_0^1 y^2\left(e^{y^3}-1\right)dy$$

$$\frac{1}{3}\int_0^1 3y^2 e^{y^3}\,dy - \int_0^1 y^2\,dy = \frac{1}{3}\int_0^1 e^t\,dt - \left(\frac{y^3}{3}\right)_0^1$$

$$= \frac{1}{3}\left(e^t\right|_0^1 - \frac{1}{3} = \frac{1}{3}\left(e^1-1\right) - \frac{1}{3}$$

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Q.4. The value of double integral $\int_0^{\pi}\int_0^{x} \frac{\sin y}{\pi - y}\, dy\, dx$. **JAM-2016**

$x = 0$, $y =$

pi

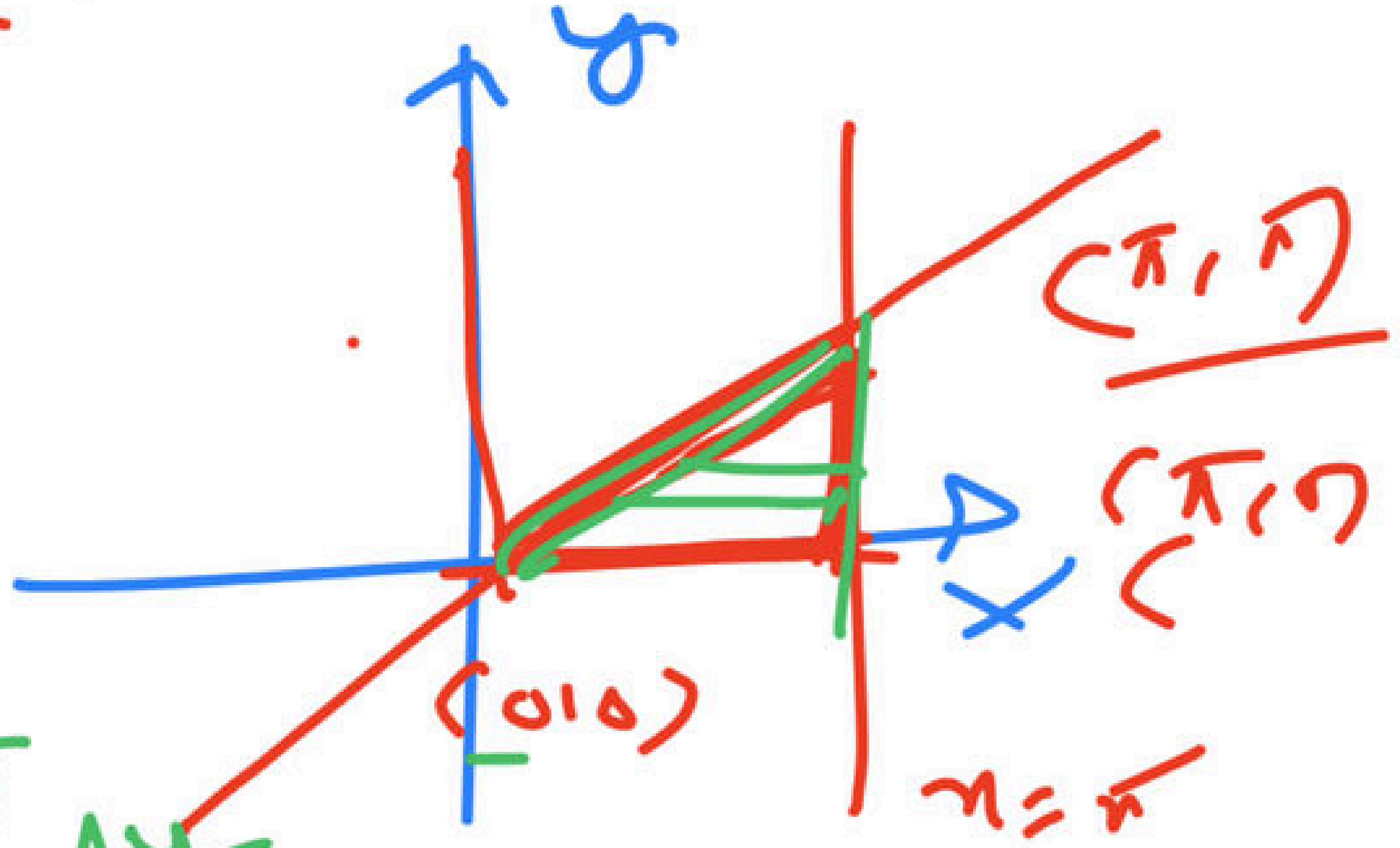
$y = \pi$

$x = 0 \;|\; y = 0$

$x = \pi \;|\; y = x$

(a) 0 (b) 1

(c) 2 (d) 2π



$$\int_{y=0}^{\pi}\int_{x=y}^{\pi} \frac{\sin y}{\pi - y}\, dx\, dy$$

$$\int_{y=0}^{\pi}\int_{y}^{\pi} \frac{\sin y}{\pi - y}\, dx\, dy = \int_0^{\pi} \frac{\sin y}{\pi - y}\,(x)_y^{\pi}\, dy$$

$$= \int_0^{\pi} \frac{\sin y}{\cancel{\pi - y}}\,\cancel{(\pi - y)}\, dy = -(\cos y)_0^{\pi}$$

$$-(\cos\pi - \cos 0) = \underline{2}$$

Q.5. The value of $\int_{0}^{4}\int_{\sqrt{4-x}}^{2} e^{y^3}\,dy\,dx$. **JAM-2012**

$y = +\sqrt{4-x}$

$x = 0$, $x = 4$ | $y = \sqrt{4-x}$, $y^2 = -(x-4)$, $y = 2$

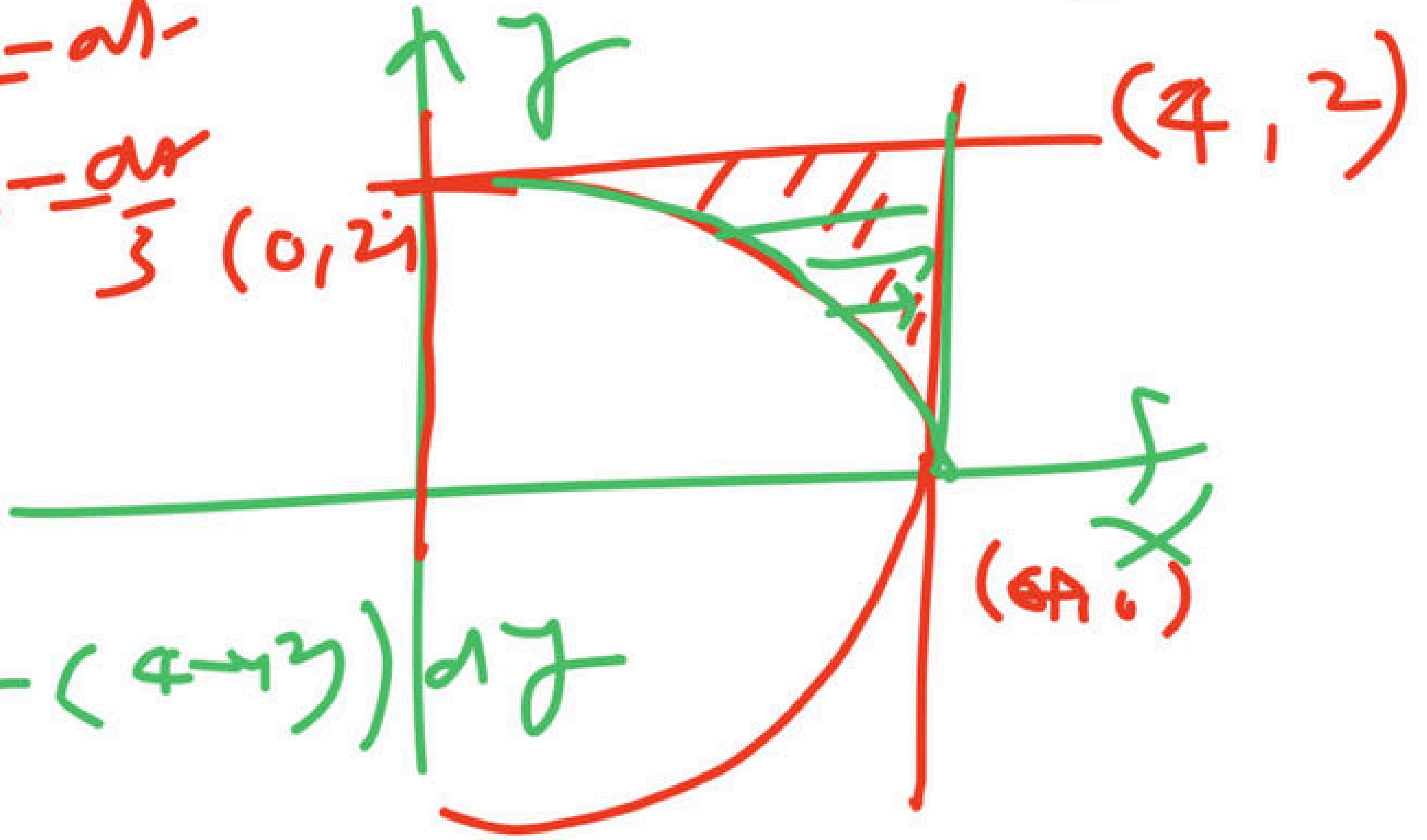
$y^3 = t$

$3y^2\,dy = dt$

$y^2\,dy = \frac{dt}{3}$

(a) $e^8 + 1$ (b) $e^8 - 1$

(c) $\frac{e^8 - 1}{2}$ (d) $\frac{e^8 - 1}{3}$



$$\int_{y=0}^{2}\int_{x=4-y^2}^{4} e^{y^3}\,dx\,dy = \int_0^2 e^{y^3}\left(4-(4-y^2)\right)dy$$

$$= \frac{1}{3}\int_0^2 3y^2 e^{y^3}\,dy$$

$$= \frac{1}{3}\int_0^8 e^t\,dt = \frac{1}{3}\left(e^t\right)_0^8 = \frac{(e^8-1)}{3}$$

Q.6. After the change of order of integration ,the double integral $\int_{0}^{8}\int_{x^{1/3}}^{2} dydx$ becomes **CUCET 2021**

(a) $\int_{x^{1/3}}^{2}\int_{0}^{8} dxdy$

(b) $\int_{0}^{2}\int_{0}^{y^3} dxdy$

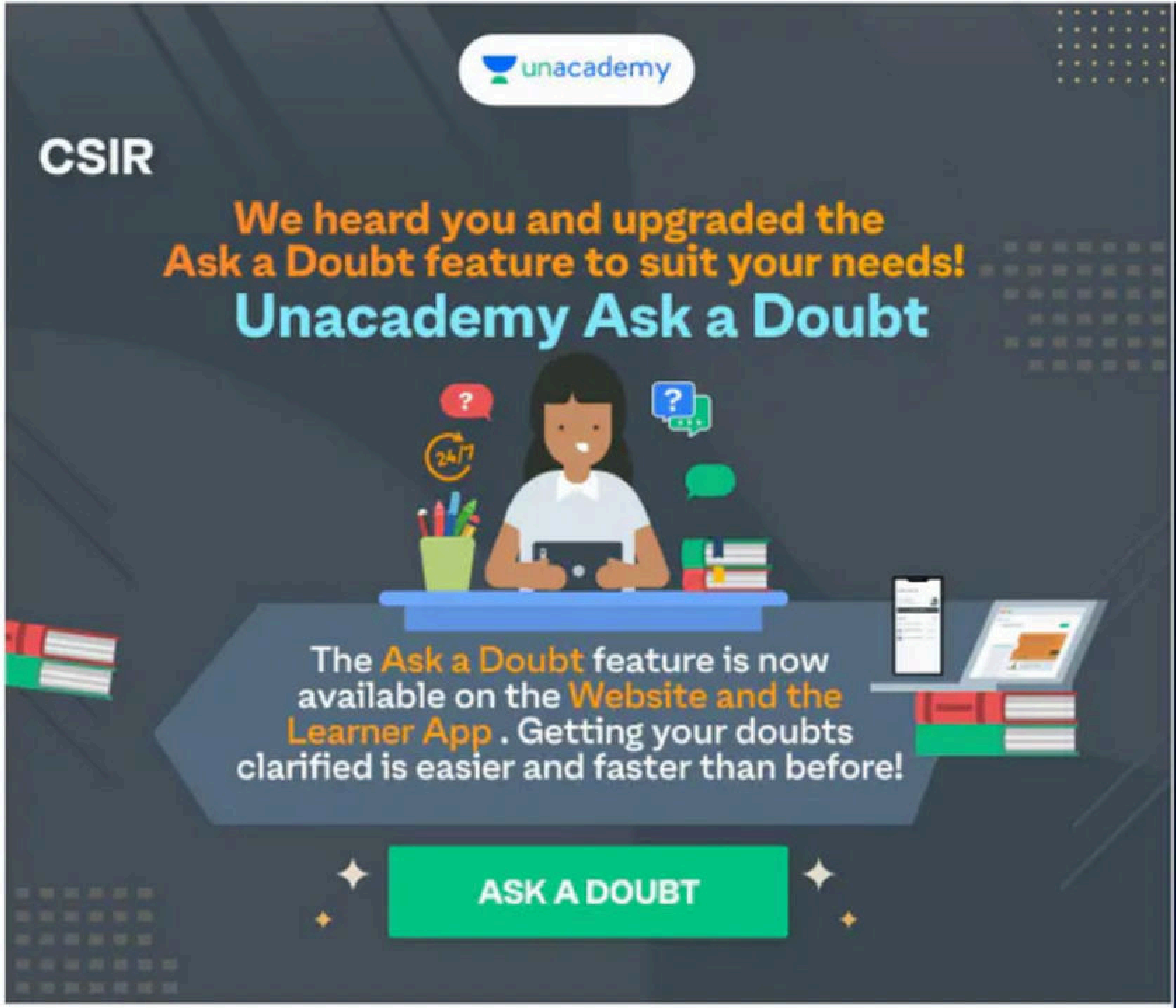
(c) $\int_{8}^{0}\int_{2}^{x^{1/3}} dxdy$

(d) $\int_{0}^{2}\int_{y^3}^{0} dxdy$

Q.7. Let f : R →R be continuous function and a > 0 then the integral $\int_0^a \int_0^x f(y)\,dy\,dx$ equals

(a) $\int_0^a y f(y)\,dy$ (b) $\int_0^a (a-y) f(y)\,dy$

(c) $\int_0^a (y-a) f(y)\,dy$ (d) $\int_a^0 y f(y)\,dy$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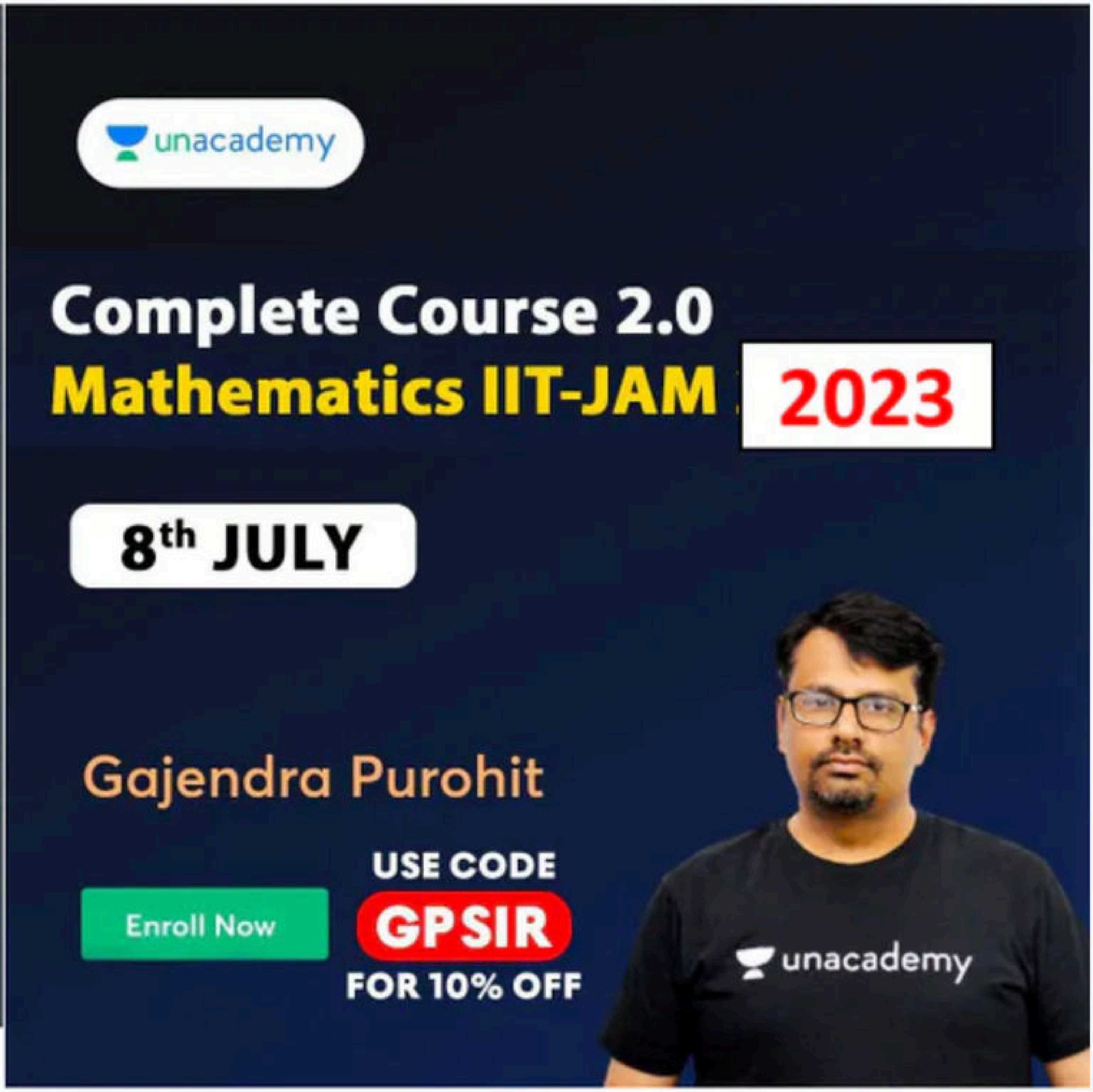
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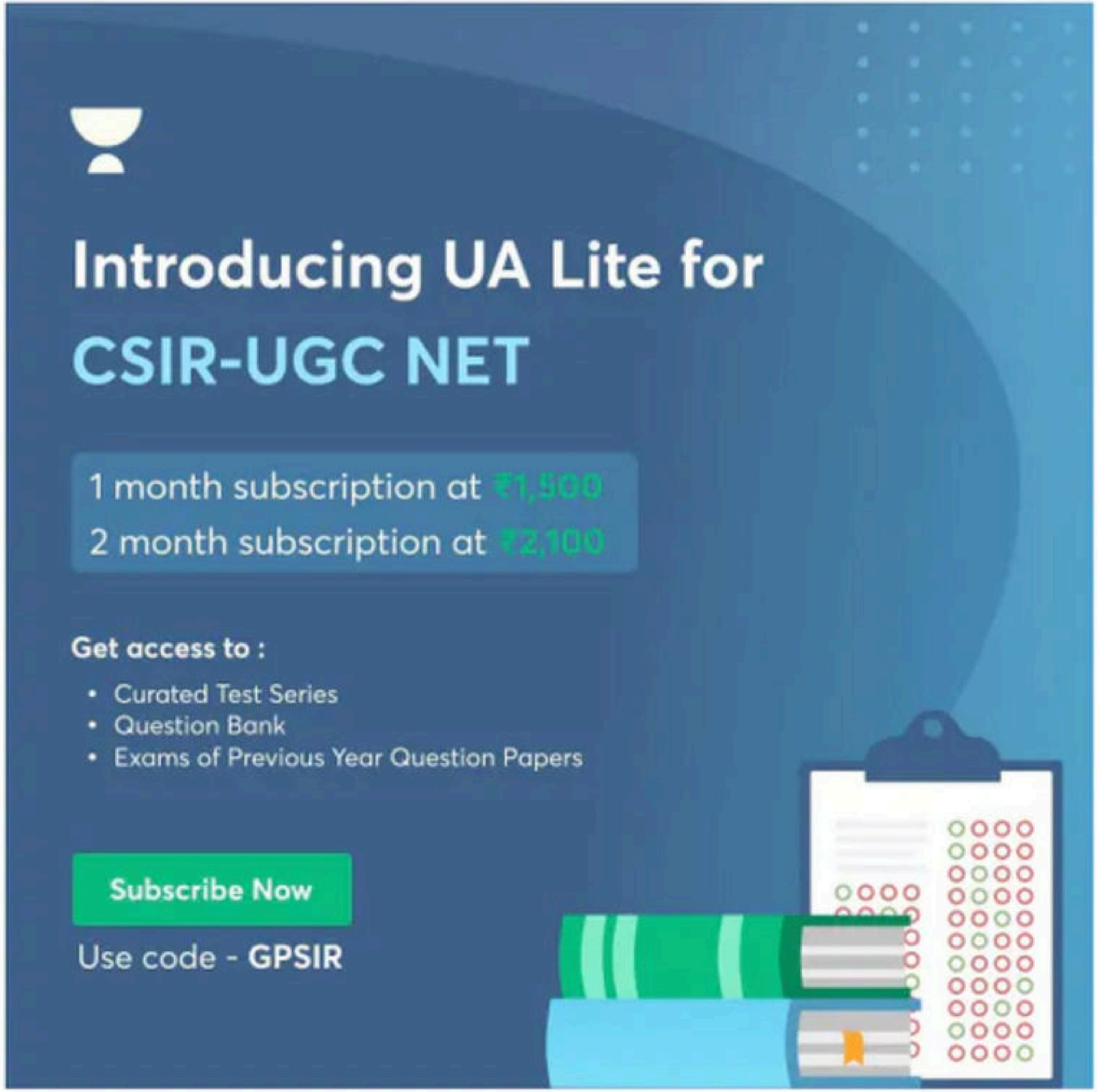
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