Riemann Integration

Detailed Course 2.0 on Function of One and Several Variable - IIT JAM, 23

Gajendra Purohit • Lesson 7 • Nov 17, 2022


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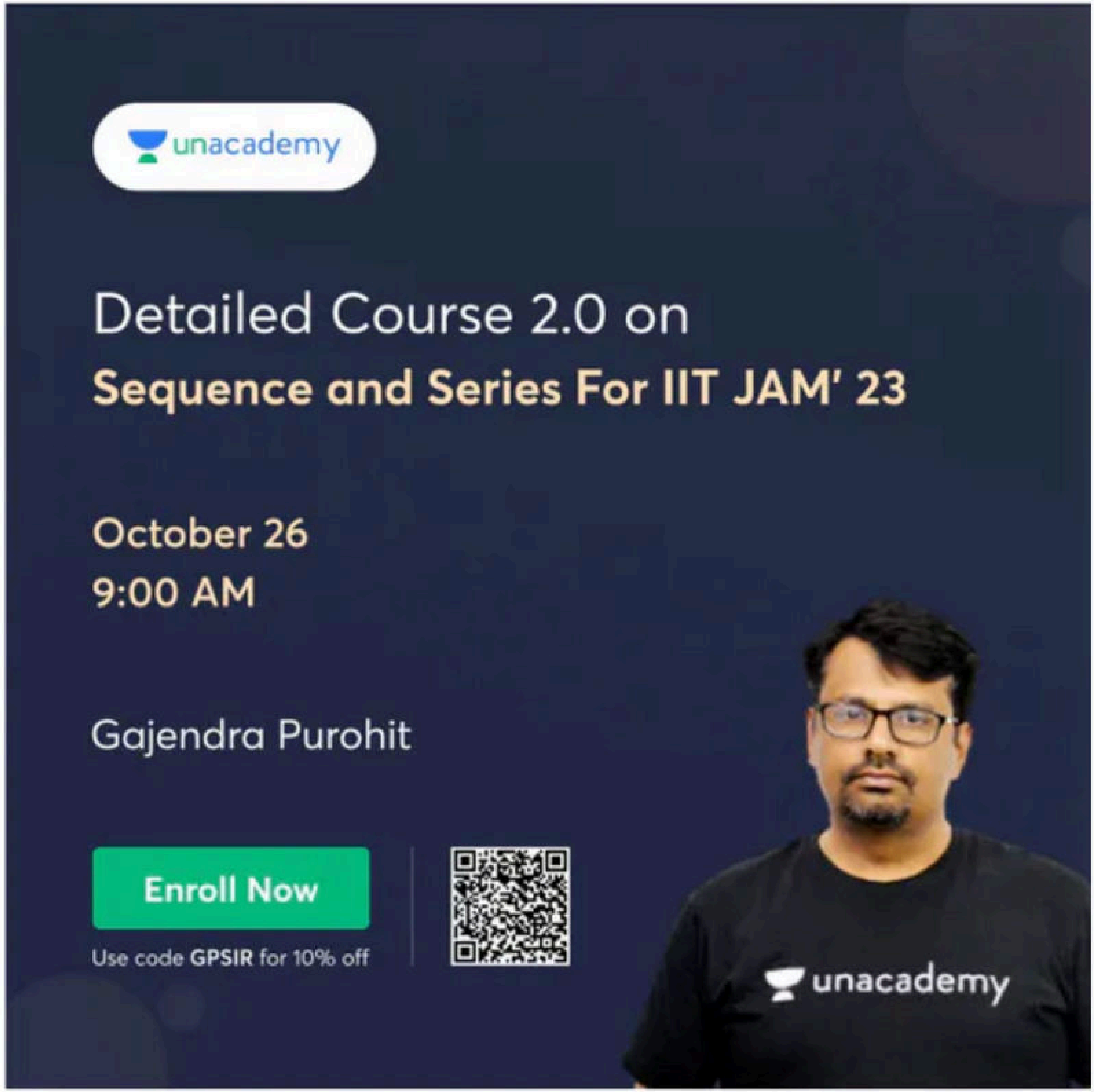
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Differentiability

Darboux Theorem :

If a function is differentiable in a closed interval [a, b] and $f'(a)$, $f'(b)$ are of opposite sign, then there exist at least one point c in the open interval (a, b) s.t. $f'(c) = 0$.

$f(x) = x^2$ $\quad [-1, 1]$

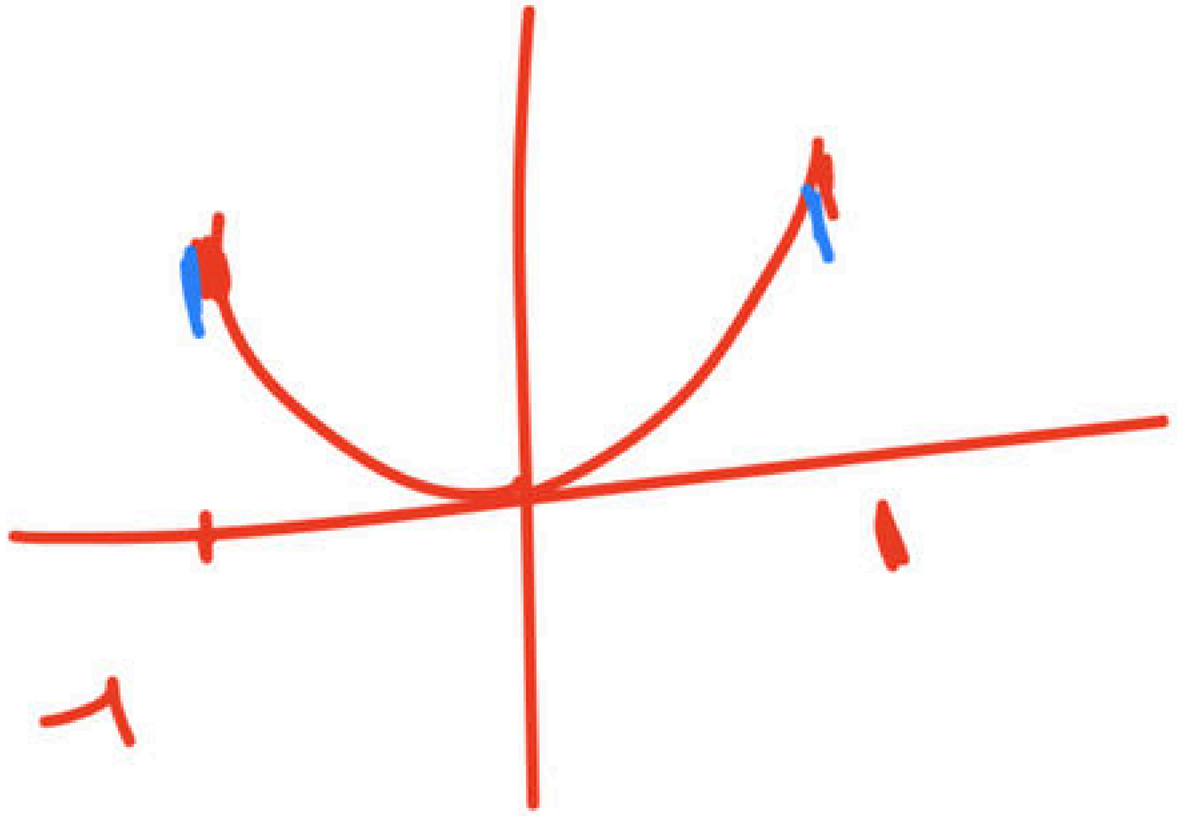
$f'(c) = 2c = 0$

$c = 0$

$f'(x) = 2x$

$f'(-1) = -2$

$f'(1) = 2$



$f(x) = \sin x \qquad x \in \underline{[0, \pi]}$

$f'(x) = \cos x$

$f'(0) = 1$

$f'(\pi) = -1$

$f'(c) = \cos c = 0$

$c = \pi/2$

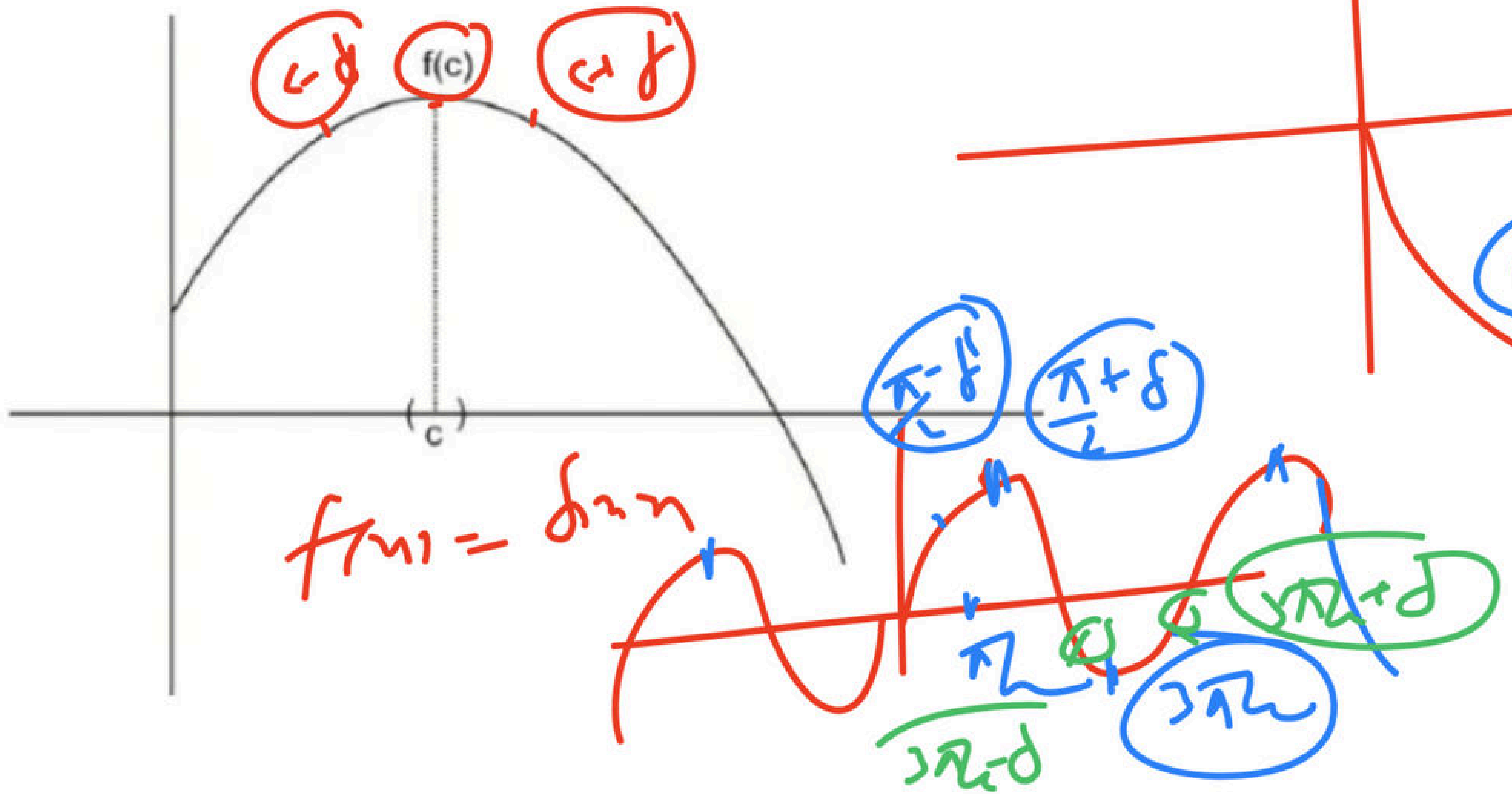
Local maxima and local minima :

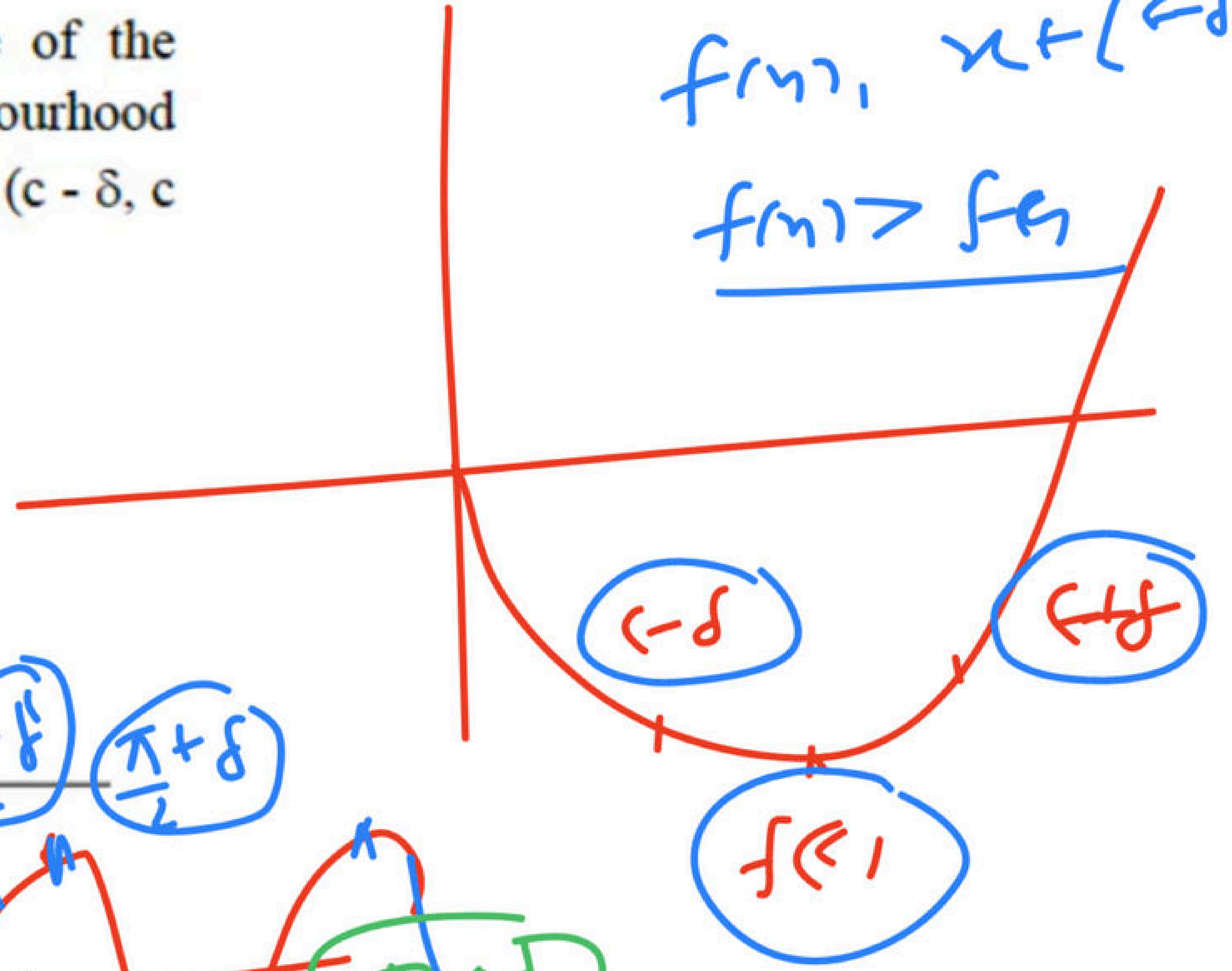
Let $f : I \to R$ and c be an interior point of the interval I, then

(i) f(c) is said to be a local maximum value of the function f, if there exist some neighbourhood $(c - \delta, c + \delta)$ of c, such that $f(c) > f(x)$, for all $(c - \delta, c + \delta)$ then c is point of local maxima.

$f(x),\ x \in (c-\delta, c+\delta)$

$f(c) > f(x)$





(2) f(c) is said to be local minimum value of the function f, if there exist some neighbourhood of $(c - \delta, c + \delta)$ of c s.t. $f(c) < f(x)$, for all $x \in (c - \delta, c + \delta)$, and c is called point of local minima.

(3) Extreme Value :

$[0, \pi]$

$y = \sin x$

$f'(x) = \cos x = 0$

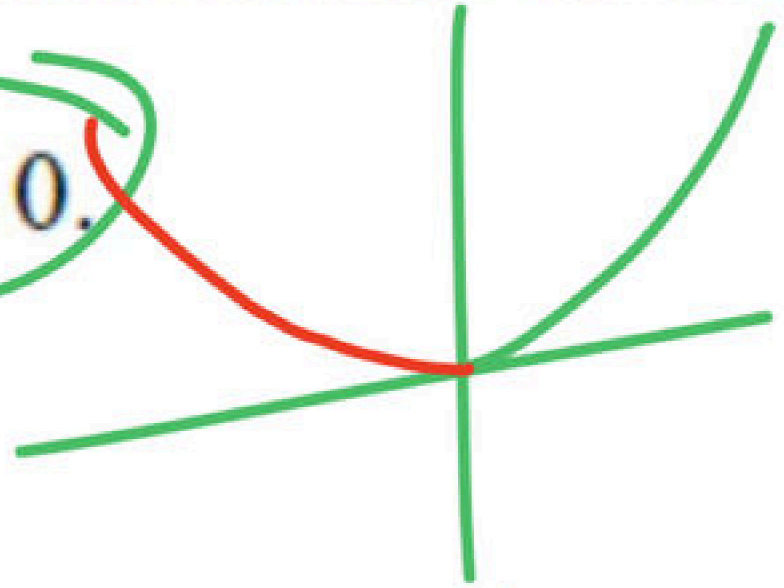
f(c) is said to be extreme value of f, if it is either a maximum or minimum value.

i.e. a point x = c is said to be extreme point if $f'(c) = 0$

$[\pi/2, 2\pi]$

Interior extremum theorem :

Let c be the interior point of the interval I at which $f : I \rightarrow R$ has an extremum value. If $f'(c) = 0$.



$f(x) = x^2 \quad [0, 4]$

$f'(x) = 2x$

$f'(x) > 0 \quad (0, 4)$

Note :

(1) If f(x) is continuous on [a, b] and $f'(x) > 0$ in (a, b), then f(x) is increasing in [a, b].

$f(x) = x^2 \quad [-4, 0]$

$f'(x) = 2x$

$f'(x) = 2x < 0 \quad (-4, 0)$

(2) If f(x) is continuous in [a, b] and $f'(x) < 0$ in (a, b) then f(x) is decreasing in [a, b].

$f(x) = \sin x$

$f'(x) = \cos x$

$f'(x) < 0$

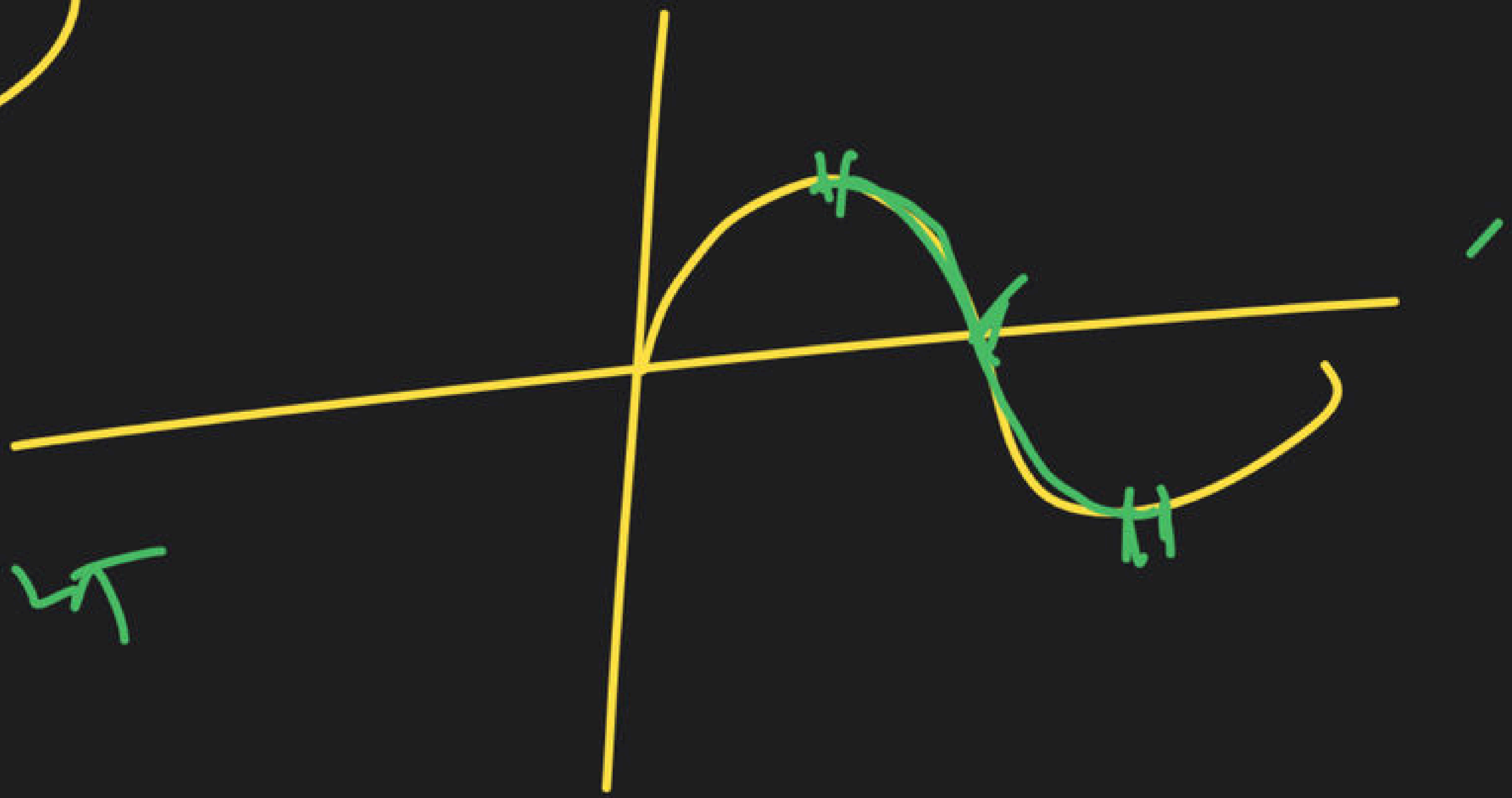
$x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$x \in \left(\frac{\pi}{2}, \pi\right)$

$\left(\frac{\pi}{2}, 2\pi\right)$

4π

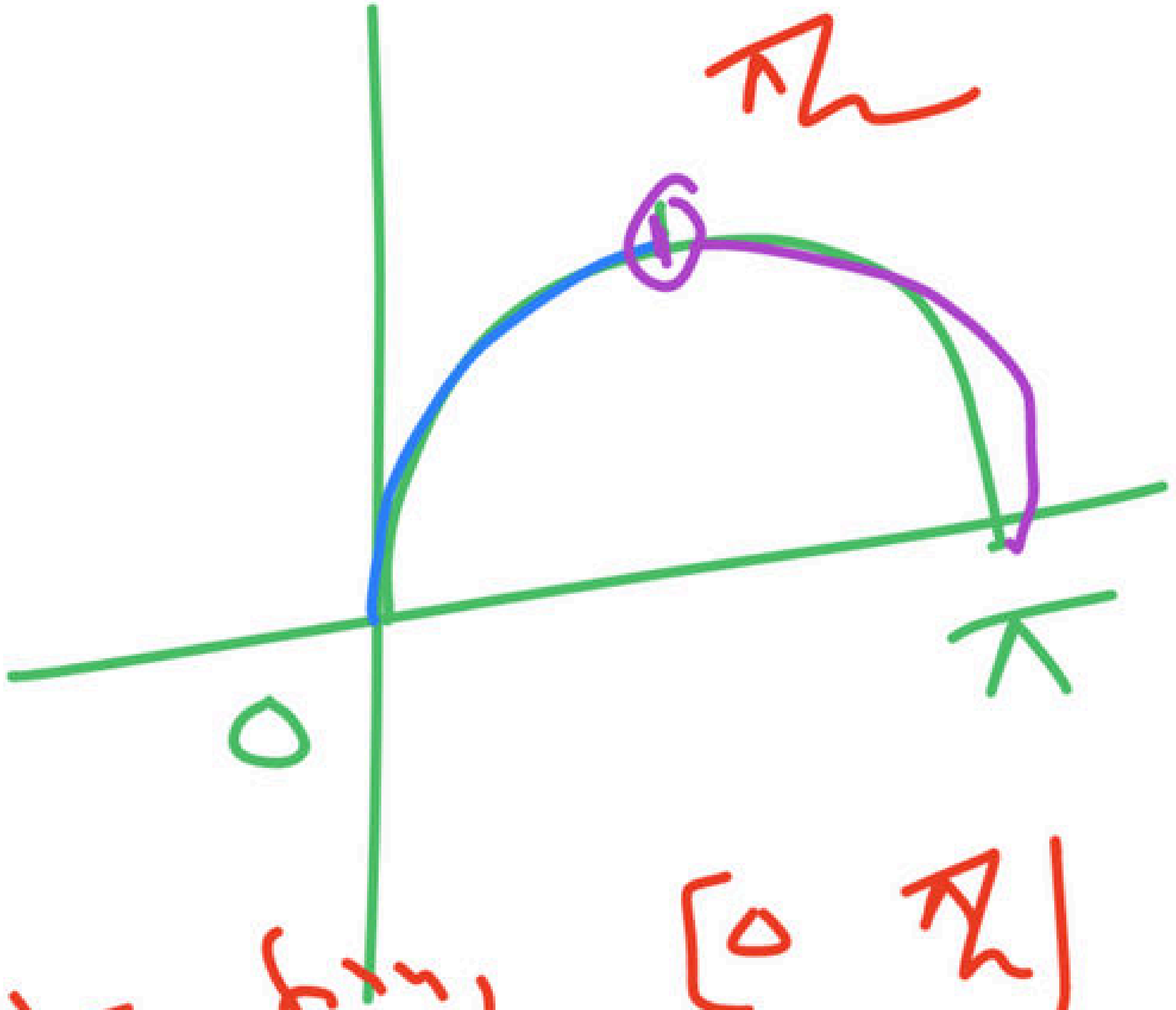
$\frac{3\pi}{2}$

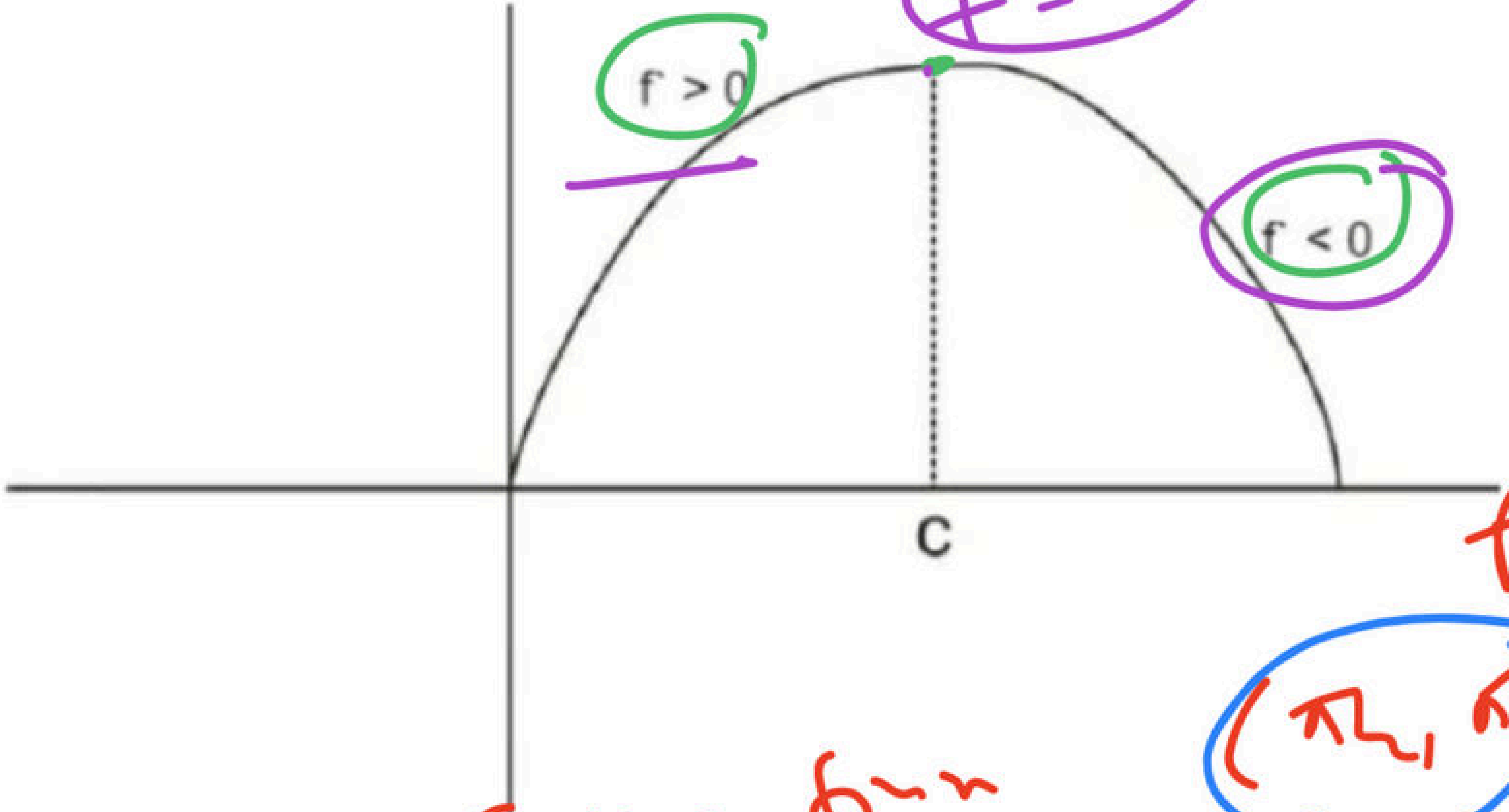


First Derivative test for extreme values :

Let a function f be differentiable in a neighbourhood of c, where f has an extreme value at c, then f(c) is a maximum value if the sign of f` change from positive to negative.

$f'(x) = \sin x$





$f(x) = \sin x, \quad [0 \ \ \pi/2]$

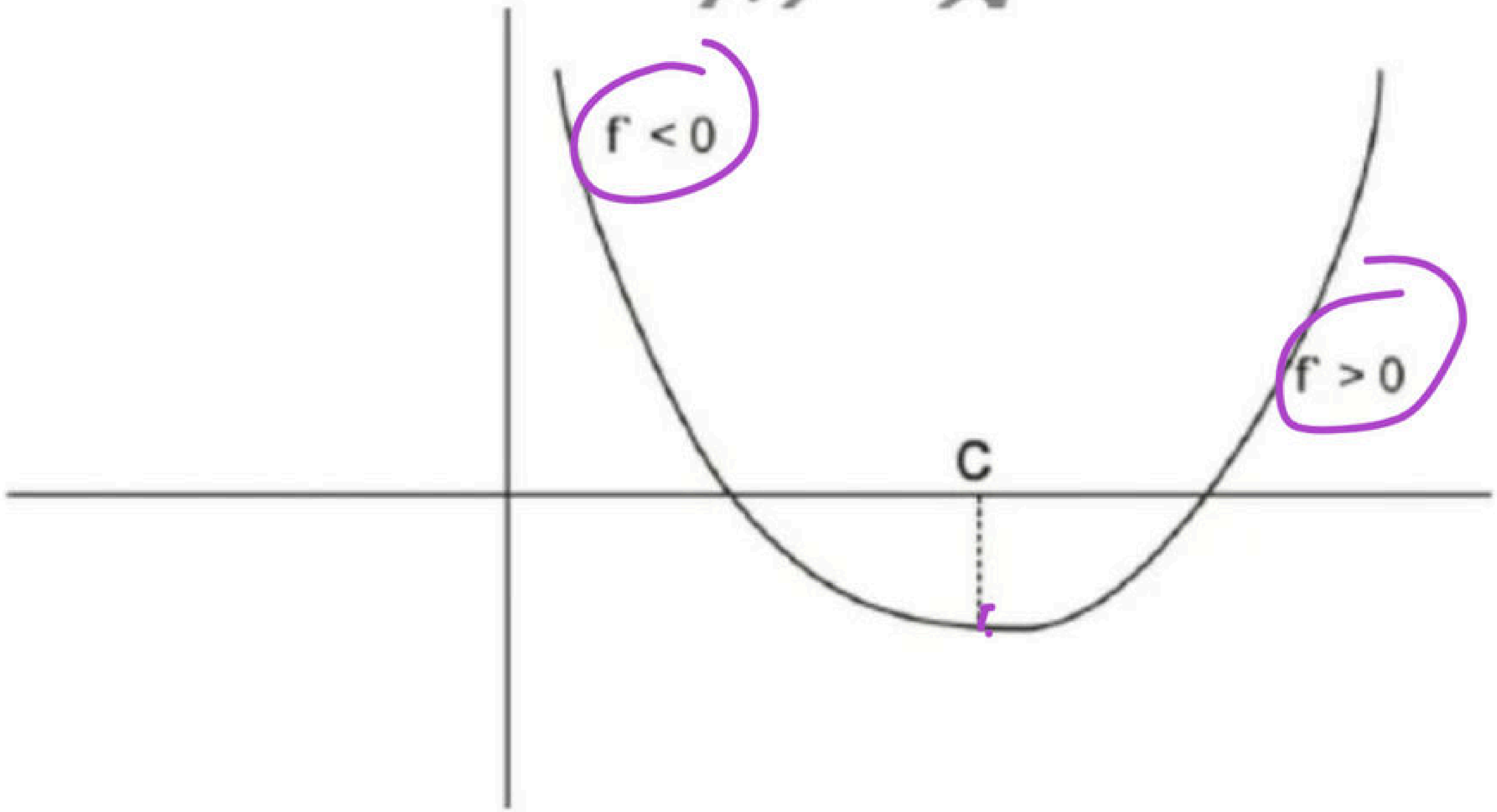
$f'(x) = \cos x$ $\quad (0, \pi/2)$

$(\pi/2, \pi)$

$f(x) = \sin x$

$f'(x) = +\cos x$

And f(c) is a minimum value of the dign of f` changes from negative to positive.



Second test for extreme value :

(a) If $f'(c) = 0$ and $f''(c) < 0$ then f has a maximum value at $x = c$.

(b) If $f'(c) = 0$ and $f''(c) > 0$ then f has a minimum value at $x = c$.

$y = f(x)$

$\frac{dy}{dx} = f'(x) = 0$

$f(x) = -2(2-x)^2$

$f'(x) = \cancel{2}4(2-x) = 0$

$x = 2$

$f''(x) = -4 < 0$

(2,0)

$f''(x) > 0$

min

$f''(x) < 0$

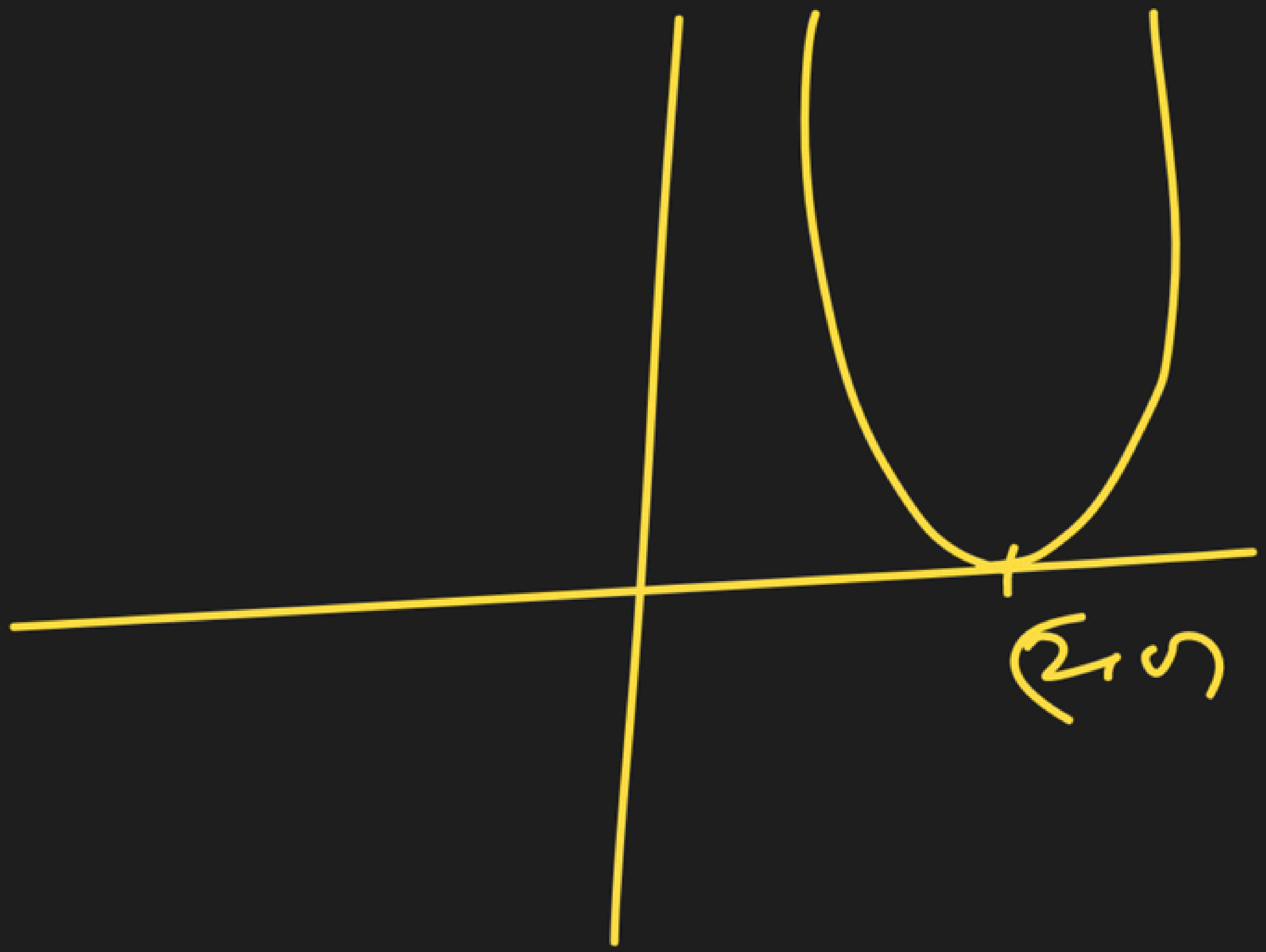
max

$f(x) = (x-2)^2$

$f'(x) = 2(x-2) = 0$

$x = 2$

$f''(x) = 2 > 0$



Q.1 Let $\varphi : R \to R$ be a differentiable function such that $\varphi`$ is strictly increasing with $\varphi`(1) = 0$. Let α and β denote t he minimum and maximum values of $\varphi(x)$ on the interval $[2, 3]$, respectively. Then which one of the following is TRUE? **IIT JAM 2017**

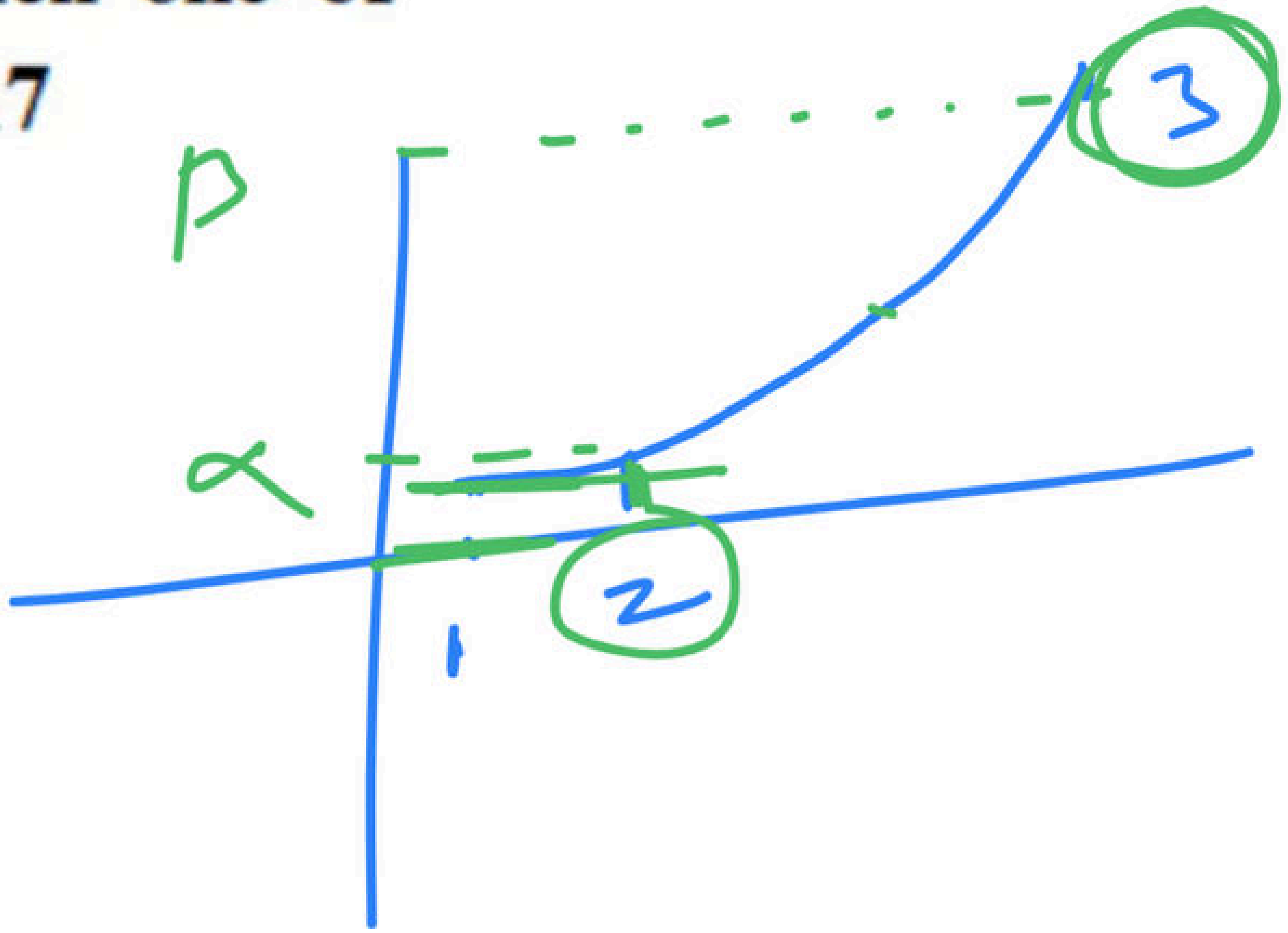
$\phi'(x) > 0$

(a) $\beta = \varphi(3)$

(b) $\alpha = \varphi(2.5)$

(c) $\beta = \varphi(2.5)$

(d) $\alpha = \varphi(3)$



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Q.2. Let $f:\left(0,\frac{\pi}{2}\right)$ be given by f(x) = $(\sin x)^{\pi} - \pi \sin x + \pi$

π. Then which of the following statements is/are TRUE? **IIT JAM 2019**

$\pi/4$

$f'(x) = \pi(\sin x)^{\pi-1}\cos x - \pi\cos x$

$= \pi\cos x \cdot \left[(\sin x)^{\pi-1} - 1\right]$

$f'(\pi/4) = \pi\frac{1}{\sqrt{2}}\left[\left(\frac{1}{\sqrt{2}}\right)^{2.14} - 1\right] < 0$

(a) f is an increasing function

(b) f is a decreasing function

(c) f(x) > 0 for all $x \in \left(0,\frac{\pi}{2}\right)$

(d) f(x) < 0 for some $x \in \left(0,\frac{\pi}{2}\right)$

$(r)^{n} < 1$

Q.3. Let $f : R \to R$ be a differentiable function with $f(0) = 0$.if for all $x \in R$, $1 < f'(x) < 2$, then which one of the following statements is true on $(0, \infty)$ **IIT JAM 2015**

$f(x) = \frac{3}{2}x$

(a) f is unbounded

(b) f is increasing and bounded

(c) f has at least one zero

(d) f is periodic

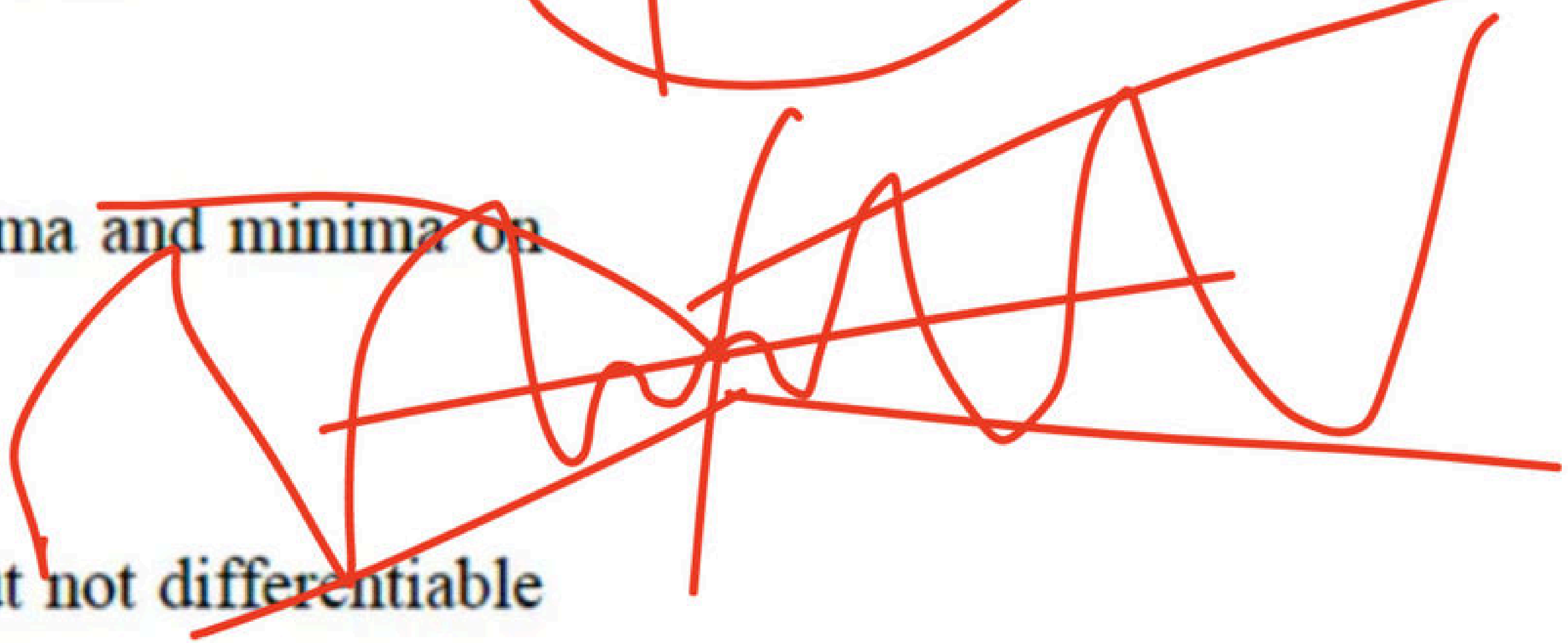
Q.4. For x ∈ R, let $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$. Then which one of the following is FALSE? **IIT JAM 2017**

(a) $\lim_{x\to 0} \frac{f(x)}{x} = 0$ (b) $\lim_{x\to 0} \frac{f(x)}{x^2} = 0$

(c) $\frac{f(x)}{x^2}$ has infinitely many maxima and minima on the interval (0,1)

(d) $\frac{f(x)}{x^4}$ is continuous at x = 0 but not differentiable at x = 0.

$x \sin \frac{1}{x}$



Q.5. Let $f(x)=\begin{cases} x+x^2\cos\left(\dfrac{\pi}{x}\right) & x\neq 0 \\ 0 & x=0 \end{cases}$ & consider the following statements :

(i) $f'(0)$ exists & is equal to 1

(ii) f is not increasing in any neighbourhood of 0

(iii) $f'(0)$ does not exist

(iv) f is increasing on R

How many of the above statement is/are true?

(a) 0 (b) 1

(c) 2 (d) 3

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{x\to 0}\frac{x+x^2\cos(\pi/x)-0}{x-0}$$

$$=\lim_{x\to 0}\left(1+x\cos\pi/x\right)$$

$$=1$$

Q.6. Let $f(x) = \begin{cases} x+1 & x < 0 \\ (x-1)^2 & x \geq 0 \end{cases}$

$y = (x-1)^2$

Which one of the following is TRUE?

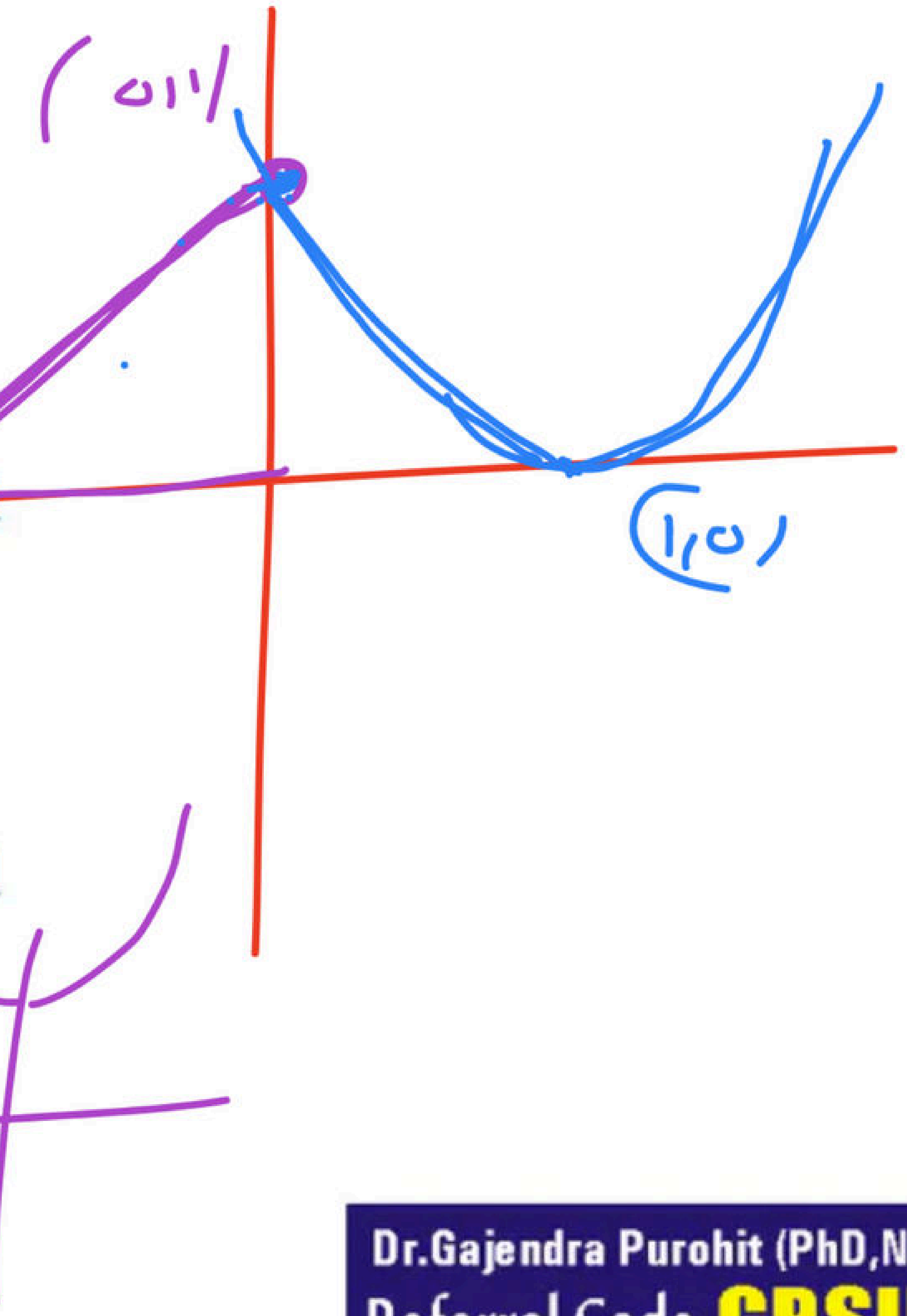
(a) f is differentiable on R

(b) f has neither a local maximum nor a local minimum in R

(c) f is bounded on R

(d) f is not differentiable at x = 0 and f(x) has local maximum at x = 0

$y = (x-1)^2$



Q.7. If f : R → R is a continuous function such that $f(x + y) = f(x) + f(y)$ for all $x, y \in R$, then

(a) f is increasing if $f(1) \geq 0$ and decreasing if $f(1) \leq 0$

(b) f is increasing if $f(1) \leq 0$ and decreasing if $f(1) \geq 0$

(c) f is not an increasing function

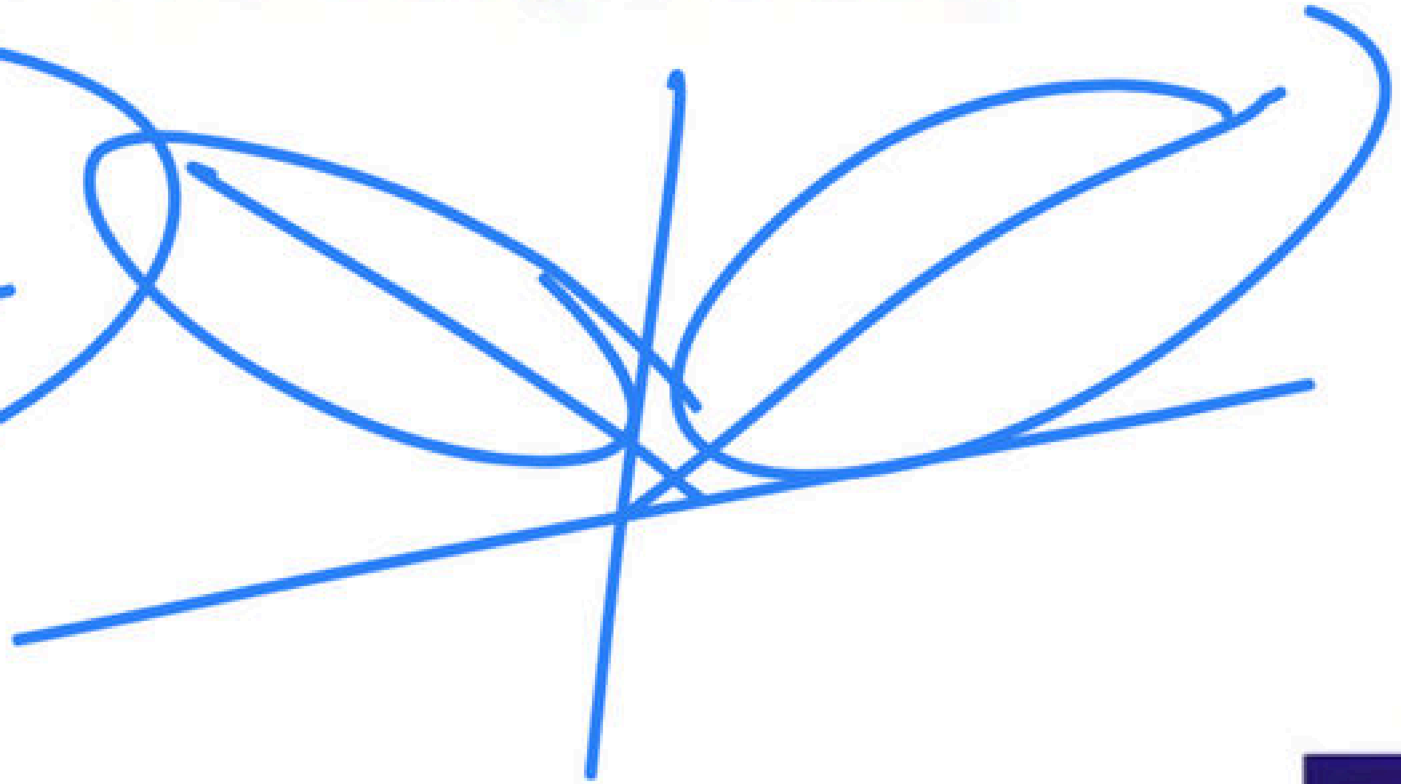
(d) f is neither an increasing nor a decreasing function

$f'(x) = k$

$f(x) = kx$

$f(1) = k \geq 0$

$f(x) = 2x$



Q.8. Let f be a twice differentiable function on R. Given t
hat $f''(x) > 0$ for all $x \in R$, then

(a) $f(x) = 0$ has exactly two solution on R.

(b) $f(x) = 0$ has a positive solution if $f(0) = 0$ & $f'(0) = 0$

(c) $f(x) = 0$ has no positive solution if $f(0) = 0$ and $f'(0) > 0$

(d) $f(x) = 0$ has no positive solution if $f(0) = 0$ and $f'(0) < 0$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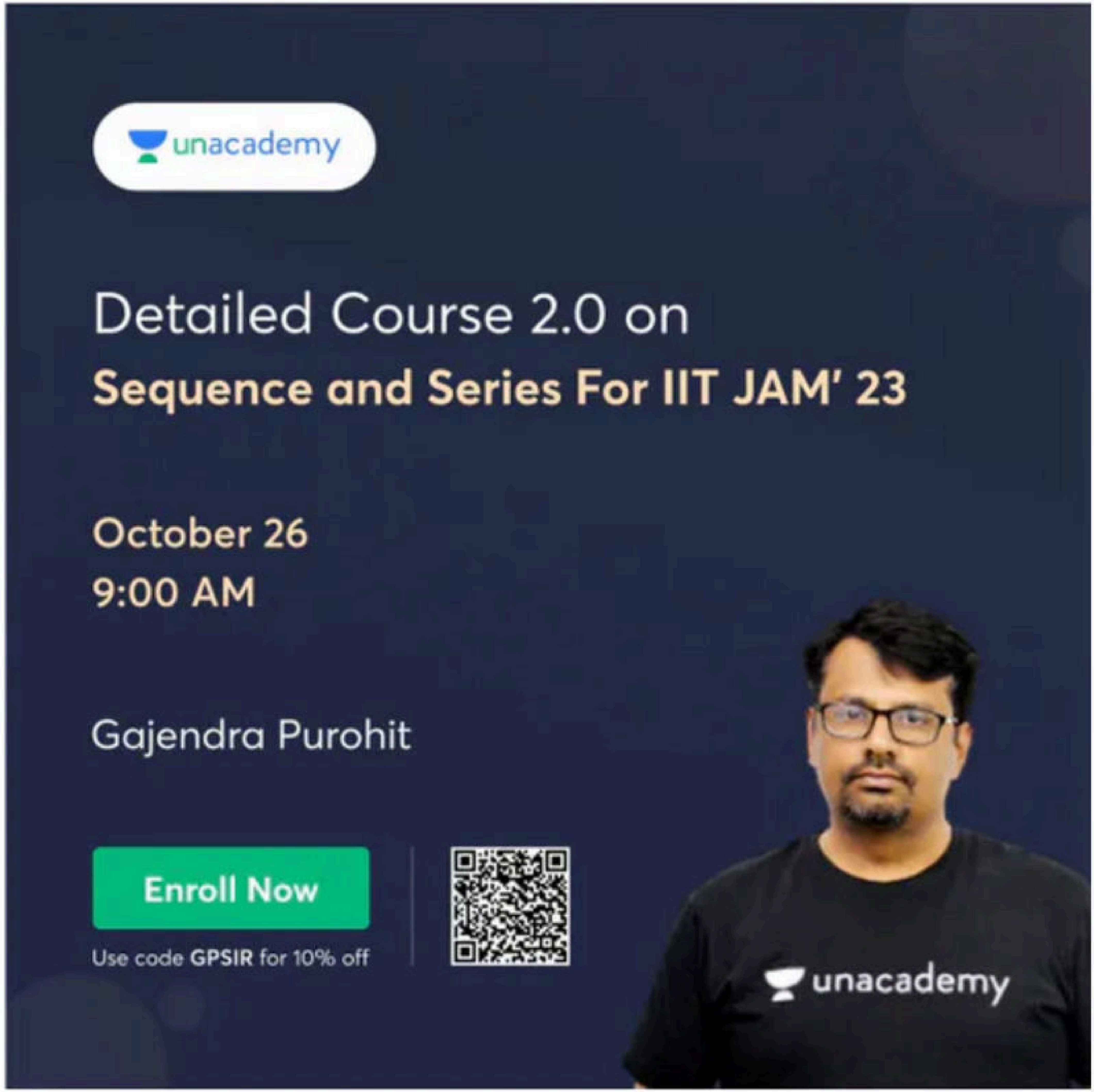
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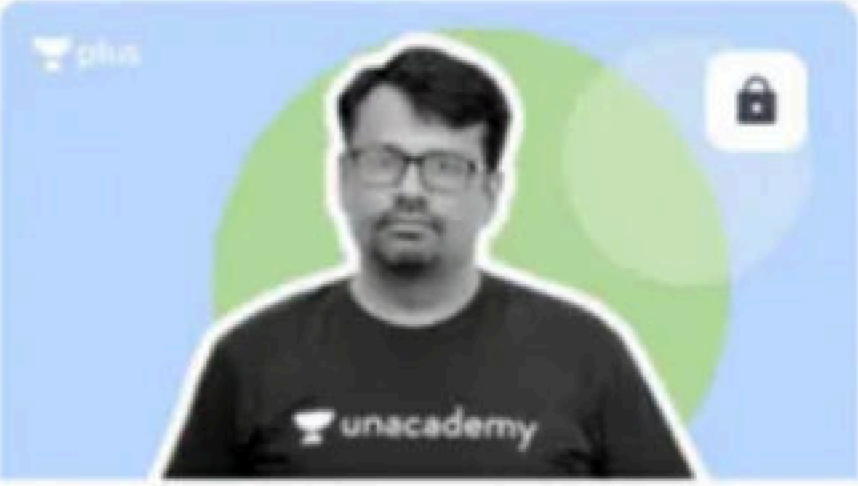

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