



Gajendra Purohit

Legend in CSIR-UGC NET & IIT-JAM

- Unlock Code : GPSIR ~ PhD, CSIR NET (Maths) | Youtuber(800K+165K Sub.)/Dr.Gajendra Purohit ( Maths), 17+ Yr. Experience, Author

50M Watch mins

3M Watch mins (last 30 days)

44K Followers

2K Dedications

→ **TOP EDUCATOR ON UNACADEMY  
FOR CSIR NET & IIT JAM**

**YouTuber with 800K Subscribers**

→ **AUTHOR OF BEST SELLER BOOK  
FOR CSIR NET & IIT JAM**

**Get  
10% Off**

**Referral Code : GP SIR**





## RANK BOOSTER COURSE UNIT 2 CSIR NET 2022

23<sup>rd</sup> AUGUST

Gajendra Purohit

[Enroll Now](#)

USE CODE  
**GPSIR**  
FOR 10% OFF



## DETAILED COURSE 2.0 LINEAR ALGEBRA FOR IIT JAM 2023

8<sup>th</sup> SEPTEMBER

Gajendra Purohit

[Enroll Now](#)

USE CODE  
**GPSIR**  
FOR 10% OFF



# FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months                    ₹ 908 / mo  
Save 67%  
Total ₹ 21,780

You get 6 months extra for free                    Offer expires 15 Jun 2022

✓ 12 months                    ₹ 1,248 / mo  
Save 54%  
Total ₹ 14,974

You get 6 months extra for free                    Offer expires 15 Jun 2022

9 months                    ₹ 1,497 / mo  
Save 45%  
Total ₹ 13,475

6 months                    ₹ 2,042 / mo  
Save 25%  
Total ₹ 12,252

3 months                    ₹ 2,269 / mo  
Save 17%  
Total ₹ 6,807

1 month                    ₹ 2,723 / mo  
Save 0%  
Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

No cost EMI available on 6 months & above subscription plans

24 months                    ₹ 817 / mo  
Save 67%  
Total ₹ 21,700 ₹ 19,602

You get 6 months extra for free                    Offer expires 15 Jun 2022

✓ 12 months                    ₹ 1,123 / mo  
Save 54%  
Total ₹ 13,477

You get 6 months extra for free                    Offer expires 15 Jun 2022

9 months                    ₹ 1,348 / mo  
Save 45%  
Total ₹ 12,128

6 months                    ₹ 1,838 / mo  
Save 25%  
Total ₹ 11,027

3 months                    ₹ 2,042 / mo  
Save 17%  
Total ₹ 6,126



After Using  
My Referral  
Code



GPSIR

Awesome! You get 10% off

Proceed to pay

## VECTOR SPACE

**Field :** A non-empty commutative set  $(F, +, \cdot)$  is a field if every non-zero element of  $F$  has multiplicative inverse.

**Note :** If  $(F, +, \cdot)$  is field then  $(F, +)$  and  $(F^*, \cdot)$  are abelian group.

**Note :**

- (1)  $\mathbb{Z}_p$  is a field iff  $p$  is prime.
- (2) Cardinality of field never divisible by two

distinct prime number.

i.e. if  $F$  is field then  $|F| = p^n$ ;  $p$  is prime.

**Internal composition :** If  $A$  be any set s.t.  $a * b \in A$ ,

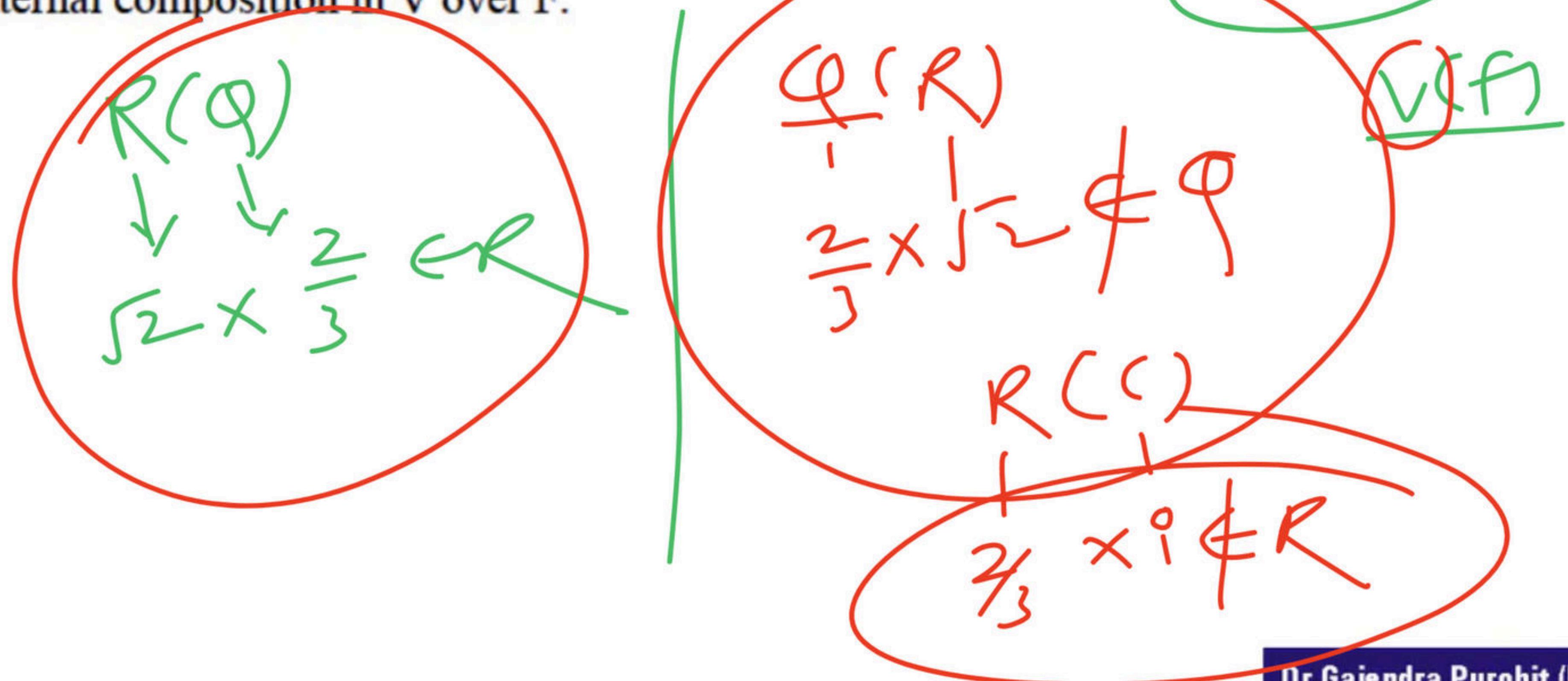
where  $a \in A$ ,  $b \in A$  then  $*$  is said to be internal composition in the set  $A$ .





**External composition** : Let  $V$  &  $F$  be any two sets.

If  $a * \alpha \in V$ , where  $a \in V$ ,  $\alpha \in F$  then '\*' is said to be external composition in  $V$  over  $F$ .



Vector Space : Let  $(F, +, \cdot)$  be a field &  $V$  be non-empty set. The elements of  $F$  are called scalars & the elements of  $V$  are called vectors. Then  $V$  is a vector space over the field if –

(i)  $(V, +)$  is abelian group.

(ii)  $V$  is closed with respect to scalar multiplication  $\alpha u \in V$ ,  
for all  $\alpha \in F, u \in V$

(iii) Scalar multiplication and vectors addition

(a)  $\alpha(u + v) = \alpha u + \alpha v$ , for all  $\alpha \in F$ , for all  $u, v \in V$

(b)  $(\alpha + \beta)u = \alpha u + \beta u$ , for all  $\alpha, \beta \in F$ , for all  $u \in V$

(c)  $(\alpha\beta)u = \alpha(\beta u)$ ; for all  $\alpha, \beta \in F$ , for all  $u \in V$

(iv)  $1.u = u$ , for all  $u \in V$ , where 1 is the unit element of the field  $F$ .

$V/F$

$V^+$

$a \in V, b \in V$

$\alpha u$

$0 \in V$

## Matrix Vector space :

Let  $V = \{[a_{ij}]_{m \times n}; a_{ij} \in \mathbb{R}\}$  and  $F = \mathbb{C}$

then  $V$  is vector space over  $F$  if  $Q$  is subset of  $P$

$$\mathbb{Q}(\sqrt{2}) = \{ a + \sqrt{2}b \mid a, b \in \mathbb{Q} \}$$

$$b=0$$

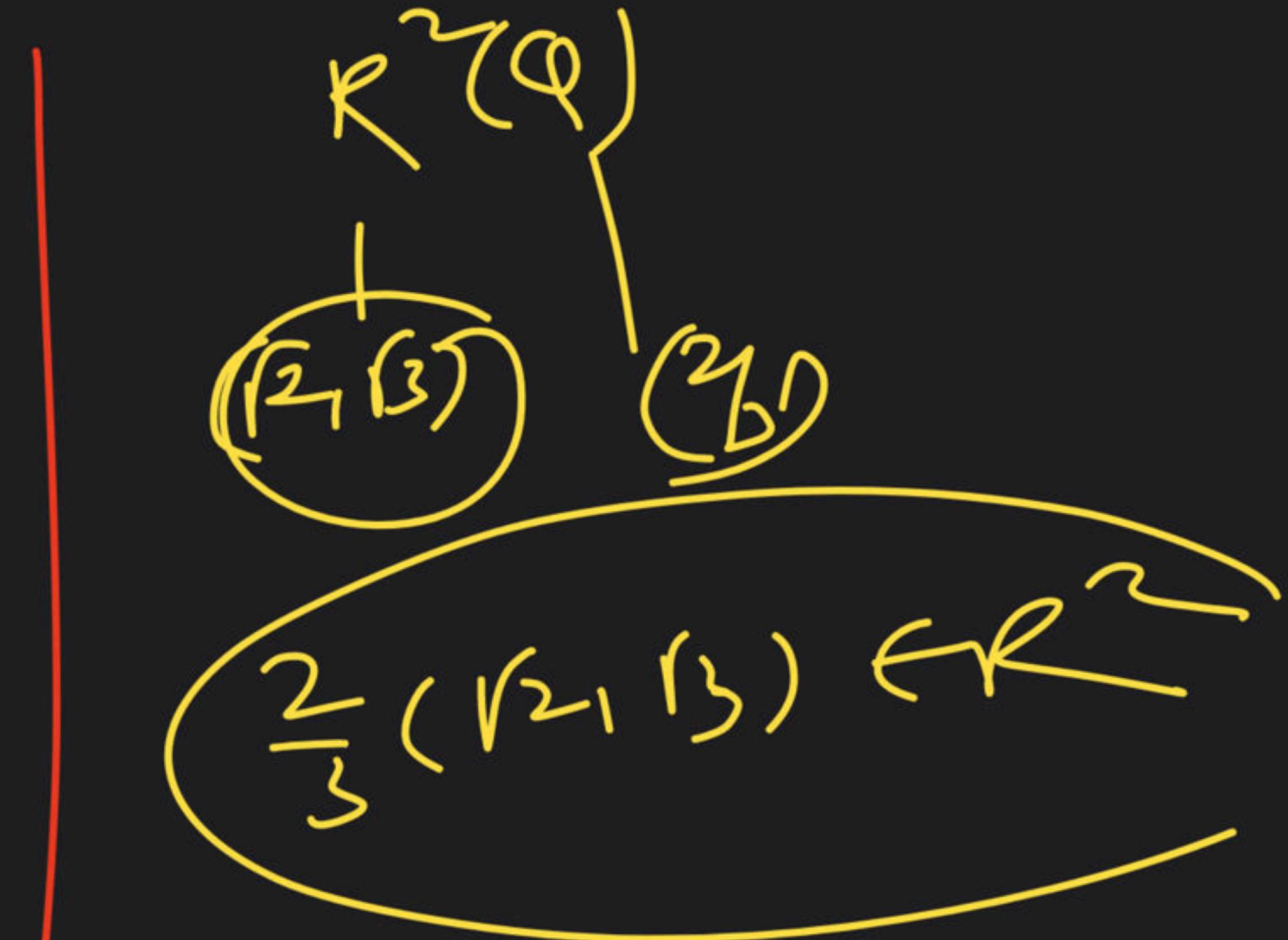
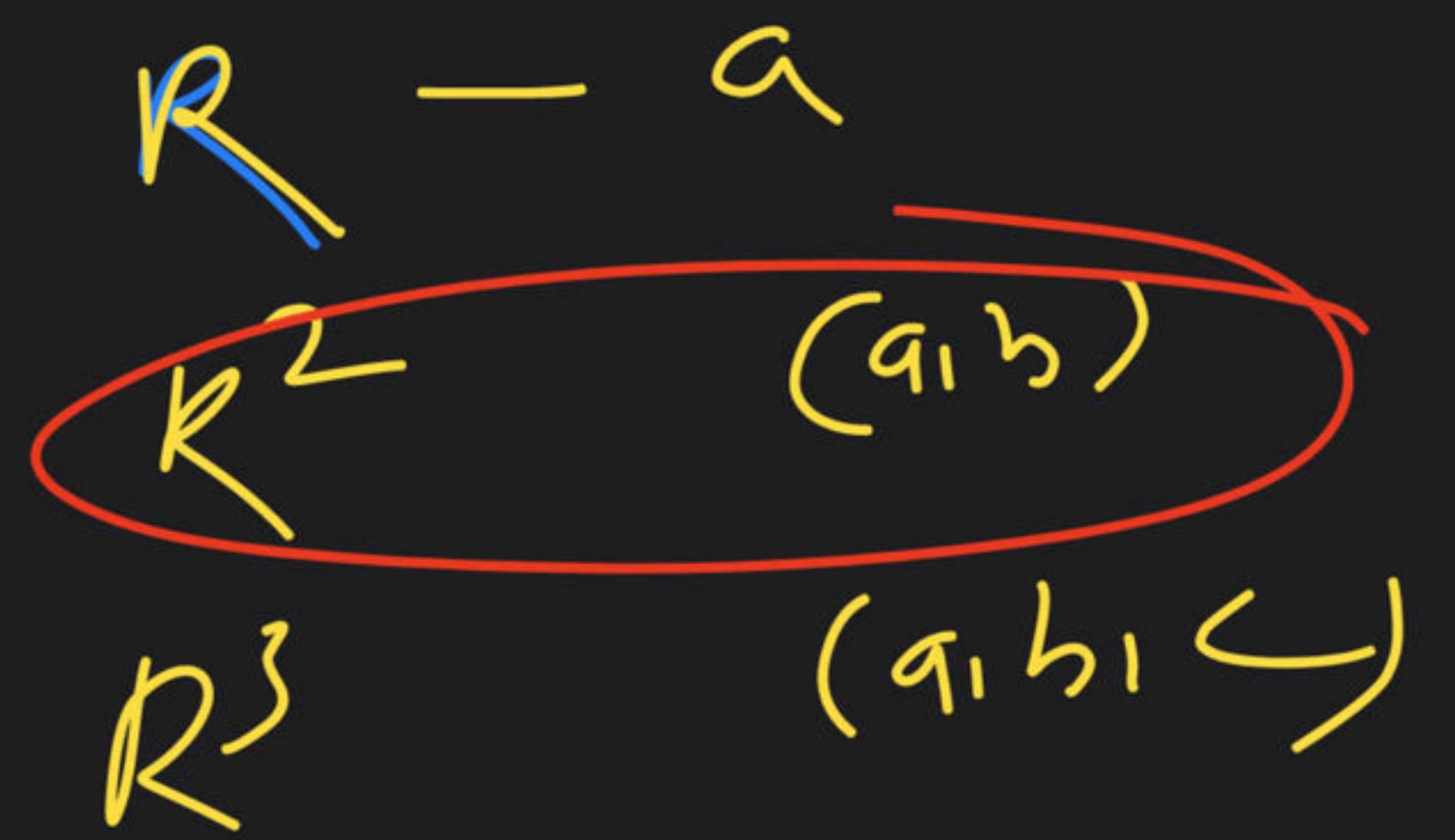
$$\mathbb{Q} \subset \mathbb{Q}(\sqrt{2})$$

$$\{ a + \sqrt{2}0 \mid a \in \mathbb{Q} \} = \mathbb{Q}$$

---

$$\mathbb{Q}\sqrt{2}(\mathbb{Q})$$





## ~~( Polynomial Vector space )~~

$$R[x] = \{ p(x) : p(x) = a_0 + a_1 x + \dots \}$$

|  $a_i \in \mathbb{R}$  |  $\cup +$

$$p(x) = 1 - 2x$$

$$q(x) = 1 + 2x$$

$$p(x) + q(x) = 1$$

$$1 - 2x + 2x$$

$$1$$

$$1 + 2x - 2x$$

## Sequence Vector Space :

Let  $V = \{\langle x_n \rangle \mid x_n \in \mathbb{R}\}$  and  $F = \mathbb{R}$

s.t.  $\langle x_n \rangle + \langle y_n \rangle = \langle x_n + y_n \rangle$  and  $\alpha \langle x_n \rangle = \langle \alpha x_n \rangle$

(Ex)

$V = \{\langle x_n \rangle \mid \langle x_n \rangle \text{ is convergent}\}$

$V = \{\langle x_n \rangle \mid \langle x_n \rangle \text{ is oscillating}\} \times$

$V = \{\langle x_n \rangle \mid \begin{cases} \lim_{n \rightarrow \infty} x_n = 0 \\ \lim_{n \rightarrow \infty} x_n = 1 \end{cases}\}$

$V = \{\langle x_n \rangle \mid \lim_{n \rightarrow \infty} x_n = 1\} \times$

$V = \{\langle x_n \rangle \mid \lim_{n \rightarrow \infty} x_n = \text{DNE}\} \times$



## Function Space :

Let  $V = \{f | f: X \rightarrow R\}$  &  $F = (R, +, \cdot)$

$V = \{f | f: X \rightarrow R$  if continuous

$V = \{f | f: X \rightarrow \mathbb{R}$  bounded

$V = \{f | f: X \rightarrow R$  and + vanishes at -

finest many points +  $\exists$

$$f(m) = 0$$

$V = \{f : K \rightarrow R \text{ and } f \text{ is monotonic}\}$

$f(x) = n^2$

$g(x) = -n$

$f(x) + g(x) = n^2 - n$

$V = \{f \mid f : X \rightarrow R\}$

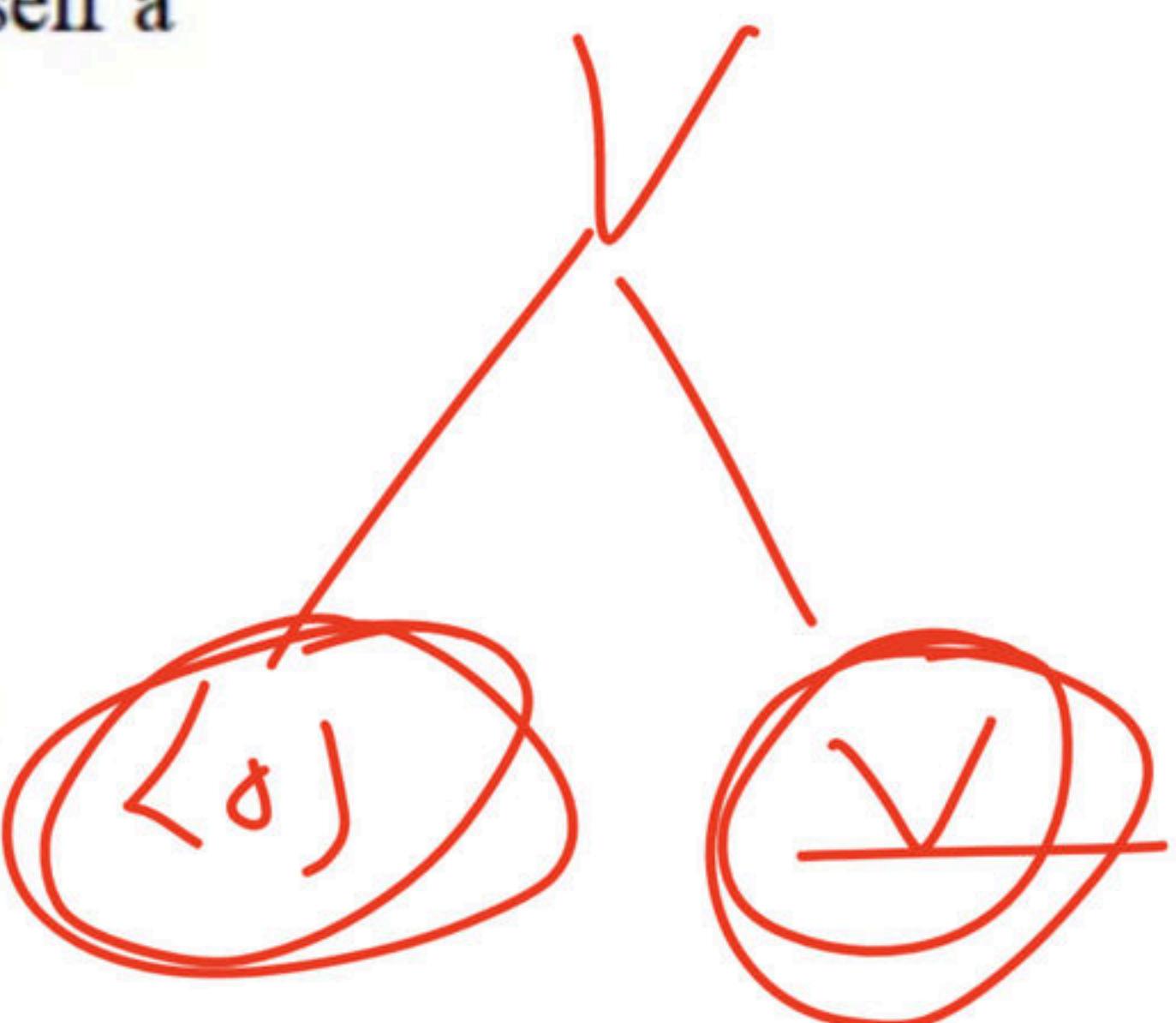
$f(x) = f(-x)$

$f(x) = -f(-x)$

**Subspace :** Let  $V(F)$  be a vector space. Let  $W$  be any non-empty subset of  $V$ , then  $W$  is called subspace of  $V$  over  $F$  if  $W$  itself a vector space with the same field and same composition.

**Note :**

- (1)  $\{0\}$  and  $V$  itself are always subspace of  $V$ .
- (2) Any subspace other than  $\{0\}$  and  $V$  known as proper subspace of  $V$ .





**COMPLETE COURSE ON**

**MATHEMATICS**

**FOR IIT-JAM 2022**

# TOPICS TO BE COVERED

- REAL ANALYSIS
- FUNCTION OF ONE & TWO VARIABLE
- LINEAR ALGEBRA
- MODERN ALGEBRA

# TOPICS TO BE COVERED

- SEQUENCE & SERIES
- INTEGRAL CALCULUS
- VECTOR CALCULUS
- DIFFERENTIAL EQUATION

# FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months                    ₹ 908 / mo  
Save 67%  
Total ₹ 21,780

You get 6 months extra for free                    Offer expires 15 Jun 2022

✓ 12 months                    ₹ 1,248 / mo  
Save 54%  
Total ₹ 14,974

You get 6 months extra for free                    Offer expires 15 Jun 2022

9 months                    ₹ 1,497 / mo  
Save 45%  
Total ₹ 13,475

6 months                    ₹ 2,042 / mo  
Save 25%  
Total ₹ 12,252

3 months                    ₹ 2,269 / mo  
Save 17%  
Total ₹ 6,807

1 month                    ₹ 2,723 / mo  
Save 0%  
Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

No cost EMI available on 6 months & above subscription plans

24 months                    ₹ 817 / mo  
Save 67%  
Total ₹ 21,700 ₹ 19,602

You get 6 months extra for free                    Offer expires 15 Jun 2022

✓ 12 months                    ₹ 1,123 / mo  
Save 54%  
Total ₹ 13,477

You get 6 months extra for free                    Offer expires 15 Jun 2022

9 months                    ₹ 1,348 / mo  
Save 45%  
Total ₹ 12,128

6 months                    ₹ 1,838 / mo  
Save 25%  
Total ₹ 11,027

3 months                    ₹ 2,042 / mo  
Save 17%  
Total ₹ 6,126



After Using  
My Referral  
Code



GPSIR

Awesome! You get 10% off

Proceed to pay

**FOUNDATION COURSE OF**

**MATHEMATICS**

**FOR CSIR-NET**

**Test for subspace** : A necessary and sufficient condition for subspace.

(1) Two step test :

(a)  $x + y \in W$  for all  $x \in W$ , for all  $y \in W$

(b)  $\alpha x \in W$  for all  $\alpha \in F$ ,  $x \in W$

(2) One step test :

$\alpha x + \beta y \in W$ ; for all  $\alpha, \beta \in F$  & for all  $x, y \in W$

Subspace in R : The only subspace in R are  $\{0\}$  &  $\mathbb{R}$

**Subspace in  $\mathbf{R}^2$**  : There are three subspace in  $\mathbf{R}^2$ .

- (i)  $W_1 = \{(0, 0)\}$
- (ii)  $W_2 = \mathbf{R}^2$
- (iii) Any line passing through an origin is also a subspace of  $\mathbf{R}^2$ .

### **Subspace in $\mathbf{R}^3$**

There are four subspace in  $\mathbf{R}^3$

- (i)  $W = \{(0, 0, 0)\}$
- (ii)  $W = \mathbf{R}^3$
- (iii) Any plane passing through an origin
- (iv) Any line passing through an origin

## Vector Matrix Space :

~~Matrix space : Let  $V = \{A = [a_{ij}]_{n \times n}, a_{ij} \in \mathbb{R}\}$  &  $F = \mathbb{R}$~~

(I)  $\omega_1 = \{A \in V : A^T A = A A^T = I\}$

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

(II)  $\omega_2 = \{A \in V : A \tilde{=} A\}$

$$A$$

(III)  $\omega_3 = \{A \in V : A \tilde{=} I\}$

$$3A$$

(IV)  $\omega_4 = \{A \in V : A = A^T\}$

$$A = A^T$$

(V)  $\omega_5 = \{A \in V : A = -A^T\}$

$$A = -A^T$$

(VI)  $\omega_6 = \{A \in V : \text{Tr}(A) = 0\}$  is Subspace

(VII)  $\omega_7 = \{A \in V : |A| = 0\}$

$w_g = \bigcup A = \bigcup a_{ij} \quad \wedge \quad$

upper triangle

Lower Tri

diag

Scam



A hand-drawn diagram in red ink on a black background illustrating vector addition. It features three points labeled A, B, and C. Point A is at the origin. Point B is located in the first quadrant. Point C is located in the second quadrant. A horizontal line segment connects A and B, representing the vector  $\vec{AB}$ . A second horizontal line segment connects A and C, representing the vector  $\vec{AC}$ . A third horizontal line segment connects B and C, representing the vector  $\vec{BC}$ . The angle between  $\vec{AB}$  and  $\vec{AC}$  is labeled  $\alpha$ , and the angle between  $\vec{AB}$  and  $\vec{BC}$  is labeled  $\beta$ .

$$A + B = C$$

## **Polynomial space :**

Q.1. Which one of the following sets of vectors

$\alpha = (a_1, a_2, \dots, a_n)$  in  $R^n$  is a subspace of  $R^{n(n \geq 3)}$ ?

- (a) all  $\alpha$  such that  $a_1 \geq 0$
- (b) all  $\alpha$  such that  $a_1 + 3a_2 = a_3$
- (c) all  $\alpha$  such that  $a_2 = a_1^2$
- (d) all  $\alpha$  such that  $a_1 a_2 = 0$

$$(1, 0, 0, \dots)$$

$$(0, 1, 0, 0, \dots)$$

~~$$(1, 1, 0, \dots)$$~~

$$(1, 0, 0, \dots)$$

$$- 2(1, 0, 0, \dots)$$

$$1 + 3 \cdot 0 - 0 = 0$$

$$p_1 = (1, 1, 0, 0, \dots)$$

$$p_2 = (0, 1, 0, 0, \dots)$$

$$p_1 + h = (2, 1, 0, 0, \dots)$$

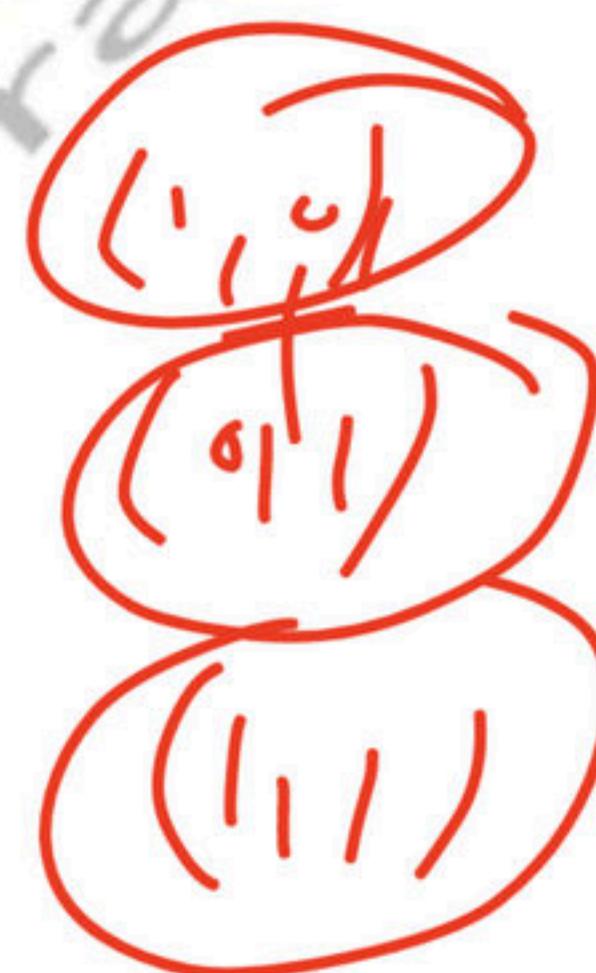
Q.2. Which of the following subsets are subspace?

(a)  $W = \{(x, y) \in \mathbb{R}^2; xy = 0\}$

(b)  $X = \{(x, y) \in \mathbb{R}^2; y = 3x\}$

(c)  $Y = \{(x, y) \in \mathbb{R}^2; x^2 - y^2 = 0\}$

(d)  $Z = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 = 0\}$



a c  
b d  
c b  
d 1 a



**CSIR-UGC NET** is giving you an opportunity to interact with Top Educators **at Unacademy Centres**

September 25 | 4 PM

**Location:** Unacademy World, 30-B, Pusa Road, Block 11, Old Rajinder Nagar, Rajinder Nagar, New Delhi

[Enroll Now](#)



Q.3 Which one of the following is a subspace of  $\mathbb{R}^3$ ?

- (a)  $(x, y, z) \in \mathbb{R}^3 \mid x + 2y = 0, 2x + 3z = 0\}$
- (b)  $\{(x, y, z) \in \mathbb{R}^3 \mid 2x + 3y + 4z - 3 = 0, z = 0\}$
- (c)  $\{(x, y, z) \in \mathbb{R}^3 \mid x \geq 0, y \geq 0\}$
- (d)  $\{(x, y, z) \in \mathbb{R}^3 \mid x - 1 = 0, y = 0\}$

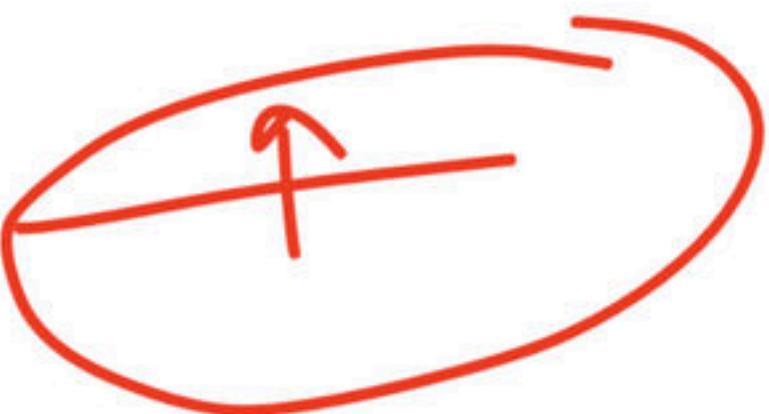
Q.4 Which of the following sets of functions from  $\mathbb{R}$  to  $\mathbb{R}$  is a vector space over  $\mathbb{R}$ ?

$$\cancel{S_1 = \left\{ f \mid \lim_{x \rightarrow 3} f(x) = 0 \right\}}$$

$$\cancel{S_2 = \left\{ g \mid \lim_{x \rightarrow 3} g(x) = 1 \right\}}$$

$$S_3 = \left\{ h \mid \lim_{x \rightarrow 3} h(x) \text{ exists} \right\}$$

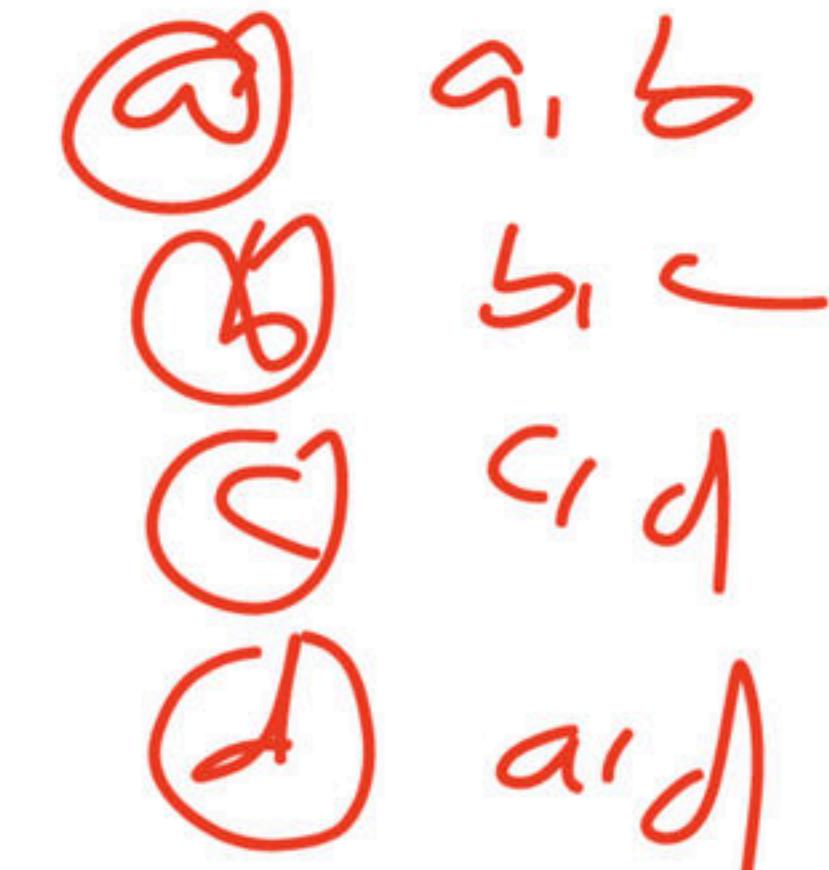
- (a) Only  $S_1$       (b) Only  $S_2$
- (c)  $S_1$  and  $S_3$  but not  $S_2$
- (d) All the three vector spaces



**Q.5** Let  $M_n$  denote the vector space of all  $n \times n$  real matrices. Among the following subsets of  $M_n$ , decide which are linear subspaces.

- (a)  $V_1 = \{A \in M_n : A \text{ is non-singular}\}$
- (b)  $V_2 = \{A \in M_n : \det(A) = 0\}$
- (c)  $V_3 = \{A \in M_n : \text{trace}(A) = 0\}$
- (d)  $V_4 = \{BA : A \in M_n\}$ , where B is some fixed matrix in  $M_n$ .

$$|A| \neq 0$$



$$\alpha(BA_1 + BA_2) \in B(M_n)$$

$\left( \alpha A_1 + \alpha A_2 \right) \in M_n$

$$PA + P^T$$

$$P(A + A^T)$$



## RANK BOOSTER COURSE UNIT 2 CSIR NET 2022

23<sup>rd</sup> AUGUST

Gajendra Purohit

Enroll Now

USE CODE  
**GPSIR**  
FOR 10% OFF



## DETAILED COURSE 2.0 LINEAR ALGEBRA FOR IIT JAM 2023

8<sup>th</sup> SEPTEMBER

Gajendra Purohit

Enroll Now

USE CODE  
**GPSIR**  
FOR 10% OFF



# Educator Profile



## Gajendra Purohit

#5 Educator in CSIR-UGC NET

[Follow](#)

Dr.Gajendra Purohit PhD, CSIR NET (Maths) | Youtuber(330K+30k Sub.)/Dr.Gajendra Purohit (Maths), 17+ Yr. Experience, Author of Bestseller

11M Watch mins

1M Watch mins (last 30 days)

22k Followers

1k Dedications



### CSIR-UGC NET

[SEE ALL](#)

HINDI MATHEMATICAL SCIENCES  
Course on Linear Algebra, Partial Diff. Equation & Calculus  
Starts on Mar 1, 2021 • 24 lessons  
Gajendra Purohit

HINDI MATHEMATICAL SCIENCES  
Course on Complex Analysis & Integral Equation  
Starts on Jan 14, 2021 • 16 lessons  
Gajendra Purohit

HINDI MATHEMATICAL SCIENCES  
Foundation Course on Mathematics for CSIR 2021  
Starts on Dec 7, 2020 • 20 lessons  
Gajendra Purohit

### Educator highlights

- 📍 Works at Pacific Science College
- 📍 Studied at M.Sc., NET, PhD(Algebra), MBA(Finance), BEd
- 📍 PhD, NET | Plus Educator For CSIR NET | Youtuber (260K+Subs.) | Director Pacific Science College |
- 📍 Lives in Udaipur, Rajasthan, India
- 📍 Unacademy Educator since

# FEE DETAILS FOR IIT JAM SUBSCRIPTION

No cost EMI available on 6 months & above subscription plans

24 months                    ₹ 908 / mo  
Save 67%  
Total ₹ 21,780

You get 6 months extra for free                    Offer expires 15 Jun 2022

✓ 12 months                    ₹ 1,248 / mo  
Save 54%  
Total ₹ 14,974

You get 6 months extra for free                    Offer expires 15 Jun 2022

9 months                    ₹ 1,497 / mo  
Save 45%  
Total ₹ 13,475

6 months                    ₹ 2,042 / mo  
Save 25%  
Total ₹ 12,252

3 months                    ₹ 2,269 / mo  
Save 17%  
Total ₹ 6,807

1 month                    ₹ 2,723 / mo  
Save 0%  
Total ₹ 2,723

To be paid as a one-time payment

Have a referral code?

Proceed to pay

No cost EMI available on 6 months & above subscription plans

24 months                    ₹ 817 / mo  
Save 67%  
Total ₹ 21,700 ₹ 19,602

You get 6 months extra for free                    Offer expires 15 Jun 2022

✓ 12 months                    ₹ 1,123 / mo  
Save 54%  
Total ₹ 13,477

You get 6 months extra for free                    Offer expires 15 Jun 2022

9 months                    ₹ 1,348 / mo  
Save 45%  
Total ₹ 12,128

6 months                    ₹ 1,838 / mo  
Save 25%  
Total ₹ 11,027

3 months                    ₹ 2,042 / mo  
Save 17%  
Total ₹ 6,126



After Using  
My Referral  
Code



GPSIR

Awesome! You get 10% off

Proceed to pay

**THANK YOU VERY MUCH EVERYONE**

**GET THE UNACADEMY PLUS SUBSCRIPTION SOON.**

**TO GET 10% DISCOUNT IN TOTAL SUBSCRIPTION AMOUNT**

**USE REFERRAL CODE: GPSIR**