Course No. – AS 5570

Principles of Guidance for Autonomous Vehicles

Assignment 2 **Due Date: September 18, 2024**

(For Computer Assignments: Sept 25, 2024)

Part I : Deviated PP (DPP)

1) Consider a δ-angle Deviated Pure Pursuit (DPP)-guided pursuer with speed V_P against a maneuvering target with speed V_T and turn rate $\dot{\alpha}_T = c_T\dot{\theta}$. Consider three cases of $\nu = V_P/V_T \lesseqgtr 1$. For each of these three cases consider $c_T \lesseqgtr 1$. For each of these 9 cases:
   - i. Obtain the trajectory on the (V_θ, V_R)-space.
   - ii. In cases of successful capture:
     - i. What are the conditions of successful capture?
     - ii. Identify the capture region in terms of V_{R_0} and V_{θ_0}.
     - iii. Derive the expression of t_f.
   - iii. In other cases (no capture):
     - i. Derive the expressions of t_{miss} and R_{miss}.
   - iv. Derive expressions of $R(\psi)$ and $a_P(\psi)$. Obtain final values of R and a_P for $\mu \geq 1$, where $\nu \cos\delta = \mu \cos\psi_{CC}$, where ψ_{CC} is the value of ψ in collision course with speed ratio ν.
   - v. Obtain the value of $a_{P_{max}}$, and find at which value of ψ, $a_P(\psi) = a_{P_{max}}$.
   - vi. Compare the set of values of ν for PP and DPP for which the final value of a_P is finite.

2) Consider the initial engagement geometry in Question 3 on Page 121-122 of NPTEL lecture series. Consider speed ratios $\nu = 0.8, 1, 1.5$. For each of these three cases consider $c_T = 0, 0.8, 1, 1.5$, where $\dot{\alpha}_T = c_T\dot{\theta}$. Solve Question 3 with all these set-ups.

   (Part A: A1, A2, A5 – **Computer assignment**; Part B: B1 – **Computer assignment**; Part C: to be done after A1-A5 and B1 – **after Computer assignment**)

Part II : Line-Of-Sight (LOS) Guidance

3) NPTEL lecture series Questions 1 (A, B) and 2 A on Page 138-139 (Consider 10 kms of downrange and 1 km of altitude). (**Computer assignment**)

4) Consider an engagement geometry: Target's altitude $h =$ constant, $\alpha_T = \pi$. Also, V_P, V_T are constants such that $V_P > V_T$. Suppose the pursuer is on the LOS guidance course and following LOS guidance command.

   i. Derive the followings:

      i. $\left(\frac{dR_P}{d\theta}\right)^2 + R_P^2 = \frac{(\nu h)^2}{\sin^4\theta}$

      ii. $\cot\theta - \cot\theta_0 = -\frac{V_T}{h}t$

      iii. $a_P = 2\frac{V_P V_T}{h}\sin^2\theta\left[1 + \frac{R_P\sin\theta\cos\theta}{\sqrt{(\nu h)^2-(R_P\sin^2\theta)^2}}\right]$

   ii. Show that

      i. $\alpha_{P_f} = \theta_f + \sin^{-1}\left(\frac{1}{\nu}\sin\theta_f\right)$

      ii. $a_{P_f} = 2\frac{{V_P}^2}{\nu h}\sin^2\theta_f\left[1 + \frac{\cos\theta_f}{\sqrt{\nu^2-\sin^2\theta_f}}\right]$

   iii. Let $V_P = 300$ m/sec, $V_T = 200$ m/sec, $h = 1500$ m, $\alpha_T = \pi$, $X_{P_0} = [0, 0]^T$ m, $X_{T_0} = [5000, 1500]^T$m. It is also desired to satisfy the followings.

      i. $a_{P_f} < 6g$, which approximately implies $0 < \theta_f < 0.24\pi$ or $0.6\pi < \theta_f < \pi$

      ii. $t_f < 20$ sec

      Then, what is the set of achievable impact angles (defined as $\alpha_{T_f} - \alpha_{P_f}$)?

5) Consider an LOS-guided pursuer with speed V_P be on LOS guidance course against a constantly maneuvering target such that its position is given as $X_T(t) = [c\cos\omega t, c\sin\omega t]^T$ and $V_P > c\omega$. Consider following three cases.

   i. $X_{P_0} = [0, 0]^T$

   ii. $X_{P_0} = [-c, 0]^T$

   Obtain pursuer's trajectory: $R_P(\theta)$, $X_P(t)$.