SEIR Disease-Spread Modeling with Physics-Informed Neural Networks

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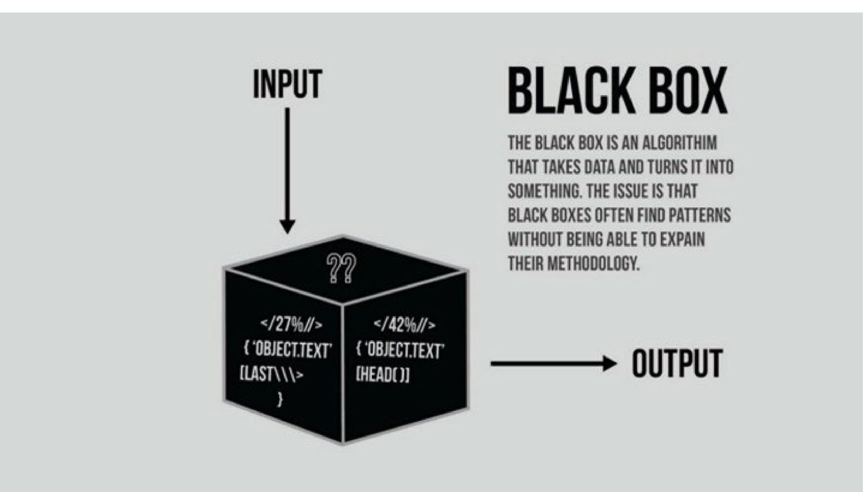
Introduction

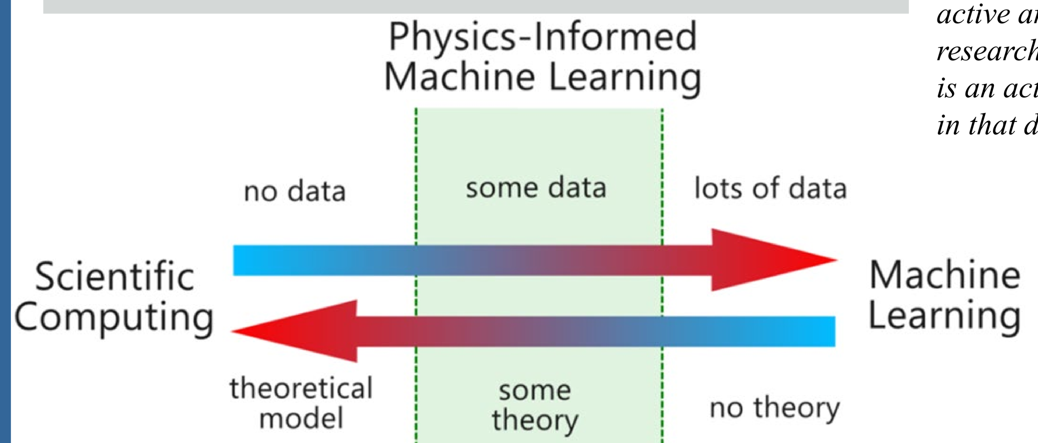
Epidemiological modeling is crucial for understanding and predicting disease transmission dynamics. Traditional SEIR (Susceptible-Exposed-Infected-Recovered) models rely on ordinary differential equations with fixed parameters, limiting their adaptability to complex real-world scenarios. This study explores the application of Physics-Informed Neural Networks (PINNs) as an innovative approach to enhance the flexibility and accuracy of SEIR modeling, giving the hybrid model additional capabilities like handling noisy data, working with variable transmission rates, and producing better accuracy.

Physics-informed neural networks (PINNs) utilize automatic differentiation and neural network function approximators to learn solutions that adhere to physics constraints. In this study, we employed PINNs as a novel method to simulate SEIR model.

The Need for PINN

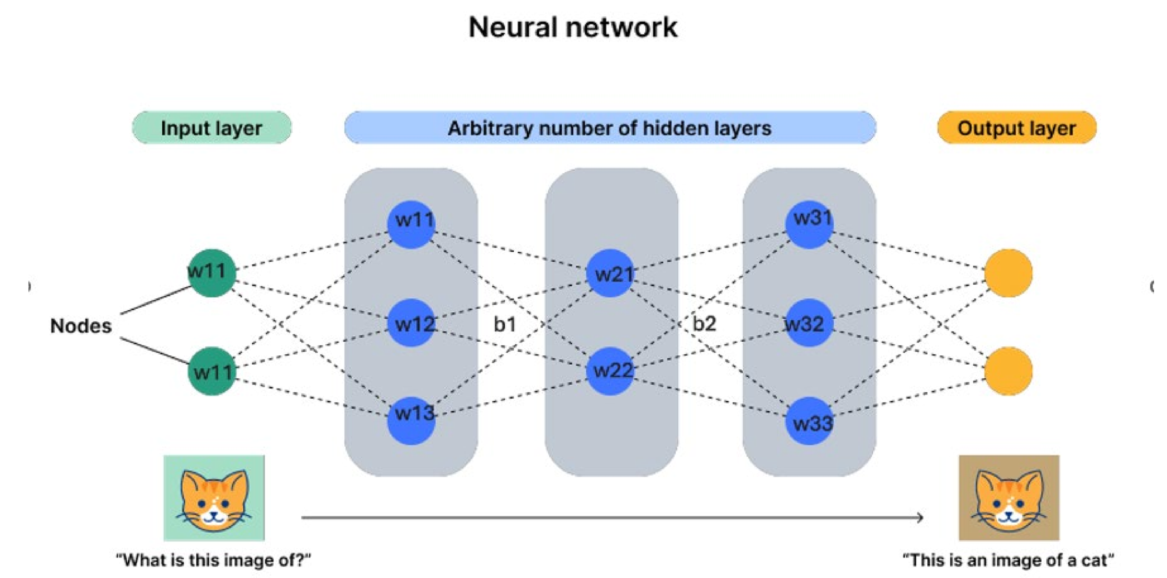
Traditional machine learning models rely heavily on data for training. During their training process, they functionally adapt their internal parameters to replicate the expected output. However, the exact way how they do this is not completely understood, which gives rise to their “**Black Box**” nature.





The black box nature of machine learning models is an active area of research. PINN is an active step in that direction.

What are Neural Networks and PINN?



The image above is a simple neural network with an input layer, an output layer, and 3 hidden layers. The network predicts whether the animal in the image is a cat.

Two important concepts related to neural networks are:

1. **Loss Function**: |Actual Value – Predicted Value|
2. **Autodifferentiation**: Adjust internal parameters using partial derivatives in forward and backward pass

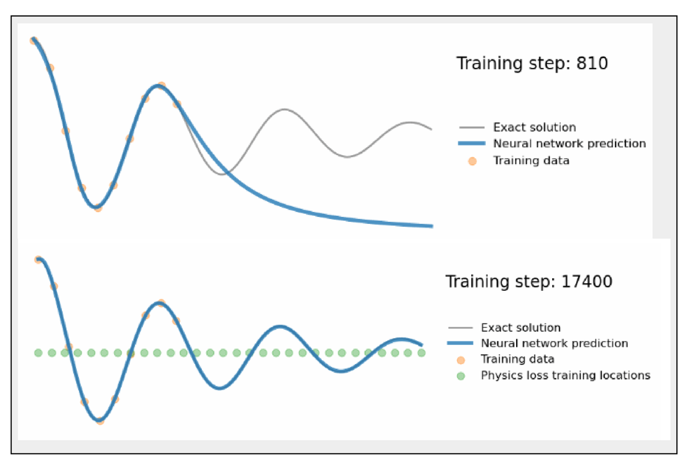
An Example:

Damped Harmonic oscillation—an introductory physics problem that can be modeled using the following ODE:

$$m\frac{d^2u}{dx^2} + \mu\frac{du}{dx} + ku = 0$$

While we can manually compute the solution through traditional mathematics, utilizing it in the context of PINNs reveals how powerful scientific machine learning can be. By implementing this ODE as a loss-function to guide the training of a PINNs, we can reproduce the traditional mathematical solution with very few data points.

More so, the accuracy of PINNs becomes even more clear when we compare traditional machine learning models (stochastic) with PINNs (physics-informed).



The top image is traditional ML models, which sees strong performance in the short-term but struggles to converge on an accurate solution when given few data points. For PINNs, with the same amount of data, they can easily extrapolate to fit the exact solution due its ability to continually gain performance over many training steps.

Hybrid SEIR Model using PINNs

Whenever an infection is introduced in a population of size N, we can divide the existing population into four mutually-exclusive compartments:

$$\text{S (Susceptible)} \xrightarrow{\lambda} \text{E (Exposed)} \xrightarrow{\sigma} \text{I (Infected)} \xrightarrow{\gamma} \text{R (Recovered)}$$

The arrows in the figure above show the flow of population from one compartment to another. The flow is governed by the following rates:

λ : Exposure Rate

σ : Incubation Rate

γ : Recovery Rate

The differential equations that model this flow are:

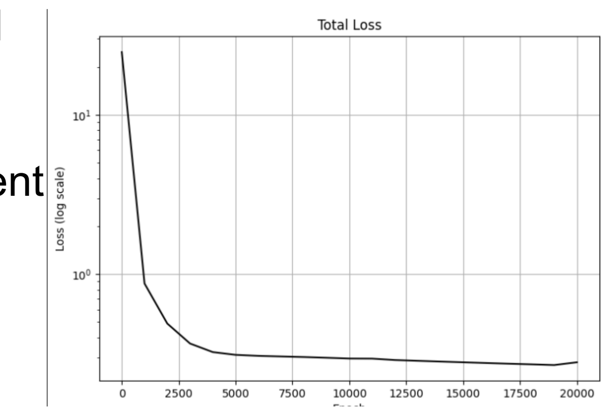
$$\frac{dS}{dt} = -\lambda S \longrightarrow \text{↓ Exposure Rate}$$

$$\frac{dE}{dt} = -\sigma E + \lambda S \longrightarrow \text{↓ Incubation Rate, ↑ Exposure rate}$$

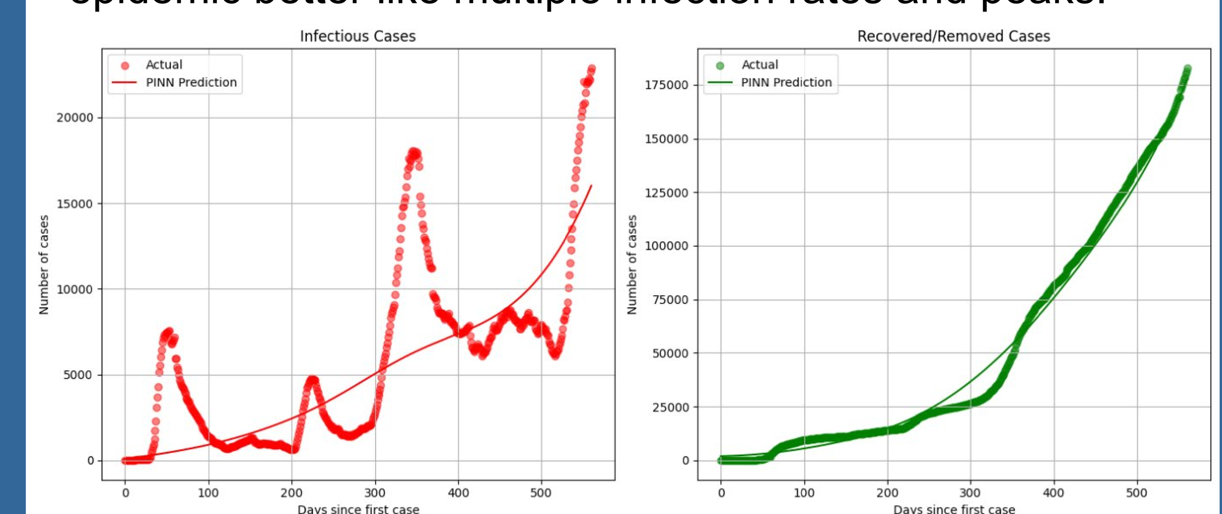
$$\frac{dI}{dt} = -\gamma I + \sigma E \longrightarrow \text{↓ Recovery Rate, ↑ Incubation rate}$$

$$\frac{dR}{dt} = \gamma I \longrightarrow \text{↑ Recovery rate}$$

For our research, we trained four neural networks by incorporating the differential equation of each compartment into their loss function. We combined the overall loss of the model to depict in the graph on the right that decreased over time.



Further, upon comparing the output of our hybrid model with the actual data in our dataset, we could observe that the hybrid model adapted the characteristics of the epidemic better like multiple infection rates and peaks.



The image on the left is the trace of the number of infected cases predicted by our model versus actual data while the one on the right is a portrayal of how better our recovery prediction fits.

Conclusion / Future Studies

In conclusion, our exploration of PINNs in modeling the SEIR epidemiological model resulted in outperforming the plain ODE curves by better adapting the spread of Covid-19 in S. Korea. The model was trained and tested on this dataset. Due to its high accuracy and its ability to model sparse-data scenario, PINNs stands as a promising and novel approach to improve traditional machine learning models.

References

- Denq, Christopher, and Dr. Liang Kong. "A Meshfree Deep Learning Approach for Numerical Solution of Differential Equations." Pre-print. Mar. 2024
- Lagaris, I. E., Likas, A., & Fotiadis, D. I. (1998). Artificial neural networks for solving ordinary and partial differential equations. IEEE Transactions on Neural Networks
- Maziar Raissi, Paris Perdikaris, & George Em Karniadakis. (2017). Physics Informed Deep Learning (Part I): Data-driven Solutions of Nonlinear Partial Differential Equations.
- Moseley, B., Markham, A., & Nissen-Meyer, T. (2020). Solving the wave equation with physics-informed deep learning. ArXiv
- Wang, H., Qiu, X., Yang, J., Li, Q., Tan, X., & Huang, J. (2023). Neural-SEIR: A flexible data-driven framework for precise prediction of epidemic disease. *Mathematical biosciences and engineering : MBE*, *20*(9), 16807–16823. https://doi.org/10.3934/mbe.2023749

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Source Code

You can find our work on the following GitHub repository and track ongoing development in it:

www.github.com/abhisoni24/seir-pinn-hybrid