Quiz 1

Course No. – AS 5570 Quiz – 1 Full Marks: 30

Principles of Guidance for Autonomous Vehicles

Date: September 4, 2024 Time: 14:00 – 15:00

Important:

1. You may **carry one A4 sheet as your supporting study material** during the test.
2. **Engagements** are **planar** unless otherwise mentioned.
3. In numerical problems, **use 3-4 digits after decimal point**, and write the **final answers inside boxes**.
4. **Cell phone and any other electronic gadgets except calculators are not allowed** in the exam.
5. Full marks will be **scaled down to 15**, and your marks will be accordingly scaled while adding to your semester marks.

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1) Answer the followings. (0.5*4 = 2)

   a. Target acquisition is normally of prime importance in which phase of an engagement?

      i. Initial phase ii. Midcourse phase iii. Endgame phase

   b. Which one below is correct ordering in terms of autonomy level?

      i. Active Human in the Loop < On-Board Decision Support < Fully Autonomous
      ii. Active Human in the Loop < Fully Autonomous < On-Board Decision Support
      iii. On-Board Decision Support < Active Human in the Loop < Fully Autonomous

   c. Collision triangle geometry is characterized by

      i. Zero range rate
      ii. Zero LOS turn rate
      iii. Both of the above

   d. Pure Pursuit geometry is characterized by

      i. Target's heading angle is same as Pursuer's heading angle
      ii. Target's heading angle is same as LOS angle
      iii. LOS angle is same as Pursuer's heading angle

2) Answer the followings.

   a. Discuss basic differences between ~~path planning,~~ navigation, guidance and vehicle control. (0.5*4=2)

   b. Mention two distinct advantages and two distinct challenges of unmanned vehicle systems. (0.5*4=2)

   c. Explain the difference between Human-in-the-loop and Human-on-the-loop autonomy levels. (0.5*2=1)

   d. Mention two reasons why the achieved lateral acceleration is, in general, not same as the commanded lateral acceleration. (0.5*2=1)

   e. In an engagement between a Pure Pursuit (PP)-guided higher speed pursuer with speed V_P and a lower speed non-maneuvering target, it is found that the initial range rate $V_{R_0} < -V_P$. Which one below is correct on the initial trend of variation of LOS angular rate $|\dot{\theta}|$? **Why?** [**Note:** $|\dot{\theta}|$ is the magnitude of LOS turn rate.] (2)

      i. Initially increasing $|\dot{\theta}|$
      ii. Initially decreasing $|\dot{\theta}|$
      iii. Cannot be inferred as it depends on the initial range

   f. Consider an engagement between a non-maneuvering pursuer with speed V_P against a non-maneuvering target with speed V_T. They attain Pure Pursuit (PP) course geometry just when they come nearest to each other. Justify whether $V_P \geq V_T$ or $V_T \geq V_P$. (2)

$\dot{\alpha}_T = 0$

$V_R \leq 0, \quad \dot{\theta} = 0$

$\alpha_T = 0$

$V_T - V_P \leq 0$

$V_T \leq V_P$

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Quiz I Quiz II/ Mid-Sem End-Semester Make-up Date : 04/09/24

Semester & Degree : Course No. AS5570 Part :

Total No. of Pages

Question No.	1	2	3	4	5	6	7	8	9	10
Marks										

11	12	13	14	15	16	17	18	19	20	Total
										12·25

Answer on both sides of the paper including the space below

1 (a) (iii) Endgame phase 0.25

(b) (iii) On Board Decision Support < Active Human in the Loop < Fully Autonomous

1.75 (c) (ii) Zero LOS turn rate

(d) (iii) LOS angle is same as Pursuer's heading angle.

2 (a) Navigation involves determination of a method for estimating the ~~state~~ configuration including position, speed, heading along the nominal path in the environment.

Guidance is determining the strategy for following a nominal path in the presence of off-nominal conditions like surrounding environments, disturbances and other uncertainties.

1.5

Vehicle control involves maintaining the speed and/or angular orientation of the UxV in compliance with the formed guidance strategy.

(b) Advantages of unmanned vehicle systems :

→ Cost effectiveness and portability, especially when large number of systems are involved

→ Reduced human risk, mainly in unknown & hazardous environment

2

Challenges of unmanned vehicle systems :

→ Improved perception of the surroundings

→ Developing robust communication channels.

(c) Human in the loop has a more active ~~task~~ responsibility for the user in making decisions for regular usage.

Human on the loop is more of a supervisory role and requires human intervention for more important decisions.

1

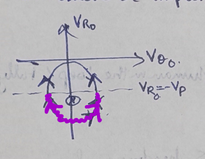
(d) Reasons why achieved lateral acceleration is generally not same as commanded lateral acceleration

— latency in control systems in receiving the command or latency in ~~control~~ systems to implement the command

1

→ disturbances from the environment can obstruct the UxV to achieve or implement the commanded state

(e)



$(V_{R_0} + V_P)^2 + V_{\theta_0}^2 = V_T^2 \rightarrow V_T < V_P$

$V_{R_0} = \dot{R}_0, \quad V_{\theta_0} = R_0\dot{\theta}_0$ R↓

$\dot{V}_R = V_\theta \dot{\theta} = \frac{V_\theta^2}{R}, \quad \dot{V}_\theta = -\dot{\theta} V_R = -\dot{\theta}\dot{R}$

$V_R < 0$ (decreasing). $|\dot{V}_\theta| > 0$

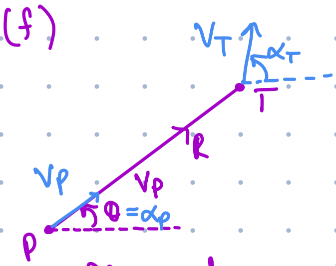
1.75

(i) Initially increasing.

$\dot{V}_{R_0} = \frac{V_{\theta_0}^2}{R} > 0 \rightarrow |V_\theta|$ increasing $\rightarrow \dot{\theta}_0 = \frac{-V_{\theta_0}}{R_0}$

$R > 0, |\dot{\theta}| > 0$

(f)



PP geometry

$V_R = V_T\cos(\alpha_T - \theta) - V_P$

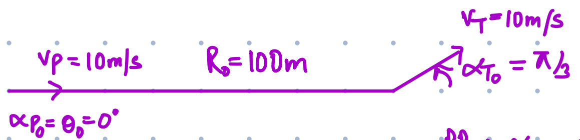
$R\dot{\theta} = V_\theta = V_T\sin(\alpha_T - \theta)$

~~$\dot{V}_R = V_\theta\dot{\theta}, \quad \dot{V}_\theta = -\dot{\theta}V_R$~~

$V_T \le V_P$.

8. Consider an engagement between a PP-guided pursuer with speed $V_P = 10$m/s, initial heading angle $\alpha_{P_0} = \theta_0 = 0$ and an equal speed target with $V_T = 10$m/s, initial heading angle $\alpha_{T_0} = \pi/3$, and turn rate $\dot\alpha_T = \dot\theta$. The initial range $R_0 =$ 100m. At which final time t_f the pursuer will intercept the target? ***Justify.*** (2)

   i. $t_f = 10$sec
   ii. $t_f = 20$sec
   iii. $t_f = \infty$ as interception will never occur



PP → $\alpha_P = \theta$

⇒ $\dot\alpha_P = \dot\theta = 0$?

$\dot\alpha_T = \dot\theta$

$V_R = \dot R = V_T\cos(\alpha_T - \theta) - V_P\cos(\alpha_P - \theta)$

$\dot V_R = -V_T\sin(\alpha_T-\theta)\cancelto{0}{(\dot\alpha_T-\dot\theta)} + V_P\sin\cancelto{0}{(\alpha_P-\theta)}\cancelto{0}{(\dot\alpha_P-\dot\theta)}$

$V_\theta = R\dot\theta = V_T\sin(\alpha_T-\theta) - V_P\sin(\alpha_P-\theta)$

$\dot V_R = 0 \to \dot R = const$

$V_{R_0} = 10\cos\pi/3 - 10\cos 0 = -5$ m/s

$\dot V_\theta = V_T\cos(\alpha_T-\theta)\cancelto{0}{(\dot\alpha_T-\dot\theta)} - V_P\cos\cancelto{1}{(\alpha_P-\theta)}\cancelto{0}{(\dot\alpha_P-\dot\theta)}$

$V_{\theta_0} = 10\sin\pi/3 - 10\sin 0 = 5\sqrt{3}$ m/s

$\dot V_\theta = 0 \to R\dot\theta = const$

$$\frac{dR}{dt} = -5$$

$$\int_{100}^{0} R = -5\int_0^{t_f} dt$$

$$(0-100) = -5(t_f - 0)$$

$$\boxed{t_f = 20s}$$

but

$$R - 100 = -5(t - 0)$$

$$R = -5t + 100$$

$$R\,d\theta = 5\sqrt{3}\,dt$$

$$\int_0^\theta d\theta = \int_0^t \frac{\cancel{5}\sqrt{3}}{-\cancel{5}t + \cancel{100}20}\,dt$$

$$\theta = -\sqrt{3}\ln\left(\frac{t-20}{20}\right)$$

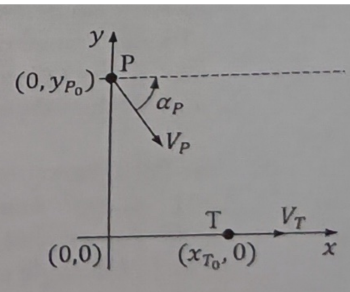
$\theta \to 0$ as $t \to 20$

$$\boxed{\therefore\ t_f \to \infty \text{ for interception}}$$

Consider $\upsilon = \frac{V_P}{V_T} = 1.25$, $x_{T_0} = 300$ unit and $y_{P_0} = 750$ unit.

a. For what value of Pursuer's heading angle α_P the miss distance (r_{miss}) is zero? What is the time required to achieve zero miss in that case? (2)

b. For other values of α_P, express the steady state LOS angle θ_{SS} in terms of α_P. (1)



(a)

$\Delta x_P = \Delta x_T$

$V_P\cos\alpha_P t = x_{T_0} + V_T t$

$\Delta y_P = \Delta y_T$

$V_P\sin\alpha_P t - y_0 = 0$

$$t = \frac{y_0}{V_P\sin\alpha_P}$$

$$(V_P\cos\alpha_P - V_T)\frac{y_0}{V_P\sin\alpha_P} = x_{T_0}$$

$$\left(\cos\alpha_p - \frac{1}{\nu}\right)\frac{y_0}{\sin\alpha_p} = x_{T_0}$$

$$\cos\alpha_p - \frac{4}{5} = \frac{2\cancel{30}}{5\cancel{750}}\sin\alpha_p$$

$$\cos\alpha_p = \frac{2}{5}\sin\alpha_p + \frac{4}{5}$$

$$1-\sin^2\alpha_p = \frac{4}{25}\left(\sin^2\alpha_p + 4\sin\alpha_p + 4\right)$$

$$29\sin^2\alpha_p + 16\sin\alpha_p - 9 = 0$$

$$\sin\alpha_p = \frac{-16 \pm \sqrt{256 + 4\times 9\times 29}}{2\times 29}$$

$$\alpha_p = \sin^{-1}(\quad)$$

(b) θ_{ss} , $\dot{\theta} = 0$

$$R = \Delta x_T - \Delta x_p = \sqrt{\left(x_{T_0} + V_T t - V_p\cos\alpha_p t\right)^2 + \left(y_0 - V_p\sin\alpha_p t\right)^2}$$

$$V_\theta = V_T\sin(\cancel{\alpha_T}^{\,0} - \theta) - V_p\sin(\alpha_p - \theta) = 0$$

$$-V_T\sin\theta_{ss} = V_p\sin(\alpha_p - \theta_{ss})$$

$$\sin\theta_{ss} = -\nu\sin(\alpha_p - \theta_{ss})$$

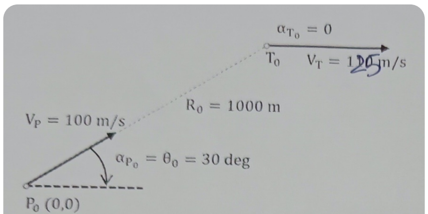
$$\sin\theta_{ss} = -\nu\left(\sin\alpha_p\cos\theta_{ss} - \cos\alpha_p\sin\theta_{ss}\right)$$

$$1 = -\nu\left(\sin\alpha_p\cot\theta_{ss} - \cos\alpha_p\right)$$

$$\frac{-1}{\nu} + \cos\alpha_p = \sin\alpha_p\cot\theta_{ss}$$

$$\theta_{ss} = \cot^{-1}\left(\cot\alpha_p - \frac{1}{\nu}\operatorname{cosec}\alpha_p\right)$$

4) Consider a Pure Pursuit (PP)-guided pursuer with speed $V_P = 100$m/s against a higher speed target with speed $V_T = 125$m/s. Initial engagement parameters: $R_0 = 1000$m, $\alpha_{P_0} = \theta_0 = \pi/6$, $\alpha_{T_0} = 0$.



a. Compute V_{R_0} and V_{θ_0}.

(1)

$$V_{R_0} = V_T\cos(\alpha_{T_0} - \theta_0) - V_p\cos(\alpha_{P_0} - \theta_0) = 125\cos -30 - 100\cos 30$$
$$= 12.5\sqrt{3}\ \text{m/s}$$

$$V_{\theta_0} = V_T\sin(\alpha_{T_0} - \theta_0) - V_p\sin(\alpha_{P_0} - \theta_0) = 125\sin -30 - 100\sin 30$$
$$= -112.5\ \text{m/s}$$

$$R_0 = 1000\text{m} \rightarrow \dot{\theta}_0 = \frac{-112.5}{1000} = -0.1125\ \text{rad/s}$$

b. **Scenario 1:** Target remains non-maneuvering.

i. Explain what happens to the states on the (V_θ, V_R)-plane by considering the equilibrium and infer whether interception would take place. (1)

ii. Express the set of points of the engagement trajectory on the (V_θ, V_R)-plane and draw the same. Highlight the direction of movement and the start and end points on this trajectory. (1.5)

iii. Schematically plot the variations of R, $\dot{\theta}$ and θ with time highlighting points of interest (start and end points, maximum and/or minimum points, etc.). Give justification. (3)

iv. Find final range rate V_{R_f}. (0.5)

$V_R = V_T\cos(\cancel{\alpha_T}^{\,0} - \theta) - V_P\cos(\cancel{\alpha_P - \theta}^{\,0})$ (PP)

$\Rightarrow V_R = V_T\cos\theta - V_P \Rightarrow \dot{V}_R = -V_T\sin\theta\,\dot{\theta} = V_\theta\dot{\theta}$ $\left(\dot\theta = \frac{V_\theta}{R}\right)$

$V_\theta = V_T\sin(\cancel{\alpha_T}^{\,0} - \theta) - V_P\sin(\cancel{\alpha_P - \theta}^{\,0})$ (PP)

$\Rightarrow V_\theta = -V_T\sin\theta \Rightarrow \dot{V}_\theta = -V_T\cos\theta\,\dot\theta = -(V_R+V_P)\dot{\theta}$ $\left(\dot\theta = \frac{V_\theta}{R}\right)$

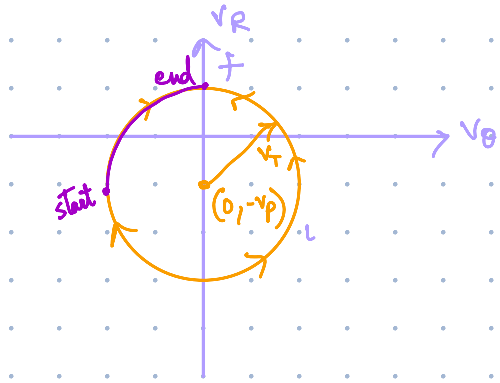
(i) eqb^m $\rightarrow$ $V_\theta = 0 \rightarrow \sin\theta = 0$

$\theta = n\pi$

(ii)

$(V_R+V_P)^2 + V_\theta^2 = V_T^2$

no interception

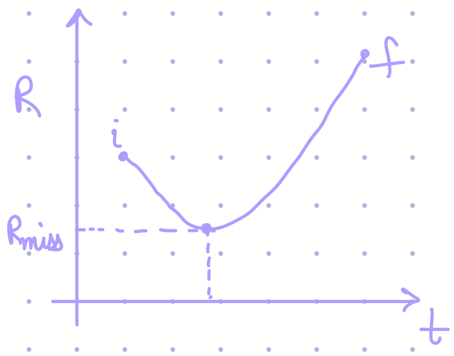


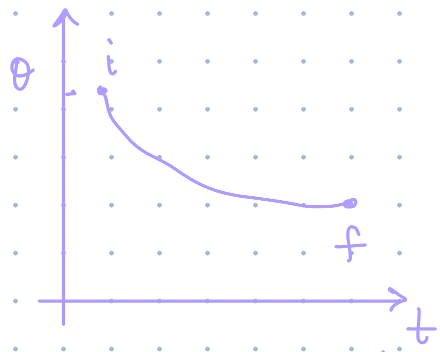
(iii)

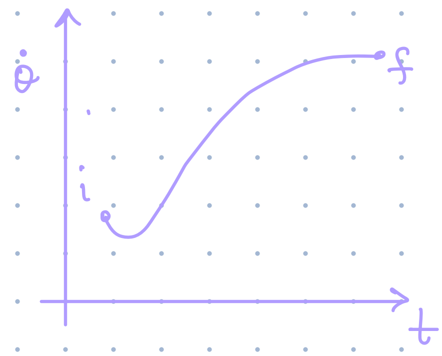
$V_R<0$ then $V_R>0$

$R\dot\theta = V_\theta < 0$ increases to zero

$\dot\theta = \dfrac{V_\theta}{R}$ ← decreasing; R ← decrease then increase







miss distance, no interception

increases to a constant

(iv) $V_{R_f} = V_T\cos\theta_f - V_P = 125\cos\theta_f - 100 = 125(-1) - 100 = -225\ \text{m/s} = V_{R_f}$

$V_{\theta_f} = 0 \Rightarrow -V_T\sin\theta_f = 0 \Rightarrow \theta_f = \pi$

c. **Scenario 2:** Target starts maneuvering with $\dot{\alpha}_T = 0.05 + 2\dot{\theta}$.

i. Explain what happens to the states on the (V_θ, V_R)-plane by considering the equilibrium and infer whether interception would take place. (2.5)

ii. Express the set of points of the engagement trajectory on the (V_θ, V_R)-plane and draw the same. Highlight the direction of movement and the start and end points on this trajectory. (1.5)

iii. Schematically plot the variations of R with time highlighting points of interest (start and end points, maximum and/or minimum points, etc.). Give justification. (1)

iv. Comment on the Pursuer's final lateral acceleration requirement. (1)

v. Final range rate V_{R_f} ?

$\dot\alpha_T = 0.05 + 2\dot\theta$

$(V_R+V_P)^2 + V_\theta^2 = V_T^2$

$V_R = -V_P \Rightarrow V_\theta\big|_{max} = V_T$

(i) (ii)

$$V_R = V_T\cos(\alpha_T-\theta) - V_P\cos(\cancel{\alpha_P-\theta}) \quad \text{(PP)}$$

$$\Rightarrow V_R = V_T\cos(\alpha_T-\theta) - V_P \longrightarrow \dot V_R = -V_T\sin(\alpha_T-\theta)\,(\dot\alpha_T-\dot\theta)$$

$$\Rightarrow \dot V_R = V_\theta(\dot\alpha_T-\dot\theta) = V_\theta(0.05+\dot\theta)$$

$$V_\theta = V_T\sin(\alpha_T-\theta) - V_P\sin(\cancel{\alpha_P-\theta})^{\,0} \quad \text{(PP)}$$

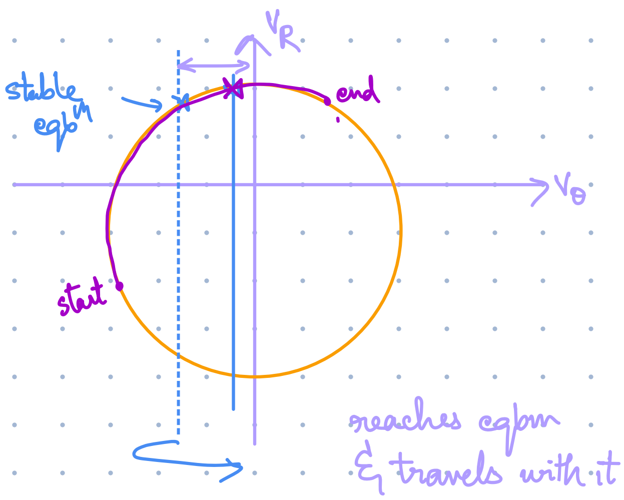
$$\Rightarrow V_\theta = -V_T\sin(\alpha_T-\theta) \longrightarrow \dot V_\theta = -V_T\cos(\alpha_T-\theta)\,(\dot\alpha_T-\dot\theta)$$

$$\Rightarrow \dot V_\theta = -(V_R+V_P)(\dot\alpha_T-\dot\theta)$$

$$\dot V_\theta = -(V_R+V_P)(0.05+\dot\theta)$$

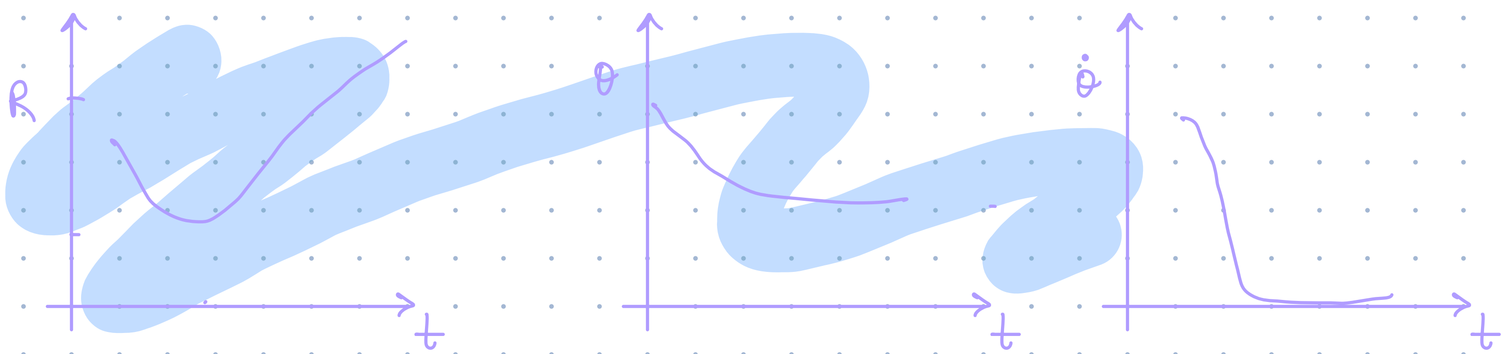
$$\longrightarrow R\dot V_\theta = -(V_R+V_P)(0.05R+\underbrace{\cancel{R\dot\theta}}_{V_\theta})$$

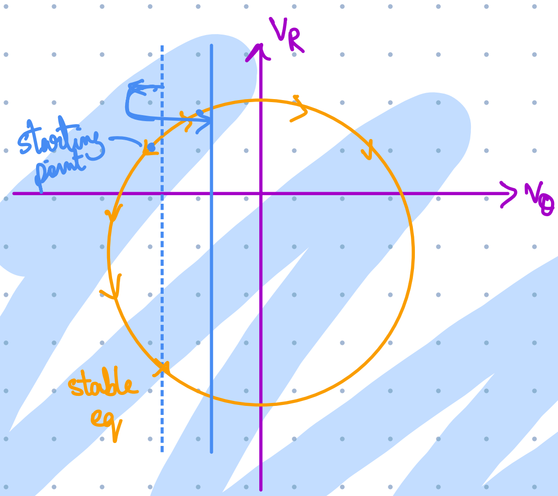
$$\longrightarrow R\dot V_R = V_\theta(0.05R+V_\theta)$$



$$R_0\dot V_{R_0} = V_{\theta_0}(0.05R_0+V_{\theta_0})$$
$$= -112.5\,(0.05\times1000 - 112.5)$$
$$= -ve \times -ve = +ve$$

(iii)





$$\dot\alpha_T = \omega_T + c_T\dot\theta$$

we want to achieve collision course before $\dot\theta = 0$., lateral acc^n spikes up

Quiz 2

1) Answer the followings: **(0.5*2+1.5*2=4)**

a. LOS Guidance is a:

i. two-agent guidance; ii. three-agent guidance; iii. Four-agent guidance

b. True Proportional Navigation (TPN) Guidance command includes:

i. Only lateral acceleration,

ii. Only longitudinal acceleration

iii. Both longitudinal and lateral accelerations

c. Which option below is right for the gain of TPN guidance command for successful capture of a non-maneuvering target? **Justify.**

i. Only positive

ii. Only negative

iii. Both positive and negative.

d. The deviation angle of Deviated Pure Pursuit (DPP) Guidance should lie in which interval? **Justify.**

look at notes

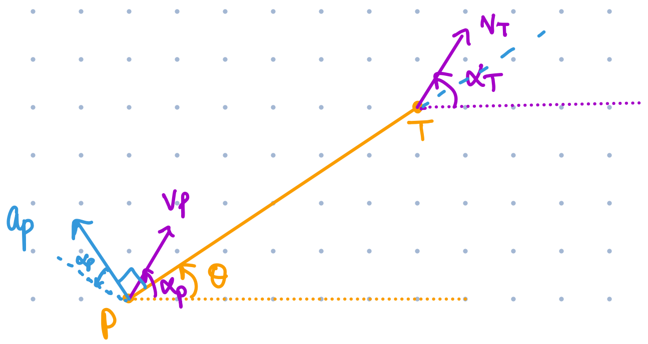
look at notes

2) Mention the differences between Beam Rider and Command-to-Line-Of-Sight implementations of LOS Guidance. **(2)**

BR	CLOS
• calculations are done onboard	• calculations are done by base station
• tracks beam & minimise distance with LOS	• base station tracks both pursuer & target
	$\ddot{\theta}_T = \ddot{\theta}_p$, $\dot{\theta}_T = \dot{\theta}_p$
$a_p^{BR} = kR_p(\theta_T - \theta_p)$	$a_p^{CLOS} = kR_p(\theta_T - \theta_p) + R_p\ddot{\theta}_T + 2\dot{R}_p\dot{\theta}_T$

3) Consider an engagement between a non-maneuvering moving target and a TPN-guided pursuer ($a_P = c\dot{\theta}$).

a. Draw the engagement geometry and write the equations of engagement dynamics. **(2)**



$$a_p = c\dot{\theta}$$

$$V_R = V_T\cos(\alpha_T - \theta) - V_p\cos(\alpha_p - \theta)$$

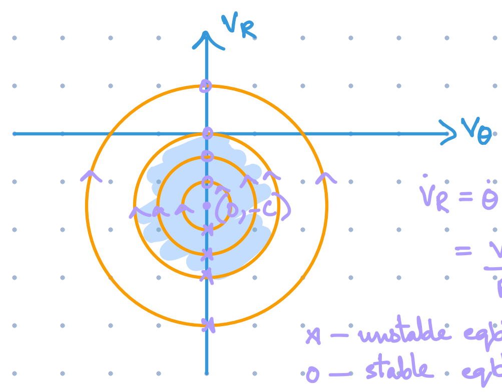
$$V_\theta = V_T\sin(\alpha_T - \theta) - V_p\sin(\alpha_p - \theta)$$

$$\dot{\alpha}_p = \frac{a_p\cos(\alpha_p - \theta)}{V_p}$$

$$\dot{V}_p = a_p\sin(\alpha_p - \theta)$$

$$\left.\begin{aligned}\dot{V}_R &= \dot{\theta}V_\theta \\ \dot{V}_\theta &= -\dot{\theta}(V_R + c)\end{aligned}\right\} \quad \frac{\dot{V}_R}{\dot{V}_\theta} = \frac{-V_\theta}{V_R + c}$$

$$(V_R + c)^2 + V_\theta^2 = (V_{R_0} + c)^2 + V_{\theta_0}^2$$



$$\dot{V}_R = \dot{\theta}V_\theta = \frac{V_\theta^2}{R} > 0$$

x – unstable eqbm.
o – stable eqbm. } $V_\theta = 0$

b. From engagement trajectories in (V_θ, V_r)-space, explain why the following conditions are necessary and sufficient for interception: (3)

i. $V_{R_0} < 0$,

ii. $c > 0$,

iii. $V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0} < 0$.

(i) we can say from (V_θ, V_R) space that any point above V_θ axis will finally reach stable eqb^m with $V_{R_f} > 0$

$\therefore V_{R_0} < 0$

(ii) $(-c, 0)$ is the centre of each trajectory in (V_θ, V_R) space

If $-c > 0$, all the stable equilibrium points will be above V_θ axis, with $V_{R_f} > 0$

$\therefore c > 0$

(iii) Any point inside the shaded region will have $V_{R_f} < 0$.

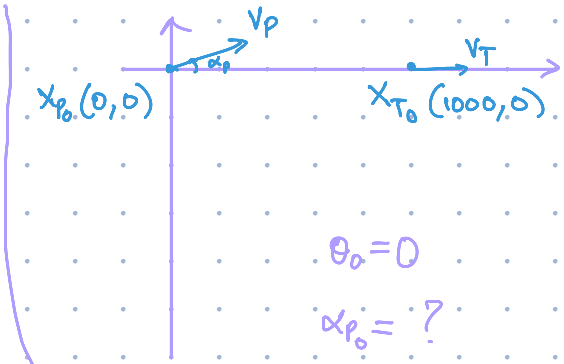
$\therefore (V_{R_0} + c)^2 + V_{\theta_0}^2 < c^2 \implies V_{R_0}^2 + V_{\theta_0}^2 + 2cV_{R_0} < 0$

c. Consider the following initial engagement geometry: $X_{P_0} = [0, 0]^T$m, $X_{T_0} = [1000, 0]^T$m, and $\alpha_{T_0} = 0$. Speeds of pursuer and target are: $V_P = 50$m/sec and $V_T = 15$m/sec.

i. Obtain the range of initial heading angles of the pursuer (α_{P_0}) that fall within the capture zone in terms of c. (1.5)

ii. Obtain the minimum value of c to ensure a non-null capture zone. (1)

iii. If $\alpha_{P_0} = \pi/3$, then justify whether it is possible to achieve interception before a final time $t_f = 100$ sec for any selected value of c? (1.5)



$\theta_0 = 0$

$\alpha_{P_0} = ?$

$V_{R_0} = \dot{R}_0 = V_T \cos(\cancel{\alpha_{T_0}} - \cancel{\theta_0}) - V_P \cos(\alpha_{P_0} - \cancel{\theta_0})$

$V_{R_0} = V_T - V_P \cos\alpha_{P_0}$

$V_{\theta_0} = R_0 \dot{\theta}_0 = V_T \sin(\cancel{\alpha_{T_0}} - \cancel{\theta_0}) - V_P \sin(\alpha_{P_0} - \cancel{\theta_0}) = -V_P \sin\alpha_{P_0}$

$(V_T - V_P\cos\alpha_{P_0})^2 + (V_P \sin\alpha_{P_0})^2 + 2c(V_T - V_P\cos\alpha_{P_0}) < 0$

$\alpha_{P_0} < \cos^{-1}\left(\dfrac{2725 + 30c}{1500 + 100c}\right)$

$V_{R_0} + 2c > 0$

$V_T - V_P\cos\alpha_{P_0} + 2c > 0$

$15 - 50\cos\alpha_{P_0} + 2c > 0$

$\alpha_{P_0} < \cos^{-1}\left(\dfrac{15 + 2c}{50}\right)$

$V_{R_0} < 0$

$V_T - V_P\cos\alpha_{P_0} < 0$

$15 - 50\cos\alpha_{P_0} < 0$

$\alpha_{P_0} < \cos^{-1}(3/10)$

$c > 0$

(ii) $(\alpha_{P_0})_{max} > 0 \rightarrow$ non-null capture zone

$$\cos^{-1}\left(\frac{2725 + 30c}{1500 + 100c}\right) > 0 \qquad (c>0)$$

$$c > 17.5 \Rightarrow c_{min} = 17.5$$

(iii) $\cos^{-1}\left(\frac{15+2c}{50}\right) > 0 \rightarrow \frac{15+2c}{50} < 1 \Rightarrow \frac{2c-35}{50} < 0 \Rightarrow c < 17.5 \Rightarrow$ no interception?

$$t_f = \frac{-R_0(V_{R_0}+2c)}{V_{R_0}^2 + V_{\theta_0}^2 + 2cV_{R_0}} = \frac{-1000\left(15-50\cos\pi/3 + 2c\right)}{\left(15-50\cos\pi/3\right)^2 + \left(-50\sin\pi/3\right)^2 + 2c\left(15-50\cos\pi/3\right)}$$

$$= \frac{-1000(-10+2c)}{10^2 + 25^2 \times 3 + 2c(-10)} = \frac{-1000(-10+2c)}{1975 - 20c} = \frac{100(2c-10)}{2c - 197.5}$$

$t_f > 0$

$V_{R_0} + 2c > 0$

$-10 + 2c > 0$

$c > 5$

$1975 - 20c < 0$

$c > 98.75$

$c > 98.75$

$t_f < 100$

$$\frac{-1000(-10+2c) - 197500 + 2000c}{1975 - 20c} < 0$$

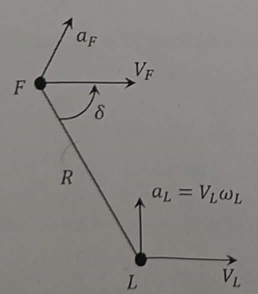
$$\frac{-187500}{1975-20c} < 0 \Rightarrow c < \frac{1975}{20} = 98.75$$

$1975 - 20c > 0$

$c > 17.5$

$c \in 17.5, 98.75$

4) As shown in the figure, consider the engagement geometry between a Leader 'L' and a Follower 'F' in a formation. Throughout the mission, L maintains constant forward speed V_L and moves in a circle with turn rate $\dot{\alpha}_L = \omega_L$, and F is driven by Deviated Pure Pursuit (DPP) guidance with a constant deviation angle δ such that the distance between L and F remains constant.



a. If F maintains a constant forward speed V_F, then write down the equation of engagement trajectory on the relative velocity (V_θ, V_R)-plane. **(0.5)**

b. Prove that F cannot maintain the constant range formation if V_F is constant. **(1)**

c. Find $\dot{V}_F$ and the overall guidance (acceleration) command. And, obtain the equilibrium configuration for V_θ, V_R and V_F for maintaining the formation. **(2.5)**

$$\dot{\alpha}_L = \frac{a_L}{V_L} = \omega_L \qquad \alpha_F = \theta + \delta$$

$$\alpha_{L_0} = 0 \Rightarrow \alpha_L = \omega_L t \qquad \dot{\alpha}_F = \dot{\theta}, \ \dot{\delta} = 0$$

$$V_R = V_L\cos(\alpha_L - \theta) - V_F\cos\delta$$

$$V_\theta = V_L\sin(\alpha_L - \theta) - V_F\sin\delta$$

$$\dot{V}_R = -V_L\sin(\alpha_L - \theta)(\dot{\alpha}_L - \dot{\theta}) - \dot{V}_F\cos\delta$$

$$\dot{V}_\theta = V_L\cos(\alpha_L - \theta)(\dot{\alpha}_L - \dot{\theta}) - \dot{V}_F\sin\delta$$

given $\dot{R} = 0 \Rightarrow V_R = 0$

$$V_L\cos(\alpha_L - \theta) = V_F\cos\delta$$

$$v_F = \frac{v_L \cos(\omega_L t - \theta)}{\cos\delta} , \quad \theta = \alpha_F + \delta$$

(a) if $v_F = \text{const}$, $\quad v_F\cos\delta \longrightarrow v_F \sin\delta = \sqrt{v_F^2 - v_F^2\cos^2\delta}$

$$\longrightarrow \left(v_L\cos(\omega_L t-\theta)\right)^2 + \left(v_F^2 - v_L^2\cos^2(\omega_L t-\theta)\right)^2 = v_F^2$$

trajectory eq^n $(v_R = 0, v_\theta)$

(b)

$$v_F = \frac{v_L \overbrace{\cos(\omega_L t - \theta)}^{\text{Time dependent}}}{\cos\delta}$$

$\hookrightarrow$ if R is constant

(c) $\dot{v}_F = \frac{v_L}{\cos\delta}\left(-\sin(\omega_L t-\theta)\,(\omega_L - \dot\theta)\right)$

$a_{F_t} = \dot v_F \qquad a_{F_n} = v_F \cancel{\dot\alpha_F}\ \dot\theta \qquad a_F = \sqrt{\dot v_F^2 + v_F^2\dot\theta^2}$

eqb^m config for $v_\theta, v_R, v_F \rightarrow \underline{\dot v_\theta = 0}$

~~$\dot v_R = 0, \dot v_F = 0$~~

$v_L\cos(\alpha_L-\theta)\,(\cancel{\dot\alpha_L}^{\ \omega_L} - \dot\theta) - \dot v_F \sin\delta = 0$

$\dot v_F \sin\delta = v_L\cos(\omega_L t-\theta)\,(\omega_L - \dot\theta)$

$-\frac{\cancel{v_L}}{\cos\delta}\sin(\omega_L t-\theta)\,\cancel{(\omega_L-\dot\theta)}\sin\delta = \cancel{v_L}\cos(\cancel{\omega_L t-\theta})\,(\omega_L - \dot\theta)$

$v_L \neq 0,\ \omega_L \neq \dot\theta \qquad -\tan\delta\tan(\omega_L t-\theta) = \phi$

eqbm $\quad \dot v_R = v_L\cos(\cancel{\alpha_L}^{\ \omega_L t} - \theta) - v_F\cos\delta$

$v_\theta = v_L\sin(\alpha_L-\theta) - v_F\sin\delta = -v_L\tan\delta$

d. For such varying V_F scenario, explain the evolution of the engagement on the (V_θ, V_R)-plane over different time-instants, and find the set of points on the engagement trajectory on the (V_θ, V_R)-plane. **(2.5)**

e. Find the expression of steady state V_F. **(1)**

f. Consider $V_L = 10\text{m/sec}$, $V_{F_0} = 10\text{m/sec}$, $R_0 = 50\text{m}$, $\delta = \pi/4$, $\omega_L = 0.1$ rad/sec. Find the steady state values of V_F and the overall guidance (acceleration) command of F. **(1.5)**