Course No. – AS 5570

Principles of Guidance for Autonomous Vehicles

<u>Assignment 4</u> **Due Date: <u>October 27, 2024</u>**
(For Computer Assignments: Nov 3, 2024)

PPN and APPN:

1) **Reading assignment**: Pages 188-191 of NPTEL lecture series on PPN.

2) Problems 1-3 on Page 211-212 of NPTEL lecture series on PPN. (**Computer assignment**)

3) Consider an initial engagement geometry as follows: $X_{P_0} = [0,0]'$ m, $X_{T_0} = [1000,100]'$ m, $\alpha_{T_0} = 0$, $V_P = 30$ m/sec, $V_T = 20$ m/sec, and $a_T = 0.2$g. Consider the following four initial headings of pursuer.

   i] $\alpha_{P_0} = \pi$, ii] $\alpha_{P_0} = 0.85\pi$, iii] $\alpha_{P_0} = 0.5\pi$, iv] $\alpha_{P_0} = 0.1\pi$

   Also, consider the pursuer's guidance be: A] Pure PN (PPN), B] Augmented Pure PN (APPN). For each of these cases,

   i. Obtain $S_\theta(t)$ and $S_R(t)$ sectors at any time t during the engagement for both PPN and APPN guidance of pursuer.

   ii. Identify the sectors $S_\theta(t_0)$, $S_R(t_0)$, $\sigma_\theta(t_0)$ and $\sigma_R(t_0)$, in which the initial engagement geometry $P(R_0, \theta_0)$ belongs to on the target-centric V_{T_0}-referenced polar plane of relative pursuit.

   iii. Also, identify whether the initial engagement configurations belong to which sectors : ${S_\theta}^+(t_0)$ or ${S_\theta}^-(t_0)$; ${S_R}^+(t_0)$ or ${S_R}^-(t_0)$; ${\sigma_\theta}^+(t_0)$ or ${\sigma_\theta}^-(t_0)$; ${\sigma_R}^+(t_0)$ or ${\sigma_R}^-(t_0)$.

   iv. Identify whether this case falls under guaranteed capture zone of PPN with $N = 2.01 + 2/\sqrt{\nu^2 - 1}$.

   v. Obtain the minimum value of navigation gain N for PPN to have this case under guaranteed capture zone of PPN.

   vi. If a capture is guaranteed by PPN with $N = 2.01 + 2/\sqrt{\nu^2 - 1}$ in this case, then does θ_f belong to which S_θ sector in each of those cases?

   vii. What are the conditions on navigation gain and augmentation parameter of APPN for achieving interception from this case without and with constraint on finiteness of lateral acceleration requirement at the endgame?

PPN-based Terminal Angle Control:

4) Consider an initial engagement geometry against stationary target: $\alpha_{P_0} = \pi/4$, $\theta_0 = 0$.

   i. What is the set of achievable terminal angles by PPN?

   ii. Suppose, in the 2 phase PPN (discussed in the class), the orientation phase follows with $N = 3/11$. Following this orientation guidance is it possible to extend the achievable terminal angles set to the entire half-space $[-\pi, 0)$?

NCC and OGL:

5) Consider a linearized engagement geometry between a pursuer and a target under the 'Near Collision Course' (NCC) assumption. Derive the expressions of $a_P(t)$ for PN and augmented PN (APN) guided pursuers in terms of effective navigation gain N', t_f and $t_{go} = t_f - t$ for the following two cases.

   i. Non-zero initial heading error, but no target maneuver

   ii. Zero initial heading error, but constant target maneuver (normal to LOS)

   Also, draw schematic block diagrams of PN and APN guidance loop

6) Show that APN with effective navigation gain $N' = 3$ is an optimal guidance law that minimizes $J = \frac{1}{2}\int_{t_0}^{t_f} a_P^2(t)dt$, subject to the followings:

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \\ a_T \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ a_T \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} a_P, \text{ where } \begin{bmatrix} y \\ \dot{y} \\ a_T \end{bmatrix} \text{ is the state vector under NCC condition;}$$

   And, $y(t_f) = 0$.