Electromagnetism and Optics

The Lab Manual for PHY 103N
Engineering Physics II Laboratory

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Contents

Preface

Welcome to Physics 103N, Engineering Physics II Lab. This class is the continuation of Physics 103M, and is a corequisite to Physics 303L, Engineering Physics II Lecture. Although this class is a corequisite to Physics 303L, the topics we discuss here are not necessarily exactly those discussed in lecture. There are several reasons for this: the first is that timing the labs with the lectures is impossible; second, you don't always need a detailed theoretical description of phenomena to measure and characterize their properties. It is this empirical approach that we want to emphasize here. Third, there are important physical phenomena that are not covered in detail in the lecture, because of a lack of time. We will examine some of these in this course. So, don't expect a mere repeat of the lectures here.

There are two essential reasons for this course. First, it should give you some general background knowledge of how experimental work is actually done. You will learn how to use equipment such as multimeters, frequency generators, and oscilloscopes among others. Further, you will see how to measure various properties of electronic circuits and optical systems. These are all very practical skills. Secondly, it should help you see that all the conjectures and calculations that you learn about in lecture do describe events in the real world. You will quantitatively verify some of the formulas derived in the lecture to check the professor and make sure you haven't been lied to. If not, then you will probably believe what else is said in lecture, whereas if you've been told lies, that makes everything else the professor expounds is liable to suspicion. So be on the lookout for discrepancies!

Most of the equipment you need will be provided in lab. You should bring a pen and pencil (sketches should always be done in pencil) and paper, a scientific calculator (*i.e.*, one with logs and trigonometric functions, not necessarily a graphing calculator), and this manual to each lab meeting.

You might want to keep a notebook instead of writing observations and calculations on loose paper. A notebook will help you to keep organized so that you don't lose your notes or confuse your data. Your reports will be turned in on the worksheets printed in this manual. So if you do use a notebook, make sure it has perforated sheets, as you will turn in any extra sheets with your worksheet. In any case, *avoid* the hardbound laboratory notebooks (the ones with the carbon paper), since they are unnecessarily expensive (>$10). We expect that you have the textbook assigned to the 303L lecture course available; the reference is

> R. A. Serway and J. W. Jewitt, *Physics for Scientists & Engineers*, 6th edition (updated), Thomson Publishing, Belmont, CA (2004).

An additional reference, that we'll refer to repeatedly in our discussion of error analysis in Chapter 0, but is by no means required reading, is

> P. R. Bevington and D.K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, 2nd. edition, McGraw-Hill, Inc., New York (1992).

On a final note, we add that this is a new lab manual, and as such, is just now meeting the tests and demands of students. Some typos, ambiguities, or other inadequacies are bound to have slipped our grasp. Please bring any errors or confusing parts of the manual to the attention of your instructor. Student input is invaluable to the production of a document that students depend on for learning. As an alternative, feel free to E-mail your comments and suggestions to 103n@physics.utexas.edu.

Chapter 0

Introduction

This introductory chapter describes some general information you need to know about how to perform laboratory procedures, how to write a lab worksheet and how to analyze data correctly. Please read these through carefully; your instructor will cover these items quickly on the first day of lab, and you should already be familiar with these ideas before you come to class.

0.1 General Lab Procedures

Physics 103N is a three hour lab designed to be self-contained. This means that your only homework is to prepare for the next lab. You *must* do this preparation. The type of laboratory report you will submit at the end of the class period is a worksheet lab. You will fill out a worksheet as you proceed through the experiment. You will spend two hours in lab collecting data and plotting graphs. At the end of the second hour, you will move to a classroom and complete your calculations and discussions for that day's lab and turn in the finished worksheet to your instructor. If you are unprepared and unable to complete the worksheet, you must hand it in by the end of the third hour anyway. We designed this format to help you focus on the physical meaning of the experiments and avoid sinking into a morass of meaningless calculations.

To help you avoid many hours of tedious number crunching, we have placed computers in the lab. You should use these to do most, if not all, of your graphing and major numerical manipulations. However, the computer cannot think for you; you have to assess whether the computer is handing you garbage or if the results are reliable. Here are some things to keep in mind about the computer

- It cannot keep track of units.
- It must be taught how to deal with uncertainties.
- It will not automatically place error bars on your graphs.

You must complete the worksheet by the end of class; therefore, your instructor may deem a quiz unnecessary, since you must be prepared to complete the worksheet on time. The possibility of a quiz exists, however, so know the rudiments of the procedure of the day. To prepare, you should read the manual over very carefully before you come to class. This means you should understand the lab goals and important equations.

0.2 Error Estimation and Propagation

Error is everywhere, and you must not only acknowledge it, but understand it and control it. Webster defines "error" as the "difference between an observed or calculated value and the true value." The problem with this definition is that we usually have no idea what the "true" answer should be. Previous experiments or theoretical calculations may give us a clue, but somehow we must extract an estimate of the true value from our data. In addition, we should also determine to what extent we should take our answer seriously. This last point embodies what we call error estimation.

An example will show why this process is so important. Suppose you measure the acceleration of a free-falling body and the answer you obtain is $10.5\ \mathrm{m/s^2}$. Have you contradicted the accepted value of $9.7990 \pm 0.0014\ \mathrm{m/s^2}$? The answer depends on the size of the error in your answer. If your result was $10.5 \pm 1.0\ \mathrm{m/s^2}$ then your answer is no, because the accepted value lies within your error. If, on the other hand, you had performed a fairly precise measurement and obtained $10.5 \pm 0.1\ \mathrm{m/s^2}$, then your answer would have to be yes; you should then start looking for what went wrong in the experiment itself, since many, many other measurements conducted over the last 300 years have established the accepted value $9.7990 \pm 0.0014\ \mathrm{m/s^2}$. So, the amount of effort and trust you put into an answer depends critically on ascertaining the correct overall uncertainty.

Rarely is what you measure directly comparable to other experimental results or theoretical calculations; typically, you must process your data through various formulas to extract a parameter you can compare with others. Therefore, it is critically important to accurately *propagate* the uncertainties in the original data through the calculations and arrive at a reasonable uncertainty for the final value. This is the process of error propagation. These are the issues that we now discuss.

0.2.1 Definition of Uncertainty

The first step involved in error estimation is to identify the possible types of errors that can occur in your experiment. There are three basic types you need to be aware of: illegitimate errors, systematic errors, and random errors. Illegitimate errors are faults in experimental procedures or calculational blunders. We will make every effort to avoid making these kinds of errors, but if we do blunder, we can easily find and correct them; *we will assume that we have eliminated all illegitimate errors from our experiments.* This is a formidable assumption since the time to perform our experiments is limited; however, the procedures are not all that complicated; so this assumption should not be a bad one. Under this assumption therefore, we cannot use illegitimate errors as reasonable explanations for any discrepancies that ultimately occur in our analysis. To discuss the other types of errors, we must more carefully distinguish between accuracy and precision.

Accuracy represents how close a measurement is to the true value. *Precision* indicates how well the results of an experiment have been determined, independently of how well the results agree with the true value. This tells us about the self-consistency of a measurement. When judging the results of an experiment, we must consider both the accuracy and the precision. In general, when we quote the uncertainty of an experimental result, we are referring to the *precision* with which the result has been determined.

Systematic errors are errors that make our results different from the true value in a reproducible way. They are usually due to the faulty calibration of equipment or some unknown bias on the part of the experimenter. They can be subtle and hard to quantify. Knowledge of the apparatus and the experimental procedure is the central manner of minimizing the impact of systematic errors in our results. Such errors affect the *accuracy* of our results, since they contribute the same amount of discrepancy each time we perform the experiment. *Random errors*, on the other hand, constitute the major source of imprecision in an experiment. These are the random fluctuations in measurements from experiment to experiment, primarily due to the finite resolution of our apparatus. To control random errors, we must perform the experiment many times and use a statistical analysis to extract our results. A given accuracy implies at least an equivalent precision; thus, accuracy depends on these fluctuations too.

To clarify the difference between these two types of errors, consider the simple experiment of determining the average speed of a rolling ball as it passes by a meter stick, by using a handheld timer. A systematic error involved in this experiment could be due to the calibration of our instruments. For instance, assume that the intervals on the meter stick are 1% larger than they should. Then, every time we record a distance of 50 cm the actual distance travelled by the ball is 50.5 cm. The distinct feature of systematic errors, such as this, is their repeatability. No matter how many times we perform the same measurement, if we are using the same instrument, we will always make the same "mistake." Random errors don't share this property. In the rolling ball experiment a random error is due to the timer operator. No matter how hard he or she tries, a human operator cannot be entirely consistent on when to press the start/stop button on the timer. Sometimes, he or she will start the timer just a little before the ball passes by the predetermined mark on the ruler, some times a little after. The same thing will happen when it is time to stop the timer. The end result is that, if we perform many attempts at the experiment (and assuming that the ball is always launched at the same speed,) the times we will get will vary randomly around the correct value. A random error is also introduced by the finite resolution of the devices we are using. Suppose that we have replaced the timer's human operator with a "perfect" photogate setup. If the smallest time increment on our timer is 0.01 s and we measure a time of 50.23 s then we only know that the actual time is between 50.225 s and 50.235 s. Every value in between these two will be rounded and displayed on our instrument as 50.23 s. If we perform the measurement many times and we always get the same reading of 50.23 s, the reasonable thing to assume is that, in every attempt, the actual time was different, but within the above-mentioned limits.

We also distinguish between what we call absolute and relative uncertainty. *Absolute uncertainty* is the uncertainty in a quantity expressed in the same units as the quantity. For example, if we write

$$g = 9.7990 \pm 0.0014 \text{ m/s}^2,$$

then the absolute uncertainty in the measurement of g is $\Delta g = 0.0014 \text{ m/s}^2$. Note that we always consider the absolute uncertainty to be a *positive* number. The *relative uncertainty* is the uncertainty expressed as a fraction or percentage of the quantity. The relative uncertainty of g in our example is

$$\frac{\Delta g}{g} = \frac{0.0014 \text{ m/s}^2}{9.7990 \text{ m/s}^2} = 0.00014 = 0.014\%.$$

0.2.2 Estimating Parameters and Their Uncertainties

Now that we know what constitutes error, we should describe how to estimate it given a data set. Let's consider the most common situation in which we need to extract an estimate for the error; this occurs when we've made several measurements of the same quantity and we want to extract an average value and a corresponding uncertainty. For concreteness, suppose we have a set of N data points x_i with uncertainty Δx_i which we do not assume is the same for each of the measurements. (We need this generality because the uncertainty can vary independently of the quantity being measured; for example, if you measure a current value on two different scales on a meter, then the uncertainties of the two measurements are different.) Then you can show (*c.f.* Bevington and Robinson, see the Preface for the reference) that, assuming a Gaussian error distribution, the **most probable value** is the mean, or average, value, $\overline{x}$:

$$\boxed{\overline{x} = \frac{\sum_{i=1}^{N} \frac{x_i}{(\Delta x_i)^2}}{\sum_{i=1}^{N} \frac{1}{(\Delta x_i)^2}}.} \tag{0.1}$$

This formula may look complicated, but all it does is give prominence to those measurements with the smallest uncertainty, a very reasonable thing to do. Notice that, if all the values have the same uncertainty, $\Delta x_i = \Delta x$ for $i = 1$ to N, this formula reduces to the usual $\overline{x} = (1/N)\sum_{i=1}^{N} x_i$. The corresponding uncertainty in the average value is the **standard deviation**, σ, and comes from the relation

$$\boxed{\frac{1}{\sigma^2} = \sum_{i=1}^{N} \frac{1}{(\Delta x_i)^2}.} \tag{0.2}$$

We can see from this formula that the uncertainty of the combination is always less than the smallest uncertainty of the component measurements. We should certainly hope so! After all, the more measurements we do, the better we should know what the answer is. In the case that all the uncertainties are equal to Δx, this expression reduces to $\sigma = \Delta x/\sqrt{N}$, showing that the uncertainty decreases at the relatively slow rate of one over the square root of the number of measurements.

Suppose you have three measurements of the resistance of a resistor that have come from three different techniques. This data appears in Table 0.1. The nominal value is the value printed on the resistor; the multimeter measurement comes from an ohmmeter measurement; and the current and voltage measurements come from a detailed analysis using Ohm's law. Each technique has its own associated uncertainty. We want to combine these separate measurements into a single estimate of the resistance. Since the errors are distributed symmetrically (*i.e.*, another measurement's value is equally likely to fall on either side of the current measurement's value), we can reliably use our formulas based on a Gaussian distribution, *i.e.*, the ones given above. We find the resistance to be

$$2.5866 \pm 0.0027 \text{ k}\Omega.$$

This answer reflects several general features of these formulas mentioned above. First, it is closest to the measurement with the smallest uncertainty. Second, the uncertainty of the combined measurements is less than the smallest measured uncertainty, but not by much. This reflects the fact that the other measurements' uncertainties were much larger than the last. This is good intuition that you should incorporate into your thinking patterns; it will help you identify the important sources of error in your experiments.

Experimental Technique	Value
Nominal Value	2.70 ± 0.14 kΩ
Multimeter	2.69 ± 0.01 kΩ
Current and Voltage Measurements	2.5785 ± 0.0028 kΩ

Table 0.1: Resistance values for a single resistor from different measurements.

For more sophisticated experiments, we don't necessarily measure the same quantity over and over. We might change one parameter and measure another, to investigate the effects of one on the other. When we do this, we are looking for correlations between the different parameters. One of the most effective ways to spot correlations is to graph the parameters and look for some functional relationship. The easiest and most reliable functional relationship to recognize and quantify is a linear one, *i.e.*, when you plot the parameters, the data points fall along a line. In fact, this type of correlation is so important that we sometimes alter the parameters that we plot to force the graphed data into a line, as in a log-log plot or a semi-log plot. Here, we don't just plot the data, but certain functions of the data that are linearly related. This can't always be done, but for our labs it can; so, we will focus solely on analyzing linear correlations. In this case, the correlation between the two quantities is described in terms of two numbers: the slope of the line and its y-intercept. For most purposes, the slope is the more important of the two, but the intercept can also contain important physical information. Once the data suggests a linear relationship, we want to extract the slope and intercept of the "best fit" line.

There are many complicated definitions of "best fit" that one can use to extract a consistent slope and intercept from a set of data. The most often used method goes by the name of *least squares* fit. The reason for the popularity of this particular method has to do with its relative simplicity and statistical significance. It will provide the slope and intercept of the most probable line to fit a set of data, assuming a Gaussian distribution for the errors. Thus you can think of this as analogous to extracting the average quantity for a larger class of measurements.

We can derive the formulas for the linear least squares fit to a set of data with the following assumptions: first, that the uncertainty is symmetrically distributed about the data values, and second, the uncertainty in the dependent variable is more significant than the uncertainty in the independent variable. These assumptions are discussed at length in Bevington and Robinson, and we will simply take them for granted. With these assumptions in mind, we can proceed as follows: given N data points $(x_i, y_i \pm \Delta y_i), i = 1, \ldots, N$, we want to determine the parameters of the line $y = ax + b$ so that the square of the vertical distance between the y coordinates of the line and the data, weighted by the uncertainty Δy_i and summed over all the data points, is a minimum. The geometry of this construction is shown in Figure 0.1. That is, we want to minimize the function of 2 variables

$$e(a,b) = \sum_{i=1}^{N} \left(\frac{1}{\Delta y_i} (y_i - a x_i - b) \right)^2$$

To minimize this function, we take the partial derivatives with respect to a

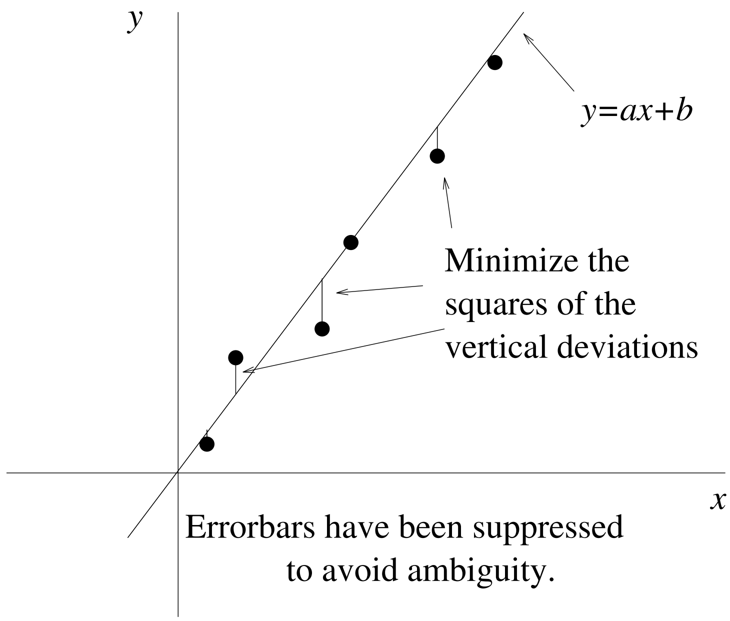


Figure 0.1: The least squares method determines the line that minimizes the square of the vertical distances between the line and the data.

and b and set them equal to zero. This yields the following system of linear equations for a and b:

$$\begin{aligned}
a \sum_{i=1}^{N} \frac{x_i^2}{(\Delta y_i)^2} + b \sum_{i=1}^{N} \frac{x_i}{(\Delta y_i)^2} &= \sum_{i=1}^{N} \frac{x_i y_i}{(\Delta y_i)^2} \\
a \sum_{i=1}^{N} \frac{x_i}{(\Delta y_i)^2} + b \sum_{i=1}^{N} \frac{1}{(\Delta y_i)^2} &= \sum_{i=1}^{N} \frac{y_i}{(\Delta y_i)^2}
\end{aligned}$$

Using your favorite technique, you can show that the solution of this linear system of equations is

$$a = \frac{1}{D} \left[\left(\sum_{i=1}^{N} \frac{1}{(\Delta y_i)^2} \right) \left(\sum_{i=1}^{N} \frac{x_i y_i}{(\Delta y_i)^2} \right) - \left(\sum_{i=1}^{N} \frac{x_i}{(\Delta y_i)^2} \right) \left(\sum_{i=1}^{N} \frac{y_i}{(\Delta y_i)^2} \right) \right],$$

$$b = \frac{1}{D}\left[\left(\sum_{i=1}^{N}\frac{x_i^2}{(\Delta y_i)^2}\right)\left(\sum_{i=1}^{N}\frac{y_i}{(\Delta y_i)^2}\right) - \left(\sum_{i=1}^{N}\frac{x_i}{(\Delta y_i)^2}\right)\left(\sum_{i=1}^{N}\frac{x_i y_i}{(\Delta y_i)^2}\right)\right], \tag{0.3}$$

where

$$D = \left(\sum_{i=1}^{N}\frac{1}{(\Delta y_i)^2}\right)\left(\sum_{i=1}^{N}\frac{x_i^2}{(\Delta y_i)^2}\right) - \left(\sum_{i=1}^{N}\frac{x_i}{(\Delta y_i)^2}\right)^2. \tag{0.4}$$

The corresponding uncertainties in the fit parameters are

$$\begin{aligned}(\Delta a)^2 &= \frac{1}{D}\sum_{i=1}^{N}\frac{1}{(\Delta y_i)^2}, \\ (\Delta b)^2 &= \frac{1}{D}\sum_{i=1}^{N}\frac{x_i^2}{(\Delta y_i)^2}.\end{aligned} \tag{0.5}$$

Let's take a look at these equations in action. Consider the data given in Table 0.2; this data came from current and voltage measurements across a resistor of unknown resistance. Ohm's law indicates that, for most resistors, the voltage is linearly related to the current, the proportionality constant being the resistance. We've plotted this data in Figure 0.2, which strongly suggests a linear relationship between current and voltage. So, it makes sense to apply our least squares equations directly to the current-voltage data. Considering the data again, we see that the errors appear symmetrically distributed around each point, and the independent variable's (the current's) uncertainty is much smaller than the corresponding uncertainty in the voltage data. So, this data satisfies both conditions for applying the least squares analysis.

We find (and you should verify with units!) the following values for the sums involved:

$$\begin{aligned}\sum_{i=1}^{10}\frac{x_i}{(\Delta y_i)^2} &= 1.5212\cdot 10^6 \\ \sum_{i=1}^{10}\frac{y_i}{(\Delta y_i)^2} &= 4.4578\cdot 10^6 \\ \sum_{i=1}^{10}\frac{x_i^2}{(\Delta y_i)^2} &= 7.0201\cdot 10^5 \\ \sum_{i=1}^{10}\frac{x_i y_i}{(\Delta y_i)^2} &= 2.0107\cdot 10^6\end{aligned}$$

Current (mA)	Voltage (V)
0.1936 ± 0.0001	0.719 ± 0.001
0.289 ± 0.001	0.813 ± 0.001
0.388 ± 0.001	1.093 ± 0.001
0.575 ± 0.001	1.620 ± 0.001
0.946 ± 0.001	2.66 ± 0.01
1.042 ± 0.001	2.93 ± 0.01
1.144 ± 0.001	3.22 ± 0.01
1.196 ± 0.001	3.37 ± 0.01
1.484 ± 0.001	4.17 ± 0.01
1.750 ± 0.001	4.93 ± 0.01

Table 0.2: Current and voltage data for computing the resistance of a resistor using Ohm's law.

$$\sum_{i=1}^{10} \frac{1}{(\Delta y_i)^2} \quad = \quad 4.0600 \cdot 10^6$$

from which we find $D = 5.3607 \cdot 10^{11}$ and then

$$\begin{aligned} a &= 2.5785 \pm 0.0028 \text{ k}\Omega \\ b &= 0.1319 \pm 0.0011 \text{ V} \end{aligned}$$

The line with this slope and intercept is the line drawn in Figure 0.2. We see that this line is a very good representation of the data. Working through the units, the slope is in kΩ and the intercept is in V. This calculation is the source of the last entry in Table 0.1. The intercept has an interesting interpretation here. It is not zero, since the average value is bigger than the error. This means that if we put the current to zero, we would still measure a voltage across the resistor. This fact should make us very suspicious about any other conclusions we might make from this data, until we have an explanation for this apparent contradiction of Ohm's law.

0.2.3 Propagating and Reporting Uncertainties

At this point, you should have a clear idea of what uncertainty is and how to estimate it in some simple cases. Once you have your estimates of the

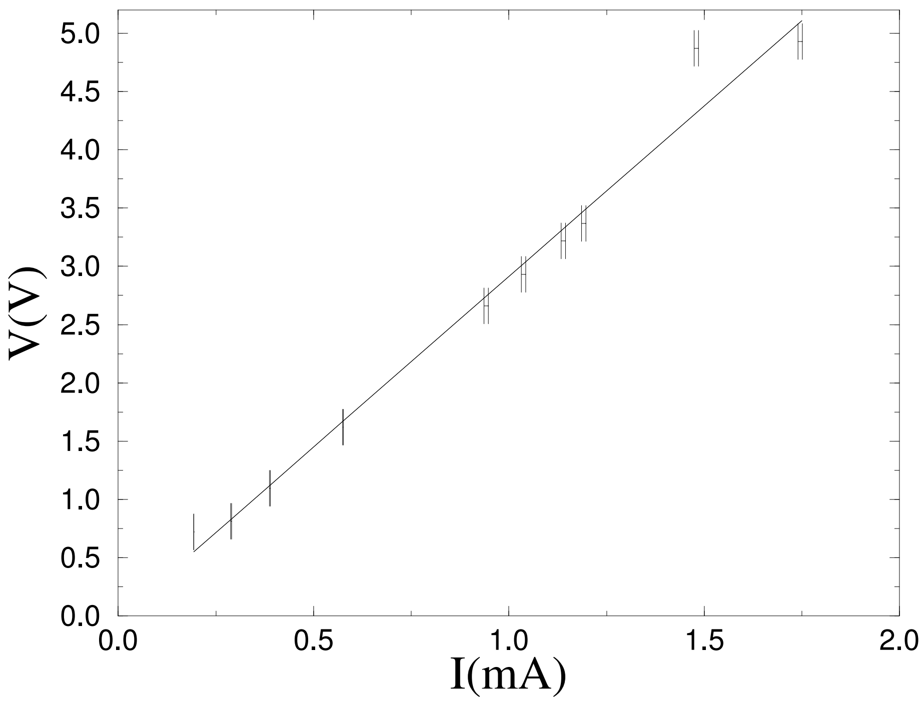


Figure 0.2: Plot of voltage versus current for the data in Table 0.2.

parameters of interest and their uncertainties, you will likely want to run them through some formulas to arrive at numbers you can compare to other people's measurements. This brings us to discuss the propagation of uncertainties through functions and formulas.

To keep things simple, we will make the assumption that the uncertainties in your parameters are symmetrically distributed about the average and that the parameters are independent of each other. That is, two measurements of different parameters are uncorrelated. This is not always true; for example, in an ideal gas at fixed pressure, the density and temperature fluctuations are linked by the equation of state. However, the added complications needed to account for these effects are typically intimately tied to the physical system you're studying, making a general treatment cumbersome. By ignoring correlations and assuming symmetry, we can reduce all the necessary error propagation down to some simple calculus.

Suppose we have a parameter with its uncertainty: $x \pm \Delta x$. The question we want to answer is "What is the uncertainty of some function, f, of this data?" Under our assumptions, the answer comes from the Taylor series expansion of f (*c.f.* Bevington and Robinson): $f(x+\Delta x) = f(x)+f'(x)\Delta x + O(\Delta x^2)$. From this we find the uncertainty Δf in the function value $f(x)$ is

$$\Delta f = \left|\frac{df}{dx}\Delta x\right|,$$

with the derivative evaluated at the point x. We can generalize this result to functions of several variables as follows: given the data $x \pm \Delta x, y \pm \Delta y, \ldots$, the function $f(x, y, \ldots)$ has the associated uncertainty

$$\Delta f = \left|\frac{\partial f}{\partial x}\Delta x\right| + \left|\frac{\partial f}{\partial y}\Delta y\right| + \ldots,$$

where all the derivatives are evaluated at the point $x, y, \ldots$. If we recall that we defined absolute uncertainties to be positive, we can write this as

$$\boxed{\Delta f = \left|\frac{\partial f}{\partial x}\right|\Delta x + \left|\frac{\partial f}{\partial y}\right|\Delta y + \ldots,} \tag{0.6}$$

From this relationship, we can derive all the familiar results of error propagation.

Example: Addition and Subtraction
Given: $f(x, y) = 3x + y - z + 5$
Find: Δf

$$\begin{aligned}\Delta f &= \left|\frac{\partial f}{\partial x}\right|\Delta x + \left|\frac{\partial f}{\partial y}\right|\Delta y + \left|\frac{\partial f}{\partial z}\right|\Delta z \\ &= |3|\,\Delta x + |1|\,\Delta y + |-1|\,\Delta z \\ &= 3\Delta x + \Delta y + \Delta z\end{aligned}$$

Example: Multiplication and Division
Given: $f(x, y) = x^2y/(5z)$
Find: Δf

$$\begin{aligned}
\Delta f &= \left|\frac{\partial f}{\partial x}\right| \Delta x + \left|\frac{\partial f}{\partial y}\right| \Delta y + \left|\frac{\partial f}{\partial z}\right| \Delta z \\
&= |2xy/(5z)|\, \Delta x + \left|x^2/(5z)\right| \Delta y + \left|-x^2 y/(5z^2)\right| \Delta z
\end{aligned}$$

Example: Ohm's Law
Given: $V = IR$
Find: ΔR

$$\begin{aligned}
\Delta R &= \left|\frac{\partial R}{\partial V}\right| \Delta V + \left|\frac{\partial R}{\partial I}\right| \Delta I \\
&= \left|\frac{1}{I}\right| \Delta(V) + \left|\frac{-V}{I^2}\right| \Delta I
\end{aligned}$$

Now that we have a clear idea of what constitutes the uncertainty of a measurement, how to estimate it, and how to propagate it, we should talk about the proper way to report the uncertainty of a measurement. This forms the subject of *significant figures*. Here is how you should determine the number of significant figures:

1. Calculate the uncertainty in the quantity.

2. Round off the uncertainty to one or two digits.

3. Express the uncertainty in the same units as the quantity measured.

4. Round off the quantity to the last decimal place of the uncertainty.

5. Always write down the final result of a calculation with the uncertainty and the units included.

Use the form

$$(2.34 \pm 0.23) \cdot 10^3 \text{ m, or } 2.34 \pm 0.23 \text{ km,}$$

not expressions such as

$$\begin{aligned}
2.34 \cdot 10^3 \text{ m} &\pm 0.23 \cdot 10^3 \text{ m,} \\
2.34 \text{ km} &\pm 23 \cdot 10^1 \text{ m,} \\
2340 \text{ m} &\pm 0.23 \cdot 10^3 \text{ m.}
\end{aligned}$$

These are the rules you will use most often in reporting your results. They become rather cumbersome, though, when you begin to make very precise measurements. Consider, for example, the charge on the electron; the best measurement we have of this number is

$$(1.60217733 \pm 0.00000049) \cdot 10^{-19} \text{ C}.$$

This is very annoying; so, we've developed a shorthand for reporting these kinds of measurements. You simply quote the result to the known uncertainty and place the uncertainty of the last few digits in parentheses after the number and before the power of ten. In this notation, the electron's charge is

$$1.60217733(49) \cdot 10^{-19} \text{ C},$$

which is much easier to deal with. If you begin to make measurements of such precision that you need to employ this convention, feel free to do so.

Finally, in various experiments we quote what are called "accepted values" for various physical parameters. These are the scientific community's best estimates of these numbers. They have been experimentally verified and checked for consistency with other measurements. Most you will find are very precise, typically 6 or 7 decimal places. You will discover in trying to do your own labs that making such high precision measurements is not easy. They also let you know that there is still some uncertainty in these parameters; they are not *exact*; but you will probably not be able to help narrow that using the equipment and techniques we have, which means they are exact as far as we can tell. So, keep in mind as you attempt to verify these numbers, that other folks had to do these measurements too.

0.3 The Lab Worksheet

The lab worksheets are preformated to be well organized, concise, and complete. The worksheets are designed to maximize your efficiency. The format of the worksheets includes the following:

- In-Lab Procedure

- In-Lab Computer Work

- Pre-Classroom Checklist
- In-Classroom Calculations & Analysis
- In-Classroom Discussion and Conclusion

Each item is described in detail below. Some may occur more than once in any given worksheet.

0.3.1 In-Lab Procedure

The In-Lab Procedure section will guide you through experimental set-up, data collection, and necessary calculations. This section will always be the first section of the worksheet, but there may be a few of them in any one worksheet. Each time you set-up a new experiment in that day's lab, a new in-lab procedure section will guide you. Since this lab is electronics and optics, you will be connecting circuits and manipulating optical equipment. Figures are provided and numbered to show you how to make connections and placements for these set-ups. The figures will become your friends in setting up the experiments. **Do not ignore them**. You cannot set- up without the figures.

After setting up the experiment, you will follow the directions given to begin data collecting. Sometimes you will be obtaining one piece of data, other times twenty. Any data collecting will be specified and organized by a table or space. Blank spaces above answers are to be used by you to **show your work** in reaching the answer below the space. This is **very important** as indicated by the **boldface**. Boldface will appear when an important point is being made vital to your grade.

When you are recording your data, be that in a table or on a line, you should present your raw data neatly and completely including **units**, uncertainties, and significant digits. Any calculations used in recording the data should be shown in the space provided above the answer. Show all calculations *keeping numbers out of the calculations until the final step*. How to do the calculations will be explained in the In-Classroom Calculations & Analysis section.

When showing your work, it is crucial that you *propagate* all units and uncertainties through the algebra. This will convince the reader that you're not trying to hide anything and it will help you check your answers as you go. Do not ignore units for several steps of a computation and then just write down what seems to be the proper units; you can loose many factors of 10 by not doing this.

Before moving on to the next section of the worksheet, double check that you have completed everything required of you. Remember that this section is not only for data recording. Just because there is no space for an answer does not mean there was nothing important to be done. You must complete everything in each section before moving on to the next one. You and your lab partner can divide the work to be more efficient but make sure you do everything.

0.3.2 In-Lab Computer Work

After collecting and recording data you will usually make a graph. You are not plotting the data to make more work for yourselves. The graphs will give you a vital piece of information that you will use in calculations, analysis, discussion or conclusion. Graphing is usually the easiest and most accurate way to get the information. You will be using the computers in lab, and that means that you must complete all the graphs in the two hour lab time. You will need them for the classroom period.

KaleidaGraph is the plotting program you will be using for all your graphs. You will spend part of the first two lab sections learning and practicing your KaleidaGraph skills. You need to become proficient in KaleidaGraph to complete the labs since you will include all graphs with your worksheet before leaving lab. You should title your graphs appropriately, include all units, and label axes. Despite the fact that the computer will be doing most of the work in graphing your data, you need to understand what the work entails.

In many labs you will have to graph sets of 5 to 15 data points and make linear fits to the data. If you graph by hand, as you will in § 0.W2, you should use graph paper of at least 4 boxes per inch and each graph should be large and clear. A full page graph of 10 points is not unreasonable. The scales on the axes should be appropriate for the data ranges, so that the data covers most of the graph. Having bunched up data points leads to difficulty in reading the graph and loss of precision in fitting lines and calculating slopes and intercepts. Do not draw your axes across a full page and choose your scale in such a way that the data points occupy only a few cm^2! Also, take care to distinguish dependent and independent variables when graphing; the quantity which is the function of the other in the experiment is conventionally plotted along the vertical axis. If you are asked to plot y vs. x, for example, you should interpret y as the dependent variable and plot it on the vertical axis. Always label the axes with their appropriate physical parameters and include the correct units in the label.

When fitting data by hand, as opposed to the least squares method discussed earlier, be sure not to obscure any data points and do not "connect the dots." Doing so has no physical basis and yields no insight into the physics at hand. Using a straight edge, "eyeball" the line that best fits the data. This line will yield the best values for the slope and intercept of your fit. Furthermore, you should draw the steepest and shallowest lines that are consistent with both the trend of your data and with the error bars. These lines will yield the uncertainty in your fit parameters, through the formulae

$$\begin{aligned}\Delta a &= \frac{|a_{\text{steep}} - a_{\text{shallow}}|}{2}\\ \Delta b &= \frac{|b_{\text{steep}} - b_{\text{shallow}}|}{2}.\end{aligned}$$

Remember that these lines represent the trend present in your data and might not pass through any data points. When calculating slopes for these lines, you want to choose two points on the lines that are not data points. The trend of the line is far more important than the slope between any two data points.

Although these instructions tell how to estimate the best fit line by hand, the computer is only doing a more sophisticated and reproducible version of this; thus, to fully employ the computer's power, you need to understand how to estimate these things independently of the computer. This will also help you check the results the computer gives you.

0.3.3 Pre-Classroom Checklist

The checklist is included for you to use as a check against what you have to take out of the lab period. You only have 2 hours in lab during which time you may do several procedures and graphs. Before leaving the laboratory, you will use this list to see that you have completed everything and have it with you to take to the classroom. Once you leave the lab, another class enters it so you can't go back to use the computers or redo a portion of the lab. The worksheet has been designed to be completed in parts. You will complete one procedure and do its computer work before moving on to the next procedure. In this manner, you will be able to do a complete analysis and discussion on at least some parts even if you don't finish the lab.

There are ◯ provided for you to physically check off. Do this. It will keep you organized when you get to the classroom. One hour is not a lot of time to finish all the calculations, discussions, and conclusion. Also make sure that each partner has her/his own data, graphs, tracings, etc.. Contact amongst you will be kept to a minimum.

0.3.4 In-Classroom Calculations & Analysis

At this point you have finished all In-Lab Procedures and In-Lab Computer Work. You have also checked all the circles in the **check list** to ensure that you have completed all the laboratory work. Once the two hour lab period is over, you will move to the classroom and finish the worksheet in **one hour**. If you complete all in-lab sections sooner than two hours you may begin the in-classroom sections.

The calculations you need to do are clearly stated with referenced equations, figures, and tables when appropriate. There is a line or space left for each answer. There is also room for you to **show your work**. You must show all the steps and reasoning behind your answers to get full credit. A sample calculation is done for you here as a future guide. Suppose the question in the in-classroom calculation & analysis section reads:

> Calculate the resistance through a circuit given the voltage, $V = 5.00 \pm 0.01\ V$, and current, $I = 20.0 \pm 0.2\ mA$, with uncertainties and units. **Show Work**.

$$\begin{aligned} V &= IR \\ R &= \frac{V}{I} \\ R &= \frac{5\,V}{20\,mA} \end{aligned}$$

$$
\begin{aligned}
R &= 250\,\Omega \\
\Delta R &= \left|\frac{1}{I}\right| \Delta(V) + \left|\frac{-V}{I^2}\right| \Delta I \\
\Delta R &= \left|\frac{1}{20\,mA}\right| 0.01\,V + \left|\frac{-5\,V}{400\,(mA)^2}\right| 0.2\,mA \\
\Delta R &= 3\,\Omega \\
R &= 250 \pm 3\,\Omega
\end{aligned}
$$

The above is what you should show and report. Note that numbers were not used until the last line of each calculation and the uncertainties were treated separately from the values. You will be performing this calculation later in lab.

The In-Classroom Calculation & Analysis sections can also ask questions that lead you to an understanding of the reasoning behind the lab and the physical principles that the lab confirms. They are rarely in yes/no format. Answer all the questions completely, stating your reasoning leading to the answer and showing any calculations or drawings necessary. At the end of this section you should understand all aspects of the lab well enough to write a concluding statement.

0.3.5 In-Classroom Discussion and Conclusion

In the final section you will bring all your analysis together to answer questions and make specific, concrete conclusions about the parameters and physics developed in the lab. This includes a clear statement of the results of the lab, *e.g.* parameters that you've measured including units and uncertainty, comments on the physical implications of these parameters, etc. You should indicate whether your results are consistent with previous efforts and discuss the internal consistency of your experiment. You should candidly address the uncertainties that arose in the lab and attempt to unambiguously and uniquely identify the key source of error. Within the paradigm that we discussed before, this cannot include things like "human error" or "errors in the calculations." You should have these illegitimate errors tightly under control. With all these ideas clearly laid out, you should then state whether you believe the experiment to be a success or not, justifying yourself by referring to your previous discussions.

Your instructor may also assign additional questions for you to ponder. You should incorporate the answers to these questions into your discussion.

They should fall naturally into your considerations, as you think about what might have gone wrong, or possible sources of discrepancy that were not in the original motivating theory. Typically, these questions have direct answers, but only after you have thought about the lab.

0.4 Using KaleidaGraph for Data Analysis

0.4.1 Introduction

What KaleidaGraph Does and Doesn't

The software package known as KaleidaGraph can be a useful tool for data analysis. Of course, it will only be useful if you learn how to tell it to do what you want it to do. This knowledge is best acquired by experience with using the software, and you will get plenty of that in the upcoming semester. That fact doesn't help you at the moment, though. Getting started is the difficult part. Hopefully, this section will help you with that.

This section is not meant to be a detailed guide to using KaleidaGraph. Instead, it hopes to demonstrate how to use those features of KaleidaGraph which will be most useful to you: plotting and fitting curves to data. To accomplish this, this section is designed to be a working example.

When you are using KaleidaGraph to analyze real data, you'll probably have to perform most of the tasks described below. In that spirit, we'll connect all the examples with...

... A Hypothetical Experiment.

Suppose an experiment has been performed that tests the well-known conjecture that the probability that a slice of toast buttered on one side will fall butter-side-up is inversely proportional to the value of the carpet on which it falls. This assertion can be represented by an equation:

$$U = a\frac{1}{V},$$

where U represents the probability of a piece of bread falling butter side up, a is a proportionality constant, and V is the price of the carpet in U.S. dollars. If you were to somehow measure the probabilities of the toast falling butter-side-up on several carpets, and plot these against the reciprocal of the price of the carpet, you'd expect to see a straight line, with a slope equal to the proportionality constant. The following examples will lead you through the process of plotting data taken in such an experiment and finding the slope of the best fit line of the plot.

Now for some details. On each run of the experiment, a helicopter dropped 100,000 slices of buttered toast on a large sample of carpet. Then, the U.S. Dept. of Parks and Wildlife flew in with their own helicopter to take an aerial photograph of the result. From the photograph, the fraction of butter-side-up slices was measured, and reported as the probability.* Only five runs were accomplished before funding ran out. The results of the experiment are

V(\$)	U
100 ± 1	0.43 ± 0.01
250 ± 1	0.15 ± 0.01
500 ± 2	0.09 ± 0.01
750 ± 2	0.06 ± 0.01
1000 ± 3	0.04 ± 0.01

0.4.2 Entering Data

Actually Entering Data

KaleidaGraph holds data in a **data window.** One of these should appear when you start; it's default name is **Data 1.** To enter data,

1. Activate the data window by clicking on it.

2. Position the cursor on the cell you want to enter data into.

3. Type in a piece of data.

4. Move to another cell using mouse, arrow keys, Tab or Return.

Enter all the Buttered Toast data, entering values for V in column "A", the uncertainties for V in column "B", the values for U in column "C" and the uncertainties for U in column "D".

*These results have met with some controversy. The Association of Premium Carpet Manufacturers has filed a lawsuit claiming that the results have been altered to make their products seem less desirable.

Note that KaleidaGraph plots data in a column vs. column fashion, so the x-coordinate and y-coordinate of a single piece of data should be placed in the same row, but different columns.

Renaming Columns of Data

This might not seem important at first, but KaleidaGraph labels the axes on plots with the name of the columns it used in the plot. The default names of columns are "A", "B", "C", etc. These appear in the **column title** row of the data window, along with a number. To change the name of the column,

1. Double-click on the column title you wish to change.

2. In the "Column Format:" dialog box which appears there will be a list of column titles. Highlight the title you wish to alter by clicking on it.

3. Type in the new title of the column (for example "A" becomes $V(\$)$).

4. When you are finished changing names, click on the button labeled **Done.**

Make sure to name each column, include uncertainties (for example "B" becomes $dV(\$)$). Don't forget the units!

The number of any column can be set to zero merely by clicking on the title cell of that column. The columns to the right then take the numbers 1, 2, 3.... The columns to the left become unnumbered.

0.4.3 Entering Formulas

Often, the raw data you enter is not immediately in the form you need for plotting. Never fear, KaleidaGraph is capable of performing mathematical operations on the numbers you have entered. One way of using this feature is to define a formula for KaleidaGraph. Formulas tell KaleidaGraph to put in one column the result of operations on data in other columns.

The syntax of formulas you define should be

$$\mathrm{c}x = \mathrm{f}(\mathrm{c}y,\ \mathrm{c}z,\ ...)$$

where x, y, and z are the numbers of the columns which contain the numbers you wish to operate on, and f(...) is the mathematical expression you wish KaleidaGraph to calculate.

For example, since you want to plot U vs. $1/V$, you need to make a column containing the reciprocal dollar values of the carpets. To actually enter and execute a formula,

1. From the **Windows** menu at the top of the screen, select the option *Formula Entry.*

2. In the "Formula Entry" window which appears, click on one of the buttons labeled **F1 - F8.**

3. Type in the formula in the space provided in the "Formula Entry" window (in your case, c4 = 1/c0).

4. Click the button marked Run in the "Formula Entry" window.

The button labeled **F1-F8** you choose corresponds to one of the function keys at the top of the keyboard. Pressing that key will bring up the "Formula Entry" window again, this time containing the formula you defined **thus saving important formulas for you**. Make sure to rename the column you just made.

Of course, the uncertainties in $1/V$ are different from the uncertainties in V. So you'll need to get KaleidaGraph to calculate an uncertainty column for you. Try to figure out the necessary formula yourself.

0.4.4 Plotting Data

Making a Scatter Plot

These two sections on plotting and fitting data require little motivation. But a few important notes will be made. First, **Never** connect-the-dots when you plot data. Fortunately, if you follow these steps, you'll never forget and accidentally do it.

1. Activate the window containing the data you want to plot by clicking on it.

2. From the **Gallery** menu at the top of the screen, select the **Linear** submenu, and from that select the **Scatter** option.

3. A dialog box will appear. In it, there will be columns of circles labeled "X" and "Y." Under "X" click on the circle in the row containing the title of the column which contains the x-coordinates of your data. A solid black circle should appear.

4. Do the same with your y-coordinates in the column labeled "Y."

5. Click on the button labeled **New Plot.**

Note the conspicuous lack of error bars.

Adding Error Bars

1. Activate the window containing your plot by clicking on it.

2. From the **Plot** menu at the top of the screen, select the option "Error bars..."

3. In the "Error Bar Variables" dialog box which appears, click on the square labeled "X Err."

4. Click and hold on one of the two rectangles labeled "% of values," and select the option "Data Column."

5. Select the column which contains the uncertainties in the x-coordinates of your plotted data.

6. Click on button labeled **OK.**

7. Now, follow the same procedure starting with the square labeled "Y Err."

8. Click on button labeled **Plot.**

0.4.5 Performing a Weighted Least-Squares Fit on Plotted Data

Weighted Fit

We come to the moment of truth. Both the slope and y-intercept of a linear plot are often important pieces of information to obtain. Of course, they are meaningless without uncertainties. Therefore, you should make sure you take into account the uncertainties in your points when calculating the fit.

1. Activate the window containing your plot by clicking on it.
2. From the **Curve Fit** Menu at the top of the screen, select the **General** submenu, and from that, select the option **fit1.**
3. In the "Curve Fit Selections:" dialog box which appears, click on the button labeled **Define...**
4. In the new dialog box that appears, click on the square labeled "Weight Data" so that an X appears in it.
5. Click on the button labeled **OK.**
6. In the "Curve Fit Selections:" dialog box, click on the square next to the column title which contains the error in the y-coordinate of the data you are plotting.
7. A new dialog box will appear called "Weight Data From Column:". By clicking on the buttons labeled $\ll$ and $\gg$, make sure the name of the column containing the uncertainties for the y-coordinate appears in the window.
8. Click on the button labeled **OK**.
9. Now click on this button window's labeled **OK**.

Now you need the numerical results of the fit. Simply choose the "Display Equations" option from the **Plot** menu, and a table containing the numbers you desire will appear. Note that, in this table, m1 is the y-intercept and m2 is the slope of the best fit line.

The Work You Should Turn In

After you have followed through the above example, you should attach the printout you have made to the worksheet that follows. This printout should at least contain: properly labeled axes, x and y error bars, and the best fit line plotted by KaleidaGraph through your plotted points. Also, somewhere on the page below the plot, you should report *in a complete sentence* what you have found to be the constant of proportionality (with uncertainty!).

Of course, your TA may require more of you.

0.W1 Error Analysis Worksheet

Name: ____________________ Day/Time: ____________

Instructions: Perform all of the following calculations using the techniques explained in Chapter 0 (Introduction) of the lab manual. Show all calculations explicitly, propagate uncertainties where appropriate, include the proper number of significant figures, and provide units.

0.W1.1 In-Lab Procedure

Perform the KaleidaGraph practice plot and analysis in the previous section § 0.4 and attach it the end of this worksheet.

0.W1.2 In-Classroom Calculations

1. Four independent measurements of the voltage supplied by a certain D-cell battery were made:

$$2.4 \pm 0.6 \text{ V}$$
$$2.96 \pm 0.08 \text{ V}$$
$$3.02 \pm 0.06 \text{ V}$$
$$2.968 \pm 0.004 \text{ V}.$$

   Referring to § 0.2.2, calculate the *most probable value* of the D-cell voltage as well as the standard deviation of the measurements using equations (0.1) and (0.2). Write your answer as you would report the final result.

2. Refer to § 0.2.3 for the calculation and propagation of uncertainty. Two lengths have been measured to be $L_1 = 4.8 \pm 1.2$ cm and $L_2 = 3.2 \pm 1.6$ cm.

   (a) Calculate the sum $L = L_1 + L_2$ and its *absolute* uncertainty, ΔL.

   Use these to calculate the *relative* uncertainty in L.

   (b) Calculate the difference $L_0 = L_1 - L_2$, as well as its absolute and relative uncertainties.

   Compare these uncertainties with those in the sum.

   (c) Now calculate the product $P = L_1 L_2$ and its absolute and relative uncertainties.

(d) Calculate the quotient $Q = L_1/L_2$, its absolute and relative uncertainties.

Compare the uncertainties to those in the product.

(e) Express L, L_0, P, and Q in proper form, i.e. with units and uncertainties.

3. The area of a square has been measured to be $A = 50 \pm 6$ cm^2. What is the length of one side of the square?

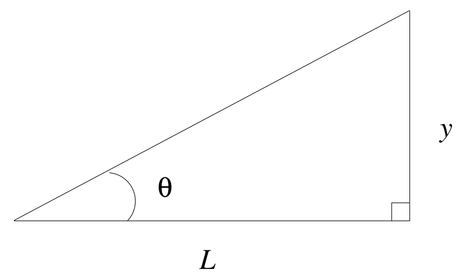
4. Two resistors, with resistances $R_1 = 540 \pm 54\ \Omega$ and $R_2 = 860 \pm 86\ \Omega$, are connected in parallel. Calculate the equivalent resistance, R_{eq}, of

the combination using the formula

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

5. For $\phi = 60 \pm 3^\circ$, calculate $\sin\phi$ and $\tan\phi$. Hint: Convert the angle to radians.

6. Given that $L = 20 \pm 4$ cm and $y = 8 \pm 2$ cm in the triangle



calculate $\sin\theta$.

Attach your KaleidaGraph plot to the end of this worksheet.

Bring graph paper next week.

End Error Analysis Worksheet

0.W2 KaleidaGraph & Graphing By Hand

Name: ________________ Day/Time: __________

0.W2.1 In-Lab Computer Work

Two Exercises for Kaleidagraph

1. Imagine that you are riding in a car with your uncle Bob, his sister Sandy, and your cousins Rob and Gary. As you head down a back road in central Florida, a brilliant blue light suddenly bathes the car, and you fall unconscious. When you wake up, you find yourself in a room with no doors and plain grey walls. An unknown source of illumination allows you to see that your only company is a spark-tape apparatus, familiar to you from your first semester lab class. You immediately realize that you can conduct a simple experiment to determine the gravitational acceleration of the planet you are on. You conduct the experiment and find the following distance fallen versus time data:

Distance (d) vs. time (t) Measurements	
d (m)	t (s)
$.20 \pm .01$	$.2 \pm .02$
$.80 \pm .04$	$.4 \pm .02$
$1.72 \pm .05$	$.6 \pm .02$
$3.16 \pm .05$	$.8 \pm .02$
$4.84 \pm .10$	$1.0 \pm .02$
$6.96 \pm .10$	$1.2 \pm .02$

Do the following

a) Plot d vs. t^2 and obtain a **weighted** least squares fit. (Remember, use **fit1** under the General option of the curve fit menu.) Error bars, properly labeled axes and units are essential! All partners should have their own plots. **Show** your work in finding $\Delta(t^2)$ below.

b) Determine the acceleration due to gravity from the results of your curve fit, including units and uncertainties. Show work.

$g =$____________________

c) Try to guess where you are....

2. In an experiment you will conduct in a few weeks, you will measure the voltage (V) of a discharging capacitor as a function of time (t). You will be required to plot

$$\ln\left(\frac{V}{V_0}\right) \quad vs. \quad t$$

where V_0 is the initial voltage (that is, the voltage measured at time $t = 0$.) Use the following sample data:

Voltage (V) vs. time (t) Measurements	
V (V)	t (ms)
$4.0 \pm .10$	$0.0 \pm .02$
$3.6 \pm .10$	$1.0 \pm .02$
$3.4 \pm .10$	$2.2 \pm .02$
$2.8 \pm .10$	$3.2 \pm .02$
$2.6 \pm .10$	$3.8 \pm .02$
$2.4 \pm .10$	$5.0 \pm .02$

Do the following

a) Plot $\ln(V/V_0)$ vs. t and obtain a **weighted** least squares fit. Again, be sure to include error bars, units, and properly labeled axes. Remember, all partners must have their own plots. Show your work in finding $\Delta(\ln(V/V_0))$.

b) Report the slope and intercept of your plot. Units and uncertainties are a must!

Slope	Intercept

Table 0.3: Slope and Intercept

0.W2.2 In-Classroom Calculation & Analysis

This half of the worksheet is your only practice in graphing by hand. You will be using KaleidaGraph to do all your graphing in Physics 103N. Before using KaleidaGraph indiscriminately, first you will learn what graphing and least squares is all about. Chapter 0 in the lab manual contains all the information required to do this worksheet, refer back to the appropriate sections.

In Lab 2 (Electron Dynamics), we will learn that an electron moving in a constant electric potential V and a constant magnetic field B (perpendicular to the electron's motion), moves in a circular path of radius R, given by the formula

$$\boxed{R^2 = \frac{2Vm}{eB^2},} \tag{0.7}$$

where m and e are the mass and charge of the electron, respectively.

Hand-fit Graphing

A former group of 103N students set the magnetic field on their apparatus to $B = 1.154 \pm 0.006$ mT (1 T = 1 Tesla = 1 kg/C s) and measured the path radius while varying the potential V:

V (V)	R (cm)
400 ± 2	5.6 ± 0.4
600 ± 2	7.4 ± 0.4
800 ± 2	8.2 ± 0.4
1000 ± 2	9.0 ± 0.4
1200 ± 2	10.2 ± 0.4

1. Plot R^2 versus V, including error bars on all points on your graph paper. Show your work in finding $\Delta(R^2)$ in the space provided here.

   Is this linear?

   Find the slope of the best line fit to the data. Use bounding lines to obtain the uncertainty in the slope. **Show all your work in the space provided below**.

Least Squares

2. Now use linear least squares to obtain the slope with uncertainty. Note that R^2 is the *dependent* variable; you'll need to fill the following table. Observe that there are two extra rows in it. One is for the units and one is for the sums of the elements of each column.

Least Squares Method							
V_i	R_i^2	$\Delta(R_i^2)$	$\frac{1}{(\Delta(R_i^2))^2}$	$\frac{V_i}{(\Delta(R_i^2))^2}$	$\frac{R_i^2}{(\Delta(R_i^2))^2}$	$\frac{V_i^2}{(\Delta(R_i^2))^2}$	$\frac{V_i R_i^2}{(\Delta(R_i^2))^2}$

Now compute the sums required to find the slope, a, and the uncertainty in slope, Δa, as in your lab manual.

How does the least squares result compare with that obtained by hand? Comment on agreement (within uncertainty), the relative uncertainties in each result, and anything else you think is important.

3. Use equation (0.7) to calculate the charge-to-mass ratio of the electron, e/m, from the value of B and each of the slopes you obtained with uncertainty. Note that the accepted value of $e/m = 1.758\,819\,62(53) \cdot 10^{11}$ C/kg

Attach your KaleidaGraph and hand plots to the end of this worksheet.

End Worksheet

Chapter 1

Electrostatics

1.1 Introduction

The principles of electrostatics play a crucial role in our understanding of everyday phenomena such as the formation of lightning in weather storms to the operation of electrical devices such as capacitors which make our stereos, computers, TVs', and many other electrical systems work. In this lab we will learn about these principles by performing a few simple experiments that will demonstrate concepts such as charge production, charge conservation, electric fields, and electric potentials (for a comprehensive overview of these subjects refer to your text book by Serway, Chapters 23, 24, and 25).

1.2 Theory

1.2.1 Electric Charge

At one time or another we have all experienced what happens when on a dry day, we close our car door, or we walk across a rug and then touch a door knob. The little shock-like feeling is a result of an electrostatic imbalance created between us and the environment. This imbalance is due to the build-up of electric charge in our bodies and the objects we come in contact with. The amount of charge that can be acquired by a particular object depends on its electrical properties.

Charge is a quantity measured in units of coulomb denoted by C. It is a conserved quantity meaning that when we speak about charges acquired by objects what we really should be saying is that charges have been moved from one place to another, as to create some imbalance of charge in that object. We cannot create nor destroy charge.

1.2.2 Coulomb's Law

Charged objects exert forces on each other. We say that like charges repel while unlike charges attract. Thus there are two kinds of charges; we call one positive, the other negative. It is possible to quantify this force by a mathematical relation known as **Coulomb's Law** which depends on the magnitudes of the charges in question, their separation, and a constant:

$$F = \frac{1}{4\pi\epsilon_o}\frac{Q_1 Q_2}{r^2} \tag{1.1}$$

The equation above is the magnitude of the force between two charges of magnitude Q_1 and Q_2 with r being their separation as shown in figure 1.1. The constant of proportionality with units is given by:

$$\frac{1}{4\pi\epsilon_o} = 8.99 \times 10^9 \text{ N} \cdot \frac{\text{m}^2}{\text{C}^2}.$$

1.2.3 Electric Field

Imagine that the two charges in figure 1.1 are positive, and that Q_1 is stationary at the origin. Lets assume that at our disposal is a device that could measure the force between these two charges. If we move Q_2 in space, than according to Eq. (1.1) our device should measure different values for the magnitude of the force as well as its direction at different points. Thus we can think of Q_2 as being a test charge for determining the force on a charge at some point in space. Plotting these different vectors in space would result in a vector field that describes the force on a test charge (see figure 1.2). In light of this it is possible to write the force on a charge as:

$$\mathbf{F} = Q\mathbf{E}. \tag{1.2}$$

E is called the electric field which is defined to be the force per unit charge. It too is a vector which points in the direction of the electric force, or opposite to it (depending on the sign of the test charge Q), and may be

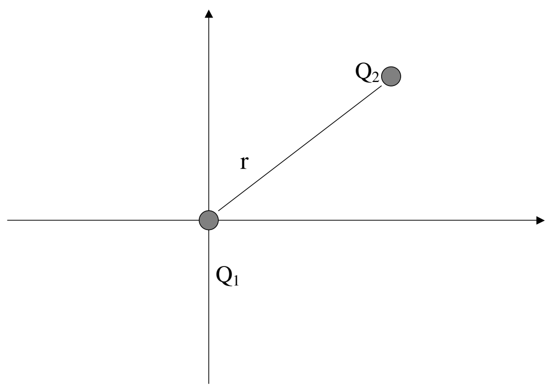


Figure 1.1: Two charges separated by a distance r.

thought of as a field in space giving rise to forces on charges. Now, the interaction among charges can be looked at as one mediated by a field.

In reality one usually encounters not point charges but continuous distribution of charges which make the concept of an electric field very useful in determining the forces by use of Eq. (1.2).

1.2.4 Gauss' Law

Determining the electric field for an arbitrary charge distribution in space can sometimes be a very difficult task. However, for charge distributions which possess certain symmetries this task can become quite easy by the use

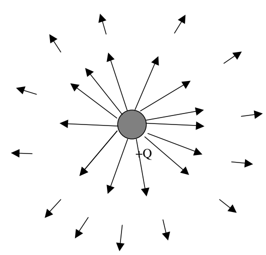


Figure 1.2: Electric field lines due to a point charge.

of Gauss' Law which is given by the following mathematical statement:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q_{enc}}{\epsilon_o}. \tag{1.3}$$

The equation above is a surface integral of the electric field dotted with a differential surface element over a Gaussian surface which encloses a charge of magnitude Q_{enc}. The left side of Eq. (1.3) is called the **electric flux** which can be thought of as measure of 'flow' of the electric field through the surface enclosing the charge. Thus for a flux equal to zero there should be no net charge inside the Gaussian surface.

By choosing the surface appropriately the integral of Eq. (1.3) can be performed to solve for **E**. Let's use it to find the electric field due to a very thin rod carrying a uniform positive charge (fig. 1.3). Positioning the rod along the z axis, figure 1.3 suggests cylindrical symmetry in which the electric field is constant along a surface of a cylinder with a radius r, and a height h. In this case we may apply equation (1.3) over the surface of the cylinder which is now our Gaussian surface:

$$\begin{aligned}\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} &= E \int \mathrm{d}A \\ &= E 2\pi r h = \frac{Q_{enc}}{\epsilon_o}.\end{aligned}$$

Note that in the second step we have used the fact that E is constant along the Gaussian surface enabling us to pull it out of the integral. The normal to the surface element is along the same direction of the field, so that their dot product is just the multiplication of their amplitudes.

Before completing the calculation we have to find what is Q_{enc}. We may define a quantity denoted by λ to be a charge density, or in this case a charge per unit length, so that Q_{enc} may be given by:

$$Q_{enc} = \lambda h.$$

With this the electric field due to a line of charge is given by:

$$E = \frac{\lambda}{2\pi\epsilon_o r}. \tag{1.4}$$

1.2.5 Electric Potential

Since an electric charge experiences a force in the presence of an electric field then a charge initially at rest will start to move under the influence of the field, and thus will gain kinetic energy. This means that the force which arises due to the field will do work on the charged particle. You may have noticed that the electrical force is very similar in form to that of a gravitational force, where, in the gravitational law, we use masses instead of charges and we don't consider repulsion among masses. Nonetheless the dependence on r and the way we define the direction of the force are exactly the same. From our mechanics course we remember that the gravitational force is conservative, and we therefore can convince ourselves that the electrical force is conservative as well. We can now use conservation of energy which tells us

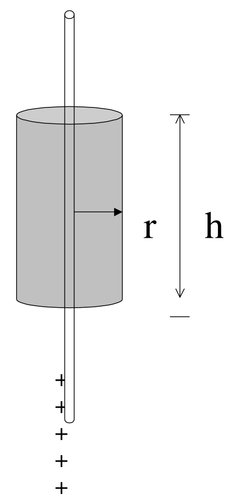


Figure 1.3: A Gaussian surface due to a line of charge

that the change in potential energy is equal to the negative change in kinetic energy, or simply put:

$$W = U_i - U_f \tag{1.5}$$

Applying this equation with the definition of work, and Eq. (1.2) we get

$$W_{i \to f} = \int_i^f \mathbf{F} \cdot \mathrm{d}\mathbf{l} = \int_i^f Q\mathbf{E} \cdot \mathrm{d}\mathbf{l}. \tag{1.6}$$

Dividing both sides by Q (remembering that Q is just a test charge) the equation above can be written as:

$$\frac{W_{i \to f}}{Q} = \frac{U_i}{Q} - \frac{U_f}{Q} = \int_i^f \mathbf{E} \cdot \mathrm{d}\mathbf{l}. \tag{1.7}$$

The quantity $\frac{U}{Q}$ is called the **electric potential** denoted by V and is defined to be the potential energy per unit charge. It is conventional to speak of an external force moving a test charge in an electric field in which case it is the external force doing the work and not the field. This introduces a minus sign in Eq. (1.7), and the potential difference of moving a test charge from point a to point b is:

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot \mathrm{d}\mathbf{l}. \tag{1.8}$$

Let us apply this formula to find the electric potential in our previous example of a very thin rod with positive charge. If we move a test charge from some point a to some point b radially to-wards the rod we can use Eq. (1.4) together with Eq. (1.8) to get

$$V_b - V_a = -\frac{\lambda}{2\pi\epsilon_o}\int_{r_a}^{r_b}\frac{\mathrm{d}r}{r} = -\frac{\lambda}{2\pi\epsilon_o}\ln\frac{r_b}{r_a}, \tag{1.9}$$

Equipotential Surfaces

Moving a charge on a surface where no work is being done by the electric field describes a charge moving on an equipotential surface which is defined as a surface where the potential is the same at every point. Since no work is being done on this surface it follows that the electric field has no components parallel to it. Thus, it is always the case that an equipotential surface is perpendicular to the field. Since every point has a unique value for the potential it follows that different equipotential surfaces can never intersect.

Finally, we note that Eq. (1.8) can be written as

$$V_b - V_a = \int_a^b \mathrm{d}V = -\int_a^b \mathbf{E} \cdot \mathrm{d}\mathbf{l}.$$

Assuming for simplicity that the displacement is only along the y axis, then the equation above can be written as

$$-\int_a^b \mathrm{d}V = \int_a^b E_y \mathrm{d}y.$$

Since the limits of integration are arbitrary points it implies that the integrands are equal, or simply put

$$E = -\frac{dV}{dy}. \tag{1.10}$$

We could have done the same thing for the x and z components, and since the potential V is a function of all the coordinates the equation above can be generalized to read

$$\mathbf{E} = -\nabla V, \tag{1.11}$$

with ∇ given by:

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Since potentials are usually easier to calculate and measure than electric fields, Eq. (1.11) provides a convenient way for finding the electric field.

1.3 Apparatus

Figure 1.4 shows the apparatus to be used in the first part of the lab.

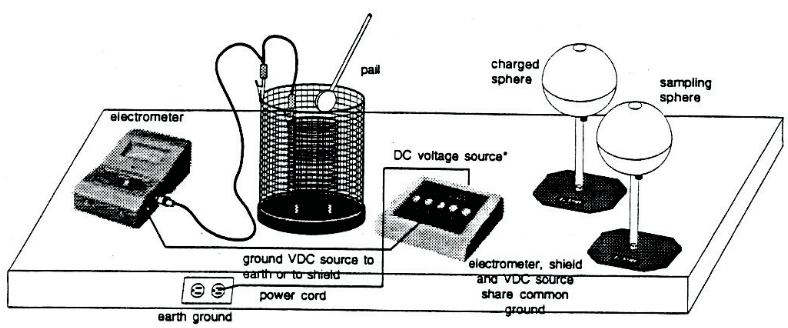


Figure 1.4: Apparatus(left to right): Electrometer, Ice Pail, Proof-plane and Charge Producers, Electrostatic Source, and Spheres.

1.3.1 The *Pasco* ES-9077 Electrostatics Voltage Source

This is a high voltage, low current power supply designed exclusively for experiments in electrostatics. It has outputs at 30 V, 1000 V, 2000 V, 3000 V, and a ground connection.

1.3.2 The *Pasco* ES-9078 Electrometer

This is a voltmeter used for direct measurements of voltage and indirect measurements of current and charge.

Never use the electrometer for measuring potentials more than 100 V.
Never touch the input leads until you have grounded yourself to an earth ground.

Operation

• Before turning on the electrometer, check that the meter reads zero. If not, turn the Mechanical Zero Adjust screw, located just below the meter face, until it does.
• Connect the test lead to the input connector of the electrometer.
• Connect the ground post of the electrometer to an earth ground by connecting to the COM on the Electrostatic Voltage Source, and plugging the AC adapter to a wall outlet.
• Push the Power button ON. One of the range switch LEDs will blink twice in quick succession.
• To zero the meter, press the ZERO button. You are now ready to use the electrometer to measure charge, current or voltage.
• Set the range switch to the desired voltage range. The range setting refers to the voltage input required to produce a full scale meter deflection. Between measurements, always press the Zero button to discard all charge from the electrometer.

1.3.3 The *Pasco* ES-9042A Faraday Ice Pail

This apparatus was originally designed by Michael Faraday. It works on the principle that any charge placed inside a conducting surface will induce an equal charge on the outside of the surface. The ice pail is a good method for sampling charges and charge distributions.

The Pasco Ice Pail consists of two wire mesh cylinders, one inside the other, mounted on a molded plastic bottom. The outer cylinder is called the shield, and when grounded helps eliminate stray charges and AC fields. The inner cylinder is the actual pail which is mounted on insulated rods. When a charged object is placed inside the pail, but without touching it, a charge of the same magnitude is induced on the outside of the pail. An electrometer connected between the pail and shield will detect a potential difference which is an indirect measurement of charge.

1.3.4 The *Pasco* ES-9075A Charge Producers and Proof Planes

Charge Producers

The charge producers consist of two wands, one with blue and one with white material attached to a conductive disk. If the blue and white surfaces are briskly rubbed together, the white surface acquires a positive charge, and the blue surface acquires a negative charge. Some guidelines are important to remember when using the charge producers.

• To get rid of excess charge and to neutralize the charge producers touch the conductive disk to ground.
• Avoid touching the neck during use. The oils from your hands will provide a path for charges to leak off.

The Proof Plane

The proof plane is an aluminum covered conductive disk attached to an insulated handle. It is used to sample the charge density on charged conductive surfaces. A Faraday Ice Pail can then be used to measure the charge density on the proof plane. By touching the proof plane to a surface, the proof plane will acquire a similar charge distribution to the section of the surface that is touched.

When a proof plane is touched to a conductive surface, the proof plane becomes part of the conductive surface. Therefore it's always best to touch the proof plane to the conductor in such a way as to minimize the distortion of the shape of the surface. For example, if the surface is a conducting sphere then the proof plane needs to be touched with its large surfaces tangent to the surface of the sphere.

1.3.5 The *Pasco* ES-9059 13-cm Spheres

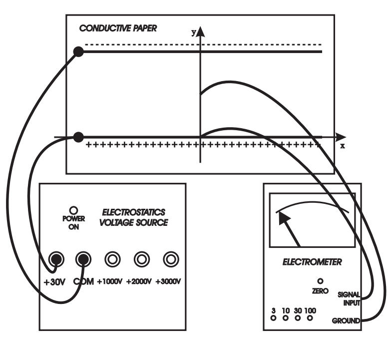
The conductive spheres are used to store electrical charge. The spheres are composed of plastic resin mold plated with a copper base, outer plating of non-sulphur brite nickel, with final plating of chrome. The spheres are mounted on insulating rods, attached to a support base. Each sphere has a thumb nut on the lower half that can be used for attaching a ground cable or a lead from a power supply. The spheres and insulating rods should be kept free of dirt, grease, and fingerprints to minimize leakage of charge from the spheres.

1.3.6 The *Pasco* High Resistance Paper and Conductive Ink

This apparatus will be used in the second part of the lab to measure voltage at different points in space due to long lines of charge. The conductive ink painted on a high resistance paper will be connected to a power source. This will create a potential distribution across the paper which will be measured by an electrometer.

The set up (see figure 1.5) of this apparatus in the experiment to follow will consist of two long parallel lines drawn by conductive ink on the high resistance paper. The lines are separated by some distance d which is relatively smaller than their length L. Now we can connect both lines to a static power source; one to the positive end, the other to the negative end. This will cause the lines to be charged with opposite polarities creating a potential difference across them. If these lines are drawn uniformly and the measurements are done close to the center of the lines (we want to stay close to the center, since near the edges the field deviates from Eq. (1.9)) then using Eq. (1.9) we can find the potential between these two lines on the plane of the paper.

Figure 1.5: High resistance paper and conductive ink.



Assume we want to know the potential at a point along the y axis (see figure 1.5) between the two lines of charge. Using Eq. (1.9) we can sum the potential due to the positive line and the negative line respectively to get

$$V(y) = \frac{\lambda}{2\pi\epsilon_o}(\ln\frac{r_o}{y} - \ln\frac{d-r_o}{d-y}).$$

The radius r_o is a reference point with respect to which we measure the potential at a point on the y axis located above the positive line of charge. Choosing our reference point r_o to be $\frac{d}{2}$ the equation above reduces to the following:

$$V(y) = \frac{\lambda}{2\pi\epsilon_o}\ln\frac{d-y}{y}, \tag{1.12}$$

which gives the potential between the two lines at a point in the same plane of the lines. Note that for our experiment Eq. (1.12) is not valid at the points $y = 0$, or $y = d$ since this equation was derived for infinitely long thin lines of charge which have no thickness. Such conditions are hard if not impossible to obtain in lab, meaning our lines of charge will be finite both in length and in thickness.

1.W Electrostatics Worksheet

Name: ____________________ **Day/Time** ____________
Partner's Name: ________________________

1.W.1 In-Lab Procedure

All measurements with the electrometer should include units and uncertainties in the space provided. Whenever you are not using the power supply you should turn it off.

Faraday Ice Pail and Charge Production

The purpose of this experiment is to investigate the relation between the charge induced on the ice pail by a charged object placed in the ice pail, and the charge of the object.
Before beginning the experiment the pail and electrometer must be momentarily grounded. To do this use the black wire with two alligator clips toward one end; one positive (red) the other negative (black). Connect the electrometer's negative clip to the shield, and the positive clip to the ice pail as shown in figure 1.4. Connect the other end (called a BNC) to the electrometer SIGNAL INPUT. Also connect the earth ground of the electrometer to the COM input of the electrostatic source, and make sure the source is connected to a wall outlet. Press the *Zero* button of the electrometer. This should remove all stray charges from the pail and the electrometer should read zero. You should also ground yourself to eliminate stray charges from your body by touching the ice pail when the *Zero* button is pressed.

A: Charging by Induction vs. Charging by Contact
Set the voltage range of the electrometer to 100V. Start with this scale for every new measurement you perform, so that you don't destroy the instrument with a high voltage. After you take an initial reading, adjust the electrometer incrementally to the finest scale that will accomodate your measurements. This will assure that you get the maximum precision. Do observe that you may have to readjust the electrometer scale several times in today's experiment.

Remove any stray charges on the charge producers by touching the necks and handles to the shield of the ice pail while the *Zero* button is pressed. Avoid touching the necks of the charge producers with your hands since oils on your hands will cause charge to leak in future use.
Rub the white and blue surfaces together to separate charges. Do this for about ten seconds. After this, place one of the charge producers far away so that it does not come in contact with any of the ice pail surfaces.
While touching the grounded shield insert the other charge producer into the ice pail all the way to the lower half of the pail, but without letting it touch the pail. Record the electrometer reading below.

$$V_{1A} = \underline{\hspace{4cm}}$$

Remove the object and again record the electrometer reading below (make sure that the handle never touches the pail when removing the object).

$$V_{2A} = \underline{\hspace{4cm}}$$

Now, press the *Zero* button to remove any residual charge. Insert the object again, but this time let it touch the ice pail. Remove the object and record the voltage reading.

$$V_{3A} = \underline{\hspace{4cm}}$$

Press the *Zero* button of the electrometer to remove all charges and insert the object again into the ice pail (again without touching the pail) and record the voltage reading.

$$V_{4A} = \underline{\hspace{4cm}}$$

B: Conservation of charge

Starting with the uncharged producers, rub the blue and white producers together. Keep them in your hands, without letting them touch each other or the ice pail. Insert the charge producers one at a time into the pail and record the voltage readings.

$V_{1B} =$__________________________

$V_{2B} =$__________________________

Remove all charge from the charge producers by grounding them (don't forget to ground the necks of the wands as well). Insert both charge producers into the pail and record the voltage reading.

$V_{3B} =$__________________________

Rub them inside the pail without touching the sides of the ice pail. Record the voltage reading.

$V_{4B} =$__________________________

Remove one of the charge producers and record the voltage reading.

$V_{5B} =$__________________________

Remove the charge producer from the pail and insert the other into the ice pail. Record the voltage reading.

$V_{6B} =$__________________________

Charge Distribution

The purpose of this procedure is to investigate the way charge is distributed over a surface, by measuring variations of charge density.
Make sure the ice pail is properly grounded with the shield connected to earth ground of the electrometer, and the electrometer is connected to the COM of the voltage source. Make sure the voltage source is connected to a wall outlet.
Place the two aluminum spheres about 50 cm apart. Connect one of the spheres to the 1000V DC outlet of the electrostatic voltage source. This sphere will be used as a charging body.
Momentarily ground the other sphere(NOT THE ONE CONNECTED TO THE VOLTAGE SOURCE) by touching the sphere with the shield of the pail.
Using the proof plane sample the uncharged sphere by touching it in few places (on the side close to the other sphere, on the side far from the other sphere, and midway between these two points on the sphere), and then placing the proof plane in the ice pail to read the voltage. Note that when touching the sphere with the proof plane it is best to minimize any distortion of the surface area of the sphere by touching the proof plane flat against the surface. You must also remember that between samplings of the uncharged sphere the proof plane must not be grounded. Record the voltage readings.

	close	**midway**	**far**
$V_{1C} =$	____________	____________	____________

Bring the 1000V DC sphere close to the uncharged sphere until they are approximately 1 cm apart. Turn the voltage source on and sample the sphere at the same points as before. Record the voltage readings.

$V_{2C} =$	____________	____________	____________

Remove the 1000V DC sphere until it is at least 50 cm away from the sampling sphere. Again repeat the sampling at the same points and record their values.

$V_{3C} =$__________ __________ __________

Electric Field and Electric Potential

In this part of the experiment we will use the high resistance paper and the conductive ink to create a charge distribution due to two lines of charge with opposite polarities separated by some distance. The purpose of this experiment is to see how the electric potential and electric field vary with this charge distribution.
Secure the high resistance paper (to be supplied by your TA) to the cork board using thumb tacks. Note that the grid marked on the high resistance paper is 1 cm $\times$ 1 cm. Measure the length of one of the lines L, and the separation distance between the two lines d and record them below.

$L =$________ $d =$________

Using the two cables which have at one end a connection to the power supply, and at the other end holes, connect the COM of the electrostatic source to one end of one line drawn by the conductive ink. Make this connection by securing the cable to the wire with a thumb tack. Make sure that the connection is made at the ends of the lines (not center), and that the wires make contact with the conductive ink. Connect the 30V output of the power supply to the other line at the end by repeating the procedure above.

Using the red and black probe wires connected to the SIGNAL INPUT, and

V	y

Table 1.1: Voltage and distance measurements.

ground connections of the electrometer respectively (in this procedure the electrometer should not be grounded), place the black probe in the center between the two lines. This will be the voltage reference point for all measurements. Use the second probe to measure the voltage with respect to the reference point starting on the y axis at 0.5 cm above the positive line of charge going up in the positive y direction. Remember that the $y = 0$ point on the y axis is determined by where the positive line of charge is located, while the positive y direction is from the positive to the negative line of charge (see figure 1.5). When taking voltage measurements it is important that the probes be placed perpendicular to the page and not slanted, as to minimize the distortion of the charge distribution across the paper. Make your measurements in 0.5 cm increments and record the value of the voltage in table 1.1 for each distance. Your last measurement should be made 0.5 cm below the top line which is the negative line of charge (see figure 1.5).

Now do the following:
a) Plot V vs. $\ln\left(\frac{d-y}{y}\right)$ in Kaleidagraph with error bars and units. All partners must have their own plots. Show your work in finding $\triangle\left(\ln\left(\frac{d-y}{y}\right)\right)$ here:

Slope	Intercept

Table 1.2: Slope and Intercept

b) Record the slope and intercept in table 1.2 with units and uncertainties.

Finally place the probe (not the one at the reference point) midway between the two lines. Move the probe 1 cm to the right and 1 cm to the left along the x axis with respect to this point. Do you see any appreciable change in voltage?
What can you say about this line on which the probe moved?

1.4.2 Pre-Classroom Check List

This check list is intended to be a guide for you to prepare yourself for the classroom work. You cannot come back to lab during this hour, collaborate with colleagues, nor hand in the worksheet late. Make sure you have completed everything.

Pre-classroom Check List

- ◯ Voltages V_{1A} through V_{3C} have units and uncertainties.
- ◯ Table 1.1 completed with units and uncertainties.
- ◯ Table 1.2 completed with units and uncertainties.
- ◯ Each student should have her/his own plot and worksheet.

1.4.3 In-Classroom Calculations & Analysis

Faraday Ice Pail and Charge Production

A: Charging by Induction vs. Charging by Contact
Using your data only from this part of the experiment answer the following

questions:

Explain why was there a potential difference between the pail and the shield only while the charged object was inside (where the object did not touch the pail).

Explain why after touching the pail with the charged object there appeared a permanent potential difference between the pail and the shield.

Explain why after zeroing the electrometer there was again a potential difference while the object was inside.

B: Conservation of charge

Using your data only from this part of the experiment answer the following questions:

After placing the charged objects one at a time into the pail what can you conclude about the magnitude of the charges, and the relation between their polarity?

Was charge conserved in this experiment? Explain your answer.

Charge Distribution

Using your data only from this part of the experiment answer the following questions:

What produced the charge distribution on the sphere that you sampled?

What can you conclude about the charge distribution on a metal sphere? Refer to both situations; first when the sampling sphere was close to the charged sphere, and second when the sampling sphere was far from the charged sphere.

Electric Field and Electric Potential

Using your data only from this part of the experiment complete the following:

Using the slope a of your graph (given in table 1.2) and Eq. (1.12) calculate the charge per unit length λ.

Now calculate the total charge Q that each line of conductive ink carried.

Use Eq. (1.10) together with Eq. (1.12) to derive the theoretical expression for the electric field E_{theo} between the two lines of charge (no uncertainty).

Finally from table 1.1 use the fourth and sixth data points $y_6 = \frac{d}{2} + 0.5$ cm and $y_4 = \frac{d}{2} - 0.5$ cm, to calculate an experimental value for the electric field at the midpoint between the two lines:

$$E_{exp} = -\frac{\partial V}{\partial y} \approx -\frac{V_6 - V_4}{y_6 - y_4} =$$

By using the theoretical expression you derived for the electric field in the third question of this section, obtain the value of the electric field at the point $\frac{d}{2}$, midway between the two lines of charge (no uncertainty). Compare this value with the quantity obtained in the previous question.

End Worksheet