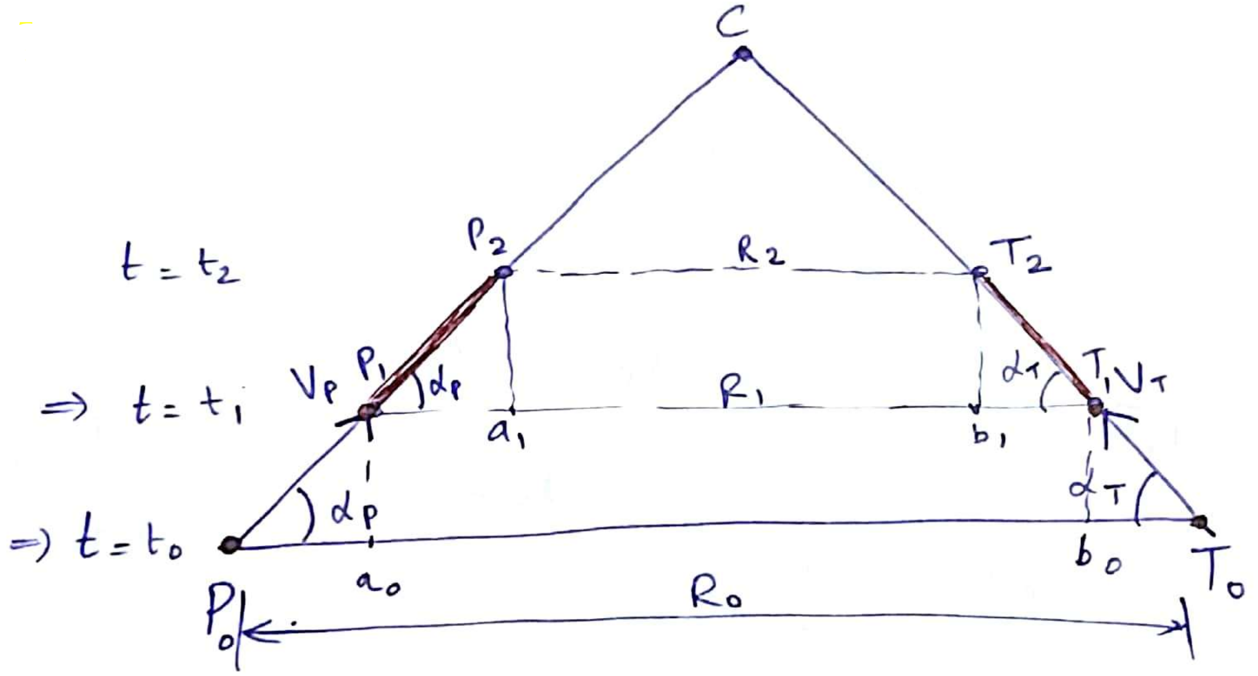
Assignment -1 hints

Part B

Q1.



V_p Si

=

$V_P, \alpha_P, V_T, \alpha_T$ are Constant

and $V_P \sin(\alpha_P - \theta) = V_T \sin(\alpha_T - \theta)$

At Time $t = t_0$ R_0 using Simillar triangles property.

$t = t_1$ R_1

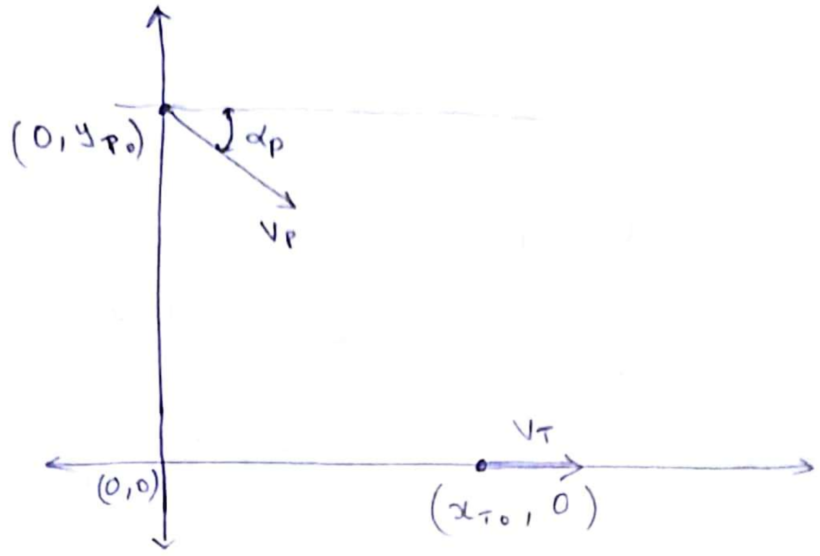
$P_0 a_0 = P_1 a_1$ and $b_0 T_0 =$

$= V_P \Delta t \cos(\alpha_P)$ $V_P \Delta t$

$$\boxed{\left.\frac{dR}{dt}\right|_{t_1} = \frac{R_0 - R_1}{\Delta t} = \frac{R_0 - (R_0 \mp P_1 a_1 + b_1 T_1)}{\Delta t} = V_P \cos(\alpha_P) + V_T \cos(\alpha_T)}$$

Same for .

Q2.



I) Position in terms of velocity.

$$(x_T(t), y_T(t)) = (V_T t + x_{T_0}, 0)$$

$$(x_p(t), y_p(t)) = (V_p \cos(\alpha_p) t, y_{P_0} + V_p \sin(\alpha_p) t)$$

II) $$R(t) = \sqrt{[x_T(t) - x_p(t)]^2 + [y_T(t) - y_p(t)]^2}$$

$$= \sqrt{a t^2 + b t + c}$$

where $a = V_T^2 - 2 V_T V_p \cos(\alpha_p) + V_p^2$

$b = 2(x_{T_0} V_T - x_{T_0} V_p \cos \alpha_p + y_{P_0} V_p \sin \alpha_p)$

$C = x_{T_0}^2 + y_{P_0}^2$

III) $R(t)$ should decrease for some time

$\dot{R}(t_0) < 0$ and $t_{miss} > 0$

$\Downarrow$

$$\theta_0 - \cos^{-1}\left(\frac{1}{v} \cos(\theta_0)\right) < \alpha_p < \theta_0 + \cos^{-1}\left(\frac{1}{v} \cos(\theta_0)\right)$$

$\frac{IV}{1-b)}$ $t = t_{min}$ when $\dot{R} = 0$

$$\Rightarrow \boxed{t_{min} = \frac{-b}{2a}}$$

$$\Rightarrow \nu \cos(\alpha_p - \theta_o) > \cos\theta_o$$

$$\Rightarrow \theta_o - \cos^{-1}\left(\frac{1}{\nu}\cos\theta_o\right) < \alpha_p < \theta_o + \cos^{-1}\left(\frac{1}{\nu}\cos\theta_o\right)$$

$$R_{min} = R(t_{min})$$

$$\boxed{R_{min} = c - \frac{b^2}{4a}}$$

$\underline{V} \Rightarrow$ For collision $\dot{\theta} = 0$ and $\dot{R}_o < 0$

$$\left.\begin{array}{l} \dot{\theta} = 0 \quad \Rightarrow \quad \alpha_p = \theta_o - \sin^{-1}\left(\frac{1}{\nu}\sin(\theta_o)\right) \\ \dot{R} = V_T\cos\theta_o - V_p\cos\left(\sin^{-1}\left(\frac{1}{\nu}\sin\theta_o\right)\right) < 0 \end{array}\right\}$$

Time for Collision $\boxed{t_f = \frac{-R_o}{V_{R_o}}}$

Q3. i) when $\gamma_T = \gamma_P = \gamma$

$$R^2 = \left(x_T(t) - x_p(t)\right)^2 + \left(y_T(t) - y_P(t)\right)^2$$

$$= R_P^2 + (R_T - R_P)^2 + 2R_P(R_T - R_P)\cos(\gamma t)$$

$$\dot{R} = 0 \Rightarrow \quad \gamma t = n\pi \ldots$$

From $\ddot{R} \Rightarrow \gamma t = 0, 2\pi \Rightarrow R_{min}$

$\gamma t = \pi \Rightarrow R_{max}$

$\boxed{R_{min} = R_T}$ won't intercept

$$\boxed{t_{min} \Rightarrow \frac{2\pi}{\gamma}}$$

ii) when $\gamma_T \neq \gamma_P$

$$R^2 = 2R_P^2 + R_T^2 + 2R_T\left[R_T\left(\cos\gamma_T t - \cos(\gamma_P - \gamma_T)t\right) - R_P\cos(\gamma_P t)\right]$$

$-2, +2$ $\qquad -1, +1$

$$\underbrace{R_T(R_T - 4R_P)} \leq R^2(t) < (2R_P + R_T)^2$$

If $R_T^2 - 4R_PR_T > 0$ then $R_{min} > 0$ No intecept

$\Downarrow$

$R_T = \frac{V_T}{\gamma_T}$ $\quad R_P = \frac{V_P}{\gamma_P}$ $\Rightarrow$ $\boxed{\frac{V_P}{V_T} > 4\gamma}$

Part C

$$\dot{R} = V_T \cos\psi - V_P$$

$$R\dot{\theta} = V_T \sin\psi$$

$$\therefore \frac{\dot{R}}{R\dot{\theta}} = \frac{V_T \cos\psi - V_P}{V_T \sin\psi}$$

$\psi = \alpha_T - \theta$

$\Rightarrow \dot{\psi} = -\dot{\theta}$

$$\boxed{R(\psi) = \frac{C_0}{2} \frac{\sin^{\nu-1}(\psi/2)}{\cos^{\nu+1}(\psi/2)}} = C_0 \frac{\tan^{\nu}(\psi/2)}{\sin\psi}$$

$$\text{And, } C_0 = \frac{R_0 \sin\psi_0}{\tan^{\nu}(\psi_0/2)}$$

ii) $\psi = \alpha_T - \theta$

$$\Rightarrow \dot{\psi} = -\dot{\theta} = -\frac{V_\theta}{R} = -\frac{V_T \sin\psi}{R}.$$

trajectory of a PP-guided pursuer in an engagement against a nonstationary non-maneuvering target remains in one half-plane

iii) $$R(\psi) = C_0 \frac{\tan^{\nu}(\psi/2)}{\sin\psi} = \frac{C_0}{2\cos^2\psi/2} = \frac{C_0}{1+\cos\psi}$$

For parabolic $r = \frac{p}{1+e\cos\theta}$, $e=1$.

$$\therefore \boxed{R(\psi) = \frac{C_0}{1+\cos\psi}}$$ is Parabolic.

$\Rightarrow$ (a) $t \to t_f$

$V_\theta = 0$

$\Rightarrow \quad \theta = \alpha_T$. $\quad$ i.e. $\psi = 0$

$$R_f(\psi) = \frac{C_0}{1+1} = \frac{C_0}{2}.$$

iv) $$a_p = V_p \dot{\alpha}_p = V_p \dot{\theta} = \frac{V_p V_\theta}{R}.$$

$$\therefore a_p = \frac{V_p V_T \sin\psi \sin\psi}{C_0 \tan^{\nu}(\psi/2)}.$$

$$= \frac{4 V_p V_T}{C_0} \cdot \frac{\cos^{\nu+2}(\psi/2)}{\sin^{\nu-2}(\psi/2)}$$

For $a_{p\,max}$:

$$\frac{d\,a_p}{dt} = 0, \quad \Rightarrow \quad \boxed{\psi_{max} = \cos^{-1}(\nu/2)}$$

$$\boxed{\therefore a_{p\,max} = \frac{V_T^2}{C_0} \frac{\nu}{4} \sqrt{(\nu+2)^{\nu+2} (2-\nu)^{2-\nu}}}$$

Part c

(5) $\dot{\alpha}_T = c_T \dot{\theta}$, $\alpha_p = \theta$

$V_R = V_T \cos(\alpha_T - \theta) - V_p$ $\quad K = 1 - c_T$

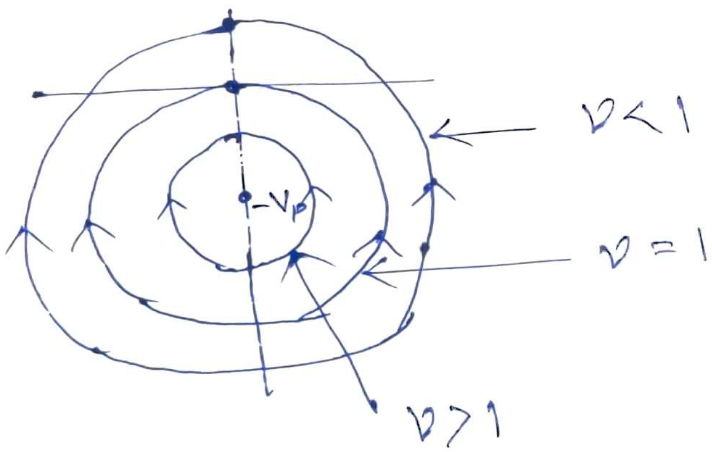
$V_\theta = V_T \sin(\alpha_T - \theta)$

$\therefore V_\theta^2 + (V_R + V_p)^2 = V_T^2$

$\dot{V}_R = K\dot{\theta} V_\theta \quad || \quad \dot{V}_\theta = -K\dot{\theta}(V_R + V_p)$

Case 1 $\quad c_T < 1, \quad K = 1 - c_T > 0$

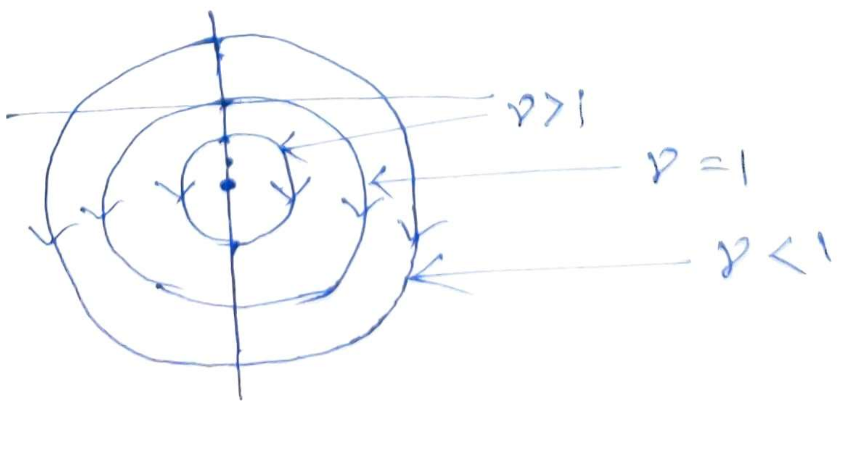
$\therefore K = +ve$.



Case 2 $\quad c_T = 1, \quad K = 1 - 1 = 0$

$\dot{V}_R = 0, \quad \dot{V}_\theta = 0$

Case 3 $\quad c_T > 1$



Condition for sucessfull capture!

Case 1! $C_T < 1$.

$$(\dot{V}_R + V_p)^2 + V_\theta^2 = V_T^2.$$

$$\gamma > 1$$

Case 2 $C_T = 1$

Every point for $V_R < 0$.

Case 3 $C_T > 1$

Entire (V_R, V_θ) Space except +ve V_R axis

$\Rightarrow$ For t_{min}, $V_R = 0$.

$$\Rightarrow \psi_{min} = \cos^{-1}(\gamma)$$

R(ψ):-

$$R(\psi) = C_0 \left[\frac{\tan^{\nu}(\psi/2)}{\sin(\psi)}\right]^{\frac{1}{1-C_T}}$$

$$a_p(\psi) = \frac{V_p V_T}{C_0} \frac{(\sin\psi)^{\frac{2-C_T}{1-C_T}}}{(\tan^{\nu}(\psi/2))^{\frac{1}{1-C_T}}}$$

$$C_0 = R_0 \left[\frac{\sin\psi_0}{\tan^{\nu}\psi_0/2}\right]^{\frac{1}{1-C_T}}$$

For a_p max:-

$$\frac{da_p}{dt} = 0$$

$$\Rightarrow \quad \psi = \cos^{-1}\left(\frac{\nu}{2-C_T}\right)$$

$$\therefore \quad a_{p_{max}} = \frac{V_p V_T}{C_o} \frac{\left(\sin^{-1}\left(\cos^{-1}\left(\frac{\nu}{2-C_T}\right)\right)\right)^{\frac{2-C_T}{1-C_T}}}{\left[\tan^{\nu}\left(\frac{1}{2}\cos^{-1}\left(\frac{\nu}{2-C_T}\right)\right)\right]^{\frac{1}{1-C_T}}}$$