DEPARTMENT OF AEROSPACE ENGINEERING, IIT MADRAS

Acoustic Instabilities in Aerospace Propulsion (AS 6320) | **Open Book** | Time: 50 Minutes
November 2, 2020 | | Marks: 20

- *The exam is open book. If any of the expressions given are wrong, feel free to use the correct expression. Make any assumptions you need to make; please write them down.*
- *At the end of the exam, please email me a SINGLE PDF FILE.*

1) Consider two acoustic sources that are in phase. Each of them individually produces an acoustic pressure amplitude (expressed in decibels) of 90 dB. What will be the acoustic pressure amplitude when both of them are on? Consider linear theory to be valid. (2)

2) Explain clearly what the real and imaginary parts of complex frequency indicate. (2)

3) What is the phase between acoustic pressure and acoustic velocity for a left running traveling wave? (2)

4) When a train goes through a tunnel, it generates acoustic waves. As a simple model for the pressure wave generated by a train traveling through a "tight" tunnel (the clearance between the train and the tunnel is minimal), we will assume a piston moving with a constant speed U ($<c$) along a duct. The piston will generate a plane wave with a wavefront propagating at the speed of sound.

The solution for the 1-D wave equation can be written as

$$p'(x,t) = f(t - x/c) + g(t + x/c)$$

Here, f and g are arbitrary functions representing waves propagating in the positive or negative x directions. These functions are determined by using the initial and boundary conditions for the problem at hand. For our train traveling through the tunnel, the acoustic wave must satisfy the condition of continuity of velocity at the piston. Assuming that the piston starts to move from x = 0 at t = 0, we have

$$u'_x(x,t) = U, t > 0$$
$$u'_x(x,t) = 0, t < 0$$

Note that $x = Ut$

Assume a reflection free (semi-infinite) duct where the medium is initially at rest, what is the value of the acoustic pressure in front of the piston for $t < 0$ and $t > 0$.

If the train is moving at 72 km/hour, calculate the resulting acoustic pressure in Pascals. Also, calculate the result in dB. (6)

5) The spherically symmetric wave equation in harmonic domain can be written as

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2\frac{dp'}{dr}\right]+k^2p'=0$$

Find the solution to acoustic pressure. [Hint: Rewrite the equation in terms of $p'r$.]

The acoustic velocity has only a radial component, which satisfies the following equation:

$$\bar{\rho}\frac{\partial u\prime}{\partial t}+\frac{\partial p\prime}{\partial r}=0$$

Derive an expression for the acoustic velocity of the outward propagating wave.

Simplify this expression in the far-field ($r \to \infty$). In the far-field, derive an expression for expressing acoustic pressure in terms of the acoustic velocity. (6)

6. The length of the S200 solid rocket motor is 25 meters, and the diameter is 3.2 meters. The temperature of the combustion products is 3400 K, and the ratio of specific heats is 1.2. Estimate the fundamental, first harmonic and second harmonic of this motor.

(2)