MA1101

Mathematics I

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Abhisruta Maity

21MS006

Indian Institute of Science Education and Research, Kolkata
Mohanpur, West Bengal, 741246, India

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§1 Construction of Numbers

§1.1 Construction of Integers

Definition 1.1 (Addition)**.** $a + b \stackrel{\text{def}}{=} [(m+p), (n+q)]$

Definition 1.2 (Multiplication)**.** $a \cdot b \stackrel{\text{def}}{=} [(mp+nq), (mq+np)]$

Theorem 1.3 (Properties of Addition)
well defined.

Proof. We first check $+$ is well-defined. Let $a = [(m,n)] = [(m',n')]$ for $m,n,m',n' \in \mathbb{N}$ and $b = [(p,q)] = [(p',q')]$ for $p,q,p',q' \in \mathbb{N}$

We now check that $+$ is commutative. We have

$$\begin{aligned} a+b &= [(m,n)] + [(p,q)] \\ &= [(m+p, n+q)] \\ &= [(p+m, q+n)] \\ &= [(p,q)] + [(m,n)] \\ &= b+a \end{aligned}$$

Let us define $x \in \mathbb{Z}$ by

$$x \stackrel{\text{def}}{=} [(n,m)]$$

Then, $a + x = [(m,n)] + [(n,m)] = [(m+n, n+m)] = [(m+n, m+n)] = [(1,1)] = \bar{0}$ ☐

Theorem 1.4 (Properties of Multiplication)
The following statements holds:

- "$\cdot$" is well defined.

Proof. We are going to check that the multiplication is well-defined. Let $a, b \in \mathbb{Z}$ with $a = [(m,n)] + [(m'+n')]$ and $b = [(p,q)] = [(p',q')]$ ☐

Recall that $\mathbb{Z} = \{[(j, 1)] : j \in \mathbb{N}, j \geq 2\} \cup$

§1.2 Construction of Rational Numbers