VIT
Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

SCHOOL OF ADVANCED SCIENCES
DEPARTMENT OF MATHEMATICS
FALL SEMESTER – 2023-24
BMAT101L – Calculus
SLOT – B1+TB1

DIGITAL ASSIGNMENT

Instructions

a) Last Date of Submission : **October 2, 2023**

b) Mention the Register Number, Name, Slot Details, Course Code and Course Title in the First Page of the Assignment. Also mention the Register Number and Name in every page of the document.

c) Submit the **Handwritten** answers for all the following questions as a single PDF document into V-Top properly on or before the mentioned last date.

d) Answer ALL the Questions.

e) Submission after the due date or through emails/ MS Teams will not be considered for the evaluation

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1) It took 14 seconds for a mercury thermometer to rise from $-19°C$ to $100°C$ when it was taken from a freezer and placed in boiling water. Find the rate of change of temperature of mercury with respect to time at some point along the way.

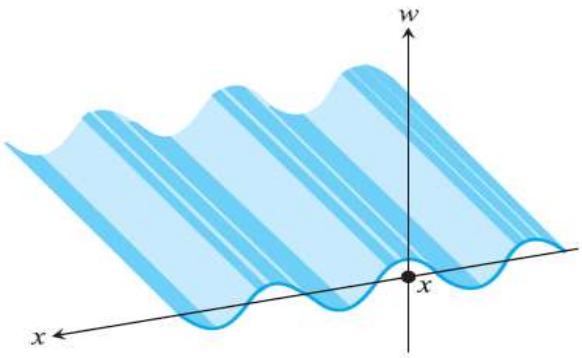
2) Find the area of the region in the first quadrant bounded on the left by the y-axis, below by the curve $x = 2\sqrt{y}$, above left by the curve $x = (y-1)^2$, and above right by the line $x = 3 - y$

3) Find the volume of the solid generated by revolving the region enclosed by the triangle with vertices (1,0), (2,1), and (1,1) about the y-axis.

4) If we stand on an ocean shore and take a snapshot of the waves, the picture shows a regular pattern of peaks and valleys in an instant of time. We see periodic vertical motion in space, with respect to distance. If we stand in the water, we can feel the rise and fall of the water as the waves go by. We see periodic vertical motion in time. In physics, this beautiful symmetry is expressed by the one-dimensional wave equation

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

where, w: wave height, x: distance variable, t: time variable, c: velocity with which the waves propagated



In this example, x: distance across the ocean's surface, but in other applications, x might be the distance along a vibrating string, distance through air (sound waves), or distance through space (light waves). The number c varies with the medium and type of wave.

Show that the following functions are the solutions of the wave equation.

a) $w = 5\cos(3x + 3ct) + e^{x+ct}$
b) $w = f(u)$, where f is a differentiable function of u, and $u = a(x + ct)$, where a is constant

5) If $u = \frac{x}{y-z}, v = \frac{y}{z-x}, w = \frac{z}{x-y}$, then prove that u, v, w are not independent and also find the relation between them.

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