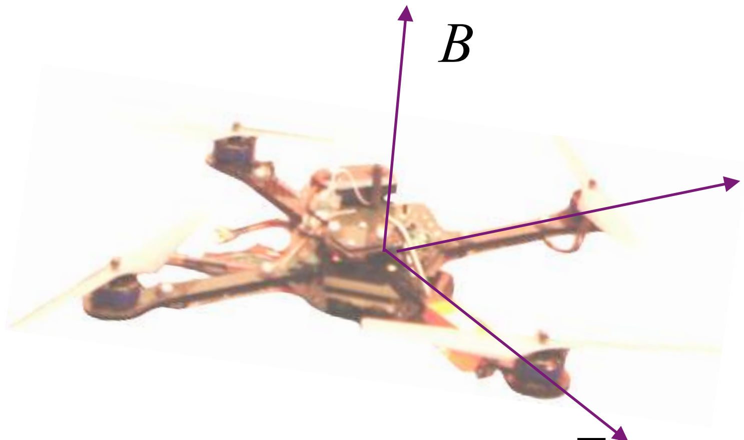
Motion Planning for Quadrotors



Newton-Euler Equations

$${}^{A}\boldsymbol{\omega}^{B} = p\,\mathbf{b}_1 + q\,\mathbf{b}_2 + r\,\mathbf{b}_3$$

Rotation of thrust vector from B to A

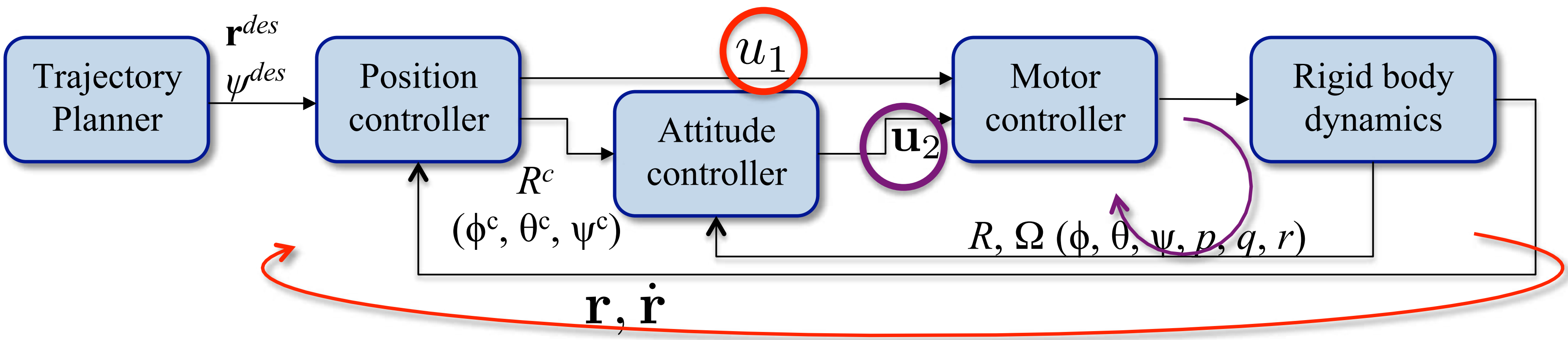
$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \underbrace{F_1 + F_2 + F_3 + F_4}_{u_1} \end{bmatrix}$$

Components in the inertial frame along $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}_{\mathbf{u}_2} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

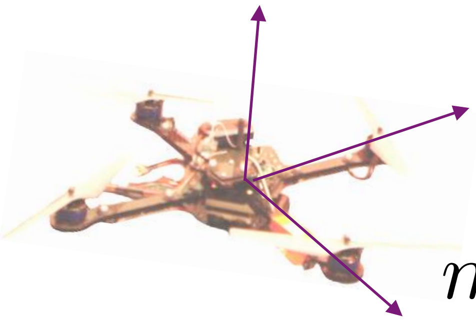
Components in the body frame along $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$, the principal axes

Position Control



Position control loop relies on an inner attitude control loop

The fourth derivative of position depends on $\mathbf{u}_2$



$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \boxed{F_1 + F_2 + F_3 + F_4} \end{bmatrix}$$

$$\boxed{R(\theta, \phi, \psi)}$$

$\boldsymbol{u}_1$

The second derivative of position depends on u_1

The second derivative of the rotation matrix depends on $\mathbf{u}_2$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

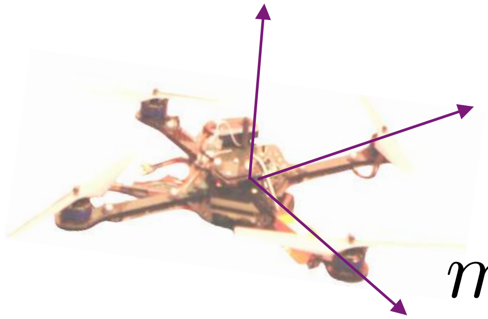
$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \boxed{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$\mathbf{u}_2$

Linearized Model

$(\theta \sim 0, \phi \sim 0, \psi \sim 0)$

$(p \sim 0, q \sim 0, r \sim 0)$



$$m\ddot{\mathbf{r}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \cancelto{I}{R} \begin{bmatrix} 0 \\ 0 \\ \boxed{F_1 + F_2 + F_3 + F_4} \end{bmatrix}$$

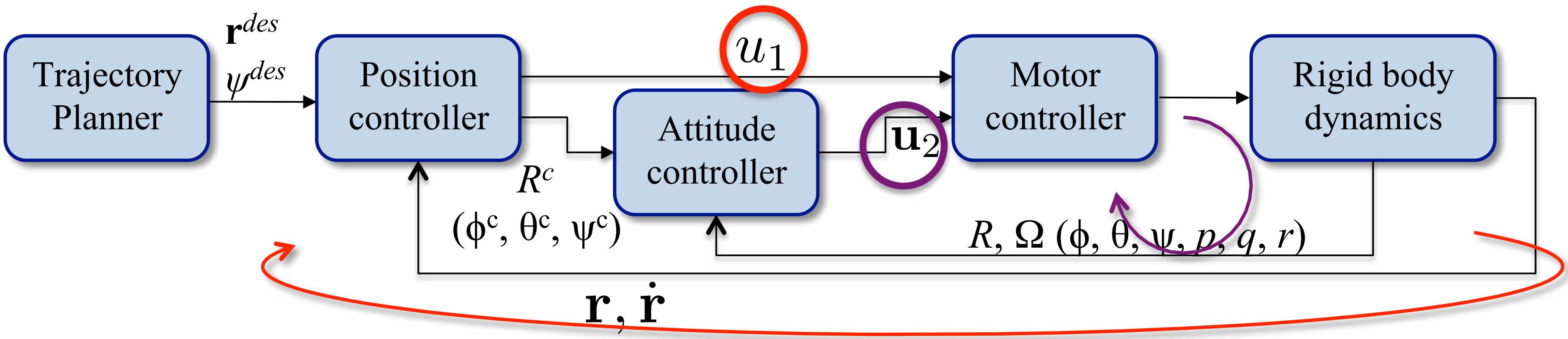
$$u_1$$

$$I \quad \boxed{R(\theta, \phi, \psi)}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \cancelto{I}{\begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix}} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underbrace{\boxed{\begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix}}}_{\mathbf{u}_2} - \cancelto{0}{\begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}}$$

Minimum Snap Trajectory



The position control system is a fourth order system

Want trajectories that can be differentiated four times

Minimum Snap Trajectory

$$x^\star(t) = \arg\min_{x(t)} \int_0^T \left(x^{(iv)}\right)^2 dt$$

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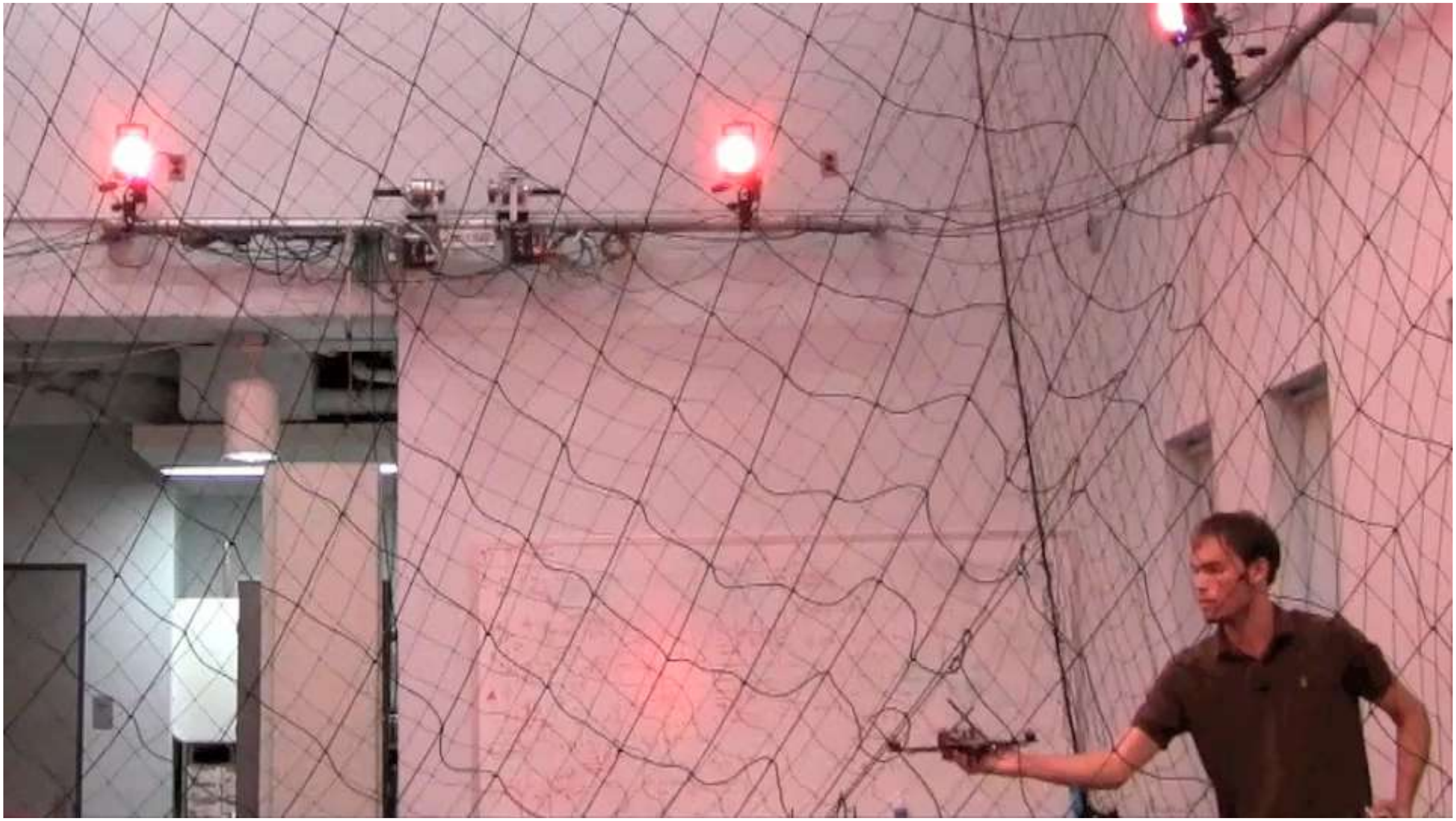
Inner Attitude Control Loop

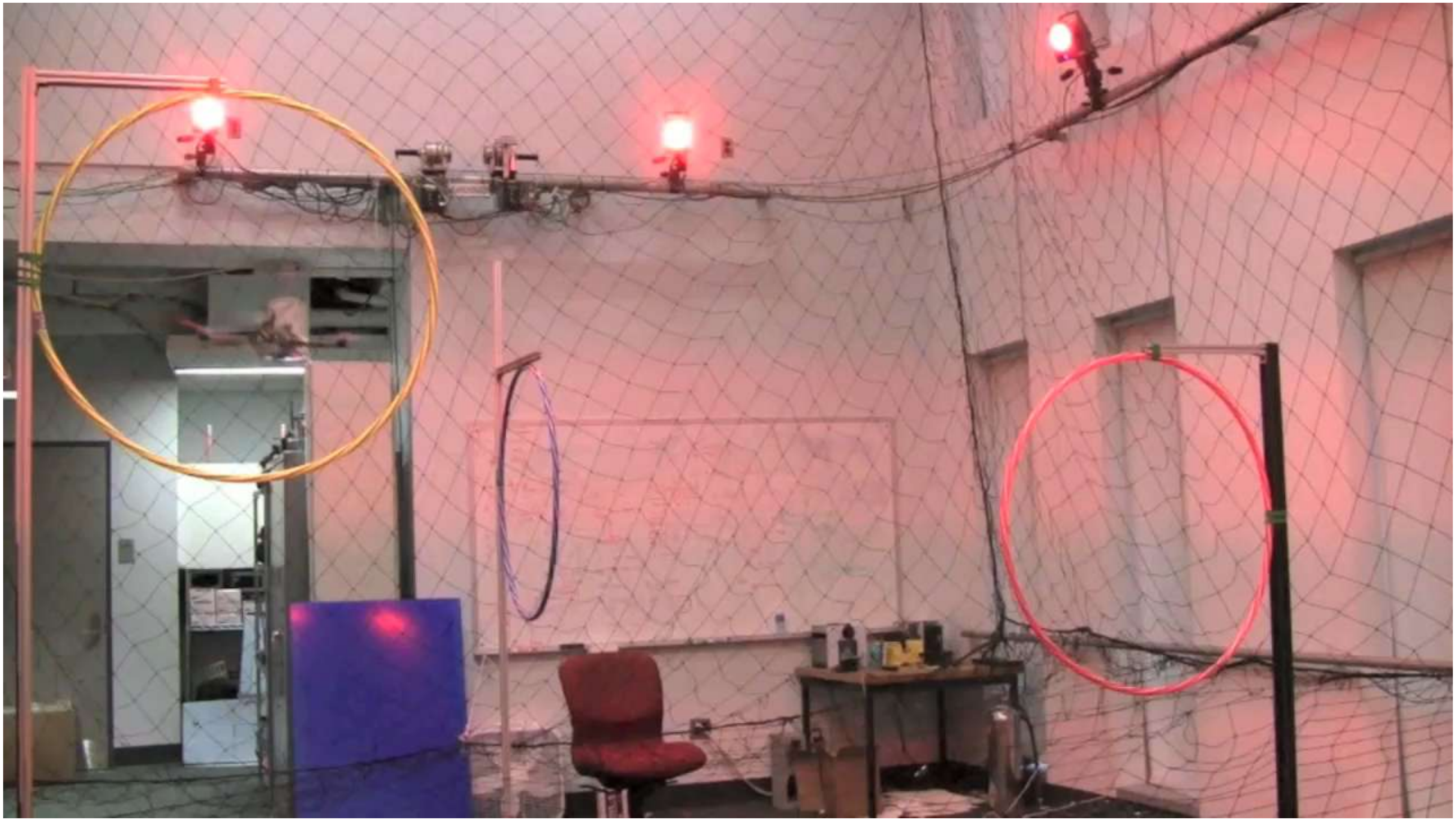


Daniel Mellinger, Nathan Michael, and Vijay Kumar. Trajectory Generation and Control for Precise Aggressive Maneuvers with Quadrotors. *International Journal of Robotics Research*, Apr. 2012.

Minimum Snap Trajectories

D. Mellinger and V. Kumar, "Minimum Snap Trajectory Generation and Control for Quadrotors," *Proc. IEEE International Conference on Robotics and Automation*. Shanghai, China, May, 2011.

Automated Synthesis of Trajectories

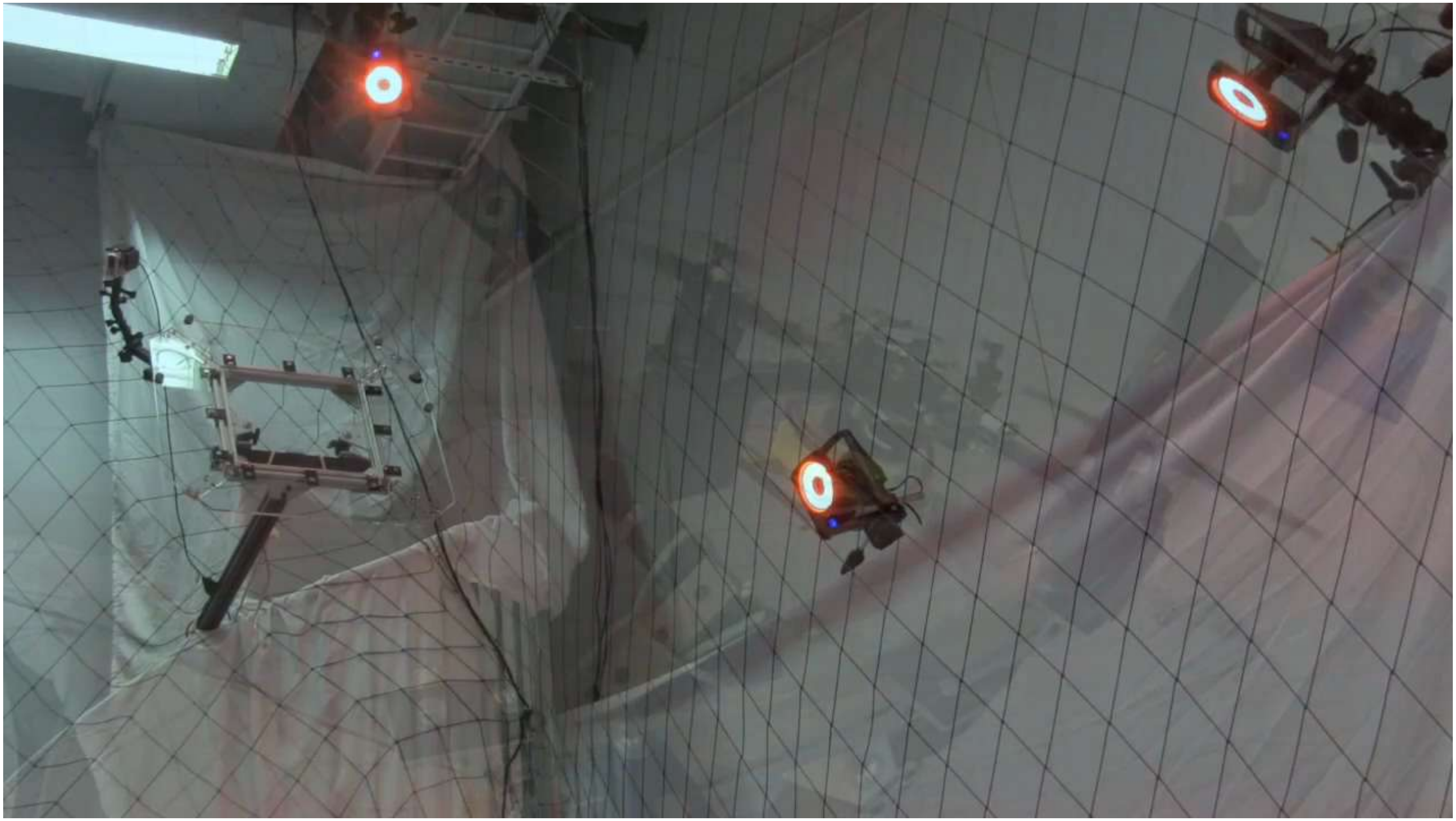


D. Mellinger and V. Kumar, "Minimum Snap Trajectory Generation and Control for Quadrotors," *Proc. IEEE International Conference on Robotics and Automation*. Shanghai, China, May, 2011.

Aerial Grasping and Manipulation

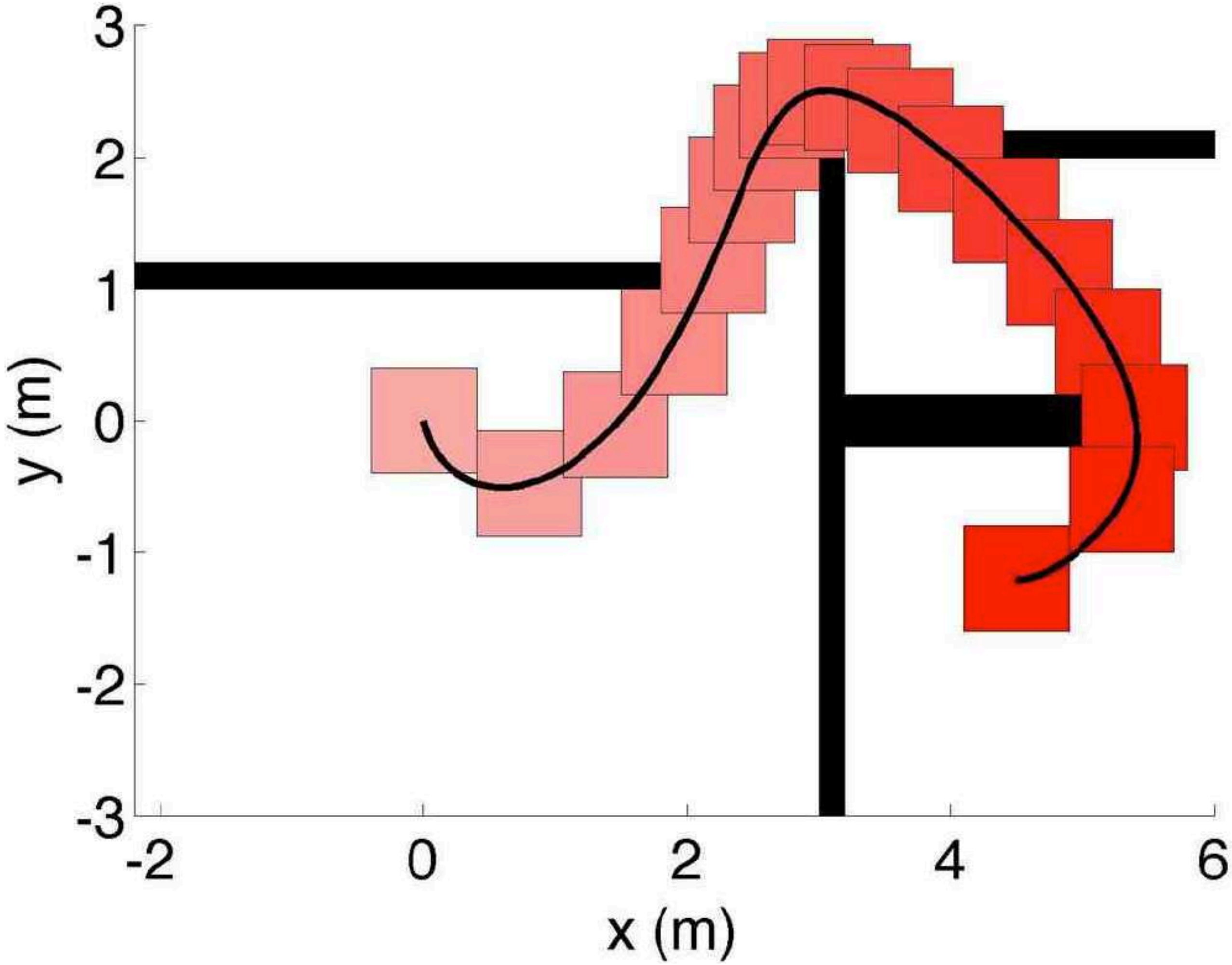
Justin Thomas, Joe Polin, Koushil Sreenath, and Vijay Kumar, "Avian-inspired grasping for quadrotor micro UAVs," *ASME International Design Engineering Technical Conference* (IDETC), Portland, Oregon, August 2013.

Perching



J. Thomas, G. Loianno, M. Pope, E. W. Hawkes, M. A. Estrada, H. Jiang, M. R. Cutkosky, and V. Kumar, "Planning and Control of Aggressive Maneuvers for Perching on Inclined and Vertical Surfaces," in *International Design Engineering Technical Conferences & Computers and Information in Engineering Conference (IDETC/CIE)*, Boston MA, August 2015.

Min Snap Trajectory with Constraints



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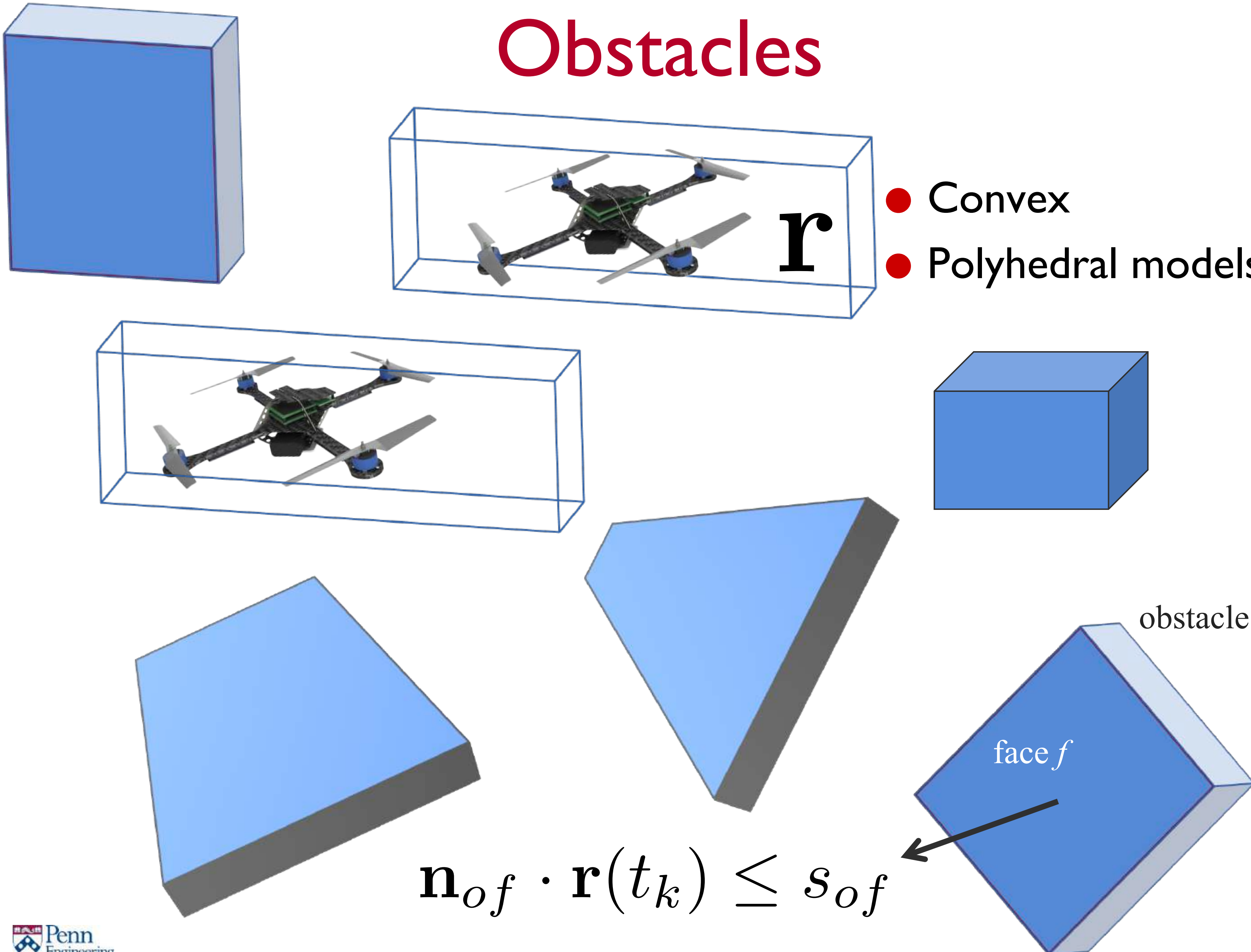
Obstacles

- Convex
- Polyhedral models

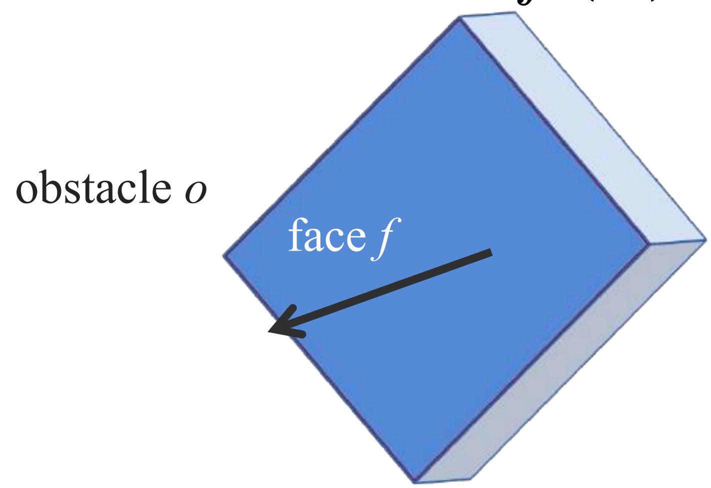


$$\mathbf{n}_{of} \cdot \mathbf{r}(t_k) \leq s_{of}$$

Integer Constraints for Obstacle Avoidance

$$\mathbf{n}_{of} \cdot \mathbf{r}(t_k) \leq s_{of} + M b_{ofk}, \ \forall f = 1, \ldots, n_f(o)$$

o	obstacle
n_f	number of faces
t_k	*kth* time instant
b_{ofk}	binary variable
M	large positive constant



$$\mathbf{n}_{of} \cdot \mathbf{r}(t_k) \leq s_{of}$$

$$\sum_{f=1}^{n_f(o)} b_{ofk} \leq n_f(o) - 1$$

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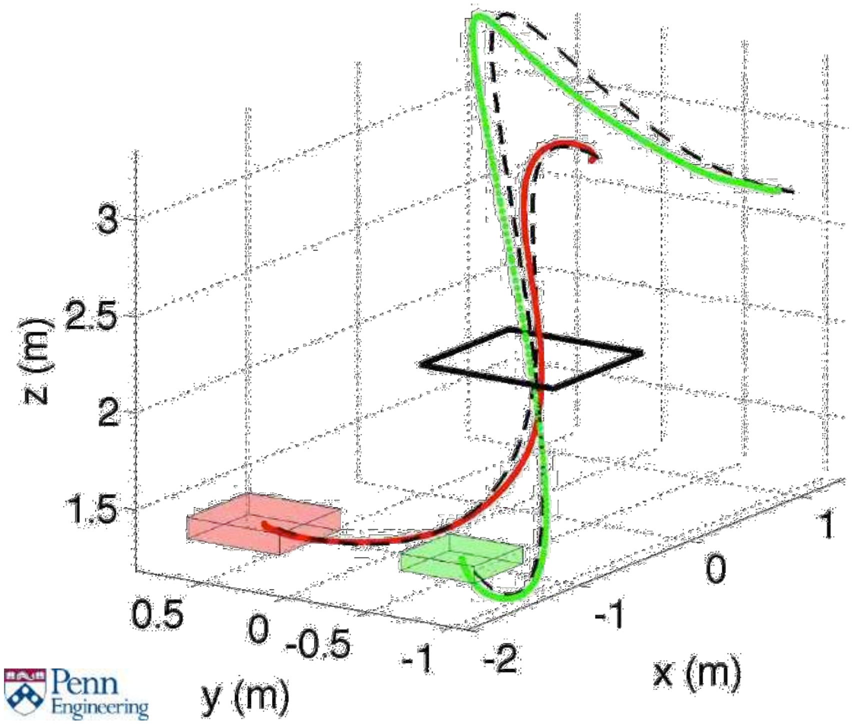
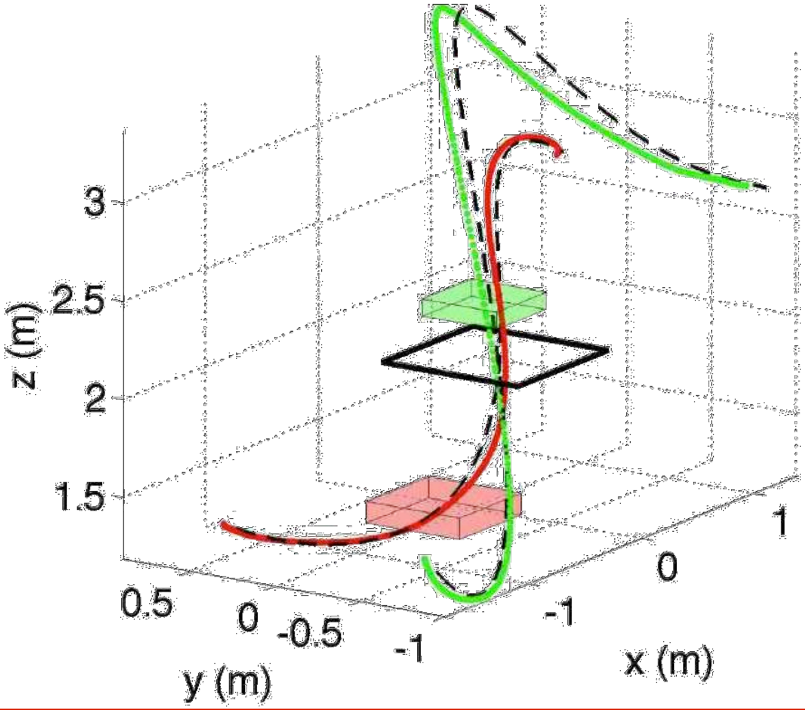
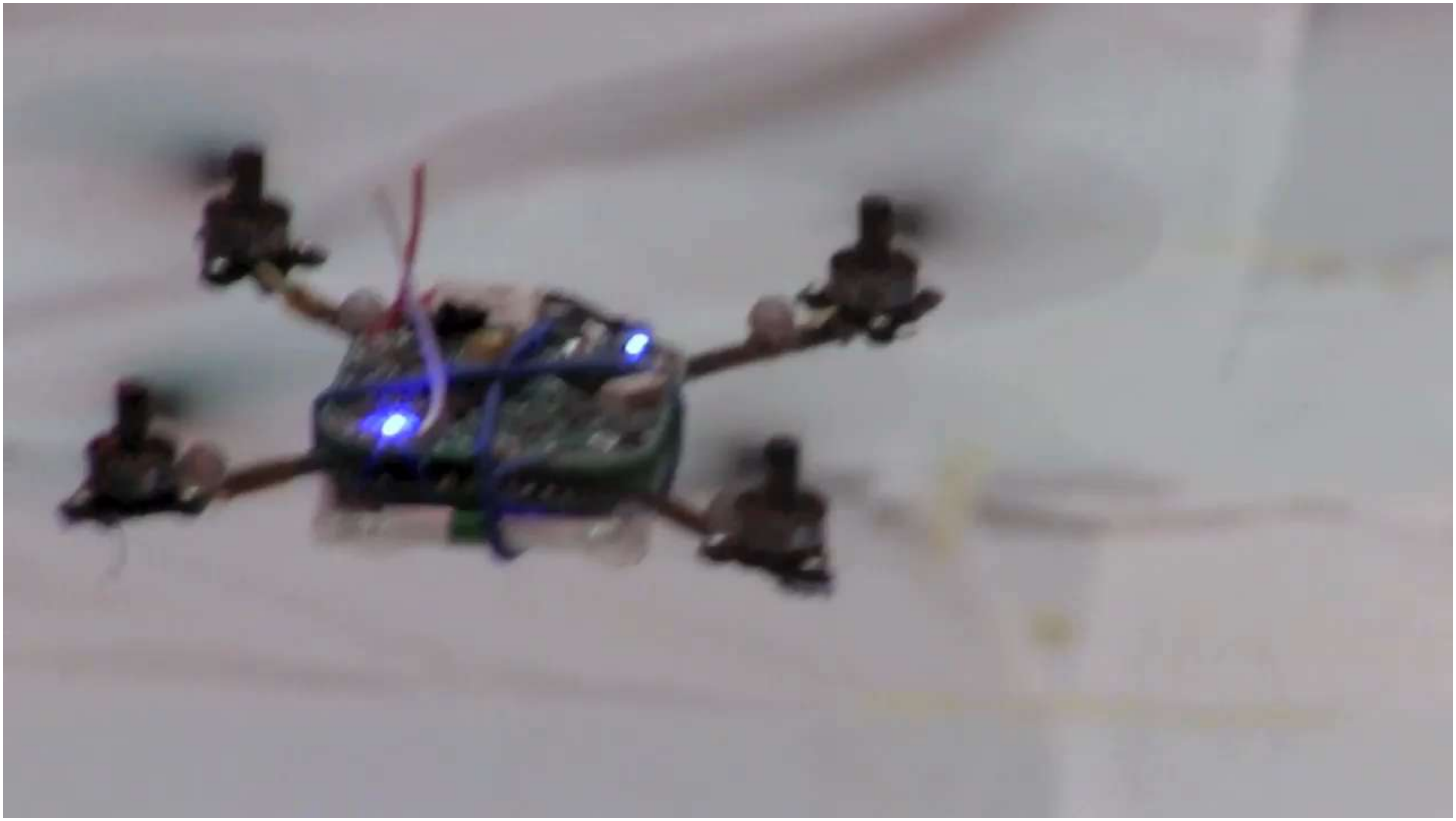
Transporting Suspended Payloads

S. Tang and V. Kumar, “Mixed Integer Quadratic Program Trajectory Generation for a Quadrotor with a Cable-Suspended Payload,” *in IEEE International Conference on Robotics and Automation*, May 2015.

Results







Aleksandr Kushleyev, Daniel Mellinger, Caitlin Powers, Vijay Kumar, “Towards a swarm of agile micro quadrotors,” *Autonomous Robots*, Vol. 35, No. 4, Pg. 287-300, 2013.