## Rates of Convergence



#### Feedback Control

#### Recall the control problem

Determine the appropriate input that will cause the error between the desired state and the actual state of a dynamical system to eventually reach 0.

$$e(t) = x^{des}(t) - x(t) \to 0 \text{ as } t \to \infty$$



### Rates of Convergence

How fast do we want this error to go to 0?

• The error exponentially converges to 0 if there exists constants  $\alpha$  and  $\beta$  and time  $t_0$ , such that for all  $t \geq t_0$ :

$$||e(t)|| \le \alpha e^{-\beta t}$$



#### Feedback Control

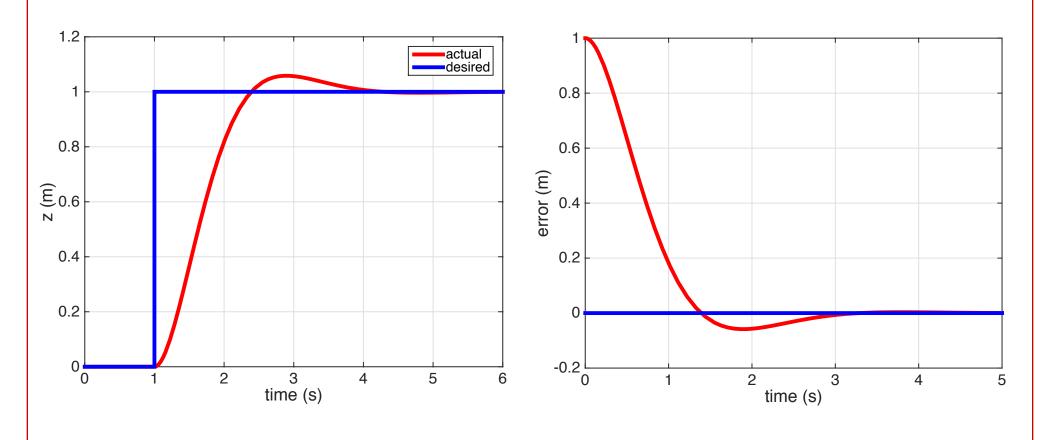
Here we will accomplish this using a PD (or PID) controller.

$$u(t) = \ddot{x}^{\text{des}}(t) + K_v \dot{e}(t) + K_p e(t)$$

Consider the controllers we used before to control the height of a quadrotor.

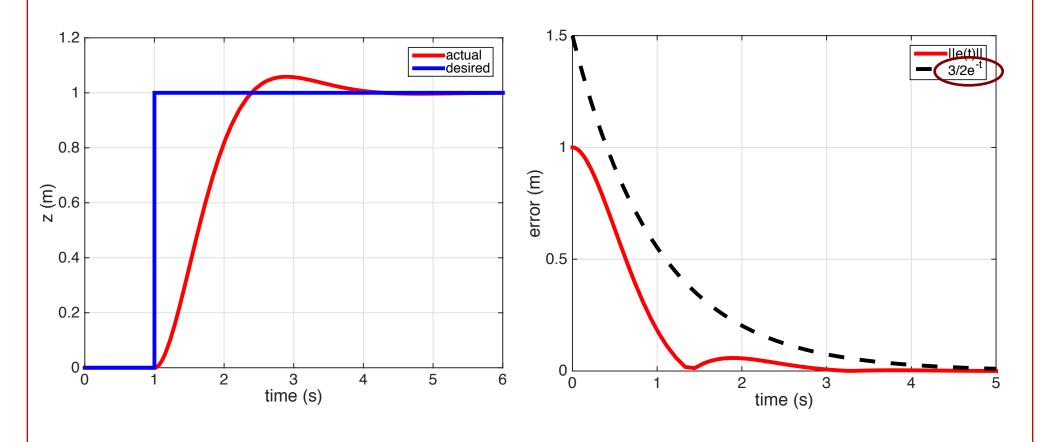






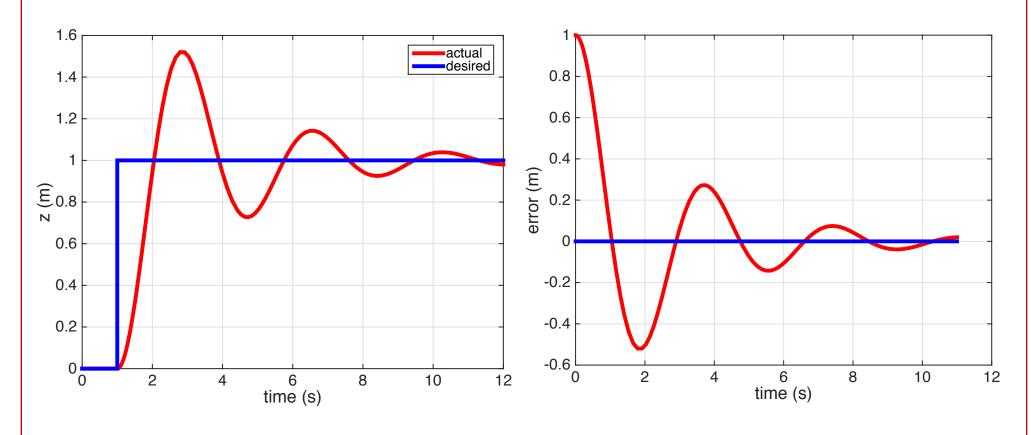


### Example I: PD Controller





# Example 2: High K<sub>p</sub>





## Example 2: High K<sub>p</sub>

