Electromagnetism and Optics

The Lab Manual for PHY 103N
Engineering Physics II Laboratory

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Contents

Preface

Welcome to Physics 103N, Engineering Physics II Lab. This class is the continuation of Physics 103M, and is a corequisite to Physics 303L, Engineering Physics II Lecture. Although this class is a corequisite to Physics 303L, the topics we discuss here are not necessarily exactly those discussed in lecture. There are several reasons for this: the first is that timing the labs with the lectures is impossible; second, you don't always need a detailed theoretical description of phenomena to measure and characterize their properties. It is this empirical approach that we want to emphasize here. Third, there are important physical phenomena that are not covered in detail in the lecture, because of a lack of time. We will examine some of these in this course. So, don't expect a mere repeat of the lectures here.

There are two essential reasons for this course. First, it should give you some general background knowledge of how experimental work is actually done. You will learn how to use equipment such as multimeters, frequency generators, and oscilloscopes among others. Further, you will see how to measure various properties of electronic circuits and optical systems. These are all very practical skills. Secondly, it should help you see that all the conjectures and calculations that you learn about in lecture do describe events in the real world. You will quantitatively verify some of the formulas derived in the lecture to check the professor and make sure you haven't been lied to. If not, then you will probably believe what else is said in lecture, whereas if you've been told lies, that makes everything else the professor expounds is liable to suspicion. So be on the lookout for discrepancies!

Most of the equipment you need will be provided in lab. You should bring a pen and pencil (sketches should always be done in pencil) and paper, a scientific calculator (*i.e.*, one with logs and trigonometric functions, not necessarily a graphing calculator), and this manual to each lab meeting.

You might want to keep a notebook instead of writing observations and calculations on loose paper. A notebook will help you to keep organized so that you don't lose your notes or confuse your data. Your reports will be turned in on the worksheets printed in this manual. So if you do use a notebook, make sure it has perforated sheets, as you will turn in any extra sheets with your worksheet. In any case, *avoid* the hardbound laboratory notebooks (the ones with the carbon paper), since they are unnecessarily expensive (>$10). We expect that you have the textbook assigned to the 303L lecture course available; the reference is

> R. A. Serway and J. W. Jewitt, *Physics for Scientists & Engineers*, 6th edition (updated), Thomson Publishing, Belmont, CA (2004).

An additional reference, that we'll refer to repeatedly in our discussion of error analysis in Chapter 0, but is by no means required reading, is

> P. R. Bevington and D.K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, 2nd. edition, McGraw-Hill, Inc., New York (1992).

On a final note, we add that this is a new lab manual, and as such, is just now meeting the tests and demands of students. Some typos, ambiguities, or other inadequacies are bound to have slipped our grasp. Please bring any errors or confusing parts of the manual to the attention of your instructor. Student input is invaluable to the production of a document that students depend on for learning. As an alternative, feel free to E-mail your comments and suggestions to 103n@physics.utexas.edu.

Chapter 0

Introduction

This introductory chapter describes some general information you need to know about how to perform laboratory procedures, how to write a lab worksheet and how to analyze data correctly. Please read these through carefully; your instructor will cover these items quickly on the first day of lab, and you should already be familiar with these ideas before you come to class.

0.1 General Lab Procedures

Physics 103N is a three hour lab designed to be self-contained. This means that your only homework is to prepare for the next lab. You *must* do this preparation. The type of laboratory report you will submit at the end of the class period is a worksheet lab. You will fill out a worksheet as you proceed through the experiment. You will spend two hours in lab collecting data and plotting graphs. At the end of the second hour, you will move to a classroom and complete your calculations and discussions for that day's lab and turn in the finished worksheet to your instructor. If you are unprepared and unable to complete the worksheet, you must hand it in by the end of the third hour anyway. We designed this format to help you focus on the physical meaning of the experiments and avoid sinking into a morass of meaningless calculations.

To help you avoid many hours of tedious number crunching, we have placed computers in the lab. You should use these to do most, if not all, of your graphing and major numerical manipulations. However, the computer cannot think for you; you have to assess whether the computer is handing you garbage or if the results are reliable. Here are some things to keep in mind about the computer

- It cannot keep track of units.
- It must be taught how to deal with uncertainties.
- It will not automatically place error bars on your graphs.

You must complete the worksheet by the end of class; therefore, your instructor may deem a quiz unnecessary, since you must be prepared to complete the worksheet on time. The possibility of a quiz exists, however, so know the rudiments of the procedure of the day. To prepare, you should read the manual over very carefully before you come to class. This means you should understand the lab goals and important equations.

0.2 Error Estimation and Propagation

Error is everywhere, and you must not only acknowledge it, but understand it and control it. Webster defines "error" as the "difference between an observed or calculated value and the true value." The problem with this definition is that we usually have no idea what the "true" answer should be. Previous experiments or theoretical calculations may give us a clue, but somehow we must extract an estimate of the true value from our data. In addition, we should also determine to what extent we should take our answer seriously. This last point embodies what we call error estimation.

An example will show why this process is so important. Suppose you measure the acceleration of a free-falling body and the answer you obtain is $10.5\ \mathrm{m/s^2}$. Have you contradicted the accepted value of $9.7990 \pm 0.0014\ \mathrm{m/s^2}$? The answer depends on the size of the error in your answer. If your result was $10.5 \pm 1.0\ \mathrm{m/s^2}$ then your answer is no, because the accepted value lies within your error. If, on the other hand, you had performed a fairly precise measurement and obtained $10.5 \pm 0.1\ \mathrm{m/s^2}$, then your answer would have to be yes; you should then start looking for what went wrong in the experiment itself, since many, many other measurements conducted over the last 300 years have established the accepted value $9.7990 \pm 0.0014\ \mathrm{m/s^2}$. So, the amount of effort and trust you put into an answer depends critically on ascertaining the correct overall uncertainty.

Rarely is what you measure directly comparable to other experimental results or theoretical calculations; typically, you must process your data through various formulas to extract a parameter you can compare with others. Therefore, it is critically important to accurately *propagate* the uncertainties in the original data through the calculations and arrive at a reasonable uncertainty for the final value. This is the process of error propagation. These are the issues that we now discuss.

0.2.1 Definition of Uncertainty

The first step involved in error estimation is to identify the possible types of errors that can occur in your experiment. There are three basic types you need to be aware of: illegitimate errors, systematic errors, and random errors. Illegitimate errors are faults in experimental procedures or calculational blunders. We will make every effort to avoid making these kinds of errors, but if we do blunder, we can easily find and correct them; *we will assume that we have eliminated all illegitimate errors from our experiments.* This is a formidable assumption since the time to perform our experiments is limited; however, the procedures are not all that complicated; so this assumption should not be a bad one. Under this assumption therefore, we cannot use illegitimate errors as reasonable explanations for any discrepancies that ultimately occur in our analysis. To discuss the other types of errors, we must more carefully distinguish between accuracy and precision.

Accuracy represents how close a measurement is to the true value. *Precision* indicates how well the results of an experiment have been determined, independently of how well the results agree with the true value. This tells us about the self-consistency of a measurement. When judging the results of an experiment, we must consider both the accuracy and the precision. In general, when we quote the uncertainty of an experimental result, we are referring to the *precision* with which the result has been determined.

Systematic errors are errors that make our results different from the true value in a reproducible way. They are usually due to the faulty calibration of equipment or some unknown bias on the part of the experimenter. They can be subtle and hard to quantify. Knowledge of the apparatus and the experimental procedure is the central manner of minimizing the impact of systematic errors in our results. Such errors affect the *accuracy* of our results, since they contribute the same amount of discrepancy each time we perform the experiment. *Random errors*, on the other hand, constitute the major source of imprecision in an experiment. These are the random fluctuations in measurements from experiment to experiment, primarily due to the finite resolution of our apparatus. To control random errors, we must perform the experiment many times and use a statistical analysis to extract our results. A given accuracy implies at least an equivalent precision; thus, accuracy depends on these fluctuations too.

To clarify the difference between these two types of errors, consider the simple experiment of determining the average speed of a rolling ball as it passes by a meter stick, by using a handheld timer. A systematic error involved in this experiment could be due to the calibration of our instruments. For instance, assume that the intervals on the meter stick are 1% larger than they should. Then, every time we record a distance of 50 cm the actual distance travelled by the ball is 50.5 cm. The distinct feature of systematic errors, such as this, is their repeatability. No matter how many times we perform the same measurement, if we are using the same instrument, we will always make the same "mistake." Random errors don't share this property. In the rolling ball experiment a random error is due to the timer operator. No matter how hard he or she tries, a human operator cannot be entirely consistent on when to press the start/stop button on the timer. Sometimes, he or she will start the timer just a little before the ball passes by the predetermined mark on the ruler, some times a little after. The same thing will happen when it is time to stop the timer. The end result is that, if we perform many attempts at the experiment (and assuming that the ball is always launched at the same speed,) the times we will get will vary randomly around the correct value. A random error is also introduced by the finite resolution of the devices we are using. Suppose that we have replaced the timer's human operator with a "perfect" photogate setup. If the smallest time increment on our timer is 0.01 s and we measure a time of 50.23 s then we only know that the actual time is between 50.225 s and 50.235 s. Every value in between these two will be rounded and displayed on our instrument as 50.23 s. If we perform the measurement many times and we always get the same reading of 50.23 s, the reasonable thing to assume is that, in every attempt, the actual time was different, but within the above-mentioned limits.

We also distinguish between what we call absolute and relative uncertainty. *Absolute uncertainty* is the uncertainty in a quantity expressed in the same units as the quantity. For example, if we write

$$g = 9.7990 \pm 0.0014 \text{ m/s}^2,$$

then the absolute uncertainty in the measurement of g is $\Delta g = 0.0014$ m/s^2. Note that we always consider the absolute uncertainty to be a *positive* number. The *relative uncertainty* is the uncertainty expressed as a fraction or percentage of the quantity. The relative uncertainty of g in our example is

$$\frac{\Delta g}{g} = \frac{0.0014 \text{ m/s}^2}{9.7990 \text{ m/s}^2} = 0.00014 = 0.014\%.$$

0.2.2 Estimating Parameters and Their Uncertainties

Now that we know what constitutes error, we should describe how to estimate it given a data set. Let's consider the most common situation in which we need to extract an estimate for the error; this occurs when we've made several measurements of the same quantity and we want to extract an average value and a corresponding uncertainty. For concreteness, suppose we have a set of N data points x_i with uncertainty Δx_i which we do not assume is the same for each of the measurements. (We need this generality because the uncertainty can vary independently of the quantity being measured; for example, if you measure a current value on two different scales on a meter, then the uncertainties of the two measurements are different.) Then you can show (*c.f.* Bevington and Robinson, see the Preface for the reference) that, assuming a Gaussian error distribution, the **most probable value** is the mean, or average, value, $\overline{x}$:

$$\boxed{\overline{x} = \frac{\sum_{i=1}^{N} \frac{x_i}{(\Delta x_i)^2}}{\sum_{i=1}^{N} \frac{1}{(\Delta x_i)^2}}.} \tag{0.1}$$

This formula may look complicated, but all it does is give prominence to those measurements with the smallest uncertainty, a very reasonable thing to do. Notice that, if all the values have the same uncertainty, $\Delta x_i = \Delta x$ for $i = 1$ to N, this formula reduces to the usual $\overline{x} = (1/N)\sum_{i=1}^{N} x_i$. The corresponding uncertainty in the average value is the **standard deviation**, σ, and comes from the relation

$$\boxed{\frac{1}{\sigma^2} = \sum_{i=1}^{N} \frac{1}{(\Delta x_i)^2}.} \tag{0.2}$$

We can see from this formula that the uncertainty of the combination is always less than the smallest uncertainty of the component measurements. We should certainly hope so! After all, the more measurements we do, the better we should know what the answer is. In the case that all the uncertainties are equal to Δx, this expression reduces to $\sigma = \Delta x/\sqrt{N}$, showing that the uncertainty decreases at the relatively slow rate of one over the square root of the number of measurements.

Suppose you have three measurements of the resistance of a resistor that have come from three different techniques. This data appears in Table 0.1. The nominal value is the value printed on the resistor; the multimeter measurement comes from an ohmmeter measurement; and the current and voltage measurements come from a detailed analysis using Ohm's law. Each technique has its own associated uncertainty. We want to combine these separate measurements into a single estimate of the resistance. Since the errors are distributed symmetrically (*i.e.*, another measurement's value is equally likely to fall on either side of the current measurement's value), we can reliably use our formulas based on a Gaussian distribution, *i.e.*, the ones given above. We find the resistance to be

$$2.5866 \pm 0.0027\ \mathrm{k}\Omega.$$

This answer reflects several general features of these formulas mentioned above. First, it is closest to the measurement with the smallest uncertainty. Second, the uncertainty of the combined measurements is less than the smallest measured uncertainty, but not by much. This reflects the fact that the other measurements' uncertainties were much larger than the last. This is good intuition that you should incorporate into your thinking patterns; it will help you identify the important sources of error in your experiments.

Experimental Technique	Value
Nominal Value	2.70 ± 0.14 kΩ
Multimeter	2.69 ± 0.01 kΩ
Current and Voltage Measurements	2.5785 ± 0.0028 kΩ

Table 0.1: Resistance values for a single resistor from different measurements.

For more sophisticated experiments, we don't necessarily measure the same quantity over and over. We might change one parameter and measure another, to investigate the effects of one on the other. When we do this, we are looking for correlations between the different parameters. One of the most effective ways to spot correlations is to graph the parameters and look for some functional relationship. The easiest and most reliable functional relationship to recognize and quantify is a linear one, *i.e.*, when you plot the parameters, the data points fall along a line. In fact, this type of correlation is so important that we sometimes alter the parameters that we plot to force the graphed data into a line, as in a log-log plot or a semi-log plot. Here, we don't just plot the data, but certain functions of the data that are linearly related. This can't always be done, but for our labs it can; so, we will focus solely on analyzing linear correlations. In this case, the correlation between the two quantities is described in terms of two numbers: the slope of the line and its y-intercept. For most purposes, the slope is the more important of the two, but the intercept can also contain important physical information. Once the data suggests a linear relationship, we want to extract the slope and intercept of the "best fit" line.

There are many complicated definitions of "best fit" that one can use to extract a consistent slope and intercept from a set of data. The most often used method goes by the name of *least squares* fit. The reason for the popularity of this particular method has to do with its relative simplicity and statistical significance. It will provide the slope and intercept of the most probable line to fit a set of data, assuming a Gaussian distribution for the errors. Thus you can think of this as analogous to extracting the average quantity for a larger class of measurements.

We can derive the formulas for the linear least squares fit to a set of data with the following assumptions: first, that the uncertainty is symmetrically distributed about the data values, and second, the uncertainty in the dependent variable is more significant than the uncertainty in the independent variable. These assumptions are discussed at length in Bevington and Robinson, and we will simply take them for granted. With these assumptions in mind, we can proceed as follows: given N data points $(x_i, y_i \pm \Delta y_i), i = 1, \ldots, N$, we want to determine the parameters of the line $y = ax + b$ so that the square of the vertical distance between the y coordinates of the line and the data, weighted by the uncertainty Δy_i and summed over all the data points, is a minimum. The geometry of this construction is shown in Figure 0.1. That is, we want to minimize the function of 2 variables

$$e(a,b) = \sum_{i=1}^{N} \left(\frac{1}{\Delta y_i} (y_i - a x_i - b) \right)^2$$

To minimize this function, we take the partial derivatives with respect to a

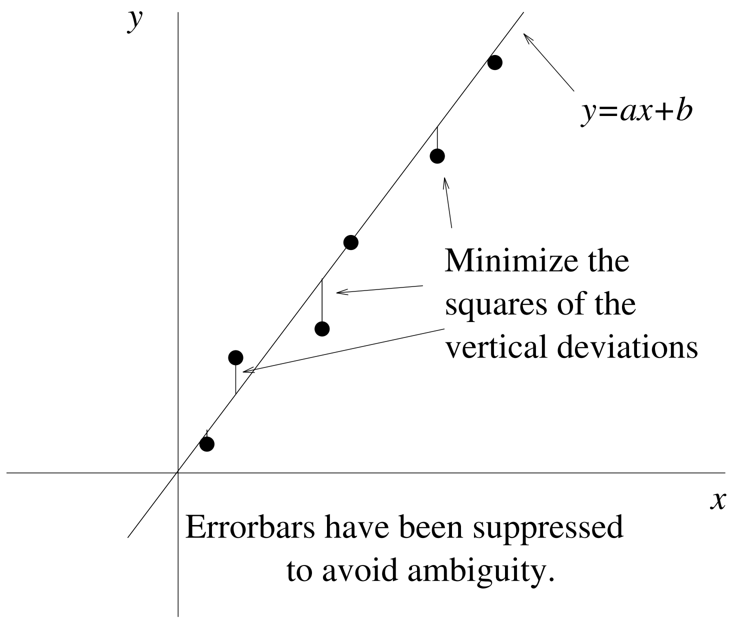


Figure 0.1: The least squares method determines the line that minimizes the square of the vertical distances between the line and the data.

and b and set them equal to zero. This yields the following system of linear equations for a and b:

$$\begin{aligned} a\sum_{i=1}^{N} \frac{x_i^2}{(\Delta y_i)^2} + b\sum_{i=1}^{N} \frac{x_i}{(\Delta y_i)^2} &= \sum_{i=1}^{N} \frac{x_i y_i}{(\Delta y_i)^2} \\ a\sum_{i=1}^{N} \frac{x_i}{(\Delta y_i)^2} + b\sum_{i=1}^{N} \frac{1}{(\Delta y_i)^2} &= \sum_{i=1}^{N} \frac{y_i}{(\Delta y_i)^2} \end{aligned}$$

Using your favorite technique, you can show that the solution of this linear system of equations is

$$a = \frac{1}{D}\left[\left(\sum_{i=1}^{N} \frac{1}{(\Delta y_i)^2} \right) \left(\sum_{i=1}^{N} \frac{x_i y_i}{(\Delta y_i)^2} \right) - \left(\sum_{i=1}^{N} \frac{x_i}{(\Delta y_i)^2} \right) \left(\sum_{i=1}^{N} \frac{y_i}{(\Delta y_i)^2} \right) \right],$$

$$b = \frac{1}{D}\left[\left(\sum_{i=1}^{N}\frac{x_i^2}{(\Delta y_i)^2}\right)\left(\sum_{i=1}^{N}\frac{y_i}{(\Delta y_i)^2}\right) - \left(\sum_{i=1}^{N}\frac{x_i}{(\Delta y_i)^2}\right)\left(\sum_{i=1}^{N}\frac{x_i y_i}{(\Delta y_i)^2}\right)\right], \tag{0.3}$$

where

$$D = \left(\sum_{i=1}^{N}\frac{1}{(\Delta y_i)^2}\right)\left(\sum_{i=1}^{N}\frac{x_i^2}{(\Delta y_i)^2}\right) - \left(\sum_{i=1}^{N}\frac{x_i}{(\Delta y_i)^2}\right)^2. \tag{0.4}$$

The corresponding uncertainties in the fit parameters are

$$\begin{aligned}(\Delta a)^2 &= \frac{1}{D}\sum_{i=1}^{N}\frac{1}{(\Delta y_i)^2}, \\ (\Delta b)^2 &= \frac{1}{D}\sum_{i=1}^{N}\frac{x_i^2}{(\Delta y_i)^2}.\end{aligned} \tag{0.5}$$

Let's take a look at these equations in action. Consider the data given in Table 0.2; this data came from current and voltage measurements across a resistor of unknown resistance. Ohm's law indicates that, for most resistors, the voltage is linearly related to the current, the proportionality constant being the resistance. We've plotted this data in Figure 0.2, which strongly suggests a linear relationship between current and voltage. So, it makes sense to apply our least squares equations directly to the current-voltage data. Considering the data again, we see that the errors appear symmetrically distributed around each point, and the independent variable's (the current's) uncertainty is much smaller than the corresponding uncertainty in the voltage data. So, this data satisfies both conditions for applying the least squares analysis.

We find (and you should verify with units!) the following values for the sums involved:

$$\begin{aligned}\sum_{i=1}^{10}\frac{x_i}{(\Delta y_i)^2} &= 1.5212\cdot 10^6 \\ \sum_{i=1}^{10}\frac{y_i}{(\Delta y_i)^2} &= 4.4578\cdot 10^6 \\ \sum_{i=1}^{10}\frac{x_i^2}{(\Delta y_i)^2} &= 7.0201\cdot 10^5 \\ \sum_{i=1}^{10}\frac{x_i y_i}{(\Delta y_i)^2} &= 2.0107\cdot 10^6\end{aligned}$$

Current (mA)	Voltage (V)
0.1936 ± 0.0001	0.719 ± 0.001
0.289 ± 0.001	0.813 ± 0.001
0.388 ± 0.001	1.093 ± 0.001
0.575 ± 0.001	1.620 ± 0.001
0.946 ± 0.001	2.66 ± 0.01
1.042 ± 0.001	2.93 ± 0.01
1.144 ± 0.001	3.22 ± 0.01
1.196 ± 0.001	3.37 ± 0.01
1.484 ± 0.001	4.17 ± 0.01
1.750 ± 0.001	4.93 ± 0.01

Table 0.2: Current and voltage data for computing the resistance of a resistor using Ohm's law.

$$\sum_{i=1}^{10} \frac{1}{(\Delta y_i)^2} = 4.0600 \cdot 10^6$$

from which we find $D = 5.3607 \cdot 10^{11}$ and then

$$a = 2.5785 \pm 0.0028 \text{ k}\Omega$$
$$b = 0.1319 \pm 0.0011 \text{ V}$$

The line with this slope and intercept is the line drawn in Figure 0.2. We see that this line is a very good representation of the data. Working through the units, the slope is in kΩ and the intercept is in V. This calculation is the source of the last entry in Table 0.1. The intercept has an interesting interpretation here. It is not zero, since the average value is bigger than the error. This means that if we put the current to zero, we would still measure a voltage across the resistor. This fact should make us very suspicious about any other conclusions we might make from this data, until we have an explanation for this apparent contradiction of Ohm's law.

0.2.3 Propagating and Reporting Uncertainties

At this point, you should have a clear idea of what uncertainty is and how to estimate it in some simple cases. Once you have your estimates of the

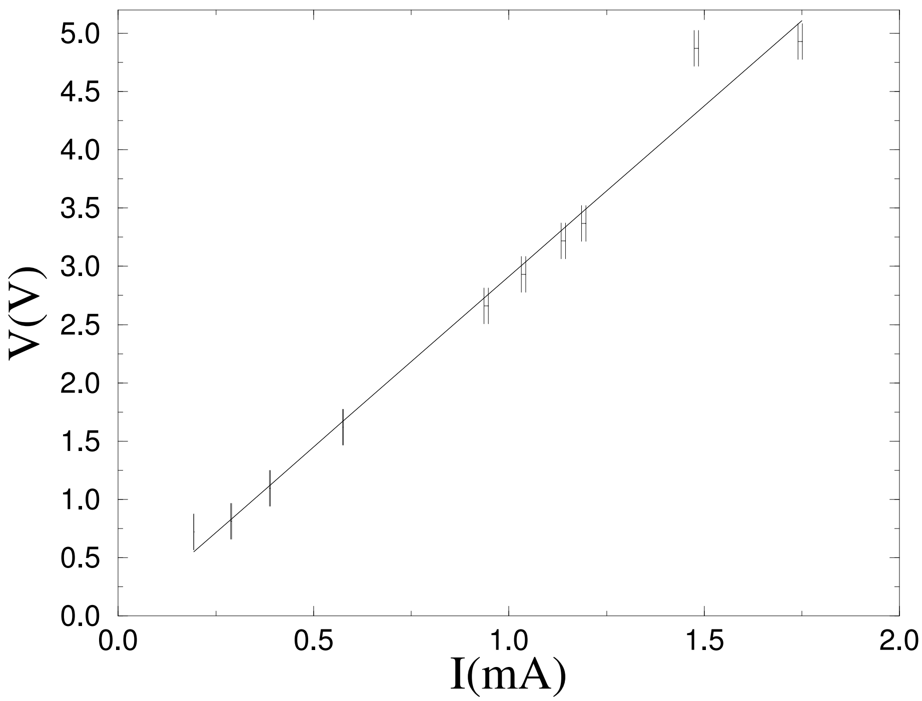


Figure 0.2: Plot of voltage versus current for the data in Table 0.2.

parameters of interest and their uncertainties, you will likely want to run them through some formulas to arrive at numbers you can compare to other people's measurements. This brings us to discuss the propagation of uncertainties through functions and formulas.

To keep things simple, we will make the assumption that the uncertainties in your parameters are symmetrically distributed about the average and that the parameters are independent of each other. That is, two measurements of different parameters are uncorrelated. This is not always true; for example, in an ideal gas at fixed pressure, the density and temperature fluctuations are linked by the equation of state. However, the added complications needed to account for these effects are typically intimately tied to the physical system you're studying, making a general treatment cumbersome. By ignoring correlations and assuming symmetry, we can reduce all the necessary error propagation down to some simple calculus.

Suppose we have a parameter with its uncertainty: $x \pm \Delta x$. The question we want to answer is "What is the uncertainty of some function, f, of this data?" Under our assumptions, the answer comes from the Taylor series expansion of f (*c.f.* Bevington and Robinson): $f(x+\Delta x) = f(x)+f'(x)\Delta x+O(\Delta x^2)$. From this we find the uncertainty Δf in the function value $f(x)$ is

$$\Delta f = \left|\frac{df}{dx}\Delta x\right|,$$

with the derivative evaluated at the point x. We can generalize this result to functions of several variables as follows: given the data $x \pm \Delta x, y \pm \Delta y, \ldots$, the function $f(x, y, \ldots)$ has the associated uncertainty

$$\Delta f = \left|\frac{\partial f}{\partial x}\Delta x\right| + \left|\frac{\partial f}{\partial y}\Delta y\right| + \ldots,$$

where all the derivatives are evaluated at the point $x, y, \ldots$. If we recall that we defined absolute uncertainties to be positive, we can write this as

$$\boxed{\Delta f = \left|\frac{\partial f}{\partial x}\right|\Delta x + \left|\frac{\partial f}{\partial y}\right|\Delta y + \ldots,} \tag{0.6}$$

From this relationship, we can derive all the familiar results of error propagation.

Example: Addition and Subtraction
Given: $f(x, y) = 3x + y - z + 5$
Find: Δf

$$\begin{aligned}\Delta f &= \left|\frac{\partial f}{\partial x}\right|\Delta x + \left|\frac{\partial f}{\partial y}\right|\Delta y + \left|\frac{\partial f}{\partial z}\right|\Delta z \\ &= |3|\,\Delta x + |1|\,\Delta y + |-1|\,\Delta z \\ &= 3\Delta x + \Delta y + \Delta z\end{aligned}$$

Example: Multiplication and Division
Given: $f(x, y) = x^2y/(5z)$
Find: Δf

$$
\begin{aligned}
\Delta f &= \left|\frac{\partial f}{\partial x}\right| \Delta x + \left|\frac{\partial f}{\partial y}\right| \Delta y + \left|\frac{\partial f}{\partial z}\right| \Delta z \\
&= |2xy/(5z)|\, \Delta x + \left|x^2/(5z)\right| \Delta y + \left|-x^2y/(5z^2)\right| \Delta z
\end{aligned}
$$

Example: Ohm's Law
Given: $V = IR$
Find: ΔR

$$
\begin{aligned}
\Delta R &= \left|\frac{\partial R}{\partial V}\right| \Delta V + \left|\frac{\partial R}{\partial I}\right| \Delta I \\
&= \left|\frac{1}{I}\right| \Delta(V) + \left|\frac{-V}{I^2}\right| \Delta I
\end{aligned}
$$

Now that we have a clear idea of what constitutes the uncertainty of a measurement, how to estimate it, and how to propagate it, we should talk about the proper way to report the uncertainty of a measurement. This forms the subject of *significant figures*. Here is how you should determine the number of significant figures:

1. Calculate the uncertainty in the quantity.
2. Round off the uncertainty to one or two digits.
3. Express the uncertainty in the same units as the quantity measured.
4. Round off the quantity to the last decimal place of the uncertainty.
5. Always write down the final result of a calculation with the uncertainty and the units included.

Use the form

$$(2.34 \pm 0.23) \cdot 10^3 \text{ m, or } 2.34 \pm 0.23 \text{ km,}$$

not expressions such as

$$
\begin{aligned}
2.34 \cdot 10^3 \text{ m} \pm 0.23 \cdot 10^3 \text{ m}, \\
2.34 \text{ km} \pm 23 \cdot 10^1 \text{ m}, \\
2340 \text{ m} \pm 0.23 \cdot 10^3 \text{ m}.
\end{aligned}
$$

These are the rules you will use most often in reporting your results. They become rather cumbersome, though, when you begin to make very precise measurements. Consider, for example, the charge on the electron; the best measurement we have of this number is

$$(1.60217733 \pm 0.00000049) \cdot 10^{-19}\ \mathrm{C}.$$

This is very annoying; so, we've developed a shorthand for reporting these kinds of measurements. You simply quote the result to the known uncertainty and place the uncertainty of the last few digits in parentheses after the number and before the power of ten. In this notation, the electron's charge is

$$1.60217733(49) \cdot 10^{-19}\ \mathrm{C},$$

which is much easier to deal with. If you begin to make measurements of such precision that you need to employ this convention, feel free to do so.

Finally, in various experiments we quote what are called "accepted values" for various physical parameters. These are the scientific community's best estimates of these numbers. They have been experimentally verified and checked for consistency with other measurements. Most you will find are very precise, typically 6 or 7 decimal places. You will discover in trying to do your own labs that making such high precision measurements is not easy. They also let you know that there is still some uncertainty in these parameters; they are not *exact*; but you will probably not be able to help narrow that using the equipment and techniques we have, which means they are exact as far as we can tell. So, keep in mind as you attempt to verify these numbers, that other folks had to do these measurements too.

0.3 The Lab Worksheet

The lab worksheets are preformated to be well organized, concise, and complete. The worksheets are designed to maximize your efficiency. The format of the worksheets includes the following:

- In-Lab Procedure

- In-Lab Computer Work

- Pre-Classroom Checklist
- In-Classroom Calculations & Analysis
- In-Classroom Discussion and Conclusion

Each item is described in detail below. Some may occur more than once in any given worksheet.

0.3.1 In-Lab Procedure

The In-Lab Procedure section will guide you through experimental set-up, data collection, and necessary calculations. This section will always be the first section of the worksheet, but there may be a few of them in any one worksheet. Each time you set-up a new experiment in that day's lab, a new in-lab procedure section will guide you. Since this lab is electronics and optics, you will be connecting circuits and manipulating optical equipment. Figures are provided and numbered to show you how to make connections and placements for these set-ups. The figures will become your friends in setting up the experiments. **Do not ignore them**. You cannot set- up without the figures.

After setting up the experiment, you will follow the directions given to begin data collecting. Sometimes you will be obtaining one piece of data, other times twenty. Any data collecting will be specified and organized by a table or space. Blank spaces above answers are to be used by you to **show your work** in reaching the answer below the space. This is **very important** as indicated by the **boldface**. Boldface will appear when an important point is being made vital to your grade.

When you are recording your data, be that in a table or on a line, you should present your raw data neatly and completely including **units**, uncertainties, and significant digits. Any calculations used in recording the data should be shown in the space provided above the answer. Show all calculations *keeping numbers out of the calculations until the final step*. How to do the calculations will be explained in the In-Classroom Calculations & Analysis section.

When showing your work, it is crucial that you *propagate* all units and uncertainties through the algebra. This will convince the reader that you're not trying to hide anything and it will help you check your answers as you go. Do not ignore units for several steps of a computation and then just write down what seems to be the proper units; you can loose many factors of 10 by not doing this.

Before moving on to the next section of the worksheet, double check that you have completed everything required of you. Remember that this section is not only for data recording. Just because there is no space for an answer does not mean there was nothing important to be done. You must complete everything in each section before moving on to the next one. You and your lab partner can divide the work to be more efficient but make sure you do everything.

0.3.2 In-Lab Computer Work

After collecting and recording data you will usually make a graph. You are not plotting the data to make more work for yourselves. The graphs will give you a vital piece of information that you will use in calculations, analysis, discussion or conclusion. Graphing is usually the easiest and most accurate way to get the information. You will be using the computers in lab, and that means that you must complete all the graphs in the two hour lab time. You will need them for the classroom period.

KaleidaGraph is the plotting program you will be using for all your graphs. You will spend part of the first two lab sections learning and practicing your KaleidaGraph skills. You need to become proficient in KaleidaGraph to complete the labs since you will include all graphs with your worksheet before leaving lab. You should title your graphs appropriately, include all units, and label axes. Despite the fact that the computer will be doing most of the work in graphing your data, you need to understand what the work entails.

In many labs you will have to graph sets of 5 to 15 data points and make linear fits to the data. If you graph by hand, as you will in § 0.W2, you should use graph paper of at least 4 boxes per inch and each graph should be large and clear. A full page graph of 10 points is not unreasonable. The scales on the axes should be appropriate for the data ranges, so that the data covers most of the graph. Having bunched up data points leads to difficulty in reading the graph and loss of precision in fitting lines and calculating slopes and intercepts. Do not draw your axes across a full page and choose your scale in such a way that the data points occupy only a few cm^2! Also, take care to distinguish dependent and independent variables when graphing; the quantity which is the function of the other in the experiment is conventionally plotted along the vertical axis. If you are asked to plot y vs. x, for example, you should interpret y as the dependent variable and plot it on the vertical axis. Always label the axes with their appropriate physical parameters and include the correct units in the label.

When fitting data by hand, as opposed to the least squares method discussed earlier, be sure not to obscure any data points and do not "connect the dots." Doing so has no physical basis and yields no insight into the physics at hand. Using a straight edge, "eyeball" the line that best fits the data. This line will yield the best values for the slope and intercept of your fit. Furthermore, you should draw the steepest and shallowest lines that are consistent with both the trend of your data and with the error bars. These lines will yield the uncertainty in your fit parameters, through the formulae

$$\begin{aligned}\Delta a &= \frac{|a_{\text{steep}} - a_{\text{shallow}}|}{2} \\ \Delta b &= \frac{|b_{\text{steep}} - b_{\text{shallow}}|}{2}.\end{aligned}$$

Remember that these lines represent the trend present in your data and might not pass through any data points. When calculating slopes for these lines, you want to choose two points on the lines that are not data points. The trend of the line is far more important than the slope between any two data points.

Although these instructions tell how to estimate the best fit line by hand, the computer is only doing a more sophisticated and reproducible version of this; thus, to fully employ the computer's power, you need to understand how to estimate these things independently of the computer. This will also help you check the results the computer gives you.

0.3.3 Pre-Classroom Checklist

The checklist is included for you to use as a check against what you have to take out of the lab period. You only have 2 hours in lab during which time you may do several procedures and graphs. Before leaving the laboratory, you will use this list to see that you have completed everything and have it with you to take to the classroom. Once you leave the lab, another class enters it so you can't go back to use the computers or redo a portion of the lab. The worksheet has been designed to be completed in parts. You will complete one procedure and do its computer work before moving on to the next procedure. In this manner, you will be able to do a complete analysis and discussion on at least some parts even if you don't finish the lab.

There are ◯ provided for you to physically check off. Do this. It will keep you organized when you get to the classroom. One hour is not a lot of time to finish all the calculations, discussions, and conclusion. Also make sure that each partner has her/his own data, graphs, tracings, etc.. Contact amongst you will be kept to a minimum.

0.3.4 In-Classroom Calculations & Analysis

At this point you have finished all In-Lab Procedures and In-Lab Computer Work. You have also checked all the circles in the **check list** to ensure that you have completed all the laboratory work. Once the two hour lab period is over, you will move to the classroom and finish the worksheet in **one hour**. If you complete all in-lab sections sooner than two hours you may begin the in-classroom sections.

The calculations you need to do are clearly stated with referenced equations, figures, and tables when appropriate. There is a line or space left for each answer. There is also room for you to **show your work**. You must show all the steps and reasoning behind your answers to get full credit. A sample calculation is done for you here as a future guide. Suppose the question in the in-classroom calculation & analysis section reads:

> Calculate the resistance through a circuit given the voltage, $V = 5.00 \pm 0.01\ V$, and current, $I = 20.0 \pm 0.2\ mA$, with uncertainties and units. **Show Work**.

$$
\begin{aligned}
V &= IR \\
R &= \frac{V}{I} \\
R &= \frac{5\,V}{20\,mA}
\end{aligned}
$$

$$
\begin{aligned}
R &= 250\,\Omega \\
\Delta R &= \left|\frac{1}{I}\right| \Delta(V) + \left|\frac{-V}{I^2}\right| \Delta I \\
\Delta R &= \left|\frac{1}{20\,mA}\right| 0.01\,V + \left|\frac{-5\,V}{400\,(mA)^2}\right| 0.2\,mA \\
\Delta R &= 3\,\Omega \\
R &= 250 \pm 3\,\Omega
\end{aligned}
$$

The above is what you should show and report. Note that numbers were not used until the last line of each calculation and the uncertainties were treated separately from the values. You will be performing this calculation later in lab.

The In-Classroom Calculation & Analysis sections can also ask questions that lead you to an understanding of the reasoning behind the lab and the physical principles that the lab confirms. They are rarely in yes/no format. Answer all the questions completely, stating your reasoning leading to the answer and showing any calculations or drawings necessary. At the end of this section you should understand all aspects of the lab well enough to write a concluding statement.

0.3.5 In-Classroom Discussion and Conclusion

In the final section you will bring all your analysis together to answer questions and make specific, concrete conclusions about the parameters and physics developed in the lab. This includes a clear statement of the results of the lab, *e.g.* parameters that you've measured including units and uncertainty, comments on the physical implications of these parameters, etc. You should indicate whether your results are consistent with previous efforts and discuss the internal consistency of your experiment. You should candidly address the uncertainties that arose in the lab and attempt to unambiguously and uniquely identify the key source of error. Within the paradigm that we discussed before, this cannot include things like "human error" or "errors in the calculations." You should have these illegitimate errors tightly under control. With all these ideas clearly laid out, you should then state whether you believe the experiment to be a success or not, justifying yourself by referring to your previous discussions.

Your instructor may also assign additional questions for you to ponder. You should incorporate the answers to these questions into your discussion.

They should fall naturally into your considerations, as you think about what might have gone wrong, or possible sources of discrepancy that were not in the original motivating theory. Typically, these questions have direct answers, but only after you have thought about the lab.

0.4 Using KaleidaGraph for Data Analysis

0.4.1 Introduction

What KaleidaGraph Does and Doesn't

The software package known as KaleidaGraph can be a useful tool for data analysis. Of course, it will only be useful if you learn how to tell it to do what you want it to do. This knowledge is best acquired by experience with using the software, and you will get plenty of that in the upcoming semester. That fact doesn't help you at the moment, though. Getting started is the difficult part. Hopefully, this section will help you with that.

This section is not meant to be a detailed guide to using KaleidaGraph. Instead, it hopes to demonstrate how to use those features of KaleidaGraph which will be most useful to you: plotting and fitting curves to data. To accomplish this, this section is designed to be a working example.

When you are using KaleidaGraph to analyze real data, you'll probably have to perform most of the tasks described below. In that spirit, we'll connect all the examples with...

...A Hypothetical Experiment.

Suppose an experiment has been performed that tests the well-known conjecture that the probability that a slice of toast buttered on one side will fall butter-side-up is inversely proportional to the value of the carpet on which it falls. This assertion can be represented by an equation:

$$U = a\frac{1}{V},$$

where U represents the probability of a piece of bread falling butter side up, a is a proportionality constant, and V is the price of the carpet in U.S. dollars. If you were to somehow measure the probabilities of the toast falling butter-side-up on several carpets, and plot these against the reciprocal of the price of the carpet, you'd expect to see a straight line, with a slope equal to the proportionality constant. The following examples will lead you through the process of plotting data taken in such an experiment and finding the slope of the best fit line of the plot.

Now for some details. On each run of the experiment, a helicopter dropped 100,000 slices of buttered toast on a large sample of carpet. Then, the U.S. Dept. of Parks and Wildlife flew in with their own helicopter to take an aerial photograph of the result. From the photograph, the fraction of butter-side-up slices was measured, and reported as the probability.* Only five runs were accomplished before funding ran out. The results of the experiment are

V($)	U
100 ± 1	0.43 ± 0.01
250 ± 1	0.15 ± 0.01
500 ± 2	0.09 ± 0.01
750 ± 2	0.06 ± 0.01
1000 ± 3	0.04 ± 0.01

0.4.2 Entering Data

Actually Entering Data

KaleidaGraph holds data in a **data window.** One of these should appear when you start; it's default name is **Data 1.** To enter data,

1. Activate the data window by clicking on it.
2. Position the cursor on the cell you want to enter data into.
3. Type in a piece of data.
4. Move to another cell using mouse, arrow keys, Tab or Return.

Enter all the Buttered Toast data, entering values for V in column "A", the uncertainties for V in column "B", the values for U in column "C" and the uncertainties for U in column "D".

*These results have met with some controversy. The Association of Premium Carpet Manufacturers has filed a lawsuit claiming that the results have been altered to make their products seem less desirable.

Note that KaleidaGraph plots data in a column vs. column fashion, so the x-coordinate and y-coordinate of a single piece of data should be placed in the same row, but different columns.

Renaming Columns of Data

This might not seem important at first, but KaleidaGraph labels the axes on plots with the name of the columns it used in the plot. The default names of columns are "A", "B", "C", etc. These appear in the **column title** row of the data window, along with a number. To change the name of the column,

1. Double-click on the column title you wish to change.

2. In the "Column Format:" dialog box which appears there will be a list of column titles. Highlight the title you wish to alter by clicking on it.

3. Type in the new title of the column (for example "A" becomes $V(\$)$).

4. When you are finished changing names, click on the button labeled **Done.**

Make sure to name each column, include uncertainties (for example "B" becomes $dV(\$)$). Don't forget the units!

The number of any column can be set to zero merely by clicking on the title cell of that column. The columns to the right then take the numbers 1, 2, 3.... The columns to the left become unnumbered.

0.4.3 Entering Formulas

Often, the raw data you enter is not immediately in the form you need for plotting. Never fear, KaleidaGraph is capable of performing mathematical operations on the numbers you have entered. One way of using this feature is to define a formula for KaleidaGraph. Formulas tell KaleidaGraph to put in one column the result of operations on data in other columns.

The syntax of formulas you define should be

$$\mathrm{c}x = \mathrm{f}(\mathrm{c}y, \mathrm{c}z, ...)$$

where x, y, and z are the numbers of the columns which contain the numbers you wish to operate on, and f(...) is the mathematical expression you wish KaleidaGraph to calculate.

For example, since you want to plot U vs. $1/V$, you need to make a column containing the reciprocal dollar values of the carpets. To actually enter and execute a formula,

1. From the **Windows** menu at the top of the screen, select the option *Formula Entry.*

2. In the "Formula Entry" window which appears, click on one of the buttons labeled **F1 - F8.**

3. Type in the formula in the space provided in the "Formula Entry" window (in your case, c4 = 1/c0).

4. Click the button marked Run in the "Formula Entry" window.

The button labeled **F1-F8** you choose corresponds to one of the function keys at the top of the keyboard. Pressing that key will bring up the "Formula Entry" window again, this time containing the formula you defined **thus saving important formulas for you**. Make sure to rename the column you just made.

Of course, the uncertainties in $1/V$ are different from the uncertainties in V. So you'll need to get KaleidaGraph to calculate an uncertainty column for you. Try to figure out the necessary formula yourself.

0.4.4 Plotting Data

Making a Scatter Plot

These two sections on plotting and fitting data require little motivation. But a few important notes will be made. First, **Never** connect-the-dots when you plot data. Fortunately, if you follow these steps, you'll never forget and accidentally do it.

1. Activate the window containing the data you want to plot by clicking on it.

2. From the **Gallery** menu at the top of the screen, select the **Linear** submenu, and from that select the **Scatter** option.

3. A dialog box will appear. In it, there will be columns of circles labeled "X" and "Y." Under "X" click on the circle in the row containing the title of the column which contains the x-coordinates of your data. A solid black circle should appear.

4. Do the same with your y-coordinates in the column labeled "Y."

5. Click on the button labeled **New Plot.**

Note the conspicuous lack of error bars.

Adding Error Bars

1. Activate the window containing your plot by clicking on it.

2. From the **Plot** menu at the top of the screen, select the option "Error bars..."

3. In the "Error Bar Variables" dialog box which appears, click on the square labeled "X Err."

4. Click and hold on one of the two rectangles labeled "% of values," and select the option "Data Column."

5. Select the column which contains the uncertainties in the x-coordinates of your plotted data.

6. Click on button labeled **OK.**

7. Now, follow the same procedure starting with the square labeled "Y Err."

8. Click on button labeled **Plot.**

0.4.5 Performing a Weighted Least-Squares Fit on Plotted Data

Weighted Fit

We come to the moment of truth. Both the slope and y-intercept of a linear plot are often important pieces of information to obtain. Of course, they are meaningless without uncertainties. Therefore, you should make sure you take into account the uncertainties in your points when calculating the fit.

1. Activate the window containing your plot by clicking on it.

2. From the **Curve Fit** Menu at the top of the screen, select the **General** submenu, and from that, select the option **fit1.**

3. In the "Curve Fit Selections:" dialog box which appears, click on the button labeled **Define...**

4. In the new dialog box that appears, click on the square labeled "Weight Data" so that an X appears in it.

5. Click on the button labeled **OK.**

6. In the "Curve Fit Selections:" dialog box, click on the square next to the column title which contains the error in the y-coordinate of the data you are plotting.

7. A new dialog box will appear called "Weight Data From Column:". By clicking on the buttons labeled $\ll$ and $\gg$, make sure the name of the column containing the uncertainties for the y-coordinate appears in the window.

8. Click on the button labeled **OK**.

9. Now click on this button window's labeled **OK**.

Now you need the numerical results of the fit. Simply choose the "Display Equations" option from the **Plot** menu, and a table containing the numbers you desire will appear. Note that, in this table, m1 is the y-intercept and m2 is the slope of the best fit line.

The Work You Should Turn In

After you have followed through the above example, you should attach the printout you have made to the worksheet that follows. This printout should at least contain: properly labeled axes, x and y error bars, and the best fit line plotted by KaleidaGraph through your plotted points. Also, somewhere on the page below the plot, you should report *in a complete sentence* what you have found to be the constant of proportionality (with uncertainty!).

Of course, your TA may require more of you.

0.W1 Error Analysis Worksheet

Name: ____________ Day/Time: ____________

Instructions: Perform all of the following calculations using the techniques explained in Chapter 0 (Introduction) of the lab manual. Show all calculations explicitly, propagate uncertainties where appropriate, include the proper number of significant figures, and provide units.

0.W1.1 In-Lab Procedure

Perform the KaleidaGraph practice plot and analysis in the previous section § 0.4 and attach it the end of this worksheet.

0.W1.2 In-Classroom Calculations

1. Four independent measurements of the voltage supplied by a certain D-cell battery were made:

$$\begin{gathered} 2.4 \pm 0.6 \text{ V} \\ 2.96 \pm 0.08 \text{ V} \\ 3.02 \pm 0.06 \text{ V} \\ 2.968 \pm 0.004 \text{ V}. \end{gathered}$$

   Referring to § 0.2.2, calculate the *most probable value* of the D-cell voltage as well as the standard deviation of the measurements using equations (0.1) and (0.2). Write your answer as you would report the final result.

2. Refer to § 0.2.3 for the calculation and propagation of uncertainty. Two lengths have been measured to be $L_1 = 4.8 \pm 1.2$ cm and $L_2 = 3.2 \pm 1.6$ cm.

   (a) Calculate the sum $L = L_1 + L_2$ and its *absolute* uncertainty, ΔL.

   Use these to calculate the *relative* uncertainty in L.

   (b) Calculate the difference $L_0 = L_1 - L_2$, as well as its absolute and relative uncertainties.

   Compare these uncertainties with those in the sum.

   (c) Now calculate the product $P = L_1 L_2$ and its absolute and relative uncertainties.

(d) Calculate the quotient $Q = L_1/L_2$, its absolute and relative uncertainties.

Compare the uncertainties to those in the product.

(e) Express L, L_0, P, and Q in proper form, i.e. with units and uncertainties.

3. The area of a square has been measured to be $A = 50 \pm 6\ \text{cm}^2$. What is the length of one side of the square?

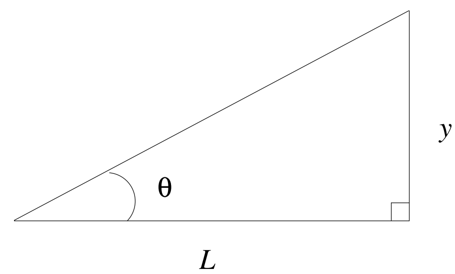
4. Two resistors, with resistances $R_1 = 540 \pm 54\ \Omega$ and $R_2 = 860 \pm 86\ \Omega$, are connected in parallel. Calculate the equivalent resistance, R_{eq}, of

the combination using the formula

$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

5. For $\phi = 60 \pm 3^\circ$, calculate $\sin\phi$ and $\tan\phi$. Hint: Convert the angle to radians.

6. Given that $L = 20 \pm 4$ cm and $y = 8 \pm 2$ cm in the triangle



calculate $\sin\theta$.

Attach your KaleidaGraph plot to the end of this worksheet.

Bring graph paper next week.

End Error Analysis Worksheet

0.W2 KaleidaGraph & Graphing By Hand

Name: ______________ Day/Time: __________

0.W2.1 In-Lab Computer Work

Two Exercises for Kaleidagraph

1. Imagine that you are riding in a car with your uncle Bob, his sister Sandy, and your cousins Rob and Gary. As you head down a back road in central Florida, a brilliant blue light suddenly bathes the car, and you fall unconscious. When you wake up, you find yourself in a room with no doors and plain grey walls. An unknown source of illumination allows you to see that your only company is a spark-tape apparatus, familiar to you from your first semester lab class. You immediately realize that you can conduct a simple experiment to determine the gravitational acceleration of the planet you are on. You conduct the experiment and find the following distance fallen versus time data:

Distance (d) vs. time (t) Measurements	
d (m)	t (s)
$.20 \pm .01$	$.2 \pm .02$
$.80 \pm .04$	$.4 \pm .02$
$1.72 \pm .05$	$.6 \pm .02$
$3.16 \pm .05$	$.8 \pm .02$
$4.84 \pm .10$	$1.0 \pm .02$
$6.96 \pm .10$	$1.2 \pm .02$

Do the following

a) Plot d vs. t^2 and obtain a **weighted** least squares fit. (Remember, use **fit1** under the General option of the curve fit menu.) Error bars, properly labeled axes and units are essential! All partners should have their own plots. **Show** your work in finding $\Delta(t^2)$ below.

b) Determine the acceleration due to gravity from the results of your curve fit, including units and uncertainties. Show work.

$g =$____________________

c) Try to guess where you are....

2. In an experiment you will conduct in a few weeks, you will measure the voltage (V) of a discharging capacitor as a function of time (t). You will be required to plot

$$\ln\left(\frac{V}{V_0}\right) \quad vs. \quad t$$

where V_0 is the initial voltage (that is, the voltage measured at time $t = 0$.) Use the following sample data:

Voltage (V) vs. time (t) Measurements	
V (V)	t (ms)
$4.0 \pm .10$	$0.0 \pm .02$
$3.6 \pm .10$	$1.0 \pm .02$
$3.4 \pm .10$	$2.2 \pm .02$
$2.8 \pm .10$	$3.2 \pm .02$
$2.6 \pm .10$	$3.8 \pm .02$
$2.4 \pm .10$	$5.0 \pm .02$

Do the following

a) Plot $\ln(V/V_0)$ vs. t and obtain a **weighted** least squares fit. Again, be sure to include error bars, units, and properly labeled axes. Remember, all partners must have their own plots. Show your work in finding $\Delta(\ln(V/V_0))$.

b) Report the slope and intercept of your plot. Units and uncertainties are a must!

Slope	Intercept

Table 0.3: Slope and Intercept

0.W2.2 In-Classroom Calculation & Analysis

This half of the worksheet is your only practice in graphing by hand. You will be using KaleidaGraph to do all your graphing in Physics 103N. Before using KaleidaGraph indiscriminately, first you will learn what graphing and least squares is all about. Chapter 0 in the lab manual contains all the information required to do this worksheet, refer back to the appropriate sections.

In Lab 2 (Electron Dynamics), we will learn that an electron moving in a constant electric potential V and a constant magnetic field B (perpendicular to the electron's motion), moves in a circular path of radius R, given by the formula

$$\boxed{R^2 = \frac{2Vm}{eB^2},} \tag{0.7}$$

where m and e are the mass and charge of the electron, respectively.

Hand-fit Graphing

A former group of 103N students set the magnetic field on their apparatus to $B = 1.154 \pm 0.006$ mT (1 T = 1 Tesla = 1 kg/C s) and measured the path radius while varying the potential V:

V (V)	R (cm)
400 ± 2	5.6 ± 0.4
600 ± 2	7.4 ± 0.4
800 ± 2	8.2 ± 0.4
1000 ± 2	9.0 ± 0.4
1200 ± 2	10.2 ± 0.4

1. Plot R^2 versus V, including error bars on all points on your graph paper. Show your work in finding $\Delta(R^2)$ in the space provided here.

   Is this linear?

   Find the slope of the best line fit to the data. Use bounding lines to obtain the uncertainty in the slope. **Show all your work in the space provided below**.

Least Squares

2. Now use linear least squares to obtain the slope with uncertainty. Note that R^2 is the *dependent* variable; you'll need to fill the following table. Observe that there are two extra rows in it. One is for the units and one is for the sums of the elements of each column.

Least Squares Method							
V_i	R_i^2	$\Delta(R_i^2)$	$\frac{1}{(\Delta(R_i^2))^2}$	$\frac{V_i}{(\Delta(R_i^2))^2}$	$\frac{R_i^2}{(\Delta(R_i^2))^2}$	$\frac{V_i^2}{(\Delta(R_i^2))^2}$	$\frac{V_i R_i^2}{(\Delta(R_i^2))^2}$

Now compute the sums required to find the slope, a, and the uncertainty in slope, Δa, as in your lab manual.

How does the least squares result compare with that obtained by hand? Comment on agreement (within uncertainty), the relative uncertainties in each result, and anything else you think is important.

3. Use equation (0.7) to calculate the charge-to-mass ratio of the electron, e/m, from the value of B and each of the slopes you obtained with uncertainty. Note that the accepted value of $e/m = 1.758\ 819\ 62(53) \cdot 10^{11}$ C/kg

Attach your KaleidaGraph and hand plots to the end of this worksheet.

End Worksheet

Chapter 1

Electrostatics

1.1 Introduction

The principles of electrostatics play a crucial role in our understanding of everyday phenomena such as the formation of lightning in weather storms to the operation of electrical devices such as capacitors which make our stereos, computers, TVs', and many other electrical systems work. In this lab we will learn about these principles by performing a few simple experiments that will demonstrate concepts such as charge production, charge conservation, electric fields, and electric potentials (for a comprehensive overview of these subjects refer to your text book by Serway, Chapters 23, 24, and 25).

1.2 Theory

1.2.1 Electric Charge

At one time or another we have all experienced what happens when on a dry day, we close our car door, or we walk across a rug and then touch a door knob. The little shock-like feeling is a result of an electrostatic imbalance created between us and the environment. This imbalance is due to the build-up of electric charge in our bodies and the objects we come in contact with. The amount of charge that can be acquired by a particular object depends on its electrical properties.

Charge is a quantity measured in units of coulomb denoted by C. It is a conserved quantity meaning that when we speak about charges acquired by objects what we really should be saying is that charges have been moved from one place to another, as to create some imbalance of charge in that object. We cannot create nor destroy charge.

1.2.2 Coulomb's Law

Charged objects exert forces on each other. We say that like charges repel while unlike charges attract. Thus there are two kinds of charges; we call one positive, the other negative. It is possible to quantify this force by a mathematical relation known as **Coulomb's Law** which depends on the magnitudes of the charges in question, their separation, and a constant:

$$F = \frac{1}{4\pi\epsilon_o}\frac{Q_1 Q_2}{r^2} \tag{1.1}$$

The equation above is the magnitude of the force between two charges of magnitude Q_1 and Q_2 with r being their separation as shown in figure 1.1. The constant of proportionality with units is given by:

$$\frac{1}{4\pi\epsilon_o} = 8.99 \times 10^9 \text{ N} \cdot \frac{\text{m}^2}{\text{C}^2}.$$

1.2.3 Electric Field

Imagine that the two charges in figure 1.1 are positive, and that Q_1 is stationary at the origin. Lets assume that at our disposal is a device that could measure the force between these two charges. If we move Q_2 in space, than according to Eq. (1.1) our device should measure different values for the magnitude of the force as well as its direction at different points. Thus we can think of Q_2 as being a test charge for determining the force on a charge at some point in space. Plotting these different vectors in space would result in a vector field that describes the force on a test charge (see figure 1.2). In light of this it is possible to write the force on a charge as:

$$\mathbf{F} = Q\mathbf{E}. \tag{1.2}$$

E is called the electric field which is defined to be the force per unit charge. It too is a vector which points in the direction of the electric force, or opposite to it (depending on the sign of the test charge Q), and may be

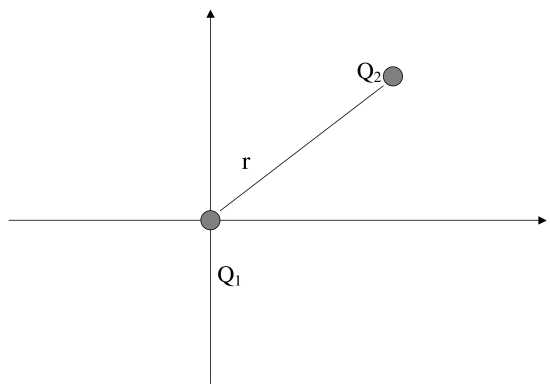


Figure 1.1: Two charges separated by a distance r.

thought of as a field in space giving rise to forces on charges. Now, the interaction among charges can be looked at as one mediated by a field.

In reality one usually encounters not point charges but continuous distribution of charges which make the concept of an electric field very useful in determining the forces by use of Eq. (1.2).

1.2.4 Gauss' Law

Determining the electric field for an arbitrary charge distribution in space can sometimes be a very difficult task. However, for charge distributions which possess certain symmetries this task can become quite easy by the use

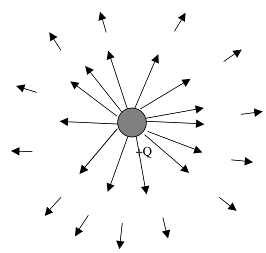


Figure 1.2: Electric field lines due to a point charge.

of Gauss' Law which is given by the following mathematical statement:

$$\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} = \frac{Q_{enc}}{\epsilon_o}. \tag{1.3}$$

The equation above is a surface integral of the electric field dotted with a differential surface element over a Gaussian surface which encloses a charge of magnitude Q_{enc}. The left side of Eq. (1.3) is called the **electric flux** which can be thought of as measure of 'flow' of the electric field through the surface enclosing the charge. Thus for a flux equal to zero there should be no net charge inside the Gaussian surface.

By choosing the surface appropriately the integral of Eq. (1.3) can be performed to solve for $\mathbf{E}$. Let's use it to find the electric field due to a very thin rod carrying a uniform positive charge (fig. 1.3). Positioning the rod along the z axis, figure 1.3 suggests cylindrical symmetry in which the electric field is constant along a surface of a cylinder with a radius r, and a height h. In this case we may apply equation (1.3) over the surface of the cylinder which is now our Gaussian surface:

$$\begin{aligned}\oint \mathbf{E} \cdot \mathrm{d}\mathbf{A} &= E \int \mathrm{d}A \\ &= E 2\pi r h = \frac{Q_{enc}}{\epsilon_o}.\end{aligned}$$

Note that in the second step we have used the fact that E is constant along the Gaussian surface enabling us to pull it out of the integral. The normal to the surface element is along the same direction of the field, so that their dot product is just the multiplication of their amplitudes.

Before completing the calculation we have to find what is Q_{enc}. We may define a quantity denoted by λ to be a charge density, or in this case a charge per unit length, so that Q_{enc} may be given by:

$$Q_{enc} = \lambda h.$$

With this the electric field due to a line of charge is given by:

$$E = \frac{\lambda}{2\pi\epsilon_o r}. \tag{1.4}$$

1.2.5 Electric Potential

Since an electric charge experiences a force in the presence of an electric field then a charge initially at rest will start to move under the influence of the field, and thus will gain kinetic energy. This means that the force which arises due to the field will do work on the charged particle. You may have noticed that the electrical force is very similar in form to that of a gravitational force, where, in the gravitational law, we use masses instead of charges and we don't consider repulsion among masses. Nonetheless the dependence on r and the way we define the direction of the force are exactly the same. From our mechanics course we remember that the gravitational force is conservative, and we therefore can convince ourselves that the electrical force is conservative as well. We can now use conservation of energy which tells us

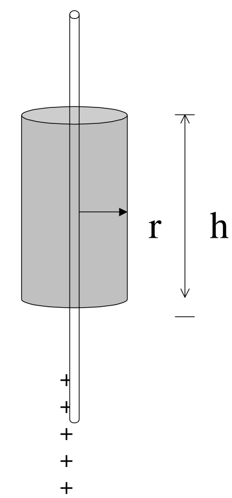


Figure 1.3: A Gaussian surface due to a line of charge

that the change in potential energy is equal to the negative change in kinetic energy, or simply put:

$$W = U_i - U_f \tag{1.5}$$

Applying this equation with the definition of work, and Eq. (1.2) we get

$$W_{i\to f} = \int_i^f \mathbf{F} \cdot \mathrm{d}\mathbf{l} = \int_i^f Q\mathbf{E} \cdot \mathrm{d}\mathbf{l}. \tag{1.6}$$

Dividing both sides by Q (remembering that Q is just a test charge) the equation above can be written as:

$$\frac{W_{i\to f}}{Q} = \frac{U_i}{Q} - \frac{U_f}{Q} = \int_i^f \mathbf{E} \cdot \mathrm{d}\mathbf{l}. \tag{1.7}$$

The quantity $\frac{U}{Q}$ is called the **electric potential** denoted by V and is defined to be the potential energy per unit charge. It is conventional to speak of an external force moving a test charge in an electric field in which case it is the external force doing the work and not the field. This introduces a minus sign in Eq. (1.7), and the potential difference of moving a test charge from point a to point b is:

$$V_b - V_a = -\int_a^b \mathbf{E} \cdot \mathrm{d}\mathbf{l}. \tag{1.8}$$

Let us apply this formula to find the electric potential in our previous example of a very thin rod with positive charge. If we move a test charge from some point a to some point b radially to-wards the rod we can use Eq. (1.4) together with Eq. (1.8) to get

$$V_b - V_a = -\frac{\lambda}{2\pi\epsilon_o}\int_{r_a}^{r_b}\frac{\mathrm{d}r}{r} = -\frac{\lambda}{2\pi\epsilon_o}\ln\frac{r_b}{r_a}, \tag{1.9}$$

Equipotential Surfaces

Moving a charge on a surface where no work is being done by the electric field describes a charge moving on an equipotential surface which is defined as a surface where the potential is the same at every point. Since no work is being done on this surface it follows that the electric field has no components parallel to it. Thus, it is always the case that an equipotential surface is perpendicular to the field. Since every point has a unique value for the potential it follows that different equipotential surfaces can never intersect.

Finally, we note that Eq. (1.8) can be written as

$$V_b - V_a = \int_a^b \mathrm{d}V = -\int_a^b \mathbf{E} \cdot \mathrm{d}\mathbf{l}.$$

Assuming for simplicity that the displacement is only along the y axis, then the equation above can be written as

$$-\int_a^b \mathrm{d}V = \int_a^b E_y \mathrm{d}y.$$

Since the limits of integration are arbitrary points it implies that the integrands are equal, or simply put

$$E = -\frac{dV}{dy}. \tag{1.10}$$

We could have done the same thing for the x and z components, and since the potential V is a function of all the coordinates the equation above can be generalized to read

$$\mathbf{E} = -\nabla V, \tag{1.11}$$

with ∇ given by:

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Since potentials are usually easier to calculate and measure than electric fields, Eq. (1.11) provides a convenient way for finding the electric field.

1.3 Apparatus

Figure 1.4 shows the apparatus to be used in the first part of the lab.

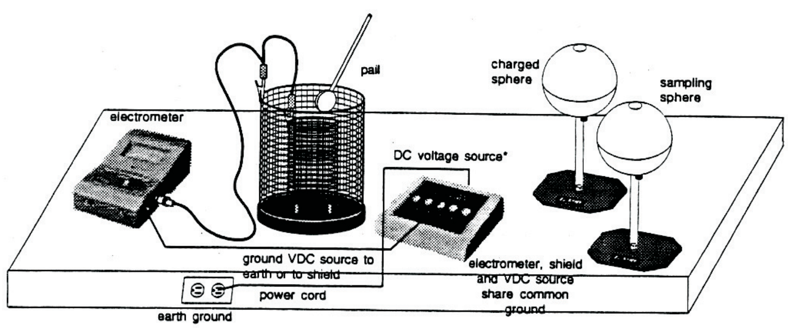


Figure 1.4: Apparatus(left to right): Electrometer, Ice Pail, Proof-plane and Charge Producers, Electrostatic Source, and Spheres.

1.3.1 The *Pasco* ES-9077 Electrostatics Voltage Source

This is a high voltage, low current power supply designed exclusively for experiments in electrostatics. It has outputs at 30 V, 1000 V, 2000 V, 3000 V, and a ground connection.

1.3.2 The *Pasco* ES-9078 Electrometer

This is a voltmeter used for direct measurements of voltage and indirect measurements of current and charge.

Never use the electrometer for measuring potentials more than 100 V.
Never touch the input leads until you have grounded yourself to an earth ground.

Operation

• Before turning on the electrometer, check that the meter reads zero. If not, turn the Mechanical Zero Adjust screw, located just below the meter face, until it does.
• Connect the test lead to the input connector of the electrometer.
• Connect the ground post of the electrometer to an earth ground by connecting to the COM on the Electrostatic Voltage Source, and plugging the AC adapter to a wall outlet.
• Push the Power button ON. One of the range switch LEDs will blink twice in quick succession.
• To zero the meter, press the ZERO button. You are now ready to use the electrometer to measure charge, current or voltage.
• Set the range switch to the desired voltage range. The range setting refers to the voltage input required to produce a full scale meter deflection. Between measurements, always press the Zero button to discard all charge from the electrometer.

1.3.3 The *Pasco* ES-9042A Faraday Ice Pail

This apparatus was originally designed by Michael Faraday. It works on the principle that any charge placed inside a conducting surface will induce an equal charge on the outside of the surface. The ice pail is a good method for sampling charges and charge distributions.

The Pasco Ice Pail consists of two wire mesh cylinders, one inside the other, mounted on a molded plastic bottom. The outer cylinder is called the shield, and when grounded helps eliminate stray charges and AC fields. The inner cylinder is the actual pail which is mounted on insulated rods. When a charged object is placed inside the pail, but without touching it, a charge of the same magnitude is induced on the outside of the pail. An electrometer connected between the pail and shield will detect a potential difference which is an indirect measurement of charge.

1.3.4 The *Pasco* ES-9075A Charge Producers and Proof Planes

Charge Producers
The charge producers consist of two wands, one with blue and one with white material attached to a conductive disk. If the blue and white surfaces are briskly rubbed together, the white surface acquires a positive charge, and the blue surface acquires a negative charge. Some guidelines are important to remember when using the charge producers.

• To get rid of excess charge and to neutralize the charge producers touch the conductive disk to ground.
• Avoid touching the neck during use. The oils from your hands will provide a path for charges to leak off.

The Proof Plane
The proof plane is an aluminum covered conductive disk attached to an insulated handle. It is used to sample the charge density on charged conductive surfaces. A Faraday Ice Pail can then be used to measure the charge density on the proof plane. By touching the proof plane to a surface, the proof plane will acquire a similar charge distribution to the section of the surface that is touched.

When a proof plane is touched to a conductive surface, the proof plane becomes part of the conductive surface. Therefore it's always best to touch the proof plane to the conductor in such a way as to minimize the distortion of the shape of the surface. For example, if the surface is a conducting sphere then the proof plane needs to be touched with its large surfaces tangent to the surface of the sphere.

1.3.5 The *Pasco* ES-9059 13-cm Spheres

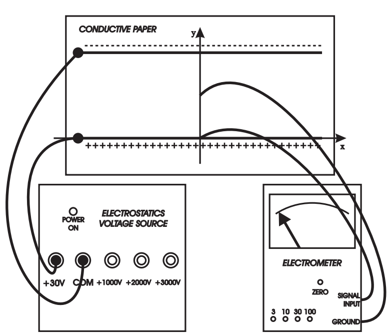
The conductive spheres are used to store electrical charge. The spheres are composed of plastic resin mold plated with a copper base, outer plating of non-sulphur brite nickel, with final plating of chrome. The spheres are mounted on insulating rods, attached to a support base. Each sphere has a thumb nut on the lower half that can be used for attaching a ground cable or a lead from a power supply. The spheres and insulating rods should be kept free of dirt, grease, and fingerprints to minimize leakage of charge from the spheres.

1.3.6 The *Pasco* High Resistance Paper and Conductive Ink

This apparatus will be used in the second part of the lab to measure voltage at different points in space due to long lines of charge. The conductive ink painted on a high resistance paper will be connected to a power source. This will create a potential distribution across the paper which will be measured by an electrometer.

The set up (see figure 1.5) of this apparatus in the experiment to follow will consist of two long parallel lines drawn by conductive ink on the high resistance paper. The lines are separated by some distance d which is relatively smaller than their length L. Now we can connect both lines to a static power source; one to the positive end, the other to the negative end. This will cause the lines to be charged with opposite polarities creating a potential difference across them. If these lines are drawn uniformly and the measurements are done close to the center of the lines (we want to stay close to the center, since near the edges the field deviates from Eq. (1.9)) then using Eq. (1.9) we can find the potential between these two lines on the plane of the paper.

Figure 1.5: High resistance paper and conductive ink.



Assume we want to know the potential at a point along the y axis (see figure 1.5) between the two lines of charge. Using Eq. (1.9) we can sum the potential due to the positive line and the negative line respectively to get

$$V(y) = \frac{\lambda}{2\pi\epsilon_o}\left(\ln\frac{r_o}{y} - \ln\frac{d-r_o}{d-y}\right).$$

The radius r_o is a reference point with respect to which we measure the potential at a point on the y axis located above the positive line of charge. Choosing our reference point r_o to be $\frac{d}{2}$ the equation above reduces to the following:

$$V(y) = \frac{\lambda}{2\pi\epsilon_o}\ln\frac{d-y}{y}, \tag{1.12}$$

which gives the potential between the two lines at a point in the same plane of the lines. Note that for our experiment Eq. (1.12) is not valid at the points $y = 0$, or $y = d$ since this equation was derived for infinitely long thin lines of charge which have no thickness. Such conditions are hard if not impossible to obtain in lab, meaning our lines of charge will be finite both in length and in thickness.

1.W Electrostatics Worksheet

Name: ____________________ **Day/Time** ____________
Partner's Name: ________________________

1.W.1 In-Lab Procedure

All measurements with the electrometer should include units and uncertainties in the space provided. Whenever you are not using the power supply you should turn it off.

Faraday Ice Pail and Charge Production

The purpose of this experiment is to investigate the relation between the charge induced on the ice pail by a charged object placed in the ice pail, and the charge of the object.
Before beginning the experiment the pail and electrometer must be momentarily grounded. To do this use the black wire with two alligator clips toward one end; one positive (red) the other negative (black). Connect the electrometer's negative clip to the shield, and the positive clip to the ice pail as shown in figure 1.4. Connect the other end (called a BNC) to the electrometer SIGNAL INPUT. Also connect the earth ground of the electrometer to the COM input of the electrostatic source, and make sure the source is connected to a wall outlet. Press the *Zero* button of the electrometer. This should remove all stray charges from the pail and the electrometer should read zero. You should also ground yourself to eliminate stray charges from your body by touching the ice pail when the *Zero* button is pressed.

A: Charging by Induction vs. Charging by Contact
Set the voltage range of the electrometer to 100V. Start with this scale for every new measurement you perform, so that you don't destroy the instrument with a high voltage. After you take an initial reading, adjust the electrometer incrementally to the finest scale that will accomodate your measurements. This will assure that you get the maximum precision. Do observe that you may have to readjust the electrometer scale several times in today's experiment.

Remove any stray charges on the charge producers by touching the necks and handles to the shield of the ice pail while the *Zero* button is pressed. Avoid touching the necks of the charge producers with your hands since oils on your hands will cause charge to leak in future use.
Rub the white and blue surfaces together to separate charges. Do this for about ten seconds. After this, place one of the charge producers far away so that it does not come in contact with any of the ice pail surfaces.
While touching the grounded shield insert the other charge producer into the ice pail all the way to the lower half of the pail, but without letting it touch the pail. Record the electrometer reading below.

$$V_{1A} = \underline{\hspace{6cm}}$$

Remove the object and again record the electrometer reading below (make sure that the handle never touches the pail when removing the object).

$$V_{2A} = \underline{\hspace{6cm}}$$

Now, press the *Zero* button to remove any residual charge. Insert the object again, but this time let it touch the ice pail. Remove the object and record the voltage reading.

$$V_{3A} = \underline{\hspace{6cm}}$$

Press the *Zero* button of the electrometer to remove all charges and insert the object again into the ice pail (again without touching the pail) and record the voltage reading.

$$V_{4A} = \underline{\hspace{6cm}}$$

B: Conservation of charge
Starting with the uncharged producers, rub the blue and white producers together. Keep them in your hands, without letting them touch each other or the ice pail. Insert the charge producers one at a time into the pail and record the voltage readings.

V_{1B} =________________________

V_{2B} =________________________

Remove all charge from the charge producers by grounding them (don't forget to ground the necks of the wands as well). Insert both charge producers into the pail and record the voltage reading.

V_{3B} =________________________

Rub them inside the pail without touching the sides of the ice pail. Record the voltage reading.

V_{4B} =________________________

Remove one of the charge producers and record the voltage reading.

V_{5B} =________________________

Remove the charge producer from the pail and insert the other into the ice pail. Record the voltage reading.

V_{6B} =________________________

Charge Distribution

The purpose of this procedure is to investigate the way charge is distributed over a surface, by measuring variations of charge density.
Make sure the ice pail is properly grounded with the shield connected to earth ground of the electrometer, and the electrometer is connected to the COM of the voltage source. Make sure the voltage source is connected to a wall outlet.
Place the two aluminum spheres about 50 cm apart. Connect one of the spheres to the 1000V DC outlet of the electrostatic voltage source. This sphere will be used as a charging body.
Momentarily ground the other sphere(NOT THE ONE CONNECTED TO THE VOLTAGE SOURCE) by touching the sphere with the shield of the pail.
Using the proof plane sample the uncharged sphere by touching it in few places (on the side close to the other sphere, on the side far from the other sphere, and midway between these two points on the sphere), and then placing the proof plane in the ice pail to read the voltage. Note that when touching the sphere with the proof plane it is best to minimize any distortion of the surface area of the sphere by touching the proof plane flat against the surface. You must also remember that between samplings of the uncharged sphere the proof plane must not be grounded. Record the voltage readings.

	close	**midway**	**far**
$V_{1C} =$	____________	____________	____________

Bring the 1000V DC sphere close to the uncharged sphere until they are approximately 1 cm apart. Turn the voltage source on and sample the sphere at the same points as before. Record the voltage readings.

$V_{2C} =$	____________	____________	____________

Remove the 1000V DC sphere until it is at least 50 cm away from the sampling sphere. Again repeat the sampling at the same points and record their values.

V_{3C} =____________ ____________ ____________

Electric Field and Electric Potential

In this part of the experiment we will use the high resistance paper and the conductive ink to create a charge distribution due to two lines of charge with opposite polarities separated by some distance. The purpose of this experiment is to see how the electric potential and electric field vary with this charge distribution.
Secure the high resistance paper (to be supplied by your TA) to the cork board using thumb tacks. Note that the grid marked on the high resistance paper is 1 cm $\times$ 1 cm. Measure the length of one of the lines L, and the separation distance between the two lines d and record them below.

L =__________ d =__________

Using the two cables which have at one end a connection to the power supply, and at the other end holes, connect the COM of the electrostatic source to one end of one line drawn by the conductive ink. Make this connection by securing the cable to the wire with a thumb tack. Make sure that the connection is made at the ends of the lines (not center), and that the wires make contact with the conductive ink. Connect the 30V output of the power supply to the other line at the end by repeating the procedure above.

Using the red and black probe wires connected to the SIGNAL INPUT, and

V	y

Table 1.1: Voltage and distance measurements.

ground connections of the electrometer respectively (in this procedure the electrometer should not be grounded), place the black probe in the center between the two lines. This will be the voltage reference point for all measurements. Use the second probe to measure the voltage with respect to the reference point starting on the y axis at 0.5 cm above the positive line of charge going up in the positive y direction. Remember that the $y = 0$ point on the y axis is determined by where the positive line of charge is located, while the positive y direction is from the positive to the negative line of charge (see figure 1.5). When taking voltage measurements it is important that the probes be placed perpendicular to the page and not slanted, as to minimize the distortion of the charge distribution across the paper. Make your measurements in 0.5 cm increments and record the value of the voltage in table 1.1 for each distance. Your last measurement should be made 0.5 cm below the top line which is the negative line of charge (see figure 1.5).

Now do the following:
a) Plot V vs. $\ln\left(\frac{d-y}{y}\right)$ in Kaleidagraph with error bars and units. All partners must have their own plots. Show your work in finding $\triangle\left(\ln\left(\frac{d-y}{y}\right)\right)$ here:

Slope	Intercept

Table 1.2: Slope and Intercept

b) Record the slope and intercept in table 1.2 with units and uncertainties.

Finally place the probe (not the one at the reference point) midway between the two lines. Move the probe 1 cm to the right and 1 cm to the left along the x axis with respect to this point. Do you see any appreciable change in voltage?
What can you say about this line on which the probe moved?

1.4.2 Pre-Classroom Check List

This check list is intended to be a guide for you to prepare yourself for the classroom work. You cannot come back to lab during this hour, collaborate with colleagues, nor hand in the worksheet late. Make sure you have completed everything.

Pre-classroom Check List

○ Voltages V_{1A} through V_{3C} have units and uncertainties.
○ Table 1.1 completed with units and uncertainties.
○ Table 1.2 completed with units and uncertainties.
○ Each student should have her/his own plot and worksheet.

1.4.3 In-Classroom Calculations & Analysis

Faraday Ice Pail and Charge Production

A: Charging by Induction vs. Charging by Contact
Using your data only from this part of the experiment answer the following

questions:

Explain why was there a potential difference between the pail and the shield only while the charged object was inside (where the object did not touch the pail).

Explain why after touching the pail with the charged object there appeared a permanent potential difference between the pail and the shield.

Explain why after zeroing the electrometer there was again a potential difference while the object was inside.

B: Conservation of charge
Using your data only from this part of the experiment answer the following questions:
After placing the charged objects one at a time into the pail what can you conclude about the magnitude of the charges, and the relation between their polarity?

Was charge conserved in this experiment? Explain your answer.

Charge Distribution
Using your data only from this part of the experiment answer the following questions:
What produced the charge distribution on the sphere that you sampled?

What can you conclude about the charge distribution on a metal sphere? Refer to both situations; first when the sampling sphere was close to the charged sphere, and second when the sampling sphere was far from the charged sphere.

Electric Field and Electric Potential

Using your data only from this part of the experiment complete the following:

Using the slope a of your graph (given in table 1.2) and Eq. (1.12) calculate the charge per unit length λ.

Now calculate the total charge Q that each line of conductive ink carried.

Use Eq. (1.10) together with Eq. (1.12) to derive the theoretical expression for the electric field E_{theo} between the two lines of charge (no uncertainty).

Finally from table 1.1 use the fourth and sixth data points $y_6 = \frac{d}{2} + 0.5$ cm and $y_4 = \frac{d}{2} - 0.5$ cm, to calculate an experimental value for the electric field at the midpoint between the two lines:

$$E_{exp} = -\frac{\partial V}{\partial y} \approx -\frac{V_6 - V_4}{y_6 - y_4} =$$

By using the theoretical expression you derived for the electric field in the third question of this section, obtain the value of the electric field at the point $\frac{d}{2}$, midway between the two lines of charge (no uncertainty). Compare this value with the quantity obtained in the previous question.

End Worksheet

Chapter 2

DC Circuits

2.1 Introduction

Electrical circuits are obviously both extremely useful and important to modern society. Society will expect you, as an engineer, to have a certain familiarity with the physical principles and phenomena responsible for making them function. You will obtain an understanding of these fundamentals in this course. This lab examines circuits in which direct current flows. Future labs will examine the extended topic of alternating currents.

We'll examine the concepts of voltage, current, and resistance; in particular, how we can go about making measurements of them. In the process, we'll learn how to use the breadboard, an important device for experimenting with circuits. We'll also take a look at the temperature dependence of resistance. Finally, we'll measure the internal resistance of a dry cell battery.

2.2 Theory

2.2.1 References

The concept of potential and potential difference is covered in Serway, Chapter 25 (Electric Potential). The notions of electric current and resistance appear in Chapter 27 (Current & Resistance), while DC circuits and Kirchoff's laws are introduced in Chapter 28 (DC Circuits). Since voltmeters and ammeters are the primary instruments we will use to investigate circuits in the lab, we strongly suggest that you read over Section 28.5 (Electrical

Instruments). Other individual sections which will be particularly useful to us are: Section 27.3 (Resistance and Ohm's Law); Section 27.3 (Resistance and Temperature), for a discussion of the temperature dependence of resistance; and Section 28.1 (Electromotive Force), for a discussion of the internal resistance of a battery.

2.2.2 Voltage, Current, and Resistance

What is an electric circuit? A succinct way of putting it is as follows:an electric circuit consists of a power source and a host of electrical components, all of which are connected together with wires. The power source provides a force which makes electrons in the wire move. These electrons in turn power the components of the circuit. After studying the concepts of voltage, current, and resistance, we'll be able to discuss power sources and resistors, so that we can understand the circuits we'll build with them.

By now, you're familiar with the notion of electric potential. A charge will experience a force due to the electric fields of other charges around it. The electric potential is defined as the potential energy per unit charge needed to move a charge (in the presence of these electric forces) from one point to another point. It is important to remember that potential is defined between two points. We can also talk about the potential at some point, but then we are referring to the potential between that point and some reference point, called the "ground." Since the electric potential is defined everywhere in space, there will be a potential defined at every point of a circuit. We define *voltage* as the difference in the potential between two points in the circuit. A battery, such as that in a car, is a power source which is said to provide "12 volts." This means that the voltage across the terminals of the battery is 12 V.

If there is a potential difference between two points of a circuit, there is also a force on the electrons in the wire which causes them to move. *Current* is the rate at which this charge flows through the circuit, *i.e.*, it is the amount of charge per unit time which passes a given point of the circuit. By *direct current*, or DC, we mean that the voltage and current values are steady; they do not vary in time. The circuits we'll be dealing with for our DC circuits will be composed of *conductors*, that is materials in which the electrons are free to move around. It's pretty obvious that we wouldn't expect very much of a current to flow in a material that wasn't a good conductor.

Given that a voltage induces a current, we can ask the question: what is the mathematical relationship between the two? For most conductors, the relationship is one of direct proportionality. Additionally, the voltage across a conductor always drops. Let's make all of this a bit clearer. Figure 2.1 shows part of a circuit containing a conductor which has a current I flowing through it and a voltage V across it (between points a and b). We denote the voltage across the conductor by

$$V = V_a - V_b,$$

where V_a is the potential difference between the point a and ground (the common point referred to earlier) and V_b is that between b and ground. By voltage drop, we mean that, in order for the current to flow in the direction shown in the figure, the potential at b must be less than that at a, so that $V_a > V_b$, and V is a positive number.

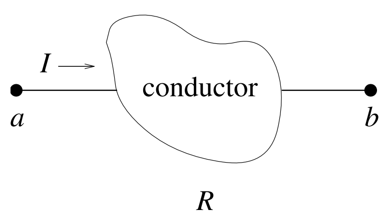


Figure 2.1: There is a voltage drop across a conductor.

The proportionality between the voltage and current is called Ohm's law, which we write as

$$\boxed{V = IR.}$$

The constant of proportionality, R, is called the *resistance* of the conductor. The resistance depends on several factors. One is the size and shape of the conductor we're dealing with and another is called the *resistivity*, which is a property of the material and is independent of any particular shape. We will not examine this dependence of resistance on size and shape of the particular material in question. What we will examine is the temperature dependence of the resistivity of a material. If you pass enough current through a conductor, you will raise the temperature of the conductor; a filament in a light bulb gets hot enough to glow. At high enough temperatures, a conductor's resistivity, and therefore its resistance, will increase. The voltage and current will no longer be proportional.

2.2.3 Diagrams and Meters

Consider a simple DC circuit consisting of voltage sources and resistors. A *resistor* is simply a conductor that has been constructed to have a particular resistance value (it's also made so that we can conveniently incorporate it into our circuits). The resistors are color-coded with the value of their resistance. Before we go further, we should establish some conventions for our diagrams. Figure 2.2 contains the schematic representations that we will use for voltage sources. We also need some notation for resistors, voltmeters, and ammeters. Figure 2.3 shows how these are represented.

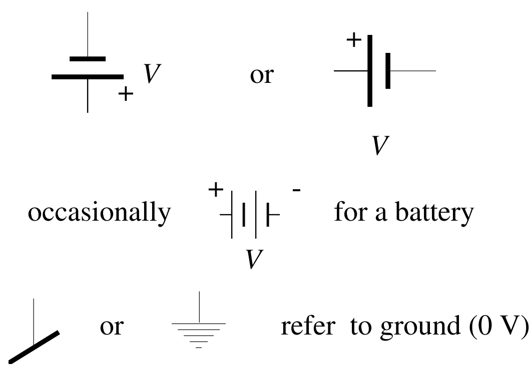


Figure 2.2: Schematic symbols for voltage sources. V refers to the voltage supplied. Don't confuse a ground with a regular voltage source.

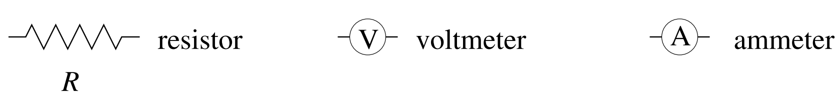


Figure 2.3: Schematic symbols for circuit elements.

We will generally use two types of meters in our circuits, voltmeters and ammeters. Serway describes these, so we'll only concentrate on where they get connected to the circuit. A voltmeter, as the name implies, measures voltage. The voltmeter is always connected *across* the part of the circuit that you want to measure the voltage across. In other words, it is connected in *parallel* with the part of the circuit you want to know the voltage across, as in Figure 2.4.

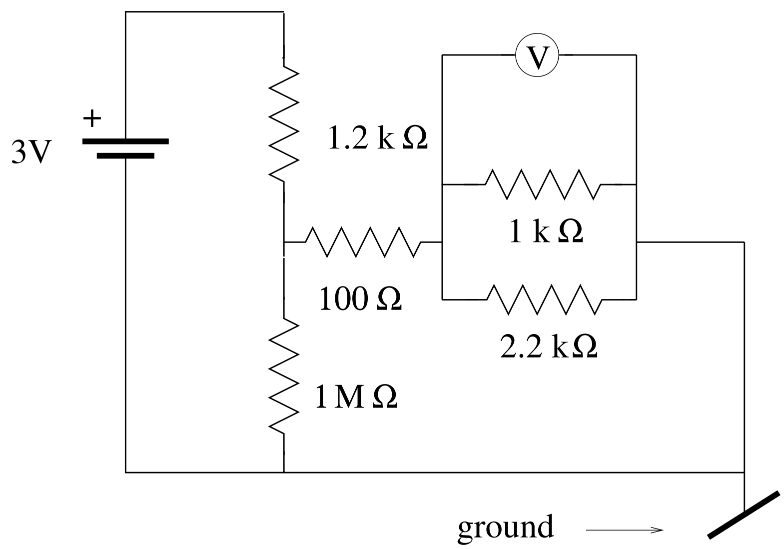


Figure 2.4: A voltmeter is always connected in parallel.

Remember that voltage is defined as the potential difference *between two points*. One of these points could be a ground, but then you will need to connect the voltmeter between a point in the circuit and ground. In any case, if you've read the section on voltmeters in Serway, you'll remember that no current will flow through an ideal voltmeter, so it cannot be in series with the circuit. The voltmeter in the figure is in a position to measure the voltage across the parallel combination of the 1 kΩ and 2.2 kΩ resistors. An ammeter on the other hand, measures current, and it must be connected in *series* with the part of the circuit you want to know the current through, as in Figure 2.5. This is because current is defined as the amount of charge which passes through a *single point*.

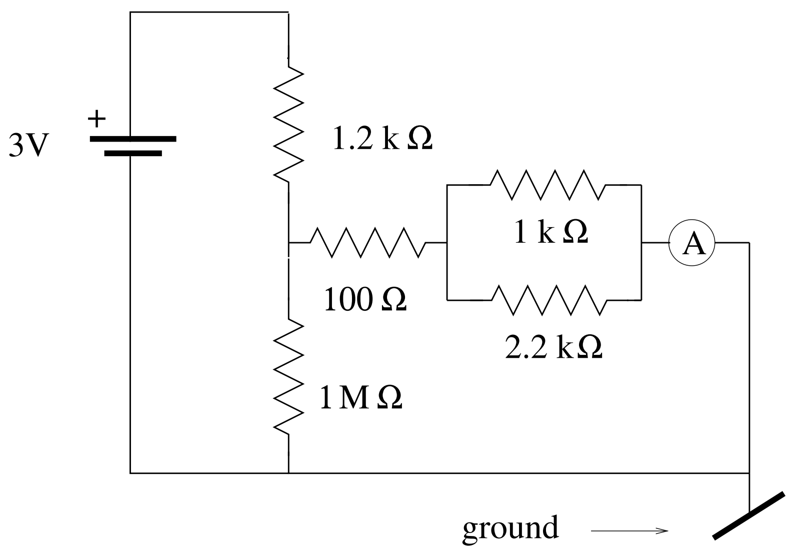


Figure 2.5: An ammeter is always connected in series.

2.2.4 Voltage Sources

The simplest DC circuit is a series combination of voltage sources as in Figure 2.6. The total voltage across the circuit, which the voltmeter shown

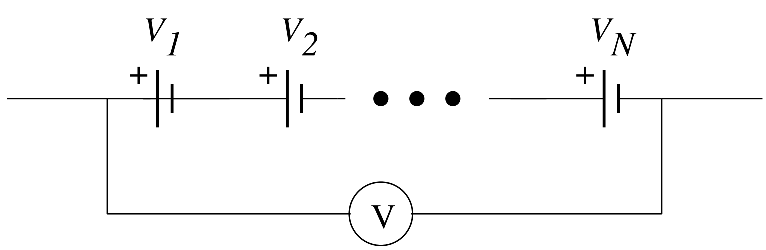


Figure 2.6: A series of voltage sources.

would display, comes from the rule that voltages in series add

$$V = V_1 + V_2 \cdots + V_N.$$

Let's apply this to some examples involving batteries. In Figure 2.7, we connect two batteries with their voltages oriented in the same direction. The

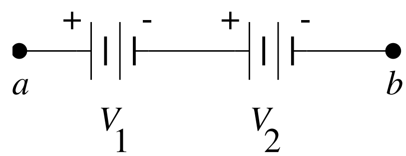


Figure 2.7: Two batteries in series.

potential across the combination is

$$V_{ab} = V_1 + V_2.$$

In Figure 2.8, two batteries have their voltages connected in opposition. The

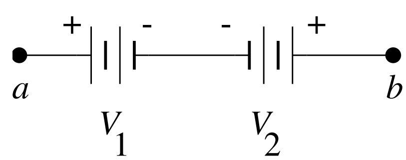


Figure 2.8: Two batteries in opposition.

potential across the combination in this case is

$$V_{ab} = V_1 + (-V_2) = V_1 - V_2.$$

We will not consider any circuits in which only voltage sources are connected in *parallel*, as shown in Fig 2.9. The reason why we would not like to build such circuits is simple. Considering that the voltage source V_1 is connected between points a and b in Figure 2.9, we would guess that the potential between a and b is V_1. But since V_2 is also connected between points a and b, the potential between them should be V_2. Unless $V_1 = V_2$, current will flow from one voltage source to another. Since most voltage sources do not like this, something will probably break! Therefore, unless we want to destroy our voltage sources (and we don't!), we should be careful not to connect them in parallel.

2.2.5 Series and Parallel Circuits with Resistors

Resistors will also be an important part of the circuits we study. In order to deal with circuits with both voltage sources and resistors (and also any

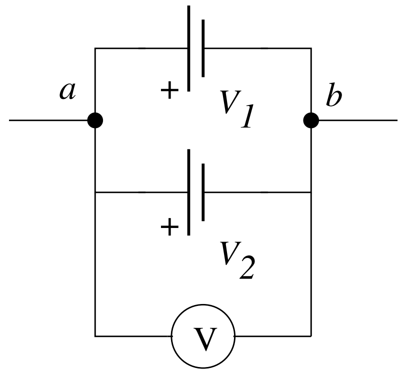


Figure 2.9: A parallel combination of voltage sources. We will never want to build such a circuit.

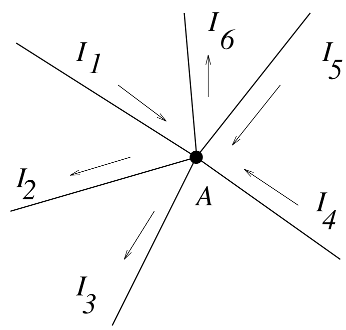


Figure 2.10: Illustration of Kirchoff's junction rule.

other components, such as capacitors or inductors) we need to use two rules, both attributed to Kirchoff. The first is the *junction law: the sum of the currents passing into a point is equal to the sum of the currents leaving the point.* Such a point is illustrated in Figure 2.10, where arrows pointing into the point A indicate that the current is ingoing and the arrows pointing out mean the current is outgoing. In the figure I_1, I_4, and I_5 are in going and I_2, I_3, and I_6 are outgoing, so that the junction rule tells us that

$$I_1 + I_4 + I_5 = I_2 + I_3 + I_6. \tag{2.1}$$

The junction rule is simply a consequence of the principle of conservation of electric charge. If a certain amount of charge passes into the point A from I_1, I_4, and I_5, it has to come out somewhere. It must leave A through the combination of I_2, I_3, and I_6, leading to the conservation statement (2.1).

Before introducing the second rule, we need to understand the concept of a voltage drop. Ohm's law tells us that, if we apply a potential V across a resistor of resistance R, the resistor will draw a current I given by

$$I = \frac{V}{R}.$$

Another way of looking at this comes from Figure 2.11. If we know that a

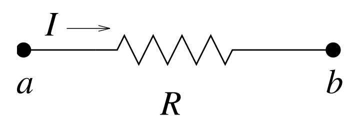


Figure 2.11: There is a voltage drop across a resistor.

current I is flowing through the resistor, then from Ohm's law, the potential difference from a to b is

$$V_b - V_a = V_{ab} = -IR.$$

Since V_b is less than V_a, we say that there is a *voltage drop* across the resistor. The second rule now is the *circuit rule: between any two points in a circuit, the potential difference between the points is given by the sum of the potentials due to any sources between the points minus the sum of any voltage drops between the points.* We can write this suggestively as an equation

$$V_{ab} = \sum(\text{sources between } a \text{ and } b) - \sum(\text{drops between } a \text{ and } b). \quad (2.2)$$

We note that the circuit rule is usually expressed as a *loop* rule: *the sum of the voltage sources minus the sum of the drops around a closed loop is zero.* We leave it as an exercise for the student to show that our rule reduces to this loop rule when applied to a closed loop. Note that the circuit law is a statement of energy conservation: the change in total energy (remember that voltage is a measure of potential energy) that occurs due to a charge moving from point a to point b is given by adding up all of the changes in its energy along its path.

Let us apply this circuit rule first to the series circuit in Figure 2.12. We note that since everything is in series, there are no points where currents branch out, so the junction rule tells us that the current through each of the elements has the same value I. Applying the circuit rule (2.2) to this we find

$$V_{ab} = (V_1 + V_2) - (IR_1 + IR_2 + IR_3) = V_1 + V_2 - I(R_1 + R_2 + R_3).$$

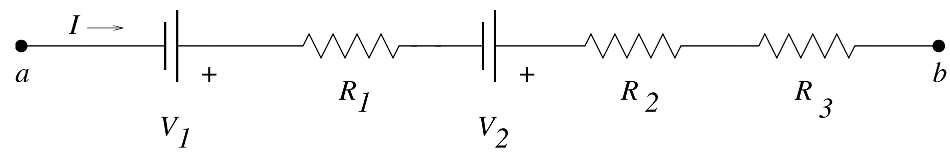


Figure 2.12: A simple series circuit.

We can also apply the circuit rule to a series combination of resistors, as at the top of Figure 2.13. Here the circuit rule tells us that

$$V_{ab} = -I(R_1 + R_2 + \cdots + R_N).$$

This series combination is the same as having a single resistor of resistance

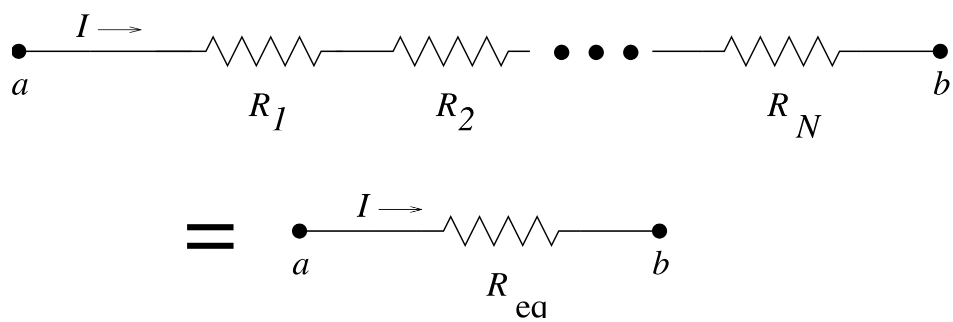


Figure 2.13: Resistors in series and equivalent resistance.

(see the bottom of Figure 2.13)

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_N \qquad \text{(resistors in series).}$$

R_{eq} is called the *equivalent resistance* of the series combination of resistors. Note that the equivalent resistance of a combination of resistors in series is always *greater* than the resistance of any of the individual resistors.

We also need to be able to deal with circuits in which some of the elements are arranged in parallel. Since we have already seen that there are problems lurking in circuits for which voltage sources are in parallel, we will consider circuits in which *only resistors* are in parallel. For concreteness, consider

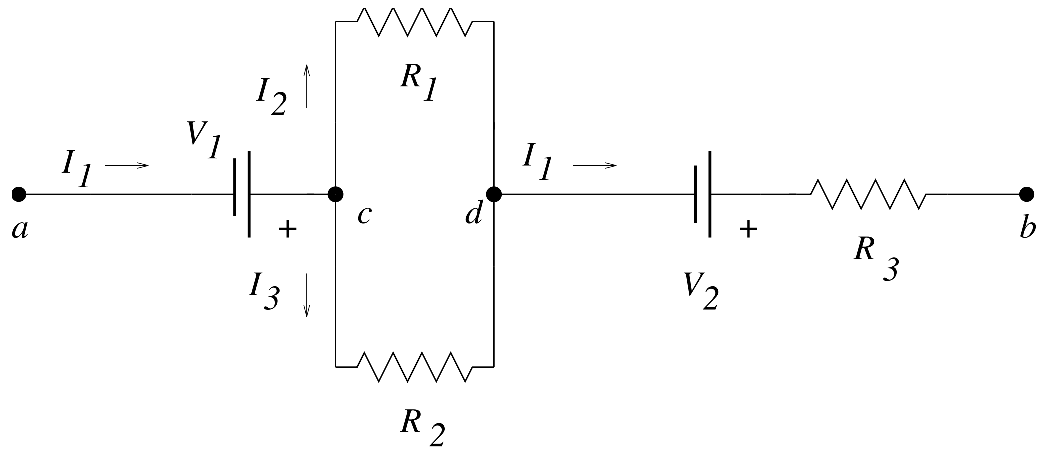


Figure 2.14: A simple parallel circuit.

a specific example of such a circuit, shown in Figure 2.14. This circuit is more complicated, since we have the junctions c and d where we must apply the junction law. At c, we have I_1 splitting up into I_2 and I_3, so that the junction rule tells us that $I_1 = I_2 + I_3$. At d, we have I_2 and I_3 joining. Since $I_2 + I_3 = I_1$, the current leaving d must be I_1. Since we know how to apply the circuit rule to series circuits, let's do the following. Note that whatever the voltage drop is across the parallel part of the circuit between c and d is, we can call it V_{cd}. Then the circuit rule gives us

$$V_{ab} = V_1 + V_{cd} + V_2 - I_1 R_3. \tag{2.3}$$

We now need to find out what V_{cd} is. For this purpose, we can simply concentrate on the relevant part of the circuit, shown in Figure 2.15. Note that we have introduced an equivalent resistor for this combination. Our goal is to determine R_{eq}. From Ohm's law we find

$$V_{cd} = -I_2 R_1 = -(I_1 - I_2) R_2 = -I_1 R_{\mathrm{eq}},$$

where we have used $I_1 = I_2 + I_3$ to eliminate I_3 in the part from R_2. These relations give us two useful equations

$$\begin{aligned} \frac{I_2}{I_1} &= \frac{R_{\mathrm{eq}}}{R_1} \\ R_{\mathrm{eq}} &= \left(1 - \frac{I_2}{I_1}\right) R_2. \end{aligned}$$

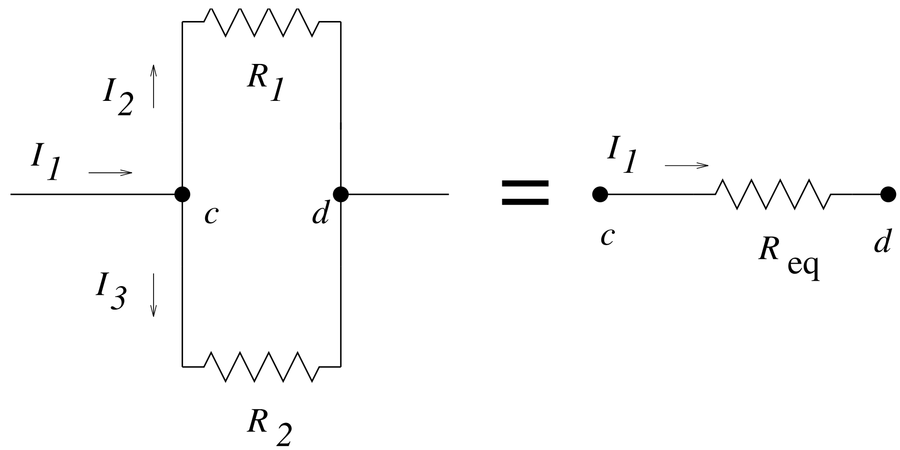


Figure 2.15: The part of Figure 2.14 contributing to V_{cd}.

Combining these, we find

$$R_{\text{eq}} = \left(1 - \frac{R_{\text{eq}}}{R_1}\right) R_2,$$

so that, after some algebra, we have

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1 + R_2},$$

or equivalently

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{2.4}$$

Therefore

$$V_{cd} = I_1 \frac{R_1 R_2}{R_1 + R_2}.$$

Then, from (2.3), for our original circuit we have

$$V_{ab} = V_1 + V_2 - I_1 \left(\frac{R_1 R_2}{R_1 + R_2} + R_3\right).$$

The reasoning we used to deal with the parallel circuit example was a bit roundabout, since we first had to learn how to approach the problem. If we knew Equation (2.4) to begin with, we would have had much less to do. We can apply the same reasoning we gave the parallel combination of resistors

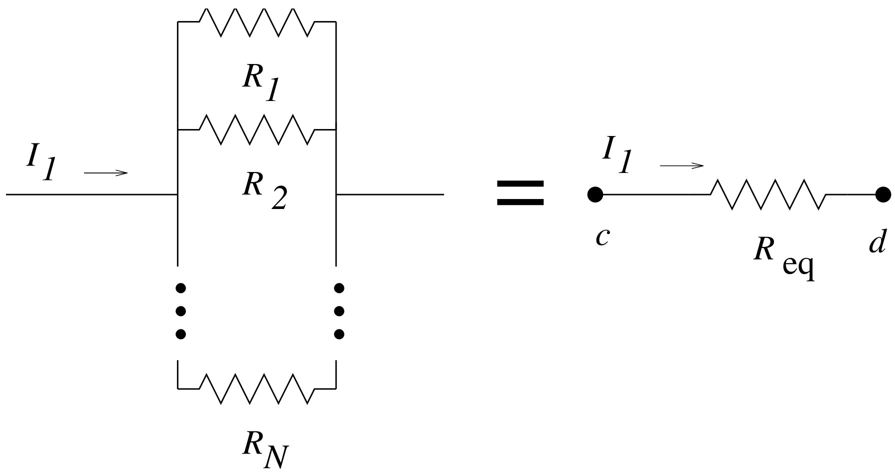


Figure 2.16: Resistors in parallel.

in the example to a general parallel combination of resistors in Figure 2.16. Since we already know what the equivalent resistance of two resistors in parallel is, we can approach this circuit by reducing a block of two resistors down to a single equivalent resistor and then repeating the step until we're only left with one resistor; see Figure 2.17. You might label the resistors

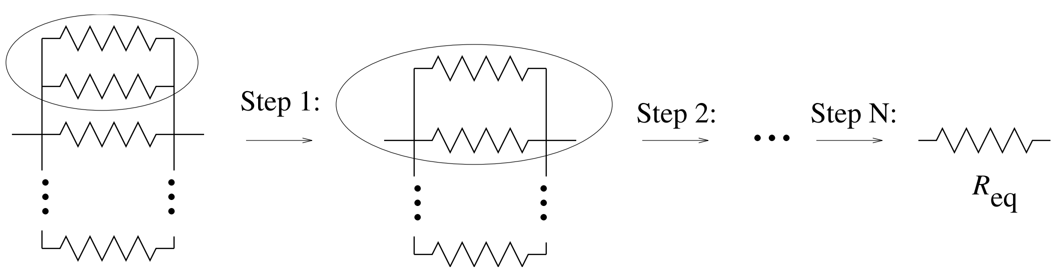


Figure 2.17: The steps used to obtain Equation (2.5).

with resistance values and work out a few steps of the algebra to convince yourself that the general result for resistors in parallel is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \qquad \text{(resistors in parallel).} \tag{2.5}$$

In contrast with the situation of resistors in series, the equivalent resistance of a parallel combination of resistors is always *less* than any of the individual resistances. Knowing these rules of thumb can come in handy when your circuit requires a resistance that isn't among the values of the resistors you have available to you. You'll be able to build series and parallel combinations of the resistors you do have to get an equivalent resistance that will be fairly close to what you need.

With the junction and circuit rules and the rules for adding resistors in series and in parallel, we can approach circuits by using equivalent resistors to reduce the circuit in steps, as in Figure 2.18. (For simplicity we consider a circuit with only resistors.) A good exercise would be to calculate the equivalent resistances needed in each step of Figure 2.18.

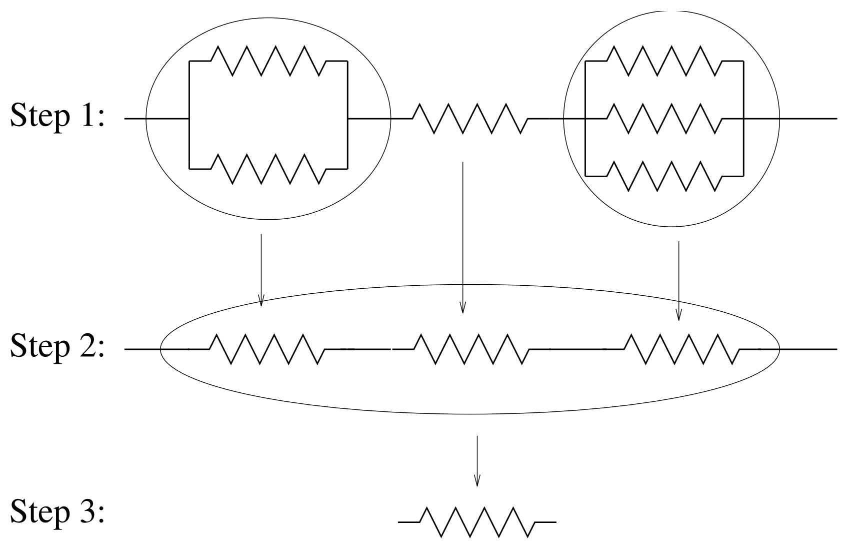


Figure 2.18: Reduce the resistor combinations in terms of equivalent resistors.

2.2.6 The Color Code for Resistors

Colored bands are drawn on the resistors to denote the resistance, as depicted in Figure 2.19. For a figure with color, see Figure 27.7 on page 747 of Serway. To read the color code of a resistor, pick up the resistor and identify the first band. The first band is the band closest to the outside of the resistor that is a color other than gold or silver. If there are no gold or silver bands, the colored bands will be clearly off-center and the first band is that closest to the outside of the resistor. Identify the number associated with the first band in Table 2.1. This is the first digit of the resistance value of the resistor. The color of the second band refers to the second digit of the resistance. The color of the third band refers to the power of ten you need to multiply the two digits by to get the resistance in Ohms. A fourth band that is gold or silver refers to the tolerance or precision with which you can trust the nominal value of resistance obtained from the color code. Resistors without a fourth band have a tolerance of $\pm 20\%$. On rare occasions a resistor will have a fifth colored band, which refers to a military specification of reliability. We will not worry about this rare option, but take note of it so that we do not confuse this band with the most important first band.

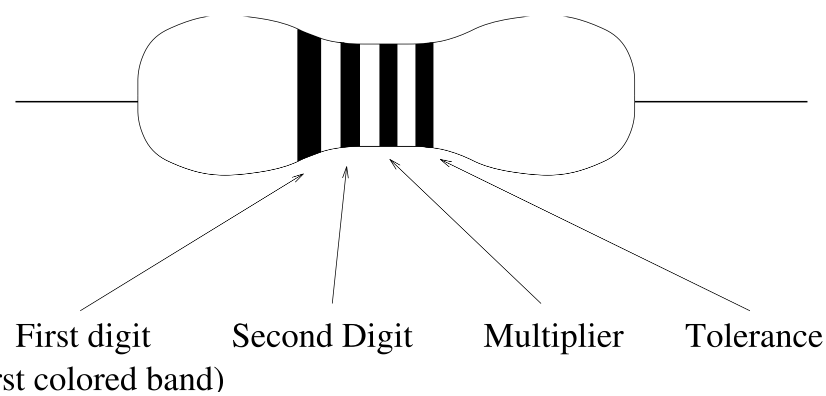


Figure 2.19: Resistors are color-coded with their resistance value.

Some examples are in order.

Colors	First Digit	Second Digit	Multiplier	Resistance	Tolerance
Yellow-Orange-Brown-Gold	4	3	10	430 Ω	$\pm 5\%$
Red-Red-Red	2	2	10^2	2.2 kΩ	$\pm 20\%$

Color	Number	Multiplier	Tolerance (%)
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold			5%
Silver			10%
Colorless			20%

Table 2.1: The resistor color code.

2.3 Apparatus

2.3.1 The Multimeter

The multimeter, illustrated in Figure 2.20, serves three functions: voltmeter, ammeter, and ohmmeter. The *ranges* for each setting indicate the maximum value of the particular quantity (voltage, current, or resistance) that can be measured in that setting. For example, the 200 mA setting will allow you to measure a current that is between 0 and 200 mA. If you attempt to measure a value higher than the maximum, the display will read "1." When this happens, it indicates that you need to try a larger scale. The *uncertainty* in a measurement with the multimeter is a ± 1 in the last decimal place; for example, if you read a current of 98 mA, then the uncertainty is ± 1 mA, so you should write down the measurement as 98 ± 1 mA. You should notice that a lower scale setting will often give you an extra significant figure (as long as the scale isn't too low), so that you can get the greatest precision in a measurement by using the smallest scale setting possible. For example, a lower scale setting with the current from our last example might give us 98.3 ± 0.1 mA, which has a relative uncertainty of 0.1%, rather than the 1% uncertainty in the first measurement. If it appears from the value of a

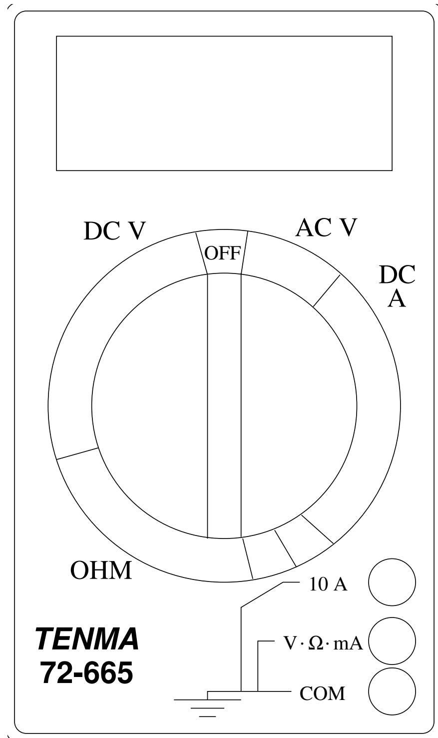


Figure 2.20: The multimeter.

measurement that you can use a smaller scale setting, and therefore improve the precision of a measurement, then be sure you do so.

The multimeter probes are color-coded: red for positive and black for ground. Always connect the red probe to the V·Ω·mA port on the multimeter and the black probe to that marked COM. If you are using an ohmmeter setting to measure resistance, the order in which you connect the probes *to the circuit* will not matter. When set as a voltmeter, the value of voltage read will be positive or negative, depending on the sign of the potential across the part of the circuit you are trying to measure. It is therefore a good idea to make a note of the order in which you connected your probes or, in a simple circuit, to follow a convention of always placing the black probe closest to ground, since the sign of the voltage is an important part of your measurement. The order is crucial when using the multimeter as an ammeter. If you do not have the current flowing from *red to black*, the ammeter will not allow the current to flow and you risk *damaging* the multimeter. Disconnect the ammeter immediately if you believe you have the connection wrong; then reexamine the circuit to determine the correct orientation.

2.3.2 The Power Supply

The power supply we'll be using is illustrated in Figure 2.21. It can supply a DC signal at a fixed voltage and can limit the current drawn by the circuit; there are both fine and coarse adjustment knobs and there are meters to display the voltage and current settings. We will make all of our voltage and current measurements with the multimeter, so use the values displayed on the power supply's meters only as a reference. The supply contacts on the front of the unit are marked + (for positive), GND (for ground), and − (for negative); our units have had the − and GND connections wired together so that they are both at ground. Make your connections to the + and GND contacts.

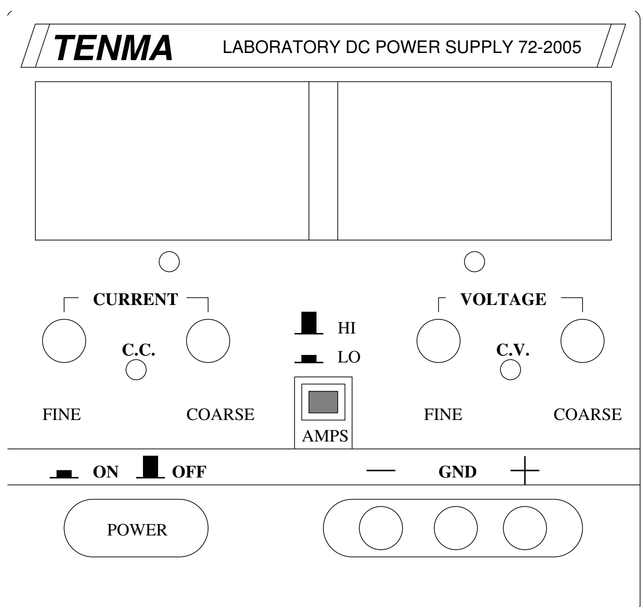
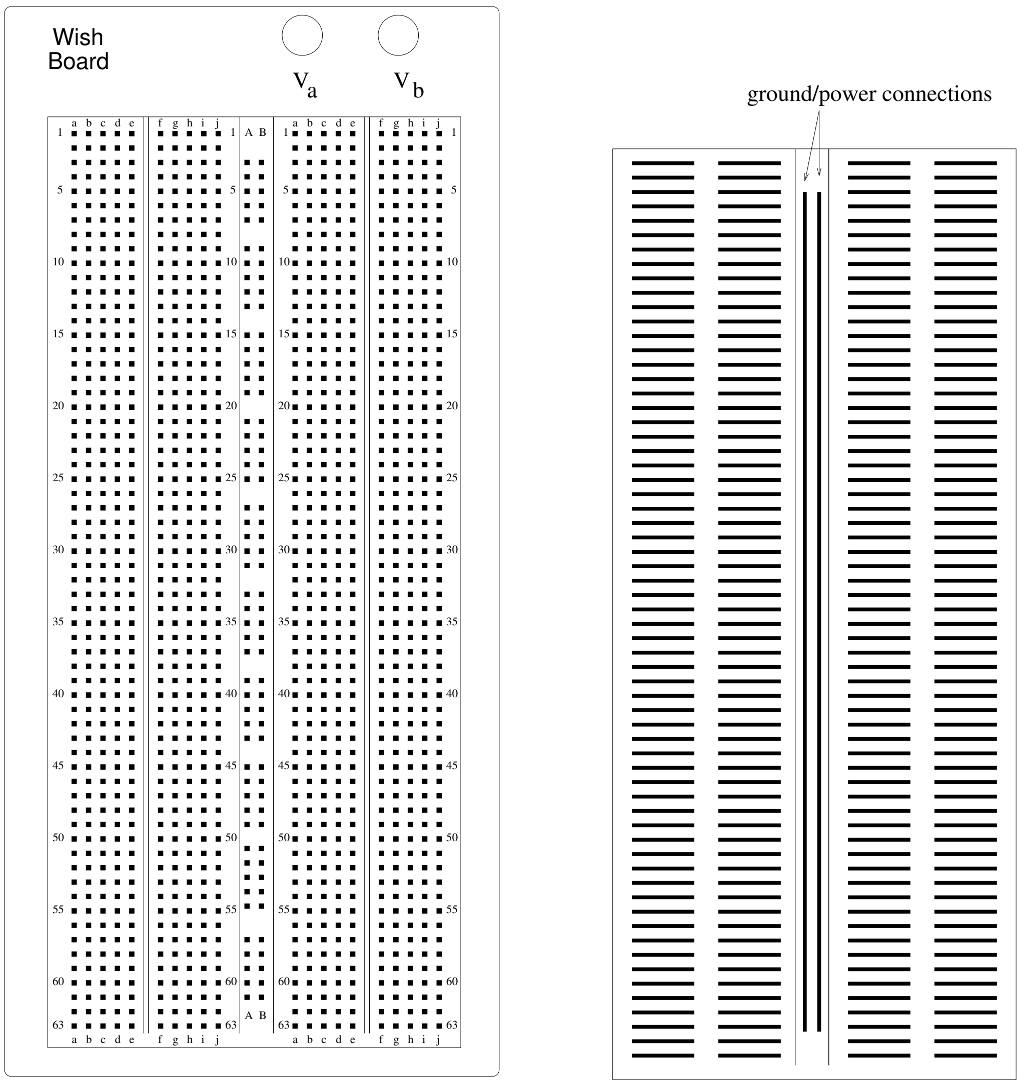


Figure 2.21: The power supply.

2.3.3 The Breadboard

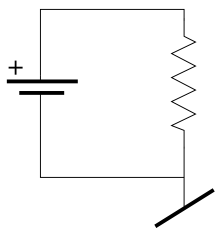
The breadboard, illustrated in Figure 2.22, conveniently provides power and electrical connections for our circuits. Figure 2.22b shows the detail of the

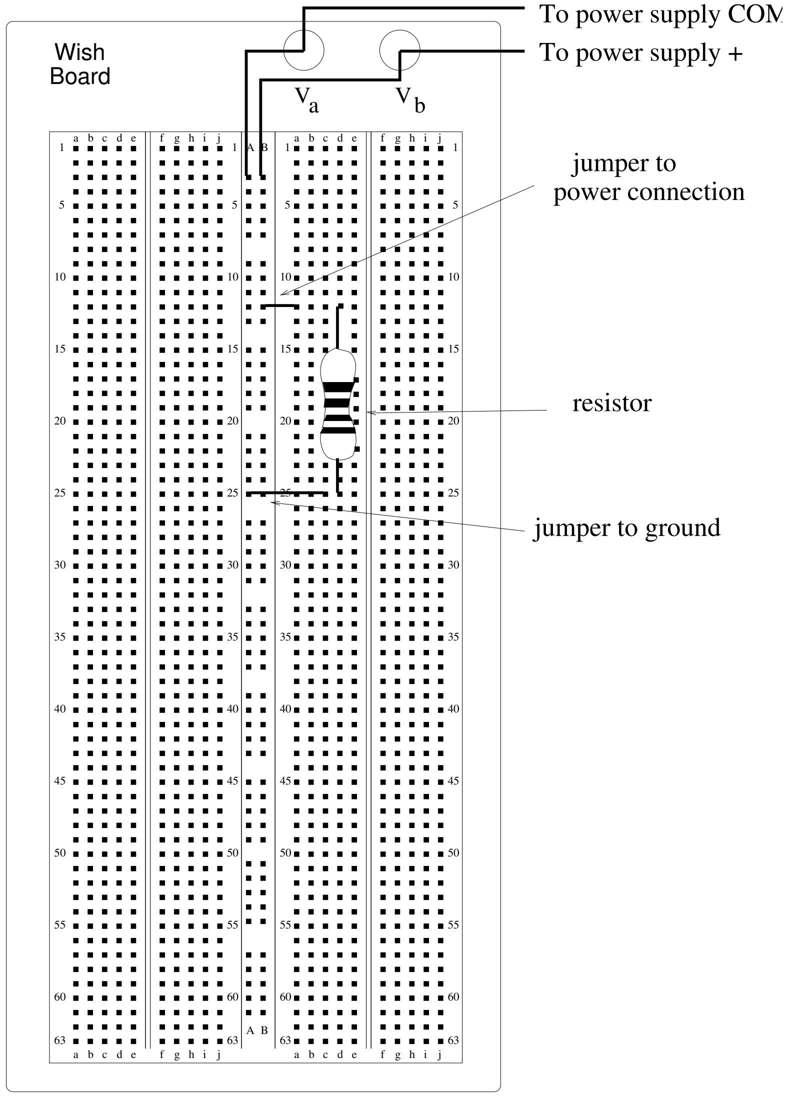


(a) (b)

Figure 2.22: (a) The breadboard. (b) The wiring connections on the board.

wiring connections on the board. The power and ground connections are contiguous, while many short “wire” sections are provided for circuit building. The best way to learn how to use the breadboard is to just start building circuits, but to better prepare you, we’ll provide an example of how to use the breadboard to power a resistor. We want to build the circuit shown in Figure 2.23a. The necessary connections are shown in Figure 2.23b. We start by connecting the power supply to the V_a and V_b connections on the board; the V_a connection will be the ground and is also connected to the A ground line on the board, the V_b connection will be the “hot” connection and will be connected to the power line on the board. A small jumper wire connects the power line to one of the wire sections. The resistor is plugged between this section and another section which is jumped to ground. Carefully compare Figure 2.23b and Figure 2.22b, so that you understand how other connections should be made. A good exercise would be to make a rough sketch of the board outline for all of the circuits listed in the Procedure and Analysis section below. You’ll need to build them when you come into lab anyway; why not prepare? A good quiz question would be to give you a circuit and a sketch of the board; you’d draw the jumper and element connections needed on the sketch…





(a) (b)

Figure 2.23: (a) The circuit schematic. (b) The necessary connections on the board.

2.W1 DC Circuits Part I Worksheet

Name: ____________________ **Day/Time:** ____________
Partner's Name: ________________________

All measurements with the multimeter should include *units* and *uncertainty*. In general, you should turn down the voltage to zero on the power supply whenever you rearrange the circuit on the breadboard. Record your observations in the spaces provided as you proceed through this worksheet. Be sure to check off each item on the check list included before you leave lab.

2.W1.1 In-Lab Procedure

Batteries in Series and Opposition

Measure the voltage supplied by each of the batteries, and enter those values into Table 2.2. Take care to distinguish between the batteries once you've made your measurements. Calculate the expected voltage value you would read on a voltmeter with uncertainties. First if they were in series and second if they were in opposition. **Space is provided here for the calculation.**

Enter these values into the spaces titled "Expected Value" in Table 2.2. Connect the batteries in series as in Figure 2.24a and measure the voltage across the combination entering your data into Table 2.2. Now connect the batteries in opposition, as in Figure 2.24b. Measure the voltage across the combination entering your data once again into Table 2.2.

Resistance Measurement

Pick two resistors and use the color code to determine their nominal resistance and tolerance. Use the ohmmeter setting of the multimeter to measure their resistance. Note that you are taking **3** measurements of **2** resistors. First you use the color code, next the ohmmeter, and finally the circuit. Using the circuit means you will make 2 measurements: you will measure voltage and current across the resistor. When you measure resistance in the

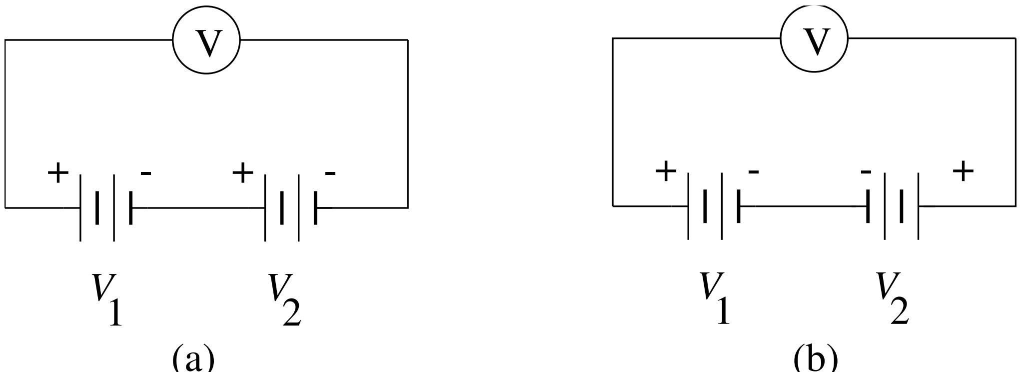


Figure 2.24: Batteries in Series and Opposition.

Battery 1 Voltage, V_1	Battery 2 Voltage, V_2
Voltage in Series, $V_{s,\text{meas}}$	Expected Value, $V_{s,\text{exp}}$
Voltage in Opposition, $V_{o,\text{meas}}$	Expected Value, $V_{o,\text{exp}}$

Table 2.2: Voltage measurements.

circuit, disconnect the power supply. When you measure **current, place the ammeter IN SERIES** as dictates Figure 2.25.

The set-up for the circuit to measure resistance is outlined in Figure 2.25. Begin by disconnecting the batteries and connecting the **power source** to the breadboard. Set the power source to a voltage of approximately 3 V. Pick one of your resistors and build the circuit in Figure 2.25. You will need to include jumper wires so that you can make the necessary connection for the current measurement. Measure the voltage across and the current through the resistor (**in series**). Use Ohm's law to calculate the resistance with uncertainty. Repeat the measurement with your other resistor.

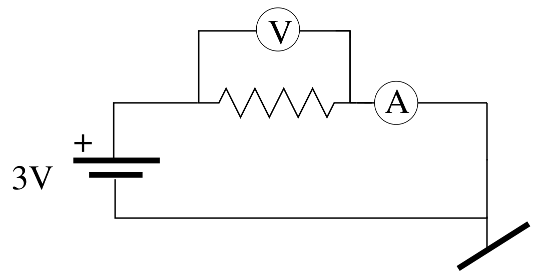


Figure 2.25: Circuit used to measure resistance.

Enter the color code resistance value (not the colors), the ohmmeter reading, the voltage and current readings, and the Ohm's law resistance with uncertainties into Table 2.3.

Resistor 1		
Color Code, $R_{1,1}$	Ohmmeter, $R_{1,2}$	Leave Blank
Voltage, V_1	Current, I_1	Ohm's Law Resistance, $R_{1,3}$
Resistor 2		
Color Code, $R_{2,1}$	Ohmmeter, $R_{2,2}$	Leave Blank
Voltage, V_2	Current, I_2	Ohm's Law Resistance, $R_{2,3}$

Table 2.3: Resistance measurements.

Resistors in Series and Parallel

Using the same resistors as above, build the series and parallel circuits in Figure 2.26. Note that you now have chosen **2** resistors and measured

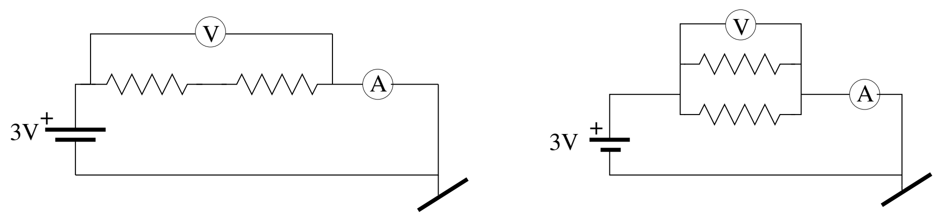


Figure 2.26: Series and parallel resistance measurements.

their resistances in **3** ways. In this section, you will use those 2 resistors to build **2** circuits, one in series and one in parallel. You are doing **2 circuits** and will need to collect data for each circuit.

Determine the resistance of the new circuits from the color code values in Table 2.3 and enter those into the Color Code section in Table 2.4 with uncertainties (*Note that the 2 resistors are in series or parallel and you will need to add the resistances appropriately*). **Show Work**.

Measure with the ohmmeter the new circuits' resistance, entering the data into Table 2.4. Measure the voltage and the current for the 2 circuits, once again being careful to place the **ammeter in SERIES** with the resistors. Enter the voltage, current, and Ohm's law resistance into Table 2.4.

Series Resistors		
Color Code, $R_{s,1}$	Ohmmeter, $R_{s,2}$	Leave Blank
Voltage, V_s	Current, I_s	Ohm's Law Resistance, $R_{s,3}$

Parallel Resistors		
Color Code, $R_{p,1}$	Ohmmeter, $R_{p,2}$	Leave Blank
Voltage, V_p	Current, I_p	Ohm's Law Resistance, $R_{p,3}$

Table 2.4: Series and parallel resistance measurements.

2.W1.2 Part I Pre-Classroom Check List

This check list is intended to be a guide for you to prepare yourself for the classroom work. You cannot come back to lab during this hour, collaborate with colleagues, nor hand in the worksheet late. Make sure you have completed everything.

Pre-classroom Check List

◯ Table 2.2 completed with units and uncertainties
◯ Table 2.3 completed with units and uncertainties
◯ Table 2.4 completed with units and uncertainties

2.W1.3 In-Classroom Calculations & Analysis

Batteries in Series and Opposition

Using Table 2.2, compare your measured results with those that you calculated as expected. Do the values match within uncertainty? For example, show that the difference is less than the sum of the uncertainties.

Resistance Measurement

For each resistor in Table 2.3, you have three values of resistance: nominal (color code), ohmmeter, and Ohm's Law. Do all the measured (ohmmeter and Ohm's Law) values agree with the nominal (color code) value within uncertainty for both resistors?

Calculate the average resistance of each resistor using equation (0.1) and the standard deviation with equation (0.2).

Discuss the validity of your results using the above calculations, i.e. what values make sense and why.

Resistors in Series and Parallel

For the series and parallel circuits in Table 2.4 you have three values for the equivalent resistance. Do the measured (Ohmmeter and Ohm's Law) agree with the nominal (color code) within uncertainty?
Series:

Parallel:

Calculate the most probable value and standard deviation of these resistances.

Series:

Parallel:

Discuss the validity of the resistance measurements using the previous calculations, i.e. which values (including previously calculated) are the most representative of the experiments.

End Part I Worksheet

2.W2 DC Circuits Part II Worksheet

Name: ____________________ **Day/Time:** ____________
Partner's Name: ________________________

2.W2.1 In-Lab Procedure

Temperature Dependence of Resistance

To begin with, turn the voltage on your power supply to *zero*. Then, devise a circuit to measure the resistance of a flashlight bulb. (Just model it on the circuits that we've been using so far.) **You are about to measure current. Ammeters are connected in SERIES**. Turn the voltage up to about 0.1 V and measure the voltage across and the current through the bulb. Make ten measurements of the voltage and current for widely spread voltage values in the region 0 V to approximately 3 V, entering the measurements into Table 2.6. You don't want to burn out the bulb, so don't make the voltage too high, but be sure that the bulb is lit for several of the measurements.

Replace the bulb with a resistor. Read the resistance with the ohmmeter and color codes as you did in §2.W1; and enter the resistance values in Table 2.5.

Resistor	
Color Code	Ohmmeter

Table 2.5: New Resistance measurements.

Make the same measurements over the same range of voltage values you used for the bulb, entering them into Table 2.7.

Light bulb	
I	V

Table 2.6: V versus I for a light bulb.

Resistor	
I	V

Table 2.7: V versus I for a resistor.

2.W2.2 In-Lab Computer Work

Using the computer, make a plot of the voltage versus the current for both sets of measurements. Make a curve fit1 to the resistor graph and write down the slope and intercept with uncertainties. You will **each** need your own light bulb and resistor plots.

Resistor	
Slope	Intercept

Table 2.8: Slope and Intercept for the resistor V vs. I plot.

2.W2.3 In-Lab Procedure

Internal Resistance of a Dry Cell

Disconnect the power supply from the breadboard and connect one of the batteries to the breadboard power contacts. Build the simple circuit shown in Figure 2.27. Make measurements of the voltage *across the battery* and the current through the resistor for ten different values of resistance; and enter these values into Table 2.9. Use different resistors and, if necessary, series and parallel combinations of the resistors to make sure you get ten data points. It isn't necessary to note the resistances you used, all we want to do is vary the load on the battery. You, however, do have to be exceedingly precise in your voltmeter measurements; therefore, choose the most precise scale.

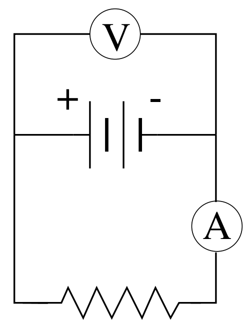


Figure 2.27: Internal resistance measurement.

Dry cell	
I	V

Table 2.9: V versus I for a dry cell.

2.W2.4 In-Lab Computer Work

Plot the voltage versus the current and obtain the slope and intercept of a curve fit1 to the plot. Once again each student needs individual printouts.

Dry cell	
Slope	Intercept

Table 2.10: Slope and Intercept for the dry cell V vs. I plot.

2.W2.5 Part II Pre-Classroom Check List

This check list is intended to be a guide for you to prepare yourself for the classroom work. You cannot come back to lab during this hour, collaborate with colleagues, nor hand in the worksheet late. Make sure you have completed everything.

Pre-classroom Check List

- ◯ Table 2.5 completed with units and uncertainties
- ◯ Table 2.6 completed with units and uncertainties
- ◯ Table 2.7 completed with units and uncertainties
- ◯ Table 2.9 completed with units and uncertainties
- ◯ 3 Plots labeled completely and correctly
- ◯ Each student has her/his own plots and worksheet

2.W2.6 In-Classroom Calculations & Analysis

The following are not yes/no types of questions; provide the reasoning that leads you to your answer. Make sure that you explain clearly how particular features of the graphs are relevant to the answers to these questions.

Temperature Dependence of Resistance

What does the slope of a line from the origin to a point on your graph measure? (Use Ohm's law to determine this.)

What does the graph for the bulb measurements tell you about the temperature dependence of resistance?

What does this experiment (including the graphs) teach you about measuring the resistance of a conductor? (Hint: The light bulb filament is a conductor.)

Is this method preferable to that used in § 2.W1.1? (Hint: You were still measuring resistances but with a different method.)

Use the slope you obtained from the curve fit1 to determine another value for the resistance of the resistor.

$$R =$$

With the two resistance measurements from § 2.W2.1, you now have two experimental values and one nominal for the resistance of your resistor. What is the average and standard deviation of the three measurements?

Is the relative uncertainty in the average smaller than that in the individual measurements? Does this make sense?

Internal Resistance of a Dry Cell

What does the V-intercept of the graph measure?

What is the internal resistance, r, of the battery?

$$r =$$

Attach plots to the worksheet.

End Part II Worksheet

Chapter 3

Electron Dynamics

3.1 Introduction

In this lab we will study the behavior of charges moving in uniform electric and magnetic fields. We will discover that electrons in a uniform magnetic field move in circular orbits, and we will determine the functional dependence of the path radius on both the strength of the magnetic field and the strength of the potential used to accelerate the electrons. Using these relationships we will calculate the charge-to-mass ratio of the electron.

This measurement of the electron charge-to-mass ratio is a great feat. We cannot put a single electron on a scale to weigh it. There is however, another simple experiment, called the Millikan oil drop experiment, which determines the electric charge of a single electron. From these two measurements, it is possible to calculate the mass of the electron; it is $9.109\ 389\ 7(57) \cdot 10^{-31}$ kg, a small number indeed.

We will also find that our relationship between the path radius, accelerating potential, magnetic field strength, and charge-to-mass ratio will lead to having several beams of different path radii if we were to use particles of varying charges. That is, if we were to send into our apparatus a beam containing particles with two distinct values of electric charge, the magnetic field would split the beam up, one species of particle would move in a circular path of some radius, while the other would move in a circle of a different radius. We will be using only electrons in our beam for this experiment and we will find only one circular orbit in the magnetic field. We may deduce from this that all electrons have the same electric charge. This is also a result of fundamental significance.

3.2 Theory

3.2.1 References

Electric and magnetic fields are introduced in Serway, Chapter 23 (Electric Fields), and Chapter 29 (Magnetic Fields). Particularly relevant are the sections on the motion of charges in uniform fields: Section 23.7 (Motion of Charged Particles in a Uniform Electric Field), and Section 29.5 (Application of the Motion of a Charged Particle in a Magnetic Field).

3.2.2 The Physical Situation

Consider the situation posed in Figure 3.1, where electrons (the e^-) are boiled off a hot filament, then accelerated through an electric potential V, and finally enter a region of uniform magnetic field $\vec{B}$. The experimental

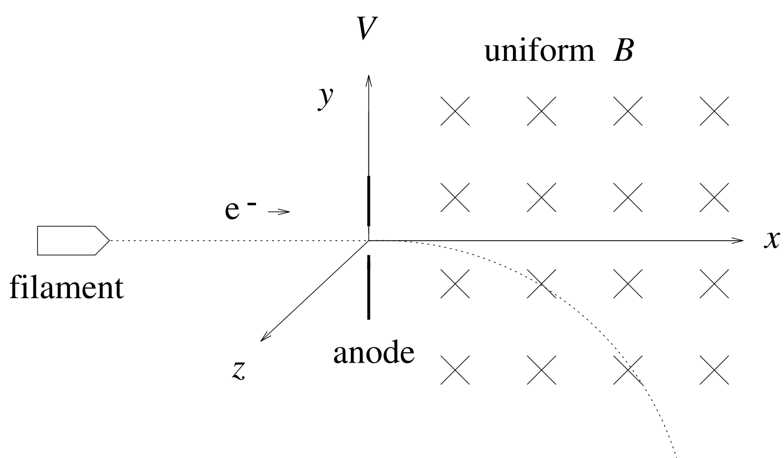


Figure 3.1: The electron beam.

apparatus we will use in the lab is designed to resemble this situation as closely as possible.

Let us assume that we may consider each electron independently. This is a big assumption, since we're neglecting any interaction between the electrons in our beam. We know that electrons have like charges, so they should repel one another. However, the accelerating potential and magnetic field in our apparatus are much stronger than this Coulomb interaction between pairs (or groups) of electrons. Therefore, we're not really making a bad assumption by treating the electrons independently. We can also gain a bit more generality if we forget for a while that we're actually dealing with electrons and we consider the more general situation of particles of some charge q in the apparatus. We can always get the specific result for the electron by taking $q = -e$.

3.2.3 Uniform Electric Field

We know that a particle of charge q and mass m in a constant electric potential V will accelerate linearly. Let the potential difference between the anode and filament in Figure 3.1 be

$$V = V_{\text{anode}} - V_{\text{filament}}.$$

The change in potential energy experienced by a particle accelerated from the filament to the anode will be

$$\Delta U = qV_{\text{anode}} - qV_{\text{filament}} = qV.$$

By conservation of energy, this change in potential energy must equal the increase in the particle's kinetic energy. If we assume that the electron came off the filament with zero initial velocity, then we have

$$\frac{1}{2}mv^2 = qV,$$

where v is the velocity the electron has when it reaches the anode. We can use this to determine the velocity from the charge, mass, and potential

$$v = \sqrt{\frac{2qV}{m}}. \tag{3.1}$$

3.2.4 Uniform Magnetic Field

The particle now enters the uniform magnetic field, which we take to be in the $-\hat{z}$ direction, into the page in Figure 3.1. The force due to the magnetic field is given by the Lorentz force law

$$\vec{F} = q\vec{v} \times \vec{B}. \tag{3.2}$$

The path of a charged particle moving through a magnetic field is illustrated in Figure 3.2. Since the velocity vector is always tangent to the particle's

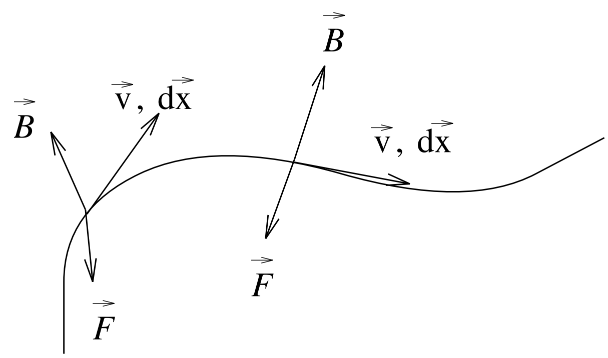


Figure 3.2: The magnetic field does no work.

path, the velocity $\vec{v}$ and the differential element $d\vec{x}$ always point in the same direction. Because of the cross product in (3.2), the force is always perpendicular to the magnetic field $\vec{B}$ and the velocity $\vec{v}$; therefore perpendicular to $d\vec{x}$ as well. Hence $\vec{F} \cdot d\vec{x} = 0$ at every point along the particle's path, so that the work done by the field between any two path points 1 and 2 is zero:

$$W_{\text{mag}} = \int_1^2 \vec{F} \cdot d\vec{x} = 0.$$

Since the magnetic field does no work, the kinetic energy of the particle cannot change once it passes the anode in Figure 3.1. Remembering the relationship between kinetic energy and velocity, we note that the magnitude of the particle's velocity must then be constant. Therefore: A magnetic field can change the *direction* of a particle's velocity, but not the *magnitude* of it's velocity.

This statement becomes more powerful in the case where the magnetic field is uniform. By uniform, we mean that the field is everywhere in the same direction and of constant magnitude. For the situation at hand, this amounts to saying that $\vec{B}$ has constant magnitude B and is in the -z-direction; or simply $\vec{B} = -B\hat{z}$, with B a positive constant. If we write out equation (3.2) in components (note the coordinates drawn in Figure 3.1), we find

$$\begin{aligned} F_x &= -qBv_y \\ F_y &= +qBv_x \\ F_z &= 0. \end{aligned}$$

Since the component of force in the z-direction is zero, if the particle starts out with its velocity along the x-axis, its motion must be confined to the (x, y)-plane; there's no force to "push" it out. The magnitude of the force

$$\begin{aligned} |\vec{F}| &= \sqrt{F_x^2 + F_y^2} \\ &= \sqrt{(qBv_y)^2 + (-qBv_x)^2} \\ &= |q|B\sqrt{v_x^2 + v_y^2} \\ F &= |q|vB \end{aligned} \tag{3.3}$$

is constant since B and v are constant. We have taken the absolute value of q because the magnitude of the force will always be positive, but the charge could be negative.

A force which is everywhere perpendicular to the motion and of constant magnitude is called a *centripetal* force. Such a force constrains the particle to move in a circle with radius R given by the condition

$$F = \frac{mv^2}{R}. \tag{3.4}$$

Figure 3.3 illustrates the circular path of a negatively charged particle (such as an electron) in a uniform magnetic field pointing into the page. A particle with positive charge would move in the opposite direction. If we set the expressions for the centripetal force (3.3) and (3.4) equal to each other, we find that the radius of the path is given by

$$R = \frac{mv}{|q|B}.$$

The velocity of the particle is just that imparted to it by the accelerating potential, given by equation (3.1). After some algebra, we find

$$R = \sqrt{\frac{2mV}{|q|B^2}}.$$

It is easy to see that, as was claimed in the introduction, particles with different charges will move in circles of different radii. What happens for particles of the same charge but different masses?

We can apply this to our particular case, where we only have electrons in our beam. Electrons all have the same charge, $q = -e = -1.602\ 177\ 33(49) \cdot$

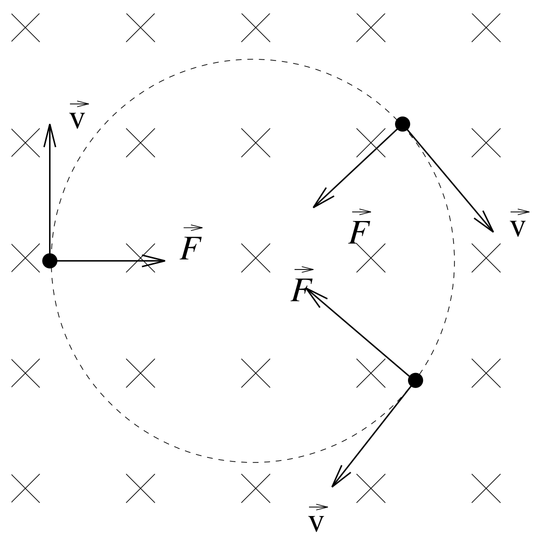


Figure 3.3: A centripetal force results in a circular orbit.

10^{-19} C, so that there will be one path, of radius

$$R = \sqrt{\frac{2mV}{eB^2}}.$$

We can equivalently write

$$\boxed{R^2 = \frac{2mV}{eB^2}} \tag{3.5}$$

which illustrates two important relationships: if we fix V, R^2 depends linearly on the quantity $1/B^2$; if we instead fix B, R^2 depends linearly on V. In either case, the slope of such a line can be used to obtain the fundamental quantity we are setting out to measure, the charge-to-mass ratio of the electron. The accepted value for this quantity is:

$$e/m = 1.758\ 819\ 62(53) \cdot 10^{11}\ \mathrm{C/kg}.$$

3.3 Apparatus

The apparatus we will use appears in Figure 3.4. We represent the electron gun schematically in Figure 3.5. The filament is heated to a temperature in excess of 1500 K. At this temperature, electrons are "boiling" off of the metal. The electrons are then accelerated by the electric potential maintained between the anode and the filament. This potential may be varied and its value can be read off on the voltmeter mounted on the apparatus. A small hole cut in the anode allows some of the electrons to pass through, where they are then focused into a beam by a set of electrodes. This electron gun is much the same as those in televisions and computer monitors. In those, the electrons are focused onto a phosphor screen, which emits light which appears as images on the screen. Here we send our electrons into a magnetic field.

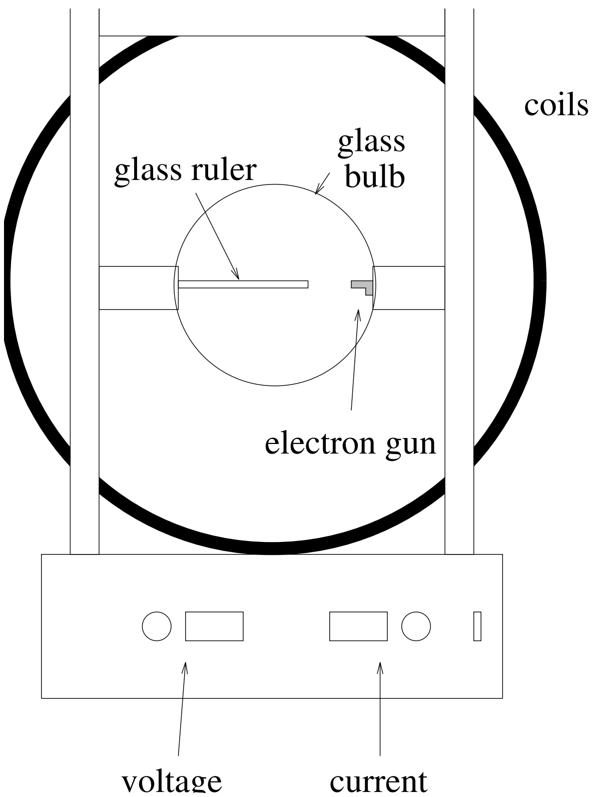


Figure 3.4: The e/m apparatus.

The electron gun is mounted within a pair of Helmholtz coils, depicted in Figure 3.6. Helmholtz coils are electromagnets in which the magnetic field is extremely uniform at the center of the coils; the Helmholtz condition is that the coils be separated by a distance equal to their radius. For the Helmholtz coils on the apparatus we're using, you will verify with a compass

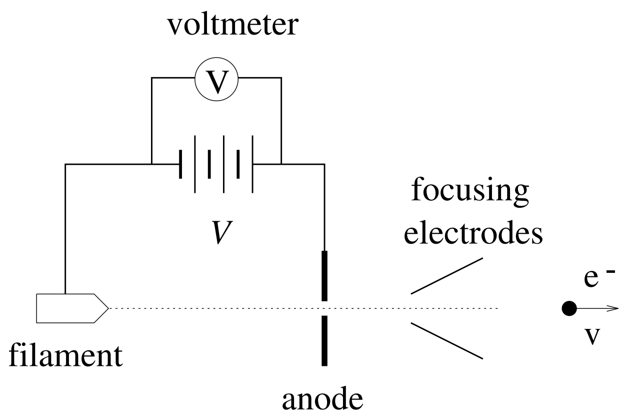


Figure 3.5: The electron gun.

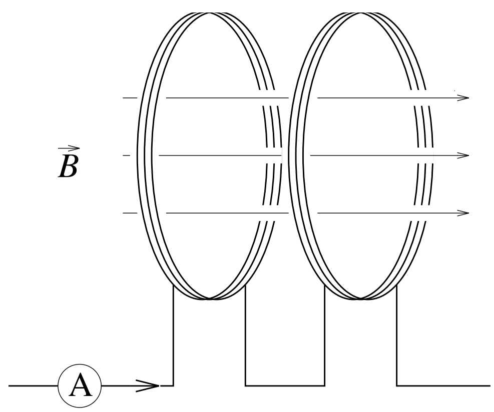


Figure 3.6: Helmholtz coils.

that the field is uniform in the region of the glass bulb, which contains the electron gun. By adjusting the current I through the coils, you can control the magnitude of the magnetic field; the current may be read off of the ammeter mounted on the apparatus. If we apply the equation for the magnetic field near the center of a coil to our Helmholtz coil apparatus, we find that the magnetic field in the region of the bulb may be calculated from the relationship

$$\boxed{B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 N I}{a},} \tag{3.6}$$

where N is the number of turns in each coil, a is the coil radius, and $\mu_0 =$

$4\pi \cdot 10^{-7}$ N/A^2.

The glass bulb is filled with helium gas. Some of the electrons in the beam collide with the helium atoms; when this happens, the atoms emit blue light. This light will allow you to see the electron path clearly. Serway, Figure 29.15, p. 817, illustrates the blue light quite nicely. A glass ruler mounted inside the bulb allows you to measure the diameter of the circular paths, from which you can calculate the radius.

3.W Electron Dynamics Worksheet

Name: ____________ **Day/Time:** ____________
Partner's Name: ____________

3.W.1 In-Lab Procedure

Remember that all measurements and calculations should include units and uncertainty.

The Magnetic Field of the Coils

Measure the radius, a, of one of the coils used in the apparatus. (You might find it easier to measure the diameter and then divide by 2.) As an estimate of the number of turns in a typical coil, we will take

$$N = 130 \pm 5.$$

What was your measurement of the coil radius with uncertainty?

$a=$ ____________

Turn on the apparatus and wait for it to warm up. Set the current through the coils to a moderate value (the maximum 3 A is preferable). Using the compass, you will make a rough sketch of the field produced by the coils. The method is explained now.

Since the Helmholtz coils are cylindrically symmetric, you should only sketch the field in the plane that forms the horizontal diameter, *i.e.*, as in the Figure 3.7. The *compass needle will point in the direction tangent to the field*, as in Figure 3.8. By sketching the compass directions as small arrows on paper, you can trace out the field lines by using the arrows as tangents to the curves. You should do this in *pencil*. When sketching the field lines, pay particular attention to the region of the glass bulb. Use Figure 3.9 to make your sketch of the magnetic field. This figure is from the viewpoint of looking down at the coils. Draw on Figure 3.9 now.

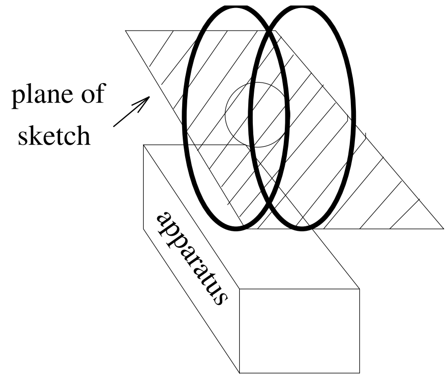


Figure 3.7: How to sketch the magnetic field.

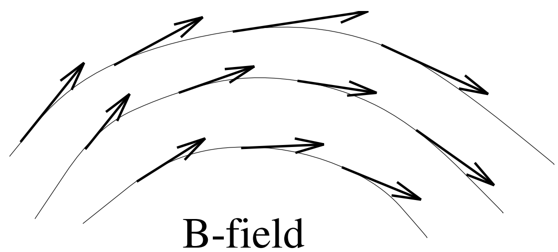


Figure 3.8: The compass needle points in the direction tangent to the field.

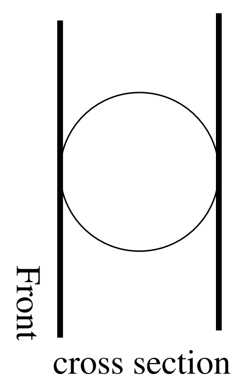


Figure 3.9: Sketch the field of the Helmholtz coils above.

Radius versus Magnetic Field

Set the accelerating voltage V to 300 V and measure the electron path radius R as a function of the magnetic field B by varying the coil current I. Remember that what you actually measure with the apparatus is the diameter. You will need to calculate the radius yourself. The magnetic field is calculated from equation (3.6) and the information gathered in § 3.W.1. Enter your 8 sets of data for current and radius (note that the ruler in the glass bulb measures diameter) into Table 3.1 with **units and uncertainties**.

Accelerating Voltage $V =$ ____________

Varied Current	
I	R

Table 3.1: I and R for varying magnetic field.

For the digital displays only, you may take the uncertainty to be a one in the last decimal place. For example, if you read a voltage as 142 V, then the uncertainty is 1 V. Report the measurement in the proper form; for the example, this would be 142 ± 1 V. You must estimate the uncertainty in the diameter measurement yourself. Be sure to take into account the precision with which you can read the scale and the width of the electron beam when doing this.

3.W.2 In-Lab Computer Work

You are now going to plot R^2 vs. $1/B^2$ and fit the line. Remember from the computer introduction classes that you can have the computer do most of the work for you. You can just enter the coil radius value from § 3.W.1, the current values, the path radius values, and all of the associated uncertainties into the computer; it will do all of the necessary calculations (calculate B, etc.) to give you this graph and will calculate the slope and R^2 intercept using a linear least squares routine.

The uncertainty for this graph is $\Delta(B^{-2})$. Perform the following calculations to help you get through the calculation of $\Delta(B^{-2})$.

Write down the formula for B.

$$B =$$

Plug in your first set of numbers from Table 3.1 and the constants to calculate a number for B now.

$$B = \underline{\hspace{3cm}}$$

Calculate $\Delta(B)$ **without numbers**. Show work.

$$\Delta(B) =$$

Plug in your first set of numbers to calculate a number for $\Delta(B)$ now.

$$\Delta(B) = \underline{\hspace{4cm}}$$

Calculate $\Delta(B^{-2})$ **without numbers**. Show work. Hint: You already know B and $\Delta(B)$.

$$\Delta(B^{-2}) =$$

Plug in your first set of numbers to calculate a number for $\Delta(B^{-2})$ now.

$$\Delta(B^{-2}) = \underline{\hspace{4cm}}$$

Now you can plug in the equation for B and $\Delta(B^{-2})$ into Kaleidagraph. You also have already done a sample calculation. **Always do a sample calculation to check the validity of what you typed into the computer.** Now plot R^2 vs. $1/B^2$ and fit the line. Write down the slope (with units and uncertainty) below.

$$S_1 = \underline{\hspace{4cm}}$$

From equation (3.5) the slope of the line should be

$$S_1 = \frac{2Vm}{e}.$$

3.W.3 In-Lab Procedure

Radius versus Potential

Now set the coil current to a fixed value (about 2 A) and measure the path radius as a function of the accelerating potential for at least 8 data points. Enter your data for voltage and radius into Table 3.2. Remember your units and uncertainties.

Current Setting $I=$ ____________

Varied Voltage	
V	R

Table 3.2: V and R for varying voltage.

3.W.4 In-Lab Computer Work

Plot R^2 vs. V and fit the line. Write down the slope (with units and uncertainty) below.

$$S_2 = \underline{\hspace{3cm}}$$

Also from equation (3.5) we have that the slope of this line should be

$$S_2 = \frac{2m}{eB^2}.$$

3.W.5 Pre-Classroom Check List

This check list is intended to be a guide for you to prepare yourself for the classroom work. You cannot come back to lab during this hour, collaborate with colleagues, nor hand in the worksheet late. Make sure you have completed everything.

Pre-classroom Check List

- ◯ Table 3.1 completed with units and uncertainties
- ◯ Table 3.2 completed with units and uncertainties
- ◯ S_1 with uncertainty and units
- ◯ S_2 with uncertainty and units
- ◯ 2 Plots labeled completely and correctly
- ◯ Each student has her/his own plots and worksheet

3.W.6 In-Classroom Calculations & Analysis

The Magnetic Field of the Coils

Is the magnetic field uniform inside the glass bulb?

From the direction of the electron velocity and the centripetal force, determine the direction of the magnetic field inside the glass bulb. Does this agree with your drawing in Figure 3.9.

Radius versus Magnetic Field

Using the slope S_1 and V with their uncertainties, calculate a value for e/m and its uncertainty. **SHOW WORK.**

$(e/m)_1 =$ ____________

Radius versus Potential

Using the slope S_2 and B (*note that you have to calculate the latter*) with their uncertainties, calculate a value for e/m and its uncertainty. **SHOW WORK.**

$(e/m)_2 =$ ____________

3.W.7 In-Classroom Discussion

Compare both values of e/m you measured to the accepted value $1.758\,819\,62(53) \cdot 10^{11}$ C/kg, as well as to each other. Also compare your observations and the results of your analysis with the objectives set out for the lab in the introduction, Section 3.1. Did you observe everything you expected? Do this in the space provided. You may write on the back of this page if necessary or staple a labeled extra sheet.

Attach plots to the worksheet.

End Worksheet

Chapter 4

Measurements with the Oscilloscope

4.1 Introduction

We've already seen some of the properties of DC circuits; as you'll recall, in a DC circuit, the current and voltage values are steady. A much more general situation is where the voltage and current values change over time, as is the case with the power supplied by the wall outlets in our homes and in our laboratory. The voltage and current supplied by a wall outlet does not change arbitrarily, however. They are examples of *periodic* signals, they repeat themselves over time; for outlets in the US, the frequency of oscillation is 60 Hz.

Since these periodic signals are so important to us (our appliances, computers, TV's, etc. use them), we will spend a few weeks studying examples of AC circuits with varying types of periodic signals. We'll learn some crucial fundamental principles along the way. It is important that we learn how to make measurements with such circuits; this will be our goal for this week's lab. The device we'll use, an oscilloscope, will display on its screen a visual representation of the signal produced by the changing voltage across a part of our circuit. We can measure the period of a signal, read off voltage values at specific times along the signal, and display two signals so that we can compare them. Viewing two signals will be important when we want to learn how the output from a specific device, such as an inductor or capacitor, depends on the characteristics of the input signal.

4.2 Theory

4.2.1 References

The properties of waves are discussed in Serway, Chapter 16 (Wave Motion).

4.2.2 The Properties of Waves

As we mentioned in the introduction, a signal which repeats itself after a certain amount of time is called *periodic*; such a signal is shown in Figure 4.1. The amount of time it takes for the signal to repeat is called the *period* of

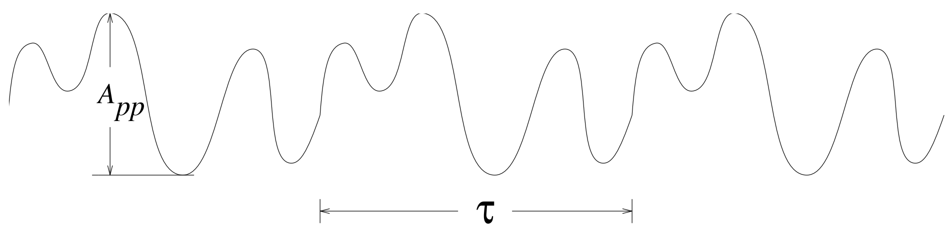


Figure 4.1: A periodic, albeit weird, signal.

the signal and is denoted T. The *rate* at which the signal repeats itself is called the *frequency* and is denoted by f. It is easy to see that $f = 1/T$. The maximum "height" between the peaks of the signal is called the *peak-to-peak amplitude* and denoted A_{pp}. What we more commonly refer to as the *amplitude* is half of A_{pp}.

The periodic signals we will study are often referred to as *waves*, due to their relationship to the physically important solutions to the *wave equation*. The first type of wave we'll discuss is the sinusoid in Figure 4.2. The mathematical functions that describe sinusoidal waves are sine and cosine. They can be expressed in the form

$$F(t) = A\sin(\omega t + \phi)$$

or

$$F(t) = A\cos(\omega t + \theta),$$

where ω is the *angular frequency*, defined by $\omega = 2\pi f$, and ϕ and θ are constants called *phase angles*. The phase angle is an artifact of the time-coordinate we choose to define the sinusoid; it tells us where the zeros of the

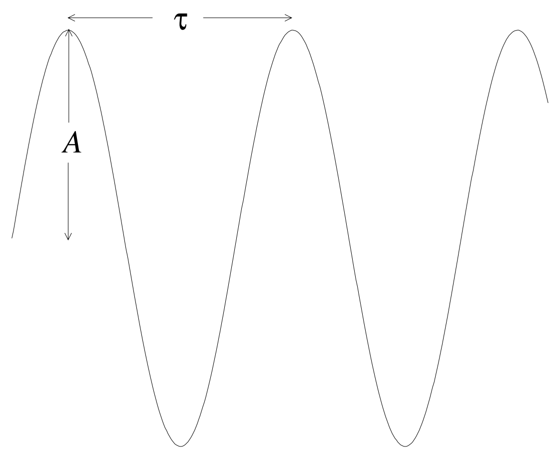


Figure 4.2: A sinusoidal wave.

wave are located. Since the sine and cosine functions take a maximum value of 1, we see that A is the *amplitude* of

the wave.

The concept of phase is an important one; we'll learn how to measure the *difference* in phase between two waves in the lab. Let's examine an example: the phase difference between sine and cosine waves of the same frequency, illustrated in Figure 4.3, where

$$\begin{aligned} F_1 &= A_1 \sin \omega t, \\ F_2 &= A_2 \cos \omega t. \end{aligned}$$

We see that these two waves differ along the time axis; that the amplitudes are different does not matter; the maxima, minima, and zeros of the two waves will always differ by a distance in *time*, Δt, called the *phase difference* between 1 and 2. We note that it is important that the two waves are of the *same* frequency. If we were dealing with waves of different frequencies, as in Figure 4.4, there's no reason for the maxima (or minima, or zeros) to be separated by a *uniform* amount. The concept of phase difference is, therefore, meaningless unless we are discussing waves of the same frequency.

Let's see how phase difference relates to the concept of phase angle. Again we'll study the sine and cosine example. Note that, from the trigonometric

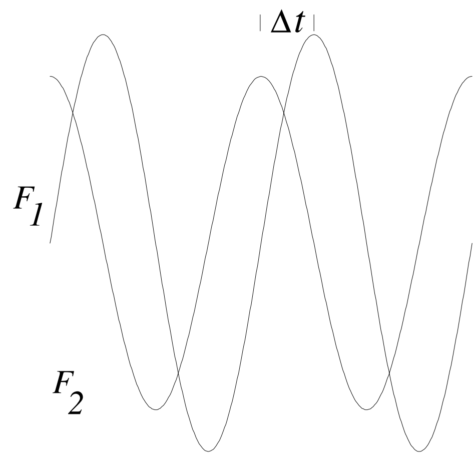


Figure 4.3: Sine and cosine waves differ in phase.

identity

$$\sin(A+B) = \sin A \cos B + \cos A \sin B,$$

we can write

$$\cos \omega t = \sin(\omega t + 90^\circ),$$

where 90° is a phase angle. We can therefore write

$$\begin{aligned} F_1 &= A_1 \sin \omega t \\ F_2 &= A_2 \sin(\omega t + 90^\circ). \end{aligned}$$

These are of the same functional form (though of differing amplitudes), but differ in that the phase angle of F_2 is $\phi_2 = 90^\circ$, while that of F_1 is $\phi_1 = 0$. In a more general case, we might have

$$\begin{aligned} F_1 &= A_1 \sin(\omega t + \phi_1) \\ F_2 &= A_2 \sin(\omega t + \phi_2); \end{aligned}$$

the *angular phase difference* between 1 and 2 is defined to be

$$\phi = \phi_2 - \phi_1.$$

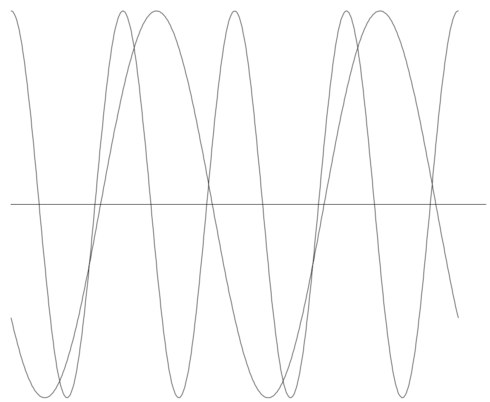


Figure 4.4: The concept of phase difference does not apply to waves of different frequencies.

We note that a similar definition exists if we have cosine, rather than sine, waves. As an exercise, the student should verify that the time shift, Δt, is related to the angular phase difference, ϕ, by

$$\boxed{\frac{\phi}{2\pi} = \frac{\Delta t}{T}.} \tag{4.1}$$

Another type of wave that we'll find important is the *square wave*, illustrated in Figure 4.5. Functionally, we can write

$$F(t) = \begin{cases} A & 0 < t < T/2 \\ -A & T/2 < t < T. \end{cases}$$

The concept of phase difference (but not phase angle, this isn't a trigonometric function!) can clearly be applied to two square waves of the same frequency, see Figure 4.6. Finally, we also have the *triangle wave* in Figure 4.7. We leave it to the interested student to write the triangle wave down in mathematical form. Note that, once again, phase difference is meaningful for triangle waves of equal frequency.

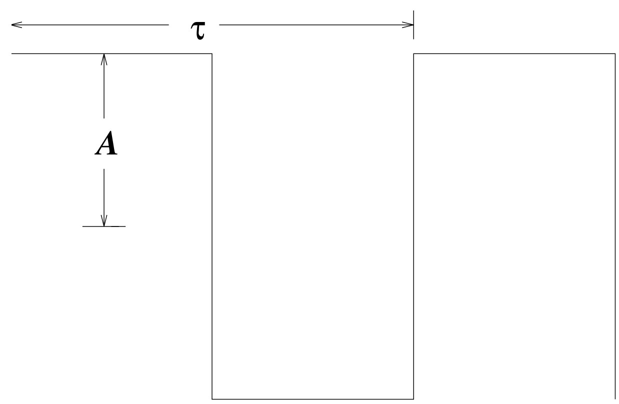


Figure 4.5: A square wave.

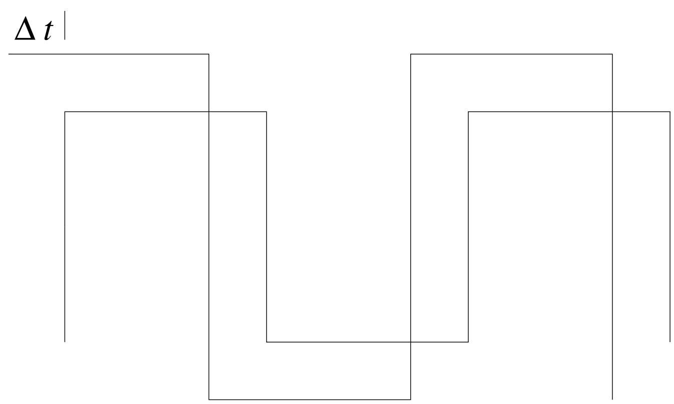


Figure 4.6: A phase difference between two square waves.

4.3 Apparatus

4.3.1 The Oscilloscope

The oscilloscope is a device which displays a voltage signal, constant or time-varying, periodic or otherwise, as a function of time. What this amounts to is an extremely enlightening picture of what is happening in our circuit. The ability to adjust the characteristics of how the oscilloscope displays our signal makes it an extremely versatile tool. An illustration of the oscilloscope we'll be using appears in Figure 4.8.

How does the oscilloscope work? Everything starts with a signal fed

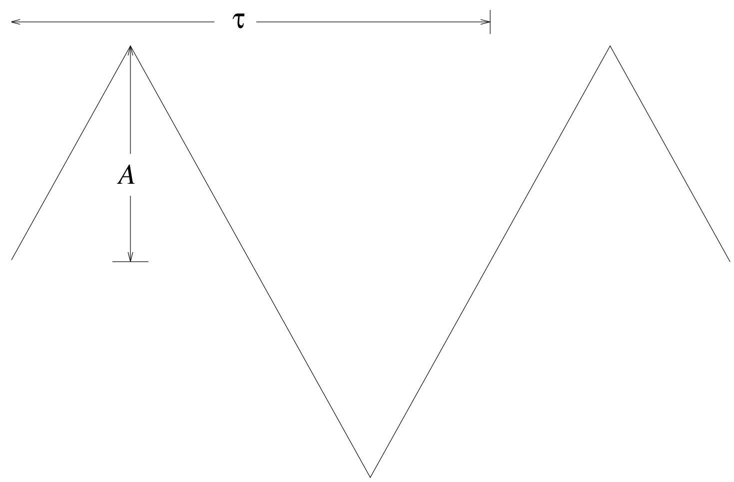


Figure 4.7: A triangle wave.

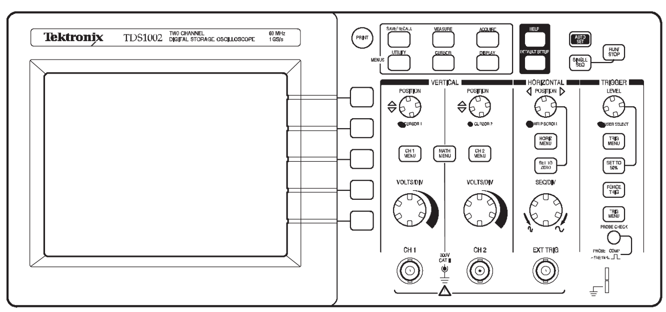


Figure 4.8: The Tektronix model TDS1002 oscilloscope.

into one of the channels of the oscilloscope. The idea is to get a voltage versus time representation of that signal on the screen of the instrument. The screen of the scope is therefore scanned from left to right at a constant speed and the voltage at each instant is displayed as a vertical deviation from the horizontal axis. Since the screen has a limited width, this scan will end in a finite amount of time. Then, to keep giving current information on our signal, the instrument has to erase the screen and start scanning again. Now imagine that happening many times in a second. In general, every scan would create a different V vs. t graph and we would just see a blur of many different waveforms displayed rapidly one after another. For periodic signals, this situation can be amended if the scanning always starts from the same position on a cycle of the wave. Imagine for instance a sinusoidal waveform. If the oscilloscope could somehow "understand" that it should always start scanning when the voltage crosses zero, going from negative to positive values, then the same waveform would be presented over and over again on the screen. We would be looking then at a stable picture, like the one in Figure 4.2. This is a picture we can study.

To make the oscilloscope "understand" where on a signal it should start scanning, we use a logic circuit, called the *trigger*. More formally, a trigger is configured to set a time origin, for each scan of the acquired signal. The signal displayed on the screen starts a set amount of time before this time origin. There are many ways to implement triggering. In this lab, we are going to be using *edge triggering*. In this method, the trigger is waiting for the signal to raise or drop to a set value, called the *trigger level*. The acquired signal is displayed on the screen, so that this event always determines the horizontal axis origin (the time $t = 0$, which doesn't necessarily coincide with the horizontal axis midpoint). The example with the sinusoidal wave (see previous paragraph) illustrates edge triggering.

How do we make measurements with the oscilloscope? The oscilloscope display, as can be seen in Figure 4.9, provides a system of axes, each one of which has large and small divisions on it. The vertical direction indicates the voltage value of the input at a given time; each of the eight large divisions indicates a number of volts given by the V/div scale factor on screen. For example, if we measure the amplitude of the signal displayed to be 3.6 div (ignoring uncertainty for the moment) and if the V/div scale factor is set at 0.5 V/div, then the amplitude, in volts, is

$$(3.6 \text{ div})(0.5 \text{ V/div}) = 1.8 \text{ V}.$$

We should always use the V/div scale factor so that we report the amplitude, and any other voltage values we measure, in units of voltage. The horizontal direction indicates time; there are ten large divisions. If we measure the distance between the maxima of a sinusoidal wave to be 5 div and the sec/div scale factor reads 5 ms/div, then the period is

$$(5 \text{ div})(5 \text{ ms/div}) = 25 \text{ ms}.$$

What about the uncertainty in our measurements made with the oscilloscope? Note that, since the oscilloscope screen is ruled, the finest scale marking may be used to estimate the uncertainty in all of the measurements we make. Estimating the uncertainty in this manner is precisely the same thing we do when we're measuring length with a ruler. Remember that the uncertainty expressed in divisions, like the quantity itself, must be multiplied by the proper scale factor to obtain an uncertainty expressed in the appropriate units.

With a digital oscilloscope, like the one we will be using, measurements can be facilitated by screen aids. In this lab, these aids take the form of pairs of horizontal and vertical lines, called *cursors*. The position of each line can be adjusted by the user, and is displayed, along with their separation, on the oscilloscope screen. By using one line as a voltage or time reference, we can use the other, to find the voltage or time at any point on the displayed signal. Since all cursor positions and distances from each other are given on the screen as digital readings, the uncertainty for each number will be its increment of change.

In this lab we will be using the Tektronix TDS1002 digital oscilloscope. The interface of this instrument with its operator comprise: a set of single function knobs (the large VOLTS/DIV and SEC/DIV knobs) and buttons (all the labeled buttons), a set of multi-function knobs (the smaller POSITION and LEVEL knobs) and buttons (the unlabeled, option buttons on the side of the screen), as well as the screen-displayed information and menus. The following description focuses on controls and display items that are going to be needed for our experiments.

1. **The Display**

   A typical oscilloscope display is shown in Figure 4.9. The indicators of interest to us are:

   3. Trigger position (time origin).
   4. Center of horizontal axis position. Observe that the time origin does not coincide with the center of the time axis. The HORIZONTAL POSITION knob can move the origin, and therefore the waveform, on this axis.
   5. Edge trigger level.
   6. Ground reference levels for each channel. If one of these indicators is missing, the corresponding channel is not displayed.
   8. Vertical scale factors for each channel. The values correspond to the large divisions of the vertical axis

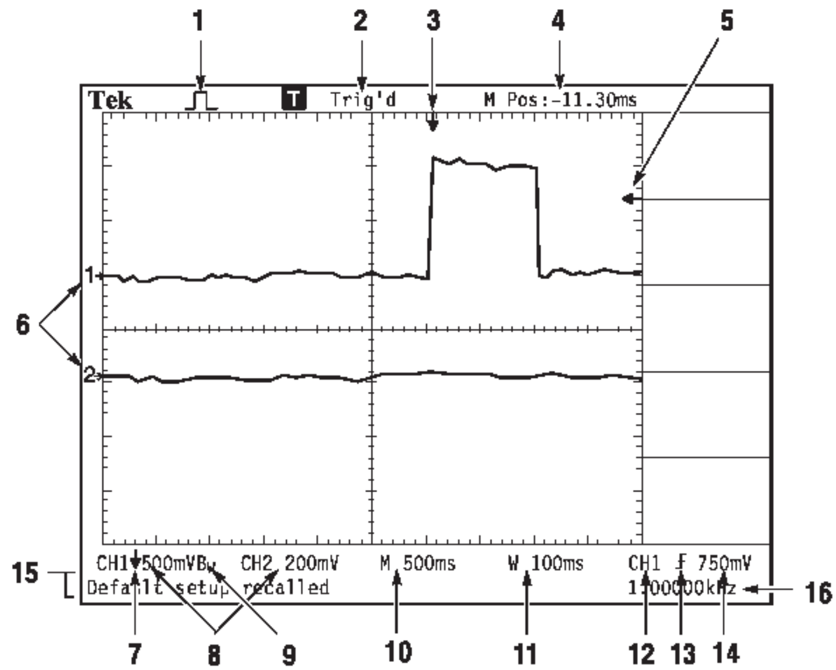


Figure 4.9: The Tektronix model TDS1002 oscilloscope display.

10. Horizontal scale factor. The value corresponds to the large divisions of the horizontal axis. Observe that the time scale is the same for both channels.
16. Trigger frequency. This should in general agree with the frequency of our signals. It may be used as a verification of our calculations, but not as a direct measurement.

On the right side of the screen, there is a dynamic menu, indicating the function of the adjacent, unlabeled, option buttons. These functions change, every time a different menu button is pressed on the control panel.

2. **Vertical Controls**

   VOLTS/DIV knobs: They adjust the vertical axis scale factor for each channel.

   POSITION - CURSOR knobs: They adjust the ground reference level, in effect moving the corresponding waveform vertically on the screen. When the cursor menu is active, the LEDs below them are illuminated, indicating that the knobs now set the cursor line positions.

CH MENU: Display the vertical menu and toggle the corresponding channel on and off. The vertical menu for each channel contains two functions that we will find useful.

(a) **Coupling:** Determines what part of a signal is shown on the screen The possible choices here are:

AC: Only the alternating part of the signal is displayed. Any constant (DC) offset is filtered out.

DC: The signal is displayed unfiltered.

GND: The input is connected to a zero voltage reference. The oscilloscope output is a horizontal line at the ground reference level.

(b) **Probe:** Sets multiplicative factors for the acquired voltages. This is done to accommodate the use of special probes. **The default setting is 10X. You must switch to a 1X setting, since we are not using a special probe for our measurements.**

3. **Horizontal Controls**

It should be noted that these controls affect settings which are common to both channels.

SEC/DIV knob: It adjusts the horizontal axis scale factor for both channels.

POSITION - HELP SCROLL: In all but the help mode, this knob adjusts the horizontal position of the displayed signals on the screen. It does that by varying the position of the center of the horizontal axis with respect to the time origin.

SET TO ZERO: Makes the time origin coincide with the center of the horizontal axis.

HORIZ MENU: Displays the horizontal menu. The only function of potential use to us here is the **Trig Knob** option. This allows us to toggle the TRIGGER LEVEL knob between its default function of setting the trigger level and a secondary function of adjusting the trigger holdoff. The trigger holdoff is the smallest possible interval between two successive triggerings of the oscilloscope. It may be used to stabilize a waveform on the screen.

4. **Trigger Controls**

   The controls we are most commonly going to use are:

   TRIGGER LEVEL: Sets the voltage that the trigger signal has to cross, for triggering to occur.

   TRIG MENU: Displays the trigger menu. There are three options that we may find useful.

   (a) **Edge/Video/Pulse:** Defines triggering type. By default, edge triggering is, and should remain, selected.

   (b) **Source:** Allows us to select the source of the triggering signal. You may have to adjust it, to make sure it corresponds to your reference channel (CH1 or CH2).

   (c) **Slope:** Determines whether the trigger level has to be crossed while the signal rises or falls. This control allows us to get the desired portion of the signal displayed.

5. **Menus**

   CURSOR: This is the only menu we are going to use from this group. The cursors are pairs of vertical or horizontal lines that allow us to perform time or voltage measurements respectively. The items on the on-screen menu are described below.

   (a) **Type:** Toggles between voltage, time, or no cursors.

   (b) **Source:** Selects the channel for which measurements are to be performed.

   (c) **Delta:** The difference in the position of the two cursors.

   (d) **Cursor 1:** Cursor 1 position. The time origin is set by the trigger, while the voltage origin is the ground reference level.

   (e) **Cursor 2:** Cursor 2 position. Time and voltage origins are the same as for Cursor 1.

6. **DEFAULT SETUP**

   The DEFAULT SETUP button should be used at the beginning of each lab as well as whenever you feel uncomfortable or unsure about the oscilloscope settings. The factory defaults provide a good start and protect you from time-consuming guesswork. You should note that, by default, the oscilloscope uses a 10X multiplier for its inputs. **YOU HAVE TO MANUALLY SELECT THE 1X SETTING FROM THE CH1 AND CH2 MENUS EACH TIME YOU RECALL THE DEFAULT SETUP.** Failure to do so will lead to erroneous measurements.

4.3.2 The Function Generator

The function generator is designed to produce waveforms of variable shape, frequency, and amplitude; an illustration of a typical one appears in Figure 4.10. The function generator controls are as follows:

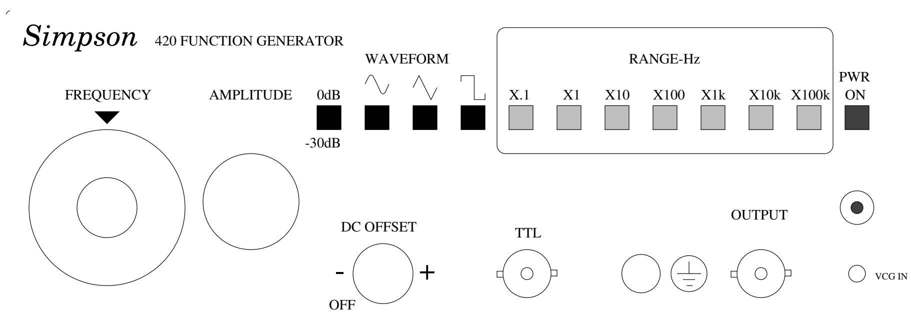


Figure 4.10: The function generator.

1. **power**: This button just turns the function generator on and off.

2. **output**: This BNC connection is the signal output.

3. **waveform**: These buttons select between sinusoidal, triangle, or square wave output.

4. **amplitude**: This adjusts the amplitude (voltage value) of the output signal.

5. **0 dB/-30 dB**: This button selects between two output voltage ranges. We'll want to keep this on the 0 dB setting, for a strong output signal.

6. **frequency**: This allows for fine tuning of the frequency output. The maximum on the dial corresponds to the value indicated by the range setting switches (see below.) The frequency read from the function generator should be regarded as a strictly nominal value. Since we generally want a frequency value that we can trust, we will *always* be sure to measure the frequency with the oscilloscope; we don't want to place very much trust in the nominal value.

7. **range**: These buttons select between available frequency ranges. In our case, from 0.2 Hz to 2 MHz.

8. **DC offset**: This provides a DC component to the output signal. Since we'll only be interested in AC signals, we'll leave this set to *off*.

4.3.3 The Phase Shifter

The phase shifter consists of a variable resistor and two capacitors, illustrated in Figure 4.11. The knob on the phase shifter box controls the amount of

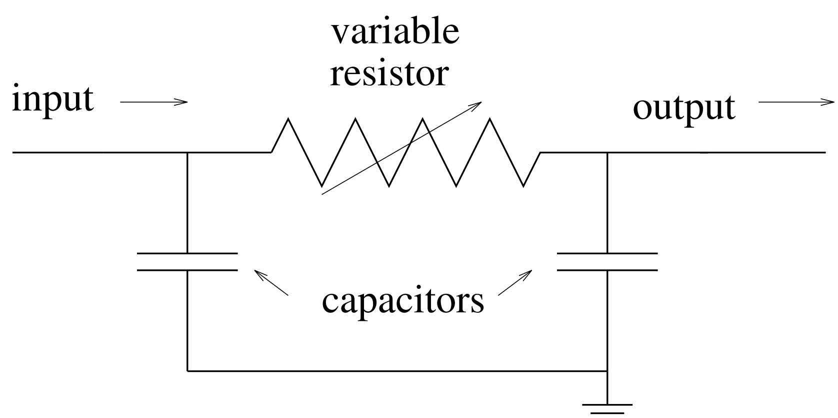


Figure 4.11: The phase shifter circuit.

resistance provided by the variable resistor. When we study RC circuits in the next lab, we'll learn that these circuits change both the *amplitude* and *phase* of an AC input, but not its frequency. For now, we can ignore the details of why this happens; we'll simply use the phase shifter to change the phase of the output signal. By examining the input and output signals together on the oscilloscope, we'll learn how to measure phase differences between signals.

4.W Oscilloscope Worksheet

Name: ____________________ **Day/Time:** ____________
Partner's Name: ________________________

4.W.1 In-Lab Procedure

Turning on the Oscilloscope

Turn on the oscilloscope and wait until the trace appears. Reset the settings of the instrument by pressing the DEFAULT SETUP button. Don't forget that, every time you do that, you have to switch the Probe setting in *each* channel's menu to 1X. The procedure may sound tedious, but it will save you time and elliminate the frustration of guessing what's wrong with the settings of your instrument. Feel free to repeat it any time the readings of the oscilloscope don't make sense.

Operating the Equipment

Now that the oscilloscope has been reset, turn the function generator on and connect it to one of the channels. Set it to produce a sinusoidal wave of approximately 1 kHz; the amplitude should be set to an arbitrary non-zero value. Make sure that the channel's Coupling option is set to DC and that you are triggering on the channel that you've connected the input to. Adjust the V/div and sec/div knobs so that a single wave fills the screen, This means that no more than two cycles (and no less than one cycle) of the signal should appear and that the vertical size of the waveform should be as large as possible on the oscilloscope screen. The controls of the instrument are versatile enough to allow you to adjust the display so that you can comfortably make measurements of the amplitude and period of the signal.
Check that the sine wave appears to be symmetric about the horizontal axis; if not, you need to adjust the vertical position. To do this, you do not need to disconnect the input. Simply set the channel's Coupling option to GND and use the vertical position knobs to realign the trace with the axis. You might need to repeat this procedure again later, since changing the V/div and sec/div scales can upset the ground. When you're done, be sure to return the Coupling option to DC.

Before making any measurements, let's experiment with the equipment a bit. Change the amplitude and frequency on the function generator and watch how the signal changes on the screen. Practice readjusting the oscilloscope scales so that you get only one or two cycles on the screen; readjust the ground when necessary. Now set the function generator to display a square wave. Again, experiment with different frequencies and amplitudes. Examine the *shape* of the signal carefully. You should have a single period filling the screen before examining the shape, i.e. a change in frequency does not constitute a change in shape.

Answer the following questions completely (not yes or no) stating your reasoning behind each answer.

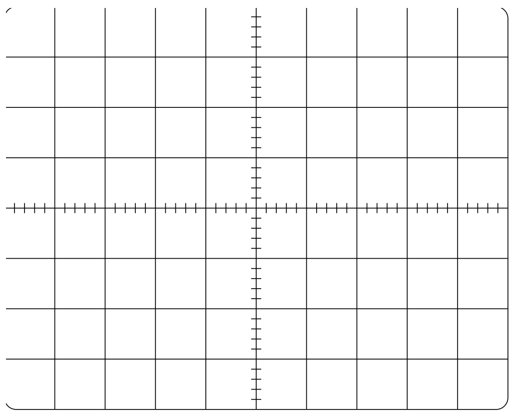
Question 1: Comparing with Figure 4.5, does the shape vary with the frequency? Illustrate as necessary.

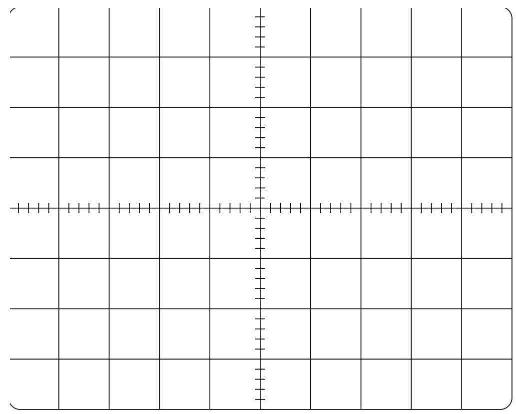
Question 2: Also examine the triangle waves produced by the function generator; compare the triangle waves to those illustrated in Figure 4.7. Does the shape vary with the frequency? Illustrate as necessary.

Question 3: Now make a definitive statement about the quality of the signals generated by your function generator. You will need to use this information as we continue with the labs.

Measuring Amplitude and Frequency

Reset the function generator to produce a sinusoidal wave and adjust the scope so that the signal fills the screen. You may find it easier to switch the coupling to AC for the remainder of this lab. Sketch the wave displayed onto the grid below. Use the second grid in case you make a mistake on the first one.





Record the frequency setting on the function generator in the **correct units** (Note that this is a nominal value and may be far from the accurate one):

Frequency setting: ______________

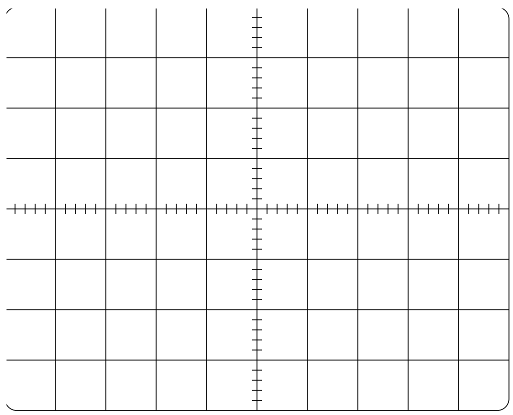
Also recored the V/div and sec/div scales.

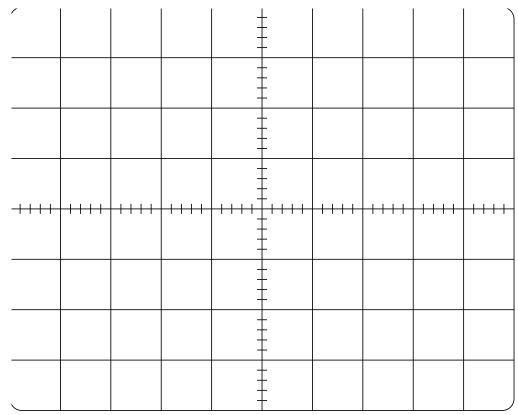
V/div: ______________ sec/div: ______________

Using the voltage and time cursors, measure the amplitude and period of the signal. The easiest and most accurate way to facilitate the cursors for your measurements is to record their distance (Delta) rather than their absolute positions. Therefore, if you are measuring period, switch the cursor Type option to time, place one cursor at the beginning of a cycle and the other at its end. Delta is then the period of the wave. The uncertainty for all cursor measurements will be the smallest increment of change in Delta. For the amplitude you should measure the *peak-to-peak amplitude* and divide by two; this eliminates any error that might creep in if the wave is not symmetric about the horizontal axis.

Amplitude: ______________ Period: ______________

Readjust both the voltage and time scales of the oscilloscope (*not* the function generator settings) so that the signal no longer fills the screen. Be sure that several cycles can be seen. Sketch the signal on the grid below.





Recored the V/div and sec/div scales.

V/div: ________________ sec/div: ________________

Measure the amplitude and period with units and uncertainties.

Amplitude: ________________ Period: ________________

Measuring Phase Difference

Let's examine the effect of the phase shifter. Use the BNC T-connector to split the output of the function generator. Connect one wire to channel 1 and the other to the input of the phase shifter. Connect the output of the phase shifter to channel 2; the circuit is illustrated in Figure 4.12. Set the

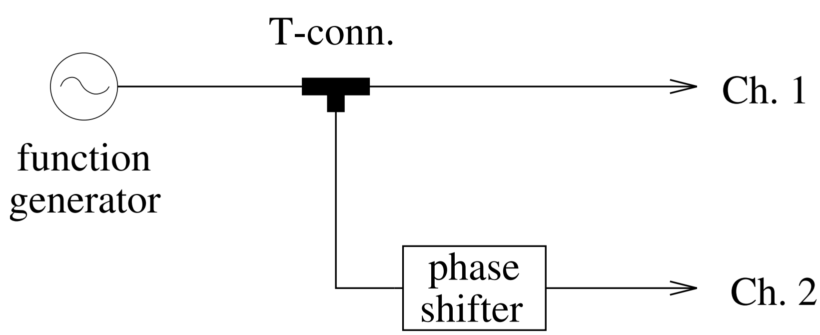
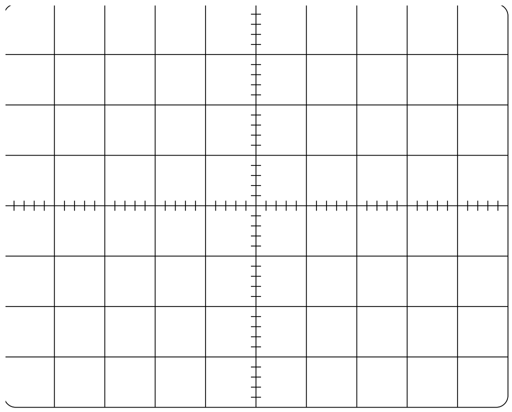
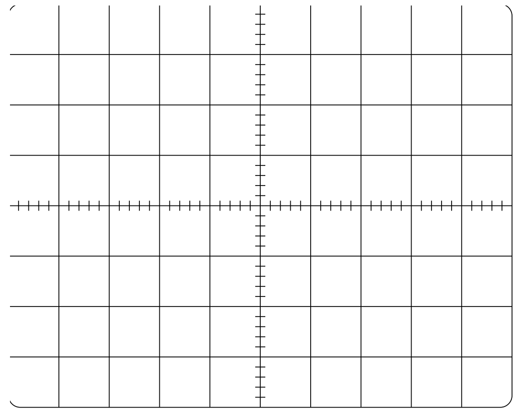


Figure 4.12: How to connect the phase shifter.

function generator to produce a sine wave at ∼5 kHz and set the phase shifter to its minimum value. Adjust the oscilloscope to display both signals and to trigger on channel 1 (the direct output of the function generator).

You'll probably have to set the V/div settings of each channel to different values so that both signals are displayed at optimal size. Sketch the two waveforms for the **minimum** setting of the phase shifter at ∼5 kHz. (Draw both on the same grid. Remember that the second grid is there just in case you make a mistake.)

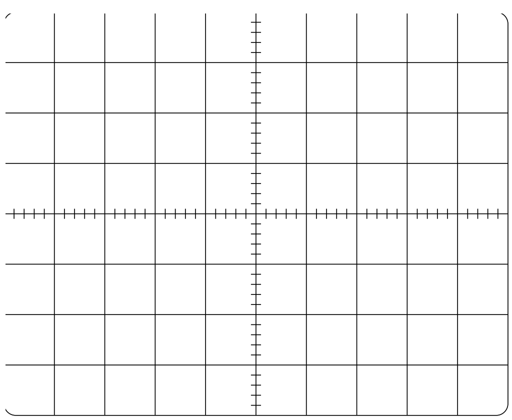
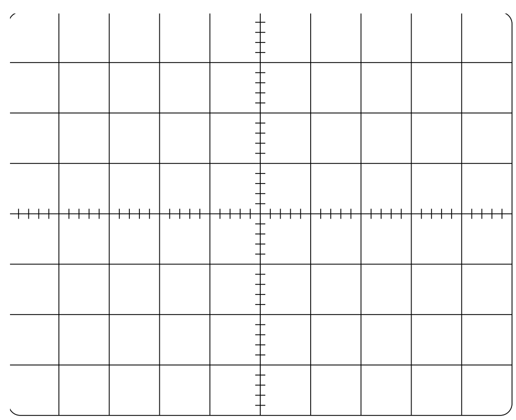



Measure the period T for both waves and the time shift Δt between them and mark them, in the proper units and with an uncertainty for each, below.

T (ch. 1): ________________ T (ch. 2): ________________

Δt: ________________

Now set the phase shifter to its **maximum** value and repeat the above sketch and measurements.

T (ch. 1): ________________ T (ch. 2): ________________

Δt: _______________

4.W.2 Pre-Classroom Check List

◯ Answered questions #1 − 3.
◯ Sketched 2 waveforms and found their respective amplitudes and periods with uncertainties.
◯ Sketched a wave and its phase shifted counterpart on the same grid for a minimum shift and found their periods and time shift.
◯ Sketched a wave and its phase shifted counterpart on the same grid for a maximum shift and found their periods and time shift.

4.W.3 In-Classroom Calculations & Analysis

Measuring Amplitude and Frequency

Calculate frequency from the period of the first waveform measured. Show your work for this and every calculation.

Frequency: _______________

Compare the measured frequency value with the nominal value from the function generator on your discussion sheet.

Repeat this calculation to determine the frequency of the second waveform measured with the contracted oscilloscope setting.

Frequency: ______________

How do these values compare to those made with the "larger" signal (the first one you sketched)? Specifically, how do the uncertainties compare; which are the more precise values?

If you require the most precise measurements possible, what should you make sure to do when you make measurements with the oscilloscope?

Measuring Phase Difference

Answer the following for the **minimum** setting of the phase shifter.
Are the periods equal, within uncertainty, *i.e.*, has the phase shifter changed the frequency of the input signal?

Now calculate the corresponding phase shift ϕ (in radians and degrees) with uncertainty using equation (4.1).

ϕ (min.) ________________

Now repeat the above for the **maximum.**

Are the periods equal, within uncertainty, *i.e.*, has the phase shifter changed the frequency of the input signal?

Calculate the phase shift ϕ (in radians and degrees) with uncertainty.

ϕ (max.) ________________

Now calculate the **range** ($\phi = \phi_{max} - \phi_{min}$) of the phase shifter with an uncertainty in radians and degrees:

Phase Shift Range: ______________

Make concluding remarks about the quality of your phase shifter. Does it function as expected? Compare the range which you calculated above to the range you expect the shifter to have. Cite all relevant data from your experiment.

End Worksheet

Chapter 5

RC Circuits and Filters

5.1 Introduction

You have probably noticed that large speakers for stereo systems have more than one actual speaker in them. Some are very big and provide the bass end, while others can be quite small, dishing out the higher frequencies. How do the electronics inside divide up the total sound signal into low and high frequency parts? Here we will find out the answer to this question and many others. The key component needed for this task is the capacitor.

First we will look at the DC response of a capacitor and find that it introduces a new parameter into the circuit that defines a time scale for charging and discharging phenomena. This time parameter is called the *RC time constant*. After thoroughly characterizing the direct current effects of the capacitor, we will then move on to oscillating circuits; here the interesting effects arise from the interplay of the two time scales, one associated with the period of the current oscillation and the other with the capacitor. This will lead us to the idea of a filter, and then we will characterize the two main types of filters, hi-pass and low-pass.

5.2 Theory

5.2.1 References

Serway discusses capacitance in Chapter 26 (Capacitance and Dielectrics), within the context of dc circuits. He discusses ac circuits in Chapter 33

(Alternating Current Circuits), and specifically capacitors in ac circuits in Section 33.4 (Capacitors in an ac Circuit). The ideas involved in filtering appear in Section 33.8 (Filter Circuits), but we will approach the subject from a different perspective. You should glance through these sections before proceeding further.

We will also run across some special types of equations, called differential equations, that we will need to solve. A good reference for the standard techniques is

> W. E. Boyce and R. C. DiPrima, *Elementary Differential Equations and Boundary Value Problems, 4th edition*, J. Wiley & Sons, New York, 1986.

5.2.2 DC Response of Capacitors

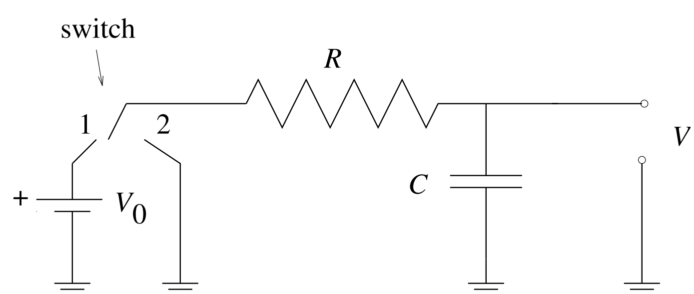


Figure 5.1: The basic DC capacitor circuit.

Consider the circuit that appears in Figure 5.1 and let the switch be in position 1. In this position, the circuit is charging the capacitor, since current will flow, carrying positive charge, from the voltage source to the capacitor; this will then force positive charges off the other plate into ground. Alternatively, electrons (negative charges) flow off of the top plate of the capacitor to the voltage source and from ground to the bottom plate; in either way of looking at it, charge builds up on the capacitor. As more and more charge accumulates on the plates of the capacitor, it will take more and more emf to push charges onto the plates. Thus, we expect this process to slow as time continues; *i.e.*, the current flowing in the circuit should decrease in time. Correspondingly, the voltage across the capacitor should approach the emf of the voltage source. Let's see if we can quantify this heuristic discussion.

Let the capacitance of the capacitor be C and the resistance of the resistor be R, with the voltage source generating an emf of V_0. We want to write down Kirchoff's laws for this circuit. If a current i flows through the resistor, then the voltage drop across it is $V_R = iR$. If charge Q develops across the capacitor, then the voltage drop across it is $V_C = Q/C$. These two voltage drops must add to give the (constant) emf of the source V_0:

$$V_0 = V_R + V_C = iR + \frac{Q}{C}.$$

Recall that the current is simply the time derivative of the charge: $i = dQ/dt$; inserting this into Kirchoff's law and rearranging yields the following first-order, linear, inhomogeneous differential equation for the charge on the capacitor:

$$\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{V_0}{R}.$$

We need to solve this equation for Q as a function of time. Before we can do this, we must specify an initial condition, *i.e.*, we must choose what the charge on the capacitor is before we turn the circuit on. The only really natural choice is obviously 0, for how could any charge get on the plates before we start to charge it! So, we have to solve the initial value problem

$$\begin{aligned} \frac{dQ}{dt} + \frac{1}{RC}Q &= \frac{V_0}{R}, \\ Q(0) &= 0. \end{aligned} \tag{5.1}$$

There are many techniques for solving linear, first-order differential equations. The easiest technique uses what is called an integrating factor. The idea behind this technique is to multiply equation (5.1) by a very carefully chosen function so that the two terms on the left combine into the derivative of a product; such a function is called an integrating factor for the equation. The integrating factor for our equation is

$$e^{t/RC};$$

see Boyce and DiPrima for a more general discussion of integrating factors and how to find them. Carrying out this procedure, we find

$$e^{t/RC}\left[\frac{dQ}{dt} + \frac{1}{RC}Q\right] = e^{t/RC}\frac{V_0}{R}$$

$$\frac{d}{dt}\left[e^{t/RC}Q\right] = \frac{V_0}{R}e^{t/RC}.$$

Integrating both sides with respect to t yields

$$e^{t/RC}Q = CV_0 e^{t/RC} + \text{const.}$$

from which we find

$$Q = CV_0 + \text{const.}e^{-t/\tau}$$

where we have introduced the *RC time constant* $\tau = RC$. Notice that CV_0 is just the charge the capacitor would store if the voltage across it were V_0. Denoting this value by Q_0, we can then determine what the unknown constant in our equation is by enforcing the condition that $Q = 0$ when $t = 0$. If this is to be true, then the constant must be $-Q_0$ and our final solution for a charging capacitor is

$$Q(t) = Q_0(1 - e^{-t/\tau}) \tag{5.2}$$

Since we can't measure charge directly, this is not an easy equation to verify. We can, however, very easily ascertain the voltage across the capacitor. At any time t this voltage is $V(t) = Q(t)/C$; so, we find from equation (5.2)

$$V(t) = V_0(1 - e^{-t/\tau}), \tag{5.3}$$

which we can detect with a multimeter or oscilloscope, if we want. We can also determine the time dependence of the current from equation (5.2) by simply taking the time derivative:

$$i(t) = \frac{dQ}{dt} = i_0 e^{-t/\tau},$$

where $i_0 = Q_0/\tau$. Thus, the voltage starts from 0 and asymptotically approaches V_0, while the current begins at i_0 and exponentially decreases to 0. In Figure 5.2, we graph the voltage and current as functions of time.

Now consider what happens if we move the switch to position 2 in Figure 5.1. The capacitor will then discharge through the resistor; how do the current and voltage behave as a function of time in this case? We can set up the analysis just as we did in the charging case. Applying Kirchoff's laws, we find that the voltage drops must sum to zero:

$$V_C + V_R = \frac{Q}{C} + iR = 0;$$

inserting the definition of current, we find that the charge on the capacitor obeys the following differential equation:

$$\frac{dQ}{dt} + \frac{1}{RC}Q = 0,$$

with $Q(0) = Q_0$ the charge on the capacitor after charging. You can solve this equation by simply separating the variables and integrating:

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_0^t \frac{dt}{RC},$$

or

$$Q(t) = Q_0 e^{-t/\tau},$$

where $\boxed{\tau = RC}$ is again the time constant of the circuit. Having the charge as function of time leads us to the voltage and current behavior, just as in the previous case:

$$\boxed{V(t) = \frac{Q(t)}{C} = V_0 e^{-t/\tau}} \tag{5.4}$$

$$i(t) = \frac{dQ}{dt} = -i_0 e^{-t/\tau},$$

where $V_0 = Q_0/C$ and $i_0 = Q_0/\tau$. Notice that the current is negative here, reflecting the fact that the charge is coming off the capacitor, rather than accumulating on it. Also notice that the time dependence is still controlled by τ. In Figure 5.3 we graph the current and voltage as functions of time. We see from this figure that both the voltage and current exponentially decay to zero as the capacitor discharges.

5.2.3 AC Response of Capacitors

Having seen that capacitors introduce a time scale into a system which otherwise had none (DC circuit), we want to examine how this intrinsic time scale interacts with an AC circuit, one that has a natural period. To carry this out, consider the circuit sketched in Figure 5.4. Suppose the input voltage V oscillates with angular frequency ω; then we can sum the voltage drops across the capacitor and the resistor and set that equal to the oscillating voltage (Kirchoff's Law again). We find, just as before, that

$$V = V_C + V_R = \frac{Q}{C} + R\frac{dQ}{dt}$$

Inserting $V = V_0 \sin(\omega t)$, we find that the charge on the capacitor obeys the relationship

$$\frac{dQ}{dt} + \frac{1}{RC}Q = \frac{V_0}{R}\sin(\omega t). \tag{5.5}$$

We can solve this differential equation using the same integrating factor approach we employed in the DC case for charging the capacitor. The integrating factor is the same, since the left-hand side is identical. After introducing this factor, equation (5.5) becomes

$$\frac{d}{dt}\left(e^{t/\tau}Q\right) = \frac{V_0}{R}e^{t/\tau}\sin(\omega t),$$

where, of course, $\tau = RC$. Integrating both sides with respect to time and enforcing $Q = 0$ when $t = 0$ yields

$$Q(t) = \frac{V_0}{R}e^{-t/\tau}\int_0^t e^{t/\tau}\sin(\omega t)dt.$$

You can evaluate the integral with a double application of integration by parts (Exercise!). The result is

$$Q(t) = \frac{V_0}{R}\frac{1}{1+(\omega\tau)^2}\left(\sin(\omega t) - \omega\tau\cos(\omega t)\right).$$

We can clean this result up by combining the trig functions using a phase angle. Consider the expansion

$$\sin(\omega t - \phi) = \sin(\omega t)\cos\phi - \cos(\omega t)\sin\phi.$$

We can use this to find $\sin\phi$ and $\cos\phi$ by matching the corresponding terms. We find initially that

$$\begin{aligned} \sin\phi &= \omega\tau, \\ \cos\phi &= 1, \end{aligned}$$

but these aren't compatible, since $\sin^2\phi + \cos^2\phi = 1$. If we divide each by $\sqrt{1+(\omega\tau)^2}$ to force the Pythagorean identity, we then see that

$$\sin\phi = \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$
$$\cos\phi = \frac{1}{\sqrt{1+(\omega\tau)^2}},$$

which are perfectly legal mathematical expressions. Thus, the phase angle can be found from

$$\tan\phi = \omega\tau. \tag{5.6}$$

Incorporating this into the solution, we find that the charge on the capacitor oscillates according to

$$Q(t) = Q_0\frac{\sin(\omega t-\phi)}{\sqrt{1+(\omega\tau)^2}}, \qquad \tan\phi = \omega\tau,$$

where $Q_0 = CV_0$, as usual. Using the same reasoning as in the DC analysis, we find the voltage across the capacitor V and current in the circuit i to be

$$V(t) = \frac{Q(t)}{C} = V_0\frac{\sin(\omega t-\phi)}{\sqrt{1+(\omega\tau)^2}} \tag{5.7}$$
$$i(t) = i_0\frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}\cos(\omega t-\phi),$$

where $i_0 = Q_0/\tau$ as in the DC case. These relations are plotted as a function of time in Figure 5.5. From these two relations, we find that the capacitor simply shifts the phase of oscillation and adjusts the amplitude according to the input frequency. This frequency dependence forms the basis for sorting out signals base on frequency content. For the case we've analyzed here, the higher the frequency (large ω) the smaller the amplitude; so, this circuit functions as a low-pass filter, meaning that lower frequencies can get through with only a slight phase shift, but higher frequencies are cut-off.

You might ask, at what frequency does this filter start to really affect the signal? The answer to this question lies in the denominator of equation 5.7. The denominator is very close to 1 for $\omega \ll 1/\tau$ and is very large for $\omega \gg 1/\tau$.

Thus, we should expect that for angular frequencies around $1/\tau$, the signal begins to change. This value is called the *cutoff* angular frequency and is typically denoted ω_c. Thus, $\omega_c = 1/\tau$. To make this clearer, consider the output amplitude, that is the stuff in front of the sine function in equation (5.7). We plot this function versus normalized frequency in Figure 5.6. We see that, for frequencies below the cutoff, the amplitude is still above 70% that of the input signal. For frequencies above the cutoff, the amplitude decays rather slowly, dropping below 20% for angular frequencies above about $5\omega_c$.

We have been discussing the voltage across the capacitor and we've found that it is greatly suppressed for frequencies above the cutoff. If, on the other hand, we examine the voltage across the resistor, we find

$$V_R(t) = i(t)R = V_0 \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} \cos(\omega t - \phi).$$

This voltage behaves quite differently, as a function of frequency, from the voltage across the capacitor. We plot its behavior in Figure 5.7. For $\omega \ll 1/\tau$ the denominator is roughly 1 while the numerator is small; so, the amplitude for low frequency waves is reduced compared to high frequency oscillations. For large frequencies, $\omega \gg 1/\tau$, the frequency dependence reduces to 1; hence, the amplitude of high frequency oscillations is unaffected by the circuit.

We can understand the physical origin of these two different behaviors by examining what the capacitor actually does. If the input voltage is larger than the voltage across the capacitor due the charge on its plates, then charge will flow onto the plates, increasing the potential across them. Oppositely, when the driving voltage falls below that on the capacitor, charge flows off the plates, decreasing the potential. Thus, the voltage on the capacitor must *follow* the input voltage, leading to a phase shift in the output. This charging and discharging process takes place on the time scale defined by τ. If the period of the input voltage oscillation is shorter than this reaction time of the capacitor, a very small amount of charge has time to build up on the plates before its drained away again. This prevents the potential from growing large and cuts off the signal. On the other hand, if the period of the input oscillation is longer than τ, then charge has time to build up on the plates, permitting the voltage to increase and allowing the signal to pass. Since the voltage across the resistor is the input voltage minus the voltage across the capacitor, it has the exact opposite behavior, but with the same cutoff.

5.3 Apparatus

The major pieces of equipment we'll need to use in this lab have already been discussed in the oscilloscope lab. We will use the oscilloscope and frequency generator, the breadboard and electrical components like resistors and capacitors. The only new item is the capacitor.

The capacitors we will use are electrolytic capacitors. Their plates have various shapes and are typically filled with a dielectric medium. All of them have a nominal value of the capacitance inscribed on them somewhere. Once we have measured the time constant for a circuit with known resistance, we can then back out the capacitance and check the nominal value. We note also that capacitors occasionally have an arrow or ring inscribed on them to denote the way they should be oriented in a circuit; the contact on the same side as the mark should be connected toward ground, see Figure 5.8.

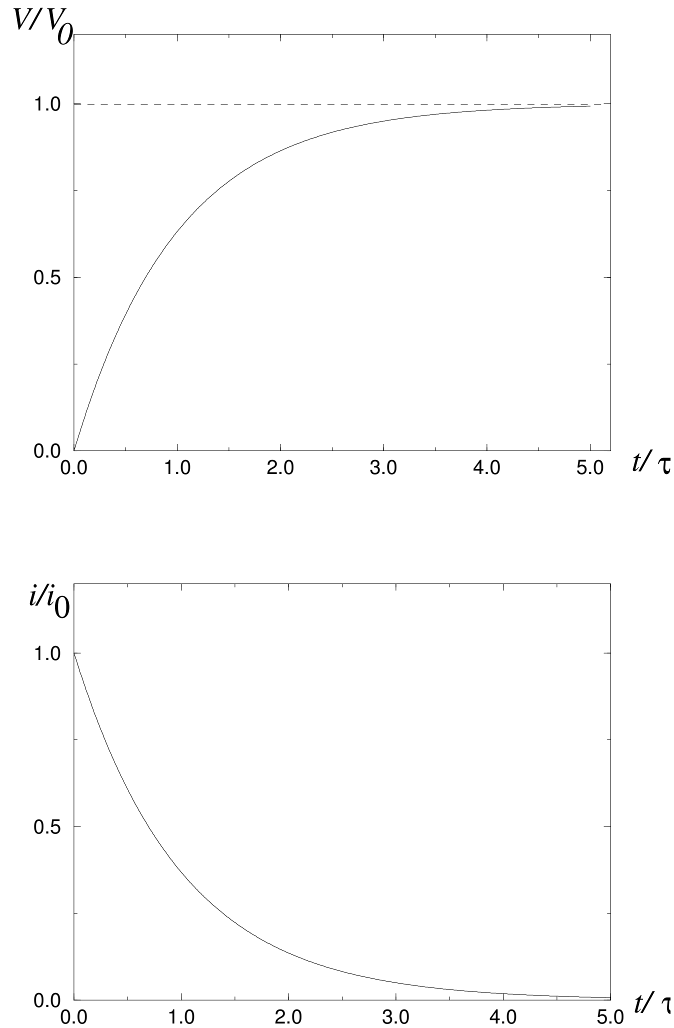


Figure 5.2: Plots of the voltage across the capacitor and the charging current as a functions of time.

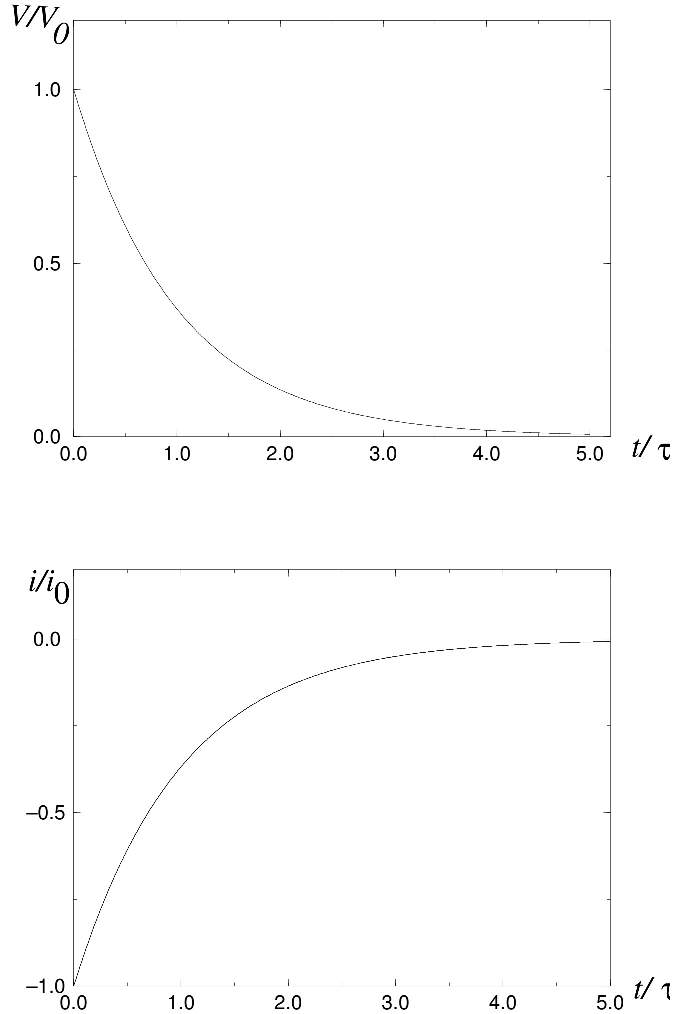


Figure 5.3: Discharging voltage and current as functions of time.

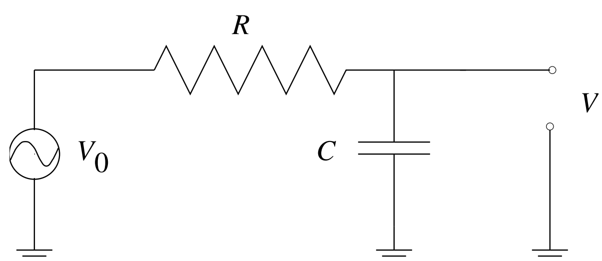


Figure 5.4: A sinusoidally driven RC circuit.

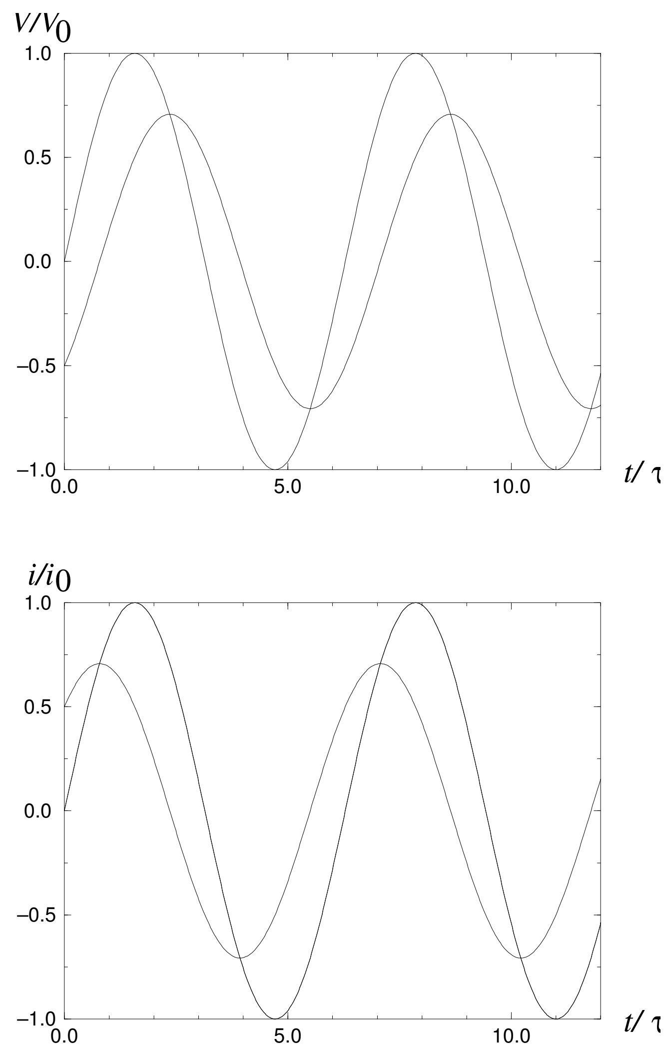


Figure 5.5: Voltage and current response of an RC circuit to a sinusoidal input voltage with $\omega\tau = 1$.

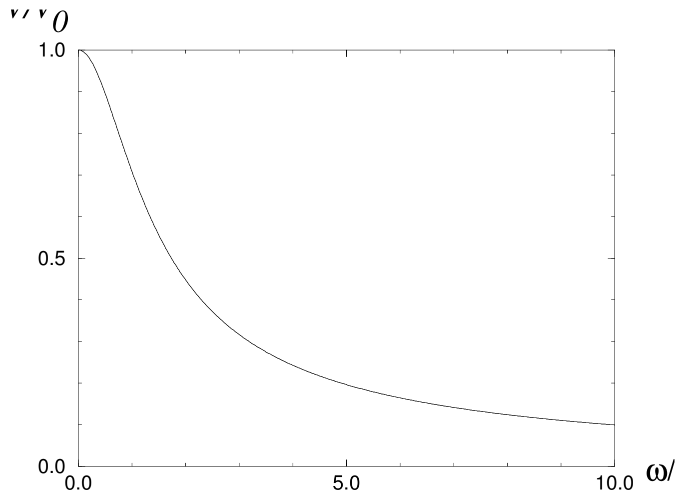


Figure 5.6: Voltage amplitude across the capacitor as a function of input frequency.

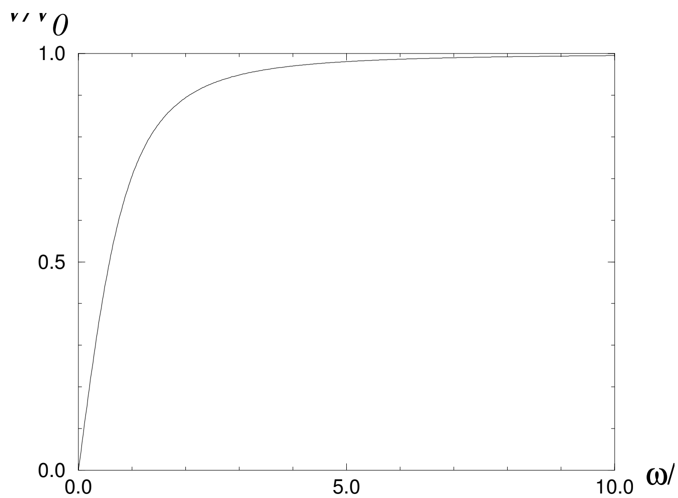


Figure 5.7: Voltage amplitude across the resistor as a function of input frequency.

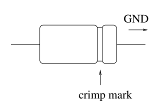


Figure 5.8: The marks which indicate the correct orientation of a capacitor.

5.W RC Circuits and Filters Worksheet

Name: ____________________ **Day/Time:** ____________
Partner's Name: ________________________

5.W.1 In-Lab Procedure

DC Response

Since most of the capacitors available to us have extremely small values of capacitance, typically on the order of μF, the time constants involved will be small fractions of a second (usually ms). This makes it difficult to use a multimeter to observe the voltage behavior during charging and discharging cycles. We will circumvent this problem by having the function generator act as a high frequency switch. We will do this by using the square wave output to toggle the circuit back and forth between the switch positions 1 and 2 in Figure 5.1. When the square wave is low, this will act like the discharge phase (position 2), and when it is high, the circuit changes to position 1 and the capacitor charges. We can choose the frequency of this switching process to allow the capacitor to completely charge and completely discharge and then repeat. We can then use the oscilloscope to examine the time dependence of the voltage in each case. In Figure 5.9 we show what the typical input and output voltages should look like.

Using the given capacitor and small resistor, construct the circuit shown in Figure 5.10 with the function generator acting as the switch. Measure the provided resistor as well as the capacitance of your capacitor with the multimeter and record its value, with uncertainties, below:

$R_{meter} =$ ____________ $C_{meter} =$ ____________

Set the function generator to provide a square wave output and choose an initial frequency of about 1 kHz. Split the signal coming out of the function generator, sending one to your circuit and the other to Channel 1 of the oscilloscope; trigger on Channel 1. We will use the T-connectors to split the output of the function generator, as illustrated in Figure 5.10. The split signal will allow us to trigger on the raw output of the function generator

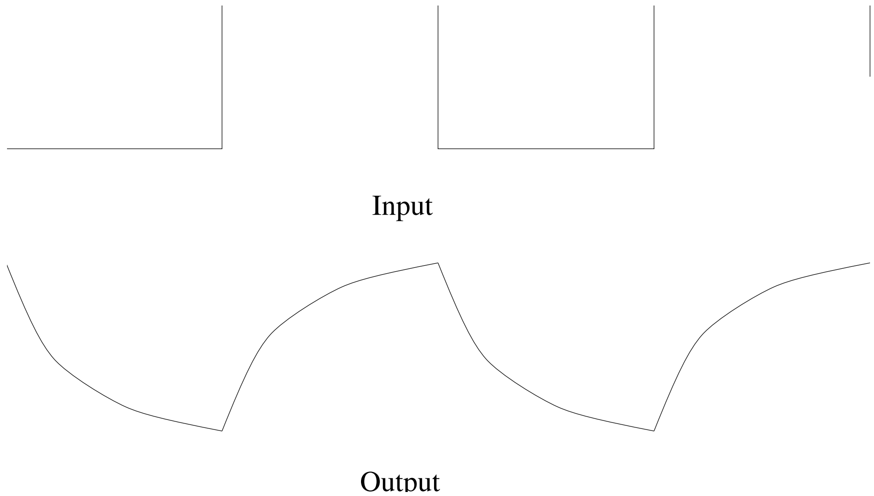


Figure 5.9: RC circuit response to a square wave input.

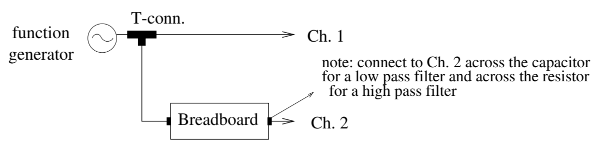


Figure 5.10: We will use the T-connector and trigger the oscilloscope on channel 1.

(on channel 1). We will use a 51 kΩ resistor and a 3.3 nF capacitor in our RC circuit. What we will set up on the board is illustrated in Figure 5.11.
Arrange the circuit in Figure 5.11 on the breadboard and set the function generator to provide a square wave at about 1 kHz. Connect the alligator clips of the BNC-to-alligator clip wire to jumpers on the board so that they are measuring the voltage across the *capacitor*, V_C; send this into channel 2 of the oscilloscope.

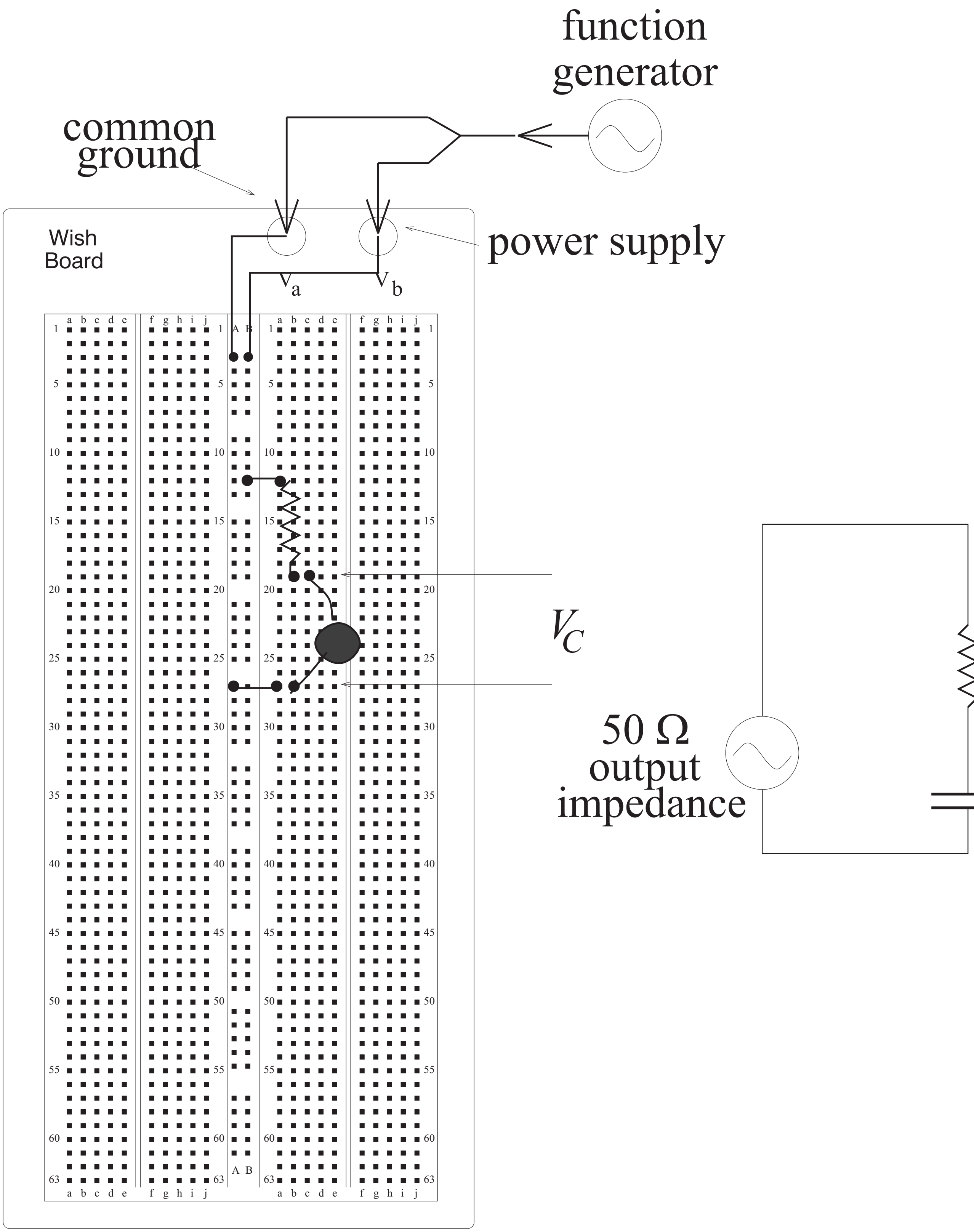


Figure 5.11: The proper breadboard connections for our RC circuit. Note that the circuit can be configured so that the oscilloscope measures the voltage across the capacitor (V_C) or across the resistor (V_R).

Trigger the oscilloscope on channel 1 and display channel 2; adjust the scope until you can see the decay/growth response illustrated as "output" at the bottom of Figure 5.9 in the manual. By adjusting the V/div and sec/div knobs of the scope, focus in on a *decay* portion of the signal. Adjust the channel 2 vertical position and the horizontal position knobs until the maximum of the decay curve lands on a vertical grid line (voltage axis) and the minimum lands on a horizontal grid line (time axis), as illustrated in Figure 5.12.

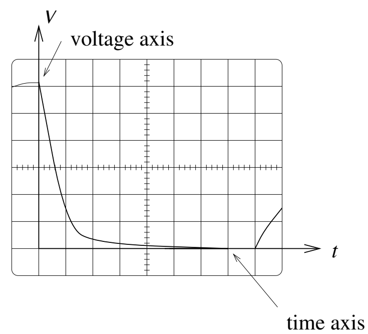


Figure 5.12: Adjust the scope until you have the decay curve aligned with a "voltage axis" and "time axis."

Now use the axes you thus created to measure the voltage-time pairs. Adjust the oscilloscope to observe the discharging phase; use the scale settings to focus the display in on one of the decay portions of the output signal. You can now read off the discharge voltage as a function of time. Take ten voltage-time pairs and enter the voltage-time pairs into Table 5.1. **Take Note:** Your first measurement should be of the voltage at the beginning of the discharging phase. (That's the peak of the curve on the screen.)

5.W.2 In-Lab Computer Work

According to equation (5.4), the quantity $\ln(V/V_0)$ should be linearly related to t. What's V_0 in this case?

$$V_0 = \underline{\hspace{4cm}}$$

Voltage and Time Coordinates				
Voltage	Time		Voltage	Time

Table 5.1: V vs. t measurements for discharging capacitor.

Graph $\ln(V/V_0)$ versus t with the set of data contained in Table 5.1 with Kaleidagraph. You must calculate the uncertainty in $\ln(V/V_0)$. Remember that both V and V_0 have uncertainties associated with them. Calculate $\Delta \ln(V/V_0)$ below. Remember that the computer will do most of the work for you. Just do **one sample calculation** and then use the computer. **SHOW WORK.**

$\Delta \ln(V/V_0)$ ______________

Find the slope and intercepts with uncertainties and record their values in Table 5.2.

From equation (5.4), determine how the slope of your plot should be related to the time constant of your circuit. With this information, calculate the time constant of your circuit, τ_{meas}, and its uncertainty from the slope of your plot.

RC Circuit	
Slope	Intercept

Table 5.2: Slope and Intercept for the resistor $ln(V/V_0)$ vs. t plot.

$\tau_{meas} =$ ______________ $\Delta(\tau_{meas}) =$ ______________

5.W.3 In-Lab Procedure

AC Response

Now adjust the function generator to provide a sinusoidal input to the circuit. Referring to the circuit diagram in Figure 5.13 for this arrangement. Use τ_{meas} to determine the natural cutoff frequency of your circuit (without uncertainty). Record this value and **Show work**:

$f_c =$ ______________

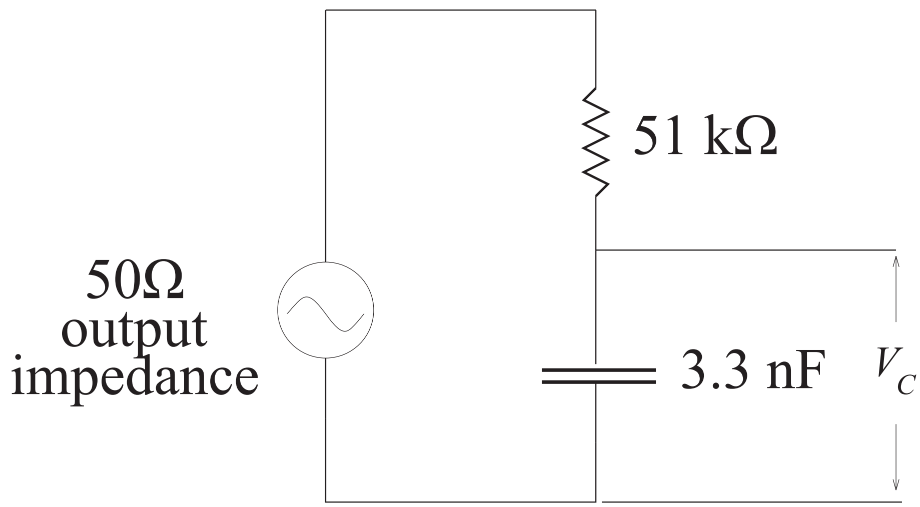


Figure 5.13: A low-pass filter arrangement.

Set the function generator to a frequency below the cutoff. You can tell that you're below the cutoff when you see that the amplitude of the output signal doesn't depend on the frequency; this will probably be somewhere in the 1 kHz range of the function generator. Set the function generator to about this frequency and compare the oscilloscope screen with Figure 5.5. Next, switch the oscilloscope to display only channel two and set the frequency generator to a frequency below the cutoff. Measure the peak-to-peak amplitude of the output waveform as a function of the frequency for ten different frequencies.

Make sure that most of your frequency values extend past the cutoff. Remember that you should always measure your frequencies from the periods of the waves on the oscilloscope. Enter your amplitude-period pairs into Table 5.3. **Take Note:**

1. Your first amplitude measurement should be taken from a signal with a frequency of around **100 Hz**.

2. Take data at your cutoff frequency f_c. You will compare that value to the theoretical value later.

3. A good majority (at least seven) of your amplitude measurements should be taken from signals with frequencies *above* the natural cutoff

frequency of the circuit.

4. Once you have taken this measurement, DO NOT adjust the amplitude knob on the function generator.

Amplitude vs. Period Measurements						
Amplitude	Period	f		Amplitude	Period	f

Table 5.3: Amplitude vs. Period Measurements.

5.W.4 In-Lab Computer Work

Now, plot the amplitude, A_{exp}, versus the frequency f with error bars using the amplitude values from Table 5.3 and calculating frequency from the period values in Table 5.3. Show a sample calculation of f and Δf here.

5.W.5 In-Lab Procedure

High-pass Filter

Now reconfigure your circuit to measure the voltage across the resistor by switching the position of the resistor and capacitor in your circuit and connecting the clips to the oscilloscope across the resistor. Scan the frequency across the cutoff and describe the behavior of the amplitude that you observe. You do not need to take any data here; just verbally express what you observe.

Question 1: Is this consistent with our predictions for a high pass filter? Why?

Question 2: Estimate the cutoff frequency and compare it to the cutoff frequency for the voltage across the capacitor.

Question 3: Does this make sense?

5.W.6 Pre-Classroom Check List

- ◯ Table 5.1 completed with units and uncertainties
- ◯ Table 5.2 completed with units and uncertainties
- ◯ Table 5.3 completed with units and uncertainties
- ◯ Plot of $ln(V/V_0)$ vs. t labeled completely and correctly
- ◯ Plot of A_{exp} vs. f labeled completely and correctly
- ◯ Each student has her/his own plots and worksheet

5.W.7 In-Classroom Calculations & Analysis

DC Response

From the measured values of the resistance and the capacitance of your circuit, calculate the expected time constant, τ_{calc}. Record this below. Be sure to include units and uncertainties. **Show your work.**

$$\tau_{calc} = \underline{\hspace{4cm}}$$

From τ_{meas}, and the measured resistance of your circuit, R_{meter}, calculate a value for the capacitance of your circuit, C_{meas}. Record this below. Be sure to include units and uncertainties. **Show your work**.

$$C_{meas} = \underline{\hspace{4cm}}$$

Explain how equation (5.4) in the lab manual leads you to expect a plot of $\ln(V/V_0)$ versus t to be linear.

Determine the expected value of the intercept of your plot and then compare this with the actual intercept value from your plot.

Compare the value of the circuit's time constant that you obtained from the plot (τ_{meas}), to its expected value (τ_{calc}).

Compare the value of the capacitance from the measured time constant (C_{meas}), with its value measured by the multimeter (C_{meter}).

AC Response

With the equation below, calculate A_{theo} at the cutoff frequency $f_c = 1/(2\pi\tau_{meas})$, with uncertainty.

$$A_{theo} = \frac{A_{in}}{\sqrt{1 + (2\pi f \tau_{meas})^2}}.$$

Use the amplitude at $f = 100\,\mathrm{Hz}$ for the input amplitude A_{in}.

Compare your measured amplitude A_{meas} at f_c to A_{theo} at f_c. To do this, you need to discuss the degree to which they agree within uncertainties. Show all your work.

5.W.8 Conclusion

Write a *brief* (that is, a one or two paragraph) conclusion for this lab below. In it, you should summarize the physical principles which were meant to be illustrated in this experiment. You should also describe the degree to which your data supported these principles.

Attach plots to the worksheet.

End Worksheet

Chapter 6

Electromagnetic Induction

6.1 Introduction

The generators that supply electricity to our homes and the motors in our cars are important devices whose mechanism of operation is typically taken for granted. Today, we'll explore the fundamental physical law responsible for their operation, Faraday's law of induction.

Faraday's law states that a changing magnetic field can produce an induced voltage in an electric circuit. The device we will study is a simple transformer, two coils of wire linked by an iron core. It operates by running an alternating current (the input) through one coil (the primary coil) to generate an alternating magnetic field in the iron. (This phenomenon is described by another fundamental law, called Ampère's law.) The magnetic field passes through the second coil (the secondary coil), where, by Faraday's law, it induces an alternating potential (the output) across the coil. This voltage can be viewed on the oscilloscope (remember that it's varying in time), so that its properties can be studied.

In particular, we will examine the output signal from the transformer when we use a sinusoidal input. We will see that the output is also sinusoidal and we will examine the dependence of the output amplitude on both the frequency of the input signal and the number of turns in the secondary coil. We will also examine the output for other input signals: triangle waves and square waves. Examining these signals will reveal that the output signals is proportional to the *time derivative* of the input signal.

6.2 Theory

6.2.1 References

Faraday's law of induction is introduced in Serway, Chapter 31 (Faraday's Law). Chapter 33 (Alternating Current Circuits) is also useful, since we're using AC circuits. The magnetic field of a solenoid is discussed in Sections 30.4 (The Magnetic Field of a Solenoid), and 30.5 (The Magnetic Field Along the Axis of a Solenoid).

6.2.2 Magnetic Flux

Whenever we have a vector field present we may speak of the flux of that field through a surface. If we have a magnetic field $\vec{B}$ which passes through an open surface S as in Figure 6.1, then we define the magnetic flux to be

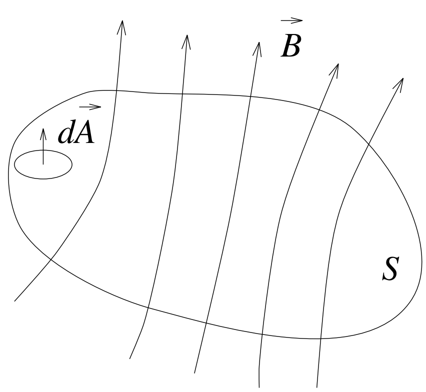


Figure 6.1: A magnetic flux Φ_M passes through the surface S.

the integral

$$\Phi_M = \int_S \vec{B} \cdot d\vec{A},$$

where $d\vec{A}$ is the perpendicularly oriented differential area element.

Let's calculate the magnetic flux which passes through a long thin solenoid, of N turns, length L, radius $a \ll L$, and carrying a current I, illustrated in Figure 6.2. We'll consider the field responsible for the flux to be that of the solenoid itself, so this is an example of *self-induction*. The magnetic field

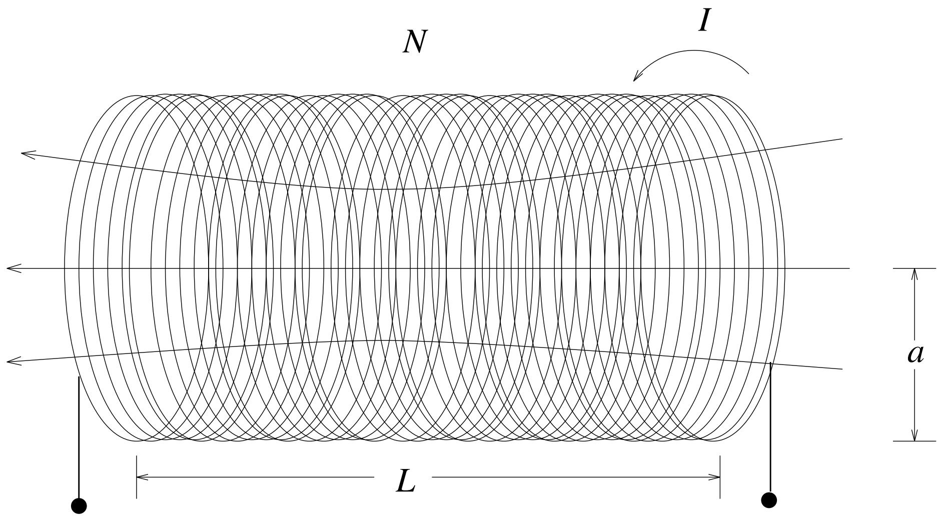


Figure 6.2: A long thin solenoid (not to scale).

along the cross section of the solenoid is roughly constant and given by

$$B = \frac{\mu_0 N I}{L};$$

for a more complete discussion, see Serway. The flux is then

$$\begin{aligned}
\Phi_M &= \int_S \vec{B} \cdot d\vec{A} \\
&= \int_0^{2\pi} d\theta \int_0^a r dr \frac{\mu_0 N I}{L} \\
&= \frac{\mu_0 N I}{L} (\pi a^2) \\
\Phi_M &= \frac{\pi \mu_0 N I a^2}{L} \qquad (6.1)
\end{aligned}$$

6.2.3 Faraday's Law

Consider again the solenoid in Figure 6.2, but this time imagine that the magnetic field is time-varying and is coming from some external source, perhaps another coil, rather than from a current running through the solenoid. Faraday's law states that the magnetic flux through the solenoid induces a voltage

$$V = -N\frac{d\Phi_M}{dt} \tag{6.2}$$

across the solenoid. The factor of N appears because the flux Φ_M links each of the N coils of the solenoid; the voltages induced by Φ_M in each turn of the coil add in series.

6.2.4 The Transformer

Let's now apply what we've learned to a transformer, illustrated in Figure 6.3. A voltage $V_{\rm in}$ is supplied to a primary coil of N_1 turns, producing a current

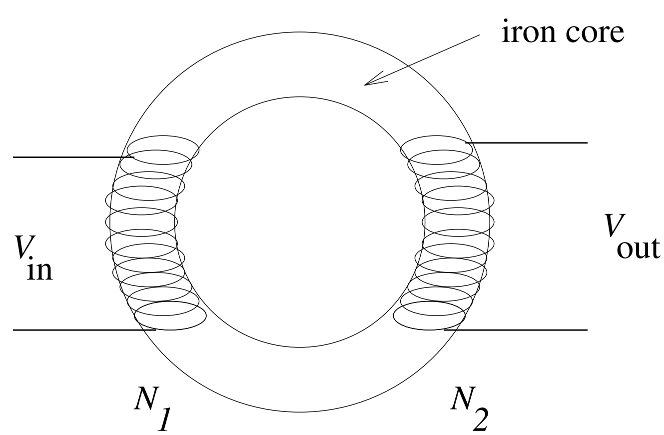


Figure 6.3: A simple transformer.

$I_{\rm in}$, which, from (6.1), generates a flux

$$\Phi = \frac{\pi\mu N_1 a_1^2}{L_1} I_{\rm in},$$

in the iron core. We assume that, since the iron is a magnetic material, it holds this flux and carries it through the secondary coil of N_2 turns; since we aren't dealing with a field in vacuo anymore, we have replaced μ_0 by μ, the permeability of iron. Faraday's law (6.2) implies that a voltage, given by

$$V_{\rm out} = -N_2\frac{d\Phi}{dt},$$

is generated across the secondary coil. We can substitute in for Φ to write

$$V_{\text{out}} = -\left(\frac{\pi\mu N_1 a_1^2}{L_1}\right) N_2 \frac{dI_{\text{in}}}{dt}.$$

Now, in our experimental setup, we will vary N_2 and I_{in}, but will not vary or measure a_1, L_1, or N_1. It will therefore be useful to define the (positive) constant $C' = \pi\mu N_1 a_1^2/L_1$ and write

$$\boxed{V_{\text{out}} = -C' N_2 \frac{dI_{\text{in}}}{dt}}, \tag{6.3}$$

which illustrates the dependences that we will study, leaving the constant C' to conveniently sum up the (unknown, but unchanging) properties of the transformer. We see that V_{out} depends linearly on N_2, the number of turns in the secondary coil, and also depends linearly on the time derivative of I_{in}. We also note that there is an important minus sign in (6.3) which corresponds to the minus sign in Faraday's law (6.2). This is called Lenz's law: the induced voltage *opposes* the change in flux.

Let's see what the relationship (6.3) predicts for the output when we use a sinusoidal input current

$$I_{\text{in}} = I_0 \sin\omega t. \tag{6.4}$$

Using (6.3) yields

$$V_{\text{out}} = -C' N_2 \omega I_0 \cos\omega t. \tag{6.5}$$

Figure 6.4 shows a plot of a sine wave versus the negative of a cosine wave. There is therefore a phase difference of 90° (that between sine and –cosine) between the input current and output voltage, while the amplitude of the latter (the part in front of the cosine function) is proportional to the amplitude and frequency of the former, as well as the number of turns in the secondary coil.

To confirm these predictions, we have to be able to monitor the current through the input coil. The problem though is that an oscilloscope (the most suitable instrument for observing a changing signal) measures voltages. To overcome this difficulty we use a monitoring resistor, R_{mon}, connected in series with the input coil. From Ohm's law, we know that a current I_{in} through this resistor will cause a voltage drop

$$V_{\text{mon}} = I_{\text{in}} R_{\text{mon}}$$

across it. For a current input of the form (6.4), the oscilloscope will measure a sinusoidal voltage signal

$$V_{\text{mon}} = I_0 R_{\text{mon}} \sin \omega t.$$

We expect then the amplitudes of the input and output signals to be related by

$$\frac{A_{\text{out}}}{A_{\text{in}}} = \frac{C' N_2 \omega I_0}{I_0 R_{\text{mon}}} = \frac{C'}{R_{\text{mon}}} N_2 \omega$$

or, if we absorb the constant resistance into a new $C = C'/R_{\text{mon}}$,

$$\boxed{A_{\text{out}} = C N_2 \omega A_{\text{in}}}. \tag{6.6}$$

The amplitude of the output voltage, A_{out}, should therefore depend linearly on both N_2 and ω.

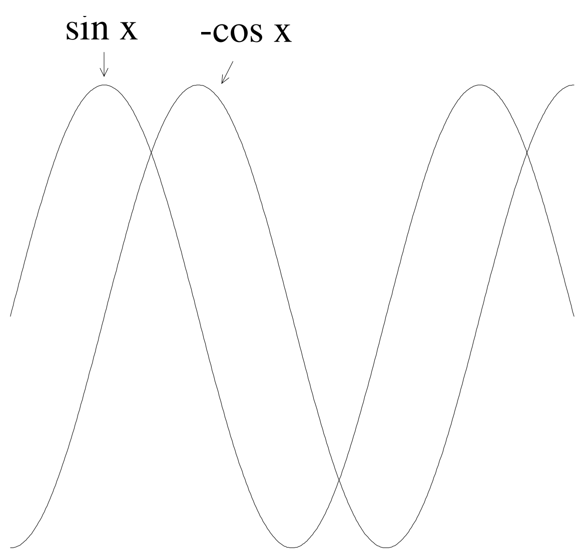


Figure 6.4: A plot of a sine wave and a –cosine wave.

6.3 Apparatus

The apparatus we will use centers around the transformer box, illustrated in Figure 6.5, which houses a transformer such as that in Figure 6.3, but which allows us to vary the number of turns in the secondary coil. Note that each setting of the knob corresponds to *four* turns in the coil. We will use the function generator to provide our input signals and the oscilloscope to view both the input and output signals.

6.W Electromagnetic Induction Worksheet

Name: ____________________ **Day/Time:** ____________
Partner's Name: ________________________

Before doing anything else, examine the transformer box. Turn it over so that you can see the coils inside. Does it resemble a transformer such as that we've been talking about, *i.e.*, that in Figure 6.3? Connect the box to the function generator and oscilloscope as in Figure 6.5.

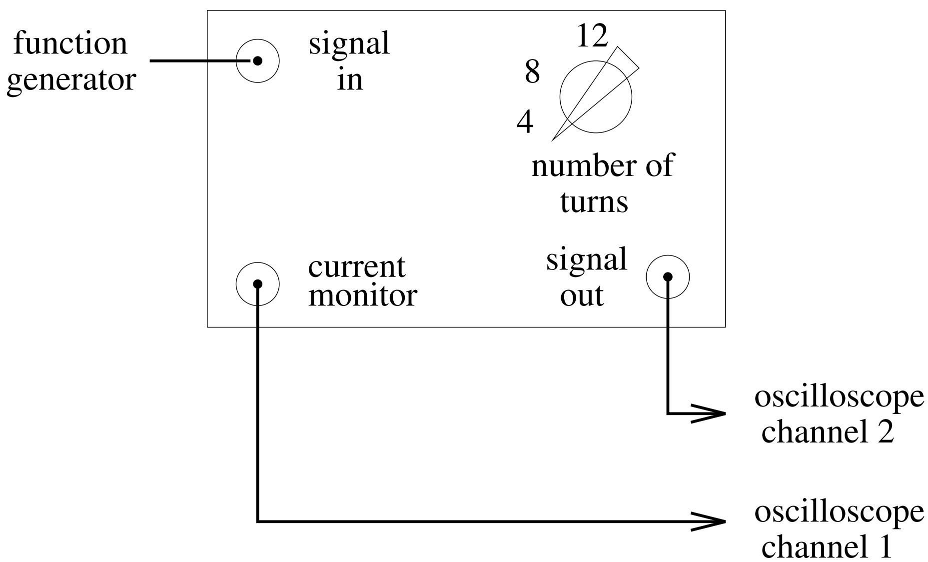
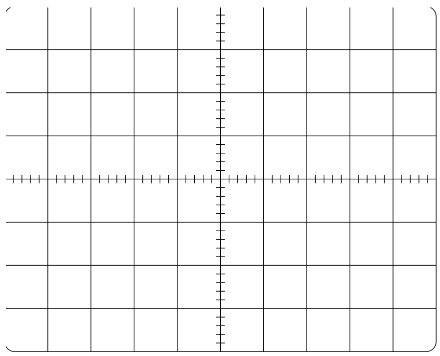
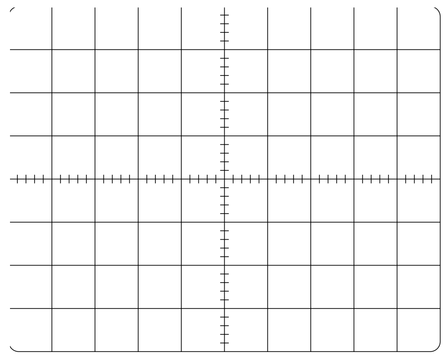


Figure 6.5: How to connect the transformer box.

Sinusoidal Input

Set the function generator to produce a good sinusoidal signal. You may need to try different frequencies (use the range selector) to get an optimal picture. Adjust the oscilloscope so that it displays both the input and output signals. Set the channel 1 and channel 2 V/div and sec/div scales so that the waveforms fill the screen. Compare the picture on the screen with that of Figure 6.4. Specifically, check the relative position of the two waveforms, to confirm the connections and operation of your setup. Sketch the two waveforms on the grid below. Sketch both the input and output on the same grid, but make sure you label each channel. Use the second grid only if you make a mistake. **Draw carefully.** You will be answering questions based on this sketch in the classroom.





Measure the phase shift, (refer to § 4.2.2) and their periods, T.

$\Delta t =$ ______________

T (ch. 1): ______________ T (ch. 2): ______________

Measure the output peak-to-peak amplitude, A_{out}, as well as the current monitor signal peak-to-peak amplitude, A_{in}, and period, T, for 10 well spaced frequencies. **DO not go above 100kHz on the function generator.** Calculate the angular frequency values, ω, by measuring the period of the input wave on the oscilloscope; *do not* trust the frequency values read off the function generator. **Show** a sample calculation for ω and $\Delta(\omega)$, as well as $A_{\text{out}}/A_{\text{in}}$ and $\Delta(A_{\text{out}}/A_{\text{in}})$, here.

Enter your measured values of A_{out}, A_{in} and T with uncertainties into Table 6.1.

Amplitude versus Frequency				
A_{out}	A_{in}	T	ω	A_{out}/A_{in}

Table 6.1: $A_{\mathrm{out}}/A_{\mathrm{in}}$ versus ω of the input wave.

Now measure the output peak-to-peak amplitude versus the number of turns in the secondary coil, N_2, for each setting that the knob allows at one fixed frequency. Enter A_{out} and N_2 into Table 6.2.

6.W.1 In-Lab Computer Work

Amplitude versus Frequency

Plot $A_{\mathrm{out}}/A_{\mathrm{in}}$ versus ω with error bars and a line fit using Kaleidagraph.

Amplitude versus Number of Turns	
A_{out}	N_2

Table 6.2: A_{out} of the output versus N_2 of the transformer.

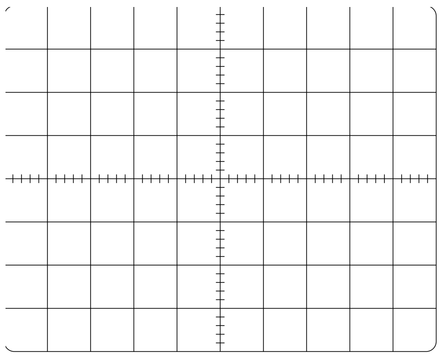
Amplitude versus Turns

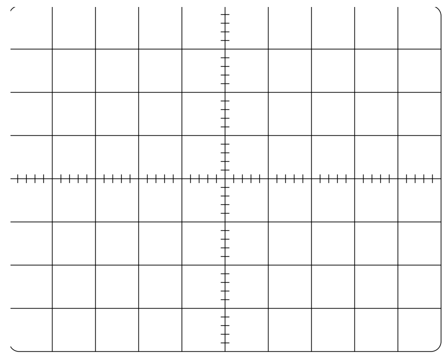
Plot A_{out} versus N_2 with error bars and line fit.

6.W.2 In-Lab Procedure

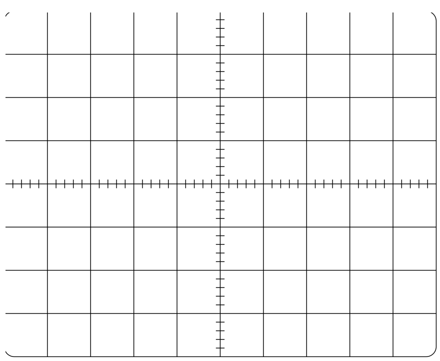
Triangle and Square Wave Input

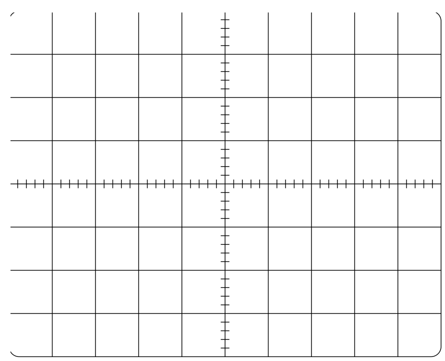
Set the function generator to produce a triangle wave and adjust the oscilloscope to display the input and output signals. Again, find the frequency at which a good quality triangular input is attained. Sketch the input and output signals **carefully**. Since the picture may be confusing, sketch the input triangle wave and the output on different, labeled grids:





Now set the function generator to produce a good quality square wave and make a sketch of the input and output signals, on different, labeled grids, as before.





6.W.3 Pre-Classroom Check List

- ○ Table 6.1 completed with units and uncertainties
- ○ Table 6.2 completed with units and uncertainties
- ○ Sketch of sinusoidal output
- ○ Sketch of triangular output
- ○ Sketch of square output
- ○ Plot and fit of $A_{\text{out}}/A_{\text{in}}$ versus ω

○ Plot and fit of A_{out} versus N_2
○ Each student has her/his own plots and worksheet

6.W.4 In-Classroom Calculations & Discussions

Sinusoidal Input

The following questions are based on the first sketch, the sinusoidal output. Answer the questions completely by stating your reasoning behind your answers and sketching when necessary.

Prove (or disprove) that the output resembles the negative of the time derivative of the input by comparing the input and output at specific angles and showing your reasoning in mathematical terms. Make sure you verify the minus sign that appears in Faraday's Law.

Are there any other characteristics of your sketch that help you make an interpretation?

Calculate the phase difference between the input and output signals with uncertainty (refer to section 4.2.2). **Show your work**.

$\phi =$ ________________

What is the phase difference predicted by Faraday's law? Explain your answer.

Compare the calculated phase difference to the predicted phase difference.

How do the two plots you made (amplitude ratio versus frequency and output amplitude versus number of turns) agree with Faraday's Law? **Be specific and use the law as a guide**.

Triangle and Square Wave Input

The following questions are based on the sketch of the triangular input. Answer the questions completely by stating your reasoning behind your answers and sketching when necessary.

Are the peaks perfectly sharp or do they round off gently, i.e. how good of a triangular wave are you studying?

Compare the output signal with the negative time derivative of the input carefully in the space provided.

The following questions are based on the sketch of the square input. Are the corners of the waves perfectly sharp, i.e. how good of a square wave are you studying?

Again, compare the output signal with the negative time derivative of the input.

6.W.5 Conclusion

Write a *brief* (that is, a one or two paragraph) conclusion for this lab below. In it, you should summarize the physical principles which were meant to be illustrated in this experiment. You should also describe the degree to which your data supported these principles.

Attach plots to the worksheet.

End Worksheet

Chapter 7

Polarization of Light

7.1 Introduction

This lab examines the phenomenon of polarized light. You will learn what polarization of light is and what the polarization axis of a polarizer is. We will see how the intensity of the light transmitted through a polarizer depends on the angle between the polarization axis of the polarizer and the polarization vector of the incident light. We will also examine various combinations of polarizers.

7.2 Theory

7.2.1 References

A very brief discussion of polarization is given in Serway, Section 38.6 (Polarization of Light Waves).

7.2.2 Polarization

In this week's lab we will explore some aspects of the wave nature of light. Light is a transverse wave of oscillating electric and magnetic fields, depicted in Figure 7.1. By transverse, we mean that the electric and magnetic fields oscillate in a plane that is transverse, or perpendicular, to the direction that the wave is moving or propagating. The wave vector $\vec{k}$ points in the direction of propagation; so transversality requires $\vec{k} \cdot \vec{E} = \vec{k} \cdot \vec{B} = 0$. Also, as is clear

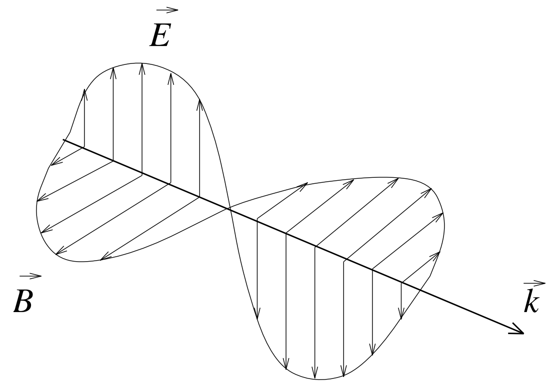


Figure 7.1: Light is a wave.

from the figure, the cross-product $\vec{E} \times \vec{B}$ points in the same direction as $\vec{k}$. The intensity of the wave is given by

$$I = \frac{\epsilon_0}{2}|\vec{E}|^2.$$

What we call polarization is the measure of how the wave is oriented in space. So far, we know that the wave vector $\vec{k}$ tells us which way the wave travels; we'd also like to know which way the oscillating fields are pointing. If we know the direction of both $\vec{k}$ and $\vec{E}$, the relation with the cross-product, namely $\vec{E} \times \vec{B} \parallel \vec{k}$, allows us to determine the direction of $\vec{B}$. As a convention, we define the *polarization vector* $\hat{\varepsilon}$ of the wave to be a unit vector in the direction of the electric field

$$\hat{\varepsilon} = \frac{\vec{E}}{|\vec{E}|}.$$

We could just as well have used $\vec{B}$ to define $\hat{\varepsilon}$, but our convention matches everyone else's and it would be nice if we could communicate with everyone else. Actually the reason for the convention is simple. For electromagnetic plane waves, the magnitudes of $\vec{B}$ and $\vec{E}$ are related by

$$E = cB,$$

where $c = 2.997\ 924\ 58 \cdot 10^8$ m/s is the speed of light. The magnitude of the force on a particle with charge q due to the electric field of the wave is $F_E = qE = qcB$, while the magnetic force has a magnitude $F_M = qvB$. The velocity of the particle will usually be much less than c, so that $v \ll c$ and $F_E \gg F_M$. In this sense, the electric field of the light wave is "stronger" than the magnetic field, so the convention is a natural one.

Since $\hat{\varepsilon}$ is a vector, we can project it into components in some coordinate system. In Figure 7.2, $\hat{\varepsilon} = \vec{\varepsilon}_x + \vec{\varepsilon}_y$. The $\vec{\varepsilon}_x$ and $\vec{\varepsilon}_y$ are not unit vectors. However, $\hat{\varepsilon}$ doesn't quite define a single direction. Since $\vec{E}$ is oscillating,

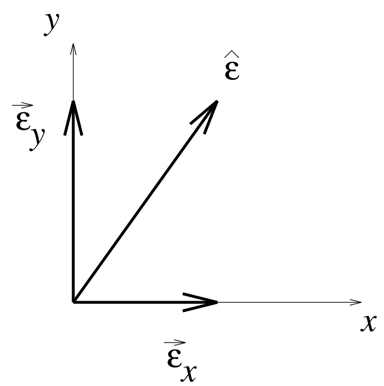


Figure 7.2: A polarization vector can be split into components.

it changes sign every half-wavelength. Therefore $\hat{\varepsilon}$ and $-\hat{\varepsilon}$ both define the same polarization, this is illustrated in Figure 7.3. When we talk about a polarization vector, we really mean the two orientations denoted by "$\hat{\varepsilon}$" in the figure. We will be specifically interested in waves for which the polarization

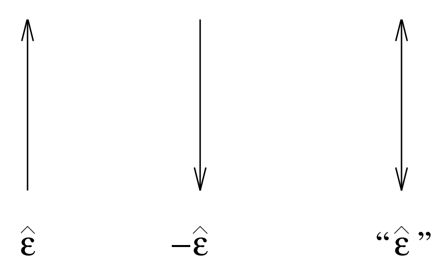


Figure 7.3: The polarization vector refers to two directions.

vector remains along the same line as the wave propagates. These waves are called linearly polarized. There are more complicated situations in which the polarization vector rotates as the wave propagates, such as in the case of circular and elliptic polarization. We will not discuss these here.

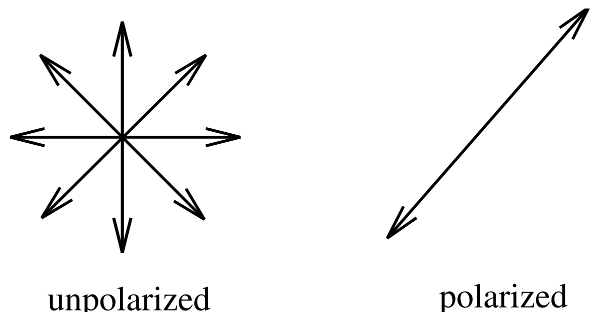


Figure 7.4: Unpolarized versus polarized light.

We have defined what we mean by polarization for a single wave. The light from most sources, such as a lamp or the sun, contains more than just one wave. The light around you is a combination of many waves. The units of light are called photons. These photons are themselves waves, so that when you put them together they obey the superposition principle and you get a more complicated wave out. Each photon will have its own polarization vector, but in general, the polarization vectors will not all point in the same direction. Superposition requires that we add the polarization vectors of all the photons up. They can add to give some overall polarization or they can add to give zero. When they add to give an overall polarization, we say that the light is polarized; when they give zero, the light is said to be unpolarized. A typical light source, such as a lamp, will generally emit unpolarized light. This is because the polarization vectors of the individual photons emitted from a lamp are typically randomly distributed. This means that if we were to pick a single photon out of the sample, it would be as probable that we would measure its polarization vector to be in one direction in the transverse plane as to find it in any other. If we were to add up all of the polarization vectors in the random distribution we would get zero total polarization. If we have polarized light, it is always most probable to find the polarization vector of an individual photon to be in the direction of the overall polarization of the light. The situation is hinted at in Figure 7.4.

7.2.3 Polarizers

How can we obtain polarized light from a source of unpolarized light? One simple method that we will make use of is to use a Polaroid filter, also called a polarizer. Polaroid consists of plastic upon which there are long strings

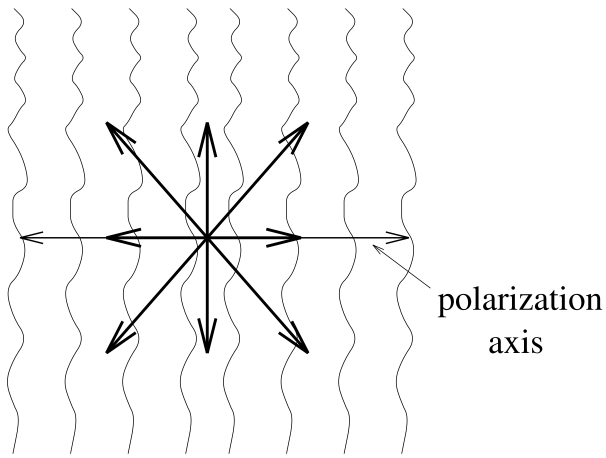


Figure 7.5: The polarizer transmits the component of the incident light with its polarization vector along the polarization axis.

of quinine iodosulfate molecules, see Figure 7.5. These molecules absorb photons with a polarization vector along the direction of the strings. This is because their electric fields oscillate along the strings and easily excite the electrons in the molecules and are therefore absorbed. Some of the remaining photons, namely those with a polarization vector close to the direction of the string, are absorbed as well. The photons that are actually transmitted are those whose polarization vector is nearly perpendicular to the strings, so that the resulting light has a strong polarization in the direction perpendicular to the strings. This perpendicular direction is called the polarization axis of the polarizer. Another way we'll use to illustrate how a polarizer works is that in Figure 7.6.

7.2.4 Malus' Law

The light that gets through the polarizer will be reduced in intensity, simply because only the photons with a component of polarization in the direction of the polarization axis get through. We would now like to get a quantitative measure of the transmitted intensity. We will first deal with polarized light incident on a polarizer. In Figure 7.7, the electric field vector $\vec{E}$ of the incident light is shown projected onto the polarization axis of the polarizer. The magnitude of the electric field for the transmitted light is just the component

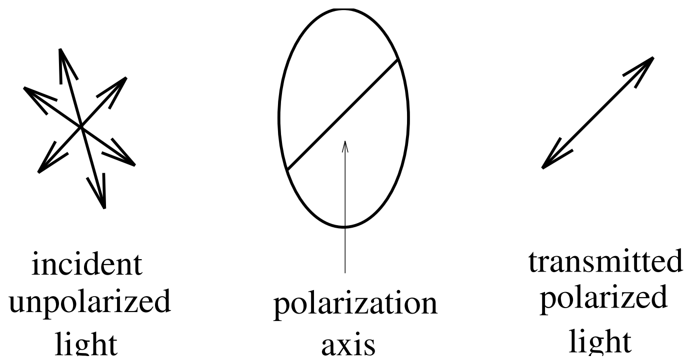


Figure 7.6: Another illustration of a polarizer.

of $\vec{E}$ along the polarization axis,

$$E_{\text{trans}} = E\cos\theta,$$

where θ is the angle between the polarization vector of the incident light and the polarization axis of the polarizer. The transmitted intensity is

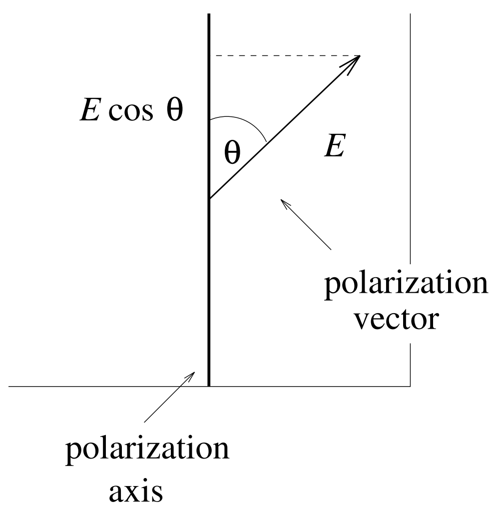


Figure 7.7: The sketch used to derive Malus' law.

$$\begin{aligned} I &= \frac{\epsilon_0}{2}|E_{\text{trans}}|^2 \\ &= \frac{\epsilon_0}{2}|E\cos\theta|^2 \end{aligned}$$

$$
\begin{aligned}
&= \left(\frac{\epsilon_0}{2}|E|^2\right)\cos^2\theta \\
&= I_0\cos^2\theta,
\end{aligned}
$$

where we have identified the intensity of the incident light as I_0.

The result:

$$\boxed{I = I_0\cos^2\theta,} \tag{7.1}$$

is called Malus' law. It is the only equation we need to discuss the results for this lab. However, as is clear from how we went about deriving it, it only applies to the case where the incident light is polarized. We would like to know what the transmitted intensity will be if the incident light is unpolarized. If we recall the discussion about unpolarized light consisting of photons with randomly distributed polarization vectors, since each photon is polarized we can average Malus' law over all of the contributions to the final intensity. Since there are 2π radians in a circle, the transmitted intensity, as given by Malus' law, is

$$
\begin{aligned}
I &= \frac{1}{2\pi}\int_0^{2\pi} I_0\cos^2\theta d\theta \\
I &= \frac{I_0}{2}.
\end{aligned}
$$

Half of the incident unpolarized light will get through the polarizer.

7.3 Apparatus

We will be using lamps as white light sources. The light should be, for the most part, unpolarized, but you will make measurements to determine if this is in fact the case. The polarizers are marked with angles. The polarization axis of the polarizers runs from the 0° marking to the 180° marking. We'll place the polarizers in magnetic mounts during use. There is a white mark at the base of the mounts which clearly indicates the angle between the polarization axis of the polarizer and vertical.

To measure the intensity of light that is transmitted through the polarizers, we will use a photodetector. The photodetector absorbs the light that falls on it and registers a voltage that you can read on a voltmeter. This voltage is directly proportional to the intensity of the light incident on the photodetector, $V \propto I$. We will only be studying *changes* in the transmitted intensity versus changes in the angle settings of the polarizers, and so we do not need to measure the constant of proportionality. A change in the voltage signals a proportional change in the intensity. An important property to note is that the photodetector must be connected to its power supply (in our case, a battery) in the proper direction, connect the *red* lead to the *positive* pole of the battery and the *black* to the *negative* pole.

7.W Polarization of Light Worksheet

Name: ____________________ **Day/Time:** ____________
Partner's Name: ________________________

7.W.1 In-Lab Procedure

It is important that you do not move the lamp during the course of data taking. Take care to leave the lamp in the same position for all measurements during a given procedure. You will probably also find it helpful to tape down the magnetic stands so that they don't move during measurements either.

Preliminaries

You should begin by connecting the photodectector to the multimeter. Set the multimeter to record voltage. Set the polarizers on the magnetic stands. There is a white mark at the base of the stand to allow you to measure the angle of the polarizer.

Set one polarizer directly in front of the photodetector. In this and all following parts of the procedure make sure that the gaps between the polarizers and the photodetector are as small as possible and that the lamp is as close as possible to the first polarizer. The set-up is illustrated in Figure 7.8

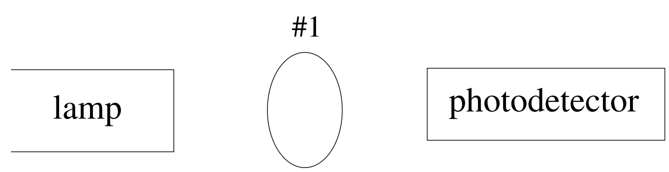


Figure 7.8: Is the light from the lamp polarized?

Record the voltage displayed. Watch the display for a few seconds. Does the voltage value change? If so, record the maximum and minimum values taken. Add half of the difference in these values to the uncertainty in your voltage values and claim the sum obtained as the uncertainty in all of the following voltage measurements. This accounts for any fluctuations in the voltage produced by the photodiode. Remember that the voltage, V, is directly proportional to the intensity, I, of the light falling on the polarizer.

For the above set-up measure the voltage, V, for five well separated polarizer angles, θ, in the range $0°$ to $90°$. Enter the voltage and angle values with uncertainties into Table 7.1.

Measurements of V vs. θ	
Voltage, V	Angle, θ

Table 7.1: V vs. θ measurements for single polarizer

7.W.2 In-Lab Computer Work

Plot V vs. θ on KaleidaGraph. Is the plot linear within error? If it is, then fit the line with curve fit1.

7.W.3 In-Lab Procedure

Polarized Light and a Polarizer

Place a second polarizer between the first polarizer and the photodetector, as in Figure 7.9.
Set the polarization angle of the first polarizer (#1), θ_1, to zero. Measure the angle that the polarization axis of the second polarizer (#2) makes with that of the first, θ_{12}, *relative* to θ_1. Measure V vs. θ_{12} for $10°$ increments from $0°$ to $90°$. Enter these values with uncertainties into Table 7.2.

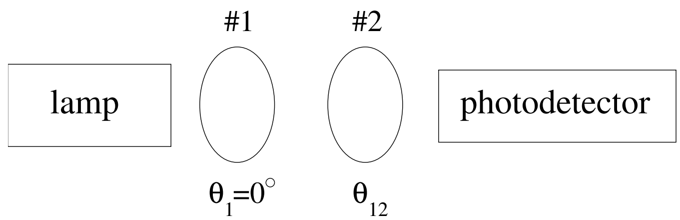


Figure 7.9: Set-up for testing Malus' law.

Measurements of V vs. θ_{12} for Two Polarizers.				
Voltage, V	Angle, θ_{12}		Voltage, V	Angle, θ_{12}

Table 7.2: V vs. θ_{12} measurements for two polarizers.

7.W.4 In-Lab Computer Work

Plot V vs. $\cos^2\theta_{12}$ with error bars. Is the plot linear? If it is, then fit the line. Show your work in obtaining the uncertainty in $\cos^2\theta_{12}$ in the space provided here.

7.W.5 In-Lab Procedure

Crossed Polarizers

Now place a third polarizer (#3) between the second polarizer and the photodetector, as in Figure 7.10. Set the angle of the polarization axis of this

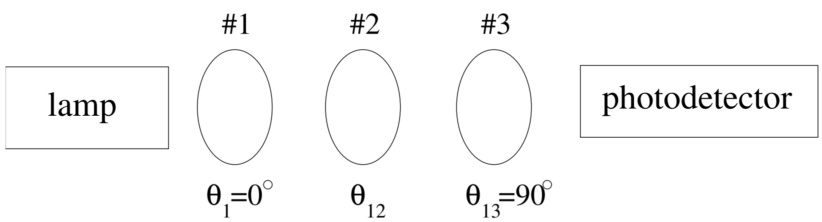


Figure 7.10: Three polarizers.

polarizer to be perpendicular (*i.e.*, at a 90° angle to) that of the first polarizer.

Again, measure V vs. θ_{12} for 10° increments from 0° to 90°. Remember that you need to vary the *middle* polarizer, #2. Enter the values for V and θ_{12} into Table 7.3.

Measurements of V vs. θ_{12} for Three Polarizers.				
Voltage, V	Angle, θ_{12}		Voltage, V	Angle, θ_{12}

Table 7.3: V vs. θ_{12} measurements for three polarizers.

7.W.6 In-Lab Computer Work

Plot V vs. θ_{12} with error bars. Is the plot linear? If it is, then fit the line.

7.W.7 Pre-Classroom Check List

- ◯ Table 7.1 completed with units and uncertainties
- ◯ Table 7.2 completed with units and uncertainties
- ◯ Table 7.3 completed with units and uncertainties
- ◯ 3 Plots labeled completely and correctly
- ◯ Each student has her/his own plots and worksheet

7.W.8 In-Classroom Discussion

Is the light from the desk lamp polarized before it encounters a polarizer? Discuss how you can determine this from the plot you made from the data you took in the first part of the procedure.

Is the data you took in the second part of the procedure consistent with Malus' Law (the theoretical relationship between the transmitted intensity and the initial intensity for polarized light passing through a polarizer at some relative angle)? Explain.

How can the intensity of the light passing through three polarizers be non-zero when the first and third polarizers are perpendicular? Assume ideal experimental conditions.

For the polarizer set-up of § 7.W.5 above, show that the correct expression for the intensity detected by the photodetector as a function of I_0 (intensity out of the lamp) and θ_{12} is

$$I = \frac{I_0}{8}\sin^2(2\theta_{12}) \tag{7.2}$$

Do this by expressing I_1 (intensity after the first polarizer) in terms of I_0, I_2 (intensity after the second polarizer) in terms of I_1 and θ_{12}, and I (final intensity) in terms of I_2 and the relative angle between polarizers 2 and 3, θ_{23}. You will need to use the fact that θ_1 and θ_{13} differ by 90°. Figure 7.11 might help.

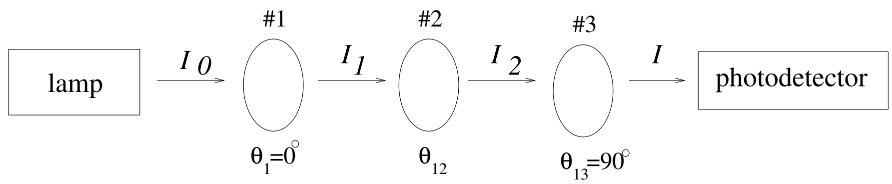


Figure 7.11: Three polarizers with the intensities referred to marked.

Show all your work

Sketch I/I_0, given by equation (7.2), versus θ_{12} from 0 to $\pi/2$ and compare this profile with the graph of V versus θ_{12} in § 7.W.5 above.

7.W.9 In-Classroom Conclusion

Write a *brief* (that is, a one or two paragraph) conclusion for this lab. In it, you should summarize the physical principles which were meant to be illustrated in this experiment. You should also describe the degree to which your data supported these principles.

Attach plots to the worksheet.

End Worksheet

Chapter 8

Refraction Optics

8.1 Introduction

You've probably seen that a pencil in a glass of water looks "bent." If you haven't, before reading any further, take a glass of water and a pencil, fill the glass half-way with water and place the pencil in. Look down from above the water's surface; the pencil will appear to be bent at the water's surface. It isn't really the pencil that's bent, the light rays coming from the pencil are bent at the air-water interface with the result that the pencil looks bent. This phenomenon of light rays being bent at the interface between two media, such as air and water, is called the *refraction of light*. We'll study several aspects and applications of it in this lab.

We'll examine the physical law, Snell's law, that describes the geometry of refraction (*i.e.*, how "bent" the pencil appears). We'll see that refraction occurs because light travels at different speeds in different media; we'll learn how to quantify this by defining the *index of refraction* of a medium. We'll also see that, for a given medium, there is a certain angle of incidence, called the *critical angle*, for which all of the incident light is reflected. This is called *total internal reflection* and it has important applications, which include the field of fiber optics.

8.2 Theory

8.2.1 References

The nature of light and refraction is covered in Serway, Chapter 35 (The Nature of Light and the Laws of Geometric Optics). Much of what we'll discuss will go a bit easier if you read the very short Section 35.1 (The Nature of Light), which describes the properties of light. Also useful are Sections 35.4 (Reflection and Refraction), 35.6 (Huygen's Principle), and 35.7 (Total Internal Reflection), which discuss Snell's law and total internal reflection.

8.2.2 Index of Refraction

At some point in your career as a student, someone will tell you that the speed of light in a vacuum is a constant, denoted by the letter c, whose value is $c = 2.997\,924\,58 \cdot 10^8$ m/s. This statement of the constancy of the speed of light has been verified by experiment and forms the basis for Albert Einstein's Theory of Relativity. We're not going to discuss relativity here, we only want to make the point that c refers to light in vacuum, that is, in a box, let's say, in which there is no air or anything else. If the light is traveling through a medium, perhaps a glass slab, or even the Earth's atmosphere, it will interact with the medium. Some of the light will "bounce off" of the atoms and molecules that make up the medium and, as a result, light will (almost) always travel slower through a medium. How much slower depends on the properties of the medium: its chemical composition, its density, temperature, etc. Also, the speed depends on the wavelength of light, a phenomenon called *dispersion*. We won't have the chance to explore this important and very interesting phenomenon, which is responsible for rainbows and the spectrum of colors emerging from prisms, but you are encouraged to read about it in Serway, Section 35.5 (Dispersion and Prisms), p. 994.

We can always express the velocity of light in a medium in terms of the vacuum value c. A convenient way of doing so is to define the *index of refraction*, denoted by n, as the ratio of c to v, the velocity of light in the medium,

$$n = \frac{c}{v}.$$

It is easy to see that n is a dimensionless number and, since v will be less than c for most materials, n is greater than one. Since v depends on the wavelength of the light, so does the index of refraction. We'll try to always report a value of n for a specific wavelength. If the index of refraction doesn't vary too much over a range of wavelengths, typically the visible band, we might quote a single value (of an appropriate number of significant figures) that is good over that range.

What are some typical values of the index of refraction? Air has an index of refraction which is very nearly equal to one. At 15°C, n_{air} varies from 1.000 325 6 (at a wavelength of 200 nm) to 1.000 273 6 (at 1000 nm) over the visible band. We'll be using red He-Ne laser light in the lab, with wavelength 632.8 nm, for which $n_{\mathrm{air}} = 1.000\ 276\ 0$. To a high degree of accuracy, we may take $n_{\mathrm{air}} \sim 1$. At 25°C, water has an index of refraction that varies from 1.452 (at 200 nm) to 1.322 (at 1000 nm). Glass is rather difficult to state a precise index of refraction for, chiefly because there are many different types and ways of making glass. In the lab we'll be using Plexiglas, which has a refractive index of roughly 1.49 and crown glass, which has $n \sim 1.52$.

Some terminology that is commonly used is to refer to media as being fast or slow. Fast media have small indices of refraction, while slow ones have larger indices of refraction. It should be clear that these are relative terms. For example, water is a fast medium compared to glass, but is slow compared to air.

8.2.3 Refraction at a Boundary - Snell's Law

We will consider a light ray which crosses the boundary between two media of indices of refraction n_1 and n_2, illustrated in Figure 8.1. The primary ray I is called the *incident ray*, R is called the *reflected ray*, and T is the *transmitted*, or *refracted ray*. All angles are defined with respect to the *normal* to the surface. We would like to know how the angle of reflection, θ_r, and the angle of refraction, θ_t, depend on the angle of incidence, θ_i, and n_1 and n_2.

We'll examine this by considering two parallel rays and using the somewhat obvious fact that points along the rays will travel at the same velocity in a given medium. Consider Figure 8.2, which illustrates two parallel rays, where the points A and B correspond to parallel points along the rays, since the line segment $\overline{AB}$ is perpendicular to both of the rays. Similarly C and D are parallel points and therefore the distances $\overline{AC}$ and $\overline{BD}$ are equal. If it takes a time t for the light to traverse the distance from A to C, then we can write $\overline{AC} = vt$. Since $\overline{AC} = \overline{BD}$, we have $\overline{BD} = vt$ as well, so that the time taken to go from B to D is also t. We can say that corresponding

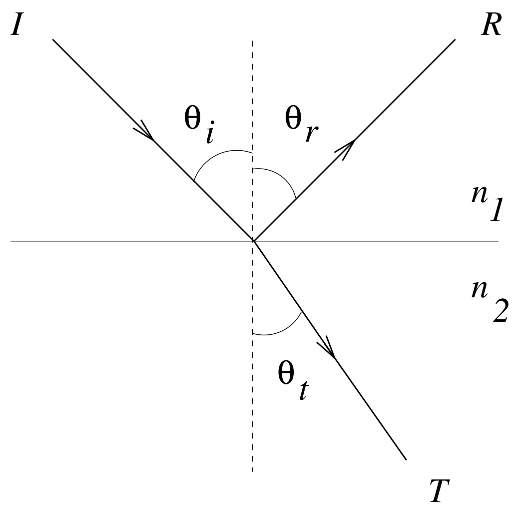


Figure 8.1: A ray of light incident on the interface between two media.

points along two parallel light beams correspond to equal times.

Let's apply this to an interface; for simplicity we'll consider the reflected and refracted rays separately. First, the reflected rays are drawn in Figure 8.3. Simple geometry tells us that the angles $\angle BAC = \theta_i$ and $\angle ACD = \theta_r$. If we take $\overline{BC} = v_1 t$ then, since $\overline{BC} = \overline{AC}\sin\theta_i$,we have

$$\overline{AC}\sin\theta_i = v_1 t.$$

By applying the equal time rule to points A and B and then C and D, we have $\overline{AD} = \overline{BC}$, so that

$$\overline{AC}\sin\theta_r = \overline{AD} = v_1 t.$$

We can then take the ratio

$$\frac{\overline{AC}\sin\theta_r}{\overline{AC}\sin\theta_i} = \frac{v_1 t}{v_1 t},$$

so that we find

$$\sin\theta_r = \sin\theta_i,$$

or simply

$$\theta_r = \theta_i.$$

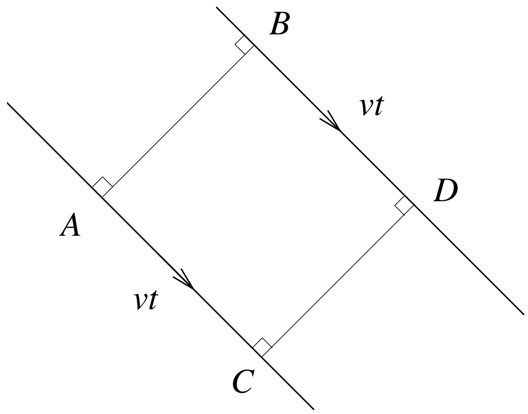


Figure 8.2: Corresponding points correspond to equal times.

This is a famous result, *the angle of reflection is equal to the angle of incidence*.

We can now look at the refracted rays, in Figure 8.4. Here we have $\angle BAC = \theta_i$ and $\angle ACD = \theta_t$ and can write $\overline{BC} = v_1 t$ and $\overline{AD} = v_2 t$ by the equal time rule. Then

$$\begin{aligned} \overline{AC} \sin\theta_i &= \overline{BC} = v_1 t \\ \overline{AC} \sin\theta_t &= \overline{AD} = v_2 t, \end{aligned}$$

which tells us that

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2},$$

which we write as

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t,} \tag{8.1}$$

which is known as Snell's law.

To summarize our results, we look back at Figure 8.1. If the light is incident at an angle θ_i, then we know that the reflected angle is equal to the incident angle, $\theta_r = \theta_i$, while the refracted angle, θ_t, may be found from Snell's law (8.1). We also note that when $n_2 > n_1$, that is, when we are passing from a fast medium to a slow medium, Snell's law tells us that the angle of refraction will be smaller than the angle of incidence. When moving from a slow medium to a fast medium, we will have $\theta_t > \theta_i$.

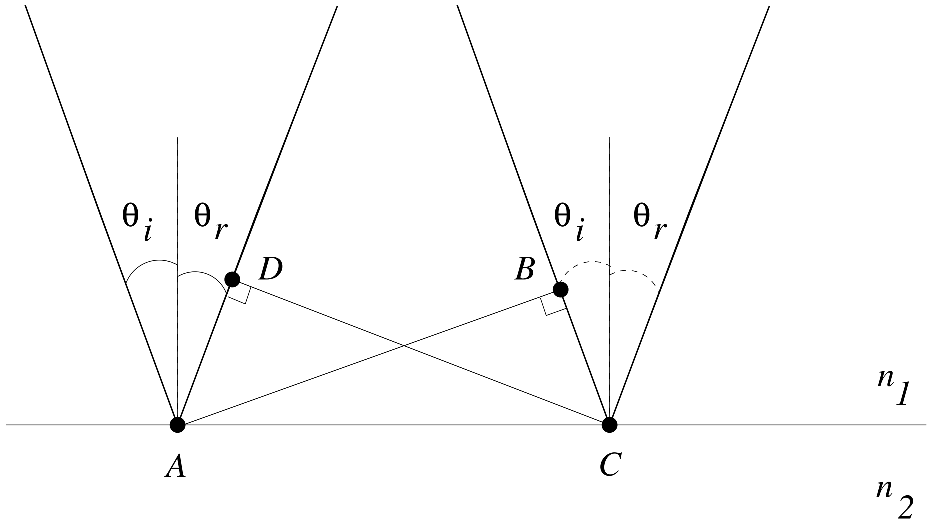


Figure 8.3: The incident and reflected rays at an interface.

Let's consider a light ray incident on a glass slab (index n) in air ($n_{\text{air}} \sim 1$), illustrated in Figure 8.5. If the faces of the slab are perfectly parallel, then the reflected ray R and the ray T_2 will be parallel. We can determine the dependence of the distance between them, s, on the incident angle and the width, D, by using Snell's law and a bit of trigonometry; the result can be determined without considering velocities or times. The student is encouraged to show as an exercise that

$$\boxed{s = \frac{2D\sin\theta_i\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}.} \tag{8.2}$$

8.2.4 Total Internal Reflection

Let's examine the possibility that, for a certain angle of incidence, the refracted angle, $\theta_t = 90°$, as shown in Figure 8.6. In other words, there is no refracted ray, since it does not exit medium 1. All of the incident light is reflected by the interface and none of it is transmitted. How can this happen?

Let's try to answer this by first noting that for $\theta_t = 90°$, $\sin\theta_t = 1$, so that Snell's law reads

$$\sin\theta_i = \frac{n_2}{n_1}, \tag{8.3}$$

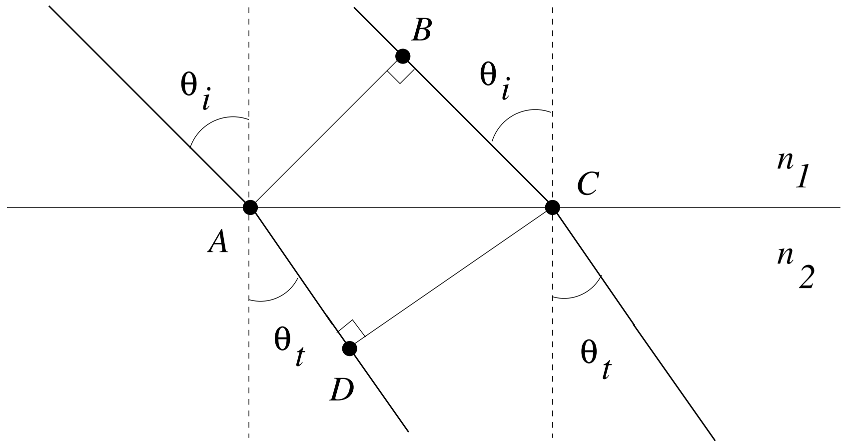


Figure 8.4: The incident and refracted rays at an interface.

or

$$\boxed{\theta_i = \sin^{-1}\left(\frac{n_2}{n_1}\right).}$$

Remember that n_1 and n_2 refer to the first medium and the second medium respectively. Now it is an extremely familiar property of the sine function that its magnitude is always less than or equal to one,

$$|\sin\theta| \le 1 \text{ for all } \theta, \quad \sin\theta = 1 \text{ only for } \theta = 90°. \tag{8.4}$$

We can consider the case that medium 1 is faster than medium 2, or $n_2 > n_1$, then equation (8.3) requires that θ_i satisfy

$$\sin\theta_i > 1.$$

This is impossible due to the fundamental property (8.4), so we conclude that there is no incident angle for which $\theta_t = 90°$, when $n_2 > n_1$.

If, instead, medium 2 is faster than medium 1, $n_2 < n_1$, then we have

$$\sin\theta_i < 1.$$

Therefore, there is *some* incident angle, we'll call it the critical angle, θ_{crit}, satisfying

$$\boxed{\theta_{\text{crit}} = \sin^{-1}\left(\frac{n_2}{n_1}\right),} \tag{8.5}$$

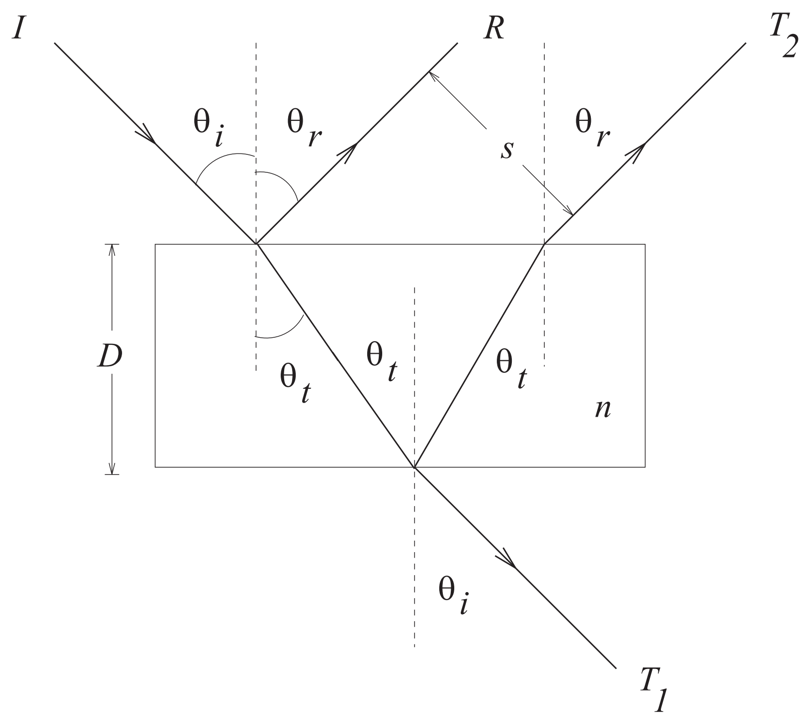


Figure 8.5: The glass slab.

for which there will be no refracted ray. Note that we have switched our point of view. We are now using the larger index of refraction as n_1 and the smaller as n_2. As an exercise the student should come up with an argument for why there will also be no refracted ray if the angle of incidence is *greater* than θ_{crit}. This phenomenon is called *total internal reflection*: *when passing from a slow medium into a faster medium, there is a critical angle of incidence, such that for incident angles greater than or equal to the critical angle, all of the light will be reflected from the interface.*

Total internal reflection has one application in the field of fiber optics. There, a glass fiber is used to carry a light beam, as in Figure 8.7. The angle of incidence for each "bounce" off of the walls of the cable is kept greater than the critical angle determined from Snell's law. As a result, the signal is kept almost completely intact over its transit through the cable, which can be hundreds of miles long. Clearly this is an efficient means of transmitting information encoded into a light signal. Serway provides a nice discussion of fiber optics in the essay on page 1009.

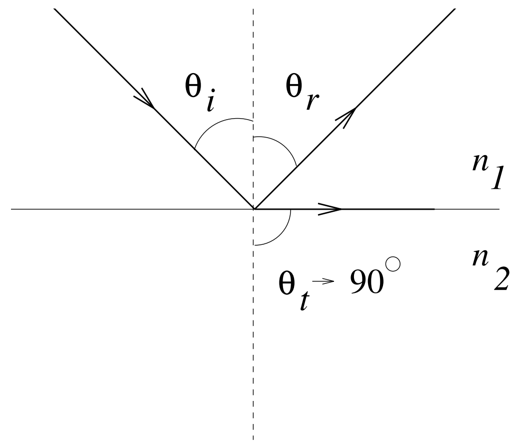


Figure 8.6: Total internal reflection occurs when the refracted ray cannot exit the primary medium.

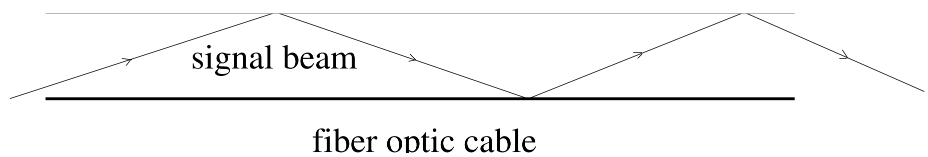


Figure 8.7: A depiction of a fiber optic cable.

8.3 Apparatus

For our refraction experiments we will be using a He-Ne laser. This produces red light with a wavelength of 632.8 nm. The laser produces an intense beam of light that is difficult to duplicate with a white light source. Using such an intense beam does not come without a price; we must be particularly cautious when using the laser.

Laser Safety Guidelines:

- Avoid looking directly into the laser beam or its reflections. The laser light is intense enough to burn the retina and permanently impair vision if it comes into direct contact with the eye.
- A "dish" with a steel rim will be provided for containing the laser beam during use. The laser should not be turned on unless it is safely pointed inside the dish.

- Do not point the laser across the room or into the air; doing so will endanger everyone else in the room.
- Do not point the laser through the lenses.

We'll use a Plexiglas slab with an index of refraction of about 1.49 to examine Snell's law. To investigate total internal reflection, we'll use a prism made of crown glass, which has an index of refraction of about 1.52.

8.W Refraction Worksheet

Name: ____________________ **Day/Time:** ____________

Partner's Name: ________________________

8.W.1 In-Lab Procedure

Ray Tracing

We are using lasers for our refraction studies so that we have clear, well defined beams of light, such as those we've been drawing in our figures. We'll use our experimental set-up to trace these beams and the objects we're using to refract the light onto a sheet of paper. This will give us a picture of what was going on, called a *ray tracing*, and will make it easy to measure angles and distances, so that we can tell if Snell's law is really valid.

What is a ray tracing? It is a projection of the light rays passing through our apparatus onto a sheet of paper. How do we make a ray tracing? First of all, we place the refracting object, say a Plexiglas slab, on top of a large sheet of computer paper that we've taped to the bottom of the laser dish. The set-up and several of the light rays are shown in Figure 8.8a. We then trace each ray individually. Using a corner of a glass block, we'll block off the beam, as shown in Figure 8.9. Be careful that you avoid looking directly into the reflection of the beam from the block. The laser beam has a finite width, so that we can partially block off the beam, if we position the glass block properly. We'll block the beam so that the glass block is approximately at the *center* of the beam, then we'll make two pencil marks at the corner of the block. What we've done is taken a point on the beam, which is above the paper, and projected it down to a point on the paper. If we move to another section of the beam and repeat this procedure, we'll have two points on the beam marked on the paper. By drawing a line through these two points, we can reproduce the path that the beam took.

Figure 8.8b illustrates how a raytracing of all the beams in Figure 8.8a will appear. All of the rays have been marked in two places by the procedure outlined; the glass slab has also been traced. By connecting the corresponding points with straight lines, we can reproduce all of the rays found outside

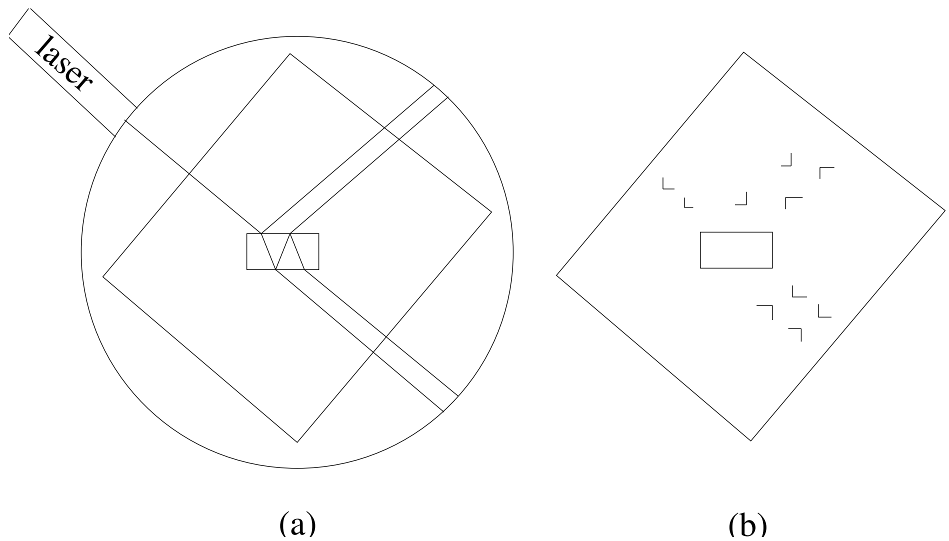


Figure 8.8: The set-up for ray tracing with a Plexiglas slab.

the slab in Figure 8.8a. By connecting the corresponding points where these rays intersect with the outline of the slab, we can draw the interior rays.

Snell's Law and a Glass Slab

Set up the laser and Plexiglas slab exactly as shown in Figure 8.8a. Perform the ray tracing as outlined in § 8.W.1. **Be sure to outline the Plexiglas block**. It will probably help you to label the marks you make during the ray tracing as the incident ray, the first reflected ray, etc. When you've traced all of the rays that you can, pick the paper up and connect the corresponding points with a straight edge. Connect the intersecting points at the slab outline to visualize the internal rays. Before continuing to the next procedure, look at your tracing and answer the following rhetorical questions. Does the resulting tracing appear as you expected it to? Do the sides of the slab appear to be parallel? How do the rays that you've traced allow you to determine this?

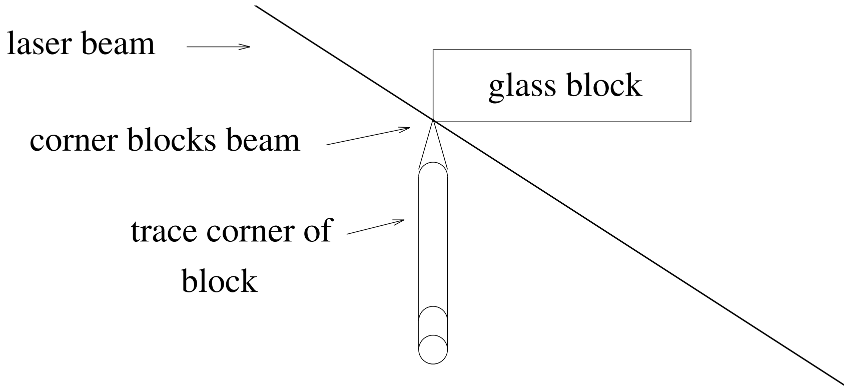


Figure 8.9: Ray tracing is done by blocking the beam and then marking off the corner of the block.

Total Internal Reflection

Place a new sheet of computer paper in the laser dish and place the prism on it. Aim the laser through one of the short sides of the prism, shown in Figure 8.10a. Start with an incident angle (θ_i in Figure 8.10b) of approximately

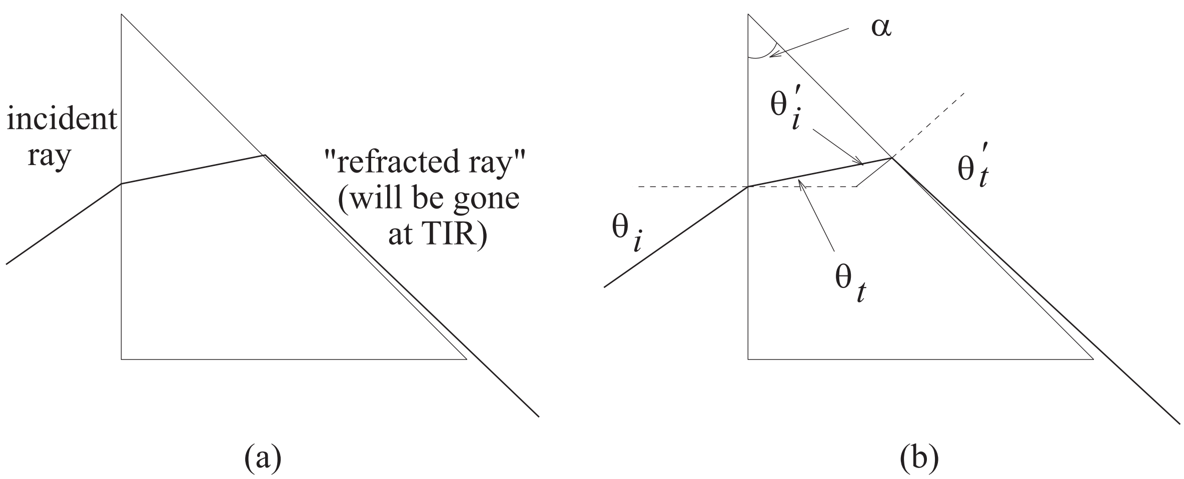


Figure 8.10: Total internal reflection with the prism.

30° and decrease the incident angle, while keeping an eye on the ray labeled "refracted ray" in Figure 8.10b. Is there an incident angle θ_i for which the refracted ray disappears? Locate the critical angle; when you find it, leave the prism in place and **trace the outline of the prism and the incident ray**.

8.W.2 Pre-Classroom Check List

- ◯ Ray tracing of slab and at least four rays per person
- ◯ Ray tracing of prism and incident ray per person

8.W.3 In-Classroom Calculations & Analysis

Snell's law and a Glass Slab

At the vertex where the incident ray, the first reflected ray and the first interior (refracted) ray meet, use a protractor to measure the angle of incidence θ_i, the angle of reflection θ_r, and the angle of refraction θ_t. Record these values (with uncertainties) below. To account for any errors introduced in the ray tracing procedure, take approximately 2° uncertainty in all of your angle measurements.

$\theta_i =$ ________________ $\theta_r =$ ________________

$\theta_t =$ ________________

Does $\theta_r = \theta_i$ within uncertainty?

Assuming that the index of refraction of air, n_{air}, is 1, use Snell's Law to determine a value for the index of refraction of the Plexiglas slab, n_{slab}, *with uncertainty.* Showing your work, record this value below.

$n_{slab} =$ ________________

How does this compare with the expected value $n \sim 1.49$?

Now, referring to Figure 8.5, measure D of the Plexiglas slab from your outline.

$$D = \underline{\hspace{4cm}}$$

Given that the rays you use to *measure* s may not be perfectly parallel (you'll have to look at your sketch to find out), determine a typical value of s from your sketch and calculate an uncertainty for it that takes into account any deviations of the rays from being parallel. This isn't as hard as it may sound, just use common sense to get a ball-park figure and let your uncertainty account for the arbitrariness in doing so.

$$s_{meas} = \underline{\hspace{4cm}}$$

Calculate the value predicted by equation (8.2) for the distance between the primary and secondary reflected rays, s_{calc}, *with uncertainty.* Make sure you use your value of n_{slab} for n in this equation. Record the calculated value below. **SHOW WORK**.

$$s_{calc} = \underline{\hspace{3cm}}$$

Total Internal Reflection

As is illustrated in Figure 8.10b, θ_i is *not* the critical angle we were referring to in our discussion of total internal reflection in § 8.2.4. Since total internal reflection is occurring at the *second* glass to air interface, the critical angle corresponds to the incident angle θ_i' in Figure 8.10b. How can we determine θ_i'? Measure the prism angle, α, and θ_i.

$$\alpha = \underline{\hspace{3cm}} \qquad \theta_i = \underline{\hspace{3cm}}$$

From Snell's law, we find at the first interface that

$$\theta_t = \sin^{-1}\left(\frac{1}{n}\sin\theta_i\right),$$

where the index of refraction of the prism, $n \sim 1.52$. By simple geometry, we can calculate θ_i' from

$$\theta_i' = \alpha - \theta_t.$$

First use this to calculate θ_t *with uncertainty*. **SHOW WORK**.

$$\theta_t = \underline{\qquad\qquad\qquad}$$

Calculate a value for θ_i' *with uncertainty.* Also, use equation 8.5 to calculate the critical angle of incidence for total internal reflection in crown glass, θ_{crit}. (Again, use $n \sim 1.52$ for the index of refraction of the prism.) Record these values below. Use the space above the recorded values to **show your work**.

$$\theta_i' = \underline{\qquad\qquad\qquad} \qquad \theta_{crit} = \underline{\qquad\qquad\qquad}$$

8.W.4 In-Classroom Discussion

Snell's Law and a Glass Slab

Do the sides of the Plexiglas slab appear to be parallel?

Explain how the rays that you've traced allow you to determine this.

Compare the distance you measured between the primary and secondary reflected rays, s_{meas} with the predicted value s_{calc}.

Total Internal Reflection

Finally, compare the value you determined for θ_i' with the value of θ_{crit} that you calculated.

8.W.5 In-Classroom Conclusion

Write a *brief* (that is, a one or two paragraph) conclusion for this lab. In it, you should summarize the physical principles which were meant to be illustrated in this experiment. You should also describe the degree to which your data supported these principles.

End Refraction Worksheet

Chapter 9

Imaging Optics

9.1 Introduction

Having studied the details of how matter bends light, now we'll examine how to use refraction to form images of various objects. Lenses are designed to refract light rays from an object and focus them into an image. We'll learn what the *focal length* of a lens is and study the relationship between the distance of an object from a lens and the distance of its image, known as the *thin lens equation*. We'll also see how to use combinations of lenses to form images, which is the principle behind microscopes and telescopes.

9.2 Theory

9.2.1 References

Geometric optics, the optics of lenses, is covered in Chapter 36 (Geometric Optics). Imaging optics and lenses are discussed in Sections 36.3 (Images Formed by Refraction) and 36.4 (Thin Lenses).

9.2.2 Imaging Optics and Lenses

We've learned that the refraction of light at a surface may be described by Snell's law. This behavior is exploited in the design of lenses, which make use of refraction to focus light rays to form images. How a lens works is fairly simple to explain, but we'll forego any detailed mathematical description; we'll also consider only convex, or converging, lenses. For a more comprehensive survey, consult the references to Serway already given.

A converging lens is designed so that parallel rays are focused into a point, called the *focal point* of the lens, illustrated in Figure 9.1. How this happens

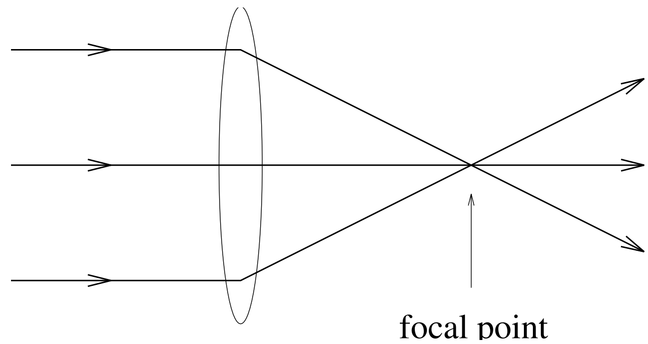


Figure 9.1: A converging lens focuses parallel rays.

can be seen by examining part of the lens; in fact we'll look at only the front surface, in Figure 9.2. If I_1 is incident on the surface with incident angle

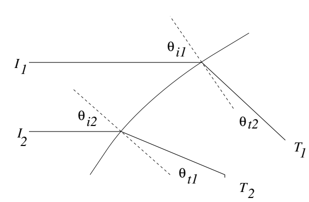


Figure 9.2: The front surface of a lens.

θ_{i1}, and I_2 is incident on the surface with incidence angle θ_{i2}, keeping in mind that the two incident rays are parallel we can use Snell's law, with the general condition that $n_{\text{lens}} > n_{\text{air}}$, to show that the resulting refracted rays T_1 and T_2 converge. It is left as another exercise to the student to develop this argument further and to draw a sketch of the second surface of the lens, where T_1 and T_2 exit the lens into the air, to show that the final rays are also converging, as we've indicated in Figure 9.1. A lens is made by carefully grinding a piece of glass (typically) to a precise curvature, so that all of the parallel rays converge at the same point, namely the focal point of the lens.

The whole reason for using lenses is to convert an object to an image. Why this is important is obvious if you consider what a camera does. It

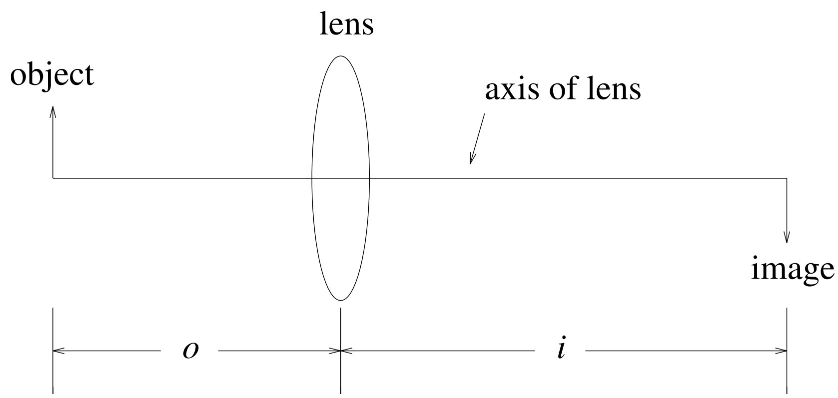


Figure 9.3: The definition of object and image distances.

takes an object that you've taken an interest in and places its image on film, which you can preserve for posterity. Also, as we'll see, the object and image will not in general be of the same size. This is important if our object is too small or too large to for us to see conveniently with the naked eye. The proper lens will produce an image that allows us to view the object easily.

Let's examine how we can determine the location of the image produced by a lens of an object placed at a known distance. We'll first learn how to do this graphically, then we'll see how to use an equation to help us. We'll illustrate the object and image with arrows, as in Figure 9.3. The object distance, o, and the image distance, i, are defined in the figure. We'll find the image by creating what is called a *ray diagram*. First, we assume that the object is *outside* of the focal length of the lens, or $o > f$, if we use f to denote the focal length. We then draw three rays, see Figure 9.4.

Ray 1 is the ray parallel to the axis of the lens, which will be focused through the opposite focal point; ray 2 is the ray which passes right through the center of the lens; while ray 3 is the opposite of ray 1, it passes through the near focal point, so that it is focused into a parallel ray. The image appears at the *intersection* of the three rays. Of course, you would be able to see the image anywhere on the right-hand side of the lens, but it would be out of focus and blurry unless you were at the image distance. This image is also called a *real* image, because you would be able to focus it onto a screen held in front of the lens. Note also that the image is *inverted*, that is, it's upside-down. If this is a bit confusing now, don't worry; we're going to examine each of these statements in the lab.

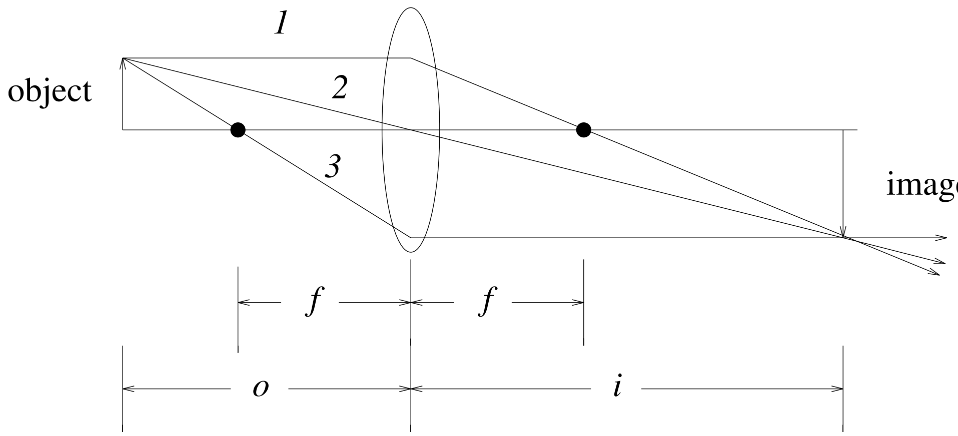


Figure 9.4: The ray diagram when the object is outside of a focal length.

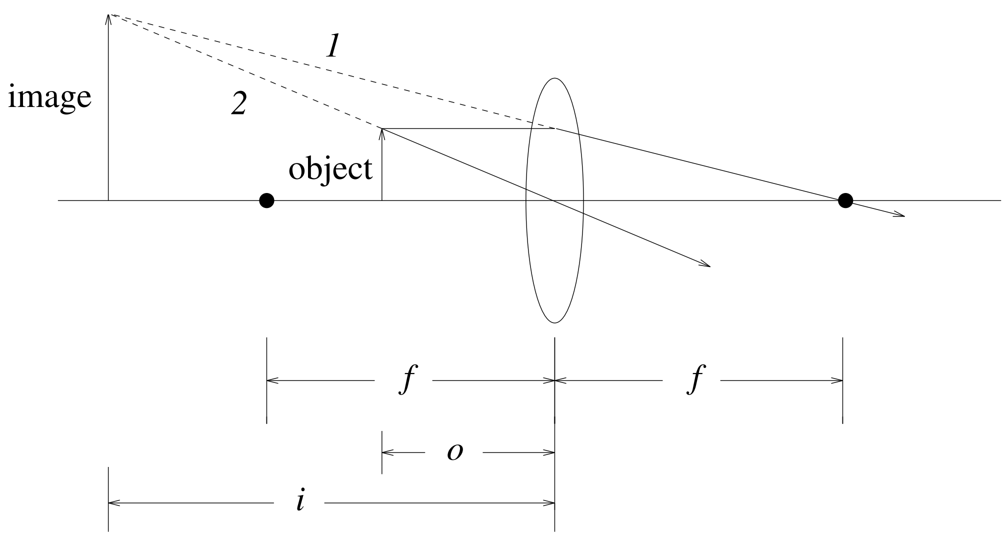


Figure 9.5: The ray diagram when the object is inside of a focal length.

If the object is *inside* of the focal length, $o < f$, things get a bit trickier. The ray diagram is drawn in Figure 9.5. We still draw the parallel ray 1, but we extend the ray *backwards*. Ray 2 is still the center ray, but again, we extend it backwards. Where rays 1 and 2 intersect marks the image. Note that the image is on the *same side* of the lens as the object. This image differs from a real image, because you can't project it on a screen, so we call it *virtual*. You can still see it by looking through the lens though. Also note that the image is now rightside-up, we say that it is "upright."

9.2.3 Compound Lenses

A compound lens is simply a set of several lenses arranged so that the light from each lens passes through the next, as in Figure 9.6. The only "new"

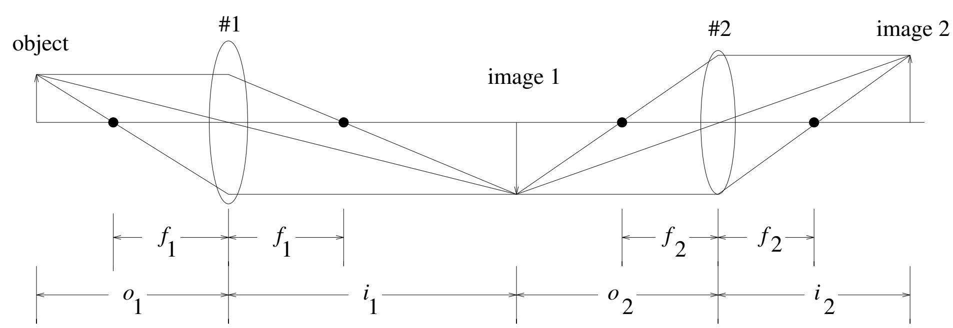


Figure 9.6: In a compound lens, the image of the preceding lens acts as the object for the following lens.

piece of information we need to deal with compound lenses is that the image produced by one of the lenses will act as the object for the next lens in the series. In Figure 9.6, the ray diagram for a two-lens system illustrates this principle nicely.

9.2.4 The Thin Lens Equation

The ray diagram technique is an excellent way to visualize how an image is formed, but we would like a faster, more convenient method of relating the image distance to the object distance. Such a method is available, in the approximation that our lenses are *thin*. This means that the distances from the object to the lens and from the lens to the image are much larger than the thickness of the lens itself. Since lenses are usually only a few millimeters thick and object and image distances are typically at least tens of centimeters, this is not a bad approximation. With the thin lens criterion, we have the *thin lens equation*

$$\boxed{\frac{1}{o}+\frac{1}{i}=\frac{1}{f},} \tag{9.1}$$

where, as you might have guessed, o is the object distance, i is the image distance, and f is the focal length of the lens.

There is a sign convention for image distances that accompanies the thin lens equation. To see why, consider the case we treated in Figure 9.5, where the object distance was less than a focal length. In this case, the image is virtual and the image distance is *negative*

$$\frac{1}{i}=\frac{1}{f}-\frac{1}{o}<0.$$

The student should verify that for a real image, *i.e.*, when $o > f$, the image distance is *positive*. It's clear from our graphical results for the two cases that we should consider a negative image distance to refer to an image on the same side of the lens as the object, while a positive value of i corresponds to a distance measured on the opposite side from the object.

9.2.5 Magnification

We've seen how to quantify object and image distances, but what about the size of the image? We'll use Figure 9.7 to define the object and image *sizes*, h_{o} and h_{i} respectively, where h_{o} is always positive and h_{i} is positive for upright and negative for inverted images. The *magnification* of a lens is defined as the ratio

$$\boxed{m=\frac{\text{image size}}{\text{object size}}=\frac{h_{\mathrm{i}}}{h_{\mathrm{o}}}.} \tag{9.2}$$

For thin lenses, we can use the thin lens equation and some geometry to express the magnification in terms of the object and image *distances*

$$\boxed{m=-\frac{i}{o}.} \tag{9.3}$$

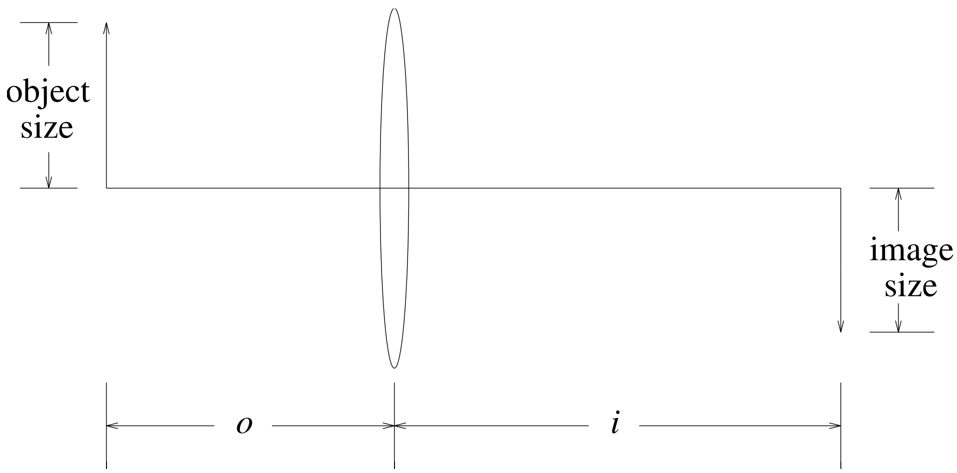


Figure 9.7: Note the difference between object and image *size* and *distance*.

We've introduced a minus sign because of the sign convention for the image distance. We leave it up to the student to verify that a *positive* magnification corresponds to an *upright* image, while a *negative* magnification corresponds to an *inverted* image.

9.3 Apparatus

For our investigations of imaging optics, we'll use a lamp as a white light source which has a crossed arrow slide as an object and two converging lenses (one having a focal length of 100 mm and the other of 200 mm). We'll use $3'' \times 5''$ note cards to find the images produced by our lenses. All these will be mounted (using lens holders for the two lenses, and a screen holder for the note card) on mobile carriages that will move on an optical bench which has a scale on one of its sides.

9.W Imaging Optics Worksheet

Name: ____________ **Day/Time:** ____________
Partner's Name: ____________

9.W.1 In-Lab Procedure

Focal Length and Magnification of a Lens

Mount the light source at one end of the optical bench followed by either one of the lenses, and the note card. When you've gotten this set-up, it should look like the illustration in Figure 9.8. Refer to the nominal focal length

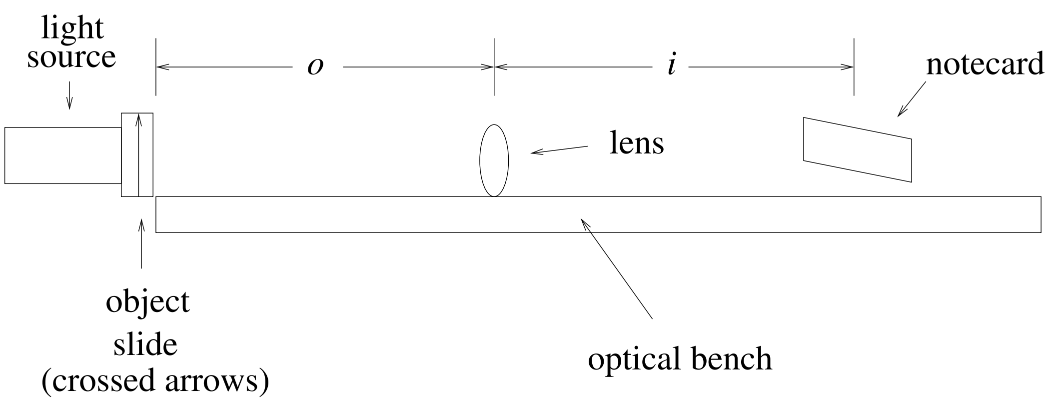


Figure 9.8: The set-up for measurement of focal length.

printed on the lens and answer the following questions.

Place the lens **HALF** its nominal focal length from the object slide. Record your answers to the following questions.

When you look through the lens, can you see a *clear* image of the object? If you can, is this image upright or inverted?

Are you able to project (through the lens) a *clear* image of the object on the note card? If you are, is this image upright or inverted?

Place the lens **TWICE** its nominal focal length from the object slide. Record your answers to the following questions.

When you look through the lens, can you see a *clear* image of the object? If you can, is this image upright or inverted?

Are you able to project (through the lens) a *clear* image of the object on the note card? If you are, is this image upright or inverted?

Is all of this consistent with the predictions we made with our ray diagrams?

Measurements of i vs. o for "100 mm" lens.			
Image Distance (i)	Object Distance (o)	Image Size	Object Size
		Leave Blank	Leave Blank
		Leave Blank	Leave Blank
		Leave Blank	Leave Blank
		Leave Blank	Leave Blank

Table 9.1: i vs. o measurements for "100 mm" lens.

Now make image distance versus object distance measurements for at least five object-image distance pairs. Record your five image distance versus object distance measurements for the 100 mm lens into Table 9.1. For the first pair of measurements, also measure the image *size* and the object *size* and record them in the designated place. Do this by picking two points on one of the arrows; measure the object size directly from the slide and measure the image size by marking the note card at the corresponding points on the image and then measuring the distance with a ruler. Size is positive for a upright image and *negative for a inverted image.* Be sure to include uncertainties.

Is the image upright or inverted?

Repeat the same i versus o measurements with the other lens entering the data into Table 9.2. Remember to make one object and image size measurement as well.

Measurements of i vs. o for "200 mm" lens.			
Image Distance (i)	Object Distance (o)	Image Size	Object Size
		Leave Blank	Leave Blank
		Leave Blank	Leave Blank
		Leave Blank	Leave Blank
		Leave Blank	Leave Blank

Table 9.2: i vs. o measurements for "200 mm" lens.

9.W.2 In-Lab Computer Work

Now, plot $1/i$ versus $1/o$ for each lens. You can show your work in calculating $\Delta(1/o)$ and $\Delta(1/i)$ in the bottom of page 243. Obtain a weighted least-squares curve fit1 for each plot. From these fits, determine the focal length of each lens. Record the slopes and intercepts on the lines below (with uncertainty).

"100 mm" lens:

Slope, a_1= ______________ Intercept, b_1= ______________

"200 mm" lens:

Slope, a_2= ______________ Intercept, b_2= ______________

9.W.3 In-Lab Procedure

A Compound Lens System

Using both of your lenses, build the configuration shown in Figure 9.9. Place the 100 mm lens at about 50 mm away from the object.

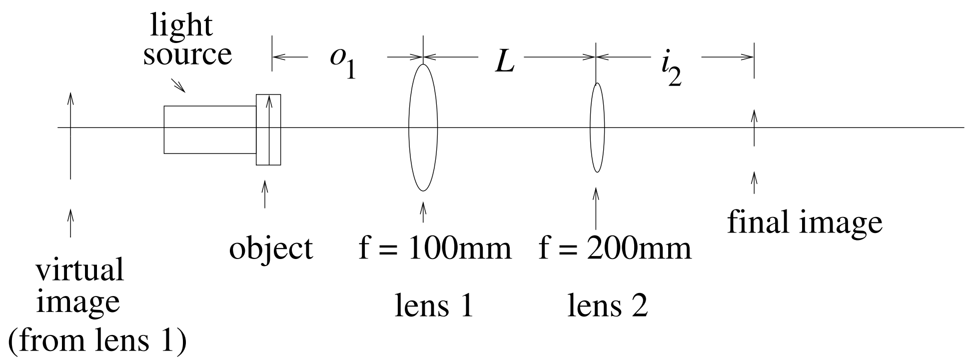


Figure 9.9: A Compound Lens System.

Measure the object distance o_1, distance L and the image distance i_2. Record these below with uncertainties.

$o_1 =$ ______________

$L =$ ______________

$i_2 =$ ______________

Measure the original object size h_o and the final image size h_i of the compound system. Record these below with uncertainties.

$h_o =$ ______________ $h_i =$ ______________

9.W.4 Pre-Classroom Checklist

- ◯ Table 9.1 with uncertainties and units
- ◯ Table 9.2 with uncertainties and units
- ◯ Plot of $1/i$ vs $1/o$ for 100 mm lens
- ◯ Plot of $1/i$ vs $1/o$ for 200 mm lens
- ◯ All questions answered

9.W.5 In-Classroom Calculations & Analysis

Focal Length and Magnification of a Lens

What is the slope and the intercept of the $1/i$ versus $1/o$ plot predicted by the thin lens equation? **SHOW WORK**.

From your slope and/or intercept, calculate the measured focal lengths of each lens, *with uncertainties*, and record them below.

$f_1 =$ ________________ $f_2 =$ ________________

From your image and object sizes, calculate the magnification you measured for each of the lenses, m_{100} and m_{200}. Record these below with uncertainties **SHOW WORK**.

$m_{100} =$ ______________ $m_{200} =$ ______________

Now, from the image and object distances in the cases where you measured the image and object sizes, calculate the *predicted* magnification of each lens, m'_{100} and m'_{200}. Record these below with uncertainties. **SHOW WORK**.

$m'_{100} =$ ______________ $m'_{200} =$ ______________

A Compound Lens System

Note: In the procedure of this section, you should use the *measured* focal lengths of the lenses. In other words, **DO NOT** use 100mm and 200mm as the focal lengths of your lenses. Instead, use the values you have just determined for f_{100} and f_{200}.

Calculate the position of the first image; that is, the image distance of the first lens. The uncertainty is given by $\Delta i = \left(\frac{1}{f} - \frac{1}{o}\right)^{-2} \left(\frac{\Delta f}{f^2} + \frac{\Delta o}{o^2}\right)$.

$i_1 =$ ________________

Is the first image real or virtual?

Calculate the object distance o_2 and the image distance i_2 of the second lens. (Hint: The *image* of the first lens act as the *object* for the second lens.) **SHOW WORK**.

$o_2 =$ ________________

$i_2^{(calc)} =$ ________________

Calculate the predicted magnification of the compound system, m'_{comp}. Note that $m'_{comp} = m'_{100} \times m'_{200}$, but you **CANNOT** use the previously determined m'_{100} and m'_{200}. You must calculate the new individual magnifications from the given and derived o_1, i_1, o_2, and i_2 of each lens in the compound system.

$$m'_{comp} = \underline{\hspace{4cm}}$$

Using the final image and original object sizes you measured, calculate the magnification of the final image, m_{comp}. Show work.

$$m_{comp} = \underline{\hspace{4cm}}$$

9.W.6 In-Classroom Discussion

Focal Length and Magnification of a Lens

Consider the image(s) you were able to observe when the lens was half its nominal focal length from the object slide. Were they real or virtual?

Consider the image(s) you were able to observe when the lens was twice its nominal focal length from the object slide. Were they real or virtual?

Compare the slopes of each of your plots to the slope predicted by the thin lens equation.

Compare the nominal focal lengths of each of your lenses to the measured values.

Compare the measured magnifications of each of your lenses to the predicted values.

Is the sign convention for magnification consistent with your observations?

A Compound Lens System

Compare the image distance you measured for the compound lens system, i_2, to the predicted image distance $i_2^{(calc)}$.

Compare the magnification you predicted for the compound lens system, m'_{comp}, with the measured magnification, m_{comp}.

9.W.7 In-Classroom Conclusion

Write a *brief* (that is, a one or two paragraph) conclusion for this lab. In it, you should summarize the physical principles which were meant to be illustrated in this experiment. You should also describe the degree to which your data supported these principles.

End Worksheet

Chapter 10

Diffraction and Interference of Light

10.1 Introduction

Maxwell's laws of electromagnetics indicate that light has wave-like properties, in fact light has dual properties being wave-like and particle-like. In this class we will be studying the wave-like properties of light. In Chapters 8 and 9, we made the *ray approximation* which states that a wave moves through a medium in a straight line. If the wave meets a barrier with an opening there are three possibilities that arise depending on the width, a, of the opening. First, for $a \gg \lambda$ where λ is the wavelength of the wave of light, the wave continues to travel in a straight line (apart from some edge affects). Second, for $a \sim \lambda$, the wave spreads out from the opening, the ray is **diffracted**; last, for $a \ll \lambda$ the opening acts like a point source of waves. The ray approximation is *invalid* for the second and third cases. These three possibilities are depicted in Fig. 10.1.

We are studying the wave-like properties of light as demonstrated by the phenomenon of diffraction, interference, and polarization (Chapter 7). For this lab, we will be using openings whose widths are of the order of the wavelength of light; therefore, we will be dealing with waves spreading out in all directions. The ray approximation we used in Chapters 8 and 9 will **no longer be valid**. Since we are treating light like a wave, we will be able to apply the properties we have learned about waves, most importantly that they obey the superposition principle and thus interfere.

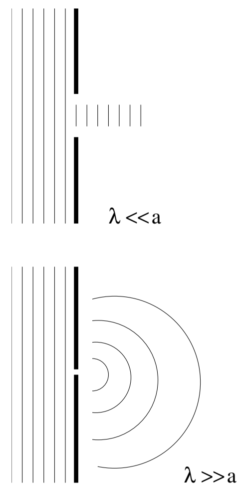


Figure 10.1: The three possible scenarios for light's path through an opening. The top left figure depicts light "unaffected" by the opening, the top right depicts diffraction effects, and the bottom depicts the opening as a point source of light.

The following theory section is divided into seven subsections. After a description of references, we will go through an introduction to diffraction and interference phenomenon. Next we will study the two-slit interference pattern in detail, ignoring diffraction effects in the limits where each slit behaves as a point source, followed by a less detailed multiple-slit interference subsection. Then the diffraction grating is introduced. The final two subsections reintroduce diffraction for the single slit and multiple slits, there we will study how diffraction determines the availability of light for our study of interference.

We will be performing four types of experiments. First, you will investigate the phenomenon of diffraction using a single slit. Second, you will use a slide with two slits to study two-slit interference and diffraction. The same slide also has some multiple slits on it which you will use to study multiple-slit interference and diffraction. Finally, you will use a diffraction grating to study the effects of having 5000 lines (or slits) per cm.

10.2 Theory

10.2.1 References

Serway addresses the interference of light waves in Chapter 37 (Interference of Light Waves), and cursorily discusses diffraction in Chapter 38 (Diffraction and Polarization). The single slit diffraction pattern appears in Section 38.2, and the diffraction grating shows up in Section 38.4. We will discuss these and other effects here.

10.2.2 Diffraction and Interference

Light is an electromagnetic *wave*; therefore, just as two waves can add together destructively or constructively, so can light. In constructive interference, the two wave amplitudes add together such that the resultant wave's amplitude is greater than either individual's amplitude. In destructive interference, the amplitude of one wave "cancels" the amplitude of the other wave. In order to observe the interference of two waves, we need coherent, monochromatic sources. Coherent means the the sources have the same phase, we will label the phase ϕ. Monochromatic means that the sources have the same wavelength, λ. Given these conditions, we will use a laser as our light source since a laser is a source of monochromatic light, and we will be illuminating a slide with slits in it thus creating sources of waves that are in phase at the slits.

If light really did travel in straight line paths (like what we investigated in Chapters 8 and 9) there would be no overlap, no interference pattern (see the top left figure in Fig. 10.1). What really happens is that light, when incident upon an opening, deviates from a straight line path and enters the region that would otherwise be in shadow. The divergence of light from its initial path of travel is called diffraction. Diffraction generally occurs when light passes through a small opening, around sharp corners, or past sharp edges. We will be assuming **very narrow slits**; therefore, we will expect diffraction.

10.2.3 Two-Slit Interference

For now only, let's assume that the two slits are so narrow that they act as two wave sources, so $a \ll \lambda$, as in the bottom picture in Fig. 10.1. With this set-up we can study the interference of two waves and then add the effects of diffraction later. We know that waves interfere constructively and destructively from our previous studies of waves in physics; therefore, we expect to see bright and dark areas, respectively, on the viewing screen.

Consider the diagram in Fig. 10.2. It shows two openings of width a whose centers are separated by a distance d. We want to calculate the intensity as a function of position on a screen a distance L away. Consider point P which makes an angle of θ with respect to the center normal. If we illuminate both slits from the left with a monochromatic wave, we can consider each slit as a source of light; to compute the intensity we need to determine the phase difference between the waves coming from each slit. Then our two-wave interference analysis will tell us the amplitude, from which we can get the intensity.

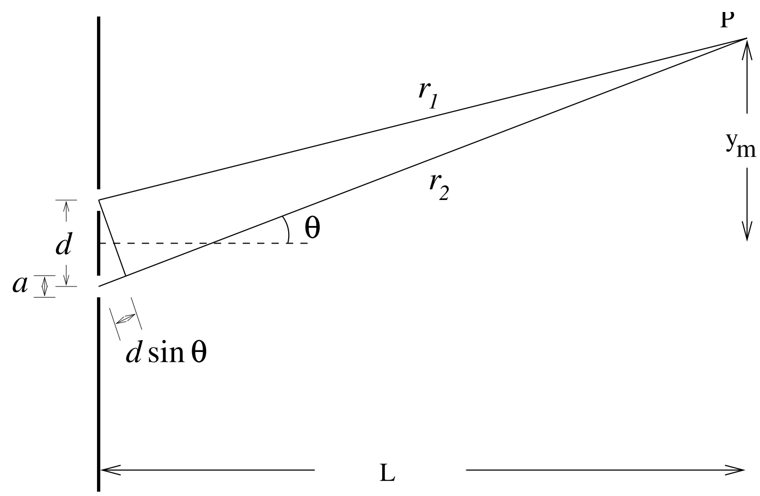


Figure 10.2: The geometry of a simple two slit interference experiment.

If the screen is far away from the slits, $d \ll L$, then the two rays between each slit and P are practically parallel. The phase difference arises from the additional distance one ray must travel over the other. From the geometry of Fig. 10.2, we see that this distance is $\delta = r_2 - r_1 = d \sin\theta$. If two waves have a path difference that is an integer multiple of λ $(\lambda, 2\lambda, \ldots)$ or

$$\delta = d \sin\theta = m\lambda \quad \text{(maxima)}, \tag{10.1}$$

where m is an integer (0,1,2,...), then they are in phase and constructive interference occurs.

For most (but not all) of the experiments we will do, the angles involved will be quite small. The leading term in the Taylor series expansion of $\sin\theta$ is simply θ; so if θ is small, the higher order terms are unimportant. So,

$$\sin\theta \sim \theta \quad (\text{for } \theta \ll 1).$$

Now, if we have a right triangle with θ as one of the acute angles, and legs

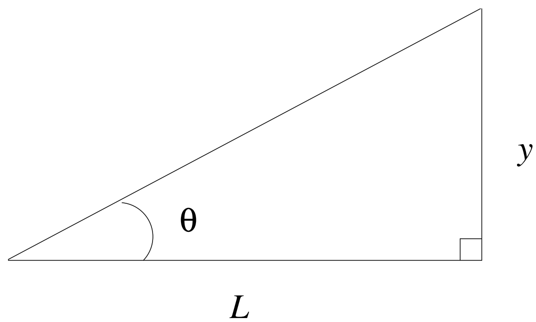


Figure 10.3: The triangle used to relate $\sin\theta$ to y and L.

with lengths y and L, $L \gg y$ (see Fig. 10.3), then we can approximate

$$\begin{aligned} \sin\theta &\sim \theta \\ \frac{y}{\sqrt{y^2+L^2}} &= \frac{y}{L}\frac{1}{\sqrt{1+\frac{y^2}{L^2}}} \sim \frac{y}{L} \end{aligned}$$

so that we arrive at the *small angle approximation* $\theta \sim y/L$. In the small angle approximation, Eq. 10.1 becomes

$$\boxed{y_m = m\lambda\frac{L}{d}} \quad \text{(maxima)} \tag{10.2}$$

where y_m is the distance from the centerline to the mth maximum. We can also predict where the minimum intensity should occur; simply set the integer multiple of δ equal to $(2m+1)\frac{\lambda}{2}$, i.e. odd multiples of half-wavelength. We can gain some quick intuition about the behavior of this interference pattern by looking at the widths of the maxima; this quantity, w, we define as the distance between successive minima:

$$w = y_{m+1} - y_m \sim \lambda\frac{L}{d}$$

which indicates that the *width grows as the distance between the slits decreases*. This is a common feature of interference and diffraction, as we shall see.

We now know the distance to the bright area but we also want to know how the intensity varies with distance, which we will determine graphically using phasors following Serway's example.

In the two-slit experiment, we have two sinusoidal waves with electric field components given by

$$E_1 = E_0 sin(\omega t) \text{ and } E_2 = E_0 sin(\omega t + \phi), \tag{10.3}$$

where E_0 is the wave amplitude, ω is the wave angular frequency, and ϕ is the phase difference. Note that since we have one source of light illuminating both slits, ω is the same for both waves and $\phi = 0$ at the slits. However, the phase difference is no longer zero at the screen. The phase difference at point P depends on the path difference, $\delta = r_2 - r_1 = d \sin\theta$. We want a relationship between phase difference, ϕ, and the angle θ. A path difference of λ for constructive interference corresponds to a phase difference of 2π radians while a path difference of $\lambda/2$ for destructive interference corresponds to a phase difference of π radians. We obtain the ratio

$$\begin{aligned} \frac{\delta}{\phi} &= \frac{\lambda}{2\pi} \\ \phi &= \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d \sin\theta. \end{aligned} \tag{10.4}$$

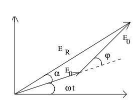


Figure 10.4: Graphical representation of the resultant E_R.

The waves in Eq. 10.3 are represented graphically in Fig. 10.4. We shall use the phasor method to graphically determine the intensity of the pattern we will see on the screen; this is obtained by drawing the second vector at an angle ϕ with respect to the first. Let's review the phasor addition of two waves such as E_1 and E_2. The resultant wave, E_R is the sum of the two waves as given in Fig. 10.4. From the geometry of Fig. 10.4 one can find

$$E_R = E_0 \cos\alpha + E_0 \cos\alpha = 2E_0 \cos\alpha \tag{10.5}$$

$$= 2E_0 \cos(\phi/2) \tag{10.6}$$

The projection of E_R on the vertical axis is E_P,

$$E_P = E_R \sin(\omega t + \alpha) = E_R \sin(\omega t + \phi/2) \tag{10.7}$$
$$= 2E_0 \cos(\phi/2) \sin(\omega t + \phi/2). \tag{10.8}$$

Knowing that the intensity of the light is $I = c\epsilon_0 E_P^2$, one can find

$$I = 4c\epsilon_0 E_0^2 \cos^2(\phi/2) \sin^2(\omega t + \phi/2); \tag{10.9}$$

however, most light detecting instruments detect the time-average intensity of light. The time average of $\sin^2(\omega t + \phi/2)$ is $1/2$ so the average intensity at P is

$$I_{avg} = I_0 \cos^2(\phi/2)$$

where I_0 is the maximum possible time average light intensity and $I_0 = 2c\epsilon_0 E_0^2$. Using Eq. 10.4 into the above equation for I_{avg},

$$\boxed{I_{avg} = I_0 \cos^2(\frac{\pi d \sin\theta}{\lambda})} \tag{10.10}$$

Indeed, the function 10.10 is a simple $\sim \cos^2 \alpha$ function, which has the simple structure as seen in the top part ($n = 2$) of Fig. 10.8 below.

For our specific example of two-slit interference, Fig. 10.5 represents the phasor diagrams for various values of ϕ (and therefore δ). Noting that I will be at a maximum when E_R is at a maximum, we see that the intensity is at a maximum at $\phi = 0, 2\pi, \ldots$ and $I = 0$ when $E_R = 0$ at $\phi = \pi, 3\pi, \ldots$.

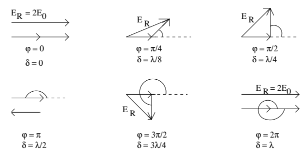


Figure 10.5: The phasor diagram for two-slits.

10.2.4 Multiple-Slit Interference

Now we consider what happens when we have more than two slits. We will determine a general formula for n slits, but let's first do the three-slit problem in some detail. Following the two-slit analysis, we once again use phasors to determine the intensity of the light that will be reaching the screen. Now we have three waves (see Fig. 10.6), namely

$$E_1 = E_0 \sin(\omega t) \tag{10.11}$$
$$E_2 = E_0 \sin(\omega t + \phi) \tag{10.12}$$
$$E_3 = E_0 \sin(\omega t + 2\phi). \tag{10.13}$$

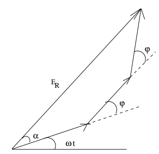


Figure 10.6: Resultant of three waves.

The phasor diagrams for various values of ϕ for the three-slit experiment is given in Fig. 10.7. These diagrams tell us that there are both primary

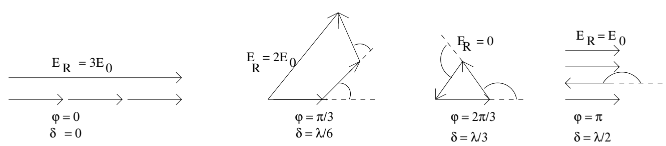


Figure 10.7: Phasor Diagram for Three Slits.

maxima and secondary maxima. The primary maxima have resultant values, $E_R = 3E_0$ when $\phi = 0, \pm 2\pi, \pm 4\pi, \ldots$. This is the case when the three phasors are aligned. The secondary maxima have $E_R = E_0$ when $\phi = \pm\pi, \pm 3\pi, \ldots$. This pattern is shown as the second plot ($n = 3$) of Fig. 10.8. At these points, one of the waves exactly cancels another leaving a third. Notice that the two-slit pattern did not have any secondary maxima while the three-slit pattern had one secondary maxima. The intensity can be found from the projection of the resultant electric field, E_R, as in the two-slit case. The result is

$$I = I_0 \frac{\sin^2\left(m\pi \frac{d}{\lambda}\sin\theta\right)}{\sin^2\left(\pi\frac{d}{\lambda}\sin\theta\right)} \tag{10.14}$$

where $I_0 = \epsilon_0 E_P^2/2$. One can show that, when $n = 2$, this takes the form (Hint: use $\sin 2x = 2\sin x\cos x$)

$$I = 4I_0\cos^2\left(\pi\frac{d}{\lambda}\sin\theta\right).$$

Figure 10.8 schematically represents the intensity pattern for 2, 3, 4, and 5 slits. You can see that the number of secondary maxima increase with increasing slit number, n.

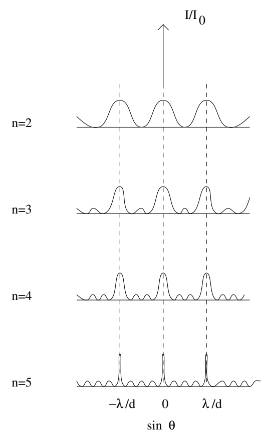


Figure 10.8: Interference pattern for 2, 3, 4, and 5 slits without diffraction.

10.2.5 Diffraction Grating

We can use our multiple-slit pattern to examine a very useful device: the diffraction grating. This is a plate that has thousands of slits cut into it. Gratings with many lines per cm have a very small slit spacing, $d = 1/(\text{lines/cm})$. A piece of the grating is illustrated in Fig. 10.9. Each slit acts

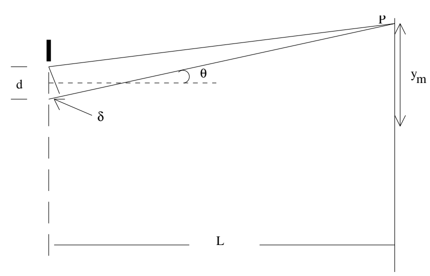


Figure 10.9: Set-up for diffraction grating.

like a source of waves; therefore, each slit produces a wave which interferes with the other waves from the other slits. As in the two-slit case, each wave is in phase at the grating, but travels, relative to its immediate neighbor, a different path length to the point P on the screen, $\delta = d \sin\theta$. When the path length difference equals a multiple of the wavelength, all the waves are in phase and a bright spot appears on the screen; therefore, the condition for maxima in the interference pattern at angle θ is

$$\boxed{d\sin\theta = m\lambda}, \tag{10.15}$$

for some integer m where m is called the *order* of the fringe. Knowing the slit spacing d, the order of the fringe and measuring $\sin\theta$ will produce a fairly precise measurement of the wavelength. The intensity of the diffraction grating versus the angle, θ, is shown in Fig. 10.10. Note the sharply peaked bright areas and the wider dark areas, we will be seeing this in the diffraction grating experiment.

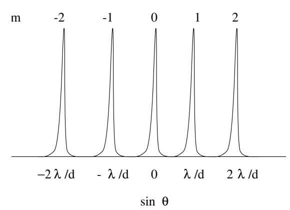


Figure 10.10: Intensity pattern for diffraction grating

10.2.6 Single-Slit Diffraction and Available Light

We have been assuming that the slit is so much smaller then the wavelength, $a \ll \lambda$ that each slit acts like a point source of light. However, in the experiments today our slits are on the order of a wavelength, $a \sim \lambda$ so we need to take diffraction into consideration. We will see (Fig. 10.13) that diffraction has the effect of diminishing the available light for the interference pattern to be seen.

We can observe the effect of diminishing light when we place a single slit in front of the laser beam. We will find a surprising result: the beam gets distorted and fringes appear in the shadow zone of the slit. To describe this phenomenon, known as diffraction, we can use Huygen's principle. According to this idea, we can treat the single slit as a bunch of really, really small single slits and let them interfere with one another. So, let our real slit have a width a and consider a small subset of this region with a width Δy located a distance y from the center, as shown in Fig. 10.11. Now, the phase difference between a ray at our miniture slit at y and one at the center is $\phi(y) = my\sin\theta$, where we have again assumed that the rays leaving are practically parallel and that

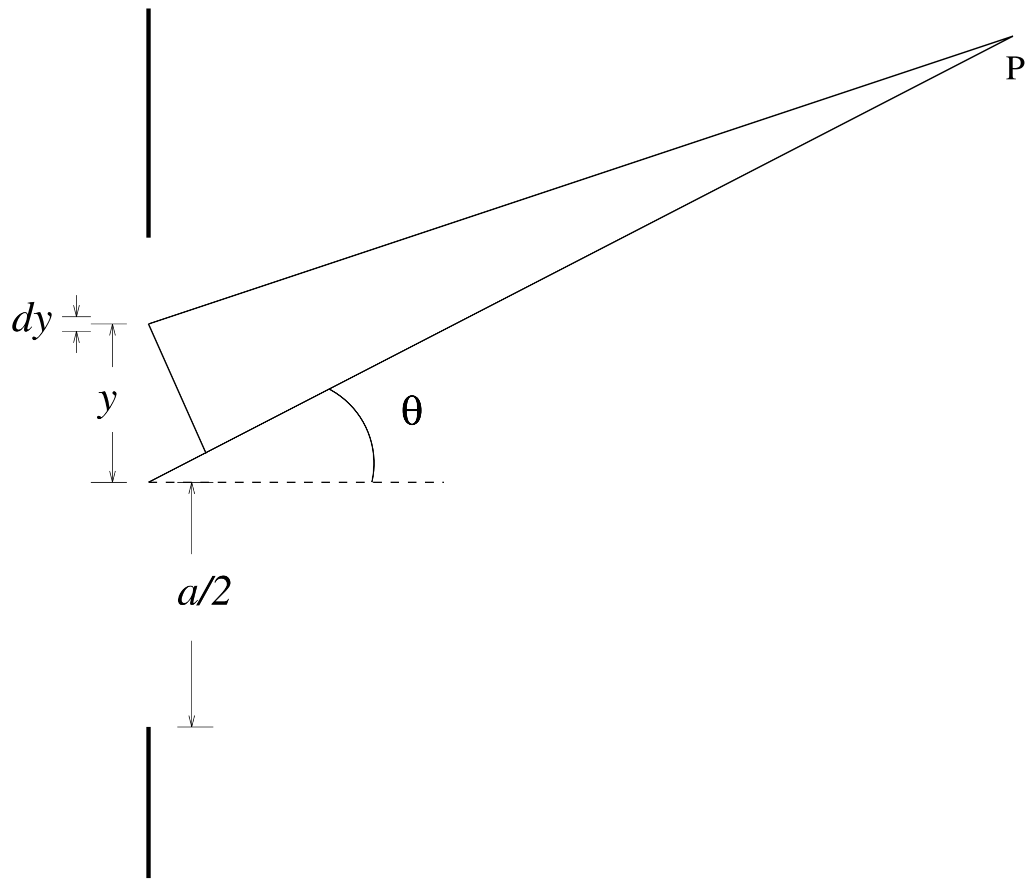


Figure 10.11: Configuration for analyzing single slit diffraction using Huygen's principle.

m is an integer. By using phasors where each portion of the slit contributes an electric field amplitude, E, one can once again solve for the intensity of the resulting diffraction pattern as seen on the screen at some point P. We will not go through this procedure since it follows the n-slit procedure. The resulting intensity is

$$I_\theta = I_0 \left[\frac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda} \right]^2, \tag{10.16}$$

which is plotted in Fig. 10.12 as a function of θ for $a = 3\lambda$. This pattern depicts the intensity of light available at various points from a single slit.

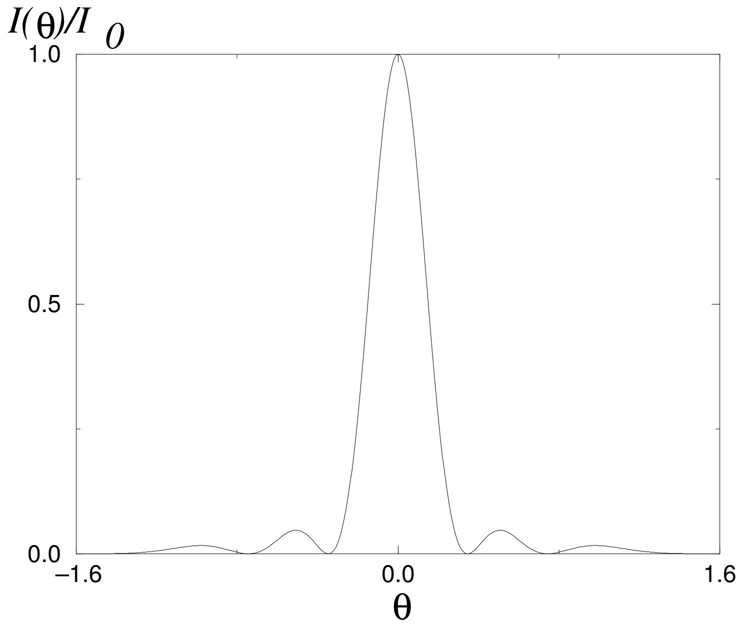


Figure 10.12: Intensity pattern for a single slit diffraction experiment.

The position of the **minima** are at angles given by

$$\sin\theta = \frac{m\lambda}{a} \quad \text{(minima)}$$

which is the same as the two-slit interference **maxima** with the slit separation d replaced by the slit width a. The $m = 1$ minima position ($\sin\theta = \frac{\lambda}{a}$) shows that the pattern expands as the slit width decreases. When $a < \lambda$, minima no longer occur and the pattern begins to resemble the point source pattern at the bottom of Fig. 10.1.

10.2.7 Multiple-Slit Diffraction

You can go through a similar calculation for multiple-slit diffraction patterns; but, we can also be a bit smarter about it and save ourselves some work. Notice that the diffraction of a single slit will not affect the treatment of interference of multiple slits because you simply add the corresponding contributions from the various slits with the appropriate phase factors. Because the single-slit diffraction pattern determines whether the light will be

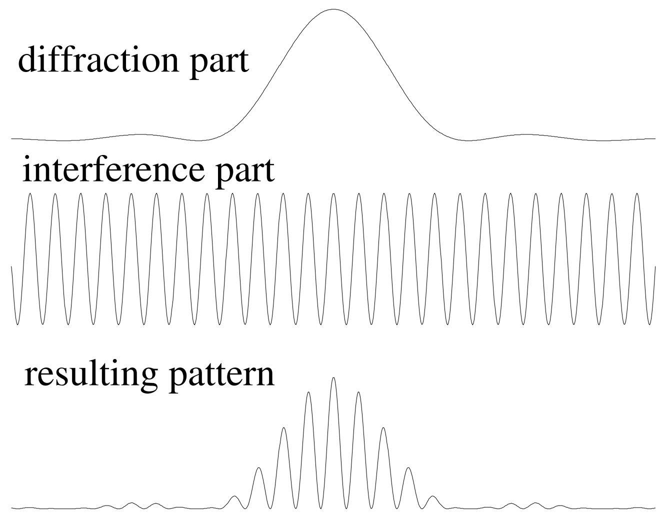


Figure 10.13: Multiple-slit diffraction pattern for $n = 2$.

available for interference at a given point, the result simply becomes the product of the single-slit diffraction pattern (Eq. 10.16) and the multiple-slit interference pattern (Eq. 10.14).

$$I = I_0 \frac{\sin^2\left(\frac{n\pi d}{\lambda}\sin\theta\right)}{\sin^2\left(\frac{\pi d}{\lambda}\sin\theta\right)} \frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2} \tag{10.17}$$

This is what you will observe in the lab. We provide a graph of this intensity in Fig. 10.13 for $n = 2$. Since $a < d$ (the slits do no overlap), the diffraction minima occur farther apart than the interference minima; this means that the diffraction pattern really forms an envelope for the interference patterns (see Fig. 10.14).

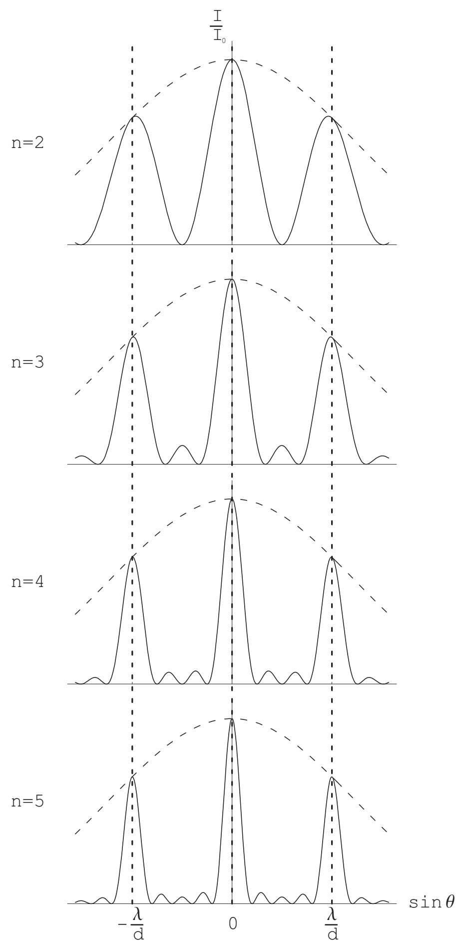


Figure 10.14: Interference pattern for 2, 3, 4, and 5 slits with diffraction

10.3 Apparatus

The apparatus for this lab is rather simple. We'll use the lasers as light sources, since we need a monochromatic beam of light to clearly illustrate the effects of interference and diffraction. Please recall the guidelines for laser safety given in Lab 8. We'll use an adjustable single slit to study single slit diffraction, a slide with multiple-slit patterns, and a diffraction grating slide.

10.W Diffraction and Interference Worksheet

Name: __________________ **Day/Time:** ____________
Partners' Names: ______________________

10.W.1 In-Lab Procedure

For most of the procedures in this lab, we will require a distance of at least 2 m between the slit assembly and the screen you'll use to observe the pattern; L must be large for the small angle approximation. As a screen, we'll use large sheets of computer paper taped to the wall; these are nice, because you can easily trace the pattern on them. However, this means that several groups of students will have to aim the lasers *across* the room. As long as everyone is careful not to aim their own lasers up at eye level and we remain aware of everyone else's lasers, there should be no problems. Be patient and understanding if your instructor or a classmate needs to walk in front of your beam when you're in the middle of making a sketch.

Single-Slit Diffraction

Find a convenient way to aim the laser at one of the walls so that there's no electrical outlet or other obstruction to taping your paper screen onto the wall; make sure that you will have at least a 2 m separation between the *slit assembly* and the wall. Place the single-slit apparatus in front of the laser and align the slit with the beam. This may take a bit of playing around; try placing the slit apparatus on its side or prop it up if necessary. You might also need to adjust the slit width; just be careful not to look directly into the laser beam or its reflections. When you have the slit set up properly, you should see a pattern of *spots*.

Question 0: What does this pattern, specifically the brightness of the spots, have to do with Fig. 10.12?

Trace **two** patterns: one with a relatively large slit width and another with a relatively small slit width; make sure that you label these appropriately. You will use these traces to answer a series of questions in the In-Classroom Calculation & Analysis section.

Multiple-Slit Diffraction/Interference Patterns

Replace the single slit assembly with the multiple-slit slide (mounted in a magnetic holder). Using a length of string and a meter-stick, measure the distance from the slide to the screen, L, and record this value below with uncertainty.

$$L = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Adjust the slide so that the two slit pattern (slit pattern, not interference pattern) is directly in front of the beam; adjust it until you get a clear interference pattern on the screen. Record the slit spacing d below.

$$d = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

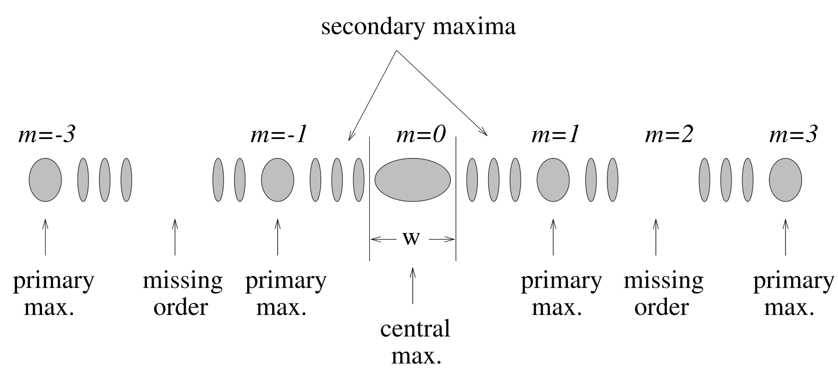


Figure 10.15: The interference/diffraction pattern for 5 slits.

Trace the **two** slit pattern (remember to label it). When doing the trace, make sure you are able to trace at least **eight primary** maxima. (Four primary maxima on each side of the central maximum is preferable). Determine the orders of each of the primary maxima on your trace of the two-slit interference pattern. Refer to Fig. 10.15 for guidance but note that this figure is of a 5-slit pattern. Measure the distances y_m for at least 8 primary maxima; remember to skip the missing orders when you do this; once again referring to Fig. 10.15 for guidance.

Measure the distances y_m from the mth primary maximum to the center of the central maximum for at least eight primary maxima. **Note: The values of y_m for negative m should be taken to be negative**. Record your eight y_m vs. m measurements into Table 10.1.

Measurements of y_m vs. m for Two-slit Diffraction Pattern.				
Distance (y_m)	Order (m)		Distance (y_m)	Order (m)

Table 10.1: y_m vs. m measurements for two-slit diffraction pattern.

Let's also examine the qualitative properties of the other slit patterns. Observe the patterns for the 3, 4, and 5 slit patterns; you don't need to sketch them. Is there anything discussed in Section 10.2.7 that you can compare this behavior with? Record below the results of your observations of the *qualitative* properties of the interference patterns resulting from 3, 4, and 5 slits. In particular, answer the following two questions.

Question 1: For each of the 3, 4, and 5 slit patterns, how many secondary maxima appear between primary maxima?

Question 2: What happens to the size of the primary maxima as you increase the number of slits?

Diffraction Grating Pattern

Replace the multiple-slit slide with the diffraction grating slide. Can you see the diffraction pattern? What if you place a sheet of paper directly in front of the grating? Adjust the grating-to-screen distance until you can see five dots, as illustrated in Fig. 10.16.

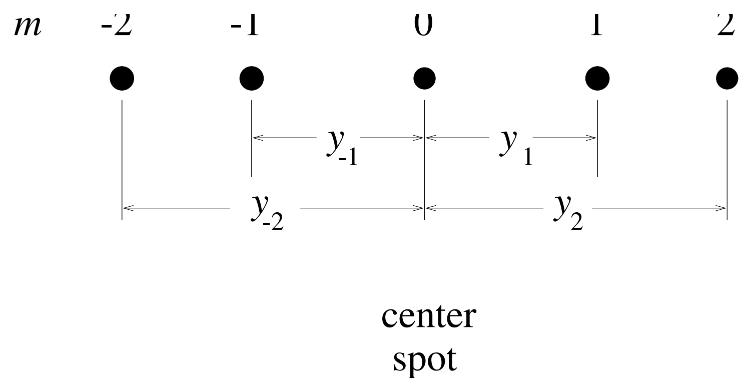


Figure 10.16: The interference pattern from the diffraction grating.

Measure the grating-to-screen distance, L_g, and write down the number of lines/cm, s, as printed on the grating slide.

$L_g =$ ______________ $s =$ ______________

Now trace the 5 bright spots that form the interference pattern. Some dim spots may appear above and/or below the bright ones; these are due to imperfections in the grating and we may ignore them.

On your trace of the diffraction grating interference pattern, assign a value m to each of the dots. Refer to Fig. 10.16 to see how. Measure the distances y_m from the mth dot to the center dot for all five of the dots. **Note:** Again, the values of y_m for negative m should be taken to be negative, see Fig. 10.17.

Record your five y_m vs. m measurements into Table 10.2.

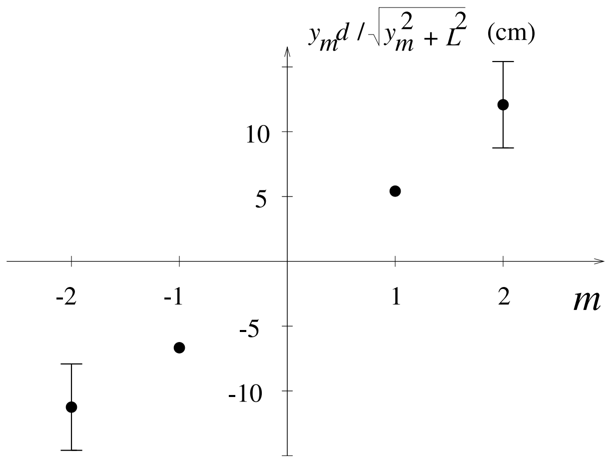


Figure 10.17: An illustration of a typical graph of the grating data.

Measurements of y_m vs. m	
Distance (y_m)	Order (m)

Table 10.2: y_m vs. m measurements for diffraction grating pattern.

10.W.2 In-Lab Computer Work

Multiple-Slit Diffraction/Interference Patterns

Using the measured values in Table 10.1, calculate $y_m d/L$ and its uncertainty for one set of data. **Show all work** in the space provided below.

Make a plot (with error bars) of $y_m d/L$ vs. m Find the slope of the weighted best fit line, and record this below with uncertainty.

S_1= ________________

Diffraction Grating Pattern

Using the measured values in Table 10.2, calculate $y_m d/\sqrt{y_m^2 + L_g^2}$ and its uncertainty for one set of data. **Show all work** in the space provided below.

Now, plot $y_m d/\sqrt{y_m^2 + L_g^2}$ vs. m with error bars. Find the slope of the weighted best fit line, and record this below with uncertainty.

S_2= ________________

10.W.3 Pre-Classroom Check List

- ◯ One partner has two labeled tracings from single slit
- ◯ One partner has a labeled tracing from two slit
- ◯ One partner has a labeled tracing from grating
- ◯ Table 10.1 completed with units and uncertainties
- ◯ Table 10.2 completed with units and uncertainties
- ◯ S_1 with uncertainty and units
- ◯ S_2 with uncertainty and units
- ◯ 2 Plots labeled completely and correctly
- ◯ Each student has her/his own plots and worksheet

10.W.4 In-Classroom Calculations & Analysis

From your value for S_1, determine the value of the wavelength of the laser light for your two-slit interference pattern, λ_1, with uncertainty.

$\lambda_1 =$ ________________

From your value of S_2, determine the value of the wavelength of the laser light for your diffraction grating interference pattern, λ_2 with uncertainty.

$\lambda_2 =$ ________________

From λ_1 and λ_2 calculate the average wavelength with standard deviation. **Note:** Use Eq. (0.1) and Eq. (0.2) in § 0.2.2. Record the average value λ_{avg} below with uncertainty.

$\lambda_{avg} =$ ______________

10.W.5 In-Classroom Discussion

Single-Slit Diffraction

Refer to the traces you made of the single-slit diffraction patterns. Is the small angle approximation valid for analyzing these traces? Explain.

What happened to the width of the spots in the single-slit diffraction pattern as you increased the slit width?

What happened to the separation between the spots in the single-slit diffraction pattern as you increased the slit width?

Do your answers to the above two questions agree qualitatively with the results obtained in § 10.2.6? Explain by citing the relevant results.

Multiple Slit Diffraction/Interference Patterns

What is responsible for the existence of missing orders in a multiple slit interference pattern? (Hint: examine Fig. 10.13).

Is the small-angle approximation valid for analyzing your trace of the two-slit interference pattern? Explain by comparing your largest y_m value to L.

Is your plot of $y_m d/L$ vs. m linear within error?

Compare your value of λ_1 to the known wavelength of He-Ne light, 632.8 nm.

Does your observation of how the sizes of the primary maxima behave as you increase the number of slits agree or disagree with the discussion in § 10.2.7?

Explain.

Referring to your trace of the two-slit interference pattern and the qualitative observations you made about the 3, 4, and 5 slit interference patterns, write an equation that relates the number of slits, n, to the number of secondary maxima, M, that occur in the corresponding interference pattern.

Diffraction Grating Pattern

Is your plot of $y_m d/\sqrt{y_m^2 + L_g^2}$ linear within error?

Compare your value of λ_2 to the known wavelength of He-Ne light, 632.8 nm.

Compare your value for λ_{avg} with the known wavelength.

Which of the two methods of measuring wavelength is the most *accurate*?

Is this expected? Why?

Do you gain anything by having made two separate measurements? What?

10.W.6 In-Classroom Conclusion

Write a *brief* (that is, a one or two paragraph) conclusion for this lab. In it, you should summarize the physical principles which were meant to be illustrated in this experiment. You should also describe the degree to which your data supported these principles.

Attach plots and tracings to the worksheet.

End Worksheet