PH1102: Experiment III

Determination of Coefficient of Viscosity of Castor Oil Using Stokes' Law

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§1 Briefing the Experiment

Analogous to *Elastic Moduli* and *Friction* related properties of solid objects, *Viscosity* is very important and one of the intrinsic properties of a liquid. It generally yields a quantitative measure of the intermolecular drag or attractive or cohesive forces of that liquid. Qualitatively, more viscous liquids shows more 'stickiness' with respect to the layers of the liquid flow.

In this nice experiment, we tried to determine coefficient of viscosity of a given liquid (high-viscous castor oil) through computational linear regression with using and manipulating the database provided to us by respected instructors.

§2 Aim of the Experiment

Our focusing goal of this experiment is to determine the coefficient of viscosity of a given liquid in our institutional laboratory using so-called *falling ball viscometer* and incorporating Stokes' Law. Technically, in these virtual sessions, the required database of the experiment is given and we are going to analyse them remotely.

§3 Underlying Theory

§3.1 Historical Background and Stokes' Law

Since the era of Newton, physicists were analysing the dynamics of the bodies in different fluid media. They carefully observed that there is a concept of friction in solid–solid contact area as well as in case of solid–fluid. Sir George Stokes, 1st Baronet (1851), mathematically derived that if a spherical object moves in a medium of infinite extent (edge–effects are zero), then the drag force (F) on that object by the internal friction of the medium is directed opposite to the velocity (v) of the object, and has the magnitude

$$F = 6\pi\eta r v \tag{1}$$

where r is the radius of the spherical object. Notice that, there is a constant in the expression known as Coefficient of Viscosity (η). This is the quantitative measure of viscosity of the medium. As we said earlier, it describes the internal friction of a moving fluid.

The force that opposes the relative motion between the adjacent layers of fluid (in terms of viscosity), is stated below.

§3.2 Origin of Coefficient of Viscosity

In case of solid deformation (shear), we know that

$$\text{Shear Stress} \propto \text{Shear Strain}$$

hence, we can define an intrinsic property of the solid as ζ being the *Shear Modulus* satisfying

$$\zeta = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

In the similar fashion, experimentalists observed that we can always find the Coefficient of Viscosity of a fluid by the following definition

$$\eta \overset{\text{def}}{=} \frac{\text{Shear Stress}}{\text{Sheer Strain Rate}} \tag{2}$$

Note that, the denomenator is *Sheer Strain Rate*, not *Sheer Strain*. The fluids that follow Equation 2 are called *Newtonian Fluids*[1]. By some physical manipulations, the internal friction force due to viscosity of the liquid comes out to be

$$\sigma = \eta \frac{\mathrm{d}v}{\mathrm{d}y} \tag{3}$$

where σ is stress and $\frac{\mathrm{d}v}{\mathrm{d}y}$ is velocity gradient. We have assumed the flow-bed to be planar here.[2]

[1] We here will analyse Newtonian liquid only. Study of non-Newtonian fluids are beyond the scope of this course.

[2] Another fomulation with circular flow-bed was found in I.E. Irodov. *Problem In General Physics.* as $\sigma = \eta r \frac{\mathrm{d}\omega}{\mathrm{d}r}$.

§3.3 Dependence upon Other Thermodynamic Variables

Viscosity depends upon temperature of the liquid. As temperature increases, the average kinetic energy of molecules of the liquid increases, so the fluctuations, consequently, cohesive forces decreases. Hence, viscosity decreases with increasing temperature.

§4 Working Formula

Stokes' Law was originally derived for motion of a spherical mass in a medium of infinite extent. But since we have finite medium in reality (liquid column here), we have to correct the formula experimentally. By virtue of various popular researches, it has been fond that, the Stokes' law takes the following form

$$F = 6\pi\eta r v \left(1 + 2.4\frac{r}{R}\right)\left(1 + 3.3\frac{r}{h}\right) \tag{4}$$

In our experimental setup, $\frac{r}{R} \approx 0.1$ and $\frac{r}{h} \approx 0.002$. We can neglect the effect of latter. Consider, the terminal velocity is achieved by the falling ball in the passage of P–Q observation, we can equate net upward and downward forces (figure below) on the ball

$$6\pi\eta r v \left(1 + 2.4\frac{r}{R}\right) + \sigma\left(\frac{4}{3}\pi r^3\right) g = \rho\left(\frac{4}{3}\pi r^3\right) g$$

A little algebraic rearrangement yields our working formula for this experiment

$$\eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{v_t(1 + 2.4\frac{r}{R})} \text{ Poise} \tag{5}$$

where η is the desired viscosity of given liquid, r cm is the radius of the spherical ball, R cm is the radius of the liquid column, ρ g/cm^3 being the density of that ball, σ g/cm^3 being the density of the given liquid, v_t cm/s denotes terminal velocity of the ball and g cm/s^2 is the gravitational acceleration due to earth's field. A similar calculation with SI units will yield η in Pa s. We have

$$1 \text{ Pa s} = 10 \text{ Poise}$$

§5 Diagram and Images of the Experiment

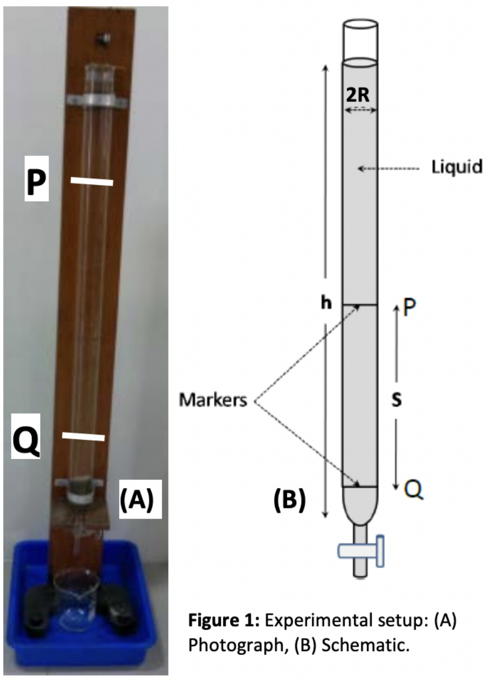


Figure 1: Experimental setup: (A) Photograph, (B) Schematic.

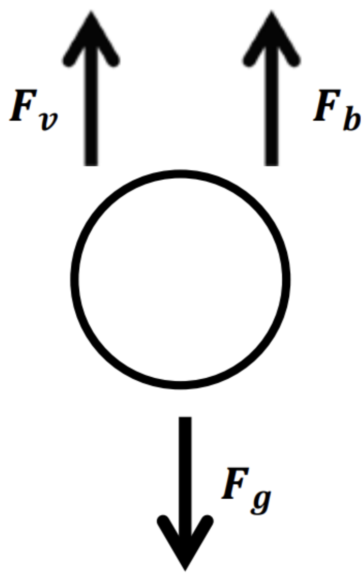


Figure 2: Force directions.

§6 Manipulating Experimental Data

§6.1 Instrumental Data and Literature Value

- Least Count of the Screw Gauge = 0.001 cm
- Least Count of Digital Balance = 0.01 g
- Least Count of Stopwatch = 0.01 s
- Least Count of Meter Scale = 1 mm = 0.001 m
- Vernier Constant of Vernier Calliper = 0.002 cm
- Distance traversed by the spherical balls = 80 cm = 0.80 m
- Density of Castor Oil = 0.961 g/cm^3 (At Room Temperature = 23 °C)

§6.2 Parameters for Ball 1 (Small)

No.	Linear Scale reading (in cm)	Circular Scale Reading	Diameter (in cm)	Mean Diameter (in cm)	Mean Radius (in cm)
1	0.3	15	0.315	0.316	0.158
2	0.3	18	0.318		
3	0.3	17	0.317		
4	0.3	15	0.315		
5	0.3	16	0.316		
6	0.3	19	0.319		
7	0.3	16	0.316		

Table 1: Determination of Radius of Ball 1 (Small) using Screw Gauge

Mass of 10 balls (in gm)	Mass of 1 ball (in gm)
1.31	0.131

Table 2: Determination of Mass of Ball 1 (Small) using Digital Balance

From the above data, we found the volume of the ball = 0.0165 cm^3.
Subsequently, we found the density = 7.94 g/cm^3.

No.	Time for Ball 1 (in s)	Avg. Time Duration (in s)
1	15.72	15.84
2	15.93	
3	15.88	
4	15.85	
5	15.75	
6	16.03	
7	15.78	

Table 3: Determination of Time Elasped by Ball 1 (Small) using Stopwatch

Hence, we found the terminal velocity of the ball = 5.05 cm/s = 0.0505 m/s.

§6.3 Parameters for Ball 2 (Medium)

No.	Linear Scale reading (in cm)	Circular Scale Reading	Diameter (in cm)	Mean Diameter (in cm)	Mean Radius (in cm)
1	0.3	48	0.348	0.348	0.174
2	0.3	46	0.346		
3	0.3	49	0.349		
4	0.3	46	0.346		
5	0.3	48	0.348		
6	0.3	49	0.349		
7	0.3	47	0.347		

Table 4: Determination of Radius of Ball 2 (Medium) using Screw Gauge

Mass of 10 balls (in gm)	Mass of 1 ball (in gm)
2.05	0.205

Table 5: Determination of Mass of Ball 2 (Medium) using Digital Balance

From the above data, we found the volume of the ball = 0.0221 cm^3.
Subsequently, we found the density = 9.28 g/cm^3.

No.	Time for Ball 1 (in s)	Avg. Time Duration (in s)
1	10.66	10.64
2	10.65	
3	10.69	
4	10.57	
5	10.66	
6	10.63	
7	10.59	

Table 6: Determination of Time Elasped by Ball 2 (Medium) using Stopwatch

Hence, we found the terminal velocity of the ball = 7.52 cm/s = 0.0752 m/s.

§6.4 Parameters for Ball 3 (Large)

No.	Linear Scale reading (in cm)	Circular Scale Reading	Diameter (in cm)	Mean Diameter (in cm)	Mean Radius (in cm)
1	0.4	24	0.424	0.426	0.213
2	0.4	26	0.426		
3	0.4	28	0.428		
4	0.4	24	0.424		
5	0.4	28	0.428		
6	0.4	26	0.426		
7	0.4	24	0.424		

Table 7: Determination of Radius of Ball 3 (Large) using Screw Gauge

Mass of 10 balls (in gm)	Mass of 1 ball (in gm)
4.47	0.447

Table 8: Determination of Mass of Ball 3 (Large) using Digital Balance

From the above data, we found the volume of the ball = 0.0405 cm^3.
Subsequently, we found the density = 11.04 g/cm^3.

No.	Time for Ball 1 (in s)	Avg. Time Duration (in s)
1	7.68	7.78
2	7.81	
3	7.72	
4	7.81	
5	7.82	
6	7.69	
7	7.90	

Table 9: Determination of Time Elasped by Ball 3 (Large) using Stopwatch

Hence, we found the terminal velocity of the ball = 10.28 cm/s = 0.1028 m/s.

§6.5 Parameter of Glass Cylinder

No.	Main Scale Reading (in cm)	Vernier Scale Reading	Diameter (in cm)
1	4.6	5	4.610
2	4.6	5	4.610
3	4.6	4	4.608

Table 10: Determination of Radius of Glass Cylinder using Vernier Caliper

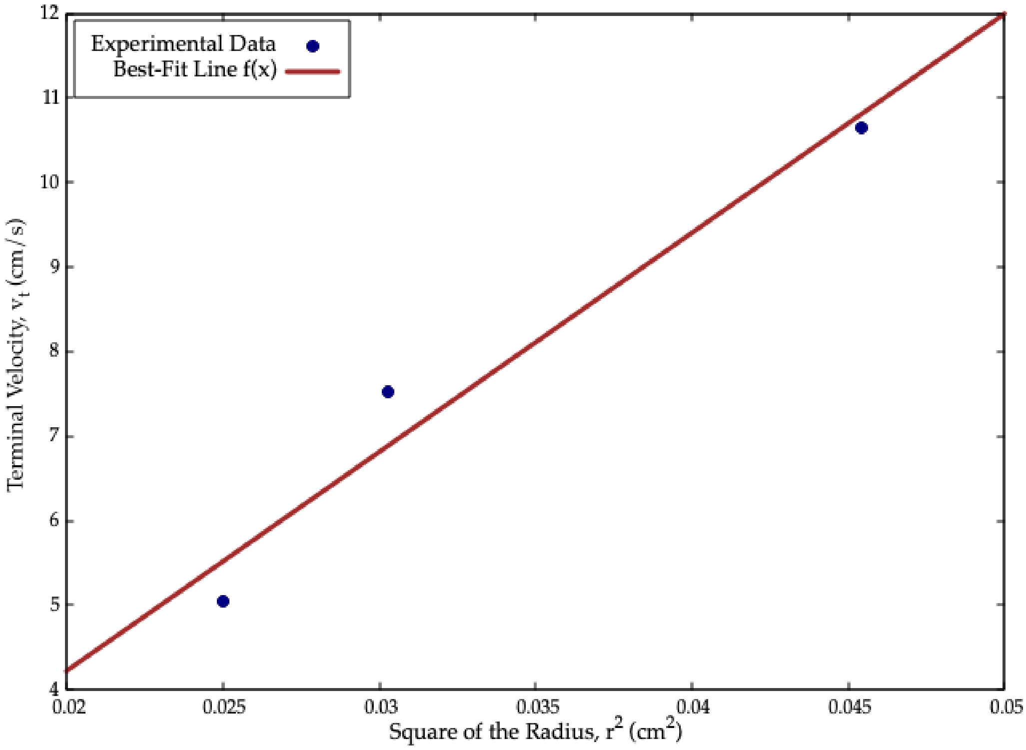
From the above data, we found the mean diameter of the glass cylinder = 4.610 cm and consequently, the mean radius = 2.305 cm.

§7 Plots

As the theory we discussed above, we expect a linear relation between v_t vs. r^2, where v_t is the terminal velocities of the respective balls and r is the radius of the radius of the respective balls.

No.	Terminal Velocity v_t (in cm/s)	Square of Radius r^2 (in cm^2)
1	5.05	0.0250
2	7.52	0.0303
3	10.64	0.0454

Table 11: Derived v_t vs. r^2 Dataset



§8 Deducing Coefficient of Viscosity

We use Equation 5 to derive viscosity of given liquid for each of three cases of the three balls. Suppose, they are η_1, η_2 and η_3 respectively. Observe that,

$$\eta_1 = \frac{2}{9} \times \frac{(0.158)^2 \times (7.94 - 0.961) \times 980}{5.05 \times (1 + 2.4 \times \frac{0.158}{2.305})} \text{ Poise} = 6.45 \text{ Poise} = 0.645 \text{ Pa s}$$

$$\eta_2 = \frac{2}{9} \times \frac{(0.174)^2 \times (9.28 - 0.961) \times 980}{7.52 \times (1 + 2.4 \times \frac{0.174}{2.305})} \text{ Poise} = 6.18 \text{ Poise} = 0.618 \text{ Pa s}$$

$$\eta_3 = \frac{2}{9} \times \frac{(0.213)^2 \times (11.04 - 0.961) \times 980}{10.28 \times (1 + 2.4 \times \frac{0.213}{2.305})} \text{ Poise} = 7.93 \text{ Poise} = 0.793 \text{ Pa s}$$

Using the above viscosity values, we take average of them to determine the final experimental value

$$\boxed{\eta = \frac{\sum_{i=1}^{3} \eta_i}{3} = 0.685 \text{ Pa s}}$$

§9 Estimation of Error

§9.1 Literature Value Error

We are given the literature value of coefficient of viscosity of Castor Oil = 0.650 Pa s. Hence,

$$\%\text{Error} = \frac{0.685 - 0.650}{0.650} \times 100 = \boxed{5.38}$$

§9.2 Instrumental Error

Due to least count of the instruments and limitations in observation strategy, there is an instrumental error whose expression can be found using logarithmic derivatives. Finally, the expression comes out to be

$$\frac{\delta\eta}{\eta} = \left|\frac{2}{r} - \frac{2.4}{R+2.4r} - \frac{9m}{4\pi r^4(\rho-\sigma)}\right|\delta r + \left|\frac{1}{x}\right|\delta x + \left|\frac{1}{t}\right|\delta t + \left|\frac{2.4r}{R(R+2.4r)}\right|\delta R + \left|\frac{3}{4\pi r^3(\rho-\sigma)}\right|\delta m$$

where, $\rho = \frac{m}{\frac{4}{3}\pi r^3}$ and $v_t = \frac{x}{t}$ with x being the column length under observation and t being time time taken by the ball to pass that column.

> **Remark 9.1.** We are taking $\delta t = 0.2$ s as conservative error bound due to neurological reaction time of human observer.

From this *pathological* expression, error in η_is comes out to be

$$\begin{aligned}\%\text{Error in } \eta_1 &= 11.78\\ \%\text{Error in } \eta_2 &= 9.28\\ \%\text{Error in } \eta_3 &= 7.13\end{aligned}$$

With the mean value of the error as

$$\langle\%\text{Error in } \eta_i\rangle = \boxed{9.40}$$

§10 Discussion on Probable Sources of Errors

1. Since handling of the stopwatch is executed by human nervous system [Observation by Eyes → Visual Cortex → Motor Neurons → Action of Start/Stop], there is an error in reading of time arises due to reaction time. If the experimentalist is very careful, then the maximum bound of this reaction time is approximately 0.2 s. This may cause inaccuracy. This can be fixed using some other (maybe computer programme-based) handling system of the stopwatch.

2. Errors also may arise due to low resolution of instruments which are used in this experiment. It can be fixed using highly precise instruments.

3. We can observe there is huge error in density of steel balls of three different sizes, which may be caused by rusting, oxidation, distorted geometry (non-spherical) etc. This may be fixed using round, carefully-reserved and pure-steel balls.

§11 Conclusion and Acknowledgement

Thanks to the instructors and teaching assistants for rewarding us this simple but beautiful experiment, where we experimentally calculated the value of coefficient of viscosity of a high-viscous liquid, namely *Castor Oil* with significantly low amount of overall error.