Principles of Guidance for Autonomous Vehicles: Class Notes

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Chapter 1

Overview of Autonomous Guidance

Autonomous guidance systems are the cornerstone of modern autonomous vehicles (AVs), enabling them to navigate towards a target or avoid obstacles without human intervention. These systems integrate several technologies, allowing vehicles to perceive their environment, plan their trajectory, and control their movements autonomously. The key aspects of autonomous guidance include:

- **Navigation and Path Planning**: Determining the optimal route to the target while considering dynamic conditions.
- **Collision Avoidance**: Avoiding obstacles in the vehicle's path by adjusting its trajectory in real-time.
- **Target Interception**: Calculating the required course to intercept a moving target.
- **Guidance Laws**: Mathematical models that dictate the vehicle's control actions to achieve mission objectives.

At its core, autonomous guidance allows the vehicle to autonomously navigate through dynamic environments. It uses an array of sensors, algorithms, and control mechanisms to make real-time decisions and adjust its path to meet its goals, all while avoiding obstacles.

1.0.1 Categories of Guidance Systems

Guidance systems are typically classified into two categories:

- **Open-Loop Systems**: These systems operate based on predefined trajectories or paths. They do not adjust for real-time feedback from the environment and are suitable for predictable or controlled environments.
- **Closed-Loop Systems**: Closed-loop systems rely on continuous feedback from sensors to adjust the vehicle's trajectory. They respond to dynamic changes in the environment, making them ideal for uncertain and unpredictable situations.

In modern autonomous systems, a combination of both open-loop and closed-loop systems is often used. This allows the vehicle to adapt to changing conditions while maintaining efficient navigation based on its pre-determined mission objectives.

1.0.2 Guidance Laws

The backbone of any autonomous guidance system is the **guidance law**, which dictates how the vehicle's control inputs (such as velocity and direction) are adjusted to ensure it reaches its goal. These laws are essential in scenarios such as target interception or avoidance, where the vehicle must adjust its path in response to moving obstacles or dynamic changes in the environment.
Several types of guidance laws are employed, each suited for different objectives:

- **Proportional Navigation**: A guidance strategy that adjusts the vehicle's trajectory based on the relative position and velocity of a target. This method is commonly used in missile defense.
- **Pure Pursuit**: A technique in which the vehicle follows a reference path by pursuing a point on the path at a constant look-ahead distance, often used in robotic path following.
- **Deviated Pure Pursuit**: An enhanced version of pure pursuit that adjusts the look-ahead distance to improve performance in certain dynamic conditions, such as avoiding sharp turns or obstacles.
- **Model Predictive Control (MPC)**: A sophisticated approach where the vehicle's future states are predicted, and control inputs are optimized over a finite horizon to ensure optimal performance, considering constraints like fuel efficiency and time.

The guidance law's primary challenge lies in managing the uncertainty of the environment. For example, unexpected changes in the vehicle's position, unforeseen obstacles, or sensor noise can disrupt the system's ability to make accurate predictions. To counteract these challenges, the system must quickly adapt to new information and adjust its course accordingly.

1.0.3 Safety, Efficiency, and Real-Time Capability

In autonomous guidance, three critical factors define the system's success:

- **Safety**: Ensuring that the vehicle avoids obstacles and maintains a collision-free path in all scenarios.
- **Efficiency**: Minimizing resource usage, such as fuel or battery, while still achieving the mission objectives.
- **Real-Time Capability**: The system's ability to process data and make decisions in real-time to adapt to changing conditions.

These factors must be carefully balanced to ensure that the vehicle operates smoothly and achieves its objectives, even in dynamic and unpredictable environments.

1.0.4 Applications of Autonomous Guidance

Autonomous guidance plays an indispensable role in a variety of industries and applications, some of the most notable being:

- **Aerial Vehicles (UAVs)**: Drones equipped with autonomous guidance systems are used for surveillance, delivery, mapping, and reconnaissance, navigating airspace safely without human control.
- **Autonomous Cars**: Self-driving cars leverage guidance systems to follow roadways, navigate intersections, adhere to traffic laws, and avoid collisions with other vehicles or pedestrians.
- **Missile Guidance**: Military applications use advanced guidance systems to track and intercept moving targets with high precision, such as in missile defense and tactical missiles.
- **Space Exploration**: Autonomous spacecraft, rovers, and landers rely on sophisticated guidance algorithms to navigate across planets and moons, even in harsh and uncharted environments.

As technology advances, particularly in fields like artificial intelligence (AI), machine learning, and sensor technologies, the precision and reliability of autonomous guidance systems continue to improve. These advancements promise to increase the capabilities of autonomous vehicles, enabling more complex missions and safer operations across a variety of domains.

Chapter 2

Guidance Related Terms

Key terms used in guidance include:

- **Line of Sight (LOS)**: The direct line between the pursuer and the target.
- **Interception Point**: The point where the pursuer meets the target.
- **Miss Distance**: The shortest distance between the pursuer and target at their closest approach.

In the study of autonomous guidance, several key terms and concepts are crucial to understanding how vehicles are controlled and directed. Some of the fundamental terms include:

- **Guidance**: The process of determining the desired path or trajectory of an autonomous vehicle, ensuring it reaches its goal or intercepts a target.
- **Navigation**: Refers to the determination of the vehicle's position, orientation, and velocity, often through the use of sensors and other input data.
- **Control**: The action taken by the vehicle to follow the guidance commands, including the adjustment of velocity, attitude, or trajectory.
- **Trajectory**: The path that the vehicle follows through space-time, which may be predetermined or dynamically adjusted based on environmental conditions and guidance laws.
- **Look-ahead Distance**: In pure pursuit guidance, this is the distance ahead of the vehicle along the reference path that the vehicle aims to follow.
- **Interception**: A task where the vehicle is required to alter its course in order to meet a moving target, typically used in missile defense or pursuit scenarios.
- **Avoidance**: The process by which a vehicle modifies its path to avoid collisions or obstacles, which could be dynamic (moving objects) or static (terrain or immovable obstacles).
- **Target**: An object or entity that the vehicle may aim to intercept or avoid, often moving in an unpredictable manner.
- **Tracking**: The process of following a target's position over time, often using sensors like radar, lidar, or cameras.

These terms form the foundation for designing and understanding autonomous guidance systems, which require a combination of sensors, algorithms, and feedback mechanisms to function effectively.

Chapter 3

Fundamentals of Avoidance and Interception

Autonomous guidance systems are often required to perform two critical tasks: avoidance and interception. Both tasks are essential for ensuring safety and achieving mission success in dynamic environments, where obstacles and moving targets are common.

3.1 Avoidance

Involves ensuring that a vehicle steers clear of an obstacle or another vehicle using guidance strategies. Avoidance refers to the capability of an autonomous vehicle to recognize and avoid obstacles in its path. This task is essential in applications such as self-driving cars and UAVs, where the vehicle must navigate safely without human intervention. The key components of avoidance are:

- **Obstacle Detection**: The ability of the vehicle to detect obstacles in its environment through sensors like cameras, radar, lidar, and ultrasonic sensors.
- **Path Planning**: Once an obstacle is detected, the vehicle must determine an alternate path to avoid the obstacle. This is often done using algorithms such as A*, RRT (Rapidly-exploring Random Tree), or Dijkstra's algorithm.
- **Reactive vs. Predictive Avoidance**: Reactive avoidance systems respond to obstacles in real-time, while predictive systems anticipate future positions of obstacles and adjust the vehicle's trajectory accordingly.
- **Collision-Free Trajectories**: The avoidance system must ensure that the new path does not cause collisions with other objects or the vehicle's own components.

The avoidance system's effectiveness depends on the quality and range of the sensors, the precision of the path planning algorithms, and the vehicle's response time.

3.2 Interception

Focuses on guiding a vehicle towards a target to meet it at a specified point in space. Interception refers to the process of directing the vehicle toward a moving target in such a way that it will meet the target at a specific point in the future. This is commonly seen in military applications, where missiles or drones are required to intercept a moving target.

- **Pure Pursuit**: A basic interception strategy where the vehicle follows a point on the target's trajectory at a fixed distance ahead of it.

- **Proportional Navigation (PN)**: A more advanced technique where the vehicle adjusts its velocity to keep the line-of-sight angle between the vehicle and the target constant, guiding it toward the target's future position.
- **Terminal Guidance**: The final phase of interception, which often involves using sensors to lock onto the target for accurate guidance as the vehicle approaches.

Effective interception strategies depend on the relative velocities, accelerations, and possible evasive actions of the target. Moreover, interception often requires real-time updates to the trajectory, which is typically done through feedback loops in the guidance system.

Chapter 4

Taxonomy of Guidance Laws

Guidance laws are the mathematical models and algorithms that govern the control of an autonomous vehicle's trajectory. They can be categorized into various classes based on their design philosophy, application, and complexity. The main types of guidance laws are:

- **Open-Loop Guidance Laws**: These laws use pre-determined or fixed trajectory profiles and do not rely on real-time feedback from the vehicle or environment. They are often used in simple, predictable environments where uncertainty is minimal.
- **Closed-Loop Guidance Laws**: These laws continuously adjust the vehicle's trajectory based on real-time feedback from sensors. Closed-loop guidance is essential for dynamic and uncertain environments, where the vehicle needs to respond to obstacles or moving targets.
- **Linear Guidance Laws**: These laws assume linear dynamics and work well in situations where the vehicle's motion is simple and can be approximated with linear equations. Proportional Navigation (PN) is a common linear guidance law.
- **Non-Linear Guidance Laws**: In more complex environments, non-linear guidance laws are employed to handle non-linearities in the vehicle's motion and external factors, such as changing speeds or maneuvering targets. These are often used in advanced missile or UAV guidance systems.
- **Geometrical Guidance Laws**: These are based on the relative geometry between the vehicle and the target. For instance, Pure Pursuit and Deviated Pure Pursuit are geometrical laws that focus on maintaining a fixed look-ahead distance or adjusting it based on the target's motion.
- **Optimal Guidance Laws**: These laws optimize certain performance criteria, such as fuel efficiency, time-to-intercept, or minimizing path deviation. Optimal control theory, such as Linear Quadratic Regulator (LQR) or Model Predictive Control (MPC), is commonly used for this purpose.

The choice of guidance law depends on factors such as the mission objectives, the vehicle's dynamics, the operational environment, and computational constraints.

We will deal with the following:

- **Geometric Guidance Laws**: Pure Pursuit, Deviated Pure Pursuit, Line-of-Sight Guidance.
- **Proportional Navigation**: Classical guidance law used in missile targeting.
- **Modern Guidance Laws**: Including augmented, sliding mode, and terminal angle control.

Chapter 5

Pure Pursuit and Deviated Pure Pursuit

R expression, time to go (R=0), time to miss and R_{miss}, lateral acceleration, maneuvering, and non-movement target for all cases

Pure Pursuit guides the pursuer directly toward the target by aligning with the line of sight. Deviated Pure Pursuit introduces a deviation angle δ to optimize the trajectory.

Deviated Pure Pursuit

In the derived pure pursuit guidance system, we define the relative velocity components of the pursuer and the target. The following equations describe the velocities in terms of the target angle θ_T and the heading angle of the pursuer θ:

$$v_R = v_P \cos(\theta_T - \theta) - v_T \cos\delta$$
$$v_I = v_P \sin(\theta_T - \theta) - v_T \sin\delta$$

Where: - v_P is the velocity of the pursuer. - v_T is the velocity of the target. - θ_T is the angle to the target. - θ is the heading of the pursuer. - δ represents the angle of the target's velocity vector.

These equations describe the motion of the pursuer relative to the target in a plane, separating their velocity components into radial (v_R) and inertial (v_I) directions. The next step is to compute the angle of pursuit:

$$\dot{\theta_P} = \frac{v_I}{R}$$

Here, $\dot{\theta_P}$ is the rate of change of the pursuer's heading angle, and R is the range or distance between the pursuer and the target.

Non-Maneuvering Target Case

For a non-maneuvering target, we assume the target maintains a constant velocity and heading. The following equations describe the evolution of the range R and the angular velocity $\dot{\theta}$:

$$\dot{R} = v_T \sin\delta - v_P \sin(\theta_T - \theta)$$
$$\dot{\theta} = \frac{v_P}{R} \sin(\theta_T - \theta)$$

Filler text: In this section, we describe the behavior of the system when the target moves in a straight line without changing its velocity. The change in distance and the rate at which the pursuer must turn to intercept the target are both affected by the geometry of the target's motion.

Measuring the Time to Collision

We can calculate the time to collision (t_c) using the following equation, which combines the target's and pursuer's velocities:

$$t_c = \frac{R_0}{v_P \cos(\theta_T - \theta) - v_T \cos\delta}$$

Where R_0 is the initial range between the pursuer and the target. This gives us an estimate of how long it will take for the pursuer to reach the target under the assumption of a constant heading.

$$Q_p = V_p \theta = K(\dot{Q}_p - \theta - \delta)$$

Illustration of δ deviation:

$$\dot{Q} = 6E\left(\frac{V_t}{V_p}\right)$$

Relative velocity components:

$$V_R = V_t \cos(\alpha_t - \theta) - V_p \cos(\delta)$$

$$V_\theta = R\dot{\theta} = V_t \sin(\alpha_t - \theta) - V_p \sin(\delta)$$

$$V_R = \dot{R} = \dot{R}_0 + \frac{d}{dt}(V_t \cos(\alpha_t - \theta) - V_p \cos(\delta))$$

For $\dot{R}$:

$$\dot{R} = V_R \cos(\delta)$$

$$V_\theta = \frac{dR}{dt} = \dot{R}_0 \cdot \tan\theta$$

Derivation of θ:

$$R^2\ddot{R} + \dot{R}^2 \cdot 2\cos(\theta) = \frac{V_t}{V_p}$$

$$V_R = \frac{V_t V_p \sin\delta}{R}$$

For maneuvering targets:

$$R = \cos^{-1}\left(\frac{\theta}{V_t \sin\delta}\right)$$

$$\ddot{R} = \frac{V_p}{R} - V_t \sin(\alpha_t - \theta) - (1 - G)\theta$$

Chapter 6

Line-of-Sight (LOS) Guidance

LOS guidance maintains a constant rate of change in the line-of-sight angle to the target, used for stable and smooth guidance.

Line of Sight Guidance

Beam Rider: missile corrects its own deviation from LOS (Line of Sight).

$$Q_p = \frac{V_R V_p}{R} = K\cos(\theta)$$

Command to Line of Sight: Base station tracks both target and missile, computing commands accordingly.

$$V_R = V_t \sin\delta - V_p \sin(\alpha_t - \theta)$$

Illustration of engagement geometries:

$$R = \frac{\sin\theta}{1 - G}$$

Where G represents gravity turn angle.

Final differential relationship for maneuvering:

$$\frac{dR}{dt} = \frac{(V_p - V_R)}{V_\theta}$$

The target is in range when:

$$H_t = -\left(R_0 - V_0 t \cdot \tan\theta + 2V_p \cos(\delta)\right)$$

$$Q_p = V_p \dot{Q}_p$$

For small angles:

$$R_t V_p \sin(\psi_p - \theta) = R_p V_p \sin(\phi - \psi_p)$$

$$V_r = V_p \cos(\alpha_T - \theta) - V_p \cos(\phi - \theta)$$

$$V_\theta = V_r \sin(\alpha_T - \theta) - V_\theta \sin(\phi - \theta)$$

Implementation for θ_r:

$$V_{Tp}\left[\frac{\sin(\phi - \psi_p)}{R_t \cos(\Delta p - \theta)}\right]$$

The Line of Sight (LOS) guidance law is used to keep the pursuer's heading aligned with the target. The guidance law suggests that the pursuer adjusts its velocity to maintain alignment along the line of sight:

$$a_P = 2v_T \frac{v_T \sin(\theta_T - \theta)}{R}$$

Where a_P is the pursuer's acceleration, directed towards the target. This equation is essential in intercept scenarios, where minimizing the miss distance is critical.

Filler text: LOS guidance ensures that the pursuer adjusts its trajectory based on the relative movement of the target, keeping the line of sight angle constant or decreasing it as needed.

Flight Control Systems and Guidance Law

To control the flight system, we use the following equations to define the dynamics of the system. The pursuer's acceleration is proportional to the rate of change of the heading angle θ_T:

$$a_P = k_P(v_P - v_T)$$

Where k_P is the proportional gain that helps tune the response of the system. This helps ensure that the flight control system is stable and responsive to changes in the target's position and velocity.

Filler text: The control system's behavior is critical for ensuring the pursuer can adjust its velocity and heading accurately. The tuning constant k_P allows the system to balance between aggressive and stable pursuit, depending on the desired performance.

Circular Pursuit Behavior

In the case of circular pursuit, where the target moves in a circular trajectory, we adjust the relative velocity calculations as follows:

$$v_R = R\dot{\theta} - v_T \sin\delta$$
$$\dot{\lambda} = \frac{v_P}{R} \sin(\theta_T - \theta)$$

Filler text: In a circular pursuit, the pursuer has to account for the continuous change in the target's position, which leads to more complex calculations for the angular velocity and relative velocity components. These equations describe the necessary adjustments.

BR Implementation

$$Q_p = V_p \sin(\psi_p - \theta) = \dot{R}_p + R_p\dot{\theta}$$
$$a_r = \frac{V_p}{R_p}\left(2\dot{\theta} + \frac{\dot{R}_p}{R_p}\right)$$

Implementation for $\dot{\theta}_p$:

$$\dot{Q}_p = K_{RP}(\theta_p - \theta_T)$$

Chapter 7

Proportional Navigation

Proportional Navigation (PN) commands a lateral acceleration proportional to the rate of change of the line-of-sight angle. Different variants include:

- **True PN (TPN)**: Based on true LOS angle rate.
- **Reduced PN (RTPN)**: A simplified version for certain kinematic scenarios.
- **Pure PN (PPN)**: PN assuming straight-line target motion.
- **Augmented PN (APN)**: Enhanced PN incorporating additional terms to compensate for target maneuvers.

True Proportional Navigation (QR to LOS)

$$Q_p = -N\frac{V_r \dot{R}_0}{R}$$

$$\dot{R} = R_0 + (m_1 z) - m_2 \cdot b$$

Velocities in relative coordinates:

$$V_r = V_T \cos(\alpha_T - \theta) - V_p \cos(\theta)$$

$$V_\theta = V_T \sin(\alpha_T - \theta) - V_p \sin(\theta)$$

For constant velocity:

$$V_r = V_{T_r} \cdot \sin(\alpha_T - \theta) + V_p \cdot \sin(\phi - \theta)$$

Resulting Relations

$$RR'' + \dot{R}^2 + 2cR = 0$$

Where R is derived as:

$$R = (R_0 - R_t^2 \cdot V_{R_0})$$

Final relationships:

$$V_{r0} = -\frac{V_\theta}{V_r}$$

Proportional Navigation (PN)

In Proportional Navigation, the acceleration command is given by:

$$a_P = N\dot{\lambda}$$

Where N is the navigation constant, and $\dot{\lambda}$ is the rate of change of the line of sight angle. This law is effective for guiding missiles or autonomous vehicles to intercept moving targets.

Pure Proportional Navigation (PPN)

For a special case of Proportional Navigation, called Pure Proportional Navigation (PPN), the acceleration is proportional to the rate of change of the angle:

$$a_P = N\dot{\lambda}$$

Filler text: Proportional Navigation is commonly used in missile guidance systems, where the objective is to minimize the miss distance by adjusting the flight path in response to the relative motion of the target.

Conclusion

In summary, the Derived Pure Pursuit and Proportional Navigation laws provide a robust framework for guiding autonomous systems or missiles towards moving targets. The interplay between the pursuer's and target's velocities and headings forms the basis of these guidance laws.

True Proportional Navigation (TPN)

The True Proportional Navigation (TPN) guidance law is given by the following equations:

$$a_P = N\dot{\lambda} = v_P\dot{\theta}$$

Where a_P represents the acceleration of the pursuer, N is the navigation constant, and $\dot{\lambda}$ is the rate of change of the line of sight angle.

Filler text: The TPN guidance law modifies the basic pursuit law to more accurately predict the behavior of the target, particularly in cases where the target performs complex maneuvers.

The radial (v_R) and tangential (v_I) components of the velocity are described as:

$$v_R = v_P\cos(\theta_T - \theta) - v_T\cos\delta$$
$$v_I = v_P\sin(\theta_T - \theta) - v_T\sin\delta$$

These two equations provide the velocity components along the radial and inertial directions, respectively.

Collision Avoidance

To ensure the pursuer avoids collision with the target, the following conditions must be satisfied:

$$v_R < 0 \quad \text{(for closing in)}$$
$$v_P\sin\delta = v_T\sin(\theta_T - \theta)$$

Filler text: In collision avoidance, it is crucial to adjust the relative velocity components such that the pursuer approaches the target safely without causing a collision.

Intercept Geometry and Stability Analysis

At the intercept point, we use the following equation to predict the geometry of the intercept:

$$R = R_0 e^{\lambda t}$$

where R_0 is the initial distance between the pursuer and the target, and λ is the rate of approach. The rate of change of distance ($\dot{R}$) is expressed as:

$$\dot{R} = -v_P \sin(\theta_T - \theta)$$

Filler text: The intercept geometry is critical for determining when and where the pursuer will meet the target. The stability of the intercept is analyzed by examining how quickly the pursuer can close the distance to the target while maintaining the correct heading.

Evaluating TPN for Different Navigation Constants

We next evaluate how the navigation constant N affects the stability of the pursuit. For different values of N, the behavior of the system changes:

$$\dot{\theta_P} = \frac{v_P}{R} \sin(\theta_T - \theta)$$

Filler text: By adjusting the navigation constant N, we can influence the stability and responsiveness of the guidance system. Higher values of N may lead to more aggressive pursuit behavior, while lower values ensure smoother trajectories.

The different cases for the velocity ratios v_P and v_T are explored to further refine the guidance:

$$N = \frac{v_P}{v_T}$$

The next step involves calculating the time to intercept under different conditions, particularly when the pursuer has a higher velocity than the target:

$$t_c = \frac{R_0}{v_P - v_T}$$

Case Analysis for Stability

For different cases of intercept stability, we evaluate the pursuer's behavior: - *Case 1:* When $N < 1$, the pursuer may not intercept the target and tends to lag behind. - *Case 2:* When $N = 1$, the pursuer has a constant closing rate. - *Case 3:* When $N > 1$, the pursuer will quickly intercept the target and may lead to an over-correction.

Filler text: The stability of the guidance system is highly dependent on the choice of N. In real-world applications, tuning this constant is critical for achieving optimal interception without overshooting or losing track of the target.

Conclusion

The analysis of True Proportional Navigation (TPN) and the conditions for collision avoidance demonstrate the importance of careful tuning in guidance systems. By controlling the navigation constant and monitoring the relative velocities of the pursuer and target, we can optimize the system for stable and effective pursuit.

Chapter 8

Modern Guidance Laws

8.1 Linearised Engagement Geometry-Based

8.1.1 Near Collision Course (NCC)

Most of the modern guidance laws work on NCC conditions at homing phase.

Near collision course is characterised as

- Angle between $\vec{R}$ and $\dot{\vec{R}}$ is 'very' small
- V_c (closing speed) is 'almost' constant, $V_c = |-V_R|$

The engagemt is very close to collision course, which imples

$$V_P sin(\alpha_{P_N} - \theta) = V_T sin(\alpha_T - \theta)$$

where α_{P_N} is nominal pursuer heading and α_P is pursuer heading.

Assumptions for Linearised Engagement

1. $\theta \leq 1 \implies \theta(t) \approx \frac{y(t)}{x(t)}$
2. $\alpha_{P_N} \approx \alpha_{P_{N_0}}$
3. $x(t) = R(t)cos(\theta(t)) \approx R(t)$

8.2 Sliding Mode-Based Guidance

Sliding mode control helps in maintaining robust guidance under varying target behaviors and uncertainties.

8.3 Terminal Angle Control

A technique used to ensure that the pursuer approaches the target at a desired terminal angle.

8.4 Time-to-Go and Final Time Control

This method calculates the remaining time before interception and adjusts the guidance commands accordingly.

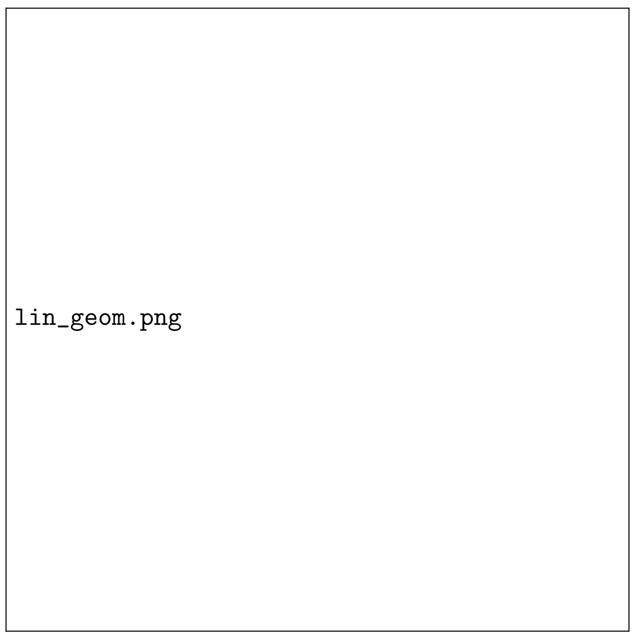

Figure 8.1: Linearised Geometry