Roll No. ....

Total No. of Questions : 9]   [Total No. of Printed Pages : 4

(1048)

B.C.A. (CBCS) RUSA IInd Semester Examination

4027

MATHEMATICS-II

Paper : BCA-0201

Time : 3 Hours]   **[Maximum Marks : 70**

Note :– Attempt *five* questions in all, selecting *one* question from each Unit–I to IV. Question No. **9** (Unit–V) is compulsory. All questions are of equal marks.

Unit–I

1. (a) Verify Lagrange's mean value theorem for the function f defined by $f(x) = x^3 - x^2 - 6x$ in the interval $[-1, 4]$.

   (b) Find nth derivative of $e^{ax} \sin (bx + C)$.

2. (a) Discuss the applicability of Cauchy's mean value theorem to $f(x)$ and $g(x)$ in $[a, b]$ where :

$$f(x) = \begin{cases} 2, & a \le x < b \\ 4, & x = b \end{cases}$$

and $g(x) = x, x \in [a, b]$.

(b) Find the nth derivative of co-efficient of $x^3 e^x \cos x$.

Unit–II

3. (a) If a and b are any *two* integers, with $b > 0$, $a \neq 0$, then there exists unique integers q and r such that $a = bq + r$, $0 \le r < |b|$.

(b) Solve the linear congruence equation :

$$11x = 2(\text{mod } 317)$$

4. (a) Find the least member and greatest member of the following set X, if they exist

$$X = \{n \in N : n^2 + 2n \le 60\}$$

(b) For any $n \in I_+$, prove that the integers $8n + 3$ and $5n + 2$ are relatively prime.

Unit–III

5. (a) Prove that the set G = $\{0, 1, 2, 3, 4\}$ is a finite abelian group of order 5 w.r.t. addition modulo 5.

(b) Show that the set Z of integers, with the ordinary addition operation, is an infinite cyclic group.

6. (a) Prove that the set G_1 of all n-rowed non-singular matrices over a field F is a non-Abelian group w.r.t. the operation of matrix multiplication.

(b) Prove that $(ab)^{-1} = b^{-1}a^{-1}\ \forall\ a, b \in G$, the inverse of the product of two elements of a group G is the product of inverse taken in reverse order.

Unit–IV

7. (a) Prove that a finite ring without zero divisors is a field.

(b) Prove that in general the set of all numbers of the form $a + (\sqrt{p})b$ where a, b are rational numbers, and p is a prime is a field w.r.t. ordinary addition and multiplication.

8. (a) Let F be a field and suppose $f(x)$ is a polynomial in $F(x)$, then prove that $x - a$ is a divisor of $f(x)$ in $F(x)$ iff $f(a) = 0$ in F.

(b) If R is a ring such that $a^2 = a\ \forall\ a \in R$, prove that :

(i) $a + a = 0 \quad \forall\ a \in R$

(ii) R is commutative ring

Unit–V

(Compulsory Question)

9. (a) State Rolle's theorem.

(b) Find nth derivative of $\dfrac{1}{ax+b}$.

(c) Find greatest common divisor of 258, 60 by using Euclidean algorithm.

(d) If a, b, c and d are integers such that $a \equiv b \pmod{m}$ and $c = d \pmod{m}$, then prove that $a + c = b + d \pmod{m}$.

(e) Prove that every cyclic group is an abelian group.

(f) Prove that the inverse of each element is unique.

(g) Show that the set Z of all integers ......, –4, –3, –2, –1, 0, 1, 2, 3, 4, ... is a group w.r.t. the operation of addition of integers.

(h) Define a ring and give two examples.

(i) If F is a field and $f(x)$ is a polynomial of degree $n \geq 1$ in F(x) then the equation $f(x) = 0$ has at most n roots in F.

(j) Prove that the set I of all integers is not a group under addition and multiplication as ring composition.

Total No. of Questions : 9] [Total No. of Printed Pages : 4

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BCA (CBCS) RUSA IInd Semester Examination

4385

MATHEMATICS-II

Paper : BCA-0201

Time : 3 Hours] [Maximum Marks : 70

Note :– Attempt *one* question from each Unit. Q. No. **9** is compulsory.

Unit–I

1. (a) Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$.

(b) Verify Lagrange's mean value theorem for the function :

$$f(x) = x(x-1)(x-2) \text{ in } \left[0, \frac{1}{2}\right]$$ 5,5

2. (a) Show that :

$$\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$$

Where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$.

5,5

(b) State and prove Leibnitz theorem.

Unit–II

3. (a) Show that for any *two* integers a and $b > 0$, there exists q_1 and r_1 such that

$$a = bq_1 + cr_1,\ 0 \le r_1 < \frac{b}{2},\ c = 1 \text{ or } -1.$$

(b) Find the remainder when 2^{340} is divided by 341. 5,5

4. (a) Solve $12x + 15 \equiv 0 \bmod 45$.

(b) If p is prime, show that $2(p - 3)! + 1$ is a multiple of p. 5,5

Unit–III

5. (a) Show that $S = \{3^n : n \in \mathbb{Z}\}$ is a commutative group with respect to multiplication.

(b) Prove that the set of matrices :

$$G = \left\{ \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} : \alpha \in \mathbf{R} \right\}$$

forms a group under matrix multiplication. 5,5

6. (a) Show that a finite semi-group G in which cancellation laws hold is a group.

(b) Prove that the group G of n n^{th} roots of unity is a cyclic group. 5,5

Unit–IV

7. (a) Prove that a ring R is commutative if and only if

$$(a + b)^2 = a^2 + 2ab + b^2 \ \forall \ a, b \in R$$

(b) Show that the ring $\mathbb{Z}_p$ of integers modulo p is a field if and only if p is prime. 5,5

8. (a) Let $f(x)$ and $g(x)$ be two non-zero polynomials in R(x), R being a ring. If $f(x) + g(x) \neq 0$, then prove that :

$$\deg(f(x) + g(x)) \leq \max(\deg f(x), \deg g(x))$$

(b) Find the sum and product of the polynomials :

$f(x) = 4x - 5$, $g(x) = 2x^2 - 4x + 2$ in $\mathbb{Z}_8[x]$ 5,5

Unit–V

(Compulsory Question)

9. (a) State Lagrange's mean value theorem.

(b) $\lim\limits_{x \to \infty} \dfrac{\sin x}{x} =$ ....

(c) Find 100th derivative of x^{101}.

(d) Find the remainder when 3^{287} is divided by 23.

(e) L.C.M. (8, 12, 15, 20, 25) = ...

(f) Give an example of an infinite abelian group.

(g) The set $\mathbb{Z}$ of all integers is an abelian group under the binary composition $a*b = a + b + 2$. Find the identity element.

(h) Prove that non-abelian groups cannot be cyclic.

(i) Define ring and ring with unity.

(j) Prove that $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$, the ring of integer modulo 6, is with zero divisors. 3×10=30