Course No. – AS 5570

Principles of Guidance for Autonomous Vehicles

Assignment 1 **Due Date: September 2, 2024**
(For Computer Assignments: Sept 7, 2024)

Part A : Introductory Topics

1) Distinguish between different levels of autonomy.

2) Distinguish between domain-dependent and domain-independent components of autonomous systems.

3) Discuss basic differences between path planning, navigation, guidance and vehicle control.

4) Draw schematic block diagram of Lateral autopilot and describe its function.

5) Draw schematic block diagram of Guidance, Navigation and Control (GNC) system and describe its function.

6) Describe different phases of guidance in an engagement.

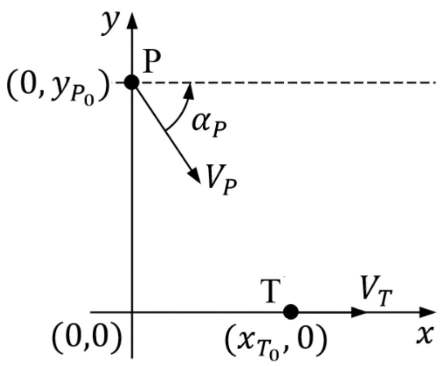
7) Describe non-homing, homing and external guidance implementation schemes.

8) What are the different categories of homing guidance in terms of seekers' modes of operation? Give examples.

Part B : Basic Engagement Scenarios

1) What is a collision triangle geometry? Prove that the closing speed of two unguided vehicles is constant when they are in a collision triangle geometry. Also, show that the time-to-go at any time t can be given as, $t_{go}(t) = -R(t)/\dot{R}(t)$, where $R(t)$ is the range between two vehicles in collision triangle geometry, and $\dot{R}(t) = dR(t)/dt$.

2) Consider the engagement between a pursuer (P) and a target (T) with speeds V_P and V_T, respectively. Both the vehicles are non-maneuvering.



i. Obtain expressions for pursuer's and target's positions $(x_P(t), y_P(t))$, $(x_T(t), y_T(t))$.

ii. Derive the expression of range between them $R(t)$.

iii. Derive the conditions on α_P for given $\upsilon = V_P/V_T$, x_{T_0} and y_{P_0} such that the range between them decreases for some finite time.

iv. Under the derived conditions above, obtain the expressions of the miss distance R_{miss} and the time t_{miss}, at which $R = R_{miss}$.

v. For what value of α_P is R_{miss} zero? What is the time required to achieve zero miss in that case?

vi. Plot the variation of R_{miss} with $\alpha_P \in [-\frac{\pi}{2}, 0]$ for a given $\upsilon = V_P/V_T$. **(Computer assignment)**

3) Consider the trajectories of two vehicles P and T be parameterized with respect to (w.r.t.) time as,

$$X_P(t) = (R_P \cos(\gamma_P t), R_P \sin(\gamma_P t)); \qquad X_T(t) = (R_P + R_T \cos(\gamma_T t), R_T \sin(\gamma_T t)),$$

where, γ_P and γ_T are the rate of change of heading angles of P and T. And, $R_P = V_P/\gamma_P$, while $R_T = V_T/\gamma_T$.

i. Let $\gamma_P = \gamma_T = \gamma$. Justify whether P and T can intercept each other. If 'Yes', what is the final time t_f? Else, if 'No', what is the miss distance R_{miss} and at which time(s) $R = R_{miss}$?

ii. Let $\gamma_P \neq \gamma_T$, and $\frac{\gamma_P}{\gamma_T} > 4\upsilon$, where $\upsilon = V_P/V_T$. Justify whether P and T can intercept each other. If 'Yes', what is the final time t_f? Else, if 'No', what is the miss distance R_{miss} and at which time(s) $R = R_{miss}$? (If not possible by hand, do the second half of the problem as computer assignment)

Part C : Pure Pursuit (PP) against Non-maneuvering and maneuvering Targets

1) For Pure Pursuit (PP) guidance,

i. Express range R in terms of angle $\psi \triangleq \alpha_T - \theta$, where α_T is the heading angle of target and θ is the line-of-sight (LOS) angle.

ii. Show that the trajectory of a PP-guided pursuer in an engagement against a non-stationary non-maneuvering target remains in one half-plane.

iii. For speed ratio $\nu = \frac{V_P}{V_T} = 1$, show that $R(\psi)$ is parabolic in nature. What is the final distance between the pursuer and target in that case?

iv. Express pursuer's lateral acceleration a_P as a function of $\psi \triangleq \alpha_T - \theta$ for $\nu > 1$.

v. For $1 < \upsilon \leq 2$, for which value of ψ is the pursuer's lateral acceleration maximum? And, what is the value of $a_{P_{Max}}$?

vi. Plot the curves showing variation of R with ψ for $R_0 = 5000$m and $\psi_0 = \pi/3$ and $\alpha_{P_0} = \theta_0$ for speed ratios $\nu = 0.8, 1$, and 1.2. (Computer assignment)

2) NPTEL lecture series Question 3 on Page 108-109 for speed ratios $\nu = \frac{V_P}{V_T} = 0.8$, 1 and 1.5. (It's mainly **Computer assignment.** However, you should practice computations as much as possible in hand.)

3) Consider a Pure Pursuit (PP)-guided pursuer with speed V_P against a maneuvering target with speed V_T and constant turn rate $\dot{\alpha}_T$. Analyze the PP trajectory on (V_θ, V_r)-space and find the conditions of successful capture. Identify the capture region in terms of V_{θ_0}, V_{r_0}.

4) NPTEL lecture series Question 3 on Page 108-109 for speed ratios $\nu = \frac{V_P}{V_T} = 0.8$, 1 and 1.5 and constant target maneuver $\dot{\alpha}_T = \pm\pi/6$ rad/sec. (It's mainly **Computer assignment.** However, you should practice computations as much as possible in hand.)

5) Consider a Pure Pursuit (PP)-guided pursuer with speed V_P against a maneuvering target with speed V_T and turn rate $\dot{\alpha}_T = c_T\dot{\theta}$. Consider three cases of $\nu = V_P/V_T \lesseqgtr 1$. For each of these three cases consider $c_T \lesseqgtr 1$. For each of these 9 cases:

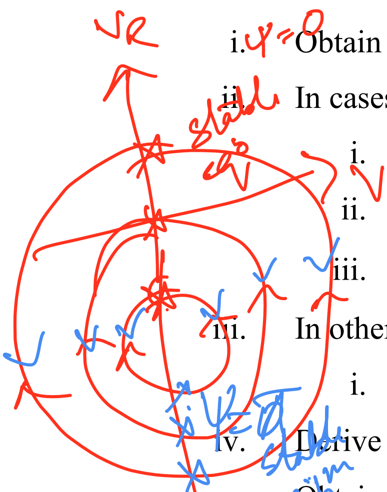
$V_R = V_T\cos(\alpha_T - \theta) - V_P$

$V_\theta = V_T\sin(\alpha_T - \theta)$

$\dot V_R = -V_T\sin(\alpha_T - \theta)(c_T - 1)\dot\theta$

$= (1 - c_T)\dot\theta V_\theta$

$\dot V_\theta = -(1 - c_T)(V_R + V_P)\dot\theta$



i. Obtain the trajectory on the (V_R, V_θ)-space.

ii. In cases of successful capture:

  i. What are the conditions of successful capture? $V_P - V_T$

  ii. Identify the capture region in terms of V_{R_0} and V_{θ_0}.

  iii. Derive the expression of t_f.

iii. In other cases (no capture): interception at lower half.

  i. Derive the expressions of t_{miss} and R_{miss}.

iv. Derive expressions of $R(\psi)$ and $a_P(\psi)$. Obtain final values of R and a_P for $\nu \geq 1$.

v. Obtain the value of $a_{P_{max}}$, and find at which value of ψ, $a_P(\psi) = a_{P_{max}}$.

$c_T = 1 \rightarrow R(\psi) = c_0$

6) Consider the initial engagement geometry in Question 3 on Page 108-109 of NPTEL lecture series. Consider speed ratios $\nu = 0.8, 1, 1.5$. For each of these three cases consider $c_T = 0.8, 1, 1.5$, where $\alpha_T = c_T\dot{\theta}$. Solve Question 3 with all these set-ups.

(Part A: A1, A2, A5 – **Computer assignment**; Part B: C1 – **Computer assignment**)

Assignment 1

Part A

Q1.
The levels of autonomy, as defined by the SAE J3016 standard, range from Level 0 (no automation) to Level 5 (full automation):
•Level 0: No automation, human driver fully responsible.
•Level 1: Driver assistance, either steering or acceleration control (not both).
•Level 2: Partial automation, control of both steering and acceleration, but human supervision is required.
•Level 3: Conditional automation, the system can drive under specific conditions, but the human must take over when needed.
•Level 4: High automation, fully autonomous in specific environments, but outside these conditions, human intervention or a safe stop is required.
•Level 5: Full automation, the vehicle is autonomous in all conditions with no human intervention needed.

Q2
Domain-Dependent Components:
•Task and environment-specific.
•Optimized for a particular domain.
•Limited transferability across domains.
•Examples: Traffic sign recognition in self-driving cars, surgical procedures in medical robots.

Domain-Independent Components:
•General-purpose and adaptable across different domains.
•Reusable in various contexts.
•Broad applicability.
•Examples: Path planning algorithms, machine learning frameworks, general control systems.

Q3
•Path Planning: The process of determining an optimal path from a starting point to a destination, considering constraints like obstacles, terrain, or energy efficiency. This step happens before the vehicle begins moving.
•Navigation: Involves determining the vehicle's current location and ensuring it follows the planned path. This includes localization, mapping, and sometimes re-planning if the environment changes.
•Guidance: Ensures the vehicle stays on the planned trajectory by calculating the required adjustments (e.g., steering angles or speed) to follow the path based on real-time data like the vehicle's position and orientation.
•Vehicle Control: The final layer that translates the guidance commands into physical actions like steering, throttle, braking, or turning. This is done through control systems that handle the vehicle's actuators to achieve the desired movements.

These components work together to enable an autonomous vehicle to operate effectively, with each layer handling progressively lower-level tasks from planning to execution.

Q4
A lateral autopilot controls the lateral movement (left or right) of an autonomous vehicle, like an aircraft or missile. The block diagram typically includes the following components:

1. **Reference Input (Command Input)**: This is the desired heading or lateral position.
2. **Error Signal Calculation**: The difference between the reference input and the actual heading or lateral position.
3. **Controller**: This could be a Proportional-Integral-Derivative (PID) controller or another control algorithm. It processes the error signal to generate a control output.
4. **Actuator Dynamics**: This block represents the dynamics of actuators like ailerons or rudders that execute the lateral control.
5. **Vehicle Dynamics**: This block represents the actual response of the vehicle to the control inputs.
6. **Feedback**: The actual heading or lateral position is fed back to calculate the error.

Function:
- The lateral autopilot controls the roll and yaw of the vehicle to maintain a specific lateral trajectory or heading.
- It ensures stability and smooth turning maneuvers by adjusting control surfaces based on the desired and actual positions.

```
[Reference Input]
       |
       v
[Error Signal Calculation] <--------- [Feedback (Actual Lateral Position)]
       |
       v
   [Controller]
       |
       v
[Actuator Dynamics (Ailerons/Rudder)]
       |
```

```
       v
[Vehicle Dynamics]
       |
       v
[Vehicle Response]
```

Q5

Here is a schematic block diagram of a Guidance, Navigation, and Control (GNC) system:

```
       [Mission Plan]
             |
             v
        [Guidance System]
             |
       [Desired Path/Trajectory]
             |
             v
      [Navigation System] <----------------- [Sensors (GPS, IMU, etc.)]
             |
       [Estimated Position/Velocity]
             |
             v
       [Control System]
             |
       [Control Commands]
             |
             v
      [Actuators/Propulsion]
             |
             v
      [Vehicle Dynamics]
             |
             v
        [Vehicle Response]
             |
             v
       [Feedback to Sensors]
```

Functions of Each Component:

1. Mission Plan: The mission objectives and desired paths or waypoints for the vehicle. This can be manually defined or autonomously generated.
2. Guidance System: The function of the guidance system is to generate a desired trajectory or path for the vehicle based on the mission plan. It calculates how the vehicle should move to achieve its objectives.
3. Navigation System: The navigation system determines the current position, velocity, and orientation of the vehicle by processing sensor data from GPS, IMU (Inertial Measurement Units), or other onboard sensors. It provides real-time updates to ensure the vehicle knows where it is in relation to the desired trajectory.
4. Control System: The control system takes the desired trajectory from the guidance system and the current state from the navigation system to calculate the necessary control actions (e.g., adjusting thrust, steering) to minimize the error between the desired and actual position/velocity.
5. Actuators/Propulsion: These are the mechanisms (e.g., engines, motors, control surfaces) that physically execute the commands from the control system to move the vehicle.
6. Vehicle Dynamics: This represents how the vehicle responds to actuator inputs based on its physical properties (e.g., mass, inertia). It describes the actual motion of the vehicle.
7. Feedback Loop: Sensor feedback from the vehicle's response is used by the navigation system to continuously update the vehicle's state, ensuring that the guidance and control systems can make necessary adjustments to stay on course.

Overall Function:
The GNC system ensures that the vehicle follows a desired trajectory or reaches a target by constantly monitoring its position, calculating the necessary adjustments, and executing commands to steer the vehicle in the correct direction. The feedback loop allows the system to adapt to disturbances or errors in real-time.

Q6

1. Boost Phase:
•Objective: Launch and accelerate the missile to the desired speed and altitude.
•Guidance Role: Minimal guidance, focused on stabilizing flight and achieving the correct trajectory with the help of the propulsion system.
2. Midcourse Phase:

•Objective: Maintain a trajectory toward the target while correcting any positional or velocity errors.
•Guidance Role: Uses inertial navigation or external updates (e.g., GPS) to adjust the missile's path. This is the longest phase of the engagement.
3. Terminal Phase:
•Objective: Guide the missile for precise impact with the moving target.
•Guidance Role: Onboard sensors (radar, infrared, etc.) track the target in real-time, making necessary adjustments for high precision as the missile closes in.
4. Endgame Phase:
•Objective: Final maneuvers to ensure a hit or optimal detonation.
•Guidance Role: Dynamic guidance, with rapid corrections to counter evasive actions or disturbances.
5. Impact/Detonation Phase:
•Objective: Achieve direct impact or detonate at the optimal distance for maximum damage.
•Guidance Role: The system may trigger detonation based on proximity or timing mechanisms.

Q7
Guidance systems are crucial in directing missiles, aircraft, and other vehicles toward their targets. These systems can be implemented in various ways, categorized into non-homing, homing, and external guidance schemes:

1. Non-Homing Guidance:
•Description: The vehicle follows a pre-determined path without directly reacting to the target's movements.
•Types:
•Preset Guidance: Follows a fixed trajectory set before launch with no in-flight corrections (e.g., unguided rockets).
•Inertial Guidance: Uses internal sensors to maintain a pre-determined trajectory.
•Beam-Riding Guidance: The vehicle follows a guidance beam projected from the launch platform.
•Advantages: Simpler and less costly.
•Disadvantages: Less accurate against moving targets.
2. Homing Guidance:
•Description: The vehicle actively tracks the target using onboard sensors.
•Types:
•Active Homing: Emits and detects its own signal (e.g., radar) to track the target.
•Semi-Active Homing: Relies on external sources to illuminate the target and tracks the reflected signal.
•Passive Homing: Detects signals emitted by the target (e.g., heat) without emitting its own.
•Advantages: High accuracy and adaptability to target movements.
•Disadvantages: More complex and expensive.
3. External Guidance:
•Description: The vehicle is guided by external sources providing real-time commands.
•Types:
•Command Guidance: Remote operators send real-time commands to adjust the vehicle's path.
•Wire-Guided Systems: Commands are sent through a physical wire connecting the vehicle to the launch platform.
•Radio-Controlled Systems: Commands are transmitted via radio signals.
•Advantages: High control by external operators.
•Disadvantages: Susceptible to jamming or communication loss.

Q8
Homing guidance systems for missiles and vehicles are categorized based on how the seeker tracks the target. The three main modes are active, semi-active, and passive homing guidance. Here's an overview:
1. Active Homing Guidance:
•Description: The missile emits its own signal (e.g., radar or sonar) toward the target and tracks the reflected signal to guide itself.
•Operation: Equipped with both a transmitter and receiver, the missile independently tracks the target.
•Examples:
•Radar-Guided Missiles: AIM-120 AMRAAM (uses radar to detect targets).
•Sonar-Guided Torpedoes: Mk 48 torpedo (uses sonar for underwater targets).
•Advantages: Independence from external sources and effectiveness at long ranges.
2. Semi-Active Homing Guidance:
•Description: The missile detects signals reflected from the target, but the signals are generated by an external source (e.g., radar or laser).
•Operation: The missile's seeker tracks the reflection of signals from an external system (e.g., radar on a ship or aircraft).
•Examples:
•Semi-Active Radar Homing Missiles: AIM-7 Sparrow (relies on external radar illumination).
•Laser-Guided Bombs (LGBs): Paveway series (guided by reflected laser energy).
•Advantages: Simpler seeker design and higher precision due to external control.
3. Passive Homing Guidance:
•Description: The missile's seeker detects natural emissions from the target, such as infrared or radar signals.
•Operation: The missile tracks emissions like heat or electromagnetic radiation from the target without emitting its own signals.
•Examples:
•Infrared (IR) Homing Missiles: AIM-9 Sidewinder (uses heat-seeking technology).
•Anti-Radiation Missiles (ARMs): AGM-88 HARM (targets radar emissions).
•Advantages: Operates passively, making it harder to detect, and allows "fire-and-forget" capability.

Part B

Q1.

Collision geometry is the mathematical and spatial definition of how two objects may collide.

$\angle 1 = \angle 3 \quad \& \quad \angle 2 = \angle 4$

$P_a = V_p t \cos 1$ $\qquad$ $P_a = D_{a'}$

$D_{a'} = V_p t \cos 3$ $\qquad$ similarly $T_b = E_{b'}$

$\frac{dR}{dt}$ for t_1, $\left.\frac{dR}{dt}\right|_{t_1} = \frac{PT - BE}{\Delta t} = \frac{Pa + bt}{\Delta t}$

$\frac{dR}{dt}$ for t_2 $\quad \left.\frac{dR}{dt}\right|_{t_2} = \frac{DE - BC}{\Delta t} = \frac{Da' + b'E}{\Delta t}$

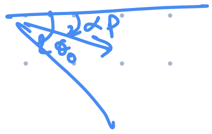
$\left.\frac{dR}{dt}\right|_{t_1} = \left.\frac{dR}{dt}\right|_{t_2} \Rightarrow \dot{R} = \text{const}$

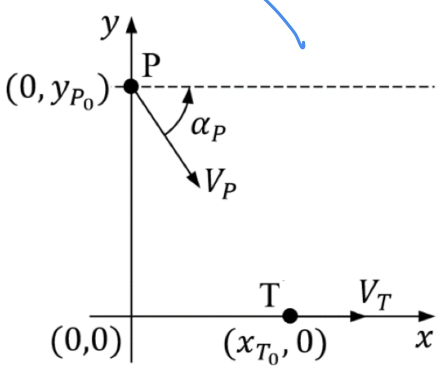
R_0

$t_{go} = \frac{R}{-\dot{R}}$ $\qquad V_c = -\dot{R}$

$a_{P_{man}} \longrightarrow (\dot{\ }) = 0$

Q2





Given both are non-maneuvering

$$x_p(t) = 0 + V_p(t)\cos\alpha_p(t)\,t \qquad \alpha_p(t) = \alpha_p$$

$$y_p(t) = y_{P_0} + V_p(t)\sin\alpha_p(t)\,t$$

follow angle coordinate system.

$$x_T(t) = x_{T_0} + V_T(t)\cos\alpha_T(t)\,t \qquad \alpha_T(t) = 0$$

$$y_T(t) = 0 + V_T(t)\sin\alpha_T(t)\,t$$

(i) Pursuer $\longrightarrow$ $(V_p\cos\alpha_p t,\ y_{P_0} + V_p\sin\alpha_p t)$

Target $\longrightarrow$ $(x_{T_0} + V_T t,\ 0)$

(ii)
$$R(t) = \sqrt{(x_p(t) - x_T(t))^2 + (y_p(t) - y_T(t))^2}$$

$$R(t) = \sqrt{(V_p\cos\alpha_p t - x_{T_0} - V_T t)^2 + (y_{P_0} + V_p\sin\alpha_p t)^2}$$

$$= \sqrt{at^2 + bt + c}$$

$a = v_T^2 - 2V_TV_p\cos\alpha_p + V_p^2$; $b = 2x_{T_0}V_T - 2X_{T_0}V_p\cos\alpha_p + 2Y_{P_0}V_p\sin\alpha_p$; $C = X_{T_0}^2 + Y_{P_0}^2$

(iii)

$$\dot{R}(t) = \frac{1}{2R(t)}\left[2(V_p\cos\alpha_p t - x_{T_0} - V_T t)(V_p\cos\alpha_p - V_T) + 2(y_0 - V_p\sin\alpha_p t)(-V_p\sin\alpha_p)\right]$$

$$\dot{R}(t) = \frac{1}{R(t)}\left[(V_p\cos\alpha_p - V_T)^2 t - x_{T_0}(V_p\cos\alpha_p - V_T) - y_0V_p\sin\alpha_p + V_p^2\sin^2\alpha_p t\right]$$

$R(t) > 0$. given $v = \frac{V_p}{V_T}$, x_{T_0}, y_{T_0}

$$\dot{R}(t) = \frac{(V_p\cos\alpha_p - V_T)^2 t - x_{T_0}(V_p\cos\alpha_p - V_T) - y_{T_0}V_p\sin\alpha_p + V_p^2\sin^2\alpha_p t}{\sqrt{(V_p\cos\alpha_p t - x_{T_0} - V_T t)^2 + (y_{T_0} - V_p\sin\alpha_p t)^2}}$$

$$= \frac{(v\cos\alpha_p - 1)^2 t - x_{T_0}/V_T(v\cos\alpha_p - 1) - y_{T_0}/V_T\, v\sin\alpha_p + v^2\sin^2\alpha_p t}{\sqrt{(v\cos\alpha_p t - \frac{x_{T_0}}{V_T} - t)^2 + (\frac{y_0}{V_T} - v\sin\alpha_p t)^2}}$$

$\dot{R}(t) < 0 \Rightarrow (v\cos\alpha_p - 1)^2 t + v^2\sin^2\alpha_p t - \frac{x_{T_0}}{V_T}\cos\alpha_p v - y_{T_0}\frac{v\sin\alpha_p}{V_T} + \frac{x_{T_0}}{V_T} < 0$

$v^2 t - 2v\cos\alpha_p t + t - v\left(\cos\alpha_p \frac{x_{T_0}}{V_T} + \frac{y_{T_0}}{V_T}\sin\alpha_p\right) + \frac{x_{T_0}}{V_T} < 0$

$$V_R = \dot{R} = V_p\cos(\alpha_p - \theta) - V_T\cos\theta$$

$$V_\theta = R\dot{\theta} =$$

$\dot{R} < 0 \longrightarrow v\cos(\alpha_p - \theta_0) - \cos\theta_0 \not< 0$

(for initially, $\dot{R}_0 < 0$)

$$\Rightarrow |\alpha_p - \theta_0| \not> \cos^{-1}\left(\frac{1}{v}\cos\theta_0\right)$$

$$\Rightarrow \theta_0 - \cos^{-1}\left(\frac{1}{v}\cos\theta_0\right) \cancel{<} \alpha_p \cancel{<} \theta_0 + \cos^{-1}\left(\frac{1}{v}\cos\theta_0\right)$$

?

$$\dot{v}_R = \dot{\theta}v_\theta, \quad \dot{v}_\theta = -\dot{\theta}v_R$$

$V_R^2 + V_\theta^2 = \text{constant} = V_T^2 - 2V_TV_p\cos\theta + V_p^2 = e^2 = V_{R_0}^2 + V_{\theta_0}^2$

$$R^2 = e^2t^2 + R_0V_{R_0} + \frac{R_0^2}{2}$$

$$R(t) = \sqrt{e^2t^2 + \sqrt{x_{T_0}^2 + y_{T_0}^2}\,V_{R_0} + \frac{x_{T_0}^2 + y_{T_0}^2}{2}}$$

$R\dot{R} = e^2t + R_0V_{R_0} \to \dot{R} = 0$

$\Rightarrow t_{miss} = \frac{-R_0V_{R_0}}{e^2}$

$$R_{miss} = \sqrt{\cancel{e^2}\frac{-R_0^2V_{R_0}^2}{e^{4}2} + R_0V_{R_0} + R_0^2}$$

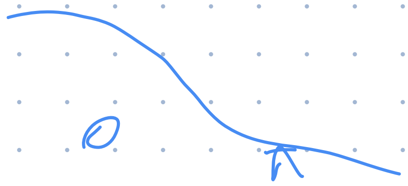
$$= \sqrt{\frac{-R_0^2V_{R_0}^2 + R_0V_{R_0}(V_{R_0}^2 + V_{\theta_0}^2) + R_0^2(\cancel{V_{R_0}^2} + V_{\theta_0}^2)}{V_{R_0}^2 + V_{\theta_0}^2}}$$

$$=$$

(iv)
$$R^2(t) = at^2 + bt + c$$

$$2R\dot{R} = 2at + b$$

$$\Rightarrow \dot{R} = 0 \Rightarrow t_{min} = \frac{-b}{2a}$$



$$R_{miss} = R(t_{miss}) = \sqrt{c - \frac{b^2}{4a}} = x_{T_0}^2 + y_{T_0}^2 - \frac{(2x_{T_0}V_T - X_{T_0}V_p\cos\alpha_p + X_{P_0}V_p\sin\alpha_p)}{4(V_T^2 - 2V_T V_p\cos\alpha_p + V_p^2)}$$

* (alternate find conditions for $t_{miss} > 0$) $t_{miss} < 0 \rightarrow$ starting is closest distance

v) $\alpha_p \longrightarrow R_{miss} = 0$

$\Rightarrow \dot{\theta} = 0$, $\dot{R}(t_0) < 0$

$\alpha_p = \theta_0 - \sin^{-1}\left(\frac{1}{\nu}\sin\theta_0\right)$

$\dot{R} = V_T\cos\theta_0 - V_p\cos^{-1}\left(\sin^{-1}\left(\frac{1}{\nu}\sin\theta_0\right)\right) < 0$

$\downarrow$ integrate

$R(t_f) = 0$ $\qquad t_f = \frac{-R_0}{V_{R_0}}$ $\leftarrow$ eqn we have already shown if in colision course

vi) CODE

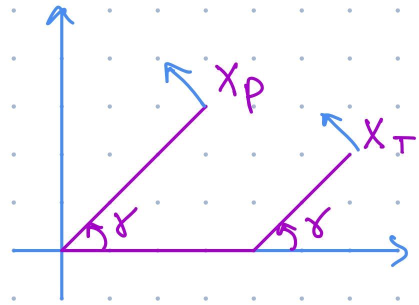
3) Consider the trajectories of two vehicles P and T be parameterized with respect to (w.r.t.) time as,

$$X_P(t) = (R_P\cos(\gamma_P t), R_P\sin(\gamma_P t)); \qquad X_T(t) = (R_P + R_T\cos(\gamma_T t), R_T\sin(\gamma_T t)),$$

where, γ_P and γ_T are the rate of change of heading angles of P and T. And, $R_P = V_P/\gamma_P$, while $R_T = V_T/\gamma_T$.

i. Let $\gamma_P = \gamma_T = \gamma$. Justify whether P and T can intercept each other. If 'Yes', what is the final time t_f? Else, if 'No', what is the miss distance R_{miss} and at which time(s) $R = R_{miss}$?

ii. Let $\gamma_P \neq \gamma_T$, and $\frac{\gamma_P}{\gamma_T} > 4v$, where $v = V_P/V_T$. Justify whether P and T can intercept each other. If 'Yes', what is the final time t_f? Else, if 'No', what is the miss distance R_{miss} and at which time(s) $R = R_{miss}$? (If not possible by hand, do the second half of the problem as computer assignment)



(i) $X_p(t_1) = X_T(t_1)$; $Y_p(t_1) = Y_T(t_1)$

$\cancel{R_p}\cos\gamma_p t_1 = \cancel{R_p} + \cancel{R_T}\cos\gamma_T t_1$ ($R_p = \frac{V_p}{\gamma_p}$, $R_T = V_T/\gamma_T$) $\Rightarrow$ $(V_p - V_T)\frac{\cos\gamma t_1}{\cancel{\gamma}} = \frac{V_p}{\cancel{\gamma}}$

$\cos\gamma t_1 = \frac{V_p}{V_p - V_T}$

$\cancel{R_p}\sin\gamma_p t_1 = \cancel{R_T}\sin\gamma_T t_1$ ($\frac{V_p}{\gamma_p}$, V_T/γ_T) $\Rightarrow$ $\frac{V_p}{\cancel{\gamma}}\sin\gamma t_1 = \frac{V_T}{\cancel{\gamma}}\sin\gamma t_1$ $\Rightarrow$ $\cancel{V_p = V_T}$ or $\sin\gamma t_1 = 0$

$\gamma t_1 = n\pi$

can be got for $R^2 = 0$ $\Downarrow$ $\dot R = 0$

if they meet $\boxed{t_f = \frac{n\pi}{\gamma}}$

$\frac{V_p}{V_p - V_T} = \pm 1$

$\Rightarrow V_T = 0$ or $V_p = \frac{V_T}{2}$

if they have to intercept

$\dot R^2 + R\ddot R = - - .$

$\ddot R =$ ____

$\ddot R > 0$, $t =$ ____

$\ddot R < 0$, $t =$ ____

if miss

$$R(t) = \sqrt{(X_p(t) - X_T(t))^2 + (y_p(t) - y_T(t))^2}$$

$$= \sqrt{\left(\frac{V_p}{\gamma}\cos\gamma t - \frac{V_p}{\gamma} - \frac{V_T}{\gamma}\cos\gamma t\right)^2 + \left(\frac{V_p}{\gamma}\sin\gamma t - \frac{V_T}{\gamma}\sin\gamma t\right)^2}$$

$$= \frac{1}{\gamma}\sqrt{V_p^2(\cos\gamma t - 1)^2 + V_T^2\cos^2\gamma t - 2V_p(\cos\gamma t - 1)V_T + (V_p^2 - 2V_pV_T + V_T^2)\sin^2\gamma t}$$

$$R(t) = \frac{1}{\gamma}\sqrt{V_p^2 + V_T^2 - 2V_p^2\cos\gamma t + V_p^2 + 2V_pV_T(\cos\gamma t - 1) - 2V_pV_T\sin^2\gamma t}$$

R_{miss} @ $\frac{dR}{dt} = 0$

$$+\cancel{2} v_p^2 \cancel{\gamma} \sin \gamma t - \cancel{2} v_p v_T \cancel{\gamma} \sin \gamma t - \overset{2}{\cancel{4}} v_p v_T \sin \gamma t \cancel{\gamma} \cos \gamma t = 0$$

$$\sin \gamma t \left(v_p - v_T - 2 v_T \cos \gamma t \right) = 0$$

$$\sin \gamma t = 0 \quad \text{or} \quad v_p = v_T \left(1 + 2 \cos \gamma t \right)$$

$$\Downarrow$$

$$\underline{\gamma t = n\pi}$$

$$R_{miss} = \sqrt{\left(\frac{\pm V_p}{-\gamma} - \frac{V_p}{\gamma} \pm \frac{V_T}{\gamma} \right)^{\cancel{2}}}$$

$$R_{miss} = \left| \frac{V_T}{\gamma} \right| \quad \text{or} \quad \left| \frac{-2V_p - V_T}{\gamma} \right| \quad ?$$

(ii) $x_p(t_1) = x_T(t_1)$; $y_p(t_1) = y_T(t_1)$

$$\underset{\frac{V_p}{\gamma_p}}{\cancel{R_p}} \cos \gamma_p t_1 = \underset{\frac{V_p}{\gamma_p}}{\cancel{R_p}} + \underset{V_T/\gamma_T}{\cancel{R_T}} \cos \gamma_T t_1 \Rightarrow \frac{V_p}{\gamma_p} \cos \gamma_p t_1 = \frac{V_p}{\gamma_p} + \frac{V_T}{\gamma_T} \cos \gamma_T t_1$$

$$\frac{V_p}{\gamma_p} \left(\cos \gamma_p t_1 - 1 \right) = \frac{V_T}{\gamma_T} \cos \gamma_T t_1$$

$$\frac{V_p}{V_T} \cdot \frac{\gamma_T}{\gamma_p} = \frac{\cos \gamma_T t_1}{\cos \gamma_p t_1 - 1}$$

$$\underset{\frac{V_p}{\gamma_p}}{\cancel{R_p}} \sin \gamma_p t_1 = \underset{V_T/\gamma_T}{\cancel{R_T}} \sin \gamma_T t_1 \Rightarrow \frac{V_p}{V_T} \cdot \frac{\gamma_T}{\gamma_p} = \frac{\sin \gamma_T t_1}{\sin \gamma_p t_1}$$

$$\frac{\sin \gamma_T t_1}{\sin \gamma_p t_1} = \frac{\cos \gamma_T t_1}{\cos \gamma_p t_1 - 1}$$

$$\tan \gamma_T t_1 \left(\cot \gamma_p t_1 - \operatorname{cosec} \gamma_p t_1 \right) = 1$$

(ii) $\gamma_p \neq \gamma_T$, $R_p = V_p/\gamma_p$, $R_T = V_T/\gamma_T$

$$R(t) = \sqrt{(X_p(t)-X_T(t))^2 + (Y_p(t)-Y_T(t))^2}$$

$$= \sqrt{(R_p\cos\gamma_p t - R_p - R_T\cos\gamma_T t)^2 + (R_p\sin\gamma_p t - R_T\sin\gamma_T t)^2}$$

$$R^2 = R_p^2(\cos\gamma_p t - 1)^2 + R_T^2\cos^2\gamma_T t - 2R_pR_T(\cos\gamma_p t - 1)\cos\gamma_T t$$
$$+ R_p^2\sin^2\gamma_p t + R_T^2\sin^2\gamma_T t - 2R_pR_T\sin\gamma_p t\sin\gamma_T t$$

$$R^2 = R_p^2 - 2R_p^2\cos\gamma_p t + R_p^2 + R_T^2 - 2R_pR_T(\cos\gamma_p t\cos\gamma_T t + \sin\gamma_p t\sin\gamma_T t)$$
$$+ 2R_pR_T\cos\gamma_T t$$

$$R^2 = 2R_p^2 + R_T^2 + 2R_pR_T(\cos(\gamma_p-\gamma_T)t - \cos\gamma_T t) - 2R_p^2\cos\gamma_p t$$

$$R^2 = 2R_p^2 + R_T^2 + 2R_p\left(R_T[(\cos(\gamma_p-\gamma_T)t) - \cos\gamma_T t] - R_p\cos\gamma_p t\right)$$

$R_{miss} = 0$

$$0 \le 2R_p^2 + R_T^2 - 2R_p(2R_T + R_p) \le R^2 \le 2R_p^2 + R_T^2 + 2R_p(2R_T + R_p)$$

$$R_T^2 - 4R_pR_T \ge 0 \Rightarrow R_T(R_T - 4R_p) \ge 0$$

$$R_T \ge 0 \ \& \ R_T - 4R_p \ge 0 \rightarrow \frac{V_T}{\gamma_T} > \frac{4V_p}{\gamma_p}$$

$$\boxed{\frac{\gamma_p}{\gamma_T} \ge 4\nu} \leftarrow \frac{\gamma_p}{\gamma_T} > 4\cancel{\frac{V_p}{V_T}}^{\,\nu}$$

∴ with given condition on heading angle, it is possible.

$$R^2 = 2R_p^2 + R_T^2 + 2R_p\left(R_T[(\cos(\gamma_p-\gamma_T)t) - \cos\gamma_T t] - R_p\cos\gamma_p t\right)$$

$R = 0$

↪ $$2R_p^2 + R_T^2 + 2R_p\left(R_T[\cos(\gamma_p-\gamma_T)t_f - \cos\gamma_T t_f] - R_p\cos\gamma_p t_f\right) = 0$$

⇓

$$\boxed{t_f = \_\_\_\_}$$

$R_{miss} \neq 0$

$$2R\dot{R} = 2R_pR_T[[-(\gamma_p-\gamma_T)\sin(\gamma_p-\gamma_T)t + \sin\gamma_T t\,\gamma_T]R_T + R_p\gamma_p\sin\gamma_p t]$$

$\dot{R} = 0$ for t_f

$$\frac{V_T}{\gamma_T} \cancel{R_T} \left[(\gamma_P - \gamma_T) \sin(\gamma_P - \gamma_T) t_f - \gamma_T \sin \gamma_T t_f \right] = \frac{V_P \cancel{R_P}}{\cancel{\gamma_P}} \cancel{\gamma_P} \sin \gamma_P t_f$$

$$\left(\frac{\gamma_P}{\gamma_T} - 1 \right) \sin(\gamma_P - \gamma_T) t_{f_{miss}} - \sin \gamma_T t_{f_{miss}} = v \sin \gamma_P t_{f_{miss}}$$

$$\boxed{t_{f_{miss}} =}$$

$$\frac{\gamma_P}{\gamma_T} = \frac{v \sin \gamma_P t_f + \sin \gamma_T t_f}{\sin(\gamma_P - \gamma_T) t_f} + 1 \quad > 4v$$

put in R_{miss}

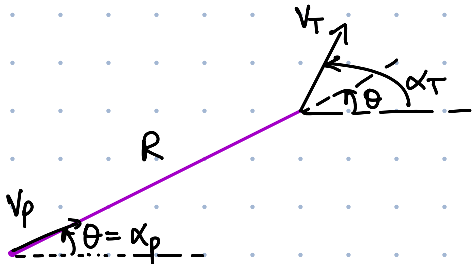
Part C

$\alpha_p = \theta$
$\alpha_p = \alpha_T - \psi$

Q1

Pure Pursuit Guidance

(i)



$$V_R = \dot{R} = V_T \cos(\overset{\psi}{\alpha_T - \theta}) - V_p$$

$$V_\theta = R\dot{\theta} = V_T \sin(\alpha_T - \theta)$$

$$\dot{V}_R = V_\theta \dot{\theta} = R\dot{\theta}^2 \qquad \dot{V}_\theta = -(V_R + V_p)\dot{\theta} = -(\dot{R} + V_p)\dot{\theta}$$

$$V_R^2 + V_\theta^2 = \text{const} \quad \text{— class notes}$$

$$R(\psi) = \overbrace{\left[\frac{R_0 \cos^{\nu+1}(\psi_0/2)}{\sin^{\nu-1}(\psi_0/2)} \right]}^{c_0} \frac{\sin^{\nu-1}(\psi/2)}{\cos^{\nu+1}(\psi/2)}$$

(ii) non-stationary $V_T \neq 0$ in one half plane?
non-maneuvering $\alpha_T = \text{const}$

$$V_R^2 + V_\theta^2 = V_T^2 + V_p^2 - 2V_T V_p \cos\psi$$

$$\psi = \alpha_T - \theta \rightarrow \dot{\psi} = -\dot{\theta} = -\frac{V_\theta}{R} = -\frac{V_T \sin\psi}{R}$$

$$\dot{\psi} = -\frac{V_T \overset{2\sin\psi/2\,\cos\psi/2}{\cancel{\sin\psi}}}{C_0 \frac{\sin^{\nu-1}\psi/2}{\cos^{\nu+1}\psi/2}} \Rightarrow \int \frac{\sin^{\nu-2}\psi/2 \, d\psi}{\cos^{\nu+2}\psi/2} = \int \frac{2}{c_0} dt$$

$$\int (\tan\psi/2)^{\nu-2} (1 + \tan^2\psi/2) \sec^2\psi \, d\psi = \frac{2}{c_0}\int dt$$

$$\int_{\psi_0}^{\psi} (\tan\psi/2)^{\nu-2} + \tan^{\nu}\psi/2 \; d(\tan\psi/2) = \frac{1}{c_0}\int_0^t dt$$

$$\frac{(\tan\psi/2)^{\nu-1}}{\nu-1} + \frac{(\tan\psi/2)^{\nu+1}}{\nu+1} = \frac{t}{c_0} + \frac{(\tan\psi_0/2)^{\nu-1}}{\nu-1} + \frac{(\tan\psi_0/2)^{\nu+1}}{\nu+1}$$

(iii)

$$R(\psi) = R_0 \cos^2\psi_0/2 \, \underset{\frac{1}{\cos^2\psi/2}}{\sec^2(\psi/2)} = \frac{2R_0\cos^2\psi_0/2}{1 + \cos\psi} \quad \therefore \text{parabolic}$$

$$r = \frac{p}{1 + e\cos\theta}, \quad e = 1$$

$$V_\theta = 0 \rightarrow \theta = \alpha_T \rightarrow \psi = 0$$

$$R_f(\psi = 0) = \frac{\cancel{2}R_0 \cos^2\psi_0/2}{\cancel{1+1}} = R_0 \cos^2\psi_0/2$$

(iv) $a_p = v_p \dot{\alpha}_p = v_p \dot{\theta} = \dfrac{v_p v_\theta}{R}$

$$a_p = \frac{v_p \cancel{v_\theta}}{R_0 \left[\dfrac{\cos^{\nu+1}(\psi_0/2)}{\sin^{\nu-1}(\psi_0/2)}\right] \dfrac{\sin^{\nu-1}(\psi/2)}{\cos^{\nu+1}(\psi/2)}} \quad v_\theta \to v_T \cancel{\sin\psi} \to 2\sin\psi/2 \cos\psi/2$$

(v) $1 < \nu < 2$

$$\frac{da_p}{d\psi} = 0 \qquad \frac{d}{d\psi}\left[\frac{\cos^{\nu+2}(\psi/2)}{\sin^{\nu-2}(\psi/2)}\right] = 0$$

$$\sin^{\nu-2}\psi/2 \; (\nu+2)\cos^{\nu+1}(\psi/2)\,\cancel{\tfrac{1}{2}}(-\sin\psi/2)$$
$$- \cos^{\nu+2}(\psi/2)\,\nu-2\,\sin^{\nu-3}(\psi/2)\,\cancel{\tfrac{1}{2}}\cos\psi/2 = 0$$

$$(\sin^{\cancel{\nu-1}\,2}\psi/2)(\nu+2)\cancel{(\cos^{\nu+1}\psi/2)} = -(\nu-2)\cancel{(\sin^{\nu-3}\psi/2)}(\cos^{\cancel{\nu+3}\,2}\psi/2)$$

$$-\tan^2\psi/2 = \frac{\nu-2}{\nu+2}$$

$$\cancel{f}\,\frac{1-\tan^2\psi/2}{(1+\tan^2\psi/2)} = -\frac{2\nu}{4}$$

$$\frac{\cos\psi}{\cancel{\cos^2\psi/2}} \cdot \cancel{\sec^2\psi/2} = \frac{\nu}{2}$$

$$\psi = \cos^{-1}\left(+\frac{\nu}{2}\right)$$

$$a_{p_{max}} = C_0 \frac{\sin^{\nu-1}\left(\dfrac{\cos^{-1}(\nu/2)}{2}\right)}{\cos^{\nu+1}\left(\dfrac{\cos^{-1}(\nu/2)}{2}\right)}$$

(vi)
$$R(\psi) = \left[\frac{R_0 \cos^{\nu+1}(\psi_0/2)}{\sin^{\nu-1}(\psi_0/2)}\right] \frac{\sin^{\nu-1}(\psi/2)}{\cos^{\nu+1}(\psi/2)}$$

② CODE

03 NOTES

Q4 CODE format of Q3

Q5.

$v_R = \dot{R} = v_T \cos(\alpha_T - \theta) - v_P \Rightarrow \dot{v}_R = -v_T(\sin(\alpha_T-\theta))(\dot{\alpha}_T - \dot{\theta}) = -v_\theta(\dot{\alpha}_T - \dot{\theta})$

$v_\theta = R\dot{\theta} = v_T \sin(\alpha_T - \theta) \Rightarrow \dot{v}_\theta = v_T(\cos(\alpha_T-\theta))(\dot{\alpha}_T - \dot{\theta}) = (v_R+v_P)(\dot{\alpha}_T - \dot{\theta})$

$\dot{\alpha}_T = c_T\dot{\theta}$

$\dot{v}_R = -v_\theta(c_T - 1)\dot{\theta}$

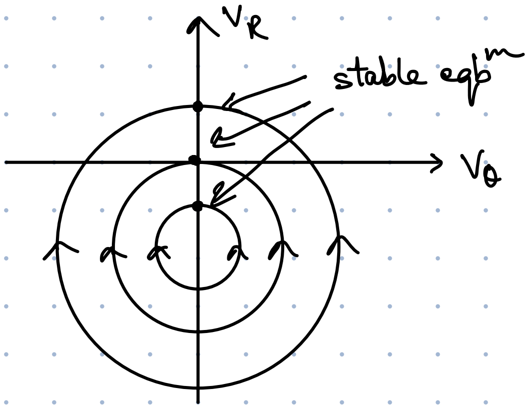
$\dot{v}_\theta = (v_R+v_P)(c_T-1)\dot{\theta}$

$(v_R+v_P)^2 + v_\theta^2 = v_T^2$

(i)

Case 1

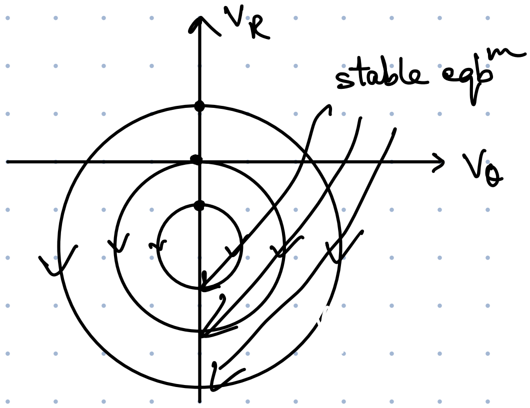
$c_T < 1$



Case 2

$c_T = 0 \longrightarrow \dot{v}_R = 0,\ \dot{v}_\theta = 0 \longrightarrow$ eqb^m at all times

Case 3

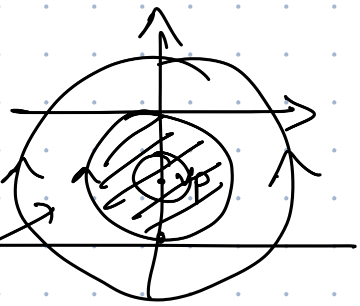


(ii)

Case 1. $c_T < 1$

$R(\psi \to 0) \longrightarrow 0$ if $\nu > 1$

$V_p > V_T$



Case 2

$\sqrt{(V_{R_0}+V_{P_0})^2 + V_{\theta_0}^2} < V_p V_{R_0} > -2V_p$

$c_T = 1$

every point for $V_R < 0$.

Case 3 entire V_R, V_θ space except V_R +ve axis

for t_{miss}, $V_R = 0$

$\Rightarrow \psi_{mis} = \cos^{-1}\nu$

$\psi = \alpha_T - \theta$

$\dot\psi = (c_T - 1)\dot\theta$

$$\frac{\dot R}{R\dot\theta} = \cot\psi - \nu \operatorname{cosec}\psi = \frac{\dot R}{R\dot\psi}(c_T - 1)$$

$$R(\psi) = \left[R_0 \frac{\cos^{\nu+1}(\psi_0/2)}{\sin^{\nu-1}(\psi_0/2)} \frac{\sin^{\nu-1}(\psi/2)}{\cos^{\nu+1}(\psi/2)}\right]^{\frac{1}{1-c_T}}$$

$$a_p = V_p \dot\alpha_p^{\nearrow \dot\theta} = \frac{V_p V_R}{R}$$

(iv)

$\cancel{t_{miss} = \frac{2V_pR_{miss} - R_0(V_{R_0}+2V_p)}{V_T^2 - V_p^2}}$

$\dot R = 0$

$$\dot v_R = -\cancel{v_\theta}^{\,R\dot\theta}(c_T - 1)\dot\theta = \ddot R$$

$$R^2\dot\theta^2 = \frac{R\ddot R}{1-c_T}$$

$$(v_R + v_p) + v_\theta^2 = v_T^2$$

$$\Rightarrow \dot R^2 + R^2\dot\theta^2 + 2v_p\dot R = v_T^2 - v_p^2$$

$$\dot R^2 + \frac{R\ddot R}{1-c_T} + 2v_p\dot R = v_T^2 - v_p^2$$

(v) $a_{p_{max}}$

↳ $\frac{da_p}{d\psi} = 0 \longrightarrow \psi = \cos^{-1}\left(\frac{\nu}{2-c_T}\right)$

$$a_{p_{max}} = \frac{v_p v_T}{c_0}\,\frac{\left(\sin^{1}\left(\cos^{-1}\left(\frac{\nu}{2-c_T}\right)\right)\right)^{\frac{2-c_T}{1-c_T}}}{\left[\tan^{\nu}\left(\frac{1}{2}\cos^{-1}\left(\frac{\nu}{2-c_T}\right)\right)\right]^{1/(1-c_T)}}$$

$$R(\psi) = \left[R_0\,\frac{\cos^{\nu+1}(\psi_0/2)}{\sin^{\nu-1}(\psi_0/2)}\,\frac{\sin^{\nu-1}(\psi/2)}{\cos^{\nu+1}(\psi/2)}\right]^{\frac{1}{1-c_T}}$$

$$a_p = v_p\cancel{\dot\chi_p}^{\,\dot\theta} = \frac{v_p v_T \sin\psi}{R} = \frac{v_p v_T \cancel{\sin\psi}^{\,2\sin\psi/2\,\cos\psi/2}}{c_0\left[\frac{\sin^{\nu-1}\psi/2}{\cos^{\nu+1}\psi/2}\right]^{1/(1-c_T)}}$$

Course No. – AS 5570

Principles of Guidance for Autonomous Vehicles

Assignment 2 **Due Date: September 18, 2024**
(For Computer Assignments: Sept 25, 2024)

Part I : Deviated PP (DPP)

1) Consider a δ-angle Deviated Pure Pursuit (DPP)-guided pursuer with speed V_P against a maneuvering target with speed V_T and turn rate $\dot{\alpha}_T = c_T\dot{\theta}$. Consider three cases of $\nu = V_P/V_T \lesseqgtr 1$. For each of these three cases consider $c_T \lesseqgtr 1$. For each of these 9 cases:
   i. Obtain the trajectory on the (V_θ, V_R)-space.
   ii. In cases of successful capture:
      i. What are the conditions of successful capture?
      ii. Identify the capture region in terms of V_{R_0} and V_{θ_0}.
      iii. Derive the expression of t_f.
   iii. In other cases (no capture):
      i. Derive the expressions of t_{miss} and R_{miss}.
   iv. Derive expressions of $R(\psi)$ and $a_P(\psi)$. Obtain final values of R and a_P for $\mu \geq 1$, where $\nu \cos\delta = \mu \cos\psi_{CC}$, where ψ_{CC} is the value of ψ in collision course with speed ratio ν.
   v. Obtain the value of $a_{P_{max}}$, and find at which value of ψ, $a_P(\psi) = a_{P_{max}}$.
   vi. Compare the set of values of ν for PP and DPP for which the final value of a_P is finite.

2) Consider the initial engagement geometry in Question 3 on Page 121-122 of NPTEL lecture series. Consider speed ratios $\nu = 0.8, 1, 1.5$. For each of these three cases consider $c_T = 0, 0.8, 1, 1.5$, where $\dot{\alpha}_T = c_T\dot{\theta}$. Solve Question 3 with all these set-ups.

   (Part A: A1, A2, A5 – **Computer assignment**; Part B: B1 – **Computer assignment**; Part C: to be done after A1-A5 and B1 – **after Computer assignment**)

Part II : Line-Of-Sight (LOS) Guidance

3) NPTEL lecture series Questions 1 (A, B) and 2 A on Page 138-139 (Consider 10 kms of downrange and 1 km of altitude). (**Computer assignment**)

4) Consider an engagement geometry: Target's altitude $h = \text{constant}$, $\alpha_T = \pi$. Also, V_P, V_T are constants such that $V_P > V_T$. Suppose the pursuer is on the LOS guidance course and following LOS guidance command.

$\nu > 1$

i. Derive the followings:

   i. $\left(\frac{dR_P}{d\theta}\right)^2 + R_P^2 = \frac{(\nu h)^2}{\sin^4\theta}$

   ii. $\cot\theta - \cot\theta_0 = -\frac{V_T}{h}t$

   iii. $a_P = 2\frac{V_P V_T}{h}\sin^2\theta\left[1 + \frac{R_P\sin\theta\cos\theta}{\sqrt{(\nu h)^2 - (R_P\sin^2\theta)^2}}\right]$

ii. Show that

   i. $\alpha_{P_f} = \theta_f + \sin^{-1}\left(\frac{1}{\nu}\sin\theta_f\right)$

   ii. $\alpha_{P_f} = 2\frac{{V_P}^2}{\nu h}\sin^2\theta_f\left[1 + \frac{\cos\theta_f}{\sqrt{\nu^2 - \sin^2\theta_f}}\right]$

iii. Let $V_P = 300$ m/sec, $V_T = 200$ m/sec, $h = 1500$ m, $\alpha_T = \pi$, $X_{P_0} = [0, 0]^T$ m, $X_{T_0} = [5000, 1500]^T$m. It is also desired to satisfy the followings.

   i. $a_{P_f} < 6g$, which approximately implies $0 < \theta_f < 0.24\pi$ or $0.6\pi < \theta_f < \pi$

   ii. $t_f < 20$ sec

   Then, what is the set of achievable impact angles (defined as $\alpha_{T_f} - \alpha_{P_f}$)?

5) Consider an LOS-guided pursuer with speed V_P be on LOS guidance course against a constantly maneuvering target such that its position is given as $X_T(t) = [c\cos\omega t\,, c\sin\omega t]^T$ and $V_P > c\omega$. Consider following three cases.

i. $X_{P_0} = [0, 0]^T$

ii. $X_{P_0} = [-c, 0]^T$

Obtain pursuer's trajectory: $R_P(\theta)$, $X_P(t)$.

Part I : Deviated PP (DPP)

$\dot{\alpha}_T = c_T \dot{\theta}$

1) Consider a δ-angle Deviated Pure Pursuit (DPP)-guided pursuer with speed V_P against a maneuvering target with speed V_T and turn rate $\dot{\alpha}_T = c_T\dot{\theta}$. Consider three cases of $\nu = V_P/V_T \lesseqgtr 1$. For each of these three cases consider $c_T \lesseqgtr 1$. For each of these 9 cases:

   i. Obtain the trajectory on the (V_θ, V_R)-space.

   ii. In cases of successful capture:

      i. What are the conditions of successful capture? → $R=0,\ V_\theta = 0$ — 2 cases.

      ii. Identify the capture region in terms of V_{R_0} and V_{θ_0}.

      iii. Derive the expression of t_f.

   iii. In other cases (no capture):

      i. Derive the expressions of t_{miss} and R_{miss}.

   iv. Derive expressions of $R(\psi)$ and $a_P(\psi)$. Obtain final values of R and a_P for $\mu \geq 1$, where $\nu\cos\delta = \mu\cos\psi_{CC}$, where ψ_{CC} is the value of ψ in collision course with speed ratio ν.

   v. Obtain the value of $a_{P_{max}}$, and find at which value of ψ, $a_P(\psi) = a_{P_{max}}$.

   vi. Compare the set of values of ν for PP and DPP for which the final value of a_P is finite.

$$V_R = V_T\cos(\alpha_T - \theta) - V_P\cos\delta$$

$$V_\theta = V_T\sin(\alpha_T - \theta) - V_P\sin\delta$$

$$\dot{V}_R = \dot{\theta}\,(V_\theta + V_P\sin\delta)\,(c_T - 1)$$

$$\dot{V}_\theta = -\dot{\theta}\,(V_R + V_P\cos\delta)\,(c_T - 1)$$

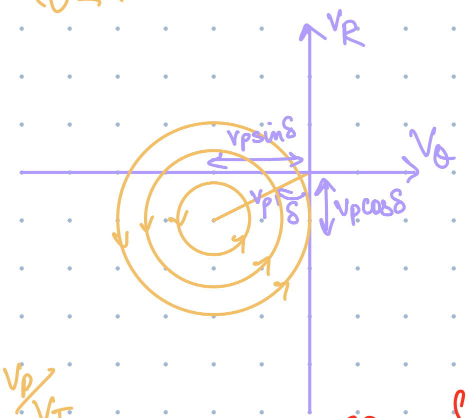
$$(V_R + V_P\cos\delta)^2 + (V_\theta + V_P\sin\delta)^2 = V_T^2$$

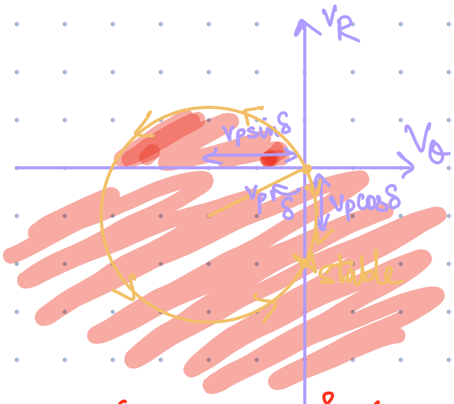
$\underline{c_T > 1}$

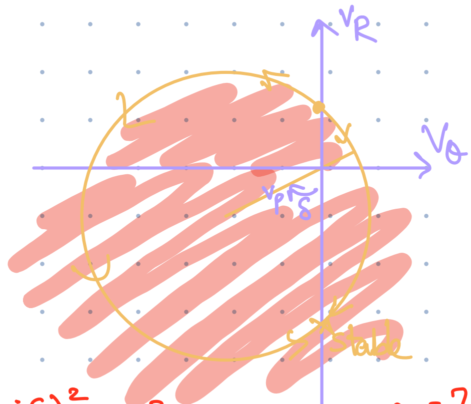
$$\dot{V}_R = -\dot{\theta}\,\underbrace{(V_\theta + V_P\sin\delta)}_{-ve,\ +ve}\,\underbrace{(c_T - 1)}_{+ve}$$

$-$: $-ve$, $\dot{\theta}$: $-ve$

$\nu < 1$ $\qquad$ $\nu > 1$ $\qquad$ $\nu = 1$







$\nu = V_P/V_T$

$$CR_{DPP} = \left\{(V_{R_0}, V_{\theta_0}) : (V_{R_0} + V_P\cos\delta)^2 + (V_{\theta_0} + V_P\sin\delta)^2 < V_T^2,\ \nu \geq 1,\ -\pi/2 < \delta < \pi/2\right\}$$

C1 : $V_T > V_P \longrightarrow \nu < 1$

C2 : $V_P\cos\delta < V_T < V_P \longrightarrow \nu > 1$

C3 : $V_P\sin\delta < V_T < V_P\cos\delta \longrightarrow \nu > 1$

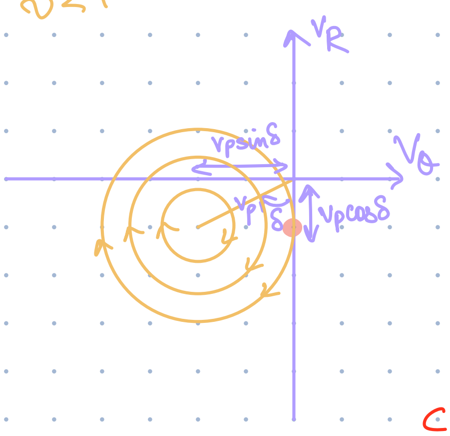
C4 : $V_T < V_P\sin\delta \longrightarrow \nu > 1$

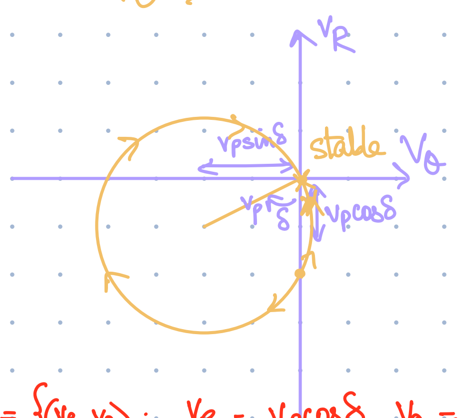
$\underline{c_T < 1}$

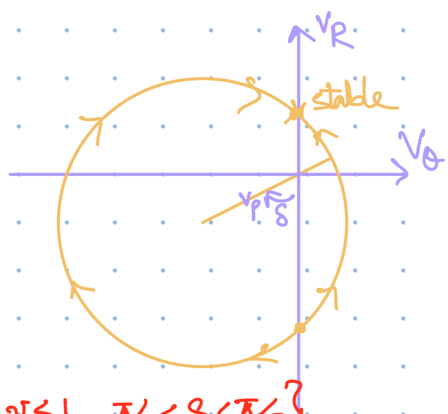
$$\dot{V}_R = -\dot{\theta}\,\underbrace{(V_\theta + V_P\sin\delta)}_{-ve,\ +ve}\,\underbrace{(c_T - 1)}_{-ve}$$

$-$: $-ve$, $\dot{\theta}$: $-ve$

$\nu < 1$ $\qquad$ $\nu > 1$ $\qquad$ $\nu = 1$

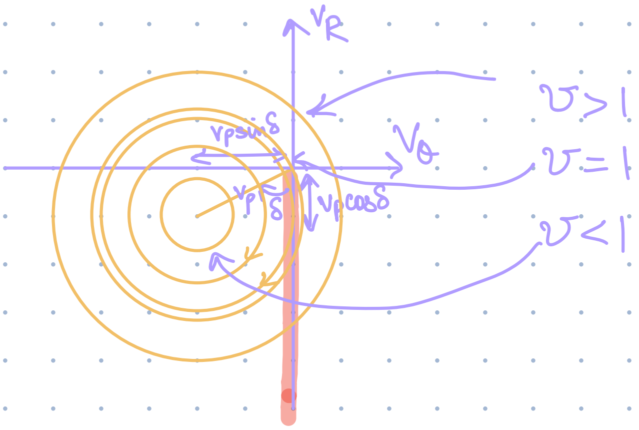






$$CR_{DPP} = \left\{(V_{R_0}, V_{\theta_0}) : V_{R_0} = -V_P\cos\delta,\ V_{\theta_0} = 0,\ \nu < 1,\ -\pi/2 < \delta < \pi/2\right\}$$

$C_T = 1 \Rightarrow \dot{V}_R = 0 \quad , \dot{V}_\theta = 0$



$CR_{DPP} = \{(V_{\theta_0}, V_{R_0}) : \; V_{\theta_0} = 0, \; V_{R_0} < 0, \; \nu \geq 1$

$-\pi/2 < \delta < \pi/2 \}$

$t_f =$

$t_f:$

$$(V_R + V_p\cos\delta)^2 + (V_\theta + V_p \sin\delta)^2 = V_T^2$$

$R = 0$

$$V_R^2 + V_\theta^2 + 2V_R V_p\cos\delta + 2V_\theta V_p \sin\delta + V_p^2 = V_T^2$$

$$R\dot{V}_\theta = R\dot{\theta}(V_R + V_p\cos\delta)(C_T-1)$$

$$R\dot{V}_\theta = -V_\theta V_R - V_\theta V_p \cos\delta \; (C_T - 1)$$

$$V_R^2 + 2V_p\cos\delta \, V_R + \cancel{V_\theta}\overset{R\dot{\theta}}{} \cancel{(V_\theta + V_p \sin\delta)}^{\dot{V}_R/\dot{\theta}(C_T-1)} + V_\theta V_p \sin\delta = V_T^2 - V_p^2$$

$$-\tan\delta\left(\frac{R\dot{V}_\theta}{C_T-1} + V_\theta\dot{R}\right) = +V_\theta V_p \sin\delta$$

$$\dot{R}^2 + \frac{R\ddot{R}}{C_T-1} + 2V_p\cos\delta\,\dot{R} + \left(\frac{R\dot{V}_\theta}{C_T-1} + V_\theta \dot{R}\right) - \tan\delta = V_T^2 - V_p^2$$

not integrable

(iii) t_{miss} , R_{miss}

$\dot{R} = 0$

$$-\dot{\theta}\,(\cancel{V_R}^{0} + V_p\cos\delta)(C_T-1)$$

$$\frac{R_{miss}\ddot{R}}{C_T - 1} + \frac{-\tan\delta \; R_{miss} \cancel{V_\theta}}{C_T - 1} = V_T^2 - V_p^2 \quad 0$$

$$\frac{R_{miss}\ddot{R}}{C_T-1} - R_{miss}\,\dot{\theta}_{miss} V_p \sin\delta = V_T^2 - V_p^2$$

$$\boxed{R_{miss} = \frac{V_T^2 - V_p^2}{\frac{\ddot{R}_{miss}}{C_T-1} - \dot{\theta}_{miss} V_p \sin\delta}}$$

t_{miss} ?

$\dot{\alpha}_T \; C_T \dot{\theta}$

(iv) $V_R = V_T\cos(\alpha_T-\theta) - V_P\cos\delta$ $\qquad \psi = \alpha_T - \theta$

$V_\theta = V_T\sin(\alpha_T-\theta) - V_P\sin\delta$ $\qquad d\psi = (G_T-1)d\theta$

$$\frac{V_R}{V_\theta} = \frac{\cos\psi - \nu\cos\delta}{\sin\psi - \nu\sin\delta} \Rightarrow \frac{dR}{(G_T-1)R} = \frac{\nu\cos\delta}{\sin\psi-\nu\sin\delta}d\psi - \frac{\cos\psi}{\sin\psi-\nu\sin\delta}d\psi$$

Define μ such that, $\boxed{\begin{aligned}\sin\psi_{cc} &= \nu\sin\delta\\ \mu\cos\psi_{cc} &= \nu\cos\delta\end{aligned}}$ $\qquad \frac{\tan\psi_{cc}}{\mu} = \tan\delta$

cc – collision course

then,

$$\frac{dR}{R} = \frac{\mu\cos\psi_{cc}}{\sin\psi - \sin\psi_{cc}}d\psi - \frac{\cos\psi}{\sin\psi-\sin\psi_{cc}}d\psi$$

$$\Rightarrow \frac{dR}{(G-1)R} = (\mu-1)\frac{\cos\left(\frac{\psi-\psi_{cc}}{2}\right)}{\sin\left(\frac{\psi-\psi_{cc}}{2}\right)}d\psi + (\mu+1)\frac{\sin\left(\frac{\psi+\psi_{cc}}{2}\right)}{\cos\left(\frac{\psi+\psi_{cc}}{2}\right)}d\psi$$

$\psi \in$ __ __

$$\frac{dR}{(G-1)R} = \frac{\mu\cos\psi_{cc}}{\sin\psi-\sin\psi_{cc}}d\psi - \frac{\cos\psi}{\sin\psi-\sin\psi_{cc}}d\psi$$

$$\mu\cos\psi_{cc} = \mu\cos\left(\frac{\psi_{cc}+\psi}{2} + \frac{\psi_{cc}-\psi}{2}\right) \qquad \cos\psi = \cos\left(\frac{\psi+\psi_{cc}}{2} + \frac{\psi-\psi_{cc}}{2}\right)$$

$$\mu\cos\psi_{cc} = \mu\cos\left(\frac{\psi_{cc}+\psi}{2}\right)\cos\left(\frac{\psi_{cc}-\psi}{2}\right) - \mu\sin\left(\frac{\psi_{cc}+\psi}{2}\right)\sin\left(\frac{\psi_{cc}-\psi}{2}\right)$$

$$\cos\psi = \cos\left(\frac{\psi+\psi_{cc}}{2}\right)\cos\left(\frac{\psi-\psi_{cc}}{2}\right) - \sin\left(\frac{\psi_{cc}+\psi}{2}\right)\sin\left(\frac{\psi_{cc}-\psi}{2}\right)$$

$$\frac{dR}{R(G-1)} = \frac{\mu\cos\left(\frac{\psi_{cc}+\psi}{2}\right)\cos\left(\frac{\psi_{cc}-\psi}{2}\right) - \mu\sin\left(\frac{\psi_{cc}+\psi}{2}\right)\sin\left(\frac{\psi_{cc}-\psi}{2}\right)}{\sin\left(\frac{\psi-\psi_{cc}}{2}\right)\cos\left(\frac{\psi+\psi_{cc}}{2}\right)}$$

$$\frac{\cos\left(\frac{\psi+\psi_{cc}}{2}\right)\cos\left(\frac{\psi-\psi_{cc}}{2}\right) - \sin\left(\frac{\psi_{cc}+\psi}{2}\right)\sin\left(\frac{\psi_{cc}-\psi}{2}\right)}{\sin\left(\frac{\psi-\psi_{cc}}{2}\right)\cos\left(\frac{\psi+\psi_{cc}}{2}\right)}$$

=

$$-\mu \cot\left(\frac{\psi_{cc}-\psi}{2}\right) + \mu \tan\left(\frac{\psi_{cc}+\psi}{2}\right)$$

$$+ \cot\left(\frac{\psi-\psi_{cc}}{2}\right) + \tan\left(\frac{\psi_{cc}+\psi}{2}\right)$$

$$\frac{dR}{R} = (\mu-1)\int -\cot\left(\frac{\psi_{cc}-\psi}{2}\right) d\psi + \mu+1 \quad \tan\left(\frac{\psi+\psi_{cc}}{2}\right) d\psi$$

$$\frac{1}{(c_T-1)} \ln R = (\mu-1) \ln\left|\sin\left(\frac{\psi-\psi_{cc}}{2}\right)\right| + (\mu+1) \ln\left|\sec\left(\frac{\psi+\psi_{cc}}{2}\right)\right|$$

$$(\mu-1)\ln\left(\operatorname{cosec}\left(\frac{\psi-\psi_{cc}}{2}\right)\right) \qquad (\mu+1)\ln\left(\sec\left(\frac{\psi+\psi_{cc}}{2}\right)\right)$$

$$\Rightarrow \boxed{R(\psi) = C_0 \frac{\sin^{(\mu-1)(c_T-1)}\left(\frac{\psi-\psi_{cc}}{2}\right)}{\cos^{(\mu+1)(c_T-1)}\left(\frac{\psi+\psi_{cc}}{2}\right)}}$$

$$a_p = v_p \dot{\alpha}_p \qquad \alpha_p = \theta + \delta \qquad v_\theta = R\dot{\theta} = \cancel{v_p\sin}(\alpha_T-\theta) - v_T \sin\delta$$

$$\dot{\alpha}_p = \dot{\theta}$$

$$= v_p \frac{\cancel{v_\theta}(v_T \sin\psi - v_p \sin\delta)}{R}$$

$$\sin\psi_{cc} = \nu \sin\delta$$

$$\mu\cos\psi_{cc} = \nu\cos\delta$$

$$= v_p \frac{\left(v_T\sin\psi - \cancel{v_p}\, v_T \sin\psi_{cc}\right)}{R}$$

$$a_p = \frac{v_p v_T \sin\left(\frac{\psi-\psi_{cc}}{2}\right)\cos\left(\frac{\psi+\psi_{cc}}{2}\right)}{R(\psi)}$$

$$a_p = v_p v_T \frac{\cos^{(\mu+2)(c_T+1)}\left(\frac{\psi+\psi_{cc}}{2}\right)}{\sin^{(\mu-2)(c_T-1)}\left(\frac{\psi-\psi_{cc}}{2}\right)}$$

$$\psi = \pi/2$$

$$\theta_1 = \pi/2 - \theta_2$$

$$\frac{\psi-\psi_{cc}}{2} = \pi/2 - \frac{\psi+\psi_{cc}}{2}$$

(v) $a_{p_{max}} = v_p v_T \left(\qquad \right)$

$$\frac{\sin^a\theta_1 \cos^{b-1}\theta_2 \cancel{\tfrac{1}{2}} = \cos^b\theta_2 \sin^{a-1}\theta_1 \cancel{\tfrac{1}{2}}}{(\sin^a\theta_1)^2}$$

$$\sin\theta_1 = \cos\theta_2$$

Q2: Simulation set up.

Q3 Simulations

1. Consider a BR missile

Consider 10 km downrange
1 km altitude

- *Assume that there is no autopilot compensation*:

(A) Guidance law is $a_M = KR_M(\theta_T - \theta_M)$. Consider K =1, 5, 10. V_T/V_M= 0.6. Plot the following quantities against time (a) missile latax (b) missile and target trajectories (c) $R_M(\theta_T - \theta_M)$ (d) R = Separation between missile and target. Find the miss distance in each case.

(B) Let $K = 10$ and V_T/V_M=0.6. Assume a pitch up constant target maneuver with a_T = 3g. Plot against time (a) missile latax (b) R (c) missile and target trajectories. Find the miss-distance.

- *Assume that there is autopilot compensation*:

$$G(s) = K\left(\frac{1+s/2}{1+s/20}\right)$$

(C) Assume a non-maneuvering target. Let $K = 10$, $V_T/V_M = 0.6$. Plot against time (a) missile latax (b) R (c) missile and target trajectories. Find the miss-distance.

(D) Assume a similar maneuvering target. Let $K = 10$, $V_T/V_M = 0.6$. Plot against time (a) missile latax (b) R (c) missile and target trajectories. Find the miss-distance.

(A)

(a) $a_M = KR_M(\theta_T - \theta_M)$

$=$

2. Consider a CLOS missile:

(A) Without autopilot compensation and $K = 1, 5, 10$, $V_T/V_M = 0.6$. The guidance law is given by $a_M = KR_M(\theta_T - \theta_M) + R_M\ddot{\theta}_T + 2\dot{R}_M\dot{\theta}_T$.

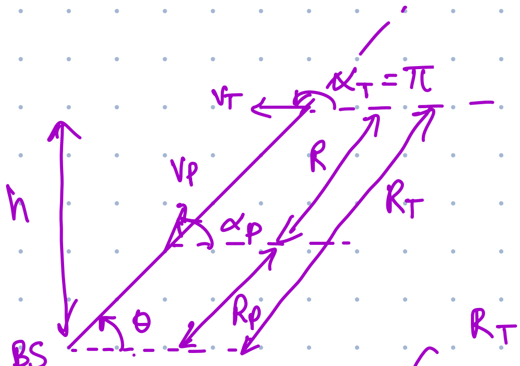
For a non-maneuvering target and then for a maneuvering target (3g pitch-up maneuver):

Plot against time (a) missile latax (b) missile target trajectories (c) R. Find the miss-distance in each case.

(B) Do the same as above for a case with autopilot compensation $G(s)$ given in the BR case.

Write overall comments comparing the results.

(4) $h = \text{constant}$, $\alpha_T = \pi$, $v_p > v_T$



$$R_p = v_p \cos(\alpha_p - \theta) \quad ?$$

$$\frac{dR_p}{d\theta} = -v_p \sin(\alpha_p - \theta)\left(\frac{d\alpha_p}{d\theta} - 1\right)$$

$$v_T \sin(\cancel{\alpha_T}^{\pi} - \theta) \qquad v_p \sin(\alpha_p - \theta)$$

$$\theta_T = \theta_p = \theta \rightarrow \frac{\cancel{v}_{\theta T}}{R_T} = \frac{\cancel{v}_{\theta p}}{R_p}$$

$$R_T \sin\theta = h$$

$$R_T = \frac{R_p v_T \sin\theta}{v_p \sin(\alpha_p - \theta)}$$

$$\frac{R_p v_T \sin^2\theta}{v_p \sin(\alpha_p - \theta)} = h$$

$$v_{R_p} = v_p \cos(\alpha_p - \theta) = \dot{R}_p$$

$$v_{\theta p} = v_p \sin(\alpha_p - \theta) = R_p \dot{\theta} \rightarrow \cot(\alpha_p - \theta) = \frac{dR_p}{R_p\, d\theta}$$

$$\frac{dR_p}{d\theta} = R_p \cot(\alpha_p - \theta) \qquad R_p = \frac{h v_p \sin(\alpha_p - \theta)}{v_T \sin^2\theta}$$

$$\text{LHS}: \left(\frac{dR_p}{d\theta}\right)^2 + R_p^2 = R_p^2 \left(\cancel{\cot^2(\alpha_p - \theta) + 1}\right) \quad \operatorname{cosec}^2(\alpha_p - \theta)$$

$$= \frac{h^2 v_p^2 \cancel{\sin^2(\alpha_p - \theta)}}{v_T^2 \sin^4\theta} \cancel{\operatorname{cosec}^2(\alpha_p - \theta)}$$

(i) $$= \left(\frac{vh}{\sin^2\theta}\right)^2 = \text{RHS}$$

$$R_T \sin\theta = h \qquad v_{\theta_T} = R_T \dot{\theta} = v_T \sin(\cancel{\alpha_T}^{\pi} - \theta)$$

$$\frac{h}{\sin\theta} \cancel{R_T} \dot{\theta} = -v_T \sin\theta \longrightarrow \sin^2\theta = \frac{h\dot{\theta}}{-v_T}$$

$$\int_{\theta_0}^{\theta} \operatorname{cosec}^2\theta \, d\theta = -\int \frac{v_T}{h} dt$$

$$\cot\theta - \cot\theta_0 = -\frac{v_T}{h} t \quad \text{(ii)}$$

$$a_p = v_p \dot{\alpha}_p \qquad R_p = \frac{h v_p \sin(\alpha_p - \theta)}{v_T \sin^2\theta} \qquad \sin^2\theta = \frac{h\dot{\theta}}{v_T}$$

$$\left(\frac{dR_p}{d\theta}\right)^2 + R_p^2 = \left(\frac{vh}{\sin^2\theta}\right)^2$$

$$R_p\dot{\theta}_p = V_{\theta p} = v_p \sin(\alpha_p - \theta)$$

$$\dot{R}_p = V_{R_p} = v_p \cos(\alpha_p - \theta)$$

$$\frac{1}{R_p}\frac{dR_p}{d\theta} = \sqrt{\left(\frac{vh}{R_p \sin^2\theta}\right)^2 - 1}$$

$$\frac{dR_p}{R_p d\theta_p} = \cot(\alpha_p - \theta)$$

$$\cot(\alpha_p - \theta) = \sqrt{\left(\frac{vh}{R_p \sin^2\theta}\right)^2 - 1}$$

$$\cos(\alpha_p - \theta) = \frac{\sqrt{(vh)^2 - (R_p \sin^2\theta)^2}}{vh}$$

$$\dot{\theta} = \frac{v_p \sin(\alpha_p - \theta)}{R_p} = \frac{v_T \sin(\overset{\pi}{\cancel{\alpha_T}} - \theta)}{\cancel{R_T}\,\frac{h}{\sin\theta}}$$

$$\alpha_p = \sin^{-1}\left(\frac{v_T \sin^2\theta\, R_p}{v_p h}\right) + \theta$$

$$\dot{\alpha}_p = \frac{1}{vh}\,\frac{1}{\sqrt{1 - \left(\frac{\cancel{v_T^2}\, R_p \sin^2\theta}{\cancel{v_p^2}\, vh}\right)^2}}\left(2\sin\theta\cos\theta\, R_p\dot{\theta} + \sin^2\theta\, \dot{R}_p\right)$$

$$\dot{R}_p = \frac{dR_p}{d\theta}\dot{\theta}$$

$$\sqrt{\frac{(vh)^2 - (R_p \sin^2\theta)^2}{\sin^2\theta}}$$

$$= \frac{1}{vh}\,\frac{1}{\sqrt{1 - \left(\frac{R_p \sin^2\theta}{vh}\right)^2}}\left(2\sin\theta\cos\theta\, R_p + \sqrt{(vh)^2 - (R_p \sin^2\theta)^2}\right)\dot{\theta}$$

$$\dot{\alpha}_p = \left(\frac{2\sin\theta\cos\theta\, R_p}{\sqrt{(vh)^2 - R_p \sin^2\theta}} + 1\right)\dot{\theta} \qquad \dot{\theta} \rightarrow \frac{v_T \sin^2\theta}{h}$$

$$a_p = \dot{\alpha}_p v_p = \frac{v_p v_T \sin^2\theta}{h}\left(1 + \frac{2\sin\theta\cos\theta\, R_p}{\sqrt{(vh)^2 - (R_p \sin^2\theta)^2}}\right)$$

(ii) (i) $\alpha_p = \sin^{-1}\left(\frac{\sin^2\theta\, R_p}{vh}\right) + \theta$

$t \to t_f$, $\theta \to \theta_f$

$R_{P_f} = R_{T_f}$

$R_{T_f}\sin\theta_f = h$

$$\alpha_{P_f} = \sin^{-1}\left(\frac{1}{v}\sin^{\cancel{2}}\theta_f \frac{\cancel{R_{P_f}}}{\cancel{h}}\right) + \theta_f \qquad \cancel{R_{T_f}\sin\theta_f}$$

$$\alpha_{P_f} = \sin^{-1}\left(\frac{1}{v}\sin\theta_f\right) + \theta_f$$

(ii) $\alpha_{P_f} = \sin^{-1}\left(\frac{1}{v}\sin\theta_f\right) + \theta_f$

$$a_p = \dot{\alpha}_p V_p = \frac{V_P V_T}{h}\sin^2\theta\left(1 + \frac{2\sin\theta\cos\theta\, R_p}{\sqrt{(vh)^2 - (R_p\sin^2\theta)^2}}\right)$$

$$a_{P_f} = \frac{V_P V_T}{h}\sin^2\theta_f\left(1 + \frac{2\cancel{\sin\theta_f}\cos\theta_f\,\cancel{R_{P_f}}}{\cancel{h}\sqrt{v^2 - (\cancel{R_{P_f}}\sin^2\theta_f)^2}}\right)$$

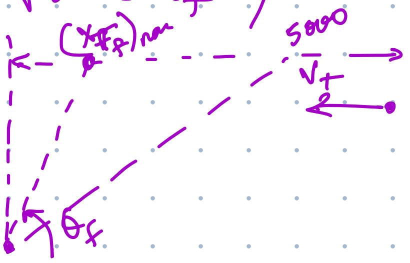
$$a_{P_f} = \frac{V_P V_T\sin^2\theta_f}{h}\left(1 + \frac{2\cos\theta_f}{\sqrt{v^2 - \sin^2\theta_f}}\right)$$

iii. Let $V_P = 300$ m/sec, $V_T = 200$ m/sec, $h = 1500$ m, $\alpha_T = \pi$, $X_{P_0} = [0, 0]^T$ m, $X_{T_0} = [5000, 1500]^T$m. It is also desired to satisfy the followings.

i. $a_{P_f} < 6g$, which approximately implies $0 < \theta_f < 0.24\pi$ or $0.6\pi < \theta_f < \pi$

ii. $t_f < 20$ sec

Then, what is the set of achievable impact angles (defined as $\alpha_{T_f} - \alpha_{P_f}$)?



$V_T t_f = 200 \times 20 = 4000$

(iii) $\beta = \cancel{\alpha_{T_f}}^{\pi} - \alpha_{P_f} \Rightarrow \pi - \alpha_{P_f} = \pi - \sin^{-1}\left(\frac{1}{v}\sin\theta_f\right) + \theta_f$

$\tan\theta_f = \frac{1500}{1000} = \frac{3}{2} \Rightarrow \theta_f = 56.3°$, 0.98

$$a_{P_f} = \frac{\cancel{300}^{20}\times\cancel{200}\sin^2\theta_f}{\cancel{1500}}\left(1 + \frac{2\cos\theta_f}{\sqrt{1.5^2 - \sin^2\theta_f}}\right) < \cancel{6}^{3}g$$

$$\sin^2\theta_f + \frac{2\sin^2\theta_f\cos\theta_f}{\sqrt{1.5^2 - \sin^2\theta_f}} < \frac{3g}{20}$$

$$\beta_{min} = \pi - \sin^{-1}\left(\frac{1}{v}\sin 0\right) + 0 = \pi$$

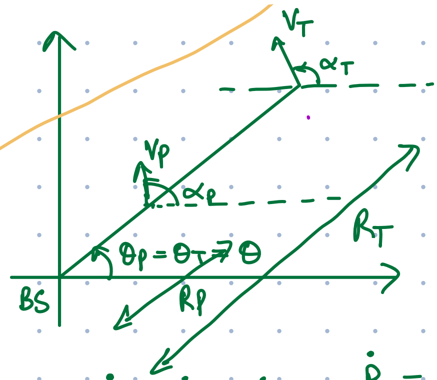
$$= \pi - \sin^{-1}\left(\frac{1}{v}\sin\pi\right) + \pi = 2\pi$$

5) Consider an LOS-guided pursuer with speed V_P be on LOS guidance course against a constantly maneuvering target such that its position is given as $X_T(t) = [c\cos\omega t, c\sin\omega t]^T$ and $V_P > c\omega$. Consider following three cases.

i. $X_{P_0} = [0,0]^T$

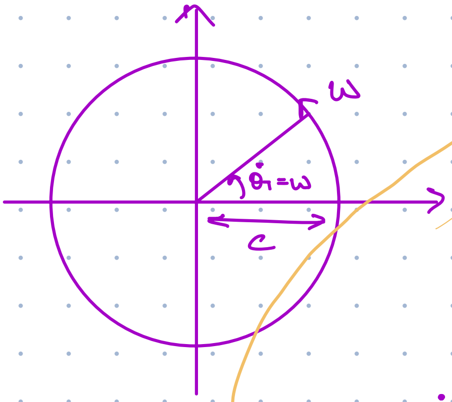
ii. $X_{P_0} = [-c, 0]^T$

Obtain pursuer's trajectory: $R_P(\theta)$, $X_P(t)$.



$$\dot\theta_T = \dot\theta_P = \dot\theta$$

$$\dot R_P = V_P\cos(\alpha_P-\theta)$$
$$R_P\dot\theta_P = V_P\sin(\alpha_P-\theta)$$
$$\dot R_T = V_T\cos(\alpha_T-\theta)$$
$$R_T\dot\theta_T = V_T\sin(\alpha_T-\theta)$$



$X_T = c\cos\omega t\,\hat i + c\sin\omega t\,\hat j$

$R_T = c \rightarrow \dot R_T = 0$

$\dot\theta_T = \omega \rightarrow R_T\dot\theta_T = c\omega$

$\dot\theta_P \rightarrow \theta_P = \omega t + \phi_0$

Similarly, $\dot R_P = V_P\cos(\alpha_P-\theta)$, $R_P\dot\theta_P = V_P\sin(\alpha_P-\theta)$

$R_P = \frac{V_P}{\omega}\sin(\alpha_P-\theta)$

$\dot R_P = \frac{V_P}{\omega}\cos(\alpha_P-\theta)(\dot\alpha_P-\dot\theta)$

$$V_P^2 = (\dot R_P)^2 + (R_P\dot\theta)^2$$
$$\hookrightarrow \left(\frac{dR_P}{d\theta}\right)^2 + R_P^2 = \frac{V_P^2}{\omega^2}$$

$\omega = \dot\theta$ initial conditions.

$$R_P = \frac{V_P}{\omega}\sin\omega t \longrightarrow \text{polar form} \qquad \theta = \omega t + \phi_0$$

$$\boxed{R_P = \frac{V_P}{\omega}\sin\theta}$$

$$\dot R_P = V_0\cos\omega t \qquad V_0 > c\omega$$

$$\dot R_P = V_P\cos\theta \quad (\theta = \omega t + \phi_0)$$

$$X = (R_P\cos\theta,\ R_P\sin\theta)$$
$$X = \left(\frac{V_P\sin 2\theta}{2\omega},\ \frac{V_P\sin^2\theta}{\omega}\right)$$
$$X_P(t) = \left(\frac{V_P\sin(2\omega t + 2\phi_0)}{2\omega},\ \frac{V_P\sin^2(\omega t+\phi_0)}{\omega}\right)$$

(i) $X_P = [0,0]$

$$\hookrightarrow \boxed{\phi_0 = 0}$$

(ii) $X_P = [-c, 0]$

$$\sin 2\phi_0 = -\frac{2c\omega}{V_P}$$

$$\boxed{\phi_0 = \frac{1}{2}\sin^{-1}\left(\frac{-2c\omega}{V_P}\right)}$$

Course No. – AS 5570

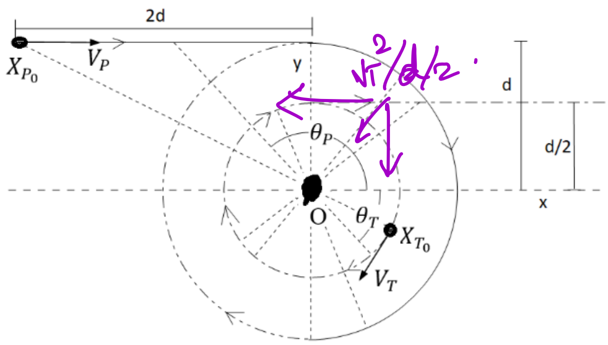
Principles of Guidance for Autonomous Vehicles

Assignment 3 **Due Date: October 3, 2024**

(For Computer Assignments: Oct 10, 2024)

Part A : Line-Of-Sight (LOS) Guidance

1) Consider an engagement between a police 'P' and a thief 'T'. Their initial locations are $X_{P_0} = (-2d, d)$ and $X_{T_0} = (\frac{d}{2}\cos\phi, \frac{d}{2}\sin\phi)$, where $\phi = -\sin^{-1}(\frac{1}{\sqrt{5}})$. [**Note:** At this location P and T cannot see each other because of the tree 'O' (serves as the origin in the inertial reference frame, see the figure)]. (Angles are considered positive CCW.)



However, from prior movement of the thief, the police realizes that the thief is hiding behind the tree 'O' (see the figure), the police starts following sequences of movement with constant speed V_P at the following three phases. And, the thief always attempts to maintain his position such that the police cannot see him because of the tree 'O'.

Phase 1: In this phase, P starts from initial location $(-2d, d)$ and reaches $(0, d)$ without any turning. During this time, T maintains same distance $\frac{d}{2}$ from the tree O and keeps on turning such that P cannot see T.

Phase 2: In this phase, P initiates and continues a clockwise turn surrounding the tree 'O' with fixed radius $'d'$ and reaches $(0, -d)$. During this time also, T maintains same distance $\frac{d}{2}$ from the tree O and keeps on turning such that P cannot see T.

Phase 3: In this phase, P continues the clockwise turn surrounding the tree 'O' with fixed radius $'d'$. During this time, T initiates a run-away motion along the straight line $y = \frac{d}{2}$ without any turn such that P cannot see T.

i. At which times Phase 1 and Phase 2 end?

ii. For each phase, obtain the expressions of $\theta_P(t)$ and $\dot{\theta}_P(t)$ in terms of V_P and d.

iii. For each phase, obtain the expressions of $V_T(t)$ and $\dot{V}_T(t)$ in terms of V_P and d.

iv. Draw schematic plots of $\theta_P(t)$ and $V_T(t)$.

after Quiz 2

Part B : TPN and RTPN Guidance

1) Derive the conditions for successful capture of a maneuvering target ($a_T = \frac{b}{V_\theta}, b > 0$) by TPN ($a_P = -N'V_{R_0}\dot{\theta}$).

2) Derive the conditions for successful capture of a maneuvering target ($a_T = \frac{b}{V_\theta}, b < 0$) by TPN ($a_P = -N'V_{R_0}\dot{\theta}$).

3) Problems 1 and 2 on Page 173-174 of NPTEL lecture series on TPN and RTPN.

   (1(b), 1(c), 2(b), 2(c) – **Computer assignment**)

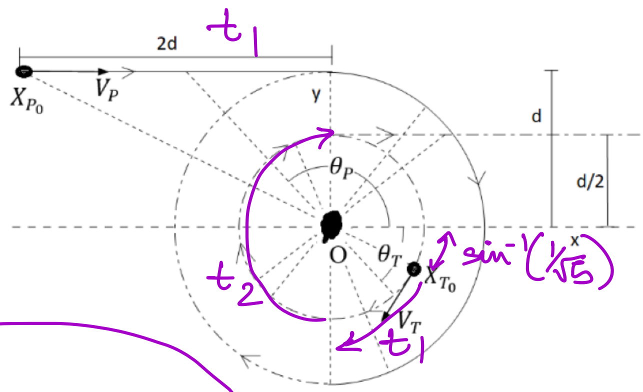
i. At which times Phase 1 and Phase 2 end?

ii. For each phase, obtain the expressions of $\theta_P(t)$ and $\dot{\theta}_P(t)$ in terms of V_P and d.

iii. For each phase, obtain the expressions of $V_T(t)$ and $\dot{V}_T(t)$ in terms of V_P and d.

iv. Draw schematic plots of $\theta_P(t)$ and $V_T(t)$.

$$\omega = \frac{V_P}{d}$$



(i) $\dot{\theta} = \dot{\theta}_P = \dot{\theta}_T = \omega$

$$\theta_P = \tan^{-1}\left(\frac{d}{-2d + V_P t}\right)$$

$$\dot{\theta}_P = \frac{1}{1 + \frac{d^2}{(-2d+V_P t)^2}} \cdot \frac{-d}{(-2d+V_P t)^2} \cdot V_P$$

$\dot{\theta}_T = \dot{\theta} = \omega$

$$\dot{\theta}_P = \frac{-dV_P}{(-2d+V_P t)^2 + d^2}$$

$$\overset{\pi/2}{\cancel{\theta_P}} = \tan^{-1}\left(\frac{d}{-2d+V_P t_1}\right)$$

$$t_1 = \frac{\pi/2 - \sin^{-1}(1/\sqrt{5})}{\cancel{\omega}\ \dot{\theta}}$$

$$t_1 = \frac{2d}{V_P}$$

$$t_2 = \frac{\pi}{\cancel{\dot{\theta}}\ \omega}$$

(ii)

PHASE 1 $\quad \theta_P = \tan^{-1}\left(\frac{d}{-2d+V_P t}\right) \qquad \dot{\theta}_P = \frac{-dV_P}{(-2d+V_P t)^2 + d^2}$

PHASE 2 $\quad \theta_P = \overset{V_P/d}{\omega}(t - t_1)$, $\quad \dot{\theta}_P = \cancel{\omega}\ V_P/d$

why there $\pi/2 -$ in answer guide?

PHASE 3 $\quad \theta_P = \underset{V_P/d}{\omega}(t - t_2)$, $\quad \dot{\theta}_P = \cancel{\omega}\ V_P/d$

$$\dot{\theta}_P = \dot{\theta}_T \rightarrow \frac{V_P \sin(\alpha_P - \theta)}{R_P} = \frac{V_T \sin(\alpha_T - \theta \pm \pi)}{R_T}$$

PHASE 1 $\quad v_T(t) = v_T = \cancel{\omega} d/2 \quad \dot\theta_p$ $\qquad \dot v_T(t) = \frac{v_T^2}{d/2} = \frac{\cancel{\omega^2}\,\dot\theta_p^2}{2d}$

$\omega = v_p/d = \frac{v_T}{d/2}$

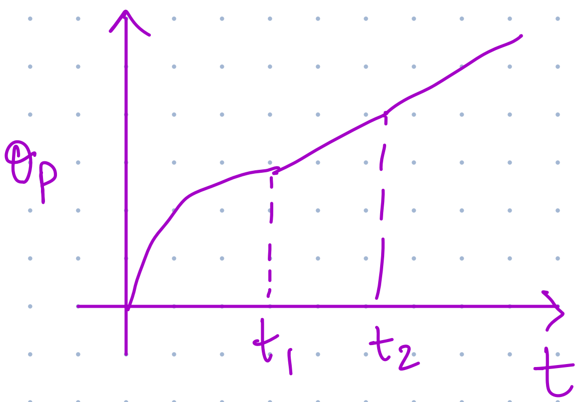
PHASE 2 $\quad v_T(t) = v_T = v_p/2 \qquad \dot v_T(t) = v_T^2/d/2 = \frac{v_p^2}{2d}$

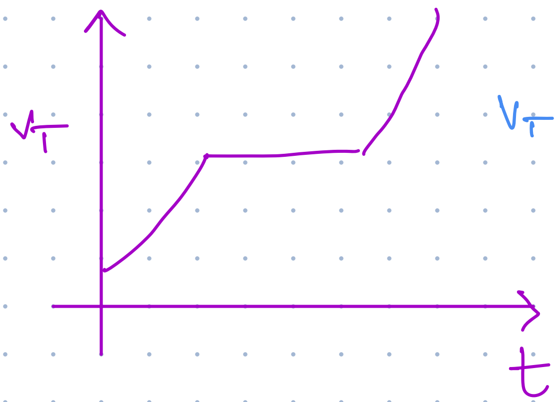
PHASE 3 $\quad v_T(t) = \frac{v_p}{2} + \frac{\omega^2 d}{2}\cos\left(\pi/2 - \theta\right) t \qquad \dot v_T(t) = \frac{\omega^2 d}{2}.$

$\pi/2 - \theta \qquad \omega^2 d/2$

check $\pi/2$ ang

(iv) $\theta_p(t)$, $v_T(t)$ $\longrightarrow$ schematic plots





v_T

Course No. – AS 5570

Principles of Guidance for Autonomous Vehicles

Assignment 4 **Due Date: October 27, 2024**
(For Computer Assignments: Nov 3, 2024)

PPN and APPN:

1) **Reading assignment**: Pages 188-191 of NPTEL lecture series on PPN.

2) Problems 1-3 on Page 211-212 of NPTEL lecture series on PPN. (**Computer assignment**)

3) Consider an initial engagement geometry as follows: $X_{P_0} = [0,0]'$ m, $X_{T_0} = [1000,100]'$ m, $\alpha_{T_0} = 0$, $V_P = 30$ m/sec, $V_T = 20$ m/sec, and $a_T = 0.2$g. Consider the following four initial headings of pursuer.

   i] $\alpha_{P_0} = \pi$, ii] $\alpha_{P_0} = 0.85\pi$, iii] $\alpha_{P_0} = 0.5\pi$, iv] $\alpha_{P_0} = 0.1\pi$

   Also, consider the pursuer's guidance be: A] Pure PN (PPN), B] Augmented Pure PN (APPN). For each of these cases,

   i. Obtain $S_\theta(t)$ and $S_R(t)$ sectors at any time t during the engagement for both PPN and APPN guidance of pursuer.

   ii. Identify the sectors $S_\theta(t_0)$, $S_R(t_0)$, $\sigma_\theta(t_0)$ and $\sigma_R(t_0)$, in which the initial engagement geometry $P(R_0, \theta_0)$ belongs to on the target-centric V_{T_0}-referenced polar plane of relative pursuit.

   iii. Also, identify whether the initial engagement configurations belong to which sectors : ${S_\theta}^+(t_0)$ or ${S_\theta}^-(t_0)$; ${S_R}^+(t_0)$ or ${S_R}^-(t_0)$; ${\sigma_\theta}^+(t_0)$ or ${\sigma_\theta}^-(t_0)$; ${\sigma_R}^+(t_0)$ or ${\sigma_R}^-(t_0)$.

   iv. Identify whether this case falls under guaranteed capture zone of PPN with $N = 2.01 + 2/\sqrt{\nu^2 - 1}$.

   v. Obtain the minimum value of navigation gain N for PPN to have this case under guaranteed capture zone of PPN.

   vi. If a capture is guaranteed by PPN with $N = 2.01 + 2/\sqrt{\nu^2 - 1}$ in this case, then does θ_f belong to which S_θ sector in each of those cases?

   vii. What are the conditions on navigation gain and augmentation parameter of APPN for achieving interception from this case without and with constraint on finiteness of lateral acceleration requirement at the endgame?

PPN-based Terminal Angle Control:

4) Consider an initial engagement geometry against stationary target: $\alpha_{P_0} = \pi/4$, $\theta_0 = 0$.

   i. What is the set of achievable terminal angles by PPN?

   ii. Suppose, in the 2 phase PPN (discussed in the class), the orientation phase follows with $N = 3/11$. Following this orientation guidance is it possible to extend the achievable terminal angles set to the entire half-space $[-\pi, 0)$?

Solve

NCC and OGL:

5) Consider a linearized engagement geometry between a pursuer and a target under the 'Near Collision Course' (NCC) assumption. Derive the expressions of $a_P(t)$ for PN and augmented PN (APN) guided pursuers in terms of effective navigation gain N', t_f and $t_{go} = t_f - t$ for the following two cases.

   i. Non-zero initial heading error, but no target maneuver

   ii. Zero initial heading error, but constant target maneuver (normal to LOS)

   Also, draw schematic block diagrams of PN and APN guidance loop

6) Show that APN with effective navigation gain $N' = 3$ is an optimal guidance law that minimizes $J = \frac{1}{2}\int_{t_0}^{t_f} a_P^2(t)dt$, subject to the followings:

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{a}_T \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ a_T \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} a_P, \text{ where } \begin{bmatrix} y \\ \dot{y} \\ a_T \end{bmatrix} \text{ is the state vector under NCC condition;}$$

And, $y(t_f) = 0$.

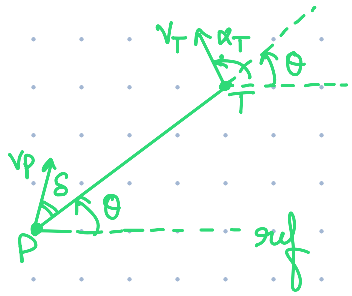
Tutorial Class

12/9

① $\dot{\alpha}_T = c_T \dot{\theta}$, $\nu = \dfrac{V_P}{V_T} \lessgtr 1$ $\quad c_T \gtrless 1$

$$V_R = V_T \cos(\alpha_T - \theta) - V_P \cos\delta$$

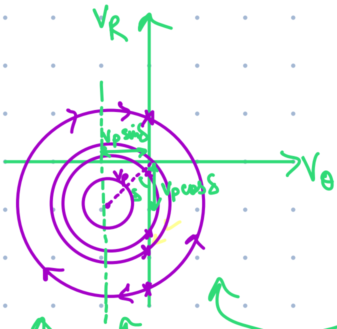
$$V_\theta = V_T \sin(\alpha_T - \theta) - V_P \sin\delta$$



$$V_T^2 = (V_R + V_P\cos\delta)^2 + (V_\theta + V_P \sin\delta)^2$$

$$\dot{V}_R = -\frac{V_T}{R} V_\theta \sin(\alpha_T - \theta)\,(c_T - 1) \qquad R > 0$$

$$\dot{V}_\theta = \frac{V_T V_\theta}{R} \cos(\alpha_T - \theta)\,(c_T - 1)$$



Case 1 : $c_T < 1$

$K = c_T - 1$

$K < 0$

$R\dot{V}_R = (-K) V_\theta \underbrace{(V_\theta + V_P \sin\delta)}$

$-K$ → +ve

$\left\{\begin{array}{l} V_\theta > 0 \\ < \end{array}\right.$, $\left.\begin{array}{l} V_\theta > -V_P\sin\delta \\ < \end{array}\right\} \rightarrow \dot{V}_R$ +ve

$V_\theta < -V_P\sin\delta$

$V_\theta > 0$

$V_\theta < 0$, $V_\theta > -V_P \sin\delta \rightarrow \dot{V}_R$ −ve

$V_T > V_P \qquad \dot{V}_R < 0$

$V_P = V_T \qquad V_\theta < 0$, $V_\theta > -V_P\sin\delta$

$V_T < V_P$

$V_T < V_P \sin\delta$

$$CR_{DPP} = \left\{ (V_{R_0}, V_{\theta_0}) : (V_{R_0} + V_P\cos\delta)^2 + (V_\theta + V_{P_0}\sin\delta)^2 = V_T^2 \right.$$
$$\left. V_T < V_P,\ -\pi/2 < \delta < \pi/2 \right\}$$

* Check for when $V_T = 0$ ← check DPP case

capture region increases in DPP why?

Case 2 : $c_T > 1$ ⟶ TRY

(iii) $t_f = ?$

$$(V_R + V_P\cos\delta)^2 + (V_\theta + V_P\sin\delta)^2 = V_T^2$$

$$\Rightarrow V_R^2 + 2V_R V_P\cos\delta + \frac{R\dot{V}_R}{-(C_T-1)} + V_\theta V_T\sin\delta = V_T^2 - V_P^2$$

$$\psi \Rightarrow \alpha_T - \theta$$

$$\hookrightarrow d\psi = -(1-C_T)\,d\theta$$

$$\dot{R} = V_T\cos(\alpha_T-\theta) - V_P\cos\delta$$
$$R\dot{\theta} = V_T\sin(\alpha_T-\theta) - V_P\sin\delta$$

$$\frac{\dot{R}}{R\dot{\theta}} = \frac{\cos\psi - \nu\cos\delta}{\sin\psi - \nu\sin\delta}$$

$$\frac{dR}{R} = -\left(\qquad\right)\frac{d\psi}{1-C_T}$$

$$R(\psi) = C_0\frac{\left[\sin\left(\frac{\psi-\psi_{cc}}{2}\right)\right]^{\frac{\mu-1}{2(1-C_T)}}}{\left[\cos\left(\frac{\psi+\psi_{cc}}{2}\right)\right]^{\frac{\mu+1}{2(1-C_T)}}}, \quad \mu\cos\psi_{cc} = \nu\cos\delta$$

$$\mu = \frac{\nu\cos\delta}{\sqrt{1-\nu^2\sin^2\delta}}$$

$$C_0 = R_0\left[\qquad\right]_0^{-1}$$

$$a_P(\psi) = V_P\dot{\alpha}_P = V_P\dot{\theta}$$
$$= \frac{V_P(V_T\sin\psi - V_P\sin\delta)}{R(\psi)}$$

$$\frac{d}{d\psi}\left(a_P(\psi)\right) = 0 \longrightarrow \underline{a_{P_{max}}} \text{ at } \underline{\psi}$$

double derivative or justify critical point is maxima.

' = simplified numerator part'

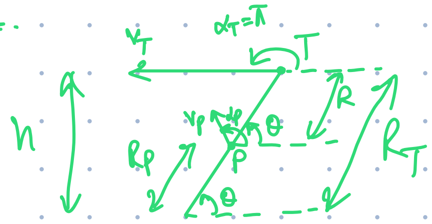
$$\left(\frac{\mu+1}{2(1-C_T)}+1\right)\cos^{-2}\left(\frac{\psi+\psi_{cc}}{2}\right)\sin(\psi+\psi_{cc}) +$$

$$\left(\frac{\mu-1}{2(1-C_T)}+1\right)\sin^{2}\left(\frac{\psi+\psi_{cc}}{2}\right)\sin(\psi-\psi_{cc}) = 0$$

$$K_1 = \mu+3-2C_T$$
$$K_2 = \mu-3+2C_T$$

ψ $$\tan\psi/2 = \left[\frac{K_1\tan^2\frac{\psi_{cc}}{2}-K_2}{K_1-K_2\tan^2\psi_{cc}/2}\right]^{1/2}$$

Q4.



$V_{R_P} =$ - - -

$V_{\theta_P} =$ - - -

$$\Rightarrow \frac{\dot{R}_P}{R_P\dot{\theta}} = \cot(\alpha_P-\theta)$$

$$\left.\begin{aligned}\theta_T = \theta_P = \theta\\ \dot{\theta}_T = \dot{\theta}_P = \dot{\theta}\\ \frac{V_{\theta_T}}{R_T} = \frac{V_{\theta_P}}{R_P}\end{aligned}\right\} LOS$$

$$\left.\begin{aligned}(R_P+R)\sin\theta = h\\ R_P^2 = \_\_\_\_\end{aligned}\right\} \text{problem specific}$$

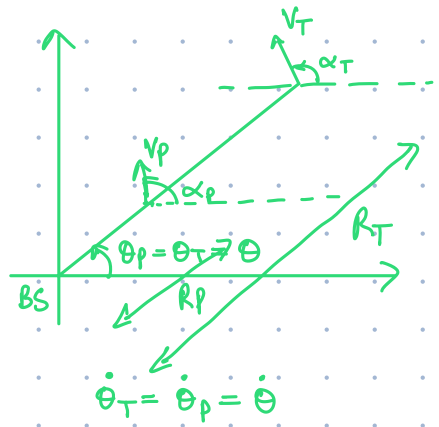
(ii) a_{P_f} // $t\to t_f$, $\theta\to\theta_f$ $\quad R_P \to \dfrac{h}{\sin\theta_f}$

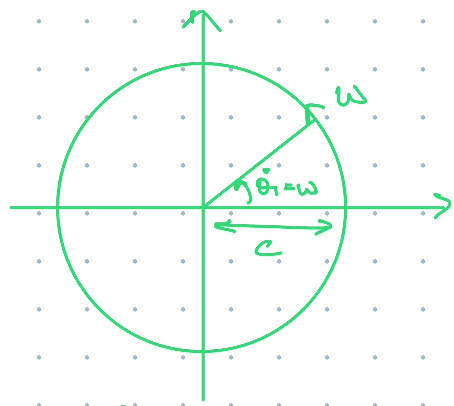
5) Consider an LOS-guided pursuer with speed V_P be on LOS guidance course against a constantly maneuvering target such that its position is given as $X_T(t) = [c\cos\omega t\,, c\sin\omega t]^T$ and $V_P > c\omega$. Consider following three cases.

i. $X_{P_0} = [0,0]^T$

ii. $X_{P_0} = [-c,0]^T$

Obtain pursuer's trajectory: $R_P(\theta)$, $X_P(t)$.





$$\dot R_P = V_0 \cos(\alpha_P - \theta) \qquad \dot R_T = V_0\cos(\alpha_T - \theta)$$

$$R_P\dot\theta_P = V_0 \sin(\alpha_P - \theta) \qquad R_T\dot\theta_T = V_0 \sin(\alpha_T - \theta)$$

$$V_0^2 = (\dot R_P)^2 + (R_P\dot\theta_P)^2$$

$$\hookrightarrow \left(\frac{dR_P}{d\theta}\right)^2 + R_P^2 = \frac{V_0^2}{\omega^2}$$

$$R_P = \frac{V_P}{\omega}\sin(\theta) \longrightarrow \text{polar form}$$

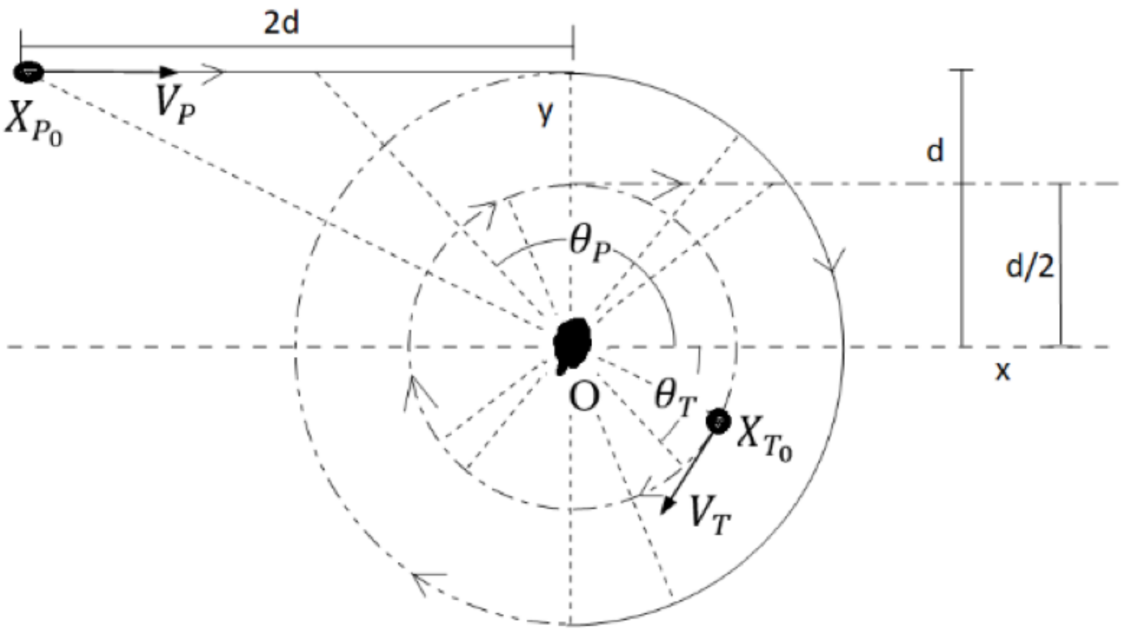
$\hookrightarrow$ check if cos( ) is valid sol^n?

$\rightarrow$ transform coordinates & solve.

$\rightarrow$ X_{P_0} serves as base station?

evasion problem.

Assigm 3



$v_p = d \cdot \omega$

$\omega = v_T / (d/2)$

$\theta_p = \theta \pm \pi$

$t_1 = \frac{1}{v_p}$

$t_2 = t_1 + \pi/\omega$

$t_3 = v_T/(d/2)$

to maintain LOS

$$\frac{v_T}{d/2} = \frac{v_p}{d} = \omega$$

I) $\theta_p = \tan^{-1}\left(\frac{d}{-2d + v_p t}\right) \xrightarrow{d/dt} \dot{\theta}_p$

II) $\theta_p = \frac{\pi}{2} - \omega(t - t_1)$ $\qquad \dot{\theta}_p = ?$

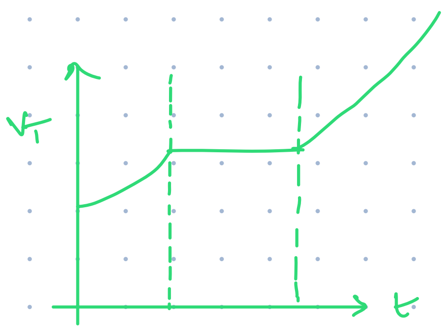
$= -\omega$

III) $v_T = \underbrace{\qquad\qquad}_{v_p,\ \theta_p,\ \theta_T}$

$$\dot{\theta}_p = \dot{\theta}_T \Rightarrow \frac{v_p \sin(\alpha_p - \theta)}{R_p} = \frac{v_T \sin(\alpha_T - \theta \pm \pi)}{R_T}$$

$\dot{v}_T = ?$

$v_T = (\qquad)$



FIND?

PPN

$-\frac{1}{K}\gamma_{n_T}(t)$

3) S_θ, S_R ||

$$S_\theta = \theta_{n_0} - \frac{1}{K}\sin^{-1}\left(\frac{1}{\gamma}\right) \le \theta_0 \le \theta_{n_0} + \frac{1}{K}\sin^{-1}\left(\frac{1}{\gamma}\right)$$

$$S_R = \theta_{n_0} + \frac{\pi}{2K} - \frac{1}{K}\sin^{-1}\left(\frac{1}{\gamma}\right) \le \theta_R \le \theta_{n_0} + \frac{\pi}{2K} + \frac{1}{K}\sin^{-1}\left(\frac{1}{\gamma}\right)$$

$n=1$	$n=0$
$\Rightarrow -82°$	$\Rightarrow -22.3°$

$\Leftarrow$ $\theta_{n_0} = -\frac{(\phi_0 + n\pi)}{K}$

$\phi_0 = -N\theta_0 + \alpha_p$

$= 67°$

$K = N - 1 = 3$

$N > 2 + \frac{2}{\sqrt{\gamma^2 - 1}}$

$\gamma = 1.5$

take any value to check

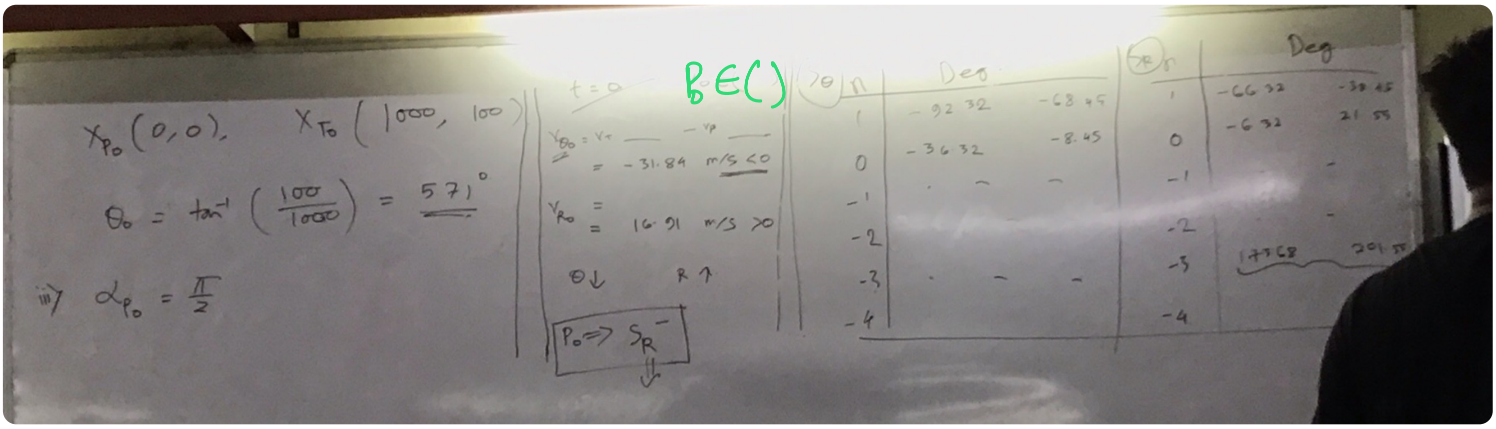
$N > 3.79$

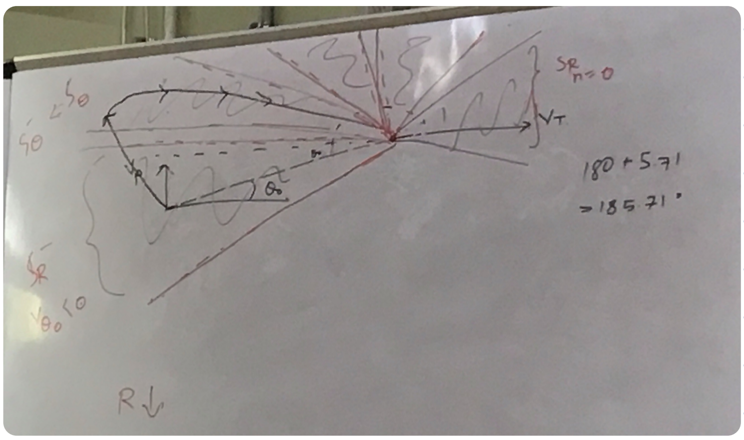
$N = 4$

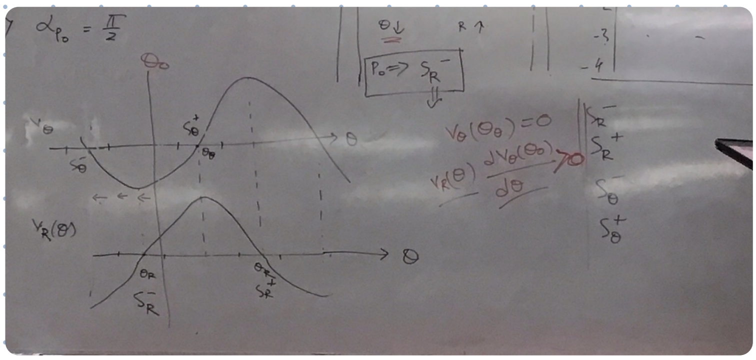
X_{P_0} (0,0) X_{T_0} (1000,100)

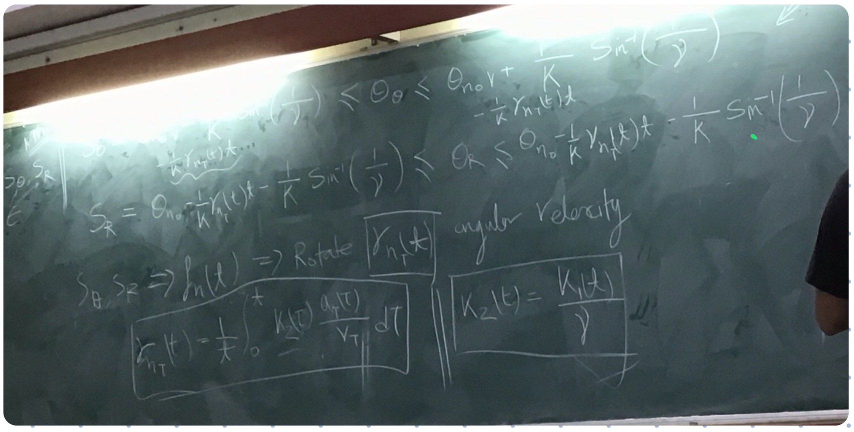
$\theta_0 = \tan^{-1}\left(\frac{100}{1000}\right) = 5.71$

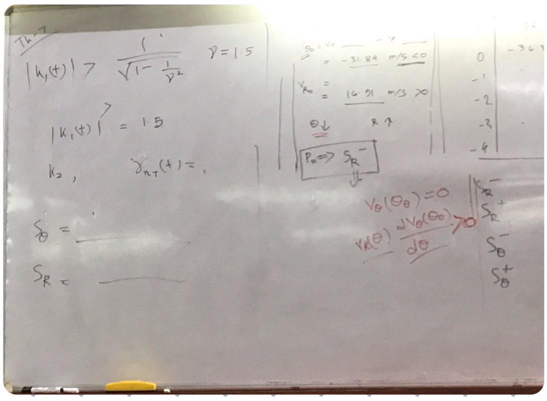
iii) $\alpha_{P_0} = \pi/2$





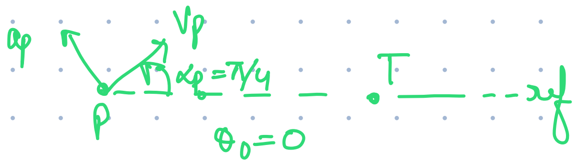






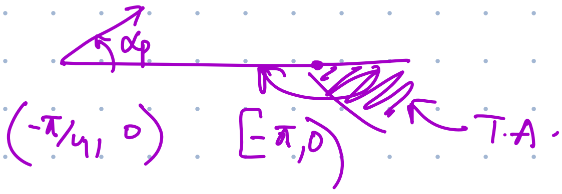
4) Terminal angle (PPN)

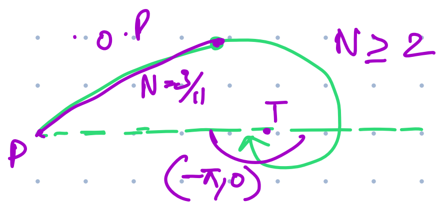
$$N \in [2, \infty)$$



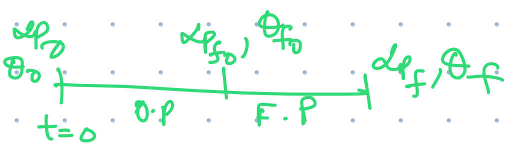
$$N \geq 2$$

$$\alpha_{P_f} \in [2\theta_0 - \alpha_{P_0}, \theta_0)$$
$$= [-\pi/4, 0)$$





$N_{ORI} = 3/11$



$$(\alpha_{P_{f0}}, \theta_0) \rightarrow (-\pi + \theta_0, \theta_0)$$

$$\alpha_{P_{f0}} \geq \pi + 2\theta_{f0}$$

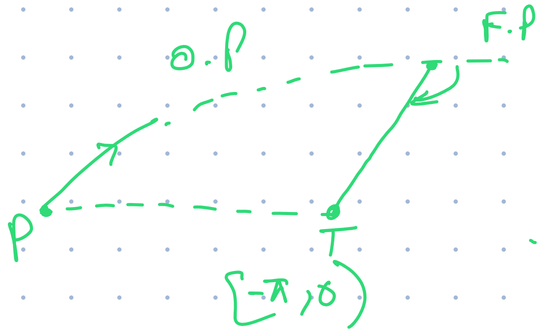
$$\rightarrow \quad N_{ORI} = \frac{\alpha_{P_{f0}} - \alpha_{P_0}}{\theta_{f0} - \theta_0}$$

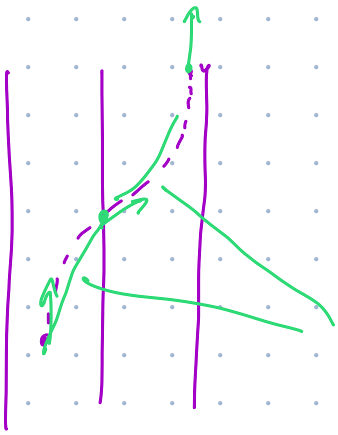
3/11 ''

$$\alpha_{P_{f0}} = \frac{3}{11}\theta_{f0} - \pi/4$$

$$\frac{3}{11}\theta_{f_0} - \pi/4 \geq \pi + 2\theta_{f_0}$$

$$\theta_{f_0} \leq -\frac{55}{76}\pi$$





Reversal of LOS Rate → not possible in PPN

split into 2 phases, assume a virtual target