

Dynamical Systems

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- $x(t)$ gives the values of these states over time

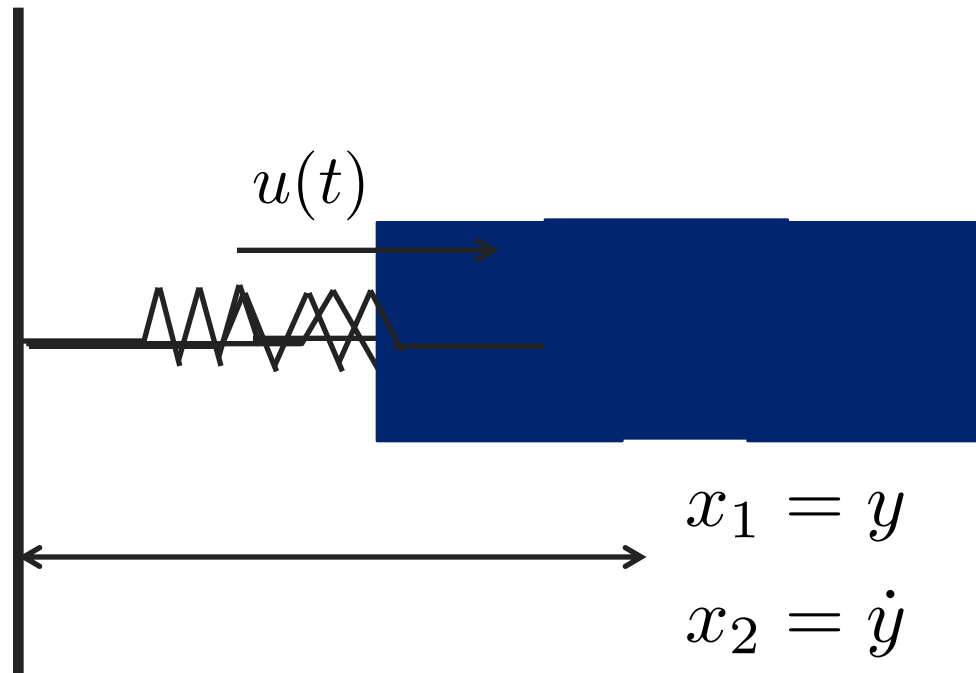
Dynamical Systems

Evolution of these states over time is often given by a set of governing ordinary differential equations

- Order: highest derivative that appears in the equations

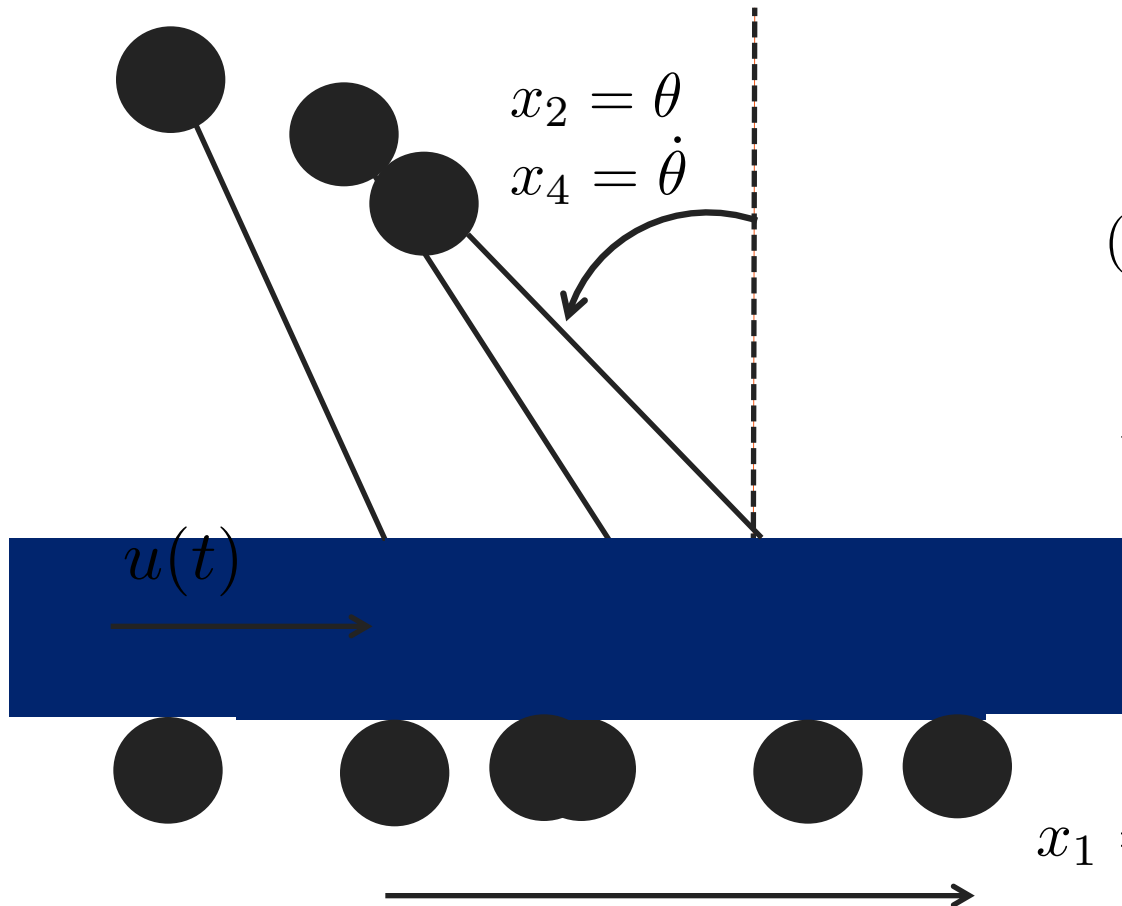
$$\ddot{x}(t) = u(t) \quad \text{Second-order system}$$

Example I: Mass-Spring System



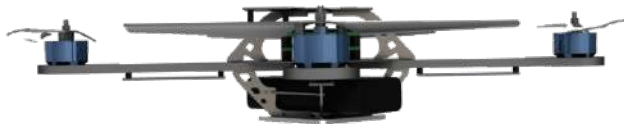
$$m\ddot{y}(t) + ky(t) = u(t)$$

Example 2: Pendulum on a Cart



$$\begin{aligned} (M + m)\ddot{y}(t) - ml \cos(\theta(t))\ddot{\theta}(t) \\ + ml \sin(\theta(t))\dot{\theta}(t)^2 = u(t) \\ - ml \cos(\theta(t))\ddot{y}(t) + (J + ml^2)\ddot{\theta}(t) \\ - mgl \sin(\theta(t)) = 0 \end{aligned}$$

Example 3: Quadrotor



$$x_1 = x \qquad x_7 = \dot{x}$$

$$x_2 = y \qquad x_8 = \dot{y}$$

$$x_3 = z \qquad x_9 = \dot{z}$$

$$x_4 = \phi \qquad x_{10} = p$$

$$x_5 = \theta \qquad x_{11} = q$$

$$x_6 = \psi \qquad x_{12} = r$$