# Number Systems and Bit Manipulation's

Tuesday, August 16, 2022 1:00 PM

**Topics covered**

1) Understanding Number Systems
2) Binary to decimal & Vice versa
3) Significance of MSB
4) Range of datatypes

5) Bitwise operators
6) Properties of bitwise operators
7) Various tips & tricks solving problems based on them

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**Decimal number** : $\{0-9\}$

$123 \Rightarrow 1 \cdot 100 + 2 \cdot 10 + 3$

$\begin{array}{ccccc} 3 & 2 & 1 & 0 & \\ 1 & 2 & 3 & & \end{array}$ $\quad 1 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0 + 0 \cdot 10^{-1}$

$\begin{array}{ccccccc} 6 & 5 & 4 & 3 & 2 & 1 & 0 \\ _ & _ & _ & _ & _ & _ & _ \\ 10^6 & 10^5 & 10^4 & 10^3 & 10^2 & 10^1 & 10^0 \end{array}$

**Number Systems**

$\{0,1\}$ : $\{0-9\}$

$\begin{array}{ccccc} 3 & 2 & 1 & 0 & \\ 1 & 0 & 1 & 1 & \end{array}$ $\quad 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$ → not valid

$\begin{array}{ccc} 2 & 1 & 0 \\ _ & _ & _ \\ 8^2 & 8^1 & 8^0 \end{array}$ $\quad 1 \cdot 8^2 + 2 \cdot 8^1 + 3 \cdot 8^0 + 4 \cdot 8^{-1} + 5 \cdot 8^{-2}$ → Decimal number

$\begin{array}{ccc} 2 & 1 & 0 \\ _ & _ & _ \\ 8^2 & 8^1 & 8^0 \end{array}$ $\quad 1 \cdot 8^2 + 2 \cdot 8^1 + 3 \cdot 8^0$, e.g.

**Hexadecimal** : $0, 1, 2$

$\begin{array}{ccc} 2 & 1 & 0 \\ _ & _ & _ \\ 16^2 & 16^1 & 16^0 \end{array}$ $\quad 2 \cdot 16^2 + 0 \cdot 16^1 + 1 \cdot 16 + 2 \cdot 16^0$

$4 \cdot 16 + 0 \cdot 16 + 2 = 82$

$\begin{array}{ccc} 2 & 1 & 0 \\ _ & _ & _ \\ 16^2 & 16^1 & 16^0 \end{array}$ $\quad 2 \cdot 16^2 + 2 \cdot 16^1 + 1 \cdot 16^0$

$4 \cdot 16 + 2 \cdot 16 + 1 = 97$, e.g.

**Binary → Decimal** ?

$\begin{array}{ccccccc} 1 & 0 & 1 & 1 & 1 & & \\ _ & _ & _ & _ & _ & & \\ 2^4 & 2^3 & 2^2 & 2^1 & 2^0 & & \end{array}$ $\quad 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$

$16 + 4 + 2 + 1 = 23$

**Decimal to Binary** : **Method**

$2 \,|\, 25 \quad -1$
$2 \,|\, 12 \quad 0$
$2 \,|\, 6 \quad 0$
$2 \,|\, 3 \quad -1$
$2 \,|\, 1 \quad -1$
$2 \,|\, 0 \quad 1$

$\Rightarrow 11011$

$N = 100 \Rightarrow \boxed{0\ 1\ 1\ 0\ 0\ 1\ 0\ 0}$

$128\ \ 64\ \ 32\ \ 16\ \ 8\ \ 4\ \ 2\ \ 1$

$\Rightarrow 1 \cdot 128 + 1 \cdot 32 + 1 \cdot 4$

$100 - 64 = 36$

$36 - 32 = 4$

$4 - 4 = 0$

**Why Binary?**

{Computer stores everything in binary}

{Switch Mech: on 1, off 0}

$128\ \ 64\ \ 32\ \ 16\ \ 8\ \ 4\ \ 2\ \ 1$

$N = 13$

$\{1, 1, 0, 1\}$

$-13 : \ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1$ ✗ Not used computer

$-13 = \frac{-13 \cdot 2^8}{2^8} = \frac{-13 \cdot 2^8}{256}$ → Magic Code 101

$-a = 2^8 + a = 256 + 1$

→ How to 2 binary number? {Code}

$\{\frac{-13 \cdot 2^8}{2^8}\} \rightarrow$ Complement

$F\ 13 : \ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 1$

$+1$

{MSB - Most significant bit}

{Most left digit 1 → -ve, Most right digit 0 → +ve}

**Note:** MSB has val & magnitude

$\begin{array}{ccccccccc} -2^8 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \hline 1 & 0 & 0 & 0 & & & & \end{array}$

$\Rightarrow -2^8$

range

$1\ 0\ 0\ 0 \rightarrow \{-2^8 : -2^8\}$

$0\ 1\ 1\ 1 \rightarrow \{0 : 2^8 - 1\}$

$-2^8\ \ 2^8$

range

$1\ 0\ 0\ 0 \rightarrow \{-2^8 : -2^8\}$

$0\ 1\ 1\ 1 \rightarrow \{0 : 2^8 - 1\}$

$-2^8\ \ 2^8$

$\begin{array}{ccc} & \text{Signed} & \text{Unsigned} \\ \hline \text{min} & -2^{N-1} & 0 \\ \text{max} & 2^{N-1} - 1 & 2^N - 1 \end{array}$

{for N, n is signed datatype}

{unsigned for n → n is unsigned datatype}

$\frac{2^N}{2^1}$ {Range complexity to expand by increase in bits}

**8 bit Signed Number**

$\begin{array}{ccccccccc} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline -2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$

$2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 - 2^7 = 127$

$\Rightarrow \text{Max} : 127 = 2^7 - 1$

$a = \frac{N^{\text{th}} \text{ term}}{a = 1,\ r = 2,\ N = 8}$

$\text{Sum of N Terms in GP} \rightarrow \left\{ a = \frac{a}{r-1} \right\} \rightarrow 1 \times \frac{2^8 - 1}{2 - 1} = 2^8 - 1$

$2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 2^8 - 1$

**8 bit unsigned Number**

$\begin{array}{ccccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline 2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \end{array}$

$\Rightarrow \text{Max} : 2^8 - 1$

$\text{Sum of N Terms in GP} \rightarrow \left\{ a = \frac{a}{r-1} \right\} \rightarrow 1 \times \frac{2^8 - 1}{2 - 1} = 2^8 - 1$

$2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 2^8 - 1$

$N = N$ - unsigned Number

$\Rightarrow \text{Max} : 2^N - 1$

$\begin{array}{ccc} & \text{Signed} & \text{Unsigned} \\ \hline \text{min} & -2^{N-1} & 0 \\ \text{max} & 2^{N-1} - 1 & 2^N - 1 \end{array}$

$\text{min} \rightarrow \text{Min} = 0$

$\text{unsigned} \rightarrow \text{Max} : 2^N - 1$

$\text{Int} \rightarrow 4B \rightarrow 32\text{bit} \rightarrow \begin{cases} \text{unsigned} \rightarrow [0, 2^{32} - 1] \\ \text{signed} \rightarrow [-2^{31}, 2^{31} - 1] \end{cases}$

$\text{long} \rightarrow 8B \rightarrow 64\text{ bit} \rightarrow \begin{cases} \text{unsigned} \rightarrow [0, 2^{64} - 1] \\ \text{signed} \rightarrow [-2^{63}, 2^{63} - 1] \end{cases}$

$\text{long int}$
$\text{long long} \rightarrow 8B$

→ {Can see the calculation later}

**Bitwise Operations** : $\{\&, |, \wedge, \sim, <<, >>\}$

$\begin{array}{ccc} a & b & a \,|\, b \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$ $\quad \begin{array}{ccc} a & b & a \,\&\, b \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$ $\quad \begin{array}{ccc} a & b & a \wedge b \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$

$\sim a \rightarrow$ 1's complement of a

$\{a \,|\, b\} \rightarrow$ Bitwise OR

$\{a \,\&\, b\} \rightarrow$ Bitwise AND

$\{a \wedge b\} \rightarrow$ Bitwise XOR

$\{\sim a\} \rightarrow$ Bitwise NOT

$-a = \sim a + 1$

$\{a << b\} \rightarrow$ Bitwise Left Shift

$\{a >> b\} \rightarrow$ Bitwise Right Shift

$\sim 25 = -26$

$\sim -25 = 24$

$\sim 20 = -21$

$\sim -20 = 19$

$\sim -25$

$\sim 25$

$\sim 20$

$\sim -20$

$\sim 25$

$\sim 20$

$\sim -20$

$\sim -25$

$\begin{array}{ccccccccc} 128 & 64 & 32 & 16 & 8 & 4 & 2 & 1 \\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{array}$

$\sim 25 : \ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$

$\sim 20 : \ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$

$\sim -20 : \ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$

$\sim -25 : \ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$

$\sim -20 : \ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1$

$\sim -25$

$\sim 25$

$\sim 20$

$\sim -20$

$\sim 25$

$\sim 20$

$\sim -20$

$\sim -25$

$\sim -20$

$\sim -25$

$\sim -20$

$\sim -25