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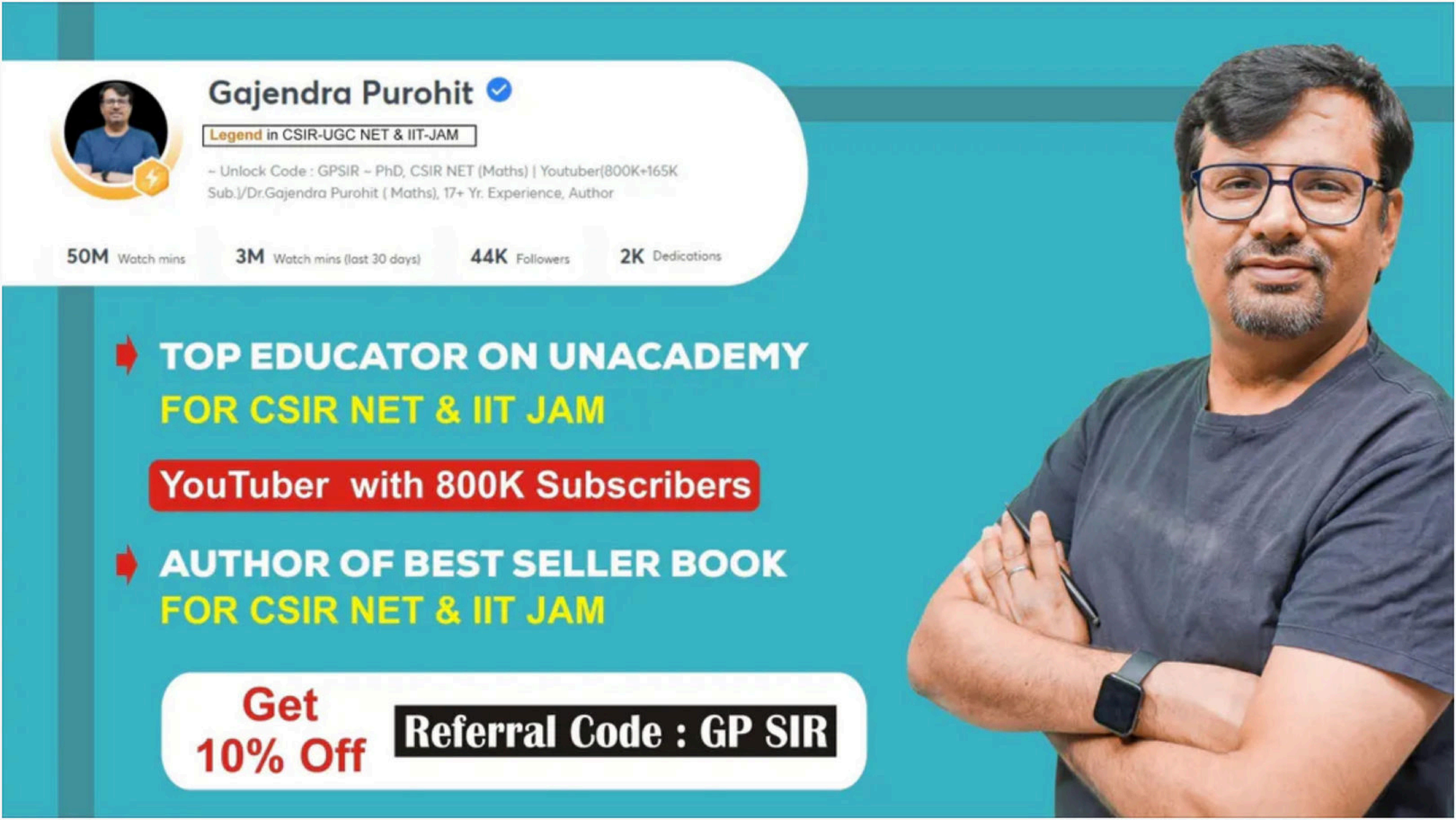
Detailed Course 2.0 on Linear Algebra For IIT JAM' 23

Gajendra Purohit • Lesson 9 • Sept 19, 2022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System of linear equation

The system of m equations and n variables

$$a_{11}x_1 + a_{12}x_2 \ldots\ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 \ldots\ldots + a_{2n}x_n = b_2$$

.

.

$$a_{m1}x_1 + a_{m2}x_2 \ldots\ldots + a_{mn}x_n = b_m$$

is called system of linear equation

It is denoted by Ax = b

$2x+3y=5$

$x-y=7$

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

$AX = B$

Homogeneous system of linear equation : The system of equation $AX = b$ is called homogeneous iff $b = 0$.

Non - homogeneous system of linear equation : The system of equation $AX = b$ is called non - homogeneous iff $b \neq 0$.

And $[A : b]$ is augmented matrix

$AX = 0$

$x - y = 0$

$2x + 3y = 0$

$AX = B$

$x - y = 5$

$3x + 2y = 7$

Consistency & inconsistency : Let AX = b be a system of equation it is called consistent if it has a solution otherwise it is called inconsistent system of equation.

Note : Homogeneous system of equation AX = 0 always has a solution. So, AX = 0 is called consistent.

3×3

Solution space of homogeneous system : Let AX = 0 be a given homogeneous system then ker(A) is called solution space of this system.

$\rho(A) = 2$

$\eta(A) = 1$

Dimension of solution space : Let $A_{m\times n}$ is a coeficient matrix of system of linear equation. Then dimension of solution space = $\eta(A) = n - \rho(A)$

$\rho(A) = 2$

$n(A) = 0$

$\rho(A) = 1$

$n(A) = 1$

$A = \begin{bmatrix} \; \end{bmatrix}_{3 \times 2}$

$\rho(A) + n(A) = 2$

$$\begin{bmatrix} 1 & 7 & 2 \\ 2 & 14 & 4 \\ 3 & 21 & 6 \\ 4 & 28 & 8 \end{bmatrix}_{4\times 3}$$

$$\begin{bmatrix} 1 & 7 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 1$

$n(A)$

$\rho(A) + n(A) = 3$

$1 + n(A) = 3$

$n(A) = 2$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}_{3 \times 4}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho(A) = 1$$

$$\rho(A) + n(A) = 4$$

$$1 + n(A) = 4$$

$$n(A) = 3$$

$x + y + z = 0$

$x + 2y + z = 0$

$x + y + 2z = 0$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \quad 3 \neq 0$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\rho(A) = 3$

$\rho(A) + n(A) = 3$

$n(A) = 0$

$AX = 0$

$\text{Ker}(A) = 0$

$x + y = 0$

$2x \; 2y = 0$

$y = k$

$x = k$

$x = -y$

$AX = 0$

$n(A) \geq 1$

$|A| = 0$

Infinite Sol'n

Non Trivial Sol's

Non Zero Sol'n

$x + y = 0$

$x - y = 0$

$x = 0$

$y = 0$

$|A| \neq 0$

$n(A) = 0$

Unique Sol'n

Trivial Sol'n

Zero Sol'n

$$x+y=0$$
$$x-y=0$$

$x=0$
$y=0$

$$\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$$

$\rho(A)=2$

$|A|\neq 0$ $\quad n(A)=0$

Unique soln

$$x+y=0$$
$$2x+2y=0$$

$y=k$
$x=-k$

$$\begin{pmatrix}1 & 1\\ 2 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$$

$$\begin{pmatrix}1 & 1\\ 0 & 0\end{pmatrix}$$

$|A|=0$

$\rho\ n-1,$ $\quad A=1$

Infinite

Note : $X = 0$ is known as trivial solution and any solution other then $X = 0$ is known as non- trivial solution

Analysis of solution of the homogeneous system of equation :

Consider the homogeneous system of equation $AX = 0$ of m-equation in n-variables.

$A = [a_{ij}]$ be $m \times n$ matrix with entries from $a_{ij} \in F$ (F is infinite field)

(i) If $\ker(A) = \{0\}$ then system has trivial solution i.e. if $\eta(A) = 0$ then system has unique (trivial) solution.

(ii) If $\ker(A) \neq \{0\}$ i.e. $\eta(A) \geq 1$ then system has infinite (non-trivial) solution.

(3) If $|A| \neq 0$ then system has unique solution.

(4) If $|A| = 0$ the system has infinite solution.

(5) If number of equation (m) < number of variables then system always has non-trivial solution.

(6) System has non-trivial solution iff it has infinite many solution.

Q.1 If system of linear equation

$$kx + y + 2z = 0$$

$$x - y - 2z = 0$$

$$x + y + 4z = 0$$

have a unique solution, then k is not equal to

(a) -1 (b) 0

(c) 1 (d) 2

$|A| \neq 0$

$$\begin{array}{c} 2 \\ \\ 4 \end{array} \begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix} \begin{array}{c} k \\ \\ 1 \end{array}$$

$(2 - 4k - 2) - (4 - 2 - 2k) \neq 0$

$-4k - 2 + 2k \neq 0$

$-2k - 2 \neq 0$

$2k \neq -2$

$k \neq -1$

Q.2. Let $M = \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{bmatrix}$, $\alpha\beta\gamma = 1$, $\alpha, \beta, \gamma \in R$ and

α

$$(1 + \alpha\beta\gamma + 1) - (\alpha + \beta + \gamma) = 0$$

$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3$. Then Mx = 0 has infinitely many solution

if trace (M) is

$$3 - (\alpha + \beta + \gamma) = 0$$

$$\alpha + \beta + \gamma = 3$$

(a) 0 (b) 1

(c) 2 (d) 3

$$x + y + z = 0$$
$$2x + y + z = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}_{2 \times 3}$$

$$\rho(A) = 2$$
$$n(A) = 1$$

$$x + y = 0$$
$$2x + y = 0$$
$$3x + 2y = 0$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}_{3 \times 2}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}$$

$$\rho(A) = 2$$
$$n(A) = 0$$

Q.3 Consider the following system of three linear equation in three unknown x_1, x_2, & x_3.

$$x_1 + x_2 + x_3 = 0$$

$$x_1 + kx_2 + 2x_3 = 0$$

$$2x_1 + 3x_2 + x_3 = 0$$

If system has infinitely many solution then value of k is

(a) 0 (b) 1

(c) 2 (d) 3

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & k & 2 \\ 2 & 3 & 1 \end{bmatrix} \begin{matrix} 1 & 1 \\ 1 & k \\ 2 & 3 \end{matrix}$$

$(3+k+4) - (1+2k+\ldots$

$7+k - (7+2k) =$

$-k = 0$

$k = 0$

Q.4. Let A be a 5×4 matrix with real entries such that $Ax = 0$ iff $x = 0$ where x is a 4×1 vector and 0 is null vector. Then the rank of A is

(a) 4 (b) 5

(c) 2 (d) 1

$n(A) = 0$

$\rho(A) + n(A) = 4$

$\rho(A) = 4$

Q.5 Consider a homogeneous system of linear equation Ax = 0 where A is an $m \times n$ real matrix and $n > m$. Then which of the following statements are always true?

(a) Ax = 0 has a solution.

(b) Ax = 0 has no non-zero solution.

(c) Ax = 0 has a non-zero solution

(d) Dimension of the space of all solutions is at least $n-m$

$[\;\;]_{m \times n}$

$\rho(A) = m$

$\rho(A) + n(A) = n$

$n(A) = n - \rho(A) = n - m$

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Q.6. Let S be the solution space of system of linear equation

$x + 2y + 3z = 0$

$3x + 4y + 4z = 0$

$7x + 10y + 12z = 0$

Then dimension of S is

(a) 0 (b) 1

(c) 2 (d) 3

$AX = 0$

$n(A) = 0$

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}_{3\times 3}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -5 \\ 0 & -4 & -5 \end{bmatrix}$$

$\rho(A) = 3, \quad n(A) = 0$

Non-homogeneous system of equation : A system of equation $Ax = b$ is called non-homogeneous iff $b \neq 0$.

$Ax = b$

Consistency & inconsistency : A non-homogeneous system $Ax = b$ is called consistency if it has a solution otherwise it is called inconsistent.

Augmented matrix : Let $Ax = b$ be a given system of equation then $[A : b]$ is called augmented matrix.

$2x + y = 3$

$x + 2y = 5$

$$\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}3\\ 5\end{pmatrix}$$

$AX = B$

$R_2 \to 2R_2 - R_1$

$$(A:B) = \left(\begin{array}{cc|c}2 & 1 & 3\\ 1 & 2 & 5\end{array}\right)$$

$$= \left(\begin{array}{cc|c}2 & 1 & 3\\ 0 & 3 & 7\end{array}\right)$$

$\rho(A:B) = 2, \ \rho(A) = 2$

$\rho(A|B) = \rho(A) = n$ $\quad n = 2$

$x + y = 7$

$2x + 2y = 8$

$$\left[\begin{array}{cc|c}1 & 1 & 7\\ 2 & 2 & 8\end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\left(\begin{array}{cc|c}1 & 1 & 7\\ 0 & 0 & -6\end{array}\right)$$

$\rho(A:B) = 2$

$\rho(A) = 1$

No sol^n

$x + y = 7$

$x + y = 14$

$$\left(\begin{array}{cc|c}1 & 1 & 7\\ 1 & 1 & 14\end{array}\right)$$

$R_2 \to R_2 - R_1$

$$\left(\begin{array}{cc|c}1 & 1 & 7\\ 0 & 0 & 0\end{array}\right)$$

$\rho(A:B) = 1$

$\rho(A) = 1$

$\rho(A|B) = \rho(A) < n$

Infinite

$$AX = B$$

$$\rho(A:B) = \rho(A)$$

Equation consistent

(i) $\rho(A:B) = \rho(A) = n$

Unique soln

(ii) $\rho(A:B) = \rho(A) < n$

infinite solns

$$\rho(A:B) \neq \rho(A)$$

Inconsistent

No. soln

Necessary and sufficient condition for solution :

$Ax = b$ has a solution iff

(i) $\rho(A : b) = \rho(A)$

(ii) b is linear combination of $c_1, c_2, \ldots, c_n$ where c_i are column of A.

Note : If $\rho(A : b) \neq \rho(A)$ then $Ax = b$ has no solution.

Analysis of solution of non-homogeneous system of equation :

(1) **Unique solution :** Let $Ax = b$ has a unique solution iff $\operatorname{Ker}(A) = \{0\}$ i.e. $\eta(A) = 0$ and $\rho(A : b) = \rho(A)$

(2) **Infinite solution :** A non-homogeneous system $Ax = b$ has infinite solution iff $\rho(A : b) = \rho(A)$ and $\eta(A) > 0$.

(3) **No solution :**

If $\rho(A : b) \neq \rho(A)$

Then system $AX = b$ has no solution.

Q.1. Consider the following system

$x + y + z + w = 4$

$x + 2y + 3z + 4w = 5$

$x + 3y + 5z + kw = 5$

If the system has no solution then k is

(a) 4 (b) 5

(c) 7 (d) 6

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & k & 5 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 2 & 4 & k-1 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & k-7 & -1 \end{array}\right)$$

$= 0 \quad \neq 0$

2 3

$k - 7 = 0$

$k = 7$

Q.3. Consider the system

$$2x + ky = 2 - k$$

$$kx + 2y = k$$

$$ky + kz = k - 1$$

in three unknowns and one real parameter k. For which of the following values of k is the system of linear equation consistent?

(a) 1 (b) 2

(c) -1 (d) -2


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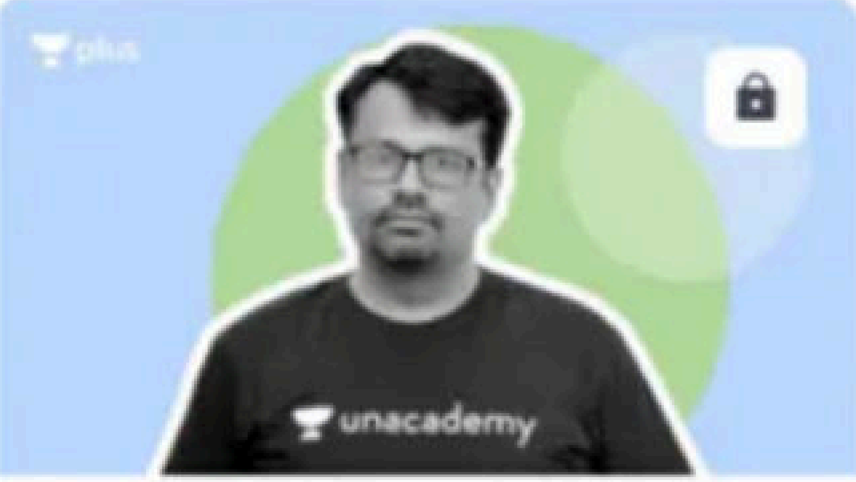

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Educator highlights

- Works at **Pacific Science College**
- Studied at **M.Sc., NET, PhD(Algebra), MBA(Finance), BEd**
- PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- Lives in **Udaipur, Rajasthan, India**
- Unacademy Educator since

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