Department of Aerospace Engineering, I.I.T. Madras

AS5580 Numerical Quiz -1 Date: 11-09-2024

4:55-5:55 PM **Pseudospectral Methods for Optimal Control** (Marks: 20)

INSTRUCTIONS:
Marks for each problem is given after the question in ( )
Submit your answers as a document along with computer programs
You earn marks for what you write and not for what you thought !

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1. Consider the function e^x for $x \in [0,\ 1]$. Using uniformly distributed nodes

   (a) Determine the interpolating polynomial and plot the max error for number of interpolating points $n = 4q+1, q = 1, 2\ldots, 6$. Take a much finer node distribution while calculating the error. (5)

   (b) Determine the derivative of e^x using the derivative matrix derived from interpolating polynomial. Compare with analytical solution (plot). (5)

2. Determine a least squares fit $p(x)$ (may use polyfit in Matlab) for e^x, $x \in [0,\ 1]$ and determine the points (not approximately from graph, but numerically) where $e^x = p(x)$? (5)

3. Chebyshev polynomials are given by $T_n(x) = \cos(n\cos^{-1}(x))$. Expand the polynomial $p(x) = \sum\limits_{k=0}^{p} x^k$ in terms of $T_j(x), j = 0, 1, \ldots, p$, that is determine b_k for $p(x) = \sum\limits_{k=0}^{p} b_k T_k(x)$. Weight function associated with $T_n(x)$ is $w(x) = \dfrac{1}{\sqrt{1-x^2}}$. First take $p = 2$ and verify the solution analytically. Then repeat with $p = 6$. You may use integral function in Matlab to evaluate integrals. Following orthogonality relation could also be useful.

$$\begin{aligned}\int_{-1}^{1} T_n(x)T_m(x)w(x)dx &= 0, m \neq n \\ &= \pi, m = n = 0 \\ &= \frac{\pi}{2}, m = n \neq 0\end{aligned}$$

(5)