Roll No. ....

Total No. of Questions : 9] [Total No. of Printed Pages : 4

(1049)

BCA (CBCS) RUSA IInd Semester Examination

4385

MATHEMATICS-II

Paper : BCA-0201

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Attempt *one* question from each Unit. Q. No. **9** is compulsory.

Unit–I

1. (a) Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$.

   (b) Verify Lagrange's mean value theorem for the function :

$$f(x) = x(x-1)(x-2) \text{ in } \left[0, \frac{1}{2}\right]$$ 5,5

2. (a) Show that :

$$\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta$$

Where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$.

(b) State and prove Leibnitz theorem. 5,5

Unit–II

3. (a) Show that for any *two* integers a and $b > 0$, there exists q_1 and r_1 such that

$$a = bq_1 + cr_1,\ 0 \le r_1 < \frac{b}{2},\ c = 1 \text{ or } -1.$$

(b) Find the remainder when 2^{340} is divided by 341. 5,5

4. (a) Solve $12x + 15 \equiv 0 \bmod 45$.

(b) If p is prime, show that $2(p - 3)! + 1$ is a multiple of p. 5,5

Unit–III

5. (a) Show that $S = \{3^n : n \in \mathbb{Z}\}$ is a commutative group with respect to multiplication.

(b) Prove that the set of matrices :

$$G = \left\{ \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} : \alpha \in \mathbf{R} \right\}$$

forms a group under matrix multiplication. 5,5

6. (a) Show that a finite semi-group G in which cancellation laws hold is a group.

(b) Prove that the group G of n n^{th} roots of unity is a cyclic group. 5,5

Unit–IV

7. (a) Prove that a ring R is commutative if and only if

$$(a + b)^2 = a^2 + 2ab + b^2 \ \forall \ a, b \in R$$

(b) Show that the ring $\mathbb{Z}_p$ of integers modulo p is a field if and only if p is prime. 5,5

8. (a) Let $f(x)$ and $g(x)$ be two non-zero polynomials in $R(x)$, R being a ring. If $f(x) + g(x) \neq 0$, then prove that :-

$$\deg(f(x) + g(x)) \leq \max\ (\deg f(x), \deg g(x))$$

(b) Find the sum and product of the polynomials :

$f(x) = 4x - 5$, $g(x) = 2x^2 - 4x + 2$ in $\mathbb{Z}_8[x]$ 5,5

Unit–V

(Compulsory Question)

9. (a) State Lagrange's mean value theorem.

(b) $\lim\limits_{x \to \infty} \dfrac{\sin x}{x} =$ ....

(c) Find 100th derivative of x^{101}.

(d) Find the remainder when 3^{287} is divided by 23.

(e) L.C.M. (8, 12, 15, 20, 25) = ...

(f) Give an example of an infinite abelian group.

(g) The set $\mathbb{Z}$ of all integers is an abelian group under the binary composition $a*b = a + b + 2$. Find the identity element.

(h) Prove that non-abelian groups cannot be cyclic.

(i) Define ring and ring with unity.

(j) Prove that $\mathbb{Z}_6 = \{0, 1, 2, 3, 4, 5\}$, the ring of integer modulo 6, is with zero divisors. $3\times10=30$