Properties of Functions

Function

A *function* is a relation that assigns each element in a set of inputs X, called the *domain*, to exactly one element in a set of outputs Y, called the *codomain* (or *range*).

$$f : X \to Y$$

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Function

$$f : X \to Y$$

One-to-one (injective): for all a, b in X, if $f(a) = f(b)$, then $a = b$

No two inputs from the domain will map to the same output in the codomain.

Onto (surjective): for all y in Y, there is an x in X such that $f(x) = y$

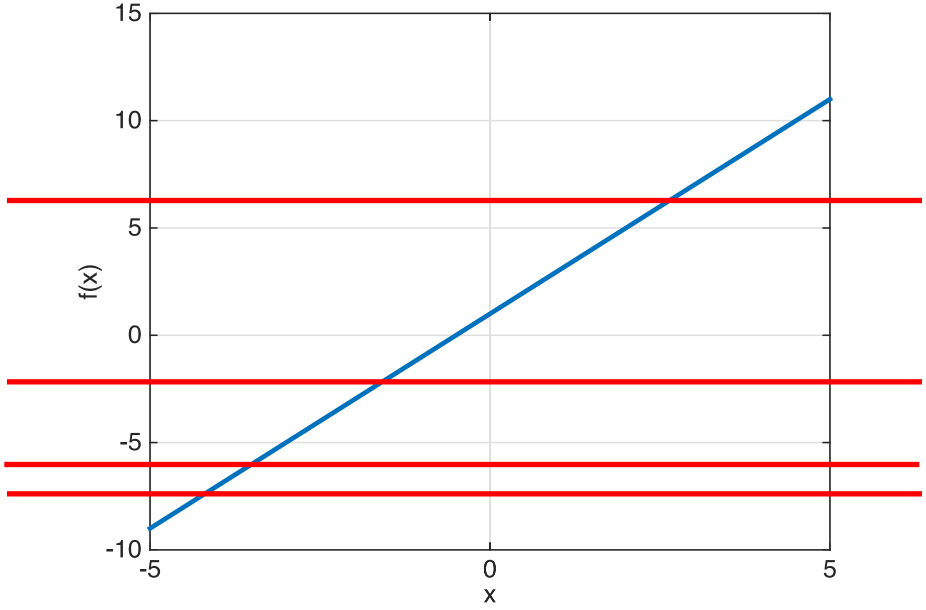
Every output in the codomain has an input in the domain that maps to it.

Example 1: One-to-one Functions

Consider:

$$f : R \to R \quad \text{such that} \quad f(x) = 2x + 1$$

This function **is** one-to-one.

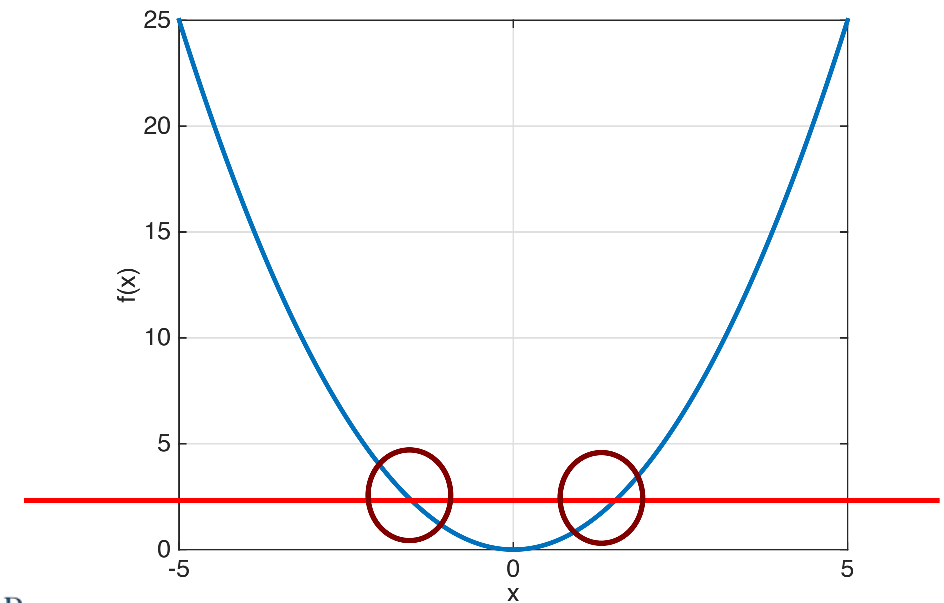


Example 2: One-to-one Functions

Consider:

$$f : R \rightarrow R \qquad \text{such that} \qquad f(x) = x^2$$

This function **is not** one-to-one.



$$f(1) = 1$$

$$f(-1) = 1$$

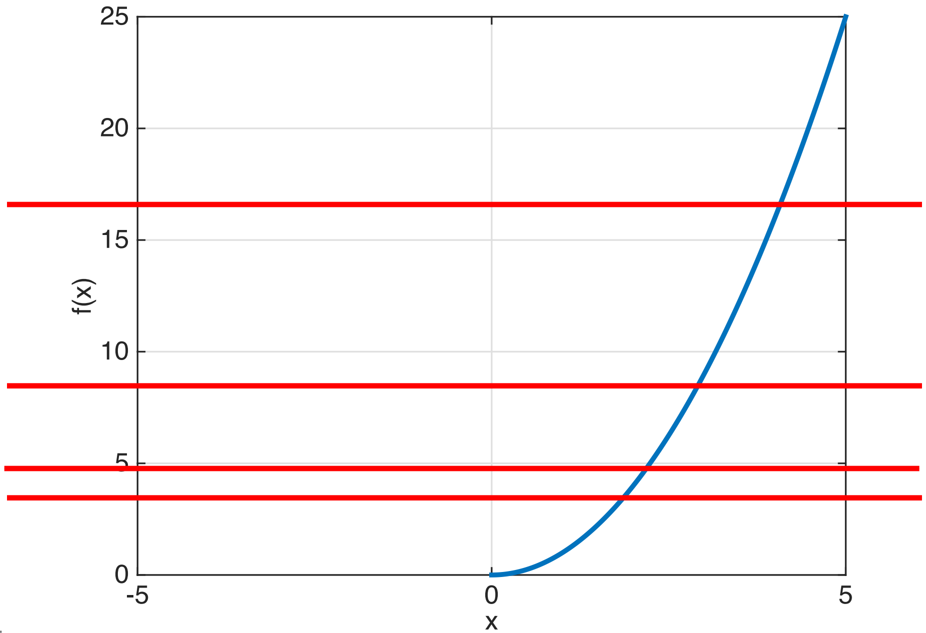
$$f(\overline{x}) = f(-x)$$

Example 2: One-to-one Functions

Consider:

$$f : \boxed{[0, \infty)} \rightarrow R \quad \textbf{such that} \quad f(x) = x^2$$

This function **is** one-to-one.



We have removed the "redundant" values of x from the domain.

Example 3: Onto Functions

Consider:

$f : R \to R$ such that $f(x) = e^x$

This function **is not** onto.

For any $y \leq 0$, there is no x such that $e^x = y$.

Example 3: Onto Functions

Consider:

$$f : R \rightarrow \boxed{(0, \infty)} \quad \text{such that} \quad f(x) = e^x$$

This function **is** onto.

The specified codomain no longer includes the values $y \leq 0$.