This worksheet demonstrates a Susceptible-Infectious-Recovered (SIR) infectious disease model, allowing you to investigate properties of the model as disease-related parameters are changed. You will also be able to compare the deterministic and stochastic versions of the model.

Using Microsoft Excel, we can formulate the model as either a set of deterministic difference equations, where the outputs are the same every time (the deterministic model), or as a set of difference equations where we incorporate stochasticity (or randomness) into the infection and removal processes. The model structure can be drawn as a schematic diagram as in the figure below.

A diagram of a number of infection

AI-generated content may be incorrect.

**The spreadsheet**

In the top left of spreadsheet, there are parameters defining the model output that you can adjust. These include the population size (N), the effective contact rate (beta), the recovery rate (gamma), and the initial number of people infected.

Below this are parameters that are derived from these input parameters. These have been locked for editing but are visible so you can see how they change with the input parameters. These include the basic reproduction number (R0, equal to beta/gamma) and the infectious period in days (1/gamma).

On the right are two charts – the top chart shows the epidemic curve using a deterministic model, and the bottom chart shows the epidemic curve using a stochastic model. Both use the same colours: blue for susceptible people, S(t), green for infected people, I(t), and pink for recovered people, R(t), through time.

As discussed in the lectures, the stochastic model will look different every time you run it. ***You can rerun the stochastic model at any time by hitting the “F9” button***.

**Practical:**

1. Start with a population size of 1000, a beta of 0.5, a gamma of 0.2, and 10 people initially infected. This is the base scenario that we will return to throughout this sheet.

First, look at the resulting parameters. What is the R0? What is the infectious period?

Next look at the model formulas in columns O:U, rows 5 and 6. Can you work out how the formulas relate to the schematic model diagram above?

1. Next, look at the deterministic model output. What is the final attack proportion? How long before the outbreak has died out?

1. Now look at the stochastic version of this model. Does it give similar results? How much does the final attack proportion and the timing of the outbreak change as you re-run the model by hitting F9? (Add rows to the table as necessary).

|  |  |  |  |
| --- | --- | --- | --- |
| Run # | Attack proportion | Peak timing | End of outbreak |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |

1. Increase beta to 0.8 (R0 = 4) and see what effect that has on the final attack proportion and the timing of the outbreak in both the deterministic and stochastic models. What has changed from the case where the reproduction number was 2? Does the stochastic model change more/less with R0 = 4?
2. Now decrease beta to 0.5 and gamma to 0.4 (R0 = 1.25, infectious period 2.5 days). What is the final attack proportion? How long does the outbreak last using the deterministic model? What about the stochastic model? Run the stochastic model a few times by pressing F9 and compare findings.
3. We will now look at the effect of the infectious period for a fixed value of R0. Increase and decrease the duration of the infectious period (1/gamma) by modifying the values of gamma and beta (to keep R0 fixed). What happens to the timing of the outbreak and to the final attack proportion?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Beta | Gamma | Infectious period | R0 | Peak timing | Final attack proportion |
| 0.8 | 0.2 | 5 days | 4 |  |  |
| 1.0 | 0.25 | 4 days | 4 |  |  |
| 1.6 | 0.4 | 2.5 days | 4 |  |  |
| 0.5 | 0.125 | 8 days | 4 |  |  |
| 0.4 | 0.1 | 10 days | 4 |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

1. Return to the case where beta = 0.5 and gamma = 0.2. Now we are going to look at the effect of the population size. Keep the initial number of people infected at 10 but decrease the population size from 1000 to 100. What happens to the deterministic model? What about the stochastic model?
2. Now we will look at the effect of the initial number of people infected. Start with our base scenario of beta = 0.5, gamma = 0.2, a population size of 1000 and 10 initial infected people. Now decrease the number of initial people infected to one. What happens to the deterministic model? Run the stochastic model a few times by hitting F9. How do things change? What is happening here? Try gradually increasing the number of initial cases and see what happens.