Course No. – AS 5570 **Quiz – 1** **Full Marks: 25**

Principles of Guidance for Autonomous Vehicles

Date: September 4, 2023 **Time: 17:00 – 18:00**

Important:

1. You may **carry any study material (books / notes in hard copy)** during the test.
2. **Engagements** are **planar** unless otherwise mentioned.
3. In numerical problems, **use 3-4 digits after decimal point**, and write the **final answers inside boxes**.
4. **Cell phone and any other electronic gadgets except your own calculators are not allowed** in the exam.
5. Full marks will be **scaled down to 15,** and your marks will be accordingly scaled while adding to your semester marks.

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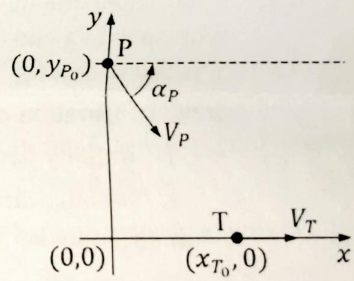
1) Prove that the closing speed of two vehicles (moving at constant speeds) is constant when they are in a collision triangle geometry. Also, show that the time-to-go at any time t can be given as, $t_{go}(t) = -R(t)/\dot{R}(t)$, where $R(t)$ is the range between two vehicles in collision triangle geometry, and $\dot{R}(t) = dR(t)/dt$. Express $\dot{R}(t)$ in terms of V_P, V_T, α_T and θ. **(2)**

2) Consider $\upsilon = \frac{V_P}{V_T} = 1.2$, x_{T_0} =500 unit and $y_{P_0} = 1000$ unit.



   a. For what value of Pursuer's heading angle α_P is the miss distance (r_{miss}) zero? What is the time required to achieve zero miss in that case? **(2)**

   b. For other values of α_P, express the steady state LOS angle θ_{SS} in terms of $\upsilon = 1.2$ and α_P. **(1)**

3) Answer the followings. **(0.5*4+2*3 = 8)**

   a. Clearing launch station periphery is of prime importance in which phase of an engagement?

      i. Initial phase ii. Midcourse phase iii. Endgame phase

   b. Negative closing speed implies

      i. Increasing distance between pursuer and target
      ii. Decreasing distance between pursuer and target
      iii. Cannot be inferred as it depends on the engagement geometry

c. For mechanizing a mixed guidance scheme over an entire mission, which option is suitable?
   i. Two-point scheme   ii. Three-point scheme   iii. All of the above

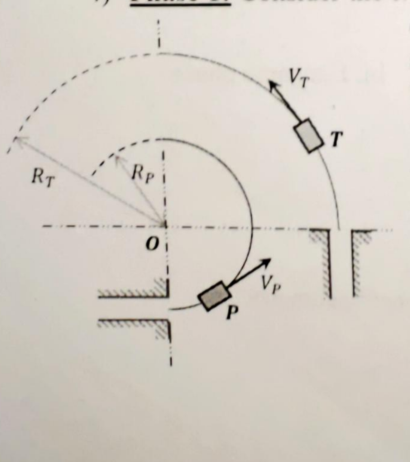
d. Which one below is correct ordering in terms of autonomy level?
   i. Active Human in the Loop < On-Board Decision Support < Fully Autonomous
   ii. Active Human in the Loop < Fully Autonomous < On-Board Decision Support
   iii. On-Board Decision Support < Active Human in the Loop < Fully Autonomous

e. Consider an engagement between a PP-guided pursuer with speed $V_P = 10$m/s, initial heading angle $\alpha_{P_0} = \theta_0 = 0$ and an equal speed target with $V_T = 10$m/s, initial heading angle $\alpha_{T_0} = \pi/3$, and turn rate $\dot{\alpha}_T = \dot{\theta}$. The initial range $R_0 =$ 200m. At which final time t_f the pursuer will intercept the target? **Justify**.
   i. $t_f = 40$sec
   ii. $t_f = 20$sec
   iii. $t_f = \infty$ as interception will never occur

f. In an engagement of a Deviated Pure Pursuit (DPP)-guided higher speed pursuer with speed V_P and deviation angle δ against a lower speed non-maneuvering target, it is found that $V_{R_0} < -V_P \cos\delta$, $V_{\theta_0} \neq 0$. Which one below is correct on the initial trend of variation of LOS angular rate $|\dot{\theta}|$? **Why?**
   i. Initially decreasing $|\dot{\theta}|$
   ii. Initially increasing $|\dot{\theta}|$
   iii. Cannot be inferred as it depends on the initial range

g. Consider an engagement between a pursuer with speed $V_P = 15$m/s, initial heading angle $\alpha_{P_0} = 3\pi/4$ and an ~~equal speed target~~ with $V_T = 10$m/s, initial heading angle $\alpha_{T_0} = \pi/3$, and initial LOS angle $\theta_0 = \pi/6$. The initial range $R_0 = 200$m. **Justify** whether the pursuer can use DPP guidance to successfully capture the target.

4) **Phase 1:** Consider the two circular paths of radii R_P and R_T, such that $R_P < R_T$, for two objects, named 'Pursuer' and 'Target', respectively, in an automated floor of a manufacturing plant as shown in the figure below. The pursuer and target travel with constant linear speeds V_P and V_T, respectively, in the inner (radius R_P) and outer (radius R_T) circle, respectively such that $V_P > V_T$. Their turning rates are denoted as ω_P and ω_T, respectively. Hence, note that $V_P = R_P\omega_P$ and $V_T = R_T\omega_T$. The positions of the pursuer and target at any time 't' in this phase are given as:



$$r_P = [x_P, y_P]^T = \left[R_P \cos\left(\omega_P t - \frac{\pi}{2}\right), R_P \sin\left(\omega_P t - \frac{\pi}{2}\right)\right]^T;$$

$$r_T = [x_T, y_T]^T = [R_T \cos(\omega_T t), R_T \sin(\omega_T t)\,]^T$$

Phase 1 continues until pure pursuit course is achieved between the target and the pursuer.

a. What is the condition of pure pursuit engagement course? **(0.5)**

b. Derive the expression of time, at which the pure pursuit course is achieved, in terms of R_P, R_T, ω_P and ω_T. **(1.5)**

c. Prove that if $\frac{R_P}{R_T} < \cos(\pi\omega_T/2\,\omega_P)$, then the pure pursuit course is achieved before the pursuer crosses the location $[R_P, 0]^T$. **(1)**

Phase 2: As the pure pursuit course is achieved between the target and the pursuer for the first time at the end of Phase 1, the pursuer is released from the inner circular path in the beginning of Phase 2 at time $t = t_0$. The pursuer subsequently follows pure pursuit (PP) guidance against the target to finally dock on the target at the end of this phase at time $t = t_f$. **Following questions are only for Phase 2**. For that, consider the following engagement parameters: $R_T = 150$ m, $R_P = 100$ m, $V_T = 15$ m/sec, $V_P = 20$ m/sec.

d. Find the time t_0, when Phase 2 is started. **(1.5)**

e. Compute pursuer's and target's heading angles α_{P_0} and α_{T_0} at the beginning of Phase 2. **(1)**

f. Compute V_{r_0} and V_{θ_0} at the beginning of Phase 2. **(1)**

g. What is the equation of the engagement trajectory on the (V_θ, V_r)-plane? Express the set of (V_θ, V_r) points traversed during the engagement in Phase 2. **(1.5)**

h. Give a schematic plot of the variation of range r Vs time t. Explain the trend. **(1)**

i. Give a schematic plot of the variation of V_θ Vs time t. Explain the trend. **(1)**

j. Explain whether the time required for docking in Phase 2, that is $t_f - t_0$, is less than or equal to or greater than $\frac{r_0(V_{r_0}+2V_P)}{\underbrace{V_P{}^2 - V_T{}^2}}$. **(2)**

$t_f \Rightarrow$ non-manuem

Course No. – AS 5570 **Quiz – 2** **Full Marks: 27**

Principles of Guidance for Autonomous Vehicles

Date: October 16, 2023 **Time: 1 hr 15 min**

Important:

1. You may **carry only one A4 size formula sheet** during the test.
2. **Engagements** are **planar** unless otherwise mentioned.
3. In numerical problems, **use 3-4 digits after decimal point**, and write the **final answers inside boxes.**
4. **Cell phone and any other electronic gadgets except your own calculators are not allowed** in the exam.
5. Full marks will be **scaled down to 15,** and your marks will be accordingly scaled while adding to your semester marks.

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1) Consider an engagement geometry: Target's altitude h = constant, $\alpha_T = \pi$. Also, V_P, V_T are constants such that $\nu = \frac{V_P}{V_T} > 1$. Initial LOS angle is θ_0. Suppose the pursuer is on the LOS guidance course and following LOS guidance command.

   a. Derive the followings:

      i. $\cot\theta - \cot\theta_0 = -\frac{V_T}{h}t$, where θ is the LOS angle from the launch station to target. **(1.5)**

      ii. $\left(\frac{dR_P}{d\theta}\right)^2 + R_P^2 = \frac{(\nu h)^2}{\sin^4\theta}$, where R_P is the distance of pursuer from the launch station. **(1.5)**

   b. From the expression $a_P = V_P\left(2\dot{\theta} + \frac{R_P\ddot{\theta}}{\dot{R}_P}\right)$, derive

   $$a_P = 2\frac{V_P V_T}{h}\sin^2\theta\left[1 + \frac{R_P\sin\theta\cos\theta}{\sqrt{(\nu h)^2 - (R_P\sin^2\theta)^2}}\right]$$ for this problem. **(2)**

   c. Show that:

      i. $\alpha_{P_f} = \theta_f + \sin^{-1}\left(\frac{1}{\nu}\sin\theta_f\right)$ **(1)**

      ii. $a_{P_f} = 2\frac{V_P{}^2}{\nu h}\sin^2\theta_f\left[1 + \frac{\cos\theta_f}{\sqrt{\nu^2 - \sin^2\theta_f}}\right]$ **(1)**

   d. Let $V_T = 200$ m/sec, $h = 1500$ m, $\alpha_T = \pi$, $X_{P_0} = [0, 0]^T$ m, $X_{T_0} = [5000, 1500]^T$ m. It is also desired to satisfy the following two conditions.

   Condition 1: $\overline{a_{P_f}} = 4\frac{V_P{}^2}{\nu h}\sin^2\theta_f < 8g$

   Condition 2: $t_f < 20$ sec

   Then,

i. Under these conditions, show that $a_{P_f} < 8g$. **(0.5)**

ii. Find the set of V_P that would sufficiently guarantee interception while also satisfying these additional engagement criteria. **(2.5)**

2) Consider an engagement between a non-maneuvering moving target and a TPN-guided pursuer ($a_P = c\dot{\theta}$).

a. Draw the engagement geometry and write the equations of engagement dynamics. **(1+1=2)**

b. From engagement trajectories in (V_θ, V_r)-space, explain why the following conditions are necessary and sufficient for interception: **(1+1+1.5=3.5)**

i. $V_{R_0} < 0$,

ii. $c > 0$,

iii. $V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0} < 0$.

c. From the condition $V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0} < 0$, explain whether $V_{R_0} + 2c$ can be negative for successful interception. **(1)**

d. Consider the following initial engagement geometry: $X_{P_0} = [0, 0]^T$m, $X_{T_0} = [1000, 0]^T$m, and $\alpha_{T_0} = 0$. Speeds of pursuer and target are: $V_P = 50$m/sec and $V_T = 15$m/sec.

i. Obtain the range of initial heading angles of the pursuer (α_{P_0}) that fall within the capture zone in terms of c. **(1.5)**

ii. Obtain the minimum value of c to ensure a non-null capture zone. **(1)**

iii. If $\alpha_{P_0} = \pi/3$, then justify whether it is possible to achieve interception before a final time $t_f = 100$ sec for any selected value of c? **(2)**

3) Consider a Pure PN (PPN)-guided pursuer with navigation gain N.

a. Consider that the <u>target is</u> <u>non-maneuvering</u>, and the speed ratio $\nu = \frac{V_P}{V_T} > 1$ and $k\nu > 1$, where $k = N - 1$.

i. Show that the roots of $V_R(\theta)$ and $V_\theta(\theta)$ alternate along the θ-axis. **(2)**

ii. Show that $V_R(\theta)\left.\frac{dV_\theta(\theta)}{d\theta}\right|_{\theta=\theta_\theta} > 0$, where θ_θ is a root $V_\theta(\theta)$. **(0.5)**

b. Draw schematic waveforms of $V_R(\theta)$ and $V_\theta(\theta)$ Vs. θ. And, prove that a successful capture of target takes place from any feasible initial condition $\left(V_{\theta_0}, V_{R_0}\right)$ except only one case $\left(V_{\theta_0} = 0, V_{R_0} > 0\right)$. **(1.5+2=3.5)**

Course No. – AS 5570 **Final Exam** **Full Marks: 58**

Principles of Guidance for Autonomous Vehicles

Date: November 11, 2023 **Time: 1:30pm – 4:10pm**

Important:

1. This exam is **open-book**. You may use any study material **except any electronic devices.**
2. **Cell phone and any other electronic gadgets except calculators are not allowed** in the exam. Ensure to **have your own calculator with yourself** as using others' calculators is strictly prohibited.
3. **Engagements** are **planar** unless otherwise mentioned.
4. **Subscript '0'** is used for **initial time** unless otherwise mentioned.
5. In numerical problems, **use 3 digits after decimal point**, and write the **final answers inside boxes**.
6. Full marks will be **scaled to 45** and your marks will be accordingly scaled while adding to your semester marks.

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1) Answer the followings. **(1.5*6 = 9)**

   a) What are domain-dependent components and domain-independent components of autonomous systems? Give examples. **(0.75*2=1.5)**

   b) Can a PPN-guided pursuer approaching a stationary point have a non-planar trajectory? Explain why. **(0.5+1 = 1.5)**

   c) Explain the difference between 'navigation' and 'guidance'. **(1.5)**

   d) Mention key differences between Beam Rider (BR) and Command-to-LOS (CLOS) realizations of LOS-Guidance? (**Note:** You do not need to give lateral acceleration expressions for BR and CLOS) **(1.5)**

   e) Which one among time interval of guidance command generation and time-constant of vehicle motion control should be higher than the other? Why? **(0.5+1 = 1.5)**

   f) Does any of the following guidance laws lead to change in pursuer's speed? Explain why. **(0.5+1 = 1.5)**

      i. LOS-Guidance ii. TPN iii. PPN

2) For Deviated Pure Pursuit (DPP) guidance against non-maneuvering target,

   a) Draw the engagement geometry and write the equations of engagement dynamics. **(0.5+1=1.5)**

   b) For a given deviation angle δ, express the capture region of DPP guidance in (V_{θ_0}, V_{R_0})-space and draw on (V_{θ_0}, V_{R_0})-plane. **(0.5+0.5=1)**

   c) Can δ be less than $-\pi/2$ or greater than $\pi/2$? Why? **(0.5+1=1.5)**

3) Consider an interception of a maneuvering target ($a_T = \frac{b}{V_\theta}, b > 0$) by a TPN-guided pursuer ($a_P = c\dot{\theta},\ c > 0$), where both a_T and a_P are applied normal to LOS.

a) Write the equations of engagement between the target and the pursuer. **(1.5)**

b) Derive the expressions of $\dot{V}_R$ and $\dot{V}_\theta$ in terms of $\dot{\theta}$, a_P and a_T. **(1.5)**

c) Derive the expression of final time (t_f) in terms of $b,\ k = V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0}$, and $p = R_0(V_{R_0} + 2c)$ as, $t_f = \frac{1}{2}\left(-k \pm \sqrt{k^2 - 4bp}\right)$. **(1.5)**

d) Derive the following necessary and sufficient conditions for interception. **(4)**

i. $V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0} < 0$, ii. $\left|V_{\theta_0}^2 + V_{R_0}^2 + 2cV_{R_0}\right| \geq 2\sqrt{bR_0(V_{R_0} + 2c)}$,

iii. $bR_0 \leq \left(\frac{2c}{3}\right)^3$

e) Consider the engagement parameters as: $R_0 = 2000$m, $\theta_0 = 0$, $V_{P0} = 300$m/sec, $\alpha_{P0} = \pi/4$, $c = 200$, $V_{T0} = 150$m/sec, $\alpha_{T_0} = \pi/2$. Find the minimum value of $b\ (> 0)$ the target should select to avoid interception. **(2.5)**

4) Consider a Pure PN (PPN)-guided pursuer with navigation gain N against a <u>maneuvering target</u>. Expressions of S_θ and S_R sectors are given in terms of N, $\phi_0 = \alpha_{P_0} - N\theta_0$ and $\nu = V_P/V_T$ as,

$$S_\theta = \left[\theta_{n_0} - \frac{1}{k}\sin^{-1}\left(\frac{1}{\nu}\right), \theta_{n_0} + \frac{1}{k}\sin^{-1}\left(\frac{1}{\nu}\right)\right];$$
$$S_R = \left[\theta_{n_0} + \frac{\pi}{2k} - \frac{1}{k}\sin^{-1}\left(\frac{1}{\nu}\right), \theta_{n_0} + \frac{\pi}{2k} + \frac{1}{k}\sin^{-1}\left(\frac{1}{\nu}\right)\right];$$

where, $k = N - 1$ and $\theta_{n_0} = -\frac{\phi_0 + n\pi}{k}$.

Consider an initial engagement geometry as follows:

$X_{P_0} = [0,0]'$ m, $X_{T_0} = [1500,0]'$ m, $\alpha_{P_0} = \pi/4$, $\alpha_{T_0} = 0$, $V_P = 30$ m/sec, $V_T = 20$ m/sec, and $a_T = 0.25$g. And, the pursuer's navigation gain $N = 3 > 1 + 1/\nu$.

i. Identify whether the initial engagement configuration belongs to which sectors : S_θ^+ or S_θ^-; S_R^+ or S_R^-; σ_θ^+ or σ_θ^-; σ_R^+ or σ_R^-. (**Hint**: Compute V_{R0}, V_{θ_0} and check.) **(0.5*4=2)**

ii. Identify the S_θ^- sector, where θ_f belongs to, and the angular sectors that P passes through on the polar plane of relative pursuit till interception. **(0.5+1=1.5)**

iii. Draw schematic trajectories of pursuer and target on real engagement plane, and plot the variations of range (R) and LOS angle (θ) time plot for the engagement. Give reasons for your answer. **(3*0.5+1=2.5)**

5) Consider a linearized engagement geometry between a pursuer and a target under the 'Near Collision Course' (NCC) assumption.

a) Against <u>maneuvering target</u>, PN and augmented PN (APN) command are given as:

$$a_P^{PN}(t) = \frac{N' a_T}{N'-2}\left[1-\left(1-\frac{t}{t_f}\right)^{N'-2}\right] \quad \text{and} \quad a_P^{APN}(t) = \frac{N' a_T}{2}\left(1-\frac{t}{t_f}\right)^{N'-2}$$

i. For $N' > 2$, obtain $\frac{a_P^{PN}{}_{Max}}{a_T}$ and $\frac{a_P^{APN}{}_{Max}}{a_T}$ in terms of N', and indicate at which time this maximum lateral acceleration is demanded. **(1*2=2)**

ii. Compare the target's maneuver induced cost $\Delta V = \int_0^{t_f} |a_P(t)|dt$ under PN and APN. **(2)**

b) Show that PN $\left(a_P(t) = \frac{N'}{t_{go}^2} ZEM(t)\right)$, with effective navigation gain $N' = 3$, where $ZEM(t)$ is zero-effort-miss at time t, is an optimal guidance law that minimizes $J = \frac{1}{2}\int_{t_0}^{t_f} a_P^2(t)dt$, subject to the followings (for <u>non-maneuvering targets</u>):

$\dot{\tilde{X}} = A\tilde{X} + Ba_P$, where $\tilde{X} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$ is the state vector under NCC condition, $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$, And, $y(t_f) = 0$. **(4)**

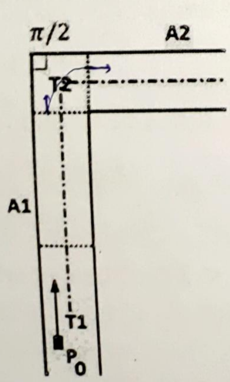
[**Recall:** 1. $\tilde{X}(t_f) = \exp[A(t_f - t)]\,\tilde{X}(t) + \int_t^{t_f} \exp[A(t_f - \tau)]\, Ba_P(\tau)d\tau$

2. Cauchy-Schwartz inequality: $\left[\int_{t_1}^{t_2} a(t)b(t)dt\right]^2 \leq \int_{t_1}^{t_2} a^2(t)dt \int_{t_1}^{t_2} b^2(t)dt$,

where $a(t)$ and $b(t)$ are integrable functions over time not uniformly equal to zero.]

6) Consider an engagement between a pursuer guided by Sliding Mode Control (SMC)-based guidance strategy based on heading error as the sliding variable against a stationary target. Show that the SMC-based guidance strategy is an augmentation of pure pursuit guidance, where the augmentation appears in the form of a switching guidance command. **(3)**

7) A part of a double-lane driving track for autonomous cars is shown in the figure beside, which consists of 2 arms - A1and A2. An autonomous vehicle P starts from the position P0 as shown in the figure. It has to satisfy the followings –



[A] Travel along the left lane in both the arms until it comes out of arm A2 (lane change is not allowed),

[B] It should not hit the outer wall.

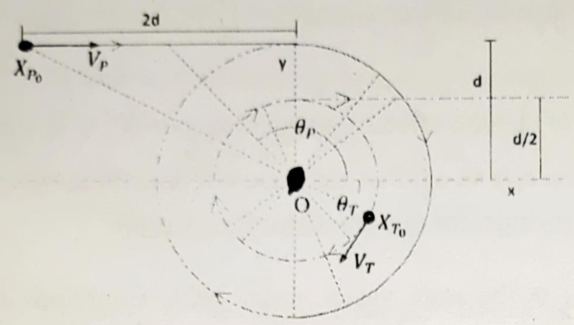
[C] In arm A2, it has to align its heading with A2 arm and at the end it has to come out of this A2 arm.

For this purpose, P needs to be programmed with a guidance strategy to control its heading direction in the track. (**Hint:** guidance to be applied especially near the junction T2 of the arms A1 and A2).

a) Design a PPN-based guidance strategy for this purpose. Find the change in headings required and corresponding navigation gain for the turning at junction T2. Also, discuss how your guidance strategy will serve the purposes [A], [B] and [C]. **(2.5)**

b) Show a schematic trajectory that P will traverse following your guidance strategy. **(0.5)**

8) Consider an engagement between a police 'P' and a thief 'T'. Their initial locations are $X_{P_0} = (-2d, d)$ and $X_{T_0} = (\frac{d}{2}\cos\phi, \frac{d}{2}\sin\phi)$, where $\phi = -\sin^{-1}(\frac{1}{\sqrt{5}})$. **[Note:** At this location P and T cannot see each other because of the tree 'O' (serves as the origin in the inertial reference frame, see the figure)]. (Angles are considered positive CCW.)



However, from prior movement of the thief, the police realizes that the thief is hiding behind the tree 'O' (see the figure), the police starts following sequences of movement with constant speed V_P at the following three phases. And, the thief always attempts to maintain his position such that the police cannot see him because of the tree 'O'.

Phase 1: In this phase, P starts from initial location $(-2d, d)$ and reaches $(0, d)$ without any turning. During this time, T maintains same distance $\frac{d}{2}$ from the tree O and keeps on turning such that P cannot see T.

Phase 2: In this phase, P initiates and continues a clockwise turn surrounding the tree 'O' with fixed radius $'d'$ and reaches $(0, -d)$. During this time also, T maintains same distance $\frac{d}{2}$ from the tree O and keeps on turning such that P cannot see T.

Phase 3: In this phase, P continues the clockwise turn surrounding the tree 'O' with fixed radius $'d'$. During this time, T initiates a run-away motion along the straight line $y = \frac{d}{2}$ without any turn such that P cannot see T.

a) At which times Phase 1 and Phase 2 end? **(0.5)**

b) For each phase, obtain the expressions of $\theta_P(t)$ and $\dot{\theta}_P(t)$ in terms of V_P and d. **(1+1+1 = 3)**

c) For each phase, obtain the expressions of $V_T(t)$ and $\dot{V}_T(t)$ in terms of V_P and d. **(1.5+1+2 = 4.5)**

d) Draw schematic plots of $\theta_P(t)$ and $V_T(t)$. **(1.5*2 =3)**

e) Consider that T cannot run at more than $2V_P$ speed, that is $V_T < 2V_P$. Then, at which value of θ_T in Phase 3, the police can see the thief? And, what should be the value of the range (distance) of weapon effectiveness of P so that P can neutralize T at that value of θ_T? (Assume that P can use his weapon as soon as he sees T) **(1.5+1.5 = 3)**

--- The End ---