⇒ ASSIGNMENT-3.

- $\gamma M \frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0.$ — (1)
- $\frac{\partial p'}{\partial t} + \gamma M \frac{\partial u'}{\partial x} = \kappa\left[\sqrt{\left|\frac{1}{3} + u'_f(t-\tau)\right|} - \sqrt{\frac{1}{3}}\right]\delta(x-x_f).$ — (2)

↓ PDE to ODE:

Galerkin technique to model exp.
↳ look up!

(1.) Expand (u', p') in terms of basis functions.
(2.) Project PDEs onto the basis functⁿs.
(3.) Indiv. eqⁿs. for all the functions.

$\Rightarrow \vec{F} = m\vec{a} = m\frac{d^2\vec{x}}{dt^2}$

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$,

$\vec{x} = x\hat{i} + y\hat{j} + z\hat{k}$;

$\left[F_x\hat{i} + F_y\hat{j} + F_z\hat{k} = m\left[\frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}\right].\hat{i}.\right.$

$F_x = m\frac{d^2x}{dt^2}$, $F_y = m\frac{d^2y}{dt^2}$, $F_z = m\frac{d^2z}{dt^2}$.

⇒ Duct open at both ends : $p'(x=0) = 0$.
$p'(x=1) = 0$,

$\Rightarrow p'(x,t) = \sum_{j=1}^{N} a_j(t)\sin(j\pi x)$.

$a_j(t) = -\frac{\gamma M}{j\pi}\dot{\eta}_j(t)$.

$\Rightarrow \gamma M\frac{\partial u'}{\partial t} = -\frac{\partial p'}{\partial x}$.

$= +\sum_{j=1}^{N}\frac{\gamma M}{j\pi}\dot{\eta}_j(t)\cos(j\pi x)$.

$u' = \sum_{j=1}^{N}\eta_j(t)\cos(j\pi x)$

$\Rightarrow p'(x,t) = \sum_{j=1}^{N} -\gamma M\frac{\dot{\eta}_j(t)}{j\pi}\sin(j\pi x)$ — (A) from (1)

$\Rightarrow u'(x,t) = \sum_{j=1}^{N}\eta_j(t)\cos(j\pi x)$.

↓

- $\frac{\partial p'}{\partial t} = -\sum_{j=1}^{N}\frac{\gamma M}{j\pi}\frac{d}{dt}(\dot{\eta}_j)\sin(j\pi x)$.
- $\frac{\partial u'}{\partial x} = -\sum_{j=1}^{N} j\pi\eta_j(t)\sin(j\pi x)$.

$$\sum_{j=1}^{N}\left[\frac{d\eta_j}{dt} + (j\pi)^2\eta_j\right]\sin(j\pi x) = -\frac{j\pi K}{8M}\left[\sqrt{\left|\frac{1}{3}+u_f(t-\tau)\right|}-\sqrt{\frac{1}{3}}\right]\delta(x-x_f)$$

↑ $\rho \times (j\pi)$?

$$\int_0^1 \times \sin(j\pi x) = \ldots$$

$$\Rightarrow \left[\frac{d\eta_j}{dt} + \omega_j^2\eta_j\right]\frac{1}{2} = -\frac{j\pi K}{8M}\left[\sqrt{\left|\frac{1}{3}+u_f(t-\tau)\right|}-\sqrt{\frac{1}{3}}\right]\sin(j\pi x_f).$$

$2\xi_j\omega_j\dot{\eta}_j$; Damping term introduced arbitrarily — for Rijke tube.

⇒ $j=1$: Fundamental mode is most significant.
↳ Get results for $j=1$ first. Then, $j=2,\ldots$

⇒ RK-4 :

⇒ $\frac{d\eta_j}{dt}$ 2 equations, 2 variables.
↳ So do not write t as an independent variable.
[Check this properly ✩
[+ Check slides ✩

⇒ Properly model the delay term ;

Lets assume $\tau = 0.2\,s$,
[Small in Rijke tube
[Much larger in rockets.

Delay Differential eq^n s. ✩

⇒ For $(t-\tau)<0$ → $u_f(t-\tau)=0$.

⇒ $\sqrt{\left|\frac{1}{3}+u_f(t-\tau)\right|}-\sqrt{\frac{1}{3}} \approx \frac{\sqrt{3}}{2}u_f(t-\tau)$.
↑ | | ? ⌊ ???

⇒ $u_f(t-\tau) \approx u_f(t) - \tau\frac{\partial u_f(t)}{\partial t}$.

⇒ TASK 1: RK-4
⇒ TASK 2: Rep.
: Ex
⇒ TASK 3: Re
⇒ TASK 4: P
⇒ BONUS:

- Introduction.
- Methodology
- Results and D
- Conclusion.
- References ✩

⇒ ② $\lambda =$
Cu
$\bar{p}$

⇒ $\cdot \frac{d\eta}{dt}$
$\cdot$ d

$\overline{X}$

⇒ **TASK 1**: RK-4 Num. int. — Perform.

⇒ **TASK 2**: Rep. figures 4–6.

: Energy eqⁿ. in paper after figures.

→ **TASK 3**: Reproduce the bifurcation diag. fig 6a.

⇒ **TASK 4**: Project Report.

⇒ **BONUS**: 6b.

MAY 2

- Introduction,
- Methodology.
- Results and Discussions,
- Conclusion.
- References.

4a ⇒ $\eta_1(0) = 0.15$.

$\eta_{i \neq 1}(0) = 0$.

⇒ $\tau = ???$

⇒ 2 $\lambda = 0.0328$.

$= 719$.

$\sqrt{\frac{1}{3}}\Big] \sin(j\pi x f)$.

· Reference

$\Rightarrow$ ② $\lambda = 0.0328$

$c_v = 719.$

$\bar{\rho} = 1.205.$

(10a) $\Rightarrow$ $\eta_1(0) = 0.15.$

$\eta_{(j+1)}(0) = 0.$

$\Rightarrow \tau = ???$

$$\Rightarrow \cdot \frac{d\eta_j}{dt} = \dot{\eta}_j$$

$$\cdot \frac{d\dot{\eta}_j}{dt} = -2\xi_j \omega_j \dot{\eta}_j - \omega_j^2 \eta_j - \frac{2j\pi K}{\gamma M}\left[\sqrt{\frac{1}{\sqrt{3}} + \omega t (t-\tau)} - \sqrt{\frac{1}{3}}\right] \sin(j\pi x)$$

$$h = (\gamma - 1)\frac{(2L_w)(T_w - \bar{T})}{5 c_v \bar{\rho}\sqrt{3}}\sqrt{\pi \lambda c_v \bar{\rho} \frac{d_w}{2} u_0}$$

$$= (\gamma - 1)\frac{(2 \times 3.6 \times (1000 - \bar{T}))}{5(399.6)\bar{\rho}\sqrt{3}}\sqrt{\pi \lambda c_v \bar{\rho} \frac{d_w}{2} u}$$

$=$

$$\cdot \xi_j = \frac{1}{2\pi}\left[c_1 \frac{\omega_j}{\omega_1} + c_2 \sqrt{\frac{\omega_1}{\omega_j}}\right]$$

$$= \frac{1}{2\pi}\left[c_1 j + \frac{c_2}{\sqrt{j}}\right]$$

$$\vec{x} = \begin{bmatrix} \eta_j \\ \dot{\eta}_j \end{bmatrix}$$

$(j+1)$

$$u^j f(t-\tau) = U^T X - \tau P^T X$$
$$= (U - \tau P)^T X .$$

$$U^T = [\cos(\pi x_f) \;\; 0 \;\; \cos(2\pi x_f) \;\; 0 \; \dots]$$
$$P^T = [0 \;\; \cos(\pi x_f) \;\; 0 \;\; \cos(2\pi x_f) \; \dots]$$

$$= [\cos(\pi x_f) \;\; -\tau\cos(\pi x_f) \;\; \cos(2\pi x_f) \;\; -\tau\cos(2\pi x_f) \; \dots] X$$

$$\cdot\ \text{RHS} = -\frac{2K}{\gamma M}\left[\frac{\sqrt{3}}{2}\right] u^j f(t-\tau) . \begin{bmatrix} 0 \\ \pi \sin(\pi x_f) \\ 0 \\ 2\pi \sin(2\pi x_f) \\ \vdots \end{bmatrix} . [\cos(\pi x_f) \;\; -\tau\cos(\pi x_f) \; \dots] X .$$

$$= -\frac{\sqrt{3}K}{\gamma M} \underbrace{\begin{bmatrix} 0 \\ \pi \sin(\pi x_f) \\ 0 \\ 2\pi \sin(2\pi x_f) \\ 0 \\ \vdots \end{bmatrix} [\cos(\pi x_f) \;\; -\tau\cos(\pi x_f) \; \dots]}_{M_1}$$

$$\boxed{K, \gamma, M, \tau, x_f, \omega_j, \xi_j}$$

$$\therefore \frac{d\vec{X}}{dt} = -\frac{\sqrt{3}K}{(\gamma M)} M_1 X - M_2 X .$$

$$\therefore \boxed{\frac{d\vec{X}}{dt} = -\left(\frac{\sqrt{3}K}{\gamma M} M_1 + M_2\right)\vec{X}}$$

=> Three damping term:

$$\Rightarrow M_2 = \begin{bmatrix} \begin{matrix} 0 & -1 \\ \omega_1^2 & 2\xi_1\omega_1 \end{matrix} & & & \\ & \begin{matrix} 0 & -1 \\ \omega_2^2 & 2\xi_2\omega_2 \end{matrix} & & \\ & & \ddots & \\ & & & \begin{matrix} 0 & -1 \\ \omega_N^2 & 2\xi_N\omega_N \end{matrix} \end{bmatrix}$$

$$\begin{array}{cccc} & 0 & 1 & 2 \\ \text{arrk2} = & [\cos(\pi x_f) & -\tau\cos(\pi x_f) & \cos(2\pi x_f) \end{array}$$

5

3 3

1 2

1

$\underline{i-1} + 1 = \frac{i+1}{2}$