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Total No. of Questions : 9] [Total No. of Printed Pages : 4

(2042)

BCA (CBCS) RUSA IInd Semester Examination

3743

MATHEMATICS-II

Paper : BCA-0201

Time : 3 Hours] **[Maximum Marks : 70**

Note :– Attempt *one* question from each Unit. Q. No. **9** *is* compulsory.

Unit–I

1. (a) Verify Lagrange's mean value theorem for the function $f(x) = (x - 2)(x - 3)(x - 4)$ in the interval $[0, 4]$. Also find the value of 'C'.

(b) If $y = (ax + b)^m$ and $y_n = \dfrac{d^n y}{dx^n}$, show that :

$$y_n = m(m - 1)(m - 2) \ldots (m - n + 1) (ax + b)^{m-n} a^n$$

5,5

2. (a) By using mean value theorem find the approximate value of $\sqrt{66}$.

(b) Apply Leibnitz theorem to find the third order derivative of :

$$f(x) = (x^2 + 1)e^{2x} \text{ at } x = 0$$ 5,5

Unit–II

3. (a) Find the greatest common divisor of 275 and 200 and express it in the from $m.275 + n.200$.

(b) Show that the relation of divisibility in the set of integer is reflexive, transitive but not symmetric. 5,5

4. (a) Prove that if a and b be two integers, then $a \equiv b(\text{mod } m)$ if and only if a and b has the same remainder when divided by m.

(b) For positive integer a and b show that :

$$\text{GCD }(a, b) \times \text{LCM }(a, b) = (a \times b) = ab$$ 5,5

Unit–III

5. (a) Show that the set of all 2×2 non-singular matrices from an infinite non-abelian group under the composition of matrix multiplication.

(b) Show that the set $G = \{1, \omega, \omega^2\}$ of cube roots of unity from a finite abelian group of order 3 under multiplication of complex number. 5,5

6. (a) Let $M_g(I) = \left\{\begin{bmatrix} a, b \\ c, d \end{bmatrix}\right\}$ where $a, b, c, d \in I$, show that $M_g(I)$ from a monoid under the operation of matrix multiplication.

(b) Define a cyclic group and prove that every cyclic group is abelian but converse is not true. 5,5

Unit–IV

7. (a) Prove that the set of integer is a ring with respect to usual addition and multiplication.

(b) Prove that < R, +, . > where R is set of all reals, is a commutative ring with unity. 5,5

8. (a) Let R be a ring. Then show that the following conditions are equivalent :

(i) R has no zero divisor

(ii) R satisfies left cancellation law

(iii) R satisfies right cancellation law

(b) Show that the set of rationals 'Q' is a field under composition of addition $\oplus$ and multiplication $\odot$, given as :

$$a \oplus b = a + b - 1 \text{ and } a \odot b = a + b - ab$$ 5,5

Unit–V

(Compulsory Question)

9. (i) Discuss applicability of Rolle's theorem for the function $f(x) = | x |$ in $[-3, 3]$.

(ii) If $f(x) = x^2 \sin 2x$, find the value of $f'''(0)$.

(iii) If a/b and a/c, then show that $a/(bx + cy)$ $\forall$ $x, y \in z$.

(iv) Solve the following congruence $3x \equiv 4 \pmod 5$.

(v) Find the LCM of (272, 1479).

(vi) Define a group.

(vii) Show that the set G = {1, 2, 3, ..., $n - 1$} does not form a group under multiplication modulo n, where 'n' is a composite number.

(viii) Let R bearing such that $x^2 = x$ $\forall$ $x \in$ R, show that R is a commutative ring.

(ix) Define a ring and a ring with unity.

(x) Prove that every field is an integral domain.

3×10=30