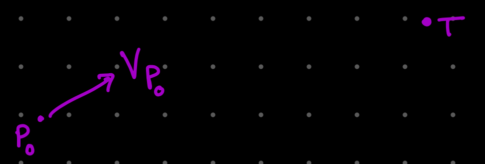
31/07

Consider u have a predefined target, u have a start point with initial vel, acct; (not dealing with dynamics, point mass model)



2 approaches
- kinematic formulation u can simulate states at different times at ideal form; we can guarantee irl, doesn't consider environment or dynamics of body; we can atleast analyse if it's good or bad algo
- Other approach is considering the body dynamics (and environment?)

<u>Scenarios where we need guidance methods</u>
- Constraints can be added on region of freedom of movement
- if multiple objects they should reach destination at same time
- object should hit the pursuer before it hits the target

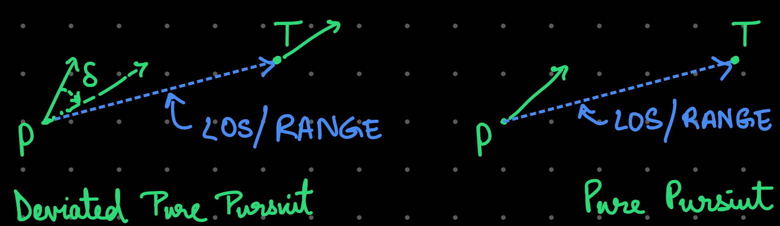
These methods should be validated on real models before implementing in the field.

Majorly required for unmanned vehicles; Classical and Modern Guidance Laws - 2 types

COURSE CONTENTS

<u>Introductory Topics</u>
1. Overview of Autonomous Systems
2. Overview of Guidance and Related Terms
3. Fundamentals of Interception and Avoidance
4. Taxonomy of Guidance Laws

<u>Classical Guidance</u>
1. Pure Pursuit (PP) and Deviated PP [velocity vector always tries to point to target is pure pursuit]
2. Line of Sight Guidance (LOS-G)
3. Proportional navigation (PN)
   A. True PN (TPN)
   B. Pure PN (PPN)
   C. Augmented PPN



<u>Modern Guidance Laws</u>
1. Optimisation-based Guidance (OGL) [we're not gonna use all tools of optimal controls, just some basics]
2. Sliding Mode-based Guidance

<u>Applications</u>
1. Terminal Angle Control
2. Time-to-go & Final Time Control

This course considers that navigation is inputted, we will decide on what path to use using guidance algos
(Static control is optimisation? Dynamic control is?)

<u>Navigation; Guidance; Control</u> (proper definition of these terms)
Navigation tell how it is moving currently
Guidance tells were/why to move
Control is low level, tells how to change movement

BOOKS/LECTURE NOTES

1. NPETL lecture series on Guidance (Guidance of Missiles one) [following basic course structure like this, but we'll do more]
2. N. A. Shneydor - "Missile Guidance and Pursuit: Kinematics, Dynamics & Control" [can be used as a first reference]
3. P. Zarchaan - "Tactical and Strategic Missile Guidance" [read only after topic is grasped]

MILESTONES

1. Quiz 1 - 15 marks
2. Quiz 2 - 15 marks
3. Endsem - 45 marks
4. Term Project - 12.5 marks (midway thru the sem, group of 3-4, replicate a paper as much as possible and simulate then if u can improve; in MATLAB or Python)
5. Computer Simulation - 12.5 marks

(Aim to do some ROS coding towards the end)

S - 85+ish (5%), ABCD - RG (16,32,30,12%)

Autonomous Systems

- Engineered systems that can 'learn', 'adapt' take decisions, act on their own.

Learn - where it is, own behaviours and wrt surroundings

Adapt - awareness to obstructions, etc

- 'Autonomy' in
  - Analysing what to do , why to do
  - Deciding when to do, how to do
  - Implementing the formed decision
- Required in scenarios
  - 'Infeasible' environment
  - 'Impossible' environment (like for very short reaction times)
  - Too expensive (deploying some system to carry out the functionality makes it cheaper)

Advancement in computing power, electronics (like chip size reduction).

SLAM - Simultaneous Localisation and Mapping, been there since 80s, but not with tech advancements it is possible now.

Velodyne is a leader in 3D LIDAR, early 2000s started out.

Advantages:

- Improved perception/sensing, reaction time, "performance"
- increased safety
- Cost-effectiveness (reduced personnel burden)
- Improved ability to act in communication-degraded/denied environment

1 vehicle highly equipped in possible an infeasible environment, better than human; further, cost-effectiveness allows for deploying multiple agents/swarm, could be a heterogenous swarm that gather different kind of data; in remote areas, swarm of agents could possibly be located that relays info to the ground station

Challenges

- Latency and Possibility of Data/Info Loss in Communication Channels

(lossless communication channels research to overcome this)

- Sensor Failure

(can be overcome by using multiple sensors)

- Reliability Issues

(cases where human user can't override the autonomous system when it knows the better/right solution)

- Robustness Issues

(can be overcome by using more sensor or actuators or their configurations, can't guarantee cause the environment is uncertain, always better to add more redundancies; consider in design)

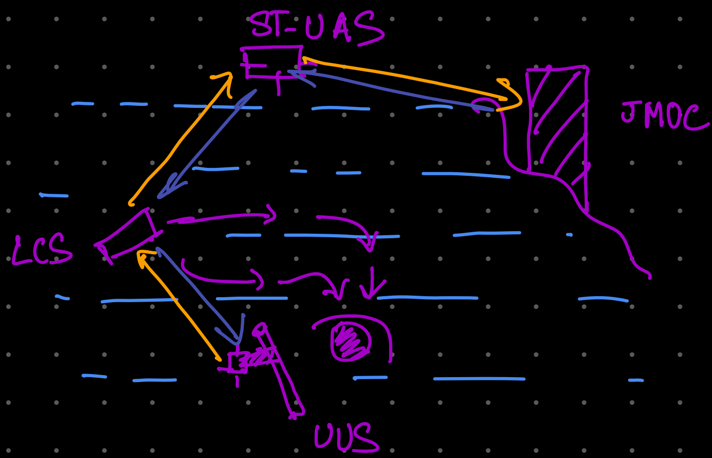
- Task Allocation Issues

(performance could be sluggish, one system does a task slower than the other and not assigned correctly?, meant for a situation when possibly there is a change in environment or constraints or available agents)

Manned-Unmanned (MUM) teamings

Littoral Oil Infrastructure

- Monitoring
- LCS (Littoral Combat Ship)
- ST-UAS (Small Tactical UAS)
- UUS (Underwater Unmanned Systems)
- JMOC (Joint Militant Operation



Interdiction

(This example is a terminal guidance thing smthn??)

LCS may not have capability to solve the issue,

ISR - Intelligence Surveillance Reconnaissance

(Used in medical industry robots, agriculture too)

Autonomy Levels

- Human - Machine Command and Control
  - Human-in-the-loop (hands on in the process)
  - Human-on-the-loop (more like supervised)
- Sophistication of Machine's Decision Making
  - Automatic (eg, like a wire trips if an certain situation occurs; simple)
  - Automated (eg, move from hostel to dept on its own; lots of rules to be followed; like self driving)
  - Autonomous (if its able to evolve its behaviour based on present conditions; like driverless cars)
- 3rd Classification
  - AL0 : Manual
  - AL1 : Manual Action On and Off-Board System Support
  - AL2 : 'Active' Human-in-the-loop
  - AL3 : Human-on-the-loop → More supervised
  - AL4 : 'Semi'-autonomous → Rarely supervised
  - AL5 : Autonomous → Unsupervised

we're technically talking about autonomy of a subsystem performing a job, cuz a human almost always is present like maybe in design process, embedding

Components

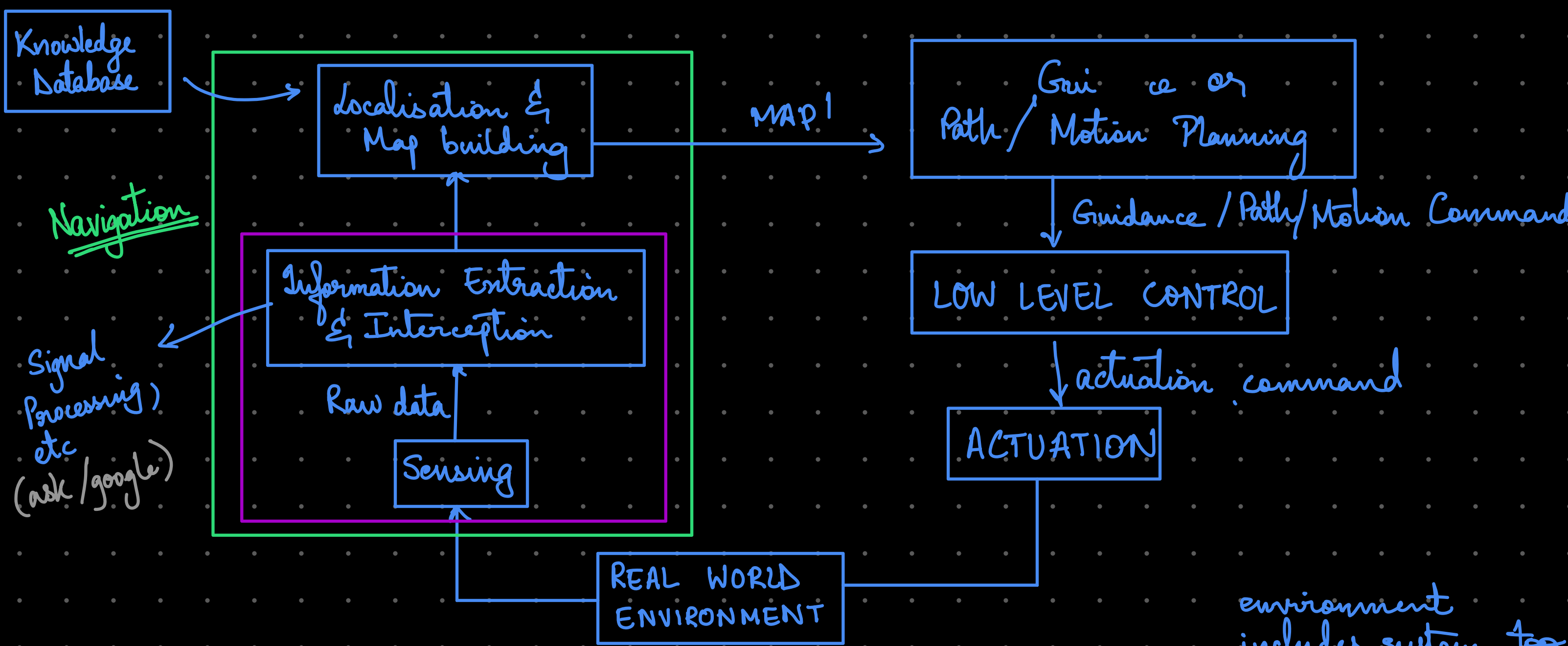
At the first/basic level, task planning

- Domain-Specific Components - if we have to specify the environment, or what the system is itself?
    - sensing
    - Vehicle and its characteristics (motion primitive)
    - Communication mechanism
    - Some more like power sources, ...
- Domain-Independent Components - vice versa
    - Algorithm itself (Task planning, Motion planning)

For eg, rover on terrain of unknown planet, use sensors to understand environment, these sensors are specific to space [environment]- sensors are domain specific

Spoofing in counter UAS systems

Schematic Diagram (online execution)



environment includes system too

At a raw level, we can do some pre planning, but its not that precise.

Invalid data from previous experience. Onboard sensors should be available to keep up with any changes.

Navigation and Guidance difference - Navigation sorta controls the movement; guidance gives path

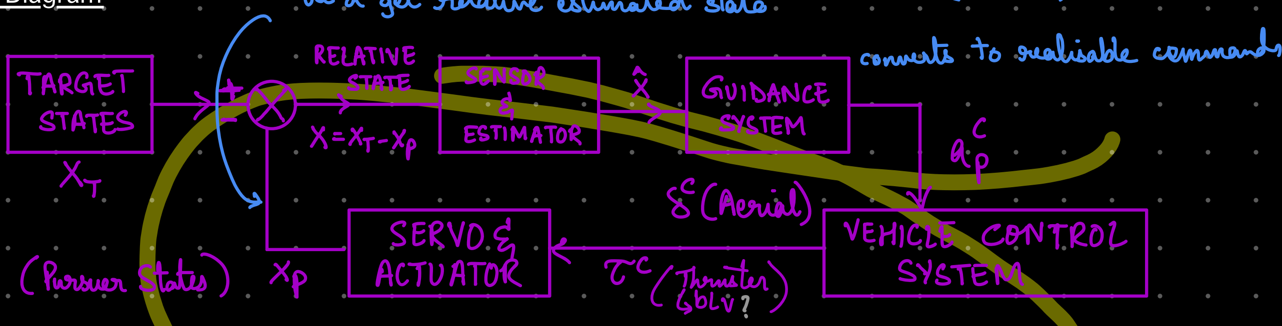
Global path planning - (offline path planning) , Global map has been built based on previous [ELABORATION??]

Pre-fed data is useful to build a preliminary path. Keep updating data with current experience.

irl, sensor & estimator fall here which gives $\hat{x}_p$ $(\hat{x}_T - \hat{x}_p)$ ← read upon this diagram.
we'd get relative estimated state

Block Diagram

converts to realisable commands



P – pursuer
t – target
X – state ?

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Schematic diagram is a flow of cognisance.

<u>Levels of cognisance</u>

- L1 - Raw Data
- L2 - Information
- L3 - Actionable/Realisable Information

Environments

- Natural
- Induced (debris after earthquake)

Major Components of UxVs (Unmanned Vehicles)

- Vehicle Frame
- Navigation Sub-system
- Guidance Sub-system
- Vehicle control Sub-system
- Actuation Sub-system

NGC / GNC System

what difference?

Apply thrust normal to velocity vector, lateral acceleration handled in this course. We are focusing on changing direction. (Aero equivalent is low angle of attack regime)

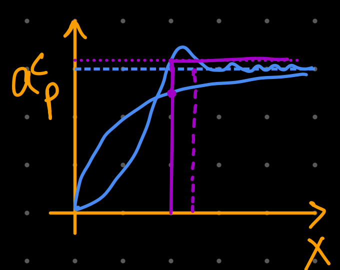
We wont be looking at longitudinal acceleration much.

Guidance Command (Latax)

- Majorly Lateral Acceleration (to be considered in this course) - responsible for mainly changing the UxVs heading direction

Downstream dynamics wont be the same/not really considered, so the guidance command given does not occur exactly n same magnitude in reality.

①



② Saturation Effect (machine not able to produce that much input)

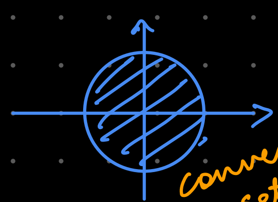
③ Lower Boundaries of Achievable Lateral Acceleration

Take any 2 points in a set and join them to get a straight line, then take any point on that line lies in the set - Convex set

If convex set, we can guarantee an optima point.

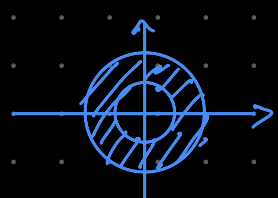
If non convex, we can use multiple approaches - split into multiple convex sets, find optima of each and merge;

$\|a_p\| < a_{p_{max}}$



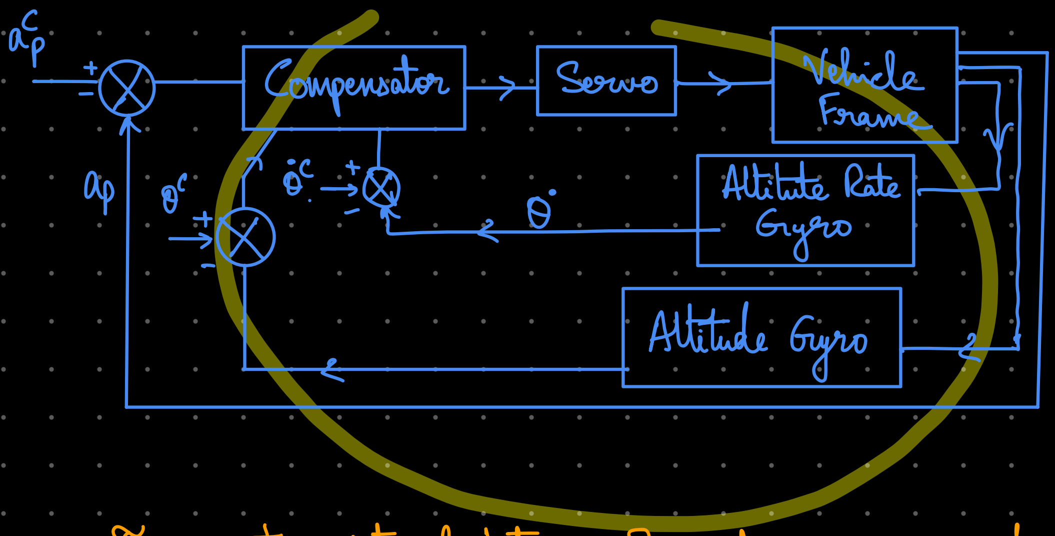
convex set

if lower boundaries exist



non convex set

Low-Level Controller Lateral Acceleration Autopilot



this system tries to correct orientation without sudden jerks (i.e large control inputs) but rather adjust it slowly

operate near an eqb^m point

τ_G — time interval between 2 guidance commands
τ_c — settling time for control input

$\tau_G > \tau_c$

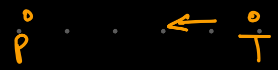
but in endgame scenarios, dynamics systems are evolving very fast so $\tau_G < \tau_c$
provision is kept by decreasing τ_c / increase computational efficiency.

Guidance Phases

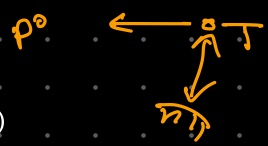
- Initial Phase
  - Pre-programmed Maneuver (mostly open loop; eg. sufficient clearance from base station for missiles)
    - Independent of Target Info
    - To clear the landing site safely
  - Boost Phase (eg. SAM missile put very high longitudinal acceleration)
    - Very high longitudinal acceleration
- Mid-Course Phase
  - To place the UxV within Target Acquisition Range
  - To bring the UxV in a ‘Favourable’ Endgame Conditions
  - Fuel-optimal/Time-optimal/…
- End-Game
  - Most crucial phase
  - ‘Locks on’ to the Target and ‘Closes in’ AND/OR reduces ‘Heading error’
  - “Miss Distance” or Terminal Separation is of prime importance (not only for missile like stuff, also in swarms, etc)

Guidance Schemes

- Non-homing [usually away from goal; initial phase usually, could be mid course phase][terrain mapping - TERCOM]
  - Inertial (close-loop - possibility of taking corrective action) [completely dependent on its own odometer data, less susceptible to jamming, but this also can have accumulation of error which may lead to huge divergence, explanation more needed
  - Pre-programmed (open-loop - explanation)
- Homing [usually in this scenario you are close to your goal; endgame phase usually, could be mid course phase]
  - Passive (the system itself does not emit any stuff, it only takes/senses from the target; susceptible to environment)
    - Sound seeker
    - Heat/IR seeker
    - Light seeker



  - Semi-active
    - Electromagnetic radiation (radar-based)



  - Active
    - Electromagnetic radiation (radar-based)

- External
  - Remote Command
  - Command-to-LOS (C-LOS)
  - Beam Rider (BR)

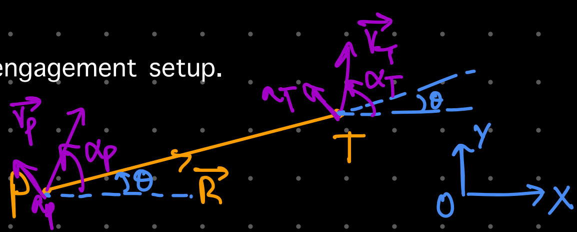
Why not to use external schemes in endgame phase - Latency in communication link, highly evolving scenario,
Midcourse phase is mostly a mixed guidance scheme.

Terminologies

this is a general pursuer-target engagement setup.

- Range/LOS (R vector)

$R = \|\vec{R}\|$

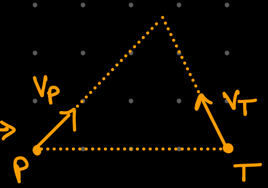


$V_P = \|\vec{V}_P\|$, $V_T = \|\vec{V}_T\|$

Heading Angle – α

Lateral acceleration – a

- Range Rate (R dot) - the rate at which the R vector is changing
- Closing speed (Vc = - R dot)
- LOS Turn Rate (or simply, LOS Rate; LOS angle θ) = θ dot
- Time-to-go; t_go - t_f (final time) - t(current time)
- Collision geometry (with this geometrical setup, collision is guaranteed; collision triangle)

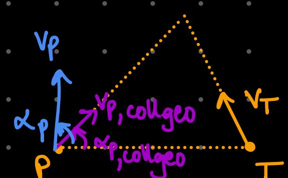


- Heading error - |α_p, collgeo - α_pl
- Relative speed along range vector

 [V_R = V_T cos(α_P- θ) - V_P cos (α_P - θ) = r dot]

- Relative speed across range vector

 [V_θ = V_T sin(α_P- θ) - V_P sin (α_P - θ) = r θdot]



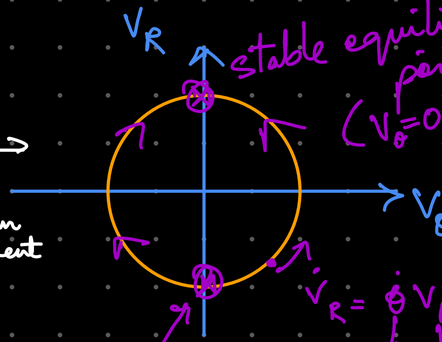
No Maneuver Scenario

$\hookrightarrow V_T, V_P \rightarrow$ constant ; R, θ — changing

$\dot{V}_R = \dot{\theta} V_\theta \quad , \quad \dot{V}_\theta = -\dot{\theta} V_R$

$V_R \dot{V}_R + V_\theta \dot{V}_\theta = 0 \implies V_\theta^2 + V_R^2 = \text{constant}$

look at these relations to confirm direction of movement of velocity vectors



stable equilibrium point ($V_\theta = 0, V_R > 0$)

$\dot{V}_R = \dot{\theta} V_\theta$

-ve $\quad R\dot{\theta}$

collision geometry corresponds to this point

diverging case; no guidance ; distance will keep on increasing

equilibruim is state from which it does not change

$\Rightarrow \dot{V}_R = 0$

$\dot{x} = f(x)$

$\dot{x}_e = f(x_e) = 0 \rightarrow \dot{\theta} = 0$

Classification of Guidance Laws.

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No Maneuver Scenario

$V_R = \dot{R} = V_T \cos(\alpha_T - \theta) - V_p \cos(\alpha_p - \theta) \rightarrow \dot{V}_R = V_T(-\sin(\alpha_T-\theta))(-\dot{\theta}) - V_p(-\sin(\alpha_p-\theta))(-\dot{\theta})$

$= V_T \sin(\alpha_T - \theta)\dot{\theta} - V_p \sin(\alpha_p - \theta)\dot{\theta} = \dot{\theta} V_\theta$

$V_\theta = R\dot{\theta} = V_T \sin(\alpha_T - \theta) - V_p \sin(\alpha_p - \theta)$ → same as above.

$a_p = 0 \qquad a_T = 0$

$\Rightarrow \dot{V}_p = 0, \ \dot{\alpha}_p = 0 \qquad \Rightarrow \dot{V}_T = 0, \ \dot{\alpha}_T = 0$

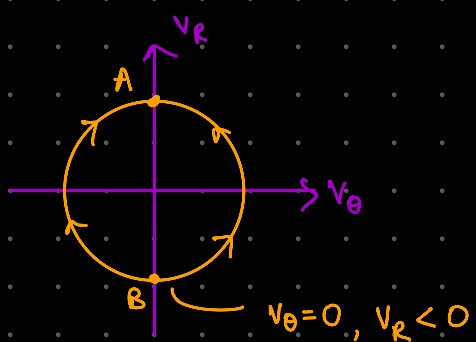
$\dot{V}_R = \dot{\theta} V_\theta \quad , \quad \dot{V}_\theta = -\dot{\theta} V_R$

$\dot{\underset{\sim}{X}} = f(\underset{\sim}{X})$

X_e is defined, as the point in the state-space, for which $\dot{\underset{\sim}{X}}\Big|_{\underset{\sim}{X}_e} = f(\underset{\sim}{X}_e) = 0$

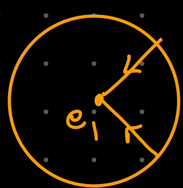
$\Rightarrow \dot{\theta} = 0$ for equilibrium

$V_R^2 + V_\theta^2 = \text{constant}$
$= V_{R_0}^2 + V_{\theta_0}^2$



globally - if for entire state-space

(locally) stable equilibrium point



Miss Distance - shortest distance by which pursuer misses the target.

$V_R^2 + V_\theta^2 = \text{constant} = e^2$ (say) → $R^2\dot{\theta}^2$

$V_R = \dot{\theta}\, \cancel{V_\theta}\, R\dot{\theta}$

$\dot{R} = \dot{\theta}^2 R$

$R\ddot{R} = \dot{\theta}^2 R^2$

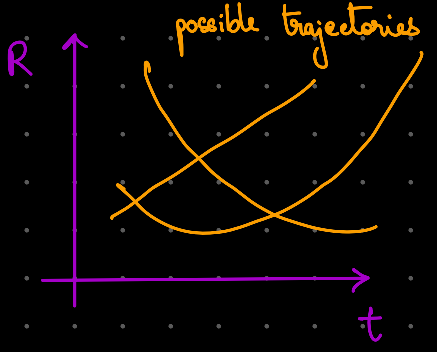
$\Rightarrow \dot{R}^2 + R\ddot{R} = e^2$

integrate wrt time $\Rightarrow R\dot{R} = e^2 t + R_0 V_{R_0}$

At miss distance, $\dot{R} = 0$

$\Rightarrow t_{MISS} = -\dfrac{R_0 V_{R_0}}{V_{R_0}^2 + V_{\theta_0}^2}$

integrate wrt time



$\frac{1}{2}R^2 \qquad \frac{e^2 t^2}{2} + R_0 V_{R_0} + \frac{R_0^2}{2}$

$$\Rightarrow \boxed{R_{miss} = R_0 \sqrt{\frac{V_{\theta_0}^2}{V_{\theta_0}^2 + V_{R_0}^2}}}$$

↳ given this will we apply guidance for $R_{miss} \neq 0$

some sort of classification

→ Classical Guidance
(in the differential geometry framework?)

→ Game Theoretic Guidance — Static
— Dynamical

Pursuit Guidance

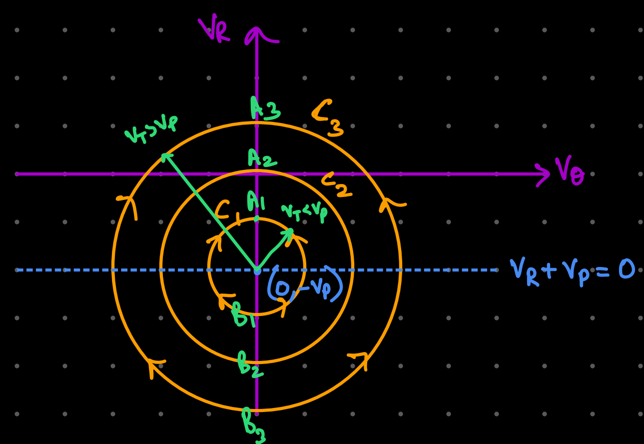
- Pure Pursuit (PP)
- Deviated PP (DPP)

Pursuit Geometry

$\alpha_p = \theta \qquad (V_T, V_P \to \text{constant})$

$$\boxed{\begin{array}{ll} V_R = V_T \cos(\alpha_T - \theta) - V_P \Rightarrow & \dot{V}_R = + V_T \sin(\alpha_T - \theta)\dot{\theta} = V_\theta \dot{\theta} \\ V_\theta = V_T \sin(\alpha_T - \theta) \Rightarrow & \dot{V}_\theta = -V_T \cos(\alpha_T - \theta)\dot{\theta} = -(V_R + V_P)\dot{\theta} \end{array}}$$

↳ $V_\theta^2 + (V_R + V_P)^2 = V_T^2$



$\dot{\theta} = 0 \Rightarrow V_\theta = 0 \longrightarrow$ WLOS

$\Rightarrow \alpha_T = \theta \;,\; \pi + \theta$

TAIL-CHASE (also $\alpha_P = 0$) HEAD-ON

At equilibrium point, $\vec{V_P}$, $\vec{V_T}$ & $\vec{R}$ are collinear

on C_1

at A_1 : $V_\theta = 0$, $V_R < 0$ (any other points lead to this)

at B_1, $V_\theta = 0$, $V_R < 0$

} Collision course

on C_3

at A_3 : $V_\theta = 0$, $V_R > 0$ (any other points lead to this)

$\downarrow$

Inverse Collision course

at B_3 , $V_\theta = 0$, $V_R < 0$

$\downarrow$

Collision course

on C_2

at A_2 : $V_\theta = 0$, $V_R = 0$

$\downarrow$

constant miss distance

$\Downarrow$

also the shortest distance (before this $V_R < 0$, so it has been decreasing only)

$$\frac{V_R}{V_\theta} = \frac{\dot{R}}{R\dot{\theta}} = \cot\Psi - \nu \operatorname{cosec}\Psi \; ; \quad \Psi = \alpha_T - \theta , \quad \nu = \frac{V_P}{V_T}$$

$$d\Psi = -d\theta$$

$\alpha_P = \theta$ (PP geometry)

once on the PP geometry, guidance command

$a_P = V_P \dot{\alpha}_P = V_P \dot{\theta}$

(to maintain)

$$\Rightarrow \frac{1}{R} dR = -(\cot\Psi - \nu \operatorname{cosec}\Psi)\, d\Psi$$

Steps.

$$\ln R = -\ln|\sin\Psi| - \nu \ln|\operatorname{cosec}\Psi + \cot\Psi| + \ln c$$

$$\ln R_0 = \ln\frac{(\operatorname{cosec}\Psi_0 + \cot\Psi_0)^{-\nu}}{\sin\Psi_0} + \ln c \Rightarrow c = \frac{R_0 \sin\Psi_0}{(\operatorname{cosec}\Psi_0 + \cot\Psi_0)^{-\nu}}$$

$$R = \frac{R_0 \sin\Psi_0}{(\operatorname{cosec}\Psi_0 + \cot\Psi_0)^{-\nu}} \cdot \frac{(\operatorname{cosec}\Psi + \cot\Psi)^{-\nu}}{\sin\Psi} = \frac{R_0\, \cancel{2}\sin\frac{\Psi_0}{2}\cos\Psi_0/2}{\left(\frac{\cancel{2}\cos^{\cancel{2}}\Psi_0/2}{\cancel{2}\sin\Psi_0/2\,\cancel{\cos\Psi_0/2}}\right)^{-\nu}} \cdot \frac{\left(\frac{\cancel{2}\cos^{\cancel{2}}\Psi/2}{\cancel{2}\sin\Psi/2\,\cancel{\cos\Psi/2}}\right)^{-\nu}}{\cancel{2}\sin\Psi/2\,\cos\Psi/2}$$

$$R(\Psi) = \left[R_0 \frac{\cos^{\nu+1}(\Psi_0/2)}{\sin^{\nu-1}(\Psi_0/2)} \right] \frac{\sin^{\nu-1}(\Psi/2)}{\cos^{\nu+1}(\Psi/2)}$$

$\Psi \to 0$, then

if $\nu > 1$, $R \to 0$

$\nu < 1$, $R \to \infty$

$\nu = 1$, $R \to$ constant $\rightarrow$

$$R(\Psi) = R_0 \frac{\cos^2(\Psi_0/2)}{\cos^2(\Psi/2)}$$

$$R(\Psi \to 0) = R_0 \cos^2(\Psi_0/2)$$

$\Psi = \alpha_T - \theta$

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1. Pure Pursuit against α_T dot = 0

Capturability analysis

- Capture zone/region of 'a' guidance law - the subset of initial conditions starting from which a pursuer is able to capture/intercept 'the' target using 'the' guidance law

$$CR_{pp}\Big|_{\dot{\alpha}_T=0} = \left\{ (V_{\theta_0}, V_{R_0}) \;\middle|\; V_{\theta_0}^2 + (V_{R_0}+V_P)^2 < V_P^2 \right\}$$

$$= \left\{ (V_{\theta_0}, V_{R_0}) \;\middle|\; V_{\theta_0}^2 + (V_{R_0}+V_P)^2 = V_T^2 \;;\; V_T < V_P \right\}$$

$R_0,\ \alpha_{P_0},\ \alpha_{T_0},\ \theta_0,\ V_P,\ V_T$

* equivalence of set in one space with another space is formally not proved in this course but can be done rigorously with maths

$\mathrm{sgn}(\dot{\psi}) = -\,\mathrm{sgn}(\psi)$ $\qquad \psi = \alpha_T - \theta \qquad (\dot{\psi} = -\dot{\theta})$

$\psi \to (0, \pi) \qquad (-\pi, 0)$

$(0,\pi)$: monotonically decrease & converge to zero

$(-\pi,0)$: monotonically increase & converge to zero

$$a_p = V_P \dot{\alpha}_P = V_P \dot{\theta}$$

$$= \frac{V_P V_T}{R} \sin(\alpha_T - \theta)$$

$$\boxed{a_p = \frac{4 V_P V_T}{C_0} \frac{\cos^{\nu+2}(\psi/2)}{\sin^{\nu-2}(\psi/2)}}$$

we have to make sure a_p is bounded

As $\psi \to 0$,

$\nu > 2 \Rightarrow a_p > \infty$

$\nu = 2 \Rightarrow a_p = \dfrac{4 V_P V_T}{C_0}$

$\nu < 2 \Rightarrow a_p < 0$

Final Time

$$\dot V_R = \dot\theta \cancel{V_\theta}\, R\dot\theta$$
$$\ddot R = \dot\theta^2 R$$
$$R\ddot R = \dot\theta^2 R^2$$

$$(\overset{\dot R}{V_R} + V_p)^2 + \overset{R\dot\theta}{V_\theta^2} = V_T^2$$

$$\Rightarrow \dot R^2 + R\ddot R + 2V_p\dot R = V_T^2 - V_p^2$$

$$\Rightarrow \frac{d}{dt}\left(R(\dot R + 2V_p)\right) = V_T^2 - V_p^2$$

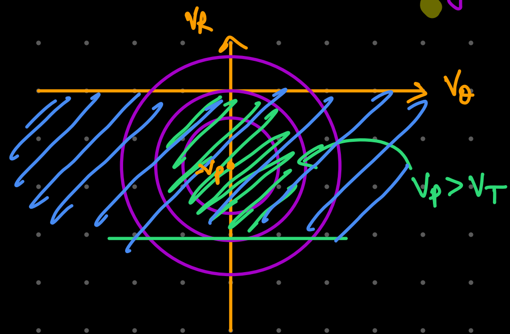
$$\int \Rightarrow R(\dot R + 2V_p) = (V_T^2 - V_p^2)t + R_0(V_{R_0} + 2V_p)$$

At capture, $R = 0$

$$\Rightarrow \boxed{t_f = \frac{R_0(V_{R_0} + 2V_p)}{V_p^2 - V_T^2}}$$

V_{R_0} max value in capture region is $-2V_p$ (bottommost point in V_R-V_θ trajectory circle)

For $V_p > V_T$, $\underline{V_{R_0} > -2V_p}$ only



For $V_p \le V_T$, there is a miss distance

At miss distance, $V_R = 0$ → *

$$\Rightarrow V_T \cos\Psi_{miss} = V_p$$

$$\Rightarrow \Psi_{miss} = \cos^{-1}(\nu)$$

$$\Rightarrow \boxed{R_{miss} = C_0 \frac{[\sin(\cos^{-1}(\nu))]^{\nu-1}}{(1+\nu)^{\nu}}}$$

*Derive

$$R(\Psi) = R_0 \frac{\cos^{\nu+1}(\Psi_0/2)}{\sin^{\nu-1}(\Psi_0/2)} \frac{\sin^{\nu-1}(\Psi/2)}{\cos^{\nu+1}(\Psi/2)}$$

$$R_{miss} = C_0 \frac{\sin^{\nu-1}\left(\frac{\cos^{-1}\nu}{2}\right)\cos^{\nu-1}\left(\frac{\cos^{-1}\nu}{2}\right)}{\left[\cos\left(\frac{\cos^{-1}(\nu)}{2}\right)\right]^{\nu+1}\cos^{\nu-1}\left(\frac{\cos^{-1}\nu}{2}\right)}$$

$$= \frac{C_0}{2^{\nu-1}} \frac{\left[\sin(\cos^{-1}\nu)\right]^{\nu-1}}{\left[\cos^2\left(\frac{\cos^{-1}\nu}{2}\right)\right]^{\nu-1}\cancel{\cos^2\left(\frac{\cos^{-1}\nu}{2}\right)}}$$

$$\left(\frac{\nu+1}{2}\right)^{\nu-1} \qquad \frac{\nu+1}{2}$$

$$= \frac{2C_0(\sin(\cos^{-1}\nu))^{\nu-1}}{(\nu+1)^{\nu}}$$

$$\Rightarrow \boxed{t_{miss} = \frac{2V_pR_{miss} - R_0(V_{R_0} + 2V_p)}{V_T^2 - V_p^2}}$$

*Derive

$\dot R = 0$

2. Pure Pursuit against α_T dot = constant

$$\left.\begin{aligned} V_R &= \dot{R} = V_T\cos(\alpha_T-\theta) - V_P \\ V_\theta &= R\dot{\theta} = V_T\sin(\alpha_T-\theta) \end{aligned}\right\} \Rightarrow V_\theta^2 + (V_R+V_P)^2 = V_T^2$$

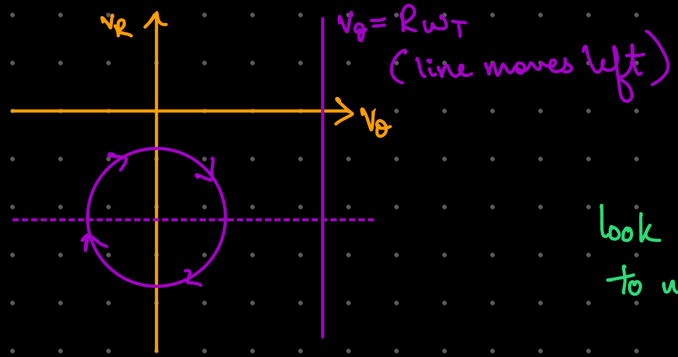
$$\dot{\alpha}_T = \omega_T = \text{constant}$$

$$\boxed{\begin{aligned} R\,\dot{V}_R &= -RV_T\sin(\alpha_T-\theta)\left[\dot{\alpha}_T-\dot{\theta}\right] = V_\theta\left[V_\theta - R\omega_T\right] \\ R\,\dot{V}_\theta &= RV_T\cos(\alpha_T-\theta)\left[\dot{\alpha}_T-\dot{\theta}\right] = -\left[V_R+V_P\right]\left(V_\theta - R\omega_T\right) \end{aligned}}$$

Equilibrium State: $V_\theta = R\omega_T$

Case 1: $V_P > V_T \Rightarrow V_R < 0$

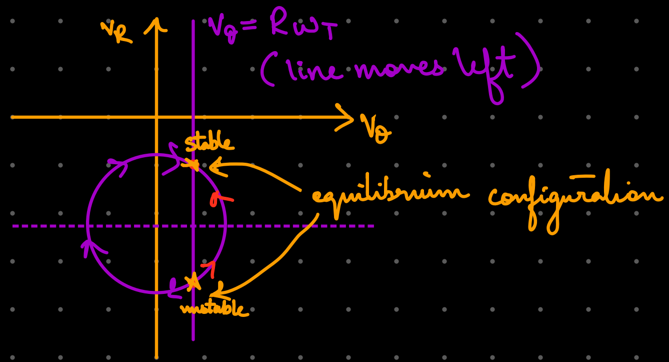
1A $V_{\theta_0} < R\omega_T$



look at signs of $\dot{V}_R$ & $\dot{V}_\theta$ to understand arrows on circle

Q. is it possible that V_θ line goes to zero before V_R reaches straight line trajectory?

1A $V_{\theta_0} < R\omega_T$



Say $t = t_1$,

$(V_\theta, V_R)_{\text{State trajectory}} = q$

$V_\theta = R\omega$ and the circle intercept at q

then, $V_{\theta_1} = V_\theta(t_1) = R(t_1)\,\omega_T = R_1\,\omega_T$ (say)

$V_{R_1} < 0$, $\dot{V}_R(t_1) = \dot{V}_\theta(t_1) = 0$

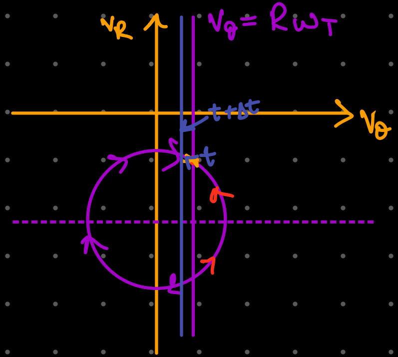
At $t = t_1 + \Delta t$,

$$v_\theta(t_1+\Delta t) = v_{\theta_1}, \quad v_R(t+\Delta t) = v_{R_1}$$

But,

$$R(t_1+\Delta t) = R_1 - \Delta R$$

$$\Rightarrow v_\theta(t_1+\Delta t) - R(t_1+\Delta t)\,\omega_T > 0$$



[This needs a written explanation abt what is happening graphically]

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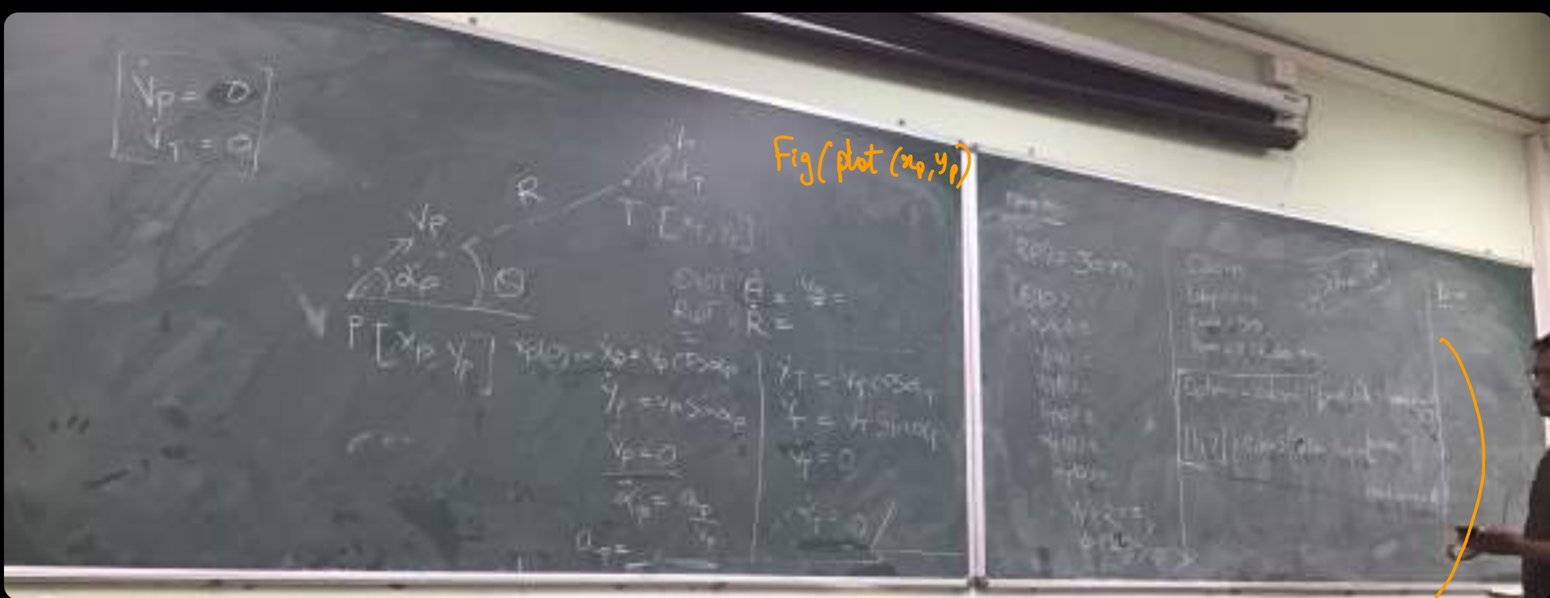
<u>Tutorial Class</u>



function dydt = kin(t,0)

TAs → AE20D412

→ $\boxed{\text{AE19D005}}$ <u>Vivek</u>.

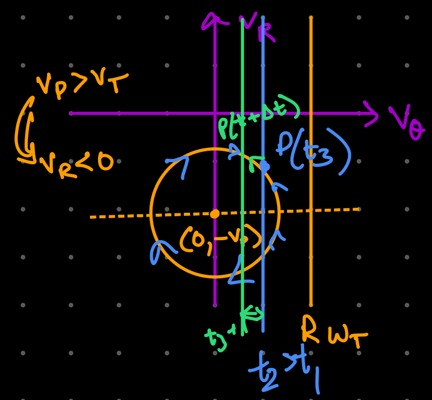
PP against $\dot{\alpha}_T = \omega_T = \text{constant}$ $(>0, \text{say})$

$\left[V_R = \text{-----}, V_\theta = \text{-----}\right] \Rightarrow (V_R+V_P)^2 + V_\theta^2 = V_T^2$

$R\dot{V}_R = (V_\theta - R\omega_T)V_\theta$

$R\dot{V}_\theta = -(V_\theta - R\omega_T)(V_R+V_P)$

At equilibrium, $V_\theta = R\omega_T$



Say at $t = t_3$, the state trajectory point touches the $V_\theta = R\omega_T$ line at point 'P'.

once V_θ becomes equal to $R\omega_T$ at some time $t = t_3$, at which $R(t_3) = R_3$, $V_\theta(t_3) = R_3\omega_T$, then onward the (V_θ, V_R) point keeps on following the point of intersection 'P' of $(V_R+V_P)^2 + V_\theta^2 = V_T^2$ and $V_\theta = R\omega_T$ till R becomes zero.

$V_P < V_T$

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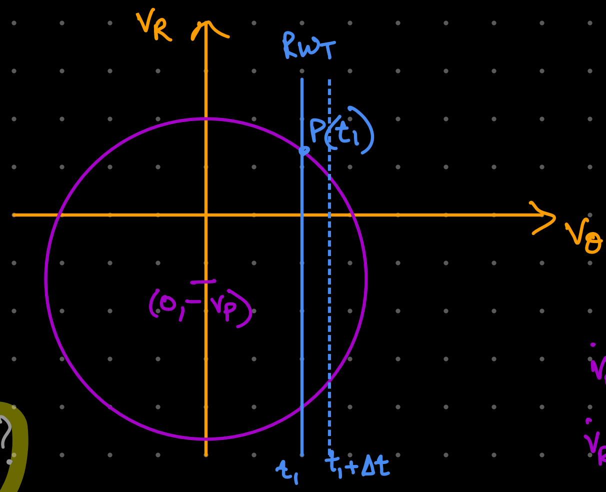
$$(V_R+V_P)^2 + V_\theta^2 = V_T^2$$

$$R\dot{V}_R = (V_\theta - R\omega_T)V_\theta$$

$$R\dot{V}_\theta = -(V_\theta - R\omega_T)(V_R+V_P)$$



$\dot{V}_\theta(t_1)=0$

$\dot{V}_R(t_1)=0$

R remains constant in a particular side? ψ?

$$\nu = \frac{V_P}{V_T}$$

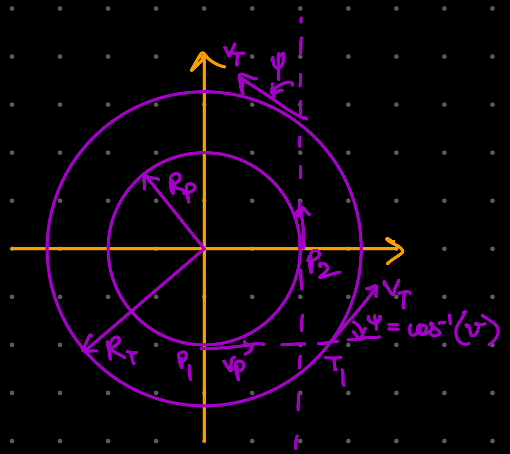
$$\frac{V_R}{V_\theta} = \frac{\cos\psi - \nu}{\sin\psi} \Rightarrow \frac{dR'}{d\psi} = \frac{(\cos\psi - \nu)R'}{\left(\frac{R}{R_T}\right) - \sin\psi}$$

at equilibrium,

$$\psi = \cos^{-1}(\nu)$$

$$R = R_T \sin(\cos^{-1}(\nu))$$

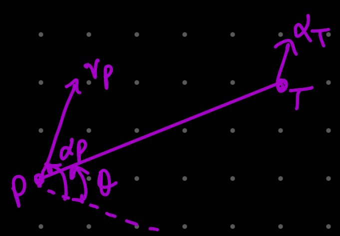
$$R = R_T\sqrt{1-\nu^2}$$



$$\dot{\psi} = \omega_T - \dot{\theta}$$

$$\dot{\theta} = \omega_T - \dot{\psi}$$

$$\frac{dR}{R} = \frac{(\omega_T - \dot{\psi})(\cos\psi - \nu)}{\sin\psi}$$



$$\dot{\alpha}_P = \dot{\theta}_P + K(\alpha_P - \theta)$$

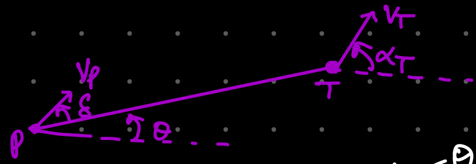
positive

$$\mathrm{sgn}(f(\alpha_P - \theta)) = -\mathrm{sgn}(\alpha_P - \theta)$$

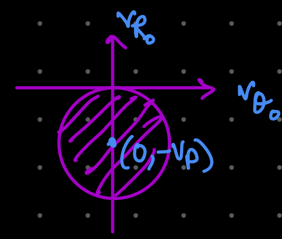
start quiz 2

Deviated Pure Pursuit (DPP)

DPP geometry



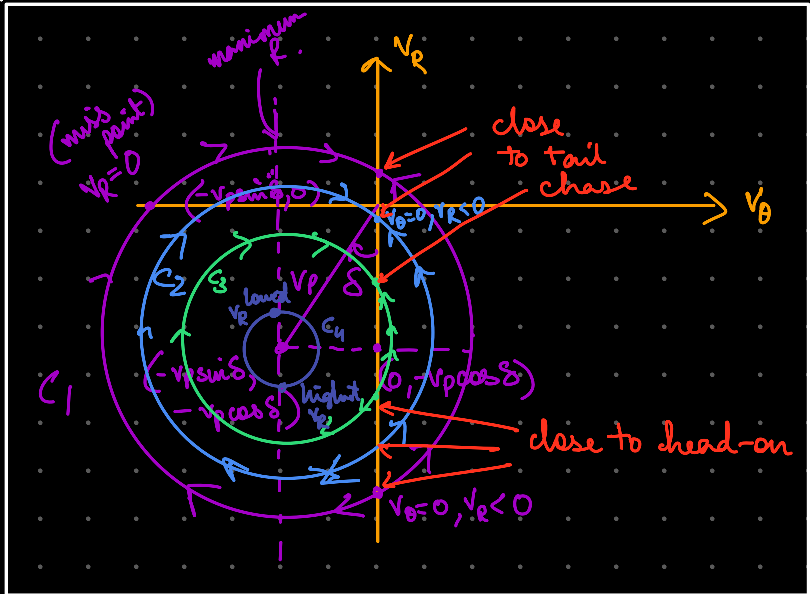
$$\alpha_p = \alpha_T - \theta$$



$$\boxed{\begin{aligned} V_R &= V_T \cos(\alpha_T - \theta) - V_p \cos\delta \\ V_\theta &= V_T \sin(\alpha_T - \theta) - V_p \sin\delta \end{aligned}}$$

$$(V_\theta + V_p \sin\delta)^2 + (V_R + V_p \cos\delta)^2 = V_T^2$$

$$\left.\begin{aligned} \dot{V}_R &= \dot{\theta}\,(V_\theta + V_p \sin\delta) \\ \dot{V}_\theta &= -\dot{\theta}\,(V_R + V_p \cos\delta) \end{aligned}\right\} \rightarrow$$



$C_1 : V_T > V_p$

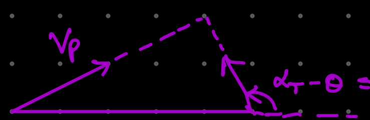
$C_2 : V_p \cos\delta < V_T < V_p$

$C_3 : V_p \sin\delta < V_T < V_p \cos\delta$

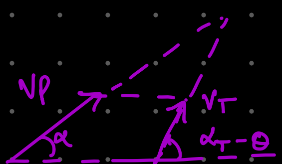
$C_4 : V_T < V_p \sin\delta$ (still there is contact because $V_R < 0$) but it never attains equilibrium.

upper bound of t_f → lowest V_R
lower bound of t_f → highest V_R

Collision course : $V_T \sin(\alpha_T - \theta) = V_p \sin\delta$

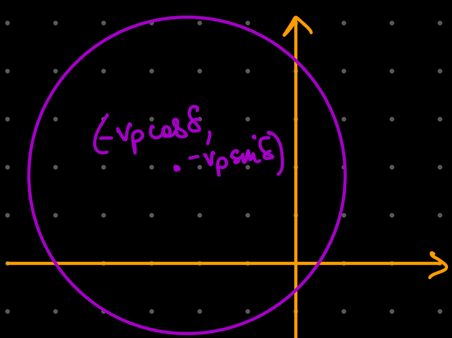


$\alpha_T - \theta = \pi - \sin^{-1}(\nu \sin\delta)$ ≈ Head-on



$\alpha_T - \theta = \sin^{-1}(\nu \sin\delta)$ ≈ Tail-chase

if $\delta > \pi/2$, ⟶ unstable equilibrium also goes into $V_R > 0$ region



$\delta = \pi/2$ → stable equilibrium goes into $V_R > 0$ region when $\delta = \pi$, practically doesn't make sense.

(facing opposite to each other)

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DPP against $\dot{\alpha}_T = 0$?,

t_f:

$$(V_R + V_p\cos\delta)^2 + (V_\theta + V_p\sin\delta)^2 = V_T^2$$

$$\Rightarrow \dot{V_R^2} + R\dot{V}_R + V_\theta V_p \sin\delta + 2V_R V_p\cos\delta = V_T^2 - V_P^2$$

$$\overset{\parallel}{-\frac{d}{dt}(V_\theta R)\tan\delta}$$

(NPTEL)

$\cancel{V_\theta\dot{R}} + R\dot{V}_\theta$

$(\cancel{-\dot{\theta}V_R} - \dot{\theta}V_p\cos\delta)$

$\dot{V}_\theta = -\dot{\theta}(V_R + V_p\cos\delta)$

$- \cancel{R\dot{\theta}} V_p\cos\delta \times \tan\delta$

V_θ

$$d(R\dot{R}) - \tan\delta \, d(V_\theta R) + 2V_p\cos\delta \, d(R) = (V_T^2 - V_p^2)\,dt$$

$$\Rightarrow R\left[V_R - V_\theta\tan\delta + 2V_p\cos\delta\right] = (V_T^2 - V_p^2)t + R_0\left[V_{R_0} - V_{\theta_0}\tan\delta + 2V_{p_0}\cos\delta\right]$$

At $t = t_f$, $R = 0$,

$$\boxed{t_f = \frac{R_0\left[V_{R_0} - V_{\theta_0}\tan\delta + 2V_{p_0}\cos\delta\right]}{V_p^2 - V_T^2}}$$

$\Psi = \alpha_T - \theta$

$d\Psi = -d\theta$

$R(\Psi)$

$$\frac{V_R}{V_\theta} = \frac{\cos\Psi - \nu\cos\delta}{\sin\Psi - \nu\sin\delta} \Rightarrow \frac{dR}{R} = \frac{\nu\cos\delta}{\sin\Psi - \nu\sin\delta}d\Psi - \frac{\cos\Psi}{\sin\Psi - \nu\sin\delta}d\Psi$$

Define μ such that, $\boxed{\begin{aligned}\sin\Psi_{cc} &= \nu\sin\delta \\ \mu\cos\Psi_{cc} &= \nu\cos\delta\end{aligned}}$

$\frac{\tan\Psi_{cc}}{\mu} = \tan\delta$

cc – collision course

then,

$$\frac{dR}{R} = \frac{\mu\cos\Psi_{cc}}{\sin\Psi - \sin\Psi_{cc}}d\Psi - \frac{\cos\Psi}{\sin\Psi - \sin\Psi_{cc}}d\Psi$$

$$\Rightarrow \frac{dR}{R} = (\mu - 1)\frac{\cos\left(\frac{\Psi - \Psi_{cc}}{2}\right)}{\sin\left(\frac{\Psi - \Psi_{cc}}{2}\right)}d\Psi + (\mu + 1)\frac{\sin\left(\frac{\Psi + \Psi_{cc}}{2}\right)}{\cos\left(\frac{\Psi + \Psi_{cc}}{2}\right)}d\Psi$$

$$\Psi =$$

$$\frac{dR}{R} = \frac{\mu\cos\Psi_{cc}}{\sin\Psi - \sin\Psi_{cc}}\, d\Psi - \frac{\cos\Psi}{\sin\Psi - \sin\Psi_{cc}}\, d\Psi$$

$$\mu\cos\Psi_{cc} = \mu\cos\left(\frac{\Psi_{cc}+\Psi}{2} + \frac{\Psi_{cc}-\Psi}{2}\right) \qquad \cos\Psi = \cos\left(\frac{\Psi+\Psi_{cc}}{2} + \frac{\Psi-\Psi_{cc}}{2}\right)$$

$$\mu\cos\Psi_{cc} = \mu\cos\left(\frac{\Psi_{cc}+\Psi}{2}\right)\cos\left(\frac{\Psi_{cc}-\Psi}{2}\right) - \mu\sin\left(\frac{\Psi_{cc}+\Psi}{2}\right)\sin\left(\frac{\Psi_{cc}-\Psi}{2}\right)$$

$$\cos\Psi = \cos\left(\frac{\Psi+\Psi_{cc}}{2}\right)\cos\left(\frac{\Psi-\Psi_{cc}}{2}\right) - \sin\left(\frac{\Psi_{cc}+\Psi}{2}\right)\sin\left(\frac{\Psi_{cc}-\Psi}{2}\right)$$

$$\frac{dR}{R} = \frac{\mu\cos\left(\frac{\Psi_{cc}+\Psi}{2}\right)\cos\left(\frac{\Psi_{cc}-\Psi}{2}\right) - \mu\sin\left(\frac{\Psi_{cc}+\Psi}{2}\right)\sin\left(\frac{\Psi_{cc}-\Psi}{2}\right)}{\sin\left(\frac{\Psi-\Psi_{cc}}{2}\right)\cos\left(\frac{\Psi+\Psi_{cc}}{2}\right)}$$

$$\frac{\cos\left(\frac{\Psi+\Psi_{cc}}{2}\right)\cos\left(\frac{\Psi-\Psi_{cc}}{2}\right) - \sin\left(\frac{\Psi_{cc}+\Psi}{2}\right)\sin\left(\frac{\Psi_{cc}-\Psi}{2}\right)}{\sin\left(\frac{\Psi-\Psi_{cc}}{2}\right)\cos\left(\frac{\Psi+\Psi_{cc}}{2}\right)}$$

$$= -\mu\cot\left(\frac{\Psi_{cc}-\Psi}{2}\right) + \mu\tan\left(\frac{\Psi_{cc}+\Psi}{2}\right) + \cot\frac{(\Psi-\Psi_{cc})}{2} + \tan\frac{(\Psi_{cc}+\Psi)}{2}$$

$$\frac{dR}{R} = (\mu-1)\int -\cot\left(\frac{\Psi_{cc}-\Psi}{2}\right)d\Psi + \mu+1\int\tan\left(\frac{\Psi+\Psi_{cc}}{2}\right)d\Psi$$

$$\ln R = \left(\frac{\mu-1}{2}\right)\ln\left|\sin\left(\frac{\Psi-\Psi_{cc}}{2}\right)\right| + \left(\mu+1\right)\ln\left|\sec\left(\frac{\Psi+\Psi_{cc}}{2}\right)\right|$$

$$\Longrightarrow \boxed{R(\psi) = \frac{C_0 \sin^{\mu-1}\left(\frac{\psi-\psi_{cc}}{2}\right)}{\cos^{\mu+1}\left(\frac{\psi+\psi_{cc}}{2}\right)}}$$

$(\mu-1)\ln\left(\operatorname{cosec}\left(\frac{\psi-\psi_{cc}}{2}\right)\right) \quad (\mu+1)\ln\left(\sec\left(\frac{\psi+\psi_{cc}}{2}\right)\right)$

As $\psi \to \psi_{cc}$,

If $\mu > 1 \longrightarrow R \to 0$

$\mu = 1 \longrightarrow R = \text{constant}$

$\mu < 1 \longrightarrow R \to \infty$

ψ_{cc} is not a physically relevant term, but helps understand if we are converging to collision course

$\dot{\alpha}_p = \dot{\theta}, \quad \alpha_p = \theta + \delta$

$a_p(\psi) = v_p \dot{\alpha}_p = v_p \dot{\theta} \qquad (\because \dot{\delta} = 0)$

$= v_p \dfrac{v_\theta}{R(\psi)} = v_p V_T \left(\dfrac{\sin\psi - \sin\psi_{cc}}{R(\psi)}\right)$

$$V_p V_T \frac{C \sin\left(\frac{\psi-\psi_{cc}}{2}\right)\cos\left(\frac{\psi+\psi_{cc}}{2}\right)}{\left(\frac{\sin^{\mu-1}\left(\frac{\psi-\psi_{cc}}{2}\right)}{\cos^{\mu+1}\left(\frac{\psi+\psi_{cc}}{2}\right)}\right)}$$

Steps

$$\boxed{a_p(\psi) = (\text{constant}) \frac{\cos^{\mu+2}\left(\frac{\psi+\psi_{cc}}{2}\right)}{\sin^{\mu-2}\left(\frac{\psi-\psi_{cc}}{2}\right)}}$$

As $\psi \to \psi_{cc}$,

If $\mu < 2 \longrightarrow a_p \to 0$

$\mu = 2 \longrightarrow a_p = \text{finite constant}$

$\mu > 2 \longrightarrow a_p \to \infty$

for finite lateral acc^n

For PP : $\nu \leq 2$

For DPP : $\mu \leq 2$

$\mu = \dfrac{\nu\cos\delta}{\sqrt{1-\nu^2\sin^2\delta}} < 2 \Rightarrow \nu^2(1-\sin^2\delta) < 4(1-\nu^2\sin^2\delta)$

$\nu^2(1+3\sin^2\delta) < 4$

* check steps.

$$\Rightarrow \quad v \leq \frac{2}{\sqrt{1+3\sin^2\delta}}$$

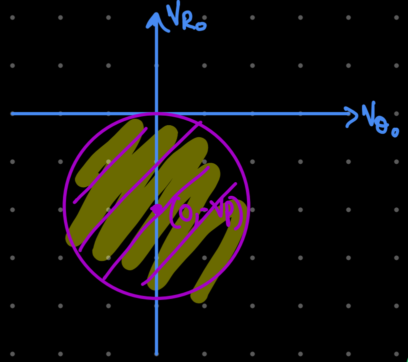
wider range for pure pursuit

$\delta_{max} = \pi/2 \rightarrow v = 1$

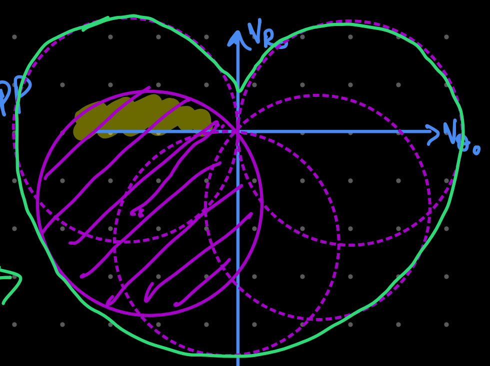
$\delta_{min} = 0 \rightarrow v = 2$

constrained as $\underline{1 < v < 2}$.

PP

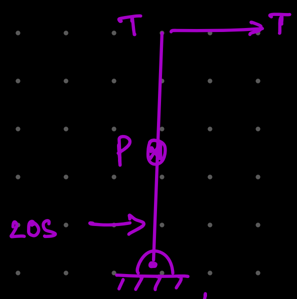


DPP



$$CR_{DPP}\Big|_{\dot{\alpha}_T=0} = \left\{ (V_{\theta_0}, V_{R_0}) \;\middle|\; (V_{R_0} + V_p\cos\delta)^2 + (V_{\theta_0} + V_p\sin\delta)^2 < V_p^2,\; -\pi/2 < \delta < \pi/2 \right\}$$

LINE-OF-SIGHT/LOS Guidance



(Launch/Base Station)

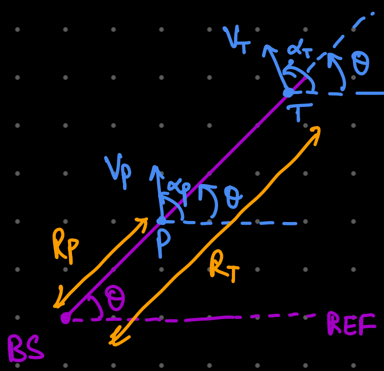
(in this course, we will consider this as stationary)

1) Beam Rider (BR)
2) C-LOS (Command-to-LOS)

→ the pursuer onboard system understand how far it is on beam, and the generates a guidance command

(not semi-active homing cuz no sensing reflection from target)

→ externally commanded guidance (deviation calculation & everything done externally)



Condition 1: P has to be on the LOS joining BS & T.

Condition 2: $\dot{\theta}$ should be same for both P–BS, T–BS, T–P engagements.

(assumption that BS is always tracking target)

$V_{R_P} = V_P \cos(\alpha_P - \theta) = \dot{R}_P \qquad V_{\theta_P} = V_P \sin(\alpha_P - \theta) = R_P \dot{\theta}$

$V_{R_T} = V_T \cos(\alpha_T - \theta) = \dot{R}_T \qquad V_{\theta_T} = V_T \sin(\alpha_T - \theta) = R_T \dot{\theta}$

$V_{R_{PT}} = V_T \cos(\alpha_T - \theta) - V_P \cos(\alpha_P - \theta)$

$V_{\theta_{PT}} = V_T \sin(\alpha_T - \theta) - V_P \sin(\alpha_P - \theta)$

Condition 2 $\Rightarrow$

$$\dot{\theta} = \frac{V_P \sin(\alpha_P - \theta)}{R_P} = \frac{V_T \sin(\alpha_T - \theta)}{R_T}$$

$$\Rightarrow R_T a_P \cos(\alpha_P - \theta) = 2 V_P V_T \sin(\alpha_T - \alpha_P)$$

$a_P = \dot{\alpha}_P V_P$

$$\Rightarrow \boxed{a_P = \frac{2 V_P V_T \sin(\alpha_T - \alpha_P)}{R_T \cos(\alpha_P - \theta)}}$$

✻ DERIVE (NPTEL)

$V_P, V_T, \alpha_T, \alpha_P, \theta, R_T$

(Compare with PP guidance)

✻ Challenge 1: Requires more state infor for a_P computation

✻ Challenge 2: (large R_T so error in θ could cause drastic effects)

$$a_p = \dot{\alpha}_p V_p$$

$$\frac{V_p \sin(\alpha_p - \theta)}{R_p} = \frac{V_T \sin(\alpha_T - \theta)}{R_T}$$

$$R_T V_p \sin(\alpha_p - \theta) = R_p V_T \sin(\alpha_T - \theta)$$

non-maneuvering

$$V_p \left[\dot{R}_T \sin(\alpha_p - \theta) + R_T \cos(\alpha_p - \theta) \cdot (\dot{\alpha}_p - \dot{\theta})\right]$$

$$= V_T \left[\dot{R}_p \sin(\alpha_T - \theta) + R_p \cos(\alpha_T - \theta) (\cancel{\dot{\alpha}_T}^{\,0} - \dot{\theta})\right]$$

a_p / V_p

$$\cancel{V_p}\left[\cancel{V_T} \cos(\alpha_T - \theta) \sin(\alpha_p - \theta) + \cancel{V_T} \frac{\sin(\alpha_T - \theta)}{\dot{\theta}} \cos(\alpha_p - \theta) (\cancel{\dot{\alpha}_p} - \dot{\theta})\right)$$

$$= \cancel{V_T}\left[\cancel{V_p} \cos(\alpha_p - \theta) \sin(\alpha_T - \theta) + \cancel{V_p} \frac{\sin(\alpha_p - \theta)}{\cancel{\dot{\theta}}} \cos(\alpha_T - \theta) (-\cancel{\dot{\theta}})\right]$$

$$\sin(\alpha_p - \cancel{\theta} - \alpha_T + \cancel{\theta}) + \frac{a_p}{V_p \dot{\theta}} \sin(\alpha_T - \theta) \cos(\alpha_p - \theta) + \sin(\alpha_T - \cancel{\theta} - \alpha_p + \cancel{\theta}) = 0$$

$$2 \sin(\alpha_p - \alpha_T) + \frac{a_p}{V_p \dot{\theta}} \sin(\alpha_T - \theta) \cos(\alpha_p - \theta) = 0$$

R_T

$$a_p = \frac{2 V_p \sin(\alpha_T - \alpha_p)}{R_T \cos(\alpha_p - \theta)}$$

initial value of $R_p = 0$

- Consider the base station (BS) on the 3rd agent be same as the pursuer's launch station (LS).

$$\Downarrow$$

- Consider LS to be stationary

$R_{p_0} = 0$

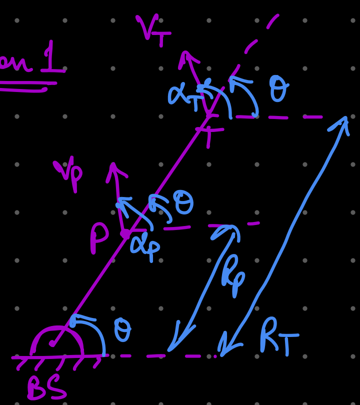
And,

$$v_p \sin(\alpha_{p_0} - \theta_0) = R_{p_0}\dot{\theta} = 0$$

$$\Rightarrow \alpha_{p_0} = \theta_0$$

this is not general case

Condition 1



$$a_p = \frac{2 v_p v_T \sin(\alpha_T - \alpha_p)}{R_T \cos(\alpha_p - \theta)}$$

$$R_{p_0} = 0 \Rightarrow a_{p_0} = 2 v_p \dot{\theta}_0$$

* Challenges 1 & 2

Alternative Forms of a_p

$$a_p = v_p \dot{\alpha}_p \qquad (\alpha_p = \theta + \psi_p)$$

$$v_{\theta_p} = R_p \dot{\theta} = v_p \sin\psi_p \Rightarrow \sin\psi_p = \frac{R_p \dot{\theta}}{v_p}$$

$$\Rightarrow \cos\psi_p \cdot \dot{\psi}_p = \frac{1}{v_p}\left[\dot{R}_p \dot{\theta} + R_p \ddot{\theta}\right] \quad \text{differentiate}$$

$v_p \sin(\alpha_p - \theta) = R_p\dot{\theta}$

$v_p \cos(\alpha_p - \theta) = \dot{R}_p$

$$a_p = v_p\left[\dot{\theta} + \dot{\psi}_p\right]$$

$$= v_p\left[\dot{\theta} + \frac{1}{\cancel{v_p \cos\psi_p}}(\dot{R}_p\dot{\theta} + R_p\ddot{\theta})\right]$$

$$\boxed{a_p = v_p\left[2\dot{\theta} + \frac{R_p \ddot{\theta}}{\dot{R}_p}\right]}$$

Challenge 3: Higher order time derivatives of θ and R_p mostly have significant noise

Challenge 4: Challenge 3 gets amplified in case of "very high" loss dynamics

preferred use cases/applications of LOS guidance — stationary or low speed moving targets

$(R_{p_0} = 0,\ \alpha_{p_0} = \theta_0)$

$\downarrow$

$(R_{p_f} = R_{T_f}, \ldots$

$\alpha_{p_f} = \alpha_{T_f})$

$R_{P_0} = 0 \Rightarrow a_{P_0} = 2 v_P \dot{\theta}_0$ — LOS guidance

$a_{P_0} = v_P \dot{\theta}_0$ — PP guidance

At intercept,

$$R_{P_f} = R_{T_f}$$

then, $v_P \sin(\alpha_{P_f} - \theta_f) = v_T \sin(\alpha_T - \theta_f)$

$$\Rightarrow \alpha_{P_f} = \theta_f + \sin^{-1}\left(\frac{1}{\nu} \sin(\alpha_T - \theta_f)\right)$$

∴ this is condition for achieving collision triangle

→ If $v_P > v_T$, then P captures T

$$t_f = \underset{t}{\mathrm{INF}} \{t : R_P(t) = R_T(t)\}$$

Proof: Let's say that P doesn't capture T

then $t_f \to \infty \Rightarrow \theta_f \to \alpha_T$

(how?)

this implies that $\alpha_{P_f} \to \theta_f \to \alpha_T$

then, $v_{R_{PT_f}} \longrightarrow V_T - V_P < 0$ ← doesn't make sense

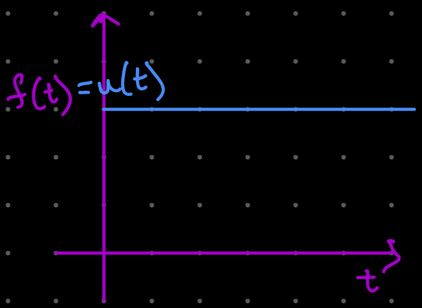
$\downarrow$

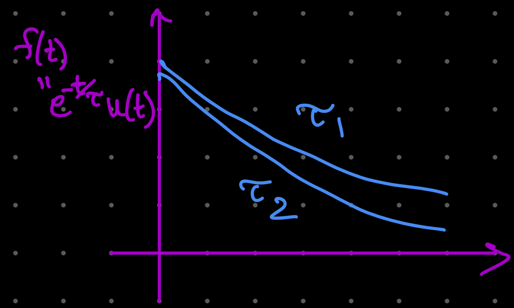
$V_T \cos(\alpha_T - \theta_f) - V_P \cos(\alpha_{P_f} - \theta_f)$

↳ range rate decreases that will lead to convergence, so intercept happens (assumption is wrong).

(this proof is not exactly solid) needs more rigorous detailing

* Laplace Transform



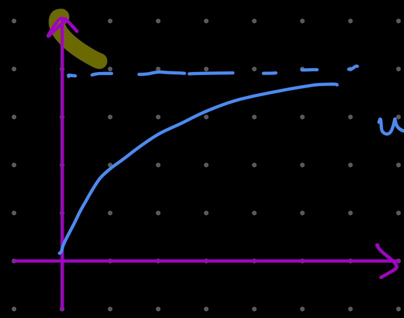


$$\frac{1}{S+\frac{1}{\tau}}$$

$$\tau_1 > \tau_2$$

$$\frac{1}{S(1+S\tau)} \xrightarrow{\text{time domain}}$$

$$\frac{1}{S} - \frac{1}{S+\frac{1}{\tau}}$$

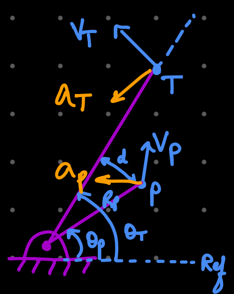


$$u(t)\left(1-e^{-t/\tau}\right)$$

$$F(s) = \int_0^\infty e^{-st}\, \cancel{f(t)}\, e^{-t/\tau}\, dt = \frac{1}{\left(s+\frac{1}{\tau}\right)}\,(\cancel{0-1})$$

Implementation of LOS-G

1) Beam Rider (BR) — both sensing & calculation done by onboard systems



$$d \approx R_p(\theta_T - \theta_p) \quad \longleftarrow \text{noisy estimate}$$

$$a_p^{BR} = k\, d = k\, R_p(\theta_T - \theta_p)$$

here, $k > 0$ $(\theta_T > \theta_p) \rightarrow a_p^{BR} > 0$

2) C-LOS (possibility of tracking target ......??)

As base station tracks both P and T, it's easier to get information about $\dot{\theta}_T$ and $\ddot{\theta}_T$
thus, additional guidance tasks that can be achieved 'more satisfactorily'

$\hookrightarrow$ $\dot{\theta}_p = \dot{\theta}_T$, $\ddot{\theta}_p = \ddot{\theta}_T$

Recall,

$$V_{\theta_T} = R\dot{\theta}_T = v_T \sin(\alpha_T - \theta_T)$$

$$\Rightarrow \dot{V}_{\theta_T} = \dot{R}_T\dot{\theta}_T + R_T\ddot{\theta}_T$$

$$= (\dot{\alpha}_T - \dot{\theta}_T)\, V_T\, \cancel{\cos}(\alpha_T - \theta_T) \quad \dot{R}_T$$

a_{T_n} is normal to LOS component

$$\ddot{\theta}_T = \frac{a_{T_n} - 2\dot{R}_T\dot{\theta}_T}{R_T}$$

$$a_{T_n} = 2\dot{R}_T\dot{\theta}_T + R_T\ddot{\theta}_T$$

Similarly,

$$a_{p_n} = 2\dot{R}_p\dot{\theta}_p + R_p\ddot{\theta}_p$$

$$\boxed{a_p^{C-LOS} = \underbrace{k R_p(\theta_T - \theta_p)}_{\text{BR-Component}} + \underbrace{R_p\ddot{\theta}_T + 2\dot{R}_p\dot{\theta}_T}_{\text{Additional Component}}}$$

$\boxed{\text{fwd forward}}$ the stuff we try to enforce/maintain

this is technically just normal component, ignored to keep it linear

$\frac{1}{\cos(\alpha_p - \theta_p)}$ factor missing

Recall,

$$a_p = 2V_p\left(\dot{\theta}_p + \frac{R_p}{2\dot{R}_p}\ddot{\theta}_p\right)$$

'Frozen Time Analysis'

Consider $\frac{R_p}{2\dot{R}_p} = T_e = \text{constant}$

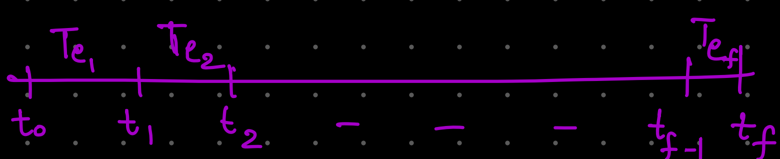
over a time-interval

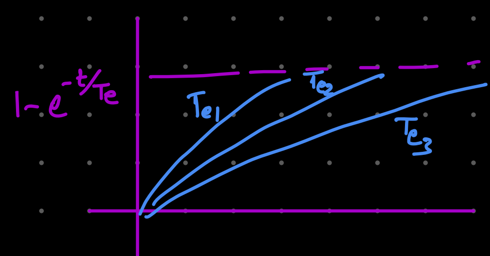
$$a_p(s) = 2V_p(s\theta_p + T_e s^2\theta_p)$$

$$\frac{\theta_p(s)}{a_p(s)} = \frac{1}{2V_p s(1+T_e s)} \longrightarrow 1 - e^{-t/T_e}$$

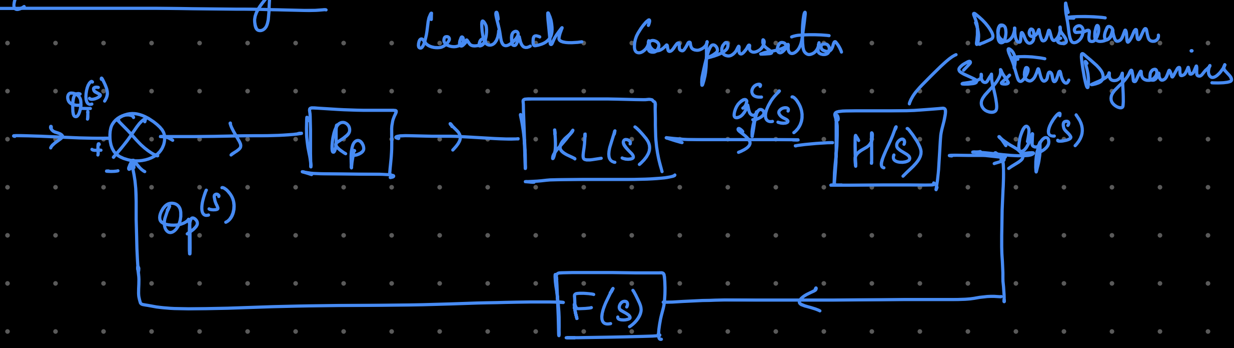
↳ more sluggish response with time even at guidance level, as T_e increases

(when we take transfer function, we consider 0 initial conditions)

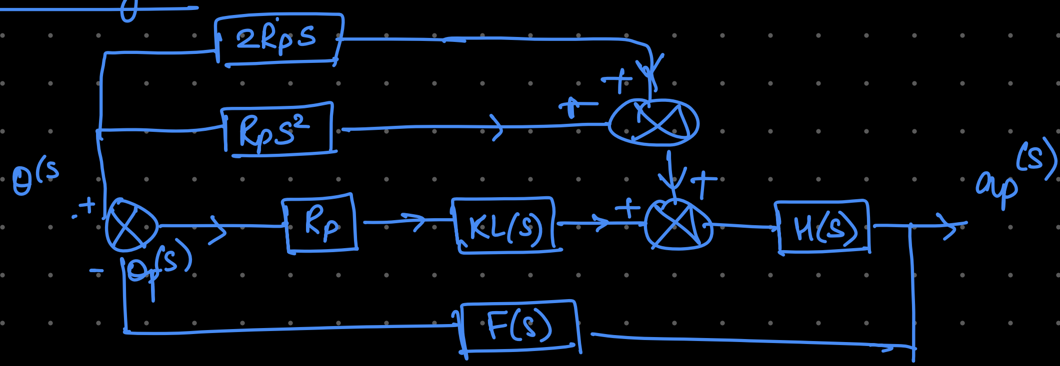




Beam Rider Block Diagram



C-LOS Block Diagram



aim to get onto LOS?
can't track target? irl implementation issues.
we have to keep maintaining geo.

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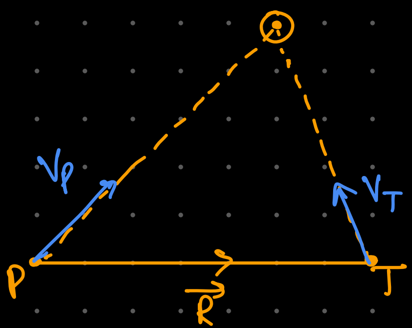
Proportional Navigation (PN)



↳ why 'navigation'?

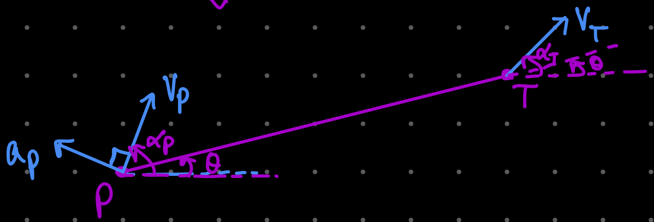
↳ why 'proportional'?

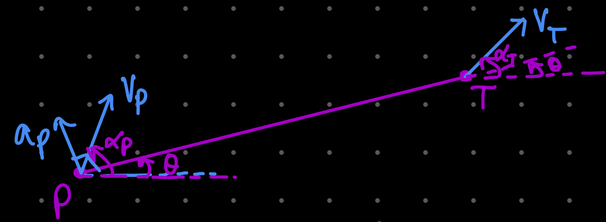
↳ major variants

- → pure PN (PPN)
- → true PN (TPN)
- → generalised TPN (GTPN)
- → ideal PN (IPN)

$\Rightarrow a_P \propto \dot{\theta}$ ← have to reduce error (if $\dot{\theta} = 0$, LOS remains same)

a_P applied normal to the pursuer's velocity vector ($\vec{V}_P$)





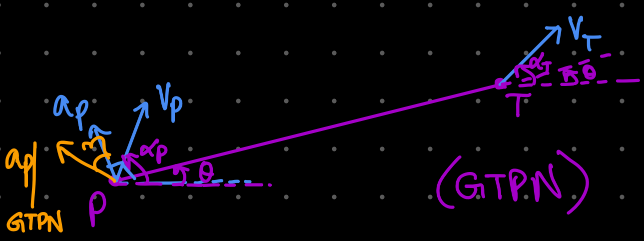
$a_P = N V_P \dot{\theta}$ (PPN)
N → constant

no speed change

$a_P = c\dot{\theta}$ (TPN)

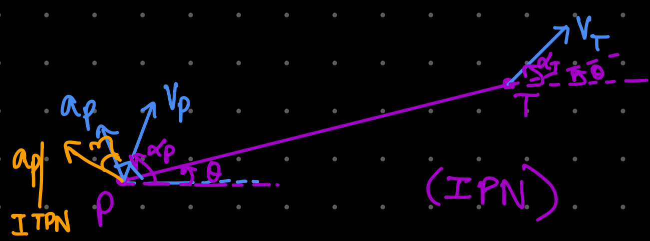
↓

some speed change
(need thrusters)
which gives longitudinal acc^n



(GTPN)

(thrusters not required?)



(IPN)

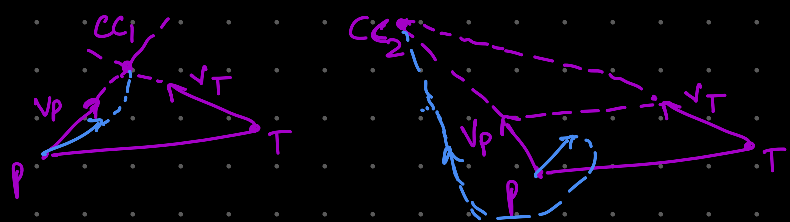
$\vec{a_P}$ applied $\perp$ $\vec{V_T} - \vec{V_P} = \vec{V}_{rel}$

it does not have to be integer gain but greater than zero.

→ Retro PN — $\vec{a}_p \perp v_p$, $a_p = N V_p \dot{\theta}$, $N < 0$ → can be applied only on low speed pursuer / high speed target

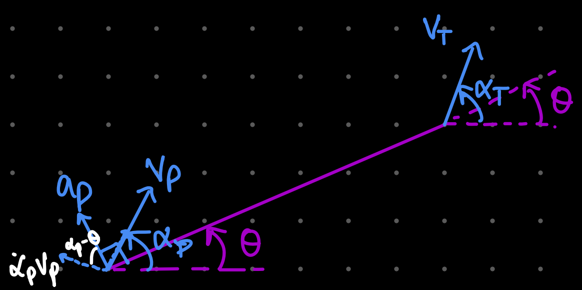
$\underline{\underline{V_p < V_T}}$

2 cases



(similar to head on) (head pursuit)

TPN ($\dot{\alpha}_T = 0$) $\dot{v}_T = 0$



$$V_R = \dot{R} = v_T \cos(\alpha_T - \theta) - V_p \cos(\alpha_p - \theta)$$

$$V_\theta = R\dot{\theta} = v_T \sin(\alpha_T - \theta) - v_p \sin(\alpha_p - \theta)$$

$$\dot{\alpha}_p = \frac{a_p \cos(\alpha_p - \theta)}{V_p}$$

$$\dot{v}_p = a_p \sin(\alpha_p - \theta)$$

$$a_p = c\dot{\theta} \text{ — guidance scheme}$$

check

[NPTEL]

$$\dot{V}_R = \dot{\theta} V_\theta$$

$$\dot{V}_\theta = -\dot{\theta}(V_R + c)$$

divide the eqn

(c is constant here)

$$V_R \dot{V}_R + V_\theta \dot{V}_\theta + c\dot{V}_R = 0$$

$$V_R^2 + V_\theta^2 + 2cV_R = \text{constant}$$

$$V_\theta^2 + (V_R + c)^2 = V_{\theta_0}^2 + (V_{R_0} + c)^2$$

$$\dot{V}_R = -v_T \sin(\alpha_T - \theta)(\dot{\alpha}_T - \dot{\theta}) + V_p \sin(\alpha_p - \theta)(\dot{\alpha}_p - \dot{\theta}) - \dot{v}_p \cos(\alpha_p - \theta)$$

$$\dot{V}_\theta = v_T \cos(\alpha_T - \theta)(\dot{\alpha}_T - \dot{\theta}) - V_p \cos(\alpha_p - \theta)(\dot{\alpha}_p - \dot{\theta}) - \dot{v}_p \sin(\alpha_p - \theta)$$

$$-v_T \sin(\alpha_T - \theta)(\dot{\alpha}_T - \dot{\theta}) + \cancel{v_p \sin(\alpha_p - \theta)}\left(\frac{\overset{c\dot{\theta}}{\cancel{a_p}} \cos(\alpha_p - \theta)}{v_p} - \dot{\theta}\right)$$

$$\cancel{-\dot{v}_p \cos(\alpha_p - \theta)}$$

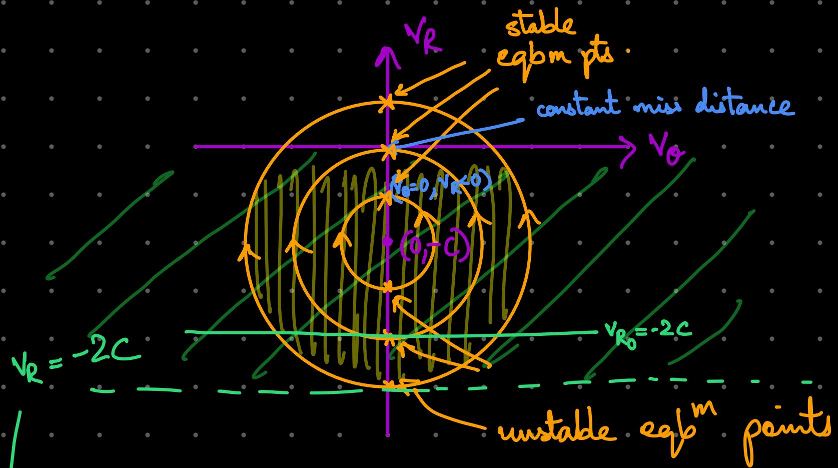
$$v_T \cos(\alpha_T - \theta)(\dot{\alpha}_T - \dot{\theta}) - v_p \cos(\alpha_p - \theta)\left(\frac{\overset{c\dot{\theta}}{\cancel{a_p}} \cos(\alpha_p - \theta)}{v_p} - \dot{\theta}\right)$$

$$-\dot{v}_p \sin(\alpha_p - \theta)$$

$$-c\dot{\theta}$$

TPN

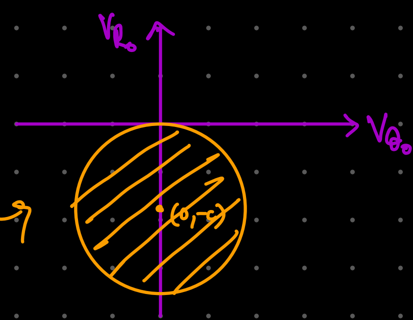
$$\dot{v}_R = \dot{\theta} v_\theta = v_\theta^2/R > 0$$
$$\dot{v}_\theta = -\dot{\theta}(v_R + c)$$
$$\rightarrow \quad v_\theta^2 + (v_R + c)^2 = v_{\theta_0}^2 + (v_{R_0} + c)^2$$



$\dot{\theta} = 0 \rightarrow$ eqb^m

similar to PP but we have control over c, no constraints over v_P & v_T

$$CR\big|_{TPN(c\dot{\theta})} = \left\{ (v_{\theta_0}, v_{R_0}) \,\middle|\, (v_{R_0}+c)^2 + v_{\theta_0}^2 < c^2 \right\}$$
$$v_{R_0}^2 + v_{\theta_0}^2 + 2c\,v_{R_0} < 0$$



$$v_R^2 + v_\theta^2 + 2c v_R = \text{constant} = K$$
$$\Rightarrow \quad R(v_R + 2c) = kt + b \quad , \quad b = R_0(v_{R_0} + 2c)$$

At interception, $R = 0$

$$t_f = -\frac{b}{k} = \frac{-R_0(v_{R_0} + 2c)}{v_{R_0}^2 + v_{\theta_0}^2 + 2v_{R_0}c} \quad \leftarrow -ve$$

$\Rightarrow$ for $t_f > 0$, $\quad v_{R_0} + 2c > 0$

$v_{R_0} > -2c \quad \longrightarrow$ if outside CR, t_f is time to reach $v_R = -2c$ line. ($v_{R_0} < -2c$)

Assumed $c > 0$.

if $c < 0$, the region shifts up, $v_R > 0$, no interception

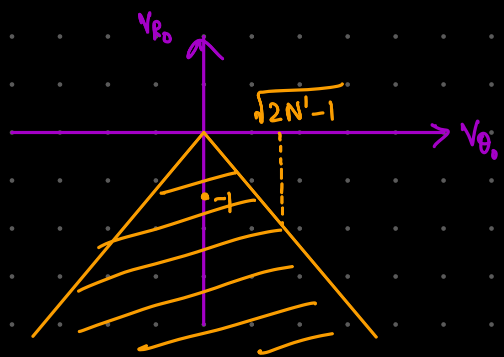
TPN $\rightarrow a_p = c\dot{\theta}$

$c > 0$

Say, $c = -N' V_{R_0}$, $N' > 0$ & $V_{R_0} < 0$

$N' > 1/2$

$$CR\Big|_{TPN\,(a_p = -N' V_{R_0}\dot{\theta})} = \left\{ (V_{\theta_0}, V_{R_0}) \;\middle|\; |V_{\theta_0}| < \sqrt{2N'-1}\,|V_{R_0}| \;,\; V_{R_0} < 0 \right\}$$



$c > 0$

Say $c = -N' V_R$, $N' > 0$, $V_R < 0$

→ Realistic TPN

$$a_p = -N' V_R \dot{\theta}$$

$$\left.\begin{aligned} \dot{V}_R &= \dot{\theta} V_\theta \\ \dot{V}_\theta &= -\dot{\theta}(1-N') V_R \end{aligned}\right\} \quad V_\theta \dot{V}_\theta + (1-N') V_R \dot{V}_R = 0$$

$$\Rightarrow V_\theta^2 + (1-N') V_R^2 = K$$
$$= V_{\theta_0}^2 + (1-N') V_{R_0}^2$$

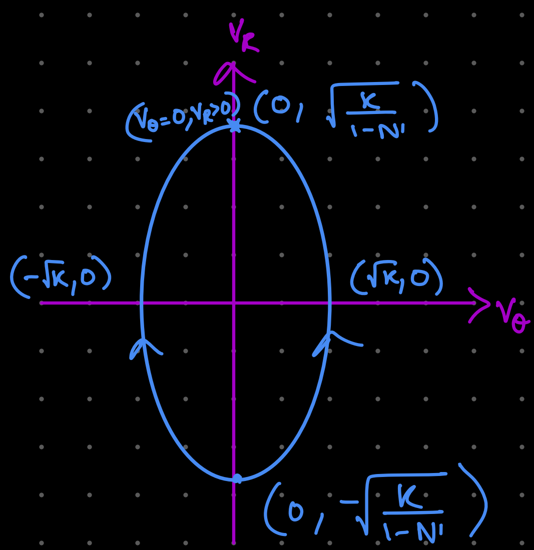
Case 1

$N' < 0$? & $0 < N' < 1$

$$\frac{V_\theta^2}{(\sqrt{K})^2} + \frac{V_R^2}{\left(\sqrt{\frac{K}{1-N'}}\right)^2} = 1$$

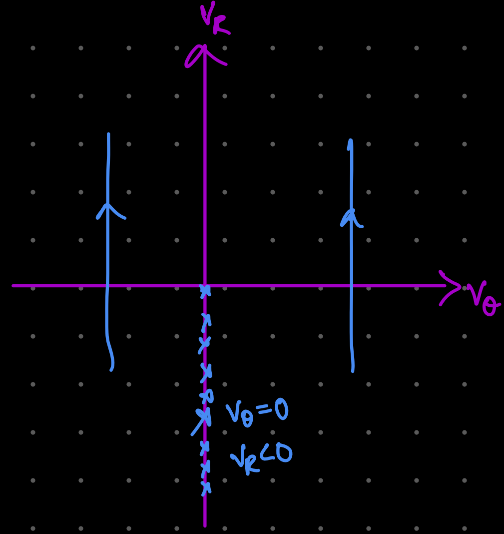
For $V_\theta = 0$, $V_R = \pm\sqrt{\dfrac{K}{1-N'}}$

$V_R = 0$, $V_\theta = \pm\sqrt{K}$



Case 2

$N' = 1 \qquad V_\theta^2 = \text{constant}$



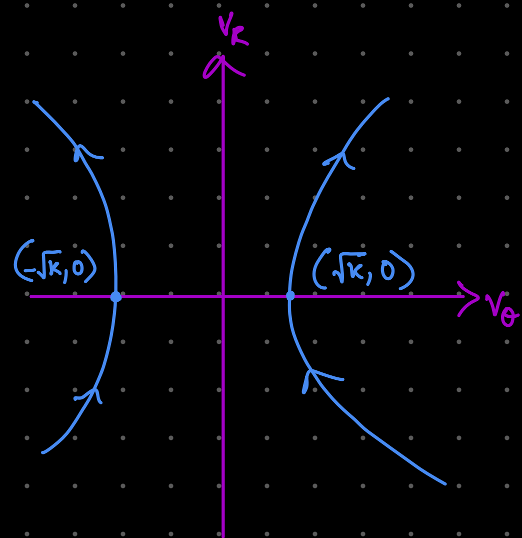
Case 3

$N' > 1$

$$V_\theta^2 - (N'-1)\,V_R^2 = K$$

3A $\quad K > 0$

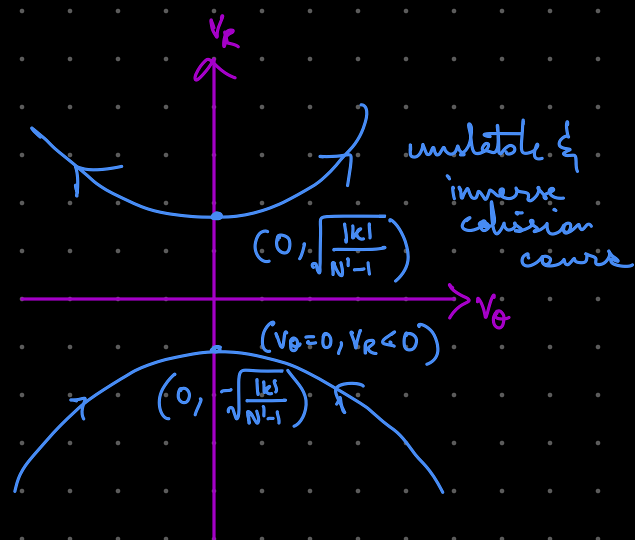
$$\frac{V_\theta^2}{(\sqrt{K})^2} - \frac{V_R^2}{\left(\sqrt{\frac{K}{N'-1}}\right)^2} = 1$$



3B $\quad K < 0$

$$(N'-1)\,V_R^2 - V_\theta^2 = |K|$$

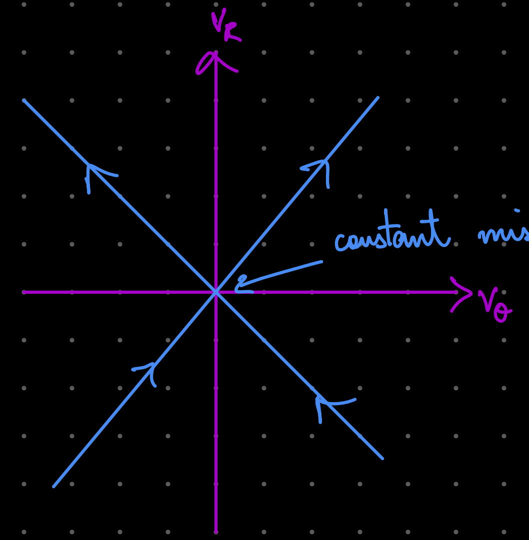
$$\Rightarrow \frac{V_R^2}{\left(\sqrt{\frac{|K|}{N'-1}}\right)^2} - \frac{V_\theta^2}{(\sqrt{|K|})^2} = 1$$



3C

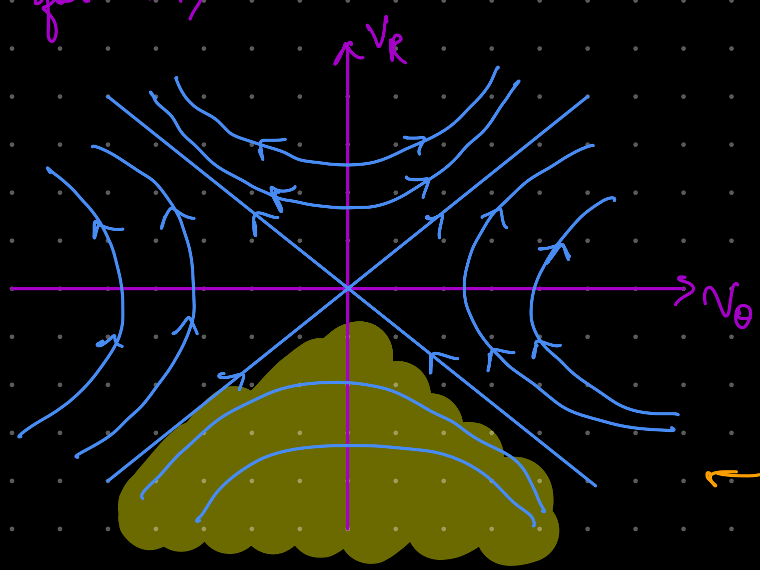
$k = 0$

$\sqrt{N'-1}\,|v_R| = |v_\theta|$



k depends on initial conditions

summarising for $N'>1$,



← $K<0$, $N'>1$
capture region

there is no non-trivial colusion sol^n ?

$$CR\big|_{RTPN} = \left\{ (V_{\theta_0}, V_{R_0}) \;\middle|\; V_{\theta_0}^2 + (1-N')V_{R_0}^2 < 0,\ V_{R_0} < 0,\ N' > 1 \right\}$$

→ (1)

$$\left(\equiv\ |V_{\theta_0}| < \sqrt{N'-1}\,|V_{R_0}|,\ N' > 1 \right)$$

→ (2) $k<0$

* we find capture region is less than TPN.

* still we cannot capture from $V_{R_0} > 0$.

we would like to know final lateral acc^n in any case

↳ so finding $\theta, \dot{\theta}$

Range and LOS Rate for TPN $(a_p = c\dot{\theta})$

Recall: $R(\dot{R}+2c) = kt+b$

where, $k = v_{\theta_0}^2 + v_{R_0}^2 + 2c\,v_{R_0}$

$b = R_0(v_{R_0}+2c)$

$R_0 = y+n$ $\quad$ $v_{\theta_0} = R_0\dot{\theta}_0 = (y+n)\dot{\theta}_0$

$v_{R_0} = m$

$v_{\theta_0}^2 + m^2 + 2c\,m = k$

$v_{\theta_0}^2 + m(m+2c) = k$

Let $R = y + mt + n$, s.t.

$m(m+2c) = k \Rightarrow m = -c \pm \sqrt{c^2+k}$

$n = \dfrac{mb}{k}$ $\qquad$ $p = m+2c = \dfrac{k}{m}$

$R = R_0\left(\mu_1 z^{-m/p} + \mu_2 z\right)$

where, $z = \dfrac{y}{y_0}$, s.T.

$g z^{-m/p} - hz = x = t + \dfrac{n}{m}$

$-\dfrac{m}{p} > 0 \rightarrow m/p < 0$

↓ m & p are of different signs

↓ $k<0$

↓ condition for capture

$g = \dfrac{R_0\mu_1}{m}$, $\quad h = \dfrac{R_0\mu_2}{p}$

$\mu_1 = \dfrac{\left(\eta + \sqrt{\eta^2+1}\right)}{2\sqrt{\eta^2+1}}$ $\qquad$ $\mu_2 = \dfrac{\left(-\eta + \sqrt{\eta^2+1}\right)}{2\sqrt{\eta^2+1}}$

$\eta = \dfrac{v_{R_0}+c}{|v_{\theta_0}|}$

As $z \to 0$, $R \to 0$

if $-m/p > 0 \Rightarrow k < 0$

$$\Rightarrow v_{\theta_0}^2 + v_{R_0}^2 + 2cv_{R_0} < 0$$

↳ already obtained

$$\dot{\theta} = \dot{\theta}_0 \frac{z^{\left(1 - \frac{3m^2}{|k|}\right)/2}}{\left(\mu_1 + \mu_2 z^{1 - \frac{m^2}{|k|}}\right)^2}$$

$\dot{\theta}$ derived from paper

↳ As $z \to 0$, $\dot{\theta} \to 0$ if $1 - \frac{3m^2}{|k|} > 0$

$$\Rightarrow |k| > 3m^2$$

<u>It implies</u>

$$\sqrt{(v_{R_0}+c)^2 + v_{\theta_0}^2} > c/2 \Rightarrow \lim_{t \to t_f} \dot{\theta} = 0$$

$$= c/2 \Rightarrow \lim_{t \to t_f} \dot{\theta} = -\left(\frac{\dot{\theta}_0}{R_0}\right)\left[\frac{c+2m}{\mu_1^3}\right]$$

? cs $\quad < c/2 \Rightarrow \lim_{t \to t_f} \dot{\theta} = \infty \underbrace{\operatorname{sgn} \dot{\theta}_0}_{\pm}$

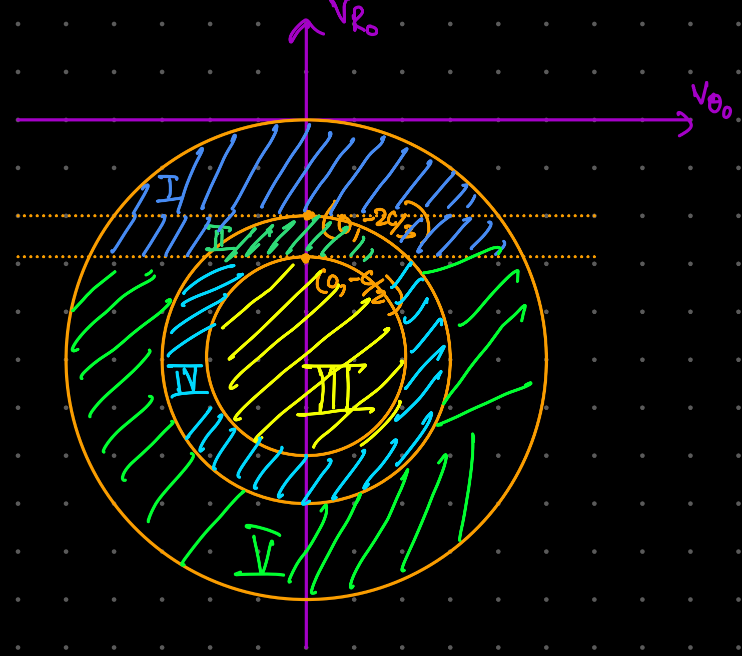
$\ddot{\theta}$

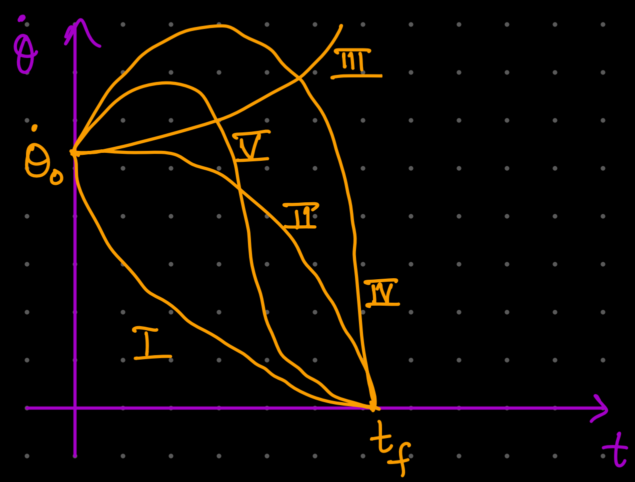
$$\sqrt{(v_R+c)^2 + v_{\theta_0}^2} > \frac{2c}{3} \Rightarrow \lim_{t \to t_f} \ddot{\theta} = 0$$

$$= \frac{2c}{3} \Rightarrow \lim_{t \to t_f} \ddot{\theta} = \text{finite}$$

? cs $\quad < \frac{2c}{3} \Rightarrow \lim_{t \to t_f} \ddot{\theta} = \infty (\operatorname{sgn} \dot{\theta}_0)$

Zone	(v_{R_0}, v_{θ_0})	$\dot\theta_f$,	$\ddot\theta_f$
I	$v_{R_0} > -c/2$ $\notin C_S, \notin C_D$	0	0
II	$v_{R_0} > -c/2$ $\notin C_S, \in C_D$	0	$\infty(-\sin\theta_0)$
III	$\in C_S, \in C_D$ $v_{R_0} < -c/2$	∞	$\infty\,\mathrm{sgn}(\dot\theta_0)$
IV	$v_{R_0} < -c/2$ $\notin C_S, \in C_D$	0	$-\mathrm{sgn}(\dot\theta_0)\infty$
V	$v_{R_0} < -c/2$ $\notin C_S, \notin C_D$	0	0





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monotonic & non-monotonic variation

$\downarrow$ I & II decreases — $v_{R_0} > -c/2$

$\rightarrow$ III, IV, V — $v_{R_0} < -c/2$

$$\dot v_\theta = -\dot\theta\,(v_R + c)$$

$$\dot\theta v_R + R\ddot\theta = -\dot\theta\,(v_R + c)$$

$$R\ddot\theta = -2\dot\theta\,(v_R + c/2)$$

$v_R + c/2 < 0$ $\rightarrow$ if $v_{R_0} < -c/2$ initial/starting point as such that there is sign change vice versa.

end of Quiz 2

TPN ($a_p = c\dot{\theta}$) against Maneuvering Target

↳ $a_T = \frac{b}{v_\theta}$, $b>0$, applied normal to LOS

as v_θ reduces to zero (and $v_R<0$), it'll be on collision course which the target wants to avoid so acc^n increase. (applied on target ⊥ LOS)

$$V_R = V_T \cos(\alpha_T - \theta) - V_P \cos(\alpha_P - \theta)$$

$$V_\theta = V_T \sin(\alpha_T - \theta) - V_P \sin(\alpha_P - \theta)$$

$$\dot{V}_P = a_P \sin(\alpha_P - \theta) \qquad \dot{V}_T = \cancel{a_T}^{\,b/v_\theta} \sin(\alpha_T - \theta)$$

$$\dot{\alpha}_P = a_P \cos(\alpha_P - \theta) \qquad \dot{\alpha}_T = \underset{b/v_\theta}{\cancel{a_T}} \cos(\alpha_T - \theta)$$

$$\dot{V}_R = \dot{\theta} V_\theta > 0 \qquad \dot{V}_\theta = -\dot{\theta} V_R + a_T - a_P$$

$$\Rightarrow V_\theta \dot{V}_\theta + V_R \dot{V}_R + a_P V_\theta = b$$

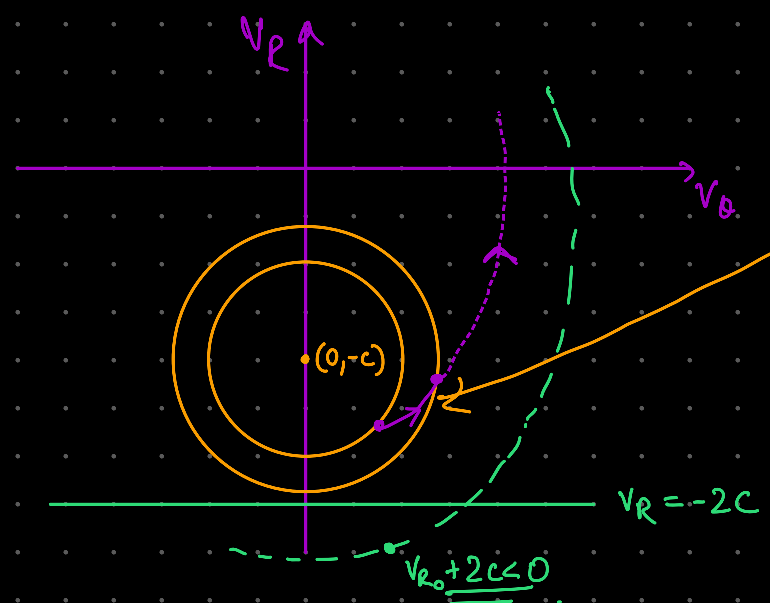
substituting $a_T = \frac{b}{v_\theta}$

$$\Rightarrow V_\theta \dot{V}_\theta + V_R \dot{V}_R + c\dot{V}_R = b$$

$$\Rightarrow V_\theta^2 + V_R^2 + 2cV_R = 2bt + K$$

$$V_\theta^2 + (V_R + c)^2 = (K + c^2) + 2bt$$

↳ circle size is increasing



if $v_\theta \to 0$, high a_T
not very implementable

$$R(v_R + 2c) = bt^2 + kt + p$$

$$p = R_0(v_{R_0} + 2c)$$

$R_0 > 0$, $v_{R_0} + 2c > 0$ $\rightarrow p > 0$ $\longrightarrow$ for capture

if $v_{R_0} + 2c < 0$, you on a circle with radius greater than c

$$t_f = \frac{-k \pm \sqrt{k^2 - 4bp}}{2b}$$

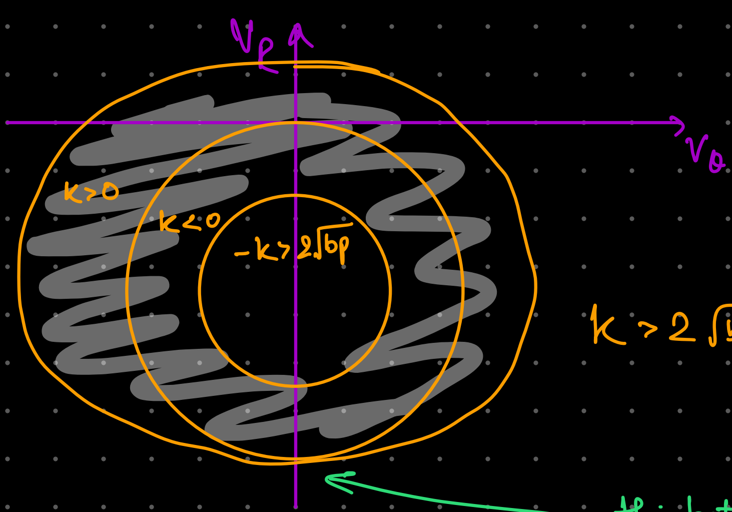
for capture to happen, t_f should be real & positive.

$$k^2 - 4bp > 0$$

$$\Rightarrow \quad |k| > 2\sqrt{bp}$$

Recall, $b, p > 0 \Rightarrow$ if $k > 0$, then $t_f < 0$ $\rightarrow$ not valid

hence, $k < 0$.



$k > 2\sqrt{bp}$

this bottom point we'll c later if they're all same or not.

$$-k \geq 2\sqrt{bp}$$

$$-v_{\theta_0}^2 - v_{R_0}^2 - 2cv_{R_0} \geq 2\sqrt{bR_0}\sqrt{v_{R_0} + 2c}$$

$$v_{\theta_0}^2 \leq \underbrace{\left(-v_{R_0}\sqrt{v_{R_0}+2c} - 2\sqrt{bR_0}\right)}\sqrt{v_{R_0}+2c}$$

to have a feasible range of v_{θ_0} satisfying capture requirement ($K \ge 2\sqrt{bK}$, $v_{R_0} < 0$)

$$|v_{R_0}|\sqrt{v_{R_0}+2c} > 2\sqrt{bR_0}$$

$$\Rightarrow \quad v_{R_0}^3 + 2c\,v_{R_0}^2 \ge 4bR_0$$

↳ if this maxima is greater than $4bR_0$, some solution will exist

is maximum at $v_{R_0} = -\frac{4c}{3}$ with maximum value of $\frac{32}{27}c^3$

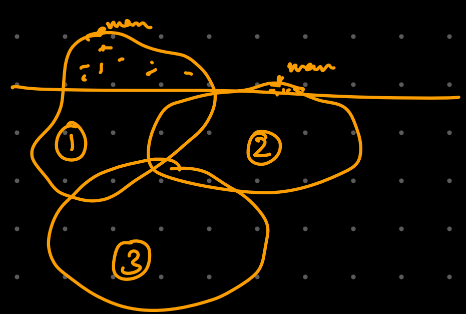
Thus, for capture, $\frac{32}{27}c^3 \ge 4bR_0$

↳ $\boxed{bR_0 \le \left(\frac{2c}{3}\right)^3}$

$\frac{bR_0}{c^3}$

- b, maneuvering high
- R_0, initial range is high
- c, TPN gain is low

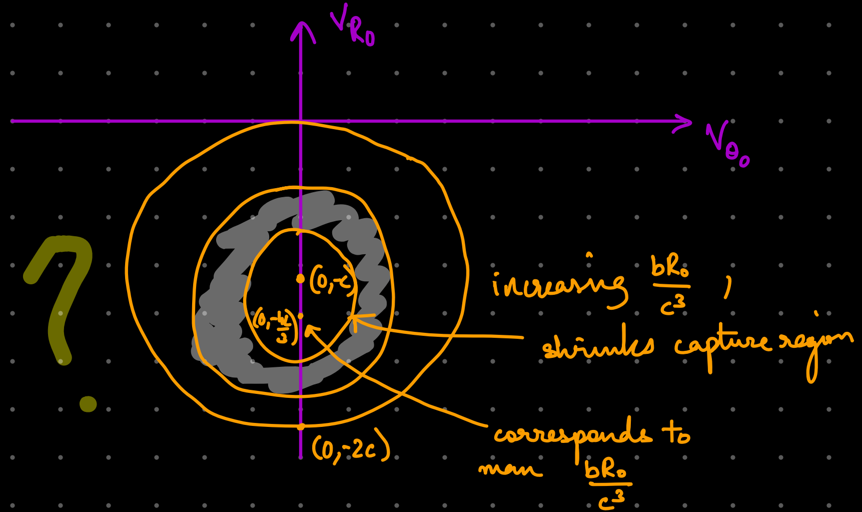
→ restricted capture region



① – decently large capture size (maxima is further from $\frac{2c}{3}$)

② – very restricted (maxima is close to $\frac{2c}{3}$)

③ – does not satisfy only



Condition for capture:

① $K < 0$, $v_{\theta_0}^2 + v_{R_0}^2 + 2c\,v_{R_0} < 0$

② $K^2 \ge 4bR_0(v_{R_0} + 2c)$

③ $bR_0 \le \left(\frac{2c}{3}\right)^3$

Pure Proportional Navigation (PPN)

+ve dir. is towards LOS

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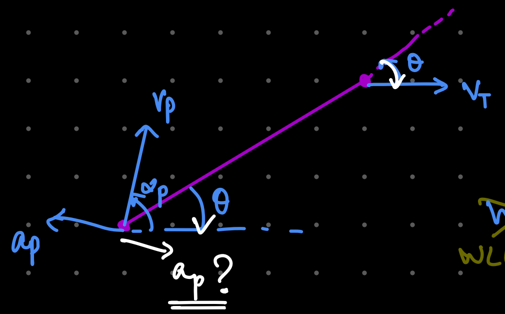
↳ $a_p \propto \dot\theta$, applied normal to $\vec{v_p}$

$\cancel{a_p = N V_p \dot\theta}$?? 60

PPN gain



~~WLOS~~ WLOG, $\alpha_T = 0$

$$\boxed{\begin{aligned} V_R &= V_T\cos(-\theta) - V_p\cos(\alpha_p - \theta) \\ V_\theta &= V_T\sin(-\theta) - V_p\sin(\alpha_p - \theta) \end{aligned}}$$

$\boxed{\dot\alpha_p = N\dot\theta}$ $\xrightarrow{\text{integrate}}$ $\alpha_p - \alpha_{p_0} = N(\theta - \theta_0)$

$k = N - 1$

$\phi_0 = \alpha_{p_0} - N\theta_0$

$\boxed{\alpha_p - \theta = k\theta + \phi_0}$ where, $\boxed{\begin{aligned} k &= N-1 \\ \phi_0 &= \alpha_{p_0} - N\theta_0 \end{aligned}}$ ← dependent on initial conditions only

$V_R = V_T\cos\theta - V_p\cos(k\theta + \phi_0) \Rightarrow \dot V_R = \dot\theta\,(V_\theta + N\,V_p\sin(k\theta + \phi_0))$

$V_\theta = -V_T\sin\theta - V_p\sin(k\theta + \phi_0) \Rightarrow \dot V_\theta = -\dot\theta\,(V_R + N V_p\cos(k\theta + \phi_0))$

Let $v_R \triangleq \dfrac{V_R}{V_p}$, $v_\theta \triangleq \dfrac{V_\theta}{V_p}$, $v \triangleq \dfrac{V_p}{V_T}$

÷ by V_T in NPETL

$$\boxed{(v_R + \cos(k\theta + \phi_0))^2 + (v_\theta + \sin(k\theta + \phi_0))^2 = v^{-2}}$$

At collision course, $v_{\theta_f} = 0$, $v_{R_f} < 0$

If $v > 1$, $v_{R_f} < 0$ iff $\cos(\alpha_{p_f} - \theta_f) \in (\sqrt{1 - v^{-2}}, 1)$ ← ≥ 0

$v_{R_f}^2 + 2v_{R_f}\cos(\alpha_{p_f} - \theta_f) + \cos^2(\alpha_{p_f} - \theta_f) + \sin^2(\alpha_{p_f} - \theta_f) = v^{-2}$

$v_{R_f}^2 + 2\cos(\alpha_{p_f} - \theta_f)\,v_{R_f} + 1 - v^{-2} = 0$

$v_{R_f} = -\cos(\alpha_{p_f} - \theta_f) \pm \sqrt{\cos^2(\alpha_{p_f} - \theta_f) - (1 - v^{-2})}$

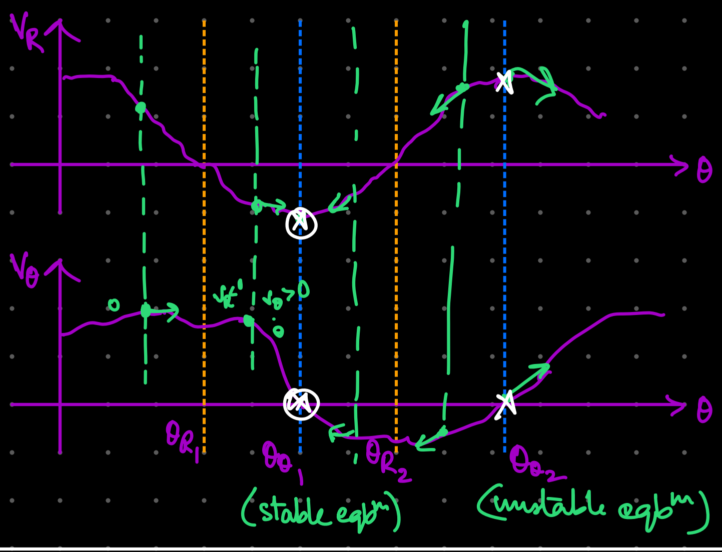
$$\frac{\dot R}{R\dot\theta} = \frac{V_R(\theta)}{V_\theta(\theta)} \Rightarrow \frac{dR}{R} = \frac{V_R(\theta)\,d\theta}{V_\theta(\theta)}$$

$$\Rightarrow \quad \boxed{R = R_0 \exp\left[\int_{\theta_0}^{\theta} \frac{v_R(\phi)}{v_\theta(\phi)}\, d\phi\right]}$$

N=1 is pure pursuit

$\boxed{\text{Lemma 1: If } v>1,\ kv>1, \text{ then } \theta_R \text{ and } \theta_\theta \text{ the Roots of } v_R(\theta) \text{ and } v_\theta(\theta), \text{ respectively, alternate along } \theta\text{-axis.}}$

Example



⊗ — collision point ($v_\theta=0$, $v_R<0$)

x — $v_\theta=0$, $v_R>0$

$\boxed{\text{Lemma 2: } v>1,\ kv>1 \Rightarrow v_R(\theta_\theta) \left.\frac{\partial v_\theta}{\partial \theta}\right|_{\theta_\theta} > 0 \text{ only at the roots of } v_\theta}$

Thus it's shown that 'P' can intercept 'T' starting from any initial engagement geometry except ($v_{\theta_0}=0$, $v_{R_0}>0$)

↓

relax from TPN where $v_{R_0}>0$ also can lead to collision

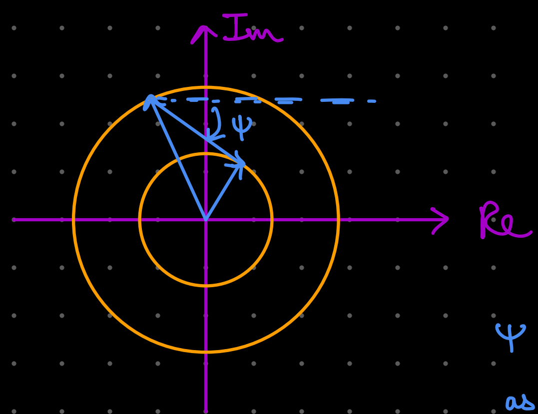
↳ relaxed from PP where speed ratio constrained for collision course

Proof:

L1: Let a complex variable

$$U \triangleq e^{-j\theta} - v e^{j(k\theta+\phi_0)}$$

$$\mathrm{Re}(U) = v_R(\theta)$$
$$\mathrm{Im}(U) = v_\theta(\theta)$$



ψ alignment alternates as it rotates around.

$\psi \triangleq \tan^{-1}\left(\frac{Im(U)}{Re(U)}\right)$

result is monotonically ____ ?

is shown by ψ varying monotonically with θ

Hello!

$$\frac{d\psi}{d\theta} = \frac{Re(U)\frac{d}{d\theta}Im(U) - Im(U)\frac{d}{d\theta}Re(U)}{(Re(U)^2 + Im(U)^2)}$$

$v_R(\theta) = \cos\theta - v\cos(k\theta+\phi_0)$

$v_\theta(\theta) = -\sin\theta - v\sin(k\theta+\phi_0)$

$= \frac{Num}{Dem}$, $Dem > 0$

* check

$v_R(-\cos\theta - vk\cos(k\theta+\phi_0)) - v_\theta(-\sin\theta + vk\sin(k\theta+\phi_0))$

$= -\cos^2\theta + v\cos\theta\cos(k\theta+\phi_0) - vk\cos\theta\cos(k\theta+\phi_0) + kv^2\cos^2(k\theta+\phi_0)$

$\quad - \sin^2\theta + vk\sin\theta\sin(k\theta+\phi_0) - v\sin\theta\sin(k\theta+\phi_0) + kv^2\sin^2(k\theta+\phi_0)$

$= -1 - v(k-1)[\cos\theta\cos(k\theta+\phi_0) - \sin\theta\sin(k\theta+\phi_0)] + kv^2$

$Num = (kv^2-1) - v(k-1)\cos((k+1)\theta+\phi_0)$

$= kv^2 \mp kv - 1 \pm v$

$kv(v\mp1) + (\pm v - 1)$

$(kv\pm1)(v\mp1)$

$\Rightarrow (kv+1)(v-1) \le Num \le (kv-1)(v+1)$

then

$v > 1,\ kv > 1 \Rightarrow Num > 0$.

Lemma 1

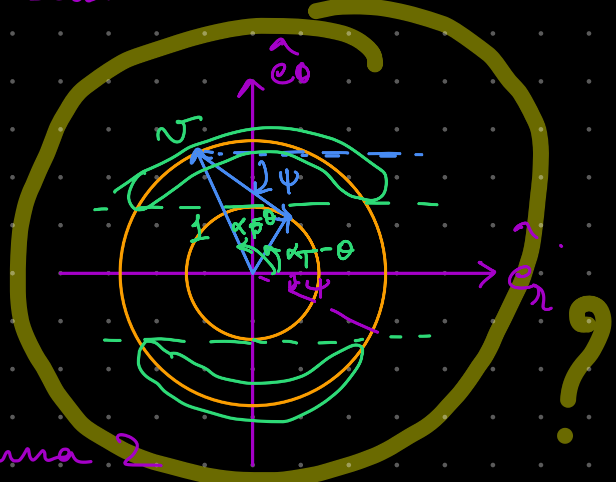
$Re(U)\frac{d}{d\theta}Im(U) - Im(U)\frac{d}{d\theta}Re(U)$

$v_R(\theta_0)\ \frac{\partial v_\theta(\theta_0)}{\partial\theta}$

this term at $\theta_0 = 0$ becomes zero.

$\Rightarrow > 0$ using Lemma 1.

Lemma 2

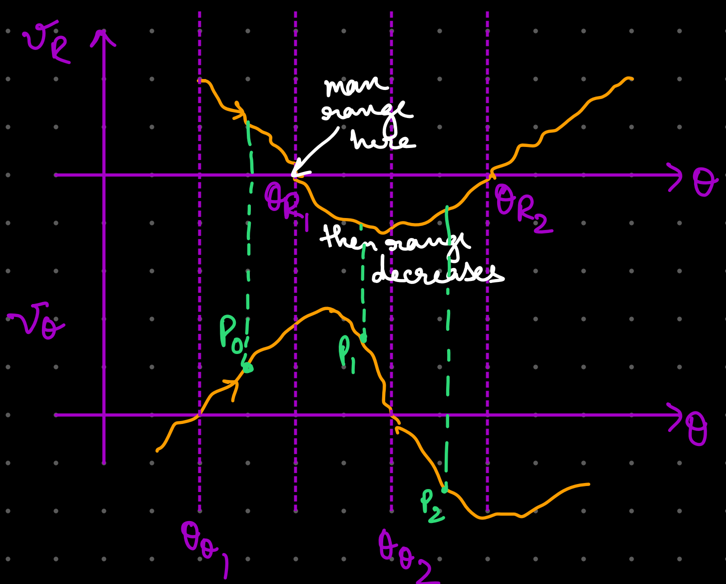


* if $v<1$, Lemma 1 holds but not Lemma 2

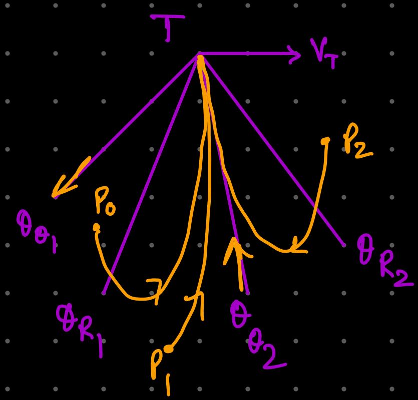
Pure Proportional Navigation (PPN)

Lemma 1: $\nu > 1,\ k\nu > 1 \Rightarrow \theta_\theta$ & θ_R alternate along θ axis

Lemma 2: $\nu > 1,\ k\nu > 1 \Rightarrow v_R(\theta_\theta) \left.\dfrac{\partial V_\theta(\theta)}{\partial\theta}\right|_{\theta_\theta} > 0$



Target-centric V_T-referenced polar plane



this is just qualitative depiction based on theta

Boundedness of ap

Lemma 3:

If $\dfrac{dV_\theta(\theta)}{d\theta} < V_R(\theta)$, then $\dot{\theta}$ is decreasing in time

Lemma 4:

If $\nu > 1$, $\left(\dfrac{N-2}{2}\right)\nu > 1$, then $\dot{\theta}$ decreases with time at the endgame

Lemma 5:

If $N > 4$, $\nu > 1$, then $\dot{\theta}$ decreases with time at the endgame

Proof:

L3: $\frac{dv_\theta}{d\theta} < v_R$

$$\Rightarrow \frac{dv_\theta}{dt} \cdot \frac{1}{\dot\theta} < v_R$$

$$\Rightarrow \frac{\dot v_\theta}{v_\theta} < \frac{\dot R}{R} \Rightarrow \frac{d|v_\theta|}{|v_\theta|} < \frac{dR}{R} \quad ?$$

Say at $t = t_1$, $\frac{dv_\theta}{d\theta} < v_R$ then

at $t = t_1 + \Delta t$, $\quad \frac{|v_\theta(t_1+\Delta t)|}{|v_\theta(t_1)|} < \frac{R(t_1+\Delta t)}{R(t_1)}$

$$\Rightarrow \dot\theta \downarrow \text{ with } t$$

!!

L4: At the endgame, $v_R < 0$

From L3, $\frac{dv_\theta}{d\theta} < v_R \Rightarrow |\theta|$ decreases with time

Therefore, sufficient to show $V_P > V_T$, $\left(\frac{N-2}{2}\right) v > 1 \Rightarrow \frac{dv_\theta}{d\theta} < v_R$

At the endgame, $\Rightarrow$ close to some θ_{θ_f}

$$\frac{dv_\theta}{d\theta} v_R < v_R^2$$

$$-\cos^2\theta + v\cos\theta\cos(k\theta+\phi_0) - vk\cos\theta\cos(k\theta+\phi_0) + kv^2\cos^2(k\theta+\phi_0)$$

$$-\cos^2\theta + 2v\cos\theta\cos(k\theta+\phi_0) - v^2\cos^2(k\theta+\phi_0)$$

$$= -2\cos^2\theta - (k-3)v\cos\theta\cos(k\theta+\phi_0) + (k-1)v^2\cos^2(k\theta+\phi_0)$$

$$\Rightarrow \lim_{\theta\to\theta_{\theta_f}} (k-1)v^2 - 2 - (k-3)v\cos[(k+1)\theta+\phi_0] > 0$$

$$\Rightarrow \left.\begin{array}{l} [(k-1)v-2][v+1] > 0 \\ [(k-1)v+2][v-1] > 0 \end{array}\right\} \Rightarrow v > 1, \underset{\substack{\downarrow \\ N-1}}{(k-1)} v > 2$$

not the only condition which is sufficient for which result is true?

clarification **needs lil more**

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$$\frac{dv_\theta}{d\theta} = \frac{d\left(|v_\theta|/\text{sgn}(\dot\theta)\right)}{dt} \cdot \frac{1}{|\dot\theta|/\text{sgn}\,\dot\theta}$$

↳ $\text{sgn}(\dot\theta)$ is sign-invariant for PPN guidance

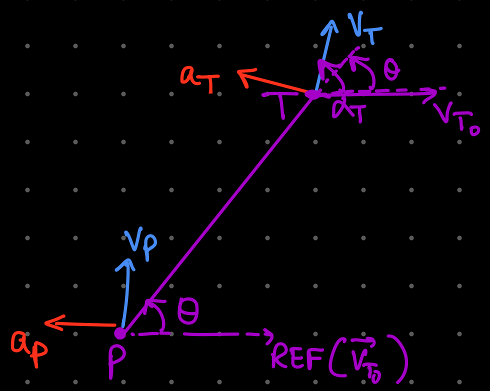
$$\frac{dv_\theta}{d\theta} = \frac{d(|v_\theta|)}{|\dot\theta|\,dt} < \dot R$$

$$\Rightarrow \frac{d(|v_\theta|)}{|\dot v_\theta|} < \frac{\dot R}{R}$$

PPN against maneuvering target

⇓

$a_p(t)$ has finite number of discontinuities in 't' (so that it is integrable)



↳ WLOG, $\vec{V}_{T_0}$ direction can be considered as reference direction
(without loss of generality)

$\Rightarrow \alpha_{T_0} = 0$

↳ then $\alpha_T(t) = \frac{1}{V_T}\int_0^T a_T\,dt = a_{n_T}(t)$

↳ Recall $\alpha_p - \theta = k\theta + \phi_0$

At time t,

$$v_R(\theta,t) = \frac{\dot R}{V_T} = \cos\left(\theta - a_{n_T}(t)\,t\right) - \nu\cos(k\theta+\phi_0)$$

$$\nu = \frac{V_p}{V_T}$$

$$v_\theta(\theta,t) = \frac{R\dot\theta}{V_T} = \sin\left(\theta - a_{n_T}(t)\,t\right) - \nu\sin(k\theta+\phi_0)$$

$\hookrightarrow$ $\boxed{L1: \text{ for a given } t = t_1,\ v > 1,\ kv > 1 \Rightarrow \theta_\theta(t_1) \text{ and } \theta_R(t_1) \text{ alternate along } \theta \text{ axis}}$

L2 ?

$$\Rightarrow \boxed{v_R(\theta, t_1)\, \frac{\partial v_\theta(\theta, t_1)}{\partial \theta}\bigg|_{\theta = \theta_\theta(t_1)} > 0}$$

Feasible range of θ to have roots of $v_\theta(\theta, t)$

$$v_\theta(\theta, t) = 0$$

$$\Rightarrow \quad t = \frac{1}{a_{n_T}(t)} \left[\theta_0 - \sin^{-1}\left(+ v \sin(k\theta_0 + \phi_0) \right) \right]$$

$\hookrightarrow$ if $a_{n_T} = 0$, becomes non-maneuvering for that instance

A real t exists iff

$$\boxed{|v \sin(k\theta_0 + \phi_0)| \leq 1}$$

$$\Rightarrow \quad \underbrace{\theta_{n_0} - \frac{1}{k}\sin^{-1}\left(\frac{1}{v}\right) \leq \theta_0 \leq \theta_{n_0} + \frac{1}{k}\sin^{-1}\left(\frac{1}{v}\right)}_{S_\theta}$$

Sector where sol^n exists

$$\theta_{n_0} = -\left(\frac{\phi_0 + n\pi}{k}\right)$$

$$n = 0, \pm 1, \pm 2, \ldots$$

Similarly, for $v_R(\theta, t)$

A real t exists iff

$$\boxed{|v \sin(k\theta_0 + \phi_0)| \leq 1}$$

$$\Rightarrow \quad \theta_{n_0} + \frac{\pi}{2k} - \frac{1}{k}\sin^{-1}\left(\frac{1}{v}\right) \leq \theta_0 \leq \theta_{n_0} + \frac{\pi}{2k} + \frac{1}{k}\sin^{-1}\left(\frac{1}{v}\right)$$

S_R

sector where sol^n exists

$\theta_{n_0} = -\dfrac{(\phi_0 + n\pi)}{k}$

$n = 0, \pm 1, \pm 2, \ldots$

* these sectors are not dependent on a_T

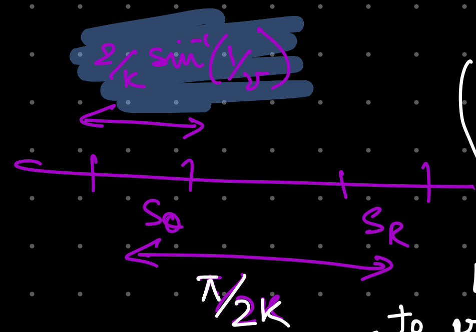
Observation

→ S_θ, S_R are time-invariant

→ S_θ, S_R don't depend on $a_T(t)$

→ Width of $S_\theta, S_R = \dfrac{2}{k}\sin^{-1}\left(\dfrac{1}{v}\right)$

→ Seperation between S_θ & consecutive S_R sector $= \dfrac{\pi}{2K}$



$\dfrac{2}{\cancel{K}}\sin^{-1}\left(\dfrac{1}{v}\right) < \dfrac{\pi}{2\cancel{K}}$

$\dfrac{1}{v} < \dfrac{1}{\sqrt{2}}$

$v > \sqrt{2}.$

to prevent overlap

⇒ Sectors are non-overlapping if $v > \sqrt{2}$

$$\boxed{\begin{array}{l}\text{L3: for any time } t = t_1, \text{ if } v > 1, \; kv > 1, \text{ then there exists one and only one value of } \theta \triangleq \theta_\theta(\theta_R) \\ \text{in each sector } S_\theta(S_R) \text{ such that } v_\theta(\theta_\theta, t_1) \; (v_R(\theta_R, t_1) = 0)\end{array}}$$

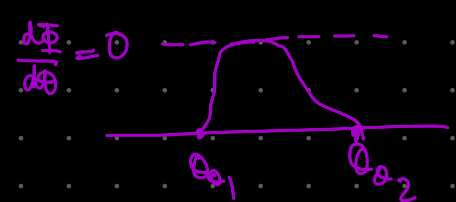
Consider S_θ :

$$\Phi(\theta) \triangleq \frac{1}{a_{n_T}(t)}\left[\theta - \sin^{-1}\left(v \sin(k\theta + \phi_0)\right)\right]$$

At $t = t_1$, $v_\theta(\theta_{\theta_1}, t_1) = v_\theta(\theta_{\theta_2}, t_1) = 0$

that is $\theta_{\theta_1}, \theta_{\theta_2} \in S_\theta$

then $\dfrac{d\Phi(\theta)}{d\theta}$ must be zero at atleast one $\theta \in (\theta_{\theta_1}, \theta_{\theta_2})$



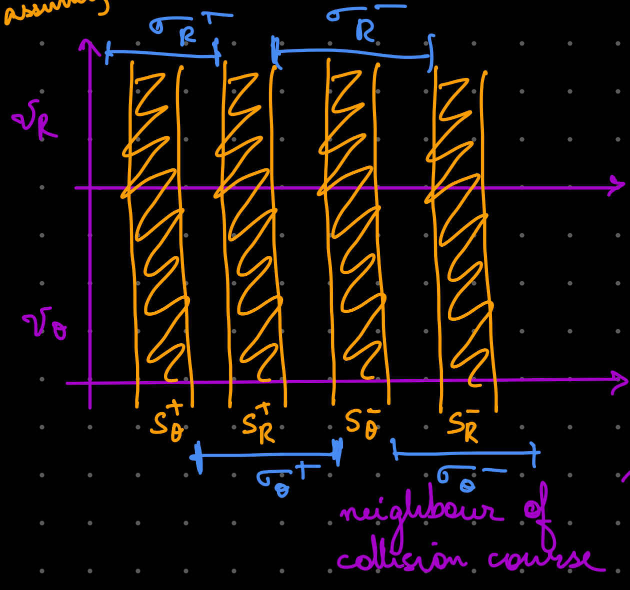
$$\frac{d\Phi(\theta)}{d\theta} = 0 \Rightarrow \cos^2(k\theta + \phi_0) = \frac{1 - v^2}{v^2(k^2 - 1)}$$

> 1 ⟹ Increases

$v > 1,\ kv > 1 \implies v^2 - 1 > v^2 - kv^2$

$$\frac{v^2 - 1}{v^2(1-k^2)} > 1$$

assuming $v > \sqrt{2}$ (non-overlapping)



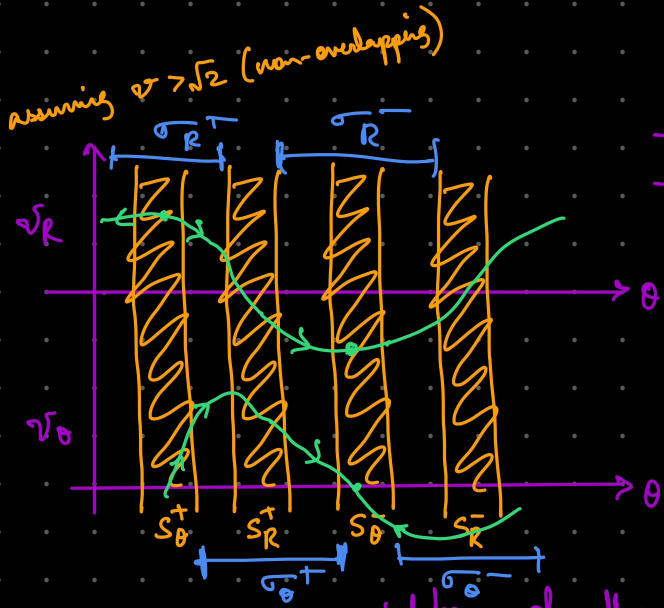
neighbourhood of inverse collision course

S_θ^+ :	$\{\theta \mid \text{for some real } t,\ v_\theta(\theta,t) = 0,\ v_R(\theta,t) > 0\}$
S_θ^- :	$\{\theta \mid \text{for some real } t,\ v_\theta(\theta,t) = 0,\ v_R(\theta,t) < 0\}$
S_R^+ :	$\{\theta \mid \text{for some real } t,\ v_\theta(\theta,t) > 0,\ v_R(\theta,t) = 0\}$
S_R^- :	$\{\theta \mid \text{for some real } t,\ v_\theta(\theta,t) < 0,\ v_R(\theta,t) = 0\}$

σ_θ^+ :	$\{\theta \mid \text{for all } t,\ v_\theta(\theta,t) > 0\}$	σ_R^+ :	$\{\theta \mid \text{for all } t,\ v_R(\theta,t) > 0\}$
σ_θ^- :	$\{\theta \mid \text{for all } t,\ v_\theta(\theta,t) < 0\}$	σ_R^- :	$\{\theta \mid \text{for all } t,\ v_R(\theta,t) < 0\}$

$S_\theta^+ \subset \sigma_R^+$	$S_R^+ \subset \sigma_\theta^+$
$S_\theta^- \subset \sigma_R^-$	$S_R^- \subset \sigma_R^-$

S_θ^- is neighbourhood of collision course.



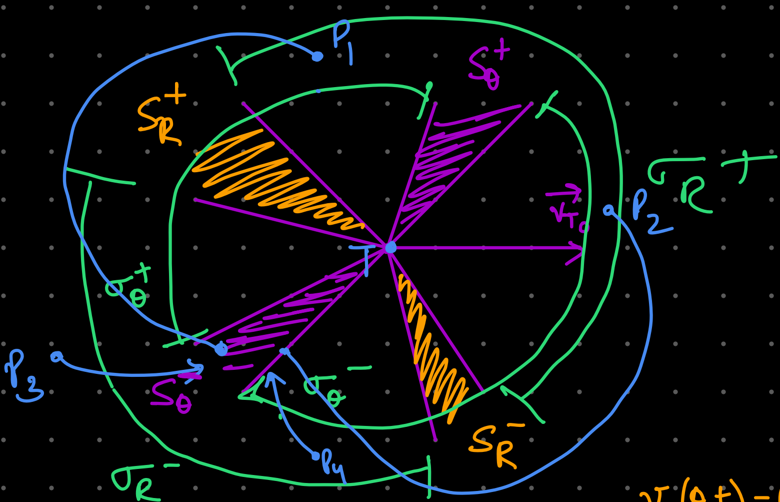
neighbour of collision course

→ $\theta_\theta(t)$, $\theta_R(t)$ alternate along θ axis

→ $v_R(\theta_\theta;t)\cdot\frac{\partial v_\theta}{\partial\theta}(\theta,t) > 0$

Theorem 1: If $v>\sqrt{2}$, $kv>1$, then P can capture T starting from any point in the interior of S_θ^+ sector.

Proof: Consider 'T'-centric $\vec{V}_{T_\theta}$-referenced polar plane.



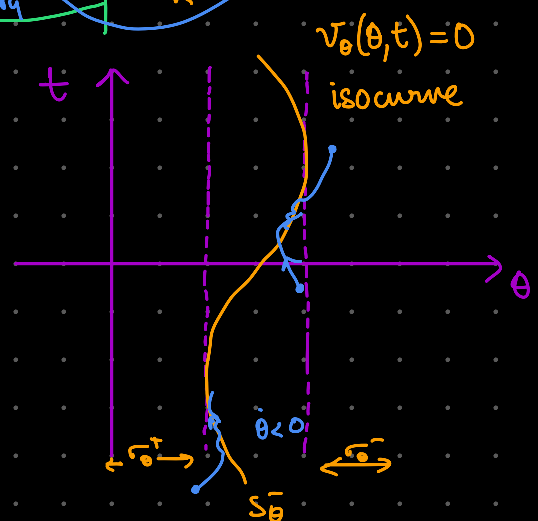
Let $P_1 \in \sigma_\theta^+ \cap \sigma_R^+$

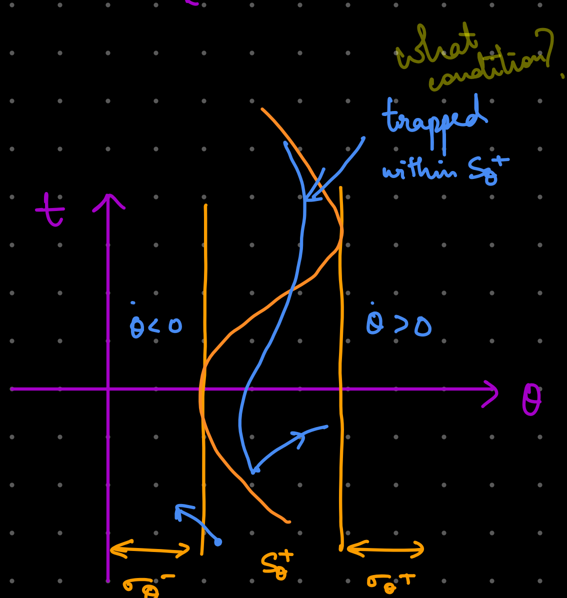
$v_R>0$ $\quad$ $\theta\uparrow$

$P_2 \in \sigma_\theta^- \cap \sigma_R^+$

$P_3 \in \sigma_\theta^+ \cap \sigma_R^-$

$P_4 \in \sigma_\theta^- \cap \sigma_R^-$





Theorem 2

If $P_0 \in S_\theta^+$, and ① $v > \sqrt{2}$, ② $N > 2 + \dfrac{2}{\sqrt{v^2 - 1}} > 1 + \dfrac{1}{v}$

③ $|\dot\theta_0| > \dfrac{|a_T|}{\underbrace{(N-2)\sqrt{v_p^2 - v_T^2} - 2v_T}_{v_e}}$, then P reaches T.

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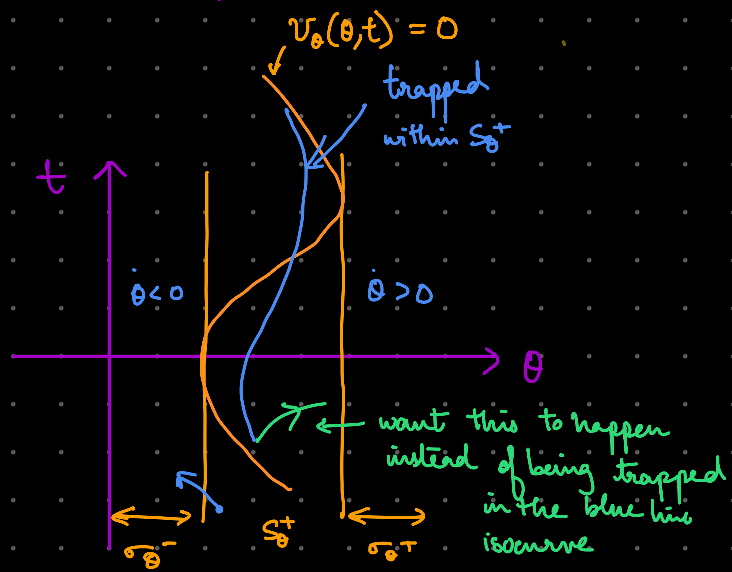
What happens when $P_0 \in S_\theta^+$?



① $v > 2$ ② $N > 2 + \dfrac{2}{\sqrt{v^2-1}} > 1 - \dfrac{1}{v}$

③ $|\dot\theta_0| > \dfrac{|a_T|}{(N-2)\sqrt{v_p^2 - v_T^2} - 2v_T}$

$\dot v_\theta = R\ddot\theta + \dot R\dot\theta =$ ______ (substitute)

$\Rightarrow R\ddot\theta = \Big[\underbrace{-(N-2)v_p\cos(k\theta + \phi_0)}_{\triangleq\, v_s}$
$- 2v_T\cos(\theta - a n_T(t)t)\Big]\dot\theta$
$+ a_T\cos(\theta - a n_T(t)t)$

$\Rightarrow R\ddot\theta = v_s\dot\theta + a n_T \cos(\theta - a n_T(t)t)$

given $a n_T(t)\,t = \dfrac{1}{v_T}\displaystyle\int_0^t a_T(\tau)\,d\tau$

$\dfrac{d}{dt}\big(a n_T(t)\,t\big) = \dfrac{a_T(t)}{v_T}$

Lemma 4: If $P \in S_\theta^+$ (S_θ^-), then

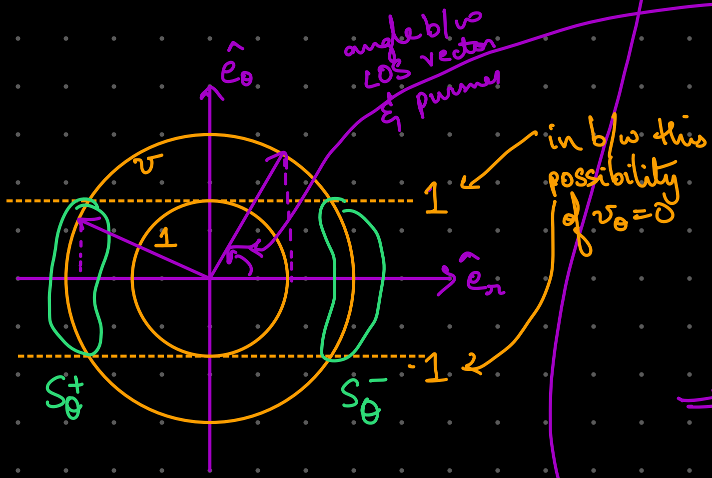
$v_s \geq v_R$ ($v_s \leq -v_e$), where

$v_e = (N-2)\sqrt{v_p^2 - v_T^2} - 2v_T$ $\quad v_e > 0$

Proof:

Let $P \in S_\theta^+$,

in S_θ region, $|\sin(k\theta + \phi_0)| < \dfrac{1}{v}$



$\overline{k\theta+\phi_0} = \alpha_p - \theta$

$v_R = 1 . \cos(\text{----}) - v\cos(\text{----})$

$v_\theta = 1 . \sin(\text{--}) - v\sin(\text{---})$

For S_θ^+, $v_R > 0$ $\quad (\because S_\theta^+ \subset \sigma_R^+)$

$\Rightarrow \cos(k\theta+\phi_0) < 0$

$\cos(k\theta+\phi_0) \leq -\frac{\sqrt{v^2-1}}{v}$

Also $\quad \cos(\theta - a_{n_T}(t)t) \leq 1$?

thus, $\quad v_s \geq v_e > 0$

Lemma 5: Let θ_1 be such that $\underbrace{v_s\dot{\theta}_1 + a_T\cos(\theta - a_{n_T}(t)t)}_{R\ddot{\theta}} = 0$,

then if $\dot{\theta}(t) > \dot{\theta}_1(t)$ $\quad (\dot{\theta}(t) < \dot{\theta}_1(t))$

At some time t, then

1. If $v_s > 0$, $\dot{\theta}$ is increasing (decreasing) with time
2. If $v_s < 0$, $\dot{\theta}$ is decreasing (increasing) with time

Proof: $\quad \dot{\theta} = \dot{\theta}_1 + \dot{\theta}_s$

If $\dot{\theta}(t) > \dot{\theta}_1(t)$, then $\dot{\theta}_s(t) > 0$

then $R\ddot{\theta} = v_s\dot{\theta} + a_T\cos(\theta - a_{n_T}(t)) = v_s\dot{\theta}_s$

(finish)

Proof of Theorem 2

For $P_0 \in S_\theta^+$, $v_s \geq v_e > 0$
(L4) (condition 2)

Consider $\dot{\theta}_1 = \dfrac{-a_T \cos(\theta - a_{nT}(t)t)}{v_s}$

Therefore, $-\dfrac{|a_T(t)|}{v_e} \leq -\dfrac{|a_T(t)|}{v_s} \leq \dot{\theta}_1(t) \leq \dfrac{|a_T(t)|}{v_s} \leq \dfrac{|a_T(t)|}{v_e}$

$\Rightarrow$ then by, L5

If $|\dot{\theta}| > \dfrac{|a_T|}{v_e}$, then

$|\dot{\theta}|$ is increasing with t

$\Rightarrow$ P will come out of S_θ^+

Theorem 3 — Boundedness of Latax

If $P_0 \in S_\theta^-$, and ① $v > \sqrt{2}$, ② $N > 2 + \dfrac{2}{\sqrt{v^2-1}} > 2 + \dfrac{2}{v}$, then

i) if $|a_p| > a_{p_1}$, $|a_p|$ decreases until $|a_p| < a_{p_1}$

ii) if $|a_p| < a_{p_1}$, then $|a_p| < a_p$ always, where

$$a_{p_1} = \frac{N}{(N-2)\sqrt{v^2-1} - 2} v |a_T|$$

Proof:

$P \in S_\theta^- \Rightarrow v_s \leq -v_e < 0$ $v_e > 0$

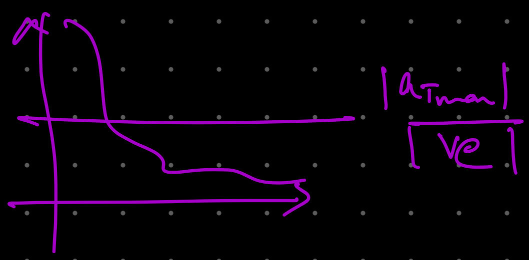
consider $\dot{\theta}_1 = -\dfrac{a_T \cos(\theta - a_{nT}(t)t)}{v_s}$

then,

$$\frac{-|a_{T_{max}}|}{v_e} \leq \frac{-|a_T|}{v_e} \leq \frac{-|a_T|}{|v_s|} \leq \phi_1 \leq \frac{|a_T|}{|v_s|} \leq \frac{|a_T|}{v_e} \leq \frac{|a_{T_{max}}|}{v_e}$$

If $|\dot{\theta}| > \dfrac{|a_{T_{max}}|}{v_R}$, then $|\dot{\theta}|$ decreases at all time t

until $|\dot{\theta}| < \dfrac{|a_{T_{max}}|}{v_e}$



Tutorial session in 1st half

* UUP stability ? — state trajectories reach some neighbourhood, some bound and not exact — convergence

Augmented Proportional Navigation (APN)

$$a_p \approx N' v_c \dot{\theta} + \underbrace{\frac{N'}{2} a_T}_{\text{TPN}}$$

(not that valid for aerodynamic forces as not $\perp$ v_p)

$$N' V_{c_0} \approx N v_p$$

$$\Rightarrow N = \frac{N' v_{c_0}}{v_p}$$

Augmented PPN (A-PPN)

$$a_p(t) = N v_p \dot{\theta}(t) + \underbrace{K_1(t)}\, a_T(t)$$

coefficient of $a_T(t)$ is time-varying (and will be found as state-dependent)

$V_R =$ _ _ _ _ _

$V_\theta =$ _ _ _ _ _

$$\dot{\alpha}_p = \frac{a_p}{V_p}, \quad \dot{\alpha}_T = \frac{a_T}{V_T}$$

$$\alpha_T = \cancel{\alpha_{T_0}}^{\,0\ \text{(WLOG)}} + \frac{1}{V_T}\int_0^t a_T(t)\,dt$$

$$\triangleq a_{n_T}(t)\, t$$

$$\dot{\alpha}_p = N\dot{\theta} + \frac{K_1(t)}{v}\dot{\alpha}_T(t) \rightarrow k_2(t)$$

$$\alpha_p = N\theta + \gamma_{n_T}(t)\, t + \phi_0$$

where, $\phi_0 = \alpha_{p_0} - N\theta_0$

$$\gamma_{n_T}(t) = \frac{1}{t}\int_0^t K_2(\tau)\,\dot{\alpha}_T(\tau)\,d\tau$$

$$v_R(\theta,t) = \cos\left(a_{n_T}(t)t - \theta\right) - v\cos\left(k\theta + \gamma_{n_T}(t)t + \phi_0\right)$$

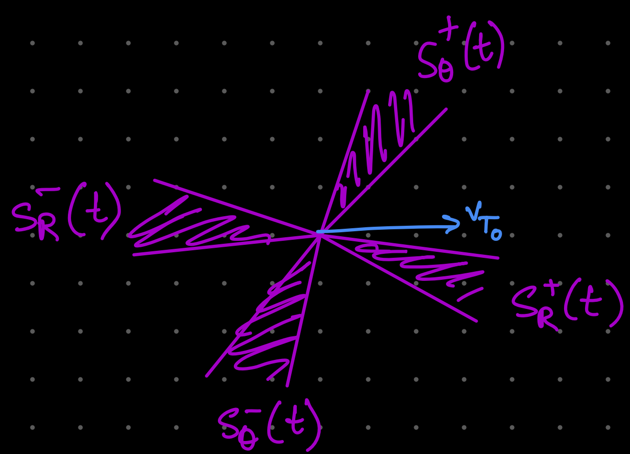
$$v_\theta(\theta,t) = \sin\left(a_{n_T}(t)t - \theta\right) - v\sin\left(k\theta + \gamma_{n_T}(t)t + \phi_0\right)$$

$$S_\theta(t) = \left[\theta_{n_0} - \frac{1}{k}\gamma_{n_T}(t)t - \frac{1}{k}\sin^{-1}\left(\frac{1}{v}\right),\ \theta_{n_0} + \frac{1}{k}\gamma_{n_T}(t)t + \frac{1}{k}\sin^{-1}\left(\frac{1}{v}\right)\right]$$

↳ time-dependent unlike PPN

$$S_R(t) = \left[\theta_{n_0} - \frac{1}{k}\gamma_{n_T}(t)t + \frac{\pi}{2k} - \frac{1}{k}\sin^{-1}\left(\frac{1}{v}\right),\ \theta_{n_0} + \frac{1}{k}\gamma_{n_T}(t)t + \frac{\pi}{2K} + \frac{1}{K}\sin^{-1}\left(\frac{1}{v}\right)\right]$$

$$\frac{d}{dt}\left(\gamma_{n_T}(t)t\right) = k_2(t)\frac{a_T(t)}{V_T}$$



sectors are rotating at angular rate, $-k_2(t)\dfrac{a_T(t)}{V_T}$

$$\dot{\theta}_{REL}(t) = \dot{\theta}(t) - \left(-\frac{k_2(t)\,a_T(t)}{V_T}\right)$$

Theorem:

Conditions for successful capture from exterior of $S_\theta^+(t_0)$

① $v > \sqrt{2}$ ② $kv > 1$ ③ $\mathrm{sgn}(k_1(t)) = \mathrm{sgn}\left(a_T(t)\,\dot{\theta}(t)\right)$

allows for faster approach to S_θ^- than PPN due to this rotating S_θ sectors

↳ less a_p requirement

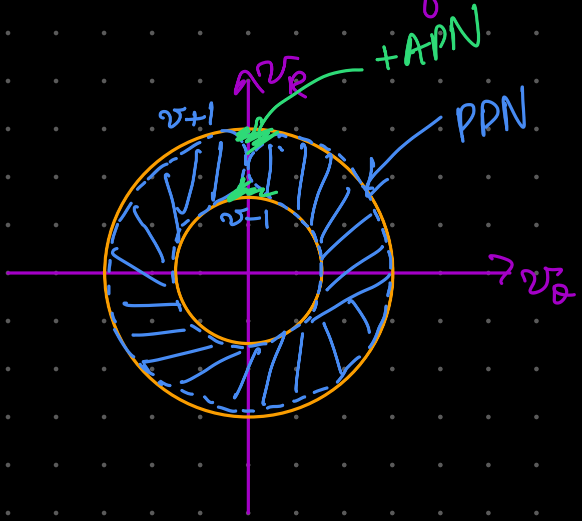
Theorem

initial starting point does not matter

If $P_0 \in S_\theta^+(t_0)$, condition for P coming out of $S_\theta^+(t)$ at some finite time $t > t_0$

① $v > \sqrt{2}$ ② $kv > 1$ ③ $\mathrm{sgn}(k_1(t)) = \mathrm{sgn}\left(a_T(t)\,\dot{\theta}(t)\right)$ ④ $|k_1(t)| > \dfrac{1}{\sqrt{1 - 1/v^2}}$

* same conditions for bounded condition for lateral acc^n of $S_{\bar{\theta}}$???



→ expansion in CR in guaranteed fashion

→ faster achievment of $S_{\bar{\theta}}$ in endgame phase

→ other numerical advantages

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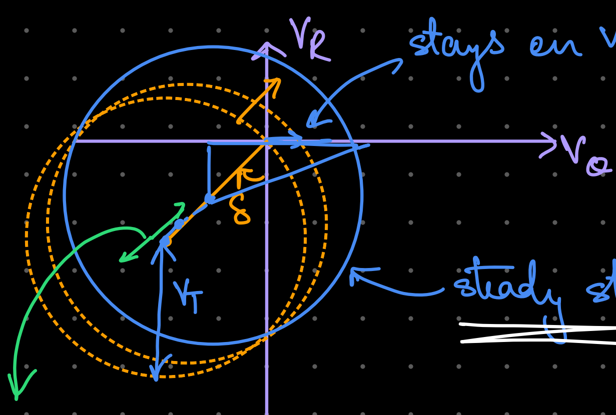
Range is constant

$\hookrightarrow V_R = V_T\cos\psi - V_p\cos\delta = 0 \quad \Rightarrow \quad V_p = \dfrac{V_T \overset{\text{const}}{} \cos\psi \leftarrow \text{varying}}{\cos\delta \leftarrow \text{const}}$ (varying)

$V_\theta = V_T\sin\psi - V_p\sin\delta \quad \rightarrow \quad \dot{\theta} = \omega_T$ — eqbm

$V_\theta = R\omega_T$

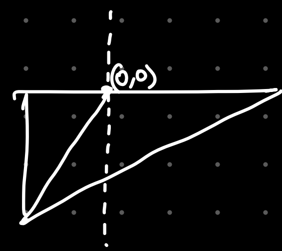
$(V_R + V_p\cos\delta)^2 + (V_\theta + V_p\sin\delta) = 0$



stays on $V_R = 0$ line (the engagement point)

steady state

??



centre will vary along the line

$\psi = \alpha_p - \theta$

$\psi = \alpha_p - \cancel{\theta}^{\,\omega_T}$

$\dot{V}_p = \dfrac{V_T}{\cos\delta} - \sin\psi \cdot \dfrac{d\psi}{dt}$

equilibrium

$V_R = 0$

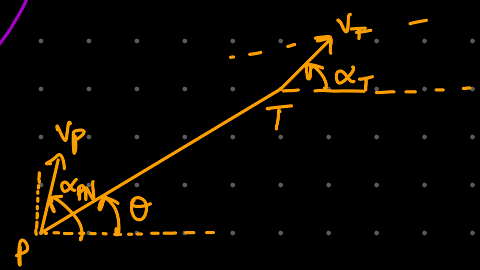
$V_\theta = V_T\sin\psi - \dfrac{V_T\cos\psi}{\cos\delta}\sin\delta$

$V_\theta = \dfrac{V_T}{\cos\delta}\left(\sin(\psi - \delta)\right)$

Linearised Engagement Geometry

↳ Near Collision Course (NCC)

Most of the modern guidance laws work on NCC condition at homing phase (Endgame)



define α_{P_N}

Characterised as

1) Angle between $\vec{R}$ and $-\dot{\vec{R}}$ is 'very' small.

2) V_C (closing speed) is 'almost' constant $\qquad V_C = -|V_R|$

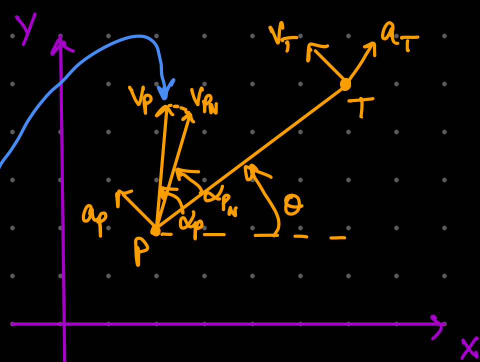
The engagement is very close to collision course.

$$V_P \sin(\alpha_{P_N} - \theta) = V_T \sin(\alpha_T - \theta)$$

α_P — pursuer heading

α_{P_N} — nominal pursuer heading

virtual pursuer (with same $\|\vec{V_P}\|$) that is in collision course with target)



$x(t) = x_T(t) - x_P(t)$

$y(t) = y_T(t) - y_P(t)$

① $\theta \leq 1 \Rightarrow \theta(t) \approx \dfrac{y(t)}{x(t)}$

② $\alpha_{P_N} \approx \alpha_{P_{N_0}}$

③ $x(t) = R(t)\cos\theta(t)$
$\approx R(t)$

④ $\dot{x}(t) \approx \dot{R}(t) = -V_C(t) = -V_C$

⑤ $\Delta\phi \ll 1$

$\Delta\phi = \alpha_{P_N} - \alpha_P$: Heading error

⑥ $\Delta\alpha_T = \alpha_T(t) - \alpha_{T_0}(t) \ll 1$

two ways of seeing this

Δt is small for high $\dot{\alpha}_T$

for a low $\dot{\alpha}_T$

$$V_C = -\left[V_T \cos(\alpha_T - \theta) - V_P \cos(\alpha_P - \theta) \right]$$

$$\cos(\alpha_T - \theta) = \cos(\alpha_T - \alpha_{T_0} + \alpha_{T_0} - \theta)$$

$$= \cos(\alpha_T - \alpha_{T_0})\cos(\alpha_{T_0} - \theta) - \sin(\alpha_T - \alpha_{T_0})\sin(\alpha_{T_0} - \theta)$$

$$= \underbrace{\cancel{\cos}}_{1}\Delta\alpha_T \cos(\alpha_{T_0} - \theta) - \sin \underbrace{\cancel{\Delta\alpha_T}}_{\Delta\alpha_T} \sin(\alpha_{T_0} - \theta)$$

$$= \Big(\underbrace{\cos\alpha_{T_0} \cancel{\cos\theta}^{1}}_{\approx \cos\alpha_{T_0}\ \approx 1} - \underbrace{\sin\alpha_{T_0}}_{\le 1}\underbrace{\sin\theta}_{\ll 1}\Big) - \underbrace{\cancel{\Delta\alpha_T}}_{\ll 1}\Big(\underbrace{\sin\alpha_{T_0}}_{\lesssim 1}\underbrace{\cancel{\cos\theta}}_{\approx 1} - \underbrace{\cancel{\sin\theta}}_{\approx 0}\underbrace{\cos\alpha_{T_0}}_{\le 1}\Big)$$

$$\approx \cos\alpha_{T_0}$$

Similarly, $\cos(\alpha_P - \theta) = ?$

then, $V_c = V_P \cos\alpha_{P_{N_0}} - V_T \cos\alpha_{T_0}$

and, $x(t) \approx R(t) = V_c t_{go}$

then, $\dot\theta(t) = \dfrac{d}{dt}\left(\tan^{-1}\left(\dfrac{y(t)}{x(t)}\right)\right)$

$$\approx \frac{d}{dt}\left(\frac{y(t)}{x(t)}\right)$$

$$\dot\theta(t) \approx \frac{\dot y}{V_c t_{go}} - \frac{y(-V_c)}{V_c^2 t_{go}^2}$$

$$= \frac{1}{V_c t_{go}^2}\underbrace{\left[y + \dot y t_{go}\right]}_{ZEM(t)}$$

in absence of a_T

Zero Effort Miss distance

target is $\underbrace{\text{maneuvering}}_{a_T}$

In presence of a_T, $\ddot y \neq 0$

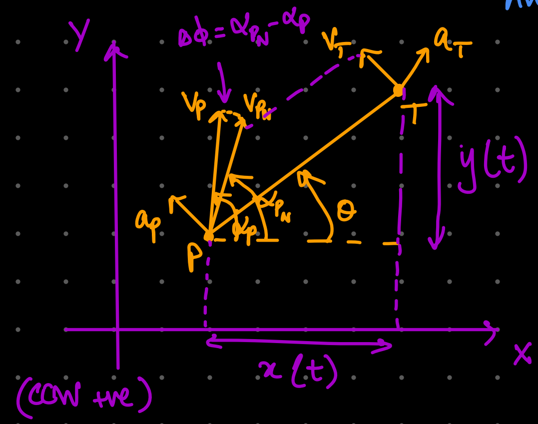
$$= y + \dot y t_{go} + \frac{1}{2}\ddot y t_{go}^2$$

~~RT~~PN : $N' V_c \dot\theta \approx \dfrac{N'}{t_{go}^2}(ZEM(t))$

APN : $\dfrac{N' V_c}{V_c t_{go}^2}\left(y + \dot y t_{go} + \frac{1}{2}\underbrace{\ddot y}_{a_T} t_{go}^2\right) \approx \underbrace{N' V_c \dot\theta}_{+\frac{N}{2} a_T}$

$\vec{R} = R\hat{e}_R$, $\dot{\vec{R}} = \dot{R}\hat{e}_R + R\dot{\hat{e}}_R = \dot{R}\hat{e}_R + R\dot{\theta}\hat{e}_\theta$

Angle between $\vec{R}', \vec{R}$ is very small $\Rightarrow$ $R\dot{\theta}\hat{e}_\theta$ is very small.



① $\theta << 1 \Rightarrow \theta(t) \approx \tan\theta(t) \approx \frac{y(t)}{x(t)}$

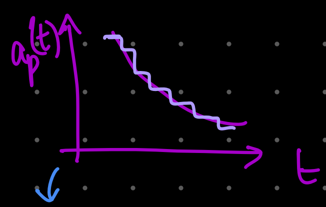
② $\alpha_{P_N} \approx \alpha_{P_{N_0}}$

③ $x(t) = R(t)\cos\theta(t) \approx R(t) \approx V_c t_{go}$

$\Rightarrow \dot{x}(t) \approx -V_c$

④ $\Delta\phi << 1$

⑤ seperate assumption : $\Delta\alpha_T(t) = \alpha_T(t) - \alpha_T(t_0) << 1$



* CHECK

$V_c \approx V_P \cos\alpha_{P_{N_0}} - V_T\cos\alpha_{T_0}$

Planning horizon : Plan the motion for an interval in a real system.

$$\dot{\theta}(t) = \frac{d}{dt}\tan^{-1}\left(\frac{y(t)}{x(t)}\right) \approx \frac{d}{dt}\left(\frac{y(t)}{x(t)}\right) \approx \frac{1}{V_c t_{go}^2}\left(\underbrace{y + \dot{y}t_{go}}_{\text{ZEM = Zero Effort Miss}}\right)$$

* CHECK

$$\dot{y}(t) = \dot{y}_T(t) - \dot{y}_P(t)$$

$$= V_T\sin(\alpha_{T_0} + \Delta\alpha(t)) - V_P\sin(\alpha_{P_{N_0}} + \Delta\phi_0 + \Delta\alpha_P(t))$$

* Derive $y(t)$

For PN,

$\Delta\alpha_P(t) = N\Delta\theta = N\theta$ ($\theta_0 = 0$ is considered)

$$\dot{y}(t) + \frac{N'}{t_{go}}y(t) = \underbrace{V_T\cos\alpha_{T_0}\,\Delta\alpha_T(t)}_{\text{Time-varying } A(t)} - \underbrace{V_P\cos\alpha_{P_{N_0}}\,\Delta\phi_0}_{B = \text{constant}}$$

consequence of initial heading error

consequence of target's maneuver

$N' = \frac{N V_P \cos\alpha_{P_{N_0}}}{V_c}$

$N'V_{c_0} \approx N V_P$

equivalence of TPN and PPN

valid if $\alpha_{P_{N_0}}$ is very very small. (Seperate assumption)

latax applied is close

Consider a constant target maneuver over a planning horizon.

$$\Rightarrow \quad A(t) = a_T \cos \alpha_{T_0} t \approx \ddot{y}_{T_0} t$$

Case 1:

If $a_T = 0$ under $\Delta\phi_0$

the constant term

$$\ddot{y}^B(t) = -\frac{V_P \cos \alpha_{P_{N_0}} \Delta\phi_0 N'}{t_f} \left(\frac{t_f - t}{t_f}\right)^{N'-2}$$

Solve for $y(t) \rightarrow$ get $\ddot{y}(t)$ $\longrightarrow$ $\ddot{y}(t) = \ddot{y}_T(t) - \ddot{y}_P(t) \approx a_T(t) - a_P(t)$

$$\Rightarrow \quad a_P^B(t) = -\ddot{y}^B(t)$$

$$= \frac{V_P \cos \alpha_{P_{N_0}} \Delta\phi_0 N'}{t_f} \left(\frac{t_f - t}{t_f}\right)^{N'-2}$$

- $a_P^B(t) \propto \Delta\phi_0$
- As $t\uparrow$, $a_P^B(t)\downarrow$ for $N' > 2$

Necessary for capturability.
- True for PN forms. Why other forms?

Case 2:

If $\Delta\phi_0 = 0$, under constant a_T

$$a_P^A(t) = \ddot{y}_{T_0} \frac{N'}{N'-2}\left[1 - \left(\frac{t_f - t}{t_f}\right)^{N'-2}\right]$$

$\ddot{y}_{T_0} \approx a_T$

$\rightarrow$ If $N' > 2$, $a_P^A(t)$ increasing

$\rightarrow$ $a_P^A(t_0) = 0$

$\rightarrow$ For $N' > 2$, as $t\uparrow$, $a_P^A(t)\Uparrow$

$$\frac{a_P^A\big|_{max}}{a_T} = \frac{N'}{N'-2}$$

↳ Recall,

Non-linear Counterpart

$$a_{p_{PN}} = \frac{N'}{t_{go}^2} ZEM(t) = \left(N' V_c \dot{\theta}\right)$$ (without taking a_T into consideration)

$$ZEM(t) = y(t) + \dot{y}(t)\, t_{go}$$

↳ when a_T is considered, $ZEM(t) = \underbrace{y(t) + \dot{y}\, t_{go}} + \frac{1}{2}\ddot{y}\, t_{go}^2$

$$a_{p_{PN}} = \frac{N'}{t_{go}^2} ZEM(t) = N' V_c \dot{\theta} + \frac{N'}{2} a_T \leftarrow \text{NL Counterpart}$$

↳ $$a_{p_{PN}}^{a_T} = \frac{N'}{N'-2}\, a_T \left[1 - \left(1 - \frac{t}{t_f}\right)^{N'-2}\right]$$

↳ $$a_{p_{APN}}^{a_T} = \frac{1}{2} N' a_T \left(1 - \frac{t}{t_f}\right)^{N'-2}$$

- finding t_{go} is pain
  - ↳ formulate equation for t_f / t_{go}, not always feasible to solve

Observations

- $a_{p_{PN}}^{a_T}$, $a_{p_{APN}}^{a_T}$ are dependent on a_T and t_{go} estimates.
- $a_{p_{PN}}^{a_T} \uparrow$ as $t \uparrow$, but $a_{p_{APN}}^{a_T} \downarrow$ as $t \uparrow$ (for $N' > 2$) → end moment high latax is risky
- $\frac{a_{p_{PN}}^{a_T}\big|_{max}}{a_T} = \frac{N'}{N'-2}$, $\frac{a_{p_{APN}}^{a_T}\big|_{max}}{a_T} = \frac{N'}{2}$

↳ Maneuver-induced Cost ← one of them

there are different measures of effectiveness

CHECK →
$$\begin{cases} \Delta V_{PN} = \int_0^{t_f} a_{p_{PN}}(t)\, dt = \int_0^{t_f} \frac{N'}{N'-2} a_T [\quad] = \frac{N' a_T t_f}{N'-2} \\ \Delta V_{APN} = \int_0^{t_f} a_{p_{APN}}(t)\, dt = \int_0^{t_f} \text{------} = \frac{N' a_T t_f}{2(N'-2)} = \frac{\Delta V_{PN}}{2} \end{cases}$$

* fix a Performance Index, optimise it —— Optimal Control
* convex function in convex domain has unique minima.
* if non convex set, can be broken into multiple convex sets and we can take union

Convex Optimisation

Linear Programming

Optimal Guidance is not a static optimisation problem.

$$\min \; J = \Phi(\underset{\sim}{X}(t_f)) + \int_{t_0}^{t_f} F(\underset{\sim}{X}, \underset{\sim}{u}, t)\, dt$$

ST $\dot{\underset{\sim}{X}} = f(\underset{\sim}{X}, \underset{\sim}{u}, t)$,

$\underset{\sim}{u} \in \mu \;\; \forall \;\; t \in [t_0, t_f]$

$g(\underset{\sim}{X}, t) = 0$ for some or all $t \in [t_0, t_f]$

$$\text{MIN} \; J = \frac{1}{2}\int a_p^2\, dt$$

sign of a_p; $|a_p|$ not handled easily

for fuel conservation, why?

S.T $\dot{\underset{\sim}{X}} = AX + B\underset{\sim}{U}$

where, $\underset{\sim}{X} = [y \;\; \dot{y} \;\; a_T]^T$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \;\; ?$$

and $y(t_f) = 0$

state transition matrix from Laplace inverse SI-At

From Linear System Theory,

$$\underset{\sim}{X}(t_f) = \Phi(t_f - t)\, \underset{\sim}{X}(t) + \int_t^{t_f} \Phi(t_f - \tau) B(\tau) u(\tau)\, d\tau$$

where, $\Phi(t) = L^{-1}[(SI - A)^{-1}] = \exp[At]$

(a is time invariant so possible)

$$\exp(At) = I + At + \frac{A^2t^2}{2!} + \frac{At^3}{3!} - - - -$$

$$= \begin{bmatrix} 1 & t & t^2/2 \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

WHAT?

Thus,

$$\begin{bmatrix} y(t_f) \\ \dot{y}(t_f) \\ a_T(t_f) \end{bmatrix} = \begin{bmatrix} 1 & t_{go} & t_{go}^2/2 \\ 0 & 1 & t_{go} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y(t) \\ \dot{y}(t) \\ a_T(t_f) \end{bmatrix} + \int_0^{t_f} \begin{bmatrix} 1 & t_f-\tau & (t_f-\tau)^2/2 \\ 0 & 1 & t_f-\tau \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} a_p(\tau)\,dt$$

Hence,

$$y(t_f) = \underbrace{y(t) + \dot{y}(t)\,t_{go} + \frac{1}{2}\ddot{y}(t)\,t_{go}^2}_{f_1(t_f-t) = ZEM(t)} - \int_t^{t_f} \underbrace{(t_f-\tau)}_{h_1(t_f-\tau)} a_p(\tau)\,d\tau$$

For $y(t_f) = 0$,

$$f_1(t_f-t) = \int_t^{t_f} h_1(t_f-\tau)\, a_p(\tau)\, d\tau$$

$$\dot{X} = AX + BU$$

$$X = \begin{bmatrix} y \\ \dot{y} \\ a_T \end{bmatrix} \qquad \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{a}_T \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ a_T \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} a_p$$

$$y(t_f) = 0$$

$$y(t_f) = \underbrace{y(t) + \dot{y}(t)\,\overset{(t_f-t)}{\cancel{t_{go}}} + \frac{1}{2}\ddot{y}(t)\,\overset{(t_f-t)^2}{\cancel{t_{go}^2}}}_{ZEM(t) = f_1(t_f-t)} - \int_t^{t_f} \underbrace{(t_f-\tau)}_{h_1(t_f-\tau)} a_p(\tau)\,d\tau$$

$$f_1(\overset{t_{go}}{\overset{\downarrow}{t_f-t}}) = \int_t^{t_f} h_1(t_f-t)\,a_p(t)\,dt$$

Using Cauchy Schwartz Inequality,

$$\int a_p^2(\tau)\,d\tau \geq \frac{f_1^2(t_f-t)}{\int_t^{t_f} h_1^2(t_f-t)\,dt}$$

at optimal value, equality holds

proportionality constant

$$a_p(\tau) = c\,h_1(t_f-t)$$

$$c^2 \int_t^{t_f} h_1^2(t_f-\tau)\,d\tau = \frac{f_1^2(t_f-t)}{\int_t^{t_f} h_1^2(t_f-\tau)\,d\tau}$$

$$c = \frac{f_1(t_f-t)}{\int_t^{t_f} h_1^2(t_f-\tau)\,d\tau} \longrightarrow h_1(t_f-\tau) = t_f-\tau$$

$$\int_{t_f}^{t} (t_f-\tau)^2\,d\tau = \frac{t_{go}^3}{3}$$

$$c = f_1(t_f-t)/(t_{go}^3/3)$$

$$a_p(t) \simeq \frac{3}{t_{go}^2}\left[y(t) + \dot{y}(t)\, t_{go} + \underline{\frac{1}{2} a_T\, t_{go}^2} \right]$$

↳ similar to PN, with $N' = 2$

there is added term for maneuvering target

but here, the optimal soln for this performance index

VC Kohn?
Automatic Control Theory
"Linear Systems and Theory"

Sliding Mode Control-based Guidance

$$\dot{R} = V_T \cos(\alpha_T - \theta) - V_p \cos(\alpha_p - \theta)$$

$$R\dot{\theta} = V_T \sin(\alpha_T - \theta) - V_p \cos(\alpha_p - \theta)$$

$$\dot{\alpha}_p = \frac{a_p}{V_p} \quad , \quad \dot{\alpha}_T = \frac{a_T}{V_T}$$

intercept $\Rightarrow$ $R \to 0$, $\dot{\theta} \to 0$ ← collision course

$V(x)$ — positive definite function

and $\underbrace{\dot{V} < 0 \text{ , then } x \to 0}_{\text{property of positive definite function}}$

sliding variable $S = \dot{\theta}$

↳ cuz we want collision course

if we want to achieve some non-zero value of x, transform the variable to $x - \hat{x}$.

$$V = \frac{1}{2} S^2$$

sliding surface $S = 0$

$$\dot{V} = S\dot{S} = \dot{\theta}\ddot{\theta}$$

$$\ddot{\theta} = \frac{1}{R}\left[-2\dot{R}\dot{\theta} + a_T \cos(\alpha_T - \theta) - \underline{a_p} \cos(\alpha_p - \theta)\right]$$

To ensure $\dot{V}$ is there is negative

so $\dot{S}$ should have $\dot{\theta}, \ddot{\theta}$?

we're not pre-fixing a_p

$$\dot{V} = \frac{\dot{\theta}}{R}\left[\qquad \qquad \right] < 0$$

a_p is under our control

To achieve this,

$$-2\dot{R}\dot{\theta} + a_T\cos(\alpha_T - \theta) - a_p\cos(\alpha_p - \theta) < \underbrace{-\mu\,\mathrm{sgn}(\dot{\theta})}_{\text{how?}} \quad , \quad \mu > 0$$

Say,

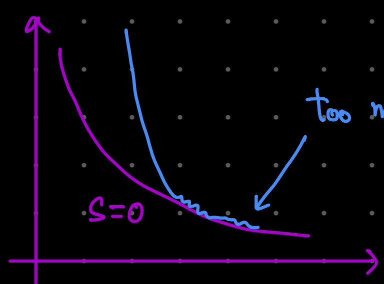
$$-2\dot{R}\dot{\theta} + a_T\cos(\alpha_T - \theta) - a_p\cos(\alpha_p - \theta) = -\mu\,\mathrm{sgn}(\dot{\theta}) \quad , \quad \mu = \eta R + \epsilon$$

$$\epsilon > 0$$

$$\Rightarrow a_p = \frac{1}{\cos(\alpha_p - \theta)}\left[\underbrace{-2\dot{R}\dot{\theta} + a_T\cos(\alpha_T - \theta)}_{\text{equivalent control}} + \underbrace{\mu\,\mathrm{sgn}(\dot{\theta})}_{\text{switching control}}\right]$$

Consider $a_T \propto \dot{\theta}$ and $|a_T| < a_{T_{max}}$, then select

$$\Rightarrow \mu_1 = \eta R + a_{T_{max}} + C \quad \text{and} \quad a_p = \frac{1}{\cos(\alpha_p - \theta)}\left[-\underset{N=2}{N}\dot{R}\dot{\theta} + \mu_1\,\mathrm{sgn}(\dot{\theta})\right]$$



too much chattering not good,
coming from input ← from control surfaces.

we would like to smoothen chattering by using sigmoid function.

APPN :

$$a_p = N V_p \dot{\theta} + K_1(t)\, a_T$$

$$|K_1(t)| > \text{---}$$

$$\mathrm{sgn}(K_1(t)) = \mathrm{sgn}(a_T\dot{\theta})$$

Similar to what we've got here.

didn't understand notion of equivalence

Terminal Angle Control by PPN

↳ applications → missiles hitting target
→ landing

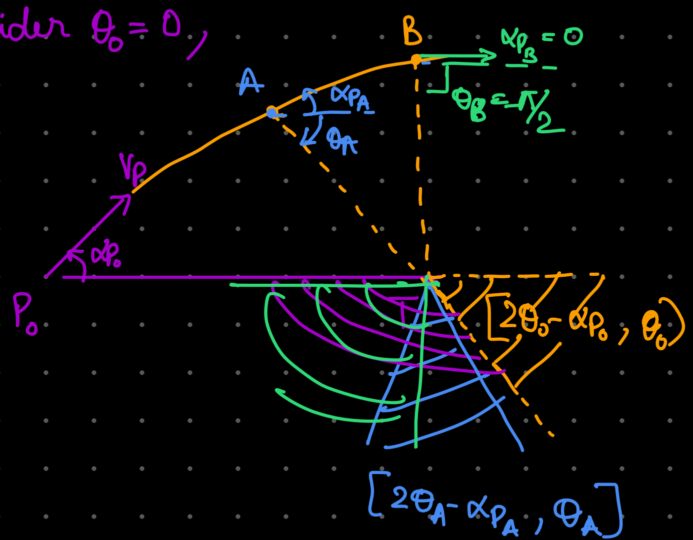
$\dot{\alpha}_p = N\dot{\theta} \Rightarrow \quad N = \dfrac{\alpha_{p_f}^D - \alpha_{p_0}}{\theta_f - \theta_0}$

For stationary target, $\alpha_{p_f} = \theta_f$, $R = 0$ (check)

For $N = 2$, $\underline{\underline{\alpha_{p_f} = 2\theta_0 - \alpha_{p_0}}}$

If $N \to \infty$, $\alpha_{p_f} \to \theta_0$

Consider $\theta_0 = 0$,



till A or B, the orientation trajectory

max value to cover lower half plane (green colour)

↳ $(\alpha_{p_0}, \theta_0) \longrightarrow \left(\alpha_{p_{ORI_f}} = \theta_0,\ \theta_{ORI_f} = \theta_0 - \pi/2\right)$

$\alpha_{p_0} > \theta_0$

$N =$

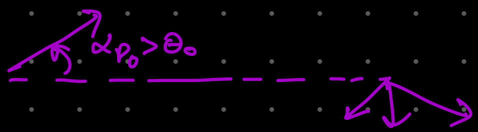
In the orientation phase, trajectory should start from (α_{p_0}, θ_0) and tend towards $\left(\alpha_{p_{ORI_f}} = \theta_0, \alpha_{p_{ORI_f}} = \theta_0 - \pi/2\right)$

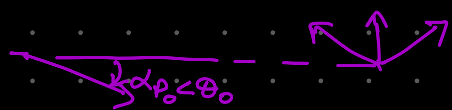
Gain for this phase,

$$N_{ORI} = \frac{\theta_0 - \alpha_{P_0}}{\theta_0 - \pi/2 - \theta_0} = \frac{2}{\pi}(\alpha_{P_0} - \theta_0)$$

$$\Downarrow$$

for both cases $\quad \frac{2}{\pi}|\alpha_{P_0} - \theta_0|$





The Composite Guidance Strategy

$a_P = N v_P \dot{\theta}$

N required

$$N = \begin{cases} (2/\pi)|\alpha_{P_0} - \theta_0|; & \text{if } (\alpha^d_{P_f} - \alpha_P)/(\alpha^d_{P_f} - \theta) < 2 \quad \text{orientation phase} \\ (\alpha^d_{P_f} - \alpha_P)/(\alpha^d_{P_f} - \theta); & \text{if } \cancel{(\alpha^d_{P_f} - \alpha_P)/(\alpha^d_{P_f} - \theta) \geq 2} \text{ else} \quad \text{standard PPN phase} \end{cases}$$

CHECK.

$\dot{N}_{Req} > 0$

Criteria to Satisfy

* should be able to attain $\pi - \theta_0$ with gain greater than 2, at end of orientation phase

$$\Downarrow$$

$$\frac{-\pi + \theta_0 - \alpha_{P_{ORI_f}}}{-\pi + \theta_0 - \theta_{ORI_f}} > 2$$

$$\Rightarrow \alpha_{P_{ORI_f}} \geq \pi - \theta_0 + 2\theta_{ORI_f}$$