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Intermittency route to thermoacoustic instability in turbulent combustors

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The dynamic transition from combustion noise to combustion instability was investigated experimentally in two laboratory-scale turbulent combustors (namely, swirl-stabilized and bluff-body-stabilized backward-facing-step combustors) by systematically varying the flow Reynolds number. We observe that the onset of combustion-driven oscillations is always presaged by intermittent bursts of high-amplitude periodic oscillations that appear in a near-random fashion amidst regions of aperiodic low-amplitude fluctuations. These excursions to periodic oscillations last longer in time as operating conditions approach instability and finally the system transitions completely into periodic oscillations. A continuous measure to quantify this bifurcation in dynamics can be obtained by defining an order parameter as the probability of the signal amplitude exceeding a predefined threshold. A hysteresis zone was observed in the bluff-body-stabilized configuration that was absent in the swirl-stabilized configuration. The recurrence properties of the dynamics of intermittent burst oscillations were quantified using recurrence plots and the distribution of the aperiodic phases was examined. From the statistics of these aperiodic phases, robust early-warning signals of an impending combustion instability may be obtained.

Key words: intermittency, turbulent reacting flows, wave–turbulence interactions

1. Introduction

The description of the dynamics underlying the onset of combustion instabilities remains a challenging problem to researchers in the field, as the dynamics may be driven by a variety of flow and combustion processes that can couple with one or more of the acoustic modes of the combustion chamber (Culick 2006). Large-amplitude pressure oscillations are easily established in confined convecting combustion environments, since only a small fraction of the available energy from heat release is sufficient to drive the oscillations, as the mechanisms of attenuation in the confinement are weak. Such combustion-driven oscillations are detrimental and result in performance losses, reduced operational range, wear and tear, and fatigue failure of the combustor walls due to increased heat transfer (McManus, Poinsot & Candel 1993; Candel 2002).

Even in the absence of large-amplitude oscillations, unsteady combustion tends to be a noisy process, and this noise, termed 'combustion noise' in the literature, is

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attributed mainly to two sources: direct combustion noise due to unsteady volumetric expansion in the reactive region, and indirect combustion noise produced when the hot combustion products traverse a region with mean flow gradients (Strahle 1978). Although there have been many studies that have focused on the characteristics of combustion noise and combustion instability, relatively little attention has been paid to the dynamics of the transition from one to the other with changes in operational parameters. An understanding of the physical mechanisms behind such a change in behaviour of the global system, comprising flow, acoustics and combustion processes, is critical to identifying robust early-warning signals of impending instabilities.

A systematic variation of the operating conditions in bluff-body and backward-facing-step combustors from stable to unstable operation was performed by Chakravarthy *et al.* (2007*a*) and Chakravarthy, Sivakumar & Shreenivasan (2007*b*). They found that, in the regimes of stable operation characterized by low-amplitude broad-band noise generation, vortex shedding and duct acoustics do not lock on. However, the broad-band noise generation gives way to the excitation of high-amplitude discrete tones, at the onset of lock-on. In a recent experimental study, Gotoda *et al.* (2011) reported that the transition to thermoacoustic oscillations happens from stochastic fluctuations to periodic oscillations through low-dimensional chaotic oscillations, when the fuel equivalence ratio was varied. In another study, Gotoda *et al.* (2012) further characterized the dynamic behaviour at combustion instability using permutation entropy and networks of radial basis functions.

The dynamic transitions in a thermoacoustic system that consists of ducted laminar premixed flames was studied by Sujith and coworkers (Kabiraj *et al.* 2012; Kabiraj & Sujith 2012; Kabiraj, Sujith & Wahi 2012*a*,*b*). Transitions to chaos were identified to occur via quasi-periodic oscillations (Kabiraj *et al.* 2012) as well as frequency-locked oscillations (Kabiraj *et al.* 2012*a*). They also found that intermittent large-amplitude irregular fluctuations were seen in unsteady pressure measurements prior to flame blow-off (Kabiraj & Sujith 2012; Kabiraj *et al.* 2012*b*). To study the rich behaviour in the dynamics of even such simple thermoacoustic systems, methods of nonlinear time-series analysis were seen to be necessary and extremely useful. The present study follows similar lines of analysis in describing the dynamic transitions in turbulent combustors from low-amplitude combustion noise to high-amplitude combustion instability with the hope of identifying robust precursors to detrimental oscillations in such combustors.

In a recent study, combustion noise was shown to be composed of chaotic fluctuations of moderately high dimensions ($d = 8$–10) by performing a series of tests for determinism and chaos (Nair *et al.* 2013). In the present study, we report the presence of intermittent dynamics in confined compressible turbulent flow environments prior to a transition to high-amplitude periodic oscillations from low-amplitude chaotic fluctuations. We show that the unsteady pressure measurements acquired at conditions immediately prior to combustion instability are composed of bursts of high-amplitude periodic oscillations embedded amidst regions of chaotic fluctuations.

Although reports of such possibly intermittent burst states are present in the literature, their dynamics have not been investigated or characterized in detail. For an unchoked fuel flow at the injector in a swirl combustor, Hong *et al.* (2008) reported the presence of pressure oscillations that alternated between a 'noisy period of 200 Hz fluctuation and a silent period with a small pressure fluctuation'. Arndt *et al.* (2010) have observed a transition in the flame dynamics between a state of stable combustion and self-excited oscillations in a premixed gas turbine model combustor using simultaneous OH^* chemiluminescence, OH^* planar laser-induced fluorescence (PLIF) and stereoscopic measurements. Bursts of pressure oscillations have also been reported close to critical transition to instability in liquid-propellant rocket engines (Clavin, Kim & Williams 1994).

The present study focuses on establishing that intermittency is a stable dynamical state in combustors distinct from the regimes classified as stable (chaotic) or unstable (periodic). The study also wishes to propose the idea that such intermittent bursts always presage an impending instability in turbulent combustors. Continuous measures that quantify intermittency in a measured signal are then sought that can serve as robust precursors to the onset of large-amplitude periodic oscillations. This involves studying the characteristics of the aperiodic phases amidst the periodic bursts and the recurrence properties of such bursts. Also, since typical measures such as the amplitude of oscillations cannot serve as measures of bifurcation in such systems with varying amplitudes, we also seek to identify suitable bifurcation measures to study intermittent transitions to instability in turbulent combustors.

The paper outline is as follows. Section 2 describes the set-up used to investigate the intermittent oscillations in turbulent combustion. The routes to instability in turbulent combustors are illustrated in § 3, outlining the methods to obtain bifurcation diagrams from unsteady pressure measurements in the presence of turbulence. The intermittency is characterized and the recurrence properties of these burst oscillations are explored in § 4. Finally, the conclusions are summarized in § 5.

2. Experimental set-up

Experiments were conducted on a backward-facing-step combustor geometry with two different flame-holding mechanisms (a fixed vane swirler, and a circular bluff body) at high Reynolds number ($Re > 14\,000$). The schematic of the set-up for the measurements reported in the current study is shown in figure 1. It consists of an upstream plenum, a burner of 40 mm diameter and a combustion chamber of cross-section 90 mm × 90 mm with extension ducts. The length of the combustor chamber along with the extension ducts was 700 mm. A support mechanism that hinges on the plenum was used to traverse a shaft of 16 mm diameter into the burner using a rack-and-pinion traverse with a least count of 1 mm. The swirler and the bluff body were then alternately attached to this shaft for the different experiments. The swirler has a length of 30 mm and has eight fixed vanes of 1 mm thickness bent at an angle (140° about the axis and then 10° clockwise along the new direction of flow) to impart radial momentum to the incoming axial flow. A centre body of length 30 mm and diameter 16 mm was located at the outer end of the swirler for flame stabilization. The centre body was positioned such that its outer edge grazes the exit plane of the burner. The bluff body was a circular disk of diameter 47 mm and thickness 10 mm and was located 50 mm from the rearward-facing step for flame stabilization. The central shaft was also used to deliver fuel into the chamber through four radial injection holes of diameter 1.7 mm, which was subsequently spark-ignited in the recirculation zone at the dump plane using an 11 kV ignition transformer. The fuel injection location was 100 mm upstream from the swirler and 160 mm upstream from the rear end of the bluff body. A circular disk of 2 mm thickness and 40 mm diameter with 300 holes of diameter 1.7 mm was inserted 30 mm downstream of the fuel-injection location to prevent flashback. A blowdown mechanism was used to supply air from high-pressure tanks, which then passed through a moisture separator before finally entering the plenum chamber.

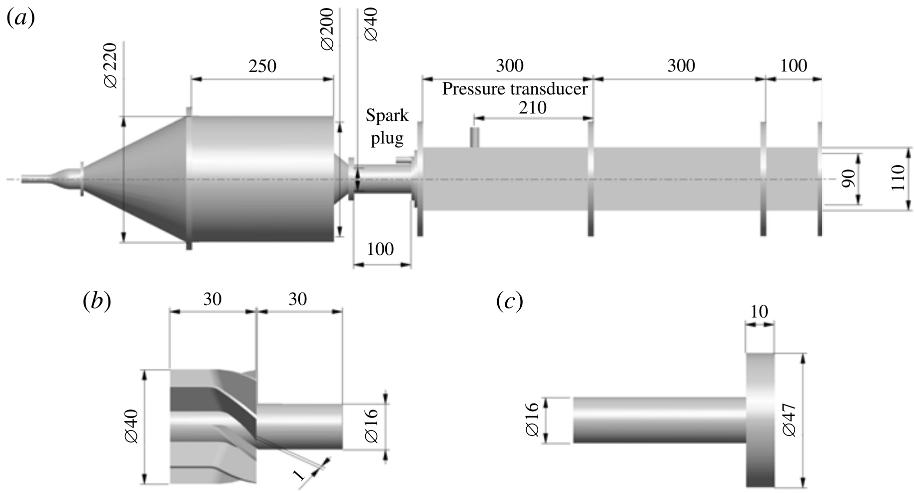

FIGURE 1. (*a*) The experimental set-up used in the current study. The length of the combustion chamber is 700 mm, with three extension ducts, two of length 300 mm and one of length 100 mm. The measurements reported in this study were acquired using a piezoelectric transducer located 90 mm from the backward-facing step. Two different flame stabilization mechanisms were used in the study: (*b*) a fixed-vane swirler with centre body, and (*c*) a circular bluff body. The design of the combustor was adapted from Komarek & Polifke (2010).

Unsteady pressure measurements (p') were acquired for 3 s at 10 kHz using piezoelectric transducers with a sensitivity of 72.5 mV kPa^{-1}, 0.48 Pa resolution and $\pm$0.64 % uncertainty. The voltage signals from the transducers were acquired using a 16 bit analogue-to-digital (A-D) conversion card (NI-643) that had a resolution of $\pm$0.15 mV and an input voltage range of $\pm$5 V. The transducers were mounted on specially made pressure ports with Teflon adapters, which were flush-mounted on the combustor wall. Semi-infinite tubes were provided to the pressure ports to prevent acoustic resonance within the mount. The configuration helps to prevent the transducers from excess heating and also ensured that the phase correction required was less than 2°.

Mass flow controllers (Alicat Scientific, MCR Series) with digital logging and monitoring capabilities were used to measure and control the supply of fuel and air into the combustion chamber and had a measurement uncertainty of $\pm$(0.8 % of reading + 0.2 % of full scale). Liquified petroleum gas (LPG) was used as the fuel, which is 60 % C_4H_{10} and 40 % C_3H_8 by volume. The fuel flow rate ($\dot{m}_f$) was held fixed and the air flow rate ($\dot{m}_a$) was gradually increased, leading to progressively increasing values of Reynolds number (decreasing equivalence ratio). At each flow condition, the flow was allowed to settle for 10 s before acquiring the pressure data to remove transients associated with the change in mass flow rate. The Reynolds number was computed using the expression $Re = 4\dot{m}D_1/(\pi\mu D_0^2)$, where $\dot{m}$ $(= \dot{m}_a + \dot{m}_f)$ is the mass flow rate of the fuel–air mixture, D_0 is the diameter of the burner, D_1 is the diameter of the circular bluff body (or the diameter of the vane swirler, D_0) and μ is the dynamic viscosity of the fuel–air mixture at the operating condition. Corrections to Reynolds number due to the change in viscosity for the varying fuel–air ratios were performed, the procedure for which can be found in Wilke (1950).

3. Intermittency in turbulent combustors

In turbulent combustors, the transition to self-sustained oscillations from regimes of stable operation can often be triggered due to the unsteadiness in the flow and combustion. Predicting the amplitude or frequency of such triggered oscillations, or even the stability margins of combustors, still remains a challenge for researchers in the field owing to the complex nature of the dynamics in combustors amongst flow, heat release and chamber acoustics (Zinn & Lieuwen 2005). The understanding of the universal features of such transitions is limited, and operators often rely on heuristic measures to prevent instability in fielded combustors.

The distributions of the pressure measurements acquired from combustors well before conditions of instability – termed 'combustion noise' – have a characteristic Gaussian distribution (Lieuwen 2002) suggestive of dynamics dictated by random processes. However, in a recent study, Nair *et al.* (2013) showed that combustion noise is deterministic chaos and therefore is not noise in the traditional sense of the word. Pressure signals acquired during combustion noise were subjected to determinism tests and were shown to be chaotic. Further, the study showed that there is a gradual loss of this chaotic behaviour as the operating conditions approach combustion instability. An objective measure was defined to capture this loss of chaos independent of the details of the geometry, fuel composition or flow parameters. However, the reason for the smooth gradual loss of chaotic behaviour was not explained in the study and requires further elaboration.

The smooth variation of the measure for chaos hinted at the existence of a dynamic regime different from chaos and limit cycle oscillations. Shown in figure 2 are two signals obtained from the two combustors at these intermediate conditions between regimes of unsteady chaotic fluctuations and large-amplitude periodic oscillations. The signals display bursts of high-amplitude oscillations amidst regions of low-amplitude fluctuations. Such an intermediate regime of intermittent oscillations was observed in all the experiments we performed when the operating conditions were close to regimes of combustion instability. Also, such states were seen to persist in time; they are not transients that eventually transform to periodic oscillations or combustion noise. In a recent study, Nair & Sujith (2013) have shown that these intermittent burst oscillations are a result of the presence of homoclinic orbits in the phase space of the underlying attractor. Such orbits connect the stable and unstable branches of a hyperbolic saddle equilibrium solution and lead to excursions of the dynamics towards a periodic state from the equilibrium state. These excursions are then seen as bursts of periodic oscillations in a measured signal. Such oscillations eventually die down to low-amplitude hydrodynamic fluctuations in the absence of self-sustaining heat release rate fluctuations resulting from combustion. The frequency of the pressure oscillations in an intermittent burst is close to the frequency of the limit cycle encountered at combustion instability. However, the time scales of appearance of individual bursts and modulation of their amplitudes are much slower and are comparable to the flow time scales. These bursts have a near-random appearance, which underscores the role of turbulence in establishing such homoclinic orbits.

An explanation for the burst oscillations was provided in the study by Clavin *et al.* (1994), where the erratic behaviour of pressure fluctuations was incorporated as a multiplicative noise term in the wave equation. The effect of such a noise term, which was used to model the effects of turbulence, in the vicinity of a sub- and supercritical Hopf bifurcation was then explored, and the corresponding probability distribution of pressure fluctuations was obtained after deriving the amplitude equations for the underlying acoustic system close to criticality. The proposed model highlights

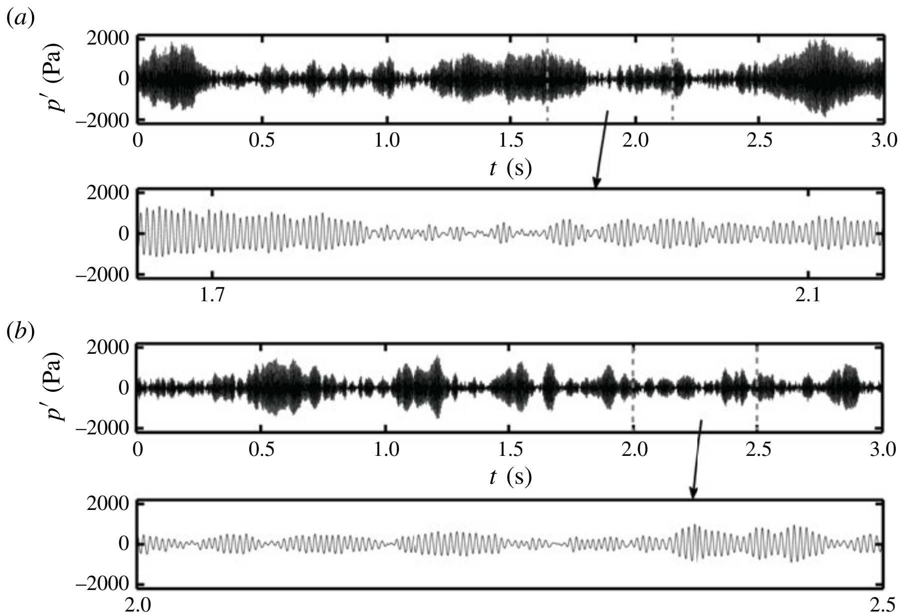

FIGURE 2. Intermittent signals obtained from the two combustors: (*a*) unsteady pressure signal from the bluff-body-stabilized backward-facing-step combustor used in the study ($Re = 1.88 \times 10^4$, $\phi = 0.85$); and (*b*) unsteady pressure measurement from a swirl-stabilized backward-facing-step combustor ($Re = 2.58 \times 10^4$, $\phi = 0.77$). Here, ϕ represents the global equivalence ratio at the operating conditions. The signal is composed of high-amplitude oscillations that appear amidst low-amplitude aperiodic fluctuations as seen in the (lower) zoomed regions of the signals. Such intermittent burst oscillations were always observed prior to the onset of instabilities in our experiments.

the need for a nonlinear approach in describing the nature of transition. However, in addition to the broad-band modulation of the pressure fluctuations, turbulence also brings with it its own dynamics, such as vortex shedding, which can have contributions over time scales close to combustion instability. To bring in the effects of turbulence as a parametric (multiplicative) noise term is to concede that it is not possible to describe or quantify the dynamics brought about by phenomena such as the formation, roll-up, coalescence and impingement of vortices. Hence, appropriately modelling these deterministic aspects of the hydrodynamics remains a continuing challenge in the field.

The dynamics of fluctuations in turbulent combustors may be better understood as a complex interplay amongst two subsystems operating over different length and time scales. Acoustics operates over time scales determined by the passage time of sound through the combustion chamber. Interaction due to hydrodynamics, on the other hand, can be spread over multiple orders of temporal magnitude owing to the broad-band nature of the underlying turbulence. At the same time, unsteady flow phenomena, such as vortex shedding, roll-up, coalescence or impingement, can give rise to dynamics over a narrow frequency band, some of which could lie close to the natural acoustic modes of the confining combustion chamber. Hydrodynamics thus operates over a broad range of time scales associated with convection and unsteadiness due to turbulence.

The major contribution to the driving received by the acoustics through combustion (other than those due to direct hydrodynamic or acoustic modulation of the flame) comes from fuel unmixedness through equivalence ratio perturbations. These equivalence ratio perturbations, which again arise due to acoustic modulation of the feed system and flow unsteadiness, are seen to influence only the magnitude of heat release rate fluctuations. Lieuwen, Neumeier & Zinn (1998) have shown that chemical time scales, being typically much smaller than flow/acoustic time scales, are unlikely to provide the necessary phase delay needed for an acoustic–chemical kinetic coupling to sustain low-frequency instabilities. Hence, it is reasonable to assume fast chemistry and essentially incorporate the effects of combustion as part of the hydrodynamic and acoustic subsystems. It should further be noted that such a description does not decouple the dynamics of acoustics and hydrodynamics; rather it emphasizes a mutual nonlinear coupling of the two subsystems.

In modelling these effects, if one were to discount the effects of turbulence or average out the equations (a mean-field description), the bifurcation of the acoustic system would be seen as a transition from a fixed-point solution to a periodic final state – a transition termed Hopf bifurcation. Such a description that decouples the two subsystems leaves no room for phenomena such as the intermittent burst observed in experiments. The intermittent oscillations can arise if the acoustic subsystem is modulated by the hydrodynamics over slower time scales (turbulent velocity fluctuations typically have an increased energy content at lower frequencies), essentially shifting the dynamics of the acoustic subsystem back and forth across the Hopf point. These back-and-forth fluctuations thus transition the dynamics from a chaotic turbulent state towards a periodic state and back, forming a homoclinic orbit in phase space. This presence of turbulence possibly also explains the near-random appearance of the burst oscillations from a background of chaotic fluctuations.

For the mechanism conjectured above to be correct, combustion instability in turbulent combustors must necessarily happen via a regime of intermittent burst oscillations. A simple phenomenological model for combustion instability in a bluff-body-stabilized combustor was developed by Nair & Sujith (2014) by incorporating both the broad-band effects of turbulence and the narrow-band effects of vortex formation and impingement into the classical Galerkin-based acoustic analysis. The model reproduces the intermittency route to instability as the flow velocity is increased and exhibits instability when the hydrodynamic forcing is at the subharmonic frequency of the fundamental acoustic mode. The results from the model lends further support to the mechanism outlined above and hints at the possibility that hydrodynamics is responsible for the intermittent phenomena observed in pressure measurements from combustion chambers.

The proposed mechanism also requires that such intermittent periodic bursts be absent when the underlying flow field is laminar, because in such cases there appear to be no possible mechanisms to provide the required low-frequency near-random modulation. Such a situation can arise, for instance, in ducted laminar flames, as long as the flame itself does not become turbulent; or in an electrical Rijke tube as long as the mean flow is laminar. For such laminar transitions, the root-mean-square (r.m.s.) levels of pressure in the system form a convenient measure that characterizes the onset of instability. In what follows, we shall describe ways to characterize intermittent burst oscillations observed in turbulent combustors.

3.1. *Bifurcation diagrams*

Typically, bifurcation diagrams of experimental data are drawn by tracking the peaks in a measured signal and plotting them as a function of the control parameter.

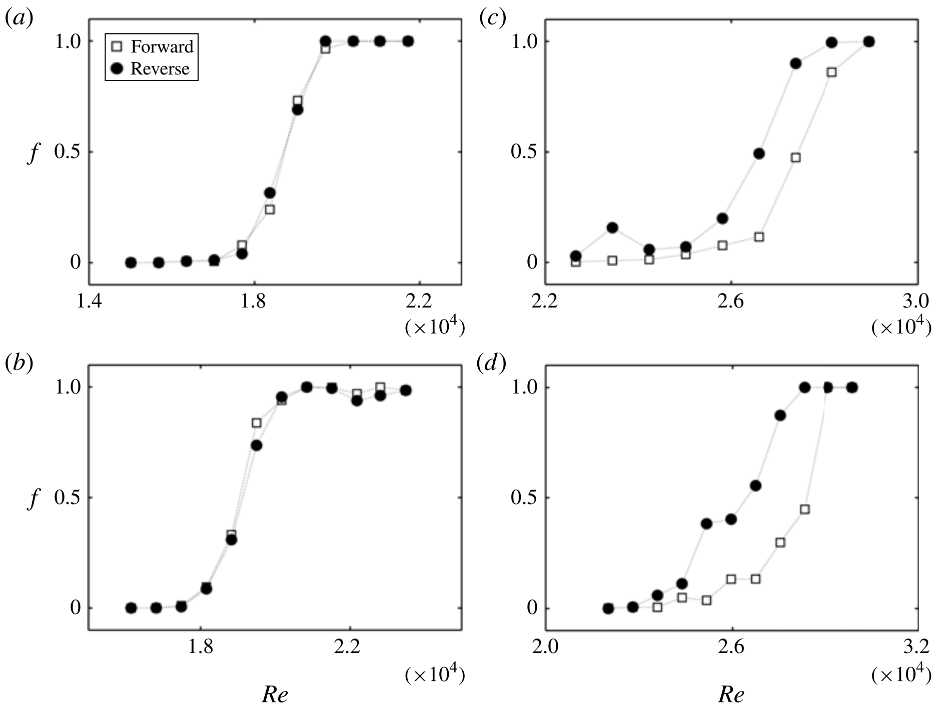

FIGURE 3. Bifurcation diagrams obtained through normalized burst count (f) for the transition from chaotic combustion noise to high-amplitude combustion instability. For the swirl-stabilized backward-facing-step combustor: (*a*) $\dot{m}_f = 0.51$ g s^{-1}; (*b*) $\dot{m}_f = 0.55$ g s^{-1}. For the bluff-body-stabilized backward-facing-step combustor: (*c*) $\dot{m}_f = 0.55$ g s^{-1}; (*d*) $\dot{m}_f = 0.59$ g s^{-1}. The shape of the forward and return trajectories resembles a sigmoid (S-shaped) curve. The threshold was set at 500 Pa.

However, the presence of turbulence shifts the peak amplitude across a range of values even during combustion instability. One simple way to bypass this issue would be to count the number of peaks (N) in the signal $\phi(j)$ for a time duration t above a fixed threshold ξ, which would correspond to acceptable levels of amplitude for the system. For the present study, ξ was chosen as 500 Pa, which roughly corresponds to the maximum amplitude levels measured during stable operation ($Re = 1.78 \times 10^4$, $\phi = 0.9$ for swirler; and $Re = 1.83 \times 10^4$, $\phi = 1.13$ for bluff body) when large-amplitude intermittent bursts are not observed. If N_{tot} is the total number of peaks that happen within that time, one can then assign the probability of the system attaining instability as

$$f = N/N_{tot}. \tag{3.1}$$

The value of f is a measure of the proximity of the system to instability. The measure also makes sense from a dynamical systems perspective as an order parameter, i.e. a parameter that measures the amount of order (order in the sense of ordered oscillations or organized behaviour) in the system (Haken 1985).

In figure 3, we have plotted values of f for various Reynolds numbers, starting from low-amplitude combustion noise to instability and back to stable operating regimes for both the combustors at two different fuel flow rates. The threshold was set at 500 Pa, which corresponds to the levels of pressure fluctuations in the system during stable regimes of operation. The values of f vary smoothly as the control parameter (Reynolds number) traverses regimes of stable operation towards combustion instability. This is because of the presence of an intermediate intermittent regime in which the pressure signals occasionally cross the threshold and leads to increased values of f. These intermittent excursions keep increasing as flow conditions approach combustion instability and finally saturate to one as instability is reached, when the dynamics becomes dominantly periodic. These values of f thus serve as an appropriate measure – a measure of the order in the signal – to draw the bifurcation diagram in systems exhibiting widely varying amplitudes in the signals.

The bifurcation diagrams further enable us to infer the nature of criticality of the bifurcation of the acoustic subsystem that leads to combustion instability in both the configurations. A hysteresis is clearly visible in this new bifurcation diagram for the bluff-body combustor (figure 3*c*,*d*) whereas there is no hysteresis for the unsteady pressure signals acquired from the swirl combustor (figure 3*a*,*b*). The nature of the graphs was found to be qualitatively similar on shifting the threshold a few tens of pascals on either side, although there is an associated quantitative change in the probability measure f. Hence, it is to be concluded that the bifurcation diagram is useful only to infer the qualitative nature of the transitions at the onset of instability. The bifurcation diagrams show that the presence of the swirler leads to a supercritical Hopf bifurcation of the acoustic subsystem whereas the bluff body causes the bifurcation of the acoustic subsystem to be subcritical.

3.2. *Precursors to combustion instability*

Although bifurcation diagrams can be drawn by computing the probability that the peaks in a measured signal exceed the levels of noise in the combustor, they cannot be used to determine the proximity of the system to an impending instability sufficiently in advance. This is because the measure f starts growing only when the amplitude levels in the combustor grow, which can in turn be conveniently measured by computing the r.m.s. levels of pressure fluctuations in the combustor.

Lieuwen (2005) has used the damping rate of the autocorrelation to predetermine the stability margin of combustors. The transition point was identified as that operating condition at which the damping of the autocorrelation of measured signals becomes zero. After computing the autocorrelation of the pressure signal, a Hilbert transform was applied on the autocorrelation to obtain the variation of the amplitude and phase of the autocorrelation. The effective damping rate was then obtained as the slope of the logarithmic decrement of the amplitude of the autocorrelation.

Shown in figure 4(*c*,*d*) is the variation of the damping rate of the autocorrelation of the pressure time traces acquired for various values of flow Reynolds number starting from regimes of combustion noise towards combustion instability for both the flame-holding configurations. The damping rate was computed for 0.04 s (roughly 10 acoustic cycles at instability), by performing a linear regression. Unlike in Lieuwen (2005), a band-pass filter was not applied to the input pressure signals. To compare the performance of this precursor measure, the variation of the r.m.s. values of the pressure time series for the various conditions are also shown in figure 4(*a*,*b*). To show the convergence of the measures, the damping rates and r.m.s. values computed for increasing intervals of data acquisition T are also shown. The regression errors associated with the straight-line fit of the logarithmic decrement for the damping rates have not been shown for the sake of clarity.

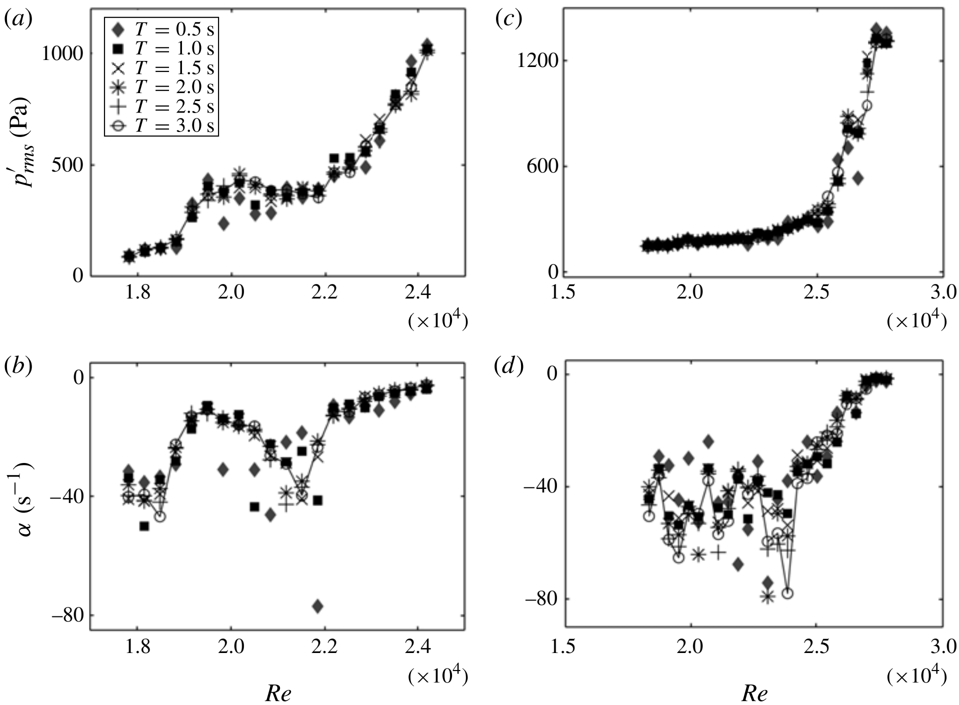

FIGURE 4. Variation of the sound levels in the combustor as measured by the r.m.s. values of unsteady pressure signal (p'_{rms}) with flow Reynolds number Re for (*a*) the swirl-stabilized configuration ($\dot{m}_f = 0.55$ g s^{-1}) and (*c*) the bluff-body-stabilized configuration ($\dot{m}_f = 0.59$ g s^{-1}). The corresponding variation of the precursor measure based on the effective damping rate of the autocorrelation of the pressure time traces (α) are shown in panels (*b*) and (*d*), respectively. Here T represents the duration of data acquisition.

It is seen that the precursors based on damping rates perform satisfactorily and show linear dependences only after the amplitude starts rising in the combustor. In other words, they were seen to have a performance comparable to the measure f used to construct the bifurcation diagrams. Further, it is seen that the decay rates fluctuate wildly and converge slowly with T for the bluff-body combustor in which the acoustic subsystem undergoes a subcritical bifurcation, for regimes prior to the sharp amplitude rise. The convergence with sampling duration is seen to be much better for the swirl combustor for which the acoustics undergoes a supercritical bifurcation; however, the variation of the damping rate was non-monotonic. The precursors based on decay rates are thus seen to perform inadequately and show non-monotonic dependences during regimes of combustion noise and the start of intermittency. This is expected since an 'effective damping rate' – which is an average measure based on a linear analysis – cannot be defined for intermittent or chaotic signals that arise due to nonlinear time-localized dynamics. In the next subsection, we introduce measures that can characterize the intermittency in a measured signal and compare the capability of these measures with the traditional methods as precursors to combustion instability.

4. Recurrence quantification

4.1. *Recurrence plots*

The temporal features of the dynamics of a measured signal can be characterized by tracking the regularity of the trajectories using recurrence plots. Recurrence is a fundamental property of dynamical systems and recurrence plots allow one to visually identify the times at which the trajectory of the system visits roughly the same area in the phase space (Marwan *et al.* 2007). The technique requires reconstruction of the mathematical phase space of evolution of the pressure fluctuations, the procedure for which is outlined in Nair *et al.* (2013). In reconstructing an appropriate phase space, knowledge of the appropriate embedding dimension d_0 and the optimum time lag τ_{opt} that is used to generate the delay vectors from the measured pressure time series (of length N_0) is necessary. A recurrence plot is constructed by computing the pairwise distances between points in the phase space. Then, a matrix of recurrences may be obtained as

$$R_{ij} = \Theta(\epsilon - \|\boldsymbol{p}'_i - \boldsymbol{p}'_j\|), \quad i, j = 1, 2, \ldots, N_0 - d_0\tau_{opt}, \tag{4.1}$$

where Θ is the Heaviside step function and ϵ is a threshold or the upper limit of the distance between a pair of points in the phase space to consider them as close or recurrent. The indices represent the various time instances when the distances are computed and the bold face represents the vector of coordinates in the phase space. The recurrence matrix is a symmetric matrix composed of zeros and ones and a recurrence plot is the two-dimensional representation of this matrix as the trajectories evolve in time. The ones in the recurrence plot are marked with black points and represent those time instants when the pairwise distances are less than the threshold ϵ. White points in the recurrence plot correspond to the zeros in the recurrence matrix and correspond to those instants when the pairwise distances exceed the threshold.

Figure 5 shows the recurrence plots drawn for the pressure signals acquired during (i) combustion noise, (ii) intermittent regime and (iii) combustion instability for pressure time series acquired from the bluff-body combustor. The data were under-sampled to a frequency F_s of 2.5 kHz and embedded in a phase space of $d_0 = 10$ with an embedding delay $\tau_{opt} = 1$ ms. The under-sampling was done to reduce the computational cost involved in obtaining the recurrence matrix. The recurrence plot for the chaotic combustion noise is seen to be grainy (figure 5*a*). This is to be expected since the dynamics is chaotic with little repeatability in the patterns. On the other hand, the recurrence plot during combustion instability displays a pattern of diagonal lines indicating high repeatability (recurrence) in the dynamics (figure 5*c*). The time duration of the signal was chosen to be 0.1 s to highlight the diagonal lines in the recurrence plot, which would otherwise not be visible. The separation between the diagonal lines gives the fundamental time period of oscillation during combustion instability (Kabiraj & Sujith 2012). The intermediate regime has a recurrence plot that consists of perforated black patches amidst white patches (figure 5*b*). The black patches represent the times when the system exhibits low-amplitude chaotic oscillations and white patches represent the higher-amplitude periodic bursts. This is a pattern typical of intermittent burst oscillations. The recurrence plots thus help to visually identify the route to instability in turbulent combustors. The transition proceeds from chaos (combustion noise) to order (combustion instability) through an intermediate intermittent regime.

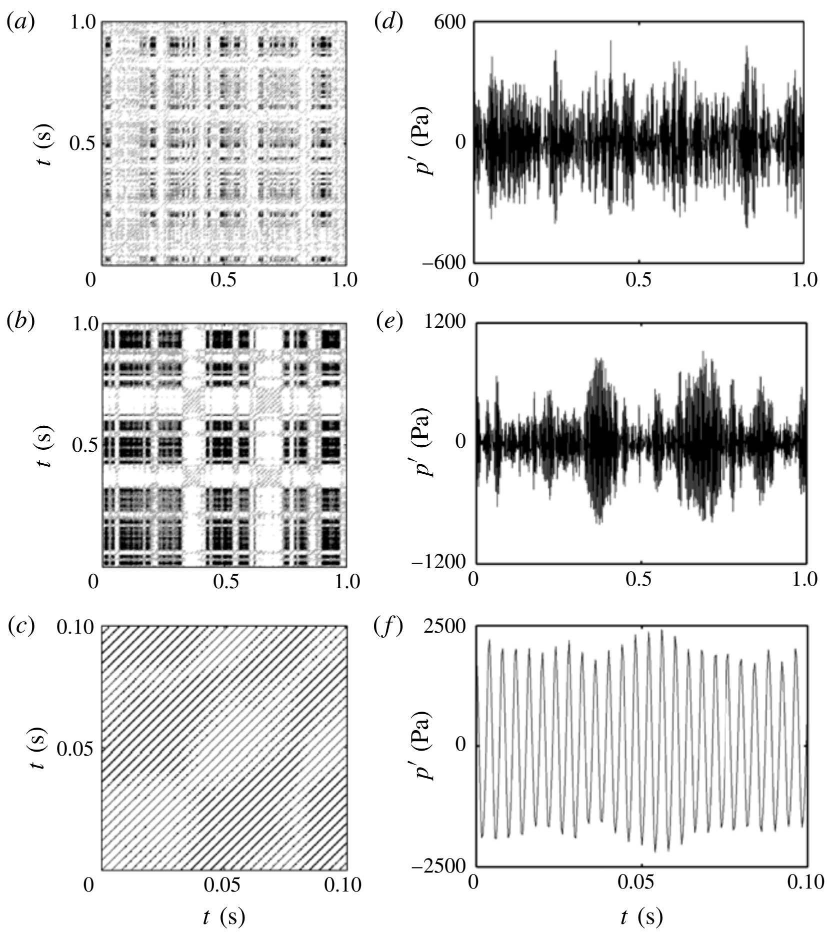

FIGURE 5. Recurrence plots and the corresponding unsteady pressure signals acquired during (*a*,*d*) combustion noise ($Re = 1.83 \times 10^4$), (*b*,*e*) intermediate intermittent regime ($Re = 2.50 \times 10^4$) and (*c*,*f*) combustion instability ($Re = 2.78 \times 10^4$) from the bluff-body-stabilized combustor. The threshold for the recurrence plot was chosen to be $\epsilon = \lambda/5$, where λ is the size of the attractor, defined as the maximum distance between pairs of points in the phase space. The black patches in the intermittent and chaotic oscillations correspond to regions of low-amplitude pressure fluctuations relative to λ. The distance between the diagonal lines in panel (*c*) corresponds to the time period of oscillation during instability. Similar results were obtained for the swirl-stabilized configuration.

4.2. *Precursors using recurrence quantification*

Several statistical measures may be constructed through a recurrence quantification analysis of a measured signal that could serve as useful measures of intermittent oscillations. These measures can further be used as precursors to an impending instability because they vary in a smooth fashion as the operating conditions traverse the intermittent regime into conditions of combustion instability. By tracking the probability distribution of black points (or white points) in such plots, measures can be constructed that can distinguish amongst the dynamically different regimes of the combustor, the procedure for which is outlined in the next subsection.

In constructing the recurrence plots of figure 5, the threshold ϵ was a relative measure, as it depended on the size of the attractor at that particular operating condition. This enables one to understand the qualitative changes in the underlying dynamics in phase space. In order to obtain quantifiable precursors across different values of *Re*, the threshold needs to be held fixed at some suitable value. Fixing the threshold allows one to compare the values of the various statistical measures obtained using recurrence quantification as the control parameter (*Re*) is varied. In what follows, the fixed threshold value (say ϵ_0) was chosen to be slightly higher than the size of the attractor obtained at the lowest operational Reynolds number. It should be mentioned that the thresholds are fixed based on the Euclidean distances between points in the phase space ($\sim\sqrt{d_0}\,|p'|$), and should not be confused with the amplitude levels in the combustor ($|p'|$).

A number of suitable markers that foretell an impending instability may be constructed by counting the number of black points in the recurrence plot. The density of black points in a recurrence plot measures the recurrence rate in the dynamics of the system and can be obtained as

$$RR = \frac{1}{(N_0 - d_0\tau_{opt}F_s)^2} \sum_{i,j=1}^{N_0-d_0\tau_{opt}F_s} R_{ij}, \tag{4.2}$$

where R_{ij} is 0 for a black point and 1 for a white point. The signal was sampled at a frequency F_s of 2.5 kHz for 3 s to give a value of N_0 of 7500 and was embedded in a phase space of $d_0 = 10$ with an embedding delay $\tau_{opt} = 1$ ms. This density of points in the recurrence plot is seen to decrease on the approach of instability (figure 6*a*,*d*). This is expected since the number of black points in the recurrence plot would come down as instability is reached because the pairwise distances now exceed the threshold more often.

This decrease in the density of black points should then correspond to a decrease in the time spent by the system in aperiodic states, which is measured by a quantity τ_0 (normalized with the time duration of evolution of the trajectory in phase space), defined as

$$\tau_0 = \frac{1}{N_0 - d_0\tau_{opt}F_s} \frac{\displaystyle\sum_{v=1}^{N_0-d_0\tau_{opt}F_s} vP(v)}{\displaystyle\sum_{v=1}^{N_0-d_0\tau_{opt}F_s} P(v)}, \tag{4.3}$$

with $P(v)$ being the frequency distribution of the vertical (horizontal) black lines of length v in the recurrence plot for a signal sampled at a frequency F_s. The quantity τ_0 also quantifies how long the system remains in a particular dynamical state (in this case, chaotic fluctuations). Hence we expect this quantity to tend towards zero as the system transitions completely into periodic oscillations (see figure 6*b*,*e*). The value of τ_0 will be equal to 1 at conditions of combustion noise.

Finally, the Shannon entropy s of the signal can be obtained from the recurrence plot through the expression

$$s = -\sum_{l=1}^{N_0-d_0\tau_{opt}F_s} p(l)\ln(p(l)), \tag{4.4}$$

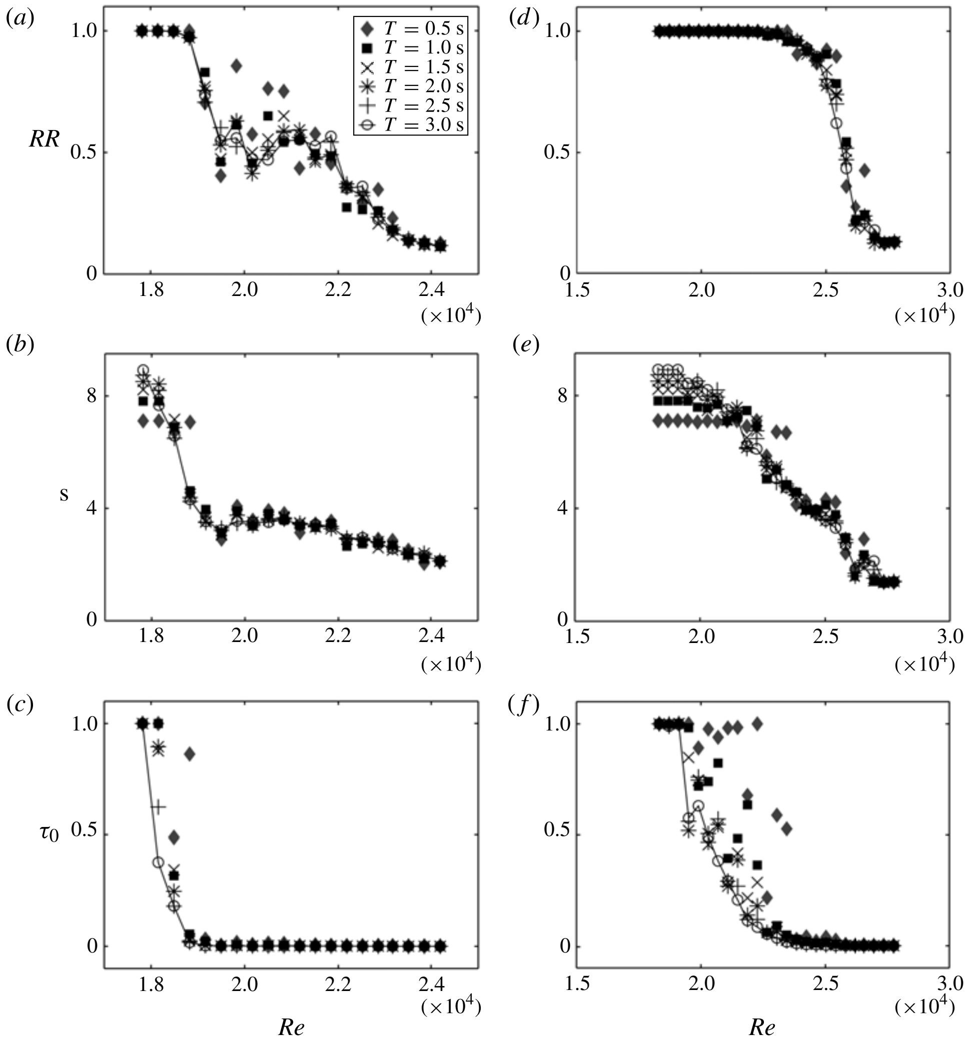

FIGURE 6. Performance of the statistical measures of intermittency obtained through recurrence quantification for (*a–c*) the swirl-stabilized configuration with $\dot{m}_f = 0.55$ g s^{-1} and (*d–f*) the bluff-body-stabilized configuration with $\dot{m}_f = 0.59$ g s^{-1}, for pressure time traces sampled at 2.5 kHz for 3 s. The measures plotted correspond to: (*a*,*d*) the recurrence rate of dynamics (*RR*), which measures the density of points in the recurrence plot; (*b*,*e*) the entropy (*s*) of the diagonal length distribution; and (*c*,*f*) the average passage time spent by the dynamics in aperiodic fluctuations (τ_0). The thresholds were chosen as $\epsilon_{0,swirl} = 1450$ Pa and $\epsilon_{0,bluff} = 1900$ Pa, which are close to the respective attractor sizes ($\lambda_{swirl} = 1437.4$ Pa, $\lambda_{bluff} = 1955.5$ Pa) in the underlying phase space at the lowest measured *Re*.

where the probability $p(l)$ that a diagonal line has length l is given by

$$p(l) = \frac{P(l)}{\displaystyle\sum_{l=1}^{N_0 - d_0 \tau_{opt} F_s} P(l)}, \tag{4.5}$$

with $P(l)$ the frequency distribution of the black diagonal lines of length l. Shannon entropy is a measure of the amount of order (disorder) in the system. We see that the Shannon entropy of the signal (s) tends towards zero at the onset of instability (figure 6*c*,*f*). A decrease in entropy indicates that the system is approaching a state of regularity or there is an emergence of order out of chaos. This makes sense intuitively, as we know that the recurrence plots for a periodic signal consist of black parallel diagonal lines and that the oscillations correspond to an ordered state. Hence, we naturally expect the entropy associated with the diagonal line distribution to come down as operating conditions approach combustion instability.

The relative merits of these measures as early-warning signals of instability were gauged by comparing their performance with the r.m.s. values of the pressure time series (figure 5*a*,*b*). The convergence of the measures with an increasing duration of data acquisition was also inspected to ensure that the precision of the measured quantities is increased. The converged measure represents its average value at the flow condition, as an ensemble average of the measure over many realizations should tend to an average measure obtained over large time durations. Since we do not have multiple realizations (pressure time traces) at the same operating conditions, we have adopted this measure of convergence to infer that uncertainties in estimation of these average measures have been minimized.

The measure RR is seen to have an inverse relationship with p'_{rms} and has good convergence as the time duration is increased from $T = 0.5$ to $T = 3$ s. However, we see that the variation in τ_0 and s starts much earlier than the regimes when amplitudes start rising in the combustor as indicated by p'_{rms}. These measures vary sooner as they quantify the time-localized statistics of the burst oscillation; for instance τ_0 measures the average duration between two successive bursts. It is interesting to note that the variation in the precursor measures are much steeper for the swirl-stabilized configuration, where the amplitudes are seen to increase gradually with Re, whereas the variation is much more gradual for the bluff-body-stabilized combustor, which has a sharp rise in amplitudes with Re.

The variability in the measures with the threshold size ϵ_0 is shown in figure 7. It is seen that the variability is more for the bluff-body-stabilized combustor as compared to the swirl-stabilized combustor. Among the precursor measures, τ_0 shows the largest variation as ϵ_0 is varied. However, the overall qualitative features are preserved even when the threshold is varied. These results indicate that knowledge of the amplitude levels in the combustor during stable operation is desirable for the optimum performance of the precursors.

Although it is possible to define additional quantifiable precursors (see Marwan *et al.* (2002) for a detailed list of statistical measures constructed using recurrence plots), our purpose in this section was merely to illustrate the power of recurrence quantification in forewarning of impending combustion instability. The reason why these precursors work is due to the presence of an intermittent regime of burst oscillations amidst chaotic combustion noise and ordered periodic oscillations. More generally, since these measures only distinguish the passage of dynamics from a chaotic to an ordered state through intermittency, such precursors can possibly be used as early-warning signals of undesirable oscillatory states in a variety of turbulent flow systems.

5. Concluding remarks

We report and characterize for the first time an intermittency route of transition from stable to unstable operation in the dynamics of turbulent combustors. In experiments,

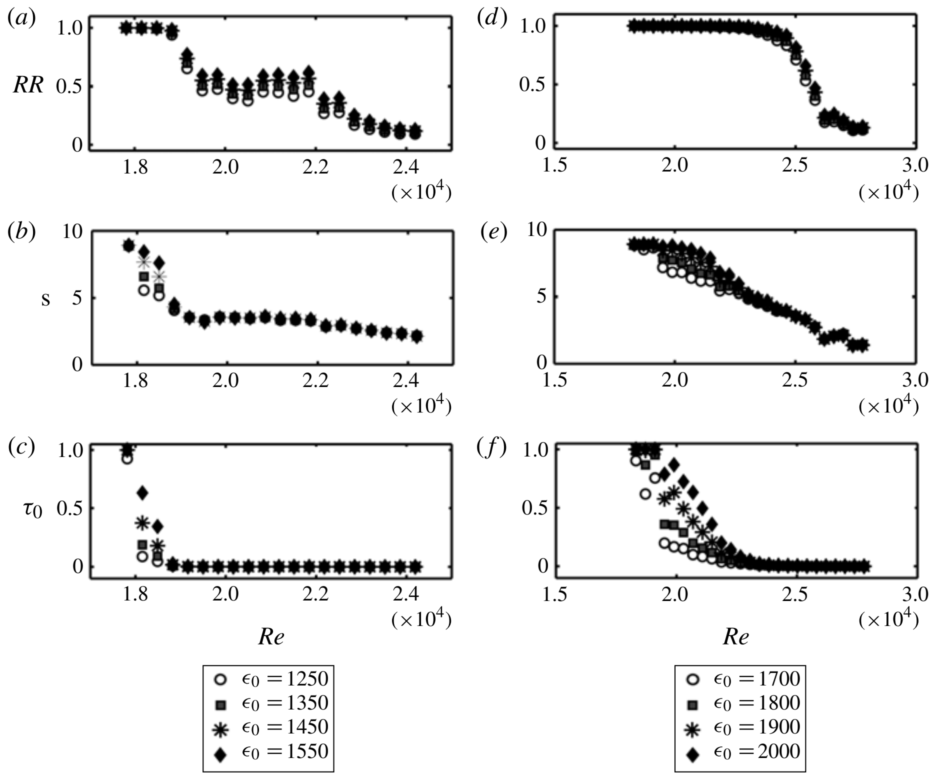

FIGURE 7. Variability in the statistical measures of intermittency for different threshold sizes ϵ_0 for (*a*–*c*) the swirl-stabilized configuration and (*d*–*f*) the bluff-body-stabilized configuration for pressure time traces sampled at 2.5 kHz for 3 s at various *Re*. The measures plotted correspond to: (*a*,*d*) the recurrence rate of dynamics (*RR*), which measures the density of points in the recurrence plot; (*b*,*e*) the entropy (*s*) of the diagonal length distribution; and (*c*,*f*) the average passage time spent by the dynamics in aperiodic fluctuations (τ_0).

these intermittent regimes were always seen to presage combustion instability and are composed of bursts of high-amplitude periodic pressure oscillations that appear in an almost random manner amidst regions of aperiodic low-amplitude fluctuations. This gives an altogether different picture from what one would expect from a mean-field description of the phenomenon, wherein the transition happens from a fixed point to a limit cycle via a Hopf bifurcation. A mechanism was proposed that considers the intermittency to arise as a result of complex nonlinear interaction amongst the flow and acoustic subsystems that operate over different time scales.

A smooth and continuous measure to plot bifurcation diagrams for parameter variation in combustors with a turbulent flow field can be obtained by counting the number of peaks in a measured signal above a predefined threshold. Hysteresis was observed for variations in the flow Reynolds number for the combustor with the bluff-body-stabilized flame using this measure. Such a hysteresis zone was absent for the swirl-stabilized configuration. This indicates that the acoustic subsystem undergoes subcritical Hopf bifurcation for the bluff-body combustor and a supercritical Hopf bifurcation for the swirl-stabilized combustor.

Precursors to an impending combustion instability can be obtained through recurrence quantification, which can warn an operator of fielded systems sufficiently in advance, so that appropriate control action may be taken to prevent detrimental oscillations. These precursors are seen to detect and warn about the onset of an oscillatory regime well in advance of other measures based on the sound levels in the combustor and effective damping rates. These measures act as effective precursors, as they act as quantifiers of the intermittency in a measured signal.

It is quite possible that combustors in fielded systems can tolerate limit cycle oscillations, provided the amplitudes are within a reasonable range. The passive control methods available in the literature work by increasing the damping, or by modifying some design or flow features to suppress the instability amplitudes, and possibly improve stability margins. Hence, they fall under a class of methods complementary to what we propose in this paper. Our methods warn the operator that oscillations are about to set in for further variations in an operating parameter. Armed with this knowledge, the decision lies with the operator as to whether to allow the operating conditions to cross over to regimes of limit cycle operation. Employing passive control measures would require knowledge of the amount of damping required to suppress the instability amplitudes when the oscillations set in whilst ensuring that the performance is not compromised. However, at present, there appear to be no reliable methods to predetermine the amplitudes of oscillation at the onset of instability in combustors. We present the operator with an alternative choice, one that is aimed at avoiding a region of unstable operation altogether.

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