DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion (AS6320)
Endsemester Exam (2020)

Deadline: Dec. 11, 21:00 hours
Marks: 40
Instructor: R. I. Sujith

Instructions:

1. This is a take-home examination. The Exam starts at 9 am on Dec. 09, 2020 (Wednesday). You must submit to sujith@smail.iitm.ac.in by December 11, 2020 (Friday) 21:00.
2. You can see your notes or books or papers in the internet. **However, you must not discuss with ANYONE. There is a viva next week to ensure this.**
3. All the questions are quite simple.
4. Pause and think for a minute before you answer.
5. *Explain what you are doing*. All the marks are for the steps and explanations. **If there is a problem with the question or if any expressions given are wrong, state why you think so in the answerscript. Correct the question if necessary.**
6. Be neat and brief. If you write too much, I may not even see the "answer" and will be searching for it!
7. If you have any questions, feel free to ask.

Relax and have fun. ☺

1. How will you get the actual pressure amplitude (what you will see, for example, in an oscilloscope) from the complex pressure amplitude in a standing wave. (1)

2. A 5 cm diameter piston driven by a shaker (vibrator) fits snugly in the end of a long tube that is filled with water. Vibrating sinusoidally at a frequency of 100 Hz, the piston produces a plane traveling wave wave that travels down the tube. An accelerometer mounted on the piston measures an acceleration amplitude of 4 m/sec^2.

   a) Find the particle velocity amplitude of the sound wave.
   b) Find the sound pressure amplitude (1)

3. An omnidirectional sound source produces in air a spherical wave at a frequency of 100 Hz. The peak acoustic amplitude is measured to be 2 Pa at 20 metres away from the source. A receiver is located at a distance of 1 km from the source. Find (a) the peak intensity (b) the peak particle displacement (c) the peak particle velocity amplitude (d) the rms pressure. [Hint: $r >> \lambda$ at 1 km; hence use the far field approximation, to make things easy.] (1)

4. The acoustic pressure in a duct can be written as $p' = A\cos\omega t \sin kx$.
   a) Rewrite this acoustic pressure field as a linear superposition of left running a right running waves.
   b) Derive an expression for the acoustic velocity field.
   c) Write the acoustic velocity field as a linear superposition of left running a right running waves. (1)

5. Consider the familiar equation for a simple harmonic operator

$$m\ddot{x} + b\dot{x} + kx = 0$$

   Rewrite this equation as a system of two first order equations. (1)

6. You have a combustor that has become unstable. You have now decided to alter the time-lag, in an effort to make it stable. What options do you have (a) if the fuel is gaseous, and the fuel and air are premixed (b) if the fuel is liquid. Make any necessary assumptions and state them. (1)

7. Consider sound propagation in a combustor with small mean Mach number (Say M = 0.01). Derive the following expression for acoustic density $\hat{\rho}$:

$$\hat{\rho} = \left(\frac{\hat{p}}{c^2}\right) + \left(\frac{i\hat{u}}{\omega}\right)\frac{d\bar{\rho}}{dx} + \frac{i(\gamma-1)}{\omega c^2}\hat{Q}$$

Where $\hat{p}$ and $\hat{u}$ are the acoustic pressure and velocity respectively, ω is the complex frequency, $\bar{\rho}$ is the mean density, γ is the ratio of specific heats, c the speed of sound and $\hat{Q}$ is the complex heat release amplitude. (1)

8. A common mechanism that causes combustion instability in Low Nox Gas Turbine (LNGT) engines that are used for power production is the equivalence ratio fluctuations. In such combustors, write an expression for the time delay between the equivalence ratio fluctuations and the pressure or velocity fluctuations at the flame, based on the travel time of acoustic and convective waves. The injector is at a distance L from the flame, and the flow has a Mach number M. (1)

9. In class, we derived the G equation and its solution, in an attempt to model premixed flames. (1) In this model, explain how will you calculate the temperature fluctuations? (2) Explain how you will calculate the heat release rate fluctuations. (1)

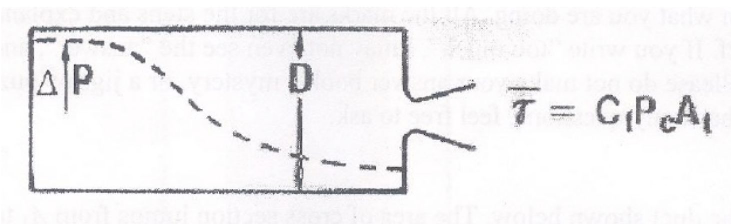
10. Consider an area jump (discontinuity) as shown in the figure. Derive relations for the acoustic pressure and the acoustic velocity jumps at the area discontinuity. Assume that there is no mean flow. The area on the left of the discontinuity is A_1 and the area on the right of the discontinuity is A_2. (2)

11. What is meant by a bifurcation? What is subcritical Hopf bifurcation? What is supercritical Hopf bifurcation? Draw bifurcations diagrams to illustrate these bifurcations.

Which one of them corresponds to triggering? Which of these bifurcations is associated with hysteresis? (2)

12. What is the difference between transition to thermoacoustic instability in a combustor with laminar flow and in a combustor with a turbulent flow? (1)

13. Consider a solid propellant rocket motor as shown in the figure. The thrust ($\bar{\tau}$) can be written in terms of the thrust coefficient c_f as $\bar{\tau} = c_f P_c A_t$, where, P_c is the chamber pressure and A_t is the throat area. D_t is the throat diameter and D is the port diameter.



Pressure and thrust oscillations in a solid rocket motor

Show that in the presence of oscillations in **fundamental mode** of amplitude ΔP, the amplitude of the thrust oscillation $\Delta\tau$ can be estimated as:

$$\frac{\Delta\tau}{\bar{\tau}} = \left(\frac{D}{D_t}\right)^2 \frac{2\Delta P_c}{\bar{P}_c}\frac{1}{c_f}$$

Assuming "typical dimensions" and operating parameters of a solid rocket motor (of any class of rockets that you may like), what would be a typical value of $\frac{\Delta\tau}{\bar{\tau}} \Big/ \frac{\Delta P_c}{\bar{P}_c}$

Write a similar expression for the amplitude of thrust oscillations $\Delta\tau$ in terms of the amplitude of pressure oscillations ΔP for the **second mode** (4)

14. In many cases the walls of a pipe are not rigid. If the walls can be described by a locally reacting impedance, we can derive a modified 1-D wave equation in a simple way.

    Assuming quiescent medium (i.e., there is no mean flow), the linearized 1-D continuity and momentum equations can be written as:

$$\frac{\partial \rho'}{\partial t} + \bar{\rho}\frac{\partial u_x'}{\partial x} = m'$$

$$\bar{\rho}\frac{\partial u_x'}{\partial t} + \frac{\partial p'}{\partial x} = 0$$

    You can see that there is an unsteady source of mass in the continuity equation. The wall vibrations in our problem u_w' can be represented as a mass injection term: $m' = -\frac{\bar{\rho} u_w' O}{A}$, where O is the circumference of the pipe and A is the cross-sectional area. The assumption of local reaction (i.e., the wall velocity is proportional to pressure fluctiations) further implies that $\frac{p'}{u_w'} = Z_w$.

a) Show that the 1-D wave equation for sound in pipes with flexible walls is

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} + \frac{\bar{\rho}}{Z_w d_h}\frac{\partial p'}{\partial t} - \frac{\partial^2 p'}{\partial x^2} = 0$$

    where $d_h = \frac{A}{O}$ is the so called hydraulic diameter, and the standard assumption of adiabatic sound propagation has been used.

b) Assuming $p' = e^{i(\omega t - kx)}$, prove that the wave number k must satisfy the following dispersion relationship.

$$k = \sqrt{\left[\frac{\omega}{c_0}\right]^2 - \left[\frac{\bar{\rho} i \omega}{Z_w d_h}\right]}$$

(2)

15) Derive an analytical solution for the linear damped oscillator given by the following equation:

$$\ddot{x} + \mu \dot{x} + k^2 x = 0$$

Under what conditions will the oscillations grow? Under what conditions will they decay? If we say that the oscillations grow/decay as $e^{\alpha t}$, find the value of α (i.e., the growth rate) in terms of the parameters in the differential equation. (3)

16. Perform modal analysis for a horizontal Rijke tube, with a heat source located at x_f. We have open boundary conditions on both ends. An electrical heater is located at a position x_f. The unsteady heat release rate from the heat source is modeled using 'n-τ' model and is of the form:

$$\dot{Q}' = nu(t - \tau)$$

Assume that the temperature is constant in the duct.

a. Derive the equations for the acoustic pressure and acoustic velocity fields in the duct.

b. Derive the relationship whose roots give the eigenvalue (i.e., the dispersion relation)

(4)

17. The transfer function gives the relationship between heat release rate and acoustic velocity. The Rayleigh Index depends on the correlation between the heat release rate and the acoustic field. Consider a "compact flame". Derive a relationship for Rayleigh Index, the admittance at the flame and the acoustic pressure amplitude at the flame.

(4)

18. Invariably, combustion instabilities will involve not just fluctuations in pressure and velocity, but also in entropy (or temperature). Let us examine the energy corollory in the presence of entropy fluctuations.

We make our usual set of assumptions:

(i) No mean flow
(ii) C_v, C_p are constants
(iii) We have a homogeneous medium. Mean density, mean pressure and mean temperature are constant everywhere.
(iv) There is a fluctuation heat source. There is no momentum or mass sources.

The following are the governing equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0$$

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right] = -\frac{\partial p}{\partial x}$$

$$\rho C_v\left[\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\right]+p\frac{\partial u}{\partial x}=\dot{Q}$$

$\dot{Q}$ is the heat release rate.

(a) Perform linearisation as we did in our class and obtain the following equations for the perturbations:

$$\frac{\partial \rho'}{\partial t}+\bar{\rho}\frac{\partial u'}{\partial x}=0$$
$$\bar{\rho}\frac{\partial u'}{\partial t}=-\frac{\partial p'}{\partial x}$$
$$\rho C_v\frac{\partial T'}{\partial t}+\bar{p}\frac{\partial u'}{\partial x}=\dot{Q}'$$

(b) Now, define the disturbance energy E (known as Chu's disturbance energy) as follows:

$$E=\frac{1}{2}\frac{c^2}{\gamma\bar{\rho}}\rho'^2+\frac{1}{2}\bar{\rho}u'^2+\frac{1}{2}\frac{\bar{\rho}C_v}{\bar{T}}T'^2$$

Derive the following energy corrollory for the evolution of the disturbances.

$$\frac{\partial}{\partial t}\iiint_v E\,dV+\oiint_{CS} p'u'\,ds=\iiint_v \frac{T'}{\bar{T}}\dot{Q}'dV$$

[Please do not fudge the result. Marks will be given *if and only if* the derivation is *crystal clear*. Any fudging anywhere will result in *Zero* marks for this entire question.]

(c) Physically, what is the condition for the disturbance energy to grow?

(4)

19. Please read the paper of Etikyala & Sujith (2017) and Sections 1-3 of Morales & Krischer (2012). These papers are attached to my email that had this question paper. Now, please answer the following questions.

1) What is Stuart Landau equation?
2) Please write an equation that gives supercritical Hopf bifurcation. Indicate the sign of the coefficients.
3) Please write an equation that gives subcritical Hopf bifurcation. Indicate the sign of the coefficients.
4) What is meant by change of criticality of a bifurcation?
5) Which bifurcation (supercritical Hopf or subcritical Hopf) is more dangerous? Why?

(5)