AS5580: Pseudospectral Methods for Optimal Control

Description:

Students should be able to apply direct (pseudospectral) methods to solve problems in optimal control.

Course Content:

Introduction to optimal control: Necessary condition for optimal control, KKT condition, Costates, overview of numerical optimization

Overview of spectral methods- Compare Galerkin, least squares and collocation methods (pseudospectral methods).

Interpolation theory: Polynomial interpolation, Runge-phenomenon, Lagrange Formula, Cauchy remainder, Weierstrass approximation theorem, Orthogonal polynomials, Gauss-Jacobi theory for integration, Legendre basis, Chebyshev polynomials, Best approximation, Equi-oscillation theorem, Fourier interpolation. Application: Differentiation Matrices, Solution of boundary value problems

Transcription of optimal control problem to nonlinear optimization, Costate mapping theorem

Textbooks:

[1] John P Boyd. Chebyshev and Fourier spectral methods. Courier Corporation, 2001.

[2] I. Michael Ross, A Primer on Pontryagin's Principle in Optimal Control, 2015

Reference Books:

[1] Fornberg, Bengt. A Practical Guide to Pseudospectral Methods. Egypt, Cambridge University Press, 1998.

[2] Interpolation and approximation, Philip J. Davis.

[3] Rivlin, Theodore J.. An Introduction to the Approximation of Functions. United Kingdom, Dover Publications, 1981.

[4] Roger Peyret. Spectral methods for incompressible viscous flow, volume 148. Springer Science & Business Media, 2013.

[5] Lloyd N Trefethen. Spectral methods in MATLAB, volume 10. Siam, 2000.

[6] Kirk, Donald E.. Optimal Control Theory: An Introduction. United States, Dover Publications, 2004.

[7] Kelly, Matthew. "An introduction to trajectory optimization: How to do your own direct collocation." SIAM Review 59.4 (2017): 849-904.

[8] Betts, John T. Practical Methods for Optimal Control and Estimation Using Nonlinear Programming. United States, Society for Industrial and Applied Mathematics, 2010.

Prerequisites: MATLAB/programming, basic calculus.
No need to have prior knowledge of control systems or aerospace engineering. However, it is expected that you can identify an optimal control problem or a boundary value problem in your field of study that can be solved using tools you learn in this course.

Assessment:
Quiz 1 and 2: 25
Final: 25
Homework :50

<u>Topics: Overview of the course</u>

1. Introduce optimal control problem: Example problem(s). Mathematical formulation. State variable, Control variable. Objective function (tf). Decision variables/unknowns. Different formulations of the problem. In an optimal control problem, we need to solve for a function that satisfies a set of ODEs and an integral that depends on the function (states, control inputs) is maximized or minimized subject to some constraints. So, we need the derivative of the function (Approximation) as well as the integral of some quantities that depend on those states and control inputs.

2. There are two approaches to solving such problems: i) Variational calculus or indirect methods and ii) Pseudospectral methods or direct methods.

3. What all we need to solve/evaluate numerically. We have differential equations, integral and algebraic constraints. The solutions we seek are functions and hence the problem to be solved involves functionals. The variational calculus approach relies on deriving conditions for optimality and then solving those differential equations which typically involves discretization. The direct methods rely on discretizing the system of equations and then solve an optimal control problem.

4. We see that the solutions or the unknowns are in fact functions that need to be determined. With an objective to develop a framework for solving such problems and accepting the fact that in most occasions we have to explore numerical computations, there should be some method where we can ‘capture’ the function using some parameters that describe the function. Some examples of curve fit you have earlier learnt are the best fit for straight line and Fourier series. Both of these in some sense reduce the overall error of the fit with the function you are approximating. However, interpolation as such typically refers to enforcing the function to agree with the fit over a few points (Collocation method).

5. Suppose we claim that all (continuous) functions can be approximated as polynomials, would that be of help to us in solving the above problem (Weierstrass Theorem)? The values of the coefficients of the polynomials are now the parameters we solve for. But 1, x, x^2, etc. is just one way to expand a polynomial (approximation to a function), you could do the same using any set of combinations of such powers of x. One such extremely useful set is that of orthogonal polynomials, e.g., Chebyshev, Legendre polynomials. Equi-oscillation theorem, optimality of Chebyshev polynomial approximation. Convergence of Fourier series.

6. The most important point here is that of finding a polynomial or an interpolant to a function (unknown). How does interpolation work? Different ways the curve is ‘fit’. Galerkin, Collocation.

7. Pseudospectral methods rely on collocation-based approach wherein we insist the approximate polynomial agrees with the function at the nodes we need and not in an overall reduction of error of the approximant and function evaluated through an integral (Galerkin). Vandermonde matrix.

8. How to fit an interpolant that agrees with the function at a fixed set of points. Does convergence always exist? Runge phenomena. Lagrange polynomial, why not Taylor series. Fourier series, Fourier interpolant, Aliasing

9. Integration: Tools you already may know: trapezoidal, Simpson's rule. To integrate a function, we integrate the approximated polynomials. So, the nth order fit is accurate up to nth order polynomial. But even better is what the Gauss-Jacobi quadrature rule provides. Quadrature is another word for integration, possibly from the earliest tasks of determining the square that has the same area as that of the circle. This is done using orthogonal polynomials as well as their roots being used as nodes. Error calculations O(N^-m). Trapezoidal rule has spectral accuracy for periodic functions, we will mathematically determine why this behavior. Integration using Clenshaw-Curtis quadrature, the nodes used therein. Why are they 'good'!?

10. Differentiation: Differentiate the polynomial to approximate the derivative of the function. Differentiation matrix from the context of finite difference. Extension of stencil to include whole domain. Fourier differentiation matrix. Equivalence of FD and Fourier D matrix. Derivation of D matrix of Chebyshev and Legendre distribution.

11. Setting up the optimal control problem using D matrix to get constraints, the integral to get objective function and include other linear or nonlinear system constraints. Optimization to determine the solution.

12. Optimization: a general introduction

13. Boundary value problems (general)