Chapter 6

Gauss Jacobi Integration

6.1 Formulation

We have an m^{th} order function $P_m(x)$ with an n^{th} order approximation using collocation, $q_n(x)$, where $m > n$.

$$P_m(x) \approx q_n(x) = \Sigma_{k=0}^{n} L_k(x) P_m(x_k)$$

where x_k are the node/collocation points, $k \in I_n = \{0, 1, ..., n\}$

Given this information, we can write the error between the approximation and the actual function as the following

$$\underbrace{P_m(x) - q_n(x)}_{(m^{th} order)} = \underbrace{[\Pi_{k \in I_n}(x - x_k)]}_{(n+1)^{th} order} \underbrace{\{R_{m-n-1}(x)\}}_{(m-n-1)^{th} order}$$

$$P_m(x) = q_n(x) + \Pi_{k \in I_n}(x - x_k) R_{m-n-1}(x)$$

Now, we integrate both sides

$$\int_{-1}^{1} P_m(x) w(x) dx = \int_{-1}^{1} \Sigma_{k=0}^{n} L_k(x) P_m(x_k) dx + \int_{-1}^{1} \Pi_{k \in I_n}(x - x_k) R_{m-n-1}(x) dx$$

For convenience, we take the weight function to be unity (w(x) = 1, i.e Legendre polynomials as base?) and the limits of integral from -1 to 1.
If we choose x_k as the roots of some basis polynomial, here of Legendre polynomial $\phi_{n+1}(x)$, then the second term goes to zero by using the orthogonality principle.

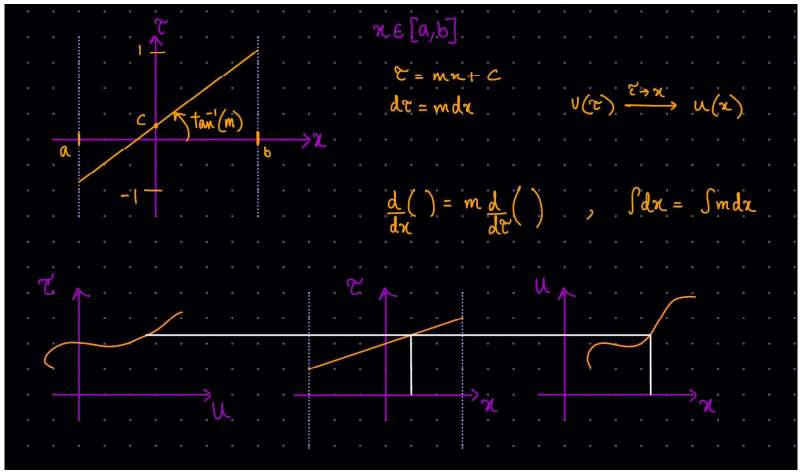
$$\int_{-1}^{1} P_m(x) w(x) dx = \int_{-1}^{1} L_k(x) P_m(x_k) dx + \int_{-1}^{1} \Pi_{k \in I_n}(x - x_k) R_{m-n-1}(x) dx$$

$$\int_{-1}^{1} P_m(x) w(x) dx = \Sigma_{k=0}^{n} P_m(x_k) \underbrace{\int_{-1}^{1} L_k(x) w(x) dx}_{w_k} \tag{6.1}$$

Thus, this is the final relation for Gauss Jacobi integration with w_k as the weight for summation for each node point.

Conditions/Prerequisites

- The function P(x) $\in P_n(x)$, where $P_n(x)$ are the set of polynomials that can be represented as as a combination of $\phi_k(x)$, $k \in I_n$
- m - n - 1 $\leq$ n $\implies$ m $\leq$ 2n + 1
- Transformation due to limits of integral being from -1 to 1 to domain limits



f(x) = P(x) $\in P_n(x) \rightarrow$ Lagrange polynomial representation is exact for 'n+1' node points for n^{th} order