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Acoustic Instablities in Aerospace Propulsion

End Semester Examination
3 hours, 40 Marks

Instructions:
1) This is an open-book exam.
2) All the questions are simple.
3) Think for a minute before you start working out the problems.
4) Explain what you are doing. All the marks are for the steps and explanations. Be brief. If you write too much, I may not even see the "answer" and will be searching for it! Please do not make your answer book a mystery.
5) If you have any questions, feel free to ask me.

I invite you to have fun ☺.

1. Let us start with solid rocket motors (SRM). The length of the PS1 booster (this is the first stage of the PSLV vehicle is 22 metres. It has 129 tonnes of solid propellant(is referred to as S129). The temperature of the burnt gases inside is 3400 K. Calculate the fundamental, first harmonic and second harmonic of this motor. (3 marks)

2. *This question has 5 parts – a, b, c, d and e. Answer all of them.* (18 marks)

This is a "simple analysis" of a complicated real-life problem. Please do not get disturbed by the length of this question. It is quite simple. You just have to patiently read it, and the answers are really simple.

Continuing with solid rocket motors – now let us take a look at the Ariane booster. A schematic of the internal geometry of Ariane-5 EAP solid rocket motor is given below.

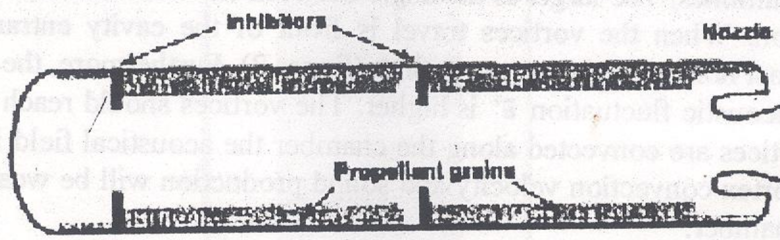


Figure 1: Internal geometry of the Ariane-5 EAP.

Large SRM have a submerged nozzle and segmented propellant grains separated by inhibitors (as in the above figure). During the combustion, the regression of the solid propellant surrounding the nozzle integration leads to the formation of a cavity whose volume varies during the launch. The hot burnt gas flow in the combustion chamber originates radially from the burning surface and then develops longitudinally before reaching the exhaust noz-zle. During the combustion, the regression of the burning surface is faster than the one of the inhibitor rings. Then, vortical flow structures may be formed from the inhibitor or from natural instability of the radial flow resulting from the propellant combustion. Such hydrodynamic manifestations drive pressure oscillations in the confined flow established within the combustion chamber of the motor. When the vortex shedding frequency is synchronized by acoustic modes of the motor chamber, resonant coupling may occur leading to self sustained oscillations. It is expected from literature that vortex-nozzle interaction is the main aeroacoustical source in this feedback loop.

Vortex nozzle interaction can be studied numerically using CFD. We however seek for a simple analytical model. In the case of solid rocket motors, it was suggested by early investigators such as Flandro that sound is produced by the "impingement" of the vortices on the wall of the nozzle inlet. When vortices approach the nozzle inlet, their transport velocity vector is significantly deviated from the acoustical streamlines (potential flow). Following Howe's theory, which we learned in class they produce sound when this occurs. Hence we should translate the "impingement" of Flandro as vortex-sound interaction localized at the nozzle inlet. We will now show that this interaction is stronger for a submerged nozzle because of the presence of the cavity.

We will make this plausible by means of an analytical model. The model is based on the vortex-sound theory for internal flows by Howe. The vortex-sound theory assumes some existing knowledge of the vortical distribution in the flow and deduces the aeroacoustical sound production from this knowledge. At low Mach numbers as occurring here, the average acoustic power P is given by:

$$P = -\bar{\rho} \left\langle \int_V (\vec{\omega} \times \vec{v}) \bullet \vec{u}' dV \right\rangle$$

where $\bar{\rho}$ is the mean density, $\vec{\omega}$ is the vorticity, $\vec{v}$ is the vortex transport velocity, $\vec{u}'$ is the acoustic velocity and V is the source volume (where $\vec{\omega} \neq 0$).

The vortex-sound theory yields the acoustic power given by Eq. (1) and representing the time average of the power transfer from the vorticity field to the acoustical field. When the vortex transport velocity vector ($\vec{v}$) is parallel to the acoustic streamlines ($\vec{u}'$), the vectorial triple product in Eq. (1) is vanishes. There is no acoustical energy production. To produce sound, the vortices should cross the acoustic streamlines, corresponding to the potential flow streamlines. The larger is the angle between the two vectors, the stronger is the acoustic production. When the vortices travel is front of the cavity entrance the acoustic velocity $\vec{u}'$ is almost normal to the vortex path $\vec{v}$ (figure 2). Furthermore, the acoustic power is stronger if the acoustic fluctuation $\vec{u}'$ is higher. The vortices should reach this point at the right time. As vortices are convected along the chamber the acoustical field is directed in the direction of the vortex convection velocity and sound production will be weak as the vortices move along the chamber.

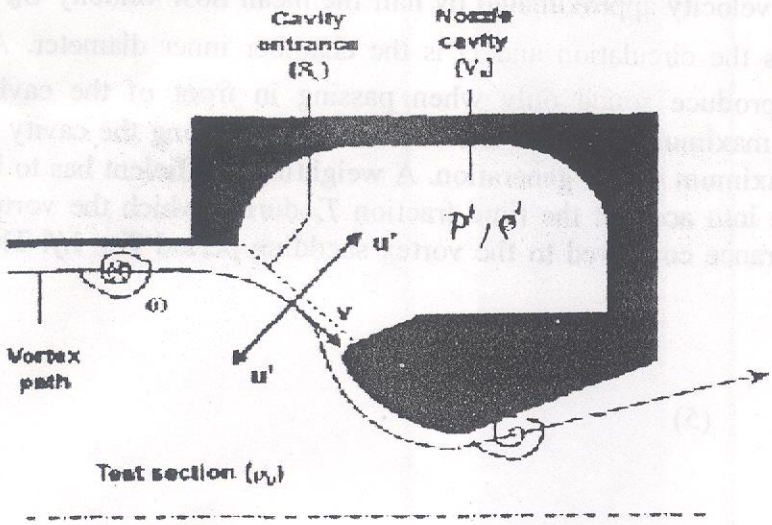


Figure 2: Theoretical modeling of the vortex nozzle interaction.

I will now walk you through the various steps involved in obtaining an expression for the acoustic pressure amplitude generated because of this phenomena. If you get different answers, please write it down. (Your analysis may be better than mine.) Also, please write down the assumptions that you are make. Even a completely different analysis is welcome, as long as you explain clearly what you are doing.

Let us consider a nozzle cavity of volume V_c limited by a cavity entrance of cross-section S_c (see Figure 2). This cavity acts as a resonator, where the "bulk pressure" in the fluid oscillates. Such a resonator is called Helmholtz resonator. The compressibility $\frac{dp'}{dt}$ of the gas in the cavity volume induces an acoustic velocity fluctuation u' in the cross-section S_c such that from mass conservation, the following equation can be written:

$$\bar{\rho}\, u' S_c = \frac{V_c}{a^2}\frac{dp'}{dt} \quad \text{, (where } a^2 = \frac{\gamma \bar{p}}{\bar{\rho}}\text{)}$$

Where we made use of the fact that the cavity is small compared to the acoustic wave length.

a) Using this expression derive and assuming a harmonically oscillating acoustic field, i.e, $u' = |u'|e^{i2\pi f t}$ and $p = |p'|e^{i2\pi f t}$, show that

$$|u'| = \frac{2\pi f V_c}{\gamma S_c}\frac{|p'|}{\bar{p}} \qquad (3)$$

The maximum acoustic power is then expressed from Eq. (1) by

$$P\max = \bar{\rho}|\vec{v}'||\vec{u}'|\int_V |\vec{\omega}|dv \approx \bar{\rho}|\vec{v}'||\vec{u}'|(\pi D\Gamma) \qquad (4)$$

where $|\vec{v}|$ is the vortex transport velocity approximated by half the mean flow velocity $U_0/2$, $|\vec{u}'|$ is given by equation 3, Γ is the circulation and D is the chamber inner diameter. As explained previously, vortices produce sound only when passing in front of the cavity entrance. We do expect that for maximum pulsation the vortex will pass along the cavity at the time corresponding to the maximum power generation. A weighting coefficient has to be introduced in equation 4 to take into account the time fraction T_c during which the vortex travels in front of the cavity entrance compared to the vortex shedding period $T = 1/f$. This coefficient is equal to

$$\frac{T_c}{T} = \frac{h_c / U_0}{1/f} = \frac{h_c f}{U_0} \qquad (5)$$

where h_c is the width of the cavity entrance.

The cross-section Sc of the cavity entrance can be assumed parallel to the test section axis: $S_c = \pi D h_c$ (see Fig. 2). Moreover we assume that the vortex structure transports all the vorticity with a velocity of the order of $U_0/2$. Therefore, the circulation Γ is given by:

$$\Gamma = l_v \frac{U_0}{2} \approx \frac{U_0^2}{4f} \qquad (6)$$

where l_v is the distance on which the vortex accumulates the vorticity, assumed to be equal to the distance between two successive vortices ($l_v \sim U_0/2f$).

b) Assuming that T_c/T is much less that 1, show that the generated acoustic power becomes:

$$P \approx \frac{\pi}{4} \overline{M}^2 f V_c |p'| \qquad (7)$$

Assuming that the acoustic losses are dominated by the radiation at the nozzle, the acoustic power loss can be expressed as:

$P' = \langle p'u' \rangle \frac{\pi D^2}{4}$ through the nozzle.

c) Given that the nozzle impedance is $Z_n = \frac{2\overline{\rho}a}{\overline{M}\,(\gamma-1)}$, show that

$$|p'| \approx \frac{4\overline{\rho}a}{(\gamma-1)D^2} \overline{M} f V_c$$

(Hint: Balance the acoustic power generated with the losses)

d) Also, using the equation $f/a = j/L$ where j is the mode number, and $V_{tot} = \pi D^2 L/4$, show that

$$\frac{|p'|}{|\bar{p}|}=\frac{\pi\gamma}{\gamma-1}jM_0\frac{V_c}{V_{tot}}$$

e) Based on the analysis that you have just performed, what are your recommendations for reducing the sound pressure level (occurring because of the vortex-nozzle cavity interaction).

Reference: This analysis was presented by
J. Anthoine, J-M. Buchlin and A. Hirschberg (2001), "Theoretical Modeling of the Effect of the Nozzle Cavity Volume on the Resonance Level in Large Solid Rocket Motors", AIAA paper 2001-2102, 7th AIAA/CEAS Aeroacoustics Conference, 28-30 May 2001, Maastricht.

3. Consider a flame sheet of infinitesimal thickness. Derive the relations for acoustic pressure and velocity jumps across the flame, when there is an oscillatory heat release. (3 Marks)

4. What is the significance of the real part of admittance? What is the significance of the imaginary part of admittance? (2 Marks)

5. What is Rayleigh's criteria? Give a physical argument substantiating it? Give a mathematical proof. (4 Marks)

6. Consider a "Horlicks bottle" or a similar glass bottle, roughly 10 cm diameter and 15 cm height. Let us now pour a little bit methanol into it. Now, we close the lid. However, the lid has a 15 mm hole in it. Now blow some air into it, so that there is a combustible mixture of air and methanol vapor in the bottle. Let us now ignite it. If we are lucky, the mixture fraction will be right (within the flammability limits) and will ignite. However, this "Horlicks bottle combustor" will work like a pulse combustor. I.e., it will produce "loud" oscillations, which you can hear, at about 40 Hz. The combustion will also be oscillatory.

   Now imagine this scenario, and explain how this works – i.e, what leads to the oscillatory pressure and oscillatory heat release. How is the Rayleigh criterion satisfied? (4 Marks)

7. In the laboratory, we demonstrated a bluff body combustor that can "self-excite" high sound pressure levels (SPL). The combustor could also be operated with much lower SPLs.

   Draw a schematic of this combustor. Explain how the SPL was adjusted? What is the principle behind?

   How would you modify this combustor to operate at different frequencies? (4 Marks)

8. What is the role of aluminium in solid rocket combustion instability? (2 Marks)

DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion (AS632) Endsemester Exam (2003)

Time: 3 hours
Marks: 40

The objective of this exercise is to show you how you can build models for combustion instability and also to teach you the concept of active control. Try having some fun. Most importantly – it is all very simple and there is absolutely nothing "high funda". So, please do not feel nervous. **If you have any questions, please ask me right away.** Relax, and have fun.

If you feel that there are any mistakes in the results that I have asked you to prove, feel free to point it out in your answer book.

Feel free to open any book.

Combustion instability mechanisms in practical combustion systems are thought to be very complex due to the coupled interactions and the inherent non-linearities associated with the phenomena involved (turbulent flow, exothermic chemical reactions, acoustic phenomena). However, we will "simplify" things and make a simple model describing the linear interaction between an unsteady heat release source and an acoustic resonator. (This model, although very simple, has been successfully applied to some real systems)

The configuration:
We consider the mean flow of a combustible mixture through a long duct (acoustic resonator) open at one end and closed at the other, with a flame stabilized at the axial location $x = a$. See the figure below:

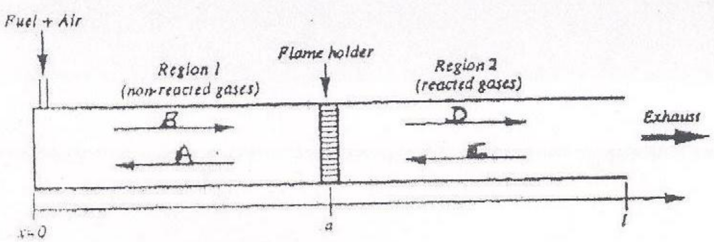


FIG. 1. Schematic diagram of the model geometry.

We assume the following:

1) The acoustic field is axial.
2) Neglect dissipation.
3) The open end of the duct (right side, $x = l$) corresponds to a pressure node and the closed end to a pressure anti-node (left side, $x = 0$).
4) The mean flow Mach number is small
5) The flame length is small compared to the acoustic wavelength so that the region of heat release may be approximated as a thin sheet located at $x = a$.

We will denote the portion of the duct upstream of the flame holder as region 1 with non-reacted gas having density ρ_1, speed of sound c_1. Region 2 will correspond to the post-combustion zone downstream of the flame holder, with a reacted gas density ρ_2 and sound speed c_2.

For longitudinal plane waves traveling along the duct, the acoustic pressure p can be expressed in the following form:

Region 1: $p_1(x,t) = Ae^{ik_1(x-a)+i\omega t} + Be^{-ik_1(x-a)+i\omega t}$ (1)

Region 2: $p_2(x,t) = Ce^{ik_2(x-a)+i\omega t} + De^{-ik_2(x-a)+i\omega t}$ (2)

$k = \omega/c$ is the wave number and A, B, C and D represent the amplitude of the acoustic traveling waves in the duct (see Figure 1).

<u>Question 1</u>
Write the expression for the acoustic velocity in regions 1 and 2.

There is an oscillatory heat release at the combustion zone which can be written as $Q'(t) = Qe^{i\omega t}$

<u>Question 2</u>
Apply the boundary conditions at $x = 0$ and $x = l$ and the "jump conditions" at the flame (i.e., $x = a$). Show that

$$(A-B) - \zeta(C-D) = (\gamma-1)\frac{Q}{c_1}, \quad (3)$$

where $\zeta = \dfrac{\rho_1 c_2}{\rho_2 c_2}$ is the specific acoustic impedance ratio.

Modeling the oscillatory heat release may well be a formidable challenge. However, we will simplify this task using an approach called a "sensitive time lag" hypothesis. This model was originally developed for the study of high-frequency combustion instabilities in liquid-fueled systems.

As before, we will assume that the combustion process in the duct may be treated as one dimensional and that the heat release occurs in an infinitesimally thin region at $x = a$. If we then suppose that the fluctuations in heat release rate are created uniquely by velocity perturbations existing at the flame holder (i.e., $u_1(x = a, t)$), but after a time lag τ, we may write the following relation.

$$\frac{(\gamma-1)}{\rho_1 c_1^2}Q' = nu_1(a.t-\tau)$$

The non-dimensional constant n is called the interaction index and describes the intensity of coupling between the velocity and heat release oscillations. τ is a time delay between the velocity and heat release fluctuations, and is assumed constant. For harmonic oscillations,

$$\frac{(\gamma-1)}{c^2}Q' = ne^{-i\omega\tau}u_1(a) \qquad (4)$$

Question 3
Substitute this into Eq. (3) and derive the following equation for the natural frequency

$$\zeta\cos(k_1 a)\cos(k_2 b) - \sin(k_1 a)\sin(k_2 b)\left(1 + ne^{-i\omega\tau}\right) = 0 \qquad (5)$$

where $k_1 = \omega/c_1$ and $k_2 = \omega/c_2$. $b = l\text{-}a$

For any given set of values of n and t, the roots of Eq. (5) will provice complex values of ω.

Question 4
a) What does the real and imaginary parts of the complex frequency signify?
b) In the n-τ model presented here, when can the imaginary part be zero?

Now, let us proceed to get the complete solution for a simplified case. Let us assume

$a=b$, $c_1 = c_2 = c$, $\rho_1 = \rho_2$ (ζ=1) !!!!

This is the case of a flame located in the center of a duct of length 2a and creating a negligible temperature jump. Neglecting the temperature jump is unrealistic; however, the resulting solution retains the same qualitative properties as that for the more complex case with a temperature jump.

Show that $\cos(2ka) - \sin^2(ka)ne^{-i\omega\tau} = 0$ is the simplified equation for natural frequency.

Now, show that if n<<1, $\cos(2ka) = 0.5ne^{-i\omega\tau}$

The solution of the problem will be considered in two steps. First we will study the case without oscillatory combustion (n=0).

Question 5
Find the resonant frequency of the first longitudinal mode, known as quarter-wave mode.

In the presence of combustion (n>0), we can expand the solution about the wave number k_0, corresponding to the frequency of oscillation ω_0, by defining $k = k_0 + k'$.

Question 6
Assuming that $k' << k_0$, and using Taylor series expansion for trignometric functions, show the following:

$$Real[k'] = -\frac{n}{4a}\cos(\omega_0\tau) \text{ and } Imaginary[k'] = \frac{n}{4a}\sin(\omega_0\tau)$$

Question 7: What is the condition on τ for instabilities to occur?

Question 8

Do this analysis for the next mode (known as three-quarter-wave mode).
We talked about passive control of oscillations in the class. Now, I shall introduce the active control. The principle behind this controller is to create a time varying acoustic boundary condition in the duct which will inhibit the amplification of longitudinal acoustic waves and minimize combustion oscillations. The control system architecture is shown in the figure below.

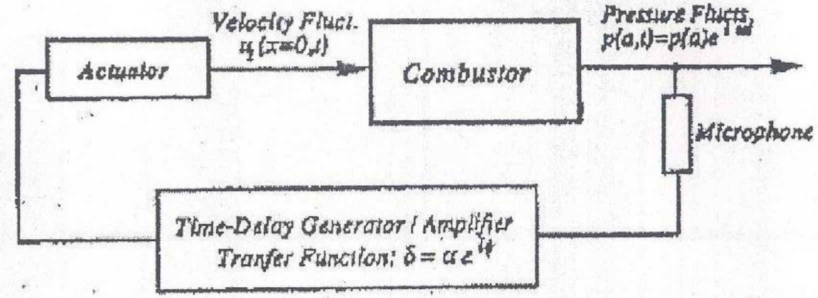


FIG. 3. Block-diagram describing model controller architecture.

The control system consists of four sensors including a microphone (sensor), an acoustic driver (actuator), a time delay generator (controller element) and an audio amplifier (controller element). The controller will be adapted to the model combustor by replacing the closed end of the duct with the acoustic driver to allow the imposition of a non-zero velocity boundary condition at x = 0. The acoustic pressure near the flame location (x = a) will be monitored by the microphone and will be used as the feedback signal. Please see the figure below.

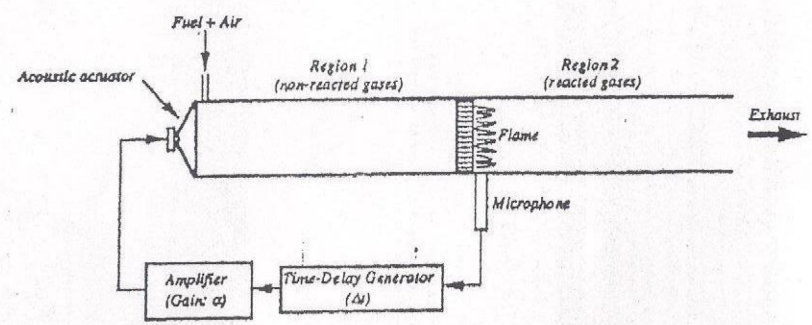


FIG. 4. Adaptation of the active control system to the model geometry.

Now we proceed to make a mathematical model for the controller. For this purpose, we define a control transfer function δ, which is simply the ratio of the time-varying velocity created by the acoustic driver to the time varying pressure at the microphone location (in dimnesionless form)

$$\delta = \frac{u_1(x=0,t)\rho_1 c_1}{p_1(x=a,t)}$$

This complex transfer function $\delta = \delta_r + \delta_{im}$(as velocity and pressure are complex, their ratio will naturally be complex) can be expressed in terms of a time-delay ϕ and gain α as

$$\delta = \alpha e^{i\phi} \text{ with } \delta_r = \alpha\cos(\phi) \text{ and } \delta_{im} = \alpha\sin(\phi)$$

<u>Questions</u>

9) Note that the boundary condition at $x = 0$ is now different. Relate $u(x = 0, t)$ to $p(x = a, t)$ and derive the following equation:

$$Ae^{-ika} - Be^{ika} = -\delta(A+B)$$

10) Show that the equation giving the natural frequency of the system now becomes

$$\zeta\cos(k_1 a)\cos(k_2 b) - \sin(k_1 a)\sin(k_2 b)\left(1 + ne^{-i\omega\tau}\right) - i\sin(k_2 b)\delta\left(1 + ne^{i\omega\tau}\right) = 0$$

11) As before we make the simplification that the flame is located at one half of the duct length and produces a negligible temperature rise. With these assumptions, show that

$$\cos(2ka) - \sin^2(ka)ne^{-i\omega\tau} - i\sin(ka)\delta\left(1 + ne^{i\omega\tau}\right) = 0$$

12) As in the earlier secion, without combustion (n=0), the first resonant frequency of the duct is $\omega_0 = k_0 c$. With combustion and active control, there is a slight frequency shift of this eigen mode and this shift can be exspressed as a shift in the wave number k_0. Thus, we will define $k = k_0 + k'$. Assuming that $k' \ll k_0$ and using Taylor series expansion of trignometric functions, and taking n and δ as small parameters, show that:

$$Real[k'] = -\frac{n}{4a}\cos(\omega_0\tau) + \frac{\delta_{im}}{2a\sqrt{2}} \text{ and } Imaginary[k'] = \frac{n}{4a}\sin(\omega_0\tau) - \frac{\delta_r}{2a\sqrt{2}}$$

As you can see, by controlling δ (by using the time delay generator and amplifier to affect the phase and magnitude of δ), we can affect the stability of the combustor.

13) Redo this analysis for the three quarter mode

I hope by now you have succeeded in learning a little bit about active control of combustion instabilities.

<u>If you are more interested in this topic, here are some references:</u>

1) K. R. McManus, T. Poinsot and S. M. Candel, (1993) A Review of Active Control of Combustion Instabilities, Progress in Energy and Combustion Science, Vol. 19, pp. 1-29.
2) W. Lang, T. Poinsot and S. Candel, Combustion and Flame (1987) Vol. 70, pp. 281-289.

Derive an expression for the normalized acoustic pressure $\frac{\hat{p}|_{x=0}}{\rho c U_0}$. **Do not** neglect the effect of mean Mach number (M). [i.e., use the wave equation and its solution in the presence of mean flow.]

(5 Marks)

4. Explain briefly how musical notes are produced in a flute? Explain the mechanism of sound production. Explain how the musician makes different notes. Also, explain, how the musician can, using the same finger position get higher octave notes. (4 Marks)

5. Briefly state what is the mechanism of combustion instability in a low NO_x gas turbine (LNGT) combustor. (1 Mark)

6. Many propulsion systems such as gas turbine engines, ramjets and liquid rocket engines use liquid fuels. These fuels are injected into the combustor as sprays which consists of small droplets. Let us now model how these liquid droplets interact with an oscillatory flow field which is established because of the onset of combustion instability.

   Let us write the momentum equation for a droplet. The rate of change of momentum of the droplet is equal to the force acting on the droplet.

$$m_p \frac{dUp}{dt} = F_D \qquad (1)$$

where, m_p is the mass of the droplet, U_p, the droplet velocity and F_D the droplet drag, and t is time.

Now, let us write a simple equation for the forces on the droplet. In this process, let us neglect gravity. (it turns out that gravity will influence only the mean droplet velocity and not the oscillatory response)

$$-F_D = \frac{\pi d^2 \rho}{8} C_D U_R |U_R| + \frac{\rho V}{2}\frac{dU_R}{dt} - \rho V \frac{dU_f}{dt} \qquad (2)$$

where V is the droplet volume, U_f is the gas velocity, U_R ($=U_p$-U_f) is the relative velocity between the droplet and the gas, d is the droplet diameter, and ρ the droplet density.

As you can easily see, the first term on the right hand side of the above equation is the drag on the droplet. The second term is the so called "added mass". This is the force required to accelerate the fluid around the droplet, as accelerating the droplet leads to accelerating the fluid around the droplet. The last term is due to the "forcing" due to the acoustic field.

(a) Assuming creeping flow ($C_D = 24/Re$), show that the equation of motion can now be reduced to

$$(\gamma+1/2)\frac{dU_p}{dt}=-\frac{18\nu U_R}{d^2}+\frac{3}{2}\frac{dU_f}{dt} \quad (3)$$

where γ is the ratio of the droplet density to the gas density and ν is the kinematic viscosity. As this equation is linear, the velocities may then be written as sums of mean values and variations from the mean; i.e.,

$$U_p=\overline{U}_p+u_p;U_f=\overline{U}_f+u_f;U_R=\overline{U}_R+u_R \quad (4)$$

(b) Show that the equation for the oscillatory particle velocity can then be written as

$$(\gamma+1/2)\frac{du_p}{dt}=-\frac{18\nu U_R}{d^2}+\frac{3}{2}\frac{du_f}{dt} \quad (5)$$

If the amplitude of the fluid oscillations is given by $u_f=\hat{u}_f\cos\omega t$, then the droplet velocity can be written as $u_p=\hat{u}_p\cos(\omega t+\phi)=\hat{\eta}\hat{u}_f\cos(\omega t+\phi)$, where $\hat{\eta}=\hat{u}_p/\hat{u}_f$ and ϕ are the entrainment factor and the phase between the particle and flow oscillations.

(c) Derive expressions for the entrainment factor $(\hat{\eta})$ and phase (ϕ) in terms of γ and Stokes number ($\sqrt{\omega/\nu d^2}$).
[Hint: Substitute the above expressions for u_f and u_p into Eq. (5) and use trignometric formulae.]

(d) Draw a rough plot of entrainment factor $(\hat{\eta})$ as a function of Stokes number($\sqrt{\omega/\nu d^2}$) for water droplets in air. (use your calculator and plot 4 or 5 points)

(e) What happens to the entrainment factor when the Stokes number is (a) 0 (b) infinite.

(f) Explain how such scenarios can occur. (12 Marks)

7. Shown in the figures are certain standing waves corresponding to the following non-dimensional admittance values. Write the appropriate values against the corresponding figure and return the next page.

a) (0.0, 0.0)

b) (-0.1, 0.0)

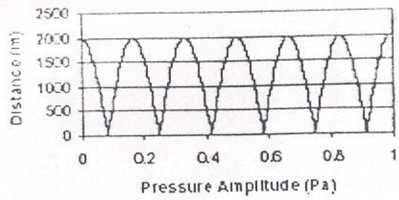
c) (-0.6, 0.0)

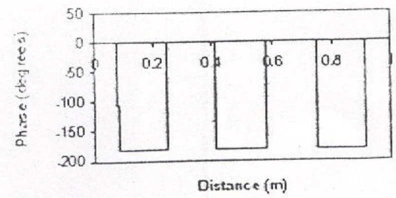
d) (0.6, 0.0)

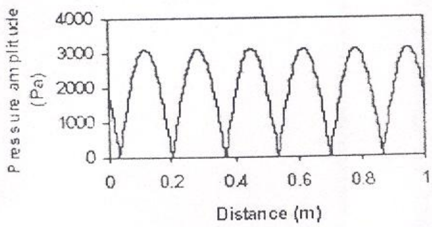
e) (0.0, -1.2)

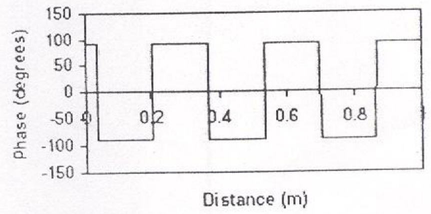
f) (0.0, 1.2)

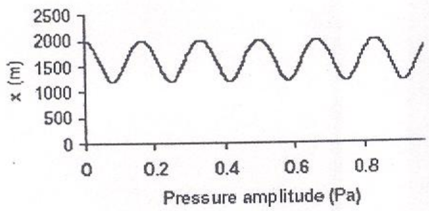
(5 Marks)

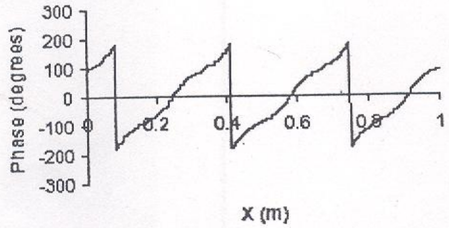


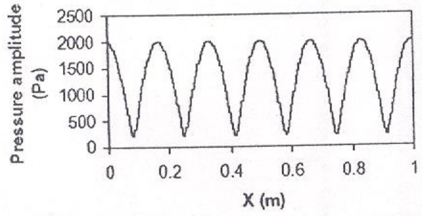




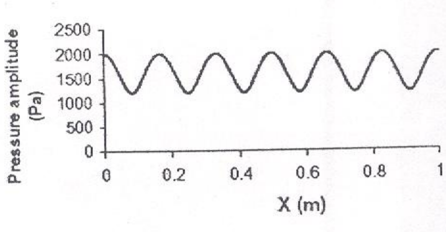


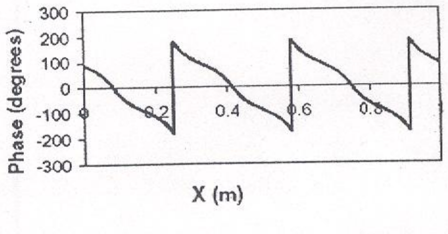






No phase plot is provided.





DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

END SEMESTER EXAMINATION (APRIL 27, 2005)

Acoustic Instabilities in Aerospace Propulsion (AS632)

Time: 3 hours
Marks: 40

Instructions:

1) This is an open notes examination. Put your notes to full use!

2) The examination is really easy. The objective of these questions is to demonstrate how we can apply the things that we learned in class to a "real" system. The steps are clearly explained. All you have to do is to follow them calmly.

3) You should be able to interpret my symbols. If there is a problem, feel free to ask.

4) If you feel that there are any mistakes in the results that I have asked you to prove, feel free to point it out in your answer book.

5) Make all the necessary assumptions and state them.

The flame front corresponds to a particular level $G(x, t) = 0$, of a scalar field G. The front consumes the fresh reactants located on $G<0$ side by moving in the normal direction at a rate given by the laminar burning velocity S_L. The front is also convected by the incident velocity field $\vec{v}$. As worked out in the class, a transport equation for the iso-line $G = 0$ yields:

$$\frac{\partial G}{\partial t} + \vec{v} \bullet \nabla G = S_L \left| \vec{\nabla} G \right|$$

We apply this equation to a conical and a V flame (see Fig. 1). For convinience, we use cylindrical co-ordinate system, and assume angular symmetry. This in this two-dimensional velocity field $\vec{v} = u\hat{i} + v\hat{j} = (u, v)$, Eq. (1) can be rewritten as

$$\frac{\partial G}{\partial t} + u\frac{\partial G}{\partial x} + v\frac{\partial G}{\partial y} = S_L\left[\left(\frac{\partial G}{\partial x}\right)^2 + \left(\frac{\partial G}{\partial y}\right)^2\right]^{1/2}$$

Question 1 (4 Marks)

Consider a uniform mean flow field $\vec{v} = (0, \bar{v})$ and let $G_0 = y - \bar{\eta}(x) = 0$ designate the steady flow position in the laboratory reference frame (x, y), as shown in the figure.

Show that the mean position $\bar{\eta}(x) = x\left[(\bar{v}/S_L)^2 - 1\right]^{1/2} = x/\tan\alpha$

Question 2 (2 Marks)

Consider now a small perturbation of the incident velocity field $\vec{v} = (u', \bar{v} + v')$ where u' and v' are quite smaller than $\bar{v}$. It is convinient to analyze the response of an inclined flame front to incident perturbations in a frame attached to the steady flame [see Figure 1A]. In this new frame, the perturbed flame position is given by $G = Y - \xi(X,t) = 0$., the velocity $\vec{v}$ has (U, V) components. Show that a first order analysis yields:

$$\frac{\partial \xi}{\partial t} + (\overline{U} + U')\frac{\partial \xi}{\partial X} = \overline{V} - S_L + V'$$

where $U = \overline{U} + U'$ and $V = \overline{V} + V'$

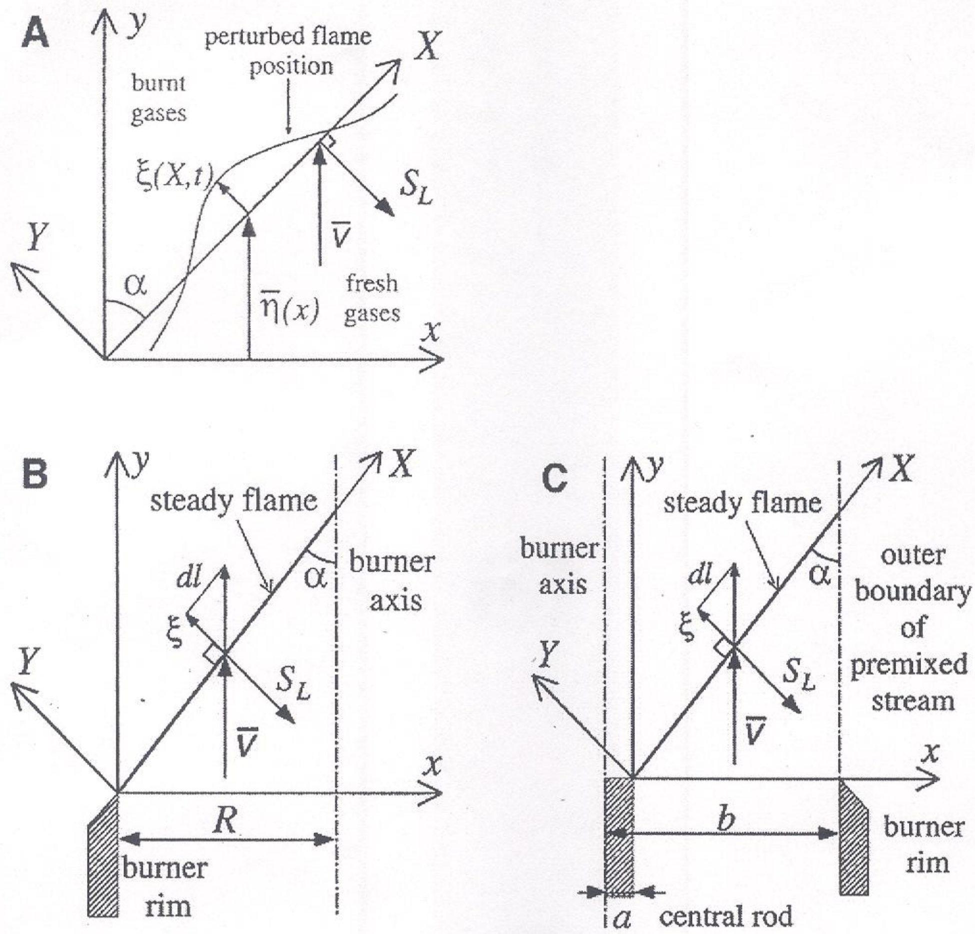


Fig. 1. (A) Laboratory (x, y) and flame (X, Y) reference frames. $\eta(x)$ steady flame position in the laboratory frame (x, y). (B) Conical flame geometry used in the determination of the transfer function. R: burner radius; α: flame angle with respect to the mean flow direction. (C) V-flame geometry used for the determination of the transfer function. a: rod radius; b: burner radius.

Question 3 (2 Marks)

Show that if U' can be neglected in comparison with $\overline{U}$,

$$\frac{\partial \xi}{\partial t} + \overline{U}\frac{\partial \xi}{\partial X} = V'$$

Question 4 (3 Marks)

What is the solution for this differential equation? [I.e., write an expression for $\xi(x,t)$]

Question 5 (3 Marks)

Now, consider a time harmonic perturbation of the form $V'(x,t) = \tilde{V}(X)e^{-i\omega t}$. The normal displacement of ξ is also harmonic $\xi(x,t) = \tilde{\xi}(X)e^{-i\omega t}$. Show that

$$\tilde{\xi}(x) = \exp\left(\frac{i\omega X}{\overline{U}}\right)\int_0^X \tilde{V}(X')\exp\left(-i\omega\frac{X'}{\overline{U}}\right)dX'$$

Question 6 (3 Marks)

The simplest perturbation model is that of a harmonic and uniform velocity modulation in the axial direction: $u' = 0$, $v' = v_1 e^{-i\omega t}$. For this case, show that

$$\frac{\tilde{\xi}(x)\cos\alpha}{R} = \frac{v_1}{\overline{v}}\frac{1}{i\omega_*}\left[\exp\left(i\omega_*\frac{x}{R}\right) - 1\right]$$

where $\omega_* = \dfrac{\omega R}{S_L\cos(\alpha)}$, where R is a characteristic scale of the flame.

Question 6 (3 Marks)

Show that the perturbation features a convective wavelength $\lambda = \dfrac{\overline{U}}{f}$ and a wave envelope, given by

$$\xi_e = \frac{2v_1\sin\alpha}{\omega}\sin\left(\frac{\omega X}{2\overline{U}}\right)$$

Question 7 (3 Marks)

Consider now the generic case of a convective velocity perturbation propagating in the vertical y direction in the laboratory frame of reference: $u' = 0$, $v' = v_1 e^{(iky - i\omega t)}$ The convective wave number is given by $k = \omega/\overline{v}$. In this flame frame, this corresponds to a normal velocity perturbation $\tilde{V}(x) = v_1\sin\alpha e^{ik\cos\alpha}$. For this case, show that:

$$\frac{\tilde{\xi}(x)\cos\alpha}{R} = \frac{v_1}{\overline{v}}\frac{1}{i\omega_*}\frac{1}{1-\cos^2\alpha}\left[\exp\left(i\omega_*\frac{x}{R}\right) - \exp\left(i\omega_*\frac{x}{R}\cos^2\alpha\right)\right]$$

Consider a conical flame anchored on the rim of a burner as in the figure. In this case, the characteristic flame scale R is the burner radius. The flame surface can be calculated by integration of the instantaneous front position over the burner radius. The instantaneous differential element of flame surface area fluctuation can be written as $dA' = 2\pi(R - x)dl$. Thus a first order estimate of the flame surface fluctuation A'(t) can be written as

$$A'(t) = \frac{2\pi}{\tan\alpha}\int_0^R (R - x)\frac{\partial\xi}{\partial x}dx$$

the above equation

As $\xi(x,t) = \tilde{\xi}(X)e^{-i\omega t}$, then $A'(t) = A'e^{-i\omega t}$, and since $\xi(0) = 0$, ~~Eq. (20)~~) reduces to:

$$A' = \frac{2\pi}{\tan\alpha}\int_0^R \tilde{\xi}(x)dx$$

Question 8

Now, show that, for this conical flame, the transfer function F can be written as

a) $F = \frac{A'/A}{v_1/\bar{v}} = \frac{2}{\omega_*^2}\left[1-\exp(i\omega_*)+i\omega_*\right]$ for the uniformly perturbed flame (3 Marks)

b) $F = \frac{A'/A}{v_1/\bar{v}} = \frac{2}{\omega_*^2}\frac{1}{1-\cos^2\alpha}\left[1-\exp(i\omega_*)+\frac{\exp\left(i\omega_*\cos^2\alpha\right)-1}{\cos^2\alpha}\right]$ for the case of axially convected velocity perturbation. (3 Marks)

Question 9 (2 Marks)

For the case of axially convected perturbations, calculate $|F|$ for the values of ω_* being 0.1, 1, 10, 100, and plot [x axis of plot has ω_* in the log scale].

Another case of practical and fundamental interest is that of a V-flame anchored on a rod of radius a placed in a burner of radius b (Fig. 1. C). The characteristic scale R is now equal to the flow channel width R = b-a. As no radial component is envisaged in the velocity models tested, the flame is assumed to extinguish at the boundary corresponding to the burner rim $x = b$. With this new flame geometry, the instantaneous differential element of flame surface area fluctuation is now given by $dA' = 2\pi x\, dl$. An integration over the burner surface leads to:

$$A'(t) = \frac{2\pi}{\tan\alpha}\int_a^b x\frac{\partial\xi}{\partial x}dx$$

Since $\tilde{\xi}(a) = 0$, Eq. (28) can be rewritten as

$$A' = \frac{2\pi}{\tan\alpha}\left(b\tilde{\xi}(R) - \int_0^R \tilde{\xi}(x')dx'\right)$$

Question 10

a) Show that for a V flame, disturbed by a uniform velocity perturbation the transfer function

$$F = \frac{A'/A}{v_1/\bar{v}} = \frac{2}{\omega_*^2}\left[\frac{b-a}{b+a}\left(\exp(i\omega_*)-1\right)+i\omega_*\left(\frac{a}{b+a}-\frac{b}{b+a}\exp(i\omega_*)\right)\right]$$ (3 Marks)

b) Show that for a V flame, disturbed by an axially convected perturbation:

$$F = \frac{A'/A}{v_1/\bar{v}} = \frac{2}{\omega_*}\frac{1}{1-\cos^2\alpha}\frac{b-a}{b+a}\left[\exp(i\omega_*)-1-\frac{\exp(i\omega_*\cos^2\alpha)-1}{\cos^2\alpha}\right]$$
$$+\frac{2i}{\omega_*}\frac{1}{1-\cos^2\alpha}\frac{b}{a+b}\left[\exp\left(i\omega_*\cos^2\alpha\right)-\exp\left(i\omega_*\right)\right]$$

(3 Marks)

Question 11 (2 Marks)
For the case of axially convected perturbations, calculate $|F|$ for the values of ω_* being 0.1, 1, 10, 100, and plot [x axis of plot has ω_* in the log scale]. Use $a = 0$, $b = R$, $\alpha = 15°$.

Question 12 (4 Marks)
Looking at the two graphs that you have plotted, what is the big difference that you see between the conical and V flame?

Instructions for Assignment Submission

Submit the assignment along with your end-semester answer-book and **NOT** at any other time.

Regarding showing the code, and compiling and running it :

Please come only during the times listed below, and **not** at any other time.

- April 29, Friday: 2-4 pm [Mr. Anil Raj will be available at DCF]
- May 2, Monday: 2-4 pm [Dr. Sujith will be available at the DCF]
- May 4, Wednesday: 2-4 pm [Mr. Anil Raj will be available at DCF]

Department of Aerospace Engineering, IIT Madras

Acoustic Instabilities in Aerospace Propulsion [Instructor: R. I. Sujith]

End Semester Examination 2006
3 hours, 40 Marks

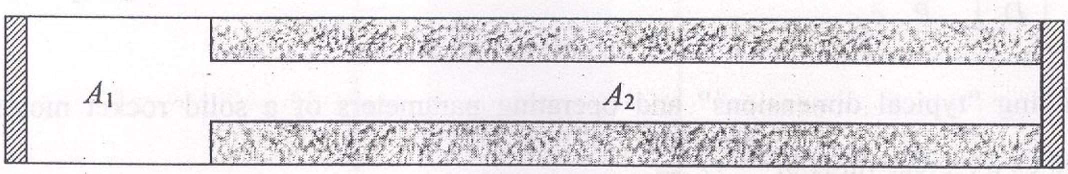
Instructions:

1) This is an open-book examination.
2) All the questions are simple. Some of them just look "longish". Let that not bother you.
3) Read the question and think for a minute before you start working out the problems.
4) If you disagree with any of the answers or with any statements in the questions, please state your disagreement, and continue to answer the questions.
5) Explain what you are doing. All the marks are for the steps and explanation.
6) Be brief. If you write "too much", I may not even see the "answer", and will be searching for it! Please do not make your answer book a mystery, or a jigsaw puzzle.
7) If you have any questions, feel free to ask.

1. Consider the duct shown below. The area of cross section jumps from A_1 to A_2 at the location $x = a$. We have rigid terminations at x = 0 and and $x = l$. The acoustic pressure amplitude at $x = 0$ is P_0.

(a) Give expressions for the acoustic pressure and velocity amplitude in the duct in terms of P_0.

(b) Derive an expression for the eigen values. How do they depend on A_1 and A_2 and the location of the area jump?



(6 Marks)

2. Consider an acoustic pressure field of the form $p' = A\cos\omega t \sin kx$. Express this field as the sum of a left running and right running wave. In other words, determine the functions $f(x-ct)$ and $g(x+ct)$. Write the acoustic velocities associated with these left and right running waves.

Hint: A useful trigonometric identity is $\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$

(3 Marks)

3. Sound from two sources of strength 100 dB that are perfectly in phase are adding up at a point. What is the net strength (in dB of course)of the acoustic field at that point. (2 Marks)

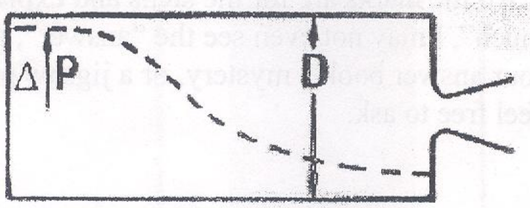
4. Show that for an acoustic horn, the equation for conservation of acoustic energy w is

$$A\frac{\partial w}{\partial t}+\frac{\partial}{\partial x}(AI)=0$$

Where I is the acoustic intensity and A is the area of cross-section. (3 Marks)

5. Consider acoustic waves in inhomogeneous quiescent media. Is it still true that $p' = c^2\rho'$? If so justify it? If not, what is the relation between p' and ρ'? (3 Marks)

6. Consider a solid propellant rocket motor as shown in the figure.



Pressure and thrust oscillations in a solid rocket motor

Show that in the presence of oscillations of amplitude ΔP, the amplitude of the thrust oscillation $\Delta\tau$ can be modelled approximately as

$$\frac{\Delta\tau}{\overline{\tau}}=\left(\frac{D}{D_t}\right)^2\frac{2\Delta P}{\overline{P}}\frac{1}{c_f}$$

Assuming "typical dimensions" and operating parameters of a solid rocket motor, what would be a typical value of $\frac{\Delta\tau}{\overline{\tau}}\Big/\frac{\Delta P}{\overline{P}}$

[Hint: With oscillations, pressures on the end wall are not equal.]

(5 Marks)

7. Consider an infinite medium (say water, density ρ_0, sound speed c_0 and specific heat at constant pressure c_p) in which a small volume V is suddenly being heated (say with a laser beam). The input energy E in the volume produces a rise in temperature ΔT according to $E = \rho_0 c_p V\Delta T$. Consequently, the volume V expands from V to $V+\Delta V$, where ΔV is related to the coefficient of thermal expansion β of the medium. [β is defined by $\beta=\frac{1}{V}\frac{\Delta V}{\Delta T}$]

(a) Show that the local change in density due to the energy input of the heat source is

$$-\delta\rho=\rho_0\left(\frac{\Delta V}{V}\right)=\left(\frac{\beta}{c_p}\right)\left(\frac{E}{V}\right) \qquad (1)$$

(b) Show that the appropriate equation of continuity is then:

$$\frac{\partial \rho}{\partial t} + \rho_0 \nabla \bullet \bar{u} = \left(\frac{\beta}{c_p}\right) q \qquad (2)$$

Where $q = E/V$ per unit time. (It represents the power density input in the medium)

(c) Combine the Eq. (2) with the usual linearized momentum and state equations to obtain the following thermoacoustic wave equation:

$$\nabla^2 p' - \frac{1}{C_0^2} \frac{\partial^2 p'}{\partial t^2} = -\frac{\beta}{c_p} \frac{\partial q}{\partial t} \qquad (3)$$

(5 Marks)

8. Let us consider low frequency combustion instability in liquid fuel gas turbine combustors, known as rumble. The acoustic field results in droplet size modulation. Therefore, for these spray flames, the oscillatory heat release should be expressed as a function of the droplet size modulations $d'/\bar{d}$ which enter the reaction zone.

I.e., $\dfrac{Q'}{\bar{Q}} = f\left(\dfrac{d'}{\bar{d}}\right)$

Let us make the following assumptions to model the transfer function of the spray flame:

- The impact of pre-vaporisation is negligible; i.e., the droplet diameters remain unaffected during the convection to the flame. This assumption is good if the carrier has low temperatures, and the convection timescales are small, leading to low pre-vaporisation.

- Combustion is controlled by droplet evaporation. Due to staged air injection with high turbulence levels, turbulent mixing and reaction are infinitely fast. Negligible interaction between the burning droplets is assumed. Each droplet burns close to its spherical surface at stoichiometric conditions. Under these conditions, the heat released by a single droplet is proportional to the instantaneous fuel evaporation rate. The evaporation process in the flame is assumed to follow the D^2 law.

- The time-dependent combustion rate of a burning droplet can be approximately represented by the average heat release over the burning time τ_b.

$$\dot{Q}_{eff} = \frac{1}{\tau_b} \int_0^{\tau_b} \dot{Q}_d(t)\, dt$$

- The heat release fluctuations can be attributed exclusively to the droplet clusted arriving at the flame. This is a simplification since the integrator effect of finite droplet burning times τ_b is not explicitly accounted for. However, for low frequencies, this assumption is justified as long as $\tau_b << 1/f$

- The impedance of the fuel supply is infinite; i.e., the total fuel flow rate $\dot{m}_f$ is constant.

- The length of the flame is small compared to the wavelengths in the combustor; i.e., the flame is compact.

The overall heat release $\dot{Q}$ is then the product of the average heat released by a single droplet $\dot{Q}_{eff}$ and the droplet number density N. Both N and $\dot{Q}_{eff}$ are functions of the droplet diameter.

(a) Show that linearization yields

$$\frac{\dot{Q}'}{\dot{Q}} = \frac{N'}{N} + \frac{\dot{Q}'_{eff}}{\dot{Q}_{eff}}$$

(b) Under conditions of constant fuel flow and monodisperse spray, show that

$$\frac{N'}{N} = -3\frac{d'}{d}$$

Let us now try to calculate the 2nd term. Following the D^2 law, the square of the droplet diameter decreases linearly with time.

$$d(t)^2 = d_0^2 - Kt$$

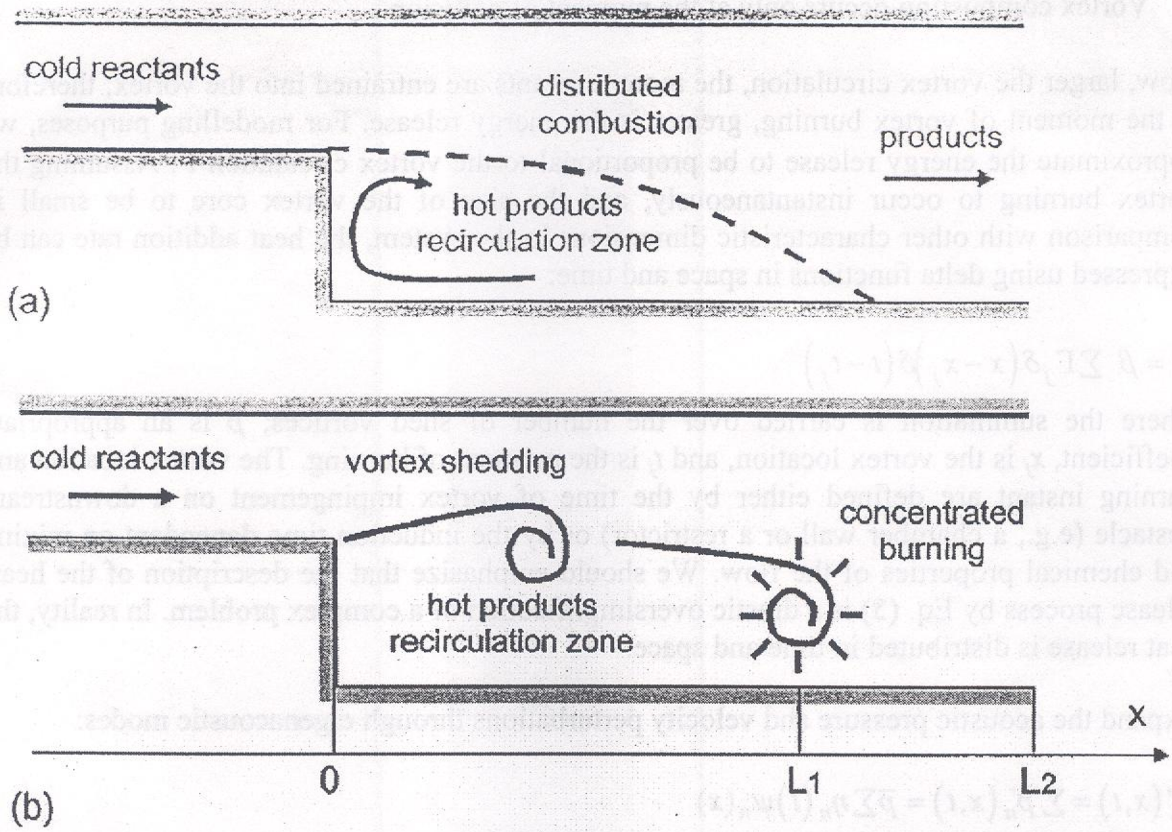
$d_0 = d(t = 0)$ is the initial droplet diameter, K is the burning rate constant. As a consequence of the above assumption about the governing effect of evaporation, the instantaneous heat release of the fuel droplet is the product of the fuel evaporation rate of the spherical droplet $\dot{m}_d$ and the specific enthalpy of reaction of the fuel h_b; i.e., $\dot{Q}_d(t) = h_b \dot{m}_d$

(c) Show that $\dot{Q}_{eff} = \frac{\pi}{6} h_b \rho_f K d_0$. [$\rho_f$ is the fuel density]. I.e., the average heat release rate of droplet combustion thus depends linearly on the initial droplet diameters.

(d) Also show that $\frac{\dot{Q}'_{eff}}{\dot{Q}_{eff}} = \frac{d'}{d}$ [now, d is the initial diameter of the droplets].

(e) Now show that $\frac{\dot{Q}'}{\dot{Q}} = -2\left(\frac{d'}{d}\right)$. (8 Marks)

9. A dump combustor is the configuration considered for studying combustion instabilities involving vortex shedding (see Figure 2).



(a) Stable and (b) unstable flow in a dump combustor

In the case of stable flow, a recirculation zone filled by hot products is formed behind the rearward facing step. Products mix with cold reactants, heating them and supporting stable distributed combustion. At certain flow velocities the shear layer at the lip becomes unstable, and vortex formation occurs. Shed vortices can impinge on structural components of a combustor downstream of the expansion; for the case shown in the figure, it is a lower wall in the chamber. That greatly enhances mixing between the cold and hot components, resulting in vigorous burning of the vortex. Alternatively, a vortex can burn intensively after a certain induction time determined by the characteristic hydrodynamic and chemical times.

Let us make the following major assumptions and formulate a mathematical model.

- Only longitudinal acoustic modes are important; that is, one-dimensional acoustic theory can be applied.
- The effects of the finite Mach numbers of the mean and oscillating flows on the acoustics are negligibly small.
- Non-uniformity of the temperature field in the combustion chamber is ignored.
- Vortex burning is the only source of unsteady heat addition, and its influence on acoustics greatly exceeds that due to collision of vorticity with a solid structure.

- During the vortex formation stage, a vortex does not move significantly in comparison with its displacement up to the impingement point after detachment.
- Vortex combustion occurs only at the moment of collision.

Now, larger the vortex circulation, the more reactants are entrained into the vortex; therefore at the moment of vortex burning, greater is the energy release. For modelling purposes, we approximate the energy release to be proportional to the vortex circulation Γ. Assuming the vortex burning to occur instantaneously, and the size of the vortex core to be small in comparison with other characteristic dimensions in the system, the heat addition rate can be expressed using delta functions in space and time:

$$\dot{Q} = \beta \sum \Gamma_j \delta\left(x - x_j\right)\delta\left(t - t_j\right)$$

where the summation is carried over the number of shed vortices; β is an appropriate coefficient, x_j is the vortex location, and t_j is the moment of burning. The vortex location and burning instant are defined either by the time of vortex impingement on a downstream obstacle (e.g., a chamber wall or a restrictor) or by the induction time dependent on mixing and chemical properties of the flow. We should emphasize that the description of the heat-release process by Eq. (5) is a drastic oversimplification of a complex problem. In reality, the heat release is distributed in time and space.

Expand the acoustic pressure and velocity perturbations through eigenacoustic modes:

$$p'(x,t) = \sum p'_n(x,t) = \bar{p}\sum \eta_n(t)\psi_n(x)$$

$$u'(x,t) = \sum u'_n(x,t) = \sum \frac{\eta_n(t)}{\gamma k_n^2}\frac{d\psi_n(x)}{dx}$$

Where $\bar{p}$ is the mean undisturbed pressure, $\eta_n(t)$ is the time-varying amplitude of the nth mode, $\psi_n(x)$ is the mode shape, γ is the gas constant and k_n is the wave number.

Show that the equation for the n$^{\text{th}}$ mode amplitude can be written as:

$$\ddot{\eta}_n + \omega_n^2 \eta_n = c\psi_n\left(x_j\right)\Gamma_j \delta\left(t - t_j\right)$$

Express c in terms of the other parameters.

(5 Marks)

References:

Question 8: J. Eckstein (2004) "On the Mechanisms of Combustion Driven Low-Frequency Oscillations in Aero-Engines", Ph.D. Thesis, TU München.

Question 9: K. I. Matveev and F. E. C. Culick (2003) "A Model for Combustion Instability Involving Vortex Shedding", Combustion Science and Technology, Vol. 175, pp. 1059-1083.

2002

DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion (AE 711)

Midterm Exam
20 Marks, 50 Minutes

Instructions:

1) This is an open notes examination.
2) The examination is really easy. We are going to work out a few simple problems. The first two are on topics that we did not cover in the class. However, you can do them using the general methodology which was introduced. The last question is to give you a feel for standing waves. Relax and enjoy.
3) You should be able to interpret my symbols. If there is a problem, feel free to ask me.

1) The spherically symmetrical wave equation (in spherical co-ordinates, naturally ☺) is given by

$$\frac{1}{r}\frac{\partial^2 rp'}{\partial r^2}-\frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2}=0$$

where, $p'(r,t)$ is the acoustic pressure, r is the radial co-ordinate, and c the speed of sound. Write down a general solution for the acoustic pressure.

The radial momentum equation in spherical co-ordinates is given by $\bar{\rho}\frac{\partial u_r'}{\partial t}=-\frac{\partial p'}{\partial r}$.
Using this, derive an expression for the acoustic velocity u_r' in the radial direction.

2) Consider the propagation of sound in a long, narrow tube as shown in the diagram. Include approximately the viscous frictional force between the tube wall and the fluid as follows. Assume plane wave propagation conditions and a net viscous force F_u per unit area of the tube wall, where F_u is proporational to the velocity of the fluid relative to the fluid wall. Assume the tube is rigid. Then $F_u = \eta u$ where η is a friction constant and u is the particle velocity.

a) Show that the momentum equation for a fluid element $A\Delta X$ is

$$\bar{\rho}\frac{\partial u_x'}{\partial t}=-\frac{\partial p'}{\partial x}-\frac{2}{a}\eta u_x'$$

b) Show that for a harmonic wave $p' = Ae^{i(k^*x+\omega t)}$, $u_x' = \hat{u}_x e^{i\omega t}$, the equations of continuity and momentum are satisfied, but the wave number k^* is complex. What are the physical consequences of the complex wave number.

$P = P_0 \,\&\, \cos kx \; e^{i\omega t}$

$u = -P_0 \sin kx \; e^{i\omega t}$

$\sin kx \quad \cos \omega t$

$\cos 90 = 0$

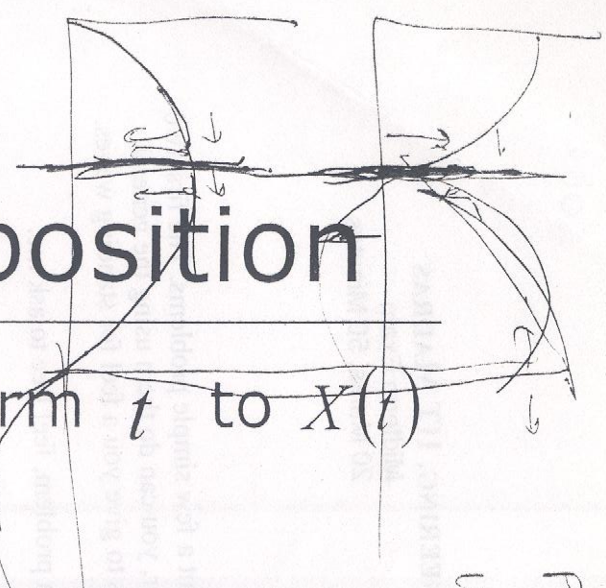
$\sin 90 = 1$

$\sin(90 - x) = \cos x$

$k = \omega / c$

$k * \lambda = 2\pi$

$(p'u')$



In terms of wave front position

Changing the independent variable form t to $X(t)$

$$\frac{du_1}{dx} + P + Qu_1 + Ru_1^2 = 0$$

☞ Riccati Equation

$\sin kx \cos \omega t$

$x = X(t)$ is the position of wave front.

$P = p/\dot{X}(t)$ $\quad Q = q/\dot{X}(t)$ and $R = r/\dot{X}(t)$

$u_1(x)$ is first derivative of gas velocity at location x.

$-\sin\theta = \sin(180 + \theta)$

$(\cos kx) \cos \omega t$

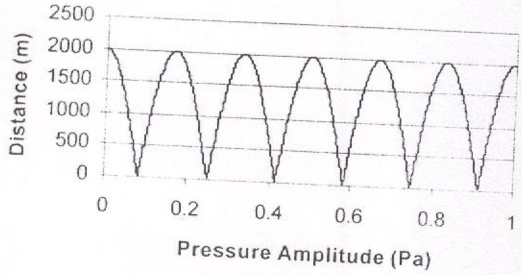
$\cos(90 - \theta) \cos \omega t = \sin\theta \cos \omega t$

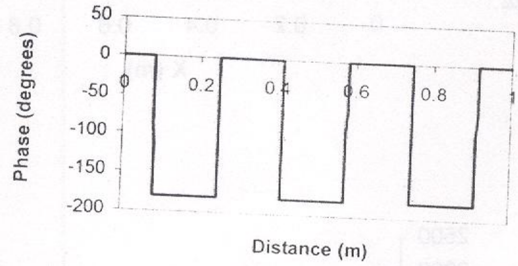
$\cos(90 + \theta) \cos \omega t = -\sin\theta \cos \omega t$

3) Shown in the figures are certain standing waves corresponding to the following non-dimensional admittance values. Write the appropriate value on the corresponding figure and return this sheet.

(0.0, 0.0): (-0.1, 0.0); (-0.6, 0.0), (0.6, 0.0);(0.0, -1.2) (0.0, 1.2)

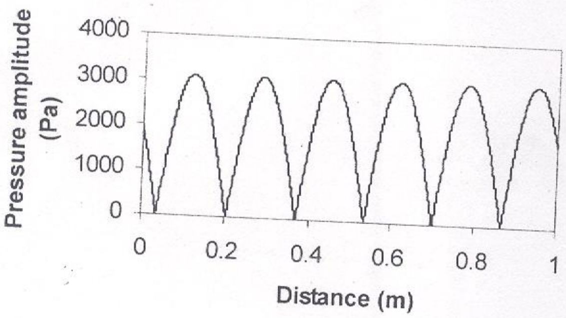
(0,0)

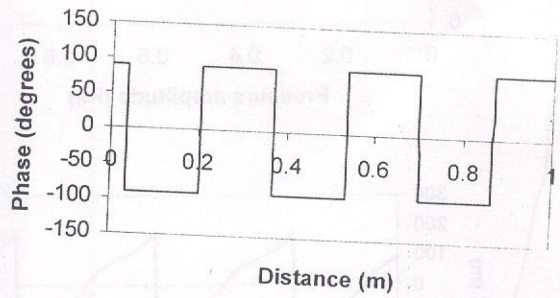




(0, -1.2)

~~(0.7,0)~~





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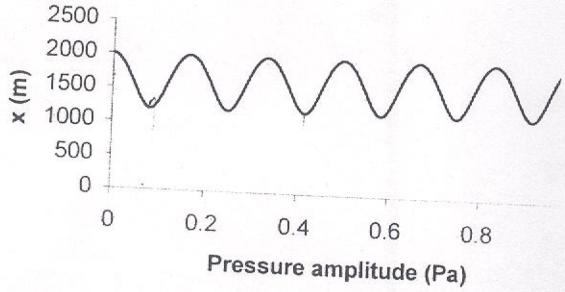

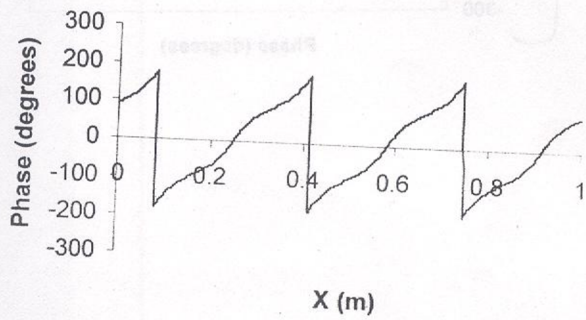

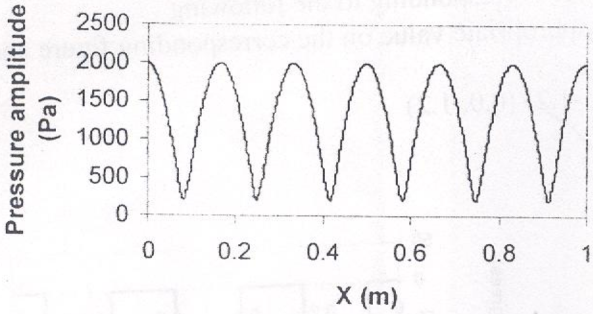

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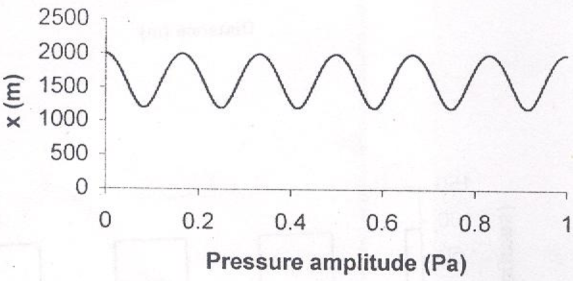

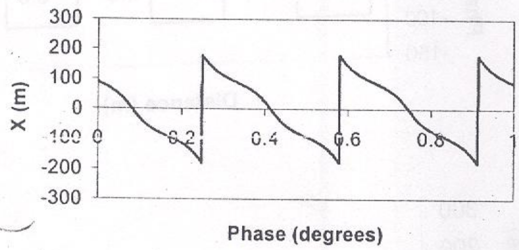

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Sidney 9-14 July 2000

DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion

Midterm Exam
20 Marks, 50 minutes

Instructions:

1) This is an open notes examination
2) The examination is really easy. We are going to work out a few simple problems. The first two problems are on topics that we did not cover in class, you can use our class room fundas and some commonsense to answer these questions. The last question is really simple.
3) You should be able to interpret my symbols. If there is a problem, feel free to ask.

1) The spherically symmetric wave equation (in spherical co-ordinates, naturally ☺) is given by

$$\frac{1}{r}\frac{\partial^2 rp'}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2}$$

where $p'(r,t)$ is the acoustic pressure, r is the radial co-ordinate, and c is the speed of sound. Write down a general solution for the acoustic pressure.

The radial momentum equation in spherical co-ordinates is given by $\bar{\rho}\frac{\partial u_r'}{\partial t} = -\frac{\partial p'}{\partial r}$. Using this equation, derive an expression for the acoustic velocity u_r' in the radial direction.

(8 marks)

2) Consider the propagation of sound in a duct having a dusty gas. Consider the dust to be a dilute suspension. The dust will introduce a drag force. Let us try to model this drag.

You must have studied in school that the drag (Stokes drag) on a small particle is $6\pi a\mu(u-u_p)$. Let us consider waves at high frequencies. At high frequencies, the dust particles will not follow the flow; i.e., $u_p = 0$. Then the drag becomes $6\pi a\mu u$. Let N be the number of dust particles per unit volume. The total drag exerted by particles in a unit volume will then be $N6\pi a\mu u$.

a) Derive the momentum equation.

b) Show that for a harmonic wave $p' = Ae^{i(k^*x+\omega t)}$, $u' = \hat{u}e^{i\omega t}$, the equations of continuity and momentum are satisfied, but the wave number k* is complex. What are the physical consequences of the complex wave number.

(8 marks)

3) We superimpose two sound sources, each of them having a sound pressure level of 0 dB. What will be the net sound pressure level? (In other words, what is 0 dB + 0dB)

(4 marks)

DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion

Midterm Exam, 2005
20 Marks, 50 minutes

Instructions:

1) This is an open notes examination
2) The examination is really easy.
3) Make any assumption necessary to solve the problems. That is what an engineer is supposed to do!
4) If the answers given in question (2) are wrong, give the correct answer.
5) The symbols used are the same as that used in the class. Use your notes or commonsense.

1. Consider one-dimensional, harmonic sound propagation in a lossless medium. Draw the variation of amplitude and phase of acoustic pressure with distance for: [6 Marks]

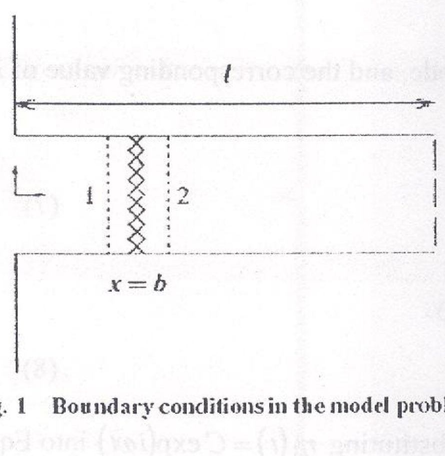
   a) when acoustic pressure is in phase with acoustic velocity.
   b) when acoustic pressure is 180° out of phase with acoustic velocity
   c) when acoustic pressure is 90° out of phase with acoustic velocity

2. Let us consider a combustor, which is a duct as shown in Fig. 1. Consider the unsteady heat input to be concentrated at a single axial plane $x = b$ [see figure] and be related to the air velocity there with a time delay τ. [14 Marks]

$$q'(x,t) = Q'(t)\delta(x-b) \qquad (1)$$

$$Q'(t) = -\left[\frac{\beta\bar{\rho}c^2}{\gamma-1}\right]u_1'(t-\tau) \qquad (2)$$

where $Q'(t)$ is the rate of heat input per unit area and subscript 1 denotes conditions just upstream of this region of heat input, i.e., $u_1'(t) = u'(b^-, t)$.
The non-dimensional number β can be expected to lie in the range from 0 to 10 and in a gas turbine system, τ is typically the convection time from fuel injection to its combustion.



A large plenum attached to the duct end at $x = 0$ and a restriction at $x = l$ as shown in the figure will give the boundary conditions that the acoustic pressure is zero at $x = 0$, and acoustic velocity is 0 at $x = l$.

Fig. 1 Boundary conditions in the model problem.

Using the jump conditions that $p'_{x=b^+} - p'_{x=b^-} = 0$ and

$$u'_{x=b^+} - u'_{x=b^-} = \left[\frac{\gamma - 1}{\bar{\rho} c^2}\right] Q'(t)$$ and the boundary conditions $p'(0) = 0$ and $u'(l) = 0$

[Assume the mean temperature & mean density in the duct to be constant.]

Show that:

a) the mode shapes are given by the following expressions:

For $x < b$, $\hat{p}(x) = A\sin(kx)$, $\hat{u}(x) = -\frac{i}{\bar{\rho} c} A\cos(kx)$ (3)

For $x > b$, $\hat{p}(x) = B\cos[k(l - x)]$, $\hat{u}(x) = \frac{i}{\bar{\rho} c} B\sin[k(l - x)]$ (4)

b) The natural frequencies are given by the implicit relation

$$\tan(kb)\tan[k(l-b)] = 1 - \beta\exp(-i\omega\tau) \tag{5}$$

We will now try to solve this problem by the Galerkin method, where we expand the pressure perturbation through a Galerkin expansion series:

$p'(x,t) = \sum_{m=1}^{\infty} \eta_m(t)\psi_m(x)$, where the functions $\psi_m(x)$ are the eigen modes of the homogeneous wave equation that satisfies the same boundary condition as p'. Instead of using a series, we will use an "<u>one-term Galerkin expansion</u>" [i.e., we use just $p'(x,t) = \eta_1(t)\psi_1(x)$].

c) Show that:

$$\frac{d^2\eta_1}{dt^2} + \omega_1^2\eta_1 = \frac{\beta c^2}{E_1}\eta_1(t - \tau)\frac{d\psi_1}{dx}(b)\psi_1(b) \tag{6}$$

d) Substitute the expression for the fundamental mode, and the corresponding value of E_1 in Eq. (6), show that:

$$\frac{d^2\eta_1}{dt^2} + \omega_1^2\eta_1 = \frac{\beta c^2\pi}{l^2}\eta_1(t - \tau)\cos\left(\frac{\pi b}{2l}\right)\sin\left(\frac{\pi b}{2l}\right) \tag{7}$$

e) Show that the frequency of oscillation is given by:

$$\omega^2 = \omega_1^2 - \left(\frac{\beta c^2\pi}{l^2}\right)\exp(-i\omega\tau)\cos\left(\frac{\pi b}{l}\right)\sin\left(\frac{\pi b}{l}\right) \tag{8}$$

(The frequency of oscillation ω can be found by substituting $\eta_1(t) = C\exp(i\omega t)$ into Eq.(7).)

DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion

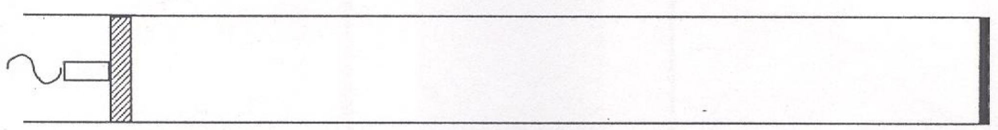
Midterm Exam, 2006
20 Marks, 50 minutes

Instructions:

1) This is an open notes examination.
2) The examination is really easy. Be calm and write the answers.
3) You should be able to interpret my symbols. If there is a problem, feel free to ask.
4) If you think there are mistakes in the question, then please correct it.

In the analysis of standing waves we performed in the class, we neglected acoustic absorption. Acoustic absorption can be considerable when the propagation is in a narrow duct, or when there are particulate matter in the media (e.g., rocket motors with Aluminized propellants). Let us see what is the effect of attenuation on the standing wave.

Consider the configuration given in the figure below. Let us assume that the termination at B is rigid. Then the amplitude of the reflected wave will be equal to that of the incident wave impinging on it from the sound source A. The sound source supplies the acoustic power which will be consumed in the acoustic absorption – therefore, there is no growth or decay of acoustic amplitudes. Hence ω is chosen to be real. The wave number will indeed be complex. However, I am expressing it as a real part k, and the complex part which represents attenuation is recast into the "attenuation coefficient" α.



A: Sound Source

B
Rigid Termination

The acoustic pressure field can then be written as

$$p' = P_0 e^{-\alpha x} e^{i(\omega t - kx)} + P_0 e^{\alpha x} e^{i(\omega t + kx)}$$

1) Derive an expression for the resulting pressure amplitude at any position along the duct. [i.e., derive an expression for $|\hat{p}| = \sqrt{\hat{p}\,\hat{p}^*}$]. (10)

2) Where are the pressure nodes located? (2)

3) Where are the pressure antinodes located? (2)

4) Make a plot of the pressure amplitude along the duct. Choose whatever values of the parameters you like. (2)

5) Derive an expression for the pressure amplitude at the nodes. (2)

6) Can you now think of a way to determine the attenuation constant α? (2)