Matrix Derivative

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Recall the following expressions from lecture:

$$\dot{R}^T R + R^T \dot{R} = 0$$

$$R\dot{R}^T + \dot{R}R^T = 0$$

What is $\dot{R}$?

Matrix Derivative

R is a matrix where each component is a function of time.

$$R = \begin{bmatrix} R_{11}(t) & R_{12}(t) & R_{13}(t) \\ R_{21}(t) & R_{22}(t) & R_{32}(t) \\ R_{31}(t) & R_{32}(t) & R_{33}(t) \end{bmatrix}$$

$\dot{R}$ is a matrix whose components are the time derivatives of the components of R .

$$\dot{R} = \begin{bmatrix} \frac{dR_{11}(t)}{dt} & \frac{dR_{12}(t)}{dt} & \frac{dR_{13}(t)}{dt} \\ \frac{dR_{21}(t)}{dt} & \frac{dR_{22}(t)}{dt} & \frac{dR_{23}(t)}{dt} \\ \frac{dR_{31}(t)}{dt} & \frac{dR_{32}(t)}{dt} & \frac{dR_{33}(t)}{dt} \end{bmatrix}$$

Matrix Derivative Properties

Properties for scalar function derivatives apply to matrix derivatives as well:

$$\frac{d}{dt}(A \pm B) = \dot{A} \pm \dot{B}$$

$$\frac{d}{dt}(AB) = \dot{A}B + A\dot{B}$$

$$\frac{d}{dt}(A(\theta(t))) = \frac{dA}{d\theta}\dot{\theta}$$

Example 1: Matrix Derivative

Consider:

$$R = \begin{bmatrix} 2t & t^2 & e^t \\ \sin(t) & \cos(t) & \tan(t) \\ 5 & ln(t) & 0 \end{bmatrix}$$

The time derivative is:

$$\dot{R} = \begin{bmatrix} 2 & 2t & e^t \\ \cos(t) & -\sin(t) & \sec^2(t) \\ 0 & \frac{1}{t} & 0 \end{bmatrix}$$

Example 2: Matrix Derivative

Consider:

$\theta(t)$ Use chain rule!

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = Rot(z, \theta)$$

$$\dot{R} = \begin{bmatrix} -\dot{\theta}\sin(\theta) & -\dot{\theta}\cos(\theta) & 0 \\ \dot{\theta}\cos(\theta) & -\dot{\theta}\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\sin(\theta) & -\cos(\theta) & 0 \\ \cos(\theta) & -\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$