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B.C.A. Ist Year (New Syllabus) Examination

MATHEMATICS–I

Paper : BCA-101

Time : Three Hours]   [Maximum Marks : $\begin{cases} \text{Regular}: 80 \\ \text{ICDEOL}: 100 \end{cases}$

The candidates shall limit their answers precisely within the answer-book (40 pages) issued to them and no supplementary/ continuation sheet will be issued.

Note : Attempt *five* questions in all, selecting *one* question from each unit. Q. No. 9 (Unit V) is compulsory. All questions carry equal marks.

UNIT–I

1. (a) If A, B and C are any three sets, then show that
    (i) $A \cup (B \cup C) = (A \cup B) \cup C$.
    (ii) $A \cap (B \cap C) = (A \cap B) \cap C$.
    (b) Find the quadratic equation with 3 and 5 as its roots.
    (c) Find the 28th term of the series $4 + 6 + 8 + 10 + \ldots$

2. (a) Write the middle term in the expansion of
    $$\left(2x^2 - \frac{1}{x}\right)^{10}.$$

(b) Prove that the coefficient of x^6 in the series

of e^{2x} is $\dfrac{4}{45}$.

(c) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 2 & 1 \\ 3 & 2 & 2 \end{bmatrix}.$$

UNIT–II

3. (a) Find the equation of the line passing through (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).

(b) Show that the points (–1, –1), (2, 3) and (8, 11) lie on a line.

(c) Find the equation of circle, the coordinates of end points of whose diameters are (3, 4) and (–3, –4).

4. (a) Find the focus and directrix of the parabola
$$y^2 = 14x.$$

(b) Find the centre, foci, eccentricity and lengths of the

axes of the ellipse $\dfrac{(x-1)^2}{25} + \dfrac{(y+3)^2}{9} \doteq 1$.

(c) Show that the equation $16x^2 - 3y^2 - 32x - 12y - 144 = 0$ represents a hyperbola.

5. (a) Prove that $\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \operatorname{cosec} A - \cot A$.

   (b) Solve $2\sin^2\theta + \sqrt{2}\cos\theta - 2 = 0$, θ being a positive acute angle.

   (c) Find the value of $\sin 36°$.

6. (a) In any triangle ABC, prove that $\dfrac{b+c}{a} = \dfrac{\cos\left(\dfrac{B-C}{2}\right)}{\sin\dfrac{A}{2}}$.

   (b) If $A + B + C = 180°$, prove that

   $$\tan A + \tan B + \tan C = \tan A \tan B \tan C.$$

   (c) The angular elevation of a tower from a point is $30°$ at a point in a horizontal line to the foot of the tower and 30 metres nearer, it is $60°$. Find the height of the tower.

UNIT-IV

7. (a) Evaluate $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{x}$.

   (b) Determine the value of λ so that the function

   $$f(x) = \begin{cases} \dfrac{x^2-2x-3}{x+1} & , \quad x \neq -1 \\ \lambda & , \quad x = -1 \end{cases}$$

   is continuous at $x = -1$.

   (c) If $y \log x = x - y$, show that $\dfrac{dy}{dx} = \dfrac{\log x}{(1+\log x)^2}$.

8. (a) Integrate $\int \log x \, dx$.

(b) Evaluate $\int\limits_{0}^{2} x e^{3x} \, dx$.

(c) Solve $(1 - x) \, dy - (1 + y) \, dx = 0$.

UNIT–V
(Compulsory Question)

9. Attempt all the following :

(a) State De'Morgan's law of sets.

(b) Prove without expansion that
$$\begin{vmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{vmatrix} = 0 .$$

(c) Find $\cos 15°$.

(d) Find $\dfrac{d}{dx} \left(\log \sqrt{x} \right)$

(e) Find $\int \dfrac{2x}{x^2 + 4} \, dx$.

(f) Solve $y \dfrac{dy}{dx} = x$.

(g) Write intercept form of a line $3y + 2x + 1 = 0$.

(h) Find the equation of a circle with centre (3, 4) and radius 5.

(i) Find the domain of the function $f(x) = \sqrt{x}$.

(j) Solve $\int\limits_{0}^{1} \dfrac{1}{x \log x} \, dx$.

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