```
In[53]:= ClearAll[Evaluate[Context[] <> "*"]]
Needs["Utilities`CleanSlate`"];
CleanSlate[];
ClearInOut[];
(*Polynomial approximation*)
P[x_] = a0  + a1 x + a2 x^2;
(*Lagrange Interpolation: 3 points---quadratic*)
coeffs = Simplify[
    Solve[P[x1] == y1 && P[x2] == y2 && P[x3] == y3, {a0, a1, a2}, Reals]];
y = P[x] /. {coeffs};
c1 = Simplify[Coefficient[y, y1]];
c2 = Simplify[Coefficient[y, y2]];
c3 = Simplify[Coefficient[y, y3]];
Pn = c1[[1]] y1 + c2[[1]] y2 + c3[[1]] y3
(*central difference from the quadratic approximation*)
D[Pn, x] /. {x1 → x2 - h, x3 → x2 + h, x → x2}

(*Solving boundary value problem u(0)=0,
u(1)=1 d^2/dx^2 u+(du/dx)^2+u d^2/dx^2 u=0*)
asol = Solve[P[0] == 0 && P[1] == 1, {a0, a1}];
Print["u(t)"];
u = P[t] /. asol
Du = D[P[t], t] /. asol;
D2u = D[Du, t] /. asol;
(*Collocation method*)
Print["d^2/dx^2 u+(du/dx)^2+u d^2u/dx^2 :"];
R = D2u (1 + u) + (Du)^2
Rc = R /. {t → 1/2};
Print["Solution to D2u (1+u)+(Du)^2=0 at t=1/2 (a0,a1)"]
a2coll = Solve[Rc == 0, {a2}]
Print["Collocation solution: a0,a1"]
ucollsol = P[t] /. {asol[[1]]} /. {a2coll[[1]]}



Du = D[P[t], t];
```

```
D2u = D[Du, t];
(*Collocation method*)
R1 = D2u (1 + u) + (Du)^2;
Rc1 = R1 /. {t → 1/2};
a2collAll = Solve[Rc1 == 0 && P[0] == 0 && P[1] == 1, {a0, a1, a2}];
Print["Collocation solution: u a0,a1,a2"]
ucollsolAll = P[t] /. a2collAll⟦1⟧
Print["Residue norm with collocation method"]
Rcnorm = Integrate[ucollsolAll^2, {t, 0, 1}]
(*least squares aapproximation*)
Print["Residue (norm LSQ)"];
Rnorm = Integrate[R D[R, a2], {t, 0, 1}]
a2lsq = NSolve[Rnorm == 0, a2];
ulsqsol = P[t] /. asol⟦1⟧ /. a2lsq⟦1⟧
Print["Residue norm with  LSQ"]
Rcnorm = Integrate[ulsqsol^2, {t, 0, 1}]
(*Exact solution*)
fx = Sqrt[3 x  + 1] – 1;
(*Evaluating solution at x=1/2*)
Pval = Simplify[Pn /.
        {y1 → 0,  y2 → fx /. {x → 1/2}, y3 → 1, x1 → 0, x2 → 1/2, x3 → 1}];
Print["Interpolation check at x=1/2"];
Simplify[Pval /. {x → 1/2}]
Print["Function solution at x=1/2"];
Simplify[fx /. {x → 1/2}]
Print["Collocation solution at x=1/2"]
Simplify[ucollsol /. {t → 1/2}]

Print["Alternate way using Collocation"]
Pnsol = c1⟦1⟧ 0 + c2⟦1⟧ uval + c3⟦1⟧ 1;
uc = Pnsol /. {x1 → 0,  x2 → 1/2, x3 → 1};
Duc = D[uc, x];
D2uc = D[Duc, x];
Rc2 = D2uc (1 + uc) + (Duc)^2;
Rcoll = Rc2 /. {x → 1/2};
```

Print["Collocation solution−2"]
usolcoll = Solve[Rcoll == 0, {uval}]

(CleanSlate) Contexts purged: {Global`}

(CleanSlate) Approximate kernel memory recovered: 125 Kb

Out[7]= $\left\{\frac{(x-x2)(x-x3)\,y1}{(x1-x2)(x1-x3)}-\frac{(x-x1)(x-x3)\,y2}{(x1-x2)(x2-x3)}-\frac{(x-x1)(x-x2)\,y3}{(x1-x3)(-x2+x3)}\right\}$

Out[8]= $\left\{-\frac{y1}{2\,h}+\frac{y3}{2\,h}\right\}$

u(t)

Out[11]=

$\left\{(1-a2)\,t+a2\,t^2\right\}$

d^2/dx^2 u+(du/dx)^2+u d^2u/dx^2 :

Out[15]=

$\left\{\left\{(1-a2+2\,a2\,t)^2+2\,a2\left(1+(1-a2)\,t+a2\,t^2\right)\right\}\right\}$

Solution to D2u (1+u)+(Du)^2=0 at t=1/2 (a0,a1)

Out[18]=

$\left\{\left\{a2\to 3-\sqrt{11}\right\},\left\{a2\to 3+\sqrt{11}\right\}\right\}$

Collocation solution: a0,a1

Out[20]=

$\left\{\left\{\left(-2+\sqrt{11}\right)t+\left(3-\sqrt{11}\right)t^2\right\}\right\}$

Collocation solution: u a0,a1,a2

Out[27]=

$\left(-2+\sqrt{11}\right)t+\left(3-\sqrt{11}\right)t^2$

Residue norm with collocation method

Out[29]=

$\frac{1}{2}-\frac{\sqrt{11}}{30}$

Residue (norm LSQ)

Out[31]=

$\left\{\left\{3+12\,a2+\frac{2\,a2^3}{5}\right\}\right\}$

Out[33]=

$1.24948\,t-0.249482\,t^2$

Residue norm with LSQ

Out[35]=

0.376988

Interpolation check at x=1/2

Out[39]=

$$\left\{-1+\sqrt{\frac{5}{2}}\right\}$$

Function solution at x=1/2

Out[41]=

$$-1+\sqrt{\frac{5}{2}}$$

Collocation solution at x=1/2

Out[43]=

$$\left\{\left\{\frac{1}{4}\left(-1+\sqrt{11}\right)\right\}\right\}$$

Alternate way using Collocation

Collocation solution−2

Out[52]=

$$\left\{\left\{\text{uval}\to\frac{1}{4}\left(-1-\sqrt{11}\right)\right\},\left\{\text{uval}\to\frac{1}{4}\left(-1+\sqrt{11}\right)\right\}\right\}$$