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Course No. – AS 5570 Quiz – 1 Full Marks: 30

Principles of Guidance for Autonomous Vehicles

Date: September 4, 2024 **Time: 14:00 – 15:00**

Important:

1. You may **carry one A4 sheet as your supporting study material** during the test.
2. **Engagements** are **planar** unless otherwise mentioned.
3. In numerical problems, **use 3-4 digits after decimal point**, and write the **final answers inside boxes.**
4. **Cell phone and any other electronic gadgets except calculators are not allowed** in the exam.
5. Full marks will be **scaled down to 15,** and your marks will be accordingly scaled while adding to your semester marks.

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1) Answer the followings. **(0.5*4 = 2)**

   a. Target acquisition is normally of prime importance in which phase of an engagement?

      i. Initial phase ii. Midcourse phase iii. Endgame phase

   b. Which one below is correct ordering in terms of autonomy level?
      i. Active Human in the Loop < On-Board Decision Support < Fully Autonomous
      ii. Active Human in the Loop < Fully Autonomous < On-Board Decision Support
      iii. On-Board Decision Support < Active Human in the Loop < Fully Autonomous

   c. Collision triangle geometry is characterized by
      i. Zero range rate
      ii. Zero LOS turn rate
      iii. Both of the above

   d. Pure Pursuit geometry is characterized by
      i. Target's heading angle is same as Pursuer's heading angle
      ii. Target's heading angle is same as LOS angle
      iii. LOS angle is same as Pursuer's heading angle

2) Answer the followings.

   a. Discuss basic differences between ~~path planning,~~ navigation, guidance and vehicle control. **(0.5*4=2)**

   b. Mention two distinct advantages and two distinct challenges of unmanned vehicle systems. **(0.5*4=2)**

c. Explain the difference between Human-in-the-loop and Human-on-the-loop autonomy levels. **(0.5*2=1)**

d. Mention two reasons why the achieved lateral acceleration is, in general, not same as the commanded lateral acceleration. **(0.5*2=1)**

e. In an engagement between a Pure Pursuit (PP)-guided higher speed pursuer with speed V_P and a lower speed non-maneuvering target, it is found that the initial range rate $V_{R_0} < -V_P$. Which one below is correct on the initial trend of variation of LOS angular rate $|\dot{\theta}|$? **Why?** [**Note:** $|\dot{\theta}|$ is the magnitude of LOS turn rate.] (2)

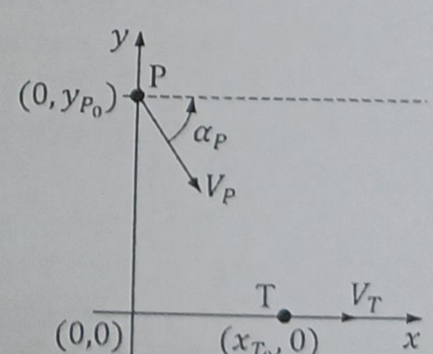
  i. Initially increasing $|\dot{\theta}|$
  ii. Initially decreasing $|\dot{\theta}|$
  iii. Cannot be inferred as it depends on the initial range

f. Consider an engagement between a non-maneuvering pursuer with speed V_P against a non-maneuvering target with speed V_T. They attain Pure Pursuit (PP) course geometry just when they come nearest to each other. Justify whether $V_P \geq V_T$ or $V_T \geq V_P$. (2)

g. Consider an engagement between a PP-guided pursuer with speed $V_P = 10\text{m/s}$, initial heading angle $\alpha_{P_0} = \theta_0 = 0$ and an equal speed target with $V_T = 10\text{m/s}$, initial heading angle $\alpha_{T_0} = \pi/3$, and turn rate $\dot{\alpha}_T = \dot{\theta}$. The initial range $R_0 = 100\text{m}$. At which final time t_f the pursuer will intercept the target? **Justify.** (2)

  i. $t_f = 10\text{sec}$
  ii. $t_f = 20\text{sec}$
  iii. $t_f = \infty$ as interception will never occur

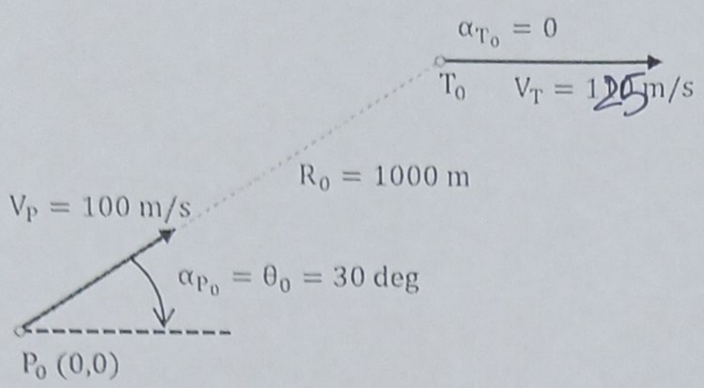
3) Consider $\upsilon = \frac{V_P}{V_T} = 1.25$, $x_{T_0} = 300$ unit and $y_{P_0} = 750$ unit.



a. For what value of Pursuer's heading angle α_P the miss distance (r_{miss}) is zero? What is the time required to achieve zero miss in that case? (2)

b. For other values of α_P, express the steady state LOS angle θ_{SS} in terms of α_P. (1)

4) Consider a Pure Pursuit (PP)-guided pursuer with speed $V_P = 100\text{m/s}$ against a higher speed target with speed $V_T = 125\text{m/s}$. Initial engagement parameters: $R_0 = 1000\text{m}$, $\alpha_{P_0} = \theta_0 = \pi/6$, $\alpha_{T_0} = 0$.



a. Compute V_{R_0} and V_{θ_0}. **(1)**

b. **Scenario 1:** Target remains non-maneuvering.

  i. Explain what happens to the states on the (V_θ, V_R)-plane by considering the equilibrium and infer whether interception would take place. **(1)**

  ii. Express the set of points of the engagement trajectory on the (V_θ, V_R)-plane and draw the same. Highlight the direction of movement and the start and end points on this trajectory. **(1.5)**

  iii. Schematically plot the variations of R, $\dot{\theta}$ and θ with time highlighting points of interest (start and end points, maximum and/or minimum points, etc.). Give justification. **(3)**

  iv. Find final range rate V_{R_f}. **(0.5)**

c. **Scenario 2:** Target starts maneuvering with $\dot{\alpha}_T = 0.05 + 2\dot{\theta}$.

  i. Explain what happens to the states on the (V_θ, V_R)-plane by considering the equilibrium and infer whether interception would take place. **(2.5)**

  ii. Express the set of points of the engagement trajectory on the (V_θ, V_R)-plane and draw the same. Highlight the direction of movement and the start and end points on this trajectory. **(1.5)**

  iii. Schematically plot the variations of R with time highlighting points of interest (start and end points, maximum and/or minimum points, etc.). Give justification. **(1)**

  iv. Comment on the Pursuer's final lateral acceleration requirement. **(1)**

  v. Final range rate V_{R_f}?

Course No. – AS 5570 Quiz – 2 Full Marks: 24

Principles of Guidance for Autonomous Vehicles

Date: October 14, 2024 Time: 1 hr

Important:

1. You may **carry only one A4 size formula sheet** during the test.
2. **Engagements** are **planar** unless otherwise mentioned.
3. In numerical problems, **use 3-4 digits after decimal point**, and write the **final answers inside boxes**.
4. **Cell phone and any other electronic gadgets except your own calculators are not allowed** in the exam.
5. Full marks will be **scaled down to 15,** and your marks will be accordingly scaled while adding to your semester marks.

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1) Answer the followings: **(0.5*2+1.5*2=4)**

   a. LOS Guidance is a:

      i. two-agent guidance; ii. three-agent guidance; iii. Four-agent guidance

   b. True Proportional Navigation (TPN) Guidance command includes:

      i. Only lateral acceleration,
      ii. Only longitudinal acceleration
      iii. Both longitudinal and lateral accelerations

   c. Which option below is right for the gain of TPN guidance command for successful capture of a non-maneuvering target? **Justify**.

      i. Only positive
      ii. Only negative
      iii. Both positive and negative.

   d. The deviation angle of Deviated Pure Pursuit (DPP) Guidance should lie in which interval? **Justify**.

2) Mention the differences between Beam Rider and Command-to-Line-Of-Sight implementations of LOS Guidance. **(2)**

3) Consider an engagement between a non-maneuvering moving target and a TPN-guided pursuer ($a_P = c\dot{\theta}$).

   a. Draw the engagement geometry and write the equations of engagement dynamics. **(2)**

b. From engagement trajectories in (V_θ, V_r)-space, explain why the following conditions are necessary and sufficient for interception: (3)

i. $V_{R_0} < 0$,

ii. $c > 0$,

iii. $V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0} < 0$.

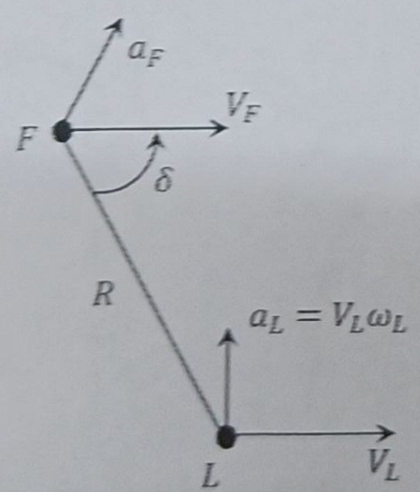
c. Consider the following initial engagement geometry: $X_{P_0} = [0,0]^T$m, $X_{T_0} = [1000,0]^T$m, and $\alpha_{T_0} = 0$. Speeds of pursuer and target are: $V_P = 50$m/sec and $V_T = 15$m/sec.

i. Obtain the range of initial heading angles of the pursuer (α_{P_0}) that fall within the capture zone in terms of c. (1.5)

ii. Obtain the minimum value of c to ensure a non-null capture zone. (1)

iii. If $\alpha_{P_0} = \pi/3$, then justify whether it is possible to achieve interception before a final time $t_f = 100$ sec for any selected value of c? (1.5)

4) As shown in the figure, consider the engagement geometry between a Leader 'L' and a Follower 'F' in a formation. Throughout the mission, L maintains constant forward speed V_L and moves in a circle with turn rate $\dot{\alpha}_L = \omega_L$, and F is driven by Deviated Pure Pursuit (DPP) guidance with a constant deviation angle δ such that the distance between L and F remains constant.



a. If F maintains a constant forward speed V_F, then write down the equation of engagement trajectory on the relative velocity (V_θ, V_R)-plane. (0.5)

b. Prove that F cannot maintain the constant range formation if V_F is constant. (1)

c. Find $\dot{V}_F$ and the overall guidance (acceleration) command. And, obtain the equilibrium configuration for V_θ, V_R and V_F for maintaining the formation. (2.5)

d. For such varying V_F scenario, explain the evolution of the engagement on the (V_θ, V_R)-plane over different time-instants, and find the set of points on the engagement trajectory on the (V_θ, V_R)-plane. (2.5)

e. Find the expression of steady state V_F. (1)

f. Consider $V_L = 10$m/sec, $V_{F_0} = 10$m/sec, $R_0 = 50$m, $\delta = \pi/4$, $\omega_L = 0.1$ rad/sec. Find the steady state values of V_F and the overall guidance (acceleration) command of F. (1.5)

Course No. – AS 5570 End-Semester Exam Full Marks: 54

Principles of Guidance for Autonomous Vehicles

Date: November 19, 2024 Time: 2pm – 4:15pm

Important:

1. You may **carry only two stapled A4 size formula sheet** during the test.
2. **Engagements** are **planar** unless otherwise mentioned.
3. In numerical problems, **use 3-4 digits after decimal point**, and write the **final answers inside boxes**.
4. **Cell phone and any other electronic gadgets except your own calculators are not allowed** in the exam.
5. Full marks will be **scaled to 45,** and your marks will be accordingly scaled while adding to your semester marks.

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1) Answer the followings. **(1*2+2*2+1.5*4 = 12)**

a. What are the inputs to and outputs from the Navigation subsystem in an autonomous system? **(1)**

b. Explain the difference miss distance and zero-effort miss. **(1)**

c. Precisely explain the following autonomy levels – Decision-support system, Human-in-the-loop, Human-on-the-loop and Semi-autonomous (rarely supervised). **(2)**

d. Consider a Pure Pursuit (PP)-guided pursuer on a PP course. Find the values of terminal a_P for $\nu \gtreqless 1$, where the speed ratio $\nu \triangleq V_P/V_T$. [Use the expression of range R in terms of angle $\psi \triangleq \alpha_T - \theta$]. **(2)**

e. Give three examples of each of domain-dependent components and domain-independent components of autonomous systems. **(0.75*2=1.5)**

f. Radars are used for which kind of homing? **Justify.** **(0.5+1 = 1.5)**

i. Passive ii. Active iii. Semi-active

g. Mention the key differences between Augmented Pure Proportional Navigation (APPN) and Augmented Proportional Navigation (APN) guidance laws. **(1.5)**

h. Does any of the following guidance laws lead to change in pursuer's speed? Explain why. **(0.5+1 = 1.5)**

i. LOS-Guidance ii. TPN iii. PPN

2) Consider an interception of a maneuvering target ($a_T = \frac{b}{V_\theta}, b > 0$) by a True Proportional Navigation (TPN)-guided pursuer $\left(a_P = c\dot{\theta},\ c > 0\right)$, where both a_T and a_P are applied normal to LOS.

a. Write the equations of engagement between the target and the pursuer. **(1.5)**

b. Derive the expressions of $\dot{V}_R$ and $\dot{V}_\theta$ in terms of $\dot{\theta}$, a_P and a_T. **(1.5)**

c. Derive the expression of final time (t_f) in terms of b, $k = V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0}$, and $p = R_0(V_{R_0} + 2c)$ as, $t_f = \frac{1}{2b}\left(-k \pm \sqrt{k^2 - 4bp}\right)$. **(1.5)**

d. Derive the following necessary and sufficient conditions for interception. **(4)**

i. $V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0} < 0$, ii. $\left|V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0}\right| \geq 2\sqrt{bR_0(V_{R_0} + 2c)}$,

iii. $bR_0 \leq \left(\frac{2c}{3}\right)^3$

e. Consider the engagement parameters in Question (2) as: $R_0 = 2500\text{m}$, $\theta_0 = 0$, $V_{P0} = 250\text{m/sec}$, $\alpha_{P0} = \pi/4$, $c = 200$, $V_{T0} = 100\text{m/sec}$, $\alpha_{T0} = \pi/2$. Find the minimum value of b (> 0) the target should select to avoid interception. **(2.5)**

3) Consider a Pure PN (PPN)-guided pursuer with navigation gain N against a maneuvering target. Expressions of S_θ and S_R sectors are given in terms of N, $\phi_0 = \alpha_{P_0} - N\theta_0$ and $\nu = V_P/V_T$ as,

$$S_\theta = \left[\theta_{\mathrm{n}_0} - \frac{1}{k}\sin^{-1}\left(\frac{1}{\nu}\right), \theta_{\mathrm{n}_0} + \frac{1}{k}\sin^{-1}\left(\frac{1}{\nu}\right)\right];$$

$$S_R = \left[\theta_{\mathrm{n}_0} + \frac{\pi}{2k} - \frac{1}{k}\sin^{-1}\left(\frac{1}{\nu}\right), \theta_{\mathrm{n}_0} + \frac{\pi}{2k} + \frac{1}{k}\sin^{-1}\left(\frac{1}{\nu}\right)\right];$$

where, $k = N - 1$ and $\theta_{\mathrm{n}_0} = -\frac{\phi_0 + n\pi}{k}$.

Consider an initial engagement geometry as follows:

$X_{P_0} = [0,0]'$ m, $X_{T_0} = [2000,0]'$ m, $\alpha_{P_0} = \pi/3$, $\alpha_{T_0} = 0$, $V_P = 30$ m/sec, $V_T = 15$ m/sec, and $a_T = 0.2g$. And, the pursuer's navigation gain $N = 3 > 1 + 1/\nu$.

a. Identify whether the initial engagement configuration belongs to which sectors : S_θ^+ or S_θ^-; S_R^+ or S_R^-; σ_θ^+ or σ_θ^-; σ_R^+ or σ_R^-. **(0.5*4=2)**

b. Identify the S_θ^- sector, where θ_f belongs to, and the angular sectors that P passes through on the polar plane of relative pursuit till interception. **(1+1=2)**

c. Draw schematic trajectories of pursuer and target on real engagement plane, and plot the variations of range (R) and LOS angle (θ) time plot for the engagement. Give reasons for your answer. **(3*0.5 + 1 = 2.5)**

4) Consider a linearized engagement geometry between a pursuer and a target under the 'Near Collision Course' (NCC) assumption.

a. From NCC assumption, obtain the expression of **(2+1.5=3.5)**

i. Closing speed V_c as, $V_P \cos\alpha_{P_{N_0}} - V_T \cos\alpha_{T_0}$, where $\alpha_{P_{N_0}}$ is the heading angle of a nominal pursuer that is in collision course with the target at the initial time.

ii. $a_P(t) = N'V_c\dot{\theta}(t)$ in terms of N', t_{go} and zero-effort-miss (ZEM) $z(t) = y + \dot{y}t_{go}$.

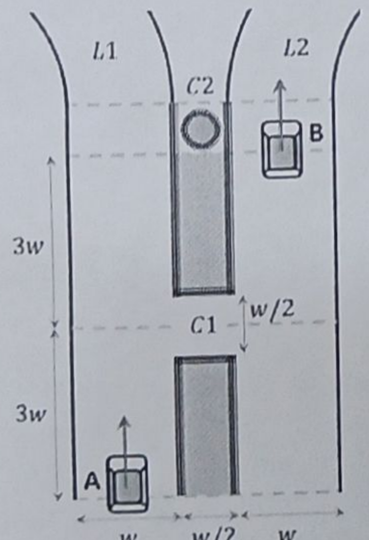
b. Show that PN $\left(a_P(t) = \frac{N'}{t_{go}^2} ZEM(t)\right)$, with effective navigation gain $N' = 3$, where $ZEM(t)$ is zero-effort-miss at time t, is an optimal guidance law that minimizes $J = \frac{1}{2}\int_{t_0}^{t_f} a_P^2(t)dt$, subject to the followings (for non-maneuvering targets):

$\dot{\tilde{X}} = A\tilde{X} + Ba_P$, where $\tilde{X} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$ is the state vector under NCC condition, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, And, $y(t_f) = 0$. (3)

[Recall: $\tilde{X}(t_f) = \exp[A(t_f - t)]\,\tilde{X}(t) + \int_t^{t_f} \exp[A(t_f - \tau)]\,Ba_P(\tau)d\tau$]

5) Consider an engagement between a pursuer guided by Sliding Mode Control (SMC)-based guidance strategy based on heading error as the sliding variable against a stationary target. Show that the SMC-based guidance strategy is an augmentation of pure pursuit guidance, where the augmentation appears in the form of a switching guidance command. (3)

6) Consider a double-lane one-way road with lane-width 'w' as shown below. The two lanes (L1 and L2) separated by a median strip of half-lane width. After C2 circle, the two lanes L1 and L2 diverge in their directions as shown. However, there is a channel C1 of half-lane-width to facilitate passage from lane L1 to L2, if needed.

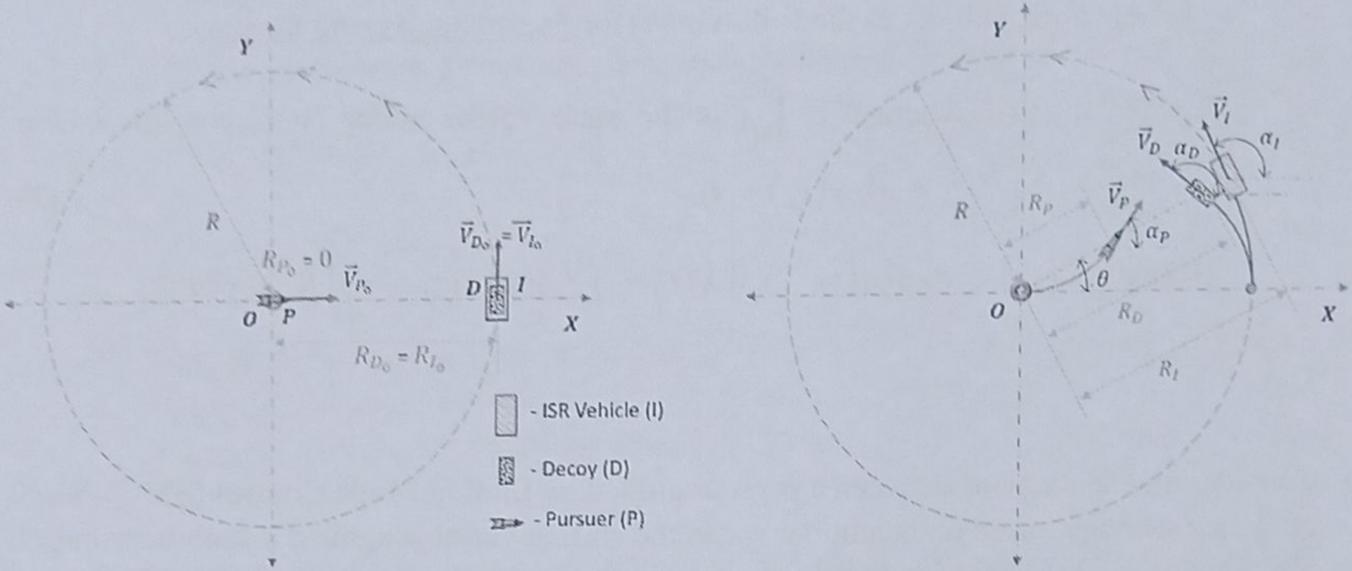


A constant-velocity autonomous car has its heading aligned with lane L1 at location A (along L1, 3w distance behind from the channel C1 center-line). It decides to pass to lane L2 through channel C1 and then align its heading with L2 when it reaches location B (along L2, 3w distance ahead from the channel C1 center-line) as shown in the figure.

a. Design a PPN-based guidance strategy for this purpose considering ideal dynamics of the vehicle and ideal information about all states. Discuss how your guidance strategy will serve the above-mentioned purpose. (4)

b. Show a schematic trajectory that P will traverse following your guidance strategy. (1)

7) Consider 'O'-centric X-Y reference frame as shown in the above figures. A vehicle 'I' has been deployed for intelligence, surveillance and reconnaissance (ISR) mission on a circular trajectory parameterized with its constant radius $R_I = R$ and the constant turn rate $\omega(= \dot{\theta})$ of the ISR vehicle 'I', where $\theta = \omega t$ is the LOS angle from O to I. The location of 'I' at time t is denoted as, $X_I(t) = [R\cos(\omega t), R\sin(\omega t)]^T$.



While performing its usual monitoring activity, the ISR vehicle identifies an anomaly (a potentially suspicious activity) at a set-up at O at time $t = 0$ and immediately launches a decoy 'D' with the same speed as that of the ISR vehicle, that is $V_D = V_I$. The decoy 'D' is to follow a trajectory such that the following two conditions are satisfied.

- The decoy D always remains on the LOS joining O and I so that the ISR vehicle can have a camouflage against the location O, that is D always lies between the LOS joining O and I, while $R_D < R_I$ for $t > 0$.
- The distance of the decoy D from O, that is R_D, decreases as time proceeds so that D can destroy the set-up at once it reaches O.

Answer the followings.

a. Show that the trajectory of D satisfies the differential equation: **(2)**

$$\left(\frac{dR_D}{d\theta}\right)^2 + R_D^2 = R^2$$

b. Obtain the decoy's trajectory: $R_D(\theta)$, $X_D(t)$. Express the rate of change of decoy's heading angle $\dot{\alpha}_D$ in terms of ω. **(0.5+0.5+1.5=2.5)**

c. At the same time, that is at $t = 0$, an LOS-guided pursuer P with speed $V_P = \nu V_D = \nu V_I$ is launched with the hope of intercepting the ISR vehicle I. However, due to the presence of the decoy D on the LOS joining O and I with $R_D < R_I$, the pursuer essentially chases the decoy D instead of the ISR vehicle I.

   i. Show that the pursuer's trajectory satisfies $R_P(\theta) = \nu R \sin\theta$. **(2)**

   ii. Obtain the pursuer's location $X_P(t)$ at time t. Express the rate of change of pursuer's heading angle $\dot{\alpha}_P$ in terms of ω. **(0.5+1.5=2)**

   iii. Find the time of interception (t_f) of the decoy by the LOS-guided pursuer in terms of speed ratio ν and ω. **(1.5)**