PH1102: Experiment I

Retrieving Young Modulus & A Glimpse on Error Analysis

Laboratory Report

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February 13, 2022

§1 Introduction

Analogous to the density of a substance, the *Young's modulus* of a solid object determines the quantitative measure of its strength, elasticity, and some other intrinsic materialistic properties. By this experiment, done in PH1102, we try to deduce the young modulus of a solid metal bar by the *flexure method*. Though its 'behind-the-scene' formula is quite tricky and intimidating, it is very convenient for practical purposes. Also, to understand experimental accuracy, we have done a brief error analysis playing with the given dataset and a little bit of statistical computation.

§2 Aim of the Experiment

To determine Young's modulus of a metal bar by the flexure method.

§3 Principle and Working Formula

A metal bar is kept horizontal with two knife-edge support at the two ends. When different suitable weights are fixed at the middle point of the bar, it gets depressing (Fig. 1). The amount of depression d is related to Young's modulus Y of the bar material. By experimentally measuring length, breadth, and thickness of the bar and depression d for different weights W, Young's modulus Y can be determined.

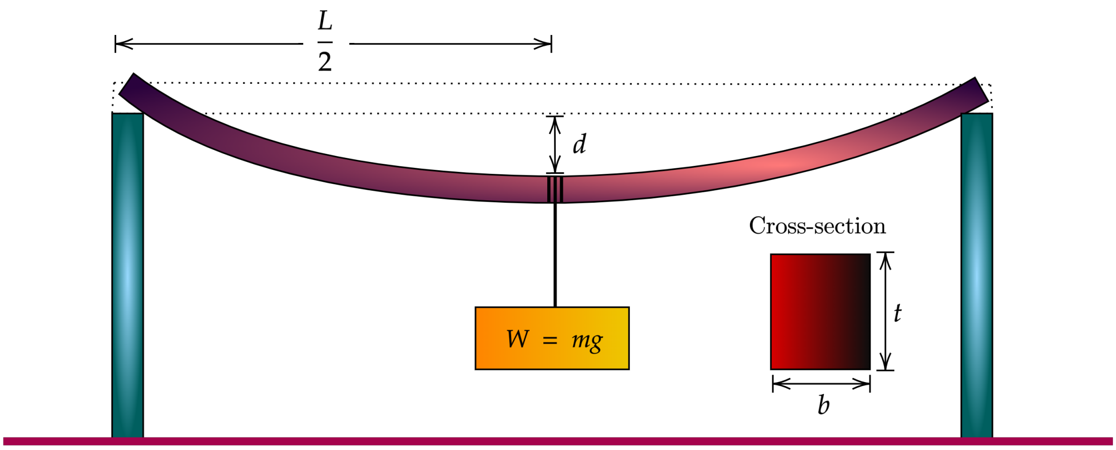

Figure 1: The experimental set-up and the cross-section of the bar

The bar in Fig. 1 can be regarded as two cantilevers, each of length $\frac{L}{2}$, fixed at the center (where weight is hanged) and loaded by $\frac{W}{2}$ at the two knife-edges. Each cantilever undergoes depression d given by

$$d = \frac{WL^3}{4Ybt^3} = \frac{mgL^3}{4Ybt^3} \implies Y = \frac{mgL^3}{4bdt^3} \tag{1}$$

Actually, it is difficult to know the absolute value of depression, because, even without any additional weight, the bar will be depressed slightly due to its own weight. So, one measures relative depression as a function of added weights and plots a graph: d vs. m, which is a straight line with a slope

$$s = \frac{gL^3}{4Ybt^3}$$

From the slope s of the d vs. m graph one can calculate Young's modulus as

$$\boxed{Y = \frac{gL^3}{4sbt^3}} \tag{2}$$

§4 Procedure

1. Measure the breadth b of the bar using a Vernier caliper and tabulate the data.
2. Measure the thickness t of the bar using a screw gauge and tabulate the data.
3. Measure the length L of bar between two knife-edges i.e. separation between two knife-edges (Fig. 1) using a meter scale and tabulate the data.
4. Mark the point on the bar exactly at the center of the two knife-edges. Put the hanger of the weights exactly at that point. The hanger has pointer on top.
5. Focus the travelling microscope such that the pointer is clearly seen in it. Position the travelling microscope such that the pointer is touching the horizontal cross wire. Note down the reading of the vertical scale of the travelling microscope.
6. Add one weight block on the hanger. Mass of all the weight blocks should be already written on them. If it is missing, weigh the block and write it down. The bar will depress and the pointer will move in the microscope field of view. Adjust microscope vertical position to align the pointer back to the horizontal cross wire. Note the reading of the vertical scale of the travelling microscope.
7. Similarly, note microscope reading for addition of 4-5 more weight blocks one after another.
8. Then start reducing the weight one by one and each time adjust microscope to align the pointer at the horizontal cross wire and take microscope reading till the hanger is empty again.
9. Record data for steps 5-8 in a table. Calculate average depression d for a given amount of mass m on the hanger.

§5 Dataset Manipulation

By proceeding along the methods above, we found: $L = 86$ cm. And we are given: Least count of the screw gauge = 0.001 cm and Vernier constant = 0.002 cm. Then we proceed to find b, t and d.

Table 1: Determination of the breadth of the bar b using Vernier Caliper

Sl. No.	MSR (cm)	VSR	VC (cm)	Breadth b [MSR + (VSR × VC)] (cm)	Average Breadth (cm)
1	2.5	40	0.002	2.58	2.55
2	2.5	16	0.002	2.532	
3	2.5	25	0.002	2.55	

Table 2: Determination of the thickness of the bar t using Screw Gauge

Sl. No.	LSR (cm)	CSR	LC (cm)	Thickness t [LSR + (CSR × LC)] (cm)	Average Thickness (cm)
1	4.5	41	0.001	4.91	4.88
2	4.5	36	0.001	4.86	
3	4.5	38	0.001	4.88	

Table 3: Determination of Average Depression for Cumulative Loading and Unloading Mass

Sl. No. (1 - 6)	Mass on hanger m (g)	Loading			Dep. d_1 (mm)	Unloading			Dep. d_2 (mm)	Average Depression $\frac{d_1+d_2}{2}$ (mm)
		MSR (mm)	VSR	x_1 (mm)		MSR (mm)	VSR	x_2 (mm)		
1	0	113	20	113.20	0	112.5	40	112.90	0	0
2	426	112	33	112.33	0.9	112	5	112.05	0.9	0.9
3	908	110.5	25	110.75	2.5	110.5	15	112.65	2.3	2.4
4	1393	109	10	109.10	4.1	109	23	109.23	3.7	3.9
5	1883	108	5	108.05	5.2	107.5	30	107.80	5.1	5.1
6	2843	105	38	105.38	7.8	105	30	105.30	7.6	7.7

In the next section, we plot a graph of Depression d against mass on the hanger m and do some computation.

§6 Computation: Linear Regression

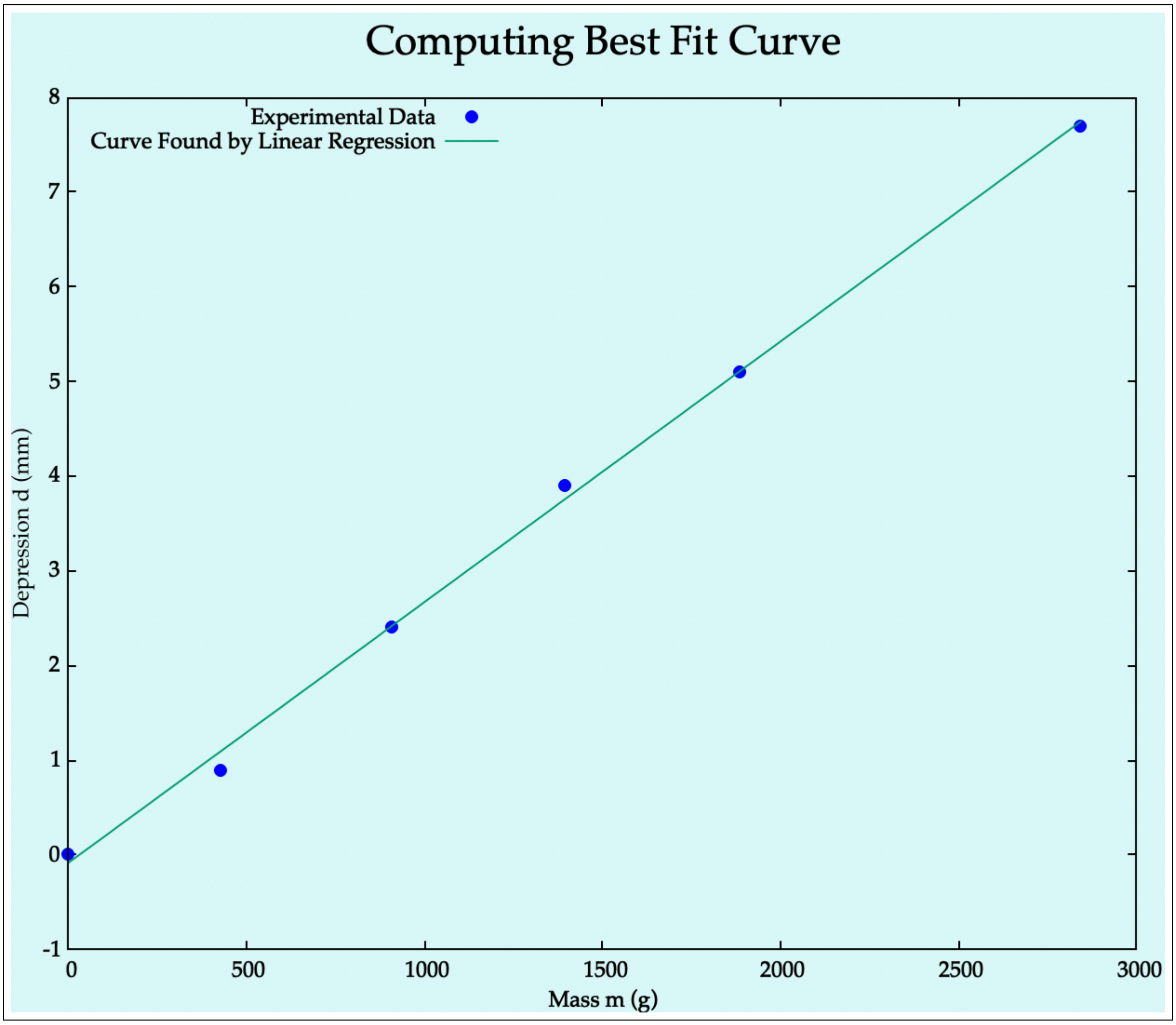

Figure 2: The graph of d vs. m

By exploring this computed graph, it really makes sense to have a quantity like Young's Modulus which is connected to the constant value of the slope.

This graph is plotted and the linear regression is computed using `gnuplot` in the terminal. After the computation, the terminal replied the following equation of the line

$$\boxed{y = 0.0027547x - 0.0884668} \tag{3}$$

upto a 'sufficient' number of significant figures.

Notice that, we have found the desired slope of the best fit line and that is

$$s = 0.0027547 \text{ mm g}^{-1}$$

> **Remark 6.1.** Units are very impotant for any measure.

§7 Calculation of Young's Modulus

From the above results, we summarise with standarizing their units:

- $L = 86$ cm 0.86 m
- $g = 9.8$ m s^{-2}
- $b = 2.55$ cm $= 0.255$ m
- $t = 4.88$ mm $= 0.00488$ m
- $s = 0.0027547$ mm g^{-1} $= 0.0027547$ m kg^{-1}

Plugging these values to Equation 2, yields

$$Y = \frac{9.8 \times 0.86^3}{4 \times 0.0027547 \times 0.0255 \times 0.00488^3} \text{ N m}^{-2} \approx \boxed{1.91 \times 10^{11} \text{ N m}^{-2}} \tag{4}$$

§8 Error Analysis

In this section, we talk about the statistical errors on doing linear regression and the experimental errors.

§8.1 Error in Regression

After computing the best fit linear curve through `gnuplot`, the output went like this:

```
gnuplot> f(x) = m*x + b
gnuplot> fit f(x) 'A1.txt' using 1:2 via m,b
iter      chisq       delta/lim  lambda   m               b
   0 1.4510726680e+07   0.00e+00  1.10e+03    1.000000e+00   1.000000e+00
   1 8.5864953832e+04  -1.68e+07  1.10e+02    7.895234e-02   9.995270e-01
   2 2.7162998711e+00  -3.16e+09  1.10e+01    2.262370e-03   9.992918e-01
   3 2.5658922668e+00  -5.86e+03  1.10e+00    2.208309e-03   9.800359e-01
   4 3.8520266730e-01  -5.66e+05  1.10e-01    2.559710e-03   2.928510e-01
   5 6.6953689010e-02  -4.75e+05  1.10e-02    2.753627e-03  -8.636259e-02
   6 6.6943997291e-02  -1.45e+01  1.10e-03    2.754703e-03  -8.846672e-02
   7 6.6943997291e-02  -4.47e-08  1.10e-04    2.754703e-03  -8.846684e-02
iter      chisq       delta/lim  lambda   m               b

After 7 iterations the fit converged.
final sum of squares of residuals : 0.066944
rel. change during last iteration : -4.47363e-13

degrees of freedom    (FIT_NDF)                        : 4
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.129368
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.016736

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 0.0027547        +/- 5.61e-05     (2.037%)
b               = -0.0884668       +/- 0.08744      (98.84%)

correlation matrix of the fit parameters:
                m      b
m               1.000
b              -0.797  1.000
```

Figure 3: The output window of `gnuplot` after computation

Fig. 3 contains some information about the errors in each data point $y_i - y$ where the set $\{(x_i, y_i) : i = 1, 2, \cdots 6\}$ represents all the points that we found from experimental dataset. These $y_i - y$ are called *residuals*. Computation yields their varience and standard deviation as

$$\sigma^2 = 0.016736; \qquad \sigma = 0.129368$$

Clearly, σ is quite low with respect to the y_is. So the above linear regression is acceptable.

§8.2 Experimental Error

The literature value of Young's Modulus (Y) of Steel[1] = 2.00×10^{11}. Combining this with Equation 4, we calculate the error in Y:

$$\Delta Y = 2.00 \times 10^{11} - 1.91 \times 10^{11}\ \mathrm{N\,m^{-2}} = 0.09 \times 10^{11}\ \mathrm{N\,m^{-2}}$$

Then, percentage relative error in Y:

$$\% \frac{\Delta Y}{Y} = \frac{0.09 \times 10^{11}\ \mathrm{N\,m^{-2}}}{2.00 \times 10^{11}\ \mathrm{N\,m^{-2}}} \times 100 = 4.5$$

Observe that, 4.5% of error is quite low for environments in general laboratories. This gives a hope to be more accurate next time we perform the experiment.

§9 Conclusion

By this nice experiment, we quantitatively measured an intensive property of a material. We also got familiar with computation, curve fitting (i.e., regression), and some statistical entities. Error analysis is also important of any experiment, which we tried to briefly understand by this simple experiment in PH1102.

§10 Acknowledgements

Thanks to the instructors of PH1102 for exposing us to IISER laboratories, handling experiments in this new era remotely and teaching how to perform computations in open-source `gnuplot` and Evan Chen, MIT for this beautiful `.sty` file for latexisation.

[1] Source: https://www.azom.com/article.aspx?ArticleID=6117