# AS5580: Runge Phenomena

Due Date: 7th September, 2024

Assignment Points: 15

Consider the function $f(x) = \dfrac{1}{1 + x^2}$ in the domain $[-5,\ 5]$.

1. Use nodes corresponding to uniform spacing and roots of Chebyshev polynomials and study the convergence of the interpolant with respect to order of the polynomial.

2. Estimate the value (numerically) of $a$ such that for $|x| < a$ the interpolate on uniform grid converges.

3. Plot the contours of $|\overline{T}_{n+1}(z)|$ and $|Q(z)|$ where $Q(z) = \prod\limits_{k=0}^{N} (z - x_k)$ where $x_k$ are uniformly spaced nodes in $[-5,\ 5]$.

4. Determine the $D$ matrix (numerically) using $\frac{dL(x)}{dx}$ for uniformly spaced nodes as well as those corresponding to Chebyshev polynomial roots.

5. Plot the error for interpolation at $x = 0$ for interpolant based on both node distributions, with respect to the order $(n)$ of the polynomial. Repeat the above for two other points (approximately $x = 3$, and $x = 4$) not coinciding with a node.

6. Compute the derivative of $f(x)$ for the above two $D$ matrices and compare with exact analytical solution at the chosen nodes.

7. Use `integral` function in MATLAB and evaluate integral of Lagrange interpolant and hence determine the integral of $f(x)$. Compare this solution with what is obtained using `integral` directly on $f(x)$.