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Total No. of Questions : 9] [Total No. of Printed Pages : 4

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BCA UG (CBCS) RUSA IIIrd Semester Examination

3600

MATHEMATICS–III

BCA–0301

Time : 3 Hours] [Maximum Marks : 70

Note :– Part–A is compulsory and of 30 marks and attempt *one* question each from Part–B, C, D and E. Marks are indicated with questions for Part–B, C, D and E.

Part–A

1. (i) Write order and degree of the differential equation :

$$\frac{d^2y}{dx^2}+\sin\left(\frac{dy}{dx}\right)+y=0$$

(ii) Every differential equation with degree one is linear differential equation. (True/False)

(iii) If $f(x)$ and $g(x)$ are two solutions of second order differential equation then their linear combination is also a solution. (True/False)

(iv) Roots of $z^2 + 2z + 1 = 0$ are complex numbers. (True/False)

(v) Find modulus and argument of complex number $1 + i$.

(vi) $\sin^2 z + \cos^2 z = 1$ holds for all complex numbers. (True/False)

(vii) Prime numbers are finite. (True/False)

(viii) The congruence $3^{100} \equiv 1 \pmod{10}$ is not true. (True/False)

(ix) Algebraic structure $(\mathbb{Z}_n, +_n, \times_n)$ is field for every positive integer n. (True/False)

(x) Characteristic of finite field is not always prime number. (True/False)

1×10=10

(xi) Find the differential equation that will represent family of circle having centre at $(a, 0)$ and radius r.

(xii) Solve :

$$\frac{d^4y}{dx^4} - y = 0$$

(xiii) Find the real part of the complex number i^i.

(xiv) Find GCD of 39 and 45 and express it as linear combination of these numbers.

(xv) Find the unit digit of 7^{45}.

4×5=20

Part–B

10 each

2. (a) Solve :

$$\frac{d^2y}{dx^2} + 2y = e^x \sin x$$

(b) Find the equation of family of the circle :

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

in the xy plane.

3. (a) Find the differential equation from the relation $ax^2 + by^2 = 1$.

(b) Solve :

$$\frac{d^3y}{dx^3} + \frac{dy^2}{dx^2} - \frac{dy}{dx} - y = \sin 2x$$

Part–C 10 each

4. (a) State and Prove De-Moivre's theorem.

   (b) Compute $(3 + 3i)^5$.

5. (a) Find the cube root of $z = -1 + i$.

   (b) Find the square root of $8 - 6i$.

Part–D 10 each

6. (a) Solve :

$$x^2 + 2x - 1 \equiv 0 \pmod 7.$$

   (b) Solve :

$$7x \equiv 2 \pmod{31}.$$

7. Find the smallest positive integers that when divided by 3, 5, 7 we get remainder 1, 4, 6, respectively.

Part–E 10 each

8. (a) Prove that $(\mathbb{Z}_7, +_7, \times_7)$ is a field.

   (b) Show that $x^2 + x + 1$ is irreducible over GF(2).

9. (a) In $(\mathbb{Z}_5, +_5, \times_5)$ solve :

$$(x^4 + 4x^3 + 3x^2 + 2) + (2x^4 + 3x^3 + 4x + 4)$$

$$\text{and } (x^2 + 2x + 4).(x^2 + 3x + 3)$$

   (b) Find idempotent and nilpotent element of $(\mathbb{Z}_8, +_8, \times_8)$.

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B.C.A. UG (CBCS) RUSA IIIrd Semester Examination

4210

MATHEMATICS-III

BCA-0301

Time : 3 Hours] [Maximum Marks : 70

Note :– Attempt *one* question from each Part of B, C, D and E. Part-A is compulsory. Marks for Part-B, C, D and E are 10 each and Part-A is of 30 marks.

Part–A

1. (i) Write order and degree of differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$$

(ii) What is particular solution of differential equation ?

(iii) Find differential equation that will represent family of straight line $y = mx + c$.

(iv) Find modulus and argument of complex number $1+\sqrt{3}i$.

(v) Find x and y if $2x + (3x + y)i = 4 + 10i$.

(vi) Show that :

$$1 + i^{10} + i^{100} + i^{1000} = 2$$ 400

(vii) The finite field of order p^n is usually denoted by GF(p^n). In this case p is prime number and n is real number. (True/False)

(viii) Only even prime number is 2. (True/False)

(ix) If $a, b, c, d \in$ I and a/b, c/d then ac/bd. (True/False)

(x) If a and b are relatively prime then any common divisor of ac and b is a divisor of c. (True/False)

604

1×10=10

(xi) Verify that $y = e^{-3x}$ is a solution of

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0.$$ P-10.

(xii) Solve :

$$\frac{dy}{dx} = \frac{-x}{y}$$

(xiii) Find square root of $-15 - 8i$. $z^2 = (\cos\theta + i\sin$

(xiv) Find g.c.d. of 35 and 56 and express it as linear combination of these numbers.

(xv) Show that $5^{48} - 1$ is divisible by 24. 713 $4 \times 5 = 20$ 480

Part–B

2. (a) Form the differential equation representing the family of curves $y = A \cos 2t + B \sin 2t$ where A and B are arbitary constant.

(b) Solve :

$$\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x$$

P-260 10

3. (a) Solve :

$$x^3 \frac{d^3y}{dx^3} + 6x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - 4y = 0$$

P-341

(b) Show that $y = (a + bx)e^{2x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$. 10

Part–C

4. (a) If $(x + iy)^3 = u + iv$ then show that

$$\frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right).$$

419

R428— (b) If 1, w, w^2 are cube roots of unity, then show that $(1 + w - w^2)^6 = 64$. 10

5. (a) If α, β are roots of $x^2 - 2x + 4 = 0$, then prove that $\alpha^6 + \beta^6 = 128$.

(b) Find the five fifth roots of unity and show that their sum vanishes. 10

Part–D

6. (a) Solve $3x^2 + 9x + 7 \equiv 0 \pmod{13}$

(b) Use Chinese Remainder Theorem solve

$$17x \equiv 9 \pmod{276}$$ 464. 10

7. Prove that the congruence $x^2 \equiv a \pmod{p}$ where p is an odd prime and g.c.d. $(a, p) = 1$ has exactly two solutions or no solutions. 10

Part–E

-818

8. (a) Prove that $(Q, +, \cdot)$ is a field.

(b) Show that $x^4 + 8 \in Q[x]$ is irreducible over Q. 10

9. (a) Let F be the field of rational numbers. Determine the degree of spliting field of the Polynomial $x^3 - 2$ over F.

(b) Show that any two finite fields having the same number of elements are isomorphic. 10

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B.C.A. UG (CBCS) RUSA IIIrd Semester Examination

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MATHEMATICS-III

BCA-301

Time : 3 Hours] [Maximum Marks : 70

Note :– There are *nine* questions in all. Section–A is compulsory for all and the students have to attempt *one* question each from Section B, C, D and E.

Section–A

1\. (A) (i) Product of the cube roots of unity is ...1...

(ii) Write the modulus of $3 + i$.

(iii) Write the additive inverse of the complex number $3 - 2i$.

(iv) What is order of the differential equation $y'' + 3y = 0$. 2

(v) What is degree of the differential equation $y' + 5y = 0$.

(vi) 2 is the smallest prime. (True/False)

(vii) What are prime factors of 231 ?

(viii) Every positive integer $a \geq 1$ can be expressed uniquely as a product of positive primes. (True/False)

(ix) Write the value of $4 \times_7 3$.

(x) Subtraction is a binary operation on the set of integers. (True/False)

$1 \times 10 = 10$

(B) (i) What is linear differential equation. Give *two* examples. 2

(ii) State Chinese Remainder Theorem. 761

(iii) State De-Moivre's theorem. 482

(iv) Find the prime factorization of 864. Write your answer using exponential notation.

(v) Add the two polynomials $f(x) = 1$ and $g(x) = x^2 + x + 1$ over the field of real numbers.

$4 \times 5 = 20$

824

Section–B

2. (a) Find the order and degree of the following differential equations :

(i) $y = x\left[\frac{dy}{dx} + \sqrt{1+\frac{dy}{dx}}\right]$. 2

(ii) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$. 3

(b) Form the differential equation from the following :

$$y = A \sin (nt + \alpha).$$ 5

3. (a) Solve :

$$\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0.$$ 5

(b) Solve :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8\sin 2x.$$ 5

Section–C

4. (a) Find modulus and argument of the following :

(i) $-1 + i$, 2.5

(ii) $1 - i$. 2.5

(b) Find the value of $\sqrt{7+24i}$ 5

5. (a) Simplify : 5

$$\frac{(\cos\theta + i\sin\theta)^3}{(\cos\theta - i\sin\theta)^2}.$$

(b) Show that the sum of the three cube roots of unity is zero. 5

Section–D

6. Explain the following :

(i) Primes Factorization.

(ii) Quadratic Congruences 5×2=10

7. Use the Chinese Remainder Theorem to find all solutions such that :

$x \equiv 2 \pmod{3}$

$x \equiv 3 \pmod{4}$

$x \equiv 4 \pmod{5}$ 10

Section–E

8. (i) Add the two polynomials $f(x) = 1$ and $g(x) = x^2 + x + 1$ over GF(2). 5

(ii) Show that the set F = {0, 1, 2}, with compositions addition and multiplication modulo 3 forms a field. 5

9. Explain the following :

(i) Finite fields

(ii) Multiplication of Polynomials over GF(2). 5×2=10

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BCA (CBCS) RUSA IIIrd Semester Examination

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MATHEMATICS–III

BCA–0301

Time : 3 Hours] [Maximum Marks : 70

Note :– Part–A is compulsory and of 30 marks and attempt *one* question each from Parts–B, C, D and E. Marks are indicated with questions for Parts–B, C, D and E.

Part–A

1. (A) (i) Write order and degree of the differential equation :

$$\left(\frac{d^3y}{dx^3}\right)+\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^3+3y=0$$

(ii) The set of solutions of homogeneous differential equation are finite. (True/False)

(iii) Find modulus and argument of complex number $-\sqrt{3}+i$.

(iv) Show that :

$$1+i^{10}+i^{100}-i^{1000}=0$$

(v) Write $\dfrac{3-i}{2+7i}$ in standard form.

(vi) Zero is an odd number. (True/False)

(vii) Any natural number n can be expressed as a product of prime numbers. (True/False)

(viii) If a, b, d, r and s are integers and d divides a, d divides b. Then d divides $(ra + sb)$. (True/False)

(ix) $(\mathbb{Z}_4, +_4, \times_4)$ is a finite field. (True/False)

(x) There exist a finite field of order 21. (True/False)

1×10=10

(B) (i) Find the square root of $8 - 7i$.

(ii) Solve :

$$\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=0$$

(iii) Solve :

$$\frac{dy}{dx} = -\frac{x^3}{y^2}$$

(iv) Find g.c.d. of 45 and 66 and express it as a linear combination of these numbers.

(v) Show that $6^{68} - 1$ is divisible by 35. 4×5=20

Part–B 10 each

2. (a) Solve :

$$\frac{d^2y}{dx^2} + 6y = e^{2x} + \sin 2x$$

(b) Solve :

$$\frac{dy}{dx} + \frac{1}{x}y = \sin x$$

3. (a) Solve :

$$\frac{d^2y}{dx^2} - y = x^2$$

(b) Form the differential equation representing the family of curves $y = e^{-x}(\mathrm{A} \cos 3t + \mathrm{B} \sin 3t)$ where A and B are arbitrary constants.

Part–C

10 each

4. (a) Compute $(5 + 5i)^3$.

(b) Find the four fourth root of unity and show that their sum vanishes.

5. (a) If 1, w, w^2 are cube roots of unity, then show that $(1 + w - 2w^2)^3 = -27$.

(b) Find the cube root of $z = -1 + 3i$.

Part–D

10 each

6. (a) Solve :

$$x^2 + 3x + 11 \equiv 0 \pmod{13}.$$

(b) Find an integer that has a remainder 3 when divided by 7 and 13.

7. State and prove Chinese Remainder Theorem.

Part–E

10 each

8. (a) Prove that $(\mathbb{Z}_5, +_5, \times_5)$ is a field.

(b) Show that $x^3 + x + 1 \in F_2[x]$ is irreducible over F_2.

9. (a) Prove that characteristic of a finite field is always prime.

(b) Find the result of $(x^5 + x^4 + x^2 + x) + (x^4 + x^3 + x^2 + x + 1)$ in $GF(2^8)$.