Functions 4th Edition Chapter 13.7 Problem 5E

Step 1 of 1

Let $f(x,y,z) = x+y+z$

We know the gradient vector ∇f is normal to the level curve of $f(x,y,z) = c$

Thus $\nabla f(x,y,z)$ is normal to the surface $x+y+z=4$

Now $\nabla f(x,y,z) = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$

$= \hat{i}+\hat{j}+\hat{k}$

Then normal vector to the surface at $(2,0,2)$ is $\nabla f(2,0,2) = \hat{i}+\hat{j}+\hat{k}$

Hence the unit normal vector is:

$$\frac{\nabla f(2,0,2)}{\|\nabla f(2,0,2)\|}$$
$$= \frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$$
$$= \frac{\sqrt{3}}{3}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

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Functions 4th Edition Chapter 13.7 Problem 7E

Step 1 of 2

Let $f(x,y,z) = \sqrt{x^2+y^2} - z$

We know the gradient vector ∇f is normal to the level surface $f(x,y,z) = c$.

Then ∇f is normal to the surface $z = \sqrt{x^2+y^2}$ $(c=0)$

Now $\nabla f(x,y,z) = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$

$$= \frac{x}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\hat{j} - \hat{k}$$

At $(3,4,5)$ the normal vector to the given surface is $\nabla f(3,4,5) = \frac{3\hat{i}}{5} + \frac{4}{5}\hat{j} - 1\hat{k}$

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Step 2 of 2

Then the unit normal vector to the surface at $(3,4,5)$ is:

$$\frac{\nabla f(3,4,5)}{\|\nabla f(3,4,5)\|}$$
$$= \frac{\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j} - \hat{k}}{\sqrt{\frac{9}{25}+\frac{16}{25}+1}}$$
$$= \frac{1}{5\sqrt{2}}\left(3\hat{i}+4\hat{j}-\hat{k}\right)$$
$$= \frac{\sqrt{2}}{10}\left(3\hat{i}+4\hat{j}-\hat{k}\right)$$

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Functions 4th Edition Chapter 13.7 Problem 15E

Step 1 of 1

Let $f(x,y,z) = x^2+y^2+z-25=0$

Then $f_x = 2x$

$f_y = 2y$

$f_z = 1$

The equation of tangent plane to the surface $x^2+y^2+z-25=0$ at $(3,1,15)$ is:

$$f_x(3,1,15)(x-3) + f_y(3,1,15)(y-1) + f_z(3,1,15)(z-15) = 0$$

i.e. $6(x-3)+2(y-1)+1(z-15)=0$

i.e. $6x+2y+z-18-2-15=0$

i.e $6x+2y+z=35$

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Functions 4th Edition Chapter 13.7 Problem 16E

Step 1 of 1

We have $z = f(x,y) = \frac{y}{x}$

Let $f(x,y,z) = z - \frac{y}{x} = 0$

Then $f_x = \frac{y}{x^2}, f_y = \frac{-1}{x}, f_z = 1$

The equation of tangent plane at $(1,2,2)$ is

$$f_x(1,2,2)(x-1) + f_y(1,2,2)(y-2) + f_z(1,2,2)(z-2) = 0$$

i.e $2(x-1)-1(y-2)+1(z-2)=0$

i.e $2x-y+z-2+2-2=0$

i.e $2x-y+z=2$

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Functions 4th Edition Chapter 13.7 Problem 19E

Step 1 of 1

We have $z = g(x,y) = x^2 - y^2$

Let $f(x,y,z) = x^2 - y^2 - z = 0$

Then $fx = 2x, \quad fy = -2y, \quad fz = -1$

The equation of tangent plane at $(5,4,9)$ is:

$$fx(5,4,9)(x-5) + fy(5,4,9)(y-4) + fz(5,4,9)(z-9) = 0$$

i.e. $10(x-5) - 8(y-4) - 1(z-9) = 0$

i.e $10x - 8y - z - 50 + 32 + 9 = 0$

i.e $10x - 8y - z = 9$

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Functions 4th Edition Chapter 13.7 Problem 21E

Step 1 of 1

Let $f(x,y,z) = e^x(\sin y + 1) - z = 0$

Then $fx(x,y,z) = e^x(\sin y + 1)$

$fy(x,y,z) = e^x \cos y$

$fz(x,y,z) = -1$

Then the equation of tangent plane at $\left(0, \frac{\pi}{2}, 2\right)$ is

$$fx\left(0,\frac{\pi}{2},2\right)(x-0) + fy\left(0,\frac{\pi}{2},2\right)\left(y-\frac{\pi}{2}\right) + fz\left(0,\frac{\pi}{2},2\right)(z-2) = 0$$

i.e $2(x) + 0\left(y - \frac{\pi}{2}\right) + (-1)(z-2) = 0$

i.e $2x - z + 2 = 0$

i.e $2x - z = -2$

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