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(2102)

BCA (CBCS) RUSA IIIrd Semester Examination

3991

MATHEMATICS–III

BCA–301

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Part–A is compulsory. Attempt *one* question each from Parts–B, C, D and E.

Part–A

(Compulsory Questions)

1. (A) Attempt all questions :

(i) Write order and degree of the differential equation :

$$\sin^2 x \frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} + y = 0$$ 2, 1

(ii) The intersection of two fields is not a field. (True/False)

(iii) $|\cos\theta + i\sin\theta| \leq 1$ (True/False)

(iv) A differential equation with degree one is a linear differential equation. (True/False)

(v) Roots of $x^2 + 1 = 0$ are purely imaginary. (True/False)

(vi) Find modulus and argument of complex number $-3i$.

(vii) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, where n is a +ve integer. (True/False)

(viii) $(\mathbb{Z}_2, +_2, \cdot_2)$ is a field. (True/False)

(ix) The algebraic structure $(\mathbb{Z}_p, +_p, \cdot_p)$ is a field, where p is a prime no. (True/False)

(x) Prime numbers are finite. (True/False)

1×10=10

(B) Attempt all questions :

(i) Solve the differential equation :

$$x\frac{dy}{dx} = y + xe^{-y/a}$$

(ii) If n is any integer, show that :

$$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos n\frac{\pi}{6}$$

(iii) Simplify :

$$\frac{(\cos\theta+\sin\theta)^6(\cos 3\theta+i\sin 3\theta)^8}{(\cos 5\theta+i\sin 5\theta)^4(\cos 2\theta+i\sin 2\theta)^7}$$

(iv) Find gcd (35, 49) and express it as linear combination of these numbers.

(v) Prove that $x^3 + 2x + 4$ is irreducible over $\mathbb{Z}_5$.

4×5=20

$x = -1$
$x = \sqrt{-1}$

Part–B

10 each

2. (a) Solve :

$$x^3 \frac{d^3y}{dx^3} + 6x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - 4y = (\log x)^2$$

(b) Solve :

$$(D^4 - 1)y = e^x \cos x$$

3. (a) Solve :

$$(x^2 + y^2 + 2x)dx + 2ydy = 0$$

(b) Solve :

$$(D^3 - 3D^2 + 3D - 1)y = (x + 1)e^x$$

Part–C

10 each

4. (a) Prove that :

$$\left(\frac{1+\sin\theta + i\cos\theta}{1+\sin\theta - i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$$

where n is any integer.

(b) If z_1, z_2 are two non-zero complex numbers, prove that :

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2[|z_1|^2 + |z_2|^2]$$

5. A triangle is formed by the points z_1, z_2, z_3 in the Argand's diagram. Prove that its :

(a) Centroid is given by :

$$\frac{z_1 + z_2 + z_3}{3}$$

(b) ~~Circum-centre is given by :~~

$$\cancel{|z - z_1| = |z - z_2| = |z - z_3|}$$

Part–D

10 each

6. Find the set of integers solutions for each of the following :

 (a) $15x \equiv 25 \pmod{25}$

 3

 (b) $9x \equiv 14 \pmod{15}$

7. Find the smallest positive integer that when divided by 3, 5, 7 we get remainder 1, 4, 6 respectively.

Part–E

10 each

8. (a) Let a and b be two elements of a finite field F. Then prove that there exist elements α and β in F such that :

$$\alpha + a\alpha^2 + b\beta^2 = 0$$

 (b) Prove that $(\mathbb{Z}_5, +_5, \cdot_5)$ is a field.

9. (a) Find all nilpotent and idempotent elements of $(\mathbb{Z}_{10}, +_{10}, \cdot_{10})$.

 (b) Construct a field extension of $\mathbb{Z}_3$ with exactly 9 elements.

F — 2, 3, 10