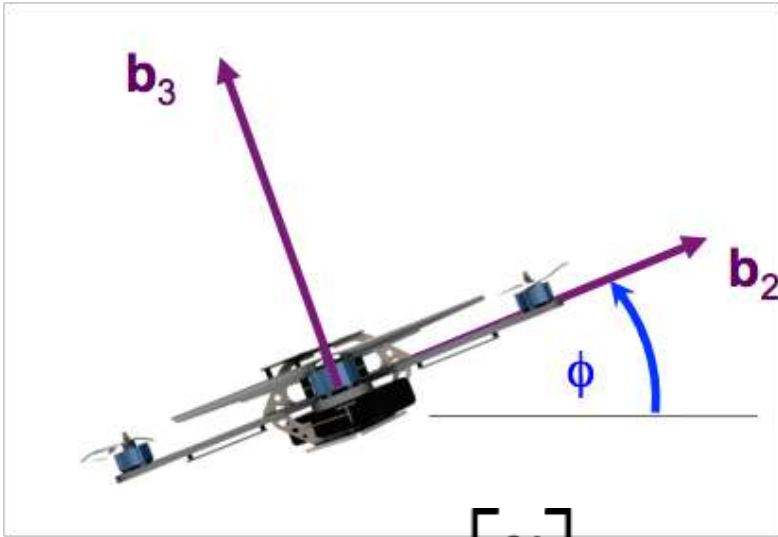


Planar Quadrotor

Planar Quadrotor Model



$$\begin{bmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} -\frac{1}{m} \sin \phi & 0 \\ \frac{1}{m} \cos \phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ z \\ \phi \\ \dot{y} \\ \dot{z} \\ \dot{\phi} \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{y} \\ \dot{z} \\ \dot{\phi} \\ 0 \\ -g \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{m} \sin \phi & 0 \\ \frac{1}{m} \cos \phi & 0 \\ 0 & \frac{1}{I_{xx}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Linearized Dynamic Model

Equations of motion

$$\begin{aligned}\ddot{y} &= -\frac{u_1}{m} \sin(\phi) \\ \ddot{z} &= -g + \frac{u_1}{m} \cos(\phi) \\ \ddot{\phi} &= \frac{u_2}{I_{xx}}\end{aligned}\quad \text{Dynamics are nonlinear}$$

Equilibrium hover configuration

$$y_0, z_0, \phi_0 = 0, u_{1,0} = mg, u_{2,0} = 0,$$

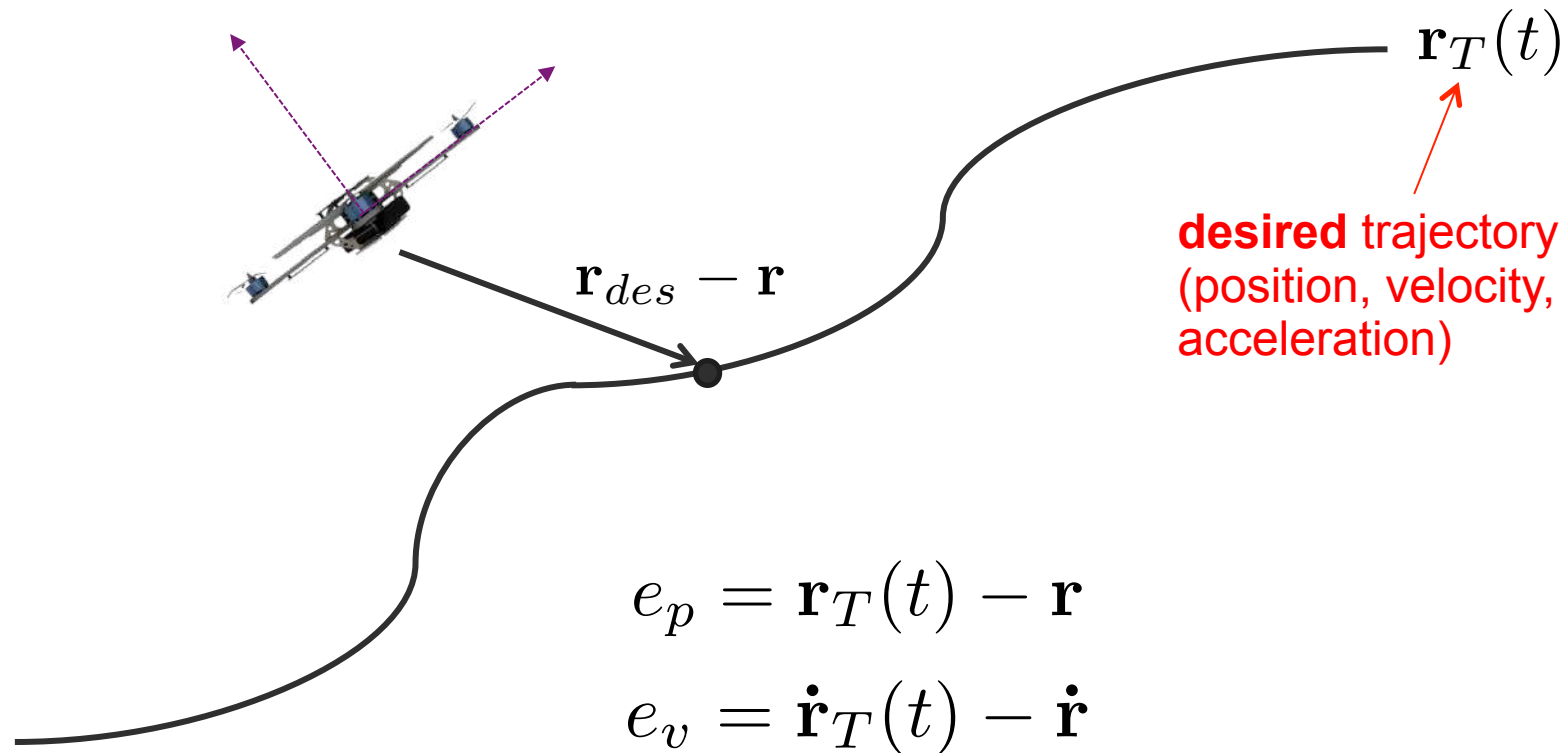
Linearized dynamics

$$\begin{aligned}\ddot{y} &= -g\phi \\ \ddot{z} &= -g + \frac{u_1}{m} \\ \ddot{\phi} &= \frac{u_2}{I_{xx}}\end{aligned}$$

Trajectory Tracking

Given $\mathbf{r}_T(t)$, $\dot{\mathbf{r}}_T(t)$, $\ddot{\mathbf{r}}_T(t)$

$$\mathbf{r}_T(t) = \begin{bmatrix} y(t) \\ z(t) \end{bmatrix}$$

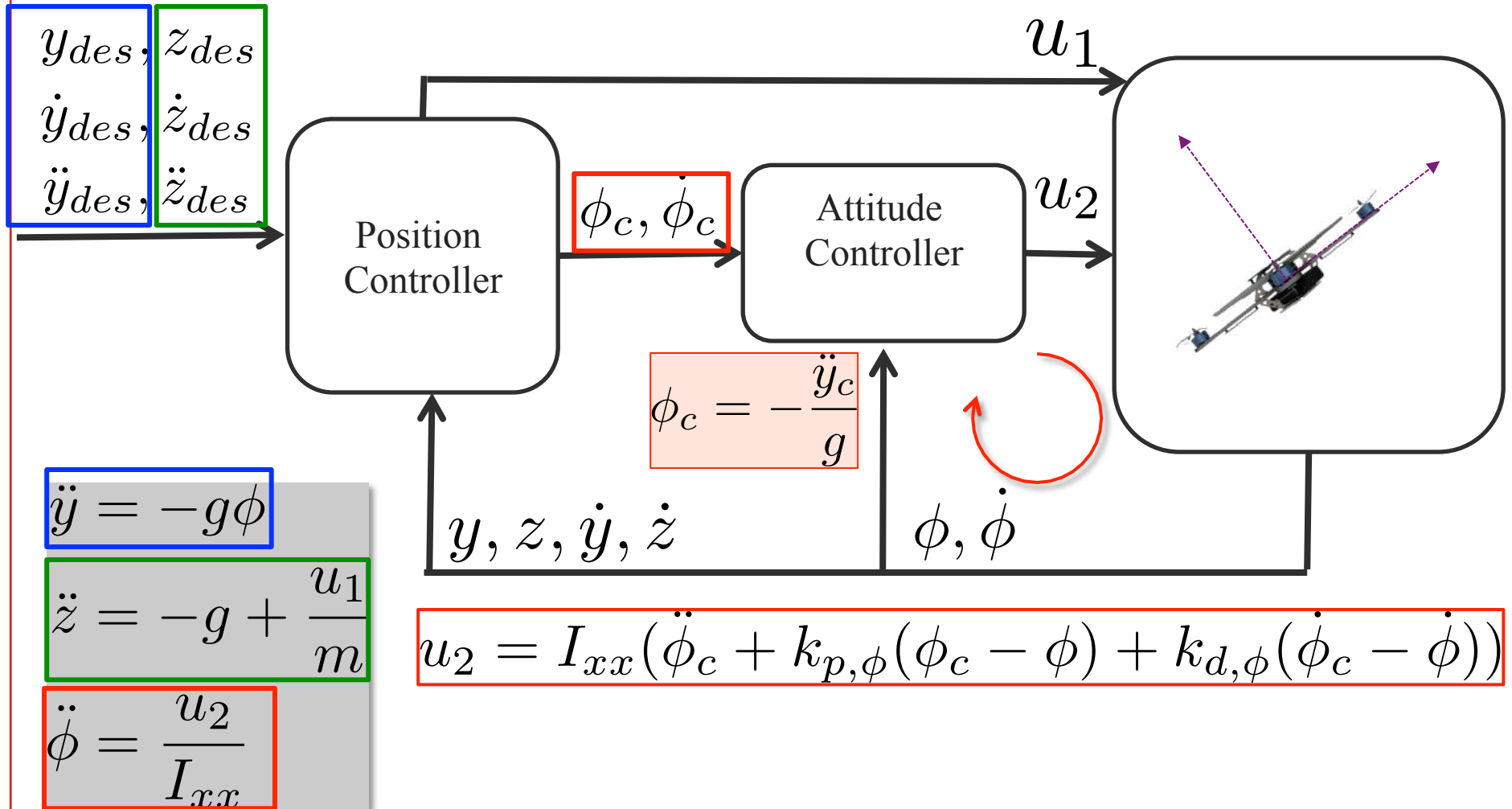


Want $(\ddot{\mathbf{r}}_T(t) - \ddot{\mathbf{r}}_c) + k_{d,x}e_v + k_{p,x}e_p = 0$

Commanded acceleration, calculated by the controller

Nested Control Structure

$$u_1 = m(g + \ddot{z}_{des} + k_{d,z}(\dot{z}_{des} - \dot{z}) + k_{p,z}(z_{des} - z))$$



Control Equations

$$u_1 = m(g + \ddot{z}_{des} + \underbrace{k_{d,z}}_{\text{damping}}(\dot{z}_{des} - \dot{z}) + \underbrace{k_{p,z}}_{\text{stiffness}}(z_{des} - z))$$

$$u_2 = \underbrace{k_{p,\phi}}_{\text{stiffness}}(\phi_c - \phi) + \underbrace{k_{d,\phi}}_{\text{damping}}(\dot{\phi}_c - \dot{\phi})$$

$$\phi_c = -\frac{1}{g}(\ddot{y}_{des} + \underbrace{k_{d,y}}_{\text{damping}}(\dot{y}_{des} - \dot{y}) + \underbrace{k_{p,y}}_{\text{stiffness}}(y_{des} - y))$$