AS5580: Optimal nodes (Assignment-1)

Assignment Points: 15, Due Date: 14th September, 2024

Consider the domain $[-1\ 1]$, polynomial $P_n(x) = \prod\limits_{k=0}^{n} (x - x_k)$ and an arbitrary polynomial $Q(x)$ (your choice) of order $2n$, e.g. $x^{2n} - 1$.

1. Use optimization (in Matlab `fmincon`) to determine the $n+1$ nodes such that the polynomial $P_n(x)$ have least norm in the following sense:

   (a) minimum uniform norm, that is minimize the maximum (absolute) value attained by the function. You may also try minimizing the maximum value of $P_n^2(x)$.

   (b) minimum norm-2 with $w(x) = 1$, that is minimize the value of $\int\limits_{-1}^{1} P_n^2(x)dx$

   (c) minimum norm-2 with weight $w(x) = \frac{1}{\sqrt{1-x^2}}$, that is minimize the value of $\int\limits_{-1}^{1} P_n^2(x)w(x)dx$

   You may use `integral` function in MATLAB to evaluate integrals. Do not use trapezoidal method. Use Gauss-Jacobi rule (if writing your own code/program). Plot each of these node distributions.

2. Determine the Lagrange fundamental polynomials ($L_k(x)$) for each of these node distributions. Use these polynomials to interpolate $Q(x)$ for these node distributions. Determine for each case the error in interpolation using all three 'norms' mentioned above. That is, if $r(x)$ is the interpolant, then for each node distribution determine (hence you will have 9 values)

   (a) $\max |Q(x) - r(x)|$

   (b) $\int\limits_{-1}^{1} (Q(x) - r(x))^2 dx$

   (c) $\int\limits_{-1}^{1} (Q(x) - r(x))^2 w(x)dx$