Roll No.

Total No. of Questions: 9]

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(2102)

# BCA (CBCS) RUSA IIIrd Semester Examination

3991

# MATHEMATICS-III BCA-301

Time: 3 Hours]

[Maximum Marks: 70

Note: Part-A is compulsory. Attempt one question each from Parts-B, C, D and E.

#### Part-A

# (Compulsory Questions)

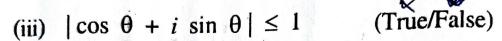
- 1. (A) Attempt all questions:
  - (i) Write order and degree of the differential equation:

$$\sin^2 x \frac{d^2 y}{dx^2} + \cos x \frac{dy}{dx} + y = 0$$

(ii) The intersection of two fields is not a field. (True/False)

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Turn Over



- (iv) A differential equation with degree one is a linear differential equation. (True/False)
- (v) Roots of  $x^2 + 1 = 0$  are purely imaginary. (True/False)
- (vi) Find modulus and argument of complex number -3i.
- (vii)  $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ , where n is a +ve integer. (True/False)
- (viii)  $(\mathbb{Z}_2, +_2, \cdot_2)$  is a field. (True/False)
- (ix) The algebraic structure  $(\mathbb{Z}_p, +_p, \cdot_p)$  is a field, where p is a prime no. (True/False)
- (x) Prime numbers are finite. (True/False)
  (B) Attempt all questions: 1×10=10
  - (i) Solve the differential equation:

$$x\frac{dy}{dx} = y + xe^{-y/a}$$

(ii) If n is any integer, show that :  $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1} \cos n \frac{\pi}{6}$ 

(iii) Simplify:

$$\frac{(\cos\theta + \sin\theta)^6(\cos 3\theta + i\sin 3\theta)^8}{(\cos 5\theta + i\sin 5\theta)^4(\cos 2\theta + i\sin 2\theta)^7}$$

- (iv) Find gcd (35, 49) and express it as linear combination of these numbers.
- (v) Prove that  $x^3 + 2x + 4$  is irreducible over  $\mathbb{Z}_5$ .

 $4 \times 5 = 20$ 

## Part-B

10 each

2. (a) Solve:

$$x^{3} \frac{d^{3} y}{dx^{3}} + 6x^{2} \frac{d^{2} y}{dx^{2}} + 4x \frac{dy}{dx} - 4y = (\log x)^{2}$$

(b) Solve

$$(D^4 - 1)y = e^x \cos x$$

3. (a) Solve :

$$(x^2 + y^2 + 2x)dx + 2ydy = 0$$

Solve: (b)

$$(D^3 - 3D^2 + 3D - 1)y = (x + 1)e^x$$
  
Part-C 10 each

4. (a) Prove that:

$$\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta+i\cos\theta}\right)^n = \cos\left(\frac{n\pi}{2}-n\theta\right)+i\sin\left(\frac{n\pi}{2}-n\theta\right)$$
where *n* is any integer.

If  $z_1$ ,  $z_2$  are two non-zero complex numbers, prove that:

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2[|z_1|^2 + |z_2|^2]$$

5. A triangle is formed by the points  $z_1$ ,  $z_2$ ,  $z_3$  in the Argand's diagram. Prove that its:

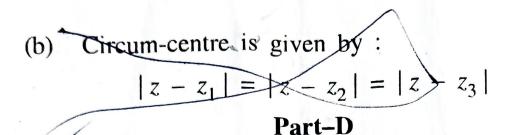
Lay Centroid is given by:

$$\frac{z_1 + z_2 + z_3}{3}$$

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(3)

Turn Over



10 each

- 6. Find the set of integers solutions for each of the following:
  - (a)  $15x \equiv 25 \pmod{25}$
  - (b)  $9x \equiv 14 \pmod{15}$
- 7. Find the smallest positive integer that when divided by 3, 5, 7 we get remainder 1, 4, 6 respectively.

## Part-E

10 each

8. (a) Let a and b be two elements of a finite field F.

Then prove that there exist elements α and β

in F such that:

$$\alpha + a\alpha^2 + b\beta^2 = 0$$

- (b) Prove that  $(\mathbb{Z}_5, +_5, \cdot_5)$  is a field.
- 9. (a) Find all nilpotent and idempotent elements of  $(\mathbb{Z}_{10}, +_{10}, \cdot_{10})$ .
  - (b) Construct a field extension of  $\mathbb{Z}_3$  with exactly 9 elements.