× Basic understanding about the subject & [illegible] and ratio (simple) problems → Question Pattern

DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion (AS 6320) **Open Notes** Time: 50 Minutes Marks: 20

1. What do you mean by combustion instability? What do you mean by combustion noise? What are the differences? Which one of them; i.e., combustion instability or combustion noise is more dangerous? (2)

2. The complex frequency (ω) of a system is $628 + 0.20\,i$ radians/second, where $i = \sqrt{-1}$. What is the frequency of the system? (cycles per second) What is the growth/decay rate? (2)

3. Consider one-dimensional, harmonic sound propagation in a lossless medium. I find that the phase between acoustic pressure and acoustic velocity is -180 degrees. Which way is the wave propagating? (2)

4. The spherically symmetric wave equation (in spherical co-ordinates, naturally ☺) is given by

$$\frac{1}{r}\frac{\partial^2 rp'}{\partial r^2} - \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} = 0$$

$\frac{\partial \psi}{\partial r}\frac{\partial r}{\partial x}$ $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

where $p'(r,t)$ is the acoustic pressure, r is the radial co-ordinate, and c the speed of sound. Write down a general solution for the acoustic pressure.

Write the corresponding Helmholtz equation (i.e., the wave equation in harmonic domain). Write down the solution for acoustic pressure and acoustic velocity in the harmonic domain. (6)

5. Consider a spring-mass oscillator system where m, k and x are the mass, spring constant and position of the oscillating mass. A force F is exerted on this system. When there is feedback, the force may depend on the position x and velocity $\frac{dx}{dt} = \dot{x}$. The equation of this spring mass system is:

$$m\frac{d^2x}{dt^2} + kx = F(x,\dot{x})$$

Derive a corollary for the total energy of the system comprising the kinetic and potential energies. [Hint: Write the above second order ODE as two first order ODEs and then derive an equation for the rate of change of total energy dE/dt. Potential energy of a spring $= \frac{1}{2}kx^2$]

Write down a criterion for the stability of this spring mass system? (6)

6. What is the significance of the real part of admittance? How is it related to the acoustic power flow, in the context of 1D wave propagation in a duct? [For 1-D propagation in a duct, acoustic power is the product of acoustic intensity and area of cross-section of a duct.] (2)

DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion

AS6320
End-semester Exam (2023)

Marks: 40, Time: 3 hrs; 14:00-17:00
Instructor: R. I. Sujith

Instructions:

1. This is an open-book examination.
2. All the questions are quite simple.
3. Pause and think for a minute before you answer.
4. *Explain what you are doing.* All the marks are for the steps and explanations. **If there is a problem with the question or if any expressions given are wrong, state why you think so in the answerscript. Correct the question if necessary.**
5. Be neat and brief. If you write too much, I may not even see the "answer" and will be searching for it!

Relax and have fun. ☺

1. We superimpose two sound sources, each of them having a sound pressure level of 0 dB. What will be the net sound pressure level? (In other words, what is 0 dB + 0dB) [1 Marks]

2. How will you get the actual pressure amplitude (what you will see, for example, in an oscilloscope) from the complex pressure amplitude in a standing wave. [1 Mark]

3. What is combustion noise? What is this state in the parlance of dynamical systems theory? How is combustion noise different from combustion instability? [1.5 Marks]

4. What is the difference between transition to thermoacoustic instability for a laminar and turbulent combustor [2 Marks]

5. What is meant by a bifurcation? What is subcritical Hopf bifurcation? What is supercritical Hopf bifurcation? Draw bifurcations and illustrate these bifurcations.
   Which one of them corresponds to triggering? Which of these bifurcations is associated with hysteresis? [2 Marks]

6. Explain with the help of a block diagram, how you can actively control thermoacoustic instability. [1 Mark]

7. What is the most common mechanism that causes combustion instability in Low NOx Gas Turbine (LNGT) engines that are used for power production? [1 Mark]

8. Consider a wave field $p' = Ae^{-ikx} + Be^{ikx}$. Which of the two terms represents the left running wave? Which one represents the right running wave? Explain clearly how you got the answer. What is the impedance at $x = 0$? What is the characteristic impedance at $x = 0$? [1 Mark]

9. Consider the following ordinary differential equation.
   $$\ddot{x} + \epsilon\dot{x} + x^3 - x = 0$$
   Rewrite this equation as a system of two first order equations. [1 Mark]

10. The acoustic pressure in a duct can be written as $p' = A\cos(\omega t)\sin(kx)$.

a. Rewrite this acoustic pressure field as a linear superposition of left running and right running waves.
b. Derive an expression for the acoustic velocity field.
c. Write the acoustic velocity field as a linear superposition of left-running and right-running waves.

[1.5 Marks]

11. What is the phase between acoustic pressure and acoustic velocity for a purely right-running traveling wave?

[1 Mark]

12. Consider acoustic waves in an inhomogeneous quiescent media. Is it still true that $p' = c^2 \rho'$? If so, justify it? If not, what is the relation between p' and ρ'? [2 Marks]

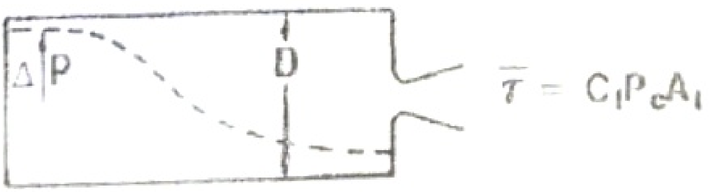
13. Consider an area jump (discontinuity) as shown in the figure below. Derive relations for the acoustic pressure and the acoustic velocity jumps at the area discontinuity. Assume that there is no mean flow. The area on the left of the discontinuity is A_1 and the area on the right of the discontinuity is A_2.

[2 Marks]

14. You must have all heard about ISRO's GSLV. The length of the main booster is 22 meters, and the diameter is 2.8 meters. It has 139 tonnes of solid propellant (Hence, the motor is called S139). The temperature of the combustion products is 3400 K, and the ratio of specific heats is 1.2. Estimate the fundamental, first harmonic and second harmonic of this rocket motor.

[2 Marks]

15. Consider a solid propellant rocket motor as shown in the figure. The thrust ($\bar{\tau}$) can be written in terms of the thrust coefficient c_f as $\bar{\tau} = c_f P_c A_t$, where, P_c is the chamber pressure and A_t is the throat area. In the figure below, D_t is the throat diameter and D is the port diameter.



Pressure and thrust oscillations in a solid rocket motor

Show that in the presence of oscillations in **fundamental mode** of amplitude ΔP, the amplitude of the thrust oscillation $\Delta \tau$ can be estimated as:

$$\frac{\Delta \tau}{\bar{\tau}} = \left(\frac{D}{D_t}\right)^2 \frac{2\Delta P_c}{\bar{P}_c} \frac{1}{c_f}$$

Assuming "typical dimensions" and operating parameters of a solid rocket motor (of any class that you may like), what would be a typical value of $\frac{\Delta \tau}{\bar{\tau}} \Big/ \frac{\Delta P_c}{\bar{P}_c}$. Choose c_f to be 1.4.

[3 Marks]

16.
(a) For an non-dimensional admittance value of (0, 0), hand-sketch the standing wave pattern in the duct, highlighting the features qualitatively. Please plot both the acoustic pressure amplitude and phase.
(b) Hand-sketch the acoustic pressure amplitude and phase, when the admittance value is (1, 0)
(c) Now sketch for admittance values of (0.5, 0) and (-0.5, 0)
(d) What is the significance of the real part of admittance? What is the significance of the imaginary part of admittance?

[2 Marks]

17. We derived the G equation and its solution, in an attempt to model premixed flames. Explain how you will calculate the heat release rate fluctuations. [1 Marks]

18. Shock waves are similar to sound waves. Sound waves have infinitesimal strength, where as shock waves have finite strength. Can we have shockwaves propagate in subsonic flows? [1 Marks]

19. Consider the 1D momentum equation for small perturbations in the following form:

$$\frac{\partial u'}{\partial t} = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x} + \nu\frac{\partial^2 u'}{\partial x^2}$$

Where ν is the kinematic viscosiy $= c\Lambda/2$, Λ is the mean free path.

In the analysis that we did in the class, we always neglected the last term; i.e., $\nu\frac{\partial^2 u'}{\partial x^2}$ [i.e., the viscous term]. Show when this assumption is justified? Show when will this assumption fail?

[It is not enough to state the answer; it **must be proved**.] [4 Marks]

20. Let us consider 1D sound propagation in a constant area duct in the presence of a constant mean flow $\bar{u}$.

(a) Derive the following wave equation:

$$\frac{\partial^2 p'}{\partial t^2} + 2\bar{u}\frac{\partial^2 p'}{\partial x \partial t} + \left(\bar{u}^2 - c^2\right)\frac{\partial^2 p'}{\partial x^2} = 0$$

(b) Derive the corresponding Helmholtz equation.

(c) Show that the solution for wave propagation can be expressed as follows:

$$p' = e^{i\omega t}\left[Ae^{i\frac{k}{1-M}x} + Be^{-i\frac{k}{1+M}x}\right]$$

Here, M is the Mach number corresponding to the mean flow velocity. Interpret this solution in terms of traveling waves

(d) Now, consider a duct that is open at $x = 0$ and $x = L$. Show that the natural frequencies are given by the formula:

$\omega = n\pi \dfrac{c}{L}\left(1 - M^2\right)$ where n can take values of 1, 2, 3....

(e) Show that the time period corresponding to the $n = 1$ mode is

$$T = \frac{L}{c - \bar{u}} + \frac{L}{c + \bar{u}}.$$

(f) Interpret the result in question (e) in terms of the travel times of the left and right running waves.
[6 Marks]

21. Most of the instabilities observed in practical systems result from a feedback between combustion and coupling modes. Schematically, a driving process generates perturbations of the flow, a feedback process couples these perturbations to the driving mechanism and produces the interaction that may lead to oscillations. These processes involve time lags because reactants introduced in the chamber at one instant are converted into burnt gases at a later time. Systems with delays are more readily unstable. This is easily shown by considering a second-order model featuring a linear damping (second term) and a restoring force with a delay (third term),

$$\frac{d^2x}{dt^2} + 2\zeta\omega_0\frac{dx}{dt} + \omega_0^2 x(t - \tau) = 0 \qquad (1)$$

a. Show that expanding Eq. (1) in a Taylor series to first order yields the following equation:

$$\frac{d^2x}{dt^2} + \omega_0(2\zeta - \omega_0\tau)\frac{dx}{dt} + \omega_0^2 x(t) = 0 \qquad (2)$$

[1 Mark]

b. Under what condition will the damping be negative, where the oscillations can grow in amplitude?
[2 Marks]